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# MODIFICATION OF AN IMPULSE-FACTORING 

ORBITAL-TRANSFER TECHNIQUE TO
ACCOUNT FOR ORBIT-DETERMINATION
AND MANEUVER-EXECUTION ERRORS
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## SUMMARY

A method has previously been developed to satisfy terminal rendezvous and intermediate timing constraints for planetary missions involving orbital operations. The method uses impulse factoring in which a two-impulse transfer is divided into three or four impulses which add one or two intermediate orbits. The periods of the intermediate orbits and the number of revolutions in each orbit are varied to satisfy timing constraints. In this paper, techniques are developed to retarget the orbital transfer in the presence of orbit-determination and maneuver-execution errors. Sample results indicate that the nominal transfer can be retargeted with little change in either the magnitude ( $\Delta \mathrm{V}$ ) or location of the individual impulses. Additionally, the total $\Delta V$ required for the retargeted transfer is little different from that required for the nominal transfer. A digital computer program developed to implement the techniques is described in the appendix.

## INTRODUCTION

The objective of one type of planetary mission might be to land a package of instruments at a preselected site on the surface of a planet to study its physical characteristics. The Viking-Mars program is a mission designed to achieve such an objective, and it will be used as an example throughout this paper. To accomplish the goal of surface reconnaissance, the Viking mission plan is to insert a lander-orbiter combination into orbit about Mars. The orbiter engine then makes up to four maneuvers (assumed here to be impulsive) to correct errors made in the Mars-orbit-insertion (MOI) maneuver, position the vehicle to allow reconnaissance of the landing site, and synchronize the spacecraft over the landing site in the proper position to make the deorbit maneuver. The lander then separates from the orbiter, deorbits, and lands while the orbiter maintains a data relay link to Earth. The sequence of orbital maneuvers may be characterized as a timefixed rendezvous from a specified initial orbit to a final orbit which has a synchronous period $P_{s}$, a periapsis radius $r_{p}$ which does not violate planetary quarantine
constraints, and a particular true anomaly $f_{\text {PER }}$ directly over the landing site. The value of $f_{\text {PER }}$ is defined by a study of the deorbit sequence. The landing-site reconnaissance may be interpreted'as an intermediate timing constraint on the rendezvous sequence.

One method of determining the proper sequence of orbital maneuvers is reported in reference 1. In this method, the problem is separated into two parts. First, a twoimpulse maneuver is found which transfers from the post-MOI orbit (initial elliptical orbit) to a final orbit which satisfies the geometry constraints ( $P_{s}, r_{p}$, and $f_{P E R}$ over the landing-site latitude $\delta^{\delta} \mathrm{PER}$ ). Then, one or both of the two impulses are factored into two or three collinear parts. For example, for the first impulse,

$$
\Delta \overline{\mathrm{V}}_{1}=\mathrm{q} \Delta \overline{\mathrm{~V}}_{1}+(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{1}
$$

By applying the first part of the impulse $\left(q \Delta \bar{V}_{1}\right)$, waiting one or more revolutions, and then applying the remainder of the impulse $\left((1-q) \Delta \bar{V}_{1}\right)$ at the same point, there is no net change in the resulting geometry. However, $q \Delta \overline{\mathrm{~V}}_{1}$ places the spacecraft in an intermediate orbit whose period is generally different from those of the two original orbits. This period difference and the number of revolutions in each orbit may be varied to satisfy the timing constraints. If the total number of impulses is limited to four, five different maneuver sequences arise. Two are three-impulse solutions (factor either the first or second impulse), and three are four-impulse solutions (factor each impulse, or factor the first or'second impulse twice).

The orbital maneuver sequences determined by the impulse-factoring method do not consider the effects of errors in the determination of the orbital elements nor of errors in the execution of each of the maneuvers. Due to errors in the determination of a spacecraft trajectory from observation data, only an estimate of the actual trajectory is available. A maneuver can be computed based on the estimate of the trajectory, but in reality it is applied to the actual trajectory. Further errors are introduced when the maneuver is applied since the maneuver cannot be executed precisely. One way to study the effects of these error sources is by a Monte Carlo analysis. In this method, the distribution of each error source is assumed. A random sample is obtained from each error distribution and the mission is simulated. By repeatedly simulating the mission with different random samples, statistics may be accumulated which represent the random process. The procedures described in this paper represent a single simulation in such a Monte Carlo analysis.

Only four-impulse transfers from the post-MOI orbit to the final orbit are considered. Since the addition of errors at any point in the sequence of maneuvers invalidates
the remainder of the original no-error sequence, the remaining maneuvers must be retargeted from the new orbit to the given final orbit. The rationale for the three-, two-, and one-impulse retargeters is presented in this paper. Sample results are given for retargeting a typical Viking-type mission. The appendix contains a brief description of the digital computer program which has been developed to implement the impulse factoring and retargeting technique.

## SYMBOLS

| f | true anomaly, deg |
| :---: | :---: |
| $\mathrm{f}_{\text {PER }}$ | true anomaly of a point in orbit directly over the landing site, deg |
| F | cost function |
| G | timing error in final orbit, $\mathrm{t}_{\mathrm{A}}{ }^{-t_{D}}$, hr |
| i | index defining reconnaissance orbit |
| I, J, K, L, M | integral revolutions in initial, first factored, transfer, second factored, and final orbits, respectively |
| k | weighting factor |
| P-- | orbital_period, hr |
| $\mathrm{P}_{\text {S }}$ | synchronous orbital period, hr |
| q, r | velocity factors |
| $\mathrm{r}_{\mathrm{p}}$ | periapsis radius, km |
| ${ }^{t}$ A | actual time of arrival over landing site, hr |
| ${ }^{t}$ D | desired time of arrival over landing site, hr |
| ${ }^{t}$ miss | target miss time, hr |

$\Delta t_{f} \quad$ time from entrance to final orbit to $f_{P E R}, \mathrm{hr}$
$\Delta \overline{\mathrm{V}} \quad$ vector describing an impulsive velocity change, $\mathrm{m} / \mathrm{s}$
$\Delta \mathrm{V} \quad$ magnitude of $\Delta \overline{\mathrm{V}}, \mathrm{m} / \mathrm{s}$
$\Delta V_{g} \quad \Delta V$ required for the two-impulse transfer which satisfies the geometry constraints on the final orbit $\Delta \mathrm{V}_{1}+\Delta \mathrm{V}_{2}, \mathrm{~m} / \mathrm{s}$
${ }^{\delta}$ PER landing-site latitude, deg

Subscripts:

1,2 refer to the first and second impulses of the geometry solution
f final orbit

Superscripts:
$\mathrm{r}, \mathrm{h}, \mathrm{n} \quad$ radial, horizontal, and normal components of $\Delta \mathrm{V}$, respectively

ANALYSIS

## Nominal Orbital Transfer

After the initial Mars orbit is established, an impulse-factoring technique can be used to find a sequence of four impulsive maneuvers which satisfy the following constraints on the Viking mission:

1. Transfer from the known initial orbit to a final orbit which has the correct period, periapsis radius, and true anomaly over the landing-site latitude.
2. Allow reconnaissance of the landing site on a particular revolution after insertion.
3. Synchronize the spacecraft with the landing site to allow the deorbit maneuver.

The first step of the impulse-factoring technique requires obtaining a "geometry solution" which satisfies the first constraint. This solution consists of two impulses: the first, $\Delta \overline{\mathrm{V}}_{1}$, places the spacecraft into a transfer orbit; the second, $\Delta \overline{\mathrm{V}}_{2}$, places the spacecraft into the geometrically correct final orbit.

In order to satisfy the second and third constraints, the two impulses may be factored such that

$$
\left.\begin{array}{l}
\Delta \overline{\mathrm{V}}_{1}=\mathrm{q} \Delta \overline{\mathrm{~V}}_{1}+(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{1}  \tag{1}\\
\Delta \overline{\mathrm{~V}}_{2}=\mathrm{r} \Delta \overline{\mathrm{~V}}_{2}+(1-\mathrm{r}) \Delta \overline{\mathrm{V}}_{2}
\end{array}\right\}
$$

After I revolutions in the initial orbit the first "factor" of $\Delta \overrightarrow{\mathrm{V}}_{1}, q \Delta \overline{\mathrm{~V}}_{1}$, is applied. This impulse places the spacecraft into an intermediate orbit between the initial and transfer orbits. Then, after $J$ complete revolutions in this intermediate orbit, the spacecraft is placed into the transfer orbit by applying the second factor, ( $1-\mathrm{q}) \Delta \bar{V}_{1}$, at the same point. Similarly, after $K$ revolutions in the transfer orbit $r \Delta \bar{V}_{2}$ is applied, placing the spacecraft into a second intermediate orbit. After $L$ complete revolutions in the second intermediate orbit the last impulse, ( $1-r$ ) $\Delta \overline{\mathrm{V}}_{2}$, places the spacecraft into the final orbit.

Since the intermediate orbits have periods which in general are different from those of the initial, transfer, and final orbits, the time spent in the two intermediate orbits serves to alter the timing of the geometry solution. By selecting proper values for $q, r, I, J, K$, and $L$, the reconnaissance and rendezvous constraints can be satisfied.

Two other four-impulse schemes are

$$
\left.\begin{array}{l}
\Delta \overline{\mathrm{V}}_{1}=\mathrm{q} \Delta \overline{\mathrm{~V}}_{1}+\mathrm{r}(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{1}+(1-\mathrm{r})(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{1}  \tag{2}\\
\Delta \overline{\mathrm{~V}}_{2}=\Delta \overline{\mathrm{V}}_{2}
\end{array}\right\}
$$

where $\Delta \overline{\mathrm{V}}_{1}$ is factored twice and $\Delta \overline{\mathrm{V}}_{2}$ is applied in full; and

$$
\left.\begin{array}{l}
\Delta \bar{V}_{1}=\Delta \overline{\mathrm{V}}_{1}  \tag{3}\\
\Delta \overline{\mathrm{~V}}_{2}=\mathrm{q} \Delta \overline{\mathrm{~V}}_{2}+\mathrm{r}(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{2}+(1-\mathrm{r})(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{2}
\end{array}\right\}
$$

where $\Delta \overline{\mathrm{V}}_{1}$ is applied in full and $\Delta \overline{\mathrm{V}}_{2}$ is factored twice.
Two three-impulse schemes are

$$
\begin{align*}
& \Delta \overline{\mathrm{V}}_{1}=\mathrm{q} \Delta \overline{\mathrm{~V}}_{1}+(1-\mathrm{q}) \Delta \overline{\mathrm{V}}_{1} \\
& \Delta \overline{\mathrm{~V}}_{2}=\Delta \overline{\mathrm{V}}_{2} \tag{4}
\end{align*}
$$

where $\Delta \overline{\mathrm{V}}_{1}$ is factored once and $\Delta \overline{\mathrm{V}}_{2}$ is applied in full; and

$$
\left.\begin{array}{l}
\Delta \overline{\mathrm{V}}_{1}=\Delta \overline{\mathrm{V}}_{1}  \tag{5}\\
\Delta \overline{\mathrm{~V}}_{2}=q \Delta \overline{\mathrm{~V}}_{2}+(1-q) \Delta \overline{\mathrm{V}}_{2}
\end{array}\right\}
$$

where $\Delta \overrightarrow{\mathrm{V}}_{1}$ is applied in full and $\Delta \overline{\mathrm{V}}_{2}$ is factored once.
The technique for selecting an optimum set of values for $q, r, I, J, K$, and $L$ for each of the factoring solutions is the subject of reference 2. There are many possibilities which must be investigated, but for the present paper it is assumed that an optimum set has been found. This set determines the nominal four-impulse transfer maneuver to which orbit-determination and maneuver-execution errors are applied.

One way to display the sequence of maneuvers is illustrated in sketch (a). Target miss time is plotted versus revolutions from MOI. The target miss time is evaluated


Sketch (a). - Nominal targeting sequence.
once each orbit as the spacecraft crosses the landing-site latitude. It is the time required for the landing site to rotate to a point directly beneath the plane of the orbit. As an aid to visualizing the maneuver sequence, the target-miss-time evaluation points are connected by straight lines. As the periods of the orbits change with each impulse, the slope of the connecting line is changed. For the Viking mission, zero slope represents a synchronous period while positive and negative slopes represent supersynchronous and sub-synchronous periods, respectively. The target miss time must be
zero both for the reconnaissance pass and when the spacecraft reaches the final orbit. The final orbit should have a synchronous period. By varying the factors and the number of revolutions in each orbit, the timing constraints can be satisfied.

## The Overall Retargeting Technique

Although a nominal orbital transfer (here restricted to four impulses) has been established, the problem is not yet solved. Since the nominal four-impulse transfer cannot be performed perfectly because of orbit-determination and maneuver-execution errors, a method must be developed to retarget the individual impulses to correct the final target parameters within acceptable tolerances. The parameters which must be controlled on the final orbit for the Viking mission are periapsis altitude, period, target miss time, and $f_{\text {PER }}$ over the landing-site latitude. A lower bound is placed on the periapsis altitude to satisfy the Mars quarantine constraint. The errors in the finalorbit period and target miss time are constrained since the spacecraft is required to be in a synchronous orbit directly over the landing site. The purpose of this requirement on the Viking mission is to allow the lander to deorbit to the landing site while the orbiter maintains a communications link with the Earth-based tracking stations.

The overall technique for determining the effect of maneuver-execution and orbitdetermination errors on the impulse sequence and final target parameters is as follows:

1. Generate a nominal four-impulse transfer from the actual initial orbit to the final orbit using the impulse-factoring technique to satisfy all constraints. The result is a sequence of four impulses applied at particular points in each orbit.
2. Perturb the actual initial orbit with orbit-determination errors, obtaining an estimate of the initial orbit.
3. Retarget the predicted four-impulse transfer from the estimated initial orbit to the known final orbit in order to remove accumulated errors.
4. Add execution errors to the first impulse of the retargeted four-impulse solution and apply this first impulse to the actual initial orbit. This step results in an actual second orbit.
5. Add orbit-determination errors to the actual second orbit, obtaining an estimate of the second orbit.
6. Using the estimated second orbit as an initial orbit, retarget a three-impulse transfer to the known final orbit in order to remove accumulated errors.
7. Repeat steps 4, 5, and 6 by applying errors to the first maneuver and first orbit in each sequence, thus retargeting two- and one-impulse transfers. The imperfect orbital transfers result in errors in the final orbit which cannot be removed completely. By
keeping track of the actual location, magnitude, and direction of each of the four applied maneuvers, the errors in the final-target conditions can be calculated. If the errors are within acceptable tolerances for a wide range of sample cases, the retargeting strategy is successful.

A typical retargeting sequence is illustrated in sketch (b). For clarity, target miss times versus revolutions from MOI are plotted as straight lines. The solid lines indicate


Sketch (b). - Typical retargeting sequence.
the intended variation in target miss time as each impulse is retargeted. The dashed line traces the actual target miss time throughout the sequence. The difference between the dotted and solid lines represents the period error of each orbit due to maneuver-execution and orbit-determination errors.

The following sections present one way of formulating each of the retargeting methods.

Four-impulse retargeting. - The four-impulse retargeting is a simple modification of the original four-impulse problem. Orbit-determination errors are added to the actual initial orbit to yield an estimate of the orbital elements. A new four-impulse sequence is generated based on the estimated initial orbit. Then, the first impulse of the new sequence is perturbed with execution errors and applied to the actual initial orbit to yield an actual second orbit.

Three-impulse retargeting. - The three-impulse retargeting is formulated as a modification to the three-impulse transfers (eqs. (4) and (5)). The initial orbit is taken
to be the actual second orbit that results from applying the first impulse of the retargeted four-impulse transfer with execution errors imposed. An estimate of this actual orbit is obtained by adding orbit-determination errors. A small timing error is allowed in the equations for the reconnaissance condition to account for accumulated errors at the time of landing-site reconnaissance. The $\Delta V$ required for the three-impulse retargeted transfer is minimized by the adaptive creeping algorithm discussed in reference 3. The velocity factor is adjusted to remove errors in the final target parameters.

Two-impulse retargeting. - The two-impulse retargeting requires a formulation different from those previously used. The actual second orbit of the retargeted threeimpulse transfer is used as the initial orbit for this portion of the trim strategy. An estimate of the initial orbit is found by adding orbit-determination errors. Since two impulses are required to transfer from this estimated initial orbit to the given final orbit, the impulse factoring technique can no longer be used. The only timing constraint which must be satisfied is the target miss time in the final orbit since the reconnaissance pass has been made on a previous orbit. A penalty -function approach is used to satisfy the final timing constraint. In this approach, the original cost function $\Delta \mathrm{V}_{\mathrm{g}}$ is augmented by a weighted function of the equality constraint to be satisfied. The augmented function $F$ is then successively minimized with increasing weights to drive the equality constraint to its desired value. That is, $F$ is minimized where

$$
F=\Delta V_{g}+\mathrm{kG}^{2}
$$

It is desired to drive $G$ to within some acceptable tolerance. By choosing a sufficiently large $k$ and minimizing $F$; the timing constraint $G$ will be satisfied at the expense of $\Delta V_{g}$. Thus, the retargeted two-impulse transfer satisfies all constraints on the final orbit.

One-impulse retargeting. - For the one-impulse retargeting, the initial orbit is taken to be the second actual orbit of the retargeted two-impulse transfer. An estimate of this initial orbit is obtained by adding orbit-determination errors:- Since only one impulse remains for the transfer to the desired final orbit, not all of the elements of the final orbit can be controlled simultaneously. Thus, a tradeoff must be made between the final timing error and the period error in the final orbit to keep both parameters within acceptable tolerances. The method used to target the final impulse is as follows:

1. Given the estimated elements of the fourth orbit and the desired final orbit along with the number of revolutions in each, a final target miss time $t_{\text {miss }}$ may be computed.
2. The miss time is nominally corrected to zero by targeting to an asynchronous final period:

$$
P_{f}=P_{S}-\frac{t_{\text {miss }}}{M+\frac{\Delta t_{f}}{P_{S}}}
$$

The new period of the final orbit has the effect of apportioning the timing error over $\mathrm{M}^{+}$ revolutions.
3. Given the new final period, a new semimajor axis of the final orbit is calculated.
4. By holding the eccentricity of the final orbit and true anomaly into the final orbit constant, a one-impulse transfer can be computed to target to the new semimajor axis on the final orbit (essentially the second impulse of the two-impulse transfer described in ref. 1). Since the true anomaly is held constant, $\Delta t_{f}$ will change and a new computed $t_{\text {miss }}$ will not necessarily be zero. If $t_{\text {miss }}$ is not within bounds, the entire procedure can be repeated from step 2 above using the new $\Delta t_{f}$. Since there is no flexibility in the one-impulse transfer, $\Delta V$ cannot be minimized.

## Application of Orbit-Determination and Maneuver-Execution Errors

Each of the retargeting methods discussed in the previous sections is applied to an initial orbit which has been corrupted by orbit-determination errors. The errors are obtained by sampling from one of two multivariate distributions which are represented by satellite-knowledge covariance matrices. The errors are in the classical orbital elements (semimajor axis, eccentricity, inclination, argument of periapsis, right ascension of the ascending node, and time of periapsis passage). One covariance matrix is valid at apoapsis and the other is valid at periapsis. The estimated true anomaly for the next maneuver determines which matrix is to be used. That is, if the maneuver true anomaly is between $-90^{\circ}$ and $90^{\circ}$ the periapsis covariance matrix is used; otherwise, the apoapsis matrix is used. These matrices are input to the program described in the appendix.

After retargeting methods are applied, the estimated sequence of maneuvers is known. However, before the next impulse is applied maneuver-execution errors are added. The following errors (assumed independent) are considered: accelerometer bias, accelerometer calibration, ignition timing error, and errors in two pointing angles (right ascension and declination of the fixed thrust vector) for each of the four trim maneuvers. These errors have the effect of perturbing the elements of the succeeding orbit in the maneuver sequence. The standard deviation of each error source is input to the program described in the appendix. The orbit-determination and maneuver-execution errors used for the following sample results are representative of those expected for the Viking mission.

The computer program developed to implement the retargeting techniques is described in the appendix. Figures 1 and 2 are a listing and flow chart of the main program. The subroutines required are also described in the appendix. A sample Viking case was computed to illustrate the retargeting techniques described herein.

Input for this case is shown in figure 3 and described in the appendix. The actual initial orbit is taken from the output of a program described in reference 4. On the final orbit, $f_{\text {PER }}$ is $-10.6^{\circ}$, the target latitude is $12.182^{\circ}$, and the final orbital period is synchronous ( 24.623 hr ). Most of the remaining input is program related and does not change from case to case (see the appendix). The nominal four-impulse maneuver sequence from the actual initial orbit to the required final orbit is given in figure 4. In figure 5 , the actual initial orbit has been perturbed by orbit-determination errors and the sequence has been retargeted to the required final orbit. In figure 6, the second orbit of figure 5 has been perturbed by orbit-determination and maneuver-execution errors and retargeted to the required final orbit. In figure 7, errors have been added to the second orbit of figure 6 and the maneuvers retargeted. In figure 8, errors have been added to the second orbit of figure 7. Here, however, a slightly asynchronous orbit of 24.62355 hours is required in order to minimize the final timing error. The following table summarizes the retargeted maneuvers:

| Maneuver | Impulses |  |  |  |  |  |  |  | Timing error, hr |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | First |  | Second |  | Third |  | Fourth |  |  |
|  | $\begin{gathered} \mathrm{f}, \\ \mathrm{deg} \end{gathered}$ | $\begin{aligned} & \Delta V, \\ & \mathrm{~m} / \mathrm{s} \end{aligned}$ | $\stackrel{\text { f }}{\mathrm{deg}}$ | $\begin{aligned} & \Delta V, \\ & \mathrm{~m} / \mathrm{s} \end{aligned}$ | $\stackrel{\mathrm{f},}{\mathrm{deg}}$ | $\begin{aligned} & \Delta \mathrm{V},- \\ & \mathrm{m} / \mathrm{s} \end{aligned}$ | $\begin{gathered} \mathrm{f}, \\ \mathrm{deg} \end{gathered}$ | $\underset{\mathrm{m} / \mathrm{s}}{\Delta \mathrm{~s}}$ |  |
| Nominal four-impulse | 110.4 | 35.3 | 112.7 | 4.4 | 113.0 | 7.7 | -152.4 | 11.1 | -0.005 |
| Retargeted four-impulse | 109.6 | 35.6 | 111.9 | 5.2 | 112.2 | 6.9 | -156.6 | 10.7 | -0.006 |
| Retargeted three-impulse |  |  | 109.2 | 3.9 | 109.4 | 8.0 | -147.7 | 11.8 | -0.0005 |
| Retargeted two-impulse |  |  |  |  | 110.3 | 7.9 | -148.0 | 12.5 | 0.0 |
| Retargeted one-impulse |  |  |  |  |  |  | -148.0 | 11.5 | 0.004 |

The small changes in the retargeted controls for this sample case demonstrate the stability of the nominal maneuver sequence in the presence of orbit-determination and maneuver-execution errors. The summary of actual end conditions in figure 8 shows that the error in the final orbital period is 7 seconds, the error in final periapsis radius is 0.75 km , and the actual final timing error is -5.3 seconds. All of these errors are well within acceptable tolerances. The actual total $\Delta V$ required for the trim is $58.5 \mathrm{~m} / \mathrm{s}$ as opposed to a nominal total $\Delta V$ of $58.4 \mathrm{~m} / \mathrm{s}$, indicating that no appreciable penalty is encountered to correct orbit-determination and maneuver-execution errors for the sample case. Experience has indicated similar behavior for a wide range of maneuver sequences.

## CONCLUDING REMARKS

A method has been presented to retarget an impulse-factoring orbital transfer to account for orbit-determination and maneuver-execution errors. Sample results indicate that the errors can be eliminated at small cost in velocity change by the retargeting techniques developed here. Additionally, the original solution for the sample case is quite stable in the presence of errors since the retargeted solutions are close to the nominal both in magnitude and location of the impulses. A digital computer program has been developed at the Langley Research Center to implement the impulse-factoring and retargeting techniques.

Langley Research Center,
National Aeronautics and Space Administration, Hampton, Va., February 4, 1974.

## APPENDIX

## PROGRAM DESCRIPTION

## General

A digital computer program has been developed to implement the impulse-factoring and impulse-retargeting technique described in the text. The program is coded in FORTRAN for the CDC 6600 computer. All of the subroutines required for MITOP (Multiple Impulse Transfer Optimization Program) are stored on the Langley Research Center data cell. The core storage required is about 70 K octal locations. Running time per case ranges from 1 to 10 sec of CPU time depending on the types of options exercised. Almost all data transfer is through two common arrays ( $C$ and IC) for floating point and integer variables. The contents of these arrays are described in tables I and II. Data associated with applying and accumulating errors for the retargeting cases are transferred through labeled COMMON TXERR described in table III. The following sections describe the main program, the primary-level subroutines, the input, and the output options.

## Main Program

A listing of the main program for MITOP is presented in figure 1. A schematic flow chart of the main program logic is given in figure 2. The main program is simply an executive routine to handle input, output, data transfer, and scheduling between the primary-level subroutines. The modular construction of the MITOP-logic flow facilitates modifications to the program without disturbing the underlying subroutines.

## Primary-Level-Subroutines -

The primary-level subroutines are those called by the main program. These special purpose subroutines in turn call the other subroutines listed on the data cell. The following is a brief description of each of the primary subroutines:

INITIAL This subroutine is called to initialize inputs or various parameters which need to be calculated only once in a run. It is called with a single integer argument (I):
$I=0 \quad$ Standard or "canned" inputs are stored (see tables I and II).

I = $1 \quad$ Calculates parameters which remain constant for a single case
$I=2 \quad$ Calculates parameters which remain constant for a single Monte Carlo sample (this option is called from the other primary-level subroutines).

STATX

RITE

## ERRORS

BURN 4, BURN 3, BURN 2, BURN 1 '

This subroutine calculates the mean and standard deviation of parameters of interest which are input in a vector. The first call initializes various storage locations and subsequent calls accumulate statistics. It is called only in the Monte Carlo mode.

This subroutine contains the various output options which are chosen by a single integer argument. The options are described in the output section.

This subroutine adds errors due to maneuver execution and orbit determination to the initial orbit involved in the current retargeting. It is called with a single integer argument which denotes the number of the maneuver being performed. The first call is with a zero to add orbit-determination errors to the initial orbit. This subroutine calls SAMPLE which generates the errors in orbital elements by sampling from covariance matrices which are input. In addition, SAMPLE keeps track of the actual performed maneuvers and computes the errors in the actual final target parameters.

These subroutines generate the optimal four-, three-, two-, and one-impulse transfers, given an initial orbit and the final target parameters. The BURN 4 subroutine is called twice. The first call is to generate a nominal four-impulse control history. The second call of BURN 4 (only in the retargeting option) is made after errors have been added to the initial orbit. A new four-impulse control history is calculated based on the perturbed initial orbit. (The same technique is used for BURN 3, BURN 2, and BURN 1.) These subroutines call several other special purpose subroutines. MODEL is the function evaluation routine which calculates the two-impulse geometry transfer (GEOM), and depending upon the mode, satisfies the timing constraints (TRIM) on every iteration, or for only the geometry minimum transfer. GPOP is the function minimization routine which uses an algorithm due to Rosenbrock (ref. 3). GUESS discretizes the independent-variable space to provide a good first guess for GPOP. RANGE determines if the reconnaiss-

APPENDIX - Continued
ance constraints are satisfied and MISS determines if the final timing constraint is satisfied.

STATOUT

This subroutine computes and outputs statistics on variables of interest. It is called only in the Monte Carlo mode. The output consists of mean and standard deviation of each parameter, the covariance matrix which indicates the degree of correlation between parameters, and histograms which indicate the distribution of selected parameters.

## Input

The primary method of input to MITOP is by the NAMELIST identified as CASE. Selected variables from the C- and IC-arrays and from the TXERR COMMON are input by name. Any other required input may be made directly to the arrays. A sample input is shown in figure 3. The input variables are equivalent to locations in the C - and IC-arrays as described in tables $I$ and $I I$. If only a single case is run, set NMC $=1$ and input the initial orbit in AO. If a Monte Carlo case is run, the individual samples are input by a tape (identified as TAPE 10) which contains many post-MOI orbits from program VEAMCOP (ref. 4). The tape is read as follows:

| First record | Total number of samples on tape (NOMC). |
| :---: | :---: |
| Second record | Five elements of the nominal initial orbit (ANOM). |
| Third and subsequent records | Five elements of the current sample initial orbit (AO), true anomaly of cutoff of the MOI maneuver (FBO), initial time bias (TBIAS), and the MOI $\Delta V$ (DVMOI). |

Multiple cases may be run with little effort. Only variables which change between cases must be input.

## Output

There are several output options available in MITOP. The output is controlled by subroutine RITE which is called at various points in the program. The subroutine has a single integer argument which controls the type of print at each call. Four input locations - IC(7 to 10) - are available to store the integer control key. Each location controls the output option chosen as follows:

| IC(7) | After nominal four-impulse targeting. |
| :--- | :--- |
| IC(8) | Prior to each retargeting. |
| IC(9) | After each retargeting. |
| IC(10) After each Monte Carlo sample. |  |

Each output control key may take values of 0 to 10 . The various values produce the following output:

0 No output.

1 Comprehensive output (explained below).

2 Minimal output consisting of one line (explained below).

3 Output the contents of the C-and IC-arrays.

4 Error summary at end of Monte Carlo sample (explained below).
$5 \quad 1+3$
$6 \quad 1+4$
$7 \quad 2+3$.

8
$2+4$.

9
$1+3+4$.

10
Available for user-supplied output.

Comprehensive output. - An example of the comprehensive output mode is given in figure 4. The labels are described as follows:

ORBIT NUMBER

A

| APPENDIX - Continued |  |
| :---: | :---: |
| E | eccentricity. |
| I | inclination. |
| W | argument of periapsis. |
| 0 | right ascension of ascending node. |
| F-IN | true anomaly of entrance to orbit. |
| F-OUT | true anomaly of exit from orbit. |
| P | semilatus rectum. |
| HP | periapsis altitude. |
| HA | apoapsis altitude. |
| PERIOD | period. |
| DELT | time from F-IN to F-OUT. |
| PER LAT | latitude directly beneath $\mathrm{f}_{\text {PER }}$ (sub-PER point). |
| RADIUS-IN | magnitude of orbital radius at F-IN. |
| RADIUS-OUT | magnitude of orbital radius at F-OUT. |
| R-OUT | $\mathrm{X}-, \quad \mathrm{Y}-$, and Z -components of orbital radius at F -OUT. |
| V-OUT | components of orbital velocity at F-OUT. |
| DV | components of $\Delta V$ required to transfer between successive orbits. |
| TIME/ORBIT | total time spent on each orbit. |

CUM TIME OUT

REVS/ORBIT

CUM REV TO FOUT

DELTA V

MOI DV

TOTAL DELTA V

RECONNAISSANCE
(OCCURS ON ORBIT i)
cumulative time from $\mathrm{F}-\mathrm{IN}$ on the initial orbit to F-OUT on each orbit.
number of revolutions on each orbit.
cumulative revolutions to F-OUT on each orbit.
magnitude of each maneuver.

Mars-orbit-insertion $\Delta V$.
sum of all maneuvers.
the reconnaissance pass occurs on the orbit identified by orbit number i.

The following reconnaissance-pass variables are labeled at three times (closest slant range approach to the target, spacecraft at $f_{P E R}$, and the spacecraft directly over the target latitude):

REV
T. A.

PHOTO TIME

TARGET RA

SUB S/C DEC and RA

TIME TO IMPULSE, LAST and NEXT

LOOK ANGLE

RANGGE

CENT-ANG
revolution from $\mathrm{F}-\mathrm{IN}$ on the initial orbit.
true anomaly.
time from $\mathrm{F}-\mathrm{IN}$ on the initial orbit.
right ascension of target at PHOTO TIME.
declination and right ascension of the spacecraft.
time between the last impulse and PHOTO TIME and between PHOTO TIME and next impulse.
angle at target between spacecraft and local vertical.
slant range between spacecraft and target.
central angle between spacecraft and target.

## APPENDIX - Continued

The following variables refer to the final orbital alinement.

TARGET DEC and RA

SUB-PER DEC and RA

ANGULAR MISS

MISS TIME AT PER
declination and right ascension of target when the spacecraft is at $f_{\text {PER }}$.
declination and right ascension of sub-PER point.
central angle between target and spacecraft at ${ }^{f} \mathbf{P E R}$.
timing error between target and spacecraft at $\mathrm{f}_{\mathrm{PER}}$.

Minimal output. - The minimal output consists of a single line of numbers which are listed from left to right as follows:

1

2,3,4,5

6

7,8

9

10

11
$12,13,14$

15

16

17

18

Monte Carlo sample number.
integer revolutions on each orbit. type of trim solution.
q and r factors applied to $\Delta \mathrm{V}$.
$\Delta \mathrm{V}$ required to satisfy geometry constraints,
$\therefore \mathrm{km} / \mathrm{s}$.
$\Delta V$ required for total trim, $\mathrm{km} / \mathrm{s}$.
true anomaly out of the initial orbit (F-OUT), deg.
components of $\Delta V_{1}, \mathrm{~km} / \mathrm{s}$.
true anomaly into the final orbit (F-IN), deg.
slant range at reconnaissance, km .
final miss time, hr .

MOI $\Delta V, \mathrm{~km} / \mathrm{s}$.

## APPENDIX - Concluded

Error summary at end of Monte Carlo sample. - This output option consists of the following variables which summarize the actual end conditions for each sample:

FINAL ORBIT

PDF

DPDF

HPF

DHPF

JDLAND

TBIASF

DVTRMA
a, e, i, $\omega, \quad \Omega, F-\mathbb{N}, F$-OUT on the final orbit. period of final orbit, hr.
difference between actual and desired PDF, s. periapsis altitude of final orbit, km .
difference between actual and desired HPF, km.
actual Julian date when spacecraft reaches $\mathrm{f}_{\mathrm{PER}}$ in final orbit, days.
timing error in the final orbit, s.
actual $\Delta V$ required for trim maneuvers, $\mathrm{km} / \mathrm{s}$.

In addition to the above print options, the following is output at the end of a Monte Carlo case:
(1) A summary of each sample consisting of case number; $\Delta V_{\text {MOI }} ; \quad \Delta V_{T R I M}$; $\Delta V_{\text {TOTAL }}$; and period, periapsis-altitude, and timing errors in the final orbit.
(2) The means and covariance matrix of $\quad \Delta \mathrm{V}_{\text {MOI }} ; \quad \Delta \mathrm{V}_{\text {GEOMETRY }} ; \quad \Delta \mathrm{V}_{\text {TRIM }} ;$ $\Delta V_{\text {TOTAL }}$; and period, periapsis-altitude, and timing errors.
(3) Histograms of $\Delta V_{\text {MOI }} \quad \Delta V_{T R I M} ; \quad \Delta V_{\text {TOTAL }} ;$ and period, periapsisaltitude, and timing errors.
(4) A listing of the summary statistics ordered on $\Delta V_{\text {TOTAL }}$.

1. Kibler, James F.; Green, Richard N.; and Young, George R.: Orbital Trim by Velocity Factoring With Applications to the Viking Mission. Proceedings of the National Space Meeting of the Institute of Navigation, Inst. Navigation, Mar. 1972, pp. 20-27.
2. Green, Richard N.; Kibler, James F.; and Young, George R.: Time-Fixed Rendezvous by Impulse Factoring With an Intermediate Timing Constraint. NASA TR R-422, 1974.
3. Rosenbrock, H. H.: An Automatic Method for Finding the Greatest or Least Value of a Function. Comput. J., vol. 3, 1960/1961, pp. 175-184.
4. Green, Richard N.; Hoffman, Lawrence H.; and Young, George R.: A Monte Carlo Error Analysis Program for Near-Mars, Finite-Burn, Orbital Transfer Maneuvers. NASA TN D-6598, 1972.

TABLE I. - DESCRIPTION OF ELEMENTS IN C-ARRAY

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| 1 | U | 42828.4 | Gravitational constant for Mars, $\mathrm{km}{ }^{3} / \mathrm{s}^{2}$. |
| 2 | RS | 3393.4 | Radius of Mars, km. |
| 3 | DR | $2 \pi / 360$ | Degrees to radians conversion factor. |
| 4 | RD | 360/2m | Radians to degrees conversion factor. |
| 5 | RHOMAX | 2000.0 | Maximum acceptable slant range from the landing site to the spacecraft at reconnaissance, km . |
| 6 | SCONV |  | No longer used. |
| 7 | CONV |  | No longer used. |
| 8 | TLAT | -30.0 | Target latitude, deg. |
| 9 | STLAT |  | Sine of target latitude. |
| 10 | CTLAT |  | Cosine of target latitude. |
| 11 to 15 | STEPLIM |  | No longer used. |
| 16 to 20 | STEPPAR |  | No longer used. |
| 21 to 25 | XIPLOW | $\begin{gathered} 0,0,0.03 \\ 0,0 \end{gathered}$ | Lower bounds on independent variables ( F -OUT, $\quad \Delta \mathrm{V}_{1}^{\mathrm{r}}, \quad \Delta \mathrm{V}_{1}^{\mathrm{h}}, \quad \Delta \mathrm{V}_{1}^{\mathrm{n}}$; $F-I N$ ) for discretization routine (GUESS). |
| 26 to 30 | XIPSTEP | $\begin{gathered} 15.0,0,0.01 \\ 0,15.0 \end{gathered}$ | Step sizes for independent variables (above) for subroutine GUESS, $\mathrm{km} / \mathrm{s}$ and deg. |
|  | For the following, $O$ refers to initial, $T$ refers to transfer, and $F$ refers to the final orbits, respectively. |  |  |
| 31, 61, 91 | AO, AT, AF |  | Semimajor axis, km. |
| 32, 62, 92 | EO, ET, EF |  | Eccentricity. |
| 33, 63, 93 | XIO, XIT, XIF |  | Inclination, deg. |

TABLE I. - DESCRIPTION OF ELEMENTS IN C-ARRAY - Continued

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| 34, 64, 94 | WO, WT, WF |  | Argument of periapsis, deg. |
| 35, 65, 95 | OO, OT, OF |  | Right ascension of the ascending node, deg. \| |
| 36, 66, 96 | FOIN, FTIN, FFIN | 0, -, - | True anomaly of entrance to the orbit, deg. |
| 37, 67, 97 | FOOUT, FTOUT, FFOUT | -, -, -10.6 | True anomaly of exit from the orbit - FFOUT is defined to be $f_{\text {PER }}$, deg. |
| 38, 68, 98 | SIO, SIT, SIF |  | Sine of inclination. |
| 39, 69, 99 | CIO, CIT, CIF |  | Cosine of inclination. |
| 40, 70, 100 | SWO, SWT, SWF |  | Sine of little omega. |
| 41, 71, 101 | CWO, CWT, CWF |  | Cosine of little omega. |
| 42, 72, 102 | SOO, SOT, SOF |  | Sine of capital omega. |
| 43, 73, 103 | COO, COT, COF |  | Cosine of capital omega. |
| 44, 74, 104 | SFOIN, SFTIN, SFFIN |  | Sine of true anomaly of entrance. |
| $45,75,105$ | C.FOIN, CFTIN, CFFIN |  | Cosine of true anomaly of entrance. |
| 46, 76, 106 | SFOOUT,'SFTOUT, SFFOUT |  | Sine of true anomaly of exit. |
| 47, 77, 107 | CFOOUT, CFTOUT, CFFOUT |  | Cosine of true anomaly of exit. |
| 48, 78, 108 | SLRO, SLRT, SLRF |  | Semilatus rectum, km. |
| 49, 79, 109 | PO, PT, PF | -, -, 24.623 | Period, hr. |
| 50, 80, 110 | DELFO, DELFT, DELFF |  | Change in true anomaly from entrance to exit, deg. |
| 51, 81, 111 | DELTO, DELTT, DELTF |  | Time from entrance to exit, hr. |
| 52, 82, 112 | HAO, HAT, HAF |  | Apoapsis altitude, km. |

TABLE I. - DESCRIPTION OF ELEMENTS IN C-ARRAY - Continued

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| 53, 83, 113 | HPO, HPT, HPF | -, -, 1500.0 | Periapsis altitude, km. <br> Time bias, hr. <br> Right ascension of the reconnaissance point, deg. <br> True anomaly of the reconnaissance point, deg. |
| 54, 84, 114 | TBO, TBT, TBF |  |  |
| 55, 85, 115 | RAOP, RATP, RAFP |  |  |
| 56, 86, 116 | FOP, FTP, FFP |  |  |
| 57, 87, 117 | TFOIN, TFTIN, TFFIN |  | Time of entrance to orbit referenced to periapsis, hr. |
| 58, 88, 118 | TFOOUT, TFTOUT, TFFOUT |  | Time of exit from orbit referenced to periapsis, hr. |
| 59, 89, 119 | TFOP, TFTP, TFFP |  | Time of reconnaissance point referenced to periapsis. hr. |
| $60,90,120$ |  |  | Not used. |
|  | For the following, $\mathbf{Q}$ and $\mathbf{R}$ refers to t | refers to the second fact | first factored orbit (q-orbit) ored orbit (r-orbit). |
| 121, 141 | AQ, AR |  | Semimajor axis, km. |
| 122, 142 | EQ, ER |  | Eccentricity. |
| 123, 143 | XIQ, XIR |  | Inclination, deg. |
| 124, 144 | WQ, WR |  | Argument of periapsis, deg. |
| 125, 145 | OQ, OR |  | Right ascension of ascending node, deg. |
| 126, 146 | FQ, FR |  | True anomaly of maneuver point, deg. |
| 127, 147 | SIQ, SIR |  | Sine of inclination. |
| 128, 148 | CIQ, CIR |  | Cosine of inclination. |
| 129, 149 | SWQ, SWR |  | Sine of little omega. |
| 130, 150 | CWQ, CWR |  | Cosine of little omega. |
| 131, 151 | SOQ, SOR |  | Sine of capital omega. |
| 132, 152 | COQ, COR |  | Cosine of capital omega. |

TABLE I. - DESCRIPTION OF ELEMENTS IN C-ARRAY - Continued

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| .133, 153 | SFQ, SFR |  | Sine of maneuver true anomaly. |
| 134, 154 | CFQ, CFR |  | Cosine of maneuver true anomaly. |
| 135, 155 | PQ, PR |  | Period, hr. |
| 136, 156 | HAQ, HAR |  | Apoapsis altitude, km. |
| 137, 157 | HPQ, HPR |  | Periapsis altitude, km. |
| 138, 158 | QV, RV |  | Factors associated with splitting $\Delta \mathrm{V}$. |
| 139, 159 | QP, RP |  | Factors associated with splitting the change in period between orbits. |
| 140, 160 |  |  | Not used. |
| 161 | DV1 |  | $\Delta V_{1}$ Magnitude of the first maneuver of the two-impulse transfer which satisfies all. geometry constraints, km/s. |
| 162 | DV2 |  | $\Delta V_{2}$ Magnitude of the second maneuver, $\mathrm{km} / \mathrm{s}$. |
| 163 | DVGEOM |  | $\Delta V_{\underset{\mathrm{g}}{ }}^{\text {Sum of }} \text { S. } \Delta V_{1}+\Delta V_{2}$ |
| 164 | DVTRIM |  | Total $\Delta V$ required to satisfy geometry and timing constraints, $\mathrm{km} / \mathrm{s}$. |
| 165, 166, 167 | DV1R, DV1H, DV1N |  | Radial, horizontal, and normal components of $\Delta V_{1}, \mathrm{~km} / \mathrm{s}$. |
| 168, 169, 170 | $\begin{gathered} \mathrm{H} 2 \mathrm{X}, \mathrm{H} 2 \mathrm{Y}, \\ \mathrm{H} 2 \mathrm{Z} \end{gathered}$ |  | $\mathrm{X}-, \mathrm{Y}-$, and Z -components of the angular momentum vector associated with the second orbit, $\mathrm{km}^{2} / \mathrm{s}$. |

TABLE I. - DESCRIPTION OF ELEMENTS IN C-ARRAY - Continued

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| 171, 172, 173 | PX, PY, PZ |  | $\mathrm{X}-, \mathrm{Y}-$, and Z -components of a unit vector pointing toward PER on the final orbit. |
| 174 to 182 | RXYZ |  | $3 \times 3$ array containing the coefficients for a coordinate transformation from the PQW to the XYZ system. |
| 183 | RATO |  | Right ascension of the landing site when the spacecraft is at periapsis on the initial orbit, deg. |
| 184 | ANGRAT | 360.0/24.623 | Angular rotation rate of Mars, $\mathrm{deg} / \mathrm{hr}$. |
| 185 | SH | 1.0/3600.0 | Seconds to hours conversion factor. |
| 186 | PMISS | 0.5 | Maximum acceptable miss time at reconnaissance, hr . |
| 187 | TMIN | 0.5 | Minimum time between reconnaissance and the closest maneuver, hr . |
| 188 | TBIAS |  | Time required for the landing site to rotate from its position when the spacecraft is at periapsis on the initial orbit to a point beneath the sub-PER right ascension, hr. |
| 189 | DELPTO |  | Difference in period from the initial to the transfer orbit PT-PO, hr. |
| 190 | DELPFT |  | Difference in period from the transfer to the final orbit PF-PT, hr. |
| 191 to 195 | STEPITR | $\begin{gathered} 5.0 ; 0.003,0.003 \\ 0.003,5.0 \end{gathered}$ | Initial step sizes in the independent variables (above) for the minimization routine, $\mathrm{deg}, \mathrm{km} / \mathrm{s}$. |

TABBLE I. - DESCRIPTION OF ELEMENTS IN C-ARRAY - Concluded

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| 196 | DVMOI |  | $\Delta V$ required for the Mars orbit <br> insertion maneuver, km/s. <br> 197 |
| 198 | RHOMISS |  | Slant range at the reconnaissance <br> point, km. <br> Difference in right ascension <br> between the loading site and the <br> spacecraft at reconnaissance, deg. |
| 199 |  |  | Not used. <br> Miss time in final orbit, hr. |
| 200 | TMISS |  |  |

TABLE II. - DESCRIPTION OF ELEMENTS IN IC-ARRAY

| Location | Variable name | Stored value | Description |
| :---: | :---: | :---: | :---: |
| 1 | NMC | 25 | Number of Monte Carlo samples. (If set to 1 , only a single case is run. If $>1$, sample input is by tape.) |
| 2 | IPHOTO | 8 | Revolution on which reconnaissance takes place. |
| 3 to 6 | IMIN, JMIN, KMIN, LMIN | 1, 2, 1, 2 | Minimum number of revolutions on the initial, first-factored, transfer, and second-factored orbits, respectively. |
| 7 to 10 |  | 0, 0, 0, 4 . | Write keys which may take values from 0 to 9 (explained in output section). |
| 11 |  | 1 | Retargeting control key: if 0 no retargeting; if 1 retarget each impulse. |
| 12 to 15 | IBEST, JBEST, KBEST, LBEST |  | Distribution of revolutions in each orbit for the most favorable total trim. |
| 16 | KKGEOM |  | Not used. |
| 17 | KEYBEST |  | Best type of trim solution. |
| 18 | MODE | 1 | Cost function to be minimized: if 1 minimize $\Delta V_{g}$; if 2 minimize $\Delta \mathrm{V}_{\text {TOTAL }}=\Delta \mathrm{V}_{\mathrm{g}}+$ penalty due to timing; if 3 minimize $\quad \Delta V_{g}+K G^{2}$. |
| 19 | IMC | 1 | Number of the current Monte Carlo sample. |
| 20 | NREV | 19 | Periapsis passage on which transfer must be complete. |
| 21 to 25 | NSTEP | 24, 1, 2, 1, 24 | Number of steps in each independent variable for subroutine GUESS. |
| 26 | ISUM |  | Number of integer revolutions in trim solution. |
| 27 | ITR | 2500 | Number of iterations for subroutine GPOP. |

TABLE II. - DESCRIPTION OF ELEMENTS IN IC-ARRAY - Concluded

| Location | Variable name | Stored value | Description |
| :---: | :--- | :--- | :--- |
| 28 | ITRIM | 0 | Key to choose factoring method: if <br> 0 factor periods; if 1 factor $\Delta V$. <br> Key to choose $1 \frac{1}{2}$ minimum revolu- <br> tions on each orbit: if 0 no constraint; <br> if 1 at least $1 \frac{1}{2}$ revolutions on initial <br> and transfer orbits. |
| 30 | IHALF | 0 | Not used. <br> 31 to 36 |
| IKEY1 to IKEY6 | $1,1,1,1,1,1$ | Keys to choose types of trim solutions: <br> if 0 skip solution; if 1 test solution. <br> 37 to 49 <br> 50 | MBEST |
| . | 2 | Not used. <br> Number of integer revolutions on final |  |

TABLE III. - DESCRIPTION OF VARIABLES IN TXERR

| Variable | Dimensions | Description |
| :---: | :---: | :---: |
| TJDCA | 1 | Julian date of periapsis passage on nominal approach hyperbola. |
| KNSAT | 1 | Satellite knowledge flag: 0 , use covariance of errors; 1, perfect knowledge. |
| VCAL | 1 | Calibrated value of velocity counting accelerometer. |
| IOSIG | 1 | Special flag to indicate use of 1-sigma level satellite orbit determination and maneuverexecution errors: 0, off; 1, on. |
| SATKN | $6 \times 6 \times 2$ | $6 \times 6$ satellite-knowledge covariance matrices valid at periapsis, $\operatorname{SATKN}(i, j, 1)$, and apoapsis, $\operatorname{SATKN}(\mathrm{i}, \mathrm{j}, 2)$. These matrices represent errors in the classical orbital elements, $\mathrm{a}, \mathrm{e}, \mathrm{i}, \omega, \Omega, \mathrm{t}_{\mathrm{p}}$. |
| OTCONT | 11 | Standard deviations of trim control parameters and spacecraft parameters $\sigma_{\delta_{\mathbf{r}}}, \sigma_{\epsilon}, \sigma_{\mathrm{t}_{\mathrm{IGN}}}$, $\sigma_{\alpha_{1}}, \quad \sigma_{\delta_{1}}, \quad \sigma_{\alpha_{2}}, \quad \sigma_{\delta_{2}}, \quad \sigma_{\alpha_{3}}, \quad \sigma_{\delta_{3}}, \sigma_{\alpha_{4}}, \quad \sigma_{\delta_{4}}$. $\delta_{r}$ is accelerometer bias; $\epsilon$ is accelerometer calibration error; ${ }^{t}{ }_{\text {IGN }}$ is ignition timing error; $\alpha$ and $\delta$ are pointing angle errors for each of a possible four trims. |
| TKACT | $7 \times 5$ | Array of values of classical orbital elements (a, e, i, $\omega, \Omega, F-I N, F$-OUT) for each of five possible actual ellipses (Post-MOI, PostOT1,... POST-OT4). |
| PDA | 5 | Period of each of the five actual ellipses mentioned above. |

TABLE III. - DESCRIPTION OF VARIABLES IN TXERR - Concluded

| Variable | Dimensions | Description |
| :---: | :---: | :---: |
| TJDPNA | 6 | Julian date of initial periapsis passage for each of the five actual ellipses and the Julian date of periapsis passage on the orbit following the orbit maneuver. |
| HPF | 1 | Periapsis altitude of the actual final orbit. |
| DVTRMA | 1 | Total actual $\Delta \mathrm{V}$. |
| DTAUF | 1 | Difference between actual final period and synchronous. |
| DHPF | 1 | Difference between actual final periapsis altitude and 1500 km . |
| TBLASF | 1 | Error in terms of Mars rotation time between the sublongitude of the PER point and the desired landing site. |

PROGRAH MITOP (INPUT, OUTPUT,TAPE5=INPUT,IAPE6=OUTPUT.IAPE10) DIMENSION AO(5), XIPLOW(5), XIPSTEP(5), NSTEP(5), XMEAN(7), XCOV(7,7), -XIN(7), ANOH(8),XIP(5),STEP(5).SAVE(6.100), KEYOPI(6)
s, CSAVE 200$)$. ICSAVE 50
COMMON C(200). IC(50)
COMMON/TXERR/TJDCA,KNSAT, VCAL, IOSIG,SATKN(6,6,2), OTCONT(11),
TKACT(7,5), PDA (5), TJOPNA (6), HPF, OVTRMA, DTAUF, DHPF, TBIASF
NAMELIST/CASE/AO, PER,TLAT.TBIAS,DVMOI, IMIN, NREV, IPHOTO, IMIN, JMIN
WKMIN,LMIN, NMC +KEYWI,KEYWZ,CONV, XIPLOW, XIPSTEP, NSIEP, MODE,ITR,IHALF
*, KEYOPT - C, IC
-TJDCA,KNSAT, IOSIG, SAIKN, OTCONT , VCAL
EQUIVALENCE (AO(1) C C(31)), (PER,C(97)), (TLAT,C(A)), (TEIAS.C(188)), -OVMOI, C(196) ), ITMIN,C(IB7)), (NREV,IC(20)), (IPHOTO,IC(2)), IIMIN, IC
 -1) (KEYW2, IC(8)), (CONV,C(7)), (XIPLOW(1),C(21)), (XIPSTEP(1),C(26)),
 -(37), (DVIR.C(165)), (OVIH.C(166)), (DVIN.C(167)), (FFIN-C(96)), (DVGE -OM.C(163)), (OVTRIM.C(184)), (ITRIM.IC(28)), (IHALF,IC(29)).(KEYOPI (I a) IC(31)

CALL INITIAL(O)
MBEST $=I C(50)=2$
1 CONTINUE
READ (5,CASE) S IF (EOF,5) 2.3
2 STOP
3 CONTINUE
IC 19 ) =?
C 19 ) $=1$
WRITE (G.CASE)
CALL INITIAL(1)
DO $160 \mathrm{I}=1.200$
CSAVE(I) $=$ C(I)
$00161 \quad 1=1.50$
16J MCSAVETI=ICI
IF (NMC.EQ.1) GO TO 5
CALL STATX (1,7,XMEAN, XCOV, XIN)
REWIND 10
READ (10) NOMC S IF (NOMC.LT. NHC) NMC=NOMC
WRITE $(6,101)$ NOMC
FORMAT(17) WRITE (6.102) ANOM
102 FORMAT (EE16.81
$001000 \mathrm{JMC}=1$, NMC
$001621=1,200$
$162 C(I)=C S A V E(I)$
$00 \quad 163 \quad I=1,50$
$163 \mathrm{IC}(\mathrm{I})=I \mathrm{ICSAVE}(\mathrm{I}$
REAO(10) AO,FRO.TEIAS,OVMOI
IMC= JMC
CALL BURN4
OVTRMA=DVTRIM
DTAUF $=0.0$
DHPF $=0.0$
FBIASF=0.0
CALL RITE(IC(7))
IFIC(11).EQ.0)GO TO 10
CALL ERRORS (O)
IC(27) $=250$
CALL BURN4
CALL BURN3
CALL ERRORS 12
CALL BURN2
CALL ERRORS 13
CALL SURNI
CALL ERRORS 141
CALL RITEIIC(10)
10
CONTINUE
$\times I N(1)=0 V M O$
IIN(2) = DVGEOM
$\times$ IN(3) $=$ OVTRMA
$\times \operatorname{IN}(4)=$ DVMOI + OVTPMA
$X I N(5)=D T A U F$
$X I N(0)=D H P F$
XIN(7) = TBIASF
SAVE (1, IMC) = DVMOI
DO $200 \quad 1=2.6$
SAVE $(I+I M C)=X I N(I+1)$
CONT INUE
CALL STATX(2.7.XMEAN,XCOV,XIN)
CALL STATOUTIXMEAN, XCOV,XIN,SAVEI
GO TO 1
5 CONTINUE
OVMOI = O.
Call burna
CALL RITEIICI71)
IF(IC(11).EQ.01GO TO 20
CALL ERRORS (O)
CALL GURN4
CALL ERRORS 11
CALL BURN3
CALL ERRORS (2)
CALL BURN2
CALL ERRORS 13
CALL BURN1
CALL ERRORS (4)
CALL RITE(IC(10))
20 CONTINUE
GO to 1
END
Figure 1. - Listing of main program.


Figure 2. - Flow chart of main program.


Figure 3. - Sample input. Note: Tape input overrides the values for AO, TBIAS, and DVMOI.

5
$2.04277099 E+04$
$7.60452835 E-01$
$3.29595516 E+01$
$3.34226681 E+01$
$1.06601094 E+02$
$2.08144857 E+02$
$-1.06000000 E+01$
$8.61459990 E+03$
$1.50000000 E+03$
$3.25686198 E+04$
$2.46230000 E+01$
$4.79824779 E+00$
$1.21824287 E+01$
$2.61472580 E+04$
$4.92973868 E+03$
$-2.83571206 E+03$
$3.89600309 E+03$
$1.04029828 E+03$
$-2.27649731 E+00$
$-2.55575120 E+00$
$1.88802482 E+00$
$5.40442478 E+01$
$1.16070880 E+01$
$2.19486853 E+00$
$1.09882261 E+01$
$5.84281154 E+01$




## m

$2.05725814 E+04$
$7.65608536 E-01$
 $3.54275891 E+01$
$1.05428446 \mathrm{E}+02$
$1.12958588 \mathrm{E}+02$
$1.12958588 \mathrm{E}+02$
$8.51383050 \mathrm{E}+03$ B. $51383050 \mathrm{E}+03$ $1.29297253 \mathrm{E}+04$
$3.29854000 \mathrm{E}+01$ $2.48854000 E+01$
0.
1.31351118E+01 $1.21389898 \mathrm{E}+04$
$1.21389898 \mathrm{E}+04$
 $-1.13885106 E+04$ $3.44380129 \mathrm{E}+03$
$9.91631974 \mathrm{E}-01$ $-1.97857475 \mathrm{E}+00$
$-2.76451263 \mathrm{E}-01$ -2. $76451263 E-01$ $00+3 E 8 S S 82 E 0^{\circ} \mathrm{S}$
$00+382 \varepsilon 88590^{\circ} \mathrm{S}$ $5.03285583 \mathrm{E}+00$
$7.46561999 \mathrm{E}+01$



80

$2.07335332 E+04$
$7.66492786 E-01$
$3.28273217 E+01$
$3.57713743 E+01$
$1.05358880 \mathrm{E}+02$
$1.12673280 E+02$
$1.12673280 \mathrm{E}+02$
$8.55235042 \mathrm{E}+03$
$1.44802958 \mathrm{E}+03$
$3.32322368 \mathrm{E}+04$
$2.51780109 \mathrm{E}+01$

## IO + 306ヶ80 <br> 2.51780109E 01

 . $21389898 E+04$ $1.21389898 E+04$$2.40772358 E+03$ $-1.13885106 E+04$ $3.44380129 E+03$
$9.93236240 E-01$ $-1.98145068 \mathrm{E}+00$ $-2.79308440 \mathrm{E}-01$
'
 1.011476
1.069904

$10$





10

4





 cum rev to fout CUM TIME OUT,DA TIME/ORAIT-HR DV.M/SEC X-COMP $\begin{array}{ll}v \text {-OUT, } & x-C O M P \\ K M / S E C & Y-C O M P\end{array}$ RADIUS-OUT, KM
R-OUT, KM X-COMP pelt.hr deg P,KM
HP,KM
HA,KM
PERIOD,HR
DELT,HR F-IN, DEG
F-OUT , DEG
P,KM
HP,KM
HA,KM
PERIOD,HR
DELT,HR
ORBIT NUMBER
PARAMETERS
号
I.DEG
W.DEG O.DEG F-IN.DEG PER LAT.DEG
RADIUS-IN,KM $r-$ COMP
$z-$ COMP $\begin{array}{ll}\text { OV.M/SEC } & \\ & X-C O M P \\ & Y-C O M P\end{array}$ $r-$ COMP
$z$-СомP REVS/OREIT CELTA V,M/SEC MOI DV.KM/SEC TOTAL DELTA V.KM/

FINAL ORBIT ALIGNMENT

| TIME TO IMPULSE | LOOK ANGLE | RANGE |  |
| :--- | :--- | :---: | :---: |
| LAST,HR NEXT,HR | DEG | KM |  |
| 23.545 | 51.166 | 6.521 | 1471.074 |
| 23.543 | 51.169 | 5.437 | 1471.422 |
| 23.531 | 51.181 | .878 | 1481.922 |


Figure 8. - Retargeted 1-impulse transfer and summary of actual end conditions.
$20-35251 E ヶ 58^{\circ} \mathrm{S}$

ORBIT NUMBER

$$
\begin{aligned}
& 01
\end{aligned}
$$

$$
\begin{aligned}
& \text { - } 3 \text { - } 5 \text { SS6682* } 6
\end{aligned}
$$

$$
\begin{aligned}
& \text { 10* 3ESZLOSLE } \\
& \text { 9SE } 1208^{\circ} \text { I }
\end{aligned}
$$

## $N$



$10+3 E S E 8905^{\circ} I$
$00+3 I S I 29 I \% 6^{\circ} \mathrm{E}$
SUB-PER DEC,DEG 12.182
$\rightarrow 80^{\circ} 9$ てt $930^{\circ} \forall 8 ~ L 398 \forall 1$ AT PER,HR

TARGET RA, OEG
MISS TIME $D P D F-S E C^{\prime}=$
DHPF: $=$

## 11 <br> $\forall W \triangle 1 \wedge 0$ <br> <br> VTRMA

 <br> <br> VTRMA}
$10+3658682^{\circ} \varepsilon$

## $00+3 \angle 6+\angle S S \angle 6^{\circ} 9$

 $7.55250675 E-01$$-5.27715324 E+00$


$7.604284 E-0$
DPDF-SEC


$$
\mathbf{I}
$$

OAR

$$
\begin{aligned}
& \text { RAMETERS } \\
& \text { A,KM } \\
& \text { E } \\
& \text { I,DEG } \\
& \text { W,DEG } \\
& \text { O.DEG } \\
& \text { F-IN:DEG } \\
& \text { F-OUT•DEG } \\
& \text { P:KM } \\
& \text { HP,KM } \\
& \text { HA,KM } \\
& \text { PERIOD,HR } \\
& \text { DELT,HR } \\
& \text { PER LAT,DE } \\
& \text { RADIUS-IN, } \\
& \text { RADIUS-OUT } \\
& \text { R-OUT,KM X }
\end{aligned}
$$

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