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# REGOLITH HISTORY FROM COSMIC-RAY-PRODUCED ISOTOPES

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# Regolith history from cosmic-ray-produced isotopes

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Abstract – A statistical model is given for soil development relating meteoroid impacts on the moon to cosmic-ray-produced isotopes in the soil. By means of this model, the average lunar mass loss rate during the past 1.4 aeons is determined to be 170 g/cm$^2$ aeon and the soil mixing rate to be ~200 cm/aeon from the Gd isotope data for the Apollo 15 and 16 drill stems. The isotope data also restrict the time variation of the meteoroid flux during the past 1.4 aeons.

## INTRODUCTION

The lunar regolith is very complicated. Each gram of soil contains millions of separate grains, each with a distinct history. The histories of a few of the larger grains (rock chips) have been traced by isotope analysis; however, it would be impossible to trace the separate histories of all the grains that have been recovered. Some similarities should exist in the grain histories because the cosmic-ray bombardment is uniform over the moon, and to a good approximation, the bombardment rate by sufficiently small meteoroids can also be considered uniform over the moon during aeons. From these two uniformity conditions and from neutron isotope data in a statistical model of soil development described by an integral equation with rigid boundary conditions, we found that the moon has been losing mass during the past aeon at a rate of approximately 80 g/cm$^2$ aeon (Fireman, 1974). We shall examine the consequences of these two uniformity conditions for a range of boundary conditions and also consider how the conditions change when the meteoroid flux increased in the past.

Cosmic-ray-produced isotopes have been measured as a function of depth to almost 3 m deep in lunar soil. It is well known that the cosmic-ray exposure age is obtained from a stable and radioactive cosmic-ray-produced isotope when the relative production rates are known. All soil samples contain large amounts of solar wind. Since appreciable amounts of stable rare-gas isotopes are in solar wind, it is difficult to measure the spallation-produced stable rare-gas isotopes normally used for exposure-age estimates. In spite of this difficulty, Bogard et al. (1973) and Pepin et al. (1974) have measured the concentrations of spallation-produced stable isotopes as a function of depth. Their interesting result is that the depth variation of the spallation-produced stable rare-gas isotopes resembles that of neutron-produced isotopes rather than that of spallation radioactivities.

The spallation-produced radioactivities $H^3$ and $Ar^{39}$ (Fireman et al., 1973) have maxima near the surface, while the neutron-produced isotopes $Ar^{37}$, $U^{235}(n, f)$, and $B^{10}(n, \alpha)$ (Stoenner et al., 1973; Stoenner and Davis, 1974; Woolum and Burnett, 1974a, b) have maxima near 100-cm depth. The neutron-produced stable isotopes of Gd and Sm were measured and have maxima at depths greater than 100 cm (Russ et al., 1972; Russ, 1973).

If no soil motion occurred during cosmic-ray irradiation, then the depth variation of the spallation-produced stable rare-gas isotopes would be identical to correspondingly produced radioactive isotopes, except for fluctuations near the surface caused by solar flares, and the depth variation of the neutron-produced isotopes of Gd and Sm could be calculated from cosmic-ray theory (Lingenfelter et al., 1972).

The depth variations of the spallation stable rare-gas isotopes and the neutron-produced isotopes of Gd and Sm are not as would be expected if there were no soil movement during the irradiation. Russ et al. (1972), Russ (1973), and Bogard et al. (1973) have proposed accretionary-type soil models that account for the differences in depth variation. Accretionary models necessitate that a large amount of irradiated soil be buried below 3 m, if the moon has been bombarded by cosmic rays for 4.5 aeons. This required thickness is at least a factor of 2 larger than the soil-thickness estimates from crater data (Shoemaker et al., 1970) and from active seismic investigations (Watkins and Kovacks, 1973), which range between 3 and 12 m. If the ancient soil has escaped from the moon, then a regolith thickness even as small as 250 cm is consistent with a cosmic-ray irradiation for 4.5 aeons (Fireman, 1974). Accretionary-type soil models are also inconsistent with the type of soil motion expected from meteoroid impacts. Since soil removed from one site and deposited at another by meteoroid impacts is generally deposited at a shallower depth, irradiated soil would be enhanced near the surface. We mention three other aspects of the statistical model: (1) A statistical model can be checked by isotope data at two different sites. The accretionary models consider each site to be so unique that there is little possibility for checking one of these models from isotope data at a different site. (2) With the statistical model, the average mass loss rate of the moon can be estimated. (3) Also, with the statistical model, isotope data can be related to meteor impact data.

## A GENERAL COSMIC-RAY REQUIREMENT FOR SOIL MODELS

The stable cosmic-ray-produced isotopes at all six Apollo sites require that irradiated soil be diluted by unirradiated (less irradiated) soil during times of the order of aeons. Such dilution is far greater than the implantation of meteors with low exposure ages would have caused, since the chemical composition of the soil limits the amount of meteoric material to less than 2% (Laul et al., 1971).

Cosmic-ray-produced isotope data are most relevant for times between 0.1 and 4 aeons and depths between ~10 and 300 cm. On the other hand, solar-wind and solar-flare-track data are relevant for much smaller times and depths. All soil grains were exposed to solar wind for approximately a thousand years, and a significant fraction of grains were exposed to solar flares for a million years. Models for soil development from cosmic-ray data can be made consistent with these solar-wind and flare data by requiring the soil grains to spend a short time during their history near the surface.

Also, occasionally, soil layers approximately 1 cm thick have a somewhat different color and texture than does the soil above and below. The most distinctive of these layers is at ~13-cm depth for an Apollo 12 location (Laul et al., 1971). This layer was essentially undisturbed for approximately 10 m.y. Models for soil development from cosmic-ray data should therefore permit a 1-cm layer of material at approximately 13-cm depth to persist undisturbed for more than 10 m.y.

From the point of view of cosmic rays, there is nothing unique about the six Apollo sites. If irradiated soil is diluted by unirradiated soil at these sites, it is natural to expect irradiated material to be diluted everywhere on the moon. If the cosmic-ray bombardment rate determined from lunar material is uniform over the moon, then the rate of dilution of irradiated material is also uniform. This requirement is certainly true when soil dilution is caused by small meteor impacts during a long time. We impose this requirement on our soil model for times between 0.1 and 4 aeons and depths between 10 and 300 cm. We examine those soil models that satisfy this uniformity requirement for consistency with the Apollo 15 and 16 isotope data.

The uniformity condition requires a relation between the dilution rate q(x) and the mass escape rate of soil from the moon, $m_e$:

$$m_e = \rho \int_0^d q(x)\, dx \qquad (\mathrm{g/cm^2\ aeon})$$

$$\frac{1}{\rho}\frac{dm_e}{dx} = q(x) \quad , \tag{1}$$

where q(x) is the fraction of mass at depth x per aeon replaced by unirradiated material, $\rho$ is the density of the soil, and d is the depth of undisturbed soil. If all the irradiated grains that are replaced by unirradiated grains return to the moon, then the moon would have more irradiated material per unit area everywhere else than at the site under consideration. Relation (1) eliminates the uniqueness of the site.

The dilution of irradiated soil affects the relation between the cosmic-ray isotope exposure age and the isotope production rate. The modification in the exposure age is discussed in the next section.

## THE SOIL MODEL

Lunar soil has developed by meteoroid impacts. Theoretically, the meteoroid impact parameters can be calculated from the soil movement and mass change if they are known. The cosmic-ray-produced isotopes in the soil and, especially, their depth variations also depend on the soil movement and its mass change. Since these isotope measurements are quite accurate, it is desirable to relate them to properties of the soil movement and the mass change for the moon. Our model formulates relations of this type.

Meteoroid impacts below a certain size range can be regarded as uniform over the moon for a time scale of the order of aeons. Cosmic-ray-produced isotopes have been measured to depths of nearly 3 m and show approximately smooth variations over this depth range. An irregularity exists in the Gd isotope measurements of the Apollo 16 drill stem in the neighborhood of 200-cm depth (Russ, 1973), but it can be attributed to a difficulty that occurred during the sample collection. Even if this irregularity is real, statistical models could encompass it.

Isotope measurements have been made at ~30-cm-depth intervals. We divide meteoroid impacts into two classes: those that remove or deposit less than ~30 cm of soil from a site, and those that remove or deposit more than ~3 m. The small impacts are assumed to be uniform over the moon during aeons; the large impacts are not. Intermediate impacts occasionally cause an irregularity in the isotope contents.

Meteoroid impacts affect the soil by removing and replacing surface material and by mixing surface material to depth. Soil layers can persist for long times even when the soil is uniformly mixed to a depth of 3 m in an aeon. If the meteoroid flux were constant in time, soil would then be uniformly mixed to 3-cm depth in 10 m.y., and a layer at 13-cm depth would be preserved for 40 m.y. Laul et al. (1971) have observed a soil layer at 13-cm depth that has an age of $10^7$ to $10^8$ yr. If the meteoroid flux increased in the past, a soil layer at 13-cm depth could be preserved longer.

A statistical soil model was mathematically formulated by an integral equation with stringent boundary conditions (Fireman, 1974). Here, we describe the same model in physical terms and relax the boundary conditions.

If the cosmic-ray-produced isotope clock at a site s is reset by a major local impact at time $T_s$, then the exposure ages $T(x)$ as a function of depth for a constant meteoroid flux are

$$T(x) = T_s e^{-T_s q(x)} + \frac{T_s}{2}\left[1 - e^{-T_s q(x)}\right] \quad \text{for} \quad T_s q(x) < 1 \quad , \tag{2a}$$

$$T(x) = T_s e^{-T_s q(x)} + \frac{1}{2q(x)}\left[1 - e^{-T_s q(x)}\right] \quad \text{for} \quad T_s q(x) \geq 1 \quad , \tag{2b}$$

if $q(x)$ is the dilution rate caused by minor impacts. The replacement time for soil at depth x is $1/q(x)$. If the meteoroid flux increased in the past, then the exposure ages would be approximately

$$T(x) = T_s e^{-T_s \overline{q(x)}} + \frac{T_s}{2}\left[1 - e^{-T_s \overline{q(x)}}\right] \quad \text{for} \quad T_s \overline{q(x)} < 1 \quad , \tag{3a}$$

$$T(x) = T_s e^{-T_s \overline{q(x)}} + \frac{1}{2\overline{q(x)}}\left[1 - e^{-T_s \overline{q(x)}}\right] \quad \text{for} \quad T_s \overline{q(x)} \geq 1 \quad , \tag{3b}$$

where $\overline{q(x)}$ is the time-averaged soil dilution rate during $T_s$. If the meteoroid flux increased exponentially in the past as $e^{2.6\,t}$, then $\overline{q(x)}$ is $[q_0(x)/2.6\ T_s]\ (e^{2.6\ T_s} - 1)$, where $q_0(x)$ is the present dilution rate. Equations (2) and (3) give the relation between the exposure age and the soil replacement parameters, $T_s$ and $q(x)$.

There are also relations between the exposure age $T(x)$ and isotope production rates $R(x)$. If soil is not mixed below depth d, then

$$T(x) = \frac{He^3}{R_3(x)} = \frac{(Gd^{158}/Gd^{157}) - (Gd^{158}/Gd^{157})_i}{[1 + (Gd^{158}/Gd^{157})]\ R_{158}(x)} \quad \text{for} \quad x > d \quad . \tag{4a}$$

$He^3$ is a frequently measured rare-gas spallation isotope. $Gd^{158}$ is a well-measured neutron-produced isotope in lunar soils, and an initial ratio $(Gd^{158}/Gd^{157})_i$ can be estimated. The relation between spallation rare-gas isotopes and exposure age is well known; that between the Gd isotope and exposure age was derived by Eugster et al. (1970). If soil is uniformly mixed to depth d, then

$$T(x) = \frac{He^3}{(1/d)\int_0^d R_3(x)} = \frac{(Gd^{158}/Gd^{157}) - (Gd^{158}/Gd^{157})_i}{[1 + (Gd^{158}/Gd^{157})]\,(1/d)\int_0^d R(x)\,dx} \quad \text{for} \quad x < d \ . \tag{4b}$$

We assume that soil is uniformly mixed from the surface to depth d, where the dilution rate is zero, or to bedrock at a rate C, where

$$d = C\,T \ . \tag{5}$$

If exposure-age relations (3) are substituted into equations (4), the soil replacement and mixing parameters are related to the cosmic-ray-produced isotopes. For example, equation (3a) is equated to (4b) to get

$$\frac{T_s}{2}\left[1 + e^{-T_s\,\overline{q(x)}}\right] = \frac{He^3}{\int_0^d R_3(x)\,dx} = \frac{(Gd^{158}/Gd^{157}) - (Gd^{158}/Gd^{157})_i}{[1 + (Gd^{158}/Gd^{157})]\,(1/d)\int_0^d R_{158}(x)\,dx} \tag{6}$$

when $x < d$ and $T_s\,\overline{q(x)} \leq 1$; and equation (3b) is equated to (4b) to get

$$T_s\,e^{-T_s\,\overline{q(x)}} + \frac{1}{2\,\overline{q(x)}}\left[1 - e^{-T_s\,\overline{q(x)}}\right] = \frac{Ar^{38}_{sp}}{\int_0^d R_{38}(x)\,dx} = \frac{(Gd^{158}/Gd^{157}) - (Gd^{158}/Gd^{157})_i}{[1 + (Gd^{158}/Gd^{157})]\,(1/d)\int_0^d R_{158}(x)\,dx} \tag{7}$$

when $x < d$ and $T_s\,\overline{q(x)} \geq 1$.

We calculate the soil parameters $\overline{q(x)}$ and $\overline{C}$ from Gd data at the Apollo 16 site with maximal and minimal values for $T_{16}$, the time of the last major impact, and then examine how well Gd ratios calculated with the same $\overline{q(x)}$ and $\overline{C}$ but with a different time, $T_{15}$, fit the Gd measurements (Russ et al., 1972) for the Apollo 15 site.

The maximum $T_{16}$ is 4.0 aeons, the solidification age of the rocks at the Apollo 16 site. Production rates $R_{158}(x)$ have been calculated by both Lingenfelter et al. (1972) and Kornblum and Fireman (1974), and the two calculations agree to better than a factor of 2. The values used in Tables 1 and 2 are essentially those given by Lingenfelter et al. (1972). The dilution rates $\overline{q(x)}$ are calculated with either equation (6) or (7) from the Apollo 16 Gd data and are given in Table 1. Two cases are considered: bedrock at 250-cm depth and a 750-cm depth for undisturbed soil obtained from setting $4\ \overline{q(250)} = 1$.

The minimum $T_{16}$ is 1.40 aeons, determined from equation (6) by setting $\overline{q(d)} = 0$ and by maximizing $(1/d) \int_0^d R_{158}(x)\, dx$. This minimal impact time, which occurs when d is approximately 250 cm, depends on the initial $Gd^{158}/Gd^{157}$ ratio. Reasonable limits for $(Gd^{158}/Gd^{157})_i$ are the terrestrial value, 1.587, and the lowest lunar value, 1.592 (Russ et al., 1972). With the initial ratio of 1.592, $T_{16}$ is 1.40 aeons. The corresponding dilution rates are given in Table 1. The bottom row in Table 1 gives the lunar mass escape rates corresponding to each of the calculated dilution rates. The Gd data at the Apollo 16 site impose three conditions on our model: (1) The time of the last major impact at the Apollo 16 site was between 1.40 and 4.0 aeons ago. (2) Soil mixes from the surface down at a rate between 60 and 180 cm/aeon. (3) The average mass escape rate of lunar soil from the moon is 171 g/cm$^2$ aeon for 1.4 aeons and ranges from 236 to 316 g/cm$^2$ aeon for 4.0 aeons.

We next examine whether any soil mixing and mass escape rates obtained from the Apollo 16 Gd data are consistent with the Apollo 15 Gd data.

## SOIL MODEL APPLIED AT THE APOLLO 15 SITE

For a constant meteoroid flux, the soil dilution and mixing rates at the Apollo 15 site are the same as those at the Apollo 16 site because of the uniformity requirement. For a meteoroid flux that increased exponentially in the past as $e^{2.6t}$, which is the largest proposed increase (Fechtig, 1971), the uniformity requirement relates the time-averaged dilution and mixing rates at the sites by

$$\overline{q_{15}(x)} = \frac{e^{2.6T_{15}} - 1}{e^{2.6T_{16}} - 1}\ \overline{q_{16}(x)} \qquad \text{and} \qquad \overline{C_{15}(x)} = \frac{e^{2.6T_{15}} - 1}{e^{2.6T_{16}} - 1}\ \overline{C_{16}(x)} \ . \tag{8}$$

From equation (4a), the exposure age below the mixing depth d for the Apollo 15 site is

$$T_{15}(x) = \frac{(Gd^{158}/Gd^{157}) - (Gd^{158}/Gd^{157})_i}{[1 + (Gd^{158}/Gd^{157})]\ [0.70\ R(x)]} \quad , \tag{9}$$

since the $Gd^{158}$ production rate at that site is approximately 0.70 times that at Apollo 16 (Lingenfelter et al., 1972). From Russ et al.'s (1972) isotope ratios, the time of the last major impact at the Apollo 15 site (exposure age at 250-cm depth) is

$$T_{15}(250) = \frac{1.596 - (Gd^{158}/Gd^{157})_i}{2.596\ (0.28 \times 10^{-2})} \quad . \tag{10}$$

Unless the initial Gd isotope ratio at the Apollo 15 site differs from that at Apollo 16, the time of the last impact at the Apollo 15 site ranges from 0.54 to 1.20 aeons.

Certain values for the soil mixing and dilution rates consistent with the Apollo 16 Gd data are inconsistent with the Apollo 15 Gd data. Our aim is to find a soil mixing rate and a mass escape rate consistent with the data at both sites or to show that none exists. We first assume a time-constant meteoroid flux and later assume a meteoroid flux that increased in the past. We require for consistency a difference of less than 0.001 between the calculated and the measured $Gd^{158}/Gd^{157}$ ratios.

The $Gd^{158}/Gd^{157}$ ratios, calculated with the dilution rates given in Table 1 for which $(Gd^{158}/Gd^{157})_i = 1.592$, are plotted as a function of depth in Fig. 1. When the meteoroid flux is constant, the dilution rate is identical for the two sites even though the major impact times at the two differ. Figure 1 shows that the soil mixing rate of 250 cm/1.4 aeons, or 178 cm/aeon, and the mass escape rate of 171 $g/cm^2$ aeon are consistent and that the other soil mixing and mass escape rates given in Table 1 are not.

The question arises as to why, with an identical statistical model, we obtain 171 $g/cm^2$ aeon for the mass escape rate here and 80 $g/cm^2$ aeon in Firemen (1974). The primary reason is that we use the neutron capture rates calculated by Lingenfelter et al. (1972) here and those calculated by Kornblum and Fireman (1974) there. This accounts for 70% of the difference. The second reason is that here we use a $(Gd^{158}/Gd^{157})_i$ of 1.592 for Apollo 16 soil in order to make Apollo 15 and 16 have the same initial ratio, while Fireman (1974) employed an initial ratio of 1.5866 for Apollo 16 soil and 1.592 for Apollo 15. Finally, there is a slight difference in the soil mixing: Here, we have the soil mix from the surface with a rate $\overline{C}$; Fireman (1974) required a square-wave type of soil mixing at all depths.

It is well established from crater information that the flux of large meteoroids increased greatly between 2.0 and 4.0 aeons ago, but it is not certain whether there has been much change in the flux of small meteoroids during the past aeon. Fechtig (1971) has presented evidence that the meteoroid flux increased for all times in the past as $e^{2.6t}$, where t is in aeons. It is of interest to examine how this meteoroid-flux change during the past 1.4 aeons would affect our soil model. With $T_{16} = 1.4$ aeons and $T_{15} = 0.54$ aeon, the time-averaged soil mixing rate at the Apollo 15 site is 18 cm/aeon (approximately 10% of the Apollo 16 rate). Since the surface soil is mixed to a depth of only 9.7 cm, the neutron capture rate is so low that the $Gd^{158}/Gd^{157}$ ratio at $x = 0$ is not significantly altered from 1.592. The soil impact parameters are not consistent with the Gd isotope data if the meteoroid flux increased for all times in the past as $e^{2.6t}$. Consistency is achieved, however, if the flux increase started 1 aeon ago.

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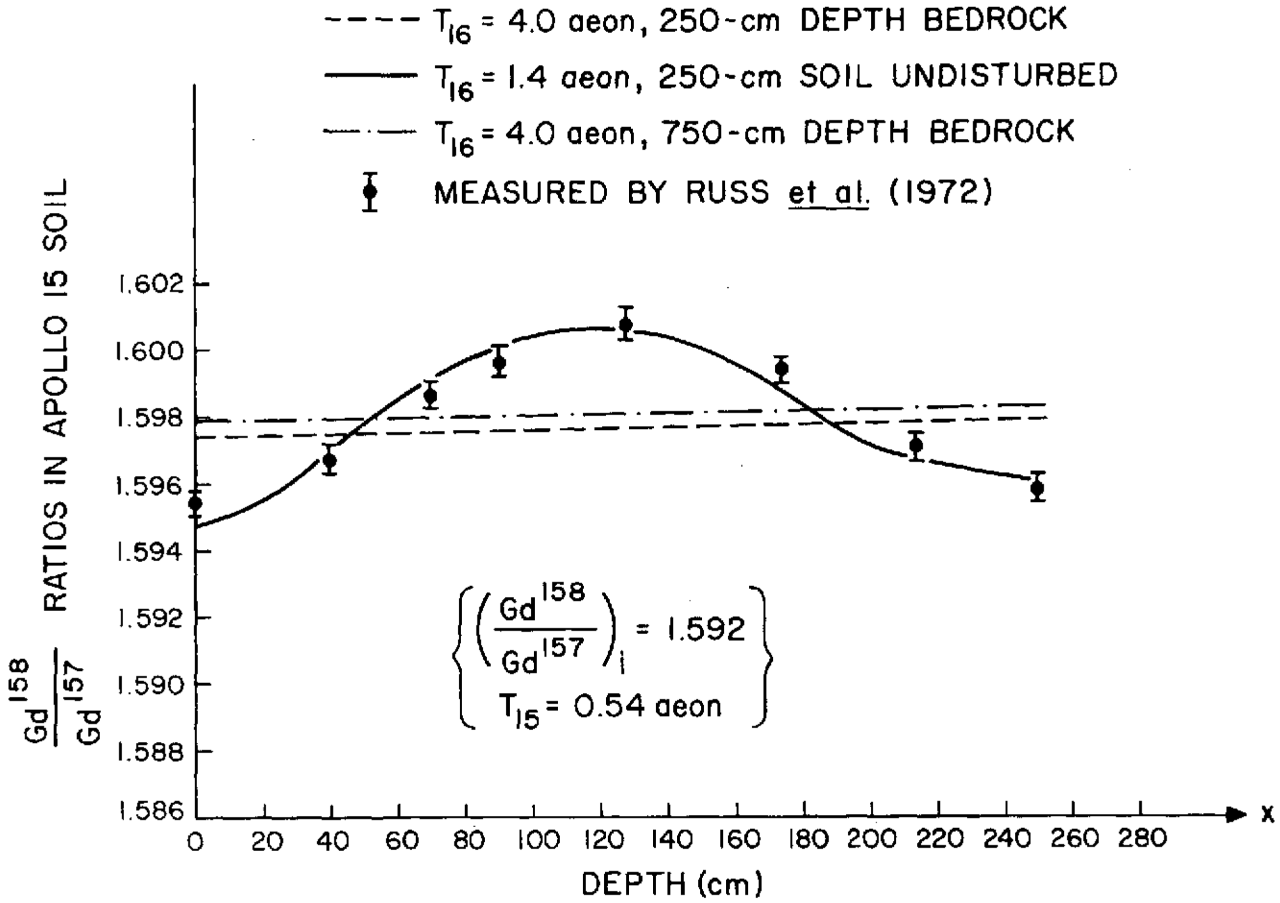

Fig. 1. Comparison of calculated and measured $Gd^{158}/Gd^{157}$ ratios.

Table 1. Dilution rates $\overline{q(x)}$ and mass escape rates determined from Apollo 16 soil $Gd^{158}/Gd^{157}$ data (Russ, 1973).

| Depth (cm) | $R_{158}(x)$ | $\dfrac{A}{\dfrac{Gd^{158}}{Gd^{157}} - 1.592}$ | $\dfrac{A/[1 + (Gd^{158}/Gd^{157})]}{\left(\frac{1}{250}\int_0^{250} R_{158}(x)\,dx\right)}$[*] | $\overline{q(x)}$[†] ($aeon^{-1}$) | $\overline{q(x)}$[‡] ($aeon^{-1}$) | $\dfrac{A/[1 + (Gd^{158}/Gd^{157})]}{\left(\frac{1}{750}\int_0^{750} R_{158}(x)\,dx\right)}$[*] | $\overline{q(x)}$[**] ($aeon^{-1}$) |
|---|---|---|---|---|---|---|---|
| 0 | $\sim 0.1 \times 10^{-2}$ | 0.011 | 0.69 | 0.85 | 1.05 | 1.38 | 0.51 |
| 35 | $0.60 \times 10^{-2}$ | 0.014 | 0.82 | 0.77 | 0.87 | 1.64 | 0.44 |
| 70 | $0.87 \times 10^{-2}$ | 0.017 | 0.96 | 0.67 | 0.71 | 1.92 | 0.38 |
| 100 | $1.00 \times 10^{-2}$ | 0.019 | 1.07 | 0.62 | 0.45 | 2.14 | 0.33 |
| 135 | $0.95 \times 10^{-2}$ | 0.020 | 1.13 | 0.60 | 0.35 | 2.26 | 0.32 |
| 170 | $0.75 \times 10^{-2}$ | 0.021 | 1.18 | 0.58 | 0.27 | 2.36 | 0.30 |
| 200 | $0.52 \times 10^{-2}$ | 0.022 | 1.26 | 0.54 | 0.17 | 2.52 | 0.28 |
| 250 | $0.38 \times 10^{-2}$ | 0.25 | 1.40 | 0.50 | 0 | 2.80 | 0.25 |
| mass escape rate ($m_e$) = $\rho\int_0^{250} q(x)\,dx \simeq \rho\sum_i \overline{q(x_i)}\,\Delta x_i$ | | | | 236 $g/cm^2$ aeon | 171 $g/cm^2$ aeon | $\rho\int_0^{250} \overline{q(x)}\,dx + \rho\int_{250}^{750} q(x)\,dx$ | $\left[129 + \rho\int_{250}^{750} q(x)\,dx\right]$ $g/cm^2$ aeon |

[*] $\frac{1}{250}\int_0^{250} R_{158}(x)\,dx = 0.68 \times 10^{-2}$; $\frac{1}{750}\int_0^{750} R_{158}(x)\,dx = 0.34 \times 10^{-2}$.

[†] $T_{10} = 4.0$ aeon; bedrock is at 250 cm, and $\overline{q(x)}$ is determined from equation (7).

[‡] $T_{16} = 1.4$ aeon; $\overline{q(250)} = 0$, which means soil *is undisturbed at* $x = 250$ cm: $\overline{q(x)}$ is determined from equation (7) for $x \geq 100$ cm and from equation (6) for $x < 100$ cm.

[**] $T_{16} = 4.0$ aeon; bedrock is at 750 cm and $\overline{q(x)}$ is determined from equation (7).

Table 2. Calculated $Gd^{158}/Gd^{157}$ ratios for Apollo 15 soil with dilution rates $\overline{q(x)}$ from Table 1.[*]

| Depth (cm) | $R_{158}(x)$ | $\overline{q(x)}^{\dagger}$ | $(Gd^{158}/Gd^{157})^{\dagger}$ | $\overline{q(x)}^{\ddagger}$ | $(Gd^{158}/Gd^{157})^{\ddagger}$ | $\overline{q(x)}^{**}$ | $(Gd^{158}/Gd^{157})^{**}$ |
|---|---|---|---|---|---|---|---|
| 0 | $\sim 0.07 \times 10^{-2}$ | 0.85 | 1.5974 | 1.05 | 1.5948 | 0.51 | 1.5979 |
| 35 | $0.42 \times 10^{-2}$ | 0.77 | 1.5975 | 0.87 | 1.5967 | 0.44 | 1.5980 |
| 70 | $0.61 \times 10^{-2}$ | 0.67 | 1.5976 | 0.71 | 1.5992 | 0.38 | 1.5980 |
| 100 | $0.70 \times 10^{-2}$ | 0.62 | 1.5977 | 0.45 | 1.6005 | 0.33 | 1.5981 |
| 135 | $0.67 \times 10^{-2}$ | 0.60 | 1.5977 | 0.35 | 1.6007 | 0.32 | 1.5981 |
| 170 | $0.527 \times 10^{-2}$ | 0.58 | 1.5978 | 0.27 | 1.599 | 0.30 | 1.5982 |
| 200 | $0.365 \times 10^{-2}$ | 0.54 | 1.5978 | 0.17 | 1.597 | 0.28 | 1.5982 |
| 250 | $0.276 \times 10^{-2}$ | 0.50 | 1.5979 | 0 | 1.596 | 0.25 | 1.5982 |

[*] Ratios calculated with $T_{15} = 0.54$ aeon and $Gd^{158}/Gd^{157} = 1.592$.

[†] Since $\overline{q(x)}\, T_{15} < 1$ and bedrock is at 250-cm depth, the ratios are calculated with equation (6) and

$$\frac{1}{250}\int_0^{250} R_{158}(x)\, dx = 0.475 \times 10^{-2}.$$

[‡] Since $\overline{q(x)}\, T_{15} < 1$ and mixed depth $\overline{C}\, T_{15} = 34$ cm, the ratios are calculated with equation (4a) for $x \geq 35$ cm; the ratio is calculated at $x = 0$ with equation (6) and $\frac{1}{34}\int_0^{34} R_{158}(x)\, dx = 0.25 \times 10^{-2}$.

[**] Since $\overline{q(x)}\, T_{15} < 1$ and soil is undisturbed below 260-cm depth in 0.54 aeon, the ratios are calculated with equation (6).