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SIMILITUDE REQUIREMENTS FOR HYPERSONIC, RAREFIED, NONEQUILIBRIUM FLOW

By William L. Hendricks
Systems Dynamics Laboratory

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16. ABSTRACT

Similitude requirements for hypersonic, rarefied flow with nonequilibrium chemistry and vibration are presented. The full Navier-Stokes equations with catalytic or noncatalytic walls and with or without slip conditions are nondimensionalized. The heat transfer coefficient is written in terms of fourteen dimensionless parameters and reduced to four by making the binary scaling assumption. The duplication of blunt and sharp nose heat transfer requires the use of air over a geometrically similar model with the same free stream velocity, wall temperature and product of free stream density and characteristic length. Estimates of this heat transfer coefficient are also presented.

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William L. Hendricks

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SIMILITUDE REQUIREMENTS FOR HYPERSONIC, RAREFIED, NONEQUILIBRIUM FLOW

I. INTRODUCTION

To determine the thermal protection system required for the Space Shuttle, accurate predictions of the heat transfer must be made from wind tunnel data. The heat transfer in the transitional flow regime, between continuum and free molecular flow, with nonequilibrium chemistry and vibration is an important part of the total heating to the Shuttle External Tank. For the ascent of the External Tank using the Rockwell International PRR Space Shuttle configuration, the preliminary estimate of the local heating rate is largest during the transitional flow regime for points on the ogival forebody. Thus, the prediction of aeroheating in the transitional zone is of great interest. If the proper similarity requirements are duplicated in the wind tunnel, the heat transfer coefficient on the flight vehicle can be obtained.

Similitude requirements have been established for inviscid, hypersonic small disturbance flows by Hayes and Probstein (Ref. 1), extended to blunted bodies with the blast wave analogy by Cheng (Ref. 2), and further extended to include real gas effects by Inger (Ref. 3). Using the thin shock layer approximation, Cheng (Ref. 4) extended his original work to include viscous effects. However, in the transitional flow regime a simple flow model is not appropriate to describe the viscous, nonequilibrium phenomenon. Jain (Ref. 5) shows that the thin layer approximation fails to give correct results in the fully merged flow regime; but, that the entire Navier-Stokes equations are appropriate from the surface of the body to the free stream for flow from the continuum regime through the fully merged regime. Thus, the entire Navier-Stokes equations will be nondimensionalized to indicate the similarity parameters.

II. SIMILARITY REQUIREMENTS

For locations near the stagnation point the basic equations for a multicomponent mixture of chemically reacting gases made up of N, O, N_2, O_2, NO, NO+, and e^- are written in spherical coordinates. By neglecting external body forces and the radiative heat flux in comparison to the convective heat flux (good for small nose radii, see Ref. 6) and using Fick's law, the equations are nondimensionalized. The boundary conditions are also nondimensionalized assuming a catalytic wall with no slip conditions. For a diatomic gas the slip conditions have little effect on the heat transfer, see Ref. 5.

The heat transfer coefficient, written in a functional relationship, for the stagnation region is

$$C_H = C_H\,(\theta;\ Re_o,\ Pr,\ Le,\ \gamma,\ M_\infty,\ T_o,\ T_w,\ \lambda_{Rj},\ K_{Rj},\ K_{cj},\ \bar{K}_{cj},\ \lambda_{vi},\ \lambda_{v2i},\ K_{vi})\quad \begin{array}{l}\text{Species } i = 1,\ \ldots.\ N\\ \text{Reaction } j = 1,\ \ldots.\ N\end{array} \qquad (1)$$

where strict similarity requires the duplication of the following: (1) Angle from the stagnation point, θ; (2) Reynolds number, $Re_o = \rho_\infty U_\infty R/\mu_o$, based on the free stream density and velocity, body radius, and viscosity corresponding to the free stream total temperature; (3) Prandtl number, Pr; (4) Lewis number, Le; (5) ratio of specific heats, γ; (6) free stream Mach number, M_∞; (7) free stream total temperature, T_o; (8) wall temperature, T_w; (9) ratio of activation temperature for chemical reactions to total temperature, λ_{Rj}; (10) ratio of residence time to chemical relaxation time, $K_{Rj} = (R/U_\infty)/(1/\rho_\infty C_{fj} T_o^{\eta_{fj}})$, where C_{fj} is the reaction rate constant in the forward direction and η_{fj} is the reaction rate temperature exponent; (11) ratio of forward to backward reaction rate for two, K_{cj}, and three body reactions, $\bar{K}_{cj} = K_{cj}/\rho_\infty$, (12) ratio of activation temperature for vibration and molecular constant to T_o, i.e., λ_{vi}, λ_{v2i}; and (13) ratio of residence time to vibrational relaxation time, $K_{vi} = (R/U_\infty)/(K_{1vi}\,T_o^{5/6}/\rho_\infty U^2_\infty)$, where K_{1vi} is a constant which depends on the physical properties of the molecule.

If air is used for the test gas; γ, Pr, and Le will be essentially the same for wind tunnel and flight conditions, due to the relative invariance of these quantities with small differences in temperature and pressure. By using a similar gas the molecular properties and the reaction constants are identical for wind tunnel and flight conditions. The remaining parameters to be duplicated are

$$C_H = C_H \left[\theta;\ Re_o\ (\rho_\infty U_\infty R/\mu_o),\ M_\infty\ (U_\infty/T_\infty^{\frac{1}{2}}),\ T_o,\ T_w,\ \lambda_{Rj}(T_o),\ K_{Rj}\ (\rho_\infty R/U_\infty\ T_o^{\eta_{fj}}),\ K_{cj}\ (T_o^{5/6})\right] \quad \begin{array}{l} \text{Species}\ \ i = 1,\ldots.\ N \\ \text{Reaction}\ j = 1,\ldots.\ N \end{array} \tag{2}$$

It is obvious from Equation (2) that, if the free stream thermodynamic state and free stream velocity are duplicated, the wind tunnel model must be the same size as the actual vehicle.

Using the free stream velocity and the stagnation temperature as similitude requirements, the free stream temperature is, ideally, also simulated and Equation (2) reduces to

$$C_H = C_H \left[\theta;\ U_\infty,\ T_o,\ T_w,\ Re_o\ \&\ K_{Rj}\ \&\ K_{vi}\ (\rho_\infty R),\ \bar{K}_{cj}\ (\rho_\infty)\right]. \tag{3}$$

By simulating the free stream velocity for hypersonic, rarefied flight, one has, in essence, simulated the total temperature. The free stream temperature may not be that of flight, but for many existing wind tunnels these low temperatures cannot be obtained. Matching the free stream temperature is relatively unimportant (Ref. 7). Then Equation (3) simplifies to

$$C_H = C_H \left[\theta;\ U_\infty,\ T_w,\ Re_o\ \&\ K_{Rj}\ \&\ K_{vi}\ (\rho_\infty R),\ \bar{K}_{cj}\ (\rho_\infty)\right]. \tag{4}$$

Once again similitude is impossible unless further relaxation of similarity requirements are made.

At high altitudes the binary scaling assumption can be made with confidence. Assuming that the nonequilibrium process is described by two body reactions is equivalent to assuming that $\bar{K}_{cj}$ is large and independent of density, which yields the final similarity relation

$$C_H = C_H\ (\theta;\ U_\infty,\ T_w,\ \rho_\infty\ R). \tag{5}$$

The last parameter in Equation (5) is proportional to the number of molecular collisions in a characteristic distance R. To duplicate a nonequilibrium relaxation the number of collisions in a distance $R_{vehicle}$ in flight must be the same as the number of collisions in a distance R_{model} in the wind tunnel. Hence, the parameter $\rho_\infty R$ is characteristic of the nonequilibrium vibration, chemistry and even radiative heat transfer (see Ref. 8).

A similar approach to determine the nonequilibrium similitude parameters for the heat transfer on a sharp cone yields essentially the same results as Equation (5), i.e., at a point L down the body

$$C_H = C_H\ (\theta_c,\ U_\infty,\ T_w,\ \rho_\infty\ L) \tag{6}$$

where θ_c is the cone half angle.

To include effects of oxygen predissociation in the nozzle, the use of an oxygen rich flow can be utilized to account for oxygen freezing and not recombining as in the equilibrium flight situation.

III. DATA CORRELATION

The heat transfer at locations downstream of the stagnation point of blunt nosed slender bodies, which still exhibit the nonequilibrium relaxation process originating from the stagnation point, will now be considered. The stagnation flow freezes in the rapid expansion around the blunt nose and its flow characteristics near the sonic line are indicative of the amount of heat transfer further downstream. When freezing occurs, the chemical enthalpy becomes unavailable for conversion to kinetic energy until much further downstream when equilibrium is achieved. The frozen situation is indicative of blunt nosed slender bodies while the equilibrium case corresponds to sharp cone flow for the same cone half angle.

An estimate is made of the reduction in heat transfer at a point immediately downstream of the stagnation point due to the nose bluntness-induced freezing of chemical energy. The estimate for the blunt nosed, conical body heat transfer coefficient in terms of the sharp cone value for the same conditions can be related to the enthalpy by

$$C_{H\ blunt}/C_{H\ sharp}\ \alpha\ h_{blunt}/h_{sharp} = h_{not\ frozen}/h_{equilibrium}. \tag{7}$$

The expressions for enthalpy are easily written for a vibrationally excited gas made up of N_2, O_2, N and O assuming that there is neither any ionization nor any electronic excitation. Two cases are examined: (1) the case where oxygen is completely dissociated, and (2) the case where oxygen and nitrogen are completely dissociated. Case (1) corresponds to rarefied flow with temperatures behind the shock from about 4,000 °K to 5,000 °K and Equation (7) gives values from 0.59 to 0.64. Case (2) corresponds to rarefied flow with temperatures from about 6,000 °K to 12,000 °K and Equation (7) gives values from 0.23 to 0.37.

The spread of $C_{H\ blunt}/C_{H\ sharp}$ from 0.23 to 0.64 for case (1) and (2) predicts the lowest value before ionization. As the flow reaches equilibrium far downstream of the nose the ratio approaches 1.0. Since more sharp cone data exists than blunt cone data for low Reynolds numbers, this range of 0.23 to 1.0 gives a rough estimate of the value for $C_{H\ blunt}$ and an idea of the degree of dissociation.

Figures 1 and 2 show the heat transfer coefficient in terms of the rarefaction parameter for sharp and blunt cones, respectively, Ref. 9 - 11. For design purposes least squares correlations of the data are obtained in the form

$$\mathrm{LOG}_{10}[C_H(1-H_w/H_o)/\mathrm{SIN}\ \theta_c] = \sum_{s=o}^{3} a_s\ (\mathrm{LOG}_{10}\ \bar{X})^s \qquad (8)$$

where H is the total enthalpy,

$$\bar{X} = \mathrm{Re}_\infty/(M_\infty^2\ \gamma_\infty\ C^*\ \mathrm{COS}\ \theta_c) = (\rho_\infty\ L)\ .\ T_r/(U_\infty\ \mu_r \mathrm{COS}\ \theta_c), \qquad (9)$$

$$\mathrm{Re}_\infty = \rho_\infty\ U_\infty\ L/\mu_\infty\ , \qquad (10)$$

$$C^* = \mu_r\ T_\infty/\mu_\infty\ T_r\ , \qquad (11)$$

$$T_r = T_w + (T_o - T_w)/2 - T_o\ \mathrm{COS}^2\ \theta_c/3 \qquad (12)$$

and for sharp cones $a_o = -0.34407400$, $a_1 = -0.34912983$, $a_2 = -0.10445498$, and $a_3 = +0.022766463$ while for blunt cones $a_o = -0.66125881$, $a_1 = -0.33391435$, $a_2 = -0.035024979$, and $a_3 = +0.0072725000$. Note that the rarefaction parameter in Equation (9) can be written in terms of the similarity parameters discussed earlier.

The estimates of the heat transfer coefficient from Equation (7) are shown in Figure 3 along with the curve fit for sharp and blunt cones. For large Reynolds numbers the nonequilibrium effects are not to prevalent; however, for values of $\bar{X} \angle 10$ the gas appears to be completely dissociated in O_2 and partially dissociated in N_2. The band of data for blunt nosed cones is all above the value $C_{H\ blunt} = .37\ C_{H\ sharp}$ and since there is very little dependence upon the amount of nose bluntness, all blunt cone values for C_H can be described by Equation (8) or the inequality $.37 \leq C_{H\ \text{blunt}}/C_{H\ \text{sharp}} \leq 1$.

IV. CONCLUSIONS

In summary, the duplication of hypersonic, rarefied blunt nose and sharp nose heat transfer requires the use of air over a geometrically similar model with the same free stream velocity, wall temperature, and product of free stream density and characteristic dimension. This is based on the binary scaling assumption.

Estimates of the heat transfer coefficient on blunt nosed cones are found to be in the range from .37 to 1.0 (as the flow relaxes) times the value on sharp nosed cones for the same half angle and distance downstream. Physically, this reduction is due to the nose bluntness-induced freezing of chemical energy. Least squares correlations are also presented for sharp and blunt cones. These correlations may be utilized for preliminary estimates of hypersonic aeroheating in the transitional flow regime.

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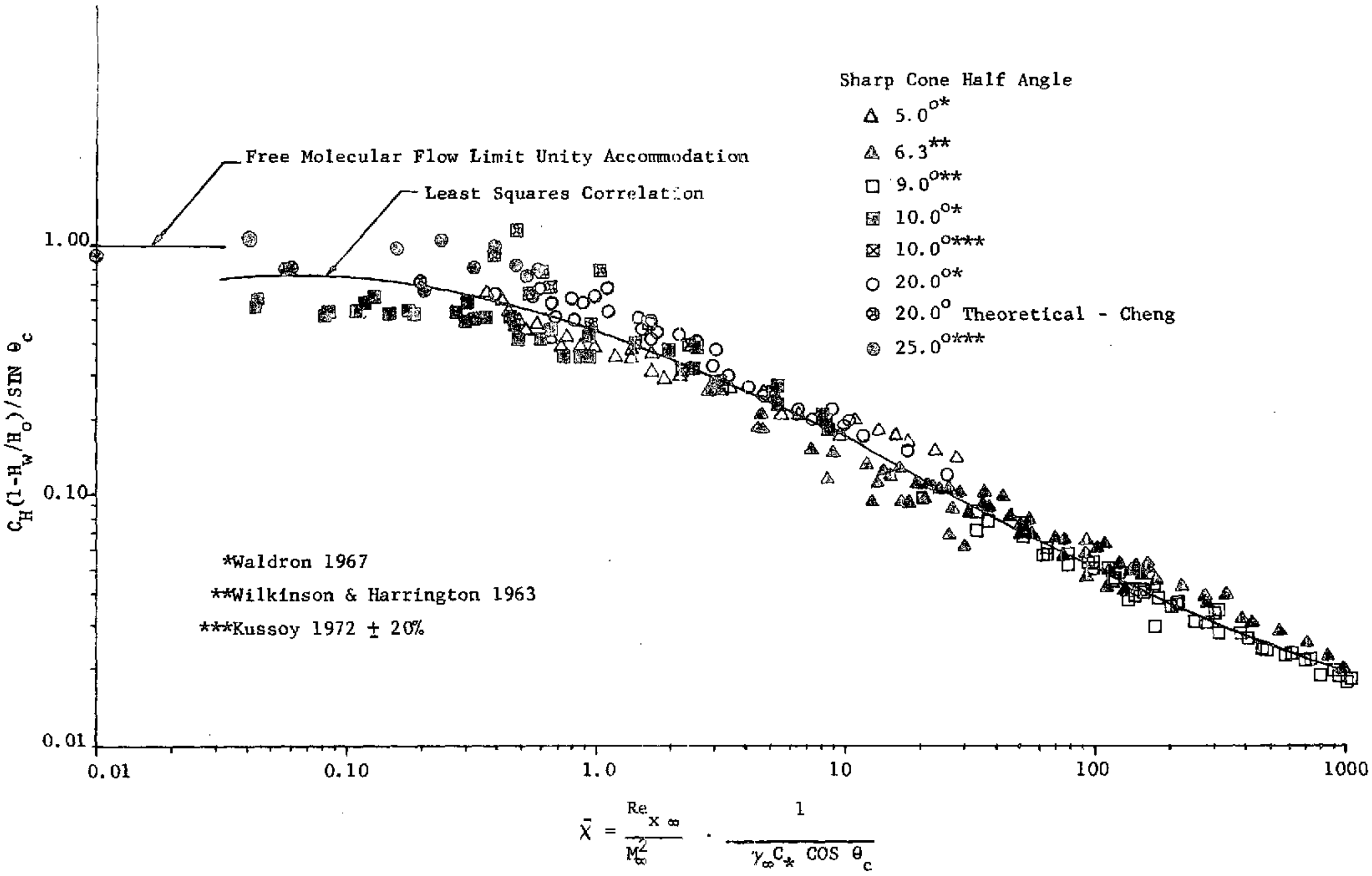

Figure 1. Sharp cone heat transfer data variation with rarefaction parameter.

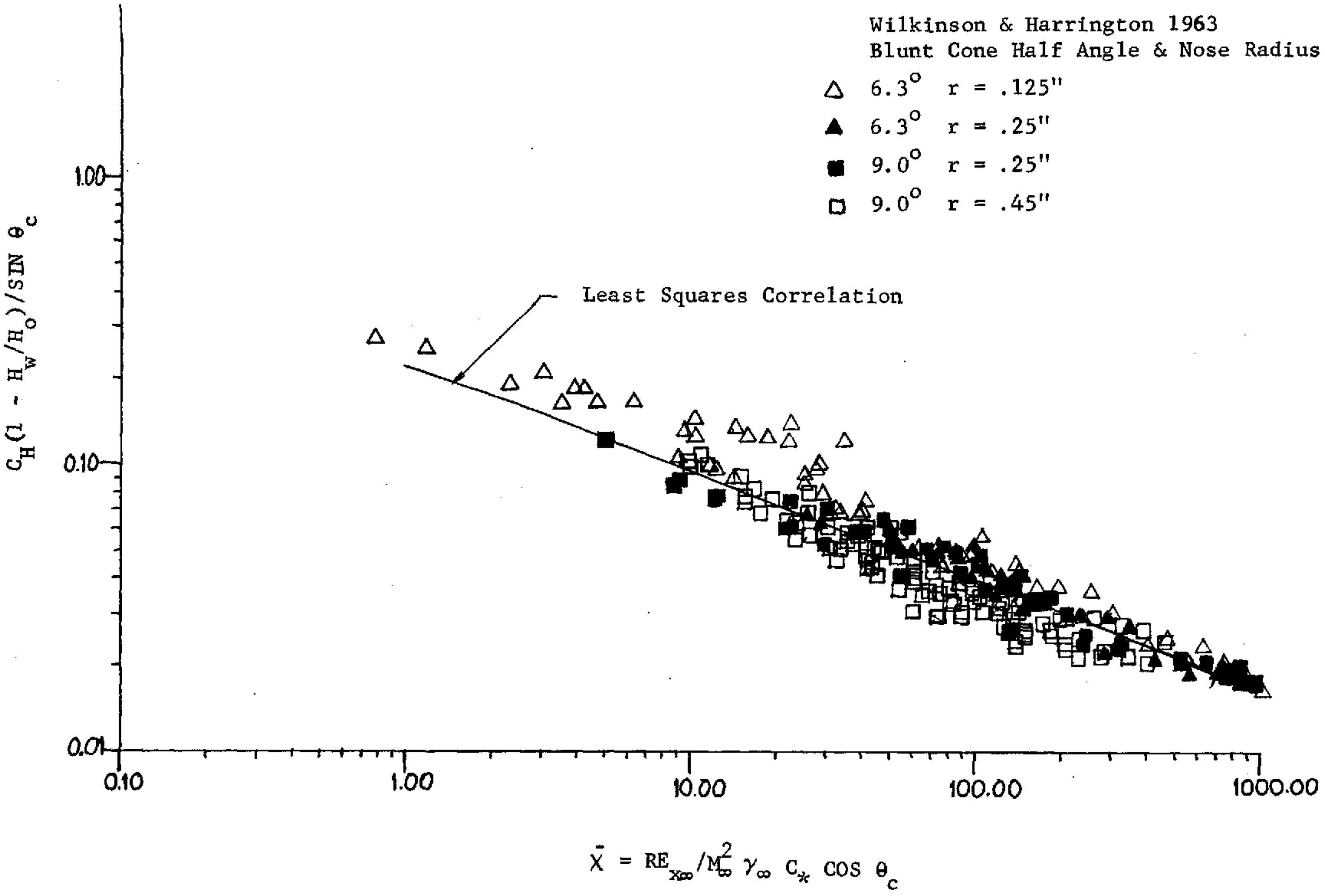

Figure 2. Blunt cone heat transfer data variation with rarefaction parameter.

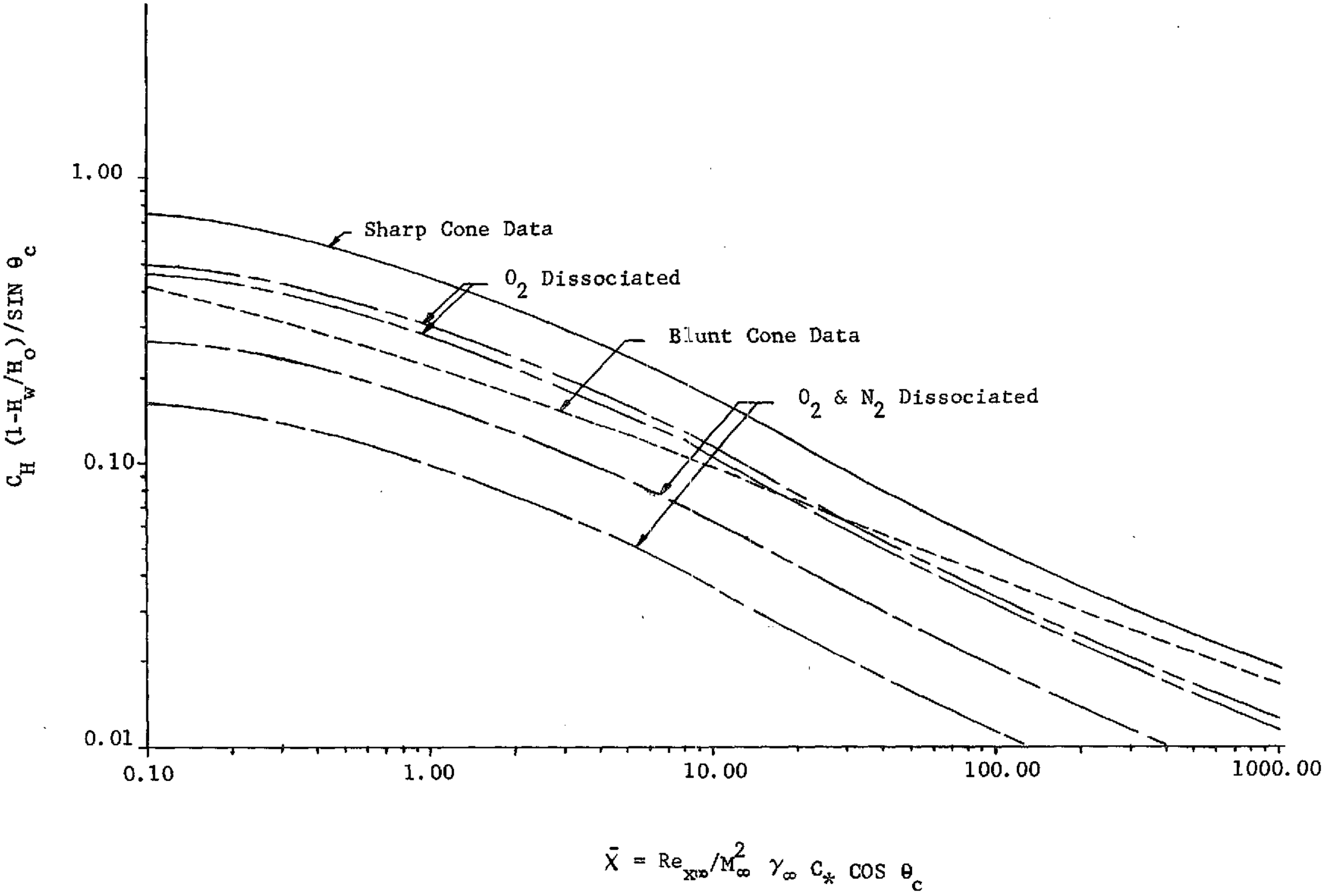

Figure 3. Comparison of blunt and sharp cone heat transfer data.

APPROVAL

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission Programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

Homer B Wilson Jr

Homer B. Wilson, Jr.
Chief, Thermal Environment Branch

Werner K. Dahm
Chief, Aerophysics Division

J. D. Lovingood
Director, Systems Dynamics Laboratory