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GEOS-C ORBIT DETERMINATION
WITH SATELLITE TO SATELLITE TRACKING

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

This report studies the feasibility of employing satellite to
satellite tracking in lieu of ground based tracking to satisfy
the orbit determination requirements of the GEOS-C mis-
sion. It is shown that with proper estimation procedures
it is possible to obtain from S. S. T. data a GEOS-C orbit
whose altitude error averages about 1 meter. The useful-
ness of this data type for geopotential recovery is also
indicated.

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GEOS-C ORBIT DETERMINATION WITH
SATELLITE TO SATELLITE TRACKING

INTRODUCTION

Satellite to satellite tracking (S. S. T.) by means of a synchronous satellite has been recognized as a promising data type for satisfying the stringent orbit determination demands of the Earth and Ocean Physics Application Program. [1,2,3,4] In this report we study the feasibility of employing this data type in lieu of ground based tracking to satisfy the orbit determination requirements of the GEOS-C mission. It is shown that with proper estimation procedures it is possible to obtain from S. S. T. data a GEOS-C orbit whose altitude error averages about 1 meter. This result is significant in itself with respect to the GEOS-C mission. It is also significant in that it suggests what the proper use of S. S. T. data can accomplish in terms of economical and accurate orbit determination of future applications satellites.

ASSUMPTIONS

The GEOS-C orbit will be circular with an altitude of approximately 850 km and an inclination of 65°. The period is 102 minutes. For this study a relay satellite is assumed at synchronous altitude and at 94° west longitude. The receiving station is Rosman. The tracking assumptions and error sources are displayed in Table 1.

With regard to range rate sum data the velocity uncertainty of the relay satellite is far more critical than position uncertainty. For this reason a serious mistake is introduced if the relay satellite is treated as an ordinary tracking station which happens to be at synchronous altitude. Under such an assumption there is no doppler shift due to relative motion between the relay satellite and the ground based station and hence the range rate sum data is left unbiased. In actuality the relay satellite is in a Keplerian orbit. Consequently an initial position error quickly propagates into a velocity error which in turn biases the range rate sum data. In this report the velocity errors in the relay satellite are correctly propagated into orbital errors of the GEOS-C satellite.

The results shown in the next section were obtained by means of covariance analysis techniques. Hence we have assumed that over the range of expected errors, perturbations of orbital estimates are approximately linear functions of perturbations of the error sources. With the use of covariance analysis it is relatively easy to isolate the effect of any subset of error sources on the orbital state estimate at any point along the arc. Such an ability is a valuable asset in

Table 1

Tracking and Error Assumptions for Error Analysis of GEOS–C
Orbit Determination With S. S. T. Data

Tracking Assumptions	
Data Type	Range Rate Sum
Data Rate	1/min.
Arc Length	5 days
Data Noise	0. 1 cm/sec
Systematic Error Assumptions	
Error Source	Standard Deviation
Rosman Survey Error	10 meters in each component
Data Bias	0. 1 cm/sec
Relay Satellite Epoch State	300 meters in each position component 3 cm/sec in each velocity component
GEOS–C Satellite Epoch State	50 meters in each position component 5 cm/sec in each velocity component
Spherical Harmonic Coefficients to Degree and Order 8	Full difference between S. A. O. 69 field and G. E. M. 4 field

determining which error sources can be ignored and which error sources must
be simultaneously adjusted along with epoch state. The mathematical develop-
ment of our covariance analysis techniques is given in the appendix.

RESULTS

The simplest estimation of GEOS–C state is obtained when the satellite epoch
vector is adjusted in a weighted least squares mode and all other parameters
are fixed at nominal values. The covariance matrix of this estimator was ob-
tained and propagated along the 5 day arc. The unadjusted error sources and
associated standard deviations are those shown in Table 1. Figure 1, is a plot
of the resultant standard deviation of the radial component of the GEOS–C orbit.
The plot was stable over the entire 5 day arc and hence only the results for the
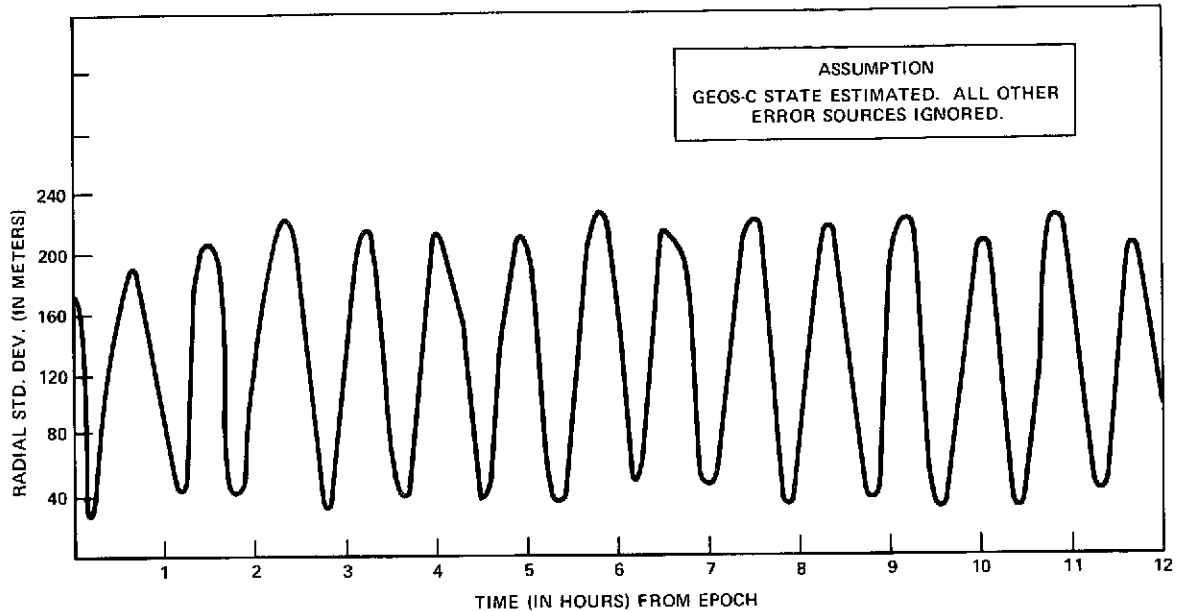first 12 hours are shown.

Figure 1. GEOS-C Radial Std. Dev. vs. Time From Epoch

As shown in the Appendix, covariance analysis techniques permit one to determine at any point in the arc, the R. S. S. contribution to the standard deviation of the estimate of a given state component due to a given unadjusted error source. These contributions are called aliasing terms. Table 2 shows the maximum aliasing along the arc in the radial, along track, and cross track directions due to data bias, Rosman survey error, relay satellite epoch state error, and geopotential. For any given point in the arc and for any given component, the aliasing due to geopotential error was obtained by computing the root sum square of the corresponding aliasing terms due to errors in individual geopotential coefficients up to and including degree and order eight. Similarly the aliasing due to relay satellite orbital error is the root sum square of six alias terms and the aliasing due to Rosman survey error is the root sum square of three alias terms. Table 2 indicates that the dominant error source for GEOS-C orbit determination is the uncertainty of the relay satellite state and that data bias and Rosman survey error are not significant error sources.

With regard to GEOS-C orbit determination, attention is focused primarily on the altitude estimation requirements imposed by its on-board altimeter. Since the instrument is capable of one meter precision it is required to determine the GEOS-C altitude with a standard deviation of one meter. From Table 2 it is clear that for this to be accomplished the relay satellite state must be simultaneously estimated along with GEOS-C state. Figure 2 shows the resultant standard deviation in the estimation of the radial component of GEOS-C position

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Table 2

Maximum Aliasing in Meters in Radial, Along Track, and Cross Track
Components of GEOS-C State Due to Data Bias, Rosman Survey Error,
Relay Satellite Epoch State Error, and Geopotential Error

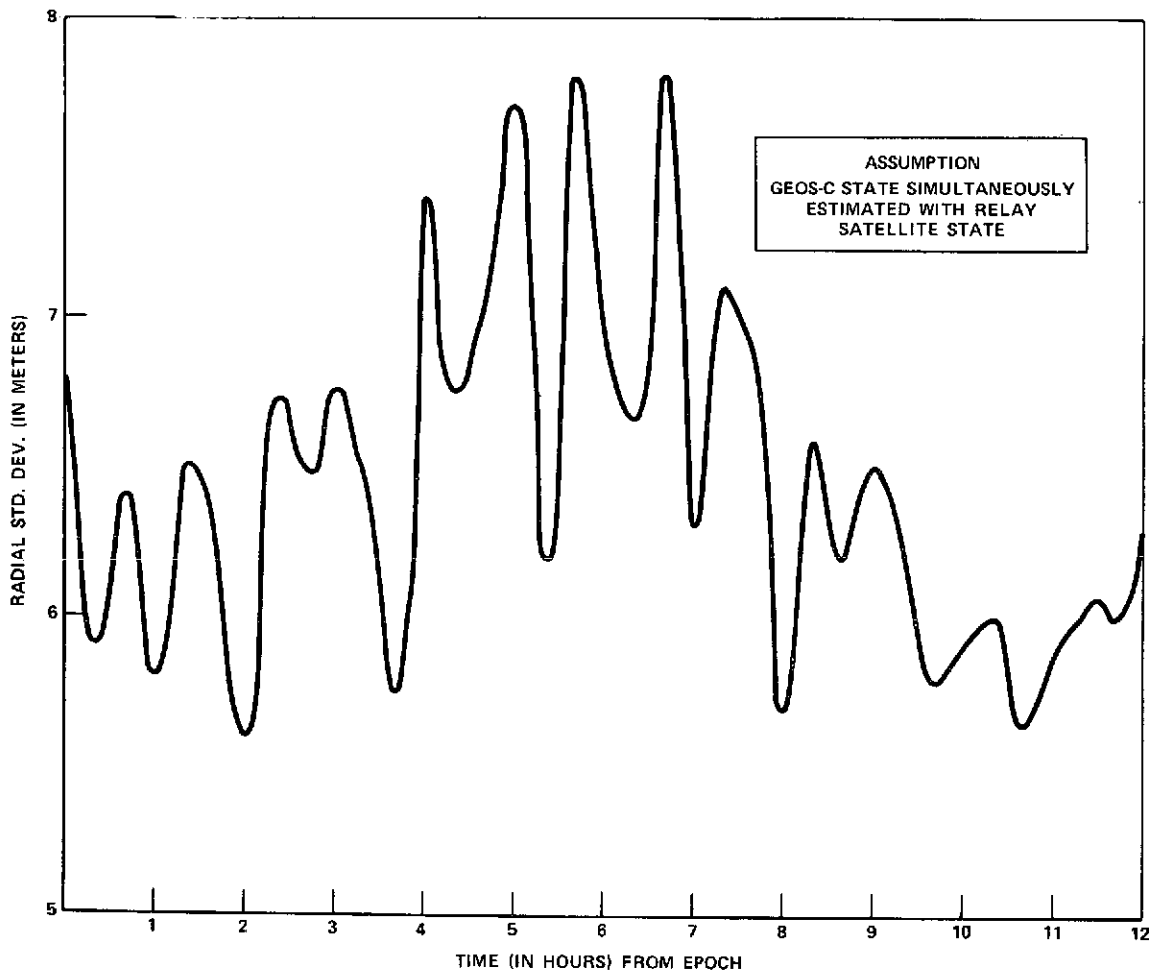|	Data Bias	Rosman Survey Error	Relay Satellite Epoch State Error	Geopotential Error
Radial	0.025	0.002	229	7.3
Along Track	3.300	0.008	1691	27.3
Cross Track	0.210	0.002	4379	17.3

Figure 2. GEOS-C Radial Std. Dev. vs. Time From Epoch

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for the first 12 hours after epoch when GEOS-C state is estimated simultaneously along with relay satellite state. The average radial uncertainty along the 5 day arc was 6.2 meters.

The object is to satisfy the constraint of one meter radial uncertainty by adding the smallest possible number of coefficients to the adjusted parameter set which will accomplish the task. Since it is not possible to examine the effects of placing each of the possible combinations of coefficients in the adjusted mode, an essentially suboptimal procedure for determining an efficient estimation strategy must be implemented. Our procedure is recursive. We choose the N + 1st coefficient to be adjusted by examining the results of adjusting N coefficients along with GEOS-C and relay satellite state and identifying as the next coefficient to be adjusted the one which produces the maximum radial aliasing along the 5 day arc. The recursive process was automated in our covariance analysis computer program and was terminated when an average radial error of one meter along the 5 day arc was achieved.

The ordering of coefficients according to their maximum radial aliasing varies as terms are added to the adjusted state. This is why the elaborate recursive procedure was necessary to identify an efficient estimation strategy. As an example, when geopotential coefficients S(7,2), S(6,6), C(7,0), C(7,6), and C(6,5) are adjusted along with GEOS-C and relay satellite state the dominant error source is the uncertainty of C(7,2) which provides a maximum radial aliasing of 2.9 meters. The uncertainty of geopotential coefficient S(4,3) is the sixth most important error source with a maximum radial aliasing of 1.9 meters. When coefficient C(7,2) is added to the adjusted state, the uncertainty of coefficient S(4,3) becomes the most significant error source with a maximum radial aliasing of 3.6 meters. Notice also that our recursive procedure was oriented toward identifying the most efficient estimation algorithm for minimizing the average radial error. If the along track or cross track or the total position errors were to be minimized a different set of geopotential coefficients would have been identified.

Table 3 displays the average error in the estimation of radial, along track, and cross track components of GEOS-C state as a function of the number of geopotential coefficients simultaneously adjusted along with GEOS-C and relay satellite state. Figures 3, 4, and 5 provide the standard deviations of the radial, along track, and cross track components of GEOS-C state when the 31 dominant geopotential coefficients are adjusted. The 31 coefficients which when adjusted yield an average error of one meter in the estimate of the radial component of GEOS-C state are given below in the order in which they were identified in the recursive procedure: S(7,2), S(6,6), C(7,0), C(7,6), C(6,5), C(7,2), S(4,3), S(5,5), C(4,4), C(4,3), S(8,1), C(6,6), C(8,7), C(6,4), C(5,5), C(2,2), S(5,4), S(6,4),

Table 3

Average Radial, Along Track, and Cross Track GEOS-C Errors in Meters
as a Function of Numbers of Estimated Geopotential Coefficients

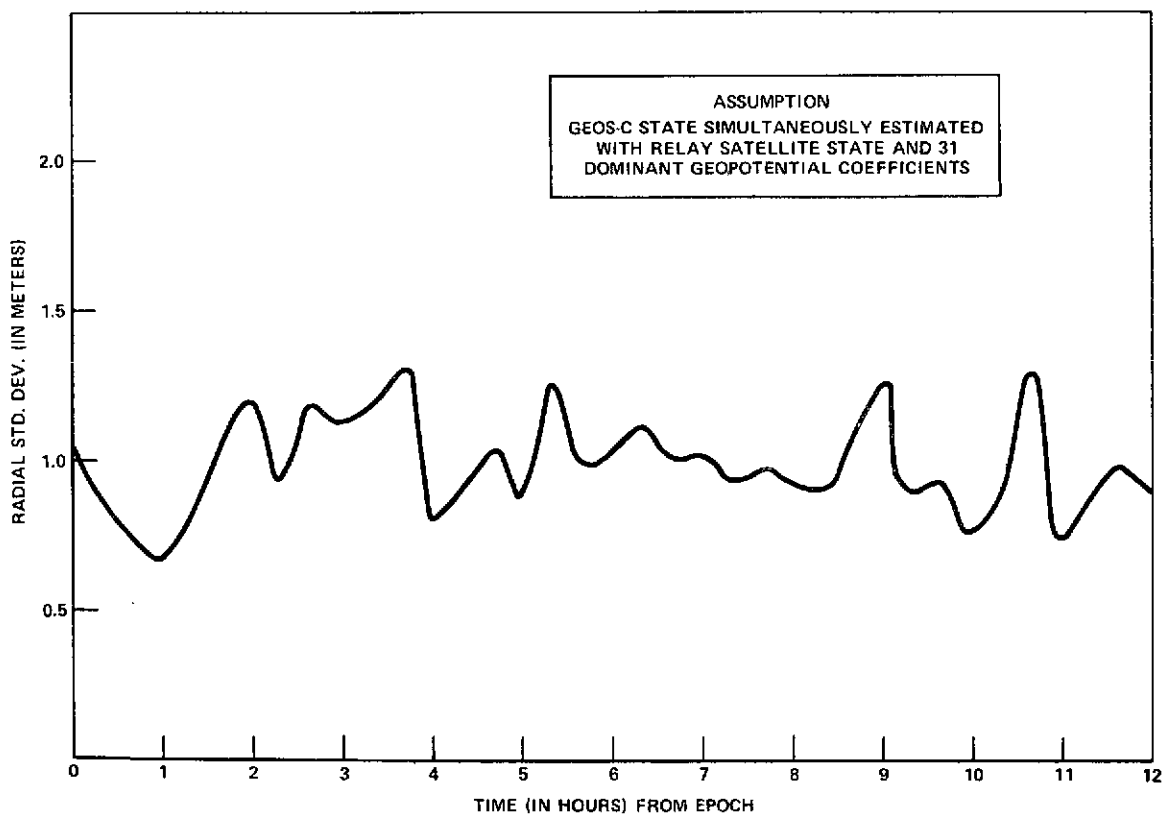| Number of Geopotential Coefficients Estimated	Average Radial Error (Meters)	Average Along Track Error (Meters)	Average Cross Track Error (Meters)
0	6.2	43.7	63.0
5	4.0	39.6	60.5
10	3.0	27.2	42.6
15	2.3	25.8	37.3
20	1.8	18.3	27.0
25	1.5	13.5	21.3
31	1.0	10.9	19.5

ASSUMPTION
GEOS-C STATE SIMULTANEOUSLY ESTIMATED
WITH RELAY SATELLITE STATE AND 31
DOMINANT GEOPOTENTIAL COEFFICIENTS

RADIAL STD. DEV. (IN METERS)

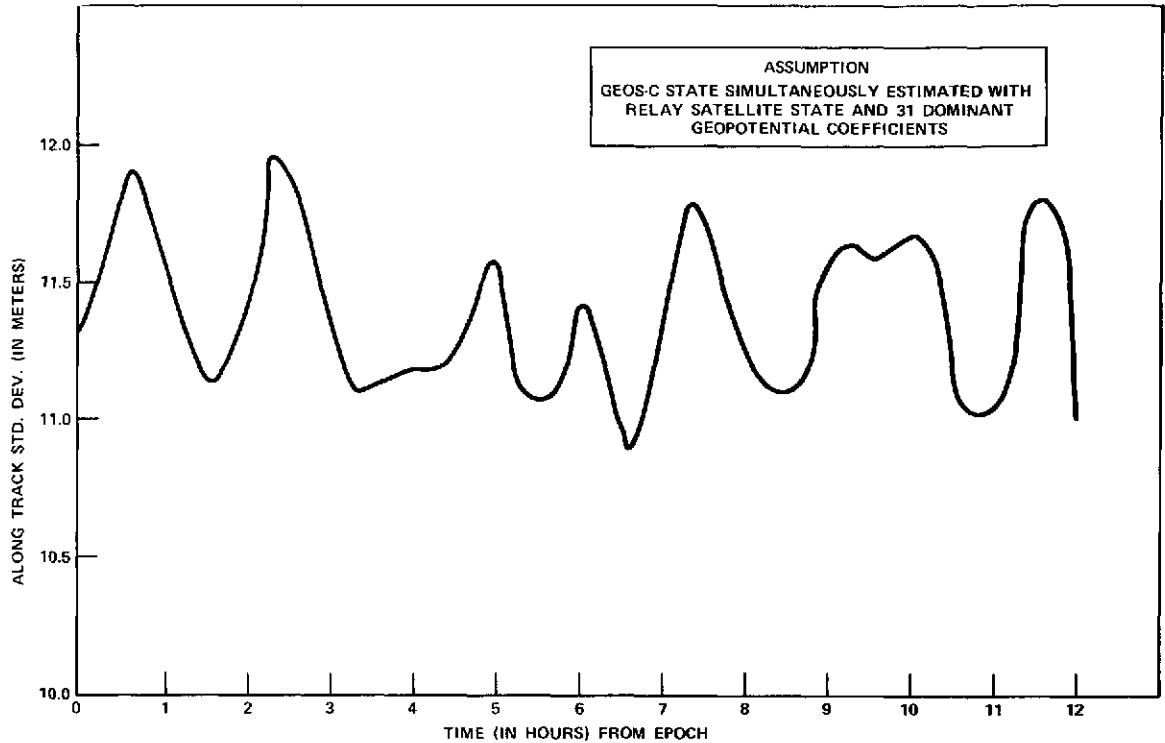TIME (IN HOURS) FROM EPOCH

Figure 3. Radial Std. Dev. vs. Time From Epoch

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Figure 4. GEOS-C Along Track Std. Dev. vs. Time From Epoch

C(7,4), S(6,5), C(7,7), C(5,3), S(4,2), S(7,3), S(7,7), S(5,3), C(5,1), C(8,4), S(6,2), C(3,2), S(5,1). The resultant estimates of these coefficients are relatively independent and are of good quality. The a priori standard deviations of the coefficients were taken to be the difference in representations in the S.A.O. 69 field and the G.E.M. 4 field. The resultant standard deviations of the estimates of the coefficients averaged about 20% of the a priori standard deviations. This suggests that satellite to satellite tracking is a useful data type for geopotential determination. The results also indicate that this estimation procedure significantly improves knowledge of relay satellite state. The total position error decreased from about 500 meters to about 150 meters. The total velocity error decreased from about 5 cm/sec to about 1 cm/sec.

CONCLUSIONS

This paper has shown the feasibility of determining application satellite orbits with satellite to satellite tracking. The uncertainty of the relay satellite state is the dominant error source. The relay satellite state must be simultaneously adjusted along with user satellite state in order to obtain satisfactory results. Relay station survey error and data bias are not significant error sources.

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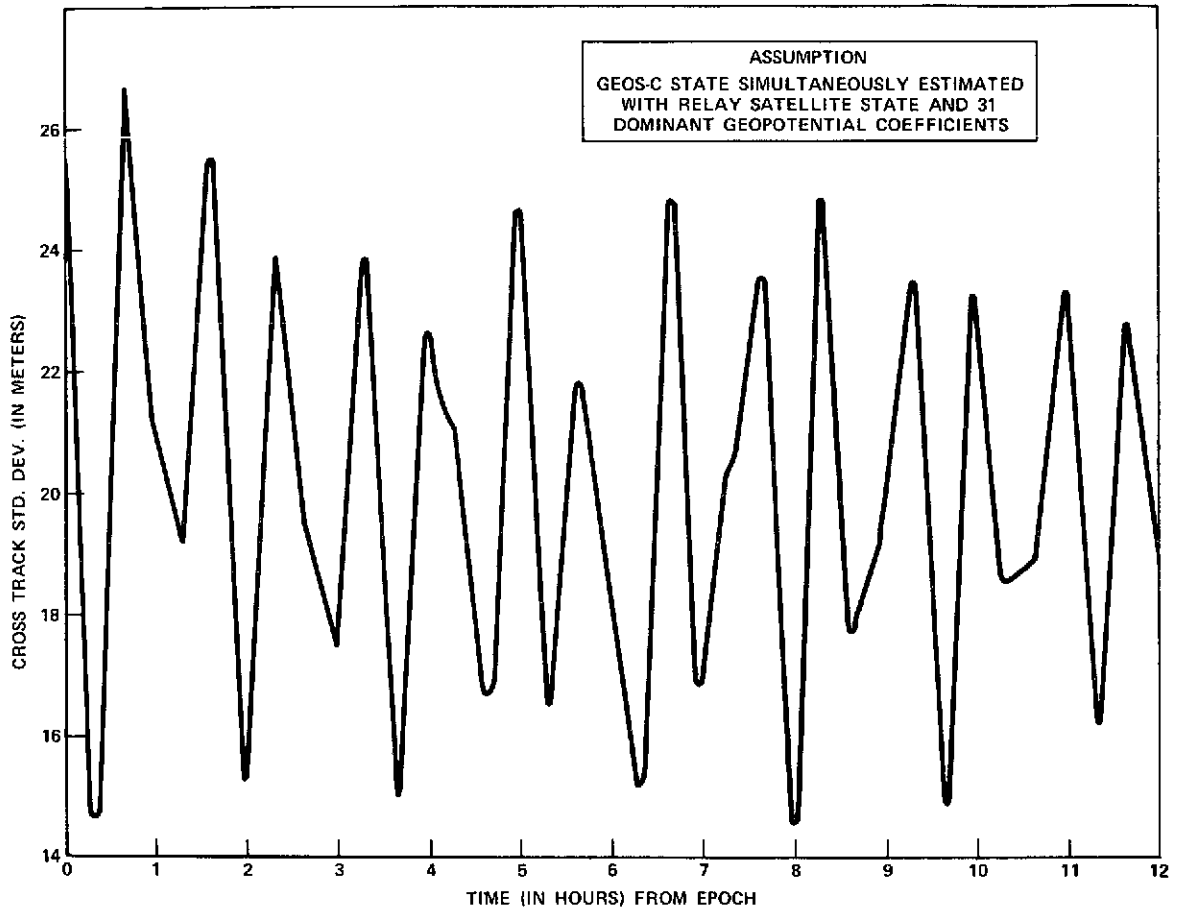
Figure 5. GEOS-C Cross Track Std. Dev. vs. Time From Epoch

A covariance analysis was performed to determine the feasibility of determining the GEOS-C orbit from 5 days of range rate sum data relayed from a single synchronous satellite. It was shown that if relay satellite state and 31 dominant geopotential coefficients are simultaneously adjusted along with GEOS-C state the radial component of state can be determined with one meter accuracy. The estimates of dominant geopotential coefficients as well as relay satellite state are also substantially improved.

ACKNOWLEDGEMENT

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APPENDIX

COVARIANCE ANALYSIS AS APPLIED TO ORBIT DETERMINATION

COMPUTING COVARIANCE MATRICES

Let $\tilde{y}(m)$ be an m dimensional vector consisting of the differences between the correct values of observations of a satellite and nominal values of the observations as determined from a nominal orbit. Also let $\tilde{z}(n)$ be an n dimensional vector of differences between actual and nominal values of the state of the satellite at an epoch and differences between actual and nominal values of parameters in the dynamic and measurement models whose associated uncertainties may limit our ability to estimate satellite state from the data. The sensitivity matrix $c(m, n)$ is defined as that matrix whose element in the ith row and the jth column is the partial derivative of $\tilde{y}(i)$ with respect to $\tilde{z}(j)$. A first order Taylor series expansion of the functional relationship between $\tilde{y}$ and $\tilde{z}$ about the nominal value of $\tilde{z}$ yields

$$\tilde{y} = c\tilde{z} \tag{A-1}$$

An orbit determination program in processing observations y of $\tilde{y}$ to obtain a least square adjustment to $\tilde{z}$ computes a so-called normal matrix defined as

$$\eta(n, n) = c^T w c \tag{A-2}$$

where w is a weighting matrix and is usually the inverse of the covariance matrix of the observations y of $\tilde{y}$. Once an orbit determination program computes and stores the normal matrix, a number of questions can be raised and answered at very little cost in terms of computation time.

The best estimate of the state of the satellite at epoch is obtained by performing a least squares adjustment of the state at epoch and all other parameters with which are associated significant uncertainties. But frequently this straightforward approach leads to severe core storage requirements. In practice some of the parameters in the dynamic and measurement models are estimated along with state and others are fixed at their nominal values and left unadjusted in the least squares process. In order to determine the consequences of estimating some parameters and ignoring others it is useful to compute the covariance matrix of such a least squares estimation procedure.

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Let $\tilde{z}$ be decomposed into two disjoint parameter sets as follows

$$\tilde{z} = \begin{bmatrix} \tilde{x}_1(n_1) \\ \tilde{x}_2(n_2) \end{bmatrix} \tag{A-3}$$

where $\tilde{x}_1$, is a set of n_1, parameters which are to be estimated in a least squares process and $\tilde{x}_2$ is a set of n_2 parameters whose nominal values are left unadjusted by the least squares process but whose uncertainties are to be considered in computing the covariance matrix of the resulting estimator. Define a matrix $A(m, n_1)$ as a matrix whose element in the ith row and jth column is the partial derivative of $\tilde{y}(i)$ with respect to x_1 (j). Analogously define $B(m, n_2)$ as the matrix whose element in the ith row and jth column is the partial derivative of $\tilde{y}(i)$ with respect to x_2 (j). For future reference notice that the normal matrix η of $\tilde{z}$ as computed and stored by an orbit determination program and defined by Equation A-2 can be written as

$$\eta = \begin{bmatrix} A^T wA & A^T wB \\ B^T wA & B^T wB \end{bmatrix} \tag{A-4}$$

Assume that there exists a priori estimates of $\tilde{x}_1$ and $\tilde{x}_2$ with properties

$$x_1' = \tilde{x}_1 + a_1, \qquad E(a_1) = \bar{0}, \qquad E(aa^T) = P_1$$

$$x_2' = \tilde{x}_2 + a_2, \qquad E(a_2) = \bar{0}, \qquad E(a_2 a_2^T) = P_2$$

and assume that the observation vector y or $\tilde{y}$ has properties

$$y = \tilde{y} + \nu, \qquad E(\nu) = \bar{0}, \qquad E(\nu\nu^T) = w^{-1}$$

The least squares estimate of $\tilde{x}_1$ is obtained as the value of $\tilde{x}_1$ which minimizes the loss function

$$L(x_1) = (y - Ax_1 - Bx_2')^T w(y - Ax_1 - Bx_2') + (x_1' - x_1)^T P_1^{-1}(x_1' - x_1) \tag{A-5}$$

The resulting least squares estimator of $\tilde{x}_1$ is well known to be

$$\hat{x}_1 = (A^T wA + P_1^{-1})^{-1} \left[A^T w(y - Bx_2') + P_1^{-1} x_1' \right] \qquad (A-6)$$

Define

$$P = \left[E \ (\hat{x}_1 - \tilde{x}_1) \ (\hat{x}_1 - \tilde{x}_1)^T \right] \qquad (A-7)$$

A series of substitutions reveals that

$$\hat{x}_1 - \tilde{x}_1 = (A^T wA + P_1^{-1})^{-1} \ (- A^T wB\alpha_2 + A^T w\nu + P_1^{-1} \alpha_1) \qquad (A-8)$$

Equation 8 yields

$$P = (A^T wA + P_1^{-1})^{-1} + (A^T wA + P_1^{-1})^{-1} A^T wBP_2 B^T wA(A^T wA + P_1^{-1})^{-1} \qquad (A-9)$$

Notice that the right side of Equation 9 can be computed if one has a priori covariance matrices P_1 and P_2, and the upper right and upper left portions of the normal matrix. To determine the covariance matrix of an estimator which estimates some subset of $\tilde{z}$ other than $\tilde{x}_1$, all that is necessary is to permute the rows and columns of η in the appropriate fashion and proceed as before. Thus if one assumes that the normal matrix defined by Equation 2 is precomputed it becomes an easy matter to obtain the resultant covariance matrix when any subset of the $\tilde{z}$ parameters are estimated in a least squares sense and the rest are ignored.

THE ALIAS MATRIX

Assume that all the data has the same variance. Hence

$$w = (I\sigma_0^2)^{-1} \qquad (A-10)$$

where σ_0^2 is the common variance of each data point. Also assume that the a priori estimates of the unadjusted parameters are independent. Under this

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assumption the covariance matrix P_2 of x_2' can be written as

$$P_2 = \begin{bmatrix} \sigma_1^2 & & & 0 \\ & \sigma_2^2 & & \\ & & \ddots & \\ 0 & & & \sigma_n^2 \end{bmatrix}$$ (A-11)

where σ_i^2 is the a priori variance of the ith unadjusted parameter. Also define a matrix $K(n_1, n_2)$ as

$$K = (A^T w A)^{-1} A^T w B$$ (A-12)

With these assumptions Equation 9 yields the following expression for the ith diagonal element of P

$$P(I, I) = \sum_{j=0}^{n_2} (\beta_{i,j} \sigma_j)^2$$ (A-13)

where $\beta_{i,0}$ is the ith diagonal element of the matrix $(A^T A)^{-1}$ (this assumes that diagonal elements of the matrix P_I^{-1} are relatively small) and

$$\beta_{i,j} = K(i, j), \quad j \geq 1$$ (A-14)

The standard deviation of the ith estimated parameter is given by

$$\sigma_i = \left(\sum_{j=0}^{n_2} (\beta_{i,j} \sigma_j)^2 \right)^{1/2}$$ (A-15)

Define the error sensitivity matrix as

$$S = \{ \beta_{i,j} \}, \quad i = 1, 2, \cdots n_1, \quad j = 0, 1, \cdots n_2$$ (A-16)

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And finally define the Alias Matrix as

$$L = \overline{S}\overline{\sigma} \qquad (A\text{-}17)$$

where

$$\overline{\sigma} = \begin{bmatrix} \sigma_0 & & & & 0 \\ & \sigma_1 & & & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & \sigma_{n_2} \end{bmatrix} \qquad (A\text{-}18)$$

The standard deviation of the ith estimated parameter is seen to be the root sum square of the terms in the ith row of the alias matrix. The elements in the first column of the alias matrix represent the RSS contribution to the standard deviation of each estimated parameter due to the data noise. The elements in the jth column, $j \geq 2$, represent the RSS contribution to the standard deviation of each estimated parameter due to the $j - 1$st unadjusted parameter.

Possession of the alias matrix reveals much of the probability structure of the postulated least squares estimator. With this information one can quickly determine which error sources are significant with regard to the estimation of a given parameter.

Propagating Covariance Matrices

Equation 9 provides the covariance matrix of the state $\widetilde{x}_1$ at some specified epoch. In many cases it is important to determine how accurately the state can be determined at some time other than epoch. In order to do this correctly it is necessary to take into proper account uncertainties in dynamic parameters. These parameters may be in an estimated mode or in an unadjusted mode and to incorporate their effect one resorts to state transition matrices which presumably have been precomputed by an orbit determination program. Let $\widetilde{x}_1$ (T) be the estimated state at time T. Assume as output from an orbit determination program the state transition matrices

$$\overline{v}_1\,(T) = \frac{\partial \widetilde{x}_1\,(T)}{\partial \widetilde{x}_1}\,, \qquad \overline{v}_2\,(T) = \frac{\partial \widetilde{x}_1\,(T)}{\partial \widetilde{x}_2} \qquad (A\text{-}19)$$

If there are no dynamic parameters in the estimation vector $\tilde{x}_1$, the matrix $\overline{v}_1$ (T) takes on the particularly simple form,

$$\overline{v}_1 \ (T) \ = \ \begin{bmatrix} \delta & 0 \\ 0 & I \end{bmatrix} \tag{A-20}$$

where δ is the six by six matrix defined as the partial derivative matrix of the state of the satellite at time T with respect to the state of the satellite at epoch. If dynamic parameters are included in the estimated state, the off diagonal matrices become non-zero and $\overline{v}_1$ (T) assumes a more complicated form. The matrix $\overline{v}_2$ (T) is the matrix of partial derivatives of the state $\tilde{x}_1$ (T) with respect to the unadjusted parameters $\tilde{x}_2$. If no dynamic parameters are in the unadjusted mode, $\overline{v}_2$ (T) is the null matrix. A first order Taylor series expansion of the function which describes the time evolution of the state $\tilde{x}_1$ (T) yields

$$\tilde{x}_1 \ (T) \ = \ \overline{v}_1 \ (T) \ \tilde{x}_1 \ + \ \overline{v}_2 \ (T) \ \tilde{x}_2 \tag{A-21}$$

Substituting $\hat{x}_1$ as obtained from Equation 6 for $\tilde{x}_1$ and x_2' for $\tilde{x}_2$ provides the best estimate $\hat{x}_1$ (T), of $\tilde{x}_1$ (T)

$$\hat{x}_1 \ (T) \ = \ \overline{v}_1 \ (T) \ \hat{x}_1 \ + \ \overline{v}_2 \ (T) \ x_2' \tag{A-22}$$

The covariance matrix of $\hat{x}_1$ (T) is given by

$$P \ (T) \ = \ \overline{v}_1 \ (T) \ P \ \overline{v}_1^T \ (T) \ + \ \overline{v}_2 \ (T) \ P_2 \ \overline{v}_2^T \ (T) \ + \ \overline{v}_1 \ (T) \ E \ [\hat{x}_1 \ x_2'] \ \overline{v}_2 \ (T)$$

$$+ \ \overline{v}_2 \ (T) \ E \ [x_2' \ \hat{x}_1^T] \ \overline{v}_1^T \ (T) \tag{A-23}$$

Equation 23 in conjunction with Equations 6 and 9 yields

$$P \ (T) \ = \ \overline{v}_1 \ (T) \ (A^T w A + P_1^{-1})^{-1} \ \overline{v}_1^T \ (T) \ + \Big[\ \overline{v}_1 \ (T) \ (A^T w A + P_1^{-1})^{-1} \ A^T w B$$

$$- \ \overline{v}_2(T) \Big] \ P_2 \Big[\overline{v}_1(T) \ (A^T w A + P_1^{-1})^{-1} \ A^T w B - \overline{v}_2 \ (T) \Big]^T \tag{A-24}$$

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Finally notice that in much the same fashion that Equation 9 was used to develop an alias matrix at epoch, Equation 24 can be utilized to develop an alias matrix for any time T.

REMARKS

If one possesses a functioning orbit determination program it becomes a relatively easy matter to add covariance analysis capability to the system. A computer program can be written which assumes as input a normal matrix and state transition matrices as generated by the orbit determination program. By permuting the rows and columns of the normal matrix and completing the matrix operations defined by Equation 9, the covariance matrix of a least square process which adjusts any subset of the parameters and ignores the rest can be computed. An alias matrix can be obtained and significant error sources can be identified. By utilizing the precomputed state transition matricies, the covariance matrix of the estimate of the state can be propagated from epoch to any other time. These operations are very simple and they consume little computer time.

Since the normal matrix and state transition matrices are computed once and permanently stored, it is possible to investigate a large number of possible estimation strategies. This can be done conveniently and cheaply. For many applications such a program is a useful and quickly developed addition to an orbit determination system.