Reports of the Department of Geodetic Science

Report No. 195

FREE GEOMETRIC ADJUSTMENT OF THE SECOR EQUATORIAL NETWORK (Solution SECOR-27)

by

Ivan I. Mueller, M. Kumar and Tomas Soler

Prepared for

National Aeronautics and Space Administration
Washington, D.C.

Contract No. NGR 36-008-093
OSURF Project No. 2514

The Ohio State University
Research Foundation
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PREFACE AND ACKNOWLEDGEMENT

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University and is under the technical direction of James P. Murphy, Special Programs, Code ES, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C., 20546.

The authors wish to express their appreciation to the Defense Mapping Agency (Topographic Center) for the SECOR data, and for other helpful information related to the analysis of the data.

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1. INTRODUCTION

The basic purpose of this experiment is to compute reduced normal equations from the observational data of the SECOR Equatorial Network (Fig. 1) obtained from DMA/Topographic Center, D/Geodesy, Geosciences Div., Washington, D.C. These reduced normal equations are to be combined with reduced normal equations of other satellite networks of the National Geodetic Satellite Program to provide station coördinates from a single least square adjustment.

An individual SECOR solution was also obtained and is presented in this report, using direction constraints computed from BC-4 optical data from stations collocated with SECOR stations. Due to the critical configuration present in the range observations [Blaha, 1971], weighted height constraints were also applied in order to break the near coplanarity of the observing stations.

Details of the SECOR network, including instrumentation, historical background, etc., are given in Rutscheid [197l].

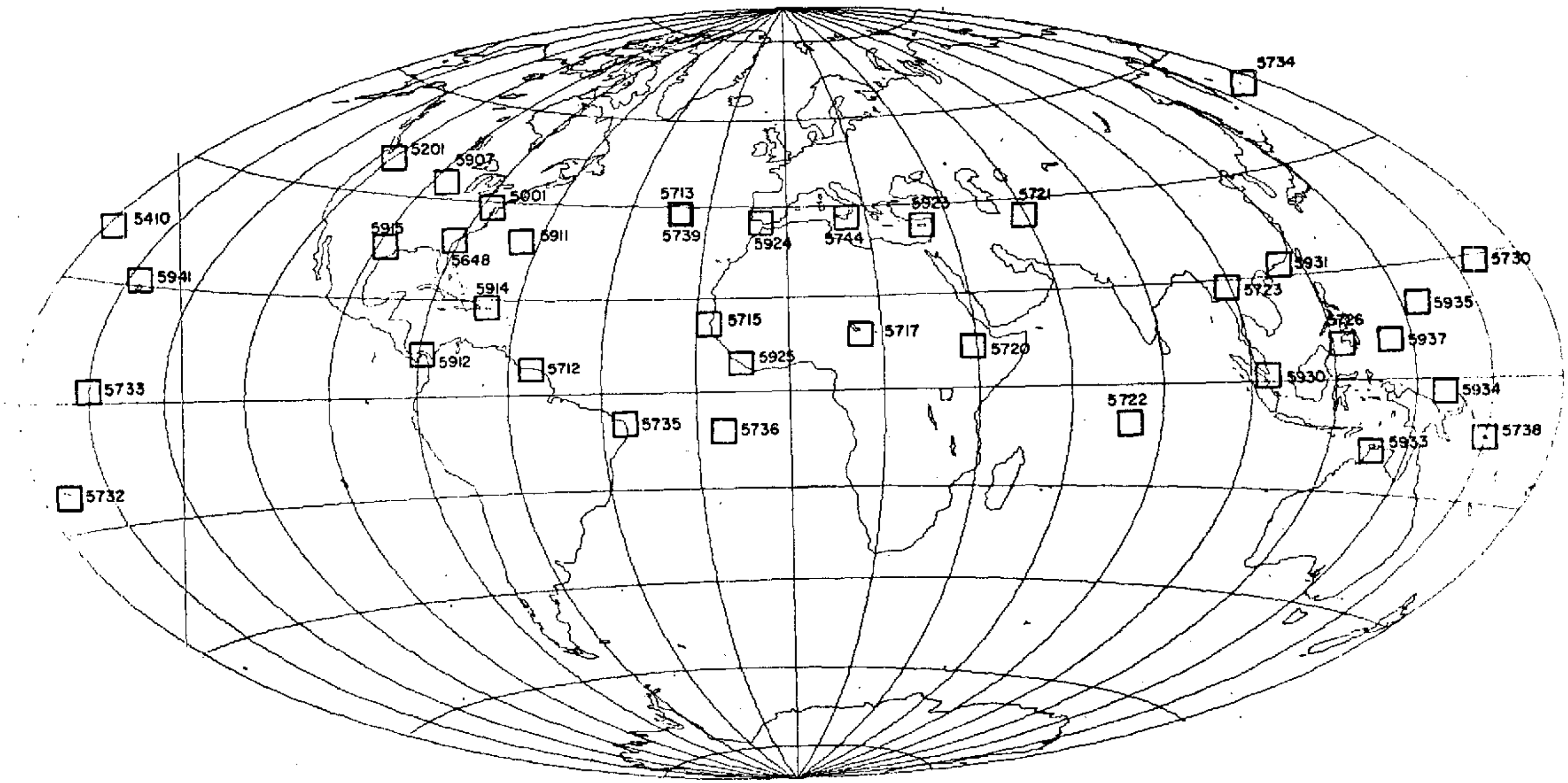


Fig. 1 SECOR Equatorial Network.

2. DATA

2.1 Terrestrial Data

Terrestrial data including survey coordinates and mean sea level heights of stations, instrument type used, etc., are given in Table 2.1-1, together with a list of geodetic datums involved (Table 2.1-2).

These survey coordinates provide the necessary relative position constraints between 13 SECOR stations and collocated BC-4 stations and in addition relative position constraint between two SECOR stations [Mueller, et al., 1973]. Constraints used in this experiment are given in Tables 2.1-3, 2.1-4 and 2.1-5. Geoidal undulations (Table 2.1-4) were computed by using formula and constants as given in [Rapp, 1973].

2.2 Satellite Observational Data and Its Handling

The magnetic tape containing SECOR data, obtained from the Defense Mapping Agency, created on the UNIVAC 1108 EXEC 8 System was translated to a 9-track BCD tape for use on the IBM 360 computer.

For checking purposes, a printout of the ranges with the first and second differences was obtained. No major blunders (besides some duplication of a few observations) were detected.

Corrections to the ranges were applied according to Figure 2.2-1 and a new data set was generated for all the simultaneous observations from four stations. This data in a new format (OSUGOP [Reilly, et al., 1972]) was transferred to a tape. A summary of these observations by quadrangle is given in Table 2.2-1.

Table 2.1-1

SURVEY INFORMATION OF OBSERVATION STATIONS

STATION		DATUM	SURVEY COORDINATES[2]			MSL[3]	INSTR. HEIGHT[4]	INSTR.	SOURCE
NO	NAME	CODE[1]	LATITUDE	LONGITUDE	ELL. H(M)	(M)	(M)	TYPE	CODE[5]
5001	HERNDON	29	38° 59' 37".697	282° 40' 16".705	129.0	127.80	9.39	SECOR	1
5201	MOSES LAKE	29	47 11 5.916	240 39 50.463	358.0	368.92	2.00	SECOR	1
5410	SAND ISLAND	27	28 12 32.061	182 37 49.531	6.0	6.10	4.13	SECOR	2
5648	FORT STEWART	29	31 55 18.405	278 26 0.260	34.0	27.80	3.90	SECOR	1
5712	PARAMARIBO	41	5 26 59.817	304 47 44.990	12.0	21.50	4.93	SECOR	1
5713	TERCEIRA	17	38 45 36.725	332 54 21.064	56.0	56.00	4.25	SECOR	1
5715	DAKAR	50	14 44 41.008	342 30 52.935	27.0	27.30	4.42	SECOR	1
5717	FORT LAMY	1	12 7 49.300	15 2 6.148	320.0	298.50	4.83	SECOR	1
5720	ADDIS ABABA	1	8 46 9.479	38 59 49.196	1881.0	1889.40	4.29	SECOR	1
5721	MASHHAD	16	36 14 30.404	59 37 40.105	962.0	994.40	4.35	SECOR	1
5722	DIEGO GARCIA	*	- 7 20 57.440	72 28 31.570	*	6.10	4.60	SECOR	2
5723	CHIANG MAI	*	18 47	99 00	*	310.80		SECOR	1
5726	ZAMBOANGA	26	6 55 26.213	122 4 3.558	14.0	13.30	4.83	SECOR	2
5730	WAKE ISLAND	49	19 17 24.100	166 36 41.206	8.0	8.10	4.29	SECOR	1
5732	PAGO PAGO	*	*	*	*	*	*	SECOR	
5733	CHRISTMAS ISLAND	12	2 0 35.622	202 35 21.962	4.0	3.50	2.29	SECOR	1
5734	SHEMYA	29	52 42 54.894	174 7 37.870	-7.0	39.30	1.50	SECOR	1
5735	NATAL	41	- 5 54 56.253	324 49 57.605	66.0	39.40	*	SECOR	1
5736	ASCENSION ISLAND	5	- 7 58 15.220	345 35 32.365	74.0	74.00	4.32	SECOR	1
5739	TERCEIRA	17	38 45 36.311	332 54 19.686	56.0	56.10	4.25	SECOR	1
5744	CATANIA	16	37 26 40.831	15 2 44.955	-4.0	11.80	4.17	SECOR	1
5907	WORTHINGTON	*	*	*	*	*	*	SECOR	
5911	BERMUDA	*	*	*	*	*	*	SECOR	
5912	PANAMA	*	*	*	*	*	*	SECOR	
5914	PUERTO RICO	*	*	*	*	*	*	SECOR	
5915	AUSTIN	*	*	*	*	*	*	SECOR	
5923	CYPRUS	*	*	*	*	*	*	SECOR	
5924	ROTA	*	*	*	*	*	*	SECOR	
5925	ROBERTS FIELD	*	*	*	*	*	*	SECOR	
5930	SINGAPORE	*	*	*	*	*	*	SECOR	

Table 2.1-1 (Cont'd)

SURVEY INFORMATION OF OBSERVATION STATIONS

STATION NO	STATION NAME	DATUM CODE[1]	SURVEY COORDINATES[2] LATITUDE	SURVEY COORDINATES[2] LONGITUDE	SURVEY COORDINATES[2] ELL. H(M)	MSL[3] (M)	INSTR. HEIGHT[4] (M)	INSTR. TYPE	SOURCE CODE[5]
5931	HONG KONG	*	*	*	*	*	*	SECOR	
5933	DARWIN	*	*	*	*	*	*	SECOR	
5934	MANUS	*	*	*	*	*	*	SECOR	
5935	GUAM	*	*	*	*	*	*	SECOR	
5937	PALAU	*	*	*	*	*	*	SECOR	
5938	GUADALCANAL	*	*	*	*	*	*	SECOR	
5941	MAUI	*	*	*	*	*	*	SECOR	
6003	MOSES LAKE	29	47 11 7.132	240 39 48.118	356.0	368.74	1.50	BC-4A	1
6004	SHEMYA	29	52 42 54.890	174 7 37.870	-9.0	36.60	1.50	BC-4	1
6007	TERCEIRA	17	38 45 36.725	332 54 21.064	53.0	53.30	1.49	BC-4	1
6008	PARAMARIBO	41	5 26 55.325	304 47 42.832	8.7	18.38	1.49	BC-4	1
6012	WAKE ISLAND I	49	19 17 23.227	166 36 39.760	4.0	3.50	1.50	BC-4	1
6015	MASHHAD	16	36 14 29.527	59 37 42.729	959.0	991.00	1.50	BC-4	1
6016	CATANIA	16	37 26 42.628	15 2 47.308	-7.0	9.24	1.50	BC-4A	1
6042	ADDIS ABABA	1	8 46 8.501	38 59 49.164	1878.0	1886.46	1.52	BC-4	1
6047	ZAMBOANGA	26	6 55 26.132	122 4 4.838	9.0	9.39	1.50	BC-4	2
6055	ASCENSION ISLAND	5	- 7 58 16.634	345 35 32.764	71.0	70.94	1.50	BC-4	1
6059	CHRISTMAS ISLAND	12	2 0 35.622	202 35 21.962	3.0	2.75	1.50	BC-4A	1
6063	DAKAR	50	14 44 44.226	342 30 55.594	26.0	26.30	1.50	BC-4A	1
6067	NATAL	41	- 5 55 37.414	324 50 6.200	66.7	40.63	*	BC-4A	1

* Data Not Available

1 Refer to Table 2.1-2

2 Geodetic Coordinates of the Instrumental Reference Point (Optical/Electronic Center, etc.) on the Local Geodetic Datum

3 Mean Sea Level Height of the Instrumental Reference Point

4 Height of Instrumental Reference Point above Survey Monument

5 Source Code:
  1 -- (CSC, 1971)
  2 -- (CSC, 1972/73)

Note: Zero in the last digit may indicate that the digit is unknown.

Table 2.1-2

GEODETIC DATUMS

CODE	DATUM	ELLIPSOID	ORIGIN	LATITUDE	LONGITUDE (E)
1	ADINDAN (ETHIOPIA)	CLARKE 1880	STATION Z5 ADINDAN	22°10' 07".110	31° 29' 21".608
5	ASCENSION IS 1958	INTERNATIONAL	MEAN OF 3 STATIONS	-07 57	345 37
12	CHRISTMAS IS ASTRO 1967	INTERNATIONAL	SAT.TRI.STA. 059 RM3	02 00 35.91	202 35 21.82
16	EUROPEAN	INTERNATIONAL	HELMERT TOWER	52 22 51.45	13 03 58.74
17	GRACIOSA IS (AZORES)	INTERNATIONAL	SW BASE	39 03 54.934	331 57 36.118
26	LUZON 1911(PHILIPPINES)	CLARKE 1866	BALANCAN	13 33 41.000	121 52 03.000
27	MIDWAY ASTRO 1961	INTERNATIONAL	MIDWAY ASTRO 1961	28 11 34.50	182 36 24.28
29	NORTH AMERICAN 1927	CLARKE 1866	MEADES RANCH	39 13 26.686	261 27 29.494
41	SOUTH AMERICAN 1969	S.AMERICAN 1969	CHUA	-19 45 41.653	311 53 55.936
49	WAKE IS ASTRO 1952	INTERNATIONAL	ASTRO 1952	19 17 19.991	166 38 46.294
50	YOF ASTRO 1967 (DAKAR)	CLARKE 1880	YOF ASTRO 1967	14 44 41.62	342 30 52.98

Table 2.1-3

RELATIVE POSITION CONSTRAINTS

STATIONS	RELATIVE COORDINATES (METERS)			WEIGHTS[1]
	Δu	Δv	Δw	($1/\sigma^2$)
5201-6003	29.55	-48.21	-25.52	1.00
5712-6008	48.95	45.97	137.68	1.00
5713-5739	8.05	33.26	9.95	20.00
5713-6007	2.08	-1.06	1.88	1.00
5715-6063	1.05	-83.72	-95.45	1.00
5720-6042	-1.87	-0.26	30.16	1.00
5721-6015	49.67	-44.84	23.59	1.00
5726-6047	30.82	24.81	3.07	1.00
5730-6012	-4.69	-41.68	26.66	1.00
5733-6059	-0.92	-0.38	0.04	1.00
5734-6004	-1.20	0.12	1.59	1.00
5735-6067	-46.20	-290.84	1257.74	1.00
5736-6055	5.82	-13.48	42.60	1.00
5744-6016	49.84	-46.49	-42.16	1.00

SOURCE: DEFENSE MAPPING AGENCY TOPOGRAPHIC CENTER

[1] APPLIED EQUALLY TO ALL THREE RELATIVE COORDINATES IN M^{-2} UNIT

Table 2.1-4

GEOIDAL UNDULATIONS AND HEIGHTS USED IN THE CONSTRAINTS

STATION		NREF[1]	HCONSTR[2]	$\sigma_{HCONSTR}$[3]
NO	NAME	(M)	(M)	(M)
5001	HERNDON	-36.87	69.67	6.0
5201	MOSES LAKE	-17.65	341.99	4.0
5410	MIDWAY ISLANDS	- 4.13	6.72	8.0
5648	FORT STEWART	-35.07	-29.10	2.5
5712	PARAMARIBO	-28.31	-40.09	4.0
5713	TERCEIRA	54.00	82.80	4.0
5715	DAKAR	27.20	20.91	4.0
5717	FORT LAMY	10.35	279.97	6.0
5720	ADDIS ABABA	- 5.78	1861.35	6.0
5721	MASHHAD	-20.67	962.23	4.0
5722	DIEGO GARCIA	-73.64	-79.68	8.0
5723	CHIANG MAI	-40.39	269.90	8.0
5726	ZAMBOANGA	62.16	79.76	8.0
5730	WAKE ISLAND	13.75	28.88	8.0
5732	PAGO PAGO	27.35	35.16	6.0
5733	CHRISTMAS ISLAND	16.07	18.52	8.0
5734	SHEMYA	6.22	48.36	8.0
5735	NATAL	-12.03	-9.55	6.0
5736	ASCENSION ISLAND	16.26	53.57	8.0
5739	TERCEIRA	54.00	82.90	4.0
5744	CATANIA	37.43	26.13	4.0
5907	WORTHINGTON	-28.11	437.93	2.5
5911	BERMUDA	-43.44	-47.06	8.0
5912	PANAMA	6.16	-11.73	6.0
5914	PUERTO RICO	-50.08	-14.72	6.0
5915	AUSTIN	-26.32	162.18	2.5
5923	CYPRUS	24.64	168.92	8.0
5924	ROTA	54.48	40.16	6.0
5925	ROBERTS FIELD	33.75	10.77	6.0
5930	SINGAPORE	8.28	13.85	6.0
5931	HONG KONG	2.32	167.12	6.0
5933	DARWIN	50.66	69.31	8.0
5934	MANUS	74.75	86.77	8.0
5935	GUAM	48.15	92.63	8.0
5937	PALAU	69.93	145.94	8.0
5938	GUADALCANAL	59.97	76.57	8.0
5941	MAUI	2.05	34.51	8.0

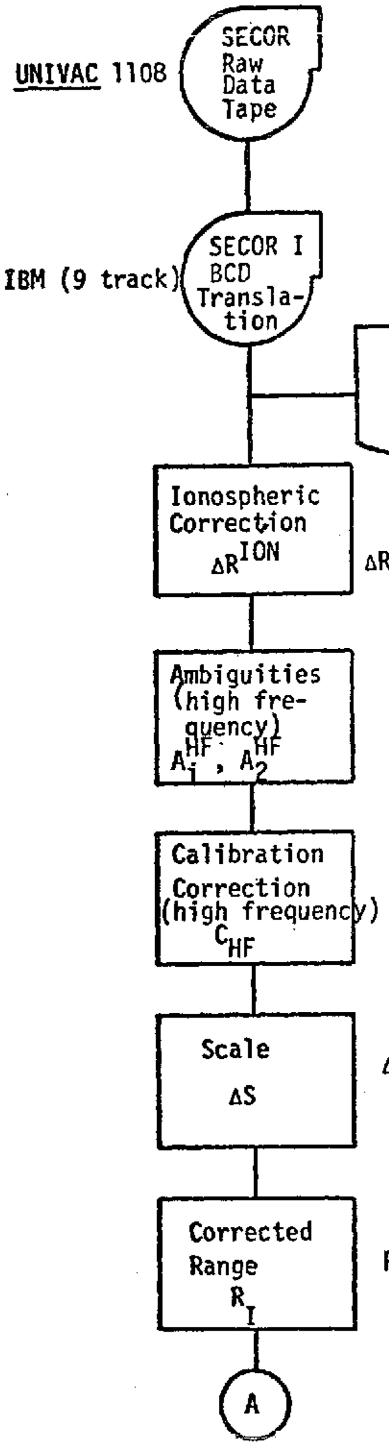
1. From [Rapp, 1973]
2. HCONSTR = MSL+NREF+ΔN, where ΔN is a correction term for the differences of position and size of the ellipsoids used [Mueller et al., 1973]
3. Used in Computing the Weights of the Height Constraints

Table 2.1-5

DIRECTION CONSTRAINTS BETWEEN BC-4 STATIONS

Station-Station	α	σ_α	β	σ_β
6003 - 6004	-67°.598	1".4	-4°.994	1".4
6003 - 6008	166.052	0.8	34.380	0.4
6004 - 6047	-95.629	1.1	40.651	1.1
6007 - 6008	74.620	1.4	47.803	1.4
6007 - 6055	-157.541	1.1	69.401	1.1
6015 - 6042	168.292	1.4	49.890	1.4
6015 - 6047	-8.781	1.2	26.323	1.2
6016 - 6042	-90.094	1.2	47.462	1.2
6016 - 6055	112.934	0.9	56.487	0.9

For the definition of the angular components α and β see section 3.43. These angles are based on station coordinates computed from the OSU WN14 solution [Mueller et al., 1973].



R_{obs} = observed range measurement

C_{HF}, C_{LF} = observed frequency channel calibration correction (high and low frequency)

DI-DC = given ionospheric correction for each range

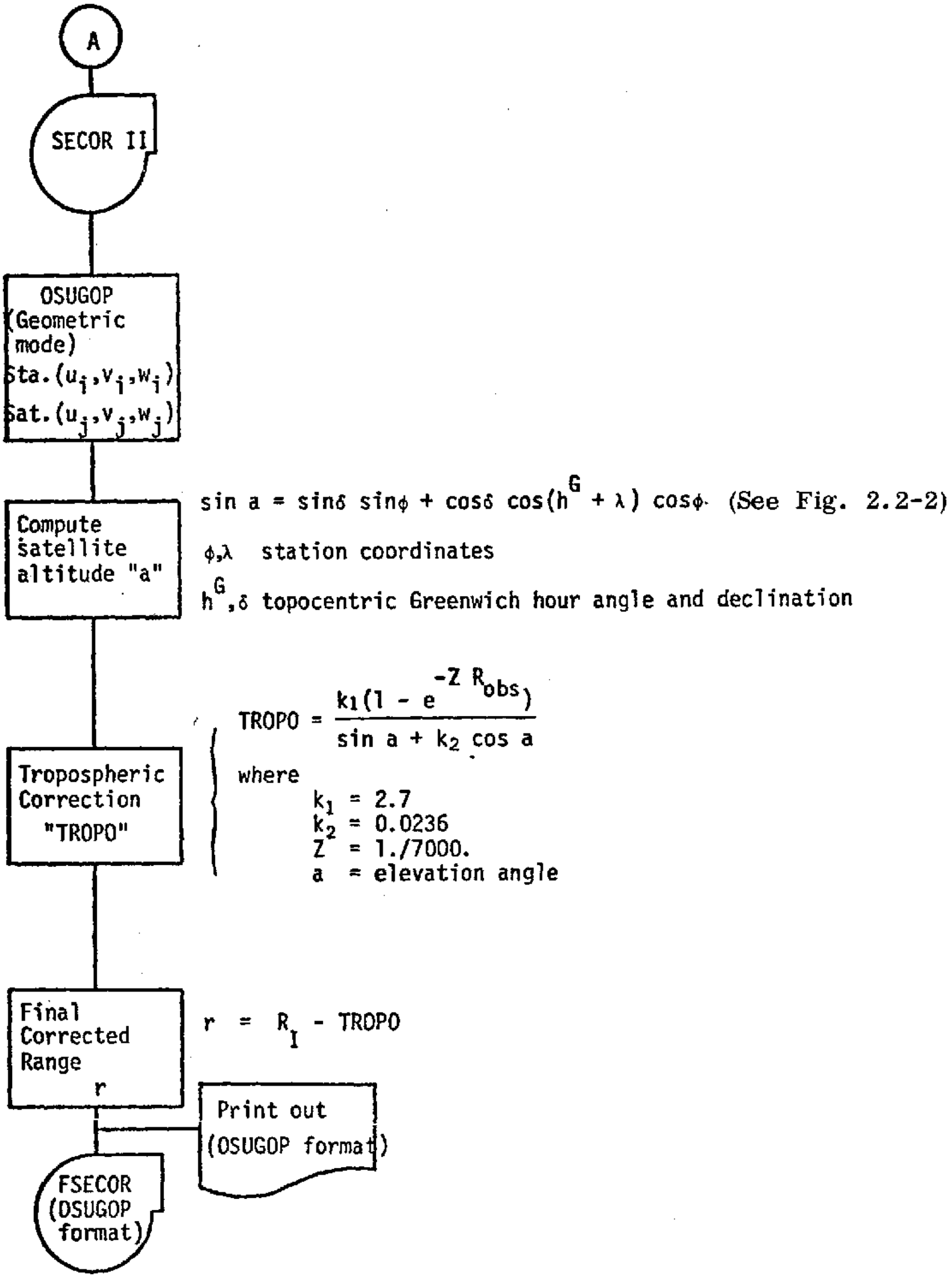
$A_i^{HF}, A_i^{LF}; A_2^{HF}, A_2^{LF}$ = given ambiguities (initial and new sets)

$$\Delta R^{ION} = -0.7125\ [(DI\text{-}IC) + A_i^{LF} + A_2^{LF} + (C_{LF} - C_{HF})]$$

$$\Delta S = .98\ \frac{R_{obs}}{10^6}$$

$$R_I = R_{obs} + \Delta R^{ION} + A_i^{HF} + A_2^{HF} + C_{HF} + \Delta S$$

Fig. 2.2-1 Scheme of SECOR preprocessing procedure at OSU.



$\sin a = \sin\delta \sin\phi + \cos\delta \cos(h^G + \lambda) \cos\phi$ (See Fig. 2.2-2)

ϕ, λ station coordinates

h^G, δ topocentric Greenwich hour angle and declination

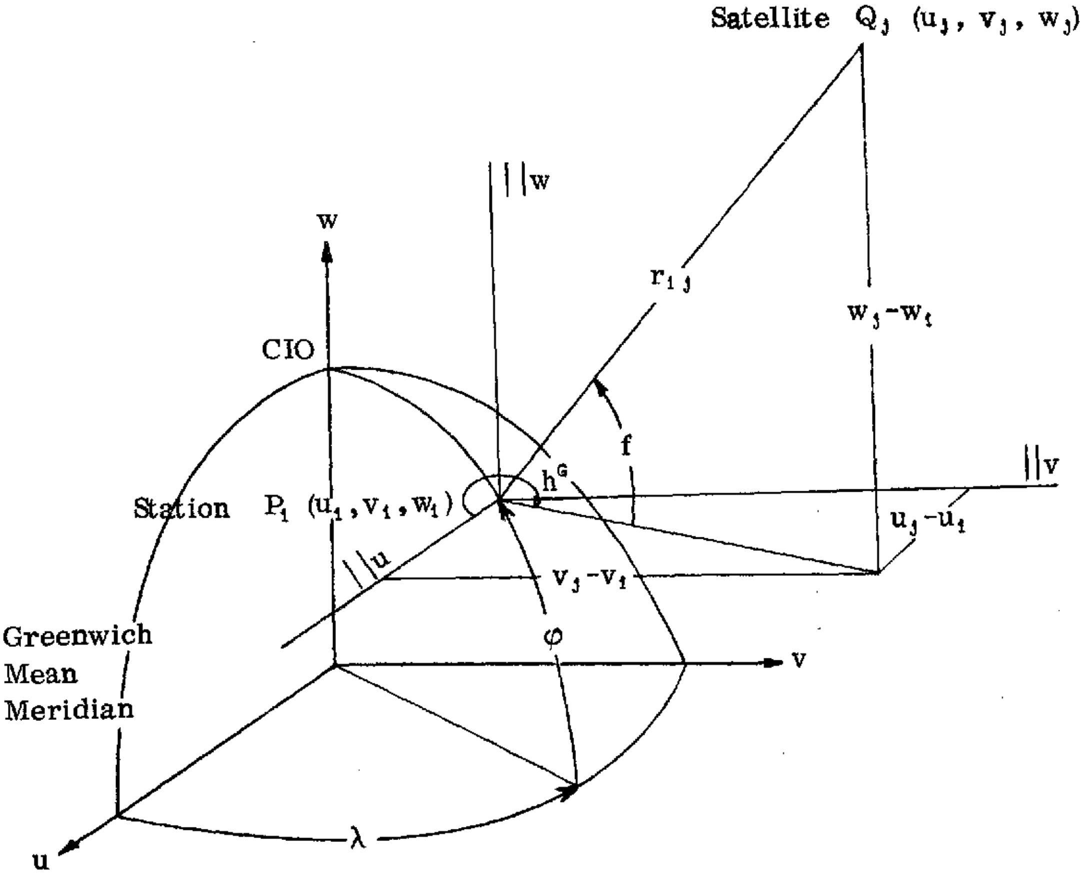
$$\text{TROPO} = \frac{k_1(1 - e^{-Z R_{obs}})}{\sin a + k_2 \cos a}$$

where

$k_1 = 2.7$
$k_2 = 0.0236$
$Z = 1./7000.$
a = elevation angle

$r = R_I - \text{TROPO}$

Fig. 2.2-1 continued



$$\tan(360^\circ - h^G) = -\tan h^G = \frac{v_j - v_i}{u_j - u_i}$$

$$\left.\begin{array}{l} \tan h^G = \dfrac{v_i - v_j}{u_j - u_i} \\[2ex] \sin f = \dfrac{w_j - w_i}{r_{ij}} \end{array}\right\} \qquad \left.\begin{array}{l} \tan \lambda = \dfrac{v_i}{u_i} \\[2ex] \tan \varphi = \dfrac{w_i}{\sqrt{u_i^2 + v_i^2}} \end{array}\right\}$$

$$\sin a = \sin f \sin \varphi + \cos f \cos(h^G + \lambda) \cos \varphi$$

Figure 2.2-2

Table 2.2-1

SUMMARY OF SECOR OBSERVATIONS BY QUADRANGLE

Quad Stations Involved	No. of Observations	Quad Stations Involved	No. of Observations
5001-5907-5648-5911	432	5726-5930-5933-5934	644
5911-5001-5648-5914	168	5726-5933-5934-5935	808
5911-5907-5915-5912	1008	5931-5726-5934-5935	1144
5911-5915-5912-5712	92	5935-5726-5934-5730	2048
5911-5907-5912-5712	260	5935-5726-5934-5937	1264
5911-5915-5912-5712	228	5730-5935-5934-5938	2216
5911-5912-5712-5713	684	5730-5935-5938-5732	1380
5713-5911-5712-5715	1220	5730-5938-5732-5733	756
5715-5713 5712-5735	548	5730-5732-5733-5411	752
5715-5739-5712-5735	288	5730-5733-5411-5410	648
5715-5712-5735-5736	660	5730-5733-5411-5734	508
5715-5735-5736-5717	640	5734-5410-5411-5201	312
5715-5736-5717-5744	28	5734-5730-5411-5201	264
5739-5715-5717-5744	384		
5715-5736-5717-5744	464		
5744-5715-5717-5923	868		
5744-5715-5717-5924	804		
5744-5715-5717-5925	612		
5923-5744-5717-5720	1236		
5923-5717-5720-5721	772		
5744-5717-5720-5721	20		
5721-5923-5720-5722	752		
5721-5720-5722-5723	296		
5923-5721-5722-5723	36		
5723-5721-5722-5930	460		
5723-5722-5930-5931	588		
5722-5723-5930-5726	68		
5931-5723-5930-5726	768		
5931-5930-5726-5933	1064		
5723-5930-5726-5933	652		

3. THEORETICAL BACKGROUND

3.1 The Mathematical Model

In the range observations mode each participating station P_i at an event $[E_j, Q_j \mid t_j]$ observes the length of the distance (P_iQ_j) i.e., the topocentric range r_{ij} from ground station P_i to satellite position Q_j (See Fig. 2.2-2).

Let (u_i, v_i, w_i) be the Cartesian coordinates of P_i and (u_j, v_j, w_j) of Q_j, with respect to an average terrestrial (tied to the solid earth) coordinate system defined by:

a) w - axis is directed toward the average north terrestrial pole as defined by the International Polar Motion Service (IPMS), commonly known as the Conventional International Origin (CIO).

b) u-w plane parallel to the mean Greenwich astronomic meridian as defined by the Bureau International de l'Heure (BIH).

Thus the mathematical model can be written as

$$r_{ij} = \left[(u_j-u_i)^2 + (v_j-v_i)^2 + (w_j-w_i)^2 \right]^{\frac{1}{2}} \qquad 3.1\text{-}1$$

or

$$F_{ij} = \left[(u_j-u_i)^2 + (v_j-v_i)^2 + (w_j-w_i)^2 \right]^{\frac{1}{2}} - r_{ij} = 0 \qquad 3.1\text{-}2$$

Thus in order to tie the satellite position points to the system only three known stations observing simultaneously are necessary and sufficient although we will not have redundant information. For redundant information at least four stations observing simultaneously are necessary, provided their configuration is not a degenerized one [Blaha, 1971a, Tsimis, 1973].

The expression for the linearized mathematical model as F is known ~~takes the form:~~

$$AX + BV + W = 0$$

where the design matrix B is a negative unit matrix and the design matrix A is formed by submatrices of the form:

$$A_{ij} = \frac{\partial F_{ij}}{\partial \vec{X}_j^o , \partial \vec{X}_i^o} = \left[\begin{array}{c:c} a_{ij} & -a_{ij} \end{array} \right]$$

where

$$a_{ij} = \left[\frac{u_j^o - u_i^o}{r_{ij}^o} \quad \frac{v_j^o - v_i^o}{r_{ij}^o} \quad \frac{w_j^o - w_i^o}{r_{ij}^o} \right]$$

and r_{ij}^o is computed from 3.1-1 using the initial approximate values for the station. and satellite coordinates, the latest coordinates resulting from a preliminary least squares adjustment (for each event j) with the observing stations held fixed. [Krakiwsky and Pope, 1967].

The unknown vector X is made up of subvectors

$$X_{ij} = \begin{bmatrix} X_j \\ \\ X_i \end{bmatrix}$$

where

$$X_j = \begin{bmatrix} du_j \\ dv_j \\ dw_j \end{bmatrix}$$

and

$$X_i = \begin{bmatrix} du_i \\ dv_i \\ dw_i \end{bmatrix}$$

The misclosure vector W is formed by the individual differences

$$W_{ij} = r_{ij}^o \text{ (computed)} - r_{ij}^b \text{ (observed)}$$

The residual vector V is composed of the individual residuals v_{ij} (in meters) corresponding to the observed ranges r_{ij}^b. Giving consideration to the characteristics of the design matrices, the final matrix equation for the linearized model can be written as:

$$AX - V + W = O$$

or

$$AX + W = V$$

3.2 The Normal Equations

The variation function for the range adjustment is similar to the optical case, namely,

$$\Phi = V'PV + X'P_XX - 2K'(AX - V + W) \qquad 3.3\text{-}1$$

where

- V is the vector of residuals corresponding to the range observations
- X is the vector of corrections to the preliminary ground and satellite positions*
- P is the weight matrix for the ranges
- P_X is the weight matrix for the ground and satellite positions
- K is the vector of correlates

The differentiation of equation 3.2-1 for the minimum condition results in the following expanded form of the normal equations:

$$\begin{bmatrix} -P_X & 0 & A' \\ 0 & -P & -I \\ A & -I & 0 \end{bmatrix} \begin{bmatrix} X \\ V \\ K \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ W \end{bmatrix} = 0 \qquad 3.2\text{-}2$$

After the elimination of the correlates and residuals, and the expansion of the A and P matrices the following expression results:

$$\left[\begin{array}{c|c} \sum_i a'_{ij} p_{ij} a_{ij} + P_j & -a'_{ij} p_{ij} a_{ij} \\ \hline -a'_{ij} p_{ij} a_{ij} & \sum_j a'_{ij} p_{ij} a_{ij} + P_i \end{array}\right] \left[\begin{array}{c} X_j \\ \hline X_i \end{array}\right] + \left[\begin{array}{c} \overset{U_j =}{\sum_i a'_{ij} p_{ij} W_{ij}} \\ \hline \overset{U_i =}{-\sum_j a'_{ij} p_{ij} W_{ij}} \end{array}\right] = 0$$

3.3 Reduced Normal Equations for Range Observations

The general form of the reduced normal equations after the elimination of X_j (corrections to the preliminary coordinates of the satellite position) can be formulated as :

$$\bar{N}X + \bar{U} = 0$$

---

* Satellite positions will be considered "nuisance" parameters and therefore eliminated from the solution.

where the 3 x 3 blocks in N are now computed using $P_i = 0$ [Mueller, 1968]:

$$\underset{3\times 3}{N_{kk}} = \sum_{j} a'_{kj}\, p_{kj}\, a_{kj} - \sum_{j} a'_{kj} p_{kj} a_{kj} \left[\sum_{j} a'_{ij} p_{ij} a_{ij}\right]^{-1} a'_{kj} p_{kj} a_{kj}$$

$$\underset{3\times 3}{N_{kl}} = -\sum_{j} \left[a'_{kj} p_{kj} a_{kj} \left(\sum_{i} a'_{ij} p_{ij}\, a_{ij}\right)^{-1} a'_{lj} p_{lj} a_{lj}\right]$$

and the vector of constant terms having the form:

$$U_k = -\sum a'_{kj}\, p_{kj} v_{kj}$$

where

v_{kj} = residual of any observed range from a particular station (resulting from a preliminary least squares adjustment of any simultaneous event with the stations held fixed).

p_{ij} = weight of any observed range r_{ij}

k, l denotes particular ground stations

j denotes particular simultaneous event

i denotes any ground station participating in an event

$\sum_{i}$ is the summation over all ground stations involved in event j.

$\sum_{j}$ is the summation over all events observed by ground station k and/or l.

3.4 Constraint's Contributions to the Normal Equations

Two alternative definitions exist for the term "constraints". The absolute constraints represent certain conditions which have to be fulfilled exactly and with no uncertainties and the relative constraints (or weighted constraints) which have the same characteristics as the observations.

In general the contribution of the functional constraint equations

$$G(X, L_C) = 0$$

to the normal equations can be found bordering the normal equation matrix

$$\begin{bmatrix} N_{n-1} & C'_n \\ C_n & -P_{C_n}^{-1} \end{bmatrix} \begin{bmatrix} X_n \\ -K_{C_n} \end{bmatrix} + \begin{bmatrix} U_{n-1} \\ W_n \end{bmatrix} = 0$$

from where after elimination of K_{C_n} it is easy to find

$$[N_{n-1} + C'_n P_{C_n} C_n] X_n + U_{n-1} + C'_n P_{C_n} W_n = 0$$

$$[N_{n-1} + N^c_n] X_n + U_{n-1} + U^c_n = 0 \qquad 3.4\text{-}1$$

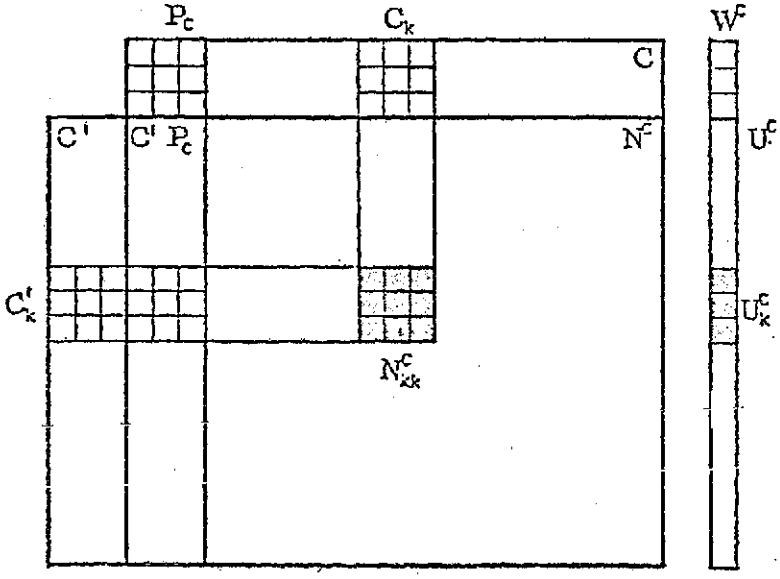
where N^c_n and U^c_n are the contributions to the coefficient matrix and constant vector of the normal equation due to the application of constraints. The coefficient n-1 represents the normal equations of the previous set (without constraints).

After the constraints are added the normal equations will take the usual form:

$$N_n X_n + U_n = 0$$

and we are in the position to obtain the contribution from a new set of constraints. Constraints can be applied between two stations k and l or to a single station. The contribution of these constraints to the matrix N (3 x 3 blocks) and U (3 x 1 blocks) can be schematically expressed in two different ways:

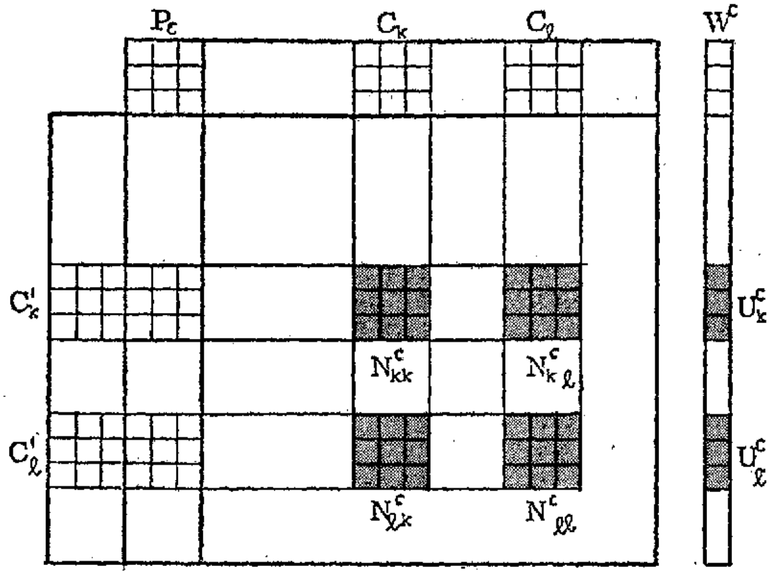
a) Contribution to the normals due to the constraint applied to station k



$$N^c_{kk} = C'_k P_c C_k \qquad 3.4\text{-}2$$

$$U^c_k = C'_k P_c W^c$$

b) Contribution to the normals due to the constraint between stations k and l



$$N^c_{kk} = C'_k P_c C_k ; \quad N^c_{k\ell} = C'_k P_c C_\ell$$
$$N^c_{\ell\ell} = C'_\ell P_c C_\ell ; \quad N^c_{\ell K} = C'_\ell P_c C_K$$
$$U^c_k = C'_k P_c W^c ; \quad U^c_\ell = C'_\ell P_c W^c \qquad 3.4\text{-}3$$

These blocks obtained as indicated above for the corresponding case will be the only ones computed and added to the original normal equations.

3.41 Relative Position Constraints

Relative position constraints are used in order to constrain "double" stations or closely situated stations of the same net. The expression for the constraints contribution to the normals can be written as follows:

$$[N + N^R] X + U + U^R = 0$$

where N^R and U^R, computed from (3.4-2), (3.4-3), are the contribution to the original normal equations ($NX + U = 0$).

If the relative position (Δu, Δv, Δw) of two stations is known, along with the standard deviation of these relative positions, the constraints can be formed. In this case the functional contraint equations are

$$u_K - u_1 = \Delta u$$
$$v_K - v_1 = \Delta v$$
$$w_K - w_1 = \Delta w$$

Therefore

$$\underset{3\times3}{C^R_K} = \underset{3\times3}{I} \; ; \; \underset{3\times3}{C^R_1} = \underset{3\times3}{-I}$$

and

$$\underset{3\times3}{N^{R}_{kk}} = I\ P_R\, I = \underset{3\times3}{P_R}$$

$$\underset{3\times3}{N^{R}_{ll}} = I\ P_R\, I = \underset{3\times3}{P_R}$$

$$\underset{3\times3}{N^{R}_{kl}} = \underset{3\times3}{N^{R}_{lk}} = I\ P_R\ (-I) = -\underset{3\times3}{P_R}$$

where

$$P_R = \begin{bmatrix} \frac{1}{\sigma^2_{\Delta u}} & 0 & 0 \\ 0 & \frac{1}{\sigma^2_{\Delta v}} & 0 \\ 0 & 0 & \frac{1}{\sigma^2_{\Delta w}} \end{bmatrix}$$

If

$$W = -\begin{bmatrix} \Delta u \\ \Delta v \\ \Delta w \end{bmatrix}$$

and

$$W^R_o = G^R(X^o, L^o_R)$$

$$W^R = W^R_o - W$$

Therefore

$$\underset{3\times1}{U^R_k} = I\ P_R\ W^R$$

$$\underset{3\times1}{U^R_l} = -I\ P_R\ W^R$$

Thus, the diagonal elements of P_R are added to each element of the diagonal of the blocks kk and ll of the matrix of the original normals N, and subtracted from the diagonal elements of the blocks kl and lk of N.

The constribution to the vector U will be obtained adding U_K^R and subtracting U_1^R to the corresponding block columns k and 1 of U.

3.42 Height Constraints

If the geodetic (ellipsoidal) height H_K of the station k is to be constrained, then

$$\underset{3\times3}{N_{KK}^H} = (C_K^H)' \, P_H \, C_K^H$$

where

$$\underset{1\times3}{C_K^H} = [\cos\varphi_K^o \cos\lambda_K^o , \quad \cos\varphi_K^o \sin\lambda_K^o , \; \sin\varphi_K^o]$$

and

$$P_H = \frac{1}{\sigma_{HK}^2} \; .$$

Here φ_K^o and λ_K^o are the approximate geodetic coordinates and σ_{HK}^2 is the variance of the height for station k.

The constant vector U_K^H can be computed from

$$U_K^H = (C_K^H)' \, P_H \, W^H$$

where

$W^H = H_K - H_K^o$, H_K^o being the approximate height.

3.43 Directional Constraints

Directional constraints are introduced when the orientation of the coordinate system is not defined through the observations (e.g., in the case of a ranging network).

The directional constraint between two stations k and 1 is accomplished by applying weights to two angles α° and β°, defining the direction between them, and computed from the approximate (u°, v°, w°) coordinates of the two stations as follows:

$$\alpha^\circ = \tan^{-1} \frac{\Delta v^\circ}{\Delta u^\circ}$$

$$\beta^\circ = \tan^{-1} \frac{\Delta w^\circ}{R^\circ}$$

where

$$\Delta u^{o} = u_{k}^{o} - u_{l}^{o}$$

$$\Delta v^{o} = v_{k}^{o} - v_{l}^{o}$$

$$\Delta w^{o} = w_{k}^{o} - w_{l}^{o}$$

and

$$R^{o} = (\Delta u^{o\,2} + \Delta v^{o\,2})^{\frac{1}{2}}$$

The matrix C^{0} of partial derivatives is then formed

$$C_{k}^{0} = \begin{bmatrix} \frac{\partial \alpha^{o}}{\partial \Delta u^{o}} \frac{\partial \Delta u^{o}}{\partial u_{k}^{o}} & \frac{\partial \alpha^{o}}{\partial \Delta v^{o}} \frac{\partial \Delta v^{o}}{\partial v_{k}^{o}} & \frac{\partial \alpha^{o}}{\partial \Delta w^{o}} \frac{\partial \Delta w^{o}}{\partial w_{k}^{o}} \\ \frac{\partial \beta^{o}}{\partial \Delta u^{o}} \frac{\partial \Delta u^{o}}{\partial u_{k}^{o}} & \frac{\partial \beta^{o}}{\partial \Delta v^{o}} \frac{\partial \Delta v^{o}}{\partial v_{k}^{o}} & \frac{\partial \beta^{o}}{\partial \Delta w^{o}} \frac{\partial \Delta w^{o}}{\partial w_{k}^{o}} \end{bmatrix}$$

where

$$\frac{\partial \alpha^{o}}{\partial \Delta u^{o}} = \cos^{2}\alpha^{o} \tan\alpha^{o} / \Delta u^{o}$$

$$\frac{\partial \alpha^{o}}{\partial \Delta v^{o}} = -\cos^{2}\alpha^{o} / \Delta u^{o}$$

$$\frac{\partial \alpha^{o}}{\partial \Delta w^{o}} = 0$$

$$\frac{\partial \beta^{o}}{\partial \Delta u^{o}} = \Delta u^{o} \cos^{2}\beta^{o} \tan^{2}\beta^{o} / R^{o\,2}$$

$$\frac{\partial \beta^{o}}{\partial \Delta v^{o}} = \frac{\partial \beta^{o}}{\partial \Delta u^{o}} \tan\alpha^{o}$$

$$\frac{\partial \beta^{o}}{\partial \Delta w^{o}} = -\cos^{2}\beta^{o} / R^{o}$$

and clearly $C_{l}^{0} = -C_{k}^{0}$.

Then the matrix

$$N^{0} = (C^{0})' P_{0} C^{0}$$

is formed where P_{0} is the weight matrix estimated from the statistics of α^{o} and

β^o in the customary way.

3.44 Inner Constraints (Free Adjustment)

Even though the definition of a coordinate system is arbitrary in the case of a minimum constraint adjustment, in the case of ranging, the selection of the six coordinates to be constrained for this purpose is very critical, since one set of constraints would give a different solution than another set. The "best" solution is arrived at in a coordinate system defined through the use of a set of constraint equations called "inner" constraints [Rinner et al., 1967]. In this sense, the "best" solution would have the smallest covariance matrix for the unknowns. Covariance matrices may be compared by means of their traces, and the inner constraint equations are characterized by the property that the trace of the covariance matrix obtained with their use is a minimum among those obtained by adjusting a given set of observations augmented by a minimal set of constraint equations. The resulting adjustment is called a "free" one. The functional inner constraints equations can be written as

$$C^I X = 0$$

where

$$C^I = \left[\begin{array}{ccc|ccc|c|ccc} 1 & 0 & 0 & 1 & 0 & 0 & & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & & 0 & 0 & 1 \end{array}\right]$$

and C^I has as many 3 x 3 unit blocks as unknown points. X is the set of corrections of the approximate coordinates of the unknown points.

In the most general application when the "best" origin, orientation and scale are sought the matrix C^I has the form

$$C^I = \begin{bmatrix} C_1^I \\ \\ C_2^I \\ \\ C_3^I \end{bmatrix} = \left[\begin{array}{ccc:ccc:c} & \underset{3x3}{I} & & & \underset{3x3}{I} & & \cdots \\ \hdashline 0 & w_1^o & -v_1^o & 0 & w_2^o & -v_2^o & \\ -w_1^o & 0 & u_1^o & -w_2^o & 0 & u_2^o & \cdots \\ v_1^o & -u_1^o & 0 & v_2^o & -u_1^o & 0 & \\ \hdashline u_1^o & v_1^o & w_1^o & u_2^o & v_2^o & w_2^o & \cdots \end{array}\right]$$

The symbols (u_i^o, v_i^o, w_i^o) denote the approximate coordinates of the i^{th} unknown point where both the ground points and the satellite positions are considered.

If we represent the normal equations with the contribution of all the constraints (except inner constraints) by

$$[N + N^R + N^H + N^D]X + U + U^R + U^H + U^D = 0 \quad \text{or}$$

$$\overline{N}X + \overline{U} = 0$$

then the inner adjustment can be obtained by bordering the coefficient matrix $\overline{N}$ of the normal equations as

$$\begin{bmatrix} \overline{N} & (C^I)' \\ C^I & 0 \end{bmatrix} \begin{bmatrix} X \\ -K_I \end{bmatrix} = \begin{bmatrix} -\overline{U} \\ 0 \end{bmatrix}$$

It can be proved [Blaha, 1971] that

$$\Sigma_X = \left\{\overline{N} + (C^I)' [C^I (C^I)']^{-1} C^I\right\}^{-1} \left\{I-(C^I)' [C^I (C^I)']^{-1} C^I\right\}$$

Upon the addition of any kind of constraint to the normal equations, it becomes necessary to consider also its contribution to $\Sigma V'PV$. The degrees of freedom change as well. In order to compute the proper variance of unit weight the latter must be taken into consideration.

4. THE SOLUTION

With the specific constraints mentioned above, particular values of which are given in Section 2.1, SECOR-27 solution was computed using the general OSUGOP program [Reilly et al., 1972].

The basic information regarding the range adjustment is presented in Table 4.1.

The coordinates of SECOR-27 solution are shown in Table 4.2 with their corresponding standard deviations and error ellipsoid parameters.

Table 4-1

General Information on the SECOR-27 Geometric Adjustment

No. of SECOR stations	37
σ of a single range observation (estimated)	3 m
Number of Constraints used:	
Relative Position Constraints	15
Height Constraints	37
Direction Constraints	10
Inner constraint defines the origin of the coordinate system	
No. of degrees of freedom	7173
$\Sigma V'PV$	14183.1
$\hat{\sigma}_0^2$ (a posteriori variance of unit weight)	1.88
$\hat{\sigma}$ of a single range observation (a posteriori)	4.1 m

Table 4.2

Cartesian and Geodetic Coordinates (Solution SECOR-27)

Sta. No	u	σ_u	v	σ_v	w	σ_w
	φ	σ_φ	λ	σ_λ	H	σ_H
		a_a	A_a	r_a		
		a_b	A_b	r_b		
		a_c	A_c	r_c		

u, v, w ·	Cartesian coordinates in meters (Orientation: u ≡ the Greenwich meridian as defined by the B.I.H.; v ≡ λ = 90° (E); w ≡ Conventional International Origin).
φ, λ	Geodetic latitude and longitude in angular units (degrees, minutes and seconds of arc) computed from the Cartesian coordinates and referred to a rotational ellipsoid of a = 6378155.00 m and b = 6356769.70 m.
H	Geodetic (ellipsoidal) height in meters referred to the same ellipsoid.
$\sigma_u, \sigma_v, \sigma_w$	Standard deviations of the Cartesian coordinates in meters.
$\sigma_\varphi, \sigma_\lambda$	Standard deviations of the geodetic coordinates in seconds of arc.
σ_H	Standard deviations of the geodetic height in meters.
a_a, A_a, r_a	Altitude (elevation angle), azimuth and magnitude of the major semi axis of the error ellipsoid, respectively. Angles in degrees, magnitude in meters. Altitude is positive above the horizon. Azimuth is positive east reckoned from the north
a_b, A_b, r_b	Same as above for the mean axis of the error ellipsoid.
a_c, A_c, r_c	Same as above for the minor axis of the error ellipsoid.

Table 4-2 continued

5001	1088828.38	9.30	-4842954.37	5.36	3991826.29	6.31
	38 59 37.09	0.24	282 40 15.47	0.40	75.52	2.94
		3.42	74.83	9.70		
		2.23	-15.30	7.25		
		-85.91	41.63	2.88		
5201	-2127764.63	10.48	-3785925.77	9.36	4656018.34	8.40
	47 11 5.43	0.37	240 39 47.37	0.52	341.78	3.99
		0.13	-0.85	11.56		
		-0.98	89.15	10.88		
		89.01	96.74	3.99		
5410	-5618727.46	4.11	-258239.91	11.66	2997266.52	7.42
	28 12 44.19	0.24	182 37 53.38	0.43	6.11	4.35
		3.72	-86.01	11.72		
		17.18	5.14	7.47		
		-72.40	-7.82	3.85		
5648	794673.60	14.25	-5360057.81	9.56	3353057.17	13.47
	31 55 18.05	0.51	278 25 59.34	0.57	-28.89	2.55
		0.08	43.04	18.90		
		3.74	-46.96	10.61		
		-86.26	-45.79	2.46		
5712	3623273.36	9.23	-5214191.74	6.34	601652.09	6.92
	5 26 57.21	0.23	304 47 41.75	0.35	-41.63	2.95
		1.82	92.96	10.82		
		3.40	2.85	6.91		
		-86.14	31.05	2.90		
5713	4433623.16	4.99	-2268166.55	8.30	3971660.02	4.28
	38 45 36.74	0.15	332 54 23.34	0.38	88.32	2.40
		0.51	89.52	9.20		
		10.24	-0.57	4.71		
		79.74	-177.69	2.29		

Table 4-2 continued

5715	5884456.27	3.63	-1853568.65	9.91	1612756.86	4.80
	14 44 39.20	0.15	342 30 56.54	0.35	21.14	2.36
		-0.77	90.46	10.33		
		13.09	0.64	4.80		
		76.89	177.15	2.15		
5717	6023411.35	4.06	1617942.91	10.16	1331652.04	6.06
	12 7 52.10	0.19	15 2 6.97	0.35	283.02	2.94
		-2.62	90.16	10.63		
		13.50	0.79	6.06		
		76.23	169.40	2.61		
5720	4900759.40	7.78	3968252.89	8.29	966350.72	6.94
	8 46 13.15	0.22	38 59 52.30	0.36	1861.24	3.32
		-4.06	90.43	11.01		
		15.15	1.54	6.98		
		74.29	165.82	2.77		
5721	2604415.22	8.37	4444129.74	5.35	3750359.77	5.47
	36 14 26.91	0.19	59 37 41.55	0.37	970.84	2.99
		1.18	-92.95	9.28		
		11.48	-2.71	5.89		
		78.45	171.26	2.80		
5722	1905138.10	12.15	6032291.16	5.63	-810717.57	7.95
	- 7 21 6.18	0.26	72 28 21.61	0.40	-84.71	4.88
		4.51	-84.72	12.43		
		8.27	5.93	8.13		
		80.56	156.96	4.70		
5723	-941701.28	11.20	5967451.72	3.62	2039344.19	5.97
	18 46 11.75	0.19	98 58 3.65	0.39	264.45	3.14
		3.36	-91.69	11.47		
		16.91	-0.87	5.97		
		72.74	167.23	2.65		
5726	-3361939.48	10.12	5365843.38	7.22	763649.99	5.66
	16 55 21.35	0.18	122 4 8.30	0.40	89.57	2.76
		2.60	-90.69	12.20		
		18.21	0.17	5.76		
		71.59	171.47	2.13		

Table 4-2 continued

5730	-5858556.40	3.75	1394470.97	11.88	2093873.15	6.74
	19 17 30.43	0.21	166 36 41.12	0.41	17.61	3.63
		2.57	-89.73	12.09		
		19.16	1.17	6.74		
		-70.66	-7.06	3.00		
5732	-6099969.36	5.57	-997356.01	13.19	-1568568.44	9.26
	-14 19 53.78	0.31	189 17 8.88	0.44	37.10	4.93
		1.15	-99.62	13.43		
		-1.00	-9.64	9.29		
		-88.47	-140.45	4.92		
5733	-5885321.54	6.71	-2448387.42	12.44	221669.14	10.15
	2 0 18.35	0.33	202 35 17.12	0.43	17.07	5.11
		-1.60	-93.89	13.22		
		8.05	-4.11	10.18		
		-81.79	7.29	4.94		
5734	-3851774.79	6.08	396407.45	10.09	5051369.83	7.02
	52 42 49.49	0.21	174 7 26.62	0.54	59.11	6.52
		3.43	-86.46	10.16		
		44.38	6.91	7.01		
		-45.42	0.05	5.98		
5735	5186342.89	7.09	-3654228.01	8.91	-653034.54	6.03
	- 5 54 58.06	0.20	324 49 55.15	0.35	0.15	3.52
		2.02	92.79	10.85		
		-3.34	2.91	6.02		
		86.10	-28.37	3.49		
5736	6118339.51	4.68	-1571766.98	10.47	-878564.03	5.81
	- 7 58 13.95	0.19	345 35 33.29	0.35	58.44	3.85
		0.33	89.37	10.78		
		1.22	-0.64	5.85		
		88.74	-165.40	3.84		
5739	4433614.77	4.99	-2268199.51	8.30	3971650.24	4.28
	38 45 36.34	0.15	332 54 21.97	0.38	88.08	2.40
		0.50	89.51	9.20		
		10.25	-0.58	4.71		
		79.74	-177.72	2.29		

Table 4-2 continued

5744	4896433.80	3.93	1316125.99	8.81	3656632.15	4.68
	37 26 37.53	0.17	15 2 42.31	0.37	19.20	2.53
		-0.72	91.87	9.07		
		8.35	1.98	5.17		
		81.62	176.99	2.44		
5907	-449437.73	9.25	-4600908.92	5.86	4380274.10	6.87
	43 38 56.58	0.28	264 25 14.84	0.41	439.06	2.31
		1.57	53.61	9.63		
		3.06	-36.48	8.31		
		-86.56	-9.25	2.26		
5911	2307970.19	8.77	-4673779.10	5.44	3394450.16	5.95
	32 21 45.28	0.20	295 20 23.35	0.37	-36.32	3.19
		2.42	82.68	9.64		
		7.93	-7.66	6.29		
		-81.70	9.53	3.07		
5912	1142624.52	11.07	-6196106.91	4.29	988310.55	8.32
	8 58 26.02	0.28	280 26 54.72	0.36	-14.80	3.88
		3.26	94.34	11.15		
		-2.22	4.47	8.44		
		-85.99	129.84	3.82		
5914	2349442.68	21.19	-5576035.64	14.34	2010318.58	18.98
	18 29 38.59	0.66	292 50 52.62	0.81	-14.77	6.09
		0.89	52.65	28.17		
		-12.02	-37.16	13.83		
		-77.94	138.47	5.47		
5915	-744112.30	10.20	-5465236.37	5.45	3192445.85	7.94
	30 13 45.29	0.30	262 14 47.91	0.38	160.55	2.30
		-0.75	82.41	10.25		
		-2.33	-7.62	9.30		
		-87.55	-169.73	2.26		
5923	4363335.16	5.84	2862257.93	7.84	3655387.25	5.23
	35 11 30.41	0.18	33 15 50.58	0.36	177.24	2.67
		-1.51	90.89	9.22		
		10.49	1.17	5.63		
		79.40	172.81	2.49		

Table 4-2 continued

5924	5093544.42	3.35	-565325.16	9.16	3784273.72	4.51
	36 37 37.26	0.16	353 40 0.28	0.37	13.36	2.60
		-0.44	90.36	9.20		
		8.44	0.43	4.93		
		81.55	177.41	2.52		
5925	6237359.96	3.26	-1140250.68	10.61	687734.03	5.53
	6 13 53.99	0.18	349 38 24.59	0.35	10.62	2.93
		-1.41	90.44	10.73		
		9.12	0.66	5.53		
		80.77	171.72	2.82		
5930	-1542545.11	11.82	6186959.64	4.80	151849.43	6.18
	1 22 24.24	0.20	103 59 58.83	0.40	20.96	3.29
		3.84	-88.31	12.37		
		17.32	2.89	6.37		
		72.23	169.60	2.69		
5931	-2423905.19	10.13	5388261.01	5.82	2394895.68	5.74
	22 11 56.43	0.18	114 13 14.02	0.39	156.14	3.38
		2.40	-92.88	11.33		
		20.69	-1.97	5.73		
		69.15	170.80	2.85		
5933	-4071567.51	9.82	4714260.45	9.28	-1366510.80	6.04
	-12 27 14.53	0.21	130 48 58.33	0.42	77.56	4.07
		2.06	-89.61	12.73		
		15.35	0.95	6.52		
		74.51	172.95	3.79		
5934	-5367655.52	7.03	3437875.44	11.01	-225394.72	6.12
	- 2 2 19.65	0.20	147 21 40.52	0.41	78.18	3.18
		1.96	-91.43	12.65		
		16.41	-0.85	6.38		
		-73.47	-8.06	2.69		
~~5935~~	~~-5059813.64~~	~~6.88~~	~~3591190.89~~	~~10.29~~	1472787.16	5.68
	13 26 22.92	0.18	144 38 5.50	0.40	96.09	3.04
		2.41	-91.38	12.12		
		19.35	-0.53	5.71		
		-70.49	-8.19	2.46		

Table 4-2 continued

5937	-4433454.80	8.57	4512935.88	9.14	809981.92	5.70
	7 20 41.10	0.18	134 29 27.56	0.40	136.63	2.83
		2.32	-91.17	12.29		
		18.44	-0.40	5.79		
		-71.40	-8.08	2.22		
5938	-5915090.05	5.67	2146866.80	12.28	-1037891.22	6.83
	- 9 25 40.37	0.23	160 3 6.35	0.42	74.03	3.86
		2.04	-92.83	12.86		
		11.16	-2.43	7.14		
		-78.65	-13.06	3.64		
5941	-5467730.74	6.62	-2381255.25	11.48	2254035.33	9.28
	20 49 55.01	0.30	203 32 1.11	0.43	40.14	5.05
		2.37	-84.24	12.43		
		14.39	6.37	9.28		
		-75.41	-3.39	4.61		
6003	-2127794.22	10.51	-3785677.57	9.39	4656043.83	8.45
	47 11 6.64	0.37	240 39 45.02	0.52	341.77	4.11
		0.17	-1.02	11.58		
		-1.04	88.98	10.91		
		88.95	98.34	4.11		
6004	-3851773.60	6.14	396407.34	10.14	5051368.22	7.08
	52 42 49.49	0.21	174 7 26.62	0.54	57.11	6.59
		3.41	-86.44	10.20		
		44.66	6.94	7.07		
		-45.13	0.13	6.05		
6007	4433621.08	5.08	-2268165.44	8.35	3971658.14	4.40
	38 45 36.74	0.15	332 54 23.34	0.38	85.30	2.60
		0.50	89.59	9.26		
		10.88	-0.51	4.79		
		79.11	-177.81	2.49		
6008	3623224.46	9.27	-5214237.73	6.40	601514.49	6.98
	5 26 52.72	0.23	204 47 39.59	0.35	-44.87	3.11
		1.85	92.99	10.84		
		3.42	2.88	6.98		
		-86.11	31.39	3.07		

Table 4-2 continued

6012	-5858551.70	3.88	1394512.64	11.93	2093846.49	6.81
	19 17 29.56	0.21	166 36 39.69	0.42	13.60	3.77
		2.57	-89.72	12.14		
		19.26	1.18	6.81		
		-70.55	-7.02	3.16		
6015	2604365.53	8.42	4444174.55	5.44	3750336.18	5.57
	36 14 26.03	0.19	59 37 44.17	0.37	967.81	3.15
		1.17	-92.94	9.34		
		11.73	-2.70	5.96		
		78.21	171.44	2.96		
6016	4896384.03	4.02	1316172.48	8.87	3856674.30	4.79
	37 26 39.35	0.17	15 2 44.66	0.37	16.25	2.72
		-0.71	91.87	9.13		
		8.97	1.98	5.25		
		81.00	177.35	2.62		
6042	4900761.23	7.82	3968254.18	8.34	966320.58	7.01
	8 46 12.18	0.22	38 59 52.27	0.36	1858.23	3.45
		-4.05	90.45	11.03		
		15.02	1.54	7.04		
		74.42	165.74	2.93		
6047	-3361970.26	10.15	5365818.57	7.29	763646.92	5.75
	6 55 21.26	0.18	122 4 9.58	0.40	84.55	2.94
		2.62	-90.69	12.23		
		18.27	0.18	5.84		
		71.53	171.44	2.35		
6055	6118333.69	4.75	-1571753.54	10.47	-878606.63	5.89
	- 7 58 15.37	0.19	345 35 33.67	0.35	55.45	3.95
		0.34	89.36	10.78		
		0.94	-0.65	5.93		
		89.00	-160.98	3.94		
6059	-5885320.62	6.79	-2448387.04	12.48	221669.11	10.20
	2 0 18.35	0.33	202 35 17.12	0.43	16.07	5.21
		-1.60	-93.87	13.26		
		8.09	-4.09	10.22		
		-81.75	7.27	5.04		

Table 4-2 continued

6063	5884455.23	3.77	-1853504.94	9.95	1612852.31	4.90
	14 44 42.42	0.16	342 30 59.20	0.35	20.15	2.57
		-0.77	90.46	10.38		
		13.26	0.64	4.90		
		76.72	177.21	2.37		
6067	5186389.10	7.16	-3653937.17	8.97	-654292.28	6.11
	- 5 55 39.22	0.20	324 50 3.75	0.35	0.87	3.67
		2.03	92.79	10.89		
		-3.22	2.90	6.10		
		86.19	-29.43	3.64		

5. COMPARISON WITH OTHER SOLUTIONS

Transformation parameters between SECOR-27 and NWL-9D [Anderle, 1973], SAO-III [Gaposchkin et al., 1973] and WN-14 solutions[Mueller et al., 1973] are included in Tables 5-1, 5-2 and 5-3 , respectively. The method of computing the parameters is described in [Kumar, 1972]. In the table the positive angles ω, ψ, and ϵ are counterclockwise rotations about the w, v, and u axes respectively, as viewed from the end of the positive axis. The scale difference factor is in units of ppM.

Tables 5-1 to 5-3 also contain the variance-covariance matrices, the correlation coefficients, and the residuals after transformation for the solutions mentioned above. The unit in the variance-covariance matrix for the elements corresponding to the rotations in the above tables is radian squared. The residuals tabulated are those of the Cartesian coordinates (u, v, w) in meters.

Table 5-1

Transformation NWL-9D - SECOR-27

```
SECOR27     -TO- NWL-9D
************************


   DU       DV        DW       DELTA      OMEGA      PSI     EPSILON
 METERS   METERS    METERS  (X1.0+6)   SECONDS   SECONDS   SECONDS

 17.61     0.96    -12.56      0.63       0.42      0.22      0.64


                    VARIANCE - COVARIANCE MATRIX
```

$\sigma_0^2=$ 1.86

```
 0.931D+01  0.212D-01  0.328D-01 -0.106D-06  0.395D-07  0.956D-07 -0.133D-07

 0.212D-01  0.140D+02  0.212D-01  0.335D-07  0.754D-07  0.196D-07 -0.252D-06

 0.328D-01  0.212D-01  0.105D+02 -0.116D-06 -0.424D-08 -0.107D-06 -0.412D-07

-0.106D-06  0.335D-07 -0.116D-06  0.547D-13  0.546D-17 -0.124D-15  0.536D-15

 0.395D-07  0.754D-07 -0.424D-08  0.546D-17  0.523D-13  0.214D-14 -0.460D-15

 0.956D-07  0.196D-07 -0.107D-06 -0.124D-15  0.214D-14  0.531D-13 -0.678D-14

-0.133D-07 -0.252D-06 -0.412D-07  0.536D-15 -0.460D-15 -0.678D-14  0.103D-12


                    COEFFICIENTS OF CORRELATION


 0.100D+01  0.186D-02  0.331D-02 -0.149D+00  0.567D-01  0.136D+00 -0.137D-01

 0.186D-02  0.100D+01  0.175D-02  0.383D-01  0.881D-01  0.228D-01 -0.211D+00

 0.331D-02  0.175D-02  0.100D+01 -0.153D+00 -0.571D-02 -0.143D+00 -0.396D-01

-0.149D+00  0.383D-01 -0.153D+00  0.100D+01  0.102D-03 -0.230D-02  0.715D-02

 0.567D-01  0.881D-01 -0.571D-02  0.102D-03  0.100D+01  0.406D-01 -0.628D-02

 0.136D+00  0.228D-01 -0.143D+00 -0.230D-02  0.406D-01  0.100D+01 -0.919D-01

-0.137D-01 -0.211D+00 -0.396D-01  0.715D-02 -0.628D-02 -0.919D-01  0.100D+01
```

Table 5-1 continued

RESIDUALS V

	V1(SECOR27)				V2(NWL-9D)			V1 - V2		
5410	-0.4	0.7	-1.1	700	7.7	-1.2	8.2	-8.0	1.9	-9.3
5648	6.3	1.4	9.3	708	-11.1	-3.6	-21.0	17.4	5.1	30.3
5713	1.3	5.6	-0.5	713	-18.1	-19.1	11.5	19.3	24.8	-12.0
5733	-22.6	27.7	7.1	733	1.7	-0.4	-0.3	-24.3	28.1	7.3
5736	-0.8	10.5	1.4	716	0.1	-0.2	-0.2	-0.9	10.7	1.5
5739	1.3	5.6	-0.5	739	-18.2	-19.2	11.2	19.5	24.8	-11.7
5915	17.7	0.0	13.5	709	-23.1	-0.1	-35.3	40.8	0.2	48.8
5923	-1.7	-9.8	0.8	719	6.6	13.4	-5.0	-8.3	-23.1	5.9
5924	0.8	3.4	-0.4	740	-25.6	-9.4	8.0	26.4	12.8	-8.4
5933	2.8	-2.8	2.3	727	-10.4	7.5	-25.7	13.2	-10.3	28.0
5934	-0.6	2.2	-0.1	729	4.6	-4.3	1.2	-5.2	6.5	-1.3
5935	1.8	1.2	1.1	728	-13.9	-2.6	-14.2	15.7	3.8	15.3
6003	-27.3	6.2	-7.7	738	4.0	-1.1	1.8	-31.3	7.3	-9.5
6004	-2.0	-15.5	-17.2	739	0.9	2.5	5.6	-2.9	-17.9	-22.7
6007	3.5	9.4	-1.6	727	-16.0	-15.7	9.9	19.5	25.2	-11.6
6008	13.1	2.7	9.5	815	-2.5	-1.1	-3.2	15.6	3.7	12.7
6012	-1.4	1.9	-2.3	708	10.6	-1.5	5.7	-12.0	3.4	-7.9
6015	-7.2	-14.8	0.3	817	1.6	8.1	-0.2	-8.8	-22.9	0.5
6016	6.8	-4.6	-1.0	812	-6.8	0.9	0.7	13.5	-5.5	-1.7
6055	-0.8	9.1	0.6	722	0.5	-1.4	-0.3	-1.3	10.5	0.9

Table 5-2

Transformation SAO-III - SECOR-27

```
 SECOR27     -TO- SAO-III
*****************************

  DU       DV       DW      DELTA     OMEGA      PSI     EPSILON
METERS   METERS   METERS  (X1.D+6)  SECONDS   SECONDS   SECONDS

 17.26    14.44   -13.93    -1.31      0.32      0.58      0.18


                      VARIANCE - COVARIANCE MATRIX
```

$\sigma_0^2=$ 1.09

```
 0.177D+02  0.164D+00 -0.120D+00 -0.310D-06  0.989D-07  0.222D-06 -0.367D-07

 0.164D+00  0.227D+02  0.725D-01  0.527D-07  0.314D-06  0.453D-07 -0.396D-06

-0.120D+00  0.725D-01  0.177D+02 -0.172D-06 -0.119D-07 -0.381D-06 -0.106D-06

-0.310D-06  0.527D-07 -0.172D-06  0.976D-13  0.315D-15  0.275D-15  0.367D-15

 0.989D-07  0.314D-06 -0.119D-07  0.315D-15  0.109D-12  0.701D-14 -0.136D-13

 0.222D-06  0.453D-07 -0.381D-06  0.275D-15  0.701D-14  0.126D-12 -0.142D-13

-0.367D-07 -0.396D-06 -0.106D-06  0.367D-15 -0.136D-13 -0.142D-13  0.193D-12

                      COEFFICIENTS OF CORRELATION

 0.100D+01  0.817D-02 -0.681D-02 -0.236D+00  0.712D-01  0.149D+00 -0.198D-01

 0.817D-02  0.100D+01  0.362D-02  0.354D-01  0.200D+00  0.268D-01 -0.189D+00

-0.681D-02  0.362D-02  0.100D+01 -0.131D+00 -0.857D-02 -0.256D+00 -0.571D-01

-0.236D+00  0.354D-01 -0.131D+00  0.100D+01  0.305D-02  0.248D-02  0.267D-02

 0.712D-01  0.200D+00 -0.857D-02  0.305D-02  0.100D+01  0.599D-01 -0.937D-01

 0.149D+00  0.268D-01 -0.256D+00  0.248D-02  0.599D-01  0.100D+01 -0.913D-01

-0.198D-01 -0.189D+00 -0.571D-01  0.267D-02 -0.937D-01 -0.913D-01  0.100D+01
```

Table 5-2 continued

RESIDUALS V

	V1(SECOR27)				V2(SAO-III)			V1 - V2		
6003	-18.2	3.6	5.3	6003	17.5	-4.3	-7.9	-35.7	7.9	13.2
6004	-0.4	0.3	-0.4	6004	8.2	-2.1	5.1	-8.7	2.5	-5.5
6007	2.2	3.9	-1.3	6007	-17.8	-11.8	14.3	20.0	15.7	-15.6
6008	5.3	-0.9	2.9	6008	-18.9	6.8	-17.8	24.3	-7.7	20.7
6012	0.1	-0.3	-2.6	6012	-1.4	0.9	21.5	1.4	-1.2	-24.2
6015	-2.2	-3.0	-0.9	6015	5.9	18.9	5.7	-8.2	-21.9	-6.7
6016	1.3	-3.9	-0.3	6016	-10.9	6.5	1.5	12.2	-10.4	-1.8
6042	-6.7	-5.9	5.5	6042	11.9	9.3	-12.2	-18.6	-15.2	17.8
6047	4.8	-0.9	-0.0	6047	-17.2	6.0	0.3	22.0	-6.8	-0.3
6055	-0.7	2.7	-0.9	6055	5.9	-4.5	4.9	-6.6	7.2	-5.8
6059	-4.4	9.8	5.3	6059	23.7	-15.7	-12.7	-28.0	25.6	18.0
6063	-0.6	6.2	-3.0	6063	2.0	-3.0	6.0	-2.6	9.2	-9.0
6067	10.6	2.2	1.1	6067	-12.1	-1.6	-1.7	22.7	3.8	2.8

Table 5-3

Transformation WN14 - SECOR-27

```
     SECOR27     -TO-   WN14
    *****************************


     DU        DV        DW       DELTA      OMEGA      PSI      EPSILON
   METERS    METERS    METERS   (X1.D+6)   SECONDS    SECONDS    SECONDS

    0.76     -5.68     -7.35      0.64      -0.25      0.29       0.48


                         VARIANCE - COVARIANCE MATRIX
```

σ_o^2 = 1.84

```
 0.155D+01   0.659D-02   0.884D-03 -0.569D-08   0.345D-09   0.721D-08 -0.222D-08

 0.659D-02   0.249D+01 -0.306D-02 -0.121D-08   0.287D-08   0.217D-08 -0.105D-07

 0.884D-03 -0.306D-02   0.177D+01 -0.814D-08   0.185D-09 -0.539D-08   0.279D-08

-0.569D-08 -0.121D-08 -0.814D-08   0.395D-14   0.754D-17 -0.411D-17   0.159D-17

 0.345D-09   0.287D-08   0.185D-09   0.754D-17   0.377D-14 -0.122D-15   0.113D-15

 0.721D-08   0.217D-08 -0.539D-08 -0.411D-17 -0.122D-15   0.358D-14 -0.111D-14

-0.222D-08 -0.105D-07   0.279D-08   0.159D-17   0.113D-15 -0.111D-14   0.518D-14


                         COEFFICIENTS OF CORRELATION


 0.100D+01   0.336D-02   0.534D-03 -0.728D-01   0.452D-02   0.968D-01 -0.247D-01

 0.326D-02   0.100D+01 -0.146D-02 -0.122D-01   0.297D-01   0.230D-01 -0.928D-01

 0.534D-03 -0.146D-02   0.100D+01 -0.973D-01   0.227D-02 -0.677D-01   0.291D-01

-0.728D-01 -0.122D-01 -0.973D-01   0.100D+01   0.195D-02 -0.109D-02   0.351D-03

 0.452D-02   0.297D-01   0.227D-02   0.195D-02   0.100D+01 -0.331D-01   0.256D-01

 0.968D-01   0.230D-01 -0.677D-01 -0.109D-02 -0.331D-01   0.100D+01 -0.257D+00

-0.247D-01 -0.928D-01   0.291D-01   0.351D-03   0.256D-01 -0.257D+00   0.100D+01
```

Table 5-3 continued

RESIDUALS V

	V1(SECOR27)				V2(WN14)			V1 - V2		
5001	15.7	3.6	4.2	5001	-2.4	-1.1	-1.4	18.1	4.8	5.6
5201	-34.0	14.6	-8.1	5201	1.6	-0.8	0.7	-35.6	15.4	-8.8
5410	-14.4	8.8	-4.2	5410	4.5	-0.5	1.0	-18.9	9.3	-5.2
5648	12.5	7.6	15.5	5648	-0.8	-0.5	-1.1	13.3	8.1	16.6
5712	6.6	6.8	9.9	5712	-0.3	-0.6	-1.8	6.9	7.4	11.7
5713	10.8	5.4	-7.2	5713	-1.7	-0.4	2.4	12.6	5.7	-9.6
5715	5.9	3.9	-1.6	5715	-1.1	-0.2	0.4	7.0	4.1	-2.0
5717	-1.4	-3.2	5.5	5717	0.3	0.1	-1.1	-1.8	-3.3	6.6
5720	-7.3	-6.9	11.9	5720	0.5	0.4	-2.0	-7.8	-7.3	13.9
5721	-1.9	-15.0	-3.2	5721	0.1	2.4	0.8	-2.0	-17.4	-4.0
5722	-4.9	-2.3	16.3	5722	0.4	1.2	-4.8	-5.3	-3.5	21.1
5723	3.1	-6.5	-0.5	5723	-0.2	2.7	0.2	3.3	-9.1	-0.6
5726	3.5	-1.8	0.8	5726	-0.2	0.2	-0.2	3.7	-2.0	1.0
5730	-6.9	4.1	-8.0	5730	2.1	-0.2	1.7	-9.0	4.3	-9.8
5732	0.4	18.1	9.4	5732	-0.2	-1.3	-1.9	0.5	19.5	11.3
5733	-9.2	21.3	8.9	5733	1.6	-1.2	-1.3	-10.8	22.5	10.2
5734	-12.1	0.9	-14.5	5734	2.4	-0.1	4.5	-14.5	1.0	-18.9
5735	-2.2	7.1	7.6	5735	0.2	-0.4	-1.3	-2.4	7.5	9.0
5736	-6.1	5.5	6.2	5736	1.5	-0.3	-1.4	-7.5	5.8	7.6
5739	10.8	5.3	-7.2	5739	-1.7	-0.4	2.4	12.5	5.7	-9.6
5744	5.2	-11.2	-3.6	5744	-1.1	0.7	0.9	6.3	-11.9	-4.5
5907	16.1	2.8	5.4	5907	-3.3	-0.8	-2.4	19.4	3.6	7.8
5911	15.1	3.9	3.1	5911	-1.3	-0.7	-0.8	16.4	4.7	3.9
5912	10.7	2.9	13.7	5912	-0.8	-1.9	-3.3	11.6	4.8	17.0
5914	5.5	8.9	12.7	5914	-1.4	-2.1	-1.5	6.9	11.0	14.1
5915	15.8	1.1	10.9	5915	-2.2	-0.5	-3.9	18.0	1.7	14.7
5923	1.5	-13.1	-0.6	5923	-0.2	0.9	0.1	1.7	-14.0	-0.7
5924	8.6	-6.1	-6.0	5924	-2.7	0.5	2.5	11.3	-6.6	-8.5
5925	0.3	5.4	2.2	5925	-0.1	-0.3	-0.7	0.4	5.7	2.9
5930	5.4	-0.3	5.9	5930	-0.3	0.1	-1.8	5.7	-0.4	7.8
5931	2.4	-9.6	-4.1	5931	-0.1	1.8	1.6	2.5	-11.4	-5.7
5933	6.6	3.3	4.9	5933	-0.7	-0.4	-1.9	7.3	3.7	6.8
5934	1.0	5.2	0.6	5934	-0.1	-0.3	-0.2	1.2	5.5	0.7
5935	-1.2	1.5	-3.3	5935	0.1	-0.1	0.8	-1.3	1.6	-4.1
5937	1.8	0.8	-0.5	5937	-0.1	-0.0	0.2	1.9	0.9	-0.7
5938	0.0	8.2	1.7	5938	-0.0	-0.5	-0.4	0.1	8.7	2.1
5941	-19.9	17.7	4.4	5941	2.9	-1.0	-0.7	-22.7	18.7	5.1

Table 5-3 continued

RESIDUALS V

	V1(SECOR27)				V2(WN14)			V1 - V2		
6003	-34.5	15.0	-8.5	6003	1.4	-0.7	0.6	-35.9	15.7	-9.1
6004	-11.8	1.1	-14.5	6004	2.3	-0.1	4.4	-14.2	1.2	-18.9
6007	12.2	6.1	-7.2	6007	-2.0	-0.4	2.3	14.1	6.5	-9.5
6008	6.7	7.0	10.3	6008	-0.4	-0.7	-1.8	7.0	7.7	12.1
6012	-6.5	3.9	-8.4	6012	2.0	-0.2	1.8	-8.4	4.1	-10.2
6015	-3.6	-16.0	-3.4	6015	0.2	2.6	0.8	-3.8	-18.6	-4.2
6016	5.6	-10.6	-3.8	6016	-1.1	0.6	0.8	6.7	-11.3	-4.7
6042	-7.5	-7.4	12.5	6042	0.5	0.5	-2.1	-8.0	-7.9	14.6
6047	4.1	-2.1	0.8	6047	-0.2	0.2	-0.2	4.4	-2.3	1.0
6055	-6.3	5.6	5.9	6055	1.5	-0.3	-1.4	-7.8	5.9	7.3
6059	-9.7	22.3	9.3	6059	1.6	-1.2	-1.3	-11.3	23.5	10.6
6063	5.5	4.5	-2.0	6063	-1.2	-0.2	0.5	6.7	4.7	-2.5
6067	-2.0	6.7	7.3	6067	0.2	-0.4	-1.3	-2.1	7.1	8.7

6. CONCLUSIONS

The average standard deviations of the coordinates and the heights for SECOR-27 solution (excluding stations 5648 and 5914) are:

$$\sigma_{\text{Position}} = \pm 7.5\text{m}$$
$$\sigma_{\text{Height}} = \pm 3.4\text{m}$$

The above values when compared with the corresponding values of WN14 solution [(Table 5.3-2) Mueller et al., 1973] show that a further significant improvement in the SECOR network determination is possible, if it is done as part of the world net.

The standard deviations of stations 5648 and 5914 (Table 4.2) indicate that these two stations are poorly determined compared to the other stations in the network -- a pattern which is also present in the WN14 solution [(Table 5.2-2) Mueller et al., 1973].

The semi-diameter of the level ellipsoid best fitting the geoid (defined through the SECOR 27 undulations) is 6378140.4 ± 7.7 m (1/f = 298.2495).

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