

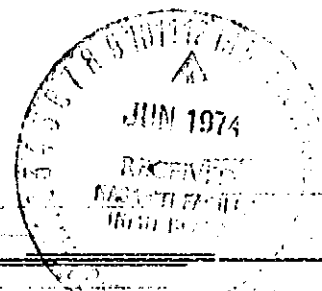
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SPACE SHUTTLE GN & C EQUATION DOCUMENT  
 No. 18  
 Station-Keeping Guidance  
 by  
 Donald E. Gustafson  
 Bernard A. Kriegsman

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**No. 18**

**Station-Keeping Guidance**

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**Donald E. Gustafson**

**Bernard A. Kriegsman**

**M. I. T. Charles Stark Draper Laboratory**

**March 1972**

**NAS9-10268**

**for**

**National Aeronautics and Space Administration**

**Systems Analysis Branch**

**Guidance and Control Division**

**Manned Spacecraft Center, Houston, Texas**

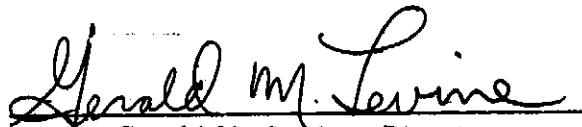
## ACKNOWLEDGEMENT

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The publication of this report does not constitute approval by the National Aeronautics and Space Administration of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

## FOREWORD

This document is one of a series of candidates for inclusion in a future revision of MSC-04217, "Space Shuttle Guidance, Navigation and Control Design Equations". The enclosed has been prepared under NAS9-10268, Task No. 15-A, "GN&C Flight Equation Specification Support", and applies to function 1 of the Station-Keeping, Docking, and Undocking Guidance Module (OG4) as defined in MSC-03690, Rev. B, "Space Shuttle Orbiter Guidance, Navigation and Control Software Functional Requirements - Vertical Flight Operations", dated 15 December 1971.

A handwritten signature in cursive script that reads "Gerald M. Levine". The signature is written in dark ink and is positioned above a horizontal line.

Gerald M. Levine, Director  
APOLLO Space Guidance Analysis Division

## TABLE OF CONTENTS

Section 1	Introduction
Section 2	Functional Flow Diagrams
Section 3	Input and Output Variables
Section 4	Description of Equations
Section 5	Detailed Flow Diagrams
Section 6	Supplementary Information

## NOMENCLATURE

### Notational Conventions

- Upper-case letters represent matrices
- Lower-case and Greek letters reserved for scalars and vectors
- Vector quantities are underlined, e. g.  $\underline{x}$
- Vectors are assumed to be column vectors unless explicitly noted

### Symbols

A	Dummy 2 x 2 matrix used in velocity-correction computations
a	Elements of A
B	Dummy 2 x 2 matrix used in velocity-correction computations
b	Elements of B
C	Dummy 2 x 2 matrix used in velocity-correction computations
c	Elements of C
d	Dummy variable used in velocity-correction computations
$h_{lc}$	Height of desired station-keeping limit cycle
$\underline{i}_{RL}$	Unit vector along $\underline{r}$ (local-vertical coordinates)

$\underline{i}_{RW}$	Unit vector along $\underline{r}$ (workshop-fixed coordinates)
$\underline{i}_{NW}$	Unit vector normal to station-keeping cone boundary (workshop-fixed coordinates)
$\underline{i}_{XWL}$ $\underline{i}_{YWL}$ $\underline{i}_{ZWL}$	Workshop-fixed frame unit vectors (local-vertical coordinates)
$k$	Midcourse-correction fractions
$M_{L-W}$	Transformation matrix from local-vertical to workshop- fixed coordinates
$M_{L-P}$	Transformation matrix from local-vertical to stable- member coordinates
$q, \dot{q}$	Dummy test variables used in boundary avoidance calculations
$\underline{r}$	Shuttle position w. r. t. workshop (stable-member coordinates)
$\underline{r}_L$	Shuttle position w. r. t. workshop (local-vertical coordinates)
$\underline{r}_W$	Shuttle position w. r. t. workshop (workshop coordinates)
$r_{XZ}$	Magnitude of component of $\underline{r}$ in workshop frame X-Z plane
$r_{MIN}$	Lower limit on $r_W$ along workshop Y-axis for which boundary-avoidance velocity corrections may be required
$\underline{r}_{DL}$	Desired target position for orbiter w. r. t. workshop at terminal time $t_F$

$s_{OPG}$	Switch used to select out-of-plane guidance mode
$s_{VCORR}$	Switch set at unity if velocity correction is required
$s_{mode}$	Switch to select current mode of operation of routine
$s_{nut_F}$	Switch set at unity when new $t_F$ is required
$t$	Current time
$t_F$	Terminal time for current guidance limit cycle
$\underline{v}_L$	Shuttle velocity w. r. t. workshop (local-vertical coordinates)
$v_{DN}$	Velocity-correction level used for boundary avoidance
$y_{min}$	Lowest part on desired limit cycle w. r. t. workshop (along vertical axis)
$z_{MAX}$	Maximum desired out-of-plane distance for orbiter
$z_{MIN}$	Minimum desired out-of-plane distance for orbiter
$z_0$	Dummy variable
$\underline{\omega}_W$	Workshop angular velocity (stable-member coordinates)
$\theta$	Dummy variable equal to $\underline{\omega}_W (t_F - t)$
$\tau$	Dummy time interval ( $t_n - t_{n-1}$ )
$\delta v_{min}$	Lower limit on computed velocity-correction magnitude
$\delta \underline{v}_L$	Required velocity correction (local-vertical coordinates)
$\theta_Z$	Station-keeping cone half angle





1. INTRODUCTION

The purpose of the station-keeping guidance system is to automatically keep one orbiting vehicle within a prescribed zone fixed with respect to another orbiting vehicle. The active vehicle, i. e. the one performing the station-keeping maneuvers, is referred to as the shuttle. The other passive orbiting vehicle is denoted as the workshop. The passive vehicle is assumed to be in a low-eccentricity near-earth orbit.

The primary navigation sensor considered is a gimballed tracking radar located on board the shuttle. It provides data on relative range and range rate between the two vehicles. Also measured are the shaft and trunnion axes gimbal angles. An inertial measurement unit (IMU) is assumed to be provided on board the orbiter. The IMU is used at all times to provide an attitude reference for the vehicle. The IMU accelerometers are used periodically to monitor the velocity-correction burns applied to the shuttle during the station-keeping mode.

The guidance system presented here is capable of station-keeping the shuttle in any arbitrary position with respect to the workshop. This objective is accomplished by periodically applying velocity-correction pulses to the shuttle. These velocity corrections are computed by the guidance routine with the objective of minimizing the average expenditure of propellant (by the shuttle) per orbit.

## 2. FUNCTIONAL FLOW DIAGRAM

A functional flow diagram for the station-keeping guidance routine is shown in Figure 1. The overall structure of the routine is simple and straight-forward. There are two basic subroutines; one is used for computing the normal velocity corrections ( $s_{mode} = 1$ ) and the small midcourse corrections ( $s_{mode} = 2$ ); the other is used for computing boundary-avoidance velocity corrections. The guidance-routine call times and mode selection are accomplished by the Station-Keeping Executive Routine (Ref. 7).

Both subroutines use relative position and velocity (shuttle w. r. t. workshop) from the Station-Keeping Navigation Routine (Ref. 6) as a basis for computing the required velocity corrections. Local-vertical coordinates are used in the normal and midcourse-correction modes, workshop fixed coordinates are used in the boundary-avoidance mode. In-plane and out-of-plane velocity corrections are computed separately in the normal and midcourse correction modes.

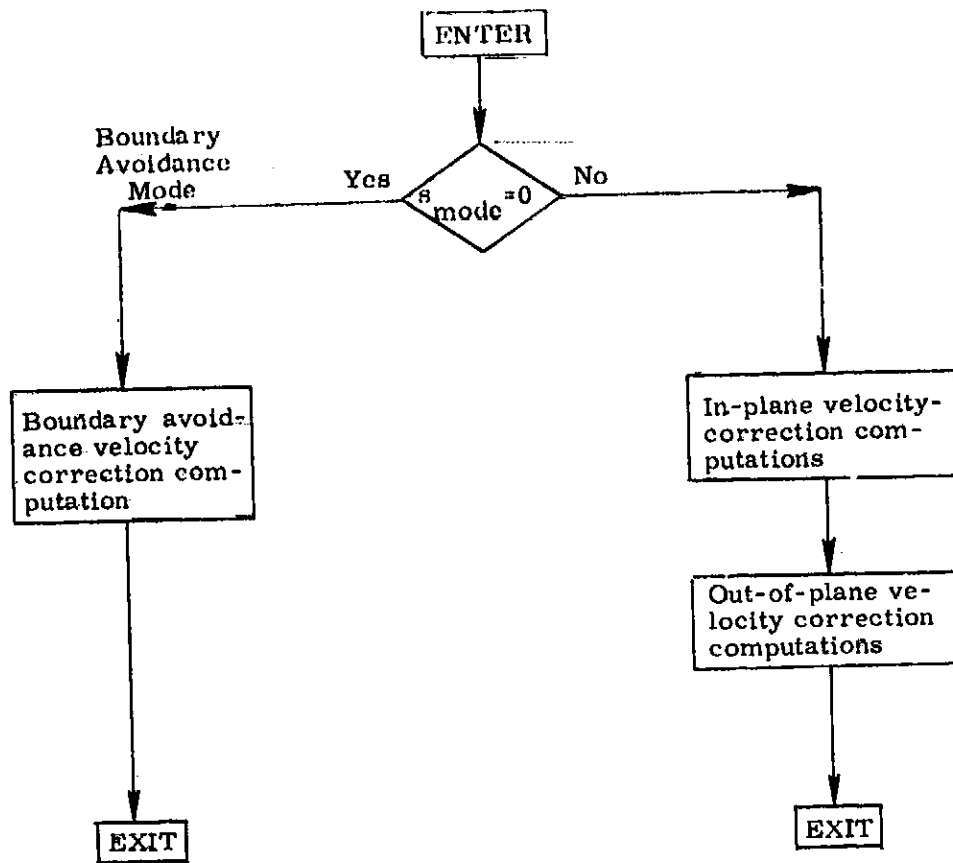


Figure 1. Station-Keeping Guidance Routine, Functional Flow Diagram

### 3. INPUT AND OUTPUT VARIABLES

#### Input Variables

$\underline{r}$	Shuttle position w. r. t. workshop (stable-member coordinates)
$\underline{r}_L$	Shuttle position w. r. t. workshop (local-vertical coordinates)
$\underline{r}_{DL}$	Terminal (or target) point position of orbiter w. r. t. workshop (local-vertical coordinates)
$\underline{v}_L$	Shuttle velocity w. r. t. workshop (local-vertical coordinates)
$M_{L-W}$	Transformation matrix from local-vertical to workshop coordinates (desired station-keeping zone unit vectors)
$M_{L-P}$	Transformation matrix from local-vertical to stable-member coordinates
$t_F$	Desired shuttle arrival time at current terminal point
$s_{nut_F}$	Switch to indicate when new $t_F$ must be computed
$t$	Present time
$\underline{\omega}_W$	Workshop angular-velocity vector (stable-member coordinates)
$s_{mode}$	Guidance routine mode selector switch
$s_{OPG}$	Out-of-plane guidance mode selector switch

#### Output Variables

$s_{VCORR}$	Switch which indicates whether a velocity correction is required (0 = no, 1 = yes)
$\delta \underline{v}$	Required station-keeping velocity correction (stable-member coordinates)

#### 4. DESCRIPTION OF EQUATIONS

##### 4.1 General Information

The station-keeping guidance routine is capable of maintaining an active vehicle (shuttle) in a small zone which may be arbitrarily located with respect to a passive orbiting vehicle (workshop). The passive vehicle is assumed to be in a low-eccentricity orbit around the earth. The station-keeping is accomplished by the periodic application of small velocity-correction pulses. The size and location of the station-keeping zone are specified as program constants and input variables (e. g.  $h_{LC}$ ,  $y_{min}$ ,  $\theta_Z$ ,  $z_{max}$ ,  $z_{min}$ ,  $r_{DL}$ ).

The guidance routine has three primary modes: (1) normal station-keeping, (2) midcourse correction, and (3) boundary avoidance. In the normal mode the velocity corrections required to hold the shuttle in the specified zone w. r. t. the workshop are computed. In typical situations these corrections are relatively small (e. g. 2-5 ft/sec or less). The magnitude and frequency of these corrections is dependent on the size and location of the station-keeping zone. The midcourse-correction mode uses essentially the same relations as the normal mode. The basic idea here is that by applying small velocity corrections in between the normal velocity-correction times, the total velocity-correction propellant expenditure may be reduced. In the boundary-avoidance mode, special tests are made to see if the shuttle is outside of the station-keeping zone and heading away from it. Appropriate velocity-correction pulses are applied to the orbiter to return it to the desired zone.

The times at which each mode of the station-keeping guidance routine is called are determined by the Station-Keeping Executive Routine (Ref. 7).

Three coordinate systems are used in the station-keeping guidance routine: (1) stable-member, (2) local-vertical, and (3) workshop-fixed coordinates. All three systems are orthogonal right-handed systems. The relationships between these frames are shown in Figure 2. The stable-member system is fixed w. r. t. the inertial measurement unit (IMU). The local-vertical system rotates with the workshop, as shown in Figure 2, with its X-axis along the local vertical and its Z-axis along the workshop-orbit angular-momentum vector. The normal-mode and midcourse-correction computations in the guidance routine are done primarily in this local-vertical frame. The workshop-fixed frame is fixed w. r. t. the desired station-keeping zone. The boundary-avoidance mode computations in the guidance routine are performed in this frame.

##### 4.2 Normal Guidance Mode

The analytical development of the basic guidance concept has been extensively documented in Refs. 1 and 3 for AAP missions, and in Ref. 2 for SSV missions.

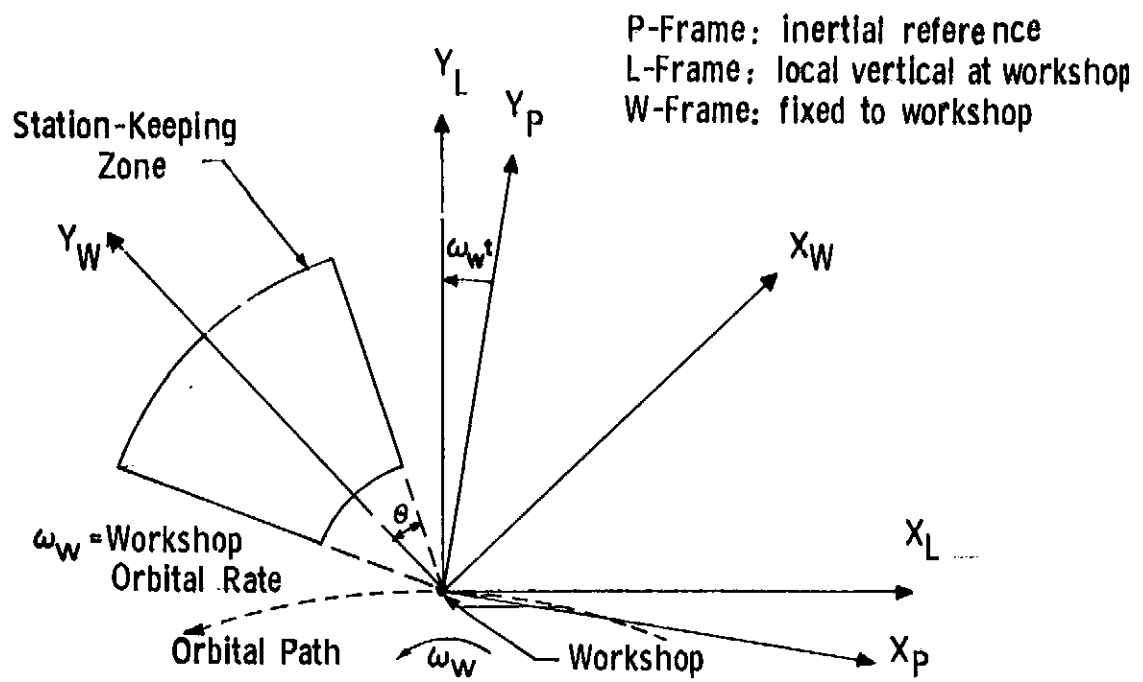


Figure 2. Station-keeping System Relative Geometry

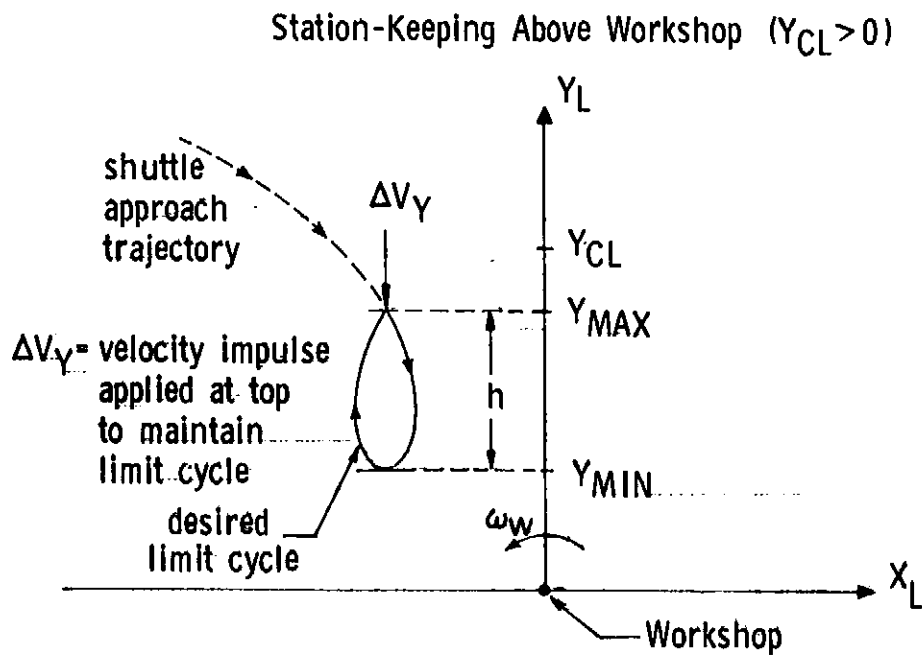


Figure 3. Geometry for Guidance Limit Cycles

Extensive performance data is given in these references. It is most convenient to consider the in-plane and out-of-plane guidance equations separately. This approach will be followed here.

The in-plane problem will be considered first. The basic idea is to put the shuttle on a trajectory that will terminate at a specified position w. r. t. the workshop ( $\underline{r}_{DL}$ ) at a fixed terminal time ( $t_F$ ). A typical limit-cycle trajectory is shown in Figure 3 for the case where the station-keeping zone is above and in front of the workshop.

The terminal time ( $t_F$ ) is based on the desired limit-cycle trajectory height ( $h_{lc}$ ) and desired minimum altitude of shuttle w. r. t. the workshop ( $y_{min}$ ). The basic relation is (Ref. 2):

$$t_F = t + \frac{2}{\omega_W} \sqrt{\frac{6 y_{lc}}{9 y_{min} + 4 h_{lc}}} \quad (1)$$

where  $t$  is the current time, and  $\omega_W$  is the workshop's angular velocity.

The required correction ( $\delta \underline{v}_L$ ) that must be made to the current shuttle velocity ( $\underline{v}_L$ ) in order for the vehicle to arrive at the position ( $\underline{r}_{DL}$ ) at the terminal time  $t_F$  is computed in a straight-forward manner (Refs. 1 and 2). The basic relation is

$$\delta \underline{v}_L = \Phi_{RV}^{-1}(t, t_F) [\underline{r}_{DL} - \Phi_{RR}(t, t_F) \underline{r}_L] - \underline{v}_L \quad (2)$$

where  $\underline{r}_L$  and  $\underline{v}_L$  represent the position and velocity of the shuttle w. r. t. the workshop, expressed in local-vertical coordinates.

The matrices  $\Phi_{RV}$  and  $\Phi_{RR}$  are submatrices of the matrix  $\Phi$ , which is used to extrapolate the shuttle state w. r. t. the workshop forward in time, using local-vertical coordinates. The relation is

$$\begin{bmatrix} \underline{r}_L(t_n) \\ \underline{v}_L(t_n) \end{bmatrix} = \begin{bmatrix} \Phi_{RR}(\tau) & | & \Phi_{RV}(\tau) \\ \Phi_{VR}(\tau) & | & \Phi_{VV}(\tau) \end{bmatrix} \begin{bmatrix} \underline{r}_L(t_{n-1}) \\ \underline{v}_L(t_{n-1}) \end{bmatrix} \quad (3)$$

where  $t_{n-1}$  and  $t_n$  are arbitrary times ( $t_n > t_{n-1}$ ). The detailed relations for  $\Phi_{RR}$ ,  $\Phi_{RV}$ ,  $\Phi_{VR}$  and  $\Phi_{VV}$  are given in Refs. (1) and (2) as a function of workshop angular velocity ( $\omega_W$ ) and the time interval from  $t_{n-1}$  to  $t_n$  (referred to as  $\tau$ ).



In the detailed flow diagram for the guidance routine (Figure 4) the required elements of  $\Phi_{RV}^{-1}$  and  $\Phi_{RV}^{-1} \Phi_{RR}$  are represented by the matrices A, B, C, and the dummy variable d.

Two out-of-plane guidance modes are provided (Ref. 2). If the desired station-keeping zone is centered in the workshop orbital plane, then Eqs. (2) and (3) can be used to compute the required velocity correction ( $\delta v_{ZL}$ ). The basic equation in this case is simply:

$$\delta v_{ZL} = -\omega_W r_{L,2} \cot \theta - v_{L,2} \quad (4)$$

where  $r_{L,2}$  and  $v_{L,2}$  are the out-of-plane components of shuttle position and velocity w. r. t. the workshop. The quantity  $\omega_W$  is the workshop's angular velocity w. r. t. the earth. The dummy variable  $\theta$  is given by:

$$\theta = \omega_W (t_F - t) \quad (5)$$

where  $t_F$  is the desired arrival time at the terminal or target point.

If, on the other hand, it is desired that the station-keeping zone be displaced from the workshop orbital plane, then the required velocity correction (Ref. 2) is given by

$$\delta v_{ZL} = \omega_W \sqrt{z_{\max}^2 - r_{L,2}^2} \omega_W \text{sign}(z_{\min}) - v_{L,2} \quad (6)$$

The parameters  $z_{\max}$  and  $z_{\min}$  specify the desired maximum and minimum displacements of the shuttle w. r. t. the workshop in the out-of-plane direction. A velocity correction is applied only if  $|r_{L,2}|$  is less than  $z_{\min}$  and the relative velocity is such as decreases  $|r_{L,2}|$  still further (i. e.  $v_{L,2} r_{L,2}$  is negative).

The boundary-avoidance guidance scheme assumes an inverted truncated cone as the desired station-keeping zone. The apex of the cone is at the workshop, and the cone's axis ( $\underline{i}_{YWL}$ ) is assumed fixed w. r. t. the workshop. The lower boundary of the zone is specified by the parameter  $r_{\min}$  which is its minimum distance from the workshop. The size of the station-keeping zone is specified by the cone half angle  $\theta_Z$ .

Two boundary-avoidance tests are made. First, if the shuttle is too close to the workshop ( $r_{W,1} < r_{min}$ ) and its velocity is taking it towards the shuttle, then a correction is required. The shuttle in this case is given a preselected velocity ( $v_{DN}$ ) away from the workshop. This is accomplished by a velocity correction ( $\delta \underline{v}_L$ ) of

$$\delta \underline{v}_L = (v_{DN} - v_{W,1}) \underline{i}_{YWL} \quad (7)$$

where  $\underline{i}_{YWL}$  is a unit vector along the workshop-frame  $\underline{Y}$ -axis (station-keeping zone cone axis), and  $r_{W,1}$  and  $v_{W,1}$  are the components of relative position and velocity along this axis.

Next, a test is made to see if the shuttle is inside the desired zone. The test quantity ( $q$ ) is computed from:

$$q = \underline{i}_{RW,1} \cdot \cos \theta_Z \quad (8)$$

A second test is now made to see if the shuttle's velocity is directed away from the zone's center line, i.e. the angle between  $\underline{r}_W$  and the cone's axis is increasing. The test quantity  $\dot{q}$  is given by:

$$\dot{q} = [ v_{W,1} - (\underline{v}_W \cdot \underline{i}_{RW,1}) \underline{i}_{RW,1} ] \quad (9)$$

If both  $q$  and  $\dot{q}$  are negative, then the shuttle's component of velocity (w. r. t. the workshop) in the direction normal to the station-keeping cone boundary ( $\underline{i}_{NW}$ ) is given a prespecified value of  $v_{DN}$ , directed in towards the cone axis. The required velocity correction to accomplish this is (Ref. 1).

$$\delta \underline{v}_W = (v_{DN} - \underline{v}_W \cdot \underline{i}_{NW}) \underline{i}_{NW} \quad (10)$$

where the required velocity correction  $\delta \underline{v}_W$  is in workshop-fixed coordinates as is the relative velocity ( $\underline{v}_W$ ).

5. DETAILED FLOW DIAGRAMS

A detailed flow diagram is shown for the Station-Keeping Guidance Routine in Figure 4. To operate this routine, navigation information is required from the Station-Keeping Navigation Routine (Ref. 6). The mode selection and routine-call times for both the Station-Keeping Guidance and Navigation Routines are controlled by the Station-Keeping Executive Routine (Ref. 7).

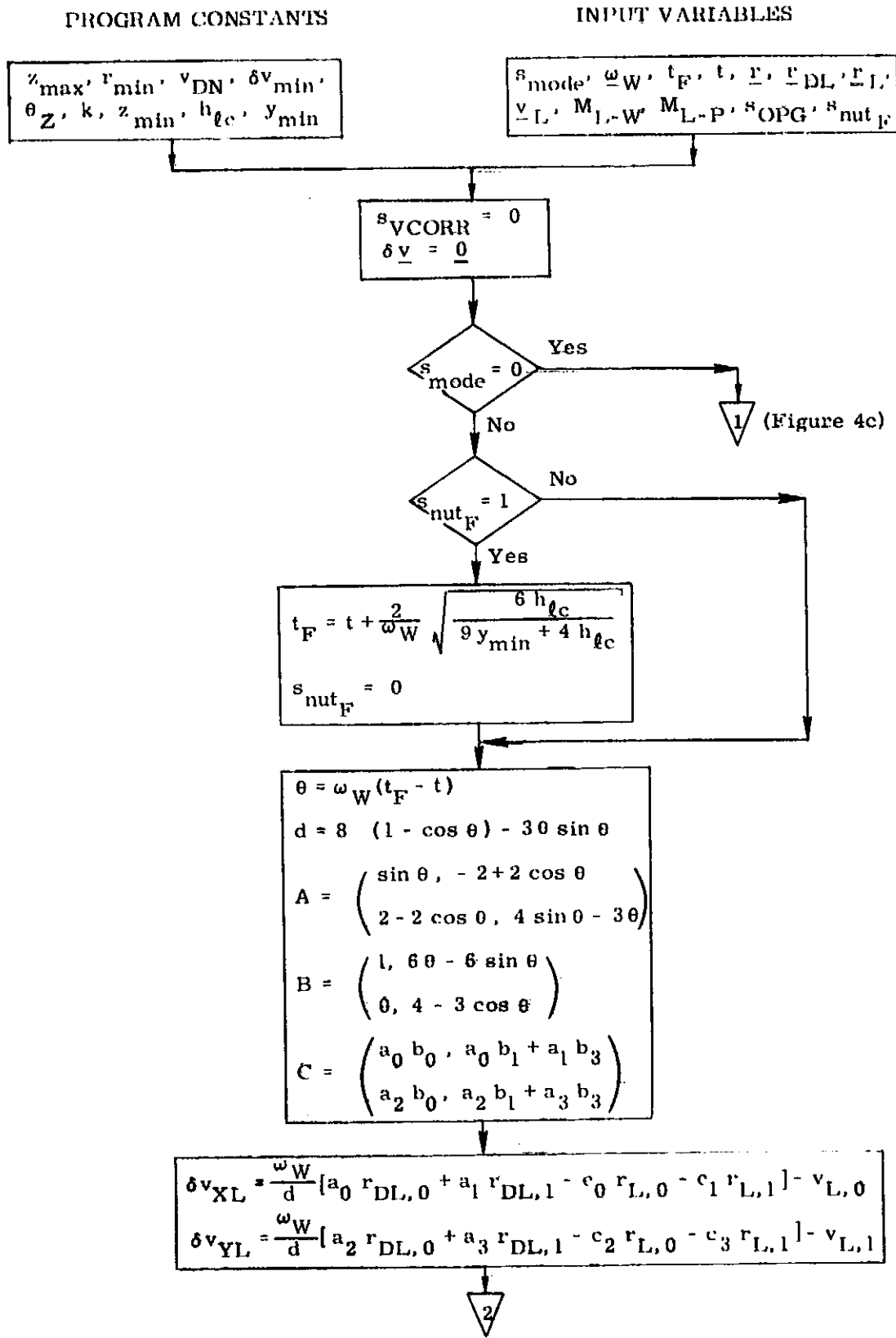
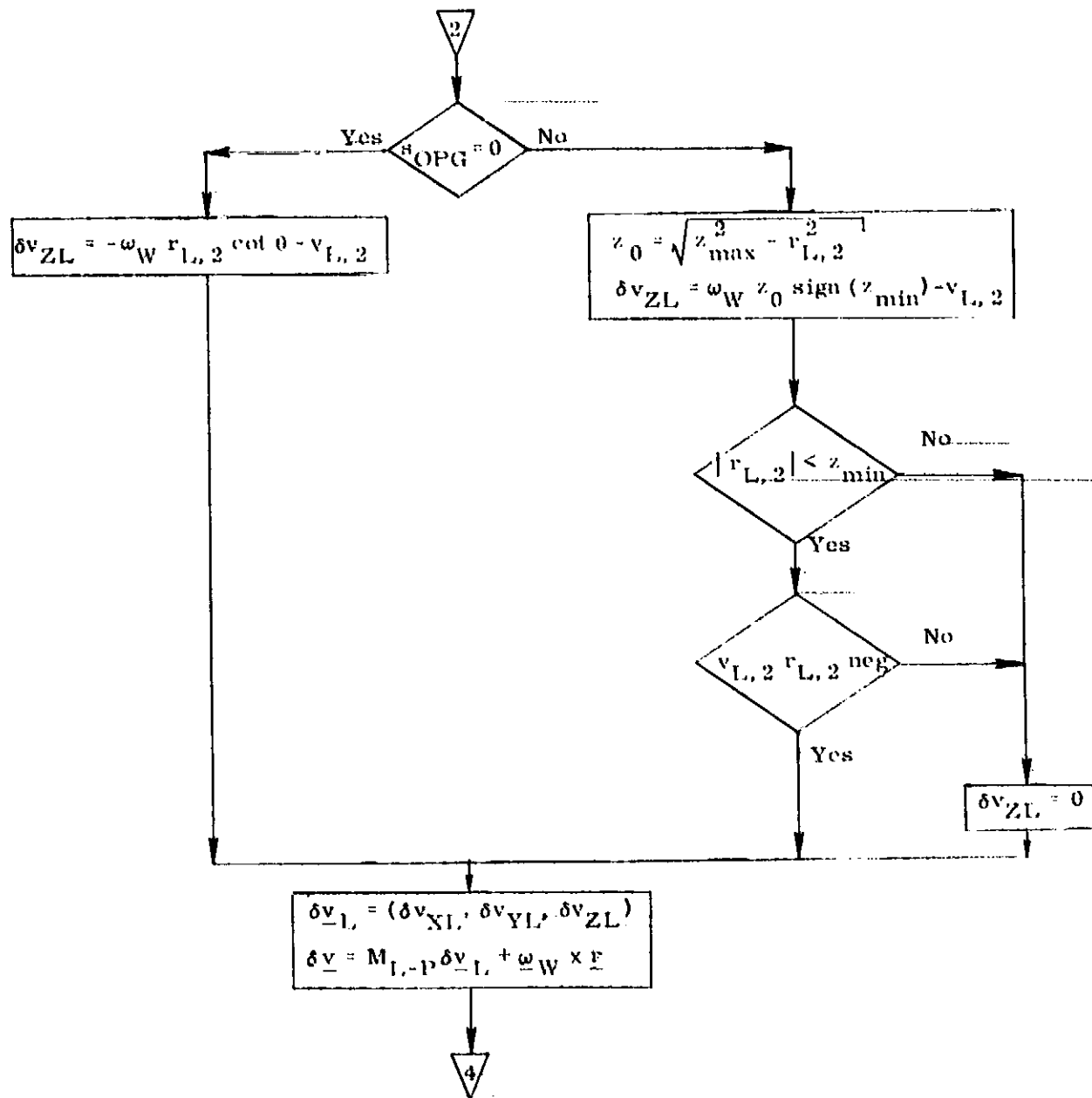


Figure 4a. Station-Keeping Guidance, Detailed Flow Diagram



(Figure 4e)

Figure 4b. Station-Keeping Guidance.  
Detailed Flow Diagram

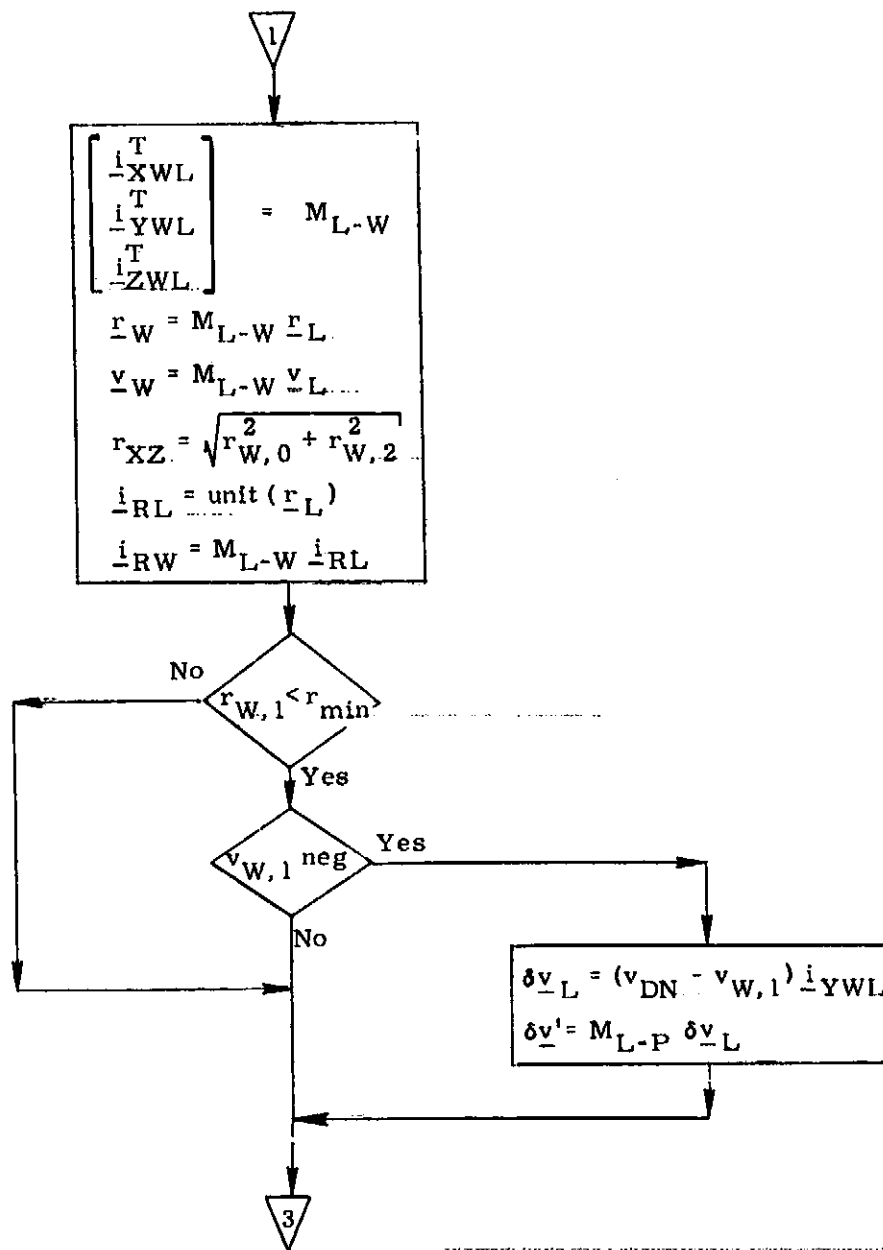


Figure 4c. Station-Keeping Guidance, Detailed Flow Diagram

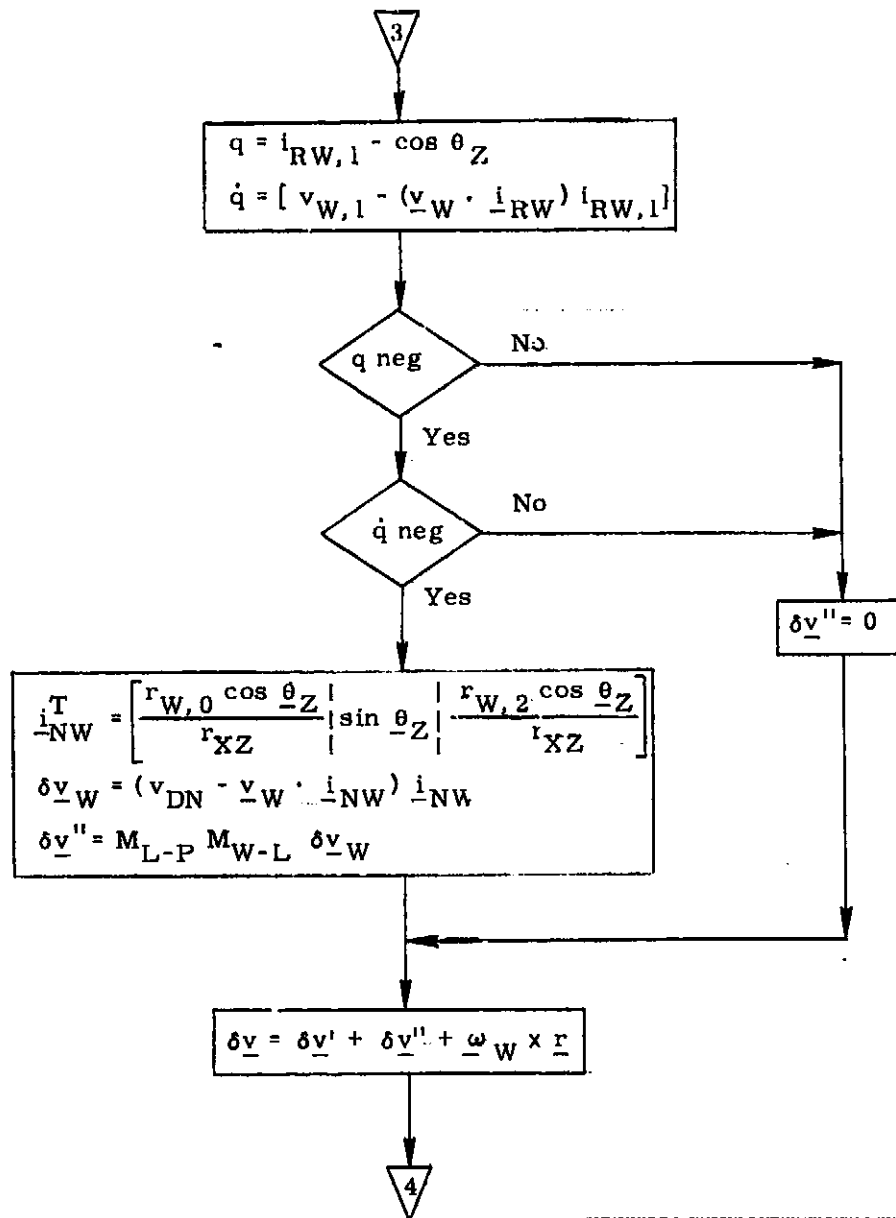


Figure 4d. Station-Keeping Guidance, Detailed Flow Diagram

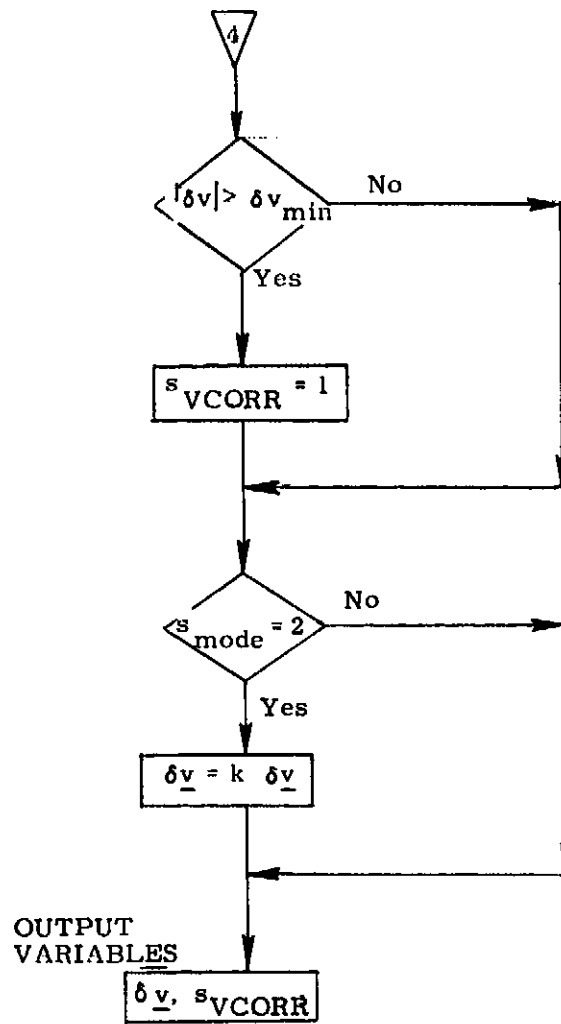


Figure 4e. Station-Keeping Guidance, Detailed Flow Diagram



6. SUPPLEMENTARY INFORMATION

The guidance concept has been extensively studied for both AAP and space-shuttle missions (Refs. 1-6). Detailed descriptions of the guidance system are given in the referenced documents along with system-performance data. For the typical station-keeping ranges studied, i. e. 1000 ft or less, the average velocity correction required was about 10 ft/sec/orbit for station-keeping above, and about 0.5 ft/sec/orbit for station-keeping in front.

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