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SYSTEM DESIGN OPTIMIZATION FOR A MARS-ROVING VEHICLE AND PERTURBED-OPTIMAL SOLUTIONS IN NONLINEAR PROGRAMMING

by

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Analysis and Design of a Capsule Landing System
and Surface Vehicle Control System for Mars Exploration

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LIST OF IMPORTANT SYMBOLS

Abbreviations

ABL	automated biological laboratory
GC-MS	gas chromatograph - mass spectrometer
JPL	Jet Propulsion Laboratory
MRV	Mars-roving vehicle
NASA	National Aeronautics and Space Administration
NLP	the nonlinear programming problem
POS	the perturbed-optimal solutions problem
RTG	radioisotope thermoelectric generator

Mars-roving vehicle (values of constants appear in parentheses)

A	equipment package surface area, m^2
a	radiometric albedo of Mars, (0.295)
$A_{alb(r)}$	radiator surface area illuminated by albedo radiation, m^2
$A_{alb(s)}$	equipment package area illuminated by albedo ratiation, m^2
A_r	total radiator surface area, m^2
A_T	surface area of the top and two sides of equipment package, m^2
A_{TR}	A_T plus area of one radiator
$A_{sun(r)}$	radiator surface area illuminated by direct solar radiation, m^2
$A_{sun(s)}$	equipment package area illuminated by direct solar radiation, m^2
a_1	ratio of radiator length to height, (3.33)

a_2	ratio of length of open side of equipment package to its height, (4.0)
a_{12}	$a_1 + a_2 + a_1 a_2$, (20.65)
D_{com}	communications antenna diameter, m
E_{batt}	battery energy, watt-hrs
E_f	depth of battery discharge divided by efficiency of recharge, (0.57)
E_{fd}	depth of battery discharge, (0.40)
e_t	efficiency of thermal transfer, (0.8)
g_m	Mars gravitational constant, (3.75 m/sec^2)
h_c	Mars forced convection transfer coefficient, (1.30 watts/K-m^2)
k_i	heat transfer coefficient of insulation, (0.0216 watts/K-m)
$K_{q1,2}$	cooling capacity, watts/K
$L_{i1,2}$	insulation thickness, m
L_w	maximum launch weight, (570 kg)
M_r	mass of total vehicle system, kg
P_{com}	power required by communications subsystem, watts
P_{cp}	power required by computation and data-handling subsystem, (90 watts)
PLR	path-length ratio
P_{mv}	power required by onboard subsystems during roving periods, watts
P_{nav}	power required by navigation subsystem, (6 watts)
P_{oa}	power required by obstacle avoidance subsystem, (15 watts)
P_{prop}	power required to propel vehicle, watts
P_{RTG}	constant power output of RTG's, watts

P_{scia}	average power required by science package, (26 watts)
P_{str}	power required by onboard subsystems during recharge mode, watts
P_{θ}	power required by thermal control subsystem, watts
$Q_{hl,2}$	heater power required at night, watts
$Q_{idl,2}$	internal heat dissipation during the day, watts
$Q_{inl,2}$	internal heat dissipation at night, watts
r_a	horizontal distance from vehicle to terrain being sensed, (30 m)
R_{com}	communications data rate, bits/sec
S_c	Mars Solar constant, (720 watts/m^2)
s^*	maximum slope for allowable vehicle traverse, degrees
S_{sci}	science stops per meter of travel
T	percentage of terrain vehicle will consider impassable
t	vehicle track, m
$T_{bdl,2}$	Temperature of equipment packages external skin during day, K
$T_{bnl,2}$	Temperature of equipment packages external skin during night, K
T_{esci}	time required for experimental science during one stop, sec
T_{hi}	worst case (high) Mars ambient temperature, (275 K)
T_{intd}	required internal temperature during day, (305 K)
T_{intn}	required internal temperature during night (295 K)
T_{low}	worst case (low) Mars ambient temperature (175 K)
T_r	time required to recharge batteries, hr
$T_{rdl,2}$	radiator surface temperature during day, K

$T_{rn1,2}$	radiator surface temperature during night, K
T_{rov}	maximum roving time between recharges, hr
T_{sci}	time required to obtain and transmit science data per stop, sec
V	time during Mars day when earth is visible from rover, (10 hrs)
v_f	roving velocity m/sec
w_b	wheelbase, front-to-back distance between wheels, m
W_{com}	earth weight of communications subsystem, kg
W_{cp}	earth weight of computation and data handling subsystem, (46.8 kg)
W_{nav}	earth weight of navigation subsystem, (15 kg)
W_{oa}	earth weight of obstacle avoidance subsystem, (5 kg)
W_p	earth weight of power subsystem, kg
W_{sci}	earth weight of science package, kg
W_v	earth weight of vehicle structure, kg
W_θ	earth weight of thermal control subsystem, kg
α_r	absorptivity of radiator to solar radiation, (0.5)
α_{ri}	absorptivity of radiator to infrared radiation, (0.8)
α_s	absorptivity of equipment package to solar radiation, (0.5)
α_{si}	absorptivity of equipment package to infrared radiation, (0.8)
$\Delta\beta$	error in detection of local vertical by navigation subsystem, (0.25°)
ϵ_r	emissivity of radiator, (0.8)
ϵ_s	emissivity of equipment package skin, (0.8)
μ_k	coefficient of kinetic friction of vehicle, (0.10)

σ	Stephan-Boltzman constant, (5.67×10^{-8} watts/m^2 - K^4)

Perturbed-optimal solutions problem

f	objective function in a nonlinear programming problem
g_i	inequality constraint in a nonlinear programming problem
h_j	equality constraint in a nonlinear programming problem
$\mathcal{L}$	the Lagrangian function in a nonlinear programming problem
u	vector of Lagrange multipliers of inequalities
w	vector of Lagrange multipliers of equalities
ν	Lagrange multiplier of the perturbation in the perturbed-optimal solutions problem
η	vector of sensitivity coefficients in the perturbed-optimal solutions problem describing the perturbation trajectory

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ABSTRACT

This thesis presents work in two somewhat distinct areas. First, the optimal system design problem for a Mars-roving vehicle is attacked by creating static system models and a system evaluation function and optimizing via nonlinear programming techniques. The second area concerns what will be termed the problem of "perturbed-optimal solutions." Given an initial perturbation in an element of the solution to a nonlinear programming problem, a linear method is determined to approximate the optimal readjustments of the other elements of the solution. Then, the sensitivity of the Mars rover designs is described by application of this method.

SECTION 1

INTRODUCTION

Unmanned planetary exploration to date has consisted in the large of Surveyor, Voyager, Viking, and Mariner missions, which while considerable technological achievements, were relatively simply fly-bys, orbiters, and stationary landers concerned with obtaining information about Mars, Venus, and Earth's moon. Apollo manned lunar missions demonstrated the gains to be derived from having increased flexibility of operations on the lunar surface (witness the tremendous increase in man's knowledge of the moon over that obtained from Surveyor data).

The desire for increased flexibility on unmanned missions along with the problems inherent in exploring further and further from Earth means that future planetary unmanned missions will be characterized by increased complexity, longer lifetime, and more dependency on autonomy [1].

One area of approach for on-surface exploration that, among other factors, promises to have applications far beyond the point of solving the present problems is the development of unmanned semi-autonomous roving vehicles for planetary exploration. The advantages of a rover over a single stationary lander are numerous. The roving capability will allow data to be gathered over a considerably greater area, geographical mapping becomes possible, and additional options as to scientific experimentation become available (there are experiments which give little information if performed at only one location, but are extremely useful if they can be performed at many sites, e.g., optical asymmetry tests). The advantage of a rover over many stationary landers is that scientific investigation need not be confined to the landing site, which is important when one considers that many locales of interest (especially geological interest) may not be feasible landing sites. In addition, the cost of numerous stationary landers would be prohibitive.

The increased capabilities of a rover require that the vehicle be more complex. Because of long round-trip communication times (8 to 41 minutes for Mars, depending on relative planetary positions), operating the vehicle "on-line, real-time" from Earth is not realistic for any location other than the moon. Even there problems arise, as evidenced by the camera tracking of lunar liftoffs with only a 3 second delay. This means that the rover will have to operate relatively independently of earth for significant periods of time, hence the designation "semi-autonomous."

At present, the National Aeronautics and Space Administration (NASA) is considering a 1979 or 1981 semi-autonomous roving vehicle mission to the Martian surface [2]. However, as recently as 1970 NASA noted that "....the technology required to implement an autonomous Martian roving vehicle capability is beyond the present state...." [3,4]. In the estimation of the author, a major stumbling block is the uncertainty that exists concerning how to govern the overall design and construction of such a complex system.

A Mars-roving vehicle (MRV) system will contain the following functional divisions, or subsystems:

1. science - collecting data and samples and performing experiments

2. communications - transmitting science and vehicle status data to earth and receiving earth commands

3. power - generating and storing the energy required by the vehicle subsystems

4. thermal control - maintaining acceptable internal temperatures onboard the vehicle

5. navigation - locating the vehicle in a set of Mars-centered coordinates

6. obstacle avoidance - sensing terrain and choosing a safe path for vehicle travel

7. computation and data-handling - conditioning sensor data, performing computations, making decisions regarding the vehicle state, and providing event sequencing

8. vehicle structure - vehicle frame, suspension, and motors.

However, the design of the MRV system requires more than the capability to build each of the subsystems. The MRV is truly a "system" in that its components (subsystems) are highly interdependent, i.e., they make demands upon each other which must be met in order to insure a functioning system. It is obvious, for example, that the power subsystem must be capable of supplying the needs of each of the other subsystems. The system design problem for an MRV is further complicated in that it is a problem dealing with limited resources. As in most aerospace systems, there are hard constraints on system weight and size corresponding to the capabilities of the launch vehicle.

The problem posed then, is determining a rational method for making design decisions involving conflicting requirements or trade-offs. More specifically, what should the weight, volume, and power allocations to each of the subsystems be; what should the operating characteristics of the subsystems be; and under what policies should the system be operated?

Intuitively, one expects that there would be more than one feasible design for the system. With a criterion for choosing one of these possibilities, the problem is now an optimal system design problem. Justification for attempting to optimize the design lies in the expected high cost of the system and mission. Because the outlay will be large, it is reasonable to attempt to maximize the "output" of the mission. The added cost of the optimization study should be negligible with respect to total cost.

Determination of the optimal design will preceed construction of the MRV. Assuming the optimum design has been specified, suppose that one or more of the design parameters is perturbed from its optimum value. Realistically, this case might occur for several reasons:

1. a decision to use "off-the-shelf" hardware
2. the addition of another constraint upon the design
3. the optimum value of a parameter being an infeasible design value (e.g., a data rate of 6.4371 bits/sec would probably appear as 6.45 or 6.5 in the final design).

Following this perturbation, there exists in general an optimum manner in which the other design parameters should readjust to the perturbation so as to minimize the effects on the total design. This new solution will be called the "perturbed-optimal solution."

The straight-forward way to handle this case would be to re-solve the optimal design problem (in effect, there is a new problem). However, as will be shown later, the solution of the optimal system design problem is complex, and requires an iterative computer solution in which there is no guarantee of convergence.

Therefore, another part of the system design problem is to develop a method to approximate the parameter readjustments and circumvent the problem of redoing the optimal design.

This research will solve the optimal system design problem for a Mars-roving vehicle and develop a method to approximate the perturbed-optimal solution subject to design parameter perturbations, applying this method to investigate the characteristics of the optimal MRV design.

The organization of the remainder of the text is as follows. Section 2 describes past work relevant to both the system optimization problem and sensitivity analysis in nonlinear programming (NLP). Section 3 is an overview of the approaches taken to solve the optimal design problem for a Mars-roving vehicle and the perturbed-optimal solutions problem in NLP. In Section 4, three alternative system models for a Mars rover are derived, and a system evaluation function is chosen. Section 5 presents optimal design results for each of the system models. The purpose of Section 6 is to present relevant NLP theory and develop the solution to the perturbed-optimal solutions (POS) problem. Section 7 contains descriptions of the sensitivity of the Mars rover designs determined in Section 5 by application of the results of Section 6.

Sections 8 includes discussion of results, conclusions, and the author's recommendations for extensions of the work presented in this thesis.

SECTION 2

HISTORICAL REVIEW

In this section, previous technical work that has bearing upon the work of this research will be discussed. Because the research contains two somewhat distinct parts, the review will be likewise divided.

The first part concerns itself with optimal system design, with specific reference to an aerospace system. The philosophy of system design for aerospace systems can best be seen by examination of design work done in the past ten years. Formal optimization has rarely been attempted for cases other than dynamic system models. In fact, only one static formulation of an aerospace system design optimization problem was found, that for launch vehicle design, and the motivation and analysis effort here was directed toward minimizing monetary cost [5,6]. Whereas this work was a problem in minimizing cost for a fixed performance level, this research seeks to identify the <u>maximum</u> performance level attainable.

Although formal optimization of large scale system designs is generally not attempted, attempts are obviously made to maximize the effectiveness of the system. As can be seen in studies relating to lunar landers and rovers [7,8], Mars fly-bys [9], Mars and Venus orbiters [10], Mars stationary landers [11,12], and the Mars-roving vehicle itself [4], system design optimization is treated heuristically. That is, project managers rely on inputs from their technical staff and a combination of experience and intuition to make decisions involving conflicting design trade-offs.

Actually, the system design optimization problem is one of optimization with constraints, and the mathematical background required has been developed. The most general formulation in an arbitrary vector space is considered in Luenberger [13], for example. However, theoretical developments have tended to come from one of two areas into which this general problem has been unfortunately divided by tradition and early applications interest. One of these is optimization of systems treated as "dynamic," where the calculus of variations [14], Pontryagin's maximum principle [15], and Bellman's dynamic programming [16] are the techniques used to solve a constrained optimization problem which is posed in infinite-dimensional Hilbert space. Concurrently, the study of the "static" optimization problem, which is generally posed in a metric space of n-tuples, has come under the general title of mathematical programming. Applications of mathematical programming have tended toward the areas of economics and finance [17,18], although there have been engineering-oriented applications [19,20,21].

The second concern of this research is with "perturbed-optimal solutions" in a nonlinear programming (NLP) problem. In general, problems where interest is in measuring the effects of a change in some system, whether physical or mathematical, are given the title of "sensitivity problems." The problem of perturbed-optimal solutions is therefore a special type of sensitivity problem. While there have been many sensitivity problems solved in the mathematical programming field, the perturbed-optimal solution problem for NLP as posed here has not been attacked. An analogue to this problem is the accessory optimal solution problem in dynamic system optimization [22], where a pre-solved problem requires a new solution because of changes in either initial or terminal conditions.

The sensitivity problems solved for the mathematical programming problem to date concerned themselves mostly with what might be termed the input to the problem. The mathematical programming problem asks: how does one extremize a given function subject to some constraints on the elements available for optimization? Sensitivity problems have posed the question: if the function to be extremized and/or the functions describing the constraints are changed, how does the optimal solution change?

One of the simplest problems where a sensitivity question arises is in the solution of simultaneous linear algebraic equations. The degree of sensitivity of the solution of the vector-matrix equation $Ax = b$, where A is an n by n matrix and b an n-vector, to perturbations in the components of A and b is discussed in Aoki [23].

Perhaps the best known sensitivity problem is the interpretation of LaGrange multipliers as sensitivity coefficients, or "shadow prices." It is known [24], that the LaGrange multipliers (λ_i) resulting from the solution of

$$\text{minimize} \quad f(x)$$

$$\text{subject to} \quad g_i(x) \leq 0 \qquad i = 1,2,\ldots,m$$

where x is an n-vector, represent the sensitivities of the optimum value of the function f with respect to "small" changes in the constraint specifications. More specifically,

$$\lambda_i = - \left. \frac{\partial f(x^*)}{\partial b_i} \right|_{b=0} \qquad i = 1,\ldots,m$$

where x* is the solution to the original problem, if the perturbations in the constraints are of the form $g_i(x) \leq b_i$.

Shetty [25] has solved a sensitivity problem for the linear programming case (i.e., the f and g_i functions above are affine) where somewhat more general perturbations are permitted. In addition, he asks: "When the value of one of the variables is changed by a given amount, what changes are necessary in the values of the other variables if the change in the value of $C(x)$ [the objective function, or $f(x)$ in the present notation] is to be a minimum?" An algorithm for the solution is developed in terms of manipulations on the final Simplex tableau. This is the only direct reference to perturbations in the components of the variable vector comprising the solution to the mathematical programming problem that the author is aware of. It is also precisely the algorithm to identify the perturbed-optimal solution in the linear case.

General sensitivity analyses with respect to perturbations in the objective function and constraints for the linear programming problem have been examined by a number of researchers, and several methods for their solution have been developed. Courtillot [26] has examined variations in the optimal solution with perturbations in the components of the matrix A and the vectors b and c in the linear programming problem:

$$\text{maximize} \quad c^T x$$

$$\text{subject to} \quad A\,x \leq b\,, \text{ where } x \text{ is an n-vector.}$$

Saaty [27] parameterized the elements of the coefficient vectors and matrix above and follows the solution as a function of the variations in these parameters. Kelley [28] shows that Saaty's approach is identically a primal-dual procedure. An interesting application of the solutions of this linear problem, which has become known as the parametric programming problem, to decision theory is by Isaacs [29]. His approach treated the perturbations as errors in probability estimations and developed theory to indicate when the errors became large enough to cause a switch in the optimal decision. Again, this work applied only to the linear case.

Boot [30] starts with the quadratic programming problem:

$$\text{maximize} \quad a^T x - \frac{1}{2} x^T B\, x$$

$$\text{subject to} \quad C^T x \leq d\,,$$

where x is an n-vector, B and C are matrices and a and d are vectors, and investigates properties of the change in the solution for small perturbations in the elements of a, B, C, and d.

Wolfe [31] solves a parametric programming-type problem in the quadratic programming case. His method obtains solutions for:

$$\text{maximize} \quad \theta\, c^T x + x^T D\, x$$

$$\text{subject to} \quad A\, x = b$$

$$x \geq 0$$

with x an n-vector, A and D matrices (D negative semi-definite), and b and c vectors, for all values of the scalar θ, $\theta \geq 0$.

The most complete sensitivity analysis to date appears in Fiacco and McCormick [32], who start with the NLP problem:

$$\begin{aligned} &\text{minimize} && f(x) && \\ &\text{subject to} && g_i(x) \geq 0 && i = 1,\dots,m \\ & && h_j(x) = 0 && j = 1,\dots,p \end{aligned}$$

with f, g_i, and h_j all nonlinear functions of the n-vector x, and consider (under certain mild conditions) the change in the optimal solution for what they term the general parametric programming problem, which is:

$$\begin{aligned} &\text{minimize} && f(x) + \epsilon_o\, a_o(x) && \\ &\text{subject to} && g_i(x) + \epsilon_i\, b_i(x) \geq 0 && i = 1,\dots,m \\ & && h_j(x) + \epsilon_{j+m}\, c_j(x) = 0 && j = 1,\dots,p \end{aligned}$$

for the case where the elements of the m + p + 1 vector, ϵ, are small.

It is important to note that with the exception of Shetty's work in the linear programming case, these sensitivity problems all concern perturbations in the constraints or objective function, while the problem of interest in this thesis is sensitivity of the solution with respect to forced perturbations in elements of the optimal solution itself. In addition, while in the linear case the perturbed-optimal solution is easily found exactly, in NLP problems only nonlinear techniques can yield the exact solution.

SECTION 3

METHODOLOGY

This section will describe the system design optimization and perturbed-optimal solutions problems more completely, and indicate proposed methods of solution.

3.1 OVERVIEW

Design, in the systems sense, is the process of specifying the information required by subsystem designers. This information consists of the operating requirements to be met by each subsystem, and all constraints under which the subsystem designer must work. For the designer of a communications subsystem for example, such information might be that a pulse-code modulated subsystem capable of 'x' data rate, not exceeding 'y' weight, and drawing 'z' watts maximum power is needed.

System analysis is the task of determining an accurate system model. Required by this definition is the examination of all design trade-offs in the context of their effect upon the operation of the system as a whole. For a system of non-trivial size, the system design is composed of many parameters under many constraints, the interrelationships between the parameters may be complex, and it is necessary to consider all parameters and constraints concurrently.

The task of optimization requires that the manner in which the design parameters react is known. It implies the use of a mathematical model of the system. In most applications, the equations of the system model are the result of work usually done by the subsystem designers. Again using the communications example, the system model can include equations relating power input, data rate, and subsystem weight. Confidence in the model equation is based upon the assumption that it should be possible to design a communications subsystem whose important parameters relate (at least approximately) as the equation predicts. This illustrates that system design is really a "closed-loop" process. Information obtained at the subsystem level of design is required to obtain a system model, which will be used eventually to specify parameters that are inputs to the subsystem design procedure. In addition, modifications or innovations which occur on the subsystem level (e.g., a new material makes it possible to reduce weight) must be used to update the model. It is important to recognize and utilize this interplay between the two levels.

It is infeasible to expect to be able to force the model to include all possible design variations. Radically different approaches to a design problem will usually have significantly different effects on how the design parameters relate. It becomes necessary then, to make certain assumptions about the system and subsystem configurations. This in turn means that optimization for a single model is not an end product simply because there are probably other design alternatives not included in that model. To claim that a system design is indeed optimal, it is necessary to first consider the models corresponding to the set of all possible input assumptions.

The search for the optimum also implies that there is a standard by which the system quality can be measured. This objective (or objective function) may or may not be unique. Generally, the objective measures how well the system is fulfilling its purpose. If there are alternate ways of describing how well the system performs, these too are inputs to the optimization process and must be separately considered.

In addition, there are assumptions that must be made about external constraints (funding, development of new techniques, time schedules,...) which may affect the design and may not be deterministic.

The many possible combinations of design assumptions, objectives, and external constraints make system design optimization an exhaustive process in the sense that the solution must be obtained for sets of inputs before confidence in the validity of the optimum is achieved. Schematically, the inputs to a single "run" of the optimization process can be represented by Figure 1, where now, for a roving vehicle, mission goals are the determining factors in formulating the system objective. The questions now are -- how does one go about determining the optimal system design, and what will the effects be of deviating from this optimal solution?

3.2 SYSTEM DESIGN OPTIMIZATION

System design is accomplished by collecting all constraints and attempting to sort out a feasible set of design parameters while keeping in mind the objective of the system. Individual subsystem designers are constrained by the requirements of other subsystem designers. The pointing error of the communications antenna will be affected

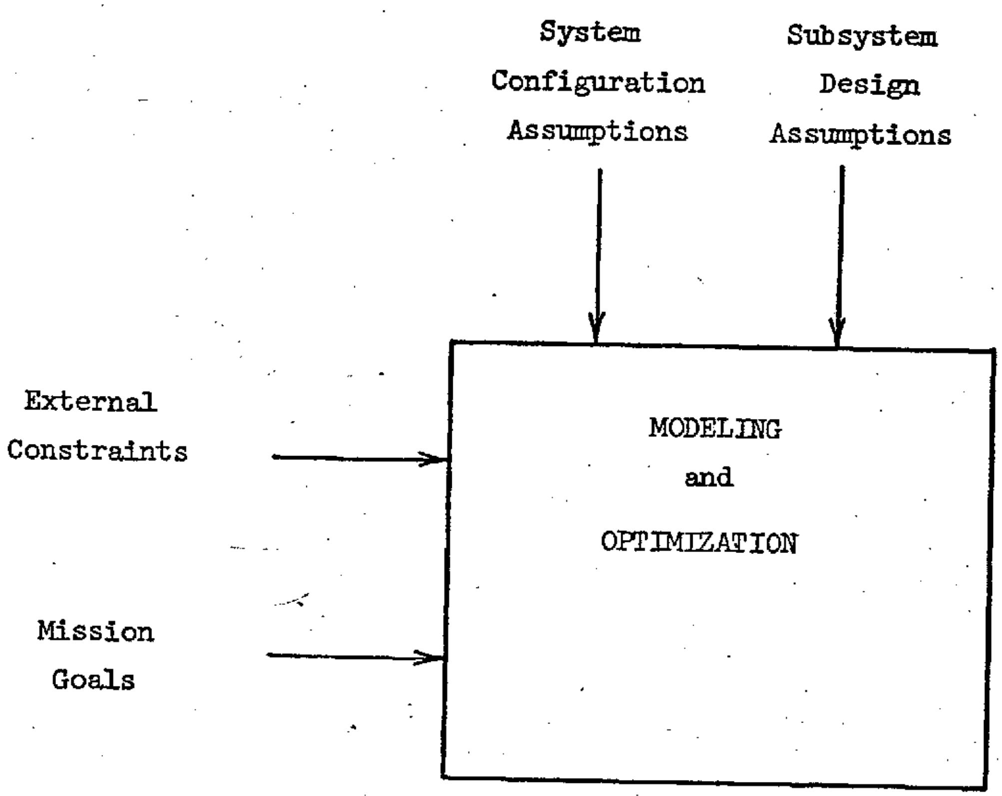


FIGURE 1
INPUTS TO THE OPTIMIZATION PROCESS

by its power and weight allocations, which must eventually depend on how much weight and power are allocated to other subsystems. Decreasing either antenna pointing power or the weight allocated to the pointing apparatus will probably have the effect of increasing power and/or weight required by the electronics section of the communications package or decreasing the performance level. The problem is to specify a set of design parameters (in this case, power and weight allocations and performance levels) that will best achieve the objective of the system.

There will normally be an infinite number of sets of parameters that will constitute a feasible and acceptable design (i.e., one that can be constructed and will operate to some measure of satisfaction). The system designer is faced with the problem of choosing one of these sets. He obviously wishes to choose that set which will maximize the effectiveness of the system.

When the system is complex (which may be a result of having many design parameters to choose and/or complex interrelationships of the parameters), the job of making this choice can be more difficult than the system modeling. Traditionally, the method has been to choose a subset of the design parameters to satisfy a system objective to some degree, and then to use the model to fix the others. If this final set is unacceptable, the designer must change some or all of his original choices and re-solve until he is "satisfied" with the system design. Unfortunately, the nagging question of whether there is a better solution remains. This drawback is inherent in this method, and as the Historical Review points out, this has been the accepted method for the overall design of large unmanned explorers. However, it is proposed here that if it is possible to describe the effectiveness of these systems as a function of the design variables, the optimal solution can most often be identified.

The nonlinear programming (NLP) problem is:

$$\begin{aligned} &\text{extremize (max or min)} \quad f(x) \\ &\text{subject to} \quad g_i(x) \geq 0 \qquad i = 1,2,\ldots,m \\ &\qquad\qquad\quad\; h_j(x) = 0 \qquad j = 1,2,\ldots,p \end{aligned}$$

where x is an n-vector of variables to be chosen by the optimization process. The f, g_i's and h_j's are all scalar functions (possibly non-linear) of the components of x.

The NLP problem is a natural way to describe the problem of optimal system design. Since the problem is now formulated as the determination of n design parameters, $f(x)$ becomes the objective function previously discussed. The g_i and h_j functions represent the physical and external constraints placed upon the design. The major advantage of this approach is that it allows all feasible designs to be identified and considered.

Thus, for a given set of assumptions, the optimization process will consist of three parts:

1. formulation of a mathematical model of the system (identification of constraints)
2. determination of the objective function in terms of the model variables
3. imbedding the problem in the nonlinear programming format and locating the optimum.

Figure 2 shows the modeling and optimization process. The optimum output appears in quotes only because it is optimal with respect to the validity of the input assumptions. The iteration is with respect to changes in these inputs. For the MRV design problem, the set of all equality relations in the model represent all physically realizable (buildable) systems. The equality and inequality relations together identify all feasible (buildable and Mars-deliverable) designs.

The feasibility of the proposed method of optimal system design for a Mars-roving vehicle is contingent upon obtaining a solution to the NLP problem resulting from system modeling and determination of the system objective function. The NLP problem has no known closed-form solution aside from some special cases, and iterative techniques do not guarantee locating the optimum. Tests on available iterative methods [33,34,35], show that attaining even a local optimum is a function of the specific problem, and that there are some problems for which a given iterative technique will not converge.

While the optimal design process may yield a solution to the problem being run, changing any of the initial assumptions will invalidate the obtained solution. For each set of assumptions there will probably be changes in the system model, and if so, there will almost definitely be a new and different solution. If solutions can be generated to major assumptions, they can be compared so as to locate a solution considered optimal independent of assumptions. However, as there appears to be a very large number of sets of assumptions, this work will concentrate on those appearing more critical to the outcome of the design.

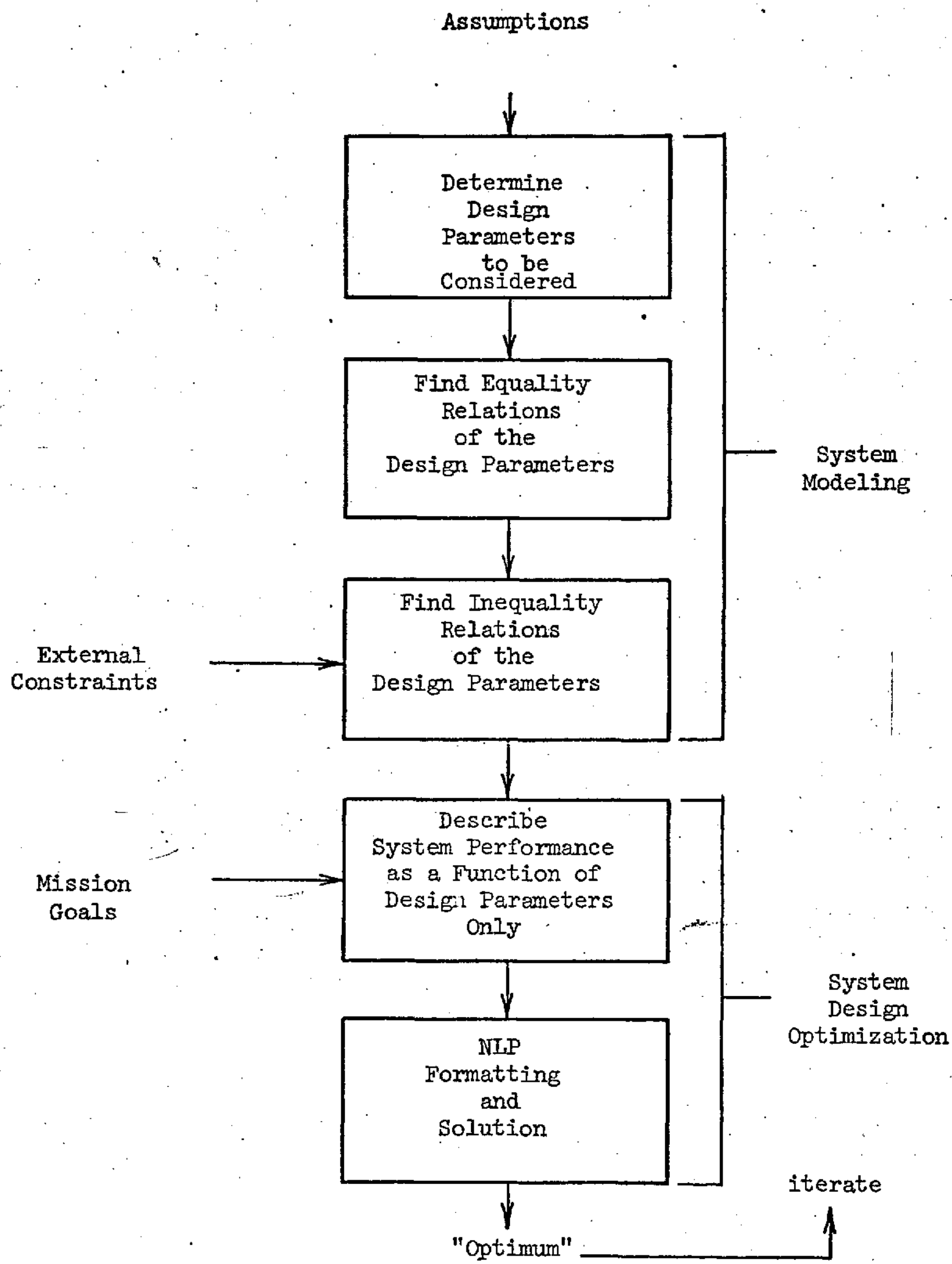


FIGURE 2

MODELING AND OPTIMIZATION PROCESS

Work toward the determination of the optimal system design for an MRV will include the following:

1. the optimal system design for certain sets of basic assumptions, namely
   a. 4-wheeled vehicle, direct Mars-earth communication
   b. 4-wheeled vehicle, communications via a Mars orbiter
   c. 6-wheeled vehicle, direct Mars-earth communication
2. in each of the categories in 1., designs for a number of different values for input variables, e.g., maximum allowable vehicle velocity, maximum allowable terrain slope to be traversed, limits on internal temperatures, relative dimensions of equipment package.

3.3 PERTURBED-OPTIMAL SOLUTIONS IN NONLINEAR PROGRAMMING

While the Introduction contains the rationale for attacking the problem of perturbed-optimal solutions, the solution of this problem is purely a mathematical concern. Given is an NLP problem which can be written as:

$$\underset{x \epsilon F}{\text{minimize}} \qquad f(x)$$

$$F : \left\{ x \,\middle|\, h_j(x)=0,\ j=1,\dots,p;\ g_i(x) \geq 0,\ i=1,\dots,m \right\}$$

and its associated solution, a specific n-vector x^*. Again, the problem to be solved is: supposing that one or more of the components of x^* is perturbed from its optimal value, how should the remaining n-1 components readjust in order to maintain the optimal property of the solution?

What is essentially being done is that at least one point is being removed from the feasible set F, and what is being sought is a new point that minimizes the change in the value of $f(x)$. If it is only known that one component will be perturbed, i.e., the magnitude and perhaps the direction of the perturbation is unspecified, the solution will consist of a line of readjustment for an arbitrarily small perturbation.

The problem will be formalized in a form such that the original perturbation is of arbitrary magnitude and direction. The aim is to create a "useful" solution, not necessarily an exact one, while considerably reducing the amount of manipulation from that required to re-solve the NLP problem.

It is most important to note that if the initial perturbation is specified in magnitude and direction, a new NLP problem in n-1 variables can be created and the perturbed-optimal solution can be found, at least theoretically, exactly. The qualification is due to the fact that the general NLP problem has no known closed-form solution and the best one can hope to do is locate local optima by iterative search techniques. Thus, the solution sought in this work has two major advantages over the "straightforward" approach. First, the perturbation will be arbitrary; second, the need to solve a new NLP problem will be circumvented. The price paid for these advantages is that the generated solution will be an approximate one whose accuracy will in general decrease with increasing magnitude of the perturbation.

Finally, an attempt will be made to extend the solution to include the case of forced perturbations of more than one design parameter.

Specifically, work on the method for the perturbed-optimal solutions problem will include:

1. development of necessary conditions for a linear approximation to the solution for small perturbations

2. analysis of conditions under which the method is applicable

3. application of these results to the MRV design problem.

The completed work contributes to the systems engineering discipline in the following manner. Determination of the optimal system design for an MRV includes the development of a method of static optimization for aerospace systems along with demonstrated feasibility of the approach. In addition, it is the solution to a complex engineering problem that has direct application to unmanned space exploration. Work on perturbed-optimal solutions in NLP will solve a problem with applications to system design that, to the author's knowledge, has not been addressed in the literature. The usefulness of the solution will be demonstrated by application to the MRV problem.

SECTION 4

SYSTEM MODELS

In this section, the system modeling for the Mars-roving vehicle is described. Three major cases are considered:

1. Four-wheeled, direct communicating rover
2. Six-wheeled, direct communicating rover
3. Four-wheeled rover, communicating via a Mars orbiter.

In 4.1, the system model equations for case 1 are developed, and the system evaluation or objective function is described and derived. Parts 4.2 and 4.3 describe the changes in the first model necessary for cases 2 and 3, respectively.

4.1 FOUR-WHEELED, DIRECT COMMUNICATING ROVER SYSTEM MODEL

4.1.1 Subsystem Models

4.1.1.1 Communication Subsystem

The Earth/Mars communication subsystem is modeled as a direct two way link in the microwave spectrum between a Mars-roving vehicle and an Earth communication station. The communication link is divided into an uplink to Mars and a downlink back to Earth. Uplink parameters associated with the rover are found to be negligible in comparison to similar downlink parameters, and were thus not considered directly.

The downlink is composed of the spacecraft transmitter, a high gain parabolic dish antenna, a standby low gain omnidirectional antenna, a free space propagation path, a high gain parabolic dish receiving antenna, and a ultra low noise receiver, as shown in Figure 3.

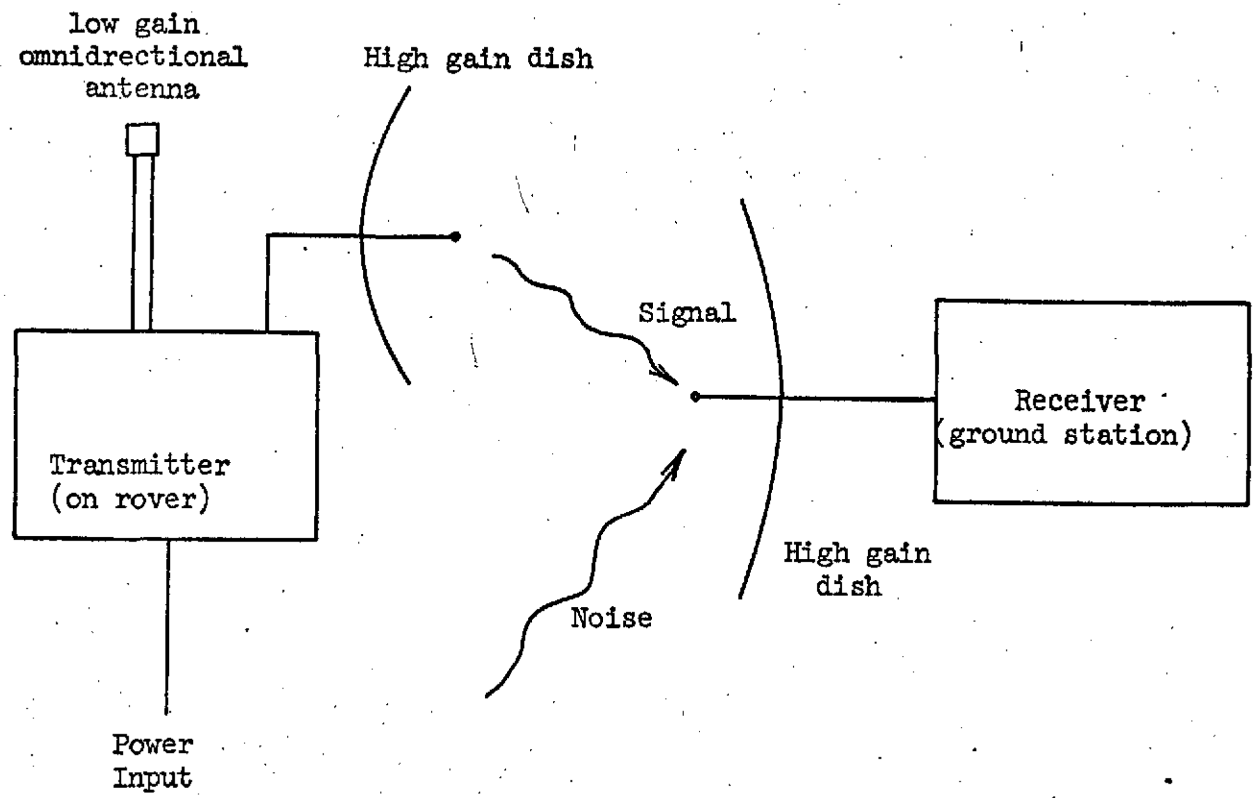


FIGURE 3

COMMUNICATION SUBSYSTEM:
DOWNLINK FUNCTIONAL DIAGRAM

The first step in the modeling task is to describe the subsystem mathematically in terms of link parameters. The list of parameters chosen to model the link is given in Table 1. The parameters can be divided into two classes: those which are fixed by nature, state of the art, or constraints; and those which are design dependent, and therefore a function of the design decisions made (e.g. link distance is fixed by nature, transmitter efficiency is fixed by the state of the art; however, data rate is free to vary over some range, as a function of the design chosen to implement the link).

Before proceeding further, it is necessary to make assumptions to specify the fixed parameters and constrain the model sufficiently to allow analysis:

1. The carrier is X-band microwaves of wavelength 3.3×10^{-2} meters, which have been shown to be especially well suited for high speed communications at Mars distances. [4]
2. The ground station antenna is a 64 meter parabolic dish [36]
3. The rover antenna is a parabolic dish with a pointing error of 1°.
4. Uplink parameters are negligible.
5. The overall r.f. efficiency of the transmitter is 20%. This figure is obtained from a 25% TWT efficiency and a very low exciter efficiency [37]
6. The worst case link distance of 5.7×10^{11} meters is used.
7. Total equivalent noise temperature for the receiving system on Earth is the sum of the galactic and receiver noise temperatures, and was assumed to be 30 K. [38]

TABLE 1

PARAMETER	SYMBOL	UNITS
Data Rate	R_{com}	bits/sec
R.F. Power Output	P_t	watts
R.F. Efficiency	e_c	-
Power Input	P_{com}	watts
Rover Antenna Diameter	D_{com}	meters
Rover Antenna Pointing Error	$\Delta \theta$	degrees
Carrier Wavelength	λ	meters
Weight (Mass)	W_{com}	kg
Volume	V_c	cubic meters
Heat Dissipation	Q_c	watts
Link Distance	L	meters
Noise Temperature	T_n	^oK
Receiver Antenna Diameter	D_r	meters
Communication Efficiency	(B_o/B)	-

COMMUNICATION SUBSYSTEM: DOWNLINK PARAMETER LIST

8. The communication efficiency, a measure of the ability of a given modulation scheme to overcome additive channel noise, is 5%. This corresponds to a 20:1 signal to noise ratio in a typical PCM system. [39]

The above assumptions specify many of the entries in the list of parameters. To further reduce the number of unspecified parameters, equations relating the various parameters can be found.

1. Conservation of energy allows two equations to be written:

$$P_t = e_c \, P_{com}$$

$$Q_c = (1-e_c) \, P_{com}$$

2. Electronics weight is obtained as a function of power input alone from data associated with various prediction efforts in Mars communication, as shown in Figure 4. [40]

$$W_c = 0.59 \text{ kg/watt } P_{com} + 34.0 \text{ kg}$$

is found to approximate the functionality for P_{com} expressed in watts.

3. The weight of the antenna and its associated steering motors can be approximated as a function of antenna diameter, D_{com}, in meters:

$$W_{ant} = 2.0 \, D_{com}^2 + 5.0 \text{ kg.} \quad [4, 41, 70]$$

At this point, note that there remain only three of the original parameters in Table 1 which have not been either specified by assumption or related to another specified parameter by the simplifying equations identified above: R_{com}, P_{com} and D_{com}. In other words, a knowledge of these three parameters alone will, in the light of the

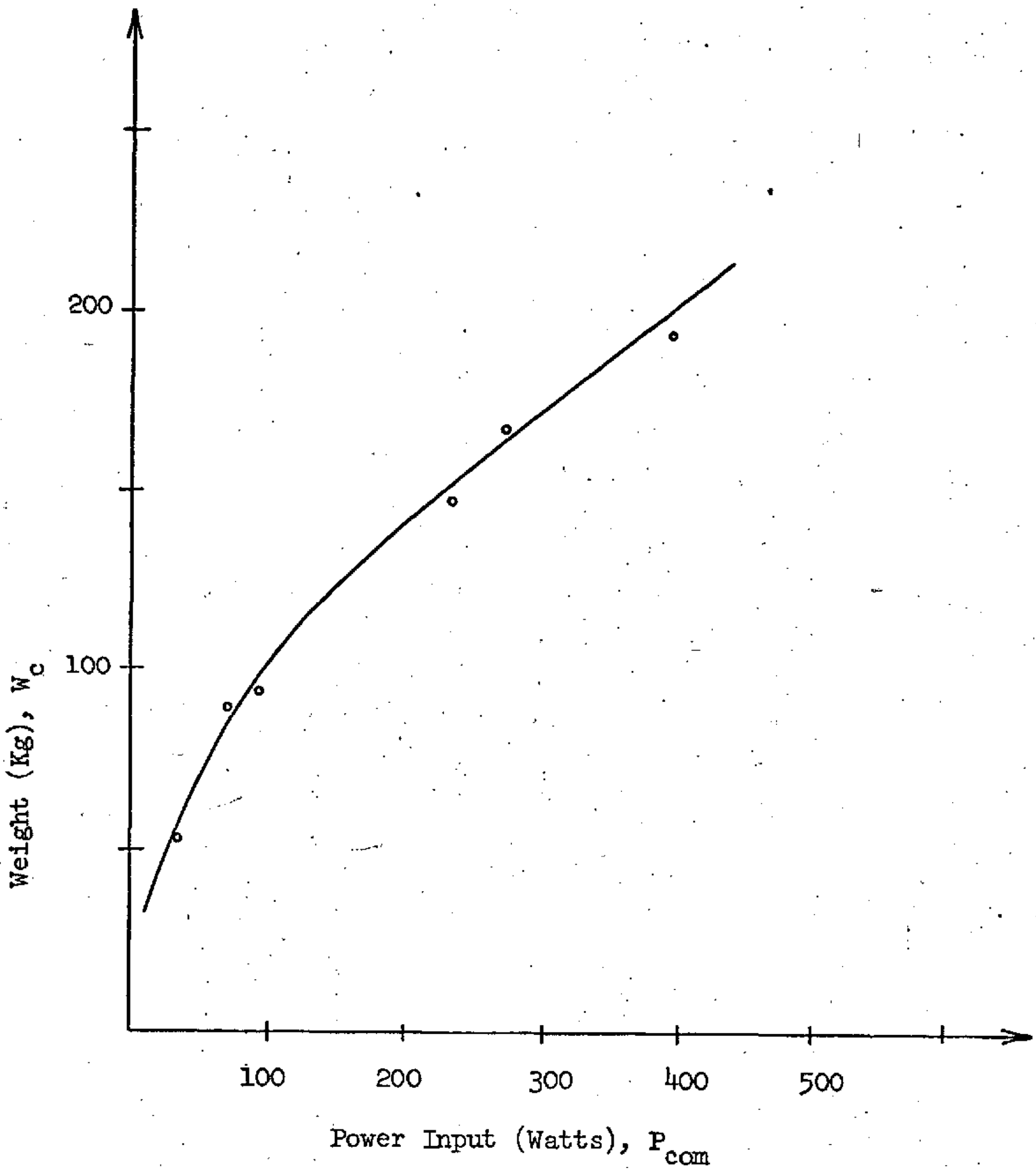


FIGURE 4

COMMUNICATION SUBSYSTEM: GRAPH OF WEIGHT VS. POWER INPUT basic assumptions listed, completely specify all of the parameters identified at the beginning of the modeling task as being necessary to uniquely describe the entire subsystem. Given these three parameters, a subsystem could be built. However, not every subsystem would satisfy the requirements which this subsystem is being asked to satisfy. In other words, not any random choice of these parameters will produce a satisfactory subsystem. There must exist another equation which will provide a relationship which the defining parameters must satisfy. The equation sought is the classic range equation for a noisy channel.

For a "successful" subsystem, the signal power received on Earth must be sufficiently large to overcome the noise. The received power is given by

$$P_r = P_t\, G_t\, G_r\, L_p \quad ,$$

where

P_r is received signal power,

P_t is transmitted signal power,

G_t is transmitting antenna gain,

L_p is the space loss attenuation,

and

G_r is the receiving antenna gain.

Substituting known parameters, for P_r and P_{com} in watts, D_{com} in meters, the received power is found to be

$$P_r = 5 \times 10^{-19}\ D^2_{com}\, P_{com} \ .$$

For the signal to overcome the noise, the following relation must be satisfied for a PCM subsystem [39]

$$P_r \geq 10^{-23} (B/B_o) \cdot T_n \cdot R_{com} ,$$

where

B/B_o is the inverse of the communication efficiency,

T_n is the system noise equivalent temperature, ^oK,

and

R_{com} is the data rate in bits/sec.

Substituting known parameters and combining the above two relations yields the relationship,

$$R_{com} \leq 42 D^2_{com} P_{com}$$

Only choices of the three variables satisfying the above relationship will specify subsystems capable of communicating successfully with Earth. Because it will obviously be advantageous to have the upper limit of the equality satisfied, the equation becomes:

$$R_{com} = 42.0 D^2_{com} P_{com} .$$

In summary, the communication subsystem can be modeled on the basis of only two chosen parameters, as the third is determined by the range equation. If any of the assumptions made at the beginning of the analysis were to be relaxed, then additional variables would be included to uniquely specify the subsystem.

For the present model, if the total weight of the communications subsystem (this includes antenna and electronics section) is denoted by

W_{com}, the weight equation is:

$$W_{com} = 0.59\ P_{com} + 2.0\ D^2_{com} + 39.0\ \text{kg}. \qquad (4.1)$$

The antenna size must be constrained to some limit:

$$D_{com} \leq D_{max} \qquad (4.2)$$

and the data transmission rate (R_{com}) is:

$$R_{com} = 42.0\ D^2_{com}\ P_{com}\ \text{bits/sec}. \qquad (4.3)$$

4.1.1.2 Science Subsystem

The purpose of an unmanned Martian roving vehicle (MRV) mission would be to gather information about the planet, as well as to develop the technology relevant to autonomous roving vehicles. A roving capability makes it possible to conduct similar tests at many different locations, or to modify tests according to present location and past experimental results. The design of the science subsystem must be based upon knowing, first, what information is sought, and second, how to endow the subsystem with the ability to gather this information.

The major thrust of a roving vehicle mission will be to determine the probability of life on the planet [12, 55]. Knowing whether life is more or less probable than was estimated before the mission would be an acceptable result. In addition, a comprehensive data-gathering program tracking Martian surface parameters (temperatures, atmospheric composition, surface gravity, seismological activity,...) will greatly increase the total knowledge of the planet's surface. Because several stationary landers will preceed an MRV mission, surface parameters will be known at some locations, and this second requirement takes on a slightly lesser priority.

Modeling the payload must result in relationships between the major parameters of the subsystem, especially those which will affect the design of other subsystems. These parameters include: 1) weight, 2) power requirement, 3) stationary science time required, and 4) data processing requirements. The only way to obtain empirical relationships between these variables is to know what equipment will be onboard. However, until the parameters of the science subsystem have been chosen, which is the result of the analysis, this information would normally not be known. What can be done is to establish a priority list for the equipment, i.e., a list of which equipment will be added to the payload as weight and power allotted to science are increased. The priority list is set by defining what tests are needed to acquire the information desired, and then ordering these tests according to which information is deemed most useful. A heavy reliance was therefore placed upon the results of an extensive literature search concerning planetary scientific exploration and exobiology [4,12,42-55].

Science priorities (descending order) were determined to be:

1. test for qualities (properties) associated with life,
2. determine Mars surface parameters at diverse locations and times, and
3. have a "general chemical laboratory" with the ability to perform varied analyses and tests under earth command.

The assumption that the Martian bio-chemistry (if any) is earth-modeled is not warranted. Free water appears to be in short supply on the surface, ultraviolet radiation (1700-3000 Å) fatal to most earth organisms is incident throughout what would be considered the biosphere, and temperatures are low (180-300 K). Some earth micro-organisms could survive on the planet, but none have been found which could grow in the Martian environment at the week rates of seasonal activity on Mars (the "wave-of-darkening," which may be biological in nature).

Life evolution normally progresses through and must exist first on molecular, microbial, and then macroorganismic stages. Therefore, life-search will be most efficient if tests are made for the qualities associated with life (attempting not to assume a specific bio-chemistry) at the lower levels.

Indications of the presence of life may be functional (dynamical and thermodynamical), morphological, and/or chemical. Testing for functional qualities can be accomplished by certain biological activity tests (radio-isotope, turbidity, pH, calorimetric) which have been shown to be adaptable to space science requirements. Morphological properties can be observed in the large (television and television microscope) or on the molecular level (optical assymmetry tests). Finally, the knowledge of what chemical constituents are present on Mars will be of importance for practically all studies, but specifically for determining the possibile bio-chemistries.

Determination of certain Mars surface parameters can be accomplished by Viking-1976-type meteorology and seismology packages [4,53]. In addition, tests for magnetic properties, surface gravity, and soil moisture should be considered.

Chemical analysis will be accomplished by the use of a gas chromatograph-mass spectrometer (GC-MS) device. The device must be capable of pyrolyzing samples prior to analysis. Ref. 46 gives details.

Certain portions of the TV microscope and chemical laboratory, seen in the literature as the automated biological laboratory (ABL, [44]), may be used to give the science package flexibility. The ABL is a general reagent laboratory, which when equipped with a minimal number of motor functions (moving samples, mixing, heating,...) will enable scientists on earth to request certain tests based upon what the MRV has observed up to that time.

Table 2 lists science equipment in order of priority as chosen by the author along with other data important to the operation of the package. Data processing requirements were not considered in this analysis (see 4.1.1.7).

Based upon the information in Table 2, two approximate relationships between subsystem parameters can be derived by plotting cumulative time and power vs. cumulative weight (i.e., total weight as equipment is added to the payload). The data points are plotted in Figures 5 and 6 along with linear approximations which are:

$$P_{sci} = 3.44\ W_{sci}$$

$$T_{esci} = 35.75\ W_{sci} - 135.0 \qquad (4.4)$$

$$T_{sci} = \left(T_{esci} + \frac{n_p \times 10^6}{R_{com}}\right) = 35.75\ W_{sci} + \frac{n_p \times 10^6}{R_{com}} - 135.0 \qquad (4.5)$$

TABLE 2

	Equipment	Performance/Science Stop		Weight (lbs)	Power (watts)
		Activities	Time required (sec)*		
1.	2 cameras	n_p pictures	$\frac{n_p \times 10^6}{R_{com}}$	14.1	12
2.	optical activity test	soil, 1 air sample	145	2	1
3.	GC-MS	test optical activity samples	400	24	60
4.	radioisotope growth test	1 test	90	6	3
5.	turbidity and pH growth test	1 test	120	4	1
6.	calorimetric	1 test	120	3	1
7.	sound detection	20 seconds	30	0.5	1
8.	magnetic properties	test soil sample (may require picture)	20	0.5	0
9.	seismometry	60 seconds	65	3.5	5
10.	meteorology	1 profile of each	180	15	1
11.	soil moisture	1 test	30	2	25
12.	surface gravity	1 test	20	3	3
13.	ABL**	no pre-programmed performance	?	75	200

SCIENCE SUBSYSTEM: EQUIPMENT PRIORITY LIST AND SOME DEVICE CHARACTERISTICS

* all entries assume that time required to sample from the atmosphere is 15 sec., and soil samples require 60 sec. Time required includes the time necessary to transmit the outcome of the activity.

** portions of the total package may be used.

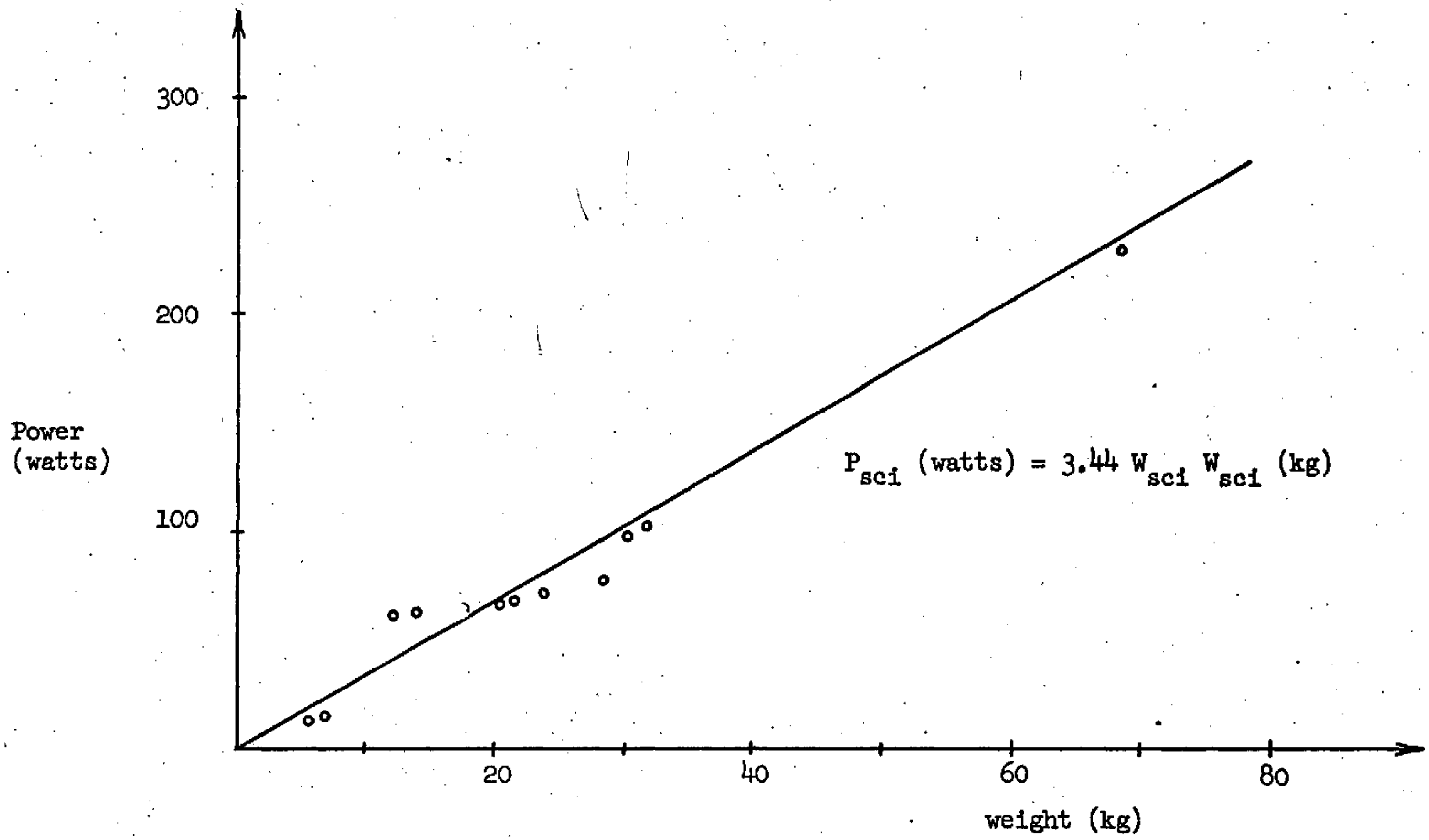


FIGURE 5

SCIENCE SUBSYSTEM: POWER REQUIRED VS. TOTAL
WEIGHT FOR MRV SCIENCE PAYLOAD

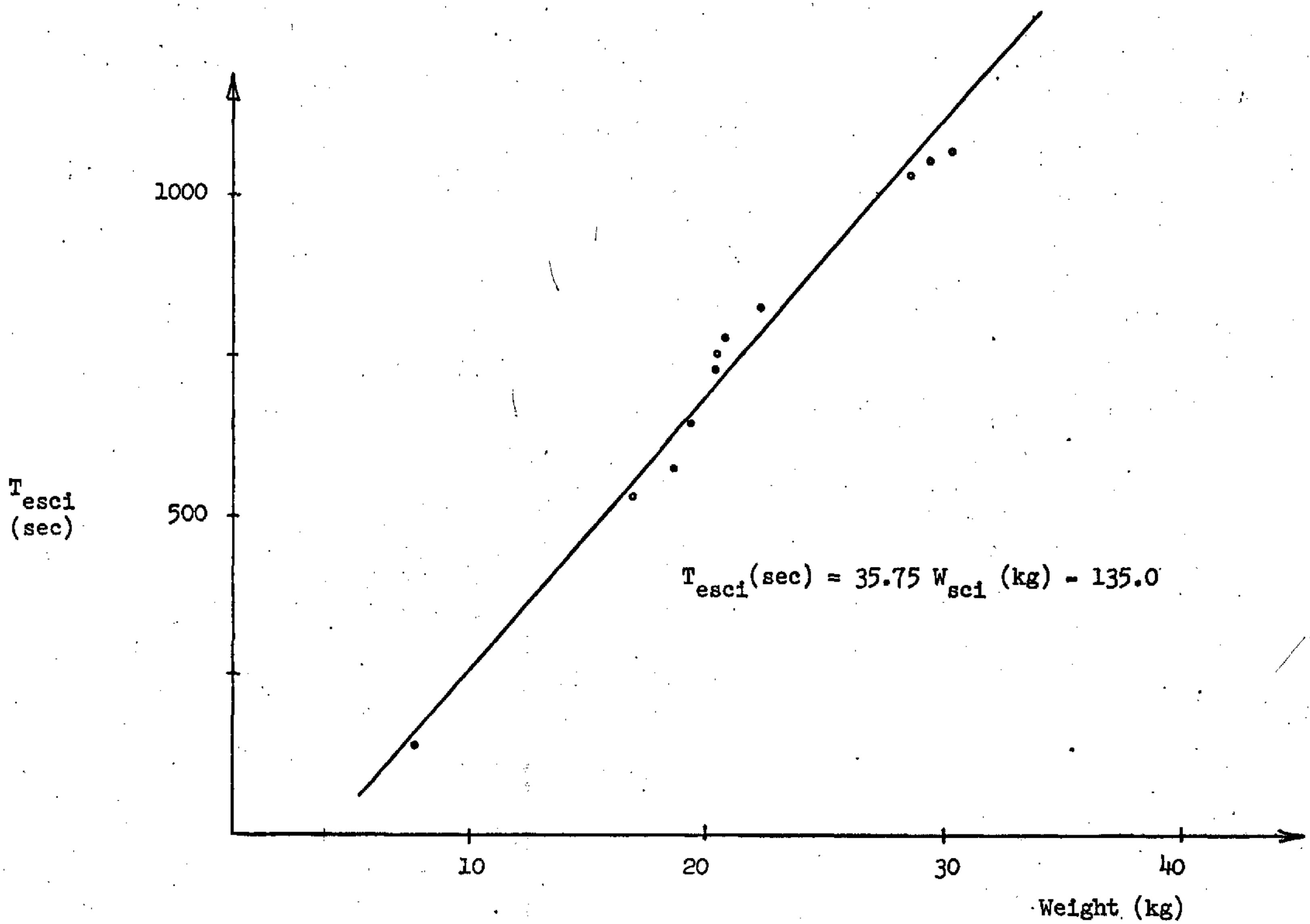


FIGURE 6

SCIENCE SUBSYSTEM: TIME REQUIRED FOR EXPERIMENTATION VS.
TOTAL WEIGHT FOR MRV SCIENCE PAYLOAD

where

W_{sci} = weight of science payload, kg

P_{sci} = power required, watts, for simultaneous operation

T_{sci} = time required to obtain and transmit science data per stop, sec

T_{esci} = time required to obtain science data per stop, sec

R_{com} = data transmission rate for science data, bits/sec

n_p = number of photographs taken/science stop.

Note the first instance of coupling between subsystems. A communications subsystem parameter can be seen to directly affect the relationship between two of the science parameters.

A more accurate indication of the power requirement for science might be the average power expended over time as a function of total weight. In other words, the average power (P_{scia}) for any weight is the sum of all the products of experiment power times experiment time, divided by the sum of all the times. Surprisingly, this number is nearly constant with total weight, and to a good approximation:

$$P_{scia} = 26 \text{ watts.} \qquad (4.6)$$

4.1.1.3 Power Generation and Storage Subsystem

In order to meet any mission requirements, the Martian roving vehicle must contain a suitable power source. Such a power generation subsystem must be capable of sustained operation in a hostile environment and under adverse loading conditions. References 56 and 57 develop a power system comprised of radioisotope thermoelectric generators (RTGs) and hermetically sealed batteries. This form will be assumed for the MRV. The power subsystem operates in a dual-mode fashion: the RTGs generate energy at a constant rate and any excess is either stored in batteries or expelled to the Mars surface as heat.

If E_{batt} represents the maximum energy capacity of the batteries (watt • hrs), and P_{RTG} is the power output of the RTGs, the time necessary to recharge the batteries (T_r, hrs.) is

$$T_r = \frac{E_f \ E_{batt}}{P_{RTG} - P_{str}} \tag{4.7}$$

where P_{str} is the power consumed by onboard vehicular subsystems while recharge is in progress, and E_f is the maximum depth of discharge of the batteries divided by the efficiency of the recharge process.

The power used to drive the vehicle P_{prop} is the result of three factors: P_a, the power used by the rover to accelerate from a stationary position to the roving velocity; P_v, the power required to maintain the velocity of the rover on level ground (the velocity is assumed constant); and P_{sl}, the power needed for slope traversal.

P_v is the power used by the rover to overcome the force of friction while traversing the planet at a constant velocity. Since on a flat plane

$P_v = Fv_f$, where v_f is velocity and F a constant

then

$P_v = \mu_k M_r g_m v_f$,

where μ_k is the coefficient of kinetic friction, M_r is the mass of the rover, and g_m is the acceleration of gravity on Mars.

The final term, P_{sl}, in the power equation is found to be

$$P_{sl} = M_r g_m v_f \sin \Psi ,$$

where Ψ is the angle of inclination of the slope being traversed. Combining the last 2 equations yields

$$P_v + P_{sl} = M_r g_m v_f (\mu_k + \sin \Psi).$$

This last equation can be modified to take into account wheel slippage; a two degree additive slope factor approximates the effect of any slippage [58] so:

$$P_v + P_{sl} = M_r g_m v_f (\mu_k + \sin (\Psi + 2^o)).$$

Because the P_a term applies only to the case where the vehicle is accelerating to v_f, and because in that case power assigned to P_v can be utilized, an approximation to P_{prop} might be:

$$P_{prop} = M_r g_m v_f (\mu_k + \sin (\Psi + 2^o)). \qquad (4.8)$$

The terms P_{mv} and P_{str} must be determined by the operating characteristics of the subsystems. They will consist of the power usages of the subsystems for the roving and recharging states. An expression for T_{rov}, total roving time between battery recharges, is highly dependent upon work done in modeling other subsystems, and its derivation is also deferred. This work is reported in 4.1.2.

The weight of the power subsystem must be found as a function of subsystem variables. Ref. 4 estimates the weight of relays, converters and shunts required for an RTG-battery configuration to be 14 kg, which should be fairly constant within the working range of the subsystem parameters. The projection of RTG technology circa 1975 is for a 5.94 watts/kg capability with practically infinite lifetime when compared to the duration of the mission [56].

Table 3 presents data on battery types considered dependable enough for space applications [59]. Silver-zinc batteries have too high a degradation rate for use on a 6-18 month mission. A conservative (more cycles, lower degradation rate) choice of NiCd was made. NiCd batteries have a 27.0 watt-hrs/kg ratio. Thus, the weight of the power subsystem (W_p) can be described by:

$$W_p = .168\ P_{RTG} + .037\ E_{batt} + 14.0 \text{ kg.} \tag{4.9}$$

4.1.1.4 Thermal Control Subsystem

The function of the thermal control subsystem is to maintain a satisfactory environment in which critical equipment can be operated. The basic assumption made in the modeling effort is that a compartment shall be temperature controlled to remain in some temperature band about 300 K despite Martian environment variations.

Variations of the Martian environment are vital inputs. Maximum temperatures occur at Martian noon and are estimated to be about 265 K, while minimum temperatures of 175 K are expected at night [60]. Other constraints affecting the subsystem design are low atmospheric pressure, thermal conductivity of the atmosphere, day/night

TABLE 3

Type	Energy capacity (watt-hr/gm)	Useful life (cycles)	Degradation (%/cycle)
NiCd	0.027	10,000	0.003
AgCd	0.053	2,000	0.015
AgZn	0.110	150	0.200

POWER GENERATION AND STORAGE SUBSYSTEM: STATISTICS ON BATTERIES FOR SPACE APPLICATIONS cyclical incident energy variations, abrasive dust storms, and limited power and weight available.

Prior to the modeling effort, it was concluded that the configuration of the subsystem would have to be specified to some extent, or the modeling task would be insurmountable. Therefore, from previous work done on choosing a thermal control configuration for a Martian laboratory [61], a preferred scheme was selected from a list of the many feasible alternatives. The choice was made on the basis of a list of desired features, such as simplicity, reliability, range of control, proven performance, insensitivity to Martian atmospheric parameters, ability to survive sterilization procedures, ease of development, resistance to dust storm damage, and required weight. The configuration chosen is an electrically heated, heat-pipe-cooled insulated compartment, as shown in Figure 7 (note that this figure defines the variables a_1 and a_2, as well as the temperatures T_a, T_b and T_r).

Having selected the configuration, a list of describing parameters can be compiled. These parameters are given in Table 4. A number of heat balance equations can be written by noting that the assumption of isothermal compartments implies that for each isothermal volume, the heat input equals the heat output. Furthermore, the heat balance is satisfied both at night and in the day. This allows six equations to be written. Also, an equation for subsystem weight can be derived.

A sample heat equation and the weight equation are shown here. For the outer skin during the day, let:

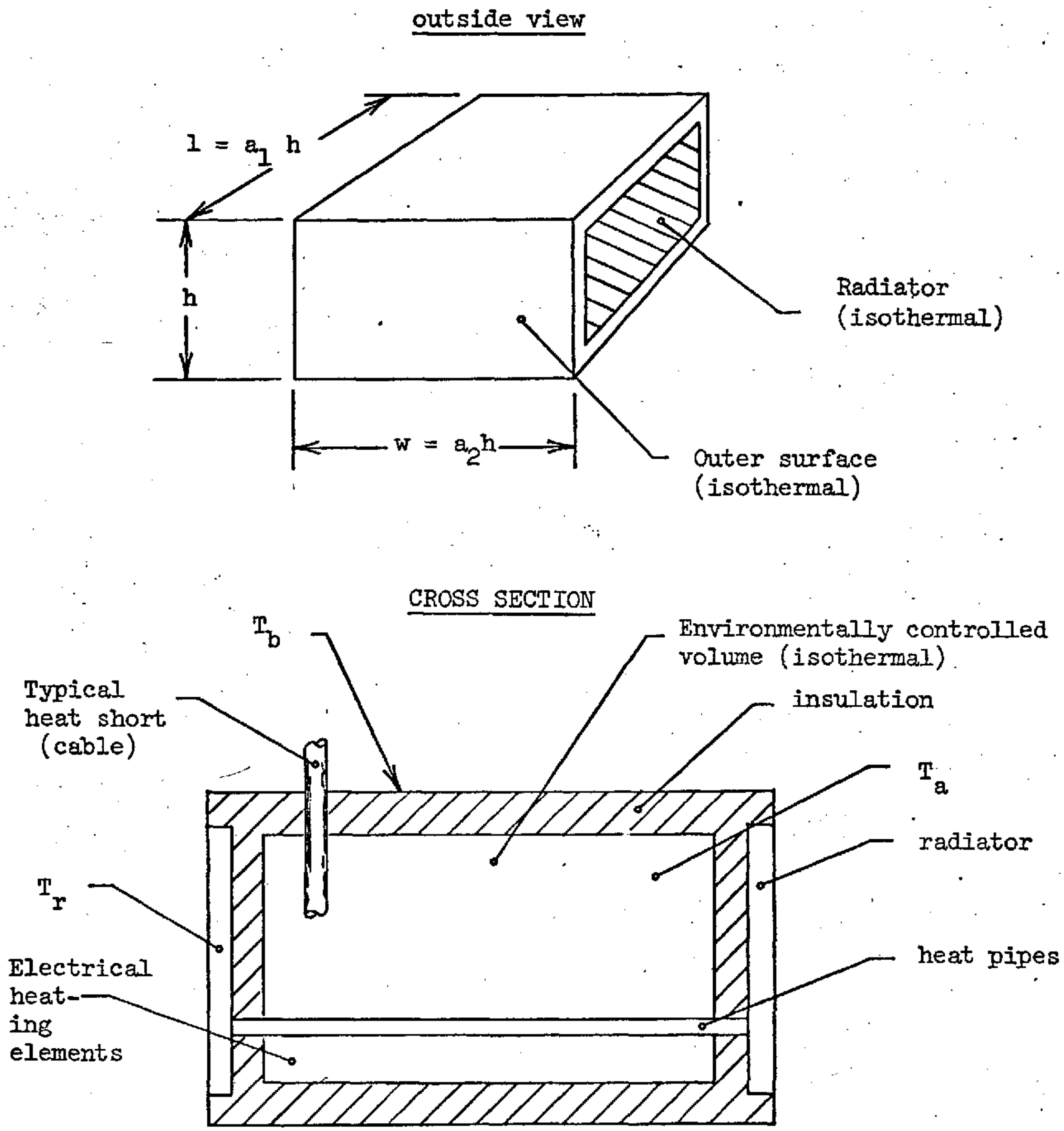


FIGURE 7

THERMAL CONTROL SUBSYSTEM: BASIC CONFIGURATION

TABLE 4

PARAMETER	SYMBOL	UNITS
Maximum Heater output	Q_h	watts
Radiator area	A_r	m^2
Insulation thickness	L_i	m
Heat pipe cooling capacity	K_q	watts/K
Weight	W_θ	kg
Night skin temp	T_{bn}	K
Day skin temp	T_{bd}	K
Night radiator temp	T_{rn}	K
Day radiator temp	T_{rd}	K

INPUT PARAMETERS

(FUNCTIONS OF OTHER SUBSYSTEM VARIABLES)

PARAMETER	SYMBOL	UNITS
Total package surface area	A	m^2
Daytime internal dissipation	Q_{id}	watts
Night internal dissipation	Q_{in}	watts

THERMAL CONTROL SUBSYSTEM: PARAMETER LIST

Mars-ambient temperature = T_m

Area of surface which radiates heat = A_{sr}

Radiometric Albedo = .295 = a

Incident solar energy = Q_{sol}

Radiated heat = Q_{rad}

Convective heat loss = Q_{conv}

Conductive heat loss = Q_{cond}

Insulation conductivity = $k_i = 0.0216 \ \frac{\text{watts}}{\text{m}\ {}^{o}\text{K}}$

Surface emissivity = $\epsilon_s = 0.8 = \epsilon_r$

Surface absorptivity = $\alpha_s = 0.5 = \alpha_r$

Incident solar energy (Solar Constant) = $S_c = 235 \ \frac{\text{BTU}}{\text{hrft}^2} = 720 \ \frac{\text{watts}}{\text{m}^2}$

Average convective transfer coefficient = h_c

Stephen - Boltzman Constant = $\sigma = 5.67 \times 10^{-8} \ \frac{\text{watts}}{\text{m}^2\text{K}^4}$

The heat transfer equations are [62,63]:

$$Q_{cond} = (A-A_r)\ (k_i/L_i)\ (T_a-T_b)\ ,$$

$$Q_{conv} = (A-A_r)\ h_c\ (T_b-T_m)\ ,$$

$$Q_{rad} = (\epsilon_s \sigma A_{sr} T_b^4) - (\epsilon_s \sigma A_{sr} T_m^4)$$

and

$$Q_{sol} = \alpha_s\ (A_{sun(s)} + a\ A_{alb(s)}) S_c\ .$$

For equilibrium the heat input must equal the heat output (i.e., zero heat build-up), therefore,

$$Q_{cond} + Q_{sol} = Q_{conv} + Q_{rad}$$

Substituting values for the variables in this equation yields the final heat balance equation:

$$(A - A_r)(k_i/L_i)\ (T_a - T_b) + \alpha_s\ (A_{sun(s)} + a\ A_{alb(s)})\ S_c$$

$$= (A - A_r)\ h_c\ (T_b - T_m) + \epsilon_s \sigma A_{sr}\ T_b^{\ 4} - \epsilon_s\ \sigma\ A_{sr} T_m^{\ 4}$$

The weight of the thermal control subsystem in kg , W_Θ , can be derived from the geometry of the package and density of its components as:

$$W_\Theta = 55.5\ AL_i + 7.28\ A_r + 3.64\ (\frac{a_1 a_2}{a_{12}})\ A + 0.1\ A_r\ K_q . \qquad (4.10)$$

Equation (4.13) defines a_{12} .

To relate equations of this form to the MRV problem, it is helpful to make some preliminary definitions. Let A_T be the area of the top and the two sides of the equipment package which do not have radiators, and A_{TR} be A_T plus the area of one radiator; then,

$$A_T = (1 - \frac{a_1 a_2}{2a_{12}})\ A - Ar \qquad (4.11)$$

$$A_{TR} = A_T + \frac{A_r}{2} \qquad (4.12)$$

where

$$a_{12} = a_1 + a_2 + a_1 a_2 . \qquad (4.13)$$

The worst-case effective incident areas of illumination of the surface not including radiators ("s") and the radiator area ("r") by direct ("sun") and reflected ("alb") solar radiation can be written as:

$$A_{sun(s)} = \frac{\sqrt{a_1^{\ 2} + a_2^{\ 2} + a_1^{\ 2}\ a_2^{\ 2}}}{2a_{12}}\ A \qquad (4.14)$$

$$A_{alb(s)} = \frac{2a_1 + a_2}{2a_{12}}\ A \qquad (4.15)$$

$$A_{sun(r)} = 0 \tag{4.16}$$

$$A_{alb(r)} = \frac{A_r}{2} \tag{4.17}$$

Note that the last two equations imply that a radiator exposed to direct solar radiation will be shut off. With these variables the six heat balances can be written.

$$A_T \epsilon_s \sigma T_{bn}^4 + (A - A_r)\, h_c\, (T_{bn} - T_{low}) =$$

$$A_T\, \alpha_{si}\, \sigma\, T_{low}^4 + (A - A_r)\, \frac{k_i}{L_i}\, (T_{intn} - T_{bn}) \tag{4.18}$$

$$A_r \epsilon_r \sigma T_{rn}^4 + A_r\, h_c\, (T_{rn} - T_{low}) =$$

$$A_r\, \alpha_{ri}\, \sigma\, T_{low}^4 + A_r\, \frac{k_i}{L_i}\, (T_{intn} - T_{rn}) \tag{4.19}$$

$$Q_h + Q_{in} = A_r\, \frac{k_i}{L_i}\, (T_{intn} - T_{rn}) + (A - A_r)\, \frac{k_i}{L_i}\, (T_{intn} - T_{bn}) \tag{4.20}$$

$$A_{TR} \epsilon_s \sigma T_{bd}^4 + (A - \frac{A_r}{2})\, \frac{k_i}{L_i}\, (T_{bd} - T_{intd}) + (A - \frac{A_r}{2})\, h_c\, (T_{bd} - T_{hi}) =$$

$$A_{TR} \alpha_{si}\, \sigma\, T_{hi}^4 + \alpha_s \left[A_{sun(s)} + a\, A_{alb(s)} \right] S_c \tag{4.21}$$

$$\frac{A_r}{2}\, \epsilon_r\, \sigma\, T_{rd}^4 + \frac{A_r}{2}\, h_c\, (T_{rd} - T_{hi}) = \frac{A_r}{2}\, \alpha_{ri}\, \sigma\, T_{hi}^4 +$$

$$\alpha_r \left[A_{sun(r)} + a\, A_{alb(r)} \right] S_c + \frac{1}{e_t}\, K_q\, (T_{intd} - T_{rd})$$

$$+ \frac{A_r}{2}\, \frac{k_i}{L_i}\, (T_{intd} - T_{rd}) \tag{4.22}$$

$$Q_{id} + \left(A - \frac{A_r}{2}\right) \frac{k_i}{L_i} (T_{bd} - T_{intd}) =$$

$$K_q(T_{intd} - T_{rd}) + \frac{A_r}{2} \frac{k_i}{L_i} (T_{intd} - T_{rd}) \qquad (4.23)$$

where

α_r = absorptivity of radiator surface (solar)

ϵ_r = emissivity of radiator surface (solar)

α_{si} = absorptivity of package surface (infrared)

α_{ri} = absorptivity of radiator surface (infrared)

T_{intd} = maximum permissible internal temperature

T_{intn} = minimum permissible internal temperature

T_{hi} = maximum Mars-ambient temperature

T_{low} = minimum Mars-ambient temperature

e_t = efficiency of heat transfer.

Total power consumption, P_θ , by the thermal control subsystem during the day is:

$$P_\theta = \frac{1}{e_t} K_q (T_{intd} - T_{rd}) , \qquad (4.24)$$

while the power requirement at night is the electrical heater requirement, Q_h .

4.1.1.5 Navigation Subsystem

Navigation is taken to mean the location of vehicle position with respect to a set of coordinates centered in Mars. The scheme considered for first analysis is one devised by a group of the RPI-MRV project at Rensselaer Polytechnic Institute.

The coordinate system is established by instruments which locate what would be the position of a true pole star of Mars [64] and the direction of local vertical [65]. The initial estimate of position is obtained by tracking an orbiter with known orbital parameters [66,67]. A direct velocity sensor [68] measures vehicle velocity relative to the surface in a body-bound frame. A system for updating the estimate of position with vehicle movement [69] has been devised.

Ideally, modeling of the navigation subsystem would include equations describing how power and weight allocations to the equipment affect the accuracy of the subsystem. In addition, the error in detecting local vertical ($\Delta\beta$) has a direct effect on the obstacle avoidance (terrain sensing and path selection) subsystem. However, because:

1. the form of these equations appears to be complex, and the time required to derive them considerable,
2. an error in position location does not directly affect the operation of any other subsystem, and
3. the error in local vertical is fairly invariant for forseeable values of the design parameters,

it was decided to allocate certain constant values of power and weight to the navigation subsystem, and make a worst case estimate of the local vertical detection error. Weight and power allocations appear in Table 5. The local vertical error is assumed to be 0.25° [65]. Thus, the navigation subsystem does not appear in any of the system model equations.

TABLE 5

Device	Weight, kg	Power, watts
pole star detector	3	1
local vertical sensor	3	2
laser (ranging to orbiter)	2	15
position update (gyrocompass, velocity sensor)	2	3
platform, motors (torquors)	5	0
Total	15 (W_{nav})	*

NAVIGATION SUBSYSTEM: POWER AND WEIGHT ALLOCATIONS FOR SUBSYSTEM COMPONENTS

* not applicable, the laser is in operation only a few seconds per day (present estimate is 3 seconds every 2.5 hours). Let $P_{nav} = 6$ watts.

4.1.1.6 Obstacle Avoidance Subsystem

The obstacle avoidance subsystem is responsible for identifying terrain hazards and choosing a safe path for travel by the vehicle. The system considered utilizes a laser rangefinder which scans the terrain in front of the moving vehicle in repeating arcs and determines the height of the terrain at the sensed points [4]. This method can be modified to estimate slopes by assuming the terrain is linear between sensed points. This information is utilized by a dual-mode routing algorithm [71]. The algorithm assumes that previous fly-by and orbiter missions have sufficiently mapped the surface so that a coarse path (segments on the order of kilometers) can be pre-programmed. Local deviations in the coarse path are achieved by following the outer contour of all obstacles encountered.

Preliminary analysis demonstrated that the errors caused by changes in power and weight allocations to the subsystem would be small compared to errors inherent in the method which are due primarily to errors in the detection of local vertical [72,73,74]. A weight allocation (W_{oa}) of 5 kg, and a continuous power draw (P_{oa}) of 15 watts were chosen.

An error in estimating the height of a portion of the terrain (Δh_t) can be written:

$$\Delta h_t = r_a \sin \Delta\beta \cong r_a \Delta\beta$$

where r_a = horizontal distance to sensed terrain point. When calculating estimates of terrain slopes, the worst case error (ϵ_{sl}) can be shown to be:

$$\epsilon_{s1} = \frac{2\, r_a\, \Delta\beta}{\delta} \quad \text{degrees,}$$

where δ = horizontal separation between the terrain points used in slope approximations, meters.

Slope segments become a real concern when their span approaches the wheelbase of the vehicle. Because the nominal separation between sensed terrain points (in the direction of vehicle travel) will be much smaller than the wheelbase, it is feasible to consider only sets of points such that:

$$\epsilon_{s1} = \frac{2\, r_a\, \Delta\beta}{w_b}\text{; i.e., } \delta = w_b$$

where w_b = wheelbase of the vehicle. For all succeeding work, it was assumed (as per Ref. 4) that r_a = 30 meters.

To find the effect of the error on vehicle travel, a model of the Mars terrain is required. Ref. 75 establishes that the probability that a terrain segment of 61 m interval will have an average slope less than or equal to s (in degrees) is:

$$P(S \leq s) = \int_0^s 0.17\, e^{-.17\lambda}\, d\lambda\ .$$

The distribution for slopes with smaller span can be assumed equivalent [76].

The percentage of terrain impassable for the vehicle on the same scale as the vehicle wheelbase (T_{act}) is a function of the maximum slope the vehicle will be allowed to traverse(s^*):

$$T_{act} = \int_{s^*}^{\infty} .17\ e^{-.17s}\ ds\ ,$$

but considering the error the vehicle will make in interpreting slopes, the percentage terrain <u>considered</u> impassable by the vehicle (T) will be:

$$T = \int_{s^*-\epsilon_{s1}}^{\infty} .17\ e^{-.17s'}\ ds \qquad (4.25)$$

where, again in this case, the error is assumed to have a worst case effect. Note that T is a function of s^*, r_a, $\Delta\beta$, w_b and the Martian terrain model.

The dual-mode routing algorithm requires that a coarse path be chosen prior to the mission. This large-scale path will be determined basically by the crater distribution on Mars. To a good approximation, it will not be a function of vehicle capability, but will simply be a path chosen to detour around craters. Recent orbiter data [76] shows that the percentage of terrain area encompassed by craters is approximately 50%. Because the average crater wall is too steep for safe vehicle travel, that portion of the terrain will be considered impassible in the large-scale case. For small-scale deviations from the large-scale path, T will be determined by slope distributions and the maximum slope the vehicle will be allowed to traverse.

Considering both these cases jointly, the modeling procedure requires a measure of how efficient the obstacle avoidance subsystem is as a function of the parameters discussed above. A useful descriptor is the path-length ratio (PLR), defined as the ratio of actual path length to straight-line (great circle) distance.

Simulation was employed to determine PLR for both cases. Given that the vehicle is at a point on the terrain and wishes to travel in the $\theta = 0^{\circ}$ direction, the probability that it will travel in the θ direction, $p(\theta)$, would have the form of Figure 8. (Theta is dimensionless; there are only a finite number of possible directions.) Briefly, this is due to the fact that the vehicle looks for a free path by considering directions in the following order: 0,1,-1,2,-2,3,..... The probability of $\theta = 0$ (i.e., the probability of traveling in the desired direction) can be assumed 1-T if the step size is not too much greater than the obstacle size. Given that $\theta = 0$ is not a free path, the probability of 1 or -1 being free is small (obstacles have size). As the scan gets further away from the known obstacle, the probability of the path being free should increase. Finally, as $|\theta|$ gets large, $p(\theta)$ should decrease because a large $|\theta|$ will only be chosen if all smaller (in $|\theta|$) paths are blocked. The problem with assuming this type of distribution is that the statistics of the "humps" are functions of statistics of the obstacles, which are unknown for Mars.

For purposes of simplification, the simulation used a distribution with $p(0) = 1 - T$ and all other probabilities equal. If $\delta\theta$ is defined as the angular deviation between possible paths, let $\delta\theta = 5^{\circ}$ be assumed (this gives a separation of approximately 3m at the maximum laser range for $r_a = 30$ m). Then,

$$p(i5^{\circ}) = \begin{cases} 1 - T, & i = 0 \\ T/70, & i = \pm 1, \pm 2, \ldots, \pm 35 . \end{cases}$$

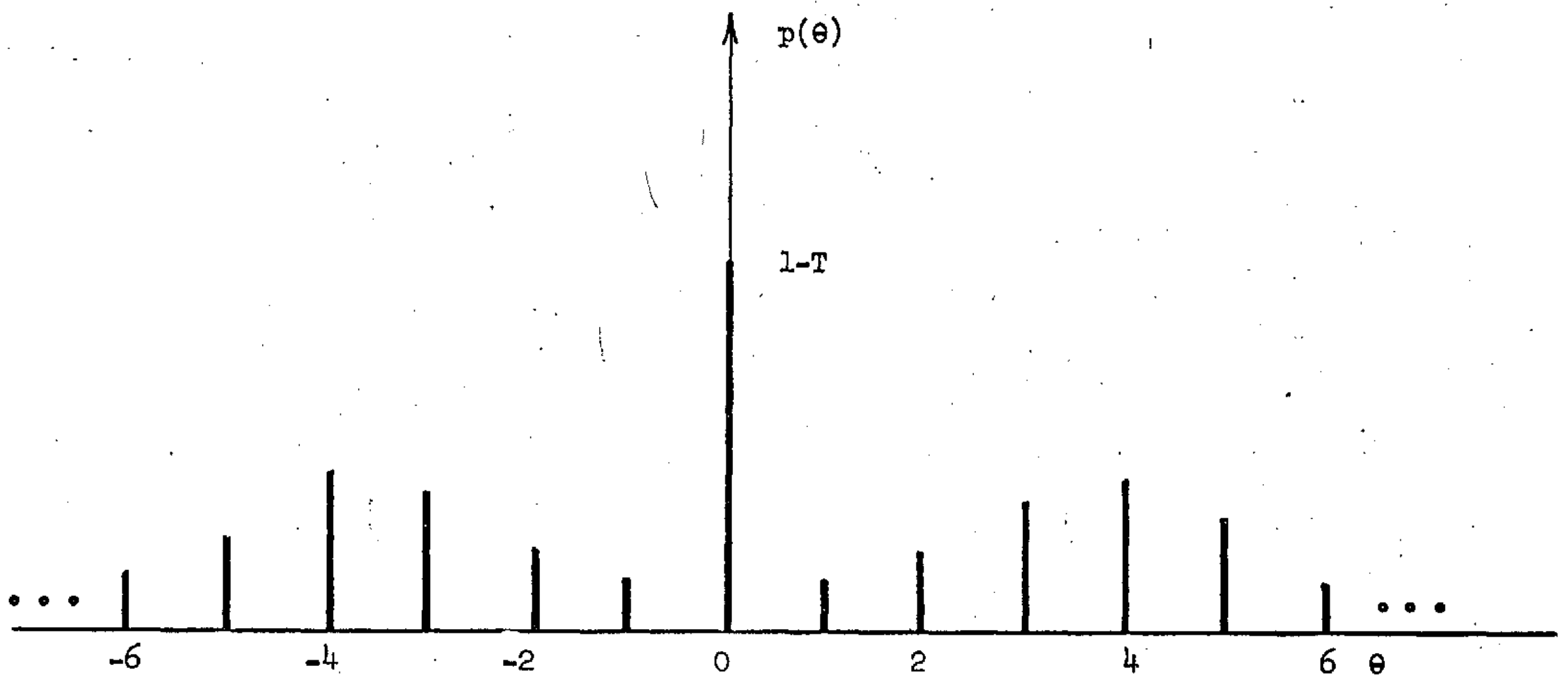


FIGURE 8

OBSTACLE AVOIDANCE SUBSYSTEM: TYPICAL CHOICE-OF DIRECTION PROBABILITY FUNCTION

A Monte-Carlo simulation computer program simulated travel from (0, 0) to (1, 0) in Cartesian coordinates. The variables in the simulation were T and r (the step size, analagous to r_a). For the large-scale path, T = 0.50 as previously established, and since the simulation required that the step size approached the average obstacle size, r = 25 km/1000 km = 0.025 (the average Mars crater is 16.3 km, with heavy debris outside the edge; the mission range will hopefully approach 1000 km). For small-scale path deviations, T is a running variable. The value of $r = w_b/25$ km, or 0.00012 if $w_b = 3$ m. Table 6 reports the results (averages) of many simulations at varying T and r.

The next step was fitting the data of Table 6 with a continuous function for use in the model. The total PLR is the product of the large-scale and small-scale PLRs. Therefore, at T = 0, PLR should be 2.0. At T = 1, PLR must approach infinity. The function

$$PLR = \frac{2}{1 - T}$$

fits this form, but was not sufficiently accurate for intermediate values. The function

$$PLR = \frac{2\ (1 + 0.05\ T + 0.167\ T^2)}{1 - T} \qquad (4.26)$$

fits all data points within 7%. Figure 9 compares the simulation results with the functional approximation.

A PLR simulation of a different approach by Eisehardt and Murtaugh [77] originally applied to a Surveyor (lunar) mission, gives small-scale PLRs which vary from deviations 4% lower at low T (.20)

TABLE 6

Large-scale

T	r	PLR
.50	.025	1.99

Small-scale
r=0.00012

T	PLR
.20	1.28
.30	1.41
.40	1.61
.50	2.08
.60	2.67
.70	3.85

OBSTACLE AVOIDANCE SUBSYSTEM: RESULTS OF PATH-LENGTH RATIO SIMULATION

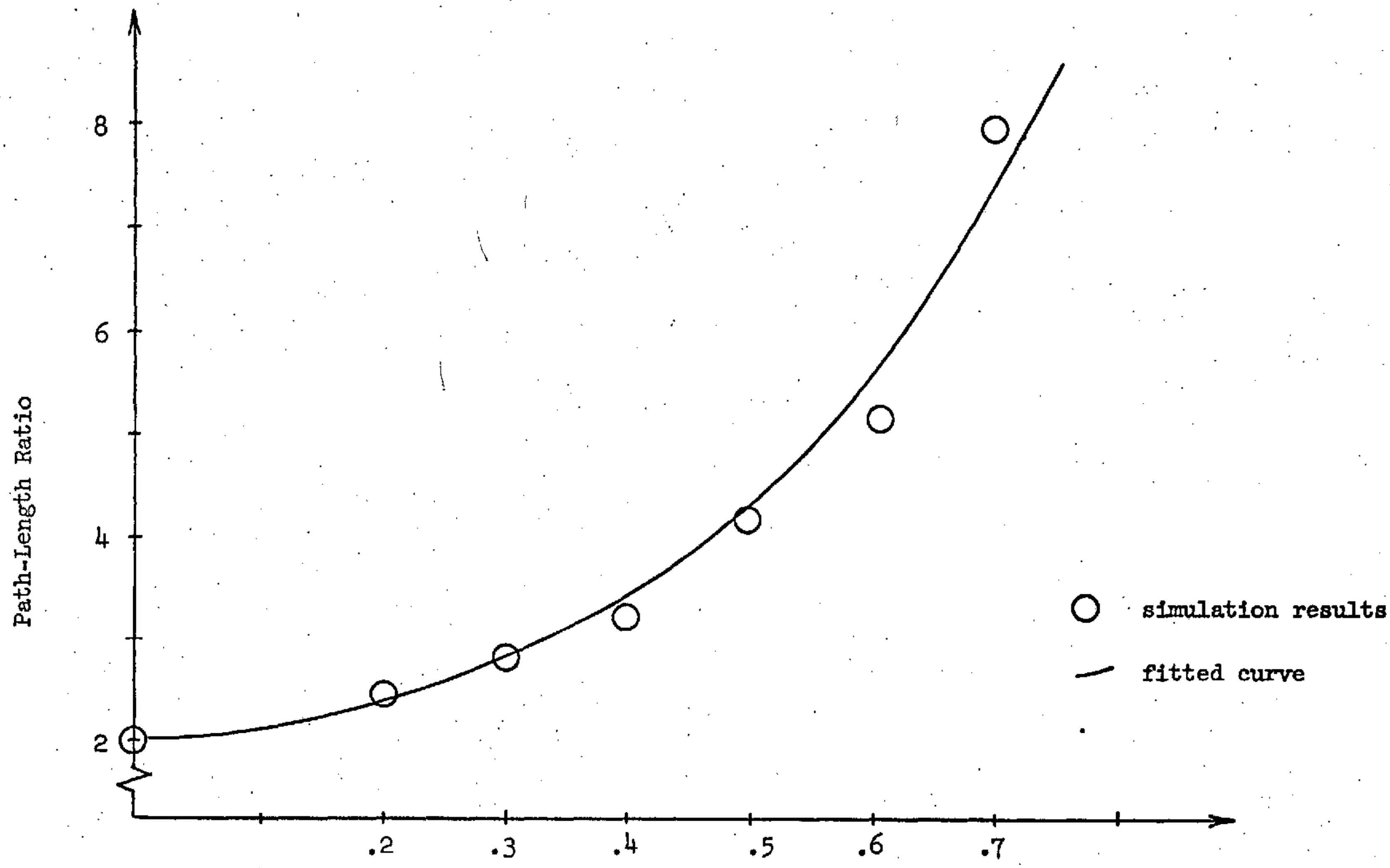


FIGURE 9

OBSTACLE AVOIDANCE SUBSYSTEM: PATH LENGTH RATIO VS. PERCENT IMPASSABLE TERRAIN (SIMULATION AND FUNCTIONAL APPROXIMATION) monotonically increasing to 15% lower at high T (.60) as compared with results presented here.

4.1.1.7 Computation and Data-Handling Subsystem

The onboard computational and data-handling requirements for a semi-autonomous MRV are succinctly stated and explained in [3]. Briefly stated, they are:

1. conditioning of onboard sensor data
2. navigation, guidance and special sensor (antenna, celestial) pointing computations
3. terrain modeling, path selection and motion control commands
4. energy bookkeeping and management functions regarding the vehicular state
5. logic for event sequencing and synchronization sequencing of the total vehicle system.

The data-handling subsystem for the Thermoelectric Outer Planet Spacecraft (TOPS) meets the MRV requirements, and exceeds the life-time requirement by a factor of ten [4]. Table 7 presents power, weight and volume data for the TOPS subsystem. These numbers will be considered constant inputs to the MRV model. Data from Refs. 78 and 79 indicates the validity of this approach.

4.1.1.8 Vehicle Structure Subsystem

There are a number of candidate vehicles for a roving exploratory Mars mission. Both 4-and 6-wheeled vehicles have been proposed. The AC Electronics Division of the General Motors Corp. [80] and McDonnell

TABLE 7

Component	Weight, kg	Power, watts	Volume, in^3
flight data subsystem	12.7	25	1000
centralized computer subsystem	22.7	50	1500
data storage subsystem	11.4	15	500
Total	46.8 (W_{cp})	90 (P_{cp})	3000 (V_{cp})

COMPUTATION AND DATA-HANDLING SUBSYSTEM: POWER AND WEIGHT ALLOCATIONS FOR SUBSYSTEM COMPONENTS

Astronautics [81] have studied 6-wheeled mobility subsystems. Work at Rensselaer Polytechnic Institute under the RPI-MRV project has led to the proposal of a 4-wheeled vehicle with an optional 3-wheeled mode [82]. It is this latter version that is considered toward formulating this system model. Figure 10 shows a simplified sketch of the concept.

Because the RPI-MRV is dynamically scaled, all major dimensions are dependent; defining

w_b = wheelbase or front-to-rear distance between wheels

t = track or side-to-side distance between wheels

W_v = weight (frame, suspension, motors)

V_v = equipment package volume,

the following relationships hold:

$$w_b = t \tag{4.27}$$

$$\left[\frac{t}{t_o}\right]^3 = \frac{W_v}{W_{v_o}} \tag{4.28}$$

$$V_{min} \leq V_v \leq V_{max} \tag{4.29}$$

where the subscript zero indicates the nominal design values, which are

t_o = 10 ft = 3.04 m

W_{v_o} = 400 lbs = 182 kg .

V_{min} and V_{max} are functions of the vehicle size and amount of equipment onboard. Their values are discussed in Section 5.

The slope climbing and other obstacle capabilities of the

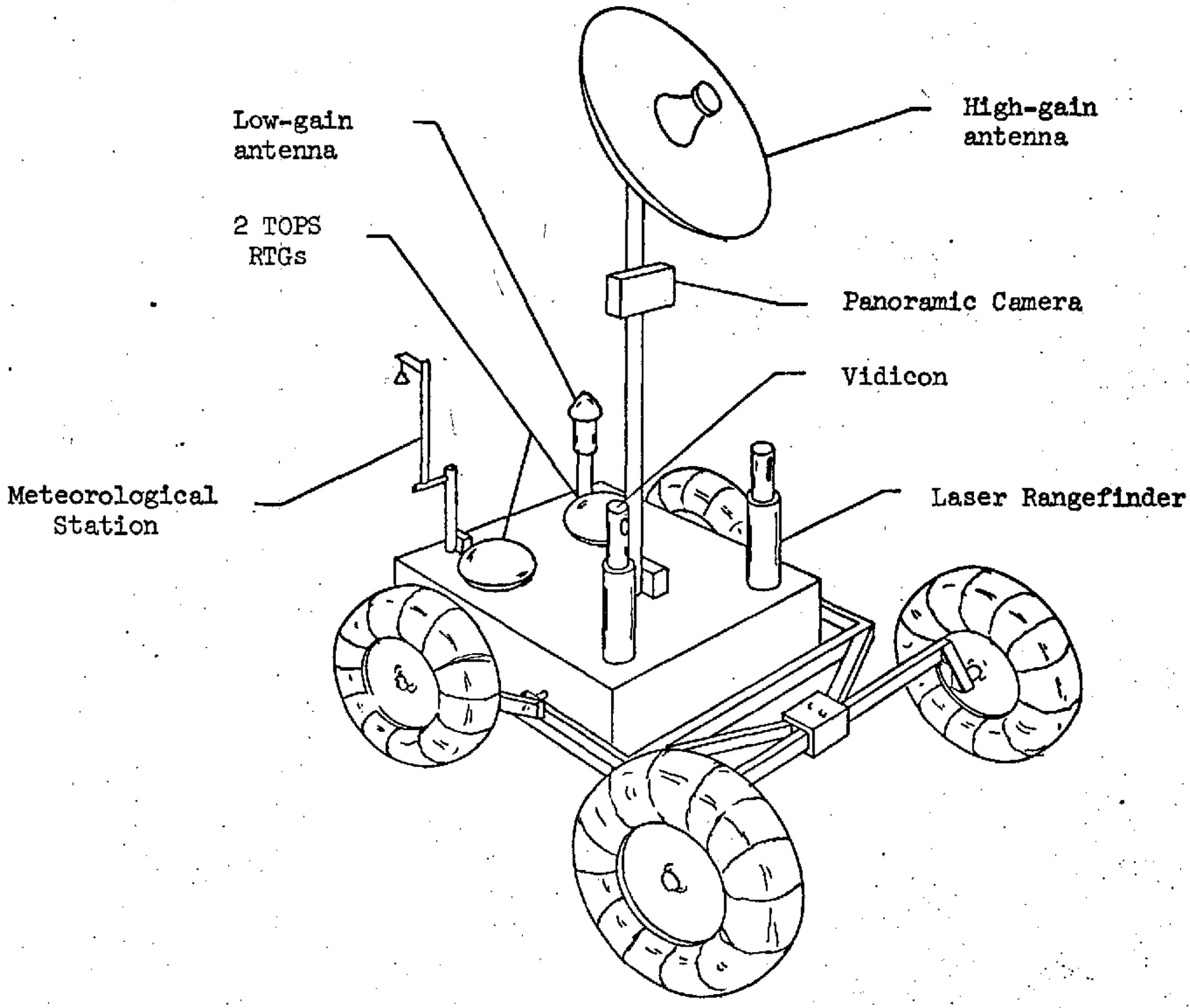


FIGURE 10

VEHICLE STRUCTURE SUBSYSTEM: SKETCH OF FOUR-WHEELED MARS-ROVING VEHICLE CONCEPT vehicle are such that they should not be the limiting factors in choosing the optimal design. The power requirements for slope climbing will probably be the limiting factor. This is a supposition which may require refinement or change during the actual optimization process.

4.1.2 System Constraints

This section regarding system constraints completes the identification and formulation of the equality and inequality constraints between the design parameters. The separation of this work from the work on subsystem models derives from the fact that the relations sought here are not basically indigenous to any one subsystem, but represent realizability and Mars-deliverability constraints that hold between parameters of different subsystems. In this section, we use previously derived relations as substitutions for the first time.

The equation for P_{prop}, eqn.(4.8), contains the total system mass (M_r) which can be written as a function of the subsystem parameters:

$$M_r = \frac{\sum \text{ weights of all subsystems in kg}}{9.806 \text{ m/sec}^2}$$

$$= 0.1020\left[W_{com} + W_{sci} + W_p + W_\theta + W_v + W_{cp} + W_{oa} + W_{nav}\right]$$

$$= 0.1020\left[W_{com} + W_{sci} + W_p + W_\theta + W_v + 66.8\right] \qquad (4.30)$$

To describe internal heat dissipation and power uses at various times, the power-use profile must be established. The vehicle system will normally operate in one of four modes:

1. rove
2. recharge
3. science and communication
4. minimal operation (idle).

Minimal operation occurs during the period when communication between Earth and the vehicle is impossible due to the Mars-Earth configuration. For early 1980's missions, this coincides with Mars night. At this time, only necessary functions (thermal control, navigation checks, computer control of system functions) and recharging are permissible. At all times, 20% of total science power is allotted toward maintaining ongoing science functions (sample treatment, experiment monitoring). Thus, minimal power consumption, which will also be the internal heat dissipation, for this period is:

$$Q_{in} = P_{cp} + 0.5\ P_{nav} + 0.2\ P_{scia}\ . \qquad (4.31)$$

To insure that thermal control functions are possible during this period, it is required that

$$P_{RTG} \geq Q_h + Q_{in}. \qquad (4.32)$$

Likewise, power consumptions during recharge (P_{str}) and rove (P_{mv}, excludes P_{prop}) are:

$$P_{str} = P_{nav} + P_{cp} + 0.2\ P_{scia} + 0.1\ P_{com} + P_{\theta} \qquad (4.33)$$

$$P_{mv} = P_{str} + P_{oa} \tag{4.34}$$

where the 10% communication power allotment is for the continuous transmission of engineering data. Internal heat dissipation (Q_{id}) during modes 1-3 can be approximated by:

$$Q_{id} = P_{cp} + 0.2\ P_{scia} + P_{nav} + 0.05\ P_{com} + 0.25\ P_{oa}\ . \tag{4.35}$$

The total roving time between recharges (T_{rov}) must be found to describe vehicle operation. First, RTG power is set at

$$P_{RTG} = P_{prop}\Big|_{s^*} + P_{mv}$$

$$= M_r\ g_m v_f\ \mu_k + \sin\ (s^* + 2^o)\ + P_{mv} \tag{4.36}$$

where s^* is the slope threshold used by the obstacle avoidance subsystem. Actually, the proper formulation of the P_{RTG} equation would be

$$P_{RTG} = \max\ (P_{prop}\Big|_{s^*} + P_{mv}\ ,\ Q_h + Q_{in})$$

but, preliminary calculations showed that roving requirements would be higher and so this equation was separated into (4.32) and (4.36). This was not true for the 6-wheel model (see part 4.2). Then, since the RTG can handle all normal roving loads, the batteries will be utilized only when the vehicle exceeds the slope threshold (s^*). From this, the relation

$$T_{rov} = \frac{\left[\dfrac{1 - e^{-.17\left(s^* + \frac{2r_a \Delta\beta}{w_b}\right)}}{e^{-.17s^*} - e^{-.17\left(s^* + \frac{2r_a \Delta\beta}{w_b}\right)}} \right] E_{fd}\, E_{batt}}{M_r g_m v_f \left(\sin\left(\frac{2r_a \Delta\beta}{w_b} + 2^\circ\right)\right)} \tag{4.37}$$

follows, where the square-bracketed term is the inverse of the percentage of the time the vehicle can expect to spend on slopes exceeding s^*. Note that T_{rov} is largely a function of errors in slope detection. The requirement that the vehicle be able to support the weight of the other subsystems can be written:

$$\frac{W_{com} + W_{sci} + W_p + W_\theta + W_{nav} + W_{oa} + W_{cp}}{W_v} \leq 2.0 , \tag{4.38}$$

where 2.0 is called the equipment weight ratio, and is a constant for any given vehicle configuration.

The constraints identified so far represent real physical limitations upon the interrelationships of the parameters. These constraints are inherent to the system itself. External constraints, those placed upon the system by influences other than those which guarantee that the system will be physically realizable, have not yet been considered, except for eqn. (4.2).

The cost of research, development, and construction of the system is a major factor, but it is outside the scope of this study. Another factor is the requirement that the system be deliverable to the surface of Mars. This imposes weight, volume, and size limitations on the vehicle system. They are:

$$\sum \text{weights of all subsystems} \leq L_w \tag{4.39}$$

$$\sum \text{volumes of all subsystems} \leq A_v \tag{4.40}$$

$$1.7\, w_b \leq A_d \quad . \tag{4.41}$$

where L_w = maximum payload weight of launch vehicle

A_v = volume of aeroshell

and A_d = horizontal diameter of aeroshell.

4.1.3 System Evaluation (Objective) Function

Any optimization process requires that the system performance be measurable with respect to some standard. When the expression of measure (hereafter called the objective, or objective function) is written as a function of the design parameters, the optimal design problem becomes one of choosing the design parameters to extremize (maximize or minimize) the value of the objective function while assuring that the parameters meet all the equality and inequality constraints of the system model.

The expression for system evaluation, i.e., the objective function, is generally not unique. There may be many different factors one would like to make large or small, each of which describes a different aspect of the system operation. Generally, it is good practice to attempt to incorporate all of the basic system functions into the objective.

An MRV has two basic functions:

1. rove the surface of the planet, and
2. obtain and transmit science data.

Note, that the second function is actually a combination of two functions, but that the system model groups these two together by considering science time as the time required to experiment and communicate the results.

The objective must express the ability of the vehicle to perform both of these functions concurrently. In formulating the objective function, one must be careful not to allow either of these measures to go to "zero." A logical form, then, is to measure the system performance by the product of experimental science time and straight-line distance roved (D_{rov}). That is, denoting the objective function as "f":

$$f = T_{esci} D_{rov} .$$

Define a cycle as comprising the activities between the ends of two recharges. The time in a cycle will then be the sum of the time spent roving, the time to recharge, and the total time spent on science and communication between recharges. The time spent on science and communication in a cycle can be expressed as:

$$T_{sci/cy} = T_{sci} S_{sci} v_f T_{rov} hr ,$$

where S_{sci} = number of science stops per meter of actual distance traveled.

The total time for a cycle (T_{cy}) will be:

$$T_{cy} = T_{sci} S_{sci} v_f T_{rov} + T_{rov} + T_r \text{ hr} .$$

If V is the number of hours in a Martian day during which communication between the vehicle and Earth is possible, the number of cycles in a Martian day (N_{cy}) is:

$$N_{cy} = \frac{V}{T_{cy}}$$

Since recharging is always possible during the shut-down operation at the end of a vehicle "day", it is reasonable that

$$T_{cy} \leq V . \qquad (4.42)$$

Because the time spent communicating the science information back to Earth is non-productive in the sense that other vehicle activities must cease, it is reasonable to wish to deal with scientific experimentation time (T_{esci}) instead of total science time. This time per cycle is:

$$T_{esci/cy} = T_{esci} S_{sci} v_f T_{rov} .$$

The straight-line distance roved in a cycle is:

$$D_{rov/cy} = \frac{v_f T_{rov}}{PLR}$$

on an average "daily" basis then

$$f = T_{esci/cy} D_{rov/cy} \frac{V^2}{{T_{cy}}^2}$$

or in terms of the parameters of the system model:

$$f = \frac{T_{esci}\, S_{sci}\, v_f^{\,2}\, T_{rov}^{\,2}\, V^2}{PLR\left[T_{sci}S_{sci}v_fT_{rov} + T_{rov} + T_r\right]^2} \tag{4.43}$$

The value of S_{sci} in the solution to the optimization problem will be part of the optimal operating policy for the vehicle. It will be the optimal manner of determining when the vehicle should stop for science investigation. This number can be pre-programmed for the mission and will have the effect of maximizing the product of distance roved and experimentation time for a vehicle designed with parameters equal to those in the optimized solution.

Note that since V is not a variable in the problem (i.e., it may take on many values according to the Earth-Mars configuration, but for any run of the problem it is considered a constant, perhaps the average over the mission lifetime) it has no effect upon the determination of the optimal design. Maximizing f is equivalent to maximizing f/V^2. But also note that this is true solely because of the form of the objective function, and it is possible that a different formulation for the system objective would result in the optimal design being dependent upon V .

4.2 SIX-WHEELED, DIRECT-COMMUNICATING ROVER SYSTEM MODEL

In this part, the modifications to the model presented in 4.1 (4-wheeled, direct communicating rover) necessary to describe the alternative case of a 6-wheeled, direct-communicating rover, are presented. The 6-wheeled rover concept has been under study by several aerospace firms (McDonnell Astronautics [81], AC Electronics [80]). Using these studies, the Jet Propulsion Laboratory has developed a 6-wheeled rover structural design [4]. It is this design that is of interest here. A sketch of the concept (courtesy of JPL) appears in Figure 11.

Model modifications are necessary for three subsystems. Obviously, equations of the vehicle structure are different. In addition, the 6-wheeled concept contains 3 equipment-carrying bays as opposed to one for the 4-wheeled concept.

One bay contains the RTGs. Another contains the science package, with the third holding all the remaining equipment. These latter two bays require temperature control, so the thermal control problem is much different in the 6-wheeled case. Finally, modifications are required to some power subsystem equations.

4.2.1 Thermal Control Modifications

As with the 4-wheeled vehicle, temperature control on each package was achieved through use of an active cooling system with two radiators on opposite sides of the rectangular compartments, and an electrical heater. In addition, it was assumed that if one package re-

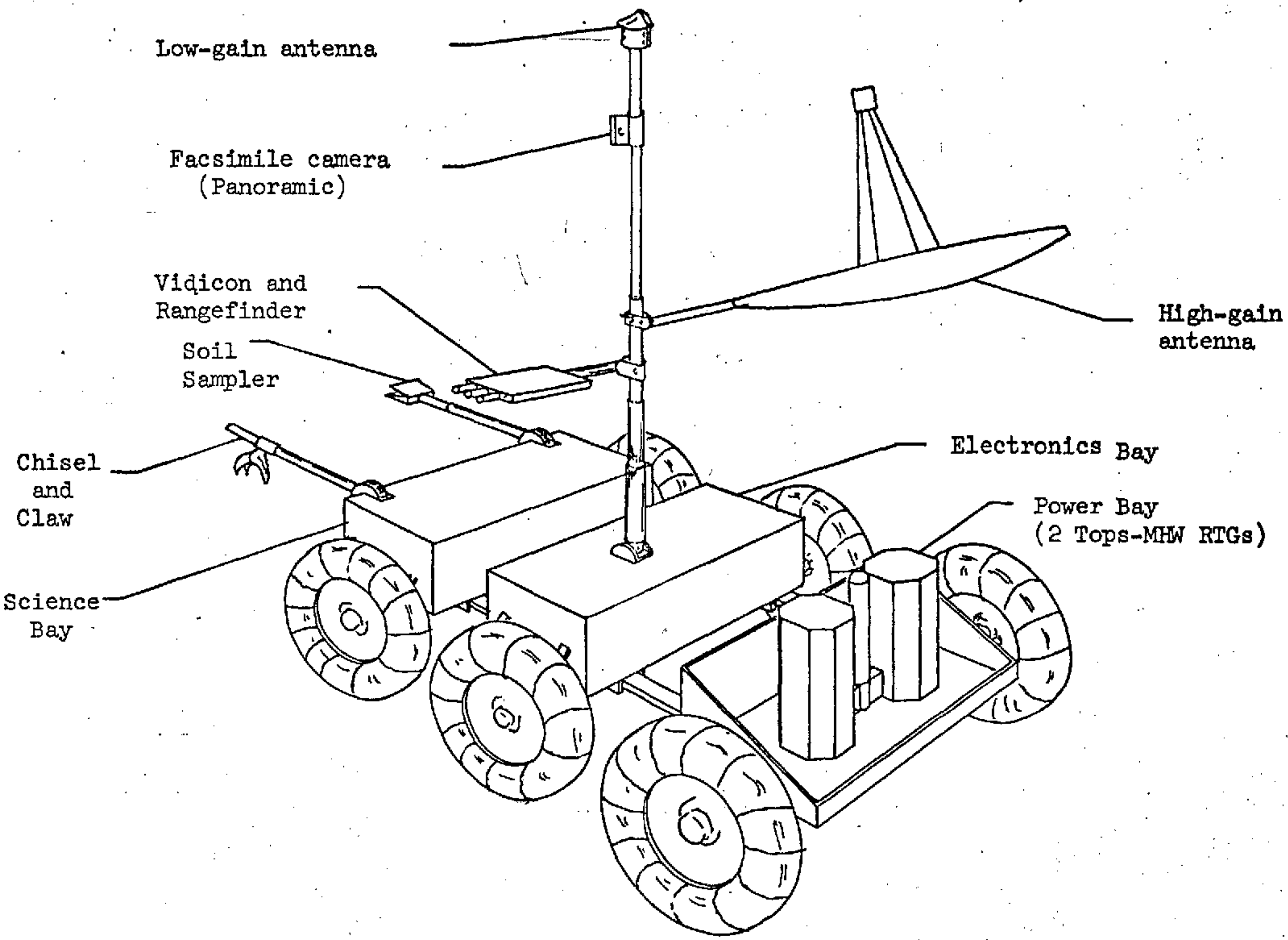


FIGURE 11

VEHICLE STRUCTURE SUBSYSTEM: SKETCH OF JPL
SIX-WHEELED MARS ROVER CONCEPT

quired heating at the same time as the other required cooling, the heat rejected from one package could be used to heat the other. Because the equations consider worst-case incident radiation effects, this heat transfer was assumed to occur with negligible loss.

Reviewing the modeling procedure for the 4-wheeled case - the steady-state equations describing the thermal control system were derived by noting the existence of three isothermal areas of the compartment: the radiators, the body skin, and the interior. There were two ambient conditions under which temperature control was required; day and night conditions. Constructing heat balances for each of the areas under both conditions yielded 6 equations (4.18-4.23).

Applying the identical procedure to each of the two compartments in the 6-wheeled case yields similar equations for each compartment (e.g. (4.18) through (4.23) can each be rewritten twice with subscripts denoting the appropriate compartments). These 12 new equations neglect cross-radiative terms which can be shown to be considerably smaller than all other terms in the equations.

The weight, power requirement, and internal heat dissipations are written analogous to the 4-wheeled case where 1 and 2 subscripts denote the separate compartments

$$W_\theta = 55.5\,A(L_{i1} + L_{i2}) + 7.28\,\frac{a_1 a_2}{a_{12}}\,A$$

$$+\ 0.2\,A_r\,(\,K_{q1} + K_{q2}\,) + 14.56\,A_r$$

$$P_\theta = \frac{1}{e_t} K_{q1}(T_{intd} - T_{rd1}) + K_{q2}(T_{intd} - T_{rd2})$$

$$Q_{id1} = 0.2 P_{scia}$$

$$Q_{id2} = P_{cp} + P_{nav} + 0.05 P_{com} + 0.25 P_{oa}$$

$$Q_{in1} = 0.2 P_{scia}$$

$$Q_{in2} = P_{cp} + 0.5 P_{nav} .$$

4.2.2 Vehicle Structure Modifications

Because an entirely different vehicle structure is considered here, equations and constants relating to the structure are modified. However, the dynamic-scaling property is maintained. Wheelbase (front-to-back distance between wheel centers for the first and last set of wheels) and track (side-to-side distance between two wheels of the same set) are no longer equal. In fact

$$w_b = \frac{90}{52} t .$$

This also requires that 'w_b' is replaced by 't' in the equations for T_{rov} (4.37) and T (4.25), since slopes will be calculated for base lengths corresponding to the smallest vehicle linear dimension. Structure weight is still described as a perturbation around a nominal design, but now

$$W_v = 182 \left(\frac{t}{1.32}\right)^3 .$$

Finally, the equipment weight ratio (ratio of equipment weight to structure weight) is lower for a 6-wheeled vehicle, so the 2.0 in (4.38) is replaced by 1.75. Again, all constants are determined from the JPL nominal design.

4.2.3 Power Subsystem Modifications

Referring back to the 4-wheeled model, the proper form of the equation setting the RTG power output was

$$P_{RTG} = \max\,(P_{prop} + P_{mv},\ Q_h + Q_{in})\ ,$$

and it was found, a posteriori, that for computer optimization in the 4-wheeled case this could be reduced to

$$P_{RTG} = P_{prop} + P_{mv}$$

$$P_{RTG} \geq Q_h + Q_{in}$$

because for all designs

$$P_{prop} + P_{mv} \geq Q_h + Q_{in}$$

Because the thermal control problem is significantly different in the 6-wheel case, the last inequality was not true for all designs and the proper simplified form of the RTG power equation varies from cast to case. Specifically, for a design where

$$P_{prop} + P_{mv} \geq Q_h + Q_{in}$$

the original equations can be used. But when

$$P_{prop} + P_{mv} \leq Q_h + Q_{in}$$

it is necessary to simplify the 'max' equation by

$$P_{RTG} \geq P_{prop} + P_{mv}$$

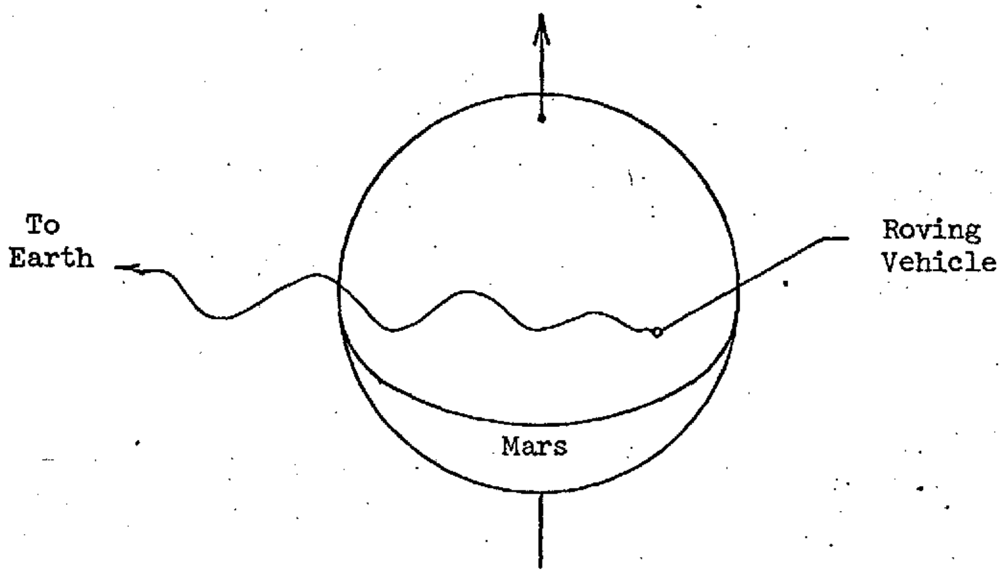
$$P_{RTG} = Q_h + Q_{in} .$$

Obviously, an alternative way to approach this is to write <u>both</u> P_{RTG} equations as inequalities, but since substitution was used to remove all equalities (i.e., eliminate as many variables as possible) before computer optimization, the procedure was chosen that would allow elimination of one additional variable.

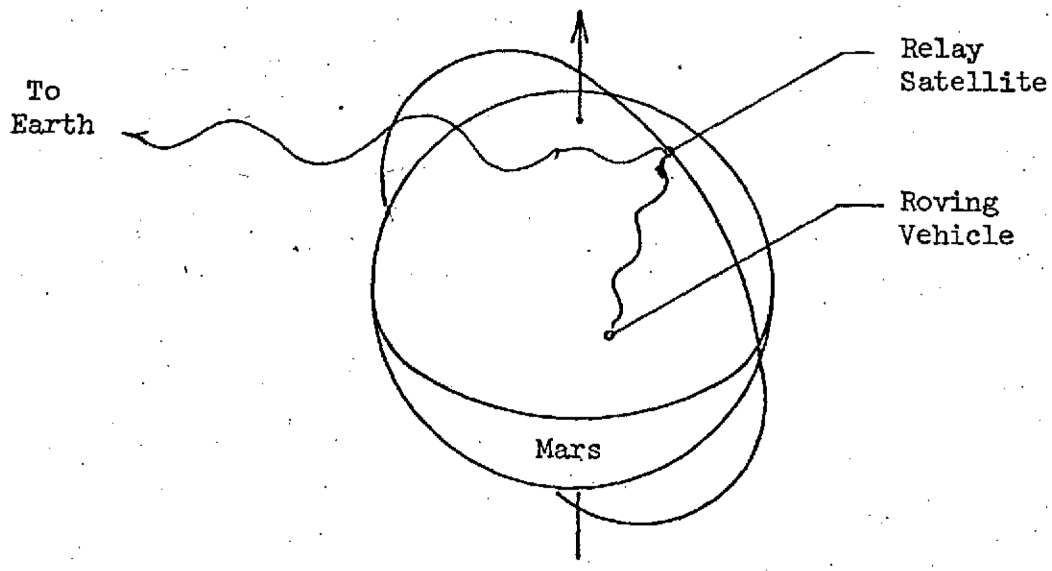
4.3 FOUR-WHEELED ROVER, COMMUNICATING VIA A MARS ORBITER, SYSTEM MODEL

An alternate model of the communications subsystem for a Mars roving vehicle mission was studied. As depicted in Figure 12-a, the originally considered direct system involved transmission directly from the vehicle to the Earth. The relay system now considered, as shown in Figure 12-b, involves transmission of the lander data to a planetary communications satellite and then to Earth. The satellite receives data from the surface sporadically at a high bit rate and re-transmits it to the Earth at a lower rate.

Aside from the modeling considerations to be employed, the relay system requires the design, orbiting and simultaneous operation of a communication satellite in addition to the landing and operation of the rover. This represents a considerable increase in complexity over the vehicle alone. The direct path scheme, on the other hand, must steer a highly directional antenna with limited power in a re-



(a) Direct System



(b) Relay System

FIGURE 12

DIRECT AND RELAY COMMUNICATION SYSTEMS
FOR A MARS ROVER

latively unknown environment characterized by high winds. Further complicating both alternatives is the fact that since the rover is on the surface of a rotating planet, it can only view and transmit to Earth (or orbiter) during a fraction of the day. The maximum view fraction of the direct link is about 0.5, with a reasonable value being about 0.33. For the relay system, the view fraction of the orbiter is at a maximum of unity, since it is possible that an orbit which does not obstruct the earth can be chosen; the view fraction for the rover-orbiter link, on the other hand, for a satellite with a circular orbit of 5000 km, is 0.118, making this leg the weaker, and therefore determining, part of the system.

Alternatively, a synchronous orbit can be chosen for the relay station. This allows a rover-orbiter view factor of 1.0 and a minimum view factor of 0.5 for the orbiter-earth path. Since this study concerns itself only with rover characteristics, this possibility was not considered as it places less constraint on the rover. Other factors that could have been considered are that the injection energy for a synchronous orbit is much higher, and that a synchronous orbit might limit the usefulness of the orbiter in other applications (mapping, scanning surface parameters, etc.).

Consequently, the major assumption of this configuration is the orbiting of a communication satellite above Mars in a 5000 km non-synchronous orbit. To enhance the received signal-to-noise ratio, the lander-to-orbiter range was kept to a minimum. The orbit chosen is circular with an altitude of five thousand kilometers above the surface, and the relay is assumed to be stationed to provide constant line-of-sight contact with the Earth for the duration of the mission. Reduction of the orbit beyond that assumed yielded a reduction of the view window with no appreciable increase in the signal-to-noise ratio.

The additional assumptions and fixing of parameter values employed are as follows:

- The carrier is x-band microwaves of wavelength 3.3×10^{-2} meters, as before.
- The orbiter antenna is a parabolic dish of diameter $d = 9$ meters.
- The rover antenna is a parabolic dish of diameter D_{com} (to be determined during system optimization) with a pointing error of less than one degree.
- Uplink parameters are considered negligible with respect to the downlink system.
- Transmitter r.f. efficiency is $e_c = 20\%$.
- Worst case Earth-Mars link distance of 5.7×10^{11} meters is used.
- Equivalent system noise temperature is $T_n = 30$ K.
- The communication efficiency $(B_o/B) = 5\%$.
- The space loss attenuation $L_p = 2.0 \times 10^{-17}$ for the orbiter at 5000 km.

Signal power at the orbiter receiver must be large enough to overcome the noise. In terms of modeling parameters, this may be written as:

$$P_r = P_t \cdot G_t \cdot L_p \cdot G_r ,$$

where

P_r is received signal power

P_t is transmitted signal power

G_t is transmitting antenna gain

L_p is the space loss attenuation

and

G_r is the receiving antenna gain.

These can be replaced with:

$$P_t = e_c \cdot P_{com} = (0.20)\ P_{com}$$

$$G_t \approx 0.54\ \left(\frac{\pi}{\lambda}\right)^2 D^2_{com} = (4.88 \times 10^3)\ D^2_{com}$$

$$L_p = 2.0 \times 10^{-17}$$

$$G_r \approx 0.54\ \left(\frac{\pi d}{\lambda}\right)^2 = 3.13 \times 10^3 .$$

These substitutions yield

$$P_r = (6.10 \times 10^{-11})\ P_{com}\ D^2_{com} .$$

An additional restriction on pulse-code modulation systems is:

$$P_r \geq 10^{-23}\ (B/B_o) \cdot T_n\ R_{com}$$

where

B/B_o is the inverse of the communication efficiency,

T_n is the system noise equivalent temperature, ^oK ,

and

R_{com} is the data rate in bits/sec.

This equation, upon substitution, yields:

$$P_r \geq (6.0 \times 10^{-21}) R_{com}$$

Combining the P_r relations and solving for the data rate, the result is:

$$R_{com} \leq (1.02 \times 10^{10}) P_{com} D^2_{com} ,$$

Clearly then, for any reasonably sized rover-based telecommunications system, the data rate can never even approach a number of the magnitude of the left-hand side of the above equation. In terms of these parameters then, the effect of introducing an active-orbiter link is high data rate for the rover at minimal power consumption.

The direct effects upon the system model are as follows. Since the rover data rate can now be very large with minimal power requirement, the time required to transmit science and engineering data becomes negligible and (4.5) is modified to read

$$T_{sci} = T_{esci}$$

i.e., the transmitting term is dropped. Because the communication subsystem parameters will now be small with respect to the other subsystems, they can be set without any great effect on the optimization results. The chosen values are

$$P_{com} = 20.0 \text{ watts}$$
$$D_{com} = 1.0 \text{ m}$$
$$W_{com} = 10.0 \text{ kg.}$$

It should be noted that since the ultimate task of this analysis was the remodeling of the vehicle's communication system, no work has been done on the orbiter-Earth link of the design. What has been done is to ask what advantages accrue from the addition of an active-orbiter communication link -- and clearly the answer is the removal of bounds on the rover's data rate. While this is a valuable result, the question that still remains to be answered is whether the end is worth the means -- is the elimination of restrictions on the data rate worth the additional expense of a Mars orbiter? The following section shows the gains in rover performance derived from the use of a communications relay. With this information, the worth of the expense and complexity of such an orbiter can be more properly evaluated.

SECTION 5

OPTIMAL DESIGNS

In Section 4, the system optimization method to be used on the Mars rover problem was described by indicating both the system evaluation (objective) function and alternative system models. In this section, the computer-generated results of that procedure will be presented.

5.1 DESCRIPTION OF COMPUTER PROCEDURE UTILIZED

The nonlinear programming problems generated in Section 4 were solved iteratively by use of Fiacco and McCormick's Sequential Unconstrained Minimization Technique (SUMT). This procedure was chosen because of its demonstrated efficiency in solving a wide class of NLP problems, and because its code contains many alternative methods of deciding on step directions.

For the general NLP problem with both equality and inequality constraints, SUMT attempts to converge to the solution by solving a series of unconstrained problems. More specifically, the scalar function

$$p(x,t') = f(x) - t' \sum_{i=1}^{m} \ln g_i(x) + \frac{1}{t'} \sum_{j=1}^{p} h_j^2(x) \qquad (5.1)$$

is created for an initial value of t' chosen by the programmer, and this function is minimized by some iterative procedure (Newton-Raphson, variable metric, Fletcher-Powell-Davidon,...). Then, the value of t' is reduced and the new p-function is minimized iteratively starting from the minimum of the old p-function. The effect of this scheme is to drive the x-vector to the interior of the region that satisfies all inequality constraints when t' is large, and then approach the equality constraints as t' gets small. The minimization at each step (each value of t') drives the search toward the minimum of the NLP problem. As $t' \rightarrow 0$, a local solution to the NLP problem should be approached, although there is no guarantee that this will occur except in special cases. None of these special cases appears in the problem of interest here.

5.2 DESCRIPTION OF PROBLEMS SOLVED

The three major cases described in Section 4 were considered separately. The optimum values of the object function are indications of the relative merit of each design. Each model required the establishment of limits on some design parameters.

Only those limits which affected the optimization procedure will be discussed. For example, there is obviously an upper limit on the vehicle size such that it will fit in the launch vehicle aeroshell. However the weight restriction is more stringent in this case, and computer results are identical with and without the size constraint.

The total launch weight (L_W), limited by the launch vehicle (Titan IIIc) was set at 570 kg. In order to insure that the equipment package volume was sufficient for all onboard equipment, the minimum volume was set. However, since the geometry of the equipment package is known, this restriction was reflected into a surface area limit for ease of computation (the surface area, A, appears in many model equations). The minimum surface area for the 4-wheeled vehicle was set at 7.35 m^2, while in the 6-wheeled case, 3.0 m^2 was required for each of the two compartments. The antenna diameter (D_{com}) is limited by the size of the aeroshell. Because this parameter always appears squared, the square of the diameter was limited ($D^2_{max} = 2.0\ m^2$). This completes the description of fixed limits.

Various "constants" and limits in the models were changed from run to run. This allowed more perception into the factors governing the optimal design of the rover. The discussion in Section 8 elaborates on the use and interpretation of the results caused by these changes. Here, the minor changes made in the models will be described:

1. S_{sci}, the number of science stops per meter, was variable in some problems and fixed for others

2. v_f (vehicle velocity) and s^* (the slope threshold used by the vehicle in determining if a slope is too large for travel) were unconstrained in some problems and limited in others

3. two alternative equipment package geometries were considered. In all cases, compartments are rectangular solids, but the relations between the linear measurements were altered

4. radiators are assumed to fill two opposite sides of equipment compartments. Radiators on both the larger (in area) and smaller faces were considered

5. two different values of the efficiency of thermal transfer were considered

6. the acceptable limits on variations in internal temperature were varied

7. (7.42) was modified in some runs to allow the maximum roving time between recharges (T_{rov}) to approach the total vehicle operational time in a Mars day (V) . That is, the equation was rewritten to read

$$T_{rov} \leq V$$

8. in almost all runs, total science weight (W_{sci}) was given a lower limit of 50 kg. In others the limit was 35 kg.

The majority of these variations (1, 3, 5-8 above) were made only in the first major case (4-wheeled vehicle, communicating directly to Earth). Five sets of limits on v_f and s^* were considered in all three major cases.

The next three parts contain design results for the three major cases. Values of all significant design parameters are reported. In addition, the values of three functions (F, P, and D) at the optimal point are recorded. F is the value of the objective function, f(x), at x^*, the solution to the NLP problem. P is the value of the augmented objective function (5.1) used by SUMT at x^*. D is the value of the objective function of the dual to the NLP problem, evaluated at x^* . At the exact solution to an NLP problem, the values of F, P and D are equal. Finally, the value of the weighting factor, t', at the last iteration is reported.

5.3 OPTIMIZATION RESULTS FOR FOUR-WHEELED DIRECT COMMUNICATING ROVER

By eliminating equalities by substitution, the optimization problem was written as an NLP problem in ten variables with thirty-seven inequality constraints. The large number of inequalities stems from the fact that almost all eliminated variables represented physical quantities that were constrained to be positive.

Results are presented in Tables 8-10. All entries are rounded-off to the nearest value of lesser precision with one exception. If a variable is approaching a direct limit placed upon it, it appears truncated instead of rounded (e.g., if a variable is limited to a maximum of 15.0, an optimal result of 14.99998 appears as 14.99).

14.99998 ?

TABLE 8

PROBLEM CODE	s* limit (deg.)	v_f limit (m/sec)	other (for 1A - 1E)
1A	20	1.5	$a_1 = 3.33$, $a_2 = 4.0$, $e_t = 0.8$
1B	15	0.5	$D^2_{com} \leq 2.0$, radiators on larger sides
1C	15	1.5	of equipment package, $295 \leq T_{int} \leq 305K$,
1D	20	0.5	$W_{sci} \geq 50$, $A_{min} = 7.35\ m^2$,
1E	none	none	S_{sci} variable , $n_p = 3$

	Constraint Description and Model Details
1F	1A with $T_{rov} \leq V$
1G	1A with $T_{rov} \leq V$, no limit on v_f
1H	1E with $v_f \leq 0.25$, $s^* \leq 10.0$
1J	1C with $T_{rov} \leq V$
1K	1E with $T_{rov} \leq V$
1L	1F with $a_1 = 3.6$, $a_2 = 5.5$, radiators on smaller sides of equipment package, $e_t = 0.5$, $290 \leq T_{int} \leq 310K$, $A_{min} = 8.0$, $D^2_{com} \leq 3.0$, $S_{sci} = 0.002$

PROBLEM CODES: CONSTRAINT DESCRIPTION AND MODEL FOR
INDIVIDUAL OPTIMIZATION RUNS; FOUR-WHEELED DIRECT COMMUNICATING ROVER

(Table 8 continued)

1M	1L with $s^* \leq 15$, $D^2_{com} \leq 2.0$
1N	1M with larger radiators
1P	1A with radiators on smaller sides, $e_t = 0.5$, $290 \leq T_{int} \leq 310$ K, $T_{rov} \leq V$
1Q	1P with larger radiators
1R	1Q with $S_{sci} = 0.01$
1S	1R with s^* and v_f constraints removed
1T	1R with $W_{sci} \geq 35$ kg
1U	1S with $W_{sci} \geq 35$ kg
1V	1R with $e_t = 0.8$
1W	1R with $295 \leq T_{int} \leq 305$ K
1X	1V with $295 \leq T_{int} \leq 305$ K
1Y	1X with v_f and s^* constraints removed
1Z	1Y with $S_{sci} = 0.005$
1AB	1Y with $s^* \leq 20$

TABLE 9

PROBLEM CODE	W_{sci} (kg)	D^2_{com} (m^2)	P_{com} (watts)	s* (deg)	v_f (m/sec)	A (m^2)	E_{batt} (watt.hr)
1A	70.1	1.99	176.8	19.82	1.499	7.36	7.60
1B	69.1	1.99	127.1	19.99	0.499	7.36	2.43
1C	61.3	1.99	132.2	14.99	1.499	7.35	15.97
1D	62.0	1.99	113.7	14.99	0.499	7.37	5.63
1E	50.0	1.98	124.1	14.42	3.154	7.35	7.82
1F	60.6	1.99	229.5	19.99	1.499	7.52	18.69
1G	50.0	1.99	216.7	19.44	2.744	7.35	22.11
1H	65.8	1.99	182.6	9.99	0.250	7.36	13.51
1J	55.5	1.99	219.8	14.99	1.499	7.35	19.99
1K	50.0	1.98	182.8	15.28	2.943	7.35	25.07
1L	50.0	2.99	150.2	19.99	1.499	8.02	15.37
1M	50.0	1.99	237.2	14.99	1.499	8.00	32.60
1N	50.0	1.99	181.6	14.98	1.499	10.47	30.91
1P	50.0	1.99	241.3	14.99	1.499	7.37	33.43
1Q	50.0	1.99	243.8	14.99	1.499	7.39	32.51
1R	50.0	1.99	245.1	14.99	1.499	7.37	33.33
1S	50.0	1.99	240.0	18.31	1.495	7.35	26.50
1T	35.0	1.99	237.7	14.99	1.499	7.50	28.61

OPTIMAL DESIGNS FOR FOUR-WHEELED DIRECT COMMUNICATING ROVER

(Table 9 continued)

PROBLEM CODE	W_{sci} (kg)	D^2_{com} (m^2)	P_{com} (watts)	s* (deg)	v_f (m/sec)	A (m^2)	E_{batt} (watt·hr)
1U	40.7	1.99	240.2	19.19	1.993	7.35	29.99
1V	50.0	1.99	274.1	14.99	1.499	7.36	33.83
1W	50.0	1.99	230.0	14.99	1.498	7.36	24.50
1X	50.0	1.99	270.6	14.99	1.499	7.36	33.71
1Y	50.0	1.99	217.6	32.22	1.924	7.42	2.17
1Z	50.0	1.99	201.9	29.76	2.345	7.35	4.92
1AB	50.0	1.99	229.9	19.99	2.353	7.35	22.68

(Table 9 continued)

PROBLEM CODE	w_b (m)	L_i (cm)	W_{com} (kg)	R_{com} (bits/sec)	T_{esci} (sec)	W_v (kg)	Q_{id} (watts)
1A	3.124	2.85	139.7	14847	2370	197.4	113.8
1B	3.122	3.46	118.9	10674	2334	197.2	111.3
1C	3.096	2.84	121.1	11109	2055	192.3	111.6
1D	3.186	3.91	113.3	9552	2082	209.6	110.6
1E	3.084	2.08	117.6	10312	1654	190.0	111.2
1F	3.097	3.23	161.7	19276	2030	192.5	116.4
1G	3.084	2.03	156.4	18142	1653	190.0	115.8
1H	3.208	3.78	142.1	15327	2218	213.9	114.1
1J	3.086	2.30	157.7	18467	1848	190.3	115.9
1K	3.085	16.3	142.2	15729	1654	190.2	114.1
1L	3.087	2.56	163.1	18931	1652	190.6	112.5
1M	3.085	2.66	164.9	19922	1653	190.1	116.8
1N	3.084	3.98	141.7	15254	1653	190.1	114.0
1P	3.084	2.27	166.6	20269	1653	190.1	117.0
1Q	3.088	2.35	167.7	20480	1653	190.7	117.1
1R	3.085	2.33	168.2	20587	1653	190.3	117.2
1S	3.084	2.22	166.1	20160	1653	190.0	117.0
1T	3.178	2.41	165.2	19966	1116	207.9	116.8

(Table 9 continued)

PROBLEM CODE	w_b (m)	L_i (cm)	W_{com} (kg)	R_{com} (bits/sec)	T_{esci} (sec)	W_v (kg)	Q_{id} (watts)
1U	3.084	1.92	166.2	20175	1320	190.0	117.0
1V	3.095	3.07	180.4	23026	1653	192.1	118.7
1W	3.088	2.34	161.9	19316	1653	190.8	116.4
1X	3.085	3.06	178.9	22726	1653	190.4	118.5
1Y	3.084	2.06	156.8	18279	1653	190.0	115.8
1Z	3.084	1.84	150.2	16962	1653	190.0	115.0
1AB	3.084	2.26	161.9	19309	1653	190.0	116.4

(Table 9 continued)

PROBLEM CODE	T	T_{sci} (sec)	PLR	K_q (watts/K)	P_θ (watts)	W_θ (kg)	P_{str} (watts)
1A	0.079	2572	2.18	-1.33	154.6	30.0	365.3
1B	0.076	2615	2.17	-1.44	153.2	32.5	358.6
1C	0.178	2325	2.47	-1.31	151.3	29.9	356.9
1D	0.174	2396	2.45	-1.51	152.5	34.4	356.6
1E	0.197	1944	2.53	-1.09	143.7	26.8	343.4
1F	0.076	2186	2.17	-7.22	185.2	31.3	309.3
1G	0.084	1819	2.20	-6.31	196.6	25.7	319.5
1H	0.405	2413	3.52	-1.50	156.4	33.8	369.5
1J	0.178	2010	2.47	-4.51	189.1	27.8	425.7
1K	0.178	1850	2.44	-2.06	148.8	84.7	357.5
1L	0.076	1811	2.18	-7.42	256.7	31.6	373.6
1M	0.178	1803	2.47	-7.18	264.4	31.9	389.3
1N	0.179	1849	2.47	-15.32	264.3	49.5	383.7
1P	0.178	1801	2.47	-16.61	283.4	26.9	408.8
1Q	0.178	1799	2.47	-10.33	272.0	27.1	397.5
1R	0.178	1798	2.47	-10.24	272.0	27.0	397.7
1S	0.102	1801	2.24	-10.08	272.5	26.5	397.7
1T	0.174	1267	2.46	-10.80	271.8	27.8	396.8

(Table 9 continued)

PROBLEM CODE	T	T_{sci} (sec)	PLR	K_q (watts/K)	P_θ (watts)	W_θ (kg)	P_{str} (watts)
1U	0.089	1468	2.21	-9.67	275.6	25.3	400.9
1V	0.178	1783	2.47	-10.82	167.9	30.0	296.4
1W	0.178	1809	2.47	-6.53	193.3	27.0	433.4
1X	0.178	1785	2.47	-6.86	187.9	29.9	316.1
1Y	0.010	1817	2.02	-6.42	196.9	26.1	319.8
1Z	0.015	1829	2.03	-6.18	198.6	24.9	320.0
1AB	0.076	1808	2.18	-6.46	194.3	26.6	318.5

(Table 9 continued)

PROBLEM CODE	P_{mv} (watts)	P_{RTG} (watts)	W_p (watts)	M_r (kg_m)	P_{prop} (watts)	T_r (hr)	T_{rov} (hr)
1A	380.3	545.4	105.9	58.14	165.1	0.024	3.76
1B	373.6	425.3	85.5	58.14	51.7	0.021	3.99
1C	371.9	500.2	98.6	58.14	128.3	0.064	3.61
1D	371.6	414.3	49.6	58.14	42.8	0.056	3.97
1E	358.4	621.5	118.7	58.14	363.1	0.016	0.755
1F	324.3	490.4	97.1	58.14	166.1	0.059	9.43
1G	334.5	632.4	121.1	58.14	297.9	0.040	5.50
1H	384.5	402.3	82.1	58.14	17.8	0.235	7.36
1J	440.7	578.0	111.8	58.14	137.3	0.075	4.21
1K	372.5	668.0	127.1	58.14	295.5	0.046	2.61
1L	388.6	554.6	107.7	58.14	166.1	0.048	7.73
1M	404.3	541.6	106.2	58.14	137.3	0.122	6.86
1N	398.7	534.4	105.0	58.14	135.7	0.117	6.56
1P	423.8	561.1	109.5	58.14	137.3	0.125	7.03
1Q	412.5	549.8	107.6	58.14	137.3	0.122	6.85
1R	412.7	550.0	107.6	58.14	137.3	0.124	6.99
1S	412.7	568.7	110.5	58.14	156.0	0.088	9.96
1T	411.8	549.1	107.3	58.14	137.3	0.105	6.16

(Table 9 continued)

PROBLEM CODE	P_{mv} (watts)	P_{RTG} (watts)	W_p (watts)	M_r (kg_m)	P_{prop} (watts)	T_r (hr)	T_{rov} (hr)
1U	415.9	629.8	120.9	58.14	213.9	0.075	9.71
1V	311.4	448.7	90.6	58.14	137.3	0.127	7.15
1W	448.4	585.6	113.3	58.14	137.1	0.092	5.17
1X	331.1	468.4	93.9	58.14	137.3	0.126	7.10
1Y	334.8	632.1	120.3	58.14	297.3	0.004	6.86
1Z	335.0	677.7	128.0	58.14	342.7	0.008	8.41
1AB	333.5	594.0	114.6	58.14	260.5	0.047	7.25

(Table 9 continued)

PROBLEM CODE	T_{rn} ^oK	T_{bn} ^oK	T_{rd} ^oK	T_{bd} ^oK	Q_h (watts)	S_{sci} (-/km)
1A	202.7	207.1	398.0	313.0	395.8	0.260
1B	199.0	203.0	190.1	313.2	326.7	0.769
1C	202.8	207.2	397.4	313.0	396.5	0.292
1D	196.9	200.6	385.9	313.3	288.5	0.847
1E	209.3	214.4	410.5	312.5	522.1	0.167
1F	200.3	204.3	325.5	313.7	360.7	0.306
1G	209.8	214.9	329.9	313.0	534.0	0.202
1H	197.5	201.2	388.4	313.3	297.4	0.350
1J	207.1	212.0	338.5	312.7	497.6	0.388
1K	181.5	182.7	362.8	314.2	11.6	0.183
1L	203.5	207.9	327.3	313.7	455.3	2.0
1M	202.9	207.3	328.4	313.7	442.7	2.0
1N	195.8	199.3	318.6	314.0	419.7	2.0
1P	206.1	211.0	318.5	314.8	463.0	2.0
1Q	205.4	210.0	323.2	314.2	450.8	2.0
1R	205.6	210.2	323.3	314.2	451.8	10.0
1S	206.6	211.3	323.5	314.1	470.3	10.0
1T	204.9	209.5	322.6	314.2	449.1	10.0

(Table 9 continued)

PROBLEM CODE	T_{rn} ^oK	T_{bn} ^oK	T_{rd} ^oK	T_{bd} ^oK	Q_h (watts)	S_{sci} (-/km)
1U	209.7	214.7	324.3	314.0	531.0	10.0
1V	200.2	204.2	322.4	314.4	349.8	10.0
1W	206.7	211.4	328.7	313.2	473.6	10.0
1X	201.3	205.4	326.9	313.6	370.1	10.0
1Y	209.4	214.4	329.5	313.0	532.2	10.0
1Z	212.0	217.3	330.7	312.8	578.5	0.5
1AB	207.5	212.3	329.1	313.1	489.0	1.0

TABLE 10

PROBLEM CODE	F	P	D	Final Value of t′
1A	15.75339	15.75352	15.75346	1.455E-06
1B	5.105966	5.105966	5.105966	6.705520E-09
1C	13.20097	13.20097	13.20097	4.440889E-09
1D	4.364834	4.364834	4.364834	6.705520E-09
1E	25.93234	25.93236	25.93234	1.8190E-07
1F	15.91536	15.91562	15.91551	3.0516E-06
1G	28.19969	28.19995	28.19981	2.9296E-06
1H	0.8920264	0.8920271	0.892069	1.757811E-06
1J	13.71691	13.71694	13.71692	3.8147E-07
1K	26.48917	26.48939	26.48931	1.757811E-06
1L	8.244371	8.244462	8.244430	1.1921E-06
1M	7.290800	7.290836	7.290823	4.7684E-07
1N	6.983361	6.986515	6.984855	3.0518E-05
1P	7.308222	7.308257	7.308245	4.7684E-07
1Q	7.320332	7.320333	7.320332	4.7684E-07
1R	1.921973	1.922002	1.921990	3.8147E-07
1S	2.110862	2.111119	2.110994	2.9300E-06
1T	2.553569	2.553594	2.553581	3.6621E-07
1U	2.591204	2.593688	2.592304	2.3437E-05
1V	1.955402	1.955432	1.955420	3.6621E-07
1W	1.901790	1.903779	1.902768	2.0000E-05
1X	1.950621	1.950648	1.950638	3.6621E-07
1Y	2.342133	2.344046	2.343071	2.0000E-05
1Z	4.438655	4.438683	4.438670	3.5095E-07
1AB	2.217680	2.217688	2.217683	8.5682E-08

VALUES OF EVALUATION FUNCTIONS AND FINAL WEIGHTING FACTOR; FOUR-WHEELED DIRECT COMMUNICATING ROVER*

* see 5.2 for definitions of variables

5.4 OPTIMIZATION RESULTS FOR SIX-WHEELED DIRECT COMMUNICATING ROVER

By eliminating equalities by substitution, the optimization problem was written as an NLP problem in eleven variables with forty inequality constraints. The large number of inequalities stems from the fact that almost all eliminated variables represented physical quantities that were constrained to be positive.

Results are presented in Tables 11-13. All entries are rounded-off to the nearest value of lesser precision with one exception. If a variable is approaching a direct limit placed upon it, it appears truncated instead of rounded (e.g., if a variable is limited to a maximum of 15.0, an optimal result of 14.99998 appears as 14.99).

TABLE 11

PROBLEM CODE	s^* limit(deg.)	v_f limit(m/sec)	maximum power requirement*	other (for all codes)
2A	20	1.5	day	$W_{sci} \geq 50$, $A_{min1,2} =$
2B	15	0.5	night	$3.0\ m^2$, $D^2_{com} \leq 2.0$,
2C	15	1.5	night	radiators on larger sides,
2D	20	0.5	night	$295 \leq T_{int} \leq 310$ K,
2E	none	none	day	$a_1 = 3.33$, $a_2 = 4.0$,
				S_{sci} variable , $n_p = 3$
2F	2A with radiators on small sides of equipment package			

PROBLEM CODES: CONSTRAINT DESCRIPTION AND MODEL DETAILS FOR INDIVIDUAL OPTIMIZATION RUNS; SIX-WHEELED DIRECT COMMUNICATING ROVER

* (see 4.2.3) day power requirement implies that $P_{prop} + P_{mv} \geq Q_h + Q_{in}$, while night power requirement implies that the inequality is reversed.

TABLE 12

PROBLEM CODE	Q_{h1} watts	Q_{h2} watts	T_{bd1} ^oK	T_{bd2} ^oK	T_{rd1} ^oK
2A	249.1	13.5	293.1	291.1	326.1
2B	235.1	26.4	292.8	291.3	324.7
2C	235.6	50.4	292.8	291.6	324.7
2D	244.3	30.6	293.0	291.3	325.6
2E	253.4	149.4	293.1	292.9	325.9
2F	30.9	196.1	287.9	291.6	316.5

PROBLEM CODE	T_{rd2} ^oK	T_{bn1} ^oK	T_{bn2} ^oK	T_{rn1} ^oK	T_{rn2} ^oK	T_{rov} hr	T_r hr
2A	363.0	215.1	193.0	209.4	190.1	3.93	0.065
2B	362.8	212.5	194.8	207.1	191.6	5.41	0.072
2C	363.5	212.5	198.4	207.1	194.7	2.94	0.096
2D	363.5	214.2	195.7	208.6	192.3	3.99	0.020
2E	368.0	214.9	212.7	209.3	207.3	1.61	0.027
2F	379.2	181.2	218.7	180.2	213.2	3.83	0.065

OPTIMAL DESIGNS FOR SIX-WHEELED
DIRECT COMMUNICATING ROVER

(Table 12 continued)

PROBLEM CODE	P_{prop} watts	M_r kg	W_p kg	P_{RTG} watts	P_{mv} watts	P_{str} watts	W_θ kg
2A	155.1	58.1	75.4	361.3	206.2	191.2	37.6
2B	42.77	58.1	75.2	359.6	209.3	194.3	37.5
2C	128.3	58.1	79.8	384.2	196.7	181.7	35.7
2D	51.74	58.1	76.9	373.1	202.6	187.6	36.2
2E	385.3	58.1	100.3	509.6	124.3	109.3	31.5
2F	155.2	58.1	69.4	325.4	170.2	155.2	58.9

PROBLEM CODE	P_θ watts	K_{q1} watts/K	K_{q2} watts	PLR	T_{sci} sec	T	Q_{idl} watts
2A	84.0	1.78	-1.37	2.53	2778	0.197	5.20
2B	86.2	1.72	-1.33	3.90	2495	0.458	5.20
2C	73.6	1.73	-1.21	3.95	2550	0.463	5.20
2D	79.2	1.76	-1.30	2.54	2719	0.200	5.20
2E	2.09	1.83	-0.624	2.54	2829	0.199	5.20
2F	47.8	-0.0004	-0.323	2.56	2773	0.205	5.20

(Table 12 continued)

PROBLEM CODE	Q_{id2} watts	W_v kg	T_{esci} sec	R_{com} bit/sec	W_{com} kg	L_{i1} m	L_{i2} m
2A	102.7	234.5	2179	5013	90.8	0.0207	0.0625
2B	103.2	237.2	1973	5748	94.4	0.0230	0.0558
2C	103.2	232.4	2033	5796	94.6	0.0230	0.0450
2D	103.4	228.1	2224	6066	95.99	0.0214	0.0528
2E	102.7	215.4	2180	4624	90.48	0.0208	0.0228
2F	102.7	219.6	2174	5008	90.7	0.2077	0.0175

PROBLEM CODE	t m	E_{batt} watt-hr	A m^2	v_f m/sec	s* deg	P_{com} watts	D_{com}^2 m^2
2A	1.436	19.28	3.005	1.499	19.99	59.8	1.99
2B	1.442	20.78	3.058	0.500	15.00	68.4	2.00
2C	1.432	34.18	3.067	1.499	15.00	69.0	1.99
2D	1.423	6.529	3.020	0.500	19.99	72.2	2.00
2E	1.396	19.28	3.067	3.697	20.25	59.8	1.84
2F	1.405	19.30	3,048	1.500	20.00	59.6	2.000

(Table 12 continued)

PROBLEM CODE	W_{sci} kg	$\frac{1}{e_t} K_{q1} (T_{intd} - T_{rd1})$	$\frac{1}{e_t} K_{q2} (T_{intd} - T_{rd2})$	S_{sci} -/km
2A	64.8	-75.1	159.0	0.244
2B	59.0	-68.8	154.3	0.670
2C	60.6	-69.2	140.4	0.270
2D	66.0	-73.9	153.4	0.739
2E	64.8	-76.5	78.2	0.097
2F	64.6	8.28E-03	47.8	0.244

TABLE 13

PROBLEM CODE	F	P	D	Final Value of t'
2A	11.43213	11.49865	11.48113	1.0000E-05
2B	2.477247	2.477260	2.477259	2.4414E-07
2C	7.333787	7.333802	7.333798	2.4414E-07
2D	4.001507	4.001537	4.001531	4.7684E-07
2E	27.60519	27.60545	27.60530	2.4414E-06
2F	11.30004	11.30677	11.30514	1.0000E-05

VALUES OF EVALUATION FUNCTIONS AND FINAL
WEIGHTING FACTOR; SIX-WHEELED
DIRECT COMMUNICATING ROVER*

* see 5.2 for definitions of variables

5.5 OPTIMIZATION RESULTS FOR FOUR-WHEELED ROVER, COMMUNICATING VIA A MARS ORBITER

By eliminating equalities by substitution, the optimization problem was written as an NLP problem in eight variables with thirty-five inequality constraints. The large number of inequalities stems from the fact that almost all eliminated variables represented physical quantities that were constrained to be positive.

Results are presented in Tables 14-16. All entries are rounded-off to the nearest value of lesser precision with one exception. If a variable is approaching a direct limit placed upon it, it appears truncated instead of rounded (e.g., if a variable is limited to a maximum of 15.0, an optimal result of 14.99998 appears as 14.99).

TABLE 14

PROBLEM CODE	s^* limit(deg.)	v_f limit(m/sec)	other (for all codes)
3A	20	1.5	$P_{com} = 20$, $W_{com} = 10$, $D_{com} = 1.0$,
3B	15	0.5	$a_1=3.33$, $a_2=4.0$, $e_t=0.8$, radiators
3C	15	1.5	on larger sides, $295 \leq T_{int} \leq 305$ K,
3D	20	0.5	$W_{sci} \geq 50$, $A_{min} = 7.35$ m^2, S_{sci}
3E	none	none	variable , $n_p = 3$

PROBLEM CODES: CONSTRAINT DESCRIPTION AND MODEL DETAILS
FOR INDIVIDUAL OPTIMIZATION RUNS;
FOUR-WHEELED ROVER, COMMUNICATING VIA A MARS ORBITER

TABLE 15

PROBLEM CODE	W_{sci} kg	s^* (deg)	v_f m/sec	A m^2	E_{batt} watts-hr	w_b m	L_i 10^{-2}m	S_{sci} $0.095/m \times 10^{-4}$
3A	92.3	19.99	1.499	7.36	6.7	3.652	4.051	2.12
3B	95.3	14.99	0.499	7.36	2.64	3.697	5.528	6.19
3C	94.0	14.99	1.499	7.68	12.08	3.675	3.019	2.10
3D	94.6	19.99	0.499	7.42	1.96	3.718	3.839	6.20
3E	54.5	17.46	13.60	7.95	15.55	3.109	0.828	0.41

PROBLEM CODE	T_{bd} °K	T_{rd} °K	T_{bn} °K	T_{rn} °K	Q_h watts	Q_{id} watts	P_{prop} watts	P_{RTG} watts
3A	313.3	383.1	199.9	196.3	276.7	105.9	169.2	522.8
3B	313.6	375.0	194.6	191.6	191.7	105.9	46.6	398.3
3C	313.0	391.2	205.8	201.5	395.2	105.9	139.9	494.0
3D	313.3	384.3	200.9	197.2	296.6	105.9	56.4	410.5
3E	310.7	467.1	238.7	232.2	1085.3	105.9	1401.3	1534.1

OPTIMAL DESIGNS FOR FOUR-WHEELED
ROVER COMMUNICATING VIA A MARS ORBITER

(Table 15 continued)

PROBLEM CODE	P_{str} watts	P_{mv} watts	PLR	M_r kg	W_p kg	W_v kg	T_{rov} hr	T_r hr x 10^{-2}
3A	338.6	353.6	2.15	58.1	102.0	315.4	4.14	2.07
3B	336.7	351.7	2.39	58.1	81.0	327.4	2.08	2.44
3C	339.0	354.0	2.39	58.1	97.4	321.4	3.14	4.44
3D	339.0	354.0	2.15	58.1	83.0	332.9	3.72	1.56
3E	117.7	132.7	2.28	58.1	272.3	194.7	0.55	0.62

PROBLEM CODE	K_g watts	W_θ kg	T	T_{sci} = T_{esci} sec
3A	-1.50	34.9	0.067	3163
3B	-1.66	40.9	0.155	3270
3C	-1.36	32.0	0.156	3224
3D	-1.48	34.3	0.066	3246

TABLE 16

PROBLEM CODE	F	P	D	Final Value of t'
3A	17.33315	17.33311	17.33311	1.5259E-06
3B	5.155109	5.155128	5.15519	2.3841E-07
3C	15.41541	15.41558	15.41550	1.9073E-06
3D	5.788019	5.788019	5.799019	2.3841E-07
3E	147.3100	147.3115	147.3106	1.2500E-05

VALUES OF EVALUATION FUNCTIONS AND FINAL WEIGHTING FACTOR; FOUR-WHEELED ROVER, COMMUNICATING VIA A MARS ORBITER*

* see 5.2 for definitions of variables

SECTION 6

PERTURBED-OPTIMAL SOLUTIONS IN NONLINEAR PROGRAMMING - THEORY

In 6.1, a brief review of mathematical programming theory relevant to the discussion of the perturbed-optimal solutions problem is presented. Section 6.2 presents the development of the solution to the perturbed-optimal sensitivity problem. In both sections, concern is with local as opposed to global properties.

6.1 REVIEW OF NONLINEAR PROGRAMMING THEORY

The general nonlinear mathematical programming problem is:

$$\text{(NLP)}\quad \begin{array}{lll} \text{minimize} & f(x) & \\ \text{subject to} & g_i(x) \geq 0 & i = 1,2,\ldots,m \\ & h_j(x) = 0 & j = 1,2,\ldots,p \end{array}$$

where x is an n-vector with real-valued components $(x \epsilon R^n)$, and f and the elements of the sets of functions $\{g_i\}$ and $\{h_j\}$ are, in general, nonlinear scalar functions of the elements of x.

At a point x^*, the set Z^* is defined as

$$Z^* = \left\{ z \mid z^T \nabla g_i^* \geq 0,\ i \epsilon A^*;\ z^T \nabla h_j^* = 0,\ j=1,\ldots,p;\ \text{and}\ z^T \nabla f^* < 0 \right\}$$

where $z \epsilon R^n$, $A^* = \left\{ i \mid g_i(x^*) = 0 \right\}$, ∇ is the gradient operator with respect to x, and for a function $q(x)$, $q(x^*)$ and q^* are used interchangeably. The condition $Z^* = \emptyset$, where $\emptyset$ is the empty set, implies that the hypotheses of the Farkas Lemma [83] are satisfied at x^* , and consequently an existence theorem for generalized Lagrange multipliers can be written [32, p.19]:

If x^* satisfies the constraints of the nonlinear programming problem (NLP), the functions f, $\{g_i\}$, $\{h_j\}$ are once differentiable, and the set Z^* is empty, there exist vectors u^* and w^* such that (x^*, u^*, w^*) satisfies

$$g_i(x) \geq 0 \qquad i = 1,\ldots,m \tag{6.1}$$

$$h_j(x) = 0 \qquad j = 1,\ldots,p \tag{6.2}$$

$$u_i g_i(x) = 0 \qquad i = 1,\ldots,m \tag{6.3}$$

$$u_i \geq 0 \qquad i = 1,\ldots,m \tag{6.4}$$

$$\nabla \mathcal{L}(x,u,w) = 0 \tag{6.5}$$

where

$$\mathcal{L}(x,u,w) = f(x) - \sum_{i=1}^{m} u_i g_i(x) + \sum_{j=1}^{p} w_j h_j(x). \tag{6.6}$$

Several "constraint qualifications" have been developed to insure that $Z^* = \emptyset$ <u>at a local minimum of (NLP)</u>. Those that relate specifically to the problem at hand are the Kuhn-Tucker, Weak Arrow-Hurwicz-Uzawa, Weak Reverse Convex, and the Modified Arrow-Hurwicz-Uzawa constraint qualifications. These constraint qualifications are conditions on the functions $\{g_i\}$ and $\{h_j\}$, and are independent of the form of the objective function f . A detailed description of each is given in Mangasarian [84] .

A sufficient condition that the Kuhn-Tucker constraint qualification hold at a point x^* , satisfying the constraints of (NLP), is that the gradients ∇g_i^* , all $i \in A^*$, and

∇h_j^* , $j = 1,\ldots,p$ be linearly independent. The other constraint qualifications involve requirements that the constraint functions be pseudoconvex or pseudoconcave, and will not be discussed here.

The First-Order Necessity Theorem (also called the Kuhn-Tucker Theorem) states [84]:

If the functions f , $\{g_i\}$, $\{h_j\}$ are differentiable at x^* and if any of the four above constraint qualifications holds at x^* , then necessary conditions that x^* be a local minimum of problem (NLP) are that there exist vectors u^* and w^* such that (x^*, u^*, w^*) satisfies (6.1-6.5).

Sufficiency conditions [32, p. 30] that a point x^* be an isolated local minimum of (NLP) where f , $\{g_i\}$, $\{h_j\}$ are twice differentiable functions, are that there exist vectors u^*, w^* such that (x^*, u^*, w^*) satisfies (6.1-6.5) and for every non-zero vector $y \in Y^*$, where

$$Y^* = \left\{ y \mid y^T \nabla g_i^* = 0, i \in B^* = \left\{ i \mid u_i^* > 0 \right\};\ y^T \nabla g_i^* \geq 0,\ i \in A^* - B^*;\ \text{and}\ y^T \nabla h_j^* = 0, j=1,\ldots,p \right\}$$

it follows that

$$y^T \left[\nabla^2 \mathcal{L}(x^*, u^*, w^*) \right] y > 0 , \quad \text{all } y \in Y^* \tag{6.7}$$

The Jacobian Condition Implying Sufficiency [32, p. 32] is that if f , $\{g_i\}$, $\{h_j\}$ are twice differentiable functions of x , and if, at x^* , the necessary conditions (6.1-6.5) hold, and if the Jacobian of (6.2), (6.3) and (6.5) with respect to (x, u, w) does not vanish at (x^*, u^*, w^*), then the sufficiency conditions above are satisfied at x^*.

6.2 THE PERTURBED-OPTIMAL SOLUTIONS PROBLEM

Suppose x^* solves (NLP). For a fixed. k, $1 \leq k \leq n$, let the k-th component of x^* be perturbed from its optimum value x_k^* by an amount δx_k, and held at this value. The solution, x', to (NLP) with this additional constraint is the solution to the perturbed-optimal solutions problem (POS). Note that a new problem is generated for each choice of k. The approach to solution will be to parameterize the solution to (POS) in δx_k (i.e., $x' = x'(\delta x_k)$) and seek conditions under which the solution exists locally (i.e., for infinitesimal δx_k), is unique, and is a continuously differentiable trajectory in δx_k. Finally, a linear approximation to the solution in an open interval about $\delta x_k = 0$ will be derived.

In the following, e^k is an n-vector of zeroes except for a '1' in the k-th place, and ν is a new generalized Lagrange multiplier associated with problem (POS). The following theorem proves, under certain conditions, the existence of a unique continuously differentiable trajectory through the solution to (NLP) that solves (POS).

Theorem 1. When

(a) the functions f, $\{g_i\}$, $\{h_j\}$ are twice differentiable

(b) (x^*, u^*, w^*) satisfies the sufficient conditions (6.1-6.5, 6.7) for (NLP)

(c) the vectors ∇g_i^*, $i \in A^*$, ∇h_j^*, $j = 1,\ldots,p$, and e^k are linearly independent

(d) "strict complementarity" of the inequality multipliers holds (i.e., $g_i = 0$ implies $u_i > 0$)

then there exists a unique continuously differentiable vector function parameterized in the scalar δx_k , $\left[x'(\delta x_k), u'(\delta x_k), w'(\delta x_k), \nu'(\delta x_k)\right]$, in an open interval D about $\delta x_k = 0$, that is the solution to (POS) for a fixed k and $\delta x_k \in D$, and $\lim_{\delta x_k \to 0} \left[x'(\delta x_k), u'(\delta x_k), w'(\delta x_k), \nu'(\delta x_k)\right] = (x^*,u^*,w^*,0)$.

<u>Proof.</u> This proof borrows the line of reasoning in Fiacco and McCormick's proof of the solution to the general parametric programming problem which is presented in the Historical Review.

First, it can be shown that u^* and w^* are unique. By (b), (x^*,u^*,w^*) satisfy (6.5), which can alternatively be written as

$$\left[\nabla f^*, -\nabla g_1{}^*, \ldots, -\nabla g_m{}^*, \nabla h_1{}^*, \ldots, \nabla h_p{}^*\right] \begin{bmatrix} 1 \\ u^* \\ w^* \end{bmatrix} = 0 \qquad (6.8)$$

Since $g_i(x^*) = 0 \Longrightarrow u_i{}^* > 0$ by assumption (d), and $g_i(x^*) > 0 \Longrightarrow u_i{}^* = 0$ by (6.3), the vector $(1,u^*,w^*)$ is of dimension $p + q + 1$, where q is the number of g_i's such that $g_1(x^*) = 0$; i.e., the number of elements in the set A^*. By assumption (c), the matrix of gradients is at least of dimension $p + q$. Since the existence conditions for the multipliers are satisfied (Z^* is empty because (c) guarantees that the Kuhn-Tucker constraint qualification holds) (6.8) has a solution, and so the dimension of the gradient matrix <u>equals</u> $p + q$ and the solution $(1,u^*,w^*)$ to (6.8) is unique.

Denote the system of nonlinear equalities

$$\nabla \mathcal{L}(x', u', w') + \nu' e^k = 0 \qquad (6.9)$$

$$u_i' \, g_i(x') = 0 \qquad i = 1, \ldots, m \qquad (6.10)$$

$$h_j(x') = 0 \qquad j = 1, \ldots, p \qquad (6.11)$$

$$x_k' - x_k^* - \delta x_k = 0 \qquad . \qquad (6.12)$$

as $F(x', u', w', \nu'; \delta x_k) = 0$. F is a continuously differentiable map from $R^{n+m+p+2}$ to $R^{n+m+p+1}$ in some open region about $(x^*, u^*, w^*, 0; 0)$ by (a), and $F(x^*, u^*, w^*, 0; 0) = 0$ because the necessary conditions for (NLP) must be satisfied since x^* solves (NLP).

The Jacobian matrix of F with respect to $(x, u, w, \;)$ at $(x^*, u^*, w^*, 0)$ is

$$\begin{bmatrix} \nabla^2 \mathcal{L}^* & -G^* & H^* & e^k \\ U^* G^{*T} & \text{diag}(g_i^*) & 0 & 0 \\ H^{*T} & 0 & 0 & 0 \\ (e^k)^T & 0 & 0 & 0 \end{bmatrix}$$

where $U^* = \text{diag}(u_i^*)$, $G^* = \left[\nabla g_1^*, \ldots, \nabla g_m^*\right]$, and $H^* = \left[\nabla h_1^*, \ldots, \nabla h_p^*\right]$. The Jacobian is invertible if (a-d) hold (a discussion of this invertibility appears later, but (a), (c), and (d) are obviously necessary). Thus, by the implicit function theorem [85], there exists a unique continuously differentiable function of δx_k, $G : D \subset R^1 \to R^{n+m+p+1}$ in the open interval D about $\delta x_k = 0$ such that

$$G(0) = (x^*, u^*, w^*, \nu^* = 0)$$

and

$$F\left[G(\delta x_k), \; \delta x_k\right] = 0, \quad \text{all} \quad \delta x_k \in D.$$

Now, it is shown that the sufficiency conditions for (POS) are satisfied along this trajectory, which is implicitly defined by $F(x',u',w',\nu';\ \delta x_k) = 0$.

The sufficient conditions for (x',u',w',ν') to solve (POS) are that (6.1-6.4) hold at (x',u',w',ν'),

$$\nabla \mathcal{L}(x',u',w') + \nu' e^k = 0 \tag{6.13}$$

and
$$y^T \left[\nabla^2 \mathcal{L}(x',u',w') \right] y > 0\ , \quad y \in Y_k' \tag{6.14}$$

where
$$Y_k' = \left\{ y \middle| y^T \nabla g_i' = 0,\ i \in B' = \left\{ i \mid u_i' > 0 \right\};\ y^T \nabla g_i' \geq 0,\ i \ni g_i(x') = 0 \text{ and } u_i' = 0;\ y^T \nabla h_j' = 0,\ j=1,\ldots,p; \text{ and } y^T e^k = 0 \text{ (or } y_k = 0) \right\}.$$

That (6.2), (6.3), and (6.13) hold at (x',u',w',ν') follows directly from (6.11), (6.10), and (6.9) respectively.

By continuity of the solution trajectory $\left[x'(\delta x_k),\ u'(\delta x_k),\ w'(\delta x_k),\ \nu'(\delta x_k) \right]$, $u_i^* > 0 \Rightarrow u_i' > 0$, for small δx_k, so (6.4) holds for $i \in A^*$. Again invoking continuity, and because the $\{g_i\}$ are differentiable and continuous, $g_i(x^*) > 0 \Rightarrow g_i(x') > 0$, for small δx_k, so (6.1) holds for $i \notin A^*$. The last two statements taken together imply that $B' = B^*$. Furthermore, by the strict complementarity requirement (d), $B^* = A^*$. By dividing (6.10) by u_i' for $i \in A^*$, (6.1) also holds for $i \in A^*$. Equation (6.4) holds at (x',u',w',ν') for $i \in A^*$ from above. For $i \notin A^*$, $g_i(x') > 0$ (also from above), so dividing (6.10) by $g_i(x')$ for $i \notin A^*$ gives the results that (6.4) holds at (x',u',w',ν') for all i, $i = 1,\ldots,m$.

What remains is to show that (6.14) holds for points on the solution trajectory.

Since $(x^*,u^*,w^*,\nu^* = 0)$ satisfies (6.7) for $y \in Y^*$, all second derivatives are assumed continuous, and $B' = B^*$,

$$y^T \left[\nabla^2 \mathcal{L}(x',u',w') \right] y > 0 \quad \text{for small} \quad \delta x_k \quad \text{and}$$

$$y \in Y' = \left\{ y \middle| y^T \nabla g_i' = 0,\ i \in B' = B^* = A^*;\ y^T \nabla h_j' = 0,\ j = 1,\ldots,p \right\} .$$

But $Y_k' \subset Y'$ so (6.14) holds at (x',u',w',ν'), for $y \in Y_k'$, and the trajectory $\left[x'(\delta x_k), u'(\delta x_k), w'(\delta x_k), \nu'(\delta x_k) \right]$ satisfies the sufficient conditions for (POS) in some open interval D about $\delta x_k = 0$.

End of proof.

Theorem 1 guarantees that, under certain conditions, there is a unique solution to (POS), and furthermore, that it is possible to find a differential approximation to the solution in an open interval about the solution to the original, unperturbed, nonlinear programming problem. See appendix for proof of Jacobian invertibility.

Again, it is important to note that, for each k, there is in effect a new perturbed-optimal solution problem. Because condition (c) in Theorem 1 depends on the value of k, it is necessary to retest the assumptions for each k. It is claimed that (a-d) imply the invertibility of the Jacobian, which is necessary to invoke the implicit function theorem to show a unique differentiable trajectory of solutions to (POS) parameterized in δx_k. However, note also that only this invertibility requirement depends on k, because it requires linear independence of e^k from other gradients, and that the rest of the proof is independent of the value of k. Therefore, it may be possible to say something about the satisfying of assumptions (a-d) relative to the value of k.

A k-dependent, directly testable condition on the invertibility of the Jacobian can be derived as follows. Write the Jacobian as

$$\begin{bmatrix} M & e^k \\ (e^k)^T & 0 \end{bmatrix}.$$

In the absence of equality constraints, it can be shown [32, p. 80] that M is invertible if (a-d) hold with e^k eliminated from (c). In similar manner, it is possible to prove that the Jacobian is invertible if the full conditions (a-d) hold, or to determine the invertibility of M independent of k (since k does not appear in M). Suppose M is invertible. Write

$$\begin{bmatrix} M & e^k \\ (e^k)^T & 0 \end{bmatrix} = \begin{bmatrix} M & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I & M^{-1} e^k \\ (e^k)^T & 0 \end{bmatrix}.$$

The first of the right-hand-side matrices is obviously invertible. Then, the Jacobian for the perturbed-optimal solutions problem is invertible iff

$$\begin{bmatrix} I & M^{-1} e^k \\ (e^k)^T & 0 \end{bmatrix}$$

is invertible, which is true if $(e^k)^T M^{-1} e^k \neq 0$, or if the k-k term of M^{-1} is non-zero.

Geometrically, it is easy to see why e^k need be linearly independent from ∇g_i^*, $i \in A^*$, ∇h_j^*, $j = 1,\ldots,p$. Intuitively, a small perturbation from x^* to x', δx ($= x' - x^*$), should satisfy $(\nabla h_j^*)^T \delta x = 0$, $(\nabla g_i^*)^T \delta x = 0$, $i \in A^*$, because it is reasonable to to expect that the perturbed-optimal solution will remain on the equalities and "active" inequalities at x^* for some small distance from x^*. That this is, in fact, the case is shown directly in Theorem 2. Now, since δx is orthogonal to these gradients, and e^k must have a non-zero projection onto δx by the definition of the perturbation, e^k must be linearly independent from the gradients at x^*.

The function $G(\delta x_k)$ is defined implicitly by (6.9-6.12), and is in general nonlinear. Next, a linear approximation to perturbed-optimal solutions about the point $(x^*, u^*, w^*, 0)$ will be derived.

Theorem 2. If the conditions (a-d) in Theorem 1 hold, a first-order approximation to the solution to (POS) is

$$\begin{bmatrix} x' \\ u' \\ w' \\ \nu' \end{bmatrix} = \begin{bmatrix} x^* \\ u^* \\ w^* \\ 0 \end{bmatrix} + \begin{bmatrix} \eta^k \\ \frac{du}{dx_k} \\ \frac{dw}{dx_k} \\ \frac{d\nu}{dx_k} \end{bmatrix} \delta x_k \qquad (6.15)$$

where the "perturbation coefficients" (elements of the vector multiplying δx_k) are solved by

$$\begin{bmatrix} \nabla^2 \mathcal{L}^* & -G^* & H^* & e^k \\ U^* G^{*T} & \text{diag}(g_i^*) & 0 & 0 \\ H^{*T} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta^k \\ \frac{du}{dx_k} \\ \frac{dw}{dx_k} \\ \frac{d\nu}{dx_k} \end{bmatrix} = 0 . \qquad (6.16)$$

Proof. Here, the linear approximation is constructed by linearization of the equalities defining the solution to (POS).

Because the solution trajectory is continuously differentiable in some open interval about $\delta x_k = 0$, and because F is continuously differentiable in an open region about $(x^*,u^*,w^*,0;\ 0)$, equations (6.8-6.11) can be expanded in first-order Taylor series' about $(x^*,u^*,w^*,0)$. Thus

$$h_j(x') = 0 \longrightarrow h_j(x^*) + \nabla h_j^T(x^*)\,\delta x = 0 \qquad j = 1,\dots,p$$

$$u_i' g_i(x') = 0 \longrightarrow u_i^* \nabla g_i^T(x^*)\,\delta x + g_i(x^*)\nabla u_i^{*T}\,\delta x = 0 \quad i = 1,\dots,m$$

$$\nabla \mathcal{L}(x',u',w') + \nu' e^k = 0 \longrightarrow \nabla \mathcal{L}(x^*,u^*,w^*) + \Big\{ \nabla^2 f(x^*) - \sum_{i=1}^{m} \nabla g_i(x^*)\nabla u_i^{*T} - \sum_{i=1}^{m} u_i^* \nabla^2 g_i(x^*) + \sum_{j=1}^{p} \nabla h_j(x^*)\nabla w_j^{*T} + \sum_{j=1}^{p} w_j^* \nabla^2 h_j(x^*) + \nabla \nu^* (e^k)^T \Big\}\,\delta x = 0 .$$

Defining $\delta_x = \eta^k \delta x_k = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_{k-1} \\ 1 \\ \eta_{k+1} \\ \vdots \\ \eta_n \end{bmatrix} \delta x_k$

and noting that

$$\nabla u_i^{*T} \eta^k = \frac{du_i}{dx_k} \qquad i = 1,\ldots,m$$

$$\nabla w_j^{*T} \eta^k = \frac{dw_j}{dx_k} \qquad j = 1,\ldots,p$$

$$\nabla \nu^*(e^k)^T = e^k \frac{d\nu}{dx_k} ,$$

equation (6.16) follows by substitution.

The vector of perturbation coefficients is a direction specifying the optimal readjustment to the perturbation in the k-th component as $\delta x_k \longrightarrow 0$, and therefore (6.15) holds in the limit.

<u>End of proof</u>.

In application of the solution with finite δx_k , bounds on the error of the approximation to (6.9-6.11) by (6.15, 6.16) can be obtained by an exact Taylor series of order two. In addition, however, there exist inequality relations that must be satisfied. These may not be, for finite δx_k . Sufficiency conditions require

$$g_i(x') \geq 0 \qquad i = 1,\ldots,m$$

and

$$u_i' \geq 0 \qquad i = 1,\ldots,m$$

A first-order approximation to the permissible values of δx_k to insure these last two conditions can be obtained from

$$g_i(x^*) + \nabla g_i^T(x^*) \eta^k \delta x_k \geq 0 \qquad i = 1,\ldots,m \qquad (6.17)$$

$$u_i^* + \frac{du_i}{dx_k} \delta x_k \geq 0 \qquad i = 1,\ldots,m. \qquad (6.18)$$

Under the assumption of strict complementarity,

$$\nabla g_i^T(x^*)\, \eta^k = 0 \qquad i \in A^*$$

from (6.16), so that (6.17) gives nontrivial relations only for i such that $g_i(x^*) > 0$, i.e., for $i \notin A^*$. Likewise, because of strict complementarity

$$u_i^* > 0 \qquad i \in A^*$$

$$u_i^* = 0 \qquad i \notin A^*$$

Then, from (6.16) it is clear that

$$\frac{du_i}{dx_k} = 0 \qquad i \notin A^*$$

and (6.18) is useful only for $i \in A^*$.

Thus, first-order tests on the permissible values of δx_k can be reduced (in number) to

$$g_i(x^*) + \nabla g_i^T(x^*)\, \eta^k\, \delta x_k \geq 0 \qquad i \notin A^* \tag{6.19}$$

$$u_i^* + \frac{du_i}{dx_k}\, \delta x_k \geq 0 \qquad i \in A^*. \tag{6.20}$$

Another case to be considered is the one with multiple initial perturbations. Consider the case of two forced perturbations, in the k-th and ℓ-th components of x^*. Then, the new solution can be written

$$x' = x^* + \delta x$$

and the aim is to approximate δx as a function of δx_k. Previously, δx was written as

$$\delta x = \eta^k \, \delta x_k = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_{k-1} \\ 1 \\ \eta_{k+1} \\ \vdots \\ \eta_n \end{bmatrix} \delta x_k ,$$

which allowed the linearized solution to be parameterized in δx_k . Now, the method derived above can be applied directly if the ratio of the forced perturbations is known. If the ratio of the perturbations is r (i.e., $\frac{\delta x_\ell}{\delta x_k} = r$) , δx can be written

$$\delta x = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_{k-1} \\ 1 \\ \eta_{k+1} \\ \vdots \\ \eta_{\ell-1} \\ r \\ \eta_{\ell+1} \\ \vdots \\ \eta_n \end{bmatrix} \delta x_k$$

which eliminates one unknown from the η vector. However, (6.9) becomes

$$\nabla \mathcal{L} + \nu_1 \; e^k + \nu_2 \, e^\ell = 0$$

introducing another unknown. Thus, the method for perturbed-optimal solutions applies directly with e^ℓ added to condition (c) in Theorem 1.

A final consideration is the idea of adjusting to a perturbation with some of the unperturbed elements of x^* held fixed. That is, it is required that some of the sensitivity coefficients (η_i's) be zero. The practical applications are obvious.

Very simply, this can be handled by adding a new constraint to the problem. If the s-th coefficient is to be forced to zero, write

$$x_s = x_s^*$$

as the $(p + 1)^{th}$ equality constraint, and return to the method for perturbed-optimal solutions with this additional constraint included in the set $\{h_j\}$. The solution is obtained directly from the theorems if the required conditions still hold.

SECTION 7

SENSITIVITY OF MARS-ROVING VEHICLE DESIGNS

Section 5 presents designs for a Mars-roving vehicle for differing assumptions and parameter values and/or limits. The number of designs totals thirty-six. For each of these designs it is possible to pose many perturbed-optimal problems, as each of the variables in each design can be individually forced from its optimum value and the corresponding perturbed-optimal solution calculated.

Because equalities can be used to eliminate variables, the system model can be adjusted until it contains the parameters it is of interest to perturb and a minimal number of other parameters. The number of equalities remaining is, of course, a function of the number of eliminations performed.

It is also of interest to note that only "active" inequalities (i.e., those whose values are identically zero at x^*) need be considered, assuming that the strict complementarity assumption holds. This is simply because for all i such that $g_i(x^*) > 0$, it is assumed that $u_i^* = 0$. Looking at (6.16) it becomes clear that this condition effectively removes inactive inequalities from consideration and forces their corresponding $\frac{du_i}{dx_k}$ to zero.

A last consideration is the need for values of the multipliers (u^*,w^*). The SUMT procedure does not return accurate multipliers, and in addition remember that all equalities were eliminated before using SUMT, so that they do not appear at all in the original NLP problem. So, it becomes necessary in this case to solve for (u^*, w^*) separately.

The $m + p$ generalized Lagrange multipliers (u^*, w^*) are found from

$$\left[\nabla f^* \,\vdots\, -\nabla g_1^* \dots -\nabla g_m^* \,\vdots\, \nabla h_1^* \dots \nabla h_p^*\right] \begin{bmatrix} 1 \\ u^* \\ w^* \end{bmatrix} = 0 \qquad (7.1)$$

and their existence is guaranteed by the Kuhn-Tucker Necessity theorem. This requires that the n by $1 + m + p$ matrix in (7.1) have a dependency relation among its columns. This dependency is guaranteed to exist only at x^*, and it is most likely that it will not exist even a very small distance away from x^*. Unfortunately, because of the iterative search nature of the NLP solution procedure and because of finite word length effects in the computer, x^* is not known exactly and consequently (7.1) has, in general, no solution.

The values (u^*, w^*) must be approximated. An approximation was made by noting that since no solution exists to the set of homogenous equations in (7.1) (since the matrix of gradients is of full rank) the problem can be posed as

$$\min \| r \| \qquad (7.2)$$

subject to

$$A \begin{bmatrix} u^* \\ \cdot \\ w^* \end{bmatrix} + \nabla f^* = r \qquad (7.3)$$

$$u_i^* \geq 0 \qquad i = 1, \dots, m \qquad (7.4)$$

where A is the sub-matrix of gradients of the equalities and inequalities and r is a residual vector. Actually, the minimization can be made over any function of r, but the norm was chosen for reasons that will be explained below.

The choice of norm is now critical. Several of the l_p norms are candidates. They are

$$\|r\|_1 = |r_1| + \ldots + |r_n| \quad .$$

$$\|r\|_2 = \sqrt{r_1^2 + \ldots + r_n^2}$$

$$\|r\|_\infty = \max_{1 \le i \le n} |r_i| \quad .$$

The l_2 norm has the disadvantage of being a nonlinear warping of the true errors r_i. The l_∞ norm will allow large errors (large r_i) for many components of r in order to reduce the largest error. Conversely, the l_1 norm does not warp the errors and considers all of them directly in its value. Thus, the norm chosen was l_1. Then, the problem specified by (7.2), (7.3), and (7.4) was written as a linear programming problem and solved by a simplex algorithm, yielding values for the multipliers (u^*,w^*).

The rest of this Section presents the investigation of the sensitivity of one of the designs determined in Section 5. The selected case is problem 1A, a four-wheeled direct communicating rover design.

After determining what design parameters it would be of interest to perturb, the system model was written in 26 design parameters with 16 equality constraints. The design variables were placed in a new x-vector

$$x = (T_{esci}, W_{sci}, T_{sci}, P_{com}, D^2_{com}, R_{com}, P_{RTG}, v_f, s^*, P_\theta,$$
$$E_{batt}, T_{rov}, w_b, A, T_{bn}, T_{rn}, T_{bd}, T_{rd}, L_i,$$
$$K_q, Q_h, T, PLR, S_{sci}, T_r, \text{roving fraction})$$

where the roving fraction is the square-bracketed term in (4.37). The 16 equalities remaining correspond to equations (4.4), (4.5), (4.3), (4.36), (4.37), (4.18), (4.19), (4.20), (4.21), (4.22), (4.23), (4.25), (4.26), (4.24), (4.7), and the roving fraction equation. They were numbered in the order above.

At the optimal point, problem 1A had 8 active inequalities. Inequalities 1 through 4 were relations (4.42), (4.32), (4.38) and (4.30), respectively. Inequalities 5 through 8 were the direct limits on D^2_{com}, A, s* and v_f, respectively. The multipliers approximated for problem 1A were:

u_1 = 0.0037197	w_5 = 0.0202090
u_2 = 0.085324	w_6 = -0.17767E-03
u_3 = 0.174217	w_7 = -0.291739E-04
u_4 = 0.0017596	w_8 = 0.96237E-04
u_5 = 0.064972	w_9 = -0.19495E-03
u_6 = 1.57424	w_{10} = -0.11319E-04
u_7 = 0.149539	w_{11} = 0.19660E-03
u_8 = 10.46755	w_{12} = -18.2746
w_1 = 0.49220E-04	w_{13} = -7.22808
w_2 = -0.0061258	w_{14} = -0.14747E-03
w_3 = 0.92256E-05	w_{15} = -4.16517
w_4 = 0.12885E-03	w_{16} = 0.11495E-02

As one notes from Theorem 2, Section 6, the output of the sensitivity analysis for <u>each</u> perturbation is the vector of "perturbation coefficients"

$$\begin{bmatrix} \eta^k \\ \frac{du}{dx_k} \\ \frac{dw}{dx_k} \\ \frac{d\nu}{dx_k} \end{bmatrix} .$$

Calculations were made for the perturbation coefficient vector for initial perturbations in W_{sci}, R_{com}, P_{RTG}, T_{rov}, w_b, L_i, and S_{sci} - a total of 7 solutions. The results are presented in Tables 17 and 18. The solutions were obtained by the use of a double precision Gaussian elimination routine to solve (6.16). It should be noted that because of round-off error, any value of magnitude less than 10^{-10} should be considered zero.

The proper interpretation of the perturbation coefficient vector is that it represents the optimal <u>direction</u> of movement from the optimal point $(x^*, u^*, w^*, 0)$ for infinitesimal values of the initial perturbation δx_k, and the magnitude of each perturbation is determined by multiplying by δx_k. Thus the value of the perturbation coefficient for any design parameter (member of x^*) indicates the sensitivity of that parameter to a forced perturbation in the k-th parameter around the optimal point. Again, this value is a linear approximation and in general becomes more inaccurate as δx_k increases due to the in-

TABLE 17

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR PARAMETER: T_{esci}	W_{sci}	T_{sci}	P_{com}	D^2_{com}	R_{com}	P_{RTG}
W_{sci}	35.75	1.0	-3.849E 02	3.680E 02	0.66E-15	3.091E 04	-1.50E 03
R_{com}	1.118E-03	3.128E-05	-2.49E-02	1.191E-02	0.102E-19	1.0	-3.723E-02
P_{RTG}	-3.004E-02	-8.402E-04	0.3355	-0.3198	-0.271E-18	-26.86	1.0
T_{rov}	-9.804	-0.2742	74.53	-73.77	-0.971E-16	-6.197E 03	2.307E 02
w_b	-1.424E 05	-3.984E 03	1.591E 06	-1.516E 06	-0.909E-12	-1.274E 08	4.742E 06
L_i	3.738E 02	10.46	-4.176E 03	3.980E 03	0.355E-14	3.343E 05	-1.245E 04
S_{sci}	7.776E 04	2.175E 03	-6.454E 05	6.327E 05	0.909E-12	5.314E 07	-1.978E 06

PERTURBATION COEFFICIENTS FOR DESIGN PARAMETERS - PROBLEM 1A

(Table 17 continued)

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR PARAMETER v_f	s^*	P_θ	E_{batt}	T_{rov}	w_b	A
W_{sci}	0.55E-14	0.35E-14	-1.187E 03	-3.053	-1.171	-2.43E-04	-0.33E-14
R_{com}	-0.936E-19	0.880E-19	-3.842E-02	-6.981E-05	-2.677E-05	-7.851E-09	-0.169E-19
P_{RTG}	-0.918E-18	-0.250E-17	1.031	1.875E-03	7.192E-03	2.109E-07	0.455E-17
T_{rov}	0.173E-14	-0.535E-15	2.380E 02	2.606	1.0	4.865E-05	0.222E-15
w_b	0.286E-10	-0.127E-10	4.893E 06	8.892E 03	3.410E 03	1.0	0.155E-10
L_i	0.264E-13	-0.355E-13	-1.284E 04	-23.34	-8.950	-2.625E-03	0.463E-13
S_{sci}	0.909E-12	0.103E-10	-2.041E 06	-1.758E 04	-6.743E 03	-0.4172	0.620E-11

(Table 17 continued)

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR PARAMETER T_{bn}	T_{rn}	T_{bd}	T_{rd}	L_i	K_q	Q_h
W_{sci}	-67.55	-60.32	21.00	-99.12	0.0925	6.156	-1.151E 03
R_{com}	-2.185E-03	-1.951E-03	6.795E-04	-3.206E-03	2.991E-06	1.991E-04	-3.723E-02
P_{RTG}	5.870E-02	5.242E-02	-1.826E-02	8.614E-02	-8.035E-05	-5.350E-03	1.000
T_{rov}	13.54	12.09	-4.211	19.87	-1.853E-02	-1.234	2.307E 02
w_b	2.783E 05	2.486E 05	-8.656E 04	4.084E 05	-3.810E 02	-2.537E 04	4.742E 06
L_i	-7.306E 02	-6.524E 02	2.272E 02	-1.072E 03	1.0	66.58	-1.245E 04
S_{sci}	-1.161E 05	-1.037E 05	3.611E 04	-1.704E 05	1.589E 02	1.058E 04	-1.978E 06

(Table 17 continued)

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR PARAMETER				
	T	PLR	S_{sci}	T_r	Roving Fraction
W_{sci}	0.49E-05	0.12E-04	0.20E-03	-0.84E-02	-0.25E-02
R_{com}	1.597E-10	4.038E-10	4.976E-09	-1.929E-07	-8.174E-08
P_{RTG}	-4.290E-09	-1.085E-08	-1.337E-07	5.182E-06	2.196E-06
T_{rov}	-9.896E-07	-2.502E-06	-1.461E-04	7.203E-03	5.065E-04
w_b	-2.034E-02	-5.143E-02	-0.6338	24.57	10.41
L_i	5.339E-05	1.350E-04	1.664E-03	-6.450E-02	-2.733E-02
S_{sci}	8.487E-03	2.146E-02	1.0	-48.58	-4.344

TABLE 18

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR MULTIPLIER: u_1	u_2	u_3	u_4	u_5	u_6
W_{sci}	-0.3692	20.84	154.3	6.995	-16.94	-1.838E 03
R_{com}	5.124E-07	-2.422E-05	-1.728E-04	-8.051E-06	2.853E-05	2.135E-03
P_{RTG}	-1.377E-05	5.339E-04	6.496E-03	2.069E-04	8.705E-04	-4.716E-02
T_{rov}	0.9322	0.5380	3.939	0.1800	-0.4969	-47.44
w_b	-65.27	2.549E 03	-4.339E 07	2.208E 05	4.128E 03	-1.866E 06
L_i	0.1713	-6.719	-80.85	-2.575	-10.83	5.920E 02
S_{sci}	98.85	-4.613E 03	-3.375E 04	-1.544E 03	4.260E 03	4.067E 05

PERTURBATION COEFFICIENTS FOR GENERALIZED LAGRANGE MULTIPLIERS - PROBLEM 1A

(Table 18 continued)

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR MULTIPLIER:						
	u_7	u_8	w_1	w_2	w_3	w_4	w_5
W_{sci}	2.052E 03	6.927E 02	-0.938E-02	-0.104E-01	-0.206E-03	1.954E 01	7.519
R_{com}	-2.386E-03	-8.654E-04	-2.498E-07	-2.821E-07	8.164E-10	-2.272E-05	-7.012E-07
P_{RTG}	5.244E-02	1.930E-02	6.707E-06	7.577E-06	1.985E-07	4.996E-04	1.884E-04
T_{rov}	52.96	19.41	5.593E-03	5.790E-03	-9.210E-06	0.5046	2.515E-02
w_b	2.475E 05	9.113E 04	31.80	35.93	94.14	2.358E 03	89.30
L_i	-6.526E 02	-2.402E 02	-8.347E-02	-9.431E-02	-2.471E-03	-6.218	-0.2344
S_{sci}	-4.543E 05	-1.793E 05	-47.96	-49.65	7.897E-02	-4.326E 03	-1.571E 02

(Table 18 continued)

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR MULTIPLIER: w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}
W_{sci}	38.48	6.319	-20.84	26.06	1.477	-26.28	-3.024
R_{com}	-4.470E-05	-7.339E-06	2.422E-05	-3.028E-05	-1.716E-06	3.055E-05	-9.767E-05
P_{RTG}	9.854E-04	1.618E-03	-5.339E-04	6.657E-04	3.774E-05	-6.719E-04	2.624E-03
T_{rov}	0.9931	0.1631	-0.5380	0.6725	3.812E-02	-0.6783	0.6164
w_b	4.704E 03	7.725E 02	-2.549E 03	3.177E 03	1.747E 02	-3.207E 03	1.244E 04
L_i	-12.40	-2.036	6.719	-8.285	-0.4696	8.362	-32.65
S_{sci}	-8.515E 03	-1.398E 03	4.613E 03	-5.766E 03	-3.269E 02	5.816E 03	-5.262E 03

(Table 18 continued)

PERTURBED PARAMETER	PERTURBATION COEFFICIENT FOR MULTIPLIER: w_{13}	w_{14}	w_{15}	w_{16}	ν
W_{sci}	-1.196	19.54	-0.2918	4.277E-02	-8.111
R_{com}	-3.863E-04	-2.272E-05	-2.213E-05	-3.989E-08	-8.107E-09
P_{RTG}	1.038E-03	4.996E-04	5.944E-03	1.071E-06	-5.923E-06
T_{rov}	0.2438	0.5044	-0.8155	1.431E-03	-1.900
w_b	4.921E 03	2.358E 03	2.818E 03	5.080	-1.332E 08
L_i	-12.92	-6.218	-7.398	-1.334E-02	-9.174E 02
S_{sci}	-2.081E 03	-4.325E 03	-1.137E 03	-8.937	-8.766E 08

herent nonlinearity of the problem. The implications of these results are discussed in Section 8.

SECTION 8

DISCUSSION AND CONCLUSIONS

This section is divided into discussions of optimal design results, the perturbed-optimal solutions problem, sensitivity of Mars-rover designs, and conclusions. The conclusions include the author's recommendations for future work.

8.1 DISCUSSION OF OPTIMAL DESIGN RESULTS

An important result of a nonlinear programming optimization is an examination of the "active" (i.e., equal to zero) inequalities at the optimal point. In all runs, the total launch weight constraint, (4.39), is active, as can be seen by noting that the mass of the rover (M_r) is at its maximum value. This is an intuitively pleasing result, for it indicates that it is optimal to use all the available weight allotted to the system.

The equipment weight ratio constraint, (4.38), establishes the equipment weight limit that a vehicle structure of given size can carry. For the 4-wheeled cases, this constraint is active when structure weight, W_v , is 190 kg. As seen in the tables of Section 5, this constraint is active for all runs for case 1 (4-wheeled, direct communicating rover), but only in problem 3E in case 3 (4-wheeled rover communicating via a Mars orbiter). This implies that when a large communications subsystem is onboard, equipment is a scarce resource in terms of the optimal design. Conversely, if the communications subsystem is small, the weight savings are not allotted entirely to other equipment, but partially to the vehicle structure. The reason for this is that a larger-wheelbase vehicle has an advantage as far as obstacle avoidance is concerned (smaller obstacles become unimportant, slopes are calculated on larger base lengths and consequently errors are smaller). However, this reasoning only holds when vehicle velocity (v_f) and the slope threshold (s^*) are limited (problems 3A - 3D). When these parameters are unconstrained (as in 3E), the weight saving optimally goes to the power subsystem so that v_f can be increased.

For the 6-wheeled vehicle (problems 2A - 2F), an active weight ratio constraint would be indicated by a vehicle structure weight of 207.3 kg. Table 12 indicates that this constraint is never active. This result is due to the fact that the 6-wheeled vehicle is more sensitive to smaller obstacles than an equivalent weight 4-wheeled rover since its track is considerably smaller than its wheelbase. Consequently, it is apparently optimal to increase the vehicle size to reduce this sensitivity, even at the expense of vehicle velocity (see results for 2E).

In subsection 4.2.3, a discussion of the RTG power (P_{RTG}) relations is presented. The RTG power output is properly found from

$$P_{RTG} = \max\,(P_{prop} + P_{mv},\ Q_h + Q_{in}).$$

In most problems, the first term is the larger. However, in some 6-wheeled problems (2B - 2D) the second term predominates. The interesting case occurs when the two terms are equal, indicating that it is optimal to utilize all power resources both during the day ($P_{prop} + P_{mv}$) and night ($Q_h + Q_{in}$). This is true in all problems for case 1 except 1K plus problems 3C and 3D. What occurs in these instances is that insulation thickness (L_i) is reduced (with corresponding weight savings) until night heater power (Q_h) is large enough to equalize the terms. Since the power is available due to the high daytime requirements, there is little cost to this maneuver as far as night operations are concerned and weight is saved. However, the thermal control problem during the day is affected by the insulation thickness reduction, and in some problems (3A, 3B, 3E) this change becomes the predominating factor. Because question of the equality of the P_{RTG} relations is highly linked with the thermal control subsystem, it is not surprising that the equality does not exist in some of the 6-wheeled rover problems (2A - 2F). The thermal control problem here is complicated by the fact that during the day one compartment (electronics) requires cooling, while the other (science) needs to be heated because of the low heat dissipation inside it. In 2A, 2E, and 2F the equality exists, but in 2B - 2D day power requirements are lower because of lower slope thresholds and vehicle velocities and the night heating problem is a more serious problem.

Another often active inequality is the relation of (4.42) which states that the time interval between the start of one battery recharge until the next recharge is required (T_{cy}) should be less than or equal to the total vehicle operational time in one Mars day (V). In all problems in which this constraint appeared, it was active. This simply means that since recharge time is "down" time for the system, an optimal design is one which maximizes the time between recharges, even at the cost of battery weight. In some problems (1F - 1AB) concerning the 4-wheeled, direct communicating rover, (4.42) was changed to limit roving time between recharges (T_{rov}) to a maximum of V. This design allows for the possibility of direct commands from Earth superceding normal vehicle operations and directing a long-distance rove. In these designs, T_{rov} was in general much higher, but the upper limit was achieved only in problems 1S and 1U, where velocity and slope threshold were unconstrained, an alternative thermal control model was used ($290 \leq T_{int} \leq 310$, $e_t = 0.5$) and S_{sci} was fixed. The combination of high slope threshold and the alternative thermal control model was the decisive factor, since less stringent temperature limits allowed less allocation to the thermal control subsystem, a corresponding increase in s^* and hence a high value for T_{rov}.

Other active constraints involved direct limits on design parameters. In all problems, upper or lower limits placed on v_f, s^*, equipment package surface area (A), and antenna diameter (D_{com}) were attained. This implies that they are critical parameters of the design. For example, if D_{com} was left unconstrained it would take a higher value, but then the antenna would not fit into the Titan IIIc aeroshell. The implication is that the upper limit placed on D_{com} should be determined accurately, and any modification to the aeroshell which would allow a larger antenna should be considered.

In some problems, v_f and s^* were left unconstrained. Particularly note the results of 3E where $v_f = 13.60$ m/sec and 1Y where $s^* = 32.22$ degrees. In instances such as these where the values are unrealistically high, it is because there are factors not considered in the model. A high velocity is unsafe because of the difficulty of stopping the vehicle quickly in an emergency situation. A high slope threshold admits the possibility of the vehicle tipping over. Whether or not either of these problems will occur is highly dependent upon the features of the Mars terrain, which are only hazily known at this time - hence the need for limits on these two parameters. The results show a high dependence upon these limits, indicating that data from future Mars-landers should be used to more accurately determine acceptable limits. However, the trade-offs remain heuristic - how "unsafe" is a specific velocity, and how much "unsafeness" can be tolerated in order to achieve greater coverage of the planet surface?

Other design implications can be culled from the results of Section 5. A comparison of results for problems 2A and 2F indicate that radiators on the larger sides of the equipment packages are marginally better for a 6-wheeled vehicle. This is also true (see 1P, 1Q) for a 4-wheeled vehicle with equipment package relative dimensions of 4 : 3.33 : 1 (these are the recommended ratios, because they allow easier correct placement of the center of gravity of the vehicle). However, if the relative dimensions go to 5.5 : 3.6 : 1 , smaller radiators become significantly better.

The results of 1R vs. 1W and 1V vs. 1X indicate that little penalty in overall system performance is paid by restricting the acceptable internal temperature range to 300 $\pm$ 5K rather than 300 $\pm$ 10K, regardless of which thermal efficiency factor is used.

Problems 1Y and 1Z show that if more frequent (in distance) science stops are desired, the only significant adjustments to the optimal design are a decrease in v_f and an increase in s^* .

As a whole the results tend to show that power is a "cheap" commodity in an MRV optimal design. That is, power consuming subsystems and activities are not normally limited by the power they require, but by their weight and/or time requirements. An exception to this general rule in problem 1K, where in a design unconstrained in vehicle velocity and slope threshold and with strong roving capability ($T_{rov} \leq V$) the high daytime roving requirements cause RTG power to be very high. This has the effect of requiring power saving, which is accomplished by increasing the insulation thickness. This in turn means that heater power requirements at night are very low.

Another indication of the results is that batteries might not be required if the slope threshold was high enough, or if a corresponding increase in path-length ratio (PLR) was acceptable (here again, the judgement cannot be made because simply not enough is yet known about the Mars surface).

Because they deal in more realistic types of designs, problems A-E in all three cases deserve closer examination. Of course, any comparison between the three major cases, as well as all others made in this section, are based upon the choice of performance index.

Except in the case where v_f and s^* are unconstrained, the performance of the 4-wheeled, direct communicating rover exceeds that of the 6-wheeled vehicle. Especially since the unconstrained case (problems 1E and 2E) is not likely to be acceptable for reasons mentioned above, the 4-wheeled concept seems clearly preferable. Six-wheeled performance is 78, 48, 55, and 92% of the 4-wheeled rover's in problems A-D respectively. The reasons are twofold. First, the equipment weight ratio is higher for the 4-wheeled rover. Second, the 6-wheeled rover has a disadvantage due to its small track and consequent problems of being concerned with smaller obstacles. The design solution was to build a large vehicle structure, which just compounds the equipment weight ratio disadvantage. Finally, because the 6-wheeled rover will be less maneuverable, due to its three articulated sections, its slope and velocity limits might have to be set lower than those for a 4-wheeled vehicle, again decreasing its relative performance. However, the 6-wheeled vehicle seems to have one overriding advantage - reliability, due to the redundancy of 6 driven wheels. Once more, the trade-off of reliability vs. performance is not a clear cut problem.

Comparing the results for the two 4-wheeled cases (direct and relay communications) shows only a small improvement in system performance in the relay case, except in the unconstrained problems (1E and 3E) which again are not realistic designs. Problems A-D show a 10, 1, 17, and 33% increase in performance respectively for the relay over the direct system. Considering the cost and complexity of establishing an orbiting communications relay satellite for Mars, the relay concept does not seem to be a viable alternative. If, however, the relay link ability could be added to another Mars-orbiter mission, or if the orbiter could be used for additional functions, the relay link concept would look more attractive by comparison.

8.2 DISCUSSION OF THE PERTURBED-OPTIMAL SOLUTIONS PROBLEM

The determinations of when a first-order approximation to a perturbed-optimal solution can be made, and the solution of the perturbed-optimal problem obtained are given in Section 6, and do not require further discussion. A few comments will be made here relative to the use of the method.

Before application, the four conditions of Theorem 1 must be satisfied. First, the objective function and constraints must be twice differentiable in the region about the optimal solution to the NLP problem. This is quite often the case, but a large class of problems is nonetheless excluded. Second, the solution must satisfy the sufficiency conditions for (NLP) presented in 6.1, which is a stronger statement that requiring that (x^*, u^*, w^*) is a local solution to (NLP). This requirement is directly testable by the Jacobian Condition Implying Sufficiency (see 6.1). If the Jacobian Condition is tested and is satisfied, the resulting inverse can be used to test the applicability of the method of perturbed-optimal solutions for a particular value of k as described in 6.2. Third, the vectors g_i^*, $i \in A^*$, h_j^*, all j, and e^k must be linearly independent. The independence of e^k from the gradients was discussed in 6.2. The independence of the gradients is not even a necessary condition for the Kuhn-Tucker Theorem to hold, but it is a sufficient condition quite often tested before the Kuhn-Tucker Necessity relation is calculated for an NLP problem. Note also that the independence of all the vectors jointly precludes the possibility of forcibly violating a direct limit on a design parameter. If the k-th element of the solution achieves an upper or lower direct limit at the optimal point, the gradient of that active inequality is e^k, which is obviously not linearly independent from itself. The question of perturbing a parameter with a direct limit is more properly viewed as changing that limit, and the problem becomes the parametric programming problem (see Historical Review). Fourth, and last, strict complementarity of the inequality multipliers must hold (i.e., $g_i = 0$ must imply $u_i > 0$). This can be alternately stated as requiring that the value of the objective function $f(x)$ be sensitive to the inequality in question. If the i-th constraint $g_i(x) \geq 0$ is rewritten as $g_i(x) \geq b_i$ then

$$u_i = \left.\frac{\partial f(x^*)}{\partial b_i}\right|_{b_i = 0} ,$$

so if $u_i = 0$ when $g_i = 0$ the objective function at x^* is not sensitive to the constraint (to first order), and it is expected that the linearization done in Theorem 2 should fail to solve the problem.

8.3 DISCUSSION OF SENSITIVITY OF MARS-ROVER DESIGNS

The perturbation coefficients of the perturbed-optimal solutions for 7 different forced parameter perturbations in problem 1A appear in Section 7. Application was limited to this problem because the usefulness of the perturbed-optimal solution method is adequately shown, and because the time and money expenditures of generating these solutions can be reduced by first reducing the number of candidate designs, which is a task outside the scope of this work.

Results confirm the design dependencies obtained by comparing the results of optimization (see 8.1). For example, the optimal design is highly sensitive to all of the perturbed parameters but the communications data rate (R_{com}). The insensitivity to R_{com} was expected from the comparison of the direct and relay link rover designs. Dependency on the value of science weight is high, as reported by the solution to (POS) and the results are supported by the comparisons of the designs of 1R to 1T and 1S to 1U. Likewise, the sensitivity to roving time between recharges, T_{rov}, given by (POS) is confirmed by the comparisons of the designs 1A to 1F, 1C to 1J and 1E to 1H. These are not direct comparisons in that for the optimization runs the constraints themselves were changed, but the same sensitivities as discussed in 8.1 can be seen.

As a further example, consider forcible perturbation of the vehicle wheelbase (w_b). Optimal design results show that increasing wheelbase makes roving a more efficient operation. Correspondingly, the perturbed-optimal solution for a perturbation in w_b shows that parameters concerned with science and communications (W_{sci}, T_{esci}, P_{com}, R_{com}) decrease in value and the frequency of science stops (S_{sci}) also decreases, while those parmeters related to the roving function (P_{RTG}, E_{batt}, T_{rov}) undergo sharp increases. As a side effect, the increase in RTG power means more available power at night for heating, so insulation thickness (L_i) decreases. These results are easily predicted, but only after the fact. In addition, the perturbed-optimal solution also gives an approximation to the proper relative magnitudes of these changes. Sensitivity analysis by perturbed-optimal solutions has provided useful information about the design adjustment procedure for an MRV after forced perturbations in design parameters.

8.4 CONCLUSIONS AND RECOMMENDATIONS

The work presented here has achieved two goals. A systematic method for examining design trade-offs has resulted in the determination of optimal designs for a Mars-roving vehicle and enabled the relationships between design parameters to be ascertained. While the mathematical tools of the method are certainly not new, the approach to the problem is an attempt to improve upon present methods for design of large aerospace systems. In addition, by derivation of a new method for examining the sensitivity of designs determined by nonlinear programming techniques, a useful tool for constructing such a system under real-world constraints has resulted.

Further investigation into the problem of determining perturbed-optimal solutions could include:

1. investigating the possibility of using second order methods, which have significantly more complex form (i.e., nonlinear equations) but should achieve greater accuracy
2. improving the accuracy of the first-order method developed here by utilizing successive approximation techniques for finite δx_k
3. determining the feasibility of exact solution to (POS) by solution of the nonlinear equations (6.9 - 6.12)

4. finding better methods of approximating the multipliers (u^*,w^*) and investigate the sensitivity of the perturbed-optimal solution to errors in (u^*,w^*)

5. considering the properties of the solution to (POS) in special cases such as geometric, convex, or quadratic programming problems.

SECTION 9

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APPENDIX

Here it is shown that the Jacobian matrix related to problem (POS) is invertible under the conditions stated in Theorem 1. This will be done by showing that there are no non-zero solutions to

$$\begin{bmatrix} \nabla^2\mathcal{L}^* & G^* & H^* & e^k \\ U^*G^{*T} & \mathrm{diag}(g_i^*) & 0 & 0 \\ H^{*T} & 0 & 0 & 0 \\ (e^k)^T & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z^1 \\ z^2 \\ z^3 \\ z^4 \end{bmatrix} = 0 \,. \qquad \text{(A1)}$$

From (A1) it is obvious that $z_i^2 = 0$ for all i such that $g_i^* > 0$. Also, $u_i^* \nabla^T g_i^* z^1 = 0$ for i such that $g_i^* = 0$ (i.e., for $i \in B^*$). From the assumption of strict complementarity, the last line also implies $\nabla^T g_i^* z^1 = 0$ for $i \in B^*$.

It is also clear that $\nabla^T h_j^* z^1 = 0$, all j, and $(e^k)^T z^1 = 0$. Premultiplying (A1) by $\left[z^{1T}, z^{2T}, z^{3T}, z^4 \right]$ gives

$$z^{1T} \nabla^2\mathcal{L}^* z^1 + z^{1T} G^* z^2 + z^{1T} H^* z^3 + z^{1T} e^k z^4$$
$$+ z^{2T} U^* G^{*T} z^1 + z^{2T} \mathrm{diag}\,(g_i^*)\, z^2 + z^{3T} H^{*T} z^1 + z^4 e^{kT} z^1 = 0 \,. \qquad \text{(A2)}$$

Using the deductions made above, all terms in (A2) but the first one are shown to be identically zero. Now, because $u_i^* = 0$ for $i \notin B^*$, the remaining (first) term can be rewritten as

$$z^{1T}\left[\nabla^2 f^* - \sum_{i \in B^*} u_i^* \nabla^2 g_i^* + \sum_{j=1}^{p} w_j^* \nabla^2 h_j^*\right] z^1 = 0.$$

Since z^1 is orthogonal to ∇g_i^* , $i \in B^*$, ∇h_j^* , all j , and from the fact that sufficiency conditions for (NLP) are satisfied

$$z^{1T}\left[\nabla^2 \mathcal{L}(x^*, u^*, w^*)\right] z^1 > 0 \quad \text{for} \quad z^1 \neq 0,$$

thus z^1 must be the zero vector.

Then, again from (A1),

$$\sum_{i=1}^{m} z_i^2 \nabla g_i^* + \sum_{j=1}^{p} z_j^3 \nabla h_j^* + e^k z^4 = 0 .$$

However, since $z_i^2 = 0$ for $i \notin B^*$ and since ∇g_i^* , $i \in B^*$, ∇h_j^* , all j , and e^k are assumed to be linearly independent, it is clear that in addition to z^1 being zero, $z^2 = 0$, $z^3 = 0$, and $z^4 = 0$.

This completes the proof since it has been shown that the only solution to (A1) is the trivial one, and thus the Jacobian in (A1) is invertible.