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DESIGN OF RETRODIRECTOR ARRAYS FOR LASER RANGING OF SATELLITES

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SUMMARY

The radar equation for laser ranging of satellites is described and the effect of the velocity aberration explained. Equations for the cross sections of cube corners and arrays of cube corners are derived. Interference effects on the distribution of the array cross section and upon range error are described. Tolerance requirements for cube corners are briefly outlined.

CONTENTS

INTRODUCTION

Optical retroreflectors of the type known colloquially as cube corners have been used to increase the reflectivity of passive targets for at least one hundred years. Probably the most familiar application is the molded plastic retrodirector array in the tail-light of nearly every car produced in the last 40 years. In 1962 the first satellite (Beacon Explorer A) equipped with a retroreflector array for laser ranging was launched by NASA. Since then, laser ranging has been developed to the point where it is the most precise method of orbit determination in existence. Because of this precision, it has become one of the primary tools of the geodocist in the study of the earth's gravity field.

Due to the high cost of the precision cube corner prisms required for the arrays and the desire to obtain maximum laser echoes with the available tracking equipment, techniques have been developed to maximize the effective cross section of the array. The purpose of this paper is to introduce some of these procedures to persons not familiar with laser tracking.

THEORY

Consider the basic radar situation shown in Figure 1. The transmitter radiates a power P_T with an antenna of gain G_T to a target at range R. The target with a cross section σ reflects radiation back to a receiver antenna with gain G_R. Neglecting losses the echo strength may be determined by the classical radar equation.

$$S = \frac{P_T G_T G_R \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (1)$$

If the target is a flat reflective plate oriented normal to the incident wave, the radar cross section is

$$\sigma = \frac{4\pi A^2}{\lambda^2} \tag{2}$$

where A is the area of the plate and λ is the wavelength. A cube corner reflector made of three mutually perpendicular conductive flats will produce the same cross section but will eliminate the need for precise orientation of the target. For reasons which will be explained later, a single large cube corner cannot be used as a satellite retroreflector and, therefore, arrays of many small cube corner prisms are used to increase the target cross section. The reason for the use of prisms rather than the triple mirror type of cube corner lies in their greater simplicity of fabrication, great strength and improved performance at oblique angles of incidence.

When a cube corner prism is tilted so that the angle between the normal to the entrance pupil and the incident wavefront normal is i, the effective area becomes

$$A(i) = 2a^2 \left[\sin^{-1}\mu - \sqrt{2}\,\mu \tan i'\right] \cos i \tag{3}$$

where

$\mu = \sqrt{1 - 2\tan^2 i'}$

i = angle of incidence

i′ = refracted angle of incidence = $\sin^{-1}\left[\frac{\sin i}{n}\right]$

n = index of refraction of prism

The geometry of the cube corner is shown in Figure 2. Using this expression, we may plot the radar cross section as a function of incidence angle utilizing Equation 2. In Figure 3, this has been done for a fused silica cube corner prism with an index of refraction of 1.46 and for a prism with a refractive index of unity which corresponds to a triple mirror type cube corner. The superiority of the prism over the triple mirror type is clearly shown for oblique incidence angles. Greater improvements in acceptance angle can be obtained with higher refractive indexes. However, high purity fused silica has proven to be the only satisfactory prism material due to its high radiation browning resistance and its low rate of change of refractive index with temperature.

An array of cube corners has an average cross section of

$$\sigma = \frac{4\pi}{\lambda^2} \sum_{m=1}^{N} [A(i)]_m^2 \tag{4}$$

Because the prisms cannot be placed with respect to each other to optical tolerances, they act as a randomly phased array with an exponentially distributed cross section. The probability density becomes

$$p(\sigma) = \frac{1}{\bar{\sigma}} e^{-\sigma/\bar{\sigma}} \tag{5}$$

as shown in Figure 4. The same random phasing causes a random ranging error with a Rician distribution.

Most of the satellites used in laser ranging experiments are gravity gradient stabilized and in near earth orbits. The retroreflector arrays are normally located on the earth facing side; and in order to obtain a good cross section at low elevation angles, the arrays are configured on spherical, conical or pyramidal surfaces. The cross section may be computed by establishing a spherical coordinate system around the satellite gravity gradient axis. The orientation of the cube corner (i.e., the outward vector normal to its entrance pupil) may then be specified by its polar coordinate angles θ_N and ϕ_N as shown in Figure 5. The line from the laser transmitter to the satellite is used as an azimuth reference so that the satellite/transmitter vector may be specified by the polar angle θ_L alone. Once defined in this manner, the incidence angle i for each cube corner is established by the following relation

$$\cos i = \sin \theta_N \sin \theta_L \cos \phi_N + \cos \theta_N \cos \theta_L \tag{6}$$

Equations 3 and 4 may then be used to compute the array cross section.

To this point, we have considered the transmitter/target relationship to be stationary with no relative motion. We will now consider the effects of the satellite motion on the radar cross section. It is this effect which places a limit on the maximum size cube corner which may be used.

In the case of the stationary target and a cube corner with circular entrance pupil oriented for an incidence angle of zero, the reflected beam intensity may be described by the familiar Airy equation

$$I(\alpha) = \left[\frac{2 J_1 (k a \alpha)}{k a \alpha}\right]^2 \frac{E_R \pi a^2}{\lambda^2} \tag{7}$$

where

k = wave number = $2\pi/\lambda$

a = radius of entrance pupil

α = angle off transmitter/target axis

E_R = total reflected energy

Using the basic definition of cross section

$$\sigma = 4\pi \left[\frac{\text{Power/Unit Solid Angle}}{\text{Power Received/Unit Area}} \right] \tag{8}$$

Equation 7 becomes

$$\sigma(\alpha) = 4\pi \left[\frac{2J_1\,(k\,a\,\alpha)}{k\,a\,\alpha} \right]^2 \left(\frac{\pi a^2}{\lambda} \right)^2$$

$$= \left[\frac{2J_1\,(k\,a\,\alpha)}{k\,a\,\alpha} \right]^2 \frac{4\pi A^2}{\lambda^2} \tag{9}$$

If the cube corner is moving with a velocity V normal to the transmitter/target line of sight, a Bradley or velocity aberration effect causes the reflected beam pattern to be angularly displaced by an amount

$$\alpha = 2V/C \tag{10}$$

where c is the speed of light. This places an upper limit on the practical size of the cube corner, since as the entrance pupil radius is increased, the central lobe of the reflected intensity will decrease in angular extent until because of the velocity aberration affect, it no longer illuminates the receiver. On the other hand, small values for the entrance pupil radius spread the central lobe over too large an area so that little energy can be collected by the receiver.

Clearly, there is an optimum size which may be computed by differentiating Equation 9 with respect to $ka\alpha$, setting the result equal to zero and substituting $\frac{2V}{C}$ for α. Doing this, we obtain for the optimum entrance pupil radius (Figure 6)

$$a = \frac{1.85\, C\, \lambda}{4\pi V} \tag{11}$$

Under this condition, the receiver will be located on the side of the central lobe of the reflected cube corner intensity pattern at a point where the intensity is 40% of the peak intensity.

For a circular orbit of altitude h, the maximum velocity aberration is

$$\alpha = \frac{2}{C}\sqrt{\frac{R_e^2\, g}{R_e + \eta}} \tag{12}$$

where R_e = radius of the earth

g = acceleration of gravity

Therefore, the optimum cube corner size becomes

$$a = \frac{1.85\, C\, \lambda}{4\pi}\sqrt{\frac{R_e + \eta}{R_e^2\, g}} \tag{13}$$

as shown in Figure 7. If the cube corner size is selected in this manner, the cross section of each cube corner in the array can never decrease below 40% of the value computed through Equation 3 and can, of course, never exceed Equation 3.

The actual value of the array cross section will be somewhere between these limits, but its value will depend upon the orientation of the spacecraft with respect to the transmitter, the orientation of each of the cube corners on the spacecraft, and the wavelength. Since calculation of these values for all possible conditions would be impractical and because there are large variations from the average cross section due to its exponential distribution, the average cross section is normally set 2 dB below its value under stationary conditions. When this is done, the maximum error is only ± 2 dB.

MANUFACTURING TOLERANCES

In order to obtain cube corners which perform as predicted by the above equations, the prisms must be essentially perfect, i.e., diffraction limited. Using the Airy criterion, this requires that the angles between the reflective faces be perpendicular to a tolerance of less than

$$\epsilon \lesssim 0.374 \frac{\lambda}{2\eta a} \qquad (14)$$

Surfaces must be flat to better than $\lambda/10$, and material of the highest homogeneity must be employed. Material homogeneity is the greatest problem in manufacture of the prisms and usually limits the maximum size of a cube corner to about 5 cm diameter due to the difficulty of obtaining large homogeneous blocks of material. The material is normally tested by polishing it into a cube before manufacture of the cube corner and tested in three mutually perpendicular directions using an interferometer.

RANDOM ERRORS

The intensity at the center of the reflected beam from an actual optical system compared to an ideal system is defined as the Strehl ratio S_R. If the errors in the reflected wavefront are assumed small relative to a wavelength, and a normal-random variable

$$S_R = 1 - \left(\frac{2\pi}{\lambda}\right)^2 \sigma_T^2 \qquad (15)$$

where σ_T is the standard deviation. Then by inversion

$$\sigma_T^2 = \left(\frac{\lambda}{2\pi}\right)^2 (1 - S_R) \qquad (16)$$

If, in turn, the errors in each of the five optical surfaces of the cube are assumed normal random variables, and if the material is assumed to add a normal wave-front error distribution, then if all of these error sources are independent

$$\sigma_T^2 = \sigma_B^2 + \sigma_n^2 + \sigma_{m1}^2 + \sigma_{m2}^2 + \sigma_{m3}^2 + \sigma_B^2 \qquad (17)$$

Where the subscripts refer to total, base, homogeneity, mirror #1, mirror #2, etc. If all surfaces are polished with equal care

$$\sigma_{m1}^2 = \sigma_{m2}^2 = \sigma_{m3}^2 \qquad (18)$$

and

$$\sigma_m^2 >> \sigma_B^2 \qquad (19)$$

So that

$$\sigma_T^2 \approx 3\sigma_m^2 + \sigma_h^2 = \left(\frac{\lambda}{2\pi}\right)^2 (1 - S_R) \qquad (20)$$

To provide satisfactory performance therefore

$$3\sigma_m^2 + \sigma_h^2 \leq \left(\frac{\lambda}{2\pi}\right)^2 (1 - S_R) \qquad (21)$$

Obviously increased polishing precision can be traded off to compensate for poor material and vice versa, but also in any case

$$\sigma_m^2 \leq \left(\frac{\lambda}{2\pi}\right)^2 \left(\frac{1 - S_R}{3}\right) \tag{22}$$

and

$$\sigma_h^2 \leq \left(\frac{\lambda}{2\pi}\right)^2 (1 - S_R) \tag{23}$$

For a Strehl ratio of 0.50 (3 dB loss in signal)

$$\sigma_m \leq \frac{\lambda}{15.3} \tag{24}$$

$$\sigma_h \leq \frac{\lambda}{8.9} \tag{25}$$

If we assume that material inhomogeneity and polishing errors each contribute an equal amount to the total error then

$$\sigma_m \leq \frac{\lambda}{30.8} \tag{26}$$

and

$$\sigma_h \leq \frac{\lambda}{17.8} \tag{27}$$

It is thus seen that the fabrication tolerances are quite severe.

CONCLUSION

The previous discussion has outlined several of the basic problems involved in design of laser retroreflector arrays. It is not intended to be a complete design procedure but rather an introductory text for the reader unfamiliar with the design procedure. The paper has outlined the importance of using the proper terminology for the range equation and the distinction between "cross section" and area.

The most common misconception in the design of retroreflector arrays in that area is the only important factor in obtaining strong echoes. In actual practice, this is not true. Beam divergence, rather plays the key role in obtaining strong echoes. It is for this reason that for equal area arrays, the array with the larger individual cube corner provides a much larger echo. Future improvements in cube corner array design lie in development of larger cube corners and improved methods for compensation of the velocity aberration.

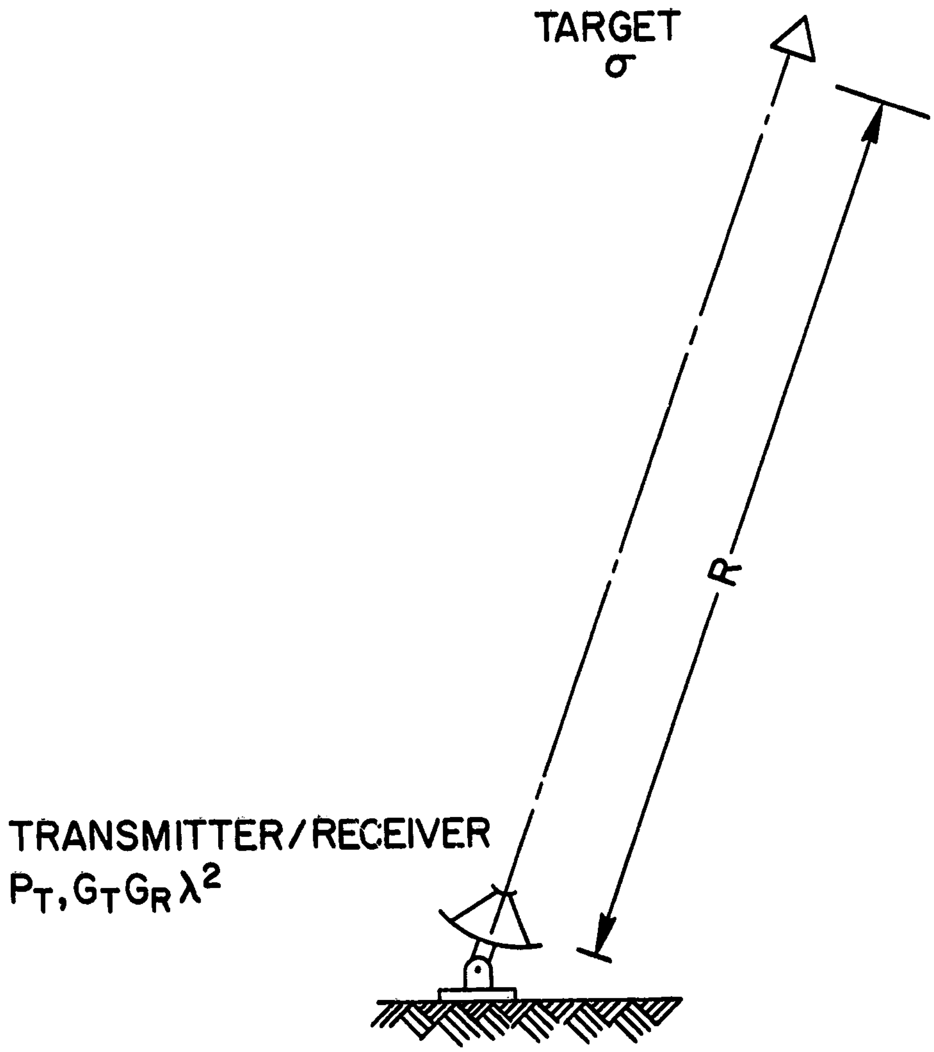

Figure 1. Basic Radar System

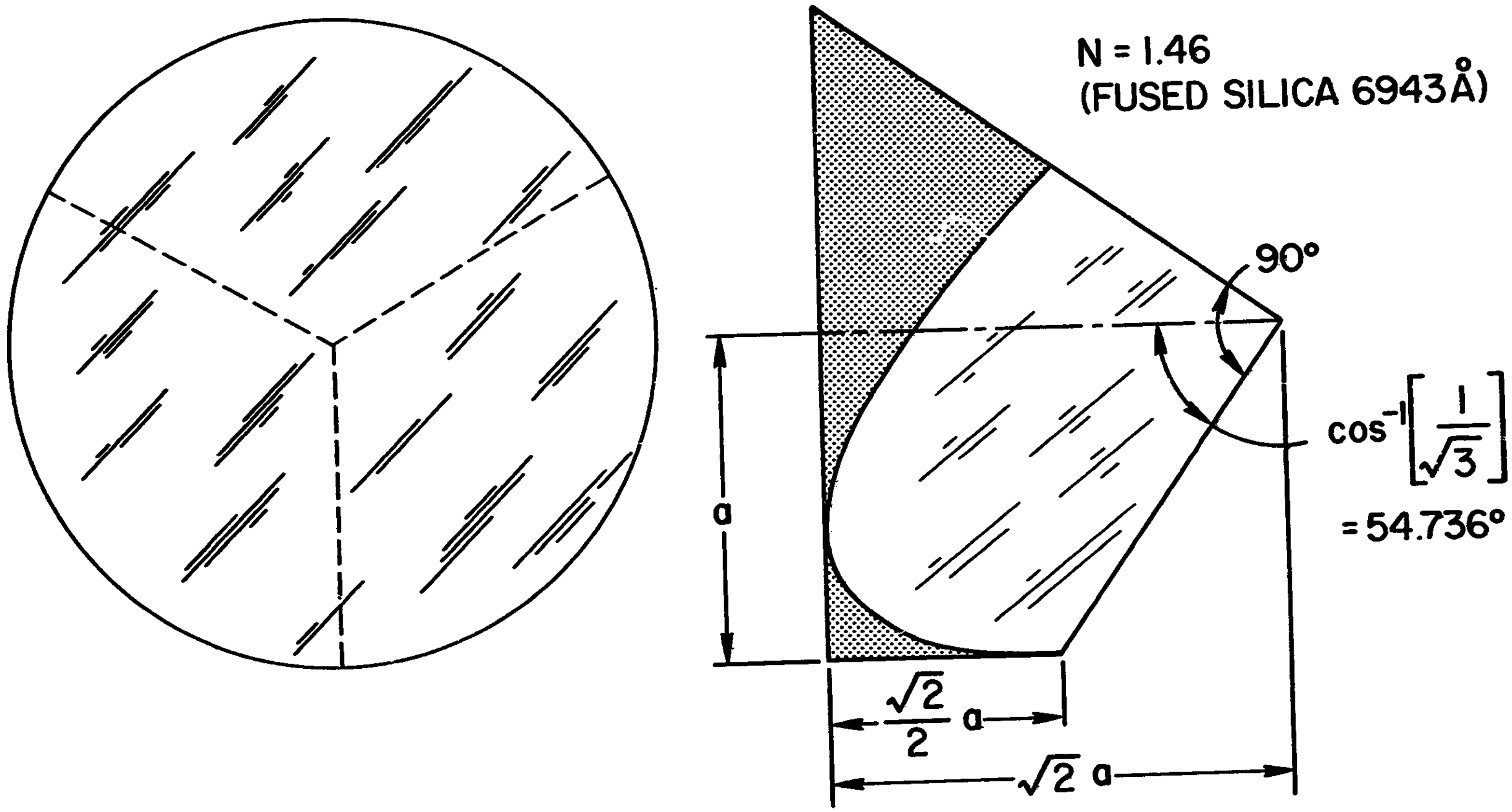

Figure 2. Cube Corner Prism Geometry.

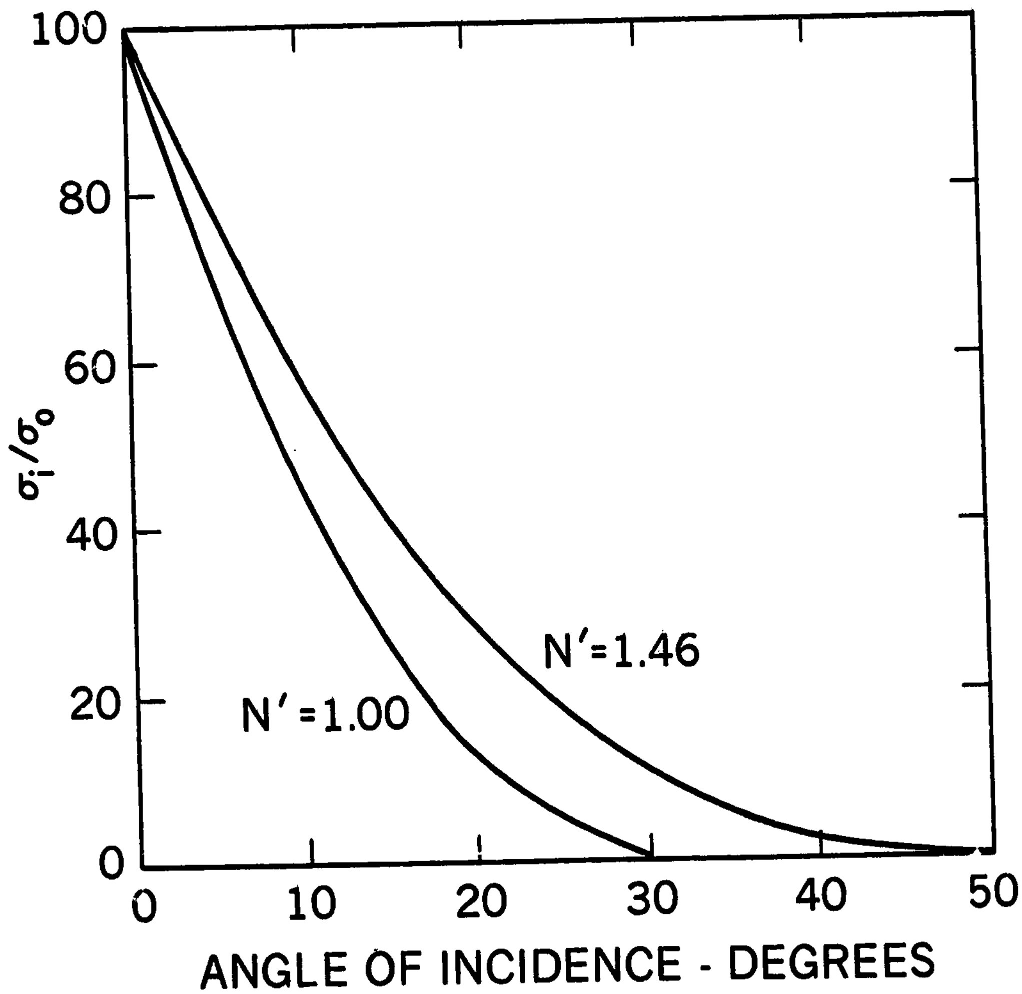

Figure 3. Normalized Cube Corner Cross Section.

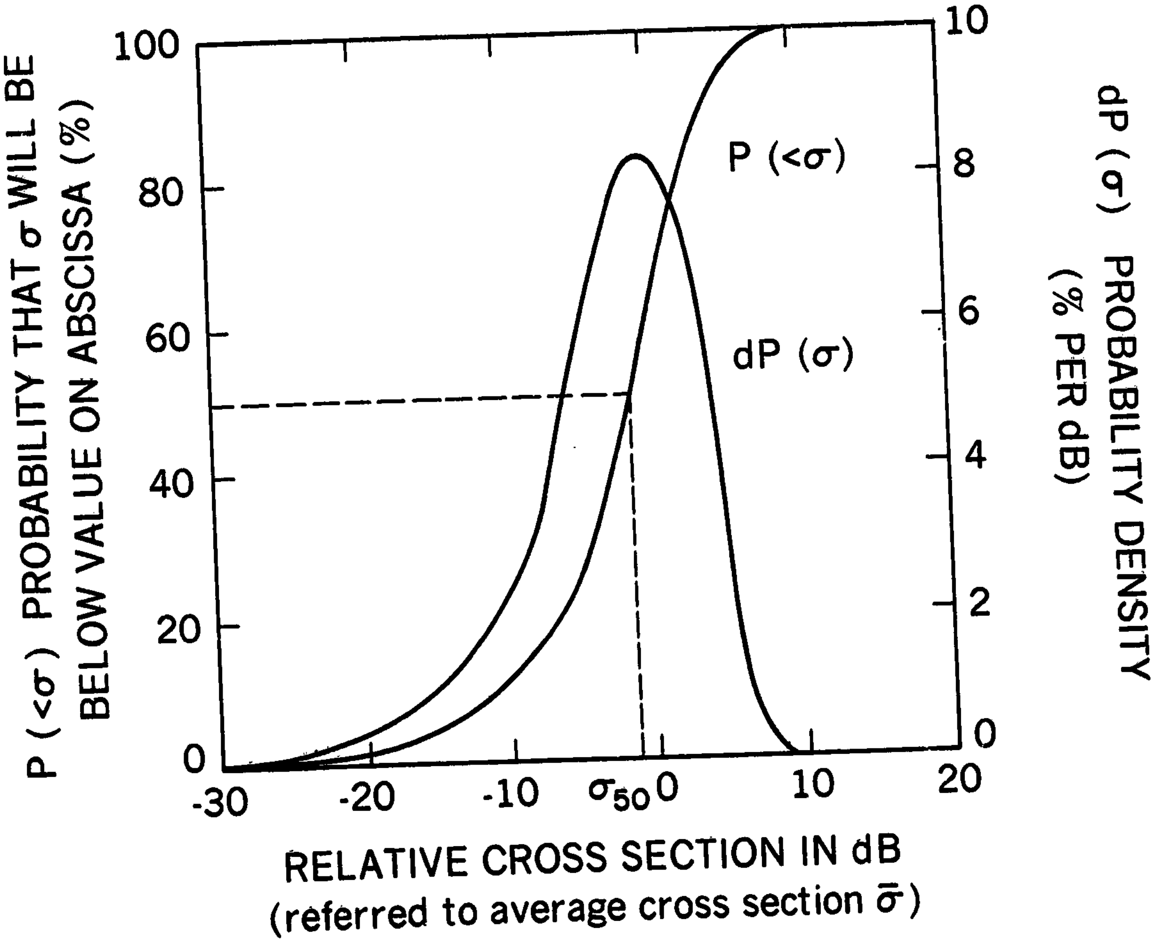

Figure 4. Exponential Distribution of Cross Section.

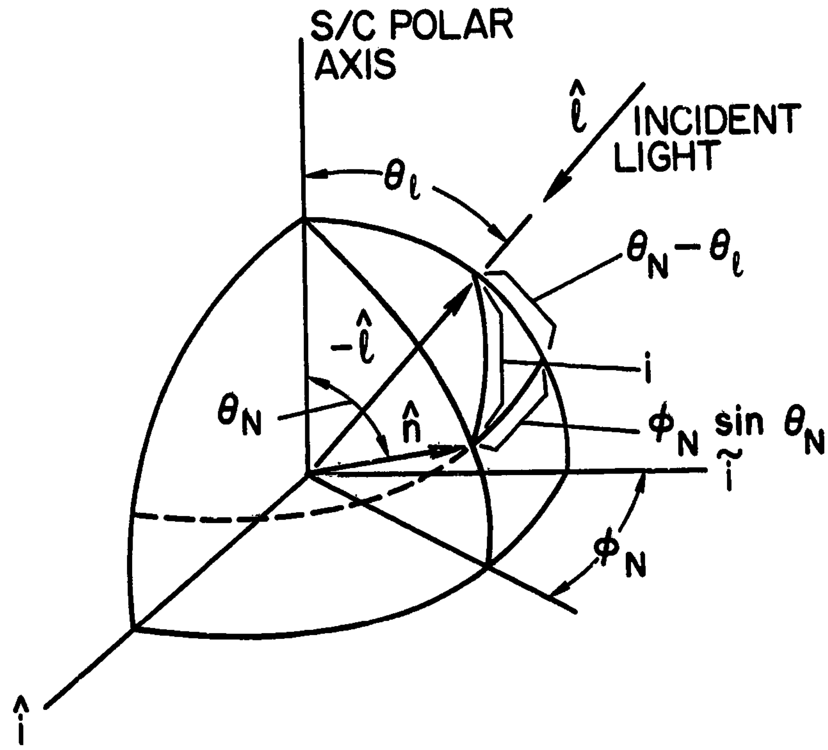

Figure 5. Coordinate System for Cube Corner Orientation.

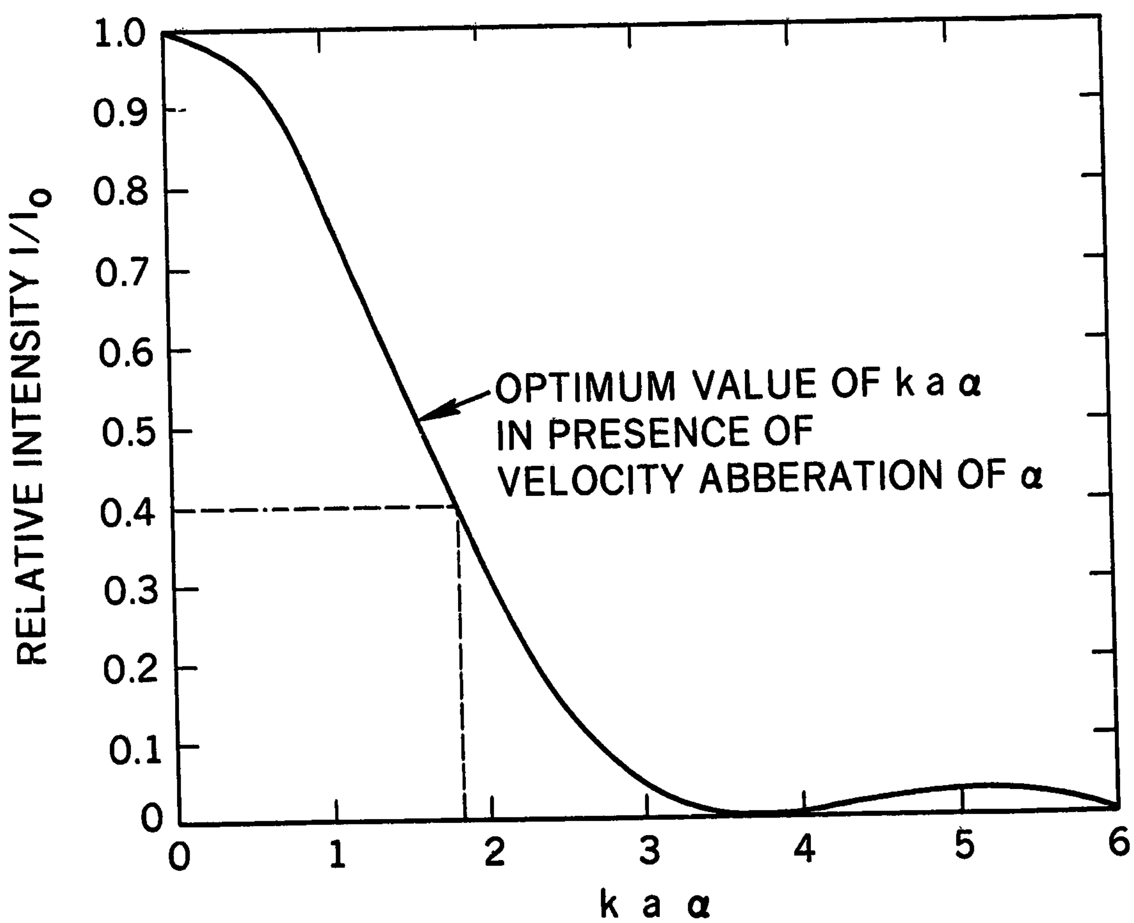

Figure 6. Fraunhofer Diffraction from a Circular Cube Corner of Radius a.

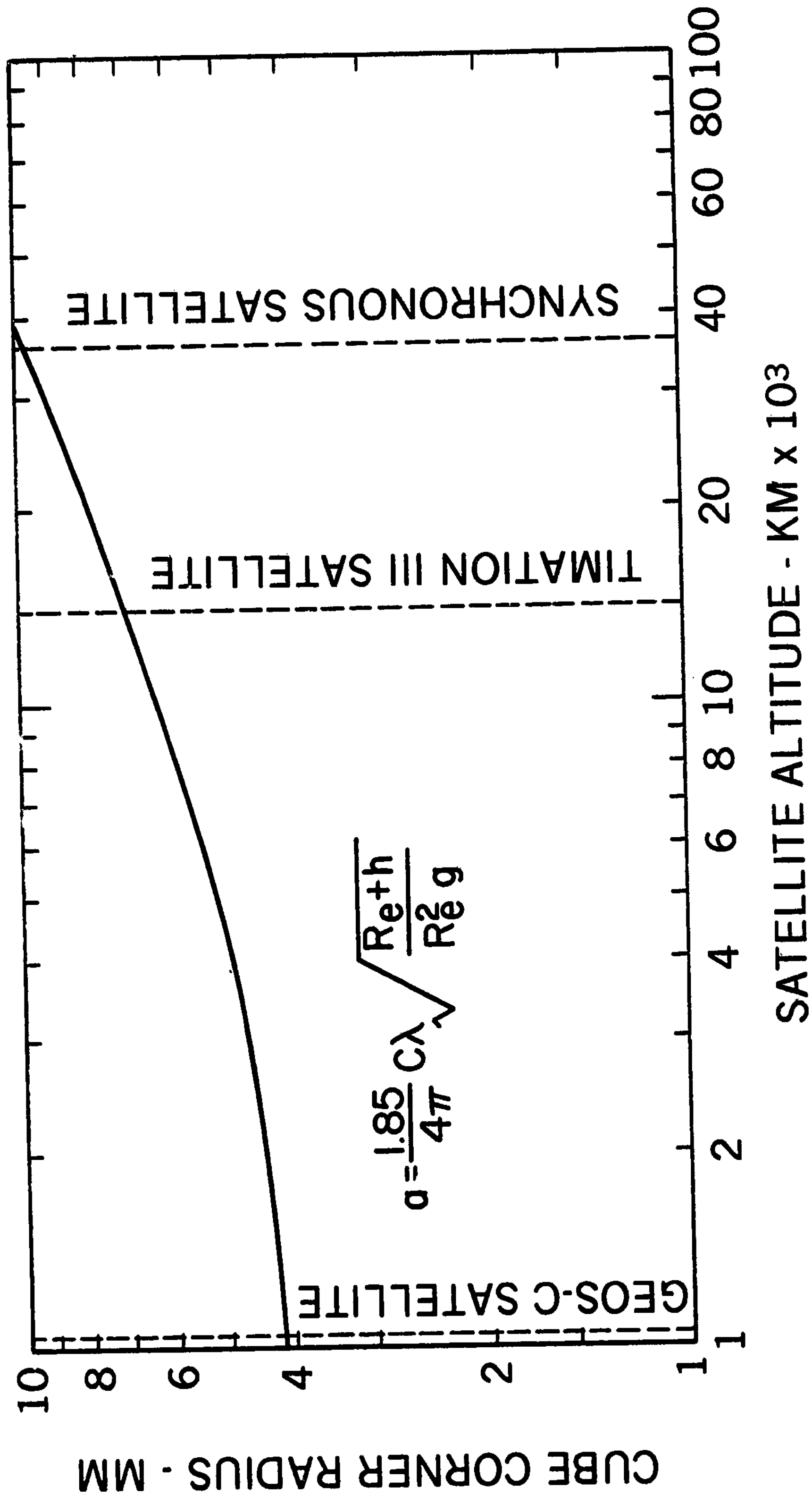

Figure 7. Optimum Cube Corner Radius vs. Satellite Altitude.