

GEOMETRICAL CORRECTION FACTORS FOR HEAT FLUX METERS

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ABSTRACT

General formulas are derived for determining gage averaging errors of strip-type heat flux meters used in the measurement of one-dimensional heat flux distributions. The local averaging error $e(x)$ is defined as the difference between the measured value of the heat flux and the local value which occurs at the center of the gage. In terms of $e(x)$, a correction procedure is presented which allows a better estimate for the true value of the local heat flux. For many practical problems, it is possible to use relatively large gages to obtain acceptable heat flux measurements.

INTRODUCTION

The aviation and power-generation industries have been concerned with heat flux measurements (refs. 1 and 2). Prior to 1960, measurements of heat flux (ref. 3) were generally concerned with average values taken over large areas. However, because of interest in prediction of local temperature gradients and thermal stresses, such as in the thermal design of turbine blades, recent work has been concerned with local heat flux measurements.

A heat flux gage measures the average heat flux across its face, rather than a local value at its center (see Fig. 1). In principle, the smallest possible heat flux gage should be used for accurate local heat flux measurements because the averaging error $e(x)$ (difference between the center value and the average value of the heat flux) will decrease for smaller gage sizes. However, the smaller

the size of the gage the greater will be the fabrication, handling, and calibration costs. Clearly, some optimum choice of gage size will generally be required, depending on the heat flux distribution and accuracy required. In choosing a heat flux meter, such as a slug-type, Gardon, or ablation calorimeter, the experimenter is faced with the choice of gage size. The motivation for this paper is to provide the experimenter with a simple means of estimating the effect of gage size on the accuracy of his experiment.

Gardon and Akfirat (ref. 4) measured the local heat transfer coefficient under impinging axisymmetrical and slotted jets. They used a small 0.9 - millimeter - diameter Gardon type heat flux meter. Because of the small size of their meter, we would naturally expect very good local measurements. However, they did experience some calibration problems with their gages reported in an earlier paper (ref. 5). On the other hand, Tabakoff and Clevenger (ref. 6) used relatively large (3.8 cm wide) heat flux measurement devices (electrical heaters) to measure their heat transfer distribution. As a result of the relatively large heater size, the actual local heat transfer coefficient at the stagnation point is higher than the average measured value.

A formula for correcting measured heat flux data to account for gage size averaging is presented. As an example, the correction procedure will be applied to the data of Tabakoff and Clevenger (ref. 6).

SYMBOLS

a	constant
C, C_0, C_1, C_2	constants
D	diameter of hole
e	probe averaging error (true value)
\hat{e}	probe averaging error (estimated value)
H	distance between holes
h	heat transfer coefficient
L	characteristic length (see eq. (2))
Nu	Nusselt number
Pr	Prandtl number
q	heat flux
\bar{q}_{meas}	measured "averaged" heat flux
x	distance from stagnation point
Δx	probe size (see fig. 2)
Z_n	distance from slot jet to surface
β	constant (see eq. (15))
η	dimensionless distance, x/L
$\Delta\eta$	dimensionless width of probe, $\Delta x/L$

ERROR ANALYSIS

Averaging Error

Consider the one-dimensional heat flux distribution $q(x)$, as shown in figure 2, which is being measured by a heat flux meter of size Δx . For the purpose of this report, we assume that the gage measures exactly the total heat flux that falls on it; that is, we assume that we can account for all losses from the heat flux meter, such as temperature gradients along the wall, and correct for them (for example- see ref. 7). In this section, we are concerned only with the problem of a large probe diameter washing out local effects.

The average heat flux measured by the gage can be expressed as

$$\bar{q}_{\text{meas}}(\eta) = \frac{1}{\Delta\eta} \int_{\eta-\Delta\eta/2}^{\eta+\Delta\eta/2} q(\eta) d\eta \quad (1)$$

where η is a dimensionless distance chosen as

$$\eta = \frac{x}{L}; \quad \Delta\eta = \frac{\Delta x}{L} \quad (2)$$

Thus, the error in the measurement of the local heat flux can be expressed as

$$e(\eta) = \frac{\bar{q}_{\text{meas}}(\eta) - q(\eta)}{q(\eta)} = \frac{\int_{\eta-\Delta\eta/2}^{\eta+\Delta\eta/2} q(\eta) d\eta}{\Delta\eta q(\eta)} - 1 \quad (3)$$

By using equation (3), we can evaluate the errors associated with some common $q(\eta)$ distributions for various dimensionless probe sizes $\Delta\eta$.

Correction Procedure

The application of corrections to raw data is commonplace in engineering. The temperature distributions on the insides of tubes are often determined from measurements taken on the outside of tubes by appropriate consideration of the thermal conductivities of the tubes and other operating parameters. In a similar manner, the heat flux measurements can also be corrected to account for the error due to probe averaging.

Rearranging the left hand side of equation (3) yields

$$q(\eta) = \frac{\bar{q}_{\text{meas}}(\eta)}{e(\eta) + 1} \quad (4)$$

where the various expressions for $e(\eta)$ will be given later in this report.

This correction technique is simple to use. Thus, relatively large heat flux meters can be used to obtain acceptable local data in many case if equation (4) is applied to the data.

General Case

Before we begin investigating specific heat flux distributions, let us consider properties of $q(\eta)$ which could lead to measurement errors. The function $q(\eta)$ can be expanded in a Taylor series about any point "a" as follows:

$$q(\eta) = q(a) + (\eta - a)q^I(a) + \frac{(\eta - a)^2}{2!} q^{II}(a) + \dots \quad (5)$$

where (I and II) represent derivatives with respect to η .

Substituting equation (5) into equation (3) and performing the integration gives

$$e(\eta) = \frac{q(a)}{q(\eta)} - 1 + \frac{q^I(a)}{q(\eta)} (\eta - a) + \frac{q^{II}(a)}{2q(\eta)} \left[(\eta - a)^2 + \frac{\Delta\eta^2}{12} \right] + \dots \quad (6)$$

The location of the parameter "a" could be at some fixed point in the coordinate system, such as $\eta = 0$, or at the center of the heat flux gage. The simplest expression results when we expand about the center of the gage ($a = \eta$). In this case, the error in the heat flux meter can be expressed as

$$e(\eta) \approx \frac{q''(\eta) \Delta\eta^2}{q(\eta) 24} \quad (7)$$

The error, therefore, is proportional to the square of the size of the heat flux meter and to the second spatial derivative of heat flux. Consideration of higher terms in the Taylor series indicates that equation (7) is valid provided that

$$\left| \frac{q''''(\eta) \Delta\eta^2}{q''(\eta) 80} \right| \ll 1 \quad (8)$$

Peaks

The manner in which the heat flux gage is used is also extremely important in sizing the gage. If the gage can be positioned at all values of x , as was done by Gardon and Akfirat in reference 4 by moving their plate and heat flux gage relative to a fixed jet, then any local heat flux peaks have a much better chance of being observed. On the other hand, if for a particular experiment the gages must be placed in a fixed position, as was done in reference 6, then it will not be possible to determine if any local peaks occur within one gage size. Also, for very small fixed gages, local peaks between the gages will also go undetected. A local peak, for example, might occur if the boundary layer changes from laminar to turbulent flow, that is, if transition occurs.

The expression for the error just derived in equation (7) is inconvenient to use when any part of the heat flux gage crosses a peak, such as might occur at a stagnation point, because of the large number of terms that would be needed to express a continuous function for the peak. In the previous analysis, it was assumed that the odd derivatives were continuous across the gage center, at $\eta = 0$, and thus did not contribute in the integration of equation (3). However, at a flux peak, symmetry is now assumed to exist for all terms in the Taylor expansion. This means that the odd power derivatives are discontinuous at the origin. Hence, equation (3) becomes

$$e(0) = \frac{2 \int_0^{\Delta\eta/2} q(\eta) d\eta}{\Delta\eta q(0)} - 1 \quad (9)$$

For this special case, the parameter "a" in equation (5) takes on the value of zero, and the Taylor series reduces to a Maclaurin series. Substituting the Maclaurin series into equation (9) and performing the specified integration gives

$$e(0) = \frac{q^I(0)}{q(0)} \frac{\Delta\eta}{4} + \frac{q^{II}(0)}{q(0)} \frac{\Delta\eta^2}{24} + \frac{q^{III}(0)}{q(0)} \frac{\Delta\eta^3}{192} + \frac{q^{IV}(0)}{q(0)} \frac{\Delta\eta^4}{1920} + \dots \quad (10)$$

or to second order

$$e(0) \approx \frac{q^I(0)}{q(0)} \frac{\Delta\eta}{4} + \frac{q^{II}(0)}{q(0)} \frac{\Delta\eta^2}{24} \quad (11)$$

As can be seen from equation (11), both the odd and even derivatives contributed to the error, in contrast to equation (7), in which only the even order terms contributed to the error. Note, if the heat flux is expanded in terms of even functions so that the first derivatives are zero at the origin, then equations (7) and (11) are identical

CURVE FITTING

The heat flux distribution can be approximated in general by a power series. Now, we will apply the general theory to a simple second-order polynomial approximation to $q(\eta)$ and an exponential approximation. These two distributions were chosen because they can be used to approximate many practical heat flux distributions.

Polynomial

In certain experiments the measured heat flux distribution can be written in terms of a polynomial of the form

$$q(\eta) = C_0 + C_1\eta + C_2\eta^2 \quad (12)$$

Since the third and higher order derivatives of equation (5) are zero, the truncated series expressions for the errors are exact. Thus, the errors are

$$e(\eta) = \frac{C_2 \Delta\eta^2/12}{C_0 + C_1\eta + C_2\eta^2} \quad \text{for } \eta > \frac{\Delta\eta}{2} \quad (13)$$

and

$$e(0) = \frac{C_1}{C_0} \frac{\Delta\eta}{4} + \frac{C_2}{12 C_0} \Delta\eta^2 \quad \text{for } \eta = 0 \quad (14)$$

Exponential

The exponential approximation for the heat flux can be expressed as

$$q(\eta) = Ce^{-\beta\eta} \quad (15)$$

In this case, the series approximations for the errors as

given in equations (7) and (11) become

$$e(\eta) = \frac{\Delta\eta^2 \beta^2}{24} \quad \text{for } \eta > \frac{\Delta\eta}{2} \quad (16)$$

$$e(0) = \frac{-\beta \Delta\eta}{4} + \frac{\beta^2 \Delta\eta^2}{24} \quad \text{for } \eta = 0 \quad (17)$$

In reference 8, a comparison of the exact solution for $e(\eta)$ to the Taylor series approximation indicated that equations (16) and (17) are valid for $\beta \leq 2$.

IMPINGING JET EXAMPLE

The results of the last section will be applied to the problem of heat transfer from impinging jets. In particular, we will examine the data of Tabakoff and Clevenger (reference 6) shown in Figure 3 for a row of hole jets impinging on a curved surface. In this experiment both the heat flux and the meters are one-dimensional; thus, the one-dimensional analysis just presented is directly applicable to this data.

As shown in Figure 4, the data of Tabakoff and Clevenger can be fitted by a polynomial of the form

$$h \propto 2.65 - 3.7 \eta + 1.8 \eta^2 \quad (18)$$

where

$$\eta = \frac{x}{7.6 \text{ cm (2 x heater width)}} \quad (19)$$

Since only three measured values are presented to the left of the stagnation point, these values are fitted exactly by a second-order

polynomial. It should be pointed out that the h given by equation (18) is not the actual distribution as represented by equation (12), but is the distribution that includes the error represented by equations (13) and (14).

We will assume that the wall-to-bulk temperature gradients are negligible; thus, the heat transfer coefficient and the heat flux will have a one-to-one correspondence in the expressions for the error, and they may be used interchangeably. Reference 3 (eq. 12) gives the added terms necessary to account for the variation in the wall-to-bulk fluid temperature differences should they be known.

If equation (18) is combined with equations (12) and (14), the expression for the error at $\eta = 0$ becomes

$$\hat{\epsilon}(0) = -0.35 \Delta\eta + 0.057 \Delta\eta^2 \quad (20)$$

In this particular case, An equals 0.5, and the error in the absolute value at the stagnation point is -16 percent. Furthermore, using equation (4) in conjunction with equation (20) gives a better estimate of the true value of the actual heat transfer coefficient at the stagnation point. The calculated value at the stagnation point is labeled "calculated peak" in Figure 4.

The error at the second station in figure 4 can be evaluated from equation (13) to be 3 percent. We see that the error in the measurement for this relatively large gage is quite acceptable. However, had any singularities or peaks existed in this range, they would have gone undetected. If the gages were movable instead of being fixed, some peaks might even be detected by this large gage.

Finally, recall as previously discussed, equation (18) was an approximation to the true h distribution. We could use the new distribution as calculated from equation (4) to reevaluate the coefficients in equation (12). Then, we could recalculate the error. However, in many cases this iteration process is not necessary.

CONCLUDING REMARKS

General formulas are derived for determining gage averaging errors of strip-type heat flux meters used in the measurement of one-dimensional heat flux distributions. In addition, a correction procedure is presented which allows a better estimate of the true value of the local heat flux. For many practical problems, it is possible to use a relatively large gage to obtain acceptable local heat flux measurements, provided that the gage is small enough to detect any peaks which might occur in the heat flux distribution.

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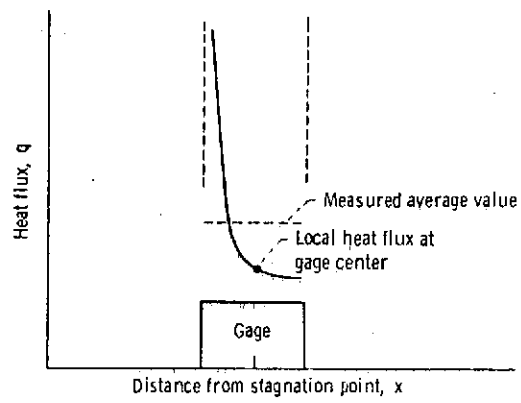


Figure 1. - Relation between local and "average" heat flux for nonlinear heat flux.

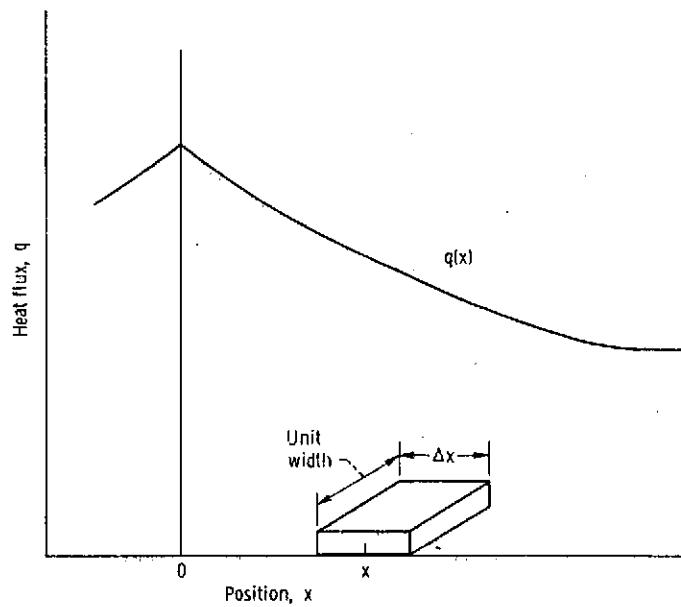


Figure 2 - Heat flux distribution and meter geometry.

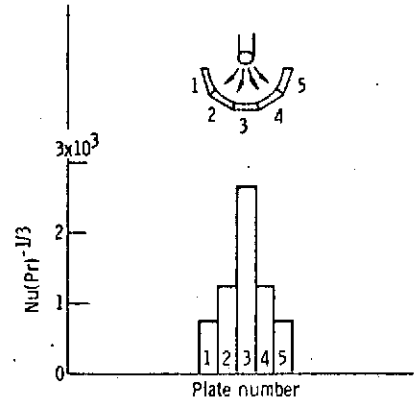


Figure 3 - Heat transfer for impinging hole jets on a concave surface. Hole diameter, D , 0.635 centimeter (1/4 in.); spacing-to-diameter ratio, H/D , 1; height-to-diameter ratio, Z_0/D , 8.8. (Data from ref. 6.)

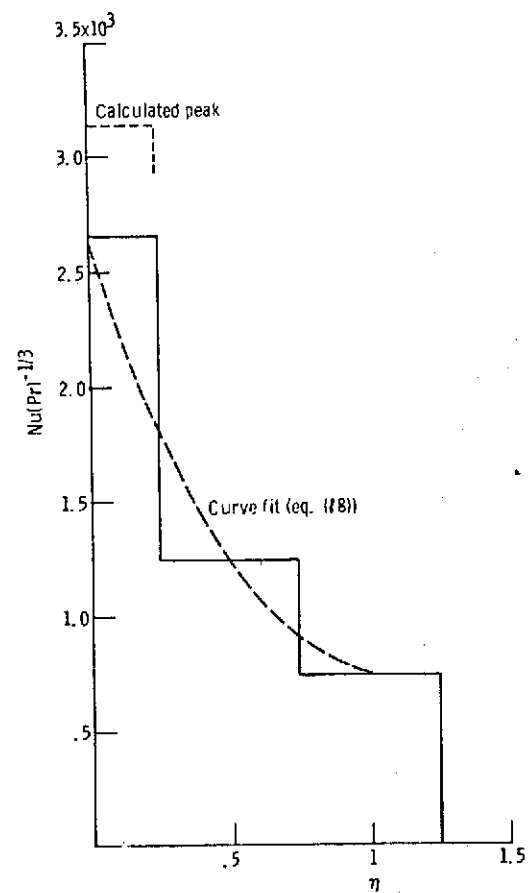


Figure 4 - Heat transfer for row of holes impinging on concave surface. (Data from ref. 6.)