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INVESTIGATION OF THE LARGE-SCALE COHERENT STRUCTURE IN A JET AND ITS RELEVANCE TO JET NOISE

by

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INTRODUCTION

Reason for Study

Significant strides have been made during the last 15 years in the reduction of aircraft noise. The flyover OASPL at 1000 ft. is more than 20 dB lower for the new higher powered L1011 aircraft than for the early, turbojet versions of the 707 and DC-8. On the basis of acoustic efficiency (noise power/thrust power ratio) the advances in technology are even more impressive amounting to a reduction by a factor of about 1000 (1% vs. 0.001%). Comparison in terms of the noise footprint is also enlightening. The 110 dB contour extended more than 2000 ft. from a typical engine (J-57) in 1956 whereas the 110 dB contour for the new CF6-6 engine extends less than 200 ft.

Basically, there have been two important theoretical concepts which have led to these advances: first, an understanding that jet noise varies strongly with velocity and, second, an understanding of the significance of pure tones generated by rotor-stator interaction. Lighthill (1952, 1954), in a pioneering study of jet noise, pointed out that acoustic power from jets varied like $\rho U^8 d^2$ whereas thrust power varies like $\rho U^3 d^2$. This observation led to the evolution of the high bypass engine which relies on a high mass rate of flow at low velocity to achieve the required thrust.

Currently, the state of the art in fan and compressor noise technology is at the point where jet noise and rotor noise are of equal significance. Unfortunately, current suppressor technology is ineffective at the lower jet velocities that are characteristic of current high bypass ratio engines. It appears at this point that further jet noise suppression will require an even more basic understanding of the mechanism of jet noise.

Over the past 20 years, numerous investigators have attempted to determine the dominant noise producing structure of a jet by a variety of measurements of the velocity and pressure fluctuations in the jet. While contributing much to our understanding of turbulent phenomena in general, and of jets in particular, these investigations have experienced only limited success in:

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a) Deducing the dominant jet structure.
b) Accounting for the radiated noise.
c) Suggesting techniques for minimizing this noise.

The first step toward a satisfactory understanding of the radiated noise is clearly an understanding of the jet turbulence structure. At the 1971 Symposium on Aerodynamic Noise at Loughborough University, sponsored by the Royal Aeronautical Society and the British Acoustical Society "delegates were divided on the ultimate possility of reducing jet noise, but it seemed to be generally considered that understanding the underlying order in the turbulence would probably be the most fruitful path for jet noise control (Fisher and Lowson, 1971)." Recent work (Crow and Champagne, 1971, and Bishop et al 1971) indicated the existence of a large scale coherent structure in both subsonic and supersonic jets. In a subsonic jet this structure has a Strouhal frequency, fd/U_o, which corresponds to the observed peak in the far field radiated noise spectrum.

This paper describes an objective method for deducing the large eddy structure in a large jet. It is also suggested that any large, coherent structure in itself is probably a weak source of sound. Sound radiation probably occurs through the interaction of finer scale turbulence with the more coherent large scale structure.

Experimental Evaluation of Lighthill's Theory

The non-homogeneous wave equation for a flow without non-steady sources of mass or applied forces was obtained by Lighthill (1952) in the form

$$\frac{\partial^2 \rho}{\partial t^2} + a_o^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \qquad (1)$$

where, in the case of a sub-sonic, high Reynolds number, turbulent flow, T_{ij} may be approximated by

$$T_{ij} \cong \rho u_i u_j \qquad (2)$$

A retarded time solution was offered by Lighthill in the form

$$(\rho-\rho_o)(\underset{\sim}{x},t) = \frac{1}{4\pi a_o^2} \frac{\partial}{\partial x_i \partial x_j} \int \frac{[T_{ij}]}{(\underset{\sim}{x}-\underset{\sim}{y})} [y,t - \frac{\underset{\sim}{x}-\underset{\sim}{y}}{a_o}] d\underset{\sim}{y} \qquad (3)$$

A more convenient expression for experimental work is the formulation of Proudman (1952):

$$\rho-\rho_o = \frac{1}{4\pi_o^4 \underset{\sim}{x}} \int [\frac{\partial^2}{\partial t^2}(\rho u_x^2)]_{t'} d\underset{\sim}{y} \qquad (4)$$

$$t' = t - \frac{\underset{\sim}{x}-\underset{\sim}{y}}{a_o} \tag{5}$$

wherein u_x is the velocity component in the direction of an observer.

Lighthill's theory has formed the basis for much additional theoretical work. Of particular interest to this study are the past experimental attempts to relate turbulence measurements to acoustic source strength, especially the distribution of sound source intensity within a jet. A direct evaluation of the Lighthill theory was first attempted by Chu (1966). He made his measurements at a single point in the flow ($x_1/d=4$, $x_2/d=0.5$) and attempted to predict the total jet noise characteristics with theoretical estimates of sound source distributions based on the similarity principles of Ribner (1958) and Powell (1959). Unfortunately, the intensity depends on the fourth time derivative of the two-point correlation of ρu_x^2. Chu's attempts to evaluate this from an experimental curve was inherently inaccurate. A similar attempt by Jones (1969) was also incomplete and only limited success was achieved.

Recently, the experimental evaluation of the Lighthill integral has taken a new direction. Lee and Ribner (1972) and more recently Seiner and Reethof (1974) report cross correlations of the fluctuating stress with the far field noise. The method is based on the Proudman form of the Lighthill integral:

$$I = \frac{\overline{p^2}}{\rho_o a_o} = -\frac{1}{4\pi a_o^3 r} \int \frac{\partial^2}{\partial t^2} \overline{\rho v_x^2 p} \; d\underset{\sim}{y} \tag{6}$$

Unfortunately the method is sensitive to phase differences in the two types of instrumentation. Lee and Ribner chose to perform their correlation in narrow bands. The data they present is questionable because of the very low value of correlation achieved. Seiner and Reethof (1974) chose to perform their correlations in octave bands which eliminated some of the problems experienced by Lee and Ribner. They also chose to reformulate the problem in terms of shear noise and self noise which meant working with both the correlation between far field pressure and velocity squared and the pressure-velocity covariance. They found that "shear noise" was the major contributor to the total power and that, oddly enough, the radial distribution of sound source strength peaked at the jet axis, even in the potential core.

An even more direct approach has been reported by Siddon and Rackl (1971) and Rackl (1973). This method depends on the cross correlation between "pseudo sound" and far field pressure using the Ribner dilation model as a basis:

$$\frac{\overline{p^2}}{\rho_o a_o} = - \frac{1}{4\pi\rho_o a_o^3 |\underset{\sim}{x}|} \int \frac{\overline{\partial^2 p^o(\underset{\sim}{y}, t')p(t)}}{\partial t^2} d\underset{\sim}{y} \tag{7}$$

Their results agree qualitatively with Lee and Ribner but are in fundamental disagreement with Seiner and Reethof (1974). The errors inherent in pressure measurement were recognized and a unique pressure probe was developed for the purpose. However, this method is sensitive to current limitations of both high and low frequency response of pressure probes as well as the still unresolved, uncertain ties associated with pressure measurement in turbulent flows.

Evidence of a Coherent Large Scale Structure

Of particular interest is the work of Crow and Champagne (1971). Their studies combined visual and hot wire investigations of jets with Reynolds numbers ranging from 2000 to 100,000. By imposing a periodic exitation on the jet with the aid of pressure transducer located in the settling chamber, they attempted to force the jet to act as a non-linear amplifier with maximum amplification occurring in the preferred modal frequency, presumably the large eddy structure. An initial series of ripples was observed to develop into a series of vortex "puffs" which could be detected even at the higher Reynolds numbers. These disturbances rapidly reached "saturation" and the final growth rate was a maximum at a Strouhal number, fd/U_o, of 0.3.

Additional confirmation of the existence of such a large coherent structure has been offered by Lau, Fisher and Fuchs (1972). Fluctuating pressures and velocities were measured in both the entrainment region and potential core of a subsonic jet. It was shown that these fluctuations have a relatively narrow band spectral content when compared to those in the mixing region. Furthermore, a high degree of correlation exists between fluctuations in the core and entrainment regions. The authors concluded that experimental results are consistent with a series of equally spaced toroidal vortices contained in the jet mixing region and convected at a speed of 0.6 times the jet efflux velocity. Similar conclusions concerning a coherent vortex ring structure are offered in papers by Fuchs (1972 a, b), Davies, Ko and Bose (1967), Scharton and White (1972), Wooldridge, Wooten and Amaro (1972) and Beavers and Wilson (1970). Mollo-Christensen (1967) observed that pressure fluctuations outside a fully developed turbulent jet came in rather well defined wave packets as though the jet column was undergoing sporadic oscillations. Similar conclusions have been made for high speed jets where there are indications that large scale pressure fluctuations may be associated with large unstable waves (Liu 1970, Bishop, Ffowcs - Williams and Smith 1971, and Tam 1972).

There is considerable evidence that a large scale coherent structure exists in a jet and, furthermore that it can play a major role in sound radiation. Cross correlation of sound with pressure fluctuations at the center of the mixing region will not necessarily isolate this structure and indeed, the cited attempts to directly evaluate the Lighthill integral have been singularly unsuccessful in this attempt.

Perturbation of the Large Eddy Structure

The possible favorable influence of perturbing the large eddy structure has been noted by Arndt, Tran and Barefoot (1972). They found that placement of a screen across the jet flow results in a substantial reduction in noise. The reduction in noise may be traced to the breaking up of the large eddy structure in the mixing zone as evidenced by the reduction in integral velocity scales. Corresponding measurement of the smaller pressure scales (via two point pressure correlations within the flow) indicate that the scale of the pressure field is similarly perturbed. (Barefoot 1972).

AN OBJECTIVE APPROACH TO THE LARGE EDDY STRUCTURE OF TURBULENT SHEAR FLOW

Townsend (1956) proposed that turbulent shear flows were characterized by the presence of large discernable, coherent structures which were designated as "large eddies." The large eddies were thought to contain no more than 20% of the turbulent energy and to exist in equilibrium with the mean flow and the smaller turbulent eddies. They have scales on the order of the scale of the mean flow and are responsible for the intermittency and spread of the turbulent-irrotational interface. By using simple arguments and measured correlations, Townsend proposed structures which semi-quantitatively accounted for the measurements available. These results were later modified and extended by Grant (1958) for turbulent wakes and by Bradshaw, et al. (1964) for the axisymmetric jet on the basis of extended measurements.

One of the major problems in determining the large eddy structure was the lack of an objective definition. The approach initiated by Townsend was the most objective in that it attempted to guess the structure which best explained the measured correlations. The other attempts to determine the large eddy structures by forcing, conditional sampling, and flow visualization while providing useful information appears to lack even this objectivity.

In 1965, Lumley proposed an objective definition of the large eddy structure (and in fact the smaller eddies as well!) based on a rational orthogonal decomposition of the turbulent velocity field. Lumley and his colleagues (Payne and Lumley 1967, Bakewell and Lumley 1967) succeeded in deducing the large eddy structure in the 2-dim. turbulent wake and in the viscous sublayer on the basis of very limited data. In spite of this success and the appeal of an objective approach to the large eddy definition, the orthogonal decomposition has been largely ignored.

In the remainder of this paper, Lumley's decomposition will be reviewed, an experimental program to apply Lumley's decomposition to the axisymmetric mixing layer of a jet will be outlined, and finally, a sketch will be given of how the radiated noise can be deduced from the characteristic eddies and their interaction.

The Orthogonal Decomposition

Lumley (1967) proposes the following approach to the large eddy identification. Suppose that we think that a candidate eddy, $\phi_i(x, t)$ is occurring in an identifiable manner in a given ensemble of random velocity fields,

$u_i(\underset{\sim}{x}, t)$. One way to test this hypothesis is to project ϕ_i on u_i and determine the extent to which they are parallel. Since we are interested only in parallelism, we remove the amplitude dependence by considering the following:

$$\alpha = \frac{\int \phi_i^*(\cdot)u_i(\cdot)d(\cdot)}{[\int \phi_j^*(\cdot)\phi_j(\cdot)d(\cdot)]^{1/2}} \tag{8}$$

where $(\cdot)$ indicates the dependence on a point in whatever space is relevant, e.g., (x_1, x_2, x_3, t), and * denotes the complex conjugate. The integral is taken over all space.

Lumley suggests that the best choice for $\phi_i(\cdot)$ is that which maximizes the quantity $\overline{|\alpha|^2}$ where the overbar denotes the ensemble average. By the methods of the calculus of variations, it can be shown that the best ϕ_i is given by the solution to the following equation:

$$\int R_{ij}(\cdot, \cdot')\phi_j(\cdot')d(\cdot') = \overline{|\alpha|^2}\ \phi_i(\cdot) \tag{9}$$

where

$$R_{ij}(\cdot,\cdot') = \overline{u_i(\cdot)u_j(\cdot')} \tag{10}$$

Finite extent velocity field. If the velocity field is of finite extent and is of finite total energy, the following hold:

A. There are not one, but a discrete set of solutions of (2) which can be written as

$$\int R_{ij}(\cdot, \cdot')\phi_j^{(n)}(\cdot')d(\cdot') = \lambda^{(n)}\phi_i^{(n)}(\cdot),\ n = 1, 2, \ldots \tag{11}$$

B. We can choose the $\phi_i^{(n)}$ so that they are orthonormal

$$\int \phi_i^{(p)}(\cdot)\phi_i^{(q)}(\cdot)d(\cdot) = \delta_{pq} \tag{12}$$

C. The velocity field u_i can be expanded in the $\phi_i^{(n)}$

$$u_i(\cdot) = \Sigma a_n\phi_i^{(n)}(\cdot) \tag{13}$$

where

$$a_n = \int u_i(\cdot)\ \phi_i^{(n)}(\cdot)d(\cdot) \tag{14}$$

and the series converges in mean square.

D. The coefficients of (13) are uncorrelated

$$\overline{a_n a_m} = \begin{cases} 0 & n \neq m \\ \lambda^{(n)} & n = m \end{cases} \tag{15}$$

E. The two point Reynolds stress tensor can be decomposed into a double series in the $\phi_i^{(n)}$

$$R_{ij}(\cdot,\cdot') = \sum_{n=1}^{\infty} \lambda^{(n)} \phi_i^{(n)}(\cdot)\phi^{(n)*}(\cdot') \tag{16}$$

and the series is uniformly and absolutely convergent.

F. The $\lambda^{(n)}$ are real and non-negative and their sum is finite and equal to the total energy in the flow.

The expansion of (13) is optimal in the sense that as little as possible is left to the remainder of each partial sum.

Lumley (1970) has shown that the number of terms necessary to adequately represent the energy variation of $u_i(\cdot)$ is

$$N \leq L/\mathit{L} \tag{17}$$

where L is a measure of the lateral extent of the velocity field and L is an estimate of the extent over which the process is correlated with itself. In a jet mixing layer $N \sim 4$, so only the first four eigenfunctions are necessary to obtain most of the turbulent energy.

It has been suggested by Lumley that the lowest order eigenfunction $\phi_i^{(1)}(\underset{\sim}{x},t)$ be identified as the large eddy. The results of Payne and Lumley (1967) and Bakewell and Lumley (1967) indicate that the large eddy derived from this approach is in excellent agreement with that predicted by Townsend on intuitive grounds although considerably more detail is present. Moreover, the decomposition can be applied to the averaged velocity equations to obtain dynamical equations for the large eddy (Lumley 1967, 1971, Payne 1968).

<u>Homogeneous Fields</u>. From equation (17) it is clear that as the extent of the field L becomes large compared to the correlation distance L, more and

more terms are necessary to represent the velocity field. For homogeneous fields the number of terms becomes infinite and the eigenvalues are not discrete but continuous. The appropriate decomposition for a homogeneous field is the familiar Fourier decomposition. If u_i is homogeneous in $\underset{\sim}{x}$ and stationary in time we can write

$$u_i(\underset{\sim}{x},t) = \int_{-\infty}^{\infty} e^{i(\underset{\sim}{k},\underset{\sim}{x} + \omega t)} \hat{u}_i(\underset{\sim}{k},\omega) d\underset{\sim}{k} d\omega \tag{18}$$

where $\hat{u}_i(\underset{\sim}{k},\omega)$ is the Fourier decomposition of $u_i(\underset{\sim}{x},t)$ and the $\hat{u}_i$ are uncorrelated at different wave numbers and frequencies.

$$\overline{\hat{u}_i(\underset{\sim}{k},\omega)\hat{u}_j(\underset{\sim}{k}',\omega')} = \Phi_{ij}(k,\omega)\delta(\omega-\omega') \tag{19}$$

where $\Phi_{ij}(\underset{\sim}{k},\omega)$ is the velocity spectrum tensor. (The $\hat{u}_i$ are random functions and (18) is written in the sense of generalized functions [c.f. Lumley 1970]. This representation is equivalent to the Fourier-Stieltjes decomposition $dZ_i(\underset{\sim}{k},\omega)$ used by Batchelor (1953) and others.)

From (18) it is clear that in a homogeneous flow the large eddy structure will not be readily identifiable from the fluctuating Fourier components unless $\hat{u}_i(\underset{\sim}{k},\omega)$ peaks strongly at a particular wave number frequency combination $(\underset{\sim}{k}^\circ,\omega^\circ)$. We will come back to this problem later under partially homogeneous flows.

Periodic Velocity Fields. If the velocity field possesses periodicities

$$u_i(\cdot,\theta) = u_i(\cdot, \theta + 2\pi) \tag{20}$$

it can be expanded into a Fourier series or other appropriate harmonic functions. This has been recognized by Michalke (1972) who expanded the pressure field of an axisymmetric jet in a Fourier series.

For the turbulent velocity field we have

$$u_i(\cdot,\theta) = \sum_{n=-\infty}^{\infty} f_i^{(n)}(\cdot)\, e^{in\theta} \tag{21}$$

where the $f_i^{(n)}(\cdot)$ are uncorrelated; that is,

$$\overline{f_i^{(m)}(\cdot)f_i^{(n)*}(\cdot')} = \begin{cases} 0; & n \neq m \\ B^{(n)}(\cdot,\cdot'); & n = m \end{cases} \tag{22}$$

The two point Reynolds stress tensor can be similarly expanded

$$\overline{u_i(\cdot,\theta)u_j(\cdot',\theta')} = \sum_{n=-\infty}^{\infty} B^{(n)}(\cdot,\cdot')e^{in(\theta-\theta')} \tag{23}$$

Fuchs (1972) has applied Michalke's decomposition to measurements of the turbulent pressure field in the jet mixing layer. Because the pressure field is strongly correlated around the jet the Fourier series representation was a natural method. The turbulent velocity correlations, however, fall off rapidly in the azimuthal direction (Bradshaw, et al. 1964). Therefore, it is not immediately obvious that the representation of equation (21) is preferable to that of equation (18) when applied to the azimuthal variable of the velocity field.

Partially Homogeneous and Stationary Fields. This special case is most often encountered in practice and is particularly relevant to the turbulent jet. Laboratory jet flows are stationary in time, homogeneous (and periodic) in the azimuthal direction, strongly inhomogeneous and of limited extent in the radial direction, and by comparison with the radial direction nearly homogeneous in the axial direction. The last statement is even more valid if the turbulence properties are normalized by the local turbulence scales. We consider here a velocity field which is homogeneous in x and z, stationary in time, and of bounded extent in y.

We first decompose the velocity field in the homogeneous directions and time according to

$$u_i(\underset{\sim}{x},t) = \int_{-\infty}^{\infty} e^{i(kx + k_3z + \omega t)} \hat{u}_i(k_1,k_3,\omega;\ y)dk_1dk_3d\omega \tag{24}$$

where the $\hat{u}_i$ are uncorrelated for different frequencies and wave numbers.

$$\overline{\hat{u}_i(k_1,k_3,\omega;\ y)\ \hat{u}_i(k_1',k_3',\omega';\ y')}$$
$$= \Phi_{ij}(k_1,k_3,\omega;\ y,y')\delta(\omega-\omega',k_1-k_1',k_3-k_3') \tag{25}$$

where Φ_{ij} is the Fourier transform of the two point Reynolds stress tensor.

We now seek an "eddy" $\phi_i(k_1,k_3,\omega;\ y)$ which maximally projects on the field $\hat{u}_i(k_1,k_3,\omega;\ y)$. The result is the following:

$$\int_{-V}^{V} \Phi_{ij}(k_1,k_3,\omega;\ y,y')\ \phi_j^{(n)}(k_1,k_3,\omega;\ y')dy'$$
$$= \lambda^{(n)}(k_1,k_3,\omega)\phi_i^{(n)}(k_1,k_3,\omega;\ y) \tag{26}$$

V denotes the finite extent of the flow in the y-direction and is large enough to contain the range of variation of Φ_{ij} - in this case the mixing layer.

As before we identify $\phi^{(1)}(k_1,k_3,\omega;\ y)$ with the large eddy. The velocity field of this eddy is given by

$$u_i^{(1)}(\underset{\sim}{x},t) = \int_{-\infty}^{\infty} e^{i(k_1x + k_3z + \omega t)} a_1(k_1,k_3,\omega)\phi^{(1)}(k_1,k_3,\ )\omega)dk_1dk_3d\omega \tag{27}$$

where the a_1 is the random coefficient. Obviously the effect of the homogeneities has been to smear out the structure of the large eddy by the introduction of a fluctuating structure in both frequency and wave number. The large eddy $u_i^{(1)}(\underset{\sim}{x},t)$ can be visualized as a group of dispersive waves propagating in the homogeneous directions whose amplitude vary in the inhomogeneous and bounded directions. This interpretation is similar to the waveguide model of Landahl (1967) and is seen to be a special case of the orthogonal decomposition.

Lumley (1967) has observed that if $\phi_i^{(1)}$ has its energy concentrated about a single spectral value, say $(k_1°,k_3°,\omega)$, then that component can be chosen to be representative of the large eddy; that is $\phi_i^{(1)}(k_1°,k_3°,\omega°,y)$. In effect, this type of assumption has been made in those experiments which attempt to force a single component of the turbulence and seek the maximal growth rate identifying it as the large eddy, e.g., Crow and Champagne (1971), Hussain and Reynolds (1971). Other heuristis models for the jet turbulence structure have also implicitly assumed that the large eddy is concentrated about a particular wave number-frequency combination; for example, the roller eddies of Davies (1972) and the vortex pairs of Laufer (1973).

The observations reported by Crow & Champagne (1971) of a puff-like structure in the periodically forced jet is a simple statement that they were able to concentrate the large eddy energy to a narrow band to the point where it was recognizable. The observations reported by Laufer (1973) of a vortex-like structure in the plane mixing layer which resembled the last stages of a Kelvin-Helmholtz instability must be interpreted in light of the low Reynolds number. It is reasonable to expect the spectral content of the large eddy to broaden as the Reynolds number is increased because of the increased nonlinear interactions. This broadening with Reynolds number will proportionately reduce discernability of the large eddy.

A final comment is in order regarding the reputed axisymmetric structure in jets and the two-dimensional structure in the plane mixing layer. It was first observed by Townsend that for an eddy to extract energy from the mean flow its plane of circulation must be tilted against the mean flow. The structures proposed to date (Bradshaw et al., 1964 excepted) are defective in this regard. The structure deduced by Payne and Lumley (1967) by applying the orthogonal decomposition to Grant's (1958) measurements in a two-dimensional wake is shown in Figure (1). It is interesting to note that the eddy pairs are at nearly 90° to that which would have been expected from the symmetry alone (aligned parallel to the cylinder). Clearly some caution must be exercised in attacking significance to geometrically appealing models.

An alternate approach to partially homogeneous flows. For partially homogeneous flows, Lumley (1967, 1970) has suggested further decomposing each of the "eddies" obtained above into characteristic eddies occurring at uncorrelated locations in time and at uncorrelated locations in the homogeneous directions. This approach is similar to the shot effect representation of Rice (1948). We write for the large eddy (or any eddy)

$$u_i^{(1)} = \iiint g_i^{(1)}(x-\xi,\ y,\ z-\zeta,\ t-\tau)\,d\sigma(\xi,\zeta,\tau) \tag{28}$$

where

$$\overline{d\sigma(\xi,\zeta,\tau)d\sigma(\xi',\zeta',\tau')} = \begin{cases} 0, & (\cdot) \neq (\cdot') \\ d\xi d\zeta d\tau, & (\cdot) = (\cdot') \end{cases} \tag{29}$$

and where the $g_i^{(1)}$ is the characteristic eddy (or group) and is deterministic. It is easy to show that

$$\overline{u_i^{(1)}(\cdot)u_j^{(1)}(\cdot')} = \int g_i^{(1)}(x,y,z,t)g_j^{(1)}(x+\xi,y',z+\zeta,t+\tau)\,d\xi d\zeta d\tau$$

where $(\cdot') = (x+\xi,\ y',\ z+\zeta,\ t+\tau)$. (30)

Defining $G_i^{(1)}(k_1,\ k_3,\ \omega,\ y)$ to be the Fourier transform of $g_i^{(1)}(\underset{\sim}{x},\ t)$ we can show that

$$\phi_i^{(1)}(k_1,\ k_3,\ \omega,\ y)\phi_j^{(1)*}(k_1,\ k_3,\ \omega,\ y')\lambda^{(1)}(k_1,\ k_3,\ \omega) = (2\pi)^3 G_i^{(1)}(k_1,\ k_3,\ \omega,\ y)G_j^{(1)*}(k_1,k_3,\omega,y') \tag{30}$$

Thus the large eddy can be identified as the inverse transform of

$$G_i^{(1)}(k_1,\ k_3,\ \omega,\ y) = \sqrt{\frac{\lambda^{(1)}(k_1,\ k_3,\ \omega)}{(2\pi)^3}}\ \phi_i^{(1)}(k_1,\ k_3,\ \omega,\ y) \tag{31}$$

or

$$g_i^{(1)}(x,y,z,t) = \int e^{ik_1x + k_3z + \omega t}\sqrt{\frac{\lambda^{(1)}(k_1,k_3,\omega)}{(2\pi)^3}}\,\phi_i^{(1)}(k_1,k_3,\omega,y)\,dk_1dk_3d\omega \tag{32}$$

Clearly if the peak of $\lambda^{(1)}$ is distinct at some value $(k_1°,\ k_3°,\ \omega°)$, the value selected at the peak is representative and the eddy (or group) is easily discernable.

EXPERIMENTAL DETERMINATION OF THE LARGE EDDIES

In the previous section we have shown that the random velocity field of a jet mixing layer can be broken into a series of characteristic eddies, the lowest order one being the large eddy. These eddies can be obtained directly from the measured velocity covariance tensor $\overline{u_i(\underset{\sim}{x},t)u_j(\underset{\sim}{x}',t)}$.

The procedure for calculating the characteristic eddies is summarized below:

1. Measure the velocity covariance tensor

   $R_{ij}(x-x', y,y', z-z', t-t')$

2. Transform to get the velocity spectrum tensor

   $\Phi_{ij}(k,m,\omega;y,y')$

3. Calculate the eigenfunctions $\phi_i^{(n)}(k,m,\omega)$ and eigenvalues $\lambda^{(n)}(k,m,\omega)$.

4. Using this information in equations, calculate the characteristic eddies.

The techniques for carrying out the computations described above are straightforward and have been described in detail by Lumley (1970). The most difficult problem is of course to obtain sufficient information on the velocity covariance tensor R_{ij}. or its transform Φ_{ij}.

We can reduce the number of measurements required by recalling that the nine components of R_{ij} are related to each other by the mass conservation equations as follows:

$$\frac{\partial}{\partial x_i{}'} R_{ij}(\underset{\sim}{x},\underset{\sim}{x}') = 0 = \frac{\partial}{\partial x_j{}'} R_{ij}(\underset{\sim}{x},\underset{\sim}{x}') \tag{31}$$

Fourier transforming in the homogeneous variables x and z and in the stationary variable t we have

$$\begin{aligned} ik_i \Phi_{ij} + \frac{\partial}{\partial y} \Phi_{2j} &= 0 \\ ik_i \Phi_{ij} + \frac{\partial}{\partial y'} \Phi_{i2} &= 0 \end{aligned} \qquad i = 1,3 \tag{32}$$

Hence we have six equations in nine unknowns. If we use two x-wire probes and measure u_1 and u_3 at each location, it is straightforward to obtain Φ_{11}, Φ_{33}, Φ_{13}, and Φ_{31}. The missing terms may be readily obtained by integration from five of the six mass conservation equations.

The properties of the covariance function can also be used to reduce the number of measurements. Since the velocity covariance depends on progressively larger eddies as the spacial separation between the probes is increased, there appears to be little merit in making equally spaced measurements. Furthermore, since the covariance falls off in a nearly exponential fashion (Tennekes and Lumley 1971), it is clear that the optimum spacing is logarithmic. This has been previously used by Bakewell and Lumley (1967) in the viscous sublayer.

For a 12 inch jet at 30 m/s, the smallest spacing used will be 5 mm (corresponding to the peak in the dissipation spectrum). In spite of the loss of small scale structure, this will cause no problem because we are interested only in the large scales. There appears, at this point at least, no reason to measure correlations at separations greater than five integral scales. In the streamwise or x-direction, 5L - 0.3 m., and in the azimuthal, z-, and cross-stream or y-directions 5L - 0.15 m. The separations to be measured are then given by the following:

x-direction (mm.)

0, 5, 10, 20, 40, 80, 160, 320

y, z-directions (mm.)

0, 5, 10, 20, 40, 80, 160

This corresponds to a measurement grid of 8 x 7 x 7 on 392 probe separations for each choice of origin, y', in the inhomogeneous direction.

To complete the specification of the velocity covariance, this grid must be remeasured for different choices of the origin in the inhomogeneous direction. By choosing the same logarithmically spaced values for y as above, we see that the origin must be moved 13 times. The net result is a total of 13 x 392 = 5096 probe spacings. Since four channels of velocity signals are being recorded, the total number of correlations to be measured is 20,384 - a formidable task, but necessary if the large eddy structure is to be found unambiguously. An automatic stepping technique can be used for probe movement.

It will be recalled that the velocity covariance spectrum tensor Φ_{ij} $(k,m,\omega; y,y')$ is sought. Therefore it remains to Fourier transform the measured covariances in the x and z directions. This will be accomplished by first curve fitting the measured covariances with a modified Gram-Charlier expansion and then Fourier transforming analytically to obtain the spectrum. Such a procedure was used with success by Payne and Lumley (1967).

Once $\Phi_{ij}(k,m,\omega; y,y')$ is obtained for i,j = 1.3 by the procedures described above, it is straightforward to integrate the mass conservation equations to obtain the five missing terms i,j = 3.

CALCULATION OF RADIATED NOISE
FROM THE ORTHOGONAL DECOMPOSITION OF THE VELOCITY FIELD

The pressure correlation for the far field can be written using the retarded time approximation to the solution of Lighthill's equation as

$$\overline{p(\underset{\sim}{x},t)p(\underset{\sim}{x},t\tau t)} \propto \frac{\partial^4}{\partial\tau^4}\iint \overline{T_{ij}(\underset{\sim}{y},t - \frac{x-y}{a_o})T_{k\ell}(\underset{\sim}{y}',t+t - \frac{x-y'}{a_o})}d\underset{\sim}{y}d\underset{\sim}{y}' \tag{33}$$

where we have for brevity left out the dependence on source distance and the directionality factor. The integrand can be abbreviated as

$$\overline{T_{ij}T_{k\ell}'} = \overline{u_i u_j u_k' u_\ell'} \tag{34}$$

where the prime denotes evaluation at y' and $t + \tau$.

Following Batchelor (1953), Kraichman (1956), Lilley (1958) and others, we assume that the 4th order moments are jointly normal and write

$$\overline{u_i u_j u_k' u_\ell'} = \overline{u_i u_j}\ \overline{u_k' u_\ell'} + \overline{u_i u_k'}\ \overline{u_j u_\ell'} + \overline{u_i u_\ell'}\ \overline{u_j u_k'} \tag{35}$$

(This will not be a valid approximation for the eddies corresponding to the scales of motion much smaller than those containing the turbulent energy.) The first term $\overline{u_i u_j}\ \overline{u_k' u_\ell'}$ is not interesting since it is independent of τ and will not contrubute to the noise. The remaining two terms are functions of τ and can contribute.

For simplicity in understanding we will restrict this discussion to completely inhomogeneous and bounded flows. The stationarity in time will, of course, introduce a fluctuating Fourier component in frequency. We define $\Phi_{ik}(\omega)$ as the cross spectral density

$$\Phi_{ik}(\omega) = F.T.[\overline{u_i u_k'}]$$

Transforming (33) with respect to τ we obtain the noise spectrum

$$S_{pp}(\underset{\sim}{x}, \omega) \propto \omega^4 \iint [\Phi_{ik} * \Phi_{j\ell} + \Phi_{i\ell} * \Phi_{jk}]d\underset{\sim}{y}d\underset{\sim}{y}' \tag{36}$$

where * denotes a convolution in frequency.

We have seen that Φ_{ik} can be decomposed into the characteristic eddies as follows

$$\Phi_{ik}(\omega;\ \underset{\sim}{y},\underset{\sim}{y}') = \sum_{n=1}^{\infty} \lambda^{(n)}(\omega)\phi_i(\underset{\sim}{y})\phi_k^*(\underset{\sim}{y}') \tag{37}$$

Using the fact that the $\lambda^{(n)}(\omega)$ are real and symmetric, substitution of (37) into (36) yields

$$S_{pp}(\underset{\sim}{x},\omega) \propto \omega^4 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \lambda^{(n)}(\omega_1)\lambda^{(m)}(\omega-\omega_1)\, d\omega_1 \tag{38}$$

$$\cdot \int \phi_i^{(n)}(\underset{\sim}{y})\phi_j^{*(m)}(\underset{\sim}{y})d\underset{\sim}{y} \cdot \int \phi_k^{*(n)}(\underset{\sim}{y}')\phi_\ell^{(m)}(\underset{\sim}{y}')d\underset{\sim}{y}'$$

+ similar term

The diagonal terms of (38) n=m clearly represent the noise generated by a particular eddy. If we take N terms to approximate the total flow energy there are N terms of what will call type 1 sources. The off-diagonal term of (38) n≠m represents the noise generated by the interaction of eddies of various sizes. Again if we take N terms in the decomposition there are N^2-N eddy interaction terms which we will call type 2 sources. The strength of these is dependent on the integrated "Reynolds stress" associated with the interacting eddies and the convolved integrated spectra of these eddies.

It is appropriate to recall at this point that Kraichman (1956), Lilley (1958) and others have treated the problem of noise generated by a homogeneous turbulence acted upon by a uniform mean shear. The result was a turbulence-interaction source and a turbulence-shear interaction source. From the above it is clear that in a bounded, inhomogeneous turbulent flow we have the same type of turbulent-shear interaction source terms as the eddies of various sizes interact with one another. This is not surprising since a large eddy appears as a quasi-uniform strain field to a smaller eddy imbedded in it. Clearly the analysis above could be extended to include the effects of a mean shear interacting with each eddy.

It has been speculated for several years by the present authors and independently by H. Tennekes (P.S.U. Acoustic Seminar, 1974) that the large eddies themselves cannot be primary sources of noise because of their long evolution times - at least at low Mach number. Rather, the short life times and high vorticities and strain rates of the dissipative scales make them ideal radiations if they were coherent over large enough distances. The type 2 mechanisms derived above provide a means by which the large eddies can interact with the smaller scales and perhaps provide the necessary coherence lengths.

These arguments are consistent with these of Townsend (1955) and Tennekes and Lumley (1972) regarding the energetics of the large eddies which are believed to obtain energy directly from the mean flow and then cascade it by means of vortex stretching to the smaller scales. It has been speculated that the larger eddies line up or orient the smaller eddies so that their principal axes of strain rate coincide. The small eddies then destroy the larger ones. It is this lining up and destructive process that will contribute to the type 2 interaction terms above.

The conclusions above should be regarded as tentative and confirmation awaits experimental evidence and further study. The important point to this discussion is the possibility that there may be a direct link between the coherence of the large eddy structure and the potentially noisy smaller scales.

Summary and Conclusions

A review of current suppressor technology indicates that further advances in jet noise reduction hinges on a more fundamental understanding of the noise generation mechanism. There has been considerable conjecture about the existence of a coherent structure in jet turbulence. Should such a structure exist, the opportunities for further jet noise reduction may be considerably enhanced.

It is suggested that Lumley's objective approach to large eddy definition may prove to be a fruitful way to deduce the jet structure in sufficient detail to allow calculation of the noise field. An experimental method for direct evaluation of these eddies has been proposed.

Several general observations on the mechanism of jet noise can be made by direct substitution of the characteristic eddy formulation into Lighthill's equation. Basically two types of noise terms are found which parallel the concept of shear noise and self noise suggested by Kraichman, Lilley, Ribner and others. Type 1 consists of the self interaction of the various eddies. Type 2 consists of interaction between eddies of various scales. Because of the preponderance of type 2 interactions and because of the possibility of obtaining both large coherence lengths and short lifetimes it is speculated that these may be the dominant source of jet noise. Thus, the existence of coherent structure in itself is not a priori evidence of a strong emitler of sound but rather such large structure may act as the catalyst for sound radiation from smaller scale, more vortical motions which feed off of the larger eddies.

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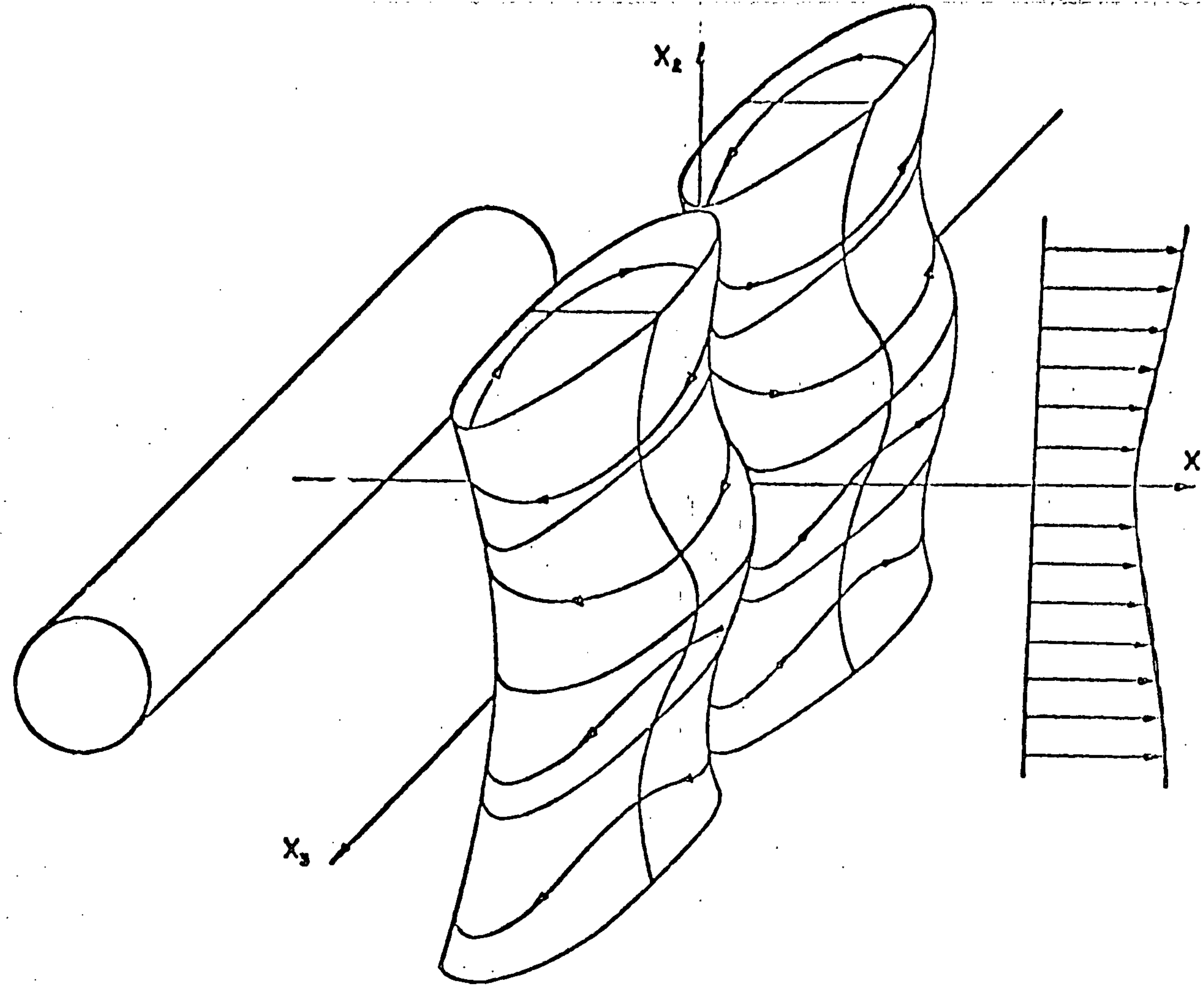

Artist's Conception of the Large Eddy Structure of a Cylinder Wake as Deduced from an Orthogonal Decomposition of the Velocity Field (Payne & Lumley 1967)