

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

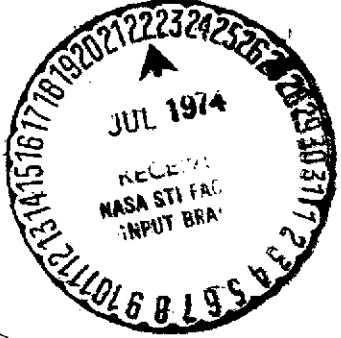
*Technical Report 32-1592*

*Attitude Dynamics Simulation Subroutines  
for Systems of Hinge-Connected  
Rigid Bodies*

G. E. Fleischer  
P. W. Likins

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SIMULATION SUBROUTINES FOR SYSTEMS OF  
HINGE-CONNECTED RIGID BODIES (Jet  
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JET PROPULSION LABORATORY  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
PASADENA, CALIFORNIA

May 1, 1974

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## **Preface**

The work described in this report was performed by the Guidance and Control Division of the Jet Propulsion Laboratory.

## Errata for Technical Report 32-1565

Since the users of this report will certainly be referring to JPL TR 32-1565 (Ref. 9) for a much more detailed treatment of hinge-connected rigid body system equations and their derivation, it seems appropriate here to add a few technical remarks that may contribute to that report's utility and to correct several significant errors in the text which have been brought to the author's attention.

(1) In the last paragraph on page 30, the hinge torques  $\tau^1$ ,  $\tau^2$ , and  $\tau^3$  are referred to. The symbol  $\tau^j$  means  $\tau^j g^j$ .

(2) On page 35, when considering the significance of Eq. (34), note that  $C^T C = (\bar{C} + \hat{C})^T (\bar{C} + \hat{C}) = (U - \hat{C})(U + \hat{C}) \cong U + \dots$  (2nd-degree terms), so orthogonality is preserved in the linear approximation. Still, for planar motion (all  $\tilde{g}$ s the same), it would be impossible to associate *an* angle of rotation with  $C$ , since the cosine = 1 and the angle is large. Thus orthogonality is preserved, but physical interpretation is jeopardized.

(3) Note the text corrections that follow.

Page	Line	Equation	Corrected Text	Comment
12	Last	—	$\epsilon_{\alpha\beta} V_\alpha = \dots$	Change subscript
13	Last	—	the set $\mathcal{B} - \{k\}$ is $\dots$	Replace boldface $k$
14	17	—	$a_{ok} \stackrel{\Delta}{=} \sum_{r \in \mathcal{B}} \sum_{j \in \mathcal{B}} \epsilon_{kj} C^{0j} \Phi^{rj} C^{rk} g^k$	Change subscript
14	18	—	$-\mathcal{M} \sum_{r \in \mathcal{B}} \sum_{j \in \mathcal{B}-k} \epsilon_{kj} (C^{0r} D^{jrT} C^{jr} D^{rj} - \dots$	Change subscript and summation index
15	12	—	$+ \mathcal{M} \sum_{j \in \mathcal{B}-k} \{\tilde{D}^{kj} C^{kj} [ \dots$	Add tilde
23	4	23	$C = \begin{bmatrix} c_2 c_3 & -c_2 s_3 & s_2 \\ s_1 s_2 c_3 + s_3 c_1 & -s_1 s_2 s_3 + c_3 c_1 & c_1 s_2 s_3 + c_3 s_1 \\ -c_1 s_2 c_3 + s_3 s_1 & -s_1 c_2 & c_1 c_2 \end{bmatrix}$	Note several changes
33	1	30b	$+ \sum_{k \in \mathcal{P}-\mathcal{A}} (\bar{a}_{ik} + \hat{a}_{ik}) \ddot{\gamma}_k$	Change summation index
33	4th from last	—	Any terminal appendage in $\dots$	Drop s from appendages
35	26	35b	$+ \sum_{k \in \mathcal{A}} \bar{a}_{ik} \ddot{\gamma}_k + \sum_{k \in \mathcal{P}-\mathcal{A}} (\bar{a}_{ik} + \dots$	Add subscript; change summation index
36	5	36b	$-\mathcal{M} \sum_{r \in \mathcal{B}} \sum_{j \in \mathcal{B}-r} \epsilon_{kj} ( \dots$	Change subscript and summation index
36	19	37b	$-\mathcal{M} \sum_{r \in \mathcal{B}} \sum_{j \in \mathcal{B}-r} \epsilon_{kj} [ \dots$	Change subscript
37	8	37e	$\dots [\bar{C}^{j0} \bar{\omega}^0 \bar{C}^{0j} + \dots$	Drop superscript $j$
37	11	37e	$\dots + \sum_{r \in \mathcal{P}-\mathcal{A}} \epsilon_{rj} \dot{\gamma}_r \bar{C}^{jr} \dots$	Add overdot
37	25	—	$\dots$ coordinate truncation are feasible.	Replace "transformation is"
82	9	A-39a	$\sum_{k \in \mathcal{P}} \sum_{r \in \mathcal{B}} [\epsilon_{kr} \Phi^{rr} - \mathcal{M} \sum_{j \in \mathcal{B}-r} \epsilon_{kj} (D^{ir} \cdot D^{rj} \mathbf{U} - D^{jr} D^{rj})] \cdot g^k \ddot{\gamma}_k = \dots$	Note several changes
82	25	A-40a	$g^s \cdot \{ \sum_{r \in \mathcal{P}} \epsilon_{sr} \sum_{j \in \mathcal{P}} [\epsilon_{jr} \Phi^{rr} - \mathcal{M} \sum_{i \in \mathcal{B}-r} \epsilon_{ji} (D^{ir} \cdot D^{ri} \mathbf{U} - D^{ir} D^{ri})] \cdot g^i \ddot{\gamma}_i \} = \dots$	Note several changes
82	29	A-40a	$\dots + \sum_{s \in \mathcal{P}} \epsilon_{sr} g^s \dot{\gamma}_s \} \times g^r \}$	Replace + by $\times$

Page	Line	Equation	Corrected Text	Comment
101	3	—	$[D^{01T}D^{10}U - D^{01}D^{10T}]$	Interchange superscripts
101	4	—	$-\gamma_1 D^{01T} \tilde{u}^1 D^{10}U - \gamma_1 D^{01T} D^{10} \tilde{u}^1 + D^{21T} D^{12}U - D^{21} D^{12T}$	Interchange superscripts
101	5	—	$-(\gamma_1 - \gamma_2) D^{21T} \tilde{u}^1 D^{12}U - \gamma_1 D^{21T} D^{12} \tilde{u}^1$	Interchange superscripts
101	9	—	$u^{1T} D^{01} D^{10} = u^{1T} D^{21} D^{12} = \dots$	Interchange superscripts
101	9	—	$D^{01T} u^1 D^{10} = D^{21T} \tilde{u}^1 D^{12} = 0$	Interchange superscripts

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## Abstract

This report describes several computer subroutines designed to provide the solution to minimum-dimension sets of discrete-coordinate equations of motion for systems consisting of an arbitrary number of hinge-connected rigid bodies assembled in a tree topology. In particular, these routines may be applied to (1) the case of completely unrestricted hinge rotations, (2) the totally linearized case (i.e., all system rotations are small), and (3) the mixed, or partially linearized, case. The use of the programs in each case is demonstrated using a five-body spacecraft and attitude control system configuration. The ability of the subroutines to accommodate prescribed motions of system bodies is also demonstrated. Complete listings and user instructions are included for these routines (written in Fortran V) which are intended as multi- and general-purpose tools in the simulation of spacecraft and other complex electromechanical systems.

# Attitude Dynamics Simulation Subroutines for Systems of Hinge-Connected Rigid Bodies

## I. Introduction

Since Hooker and Margulies (Ref. 1) published their derivation of the vector-dyadic equations of motion for an arbitrary number of point-connected rigid bodies in a topological tree, a number of computer programs (Refs. 2-7) based on these equations and similar generic formulations have come into wide use in the aerospace industry. Meanwhile, Hooker (Ref. 8) has devised a procedure for reducing the original Hooker-Margulies equations to a minimum-dimension set (i.e., equal to the number of degrees of rotational freedom) by summing selected subsets of the equations (paralleling Russell's approach—Ref. 2) and dot-multiplying by unit vectors associated with the connecting joint geometry.

In Reference (9) a variation of Hooker's procedure was used to derive a minimum-dimension set of equations of motion for a *hinge-connected* system of rigid bodies. (It is always possible to substitute a hinge-connected system for a point-connected system by introducing auxiliary bodies and without changing the dimension of the problem.) The objective was, having derived this set of *discrete-coordinate* equations in a highly systematic and detailed way, to (1) isolate certain subsets of the system's rigid bodies that may be considered collectively as a flexible substructure, (2) linearize in the variables describing relative motions of contiguous bodies within the substructure, and (3) transform from substructure discrete variables to distributed (modal) coordinates which may then be truncated for computational efficiency. The result is, in general, a new kind of *hybrid-coordinate* (discrete + distributed) formulation which can accommodate large gross deformations of an appendage even though its local deformations are small.

However, while computer subroutines which solve the hybrid-coordinate equations of Ref. 9 will be discussed in a subsequent report, this report will confine itself to descriptions of subroutines which solve only the discrete-coordinate equations of motion for the hinge-connected system. Specifically, it will describe and apply these programs as dynamic simulation tools for three possible situations: (1) the case in which all the system bodies are permitted large or *unrestricted* rotations (Section II), (2) the case in which all system rotations (and their derivatives) are small, allowing total linearization of the equations (Section III), and (3) the mixed case of large and small rotations for which partial linearization is appropriate (Section IV).

The programs, since they are based on the discrete-coordinate equations of Ref. 9, will differ from those developed in Ref. 4 in several significant respects. First, they will be of minimum dimension. Also, interbody coordinate transformations are accomplished internal to the programs, the user being required only to arbitrarily specify some nominal system configuration. Input has been simplified. Pains have been taken to minimize internal coordinate transformations and other computations to improve speed. And, finally, the user may, if he desires, *prescribe* the motion of some system bodies rather than allow the subroutine to calculate it.

The *subroutine* approach to solving attitude dynamics equations is a logical consequence of the approach taken historically at JPL to develop computer programs (analog or digital) specialized to each particular control mode or portion of a control mode under study, e.g., the thrust vector control mode during propulsive trajectory corrections. In contrast, however, to the narrow specialization of each simulation program to specific control problems has been the extensive use of *general-purpose simulation languages* to facilitate the creation of such programs. Over the past decade, such digital simulation languages as MIDAS, MIMIC, DSL/90, and most recently, CSSL III (Continuous System Simulation Language) have proved extremely valuable since they relieve the programmer of the worry over numerical integration routines, and since they can properly sort the many, at times highly interrelated, computational statements which occur and thereby help to ensure a correct solution.

Many of these "simulators" also provide a powerful array of useful special functions, in addition to the standard Fortran library of mathematical and intrinsic functions, such as hysteresis loops, noise generators, time delays, pulse generators, logic gates, etc. In fact, provision is also made for inclusion into the language of any *user-created* functions or subroutines. An example of an extremely useful subroutine that has been made a part of the CSSL III processor is HCK (Hamilton-Cayley-Klein), a rigid-body attitude dynamics package, written by E. H. Kopf. HCK calculates the angular acceleration components of a single rigid body, given its angular velocities, applied torques, and inertial properties. The HCK parameters (i.e., Euler parameters) are used in the necessary kinematic calculations and provide body-to-inertial (or vice versa) coordinate transformations as desired. Euler angles may also be requested to indicate the angular position of the body with respect to the inertial reference frame.

Just like HCK, the subroutines to be described here calculate angular accelerations, except that they do so for *every* body in a *system* of hinge-connected rigid bodies. All integration operations are confined to the main CSSL III simulation program (which calls its centralized numerical integration routines as required). Thus,

performing no numerical integration and required only to compute angular acceleration components (given the necessary system states, forcing functions, and physical properties), the routines are intended simply as very specialized and yet powerful simulation tools for application to a wide variety of current space vehicle attitude control problems.

## II. Unrestricted Systems

### A. System Mathematical Model

Figure 1 illustrates a set of eleven hinge-connected rigid bodies, labeled according to a convention designed to facilitate the processing of the equations of motion. This example will be useful in illustrating and interpreting the general mathematical model and the labeling conventions adopted here for its description. (In reading the list of conventions and symbol definitions, frequent reference should be made to Fig. 1.) Only after entertaining the complete cast of symbols used in what follows will consideration be given to the much smaller list of symbols to which values must be assigned by the user of the computer programs.

The mathematical model consists of a set of  $n + 1$  rigid bodies interconnected by  $n$  hinges<sup>1</sup>; the indicated numbers of bodies and hinges imply a tree topology. Any interbody connection forces in addition to those at the  $n$  hinges must be treated as forces external to the entire system. (If there were a linear spring connecting the mid-point of body  $\mathcal{L}_6$  in Fig. 1 to some point on body  $\mathcal{L}_0$ , for example, it would be necessary to replace the spring with a pair of equal and opposite forces applied to the spring attachment points on  $\mathcal{L}_6$  and  $\mathcal{L}_0$ ; the magnitude and direction of these forces would have to be expressed in terms of the unknown kinematical variables of the system.)

#### 1. Definitions of fundamental symbols

*Def. 1.* Let  $n$  be the number of hinges interconnecting a set of  $n + 1$  bodies.

*Def. 2.* Define the integer sets  $\mathcal{B} \triangleq \{0, 1, \dots, n\}$  and  $\mathcal{P} \triangleq \{1, \dots, n\}$ .

*Def. 3.* Let  $\mathcal{L}_0$  be a label assigned to one body chosen arbitrarily as a reference body, and let  $\mathcal{L}_1, \dots, \mathcal{L}_n$  be labels assigned to the remaining bodies in such a way that if  $\mathcal{L}_j$  is located between  $\mathcal{L}_0$  and  $\mathcal{L}_k$ , then  $0 < j < k$ .

*Def. 4.* Define the dextral orthogonal sets of unit vectors  $\mathbf{b}_1^k, \mathbf{b}_2^k, \mathbf{b}_3^k$  so as to be imbedded in  $\mathcal{L}_k$  for  $k \in \mathcal{B}$ , and such that in some arbitrarily selected nominal configuration of the system  $\mathbf{b}_\alpha^k = \mathbf{b}_\alpha^j$  for  $\alpha = 1, 2, 3$  and  $k, j \in \mathcal{B}$ .

*Def. 5.* Define  $\{\mathbf{b}^k\}$  as the column array of unit vectors<sup>2</sup>

<sup>1</sup>The word "hinge" as used here implies a connection which maintains a line common to both bodies; such a connection is sometimes called a *line hinge*.

<sup>2</sup>Note that braces  $\{ \}$  are used in this report both to identify column arrays of vectors and to enclose sets of integers; the distinction is apparent as soon as the objects within the braces are identified as vectors (boldface) or scalars.

$$\{b^k\} \triangleq \begin{Bmatrix} b_1^k \\ b_2^k \\ b_3^k \end{Bmatrix}, \quad \text{for } k \in \mathcal{B}$$

Def. 6. Let  $\omega^0 \triangleq \{b^0\}^T \omega^0$  be the inertial angular velocity vector of  $\mathcal{B}_0$ , so that  $\omega^0$  is the corresponding 3-by-1 matrix in vector basis  $\{b^0\}$ .

Def. 7. Let  $c_k$  be the mass center of  $\mathcal{B}_k$  for  $k \in \mathcal{B}$  (see Fig. 1).

Def. 8. Let  $p_k$  be a point on the hinge axis common to  $\mathcal{B}_k$  and  $\mathcal{B}_j$  for  $j < k$  and  $k \in \mathcal{P}$  (see Fig. 1).

Def. 9. Define the  $k$ th neighbor set  $\mathcal{B}_k$  for  $k \in \mathcal{B}$  such that  $j \in \mathcal{B}_k$  if  $\mathcal{B}_j$  is attached to  $\mathcal{B}_k$ .

Def. 10. For  $j \in \mathcal{B}_k$ , let  $p^{kj} \triangleq \{b^k\}^T p^{kj}$  be the position vector with respect to  $c_k$  of the labeled point on the hinge axis (either  $p_k$  or  $p_{jk}$ ) which is common to  $\mathcal{B}_k$  and  $\mathcal{B}_j$ , so that  $p^{kj}$  is the corresponding 3-by-1 matrix in vector basis  $\{b^k\}$ . (See Fig. 1, and note the special cases  $p^{21} = p^{23} = p^{32} = p^{34} = p^{43} = 0$ .)

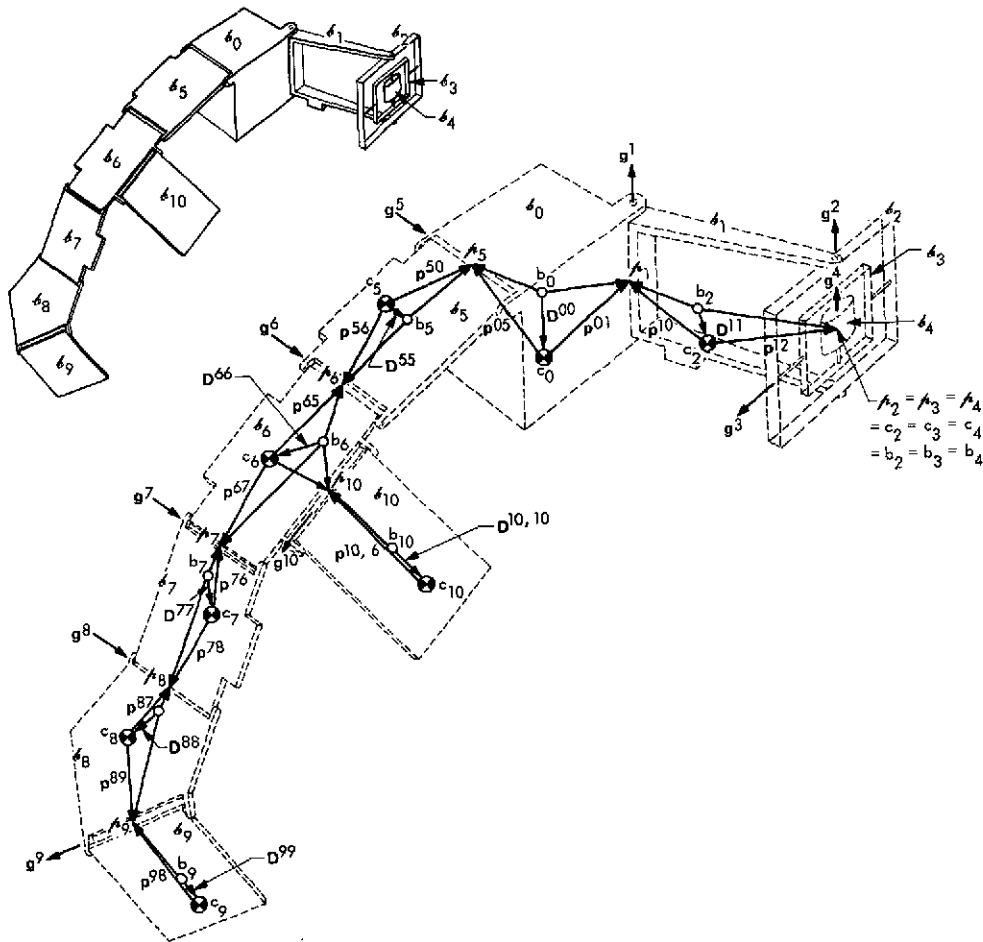


Fig. 1. Eleven rigid bodies interconnected by ten hinges

Def. 11. Let  $\mathbf{g}^k \triangleq \{\mathbf{b}^k\}^T \mathbf{g}^k$  be a unit vector parallel to the hinge axis through  $\mathcal{A}_k$ , so that  $\mathbf{g}^k$  is the corresponding 3-by-1 matrix in vector basis  $\{\mathbf{b}^k\}$ .

Def. 12. For  $k \in \mathcal{P}$ , let  $\gamma_k$  be the angle of a  $\mathbf{g}^k$ -rotation of  $\mathcal{L}_k$  with respect to the body attached at  $\mathcal{A}_k$ , that is, a rotation during which a right-handed screw fixed in  $\mathcal{L}_k$  with its axis parallel to  $\mathbf{g}^k$  advances in the direction of  $\mathbf{g}^k$ . Assign the value zero to  $\gamma_k$  for  $k \in \mathcal{P}$  when the system is in its arbitrarily chosen nominal configuration, for which  $\mathbf{b}_\alpha^k = \mathbf{b}_\alpha^j$ ,  $\alpha = 1, 2, 3$ ;  $k, j \in \mathcal{B}$ .

Def. 13. Let  $m_k$  be the mass of  $\mathcal{L}_k$  for  $k \in \mathcal{B}$ .

Def. 14. Let  $\mathbf{I}^k \triangleq \{\mathbf{b}^k\}^T \mathbf{I}^k \{\mathbf{b}^k\}$  be the inertia dyadic of  $\mathcal{L}_k$  for  $c_k$ , so that  $\mathbf{I}^k$  is the corresponding inertia matrix in vector basis  $\{\mathbf{b}^k\}$ , i.e.,

$$\mathbf{I}^k = \begin{bmatrix} I_{11}^k & I_{12}^k & I_{13}^k \\ I_{12}^k & I_{22}^k & I_{23}^k \\ I_{13}^k & I_{23}^k & I_{33}^k \end{bmatrix}$$

Def. 15. Let  $\mathbf{F}^k \triangleq \{\mathbf{b}^k\}^T \mathbf{F}^k$  be the resultant vector of all forces applied to  $\mathcal{L}_k$  *except for those due to interbody forces transmitted at hinge connections*, so that  $\mathbf{F}^k$  is the corresponding 3-by-1 matrix in vector basis  $\{\mathbf{b}^k\}$ . (Thus, any interbody forces due to spring connections, etc., which are separate from the  $n$  hinge connections would contribute to  $\mathbf{F}^k$ , as would all forces from sources external to the system.)

Def. 16. Let  $\mathbf{T}^k \triangleq \{\mathbf{b}^k\}^T \mathbf{T}^k$  be the resultant moment vector with respect to  $c_k$  of all forces applied to  $\mathcal{L}_k$  *except for those due to interbody forces transmitted at hinge connections*, so that  $\mathbf{T}^k$  is the corresponding 3-by-1 matrix in vector basis  $\{\mathbf{b}^k\}$ .

Def. 17. Let  $\tau_k$  be the scalar magnitude of the torque component applied to  $\mathcal{L}_k$  in the direction of  $\mathbf{g}^k$  by the body connected at  $\mathcal{A}_k$ .

Note that Def. 3 allows a certain freedom in labeling the system bodies after the reference body  $\mathcal{L}_0$  has been chosen, the requirement always being that label values increase as one progresses outward from  $\mathcal{L}_0$  along any chain of bodies. Figure 2 shows, for example, two equally acceptable ways of labeling a seven-body system.

Def. 18. Let the tilde symbol ( $\sim$ ) signify in application to a 3-by-1 matrix  $\mathbf{V}$  with elements  $V_\theta$  ( $\theta = 1, 2, 3$ ) transformation to a skew-symmetric 3-by-3 matrix  $\tilde{\mathbf{V}}$ . Thus, the matrix  $\tilde{\mathbf{V}}$  has the expanded form

$$\tilde{\mathbf{V}} \triangleq \begin{bmatrix} 0 & -V_3 & V_2 \\ V_3 & 0 & -V_1 \\ -V_2 & V_1 & 0 \end{bmatrix}$$

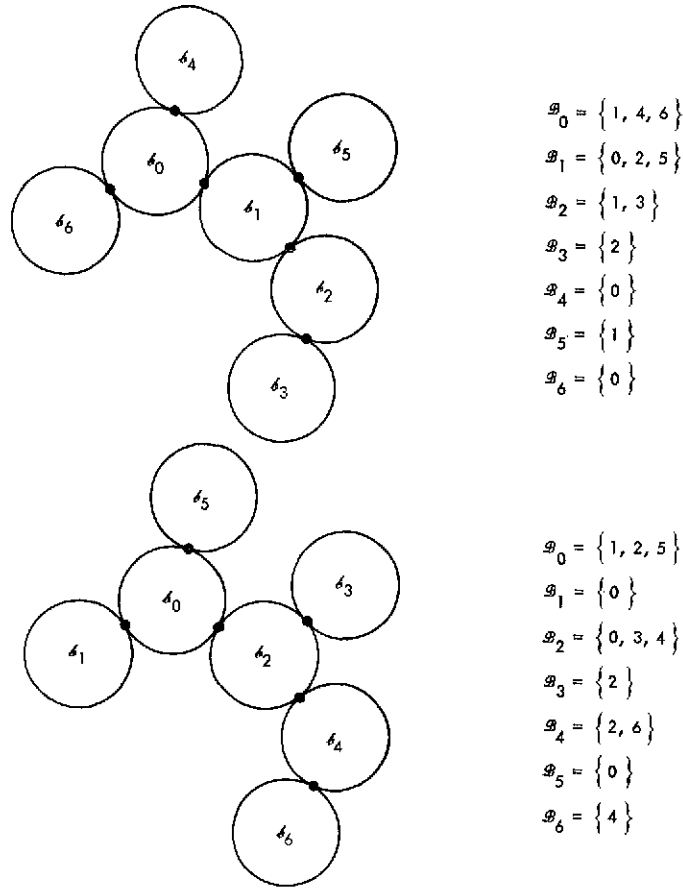


Fig. 2. Alternate labeling schemes for a system of seven bodies

## 2. Definitions of derived symbols

Def. 19. Define the scalar  $\epsilon_{sk}$  such that for  $k \in \mathcal{B}$  and  $s \in \mathcal{P}$ ,

$$\epsilon_{sk} \triangleq \begin{cases} 1 & \text{if } \mathcal{A}_s \text{ lies between } \mathcal{A}_0 \text{ and } \mathcal{A}_k \\ 0 & \text{otherwise} \end{cases}$$

The term *path elements* will be applied to the scalars  $\epsilon_{sk}$ , which are  $n(n+1)$  in number.

Def. 20. Let  $C^{rj}$  be the direction cosine matrix defined by

$$\{\mathbf{b}^r\} \triangleq C^{rj} \{\mathbf{b}^j\}$$

for  $r, j \in \mathcal{B}$ . In the nominal state,  $\gamma_k = 0$  for  $k \in \mathcal{P}$ , and all unit vectors of corresponding subscript are aligned, so that  $C^{rj}$  then is  $U$ , the 3-by-3 unit matrix.

Def. 21. Define  $\mathcal{M} \triangleq \sum_{k \in \mathcal{B}} m_k$ , so that  $\mathcal{M}$  is the total system mass.



Def. 22. Define  $\mathcal{M}_{kj} \stackrel{\Delta}{=} \text{the total mass on the branch of the system attached to } \mathcal{A}_k \text{ and commencing with } \mathcal{A}_j.$

Def. 23. For  $j \in \{\mathcal{B} - k\}$ , define  $\mathbf{L}^{kj} \stackrel{\Delta}{=} \text{the vector from } c_k \text{ to the hinge point attached to } \mathcal{A}_k \text{ and on the path leading from } \mathcal{A}_k \text{ toward } \mathcal{A}_j. \text{ (In Fig. 1, for example, } \mathbf{L}^{56} = \mathbf{L}^{57} = \mathbf{L}^{58} = \mathbf{p}^{56} \text{ and } \mathbf{L}^{3j} = 0 \text{ for } j \in \mathcal{B}.) \text{ (Define } \mathbf{L}^{kk} \stackrel{\Delta}{=} 0.)$

Def. 24. Define  $\{\mathbf{b}^k\}^T \mathbf{D}^{kk} \stackrel{\Delta}{=} \mathbf{D}^{kk} \stackrel{\Delta}{=} - \sum_{j \in \mathcal{B}} \mathbf{L}^{kj} m_j / \mathcal{M}$  for  $k \in \mathcal{B}$ .

Def. 25. Let  $b_k$  be a point called the *barycenter* of  $\mathcal{A}_k$  and fixed relative to  $\mathcal{A}_k$  such that  $\mathbf{D}^{kk}$  is the position vector of  $c_k$  with respect to  $b_k$ .

Def. 26. Define  $\{\mathbf{b}^k\}^T \mathbf{D}^{kj} \stackrel{\Delta}{=} \mathbf{D}^{kj} \stackrel{\Delta}{=} \mathbf{D}^{kk} + \mathbf{L}^{kj}$  for  $k, j \in \mathcal{B}$ . Thus,  $\mathbf{D}^{kj}$  is the vector from  $b_k$  to the hinge point fixed in  $\mathcal{A}_k$  and on the path leading to  $\mathcal{A}_j$ .

Def. 27. Define the dyadic

$$\Phi^{kk} \stackrel{\Delta}{=} \mathbf{I}^k + m_k (\mathbf{D}^{kk} \cdot \mathbf{D}^{kk} \mathbf{U} - \mathbf{D}^{kk} \mathbf{D}^{kk}) + \sum_{j \in \mathcal{B}_k} \mathcal{M}_{kj} (\mathbf{D}^{kj} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{kj} \mathbf{D}^{kj})$$

where  $\mathbf{U}$  is the unit dyadic.

Def. 28. For  $j \in \{\mathcal{B} - k\}$ , define the dyadic

$$\Phi^{kj} \stackrel{\Delta}{=} -\mathcal{M} (\mathbf{D}^{jk} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{jk} \mathbf{D}^{kj})$$

**3. Augmented bodies and barycenters.** The *barycenter*  $b_k$  of body  $\mathcal{A}_k$  is defined in Def. 25 in mathematical terms which admit an interesting interpretation as a physical property of an abstraction called the *augmented body*. If one imagines the body  $\mathcal{A}_k$  to be augmented by the addition of a particle of mass  $\mathcal{M}_{kj}$  at the connection point of  $\mathcal{A}_k$  with  $\mathcal{A}_j$  (for  $j \in \mathcal{B}_k$ ), then the result is called the *kth augmented body*, here referred to as  $\mathcal{A}'_k$ . Since  $\mathcal{M}_{kj}$  is the total mass of the branch of bodies attached to  $\mathcal{A}_k$  commencing with  $\mathcal{A}_j$ , the mass of *each* augmented body is  $\mathcal{M}$ , the total system mass.

Definition 24 provides

$$-\mathcal{M} \mathbf{D}^{kk} = \sum_{j \in \mathcal{B}} m_j \mathbf{L}^{kj}$$

which reveals the fact that  $-\mathbf{D}^{kk}$  is the position vector of the mass center of  $\mathcal{A}'_k$  with respect to the mass center  $c_k$  of  $b_k$ . Thus, *the barycenter  $b_k$  is the mass center of the kth augmented body  $\mathcal{A}'_k$ .* With this interpretation comes the relationship

$$\sum_{j \in \mathcal{B}} m_j \mathbf{D}^{kj} = m_k \mathbf{D}^{kk} + \sum_{j \in \mathcal{B} - k} m_j \mathbf{D}^{kj} = 0$$

Since  $\mathbf{D}^{kk}$  is the position vector from the barycenter  $b_k$  to the mass center  $c_k$  of  $\mathcal{A}_k$ , and (from Def. 26)  $\mathbf{D}^{kj}$  is the position vector from  $b_k$  to a particle of mass  $\mathcal{M}_{kj}$  located at the connection point of  $\mathcal{A}_k$  leading to body  $\mathcal{A}_j$ , for  $j \in \mathcal{B}_k$ , one can interpret  $\Phi^{kk}$  from Def. 27 as the inertia dyadic of the augmented body  $\mathcal{A}'_k$  with respect to the barycenter  $b_k$ . Such physical interpretation is not possible for the dyadic  $\Phi^{kj}$  defined in Def. 28, since this dyadic involves vector  $\mathbf{D}^{jk}$  fixed in  $\mathcal{A}_j$  and vector  $\mathbf{D}^{kj}$  fixed in  $\mathcal{A}_k$ .

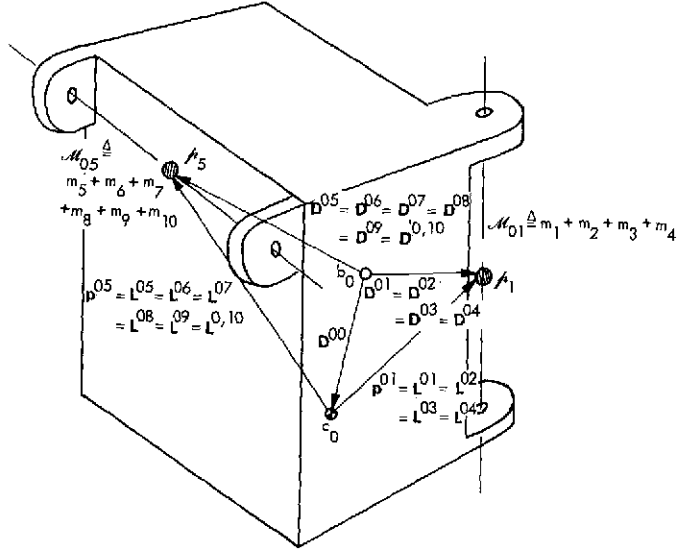


Fig. 3. Augmented  $\mathcal{B}'_0$  from Fig. 1

In illustration of the concepts just described, Fig. 3 shows the augmented body  $\mathcal{B}'_0$  associated with body  $\mathcal{B}_0$  in the system of Fig. 1.

### B. The Dynamical Equations

Using the symbols defined in the preceding section, the minimum-dimension set of equations for the rotational motion of the hinge-connected system (i.e., with  $n$  line hinges) can be written in vector-dyadic form as

$$\sum_{k \in \mathcal{B}} [\mathbf{T}^k + \sum_{j \in \mathcal{B}} \mathbf{D}^{kj} \times \mathbf{F}^j - \Phi^{kk} \cdot \dot{\omega}^k - \omega^k \times \Phi^{kk} \cdot \omega^k + \mathcal{M} \sum_{j \in \mathcal{B}-k} \mathbf{D}^{kj} \times \ddot{\mathbf{D}}^{jk}] = 0 \quad (1a)$$

$$\tau_r + \mathbf{g}^r \cdot \sum_{k \in \mathcal{P}} \varepsilon_{rk} [\mathbf{T}^k + \sum_{j \in \mathcal{B}} \mathbf{D}^{kj} \times \mathbf{F}^j - \Phi^{kk} \cdot \dot{\omega}^k - \omega^k \times \Phi^{kk} \cdot \omega^k + \mathcal{M} \sum_{j \in \mathcal{B}-k} \mathbf{D}^{kj} \times \ddot{\mathbf{D}}^{jk}] = 0 \quad r \in \mathcal{P} \quad (1b)$$

This result is, of course, the same as that derived by Hooker (Ref. 8) and proved in Ref. 9. Equation (1b) provides  $n$  scalar equations, and the vector equation (1a) completes the minimum-dimension set of  $n + 3$  scalar equations. By substituting the following kinematic expansions,

$$\ddot{\mathbf{D}}^{jk} = \dot{\omega}^j \times \mathbf{D}^{jk} + \omega^j \times (\omega^j \times \mathbf{D}^{jk}) \quad (2)$$

and

$$\dot{\omega}^j = \dot{\omega}^0 + \sum_{s \in \mathcal{P}} \varepsilon_{sj} (\mathbf{g}^{s*} \ddot{\gamma}_s + \omega^s \times \mathbf{g}^s \dot{\gamma}_s) \quad (3)$$

the vector-dyadic equations can be written in the form:

$$\mathbf{a}^{00} \cdot \dot{\omega}^0 + \sum_{s \in \mathcal{P}} \mathbf{a}^{s*} \ddot{\gamma}_s = \sum_{k \in \mathcal{B}} \mathbf{A}^k \quad (4a)$$

and (for  $r \in \mathcal{P}$ )

$$\mathbf{a}^{r0} \cdot \dot{\omega}^0 + \sum_{s \in \mathcal{P}} \mathbf{a}_{rs} \ddot{\gamma}_s = \mathbf{g}^r \cdot \sum_{k \in \mathcal{B}} \varepsilon_{rk} \mathbf{A}^k + \tau_r \quad (4b)$$

where

$$\text{Def. 29. } \mathbf{a}^{00} \triangleq \sum_{k \in \mathcal{B}} [\Phi^{kk} - \mathcal{M} \sum_{j \in \mathcal{B}-k} (\mathbf{D}^{jk} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{jk} \mathbf{D}^{kj})] = \sum_{k \in \mathcal{B}} \sum_{j \in \mathcal{B}} \Phi^{kj}$$

$$\begin{aligned} \text{Def. 30. } \mathbf{a}^{0s} &\triangleq \sum_{k \in \mathcal{B}} [\varepsilon_{sk} \Phi^{kk} - \mathcal{M} \sum_{j \in \mathcal{B}-k} \varepsilon_{sj} (\mathbf{D}^{jk} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{jk} \mathbf{D}^{kj})] \cdot \mathbf{g}^s \\ &= \sum_{k \in \mathcal{B}} \sum_{j \in \mathcal{B}} \varepsilon_{sj} \Phi^{kj} \cdot \mathbf{g}^s \end{aligned}$$

$$\begin{aligned} \text{Def. 31. } \mathbf{a}^{r0} &\triangleq \mathbf{g}^r \cdot \sum_{k \in \mathcal{P}} \varepsilon_{rk} [\Phi^{kk} - \mathcal{M} \sum_{j \in \mathcal{B}-k} (\mathbf{D}^{jk} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{jk} \mathbf{D}^{kj})] \\ &= \mathbf{g}^r \cdot \sum_{k \in \mathcal{P}} \sum_{j \in \mathcal{B}} \varepsilon_{rk} \Phi^{kj} \end{aligned}$$

$$\begin{aligned} \text{Def. 32. } a_{rs} &\triangleq \mathbf{g}^r \cdot \sum_{k \in \mathcal{P}} \varepsilon_{rk} [\Phi^{kk} - \mathcal{M} \sum_{j \in \mathcal{B}-k} (\mathbf{D}^{jk} \cdot \mathbf{D}^{kj} \mathbf{U} - \mathbf{D}^{jk} \mathbf{D}^{kj})] \cdot \varepsilon_{sj} \mathbf{g}^s \\ &= \mathbf{g}^r \cdot \sum_{k \in \mathcal{P}} \sum_{j \in \mathcal{B}} \varepsilon_{rk} \varepsilon_{sj} \Phi^{kj} \cdot \mathbf{g}^s \end{aligned}$$

$$\begin{aligned} \text{Def. 33. } \mathbf{A}^k &\triangleq \mathbf{T}^k + \sum_{j \in \mathcal{B}} \mathbf{D}^{kj} \times \mathbf{F}^j - \boldsymbol{\omega}^k \times \Phi^{kk} \cdot \boldsymbol{\omega}^k \\ &\quad + \mathcal{M} \sum_{j \in \mathcal{B}-k} \{\mathbf{D}^{kj} \times [\boldsymbol{\omega}^j \times (\boldsymbol{\omega}^j \times \mathbf{D}^{jk})]\} \\ &\quad - \sum_{j \in \mathcal{B}} \Phi^{kj} \cdot \sum_{s \in \mathcal{P}} \varepsilon_{sj} \dot{\gamma}_s (\boldsymbol{\omega}^s \times \mathbf{g}^s) \end{aligned}$$

A matrix equivalent to Eqs. (4a) and (4b) may be obtained by defining the following symbols:

$$\text{Def. 34. Define the 3-by-3 matrix in vector basis } \{\mathbf{b}^0\} \text{ as } a_{00} \triangleq \{\mathbf{b}^0\} \cdot \mathbf{a}^{00} \cdot \{\mathbf{b}^0\}^T.$$

$$\begin{aligned} \text{Def. 35. For } s \in \mathcal{P}, \text{ define the 3-by-1 matrices in basis } \{\mathbf{b}^0\} \text{ as } a_{0s} &\triangleq \{\mathbf{b}^0\} \cdot \mathbf{a}^{0s}, \text{ and} \\ \text{let } a_{s0} &\triangleq a_{0s}^T. \end{aligned}$$

$$\text{Def. 36. For } k \in \mathcal{B}, \text{ define the 3-by-1 matrices in basis } \{\mathbf{b}^0\} \text{ as } E_k \triangleq \{\mathbf{b}^0\} \cdot \mathbf{A}^k.$$

In terms of these symbols, the system matrix equation can be written as

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & \cdots & a_{0n} \\ a_{10} & a_{11} & a_{12} & \cdots & a_{1n} \\ a_{20} & a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n0} & a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} \dot{\omega}^0 \\ \ddot{\gamma}_1 \\ \ddot{\gamma}_2 \\ \vdots \\ \ddot{\gamma}_n \end{bmatrix} = \begin{bmatrix} \sum_{k \in \mathcal{B}} E_k \\ \mathbf{g}^{1T} C^{10} \sum_{k \in \mathcal{P}} \varepsilon_{1k} E_k + \tau_1 \\ \mathbf{g}^{2T} C^{20} \sum_{k \in \mathcal{P}} \varepsilon_{2k} E_k + \tau_2 \\ \vdots \\ \mathbf{g}^{nT} C^{n0} \sum_{k \in \mathcal{P}} \varepsilon_{nk} E_k + \tau_n \end{bmatrix} \quad (5)$$

The coefficient matrix on the left side of (5) is a real, nonsingular, symmetric matrix which varies with time as the kinematical variables  $\gamma_1, \dots, \gamma_n$  change in

value. In general, then, the solution to this set of differential equations requires (e.g., when using numerical integration) repeated solutions of the matrix equation, (i.e., at every integration step).

If one wishes to replace any variable  $\gamma_j$  ( $j \in \mathcal{P}$ ) by a known function of time, then one can remove  $\ddot{\gamma}_j$  from the angular acceleration matrix on the left side of Eq. (5), removing also the corresponding row and column of its coefficient matrix, and then rewrite on the right side those terms just deleted by the removal of the column on the left. This will be performed automatically by the subroutines if the user desires.

Equation (5) constitutes a complete set of *rotational dynamics equations*, but they are not fully descriptive of the system motion until they are augmented by control equations specifying  $\tau_1, \dots, \tau_n$  and certain external forces and torques, and augmented also by kinematical equations as provided in the next section.

### C. Kinematical Equations

The kinematical variables adopted in the preceding sections are as follows:  $\gamma_k$  for  $k \in \mathcal{P}$  (Def. 12);  $C^{rj}$  for  $r, j \in \mathcal{B}$  (Def. 20); and  $\omega^0 \triangleq \{\mathbf{b}^0\} \cdot \boldsymbol{\omega}$  (Def. 6). Although the equations of motion have been expressed in terms of these quantities, the latter are not all independent. Relationships among kinematical variables developed in this section must therefore either be considered in conjunction with the dynamical equations or be substituted into them to remove redundant variables whenever a solution is sought.

The direction cosine matrix  $C^{rj}$  (Def. 20) relates sets of orthogonal unit vectors fixed in  $\mathcal{B}_r$  and  $\mathcal{B}_j$ , and hence depends upon those angles  $\gamma_\alpha$  for which  $\mathbf{a}_\alpha$  lies between  $\mathcal{B}_r$  and  $\mathcal{B}_j$ , and also upon the corresponding unit vectors  $\mathbf{g}^\alpha$  defining the intervening hinge axes. For the special case in which  $\mathcal{B}_r$  and  $\mathcal{B}_j$  are *contiguous* and  $j < r$ , it is always possible (see Ref. 9) to express  $C^{rj}$  (and  $C^{jr}$ ) in terms of the single angle  $\gamma_r$  and the single matrix  $g^r$ , as follows:

$$C^{rj} = U \cos \gamma_r - \tilde{g}^r \sin \gamma_r + \tilde{g}^r g^{r^2} (1 - \cos \gamma_r) \quad (6)$$

and

$$C^{jr} = U \cos \gamma_r + \tilde{g}^r \sin \gamma_r + \tilde{g}^r g^{r^2} (1 - \cos \gamma_r) = (C^{rj})^T \quad (7)$$

Definitions 34, 35, and 36 make clear that all  $n + 3$  scalar equations in (5) are described in the vector basis  $\{\mathbf{b}^0\}$  of the reference body. This is for the purpose of minimizing the number of coordinate transformation computations that must be performed by the subroutines. Thus, it becomes unnecessary to calculate  $C^{rj}$  for all  $r \in \mathcal{B}, j \in \mathcal{B}$ . It is only required that  $C^{rj}$  be determined where  $\mathcal{B}_r$  and  $\mathcal{B}_j$  are contiguous and, since  $C^{0r} = C^{0j} C^{jr}$ , to then derive matrices  $C^{0r}$  for  $r \in \mathcal{P}$ . An algorithm for accomplishing this task is described in Appendix A.

### D. System Specification

Once the appropriate equations from the three preceding sections are embodied in a computer program, the user need not deal directly with all of the symbols and concepts introduced in the formulation of these equations. He can provide the computer with the very limited body of input required to specify his system, and then concentrate on interpreting the numerical integration output.

It is assumed that the engineer in his wisdom and experience has devised for his physical system a mathematical model consisting of a system of  $n + 1$  bodies interconnected by  $n$  hinges (such as that illustrated in Fig. 1). He has labeled the bodies as indicated by Def. 3 (Section IIA), and fixed in his mind a nominal configuration (for which  $\gamma_k = 0$  for  $k \in \mathcal{P}$  and  $\mathbf{b}_\alpha^k = \mathbf{b}_\alpha^j$  for  $j, k \in \mathcal{B}$  and  $\alpha = 1, 2, 3$ ). One final aid, however, will be needed to communicate the system's interconnection pattern to the computer subroutine.

*Def. 37.* For  $k \in \mathcal{P}$ , let  $h_k$  denote the index of the body attached to  $\mathcal{A}_k$  and on the path leading to  $\mathcal{A}_n$ . The scalars  $h_k$  will be termed "connection elements." (In Fig. 1,  $h_1 = 0$ ,  $h_2 = 1$ ,  $h_3 = 2$ ,  $h_4 = 3$ ,  $h_5 = 0$ ,  $h_6 = 5$ ,  $h_7 = 6$ ,  $h_8 = 7$ ,  $h_9 = 8$ , and  $h_{10} = 6$ .)

With the definition of  $h_k$ , it is now possible to specify a complete set of input information for the subroutine:

*Subroutine input (required)*

- (1) The integer  $n$  (see Def. 1).
- (2) The  $n$  connection elements  $h_k$  ( $k \in \mathcal{P}$ ) (see Def. 37).
- (3) For  $k \in \mathcal{B}$  and  $j \in \mathcal{B}_k$ , the 3-by-1 matrices  $p^{kj}$  (see Def. 10).
- (4) The  $n$  3-by-1 matrices  $g^k$  for  $k \in \mathcal{P}$  (see Def. 11).
- (5) The  $n + 1$  masses  $m_k$  for  $k \in \mathcal{B}$  (see Def. 13).
- (6) The  $n + 1$  3-by-3 inertia matrices  $I^k$  for  $k \in \mathcal{B}$  (see Def. 14).
- (7) For  $k \in \mathcal{B}$ , the  $2(n + 1)$  3-by-1 matrices  $F^k$  and  $T^k$  (see Defs. 15 and 16).
- (8) For  $k \in \mathcal{P}$ , the  $n$  scalar values  $\tau_k$  (see Def. 17).
- (9) Values for the  $2n$  scalars  $\gamma_k$  and  $\dot{\gamma}_k$  for  $k \in \mathcal{P}$  (see Def. 12) and the three elements of the angular velocity matrix  $\omega^0$  (see Def. 6).

*Subroutine input (optional)*

Any kinematically prescribed variables, such as  $\gamma_j(t)$  for some  $j \in \mathcal{P}$ . As noted in Section IIB, it is possible to modify Eq. (5) so as to accommodate the substitution of an explicit function of time for a variable, such as  $\gamma_j(t)$ , which in Eq. (5) is treated as an unknown. If this option is elected, the computer program input must include the specification of variables to be prescribed and appropriate values for  $\gamma_j$ ,  $\dot{\gamma}_j$ , and  $\ddot{\gamma}_j$ .

No additional input is required. The subroutine is programmed to construct numerically all intermediate concepts (such as the path elements  $\varepsilon_{sk}$  for  $k \in \mathcal{B}$ ,  $s \in \mathcal{P}$ , the barycentric position matrices  $D^{kk}$  and  $D^{kj}$  for  $k, j \in \mathcal{B}$ , the augmented body inertia matrices  $\Phi^{kk}$ , etc. It can then evaluate numerically the functions displayed in Eq. (5) and provide the solution for  $\dot{\omega}^0$  and  $\ddot{\gamma}_1, \dots, \ddot{\gamma}_n$ . The subroutine and explicit details of its use are described in the following section.

### E. Subroutine MBDY Usage

The computer subroutine, called MBDY, which provides the solution to Eq. (5), i.e., the solution to the unrestricted rotations of an arbitrary, hinge-connected, rigid-body tree system, may be exercised by means of either of two call statements.

The first of these initializes the program, supplying it with data which will remain constant throughout the dynamic simulation run.

*Initializing Call Statement*

CALL MBDY(NH,H,MB,MA,PB,PA,G,PI)

where

NH = the integer  $n$  = number of system hinges.

H(k) = array containing the connection elements  $h_k, k \in \mathcal{P}$   
(H(1) =  $h_1, H(2) = h_2, \dots, H(n) = h_n$ )

MB(j) = array of reference body ( $\mathcal{B}_0$ ) inertial constants;  $j = 1, \dots, 7$   
(Specifically: MB(1) =  $I_{11}^0, MB(2) = I_{22}^0, MB(3) = I_{33}^0,$   
MB(4) =  $-I_{12}^0, MB(5) = -I_{13}^0, MB(6) = -I_{23}^0,$  and MB(7) =  $m_0$ .)

MA(i,j) = array of appendage body ( $\mathcal{B}_1, \dots, \mathcal{B}_n$ ) inertial constants;  $i \in \mathcal{P},$   
 $j = 1, \dots, 7$  (Thus, MA(i,1) =  $I_{11}^i, MA(i,2) = I_{22}^i, \dots, MA(i,7) = m_i$ .)

PB(i,j) = array containing elements of  $p^{0j}; i \in \mathcal{B}_0, j = 1, 2, 3$

PA(i,j,k) = array containing elements of  $p^{ij}; i \in \mathcal{P}, j \in \mathcal{B}_i, k = 1, 2, 3$   
(Exception!! If  $j < i$ , set PA(i,j,k) =  $p^{ji}$ . Example: PA(3,3,1) =  $p_1^{32}$ .  
All PA(i,j,k), where  $j < i$ , will be ignored.)

G(i,j) = array containing elements of  $g^i; i \in \mathcal{P}, j = 1, 2, 3$

PI(i) = array of prescribed variable indicators;  $i \in \mathcal{P}$  (If  $\gamma_i$  is a prescribed variable, PI(i) = 1. Otherwise, PI(i) = 0.)

The statement CALL MBDY(NH,H,...) need only be executed *once* prior to a simulation run, but subsequently the subroutine must be entered (at every integration step) to compute the angular accelerations  $\ddot{\omega}^0, \ddot{\gamma}_1, \dots, \ddot{\gamma}_n$ , i.e., the system's dynamic response to its forcing functions. This is accomplished by executing the "dynamic" call statement.

*Dynamic Call Statement*

CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,WO,WDOT)

where

NH = the integer  $n$  = number of system hinges

TH(i) = array containing the hinge torques,  $\tau_i; i \in \mathcal{P}$

TB(j) = array containing the elements of  $T^0; j = 1, 2, 3$

TA(i,j) = array containing the elements of  $T^i; i \in \mathcal{P}, j = 1, 2, 3$

FB(j) = array containing the elements of  $F^0; j = 1, 2, 3$

FA(i,j) = array containing the elements of  $F^i; i \in \mathcal{P}, j = 1, 2, 3$

GM(i) = array of the angles,  $\gamma_i; i \in \mathcal{P}$

GMD(i) = array of the angular velocities,  $\dot{\gamma}_i; i \in \mathcal{P}$

GMDD(i) = array of the *prescribed* angular accelerations,  $\ddot{\gamma}_i; i \in \mathcal{P}$

$WO(j) =$  array containing the components of  $\omega^0$ ;  $j = 1, 2, 3$   
 $WDOT(j) =$  solution vector containing the elements of  $\dot{\omega}^0$  and  $\ddot{\gamma}_1, \dots, \ddot{\gamma}_n$ ;  
 $j = 1, \dots, n + 3$  ( $WDOT(1) = \dot{\omega}_1^0, WDOT(2) = \dot{\omega}_2^0, WDOT(3) = \dot{\omega}_3^0,$   
 $WDOT(4) = \ddot{\gamma}_1, WDOT(5) = \ddot{\gamma}_2, \dots, WDOT(n + 3) = \ddot{\gamma}_n$ )

The solution vector array, WDOT, is returned to the main calling program by the subroutine and contains the current computed value for the reference body's angular acceleration relative to the inertial frame  $\dot{\omega}^0$  and the relative angular accelerations  $\ddot{\gamma}_i$  of the remaining appendage bodies.

To summarize, the call to MRATE supplies the subroutine with current instantaneous values for hinge torques and externally applied torques and forces on the system. Explicit expressions for computing the  $T^i$ ,  $F^i$ , and  $\tau_i$ , often as functions of the  $\gamma_i$  and  $\dot{\gamma}_i$  and other system or control variables, are located in the main simulation program (see sample problem which follows). Current values of  $\omega^0$ ,  $\gamma_i$ , and  $\dot{\gamma}_i$  are continuously produced by the main program's numerical integration operators (operating on the solution vector components, WDOT( $j$ )) and are therefore always available for input to the subroutine. The  $\ddot{\gamma}_i$  inputs to the subroutine are used only when the motion of certain system bodies is prescribed.

When the MBDY subroutine is used, the main calling program must contain Fortran V (or IV) statements which specify "type" and allocate storage for the variables and arrays being used and assign values to these variables and arrays. Some examples of this will be shown in the next section. However, the *mandatory* specification statements are listed here.

#### Required Specification Statements

```

INTEGER  NH, H(n), PI(n)

REAL     MB(7), MA(n,7), PB(n,3), PA(n,n,3), C(n,3), TH(n),
         TB(3), TA(n,3), FB(3), FA(n,3), GM(n), GMD(n),
         GMDD(n), WO(3)

DOUBLE PRECISION  WDOT(n + 3)
  
```

Also, in order that storage allocation for arrays internal to the subroutine MBDY be minimized (i.e., made consistent with  $n$  hinges and no more), the following Fortran V (or IV) statement must appear *in the subroutine*:

```
PARAMETER  Q = n
```

The proper placement of this statement in MBDY is shown in the subroutine listing (Appendix A).

#### F. A Five-Body System Simulation

To illustrate the application of the MBDY subroutine for the unrestricted motion case, the space vehicle configuration shown in Fig. 4 will be simulated. The vehicle

is idealized to consist of a rigid central body, which will be designated as the reference body  $\mathcal{B}_0$ , including the bus, solar panels, gas jets, sensors, communications equipment, etc.

Hinged to the reference body is a rotating hub, labeled  $\mathcal{B}_1$ , which in turn is hinged to platform  $\mathcal{B}_2$ , carrying a television camera. The platform hinge axes located by  $\mathbf{g}^1$  and  $\mathbf{g}^2$  are orthogonal,  $\mathbf{g}^1 = \mathbf{b}_3^0$  (+Z-direction), and the rotation angles  $\gamma_1$  and  $\gamma_2$  are arbitrarily defined to be zero when the TV boresight unit vector  $\mathbf{u}$  is in the Y-direction. Also,  $\mathbf{u}$  is defined perpendicular to  $\mathbf{g}^2$ . Thus, we may write

$$\begin{aligned}\mathbf{u} &= -\sin \gamma_1 \cos \gamma_2 \mathbf{b}_1^0 + \cos \gamma_1 \cos \gamma_2 \mathbf{b}_2^0 - \sin \gamma_2 \mathbf{b}_3^0 \\ \mathbf{g}^1 &= \mathbf{b}_3^1 \\ \mathbf{g}^2 &= -\mathbf{b}_1^2\end{aligned}$$

A large instrumented boom,  $\mathcal{B}_4$ , is also hinged to the reference body by the mutually orthogonal and coplanar axes defined by  $\mathbf{g}^3$  and  $\mathbf{g}^4$ . (In this case, the intermediate rigid body,  $\mathcal{B}_3$ , will be assigned zero mass.) Therefore,

$$\begin{aligned}\mathbf{g}^3 &= \mathbf{b}_1^3 \\ \mathbf{g}^4 &= \mathbf{b}_3^4\end{aligned}$$

The following list of system parameters will be used to define the system and initialize the MBDY subroutine:

- (1)  $n = 4$
- (2)  $h_1 = 0, h_2 = 1, h_3 = 0, h_4 = 3$
- (3)  $p^{01} = [0 \ 0 \ -1.5]^T, p^{03} = [0 \ -1.20 \ 0]^T$  meters  
 $p^{10} = [0 \ 0 \ -0.75]^T, p^{12} = [0 \ -0.10 \ -0.75]^T$  meters  
 $p^{21} = [0 \ -0.22 \ 0.20]^T$  meters  
 $p^{30} = [0 \ 0 \ 0]^T, p^{34} = [0 \ 0 \ 0]^T$  meters  
 $p^{43} = [0 \ 3.3 \ 0]^T$  meters
- (4)  $\mathbf{g}^1 = [0 \ 0 \ 1]^T$  meters  
 $\mathbf{g}^2 = [-1 \ 0 \ 0]^T$  meters  
 $\mathbf{g}^3 = [1 \ 0 \ 0]^T$  meters  
 $\mathbf{g}^4 = [0 \ 0 \ 1]^T$  meters
- (5)  $m_0 = 410$  kg  
 $m_1 = 6.8$  kg  
 $m_2 = 57.5$  kg  
 $m_3 = 0$   
 $m_4 = 10.7$  kg



$$\begin{aligned}
 (6) \quad I^0 &= \begin{bmatrix} 115 & -14 & 14 \\ & 316 & -34.6 \\ \text{sym.} & & 440 \end{bmatrix}, & I^1 &= \begin{bmatrix} 0.35 & 0 & 0 \\ & 0.35 & 0 \\ \text{sym.} & & 0 \end{bmatrix} \text{ kg-m}^2 \\
 I^2 &= \begin{bmatrix} 4.85 & 0.41 & -0.07 \\ & 2.20 & 0.54 \\ \text{sym.} & & 5.50 \end{bmatrix}, & I^3 &= \begin{bmatrix} 0 & 0 & 0 \\ & 0 & 0 \\ \text{sym.} & & 0 \end{bmatrix} \text{ kg-m}^2 \\
 I^4 &= \begin{bmatrix} 27.2 & 0 & 0 \\ & 0.2 & 0 \\ \text{sym.} & & 27.2 \end{bmatrix} \text{ kg-m}^2
 \end{aligned}$$

This completes the list of constants that are needed to characterize the system. It remains then to develop expressions for the dynamic forcing functions  $T^k$ ,  $F^k$ , and  $\tau_k$ .

Control of the vehicle shown in Fig. 4 is to be maintained by firing pairs of gas jets located on the reference body to produce, ideally, a pure torque couple on that body. It will be assumed in the simulations to follow that this is the case and that no net force is applied to any of the system bodies; i.e.,  $F^k = 0$  for  $k \in \mathcal{B}$ . The control torque  $T^0$  can be expressed by

$$T^0 = [L_1 f_1 \quad L_2 f_2 \quad L_3 f_3]^T$$

where the control functions  $f_1$ ,  $f_2$ , and  $f_3$  each may take on the values  $\pm 1$  or 0, depending on the single-axis control law defined in Fig. 5. (All other  $T^k$  for  $k \in \mathcal{P}$  are assumed to be zero.) The control law is based on position feedback signals derived from celestial sensors (i.e., sun sensors, star trackers) as well as rate feedback derived from gyro measurements. If the sum of these feedback errors is

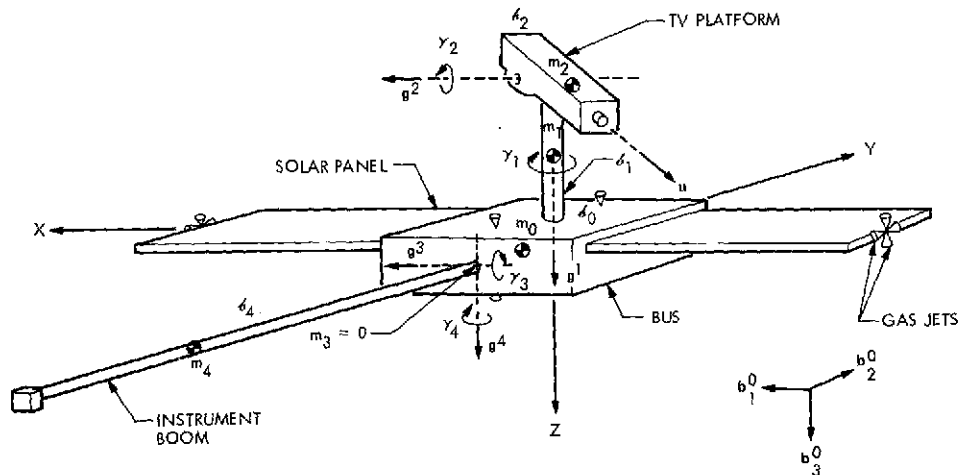


Fig. 4. A space vehicle idealized as a system of five hinge-connected rigid bodies

greater than some arbitrary value ( $\pm\theta_{DB}$  in Fig. 5), a switching amplifier fires the gas jets to apply corrective torques. A list of values assigned to the reference body attitude control parameters follows:

$$\begin{aligned} \theta_{DB} &= 0.005 \text{ rad} & L_1 &= 0.23 \text{ n-m} \\ \tau_R &= 2.0 \text{ s} & L_2 &= 0.21 \text{ n-m} \\ & & L_3 &= 0.31 \text{ n-m} \\ & & TM &= 0.020 \text{ s} \end{aligned}$$

Finally, it is necessary to characterize the hinge torques  $\tau_k (k \in \mathcal{P})$ . Since the boom will not be articulated, the boom hinge torques can be described by

$$\begin{aligned} \tau_3 &= -K_3\gamma_3 - B_3\dot{\gamma}_3 \\ \tau_4 &= -K_4\gamma_4 - B_4\dot{\gamma}_4 \end{aligned}$$

where

$$\begin{aligned} K_3 &= 2000 \text{ n-m/rad}, & B_3 &= 10 \text{ n-m-s/rad} \\ K_4 &= 2000 \text{ n-m/rad}, & B_4 &= 10 \text{ n-m-s/rad} \end{aligned}$$

However, since the TV platform will be articulated relative to its nominal pointing direction, it is convenient to introduce the variables  $\gamma_{1c}$  and  $\gamma_{2c}$  to represent the *commanded* position of the platform. The hinge torques can then be written as

$$\begin{aligned} \tau_1 &= -K_1(\gamma_1 - \gamma_{1c}) - B_1\dot{\gamma}_1 \\ \tau_2 &= -K_2(\gamma_2 - \gamma_{2c}) - B_2\dot{\gamma}_2 \end{aligned}$$

where

$$\begin{aligned} K_1 &= 3500 \text{ n-m/rad}, & B_1 &= 20 \text{ n-m-s/rad} \\ K_2 &= 3500 \text{ n-m/rad}, & B_2 &= 20 \text{ n-m-s/rad} \end{aligned}$$

**1. Case 1—large platform maneuver.** The first simulation of the five-body system of Fig. 4 and its controller will show its dynamic response to a large platform

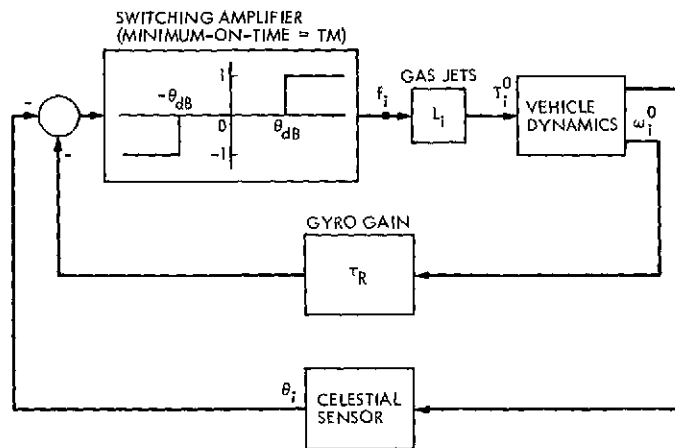


Fig. 5. Single-axis attitude control loop

maneuver. Initially, the platform is positioned at  $\gamma_1 = 218$  deg and  $\gamma_2 = -30$  deg. All other system variables, i.e., angular positions and rates, are assumed initially at zero. At  $t = 0$ , the command is issued to rotate the platform (about both axes simultaneously) at the constant rate of 1 deg/s to the position  $\gamma_1 = 168$  deg,  $\gamma_2 = 20$  deg. The program used to simulate this maneuver is shown in Fig. 6.

Note that the required MBDY specification statement REAL MB(7), . . . has been replaced by its equivalent in the CSSL III language, i.e., ARRAY MB(7), . . .

Also, note that the HCK routine mentioned in Section I is used to establish the position of the reference body in the inertially fixed frame, and it thereby provides a means for calculating  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ , the attitude angles measured by celestial sensors fixed to the reference body.

Three other CSSL III built-in functions are also used in the simulation. The function "SWAMP" simulates the switching amplifier shown in Fig. 5. "STEP" provides the unit step function when the independent variable, TIME, is greater than the specified constant. And finally, "INTEG( $a_1, a_2$ )" signifies the integration operation, where  $a_1$  is the integrand and  $a_2$  is the initial condition.

All arithmetic statements are in Fortran, although CSSL III allows several statements on a single line if separated by a "\$." Variables to be plotted at every communication interval CI are listed in the PREPAR statement. Printed variables are listed in the OUTPUT statement.

The statement "CALL MBDY(NH, . . .)" is located in the INITIAL section and is therefore executed only once, i.e., prior to the dynamic calculations. However, "CALL MRATE(NH, . . .)" is in the DERIVATIVE section and is thus executed at every integration step, as required.

Results of the simulation described in Fig. 6 are shown in the computer plots of Fig. 7. All variables plotted are in terms of radians, radians per second, and seconds.

Notice that all three components ( $W1, W2, W3$ ) of the reference body angular velocity  $\omega^0$ , show evidence of platform hinge vibrations immediately following platform acceleration or deceleration (i.e., at  $t = 0$  and  $t = 50$  s). Only  $W1$  and  $W3$  show any significant evidence of the lower-frequency boom vibrations, since the boom is nominally aligned with the reference body's "2" axis. Attitude angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  (THET1, THET2, THET3) have obviously been effectively confined to the  $\pm 0.005$ -rad deadband by the control system. Platform and boom angular rate and position responses clearly show the transients at  $t = 0$  and  $t = 50$  s. Some platform vibration is discernible in the boom responses.

Figure 8 repeats the transient solutions for  $W1, W2$ , and  $W3$  over just the first 10 s in order to display the response in better detail. The  $W2$  response indicates that control jets began pulsing at about  $t = 4.6$  s in that axis, while pulsing in the other axes was not required until much later.

**2. Case I with prescribed motion.** In order to demonstrate the utility of the MBDY subroutine's option for prescribed motion, Case I was rerun with the platform rotations  $\gamma_1$  and  $\gamma_2$  prescribed. As indicated in Section IIE, the motion of any

```

PROGRAM 5-BODY SYSTEM SIMULATION USING MBDY --- CASE 1
      INTEGER NH,P1(4),H(4)
      ARRAY MB(7),MA(4,7),PB(4,3),PA(4,4,3),G(4,3),TB(3),TA(4,3)
      ARRAY FB(3),FA(4,3),TH(4),GM(4),GMD(4),GMD(4),WO(3)
      DOUBLE PRECISION WDOT(7)
DATA H/O,1,0,3/P1/O,0,0,0/
DATA MB/115.,316.,440.,14.,-14.,34.6,410./
DATA MA(1,1)/.35/MA(1,2)/.35/MA(1,7)/6.8/
DATA MA(2,1)/4.85/MA(2,2)/2.2/MA(2,3)/5.5/MA(2,4)/-.41/MA(2,6)/.67/
DATA MA(2,6)/-.54/MA(2,7)/57.5/
DATA MA(4,1)/27.2/MA(4,2)/.2/MA(4,3)/27.2/MA(4,7)/10.7/
DATA PB(1,3)/-1.5/PB(3,2)/-1.2/
DATA PA(1,1,3)/-.75/PA(1,2,2)/-.10/PA(1,2,3)/-.75/
DATA PA(2,2,2)/-.22/PA(2,2,3)/.20/
DATA PA(4,4,2)/3.3/
DATA G(1,3)/1./G(2,1)/-1./G(3,1)/1./G(4,3)/1./
      CONSTANT TM=.020,DB=.005,TK=2.0
      CONSTANT TFINAL=60.,CLKTIM=1100.
      CONSTANT L1=.23,L2=.21,L3=.31
      CONSTANT SLRAT=.01745329
      CONSTANT PHIZ=0.,THETZ=0.,PSIZ=0.
      CONSTANT W1=0.,W2=0.,W3=0.
      CONSTANT K1=3500.,K2=3500.,K3=2000.,K4=2000.
      CONSTANT B1=20.,B2=20.,B3=10.,B4=10.
      CONSTANT IC1=218.,IC2=-30.
INITIAL
      NH=4
      RD=57.2957795 $ DR=1./RD $ GM(1)=IC1/RD $ GM(2)=IC2/RD
      GM(1)=IC1/RD $ GM(2)=IC2/RD
      CALL MBDY(NH,H,MB,MA,PB,PA,G,P1)
      PZ1,PA1,PB1,PC1=HCK(MATRIX,IL,PHIZ,THETZ,PSIZ)
END
DYNAMIC
      IF(TIME.GE.TFINAL) GO TO S1
DERIVATIVE BODYS
      VARIABLE TIME=0.
      CINTERVAL CI=.06
      XERROR W1=1.E-6,W2=1.E-6,W3=1.E-6,GM3=1.E-4
      STPCLK CLKTIM
      OUTPUT 50,W1D,W2D,W3D,W1,W2,W3,THET1,THET2,THET3,...
      IN1,IN2,IN3,GM1,GM2,GM3,GM4,PL1,PL2
      PREPAR W1,W2,W3,THET1,THET2,THET3,GM1D,GM2D,...
      GM1,GM2,GM3D,GM4D,GM3,GM4
NOSORT
      WO(1)=W1 $ WO(2)=W2 $ WO(3)=W3
      PL1=RD*GM1 $ PL2=RD*GM2
      GM(1)=GM1 $ GM(2)=GM2 $ GM(3)=GM3 $ GM(4)=GM4
      GMD(1)=GM1D $ GMD(2)=GM2D $ GMD(3)=GM3D $ GMD(4)=GM4D
COMMENT
      COMMENT CELESTIAL-SENSOR-MEASURED ANGLES
COMMENT
      D1,D2=HCK(MATRIX,PZO,PAQ,PBO,PCO)
      NXO,NYO,NZO=HCK(ITOB,0.,0.,1.,D1,D2)
      LXO,LYO,LZO=HCK(ITOB,1.,0.,0.,D1,D2)
      THET1=ATAN2(NYO,NZO) $ THET2=ATAN2(-NXO,NZO)
      THET3=ATAN2(-LYO,LXO)

```

Fig. 6. Five-body system simulation using MBDY (Case 1)

```

COMMENT
COMMENT          RATE FEEDBACK SIGNAL AND SWITCHING AMP. DYNAMICS
COMMENT
IN1=-THET1-TR*W1 $ IN2=-THET2-TR*W2 $ IN3=-THET3-TR*W3
AMP1=SWAMP(TIME,TH,UB,IN1)
AMP2=SWAMP(TIME,TH,UB,IN2)
AMP3=SWAMP(TIME,TH,UB,IN3)

COMMENT
COMMENT          PLATFORM POSITION COMMANDS
COMMENT
GM1CD=SLRAT*(-1.+STEP(50,0,TIME)) $ GM1C=INTEG(GM1CD,GM1C)
GM2CD=SLRAT*(1.-STEP(50,0,TIME)) $ GM2C=INTEG(GM2CD,GM2C)

COMMENT
COMMENT          REFERENCE BODY EXTERNAL TORQUE
COMMENT          HINGE TORQUES
COMMENT
TB(1)=L1*AMP1 $ TB(2)=L2*AMP2 $ TB(3)=L3*AMP3
TH(1)=-K1*(GM1-GM1C)-B1*GM1D
TH(2)=-K2*(GM2-GM2C)-B2*GM2D
TH(3)=-K3*GM3-B3*GM3D
TH(4)=-K4*GM4-B4*GM4D

COMMENT
COMMENT          SYSTEM ANGULAR ACCELERATIONS
COMMENT
CALL MRATE(INH,TH,TB,TA,FB,FA,GM,GMD,GMDD,#0,WDOT)
W1D=WDOT(1) $ W2D=WDOT(2) $ W3D=WDOT(3)

COMMENT
COMMENT          SYSTEM ANGULAR RATES AND POSITIONS
COMMENT
W1=INTEG(WDOT(1),W1)
W2=INTEG(WDOT(2),W2)
W3=INTEG(WDOT(3),W3)
GM1D=INTEG(WDOT(4),0.) $ GM2D=INTEG(WDOT(5),0.)
GM3D=INTEG(WDOT(6),0.) $ GM4D=INTEG(WDOT(7),0.)
GM1=INTEG(GM1D,GM1) $ GM2=INTEG(GM2D,GM2)
GM3=INTEG(GM3D,0.) $ GM4=INTEG(GM4D,0.)

COMMENT
COMMENT          HCK PARAMETER RATES AND POSITIONS
COMMENT
PZOD,PAOD,PBOD,PCOD=HCK(HCK,PZO,PAO,PBO,PCO,W1,W2,W3)
PZO=INTEG(PZOD,PZ)
PAO=INTEG(PAOD,PA)
PBO=INTEG(PBOD,PB)
PCO=INTEG(PCOD,PC)

END
END
END
TERMINAL
SI** CONTINUE
END
END

```

Fig. 6 (contd)

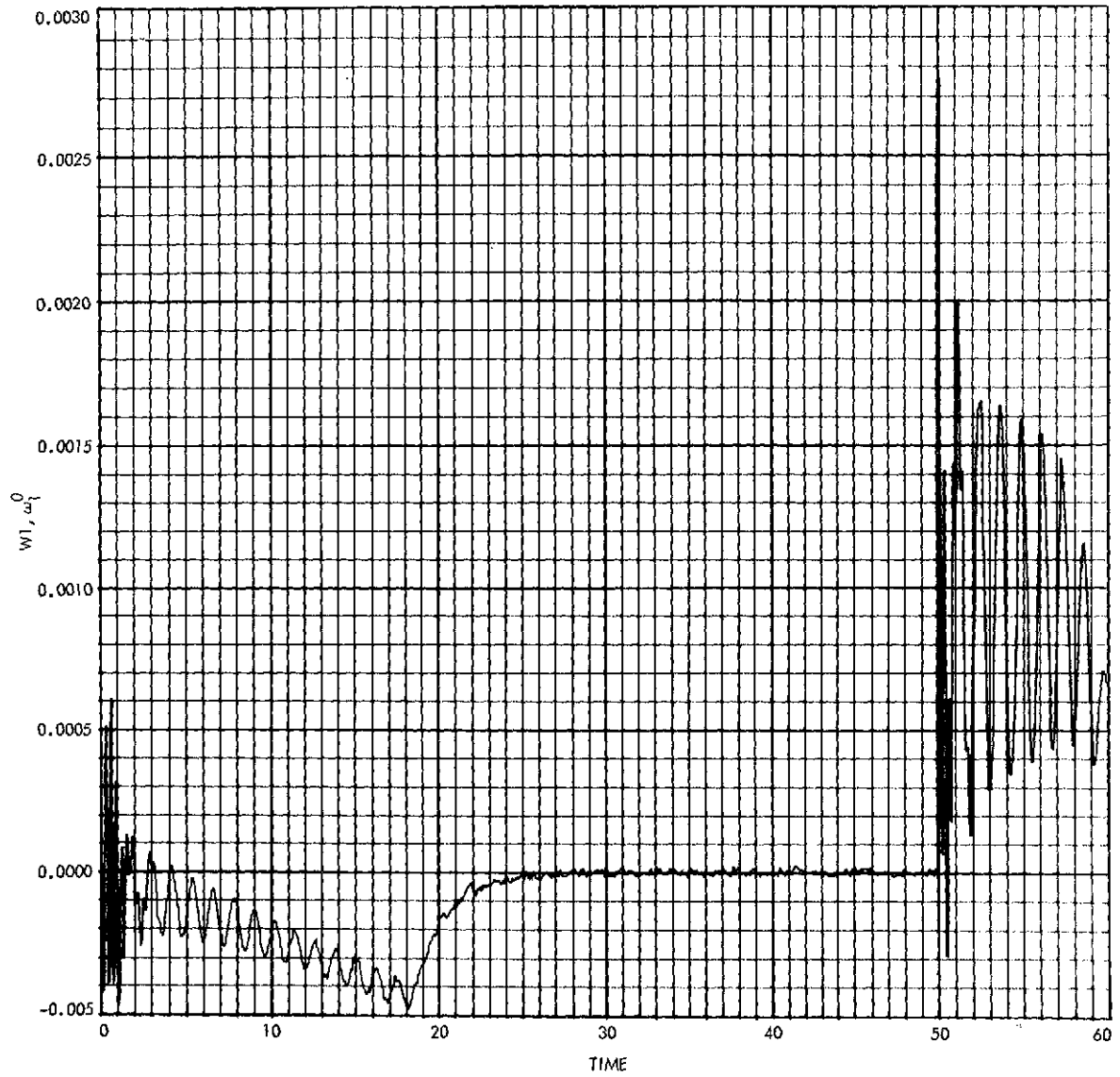


Fig. 7. Results of Case 1 simulation using MBDY

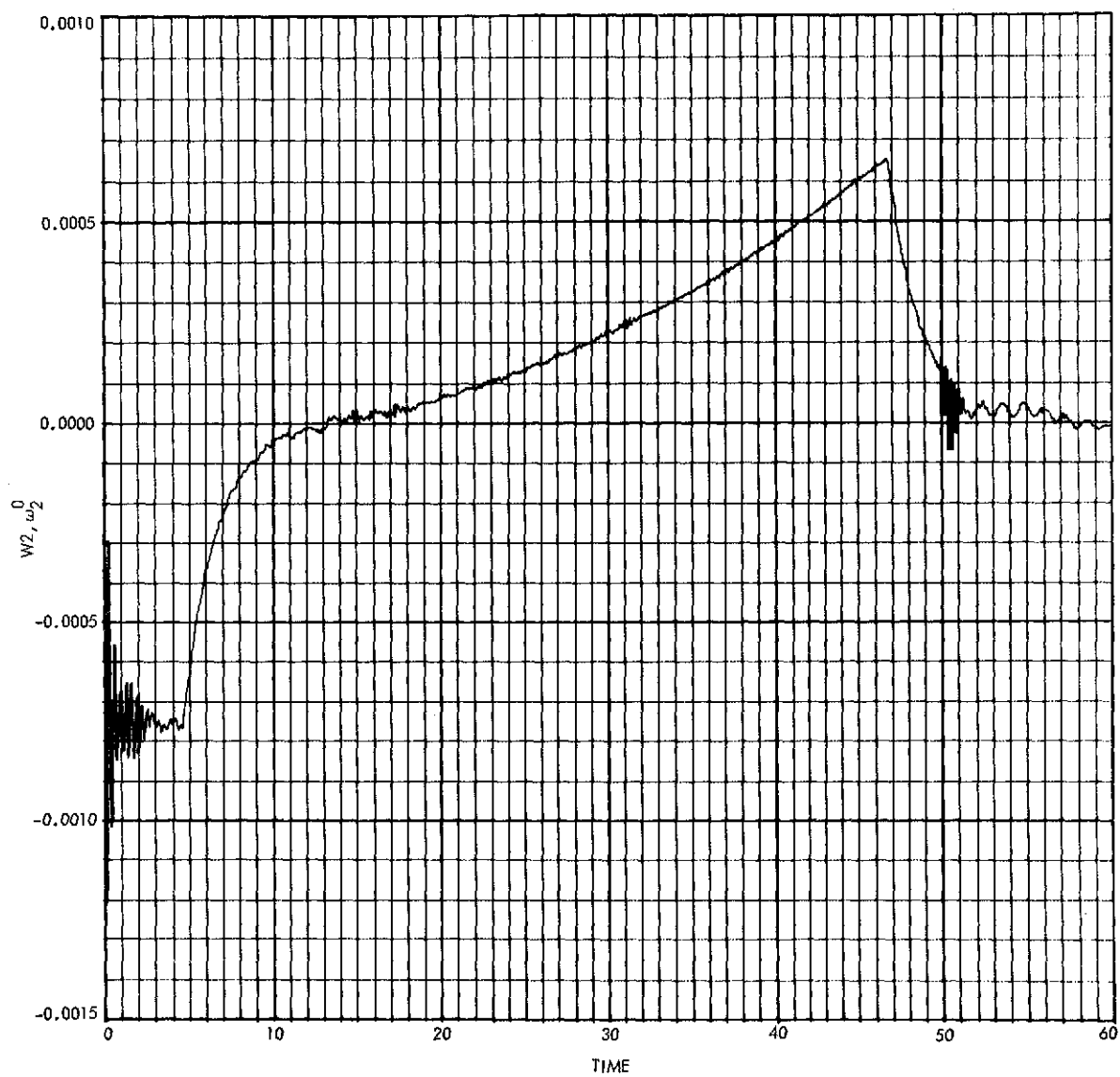


Fig. 7 (contd)

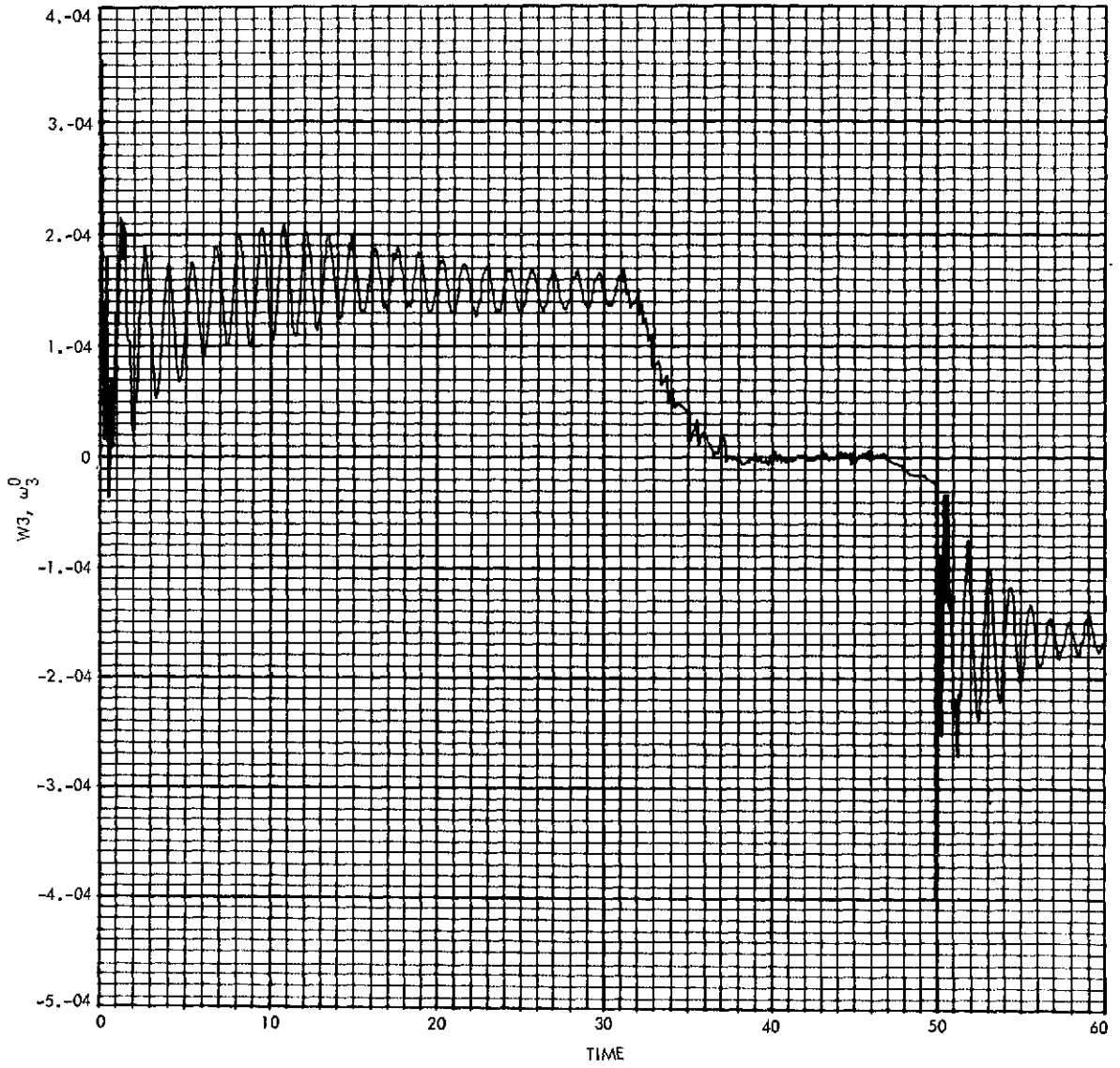


Fig. 7 (contd)



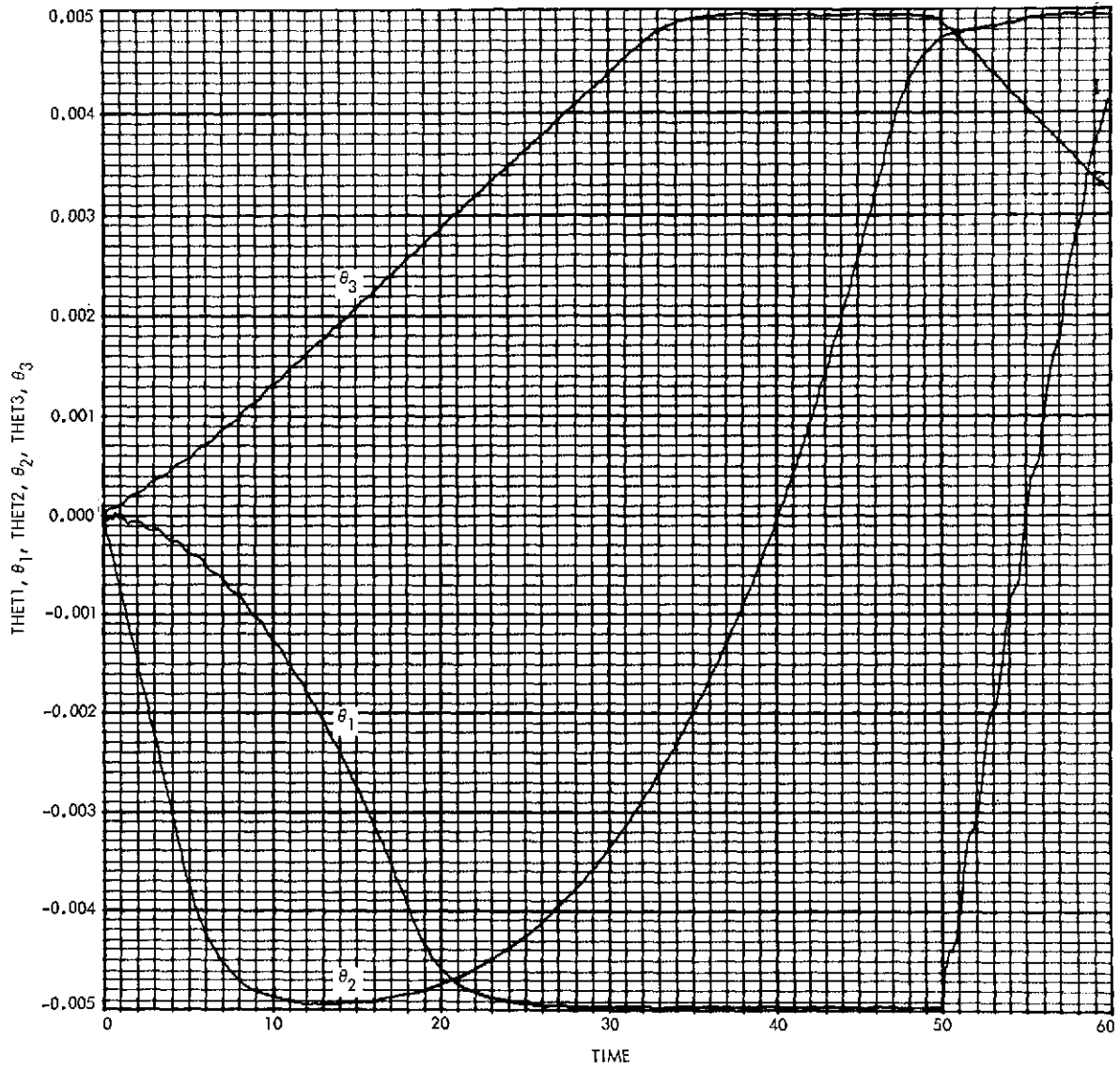


Fig. 7 (contd)

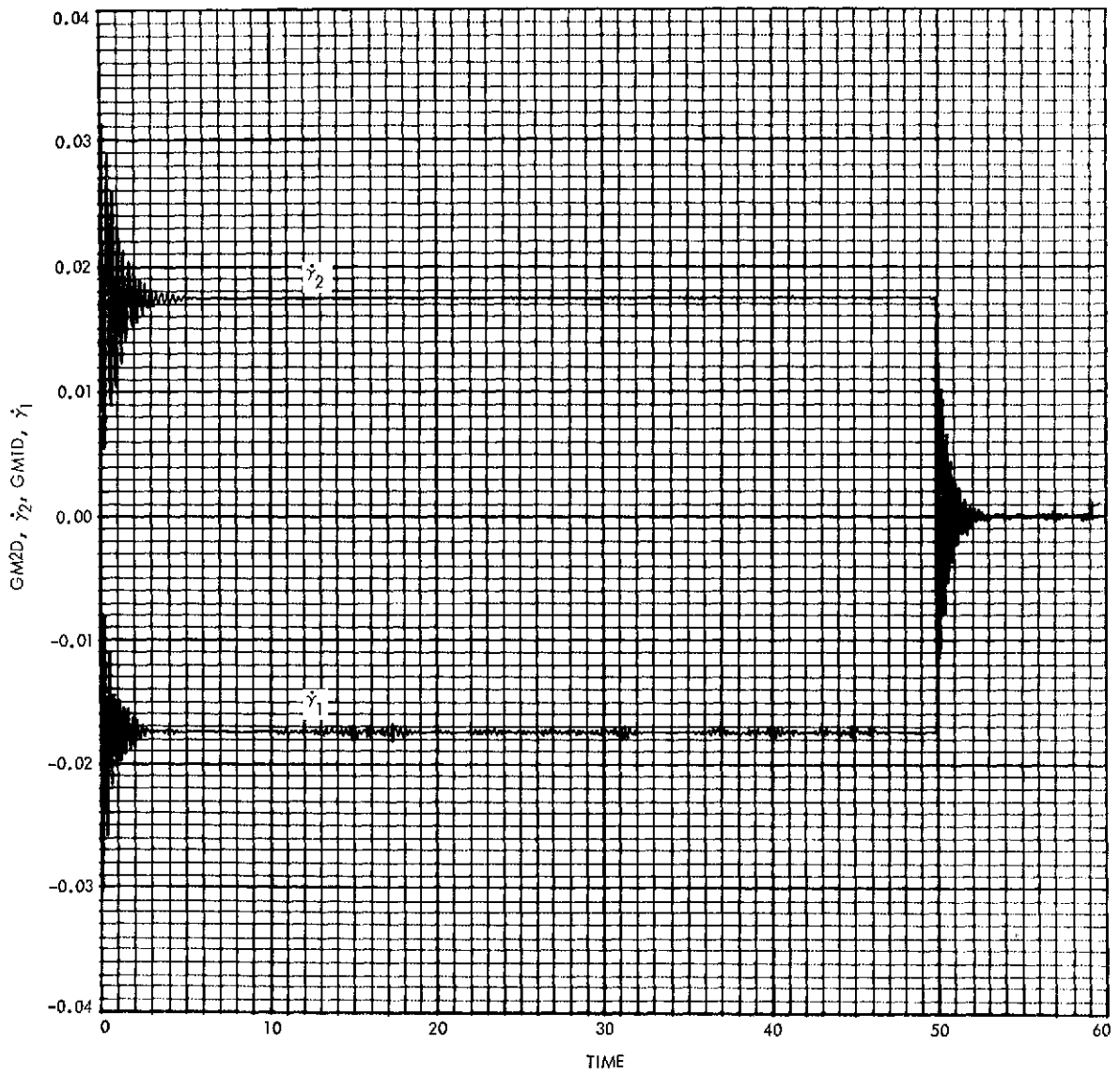


Fig. 7 (contd)

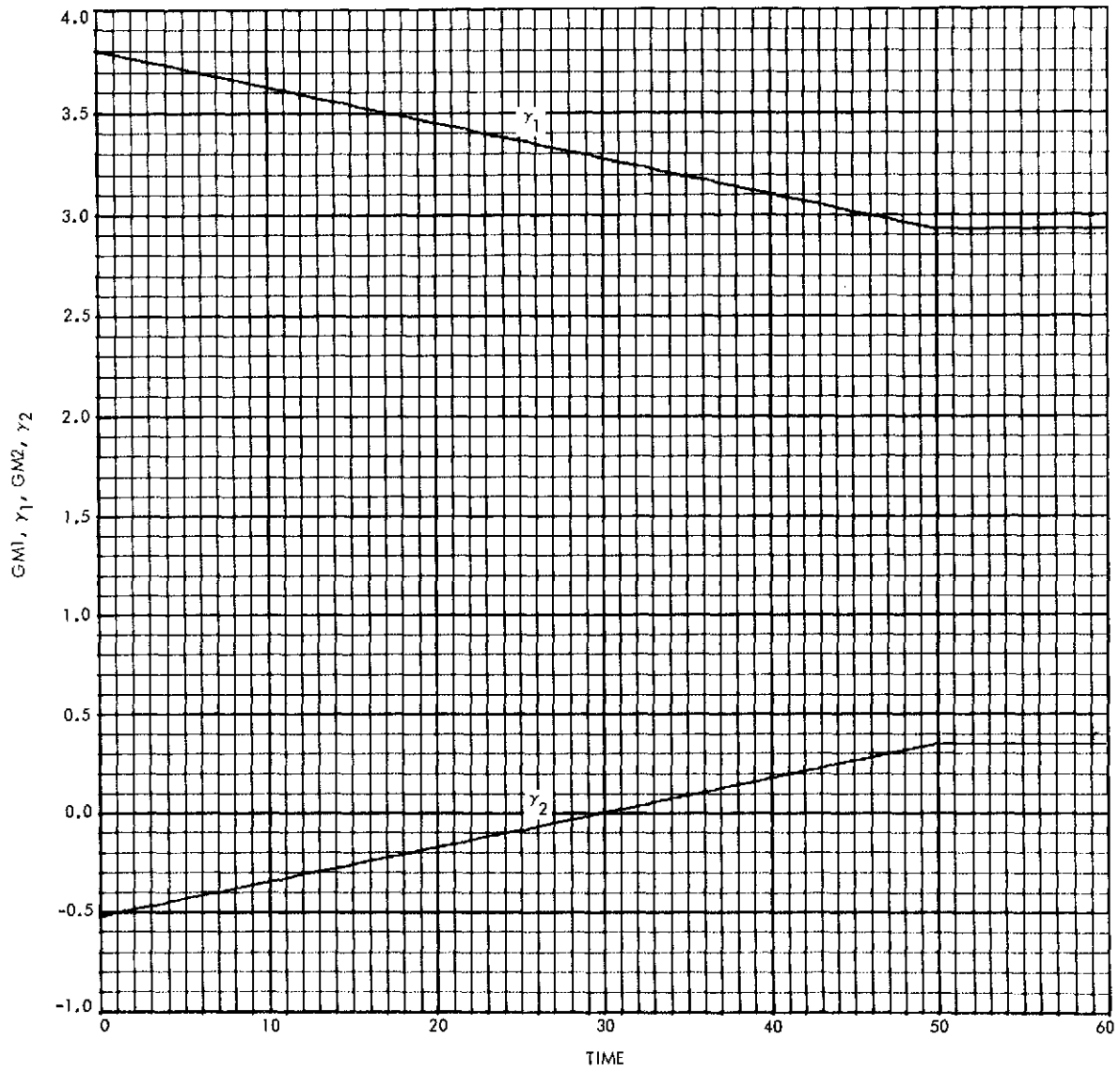


Fig. 7 (contd)

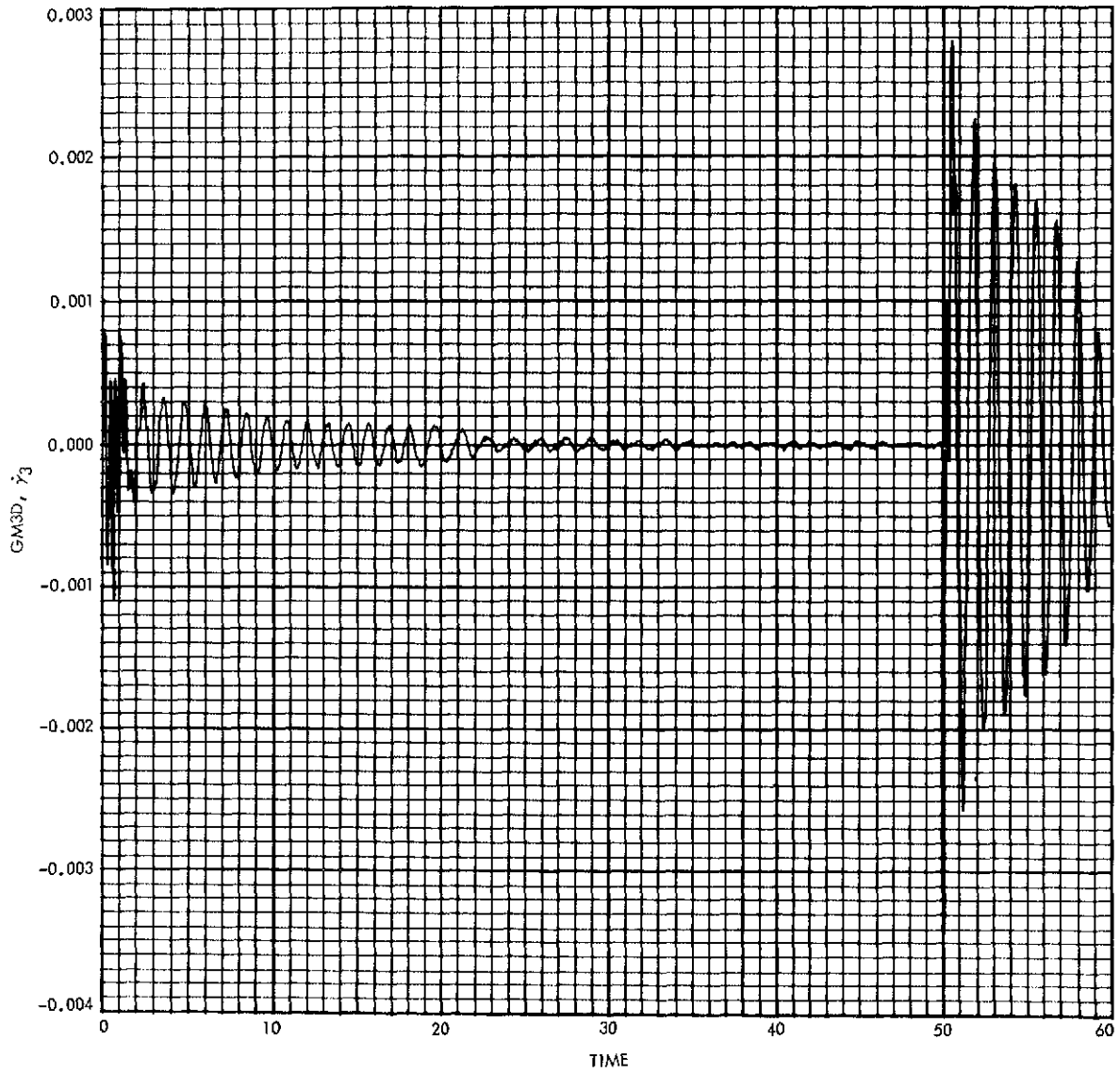


Fig. 7 (contd)

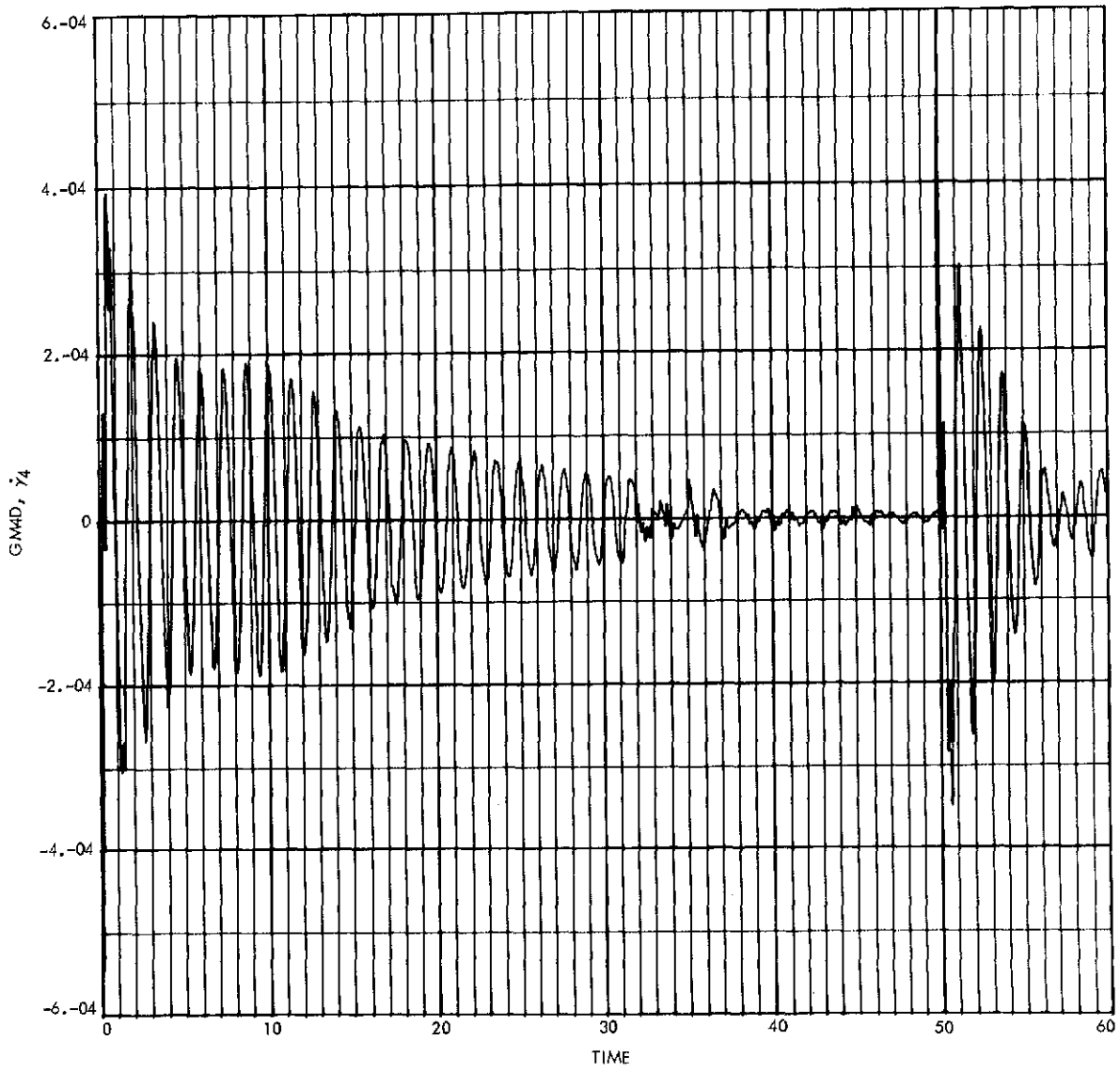


Fig. 7 (contd)

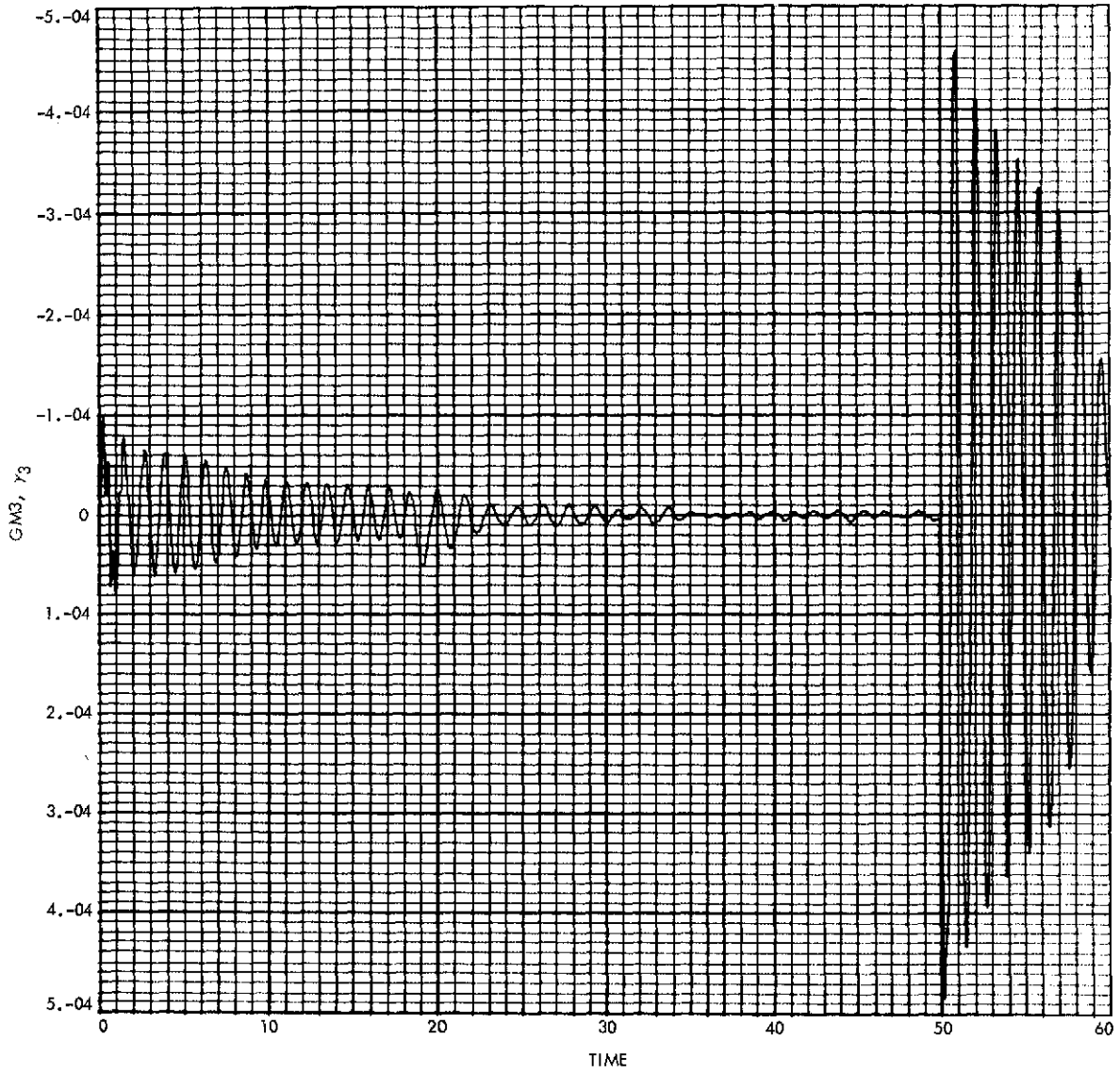


Fig. 7 (contd)

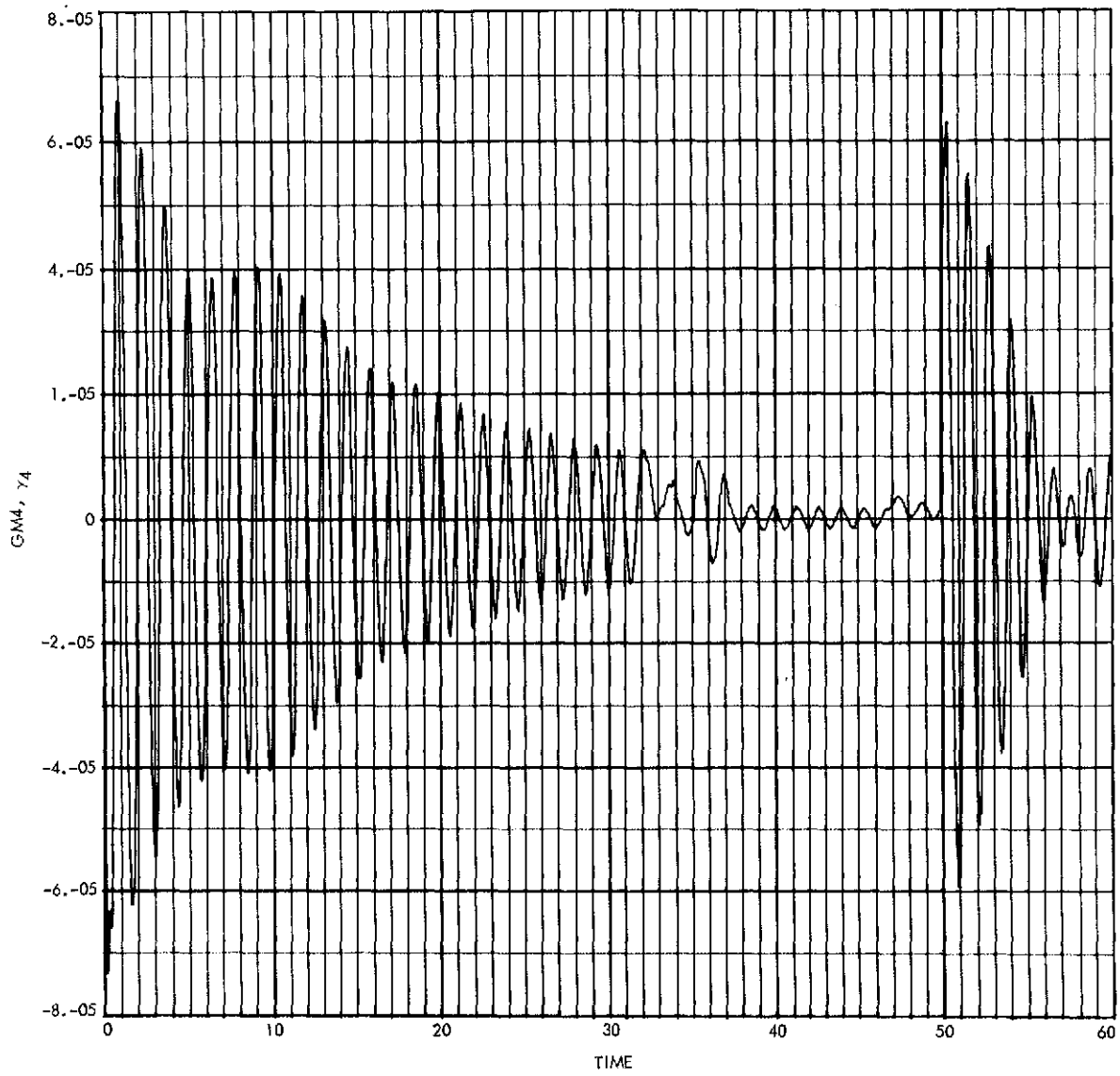


Fig. 7 (contd)

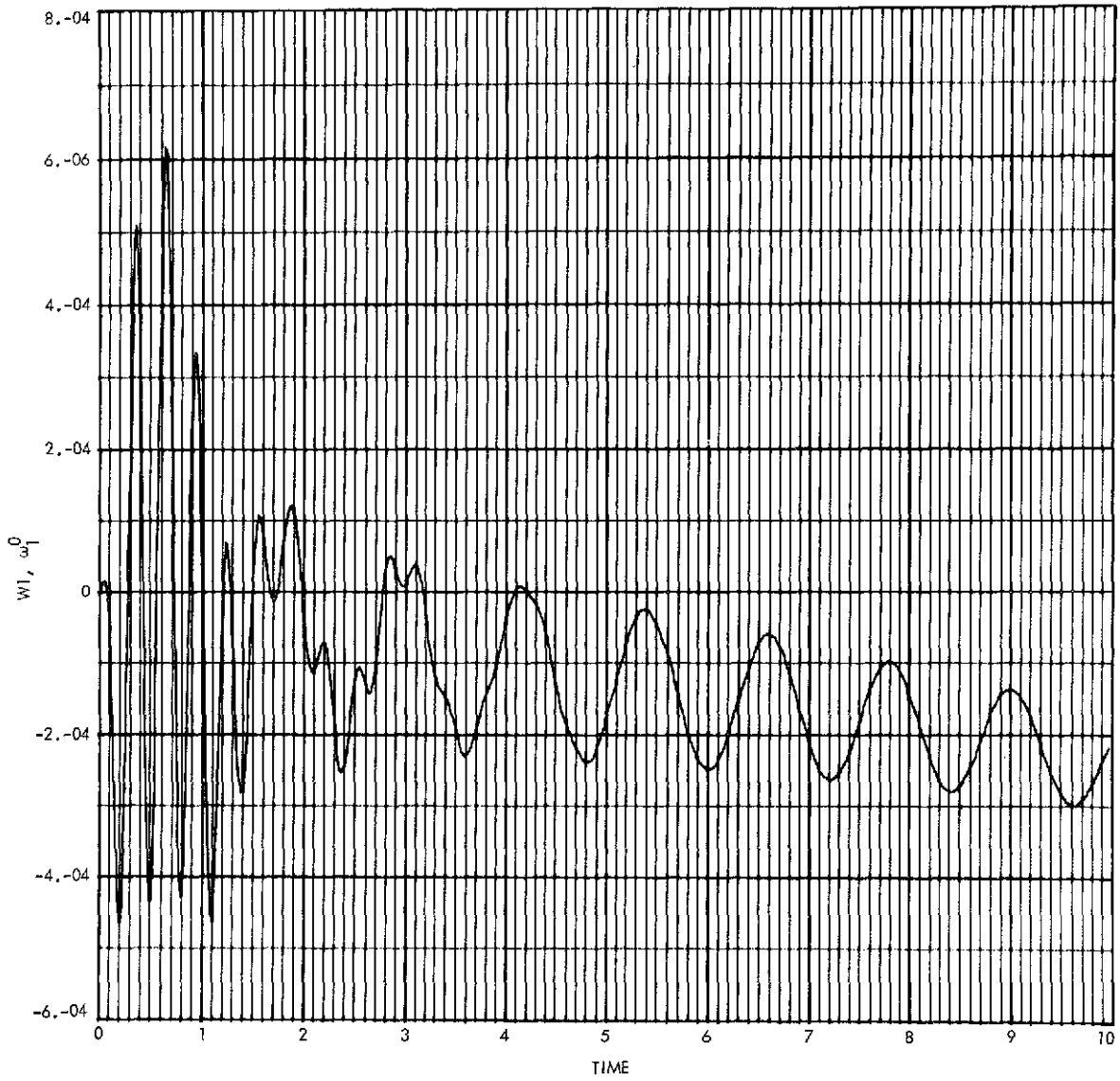


Fig. 8. Results of Case 1 simulation using MBDY (components of  $\omega^0$ ; first 10 s)



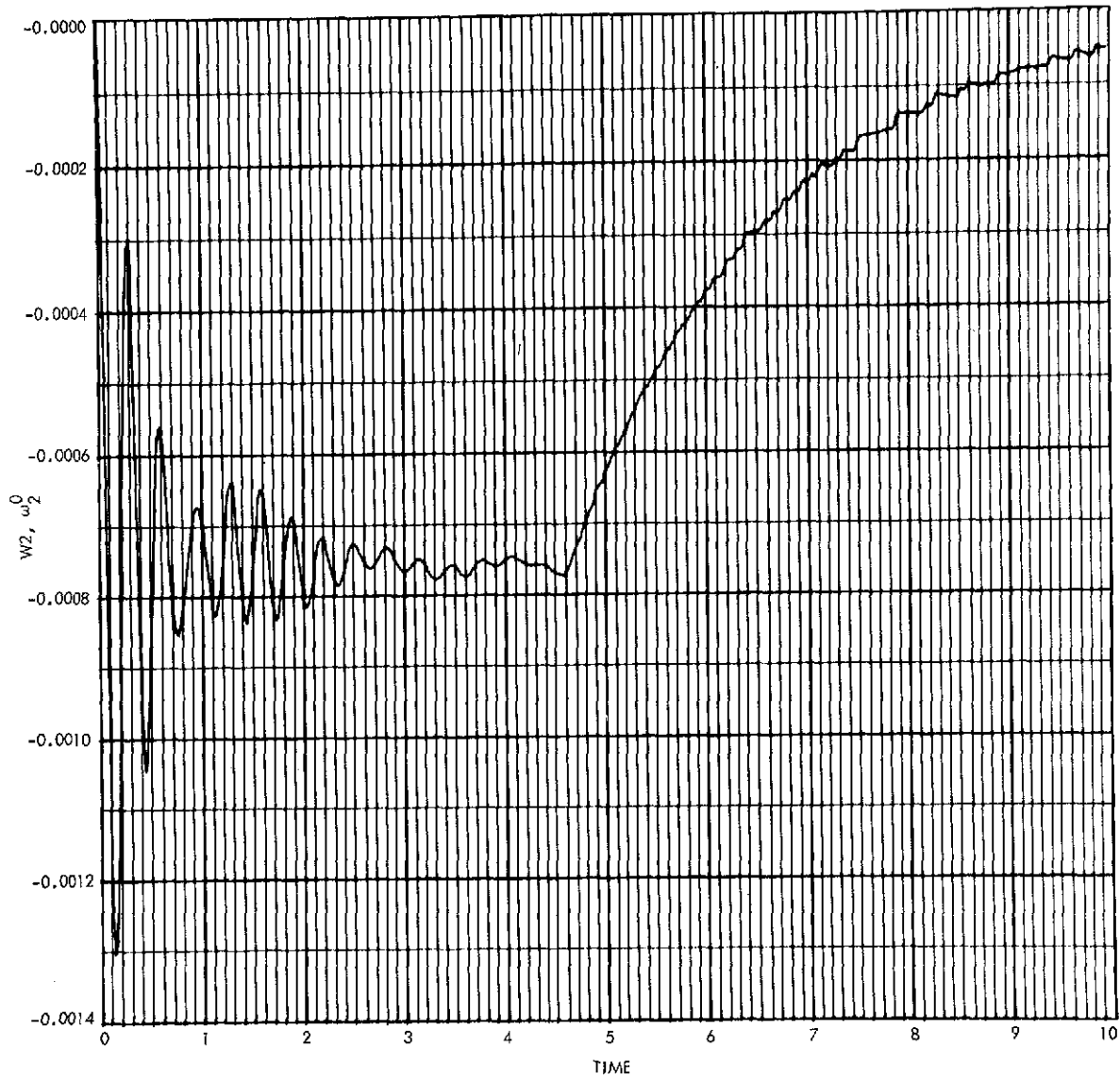


Fig. 8 (contd)

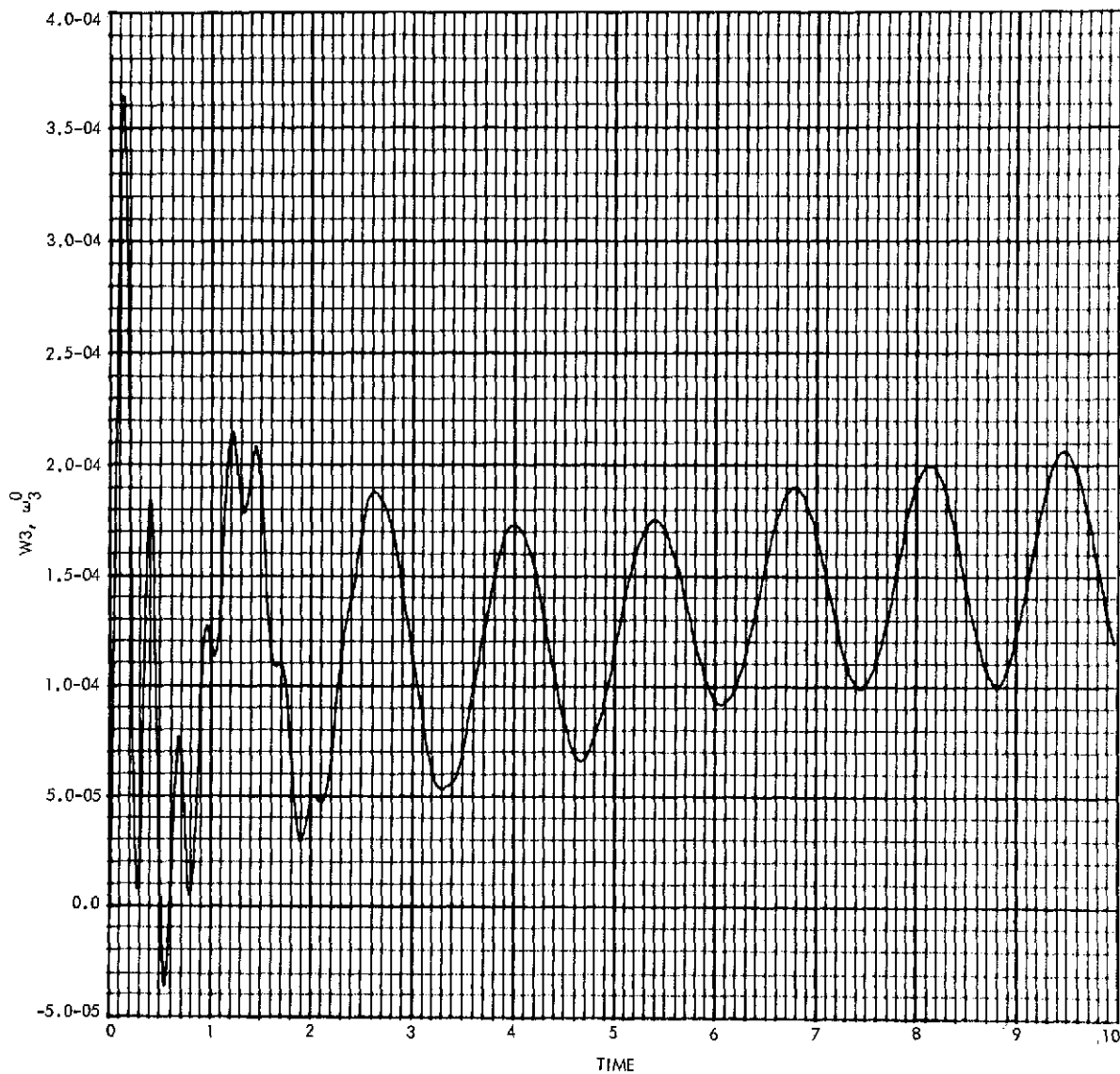


Fig. 8 (contd)

body  $\mathcal{L}_i$ , ( $i \in \mathcal{P}$ ) may be prescribed relative to an adjoining body  $\mathcal{L}_j$ , where  $j = h_i$  (Def. 37), by setting  $PI(i) = 1$  and supplying the appropriate values or function for  $\ddot{\gamma}_i(GMDD(i))$ . This was done for the platform bodies  $\mathcal{L}_1$  and  $\mathcal{L}_2$ , resulting in the simulation program of Fig. 9. Notice that the platform command angles have been replaced by time functions for GMDD(1) and GMDD(2) (using the CSSL III "RAMP" function). The GMDD(1) function is plotted in Fig. 10.

The expressions for the hinge torques  $\tau_1$  and  $\tau_2$  (TH(1), TH(2)) have also been removed from the original program (although it was not necessary) to emphasize the fact that they are not used when  $\gamma_1$  and  $\gamma_2$  are prescribed.

Figure 11 shows the simulation results. It is clear that the overall system response is virtually the same as that obtained from the first simulation (Fig. 7), *except* for the absence of the characteristic platform hinge vibrations. Since the simulation with prescribed platform motion took only 4½ min of computation time, compared with 7½ min for the first run, the usefulness of the prescribed motion option is well demonstrated. In this case, not only was the order of the system equation reduced, but the absence of the high-frequency platform vibrations helped to speed up the numerical integration process.

Of course, some caution must also be exercised when employing prescribed variables to make sure that the dynamics being eliminated are not important. In the sample problem considered here, the platform hinge vibrations could well have triggered gas jet firings before damping out if  $\tau_R$  had been larger and/or the initial conditions had been different. The absence of these vibrations might then have resulted in an entirely different time history of the reference body attitude angles.

One additional simulation of the Case 1 platform maneuver was performed *without* the application of control torques. In the absence of external torques, the system angular velocities should return to their initial values (i.e., zero) after the platform maneuver is completed (Conservation of Angular Momentum). Of particular interest are the reference body angular rates, which are given in Fig. 12. Although vibrations are still in evidence at  $t = 60$  s, it is clear that the final *average* rates are zero, as they should be. (The small high-frequency disturbances which seem to occur randomly, particularly in the  $W_2$  and  $W_3$  responses, are due to numerical integration errors.)

**3. Case 2—small platform maneuver.** The subroutine MBDY, since it solves the complete and exact set of equations of motion, must certainly be used when relatively large angles and angular rates are expected at each hinge. However, if the hinge rotations or both the rotations and rotational rates are "small" at any hinge, some linearization of the equations of motion may be effected, often to good advantage in simplifying and speeding the computational task.

In subsequent sections of this report, modified versions of MBDY, employing some form of linearization, will be described. One version, MBDYTL, employs total linearization; i.e., all angular position and velocity responses in the system are assumed "small" in some sense. A sample simulation, using the five-body configuration of Fig. 4, will be performed later using MBDYTL, and it is therefore of interest to have for comparison purposes an "exact" solution to the same problem using MBDY.

```

PROGRAM 5-BODY SYSTEM SIMULATION USING MBDY --- CASE 1
  INTEGER NH,PI(4),HI(4)
  ARRAY MB(7),PA(4,7),PB(4,3),PA(4,4,3),G(4,3),TB(3),TA(4,3)
  ARRAY FB(3),FA(4,3),TH(4),GM(4),GMD(4),GMDD(4),W0(3)
  DOUBLE PRECISION WDOT(7)
DATA H/0,1,0,3/PI/1,1,0,0/
DATA MB/115.,316.,44C.,14.,-14.,34.6,410./
DATA MA(1,1)/.35/MA(1,2)/.35/MA(1,7)/6.8/
DATA MA(2,1)/4.85/PA(2,2)/2.2/MA(2,3)/5.5/MA(2,4)/-.41/MA(2,5)/.07/
DATA MA(2,6)/-.54/MA(2,7)/57.5/
DATA MA(4,1)/27.2/PA(4,2)/.2/MA(4,3)/27.2/MA(4,7)/10.7/
DATA PB(1,3)/-1.5/PB(3,2)/-1.2/
DATA PA(1,1,3)/-.75/FA(1,2,2)/-.10/PA(1,2,3)/-.75/
DATA PA(2,2,2)/-.22/PA(2,2,3)/.20/
DATA PA(4,4,2)/3.3/
DATA G(1,3)/1./G(2,1)/-1./G(3,1)/1./G(4,3)/1./
  CONSTANT TM=.020,DB=.005,TR=2.0
  CONSTANT TFINAL=60.,CLKTIM=420.
  CONSTANT L1=-23,L2=.21,L3=.31
  CONSTANT SLRAT=.01745329
  CONSTANT PHIZ=0.,THET2=0.,PSIZ=0.
  CONSTANT W1I=0.,W2I=0.,W3I=0.
  CONSTANT K1=3500.,K2=3500.,K3=2000.,K4=2000.
  CONSTANT B1=20.,B2=20.,B3=10.,B4=10.
  CONSTANT IC1=218.,IC2=-30.
INITIAL
  NH=4
  RD=57.2957795 $ DR=1./RD $ GM1CI=IC1/RD $ GM2CI=IC2/RD
  GM1I=IC1/RD $ GM2I=IC2/RD
  CALL MBDY(NH,H,MB,MA,PB,PA,G,PI)
  PZI,PAI,PBI,PCI=HCK(INITL,PHIZ,THET2,PSIZ)
END
DYNAMIC
  IF(TIME.GE.TFINAL) GO TO S1
DERIVATIVE BODY5
  VARIABLE TIME=0.
  CINTERVAL CI=.06
  XERROR W1=1.E-6,W2=1.E-6,W3=1.E-6,GM3=1.E-4
  STPCLK CLKTIM
  OUTPUT 50,W1D,W2D,W3D,W1,W2,W3,THET1,THET2,THET3,...
  IN1,IN2,IN3,GM1,GM2,GM3,GM4,PL1,PL2
  PREPAR W1,W2,W3,THET1,THET2,THET3,GM1D,GM2D,...
  GM1,GM2,GM3D,GM4D,GM3,GM4
NOSORT
  W0(1)=W1 $ W0(2)=W2 $ W0(3)=W3
  PL1=RD*GM1 $ PL2=RD*GM2
  GM(1)=GM1 $ GM(2)=GM2 $ GM(3)=GM3 $ GM(4)=GM4
  GMD(1)=GM1D $ GMD(2)=GM2D $ GMD(3)=GM3D $ GMD(4)=GM4D
COMMENT
COMMENT CELESTIAL-SENSOR-MEASURED ANGLES
COMMENT
  D1,D2=HCK(MATRIX,PZO,PAC,PBO,PCO)
  NX0,NY0,NZ0=HCK(ITOB,0.,0.,0.,1.,D1,D2)
  LX0,LY0,LZ0=HCK(ITOB,1.,0.,0.,D1,D2)
  THET1=ATAN2(NY0,NZ0) $ THET2=ATAN2(-NX0,NZ0)
  THET3=ATAN2(-LY0,LX0)

```

Fig. 9. Case 1 simulation program using MBDY, with prescribed platform motion

```

COMMENT
COMMENT   PSEUDC-RATE FEEDBACK SIGNAL AND SWITCHING AMP. DYNAMICS
COMMENT
      IN1=-THET1-IR*W1 $ IN2=-THET2-IR*W2 $ IN3=-THET3-IR*W3
      AMP1=SWAMP(TIME,1M,08,IN1)
      AMP2=SWAMP(TIME,1M,08,IN2)
      AMP3=SWAMP(TIME,1M,08,IN3)
COMMENT
COMMENT   PRESCRIBED PLATFORM ACCELERATIONS
COMMENT
      GMDD(1)=6.981316*(-RAMP(0.,TIME)+2.*RAMP(1.05,TIME)-...
      RAMP(1.1,TIME)+RAMP(150.,TIME)-2.*RAMP(150.05,TIME)+RAMP(150.10,TIME))
      GMDD(2)=-GMDD(1)
COMMENT
COMMENT   REFERENCE BODY EXTERNAL TORQUE
COMMENT   HINGE TORQUES
COMMENT
      TB(1)=L1*AMP1 $ TB(2)=L2*AMP2 $ TB(3)=L3*AMP3
      TH(3)=-K3*GM3-B3*GM3D
      TH(4)=-K4*GM4-B4*GM4D
COMMENT
COMMENT   SYSTEM ANGLLAR ACCELERATIONS
COMMENT
      CALL HRATE(NH,TH,TB,TA,FB,FA,GM,GMDD,W0,WDOT)
      W1D=WDOT(1) $ W2D=WDOT(2) $ W3D=WDOT(3)
COMMENT
COMMENT   SYSTEM ANGULAR RATES AND POSITIONS
COMMENT
      W1=INTEG(WDOT(1),W1I)
      W2=INTEG(WDOT(2),W2I)
      W3=INTEG(WDOT(3),W3I)
      GM1D=INTEG(WDOT(4),0.) $ GM2D=INTEG(WDOT(5),0.)
      GM3D=INTEG(WDOT(6),0.) $ GM4D=INTEG(WDOT(7),0.)
      GM1=INTEG(GM1D,GM1I) $ GM2=INTEG(GM2D,GM2I)
      GM3=INTEG(GM3D,0.) $ GM4=INTEG(GM4D,0.)
COMMENT
COMMENT   HCK PARAMETER RATES AND POSITIONS
COMMENT
      PZOD,PAOD,PBOD,PCOD=HCK(HCK,PZ0,PA0,PB0,PC0,W1,W2,W3)
      PZ0=INTEG(PZOD,PZI)
      PA0=INTEG(PAOD,PAI)
      PB0=INTEG(PBOD,PBI)
      PC0=INTEG(PCOD,PCI)
END
END
END
TERMINAL
SI.. CONTINUE
END
END

```

Fig. 9 (contd)

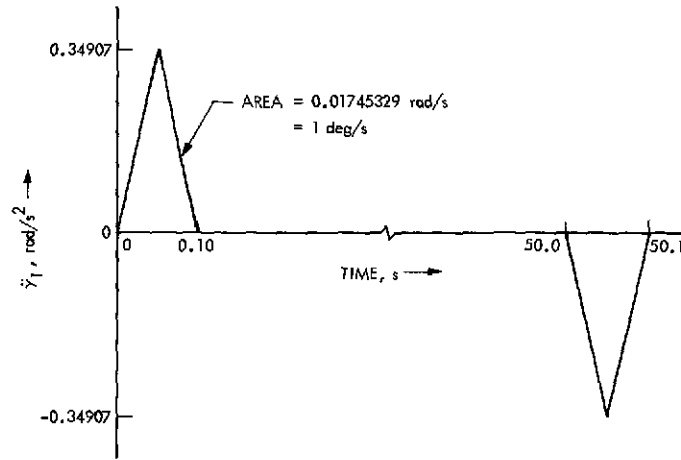


Fig. 10.  $\ddot{\gamma}_1$  prescribed function of time ( $\ddot{\gamma}_2 = -\ddot{\gamma}_1$ )

In Case 1, we have already seen that the reference body and boom responses are relatively small even though the platform angular excursions are substantial. If the platform maneuvers were also restricted to small values, this would seem the ideal case for which to apply MBDYTL and to obtain a check solution using MBDY. Results of the latter will be presented here.

In this case, it will be assumed that the platform again starts from the position  $\gamma_1 = 218$  deg and  $\gamma_2 = -30$  deg and that at  $t = 0$  the command is issued to rotate to  $\gamma_1 = 220$  deg at a commanded rate of 1 deg/s. Then, at  $t = 5$  s, the command is issued to rotate about the other axis to the position  $\gamma_2 = -25$  deg, at the same rate. A CSSL III program to simulate this maneuver sequence is listed in Fig. 13.

The simulation results are shown in Fig. 14. Obviously no gas jets were fired. These solutions will be compared in following sections to those obtained from linearized versions of MBDY.

### III. Totally Linearized Systems

#### A. The Equations

When any justification can be found for the linearization of any of the kinematical variables appearing in Eq. (5), this step should be taken, since the mathematical and computational benefits of linearization are quite substantial. The primary immediate benefit is the elimination of variable terms from the coefficient matrix on the left side of Eq. (5). If  $\omega^0$  and *all* angles  $\gamma_1, \dots, \gamma_n$  and their time derivatives can be assumed to remain *arbitrarily* small, then the indicated coefficient matrix is formally constant, and much computer simulation time can be gained by computing its inverse only once, in advance of numerical integration. When the angles are expected to be small in an engineering sense, less rigorous arguments can be marshalled to gain the same computational advantage. However, more than sufficient reason for employing linearization can very often be found by experimental demonstration.

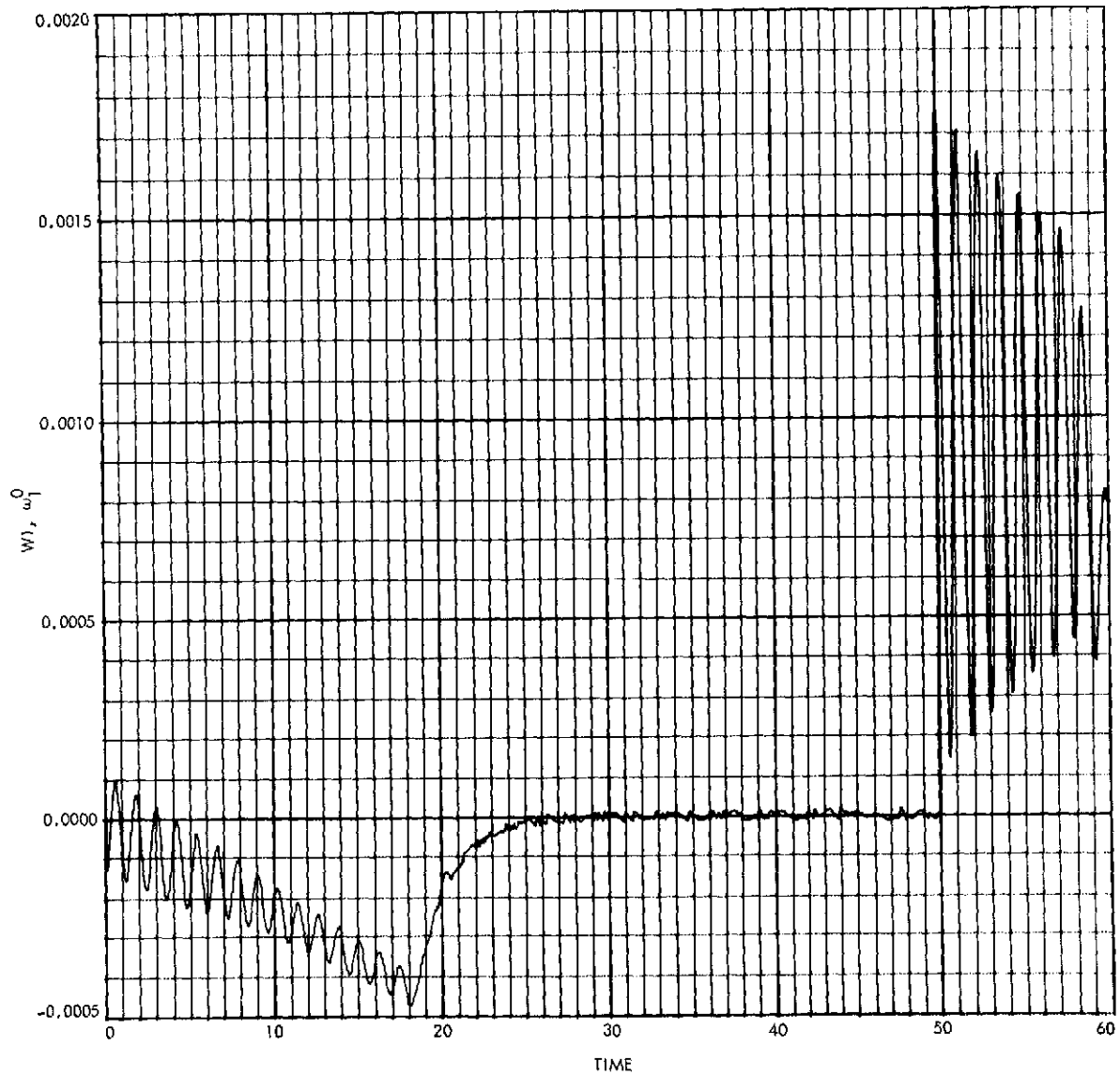


Fig. 11. Results of Case 1 simulation with prescribed platform motion

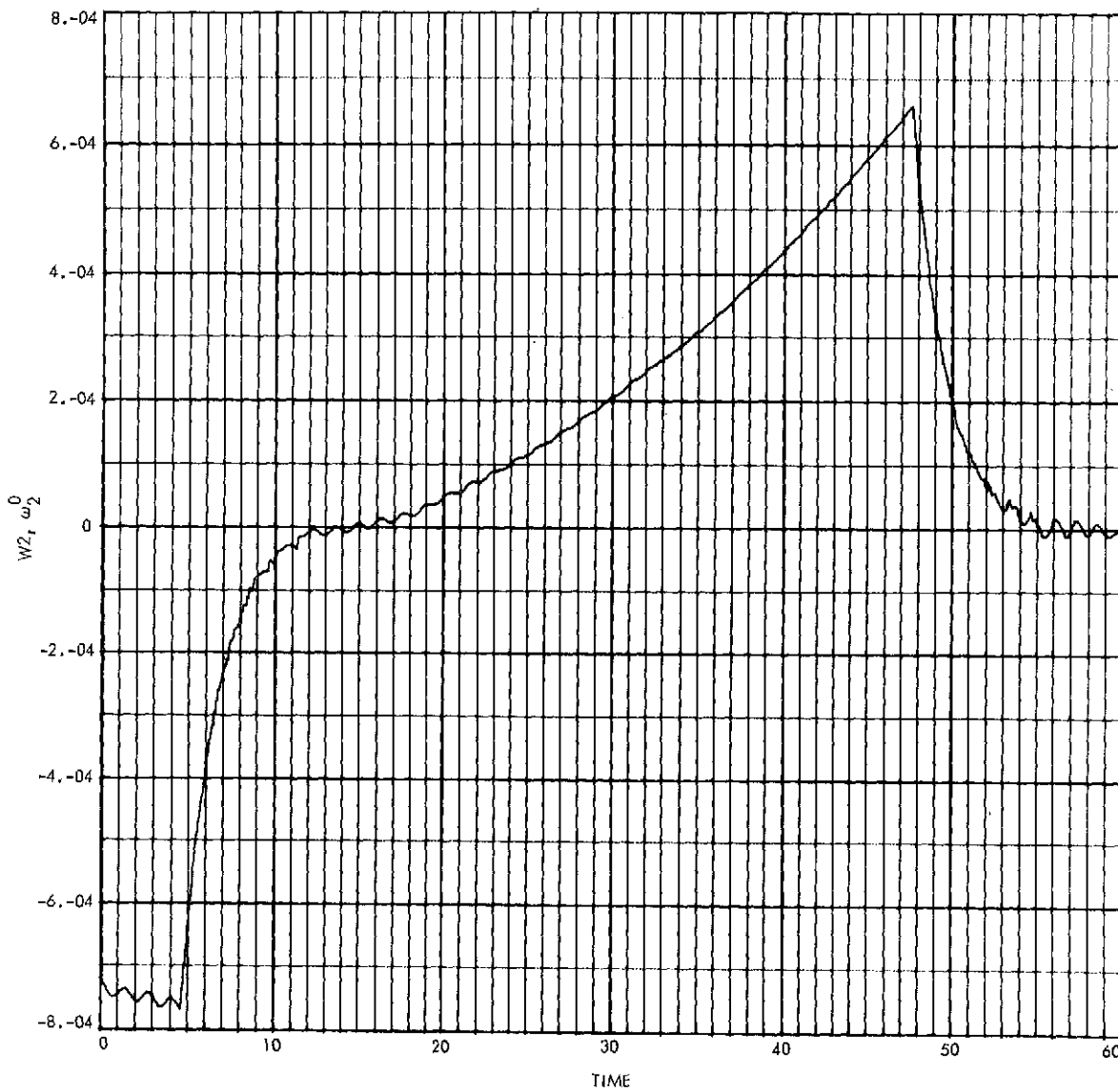


Fig. 11 (contd)



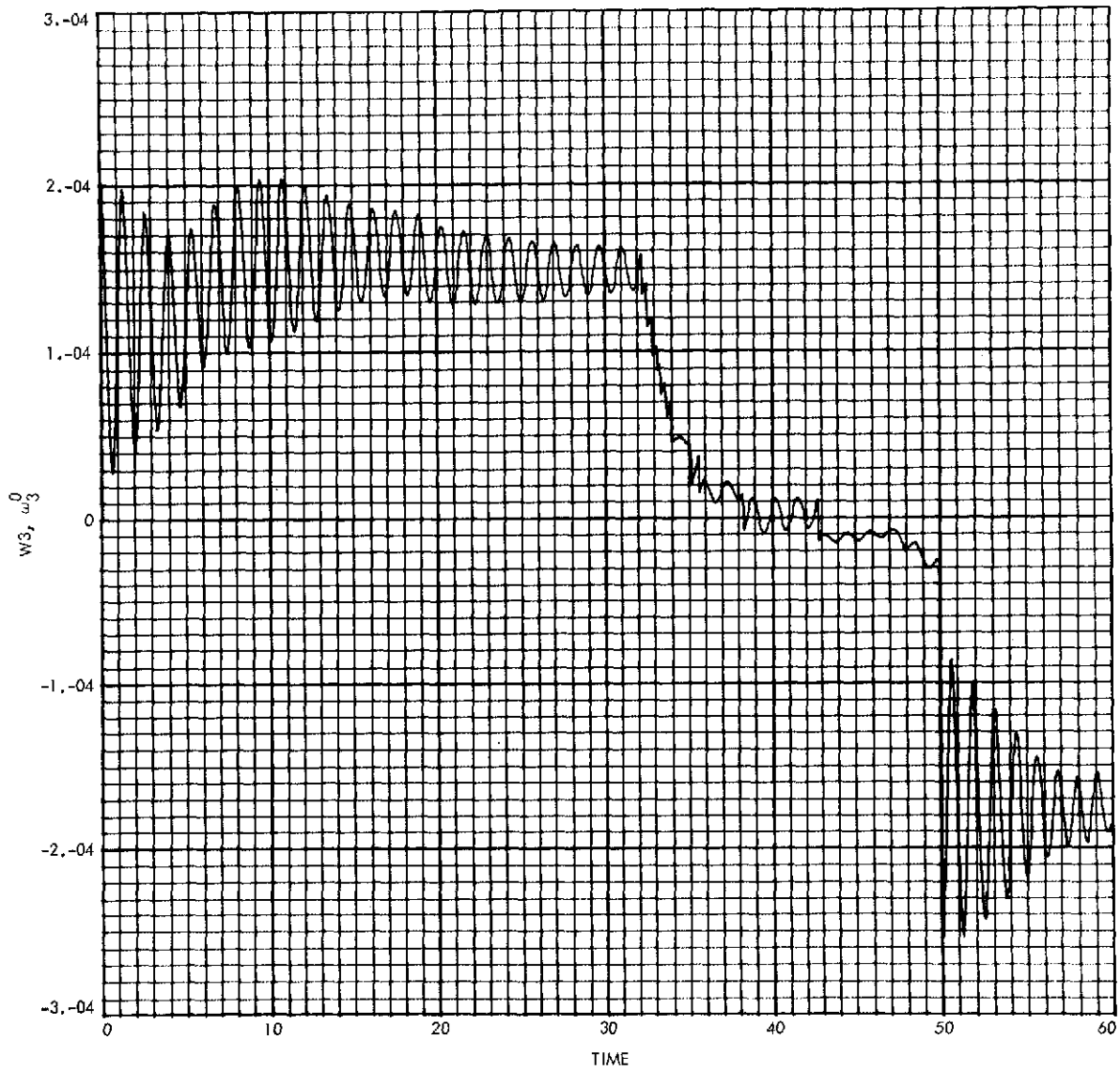


Fig. 11 (contd)

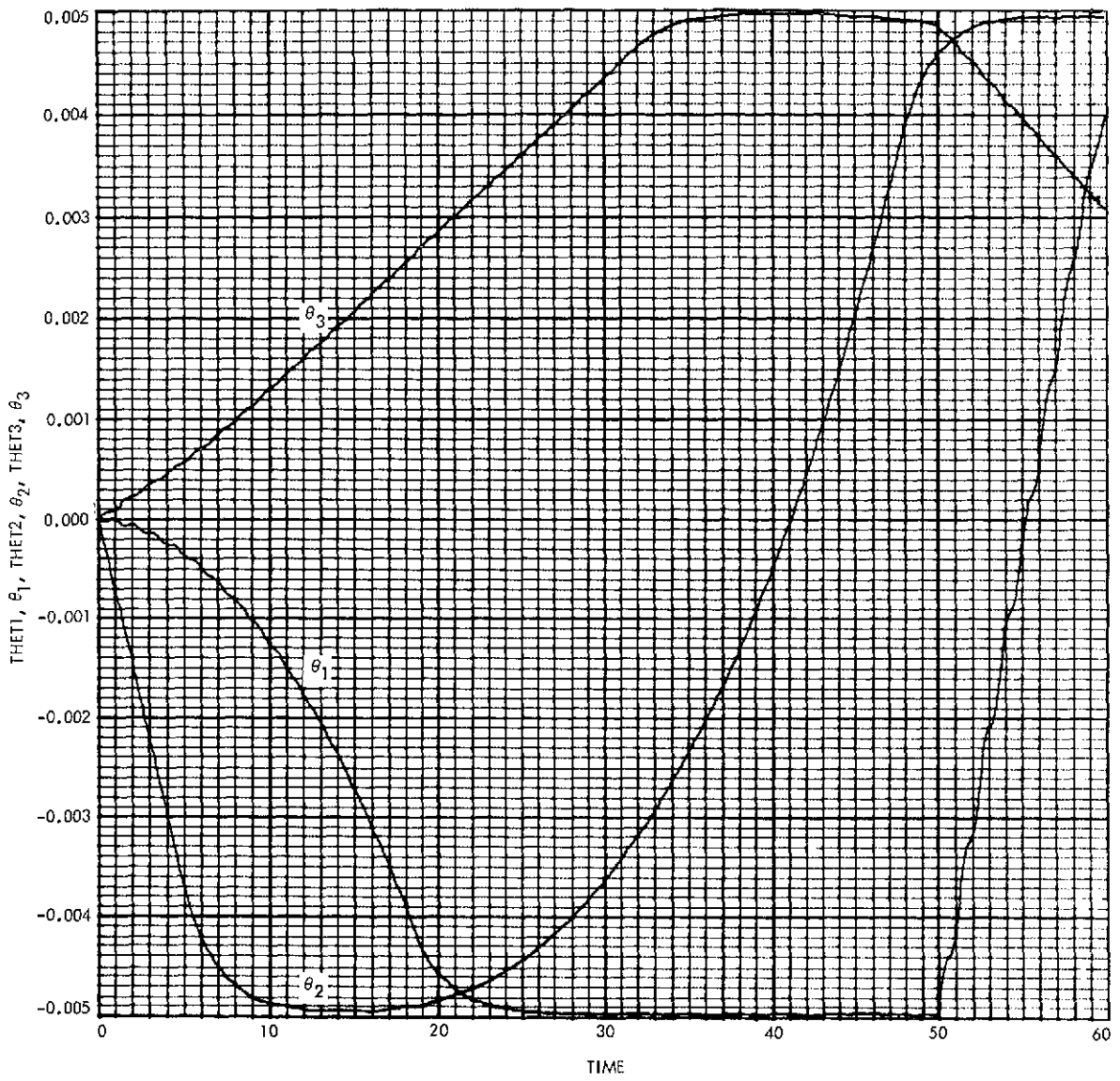


Fig. 11 (contd)

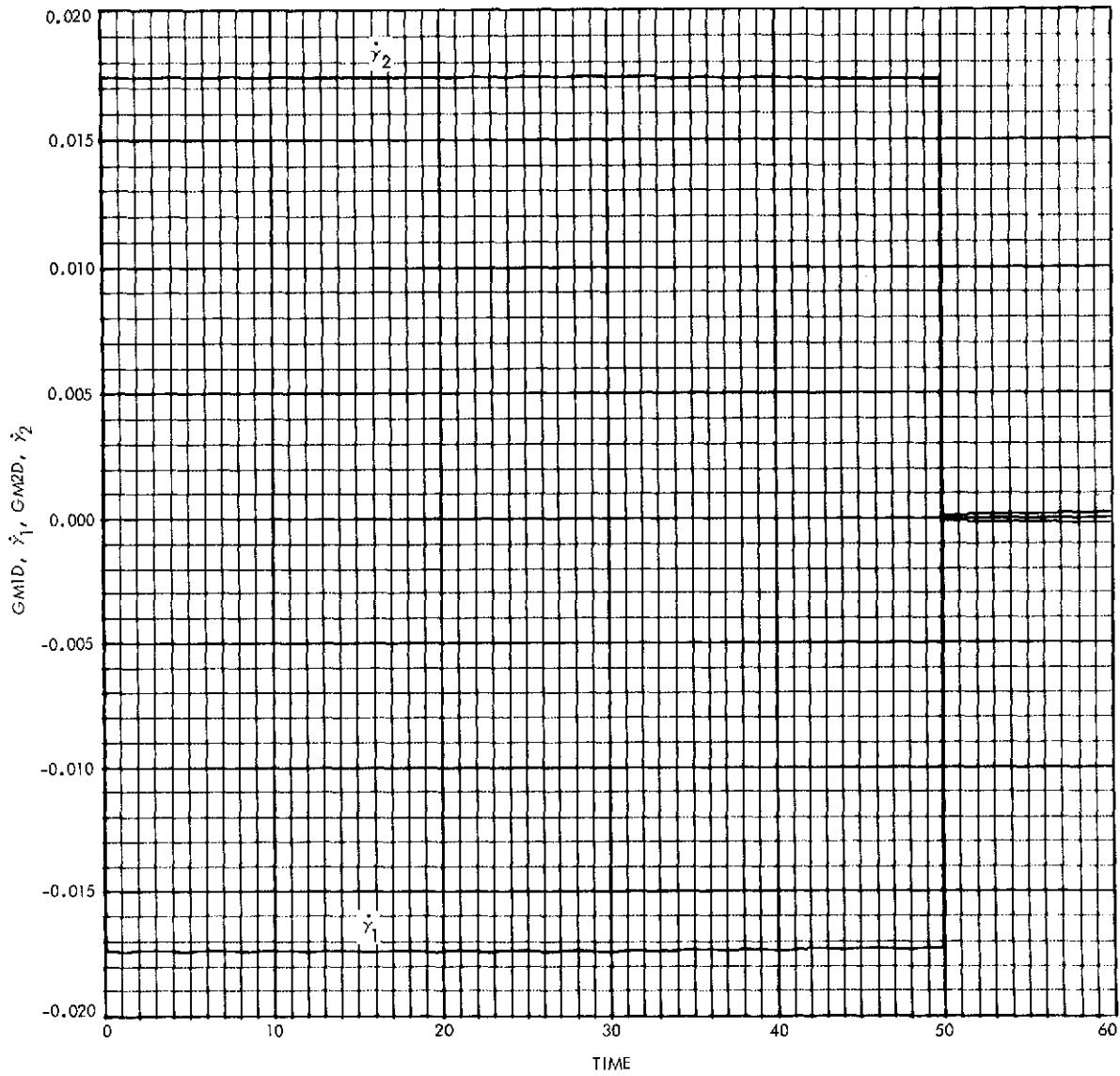


Fig. 11 (contd)

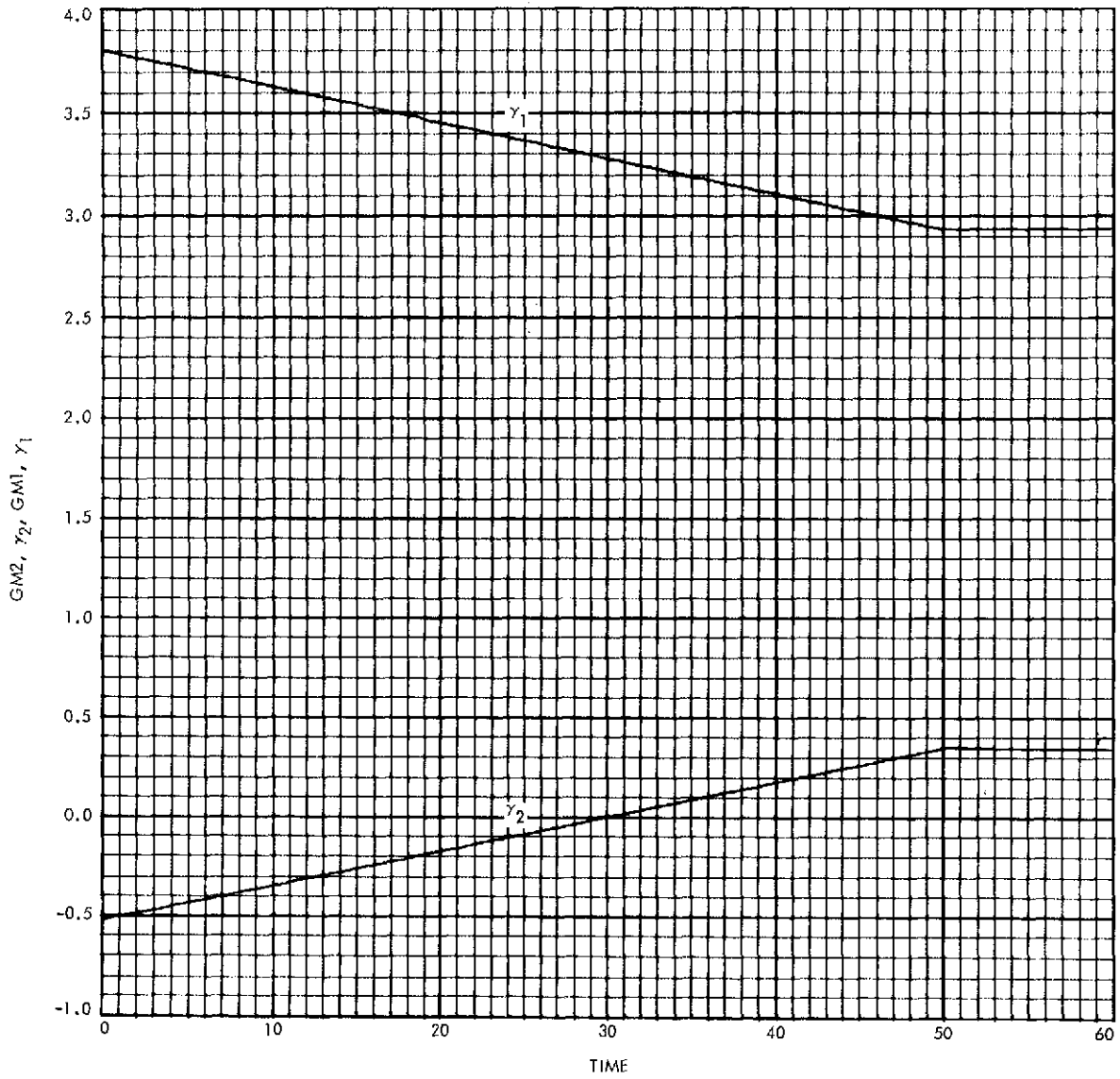


Fig. 11 (contd)

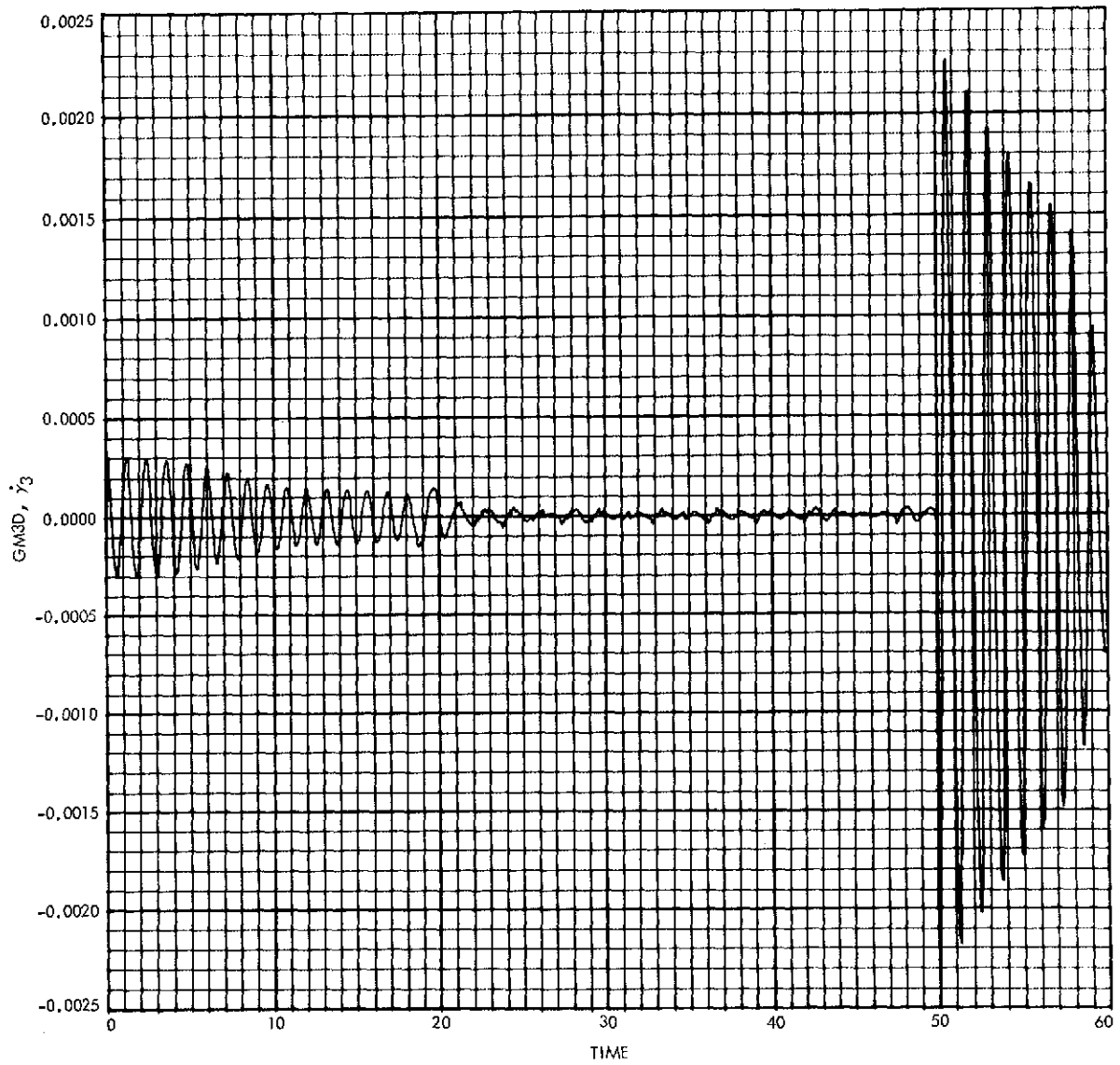


Fig. 11 (contd)

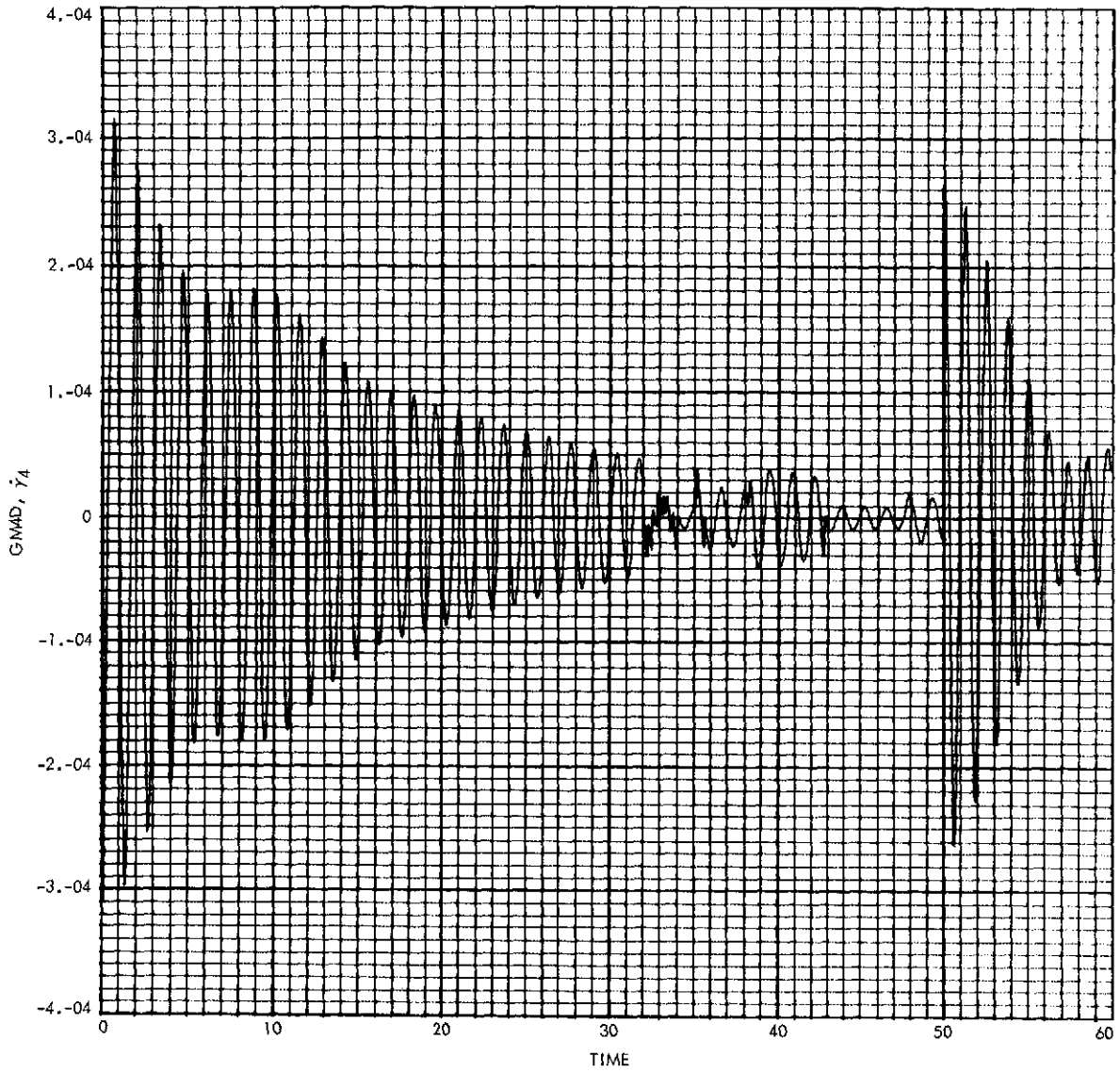


Fig. 11 (contd)

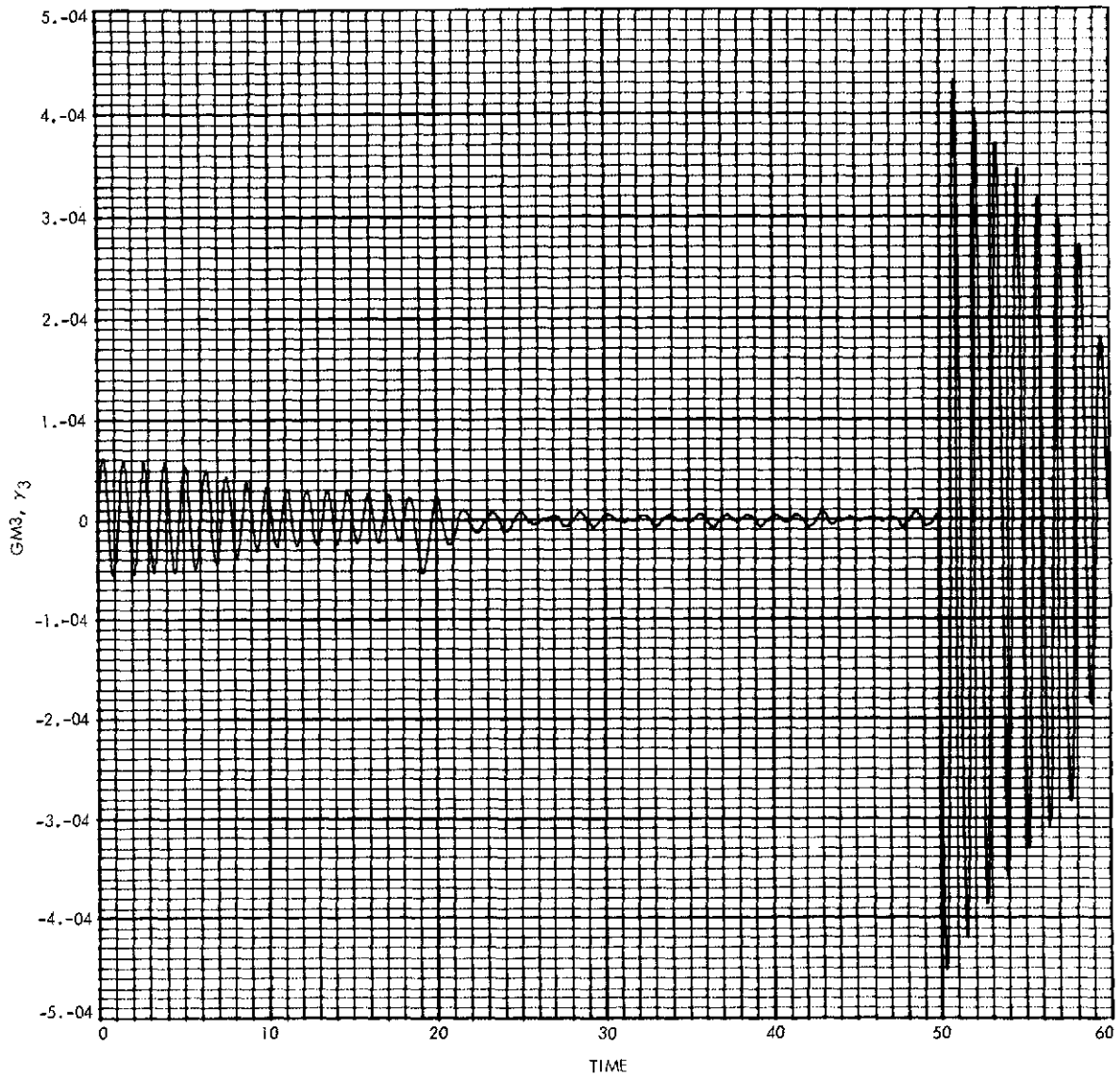


Fig. 11 (contd)

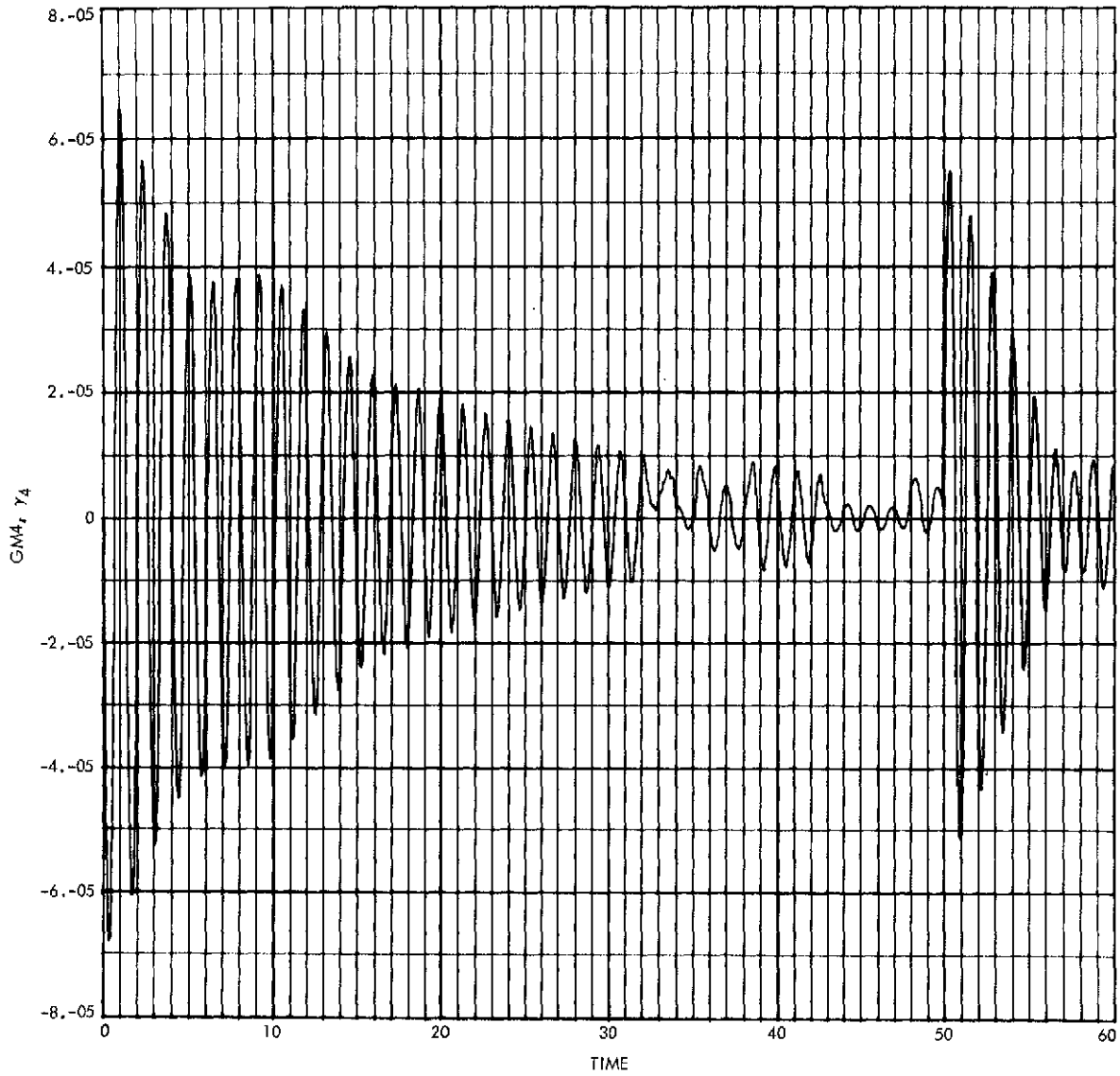


Fig. 11 (contd)



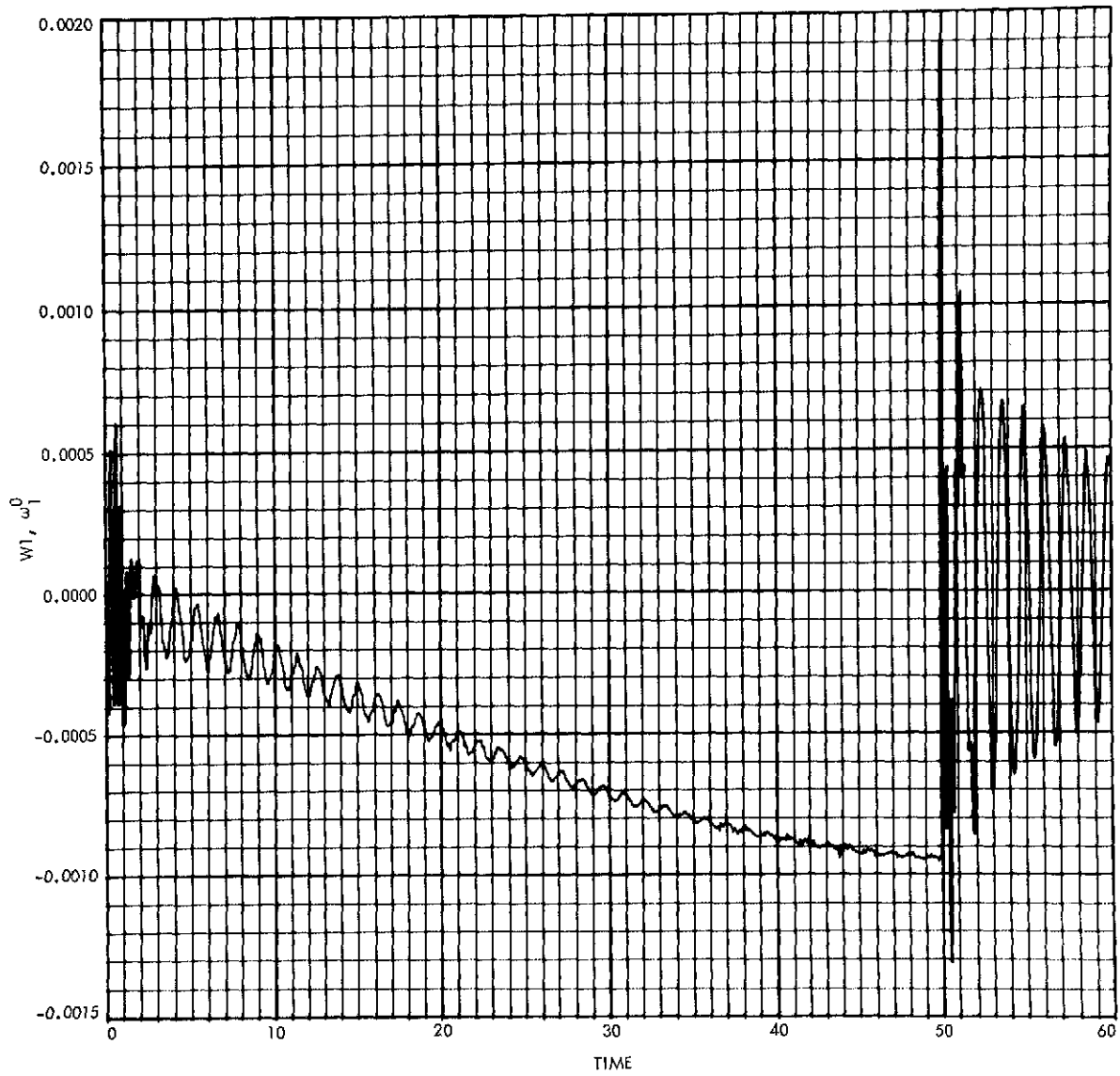


Fig. 12. Results of Case 1 simulation without control torques

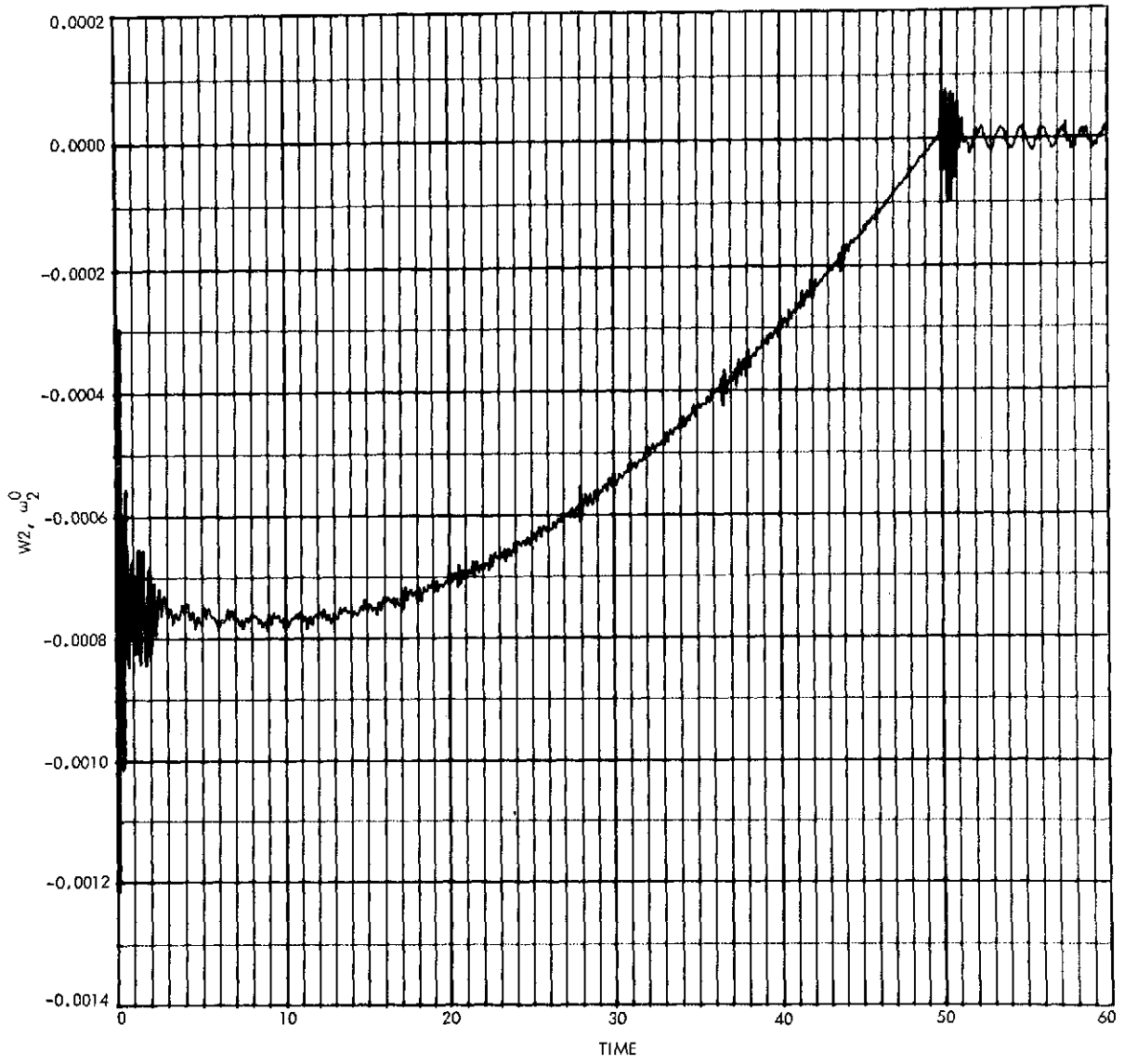


Fig. 12 (contd)

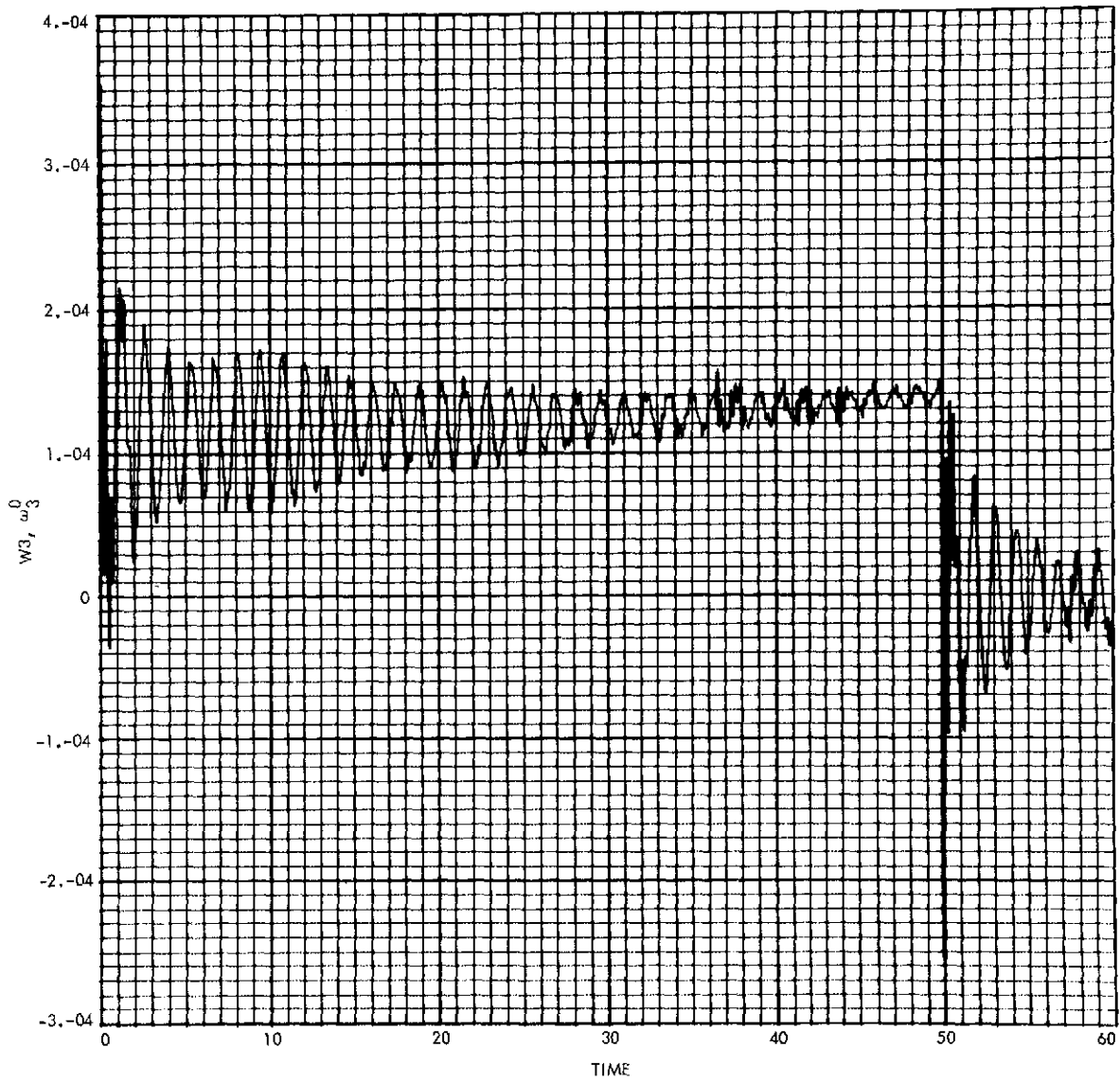


Fig. 12 (contd)

```

PROGRAM 5-BODY SYSTEM SIMULATION USING MBDY --- CASE 2
  INTEGER NH,PI(4),H(4)
  ARRAY MB(7),MA(4,7),PB(4,3),PA(4,4,3),G(4,3),TB(3),TA(4,3)
  ARRAY FB(3),FA(4,3),TH(4),GM(4),GMD(4),GMD(4),W(3)
  DOUBLE PRECISION WDC(7)
  DATA H/D=1.0,3/PI/D=0.0,0.0/
  DATA MB/15.,316.,44C.,14.,-14.,34.6,410./
  DATA MA(1,1)/.35/MA(1,2)/.35/MA(1,7)/6.8/
  DATA MA(2,1)/4.85/MA(2,2)/2.2/MA(2,3)/5.5/MA(2,4)/-.41/MA(2,5)/.07/
  DATA MA(2,6)/-.54/MA(2,7)/57.5/
  DATA MA(4,1)/27.2/MA(4,2)/.2/MA(4,3)/27.2/MA(4,7)/10.7/
  DATA PB(1,3)/-1.5/PB(3,2)/-1.2/
  DATA PA(1,1,3)/-.75/FA(1,2,2)/-.10/PA(1,2,3)/-.75/
  DATA PA(2,2,2)/-.22/PA(2,2,3)/.20/
  DATA PA(4,4,2)/3.3/
  DATA G(1,3)/1./G(2,1)/-1./G(3,1)/1./G(4,3)/1./
  CONSTANT TM=.020,DB=.005,TR=2.0
  CONSTANT TFINAL=20.,CLKTIM=1100.
  CONSTANT L1=.23,L2=.21,L3=.33
  CONSTANT SLRAT=.01745329
  CONSTANT PHIZ=0.,THETZ=0.,PSIZ=0.
  CONSTANT W1=0.,W2=0.,W3=0.
  CONSTANT K1=3500.,K2=3500.,K3=2000.,K4=2000.
  CONSTANT B1=20.,B2=20.,B3=10.,B4=10.
  CONSTANT IC1=218.,IC2=-30.
INITIAL
  NH=4
  RD=57.2957795 $ DR=1./RD $ GM1CI=IC1/RD $ GM2CI=IC2/RD
  GM1I=IC1/RD $ GM2I=IC2/RD
  CALL MBDY(NH,H,MB,MA,PB,PA,G,PI)
  PZI,PAI,PBI,PCI=HCK(INITL,PHIZ,THETZ,PSIZ)
END
DYNAMIC
  IF (TIME.GE.TFINAL) GO TO S1
DERIVATIVE BODY5
  VARIABLE TIME=0.
  CINTERVAL CI=.02
  XERROR W1=1.E-6,W2=1.E-6,W3=1.E-6,GM1=1.E-4
  STPCLK CLKTIM
  OUTPUT 50,W1D,W2D,W3D,W1,W2,W3,THET1,THET2,THET3,...
  IN1,IN2,IN3,GM1,GM2,GM3,GM4,PL1,PL2
  PREPAR W1,W2,W3,THET1,THET2,THET3,GM1D,GM2D,...
  GM1,GM2,GM3D,GM4D,GM3,GM4
NOSORT
  W(1)=W1 $ W(2)=W2 $ W(3)=W3
  PL1=RD*GM1 $ PL2=RD*GM2
  GM(1)=GM1 $ GM(2)=GM2 $ GM(3)=GM3 $ GM(4)=GM4
  GMD(1)=GM1D $ GMD(2)=GM2D $ GMD(3)=GM3D $ GMD(4)=GM4D
COMMENT
COMMENT CELESTIAL-SENSOR-MEASURED ANGLES
COMMENT
  D1,D2=HCK(MATRIX,PZC,PAC,PBC,PCO)
  NXO,NYO,NZO=HCK(ITOB,.0.,0.,1.,D1,D2)
  LXO,LYO,LZO=HCK(ITOB,1.,0.,0.,D1,D2)
  THET1=ATAN2(NYO,NZO) $ THET2=ATAN2(-NXO,NZO)
  THET3=ATAN2(-LYO,LXC)

```

Fig. 13. Case 2 simulation program using MBDY

```

COMMENT
COMMENT RATE FEEDBACK SIGNAL AND SWITCHING AMP. DYNAMICS
COMMENT
IN1=-THET1-TR*W1 $ IN2=-THET2-TR*W2 $ IN3=-THET3-TR*W3
AMP1=SWAMP(TIME,TH,OB,IN1)
AMP2=SWAMP(TIME,TH,OB,IN2)
AMP3=SWAMP(TIME,TH,OB,IN3)
COMMENT
COMMENT PLATFORM POSITION COMMANDS
COMMENT
GM1CD=SLRAT*(1.-STEP(2.,TIME)) $ GM1C=INTEG(GM1CD,GM1C)
GM2CD=SLRAT*(STEP(5.,TIME)-STEP(10.,TIME)) $ GM2C=INTEG(GM2CD,GM2C)
COMMENT
COMMENT REFERENCE ECDY EXTERNAL TORQUE
COMMENT HINGE TORQUES
COMMENT
TB(1)=L1*AMP1 $ TB(2)=L2*AMP2 $ TB(3)=L3*AMP3
TH(1)=-K1*(GM1-GM1C)-B1*GM1D
TH(2)=-K2*(GM2-GM2C)-B2*GM2D
TH(3)=-K3*GM3-E3*GM3D
TH(4)=-K4*GM4-B4*GM4D
COMMENT
COMMENT SYSTEM ANGULAR ACCELERATIONS
COMMENT
CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,W0,WDOT)
W1D=WDOT(1) $ W2D=WDOT(2) $ W3D=WDOT(3)
COMMENT
COMMENT SYSTEM ANGULAR RATES AND POSITIONS
COMMENT
W1=INTEG(WDOT(1),W1)
W2=INTEG(WDOT(2),W2)
W3=INTEG(WDOT(3),W3)
GM1D=INTEG(WDOT(4),0.) $ GM2D=INTEG(WDOT(5),0.)
GM3D=INTEG(WDOT(6),0.) $ GM4D=INTEG(WDOT(7),0.)
GM1=INTEG(GM1D,GM1) $ GM2=INTEG(GM2D,GM2)
GM3=INTEG(GM3D,0.) $ GM4=INTEG(GM4D,0.)
COMMENT
COMMENT HCK PARAMETER RATES AND POSITIONS
COMMENT
PZCD,PAOD,PBOD,PCCD=HCK(HCK,PZ0,PA0,PB0,PC0,W1,W2,W3)
PZ0=INTEG(PZOD,PZ)
PA0=INTEG(PAOD,PA)
PB0=INTEG(PBOD,PB)
PC0=INTEG(PCOD,PC)
END
END
END
TERMINAL
S1.. CONTINUE
END
END

```

Fig. 13 (contd)

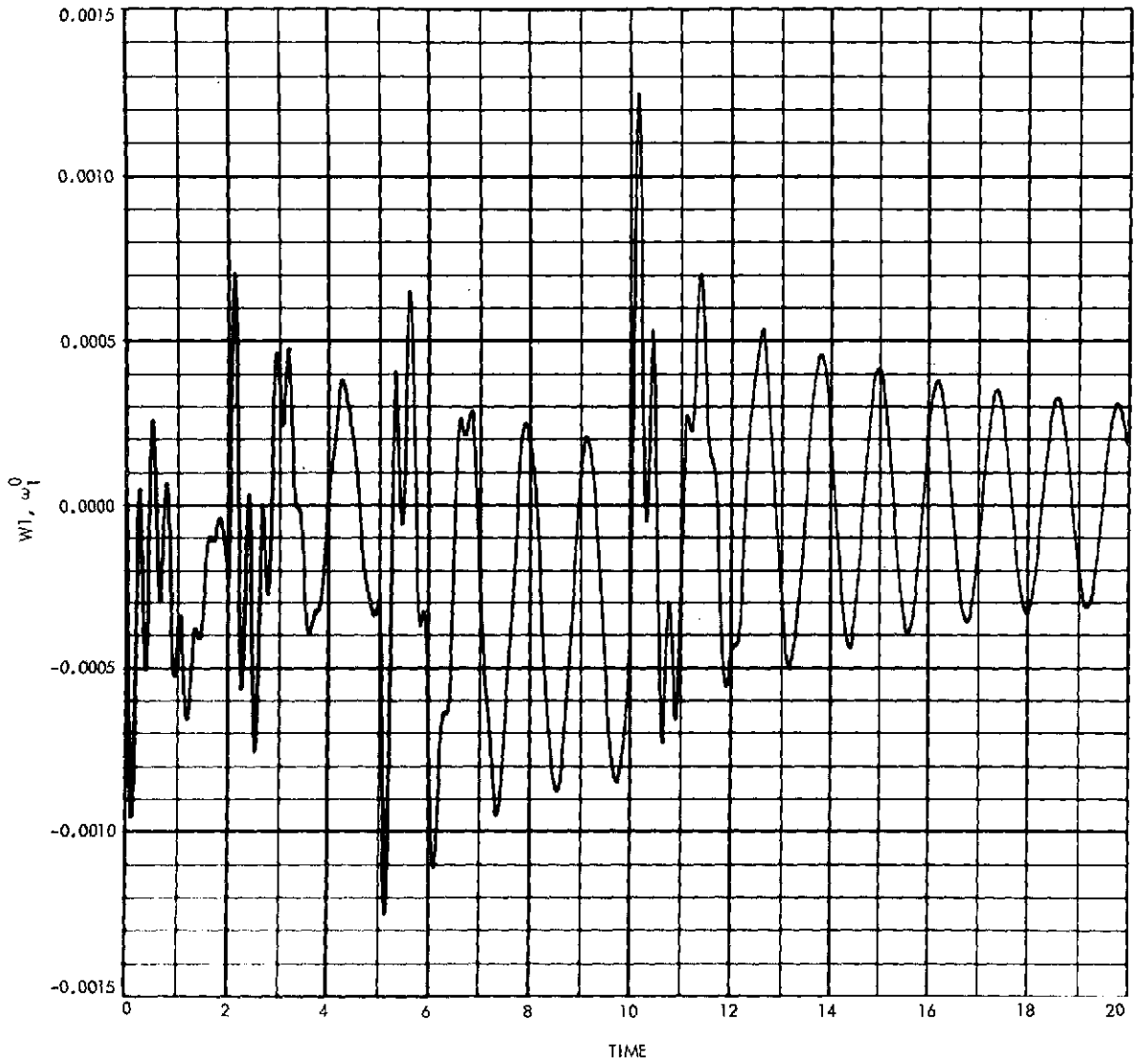


Fig. 14. Results of Case 2 simulation

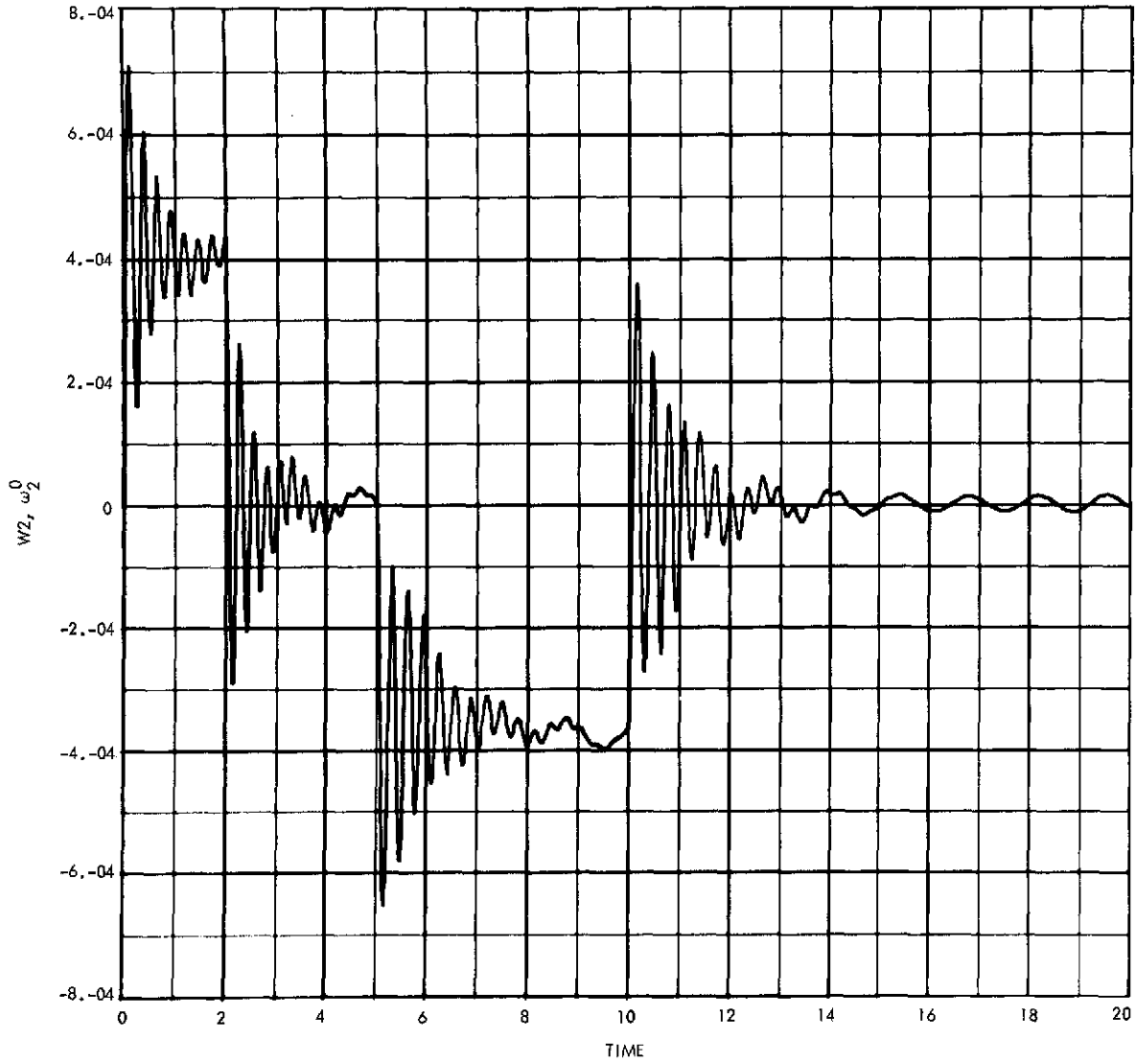


Fig. 14 (contd)

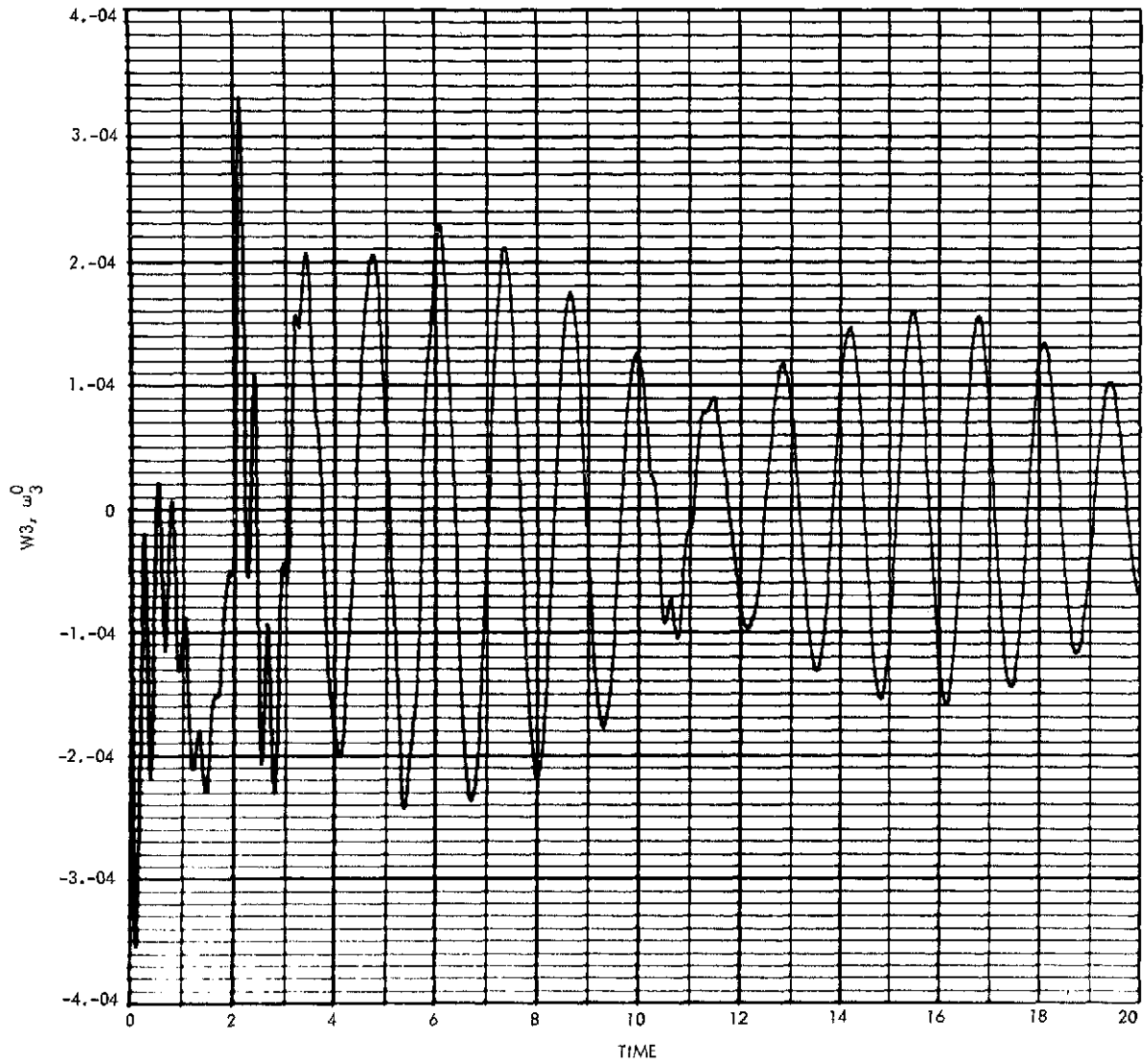


Fig. 14 (contd)



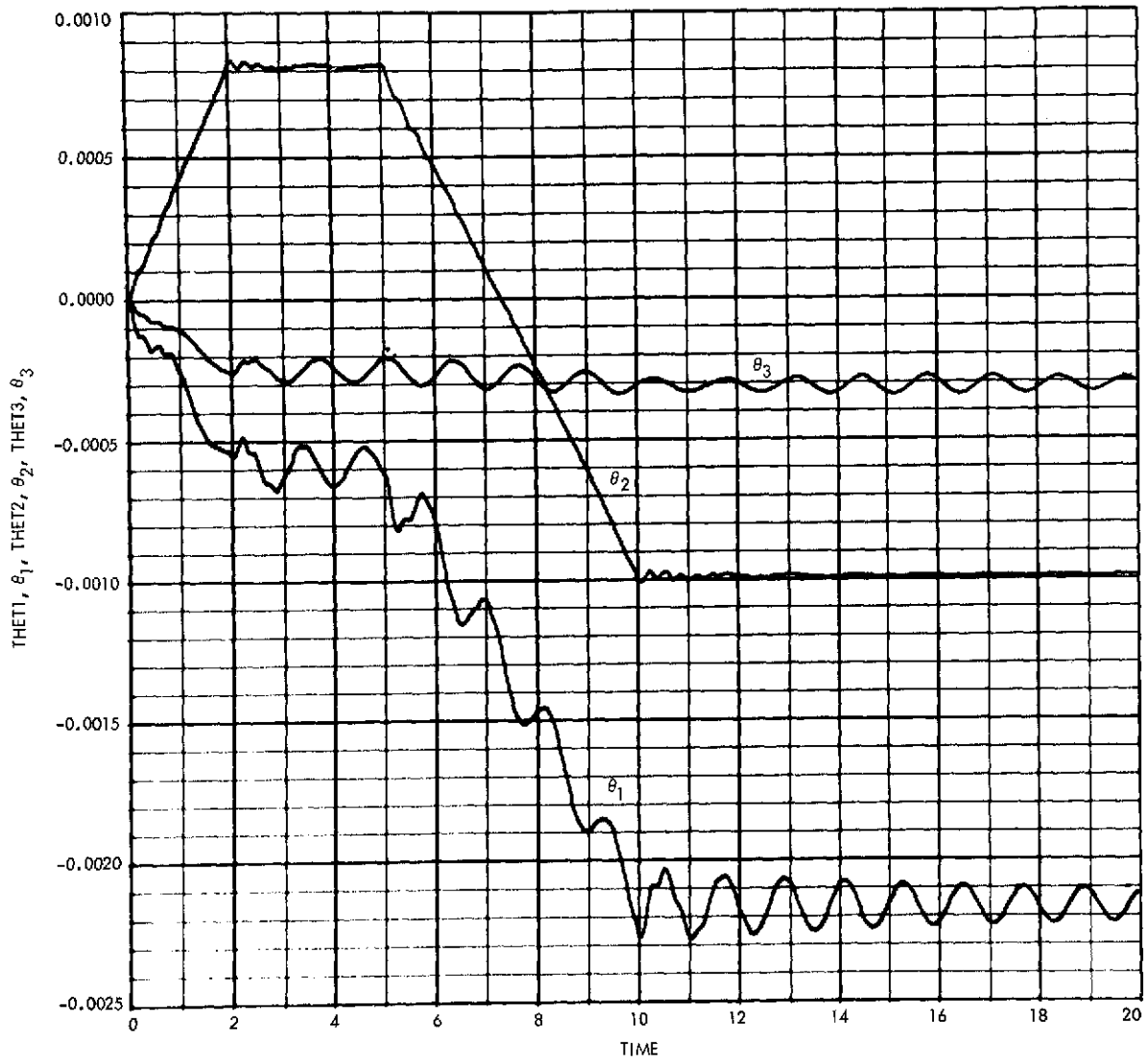


Fig. 14 (contd)

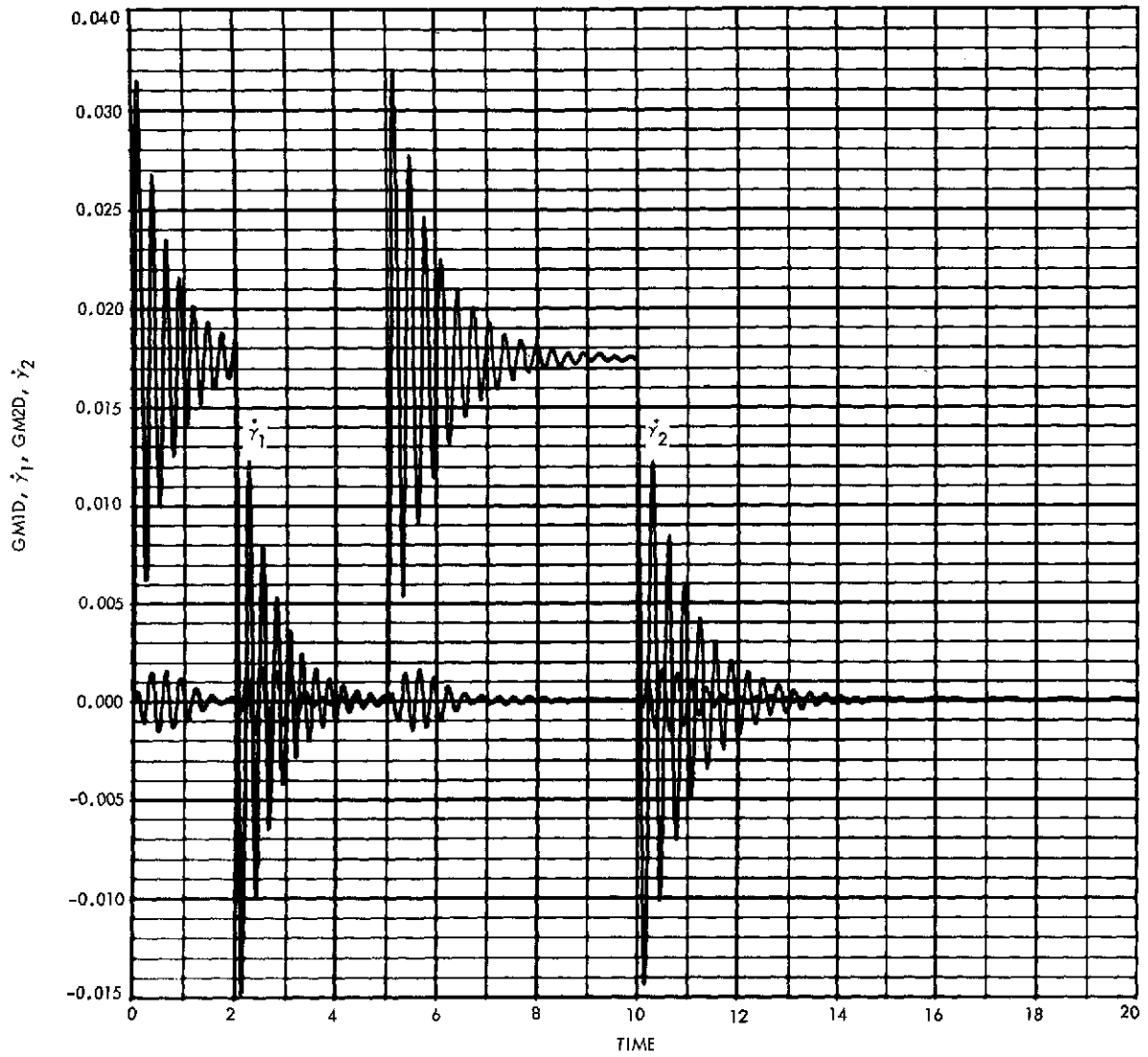


Fig. 14 (contd)

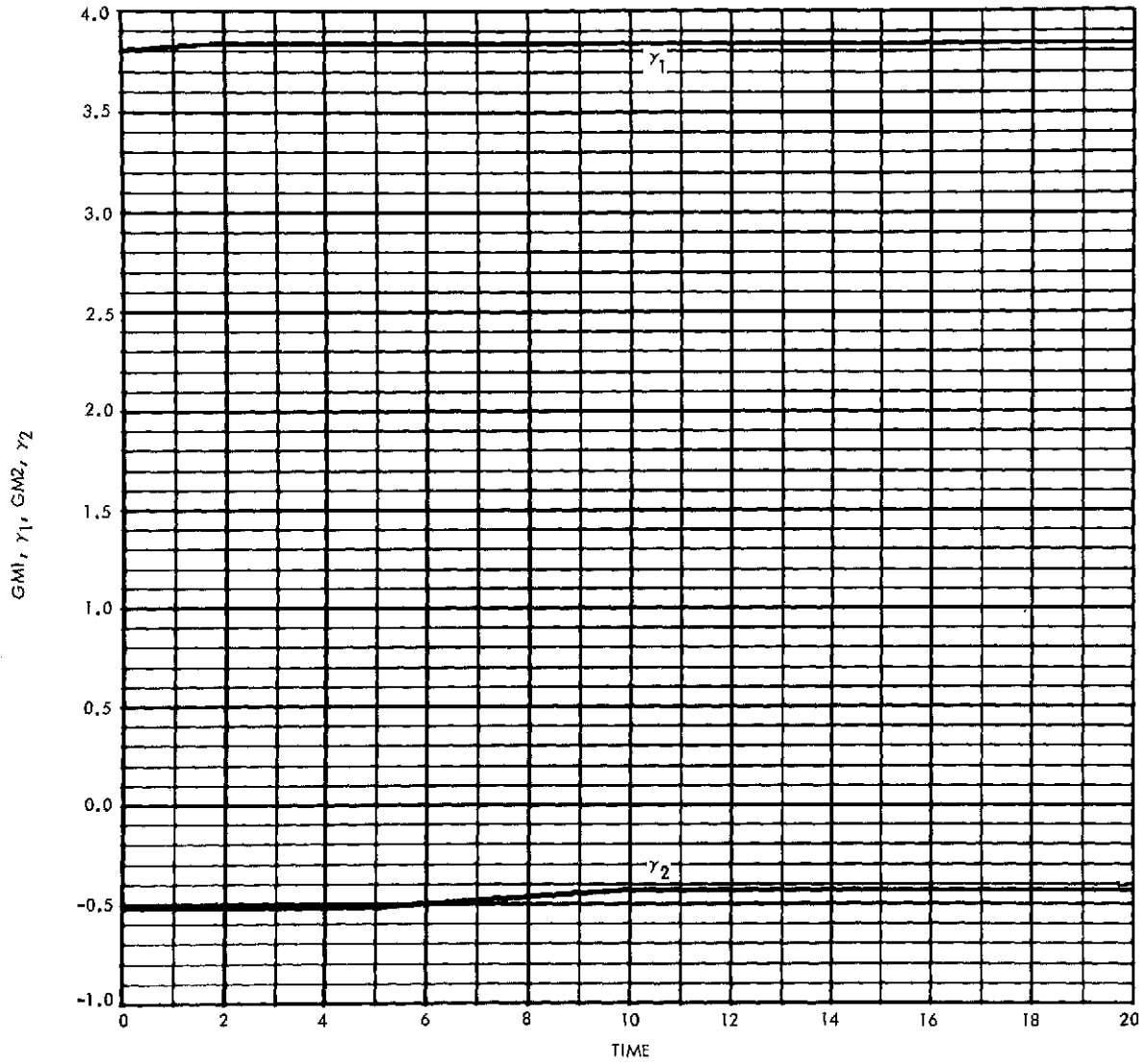


Fig. 14 (contd)

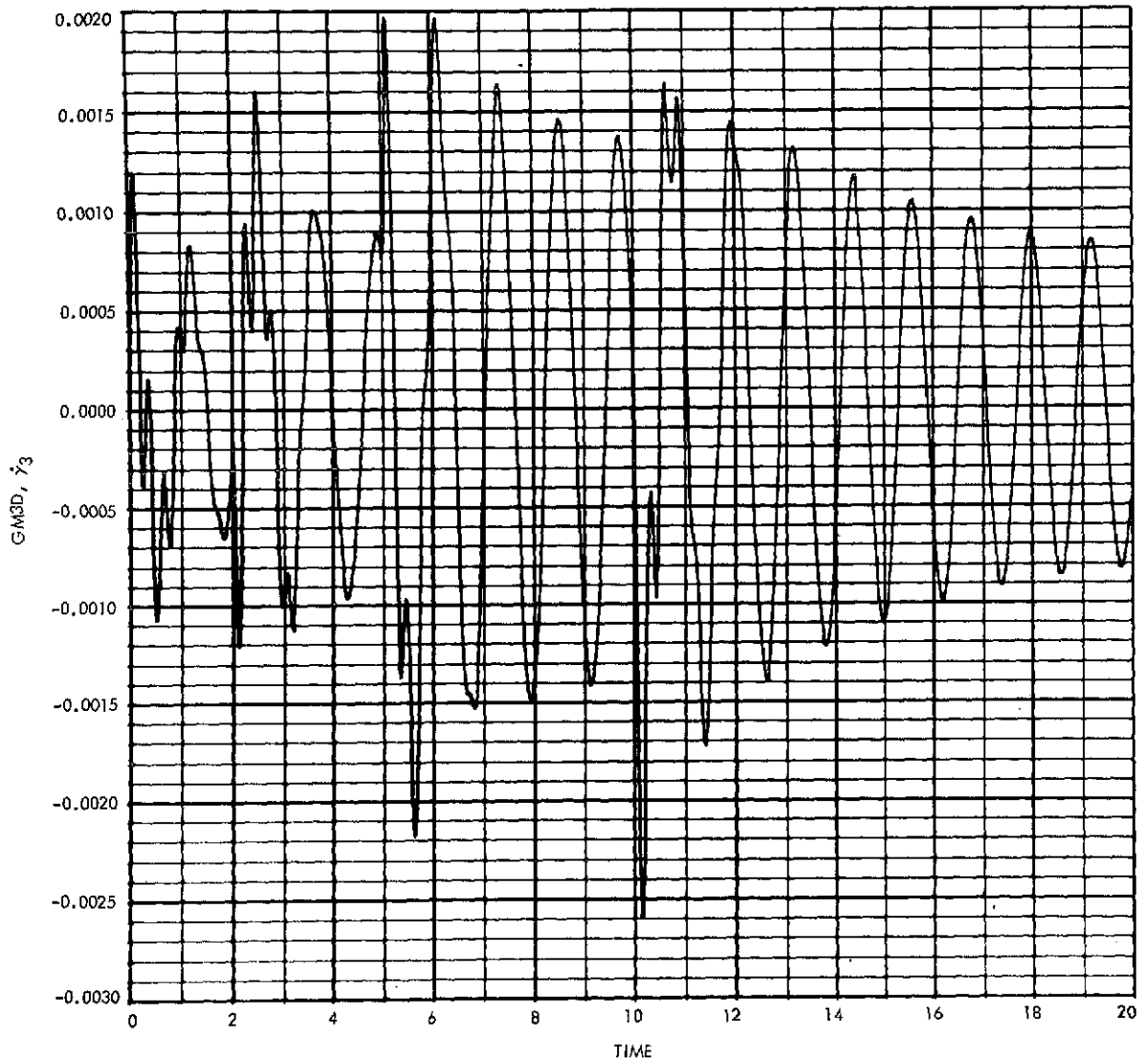


Fig. 14 (contd)

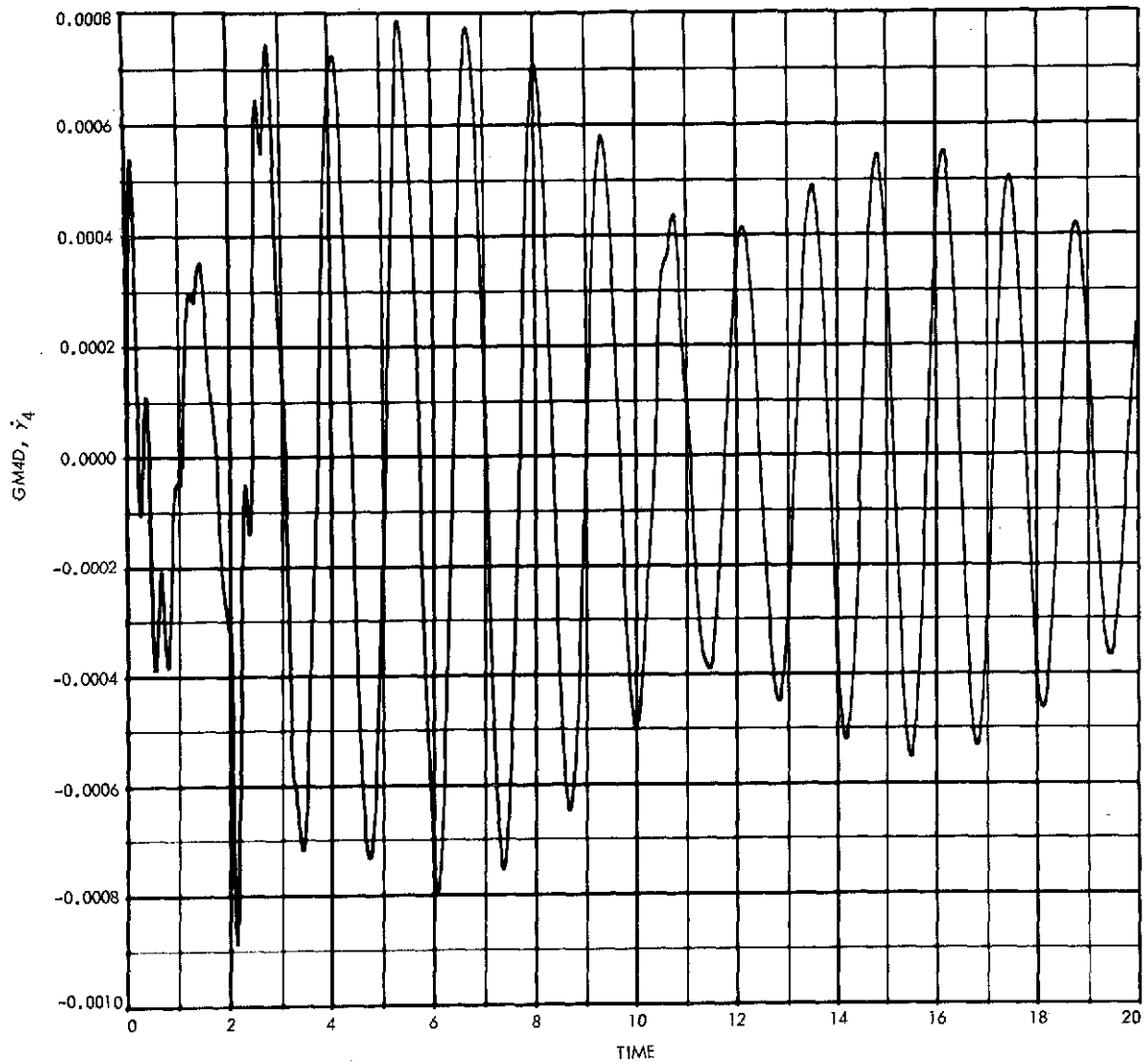


Fig. 14 (contd)

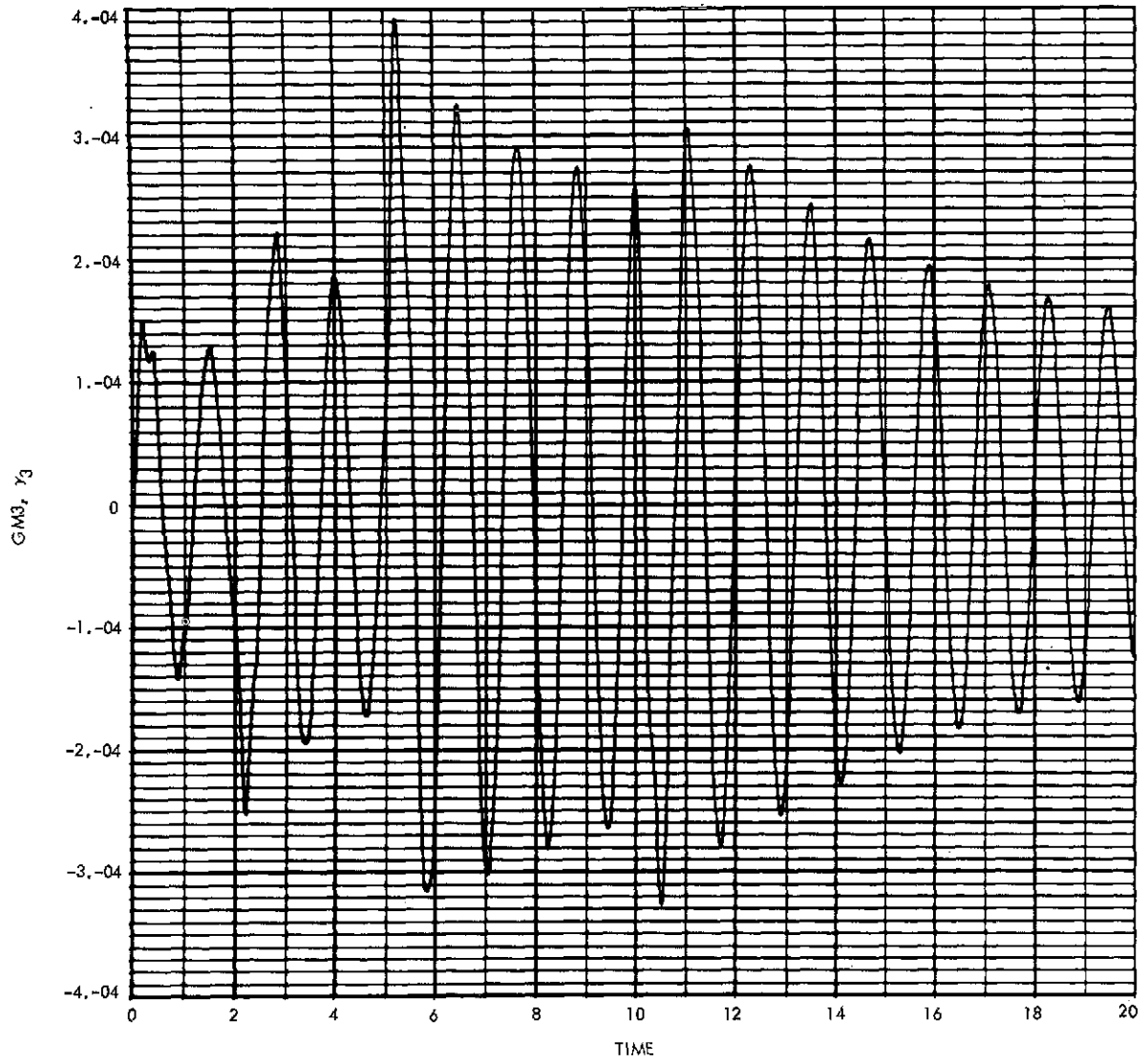


Fig. 14 (contd)

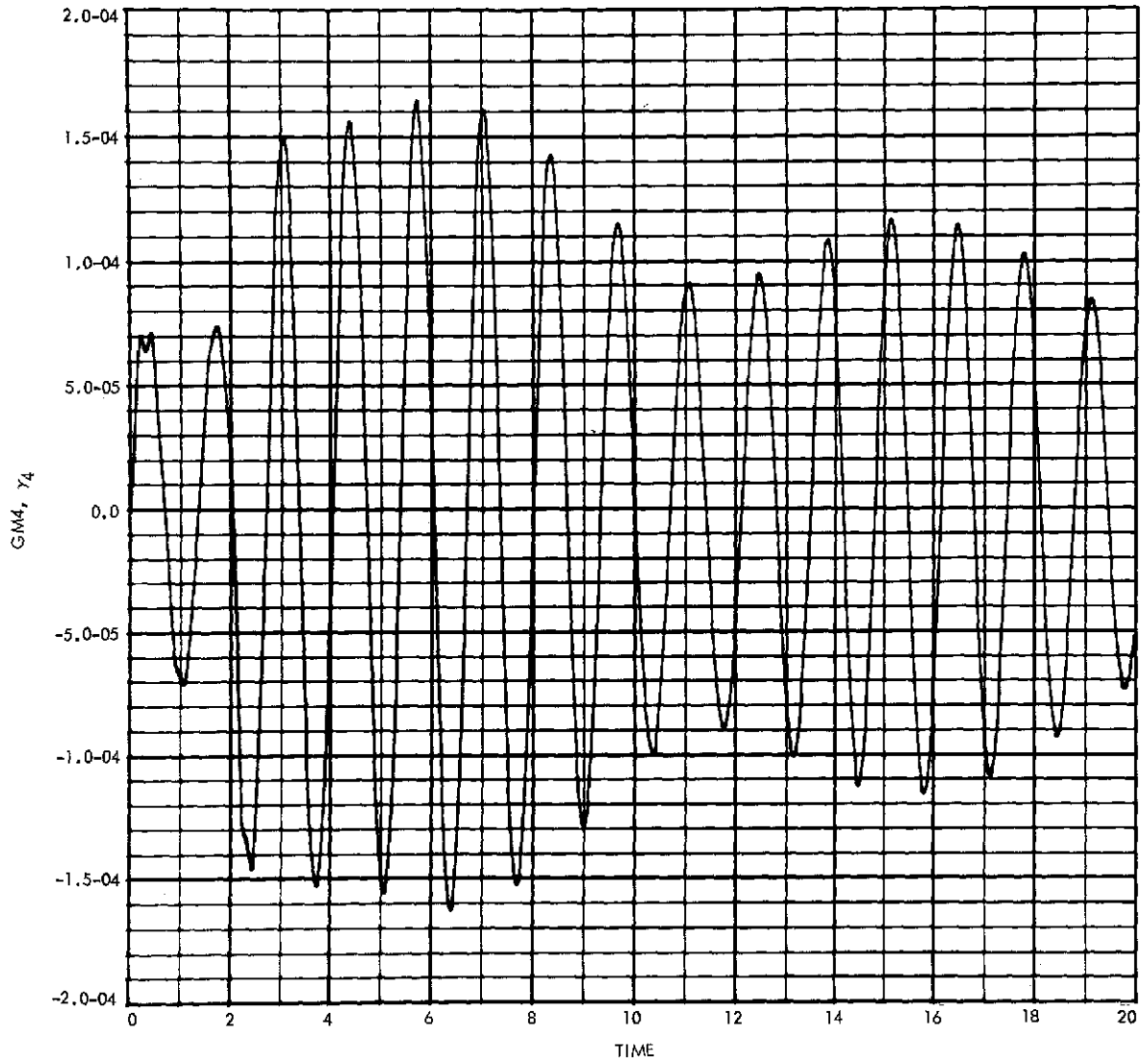


Fig. 14 (contd)

In any case, Eqs. (5) can be rewritten in the form

$$a_{00}\dot{\omega}^0 + \sum_{k \in \mathcal{P}} a_{0k}\ddot{\gamma}_k = \sum_{k \in \mathcal{B}} E_k \quad (8a)$$

and

$$a_{i0}\dot{\omega}^0 + \sum_{k \in \mathcal{P}} a_{ik}\ddot{\gamma}_k = g^{iT} C^{i0} \sum_{k \in \mathcal{P}} \varepsilon_{ik} E_k + \tau_i \quad (i \in \mathcal{P}) \quad (8b)$$

The next step is the linearization of Eq. (8) in the variables  $\omega^0, \gamma_k$  and their time derivatives. To this end, we expand each symbol in Eq. (8) which may involve such variables into three parts, the first being free of these variables (indicated by overbar), the second being linear in the variables (indicated by overcaret), and the third containing terms above the first degree in the variables (indicated by three dots). In particular, we write (for any  $i, k$ )

$$C^{i0} = \bar{C}^{i0} + \hat{C}^{i0} + \dots \quad (9a)$$

$$\tau_\alpha = \bar{\tau}_\alpha + \hat{\tau}_\alpha + \dots \quad (9b)$$

$$a_{ik} = \bar{a}_{ik} + \hat{a}_{ik} + \dots \quad (9c)$$

$$E_k = \bar{E}_k + \hat{E}_k + \dots \quad (9d)$$

and then determine explicit expressions for the new barred and caret symbols from the definitions of  $a_{ik}$  (Defs. 32, 34, 35),  $E_k$  (Def. 36), and the expansion for  $C^{i0}$  (Appendix A). In terms of the symbols introduced in Eq. (9), the linearized form of Eq. (8) becomes

$$\bar{a}_{00}\dot{\omega}^0 + \sum_{k \in \mathcal{P}} \bar{a}_{0k}\ddot{\gamma}_k = \sum_{k \in \mathcal{B}} (\bar{E}_k + \hat{E}_k) \quad (10a)$$

$$(i \in \mathcal{P}) \quad \bar{a}_{i0}\dot{\omega}^0 + \sum_{k \in \mathcal{P}} \bar{a}_{ik}\ddot{\gamma}_k = g^i \sum_{k \in \mathcal{P}} \varepsilon_{ik} [\bar{C}^{i0} (\bar{E}_k + \hat{E}_k) + \hat{C}^{i0} \bar{E}_k] + \bar{\tau}_i + \hat{\tau}_i \quad (10b)$$

In deriving explicit expressions for  $\bar{C}^{i0}$  and  $\hat{C}^{i0}$ , we may begin by recognizing that for any  $C^{rj}$ , where  $r$  and  $j$  are labels of *adjacent* bodies,

$$\bar{C}^{rj} = \bar{C}^{jr} = U = 3 \times 3 \text{ identity matrix}$$

and

$$\hat{C}^{rj} = -\gamma_r \tilde{g}_r, \quad (r > j)$$

$$\hat{C}^{jr} = \gamma_r \tilde{g}^r = (\hat{C}^{rj})^T$$

It is clear that  $\hat{C}^{i0}$  must then be given by

$$\hat{C}^{i0} = -\sum_{s \in \mathcal{P}} \varepsilon_{si} \tilde{g}^s \gamma_s, \quad \hat{C}^{0i} = (\hat{C}^{i0})^T = \sum_{s \in \mathcal{P}} \varepsilon_{si} \tilde{g}^s \gamma_s$$

Also,  $\bar{C}^{i0} = U$ . In general,

$$(r, j \in \mathcal{B}) \quad \hat{C}^{rj} = \sum_{s \in \mathcal{P}} (\varepsilon_{sj} - \varepsilon_{sr}) \tilde{g}^s \gamma_s \quad (11a)$$

and

$$\bar{C}^{rj} = U \quad (11b)$$



From Def. 36,  $\bar{E}_k$  and  $\hat{E}_k$  are given by

$$\bar{E}_k = \bar{C}^{0k} [\bar{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \bar{C}^{kj} \bar{F}^j] = \bar{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \bar{F}^j \quad (12a)$$

$$\begin{aligned} \hat{E}_k &= \bar{C}^{0k} [\hat{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \hat{C}^{kj} \bar{F}^j + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \bar{C}^{kj} \hat{F}^j] + \hat{C}^{0k} [\bar{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \bar{C}^{kj} \bar{F}^j] \\ &= \hat{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \hat{C}^{kj} \bar{F}^j + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \hat{F}^j + \hat{C}^{0k} [\bar{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \bar{F}^j] \end{aligned} \quad (12b)$$

Of course, the most important result of the complete linearization is the absence of any time-varying elements in  $\bar{a}_{00}$ ,  $\bar{a}_{0k}$ ,  $\bar{a}_{i0}$ , and  $\bar{a}_{ik}$ . These *constant* coefficients now allow the square matrix on the left side of (5) to be inverted once and for all, in advance of numerical integration.

It would remain then to determine  $\bar{T}^k$ ,  $\hat{T}^k$ ,  $\bar{F}^k$ ,  $\hat{F}^k$ ,  $\bar{\tau}_i$ , and  $\hat{\tau}_i$  for the specific system under study and, together with the relations in (11) and (12), to carry out the computations implied by (10). However, in constructing a subroutine to perform these computations, it was found to be much easier to directly manipulate the combined form:

$$\bar{a}_{00} \dot{\omega}^0 + \sum_{k \in \mathcal{P}} \bar{a}_{0k} \ddot{\gamma}_k = \sum_{k \in \mathcal{B}} \hat{E}_k \quad (13a)$$

$$\bar{a}_{i0} \dot{\omega}^0 + \sum_{k \in \mathcal{P}} \bar{a}_{ik} \ddot{\gamma}_k = g^{iT} \hat{C}^{i0} \sum_{k \in \mathcal{P}} \epsilon_{ik} \hat{E}_k + \hat{\tau}_i \quad (i \in \mathcal{P}) \quad (13b)$$

where

$$\begin{aligned} \hat{E}_k &= \hat{C}^{0k} [\hat{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \hat{C}^{kj} \hat{F}^j] \\ \hat{C}^{i0} &= \bar{C}^{i0} + \hat{C}^{i0} \\ \hat{C}^{0k} &= \bar{C}^{0k} + \hat{C}^{0k} \\ \hat{T}^k &= \bar{T}^k + \hat{T}^k \\ \hat{F}^j &= \bar{F}^j + \hat{F}^j \\ \hat{\tau}_i &= \bar{\tau}_i + \hat{\tau}_i \end{aligned}$$

By dealing only with the variables  $\hat{E}_k$ ,  $\hat{C}^{0k}$ , etc., rather than separating them into the parts  $\bar{E}_k$ ,  $\hat{E}_k$ , etc., and explicitly forming all the separate terms in (10) and (12), the computation becomes much more straightforward and efficient even though some second-order terms in the linearized variables are retained.

One further form of the system equations, which goes beyond strict linearization in order to simplify and speed the computations, is the following:

$$\bar{a}_{00} \dot{\omega}^0 + \sum_{k \in \mathcal{P}} \bar{a}_{0k} \ddot{\gamma}_k = \sum_{k \in \mathcal{B}} E'_k \quad (14a)$$

$$\bar{a}_{i0} \dot{\omega}^0 + \sum_{k \in \mathcal{P}} \bar{a}_{ik} \ddot{\gamma}_k = g^{iT} \sum_{k \in \mathcal{P}} \epsilon_{ik} E'_k + \hat{\tau}_i \quad (14b)$$

where

$$E'_k = \bar{C}^{0k} [\hat{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \bar{C}^{kj} \hat{F}^j] = \hat{T}^k + \sum_{j \in \mathcal{B}} \bar{D}^{kj} \hat{F}^j$$

Equations (14) obviate the computation of any body-to-body coordinate transformation matrices,  $\bar{C}^{rj}$ , at all.

Subroutines which provide solutions to both Eqs. (13) and (14) will be described in the next section.

## B. Subroutines MBDYTL and MBDYL

The subroutines MBDYTL and MBDYL provide the solutions to Eqs. (13) and (14) of the preceding section, respectively. As in the case of MBDY, they each may be exercised by either of two call statements, the first of which initializes the program with the system constants.

### Initializing Call Statements

CALL MBDYTL(NH,H,MB,MA,PB,PA,G,PI)

or

CALL MBDYL(NH,H,MB,MA,PB,PA,G,PI)

Except for the call statement name, the statements are exactly the same as that for MBDY. All the arguments are defined exactly as given in Section IIE.

To compute the angular accelerations  $\ddot{\omega}^0, \ddot{\gamma}_1, \dots, \ddot{\gamma}_n$  the subroutine MBDYTL (or MBDYL) must be entered at every integration interval. This is accomplished by executing the dynamic call statement.

### Dynamic Call Statement

CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,WO,WDOT)

Again, the dynamic call statement arguments are the same as those defined for MBDY in Section IIE, with the following *exceptions*:

TH(*i*) = array of hinge torques,  $\hat{\tau}_i; i \in \mathcal{P}$

TB(*j*) = array of elements of  $\hat{T}^0; j = 1, 2, 3$

TA(*i,j*) = array of elements of  $\hat{T}^i; i \in \mathcal{P}, j = 1, 2, 3$

FB(*j*) = array of elements of  $\hat{F}^0; j = 1, 2, 3$

FA(*i,j*) = array of elements of  $\hat{F}^i; i \in \mathcal{P}, j = 1, 2, 3$

Even though the arguments GMD and WO are not used by either MBDYTL or MBDYL, and GM is not used by MBDYL, these "dummy" arguments are retained in the call statements for the sake of uniformity and to allow easy interchange of subroutines if desired.

The Fortran "type" and storage allocation statements which must appear in the main program that calls MBDYTL (or MBDYL) are identical to those required for MBDY.

### Required Specification Statements

INTEGER NH, H(*n*), PI(*n*)

REAL MB(7), MA(*n*,7), PB(*n*,3), PA(*n*,*n*,3), G(*n*,3), TH(*n*),  
TB(3), TA(*n*,3), FB(3), FA(*n*,3), GM(*n*), GMD(*n*),  
GMDD(*n*), WO(3)

DOUBLE PRECISION WDOT(*n* + 3)

Also, the statement

PARAMETER  $Q = n$

must appear *within* the subroutine MBDYTL (or MBDYL) to allocate the proper amount of internal storage. Complete listings of the MBDYTL and MBDYL programs are given in Appendix B.

### C. Application of MBDYTL and MBDYL to the Five-Body System

*1. Small platform maneuver simulation.* We may now proceed to apply the linearized subroutines to the same vehicle dynamics/control configuration examined in Section IIF. The vehicle is shown in Fig. 4 and the control system in Fig. 5. Expressions previously given for  $T^0$ ,  $\tau_1$ ,  $\tau_2$ ,  $\tau_3$ , and  $\tau_4$  may now be re-interpreted as describing  $\hat{T}^0$ ,  $\hat{\tau}_1$ ,  $\hat{\tau}_2$ ,  $\hat{\tau}_3$ , and  $\hat{\tau}_4$ , respectively, since they contain no second-order or higher terms in the variables  $\omega^0$ ,  $\gamma_k$  and their derivatives. In fact, in order to again simulate Case 2, i.e., the small platform maneuver from  $\gamma_1 = 218$  deg to  $\gamma_1 = 220$  deg (at  $t = 0$ ) and  $\gamma_2 = -30$  deg to  $\gamma_2 = -25$  deg (at  $t = 5$  s), it is necessary to replace the statement "CALL MBDY(NH, . . .)" with the statement "CALL MBDYTL(NH, . . .)" and to modify the arrays MA(2,i), PA(1,2,j), PA(2,2,j), and G(2,j).

The reason for the required modifications to these arrays stems from Defs. 4 and 12, which require all local, body-fixed coordinate frames to coincide (i.e.,  $\mathbf{b}_\alpha^k = \mathbf{b}_\alpha^j$  for  $\alpha = 1, 2, 3$  and  $k, j \in \mathcal{B}$ ) when the system is in its nominal state, i.e., when  $\gamma_k = 0$  for all  $k \in \mathcal{P}$ . When the routines MBDYTL and MBDYL are used, the variables  $\gamma_k$  are, by definition, required to remain very close to the nominal state  $\gamma_k = 0$ . Otherwise, the linearization process is meaningless. It is clear, then, that to simulate Case 2 with the linearized programs, the new linearized variables  $\hat{\gamma}_k$  ( $k = 1, 4$ ) must be redefined in terms of the old  $\gamma_k$  such that, for example,

$$\begin{aligned}\hat{\gamma}_1 &= \gamma_1 - 219 \text{ deg} \\ \hat{\gamma}_2 &= \gamma_2 + 27.5 \text{ deg} \\ \hat{\gamma}_3 &= \gamma_3 \\ \hat{\gamma}_4 &= \gamma_4\end{aligned}$$

And now it is also clear that bodies  $\mathcal{B}_1$  and  $\mathcal{B}_2$  have different orientations in their own body-fixed frames, implying different inertia matrices  $I^1, I^2$  and different components for the vectors  $\mathbf{p}^{10}, \mathbf{p}^{12}, \mathbf{p}^{21}, \mathbf{g}^1, \mathbf{g}^2$  in their respective frames,  $\{\mathbf{b}^1\}$  and  $\{\mathbf{b}^2\}$ .

In this particular vehicle configuration, the platform hub,  $\mathcal{B}_1$ , has symmetrical mass properties about its axis of rotation, which in turn is aligned to one of the body-fixed coordinate axes. Thus,  $I^1$ , and therefore MA(1,j),  $j = 1 - 7$ , remains unchanged, as does  $\mathbf{p}^{10}$  (PA(1,1,j),  $j = 1 - 3$ ) and  $\mathbf{g}^1$  (G(1,j),  $j = 1 - 3$ ). However, it is apparent that  $\mathbf{p}^{12}, \mathbf{p}^{21}$ , as well as  $\mathbf{g}^2$ , and  $I^2$  must change, since they were previously defined for  $\gamma_1 = 0, \gamma_2 = 0$  and now must reflect the configuration defined by  $\hat{\gamma}_1 = 0, \hat{\gamma}_2 = 0$  ( $\gamma_1 = 219$  deg,  $\gamma_2 = -27.5$  deg).

The components of a vector described in the old frame fixed in  $\mathcal{E}_2$  can be related to its components in the new frame by the sequence of rotations  $\gamma_1$  and  $\gamma_2$ , as follows:

$$\begin{Bmatrix} v_1^N \\ v_2^N \\ v_3^N \end{Bmatrix} = \begin{bmatrix} c\gamma_1 & -s\gamma_1 c\gamma_2 & -s\gamma_1 s\gamma_2 \\ s\gamma_1 & c\gamma_1 c\gamma_2 & c\gamma_1 s\gamma_2 \\ 0 & -s\gamma_2 & c\gamma_2 \end{bmatrix} \begin{Bmatrix} v_1^0 \\ v_2^0 \\ v_3^0 \end{Bmatrix} = C^{N0} \begin{Bmatrix} v_1^0 \\ v_2^0 \\ v_3^0 \end{Bmatrix}$$

$$(s\gamma_1 = \sin \gamma_1, c\gamma_1 = \cos \gamma_1, \text{ etc.})$$

Therefore, the components of  $g^2$  in a simulation linearized about  $\gamma_1 = 219$ ,  $\gamma_2 = -27.5$  deg become:

$$\begin{Bmatrix} g_1^2 \\ g_2^2 \\ g_3^2 \end{Bmatrix} = C^{N0} \begin{Bmatrix} -1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -c\gamma_1 \\ -s\gamma_1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.77715 \\ 0.62932 \\ 0 \end{Bmatrix}$$

Similarly, the components of  $p^{21}$  become

$$\begin{Bmatrix} p_1^{21} \\ p_2^{21} \\ p_3^{21} \end{Bmatrix} = C^{N0} \begin{Bmatrix} 0 \\ -0.22 \\ 0.20 \end{Bmatrix} = \begin{Bmatrix} 0.22 s\gamma_1 c\gamma_2 - 0.20 s\gamma_1 s\gamma_2 \\ 0.20 c\gamma_1 s\gamma_2 - 0.22 c\gamma_1 c\gamma_2 \\ 0.22 s\gamma_2 + 0.20 c\gamma_2 \end{Bmatrix} = \begin{Bmatrix} -0.18092 \\ 0.22342 \\ 0.07582 \end{Bmatrix}$$

The vector  $p^{12}$  only changes due to the  $\gamma_1$  rotation, i.e.,

$$\begin{Bmatrix} p_1^{12} \\ p_2^{12} \\ p_3^{12} \end{Bmatrix} = \begin{bmatrix} c\gamma_1 & -s\gamma_1 & 0 \\ s\gamma_1 & c\gamma_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.10 \\ -0.75 \end{Bmatrix} = \begin{Bmatrix} 0.1 s\gamma_1 \\ -0.1 c\gamma_1 \\ -0.75 \end{Bmatrix} = \begin{Bmatrix} -0.06293 \\ 0.07772 \\ -0.75 \end{Bmatrix}$$

Finally, the modified value of  $I^2$  is given by

$$I^2 = C^{N0} \begin{bmatrix} 4.85 & 0.41 & -0.07 \\ 0.41 & 2.20 & 0.54 \\ -0.07 & 0.54 & 5.50 \end{bmatrix} (C^{N0})^T = \begin{bmatrix} 3.52 & 1.25 & -0.75 \\ 1.25 & 3.79 & 0.73 \\ -0.75 & 0.73 & 5.24 \end{bmatrix}$$

Having determined these changes in the system configuration description, the appropriate values of MA(2,i),  $i = 1 - 6$ , PA(1,2,i), PA(2,2,i), G(2,i) may be entered to produce the simulation program of Fig. 15. Notice in Fig. 15 that it was also necessary to change IC1 and IC2 to reflect proper initial values of the new variables  $\hat{\gamma}_1$  and  $\hat{\gamma}_2$  (i.e.,  $\hat{\gamma}_1(0) = -1$  deg,  $\hat{\gamma}_2(0) = -2.5$  deg; these variables correspond to  $\gamma_1(0) = 218$  deg,  $\gamma_2(0) = -30$  deg). Using this program and the MBDYTL subroutine, Case 2 was simulated again, and the results are shown in Fig. 16. They may be directly compared to the dynamic responses of Fig. 14. While there are some very slight differences visible in the angular rate responses, the overall result is virtually identical with that obtained using MBDY, and is obtained in *one third the time used by MBDY* (MBDY used 140 s, MBDYTL 48 s on the Univac 1108). Further, the substitution of MBDYL for MBDYTL gives exactly the same results as those shown in Fig. 16, since no *external* forces or torques were applied to the system, and calculates them in only 42 s.

Obviously, the gain in computational speed when the linearized routines are used is quite significant, even for this relatively simple example. In Ref. 7, a 17-degree-of-freedom system is simulated using MBDYTL exclusively, since it gave a speed advantage over MBDY of about 25:1 with more than adequate solution accuracy.

```

PROGRAM S-900Y SYSTEM SIMULATION USING MBDYTL --- CASE 2
  INTEGER NH,PI(4),H(4)
  ARRAY MB(7),MA(4,7),PB(4,3),PA(4,4,3),G(4,3),TH(3),TA(4,3)
  ARRAY FB(3),FA(4,3),TH(5),GM(4),GMD(4),GMDD(4),WO(3)
  DOUBLE PRECISION WDOT(7)
  DATA H/5.,1.,0.,3/PI/0.,0.,0.,0./
  DATA MB/115.,315.,440.,14.,-14.,34.,6.,410./
  DATA MA(1,1)/.35/MA(1,2)/.35/MA(1,7)/6.8/
  DATA MA(2,1)/3.52/MA(2,2)/3.73/MA(2,3)/5.24/MA(2,4)/-1.25/MA(2,5)/.75/
  DATA MA(2,6)/-.77/MA(2,7)/57.5/
  DATA MA(4,1)/27.2/MA(4,2)/.2/MA(4,3)/27.2/MA(4,7)/10.7/
  DATA PB(1,3)/-1.5/PB(3,2)/-1.2/
  DATA PA(1,1,3)/-.75/PA(1,2,1)/-.06293/PA(1,2,2)/.07772/PA(1,2,3)/-.75/
  DATA PA(2,1,1)/-.18032/PA(2,2,2)/.22342/PA(2,2,3)/.07582/
  DATA PA(4,4,2)/3.3/
  DATA G(1,3)/1.76/2,1)/.77715/3,2)/.62932/3,1)/1./4,3)/1./
  CONSTANT IM=.020,OB=.005,IR=2.0
  CONSTANT IFINAL=20.,CLKTIM=1100.
  CONSTANT L1=.23,L2=.21,L3=.31
  CONSTANT SLRAT=.01745328
  CONSTANT PHIZ=0.,THETZ=0.,PSIZ=0.
  CONSTANT IC1=-1.,IC2=-2.5
  CONSTANT W1I=0.,W2I=0.,W3I=0.
  CONSTANT K1=3500.,K2=3500.,K3=2000.,K4=2000.
  CONSTANT B1=20.,B2=20.,B3=10.,B4=10.
  INITIAL
    NH=4
    PD=57.2957795 $ DR=1./RO $ GM1CI=IC1/RO $ GM2CI=IC2/RO
    GM1I=IC1/RO $ GM2I=IC2/RO
    CALL MBDYTL(NH,H,MB,MA,PB,PA,G,PI)
    PZI,PAI,PBI,PCI=4CK(IINITL,PHIZ,THETZ,PSIZ)
  END
  DYNAMIC
    IF (TIME.GE.TFINAL) GO TO S1
  DERIVATIVE 900YS
    VARIABLE TIME=0.
    CINTERVAL C1=.02
    XERROR W1=1.E-6,W2=1.E-6,W3=1.E-6,GM3=1.E-4
    S1PCLK CLKTIM
    OUTPUT 50,W1D,W2D,W3D,W1,W2,W3,THET1,THET2,THET3,...
    IN1,IN2,IN3,GM1,GM2,GM3,GM4,PL1,PL2
    PREPAR W1,W2,W3,THET1,THET2,THET3,GM1D,GM2D,...
    GM1,GM2,GM3D,GM4D,GM3,GM4
  NOSORT
    WO(1)=W1 $ WO(2)=W2 $ WO(3)=W3
    PL1=RO*GM1 $ PL2=RO*GM2
    GM(1)=GM1 $ GM(2)=GM2 $ GM(3)=GM3 $ GM(4)=GM4
    GMD(1)=GM1D $ GMD(2)=GM2D $ GMD(3)=GM3D $ GMD(4)=GM4D
  COMMENT
    COMMENT CELESTIAL-SENSOR-MEASURED ANGLES
  COMMENT
    O1,O2=HCK(MATRIX,PZO,PAO,PBO,PCO)
    NXO,NYO,NZO=HCK(IIOB,G.,O.,1.,O1,O2)
    LYO,LZO=HCK(IIOB,1.,O.,O.,O1,O2)
    THET1=ATAN2(NYO,NZO) $ THET2=ATAN2(-NXO,NZO)
    THET3=ATAN2(-LYO,LZO)

```

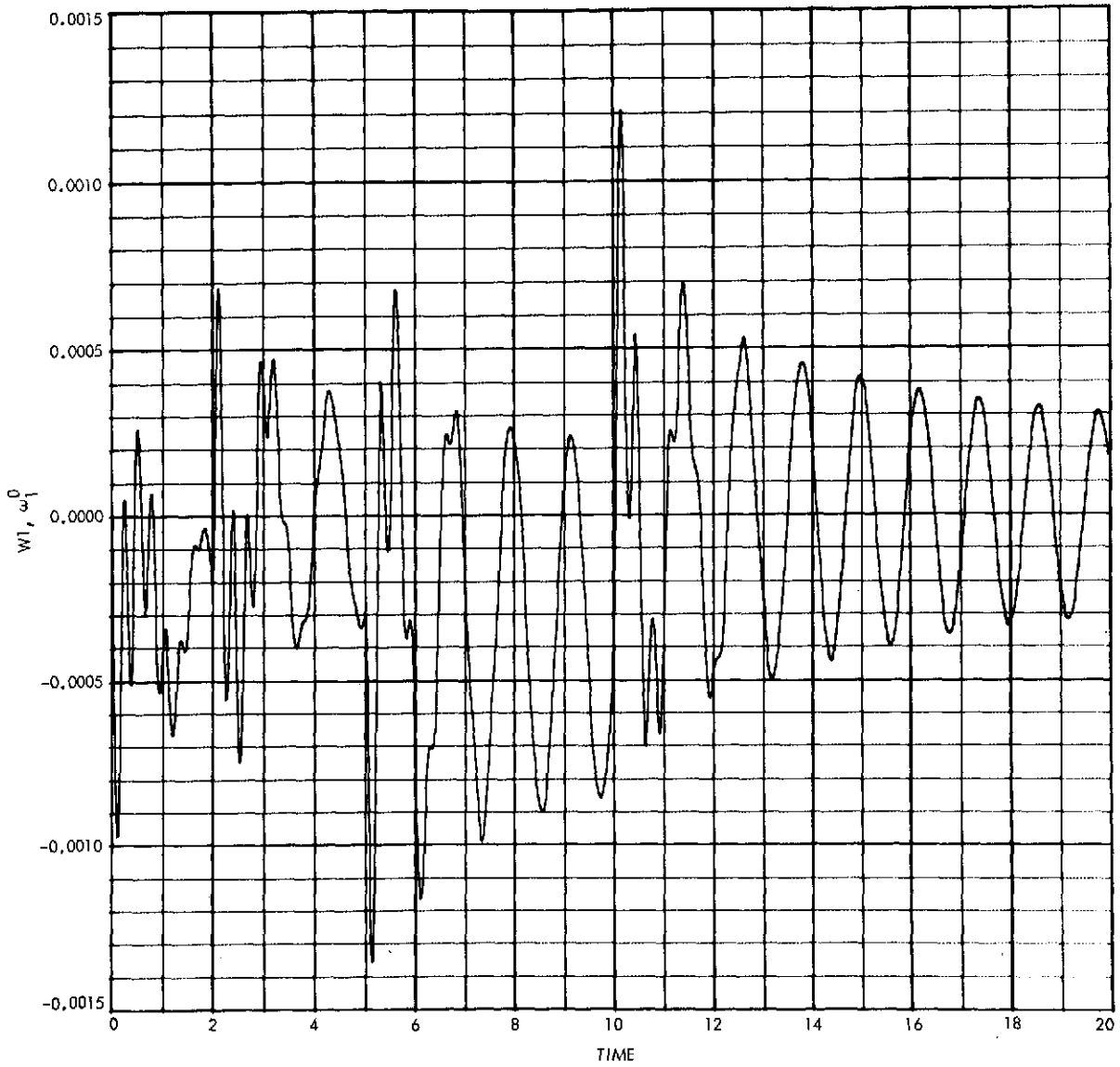
Fig. 15. Case 2 simulation using MBDYTL

```

COMMENT
COMMENT RATE FEEDBACK SIGNAL AND SWITCHING AMP. DYNAMICS
COMMENT
IN1=-THE11-TR*W1 $ IN2=-THE12-TR*W2 $ IN3=-THE13-TR*W3
AMP1=SWAMP(TIME, TM, DB, IN1)
AMP2=SWAMP(TIME, TM, DB, IN2)
AMP3=SWAMP(TIME, TM, DB, IN3)
COMMENT
COMMENT PLATFORM POSITION COMMANDS
COMMENT
GM1C=SLRAT*(1.-STEP(2.,TIME)) $ GM1C=INTEG(GM1C,GM1C)
GM2C=SLRAT*(STEP(5.,TIME)-STEP(10.,TIME))$ GM2C=INTEG(GM2C,GM2C)
COMMENT
COMMENT REFERENCE BODY EXTERNAL TORQUE
COMMENT HINGE TORQUES
COMMENT
TB(1)=L1*AMP1 $ TB(2)=L2*AMP2 $ TB(3)=L3*AMP3
TH(1)=-K1*(GM1-GM1C-B1)*GM1D
TH(2)=-K2*(GM2-GM2C-B2)*GM2D
TH(3)=-K3*GM3-B3*GM3D
TH(4)=-K4*GM4-B4*GM4D
COMMENT
COMMENT SYSTEM ANGULAR ACCELERATIONS
COMMENT
CALL MRATEINH,TH,TB,TA,FB,FA,GM,GM0,GMDD,W0,WDOT)
W1D=WDOT(1) $ W2D=WDOT(2) $ W3D=WDOT(3)
COMMENT
COMMENT SYSTEM ANGULAR RATES AND POSITIONS
COMMENT
W1=INTEG(WDOT(1),W1)
W2=INTEG(WDOT(2),W2)
W3=INTEG(WDOT(3),W3)
GM1D=INTEG(WDOT(4),0.) $ GM2D=INTEG(WDOT(5),0.)
GM3D=INTEG(WDOT(6),0.) $ GM4D=INTEG(WDOT(7),0.)
GM1=INTEG(GM1D,GM1) $ GM2=INTEG(GM2D,GM2)
GM3=INTEG(GM3D,0.) $ GM4=INTEG(GM4D,0.)
COMMENT
COMMENT HCK PARAMETER RATES AND POSITIONS
COMMENT
PZ0Q,PA0Q,PB0Q,PC0Q=HCK(HCK,PZ0,PA0,PB0,PC0,W1,W2,W3)
PZ0=INTEG(PZ0Q,PZ0)
PA0=INTEG(PA0Q,PA0)
PB0=INTEG(PB0Q,PB0)
PC0=INTEG(PC0Q,PC0)
END
END
END
TERMINAL
SI.. CONTINUE
END
END

```

Fig. 15 (contd)



**Fig. 16. Results of Case 2 simulation using MBDYTL**

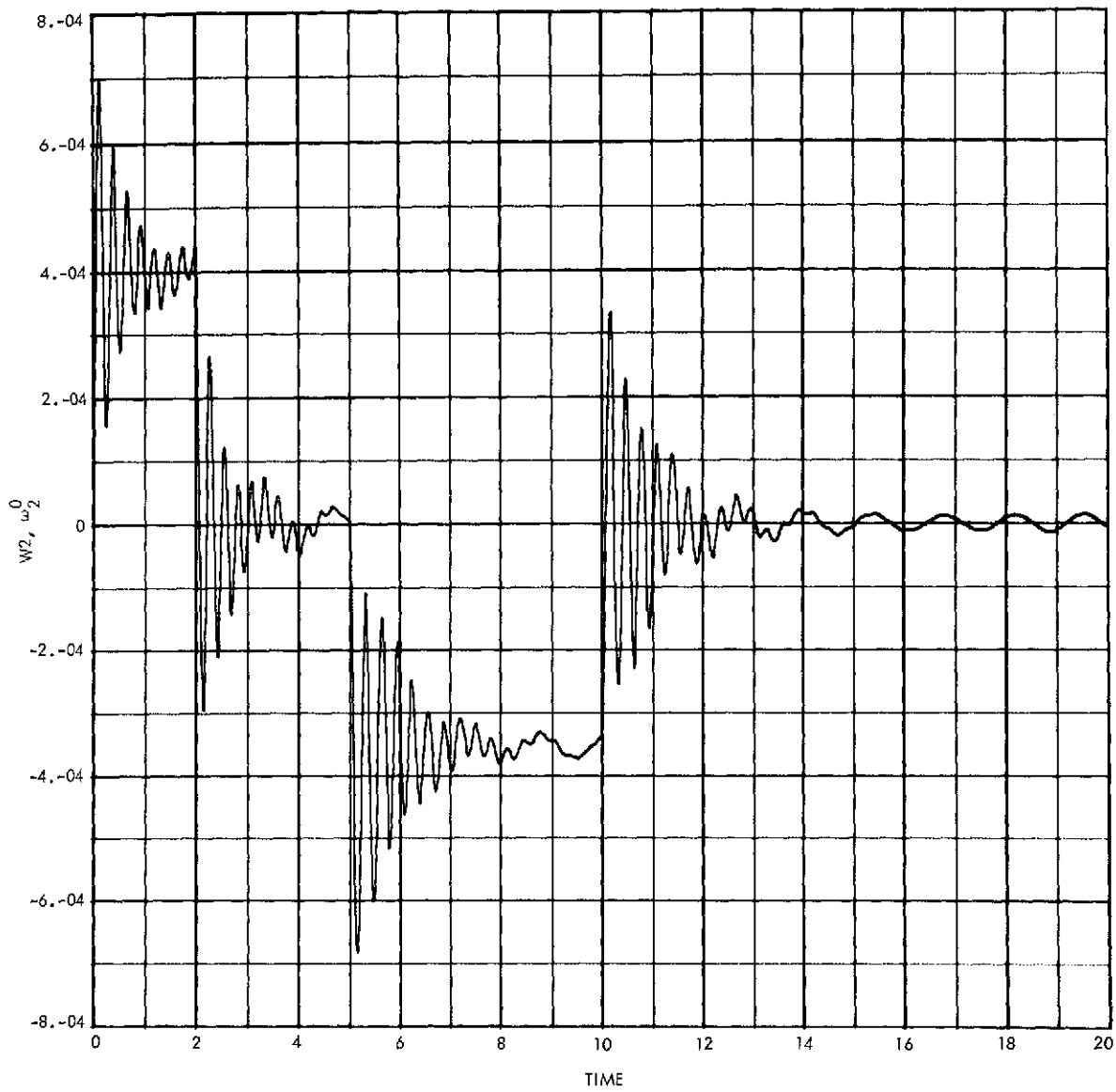


Fig. 16 (contd)



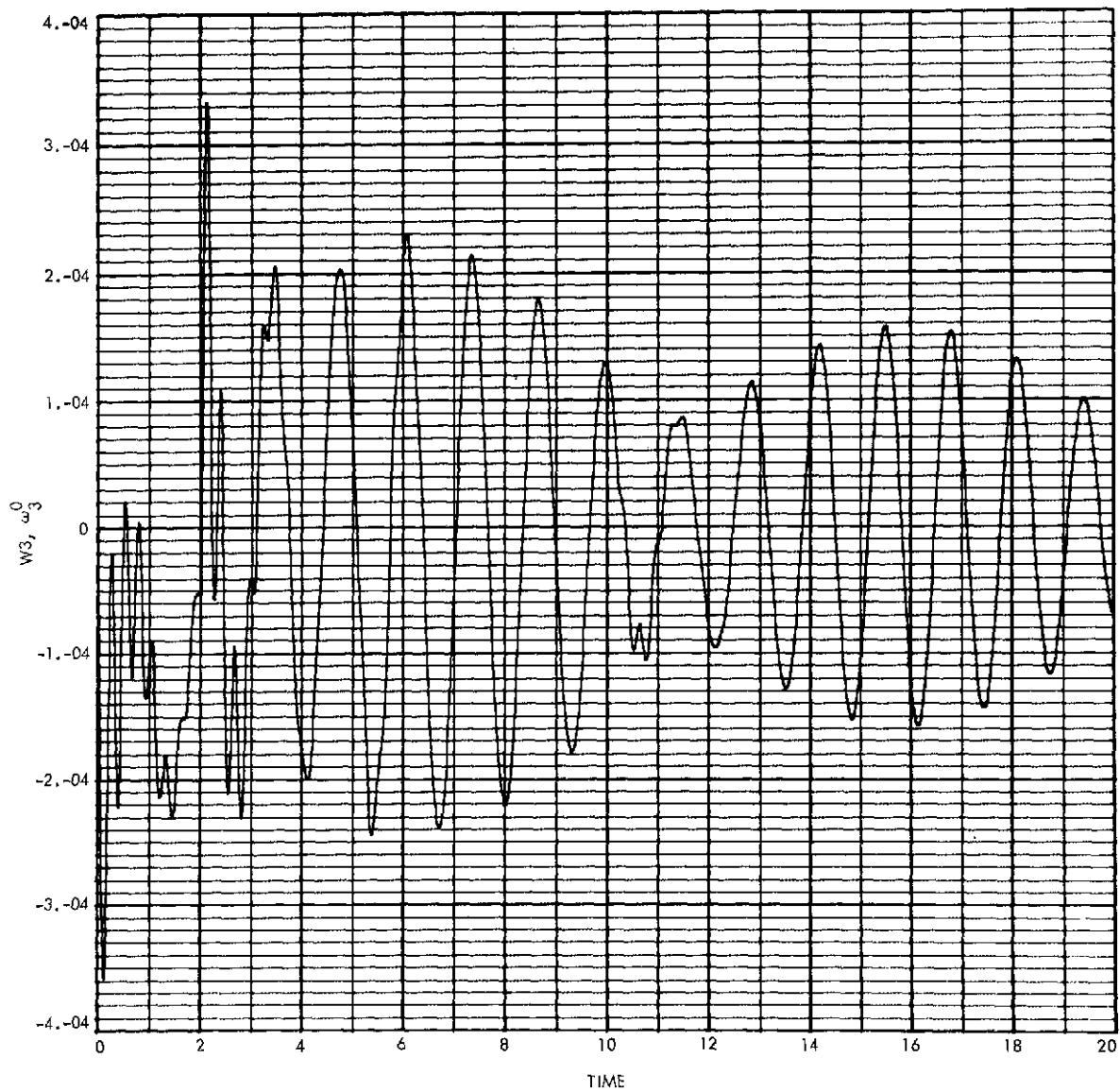


Fig. 16 (contd)

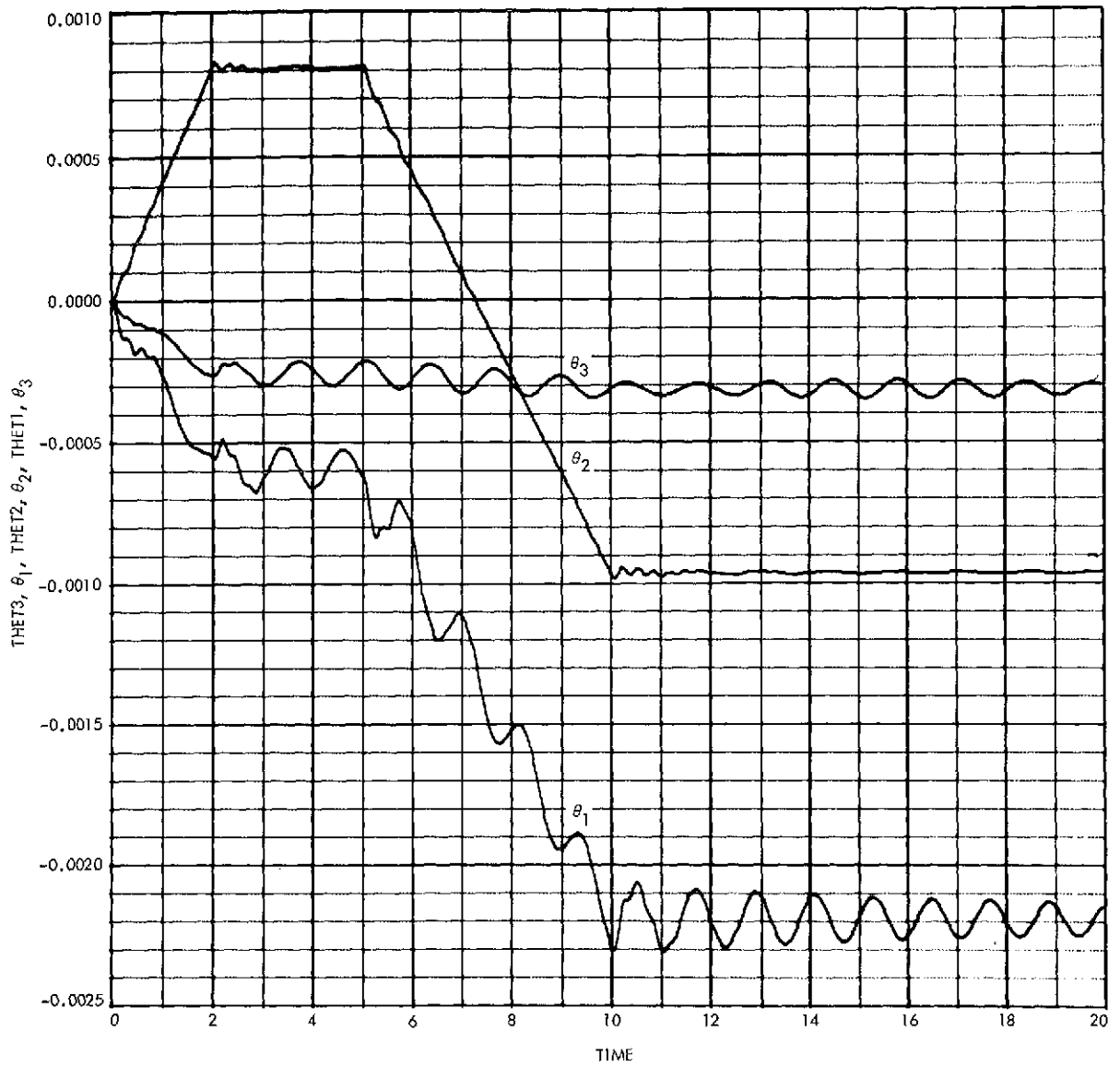


Fig. 16 (contd)

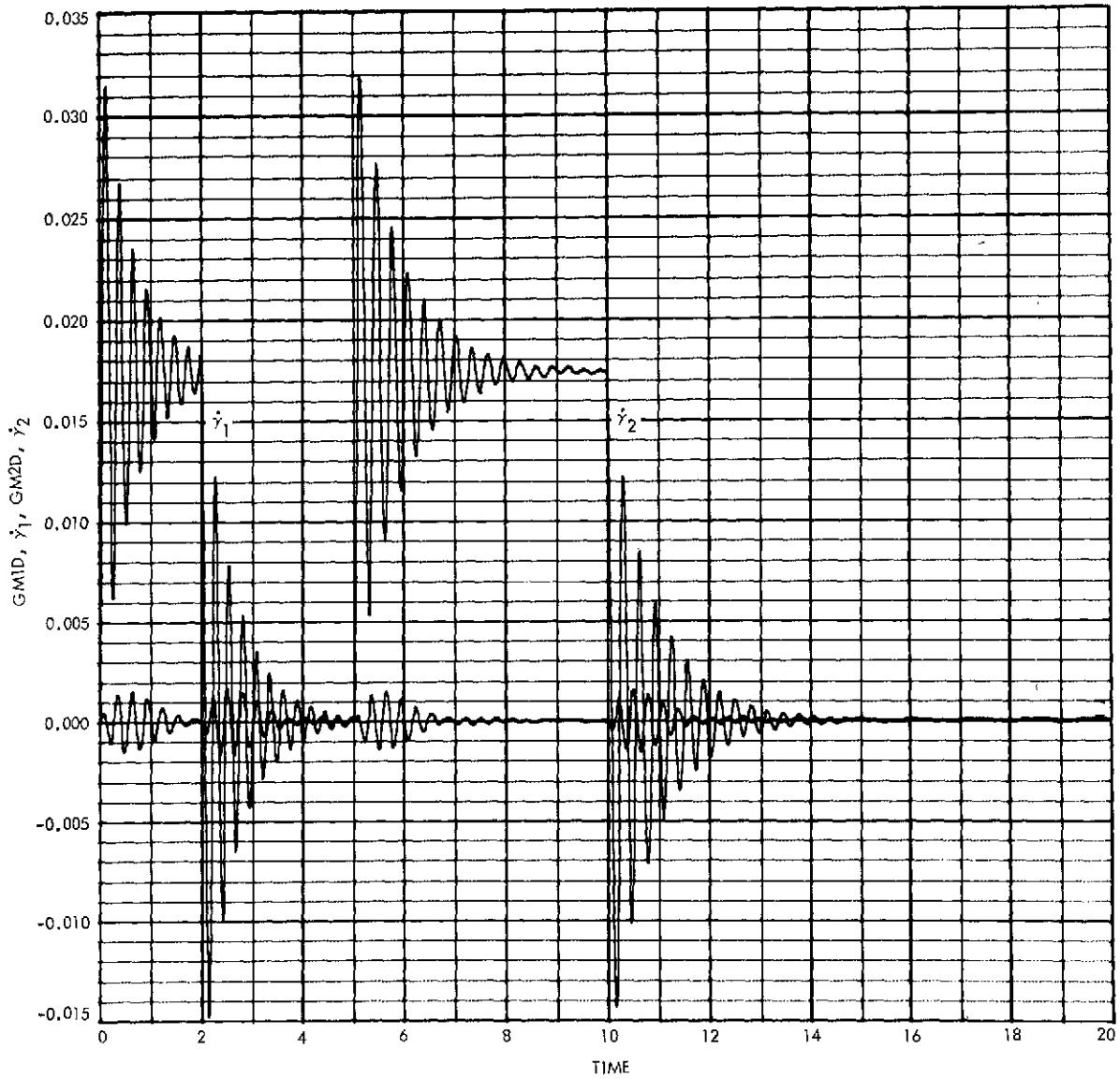


Fig. 16 (contd)

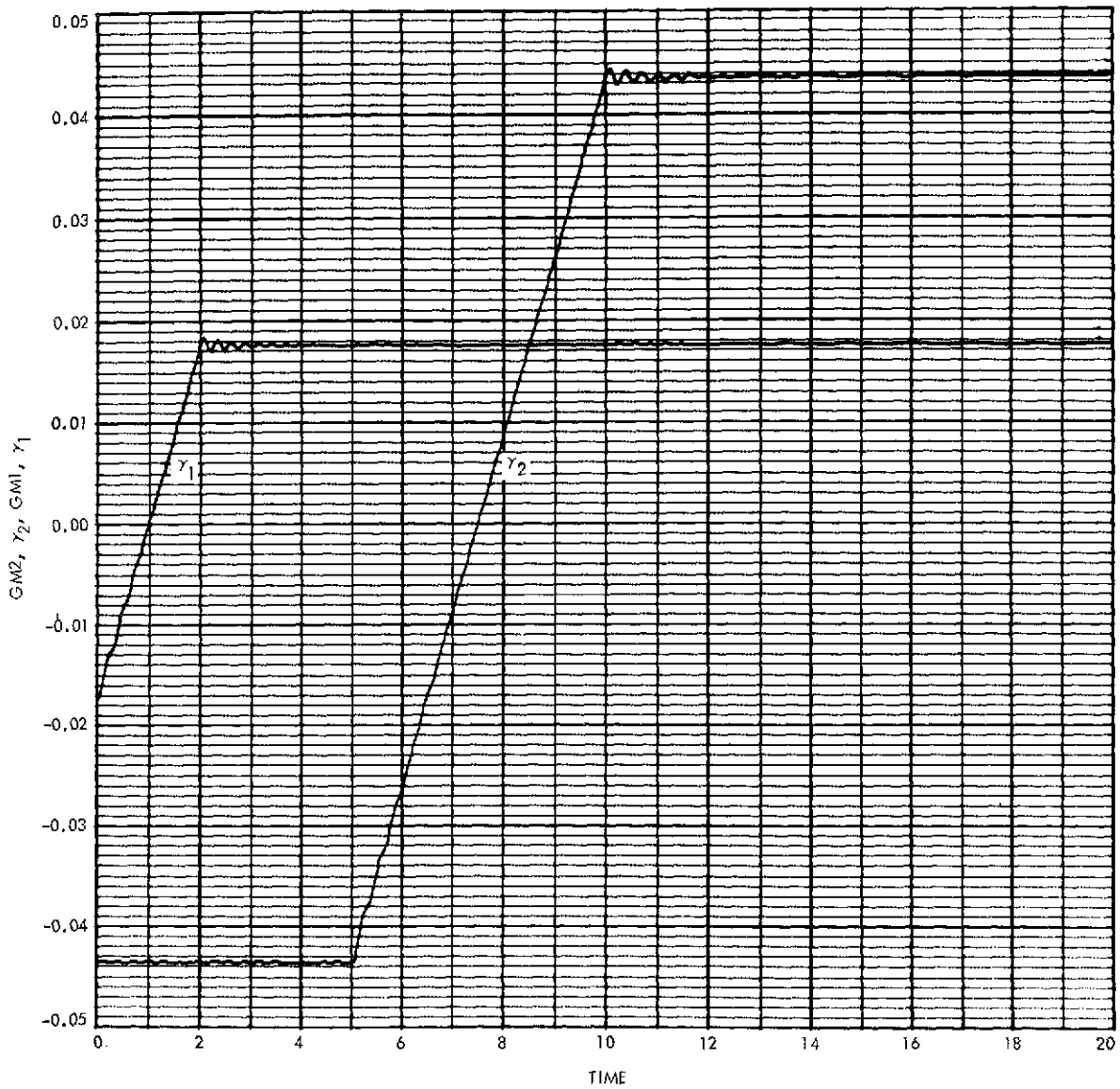


Fig. 16 (contd)

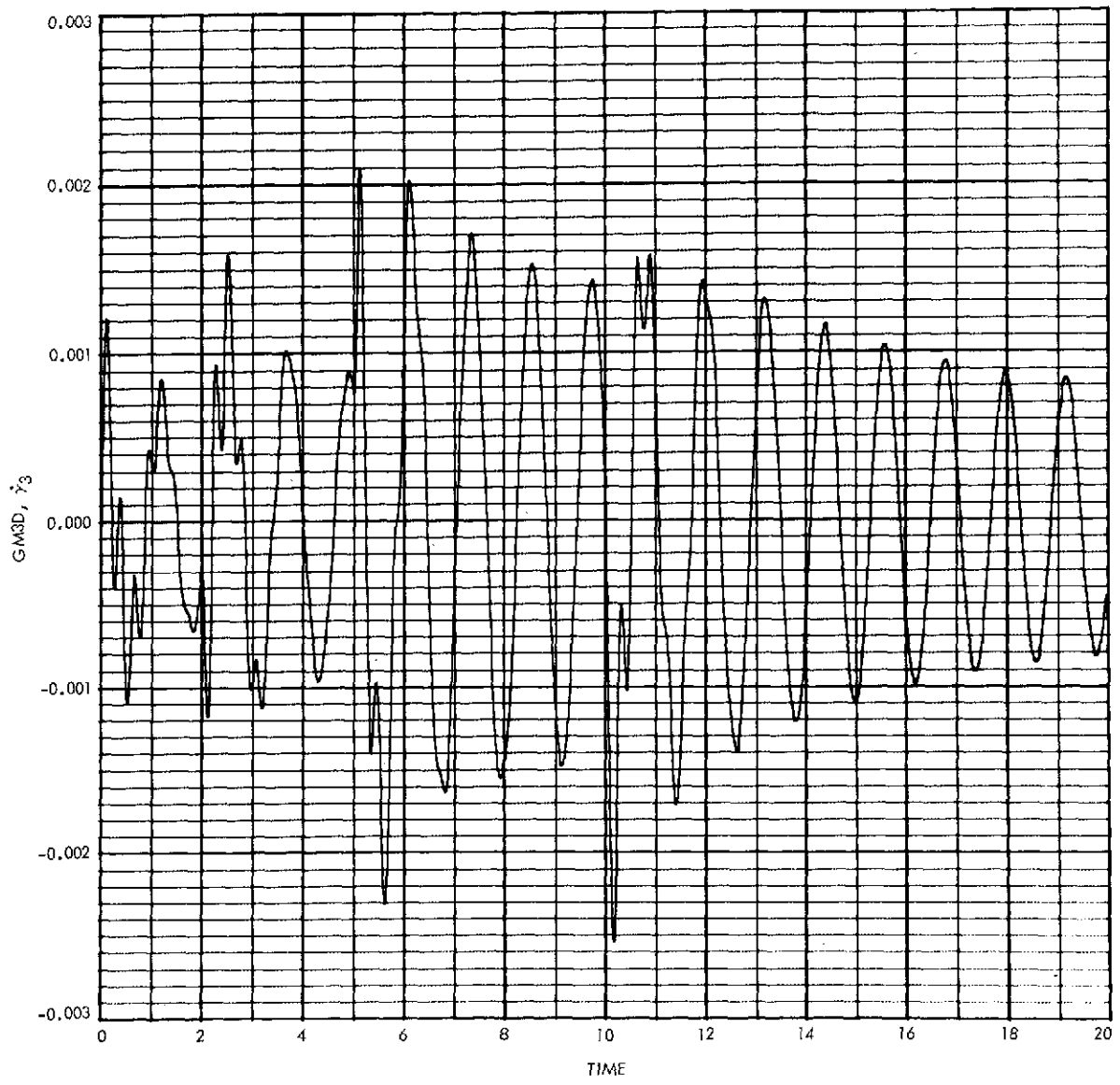


Fig. 16 (contd)

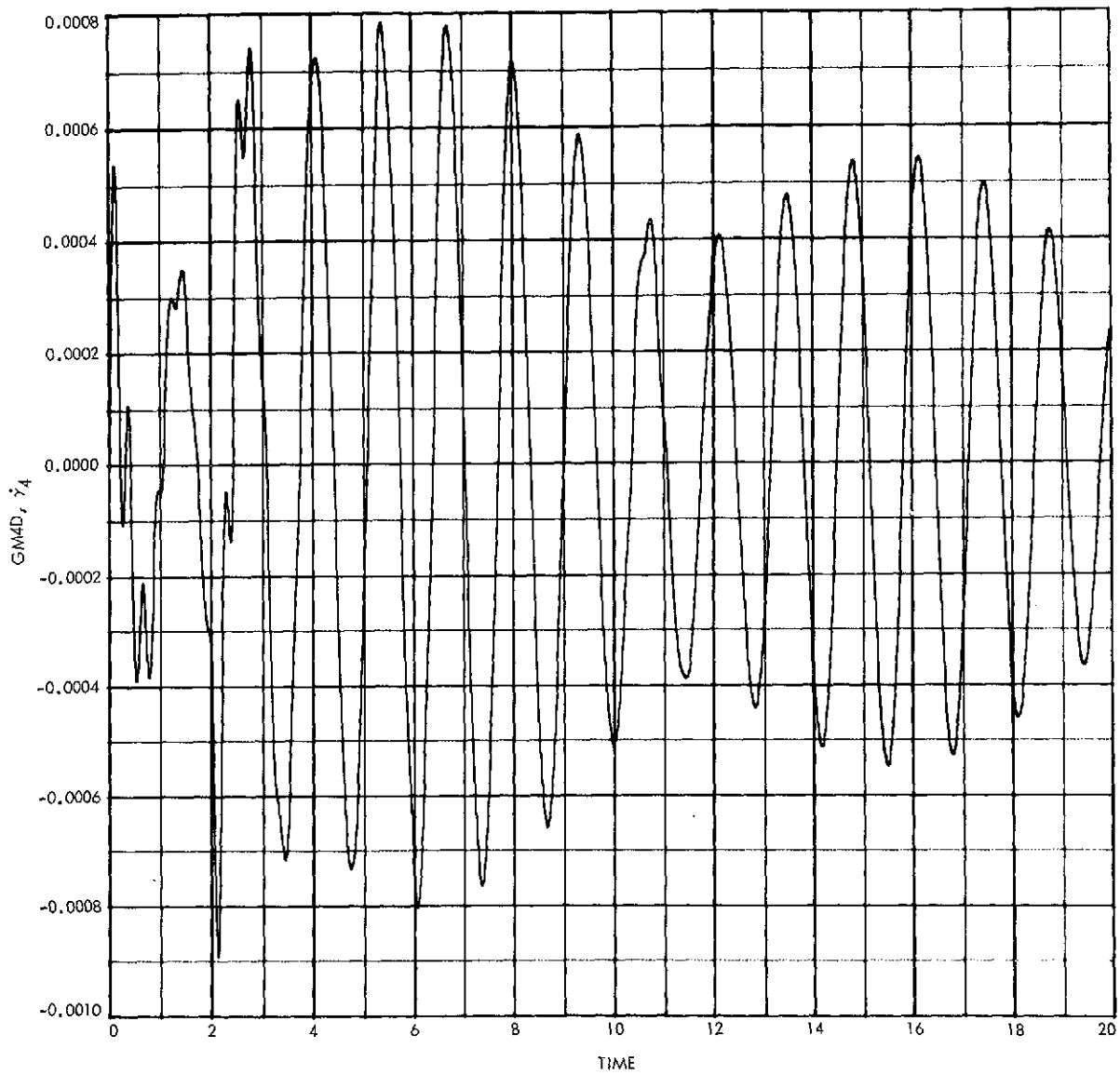


Fig. 16 (contd)

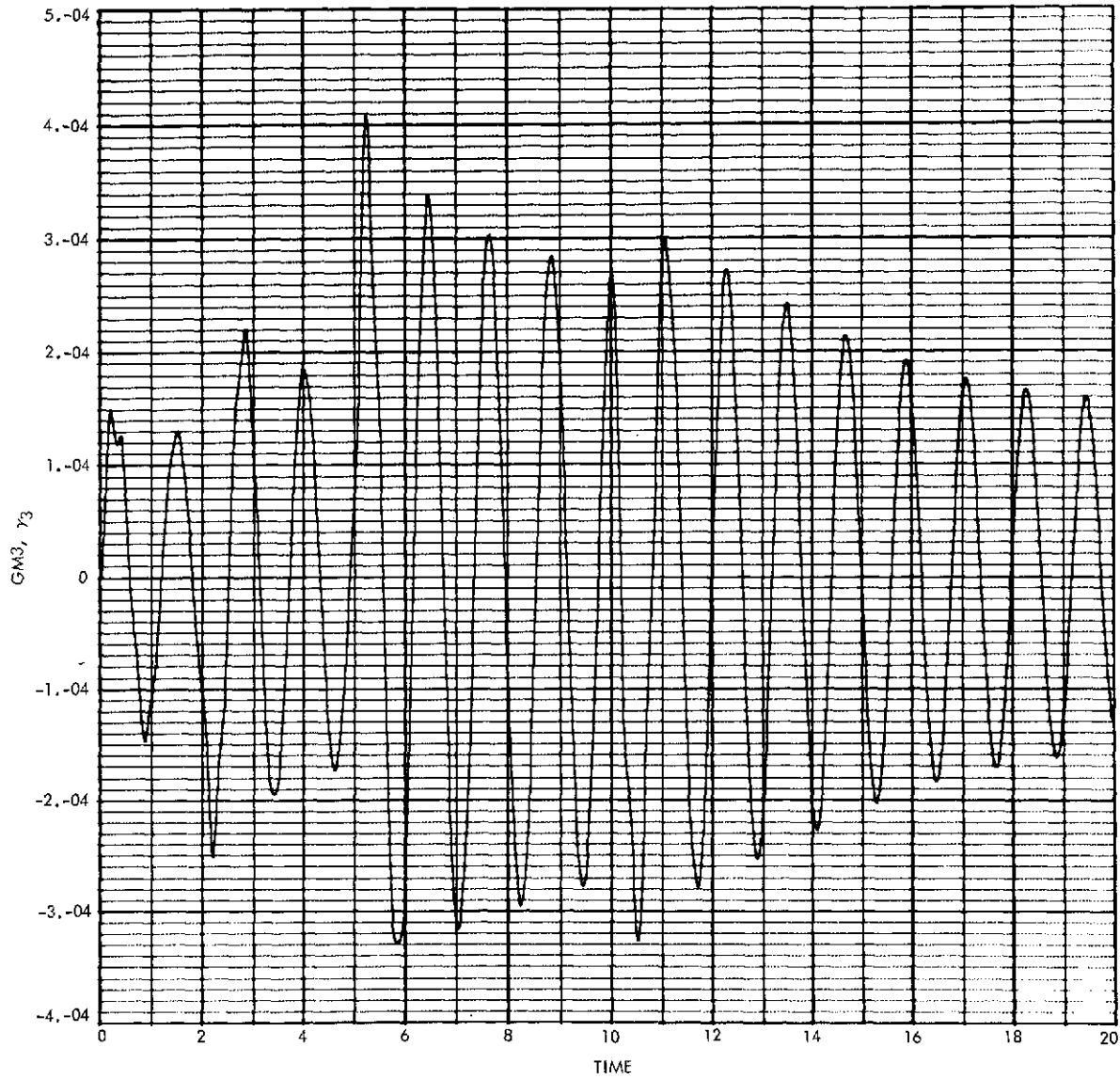


Fig. 16 (contd)

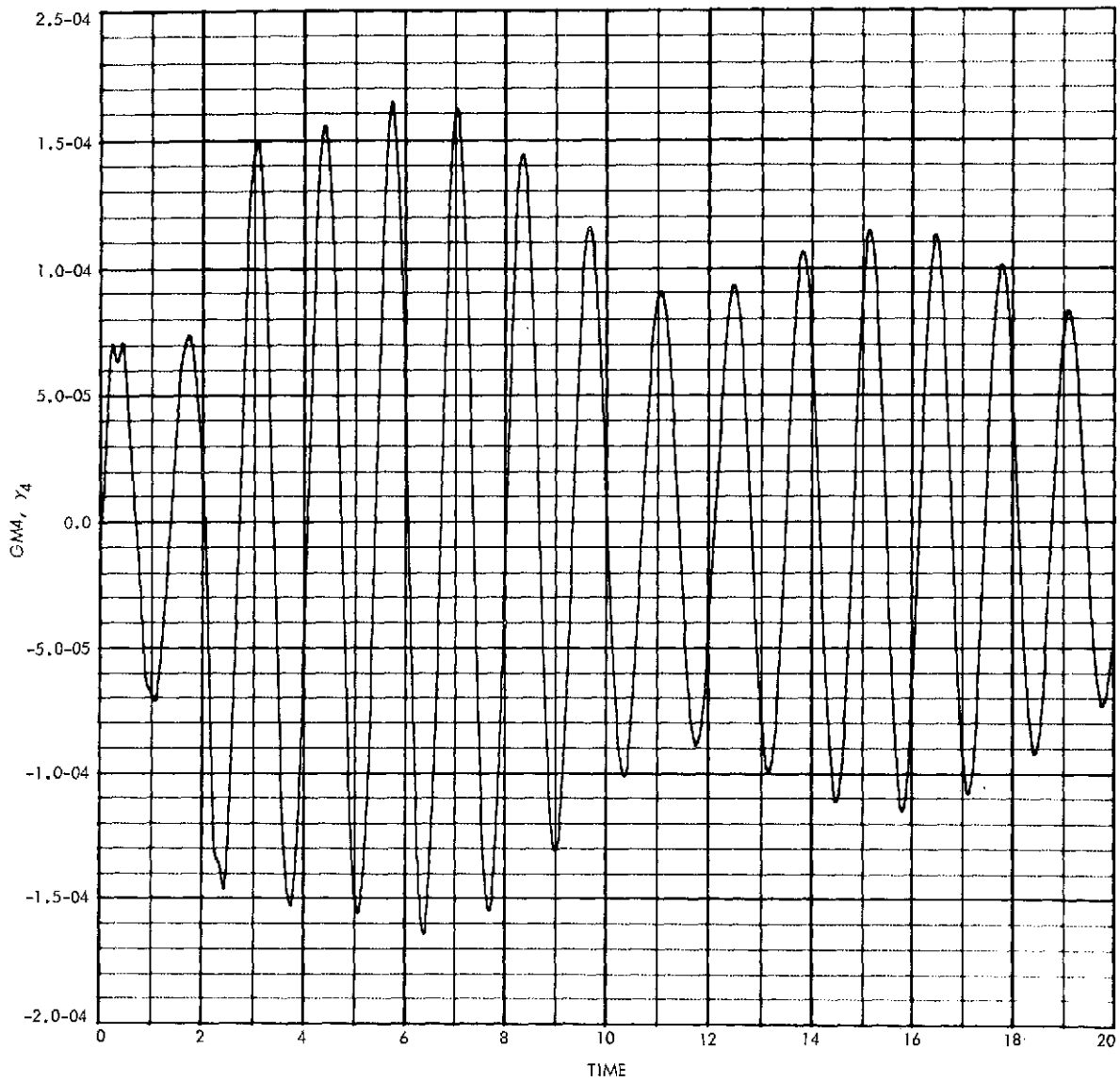


Fig. 16 (contd)



## IV. Partially Linearized Systems

### A. The Equations

One further distinction that can certainly be made in characterizing systems of hinge-connected rigid bodies (somewhere between those systems where all hinges are permitted *unrestricted* rotation and those systems whose hinge and reference body rotations are all "small") is the partially linearized system. In this report, the term "partially linearized" will refer to the case in which the reference body rotations are unrestricted and some, but not all, of the  $\gamma_k$  ( $k \in \mathcal{P}$ ) and their derivatives are considered to be "small."

The indices of those angles for which linearization is in some sense justified may be collected in the set  $\mathcal{A}$ , and, using the expansions of Eq. (9), the linearized form of Eq. (8) becomes

$$(\bar{a}_{00} + \hat{a}_{00}) \dot{\omega}^0 + \sum_{k \in \mathcal{A}} \bar{a}_{0k} \ddot{\gamma}_k + \sum_{k \in \mathcal{P} - \mathcal{A}} (\bar{a}_{0k} + \hat{a}_{0k}) \ddot{\gamma}_k = \sum_{k \in \mathcal{B}} (\bar{E}_k + \hat{E}_k) \quad (15a)$$

$$\begin{aligned} (\bar{a}_{i0} + \hat{a}_{i0}) \dot{\omega}^0 + \sum_{k \in \mathcal{A}} \bar{a}_{ik} \ddot{\gamma}_k + \sum_{k \in \mathcal{P} - \mathcal{A}} (\bar{a}_{ik} + \hat{a}_{ik}) \ddot{\gamma}_k \\ = g^{iT} \sum_{k \in \mathcal{P}} \varepsilon_{ik} [\bar{C}^{i0} (\bar{E}_k + \hat{E}_k) + \hat{C}_{i0} \bar{E}_k] + \bar{\tau}_i + \hat{\tau}_i \end{aligned} \quad (15b)$$

It should be noted that, for a wide class of systems, it can be recognized in advance that  $\varepsilon_{ik} = 0$  for  $i \in \mathcal{A}$  and  $k \in \mathcal{P} - \mathcal{A}$ . This is the case whenever none of the hinge points whose indices comprise  $\mathcal{A}$  lies on a path from  $\mathcal{A}_0$  to any hinge point whose index is not in  $\mathcal{A}$ . In physical terms, this means that the small-angle rotations are confined to *terminal appendages*, each of which is attached to only one body whose index lies outside of  $\mathcal{A}$ . In fact, primary attention is focused on those scalar elements  $a_{ik}$  in (15b) where  $i, k \in \mathcal{A}$  and  $i, k$  are members of the same terminal appendage set. It can be shown that such elements  $a_{ik}$  are all constants.

*Def. 38.* For  $v \in \mathcal{B}$ , let  $\mathcal{A}_v$  denote a "terminal appendage set." Thus  $\mathcal{A}_v$  is a subset of  $\mathcal{A}$ , containing the body indices of one or more terminal appendages which share a common base body,  $v$ .

By appropriate exchanges of rows and columns of the symmetric system matrix, the constant  $\bar{a}_{ik}$ 's may be isolated in an  $r \times r$  symmetric matrix,  $A_c$ , forming the lower right-hand corner of the original matrix. One can then employ a matrix partitioning scheme, where the constant matrix  $A_c$  is inverted just once, and a time-varying matrix equation, now of order  $(n + 3 - r)$ , is solved at every integration step. Clearly, the size of  $A_c$  should be maximized for the greatest computational advantage, and, since only one terminal appendage set (there may be several) may be used to form  $A_c$ , the largest terminal appendage set should be used.

With this approach, i.e., a single partitioning of the system matrix into constant and time-varying parts, another version of the MBDY subroutine, called MBDYPL, was written to solve the partially linearized case. However, in a move to further simplify and speed up the computational task, Eq. (15) was modified as follows:

$$\hat{a}_{00} \dot{\omega}^0 + \sum_{k \in \mathcal{P}} \hat{a}_{0k} \ddot{\gamma}_k = \sum_{k \in \mathcal{B}} \hat{E}_k \quad (16a)$$

$$\hat{a}_{i_0}\dot{\omega}^0 + \sum_{k \in \mathcal{P}-\mathcal{A}} \hat{a}_{ik}\ddot{\gamma}_k + \sum_{k \in \mathcal{A}} \bar{a}_{ik}\ddot{\gamma}_k = g^{ix} \hat{C}^{i_0} \sum_{k \in \mathcal{P}} \epsilon_{ik} \hat{E}_k + \hat{\tau}_i \quad (16b)$$

where

$$\begin{aligned} \hat{a}_{00} &= \bar{a}_{00} + \hat{a}_{00} + \dots \\ \hat{a}_{0k} &= \bar{a}_{0k} + \hat{a}_{0k} + \dots \\ \hat{a}_{i_0} &= \bar{a}_{i_0} + \hat{a}_{i_0} + \dots \\ \hat{a}_{ik} &= \bar{a}_{ik} + \hat{a}_{ik} + \dots \end{aligned}$$

The intent of Eq. (16) is to avoid having to form the *separate* quantities  $\bar{a}_{00}$ ,  $\hat{a}_{00}$ ,  $\bar{a}_{0k}$ ,  $\hat{a}_{0k}$ , etc., and instead, to compute only the combined forms  $\hat{a}_{00}$ ,  $\hat{a}_{0k}$ , etc., using the combined form of the coordinate transformation,  $\hat{C}^{0k}$ . This approach saves overall computation time, although it allows the retention of some second-order terms in the linearized variables that would have disappeared otherwise. Thus, Eq. (16) is the matrix equation actually solved by MBDYPL, a somewhat more accurate representation than that of Eq. (15).

## B. Subroutine MBDYPL Usage

Just as in the other subroutines, MBDYPL is entered by means of either of two call statements, one of which initializes the program with the system constants.

### *Initializing Call Statement*

CALL MBDYPL(NH,H,MB,MA,PB,PA,G,PL,LI,TI)

The call statement arguments are exactly as defined previously, with exception of the two additions

LI(*i*) = array of linearized variable indicators;  $i \in \mathcal{P}$  (If  $\gamma_i$  and its derivatives are assumed small and therefore are to be linearized, LI(*i*) = 1. Otherwise, LI(*i*) = 0.)

TI(*i*) = array of terminal appendage body indicators;  $i \in \mathcal{P}$  (If TI(*i*) = 1,  $\mathcal{A}_i$  is a member of the *largest* terminal appendage in the system. If TI(*i*) = 0, it is not, although  $\gamma_i$  could still be a linearized variable.)

It should be made clear that LI(*i*) is an array to be supplied *by the user*; i.e., it is an *input* array. On the other hand, TI(*i*) is calculated *by the subroutine* for the user's information, and is an *output* quantity. As mentioned previously, it is desirable to identify the *largest* terminal appendage of the system and its member bodies. An algorithm to do just that is located in the initializing portion of the subroutine, and the results of its calculations are later used for optimum rearrangement and partitioning of the system matrix, as well as made available to the user in TI.

As before, MBDYPL is executed at every numerical integration step using the dynamic call statement.

### *Dynamic Call Statement*

CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,WO,WDOT)

All arguments of the dynamic call statement are exactly as defined for MBDY, with the understanding that  $TH(i)$ ,  $TA(i,j)$ , and  $FA(i,j)$  refer to  $\hat{\tau}_i$ ,  $\hat{T}^i$ , and  $\hat{F}^i$ , respectively, when  $i \in \mathcal{A}$ . In general, *any* torque or force expressions should be free of second-order terms in the linearized variables (for which  $LI(i) = 1$ ) or their derivatives.

The necessary specification statements for MBDYPL are the same as for MBDY, with the addition of the integers LI and TI.

*Required Specification Statements*

```

INTEGER  NH, H(n), PI(n), LI(n), TI(n)

REAL     MB(7), MA(n,7), PB(n,3), PA(n,n,3), G(n,3), TH(n),
         TB(3), TA(n,3), FB(3), FA(n,3), GM(n), GMD(n),
         GMDD(n), WO(3)

DOUBLE PRECISION  WDOT(n + 3)

```

Again, the statement,

```
PARAMETER  Q = n
```

must appear within the subroutine MBDYPL to properly allocate internal storage. A complete listing of MBDYPL is provided in Appendix C.

### C. Application of MBDYPL to the Five-Body System

1. *Large platform maneuver.* Since MBDYPL is designed to accommodate both large and small rotations in a hinge-connected system, Case 1 would be the obvious dynamic maneuver with which to test MBDYPL.

Recall that the maneuver consists of rotating the platform from its initial position ( $\gamma_1(0) = 218$  deg,  $\gamma_2 = -30$  deg) to the position  $\gamma_1 = 168$  deg,  $\gamma_2 = 20$  deg by simultaneous slewing about both axes at the commanded rate of 1 deg/s (starting at  $t = 0$ ).

The main simulation program listed in Fig. 6 can be used virtually "as is" by MBDYPL, with only the addition of LI(4) and TI(4) to the INTEGER statement and LI input elements to the DATA statement. In this case, linearization will be requested for  $\gamma_3$  and  $\gamma_4$  so that  $LI(1) = 0$ ,  $LI(2) = 0$ ,  $LI(3) = 1$ , and  $LI(4) = 1$ . Examination of TI, after executing "CALL MBDYPL (NH, . . .)," would show it to be identical to LI; i.e., bodies  $\mathcal{L}_3$  and  $\mathcal{L}_4$  would be identified as members of the largest system terminal appendage. The modified simulation program using MBDYPL is listed in Fig. 17.

Results of the platform maneuver, Case 1, using the partially linearized approach are shown in Fig. 18. Comparison of these to the results produced by MBDY in Fig. 7 shows the differences to be indistinguishable. Figure 19 expands the plots of  $\omega^o$  components for just the first 10 s for closer comparison (with Fig. 8).

While the accuracy of the MBDYPL run in this case was apparently excellent, the speed advantage over MBDY was only very slight, i.e., 430 versus 440 s of computation time for this four-hinge system. Since MBDYPL does perform a number of manipulations associated with partitioning and rearranging the system matrix as well as matrix multiplication to solve the partitioned system, some tradeoff between

the time *used* to accomplish these tasks and the time *saved* by recognizing certain constants and advanced matrix inversion should be expected. Just exactly where the "break-even" point between MBDY and MBDYPL speed is depends on both the number of hinges and the number of linearized hinges. However, in the case simulated here, we seem to be quite near the break-even point. In contrast, the system simulation in Ref. 7 with 14 hinges (10 linearized) was speeded up by a factor of two using MBDYPL.

It should also be noted that the subroutine MBDYPL does accommodate *prescribed* motions at *any* of its hinges, as do the other subroutines, whether the hinge is linearized or not.

## V. Summary and Conclusions

This report confines itself to the consideration of mathematical models useful for the description of hinge-connected rigid body system dynamics and, further, the computer simulation of those idealizations using general-purpose subroutines. As the sequel to Ref. 9, it documents in explicit detail the application of each of these subroutines under the circumstances for which they were especially designed, i.e., MBDY for unrestricted systems, MBDYTL and MBDYL for totally linearized systems, and MBDYPL for partially linearized systems. In each case, the subroutines, which are written in Fortran V, serve as the powerful tool that is called upon by the main simulation program (written here in CSSL III) to calculate vehicle attitude dynamics.

In the process of drawing conclusions about the relative utility of these programs, one must certainly begin with the obvious conclusion that MBDY, since it solves the exact and unrestricted equations of motion for the system, serves as the ultimate standard of accuracy for the rest. However, MBDY need not be restricted in its use to just the role of a "check routine." For small systems, it will run faster than MBDYPL, and one can certainly envision system rotations large enough to make the linearized routines completely unsuitable for the simulation accuracies required. Further, the judicious use of the option available in all the routines for *prescribed* hinge rotations can speed the running time of a simulation using MBDY remarkably. Thus, MBDY should be used often and to good advantage.

Of course, the utility of the completely linearized dynamic simulation subroutines, MBDYTL and MBDYL, was well exposed in Section III. For large or small systems of bodies in which all the hinge and reference body rotations are deemed "small," either due to fortunate circumstance or deliberately controlled constraint, the use of MBDYTL or MBDYL can offer tremendous speed advantages over MBDY and still maintain acceptable accuracies of the dynamic response. This totally linearized approach has proved to be of the greatest value in analyzing control problems at JPL, since most systems fall, or can be forced into, this category. In fact, the accuracy of MBDYL has been more than adequate for a large variety of space vehicle control problems studied at JPL. Of course, if the vehicle *control* dynamics can also be linearized, the inverted attitude dynamics matrix used by MBDYTL or MBDYL can be used directly in control stability analyses, i.e., to compute overall system root loci and/or eigenvalues.

Finally, the utility of MBDYPL is perhaps the least of the four routines, although it still offers some advantages, particularly for large systems. Where there are a

few, very large hinge rotations to be simulated but the large majority of system hinges exhibit only small excursions, MBDYPL may offer the only chance to achieve the desired accuracies in a reasonable simulation time. In similar circumstances, it may serve as a more *economical* reference or check solution source than MBDY. All in all, the partitioning scheme employed in MBDYPL, while not offering the tremendous speed advantages of MBDYL, at least provides some gain in efficiency in certain specialized instances.

As for the general-purpose character of all four subroutines, it is felt that their presentation in Fortran will make them widely useful, both to those who are comfortable with special-purpose simulation languages and to those who prefer to write their own simulation programs and integration subroutines. In order to make the routines more easily accessible, the entire package has been submitted to COSMIC (Computer Software Management and Information Center), University of Georgia, Athens, GA 30601, for evaluation and dissemination to interested governmental agencies as well as industrial, educational, and research institutions. While many may wish to further specialize one or more of these programs to their own particular needs, the four programs, as they are described here, can serve as a reliable and valuable addition to any simulation subroutine library.

```

PROGRAM 5-BODY SYSTEM SIMULATION USING MBDYPL --- CASE 1
  INTEGER NH,PI(4),H(4),LI(4),TI(4)
  ARRAY MB(7),MA(4,7),PB(4,3),PA(4,4,3),G(4,3),TB(3),TA(4,3)
  ARRAY FB(3),FA(4,3),TH(4),GM(4),GMD(4),GMDD(4),WO(3)
  DOUBLE PRECISION WOOT(7)
DATA F/O,1,0,3/PI/O,0,0,0/LI/O,0,1,1/
DATA MB/115.,315.,440.,14.,-14.,34.6,410./
DATA MA(1,1)/.35/MA(1,2)/.35/MA(1,7)/6.8/
DATA MA(2,1)/4.85/MA(2,2)/2.2/MA(2,3)/5.5/MA(2,4)/-.41/MA(2,5)/.07/
DATA MA(2,6)/-.54/MA(2,7)/57.5/
DATA MA(4,1)/27.2/MA(4,2)/.2/MA(4,3)/27.2/MA(4,7)/10.7/
DATA PB(1,3)/-1.5/PB(3,2)/-1.2/
DATA PA(1,1,3)/-.75/PA(1,2,2)/-.10/PA(1,2,3)/-.75/
DATA PA(2,2,2)/-.22/PA(2,2,3)/.20/
DATA PA(4,4,2)/3.3/
DATA G(1,3)/1./G(2,1)/-1./G(3,1)/1./G(4,3)/1./
  CONSTANT TM=.020,DS=.005,TR=2.0
  CONSTANT TFINAL=60.,CLKTIM=1100.
  CONSTANT L1=.23,L2=.21,L3=.31
  CONSTANT SLRAT=.01745329
  CONSTANT PHIZ=0.,THETZ=0.,PSIZ=0.
  CONSTANT W1I=0.,W2I=0.,W3I=0.
  CONSTANT K1=3500.,K2=3500.,K3=2000.,K4=2000.
  CONSTANT B1=20.,B2=20.,B3=10.,B4=10.
  CONSTANT IC1=218.,IC2=-30.
INITIAL
  NH=4
  RD=57.2957795 $ DR=1./RD $ GM1CI=IC1/RD $ GM2CI=IC2/RD
  GM1I=IC1/RD $ GM2I=IC2/RD
  CALL MBDYPL(NH,H,MB,MA,PB,PA,G,PI,LI,TI)
  PZI,PAI,PBI,PCI=HCK(INITL,PHIZ,THETZ,PSIZ)
ENC
DYNAMIC
  IF(TIME.GE.TFINAL) GO TO S1
DERIVATIVE BODYS
  VARIABLE TIME=0.
  CINTERVAL CI=.06
  XERROR W1=1.E-6,W2=1.E-6,W3=1.E-6,GM3=1.E-4
  STPCLK CLKTIM
  OUTPUT 50,W1D,W2D,W3D,W1,W2,W3,THET1,THET2,THET3,....
  IN1,IN2,IN3,GM1,GM2,GM3,GM4,PL1,PL2
  PREPAR W1,W2,W3,THET1,THET2,THET3,GM1D,GM2D,....
  GM1,GM2,GM3D,GM4D,GM3,GM4
NOSOPT
  WO(1)=W1 $ WO(2)=W2 $ WO(3)=W3
  PL1=RD*GM1 $ PL2=RD*GM2
  GM(1)=GM1 $ GM(2)=GM2 $ GM(3)=GM3 $ GM(4)=GM4
  GMD(1)=GM1D $ GMD(2)=GM2D $ GMD(3)=GM3D $ GMD(4)=GM4D
COMMENT
COMMENT CELESTIAL-SENSOR-MEASURED ANGLES
COMMENT
  D1,D2=HCK(MATRIX,FZO,PAC,PBO,PCO)
  NX0,NY0,NZ0=HCK(ITOB,0.,0.,1.,D1,D2)
  LX0,LY0,LZ0=HCK(ITOB,1.,0.,0.,D1,D2)
  THET1=ATAN2(NY0,NZ0) $ THET2=ATAN2(-NX0,NZ0)
  THET3=ATAN2(-LY0,LX0)

```

Fig. 17. Case 1 simulation program using MBDYPL

```

COMMENT
COMMENT   RATE FEEDBACK SIGNAL AND SWITCHING AMP. DYNAMICS
COMMENT
      IN1=-THET1-TR*W1 $ IN2=-THET2-TR*W2 $ IN3=-THET3-TR*W3
      AMP1=SWAMP(TIME,TH,DB,IN1)
      AMP2=SWAMP(TIME,TH,DB,IN2)
      AMP3=SWAMP(TIME,TH,DB,IN3)
COMMENT
COMMENT   PLATFORM POSITION COMMANDS
COMMENT
      GM1CD=SLRAT*(-1.*STEP(50.0,TIME)) $ GM1C=INTEG(GM1CD,GM1CI)
      GM2CD=SLRAT*(1.-STEP(50.,TIME)) $ GM2C=INTEG(GM2CD,GM2CI)
COMMENT
COMMENT   REFERENCE BODY EXTERNAL TORQUE
COMMENT   HINGE TORQUES
COMMENT
      TB(1)=L1*AMP1 $ TB(2)=L2*AMP2 $ TB(3)=L3*AMP3
      TH(1)=-K1*(GM1-GM1C)-B1*GM1D
      TH(2)=-K2*(GM2-GM2C)-B2*GM2D
      TH(3)=-K3*GM3-B3*GM3D
      TH(4)=-K4*GM4-B4*GM4D
COMMENT
COMMENT   SYSTEM ANGULAR ACCELERATIONS
COMMENT
      CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,W0,WDOT)
      W1D=WDOT(1) $ W2D=WDOT(2) $ W3D=WDOT(3)
COMMENT
COMMENT   SYSTEM ANGULAR RATES AND POSITIONS
COMMENT
      W1=INTEG(WDOT(1),W1I)
      W2=INTEG(WDOT(2),W2I)
      W3=INTEG(WDOT(3),W3I)
      GM1D=INTEG(WDOT(4),0.) $ GM2D=INTEG(WDOT(5),0.)
      GM3D=INTEG(WDOT(6),0.) $ GM4D=INTEG(WDOT(7),0.)
      GM1=INTEG(GM1D,GM1I) $ GM2=INTEG(GM2D,GM2I)
      GM3=INTEG(GM3D,0.) $ GM4=INTEG(GM4D,0.)
COMMENT
COMMENT   HCK PARAMETER RATES AND POSITIONS
COMMENT
      PZOD,PAOD,PBOD,PCOD=HCK(HCK,PZO,PAO,PBO,PCO,W1,W2,W3)
      PZO=INTEG(PZOD,PZI)
      PAO=INTEG(PAOD,PAI)
      PBO=INTEG(PBOD,PBI)
      PCO=INTEG(PCOD,PCI)

END
END
END
TERMINAL
S1.. CONTINUE
END
END

```

Fig. 17 (contd)

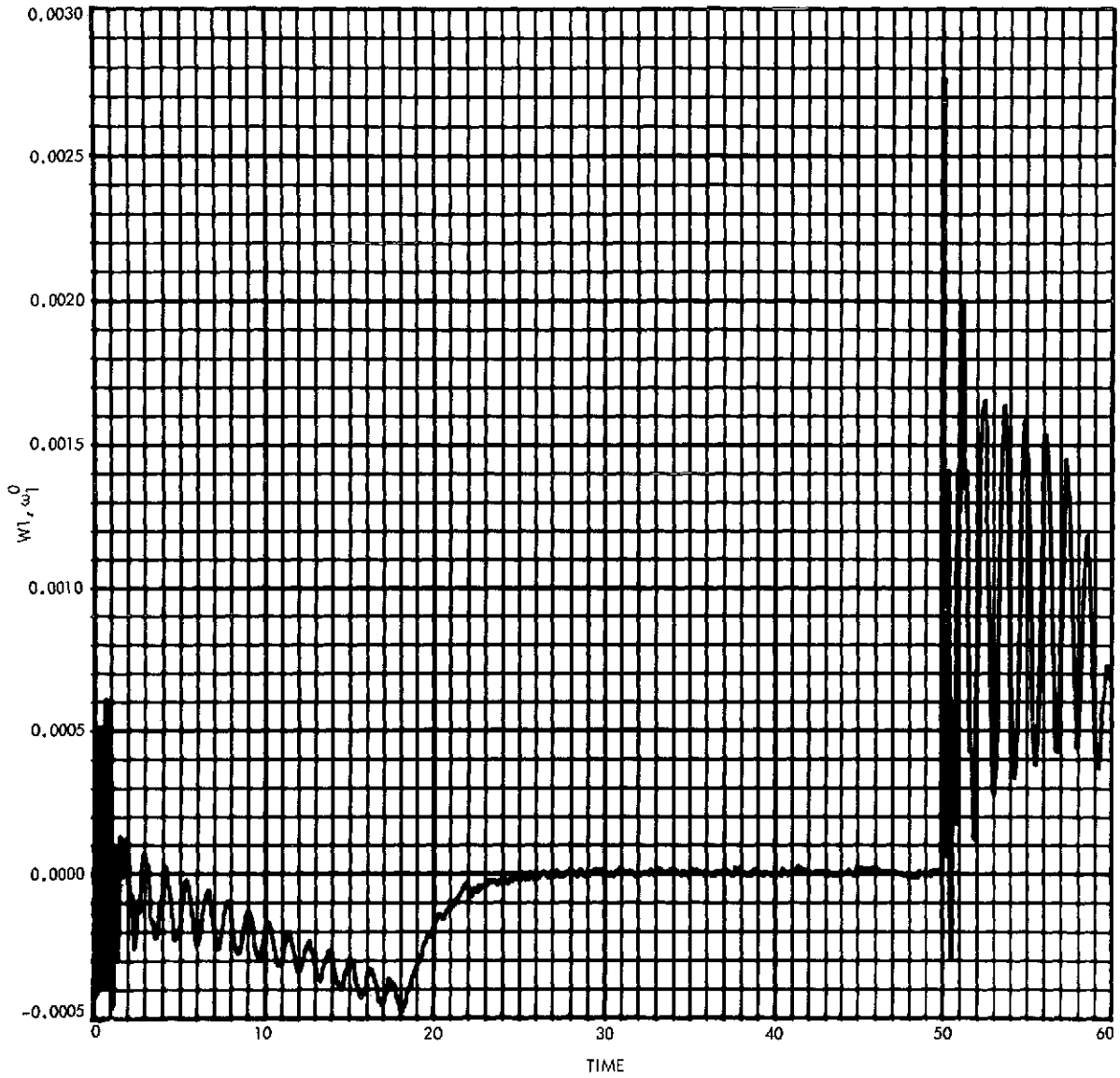


Fig. 18. Results of Case 1 simulation using MBDYPL



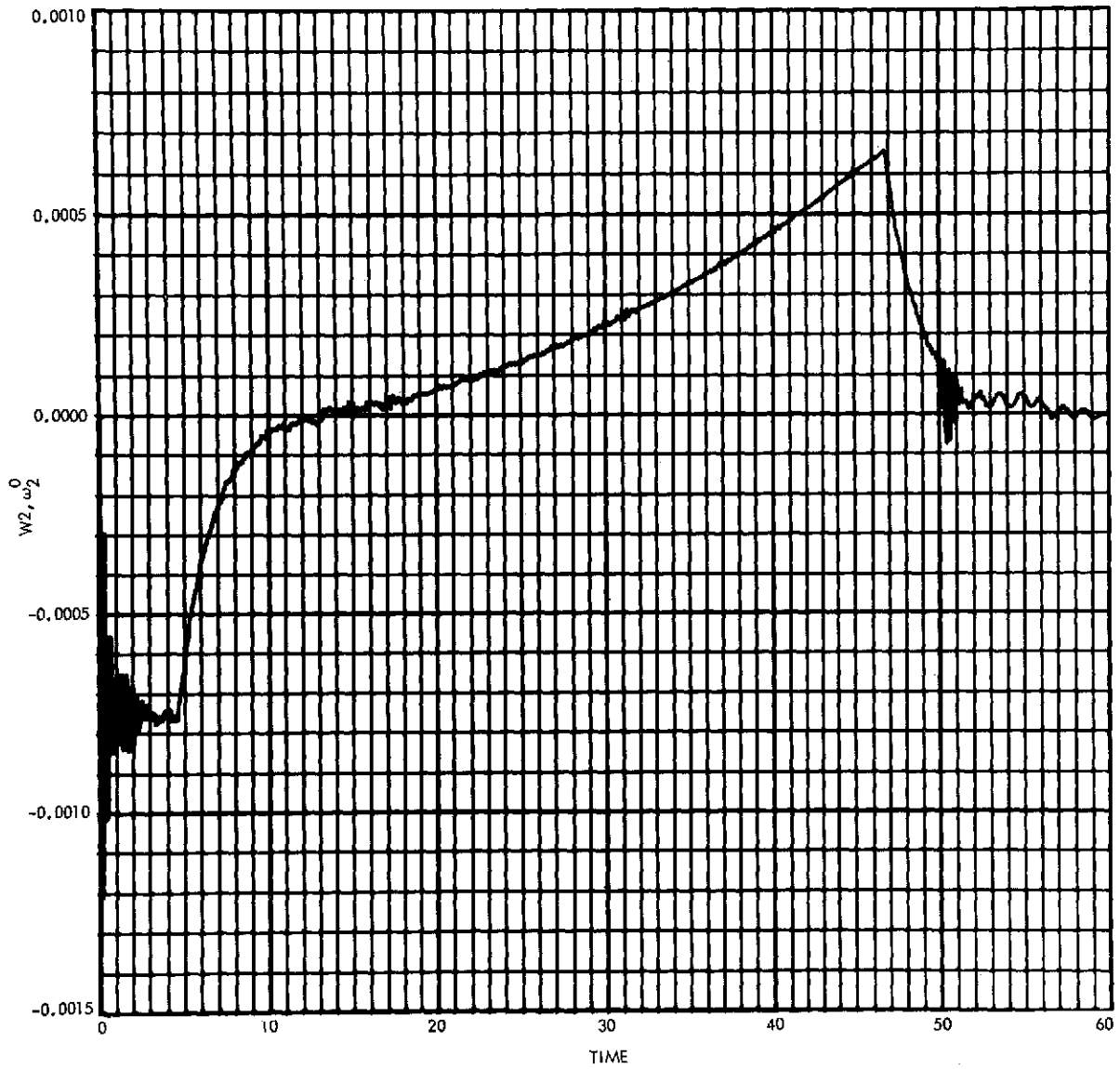


Fig. 18 (contd)

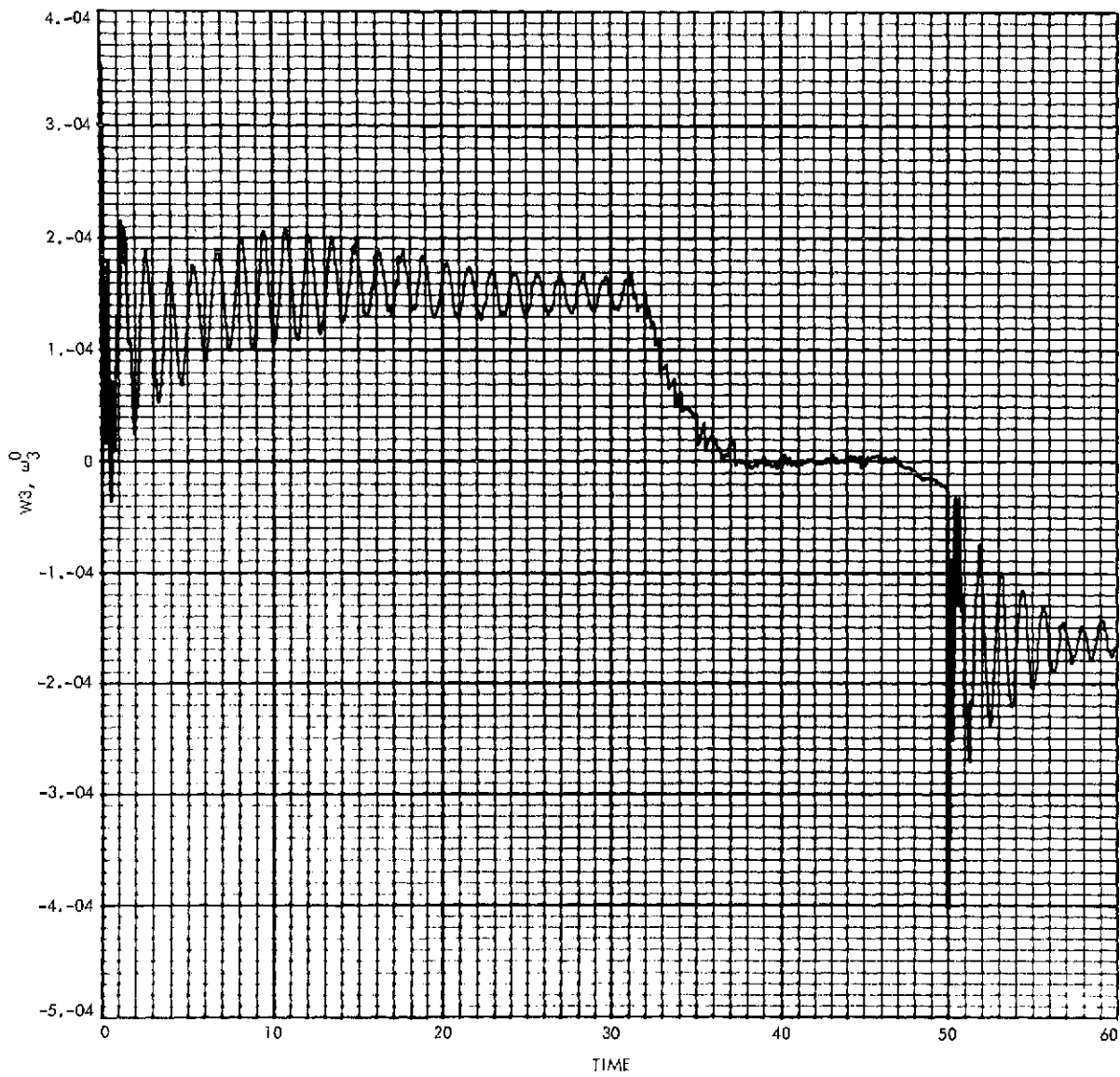


Fig. 18 (contd)



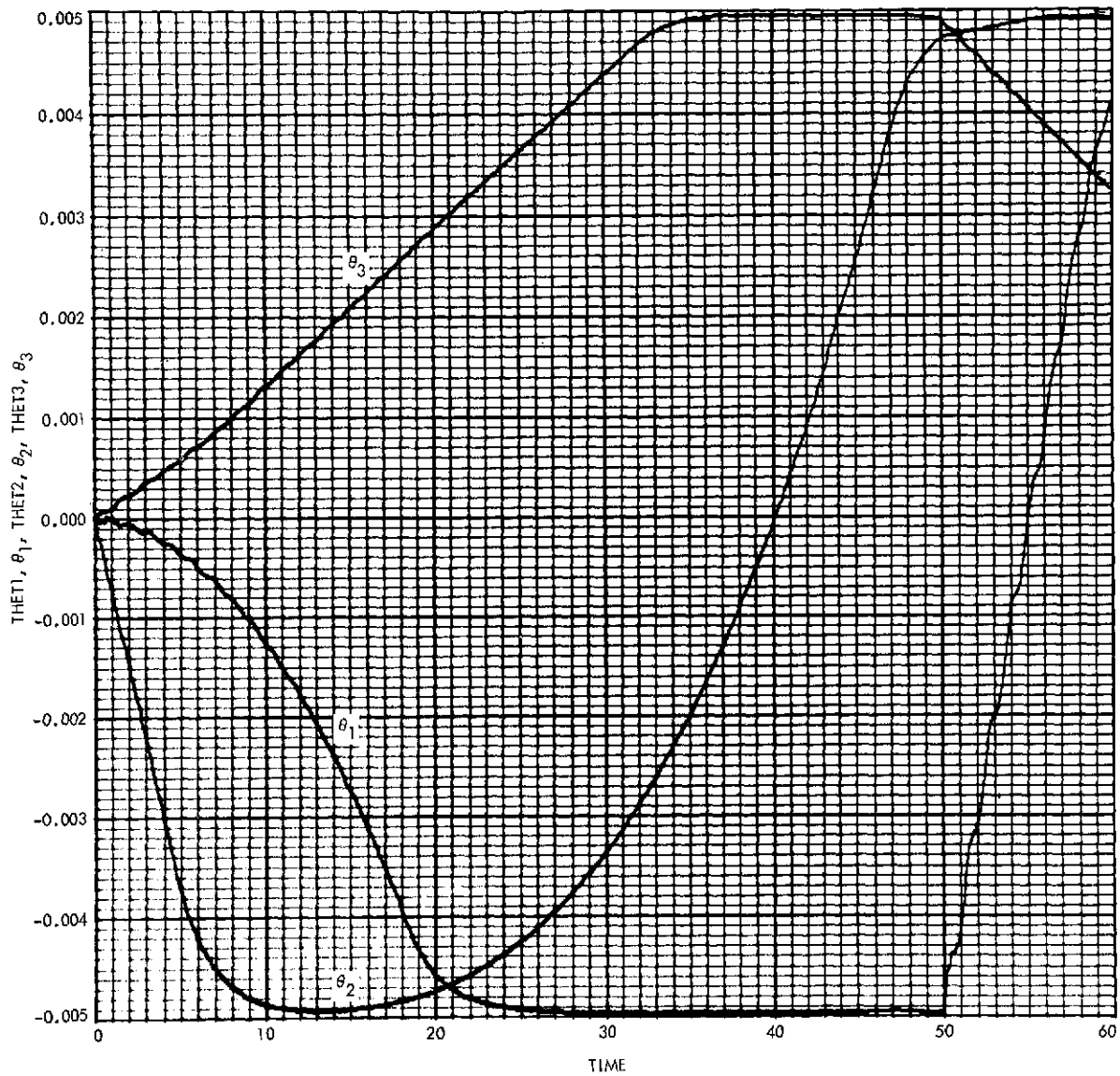


Fig. 18 (condt)

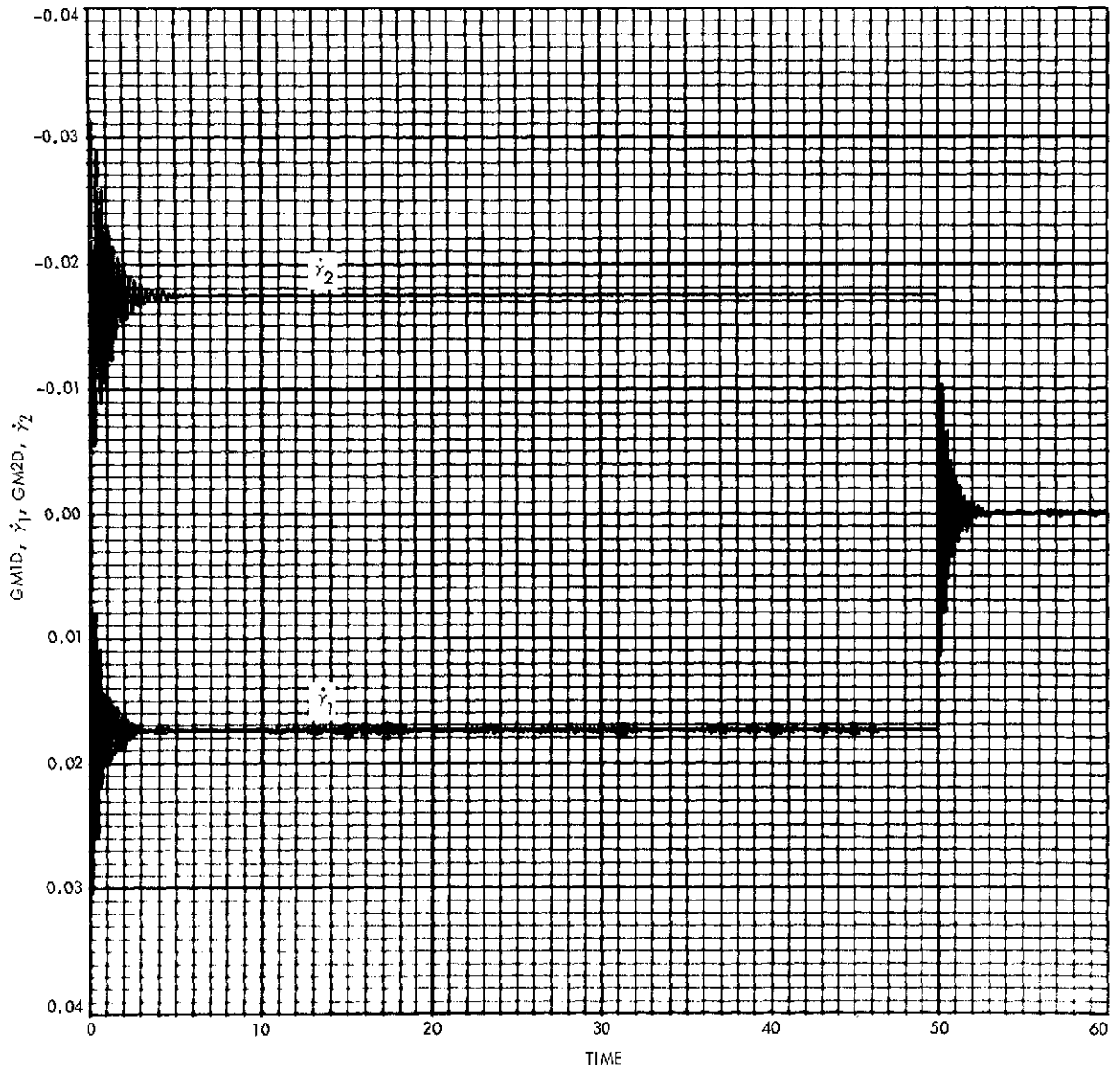


Fig. 18 (contd)

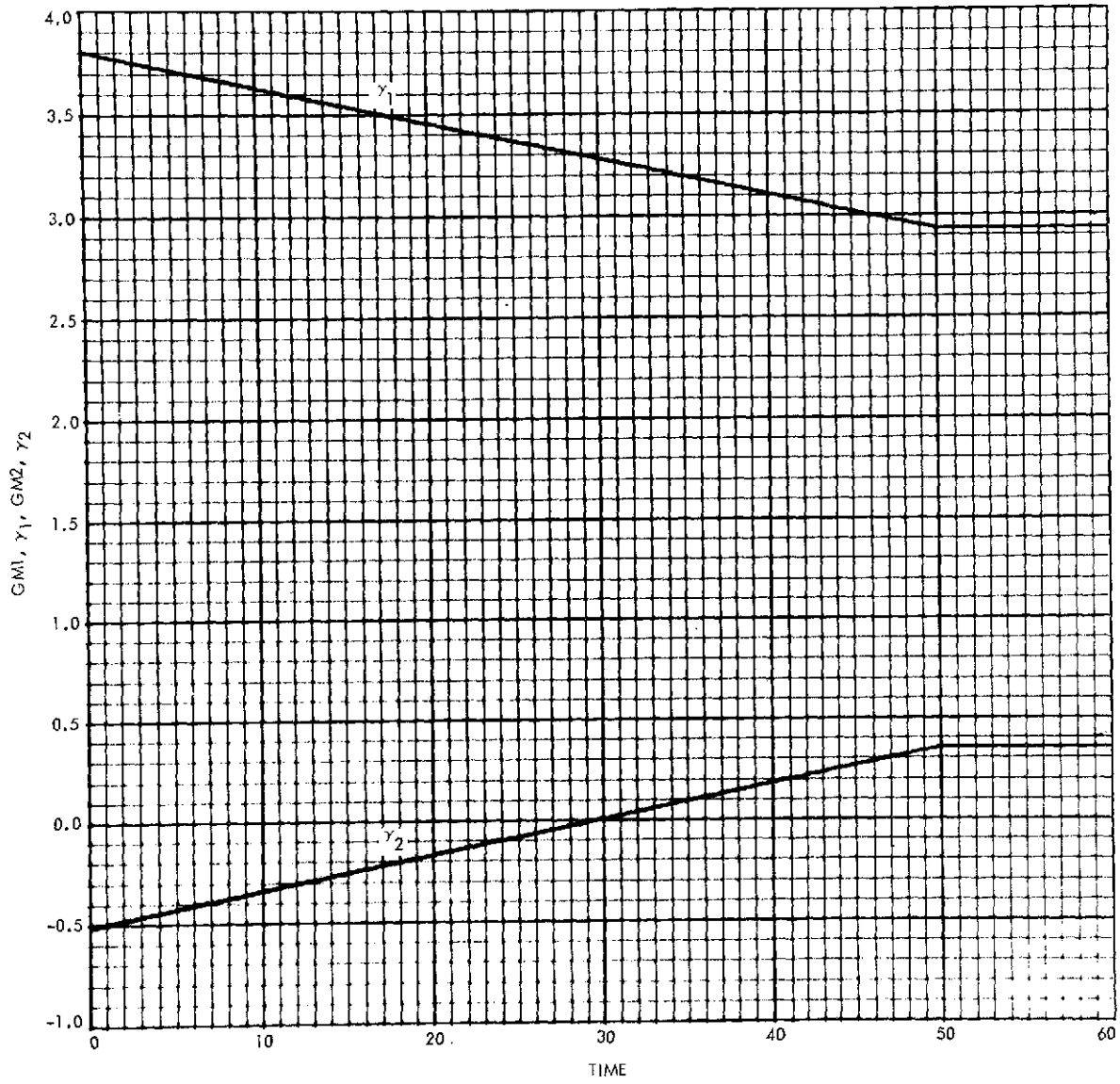


Fig. 18 (contd)

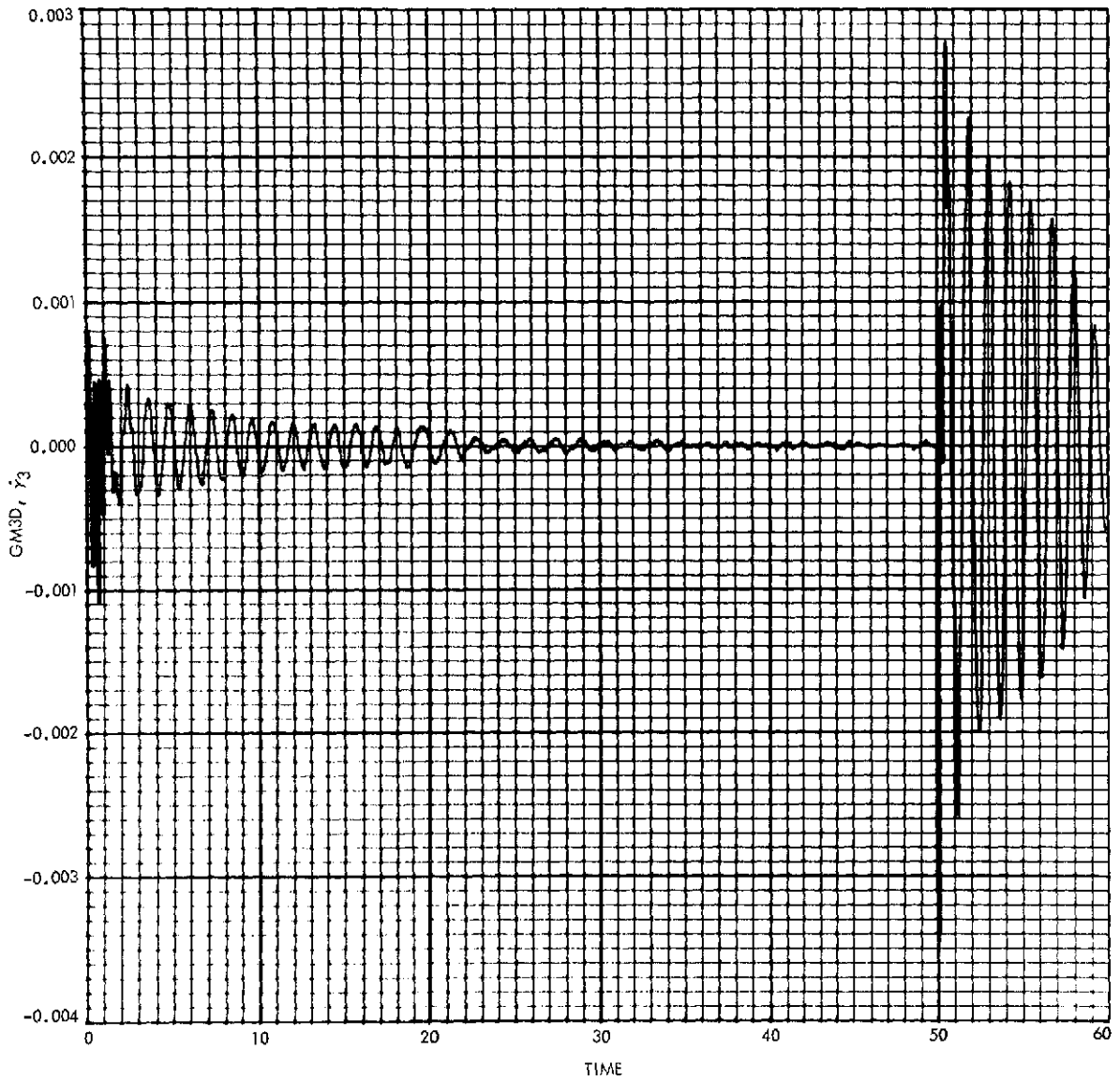


Fig. 18 (contd)

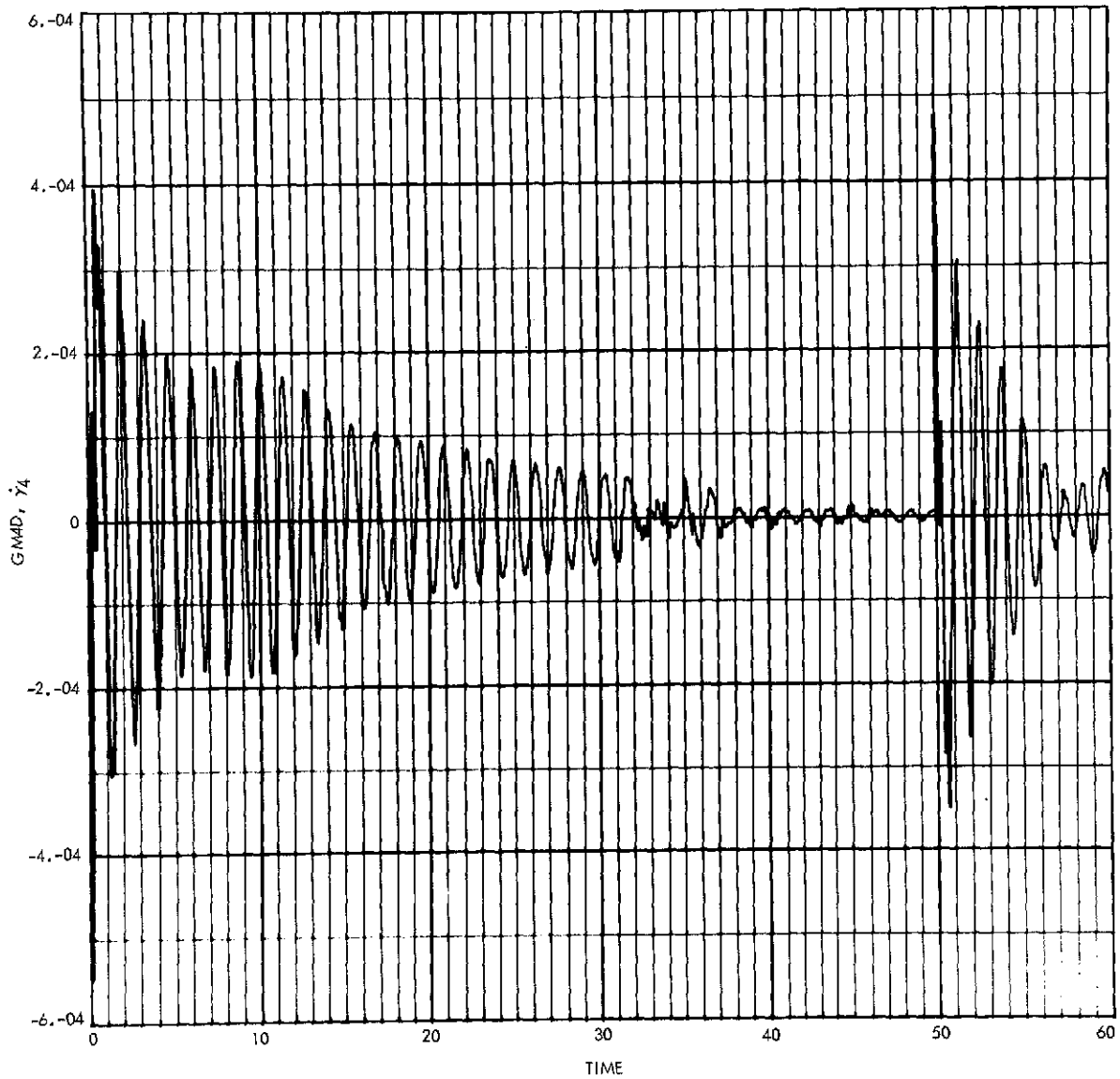


Fig. 18 (contd)

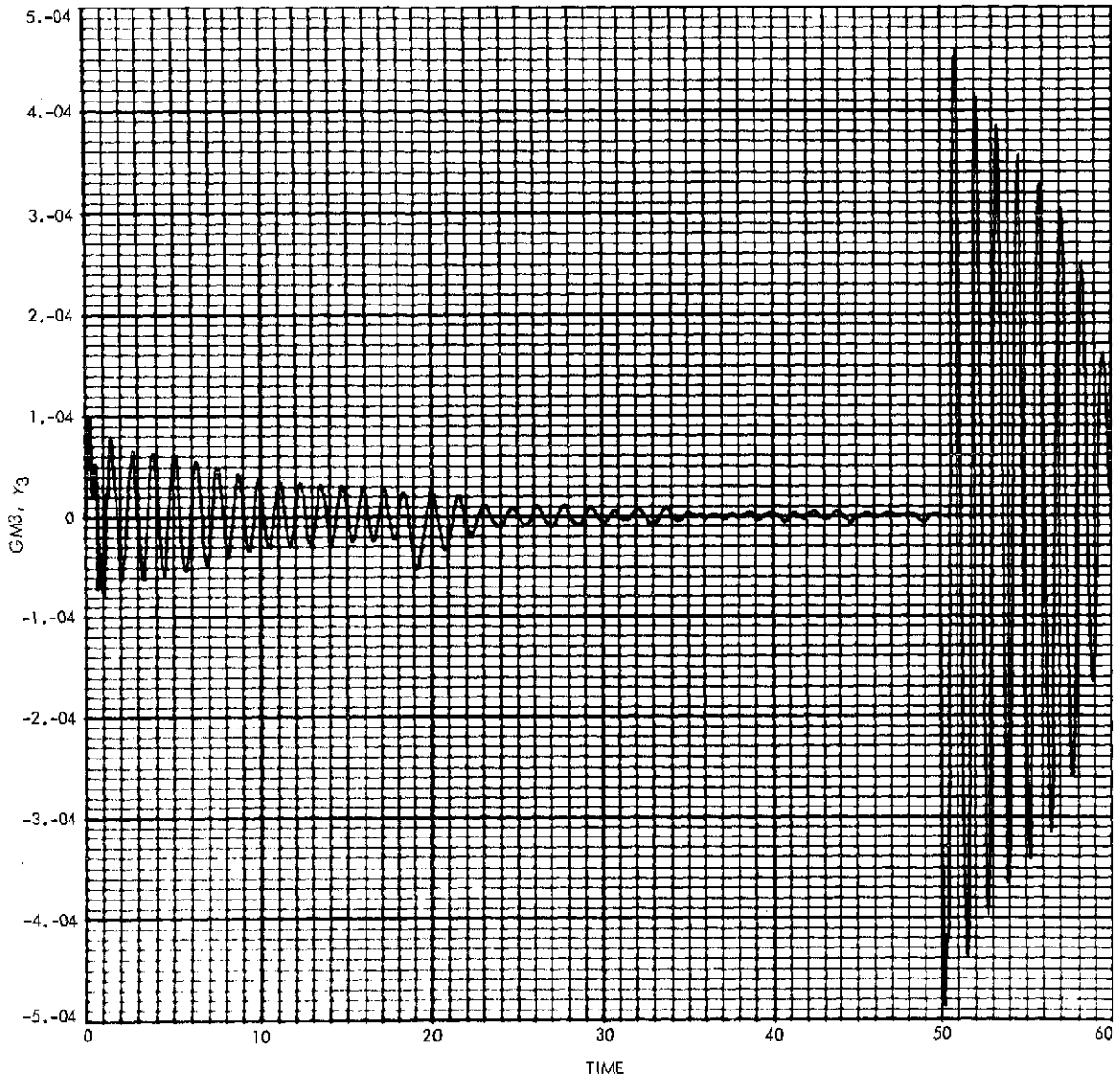


Fig. 18 (contd)



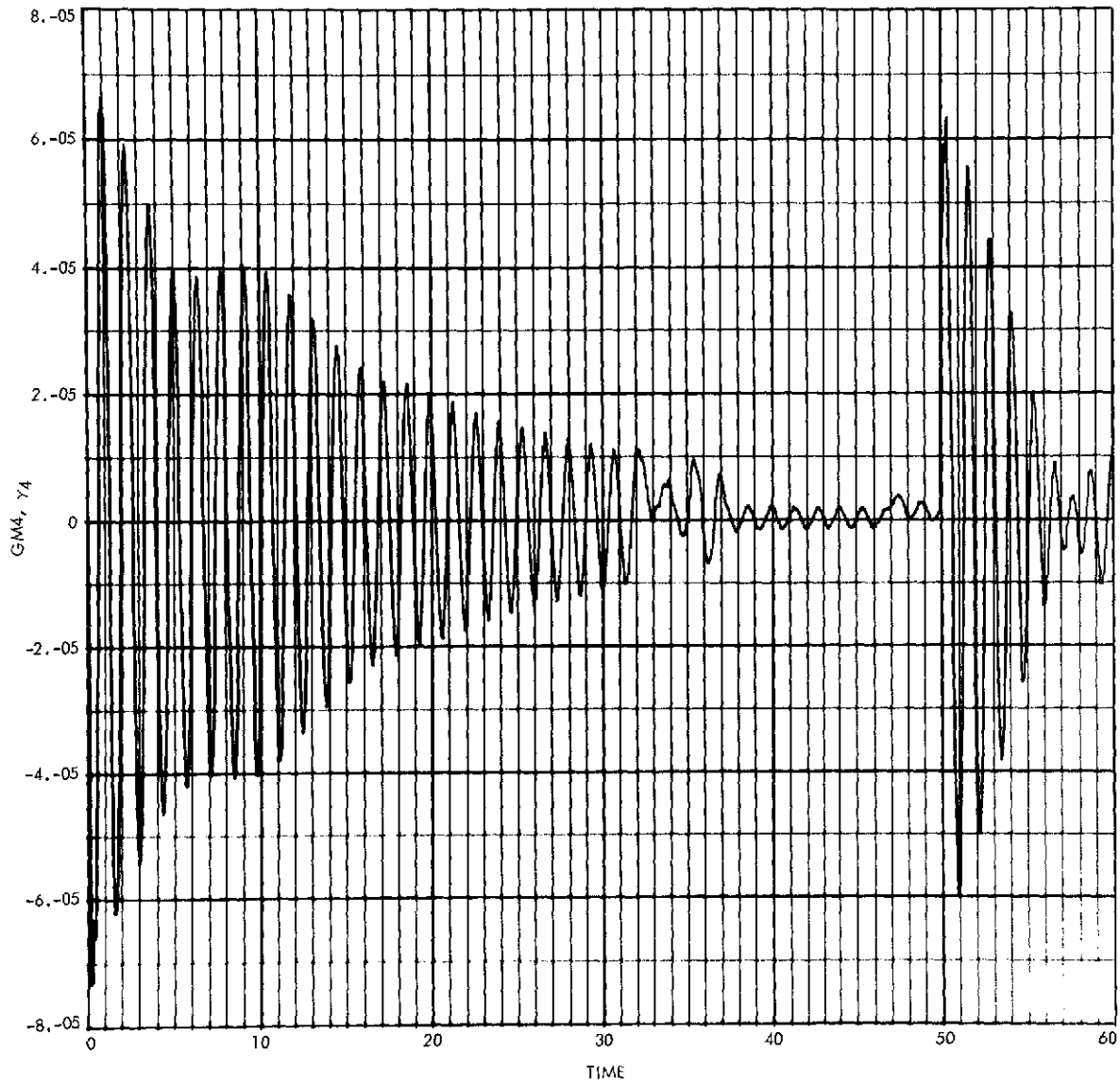


Fig. 18 (contd)

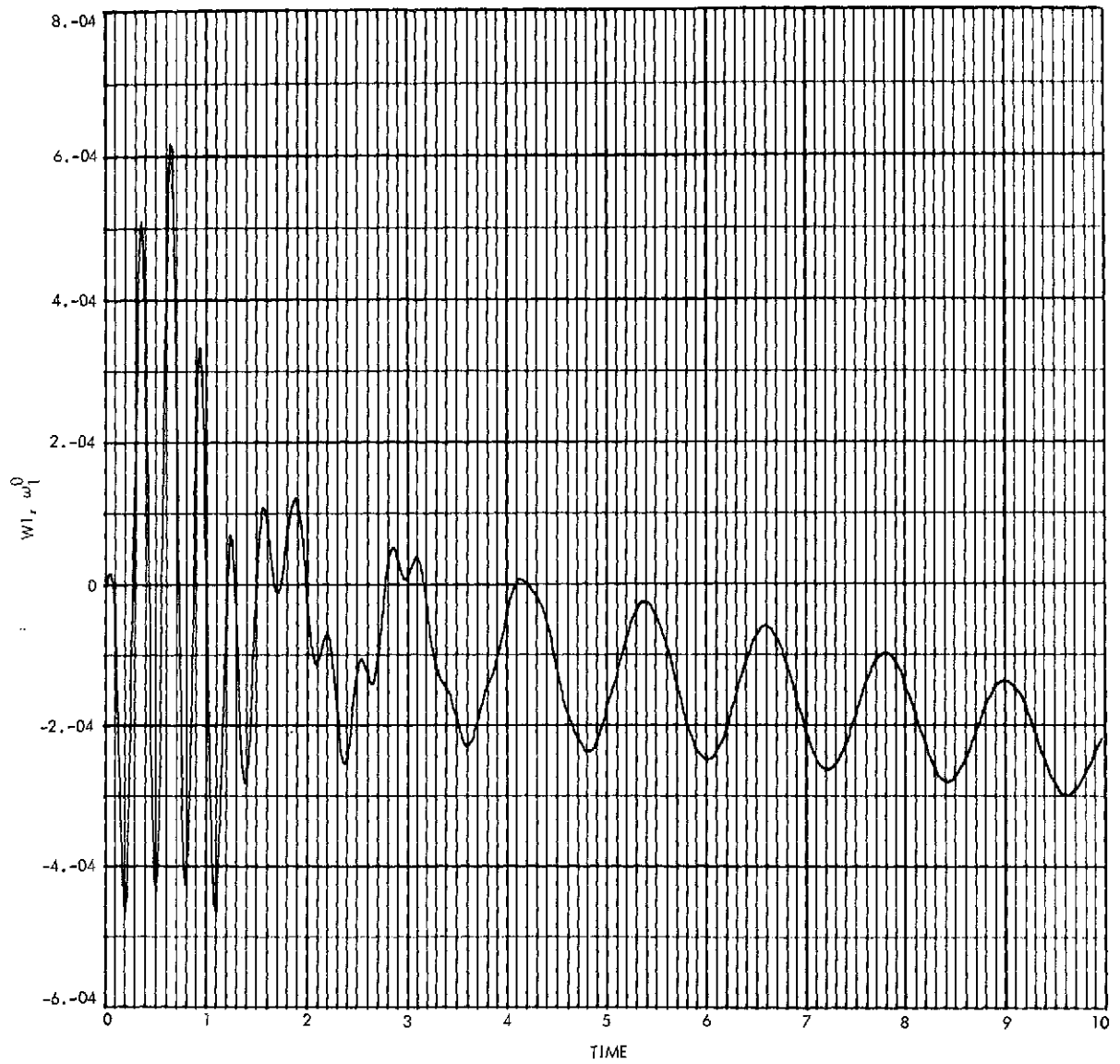


Fig. 19. Results of Case 1 simulation using MBDYPL (components of  $w^0$ ; first 10 s)

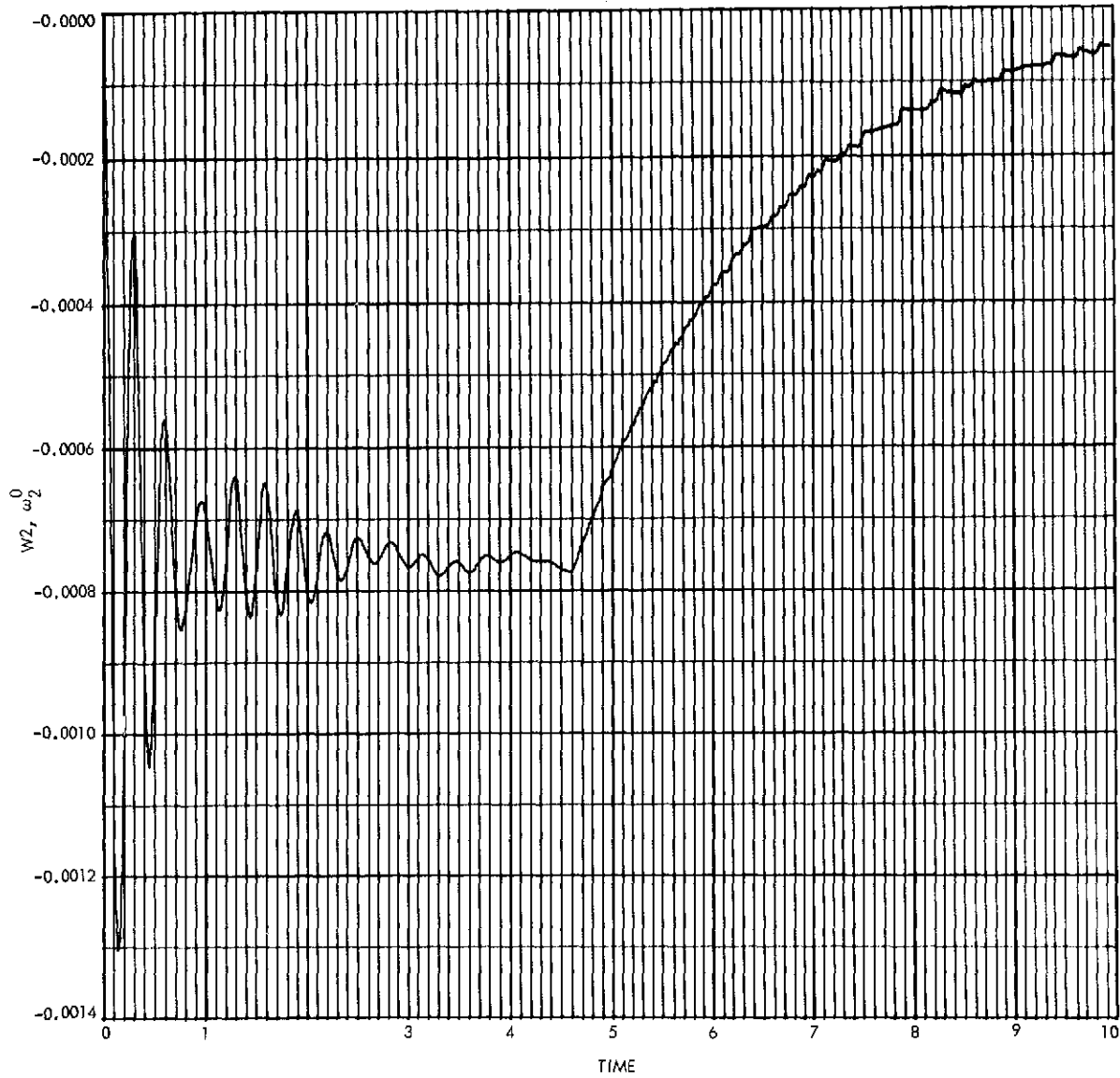


Fig. 19 (contd)

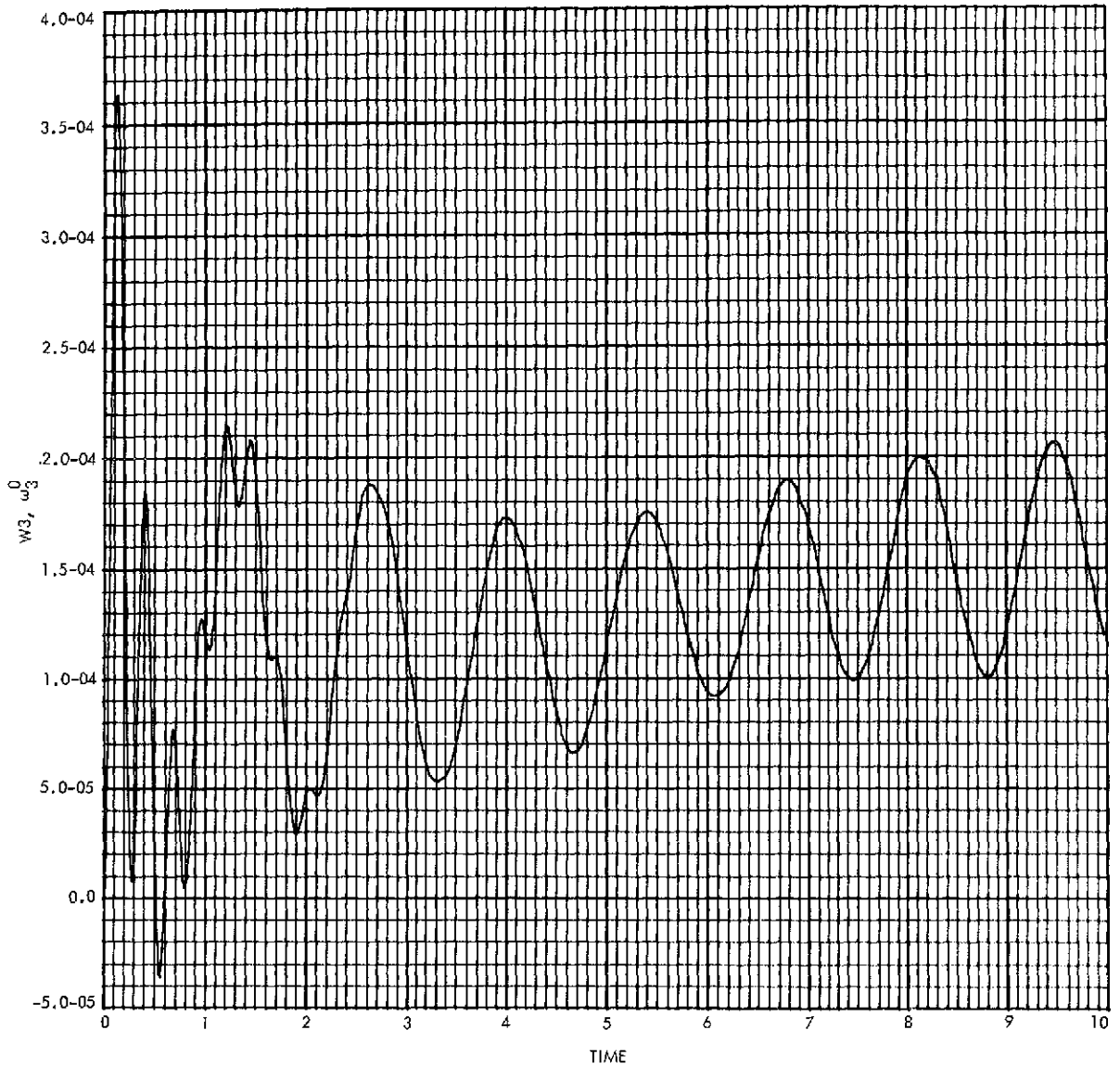


Fig. 19 (contd)

# Appendix A

## Subroutine MBDY Description

### I. Algorithm for Calculating Direction Cosine Matrices, $C^{or}$ ( $r \in \mathcal{P}$ )

As was pointed out in Section IIC, *all* the coordinate transformation, or direction cosine, matrices,  $C^{rj}$  ( $r, j \in \mathcal{B}$ ) need not be calculated if all the system equations are expressed in the reference body ( $\mathcal{A}_0$ ) coordinate frame. Thus, the objective is simply to obtain  $C^{or}$  for all  $r \in \mathcal{P}$ . An algorithm for accomplishing this is incorporated in MBDY and in MBDYTL and MBDYPL as well (MBDYL uses no direction cosine transformations). The computation procedure is as follows:

- Step 1. Let  $r = 1$
- Step 2. Calculate elements of the direction cosine matrix  $C^{jr}$ , where  $j = h_r$  (see Eq. 6)
- Step 3. If  $j = 0$ , jump to Step 5
- Step 4. Calculate elements of  $C^{or}$  from relation  $C^{or} = C^{oj}C^{jr}$
- Step 5. Let  $r = r + 1$
- Step 6. If  $r < n + 1$ , go back to Step 1. Otherwise, stop.

The calculations involved in Step 2, of course, may include the use of  $\sin \gamma_r$  and  $\cos \gamma_r$ , or they may use approximations thereof (i.e.,  $\gamma_r = \sin \gamma_r$ ,  $\cos \gamma_r = 1$ ), depending upon whether linearization is being employed or not. This sort of bootstrapping calculation of  $C^{or}$ , by way of Step 4, is made possible by the fact that  $j$  is always less than  $r$ , and therefore  $C^{oj}$  and  $C^{jr}$  must necessarily have been computed before coming to  $C^{or}$ .

### II. Summary of MBDY User Requirements

#### Subroutine Entry Statements

```
CALL MBDY(NH,H,MB,MA,PB,PA,G,PI)
```

```
CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,WO,WDOT)
```

#### Input/Output Variable Type and Storage Specifications—Fortran V

```
INTEGER NH, H(n), PI(n)
```

```
REAL MB(7), MA(n,7), PB(n,3), PA(n,n,3), G(n,3), TH(n),  
TB(3), TA(n,3), FB(3), FA(n,3), GM(n), GMD(n),  
GMDD(n), WO(3)
```

```
DOUBLE PRECISION WDOT(n+3)(where  $n$  = number of sys-  
tem hinges)
```

#### External Subroutines Called

SOD—double precision matrix equation solving subroutine; user-supplied (see line 437 in MBDY listing)

This subroutine solves the matrix equation  $A\dot{x} = EC$ , where  $A$  is of order  $IV \times IV$  and  $IV \cong 3$ . The solution is returned in  $EC$ .

*Subroutine Setup*

Insert the Fortran V statement (see listing)

```
PARAMETER Q = n
```

where  $n$  = number of system hinges.

*Data Restrictions*

$n \cong 1$

*Core Storage Required*

Code: 2000 words

Data: 500 words (minimum; increases with  $n$ )

*Listing*

```

1*      SUBROUTINE MBDY(NH,H,MB,MA,PB,PA,G,PI)
2*      C
3*      C      ADJUSTABLE DIMENSIONS
4*      C
5*      INTEGER H(1),PI(1)
6*      REAL MB(1),MA(NH,7),PB(NH,3),PA(NH,NH,3),G(NH,3)
7*      PARAMETER G=4
8*      PARAMETER S=Q+1,V=Q+3,V4=4*V,S3=3*S
9*      C
10*     C      ADDITIONAL DIMENSIONED VARIABLES
11*     C
12*     C      DOUBLE PRECISION A(V,V),WRK(V4),BMASS(S)
13*     C      DIMENSION PH(S,3,3),DX(S,S),DY(S,S),DZ(S,S),DXO(S,S),DYO(S,S),DZO(
14*     C      S,S),LX(S,S),LY(S,S),LZ(S,S),FEXO(S),FEYO(S),FEZO(S),GO(4,3),TXO(S
15*     C      S),TYO(S),TZO(S),PS(S,S,3,3),AS(Q,Q),AV(Q,3),T(1,S,3,3),A0(3,3),AB
16*     C      S(3,3),AIS(3),EA(3),CW(3),CE(3)
17*     C      INTEGER KK(Q),EPSI(Q,S)
18*     C      REAL LX,LY,LZ,IXX(S),IYY(S),IZZ(S),IXY(S),IXZ(S),IYZ(S)
19*     C      EQUIVALENCE (A,PS),(LX,DXO,AS),(LY,DYO),(LZ,DZO)
20*     C      NE=NH+1
21*     C
22*     C      INITIALIZE EPSI(K,J) WITH ZEROS AND DEFINE KK(K)
23*     C
24*     C      DO 85 K=1,NH
25*     C      KK(K)=K+1
26*     C      DO 85 J=1,NB
27*     85  EPSI(K,J)=0
28*     C
29*     C      DEFINE EPSI(K,J) USING H(IJ)
30*     C
31*     C      DO 86 K=1,NH
32*     C      DO 86 J=2,NB
33*     C      IF(K.EQ.(J-1)) EPSI(K,J)=1
34*     C      IF(K.LT.(J-1)) GO TO 87
35*     C      GO TO 86
36*     87  CCONTINUE
37*     C      JO=K+1
38*     C      JI=-1
39*     C      DO 89 L=JO,JI
40*     C      IF(K.GT.(L-1)) GO TO 89
41*     C      IF((EPSI(K,L).EQ.1).AND.(H(J-1).EQ.(L-1))) EPSI(K,J)=1
42*     89  CCONTINUE
43*     86  CCONTINUE
44*     C
45*     C      INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.O.S
46*     C      AND HINGE POINTS
47*     C
48*     C      IXX(1)=MB(1)
49*     C      IYY(1)=MB(2)
50*     C      IZZ(1)=MB(3)
51*     C      IXY(1)=MB(4)
52*     C      IXZ(1)=MB(5)
53*     C      IYZ(1)=MB(6)
54*     C      BMASS(1)=MB(7)
55*     C      TP=BMASS(1)
56*     C      DO 35 J=2,NB
57*     C      IXX(J)=MA(J-1,1)
58*     C      IYY(J)=MA(J-1,2)
59*     C      IZZ(J)=MA(J-1,3)
60*     C      IXY(J)=MA(J-1,4)
61*     C      IXZ(J)=MA(J-1,5)
62*     C      IYZ(J)=MA(J-1,6)
63*     C      BMASS(J)=MA(J-1,7)
64*     35  TP=TP+BMASS(J)
65*     C      DO 149 I=1,NB

```

```

66*      I1=I-1
67*      DC 149 J=1,NB
68*      J1=J-1
69*      IF(I.EQ.J) GO TO 163
70*      IF(I.GT.J) GO TO 70
71*      IF(I.EQ.1) GO TO 80
72*      IF(EPS(I1,J).EQ.1) GO TO 400
73*      70  LX(I,J)=PA(I1,I1,1)
74*          LY(I,J)=PA(I1,I1,2)
75*          LZ(I,J)=PA(I1,I1,3)
76*          GO TO 149
77*      400  CONTINUE
78*          DO 600 K=I,J1
79*          IF(EPS(K,J).EQ.1) GO TO 500
80*          600  CONTINUE
81*              GC TO 149
82*          500  LX(I,J)=PA(I1,K,1)
83*              LY(I,J)=PA(I1,K,2)
84*              LZ(I,J)=PA(I1,K,3)
85*              GC TO 149
86*          80  DO 90 L=1,J1
87*          IF(EPS(L,J).EQ.1) GO TO 101
88*          90  CONTINUE
89*              GC TO 149
90*          101  LX(I,J)=PB(L,1)
91*              LY(I,J)=PB(L,2)
92*              LZ(I,J)=PB(L,3)
93*              GC TO 149
94*          163  LX(I,J)=0.
95*              LY(I,J)=0.
96*              LZ(I,J)=0.
97*          149  CONTINUE
98*              DO 13 N=1,NB
99*              DC 13 J=1,NB
100*              DX(N,J)=LX(N,J)
101*              DY(N,J)=LY(N,J)
102*              DZ(N,J)=LZ(N,J)
103*              DO 13 K=1,NB
104*              DX(N,J)=DX(N,J)-(BMASS(K)/TM)*LX(N,K)
105*              DY(N,J)=DY(N,J)-(BMASS(K)/TM)*LY(N,K)
106*          13  DZ(N,J)=DZ(N,J)-(BMASS(K)/TM)*LZ(N,K)
107*          C
108*          C  CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
109*          C
110*          DC 31 N=1,NB
111*          PH(N,1,1)=IXX(N)
112*          PH(N,1,2)=-IXY(N)
113*          PH(N,1,3)=-IXZ(N)
114*          PH(N,2,2)=IYY(N)
115*          PH(N,2,3)=-IYZ(N)
116*          PH(N,3,3)=IZZ(N)
117*          DO 30 J=1,NB
118*          PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
119*          PH(N,1,2)=PH(N,1,2)-BMASS(J)*DX(N,J)*DY(N,J)
120*          PH(N,1,3)=PH(N,1,3)-BMASS(J)*DX(N,J)*DZ(N,J)
121*          PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
122*          PH(N,2,3)=PH(N,2,3)-BMASS(J)*DY(N,J)*DZ(N,J)
123*          30  PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)
124*          PH(N,2,1)=PH(N,1,2)
125*          PH(N,3,1)=PH(N,1,3)
126*          31  PH(N,3,2)=PH(N,2,3)
127*          RETURN
128*          ENTRY MRATE(NH,TH,IB,TA,FB,FA,GM,GMD,GMD,WC,EC)
129*          DIMENSION TB(3),TA(NH,3),FB(3),FA(NH,3),GM(1),GMD(1),GMD(1),
130*          TM(NH),WD(3),WXC(S),WYC(S),WZC(S),E(5,1)
131*          DOUBLE PRECISION ECIV)
132*          C
133*          C  BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES
134*          C
135*          DO 335 J=1,NH
136*          K=H(J)

```



```

137*      SGM=SIN(GM(J))
138*      CGM=COS(GM(J))
139*      CGM1=1.-CGM
140*      G1=CGM1*G(J,1)
141*      G2=CGM1*G(J,2)
142*      G3=CGM1*G(J,3)
143*      SG1=SGM*G(J,1)
144*      SG2=SGM*G(J,2)
145*      SG3=SGM*G(J,3)
146*      G1S=G1*G(J,1)
147*      G2S=G2*G(J,2)
148*      G3S=G3*G(J,3)
149*      G12=G1*G(J,2)
150*      G13=G1*G(J,3)
151*      G23=G2*G(J,3)
152*      AB(1,1)=CGM*G1S
153*      AB(1,2)=SG3*G12
154*      AB(1,3)=-SG2*G13
155*      AB(2,1)=-SG3*G12
156*      AB(2,2)=CGM*G2S
157*      AB(2,3)=SG1*G23
158*      AB(3,1)=SG2*G13
159*      AB(3,2)=-SG1*G23
160*      AB(3,3)=CGM*G3S
161*      IF(K.EQ.0) GO TO 3350
162*      DO 334 L=1,3
163*      DC 334 M=1,3
164*      T(1,J+1,L,M)=0.
165*      DC 334 I=1,3
166*      334  T(1,J+1,L,M)=T(1,J+1,L,M)+AB(L,I)*T(1,K+1,I,M)
167*      GC TO 335
168*      3350 CONTINUE
169*      DC 3351 L=1,3
170*      DO 3351 M=1,3
171*      3351  T(1,J+1,L,M)=AB(L,M)
172*      335 CONTINUE
173*      C
174*      C   COORD. TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
175*      C
176*      DO 362 I=1,NH
177*      DC 362 J=1,3
178*      GO(I,J)=0.
179*      DC 362 K=1,3
180*      GO(I,J)=GO(I,J)+T(1,I+1,K,J)*G(I,K)
181*      362 CONTINUE
182*      C
183*      C   ANG. VELOCITY COMPONENTS OF EACH BODY (IN REF. BODY FRAME)
184*      C
185*      DC 361 J=1,NB
186*      WX(J)=W(1)
187*      WY(J)=W(2)
188*      WZ(J)=W(3)
189*      DC 36 K=1,NH
190*      IF(EPS(K,J).EQ.0) GO TO 36
191*      WXC(J)=WX(J)+GMD(K)*GO(K,1)
192*      WY(J)=WY(J)+GMD(K)*GO(K,2)
193*      WZ(J)=WZ(J)+GMD(K)*GO(K,3)
194*      36 CONTINUE
195*      361 CONTINUE
196*      C
197*      C   TRANSFORM AUGMENTED BODY INERTIA DYADICS TO REF. BODY FRAME
198*      C
199*      DC 363 I=2,NB
200*      DO 364 J=1,3
201*      DC 364 K=1,3
202*      AB(J,K)=0.
203*      DC 364 L=1,3
204*      AB(J,K)=AB(J,K)+PH(I,J,L)*T(1,I,L,K)
205*      364 CONTINUE
206*      DO 365 J=1,3
207*      DC 365 K=1,3
208*      PS(I,I,J,K)=0.

```

```

209*      DC 365 L=1,3
210*      PS(I,I,J,K)=PS(I,I,J,K)+T(I,I,L,J)*AB(I,L,K)
211*      365  C(NTINUE
212*      363  CONTINUE
213*      FEYC(1)=FB(1)
214*      FEYC(2)=FB(2)
215*      FEZC(1)=FB(3)
216*      TXC(1)=TB(1)
217*      TYC(1)=TB(2)
218*      TZC(1)=TB(3)
219*      DO 17 I=2,NB
220*      K=I-1
221*      FEYC(I)=T(I,I,I,1)*FA(K,1)+T(I,I,2,1)*FA(K,2)+T(I,I,3,1)*FA(K,3)
222*      FEYC(I)=T(I,I,I,2)*FA(K,1)+T(I,I,2,2)*FA(K,2)+T(I,I,3,2)*FA(K,3)
223*      FEZC(I)=T(I,I,I,3)*FA(K,1)+T(I,I,2,3)*FA(K,2)+T(I,I,3,3)*FA(K,3)
224*      TYC(I) =T(I,I,I,1)*TA(K,1)+T(I,I,2,1)*TA(K,2)+T(I,I,3,1)*TA(K,3)
225*      TYC(I)= T(I,I,I,2)*TA(K,1)+T(I,I,2,2)*TA(K,2)+T(I,I,3,2)*TA(K,3)
226*      TZC(I)= T(I,I,I,3)*TA(K,1)+T(I,I,2,3)*TA(K,2)+T(I,I,3,3)*TA(K,3)
227*      DO 17 J=1,NB
228*      DXC(I,J)=T(I,I,I,1)*DX(I,J)+T(I,I,2,1)*DY(I,J)+T(I,I,3,1)*DZ(I,J)
229*      DYC(I,J)=T(I,I,I,2)*DX(I,J)+T(I,I,2,2)*DY(I,J)+T(I,I,3,2)*DZ(I,J)
230*      DZC(I,J)=T(I,I,I,3)*DX(I,J)+T(I,I,2,3)*DY(I,J)+T(I,I,3,3)*DZ(I,J)
231*      17  CONTINUE
232*      DC 367 I=1,NB
233*      DXC(1,I)=DX(1,I)
234*      DYC(1,I)=DY(1,I)
235*      367  DZC(1,I)=DZ(1,I)
236*      C
237*      C      ADDITIONAL AUGMENTED INERTIA DYADICS (IN REF. BODY FRAME)
238*      C
239*      DO 37 I=1,NB
240*      DC 37 J=1,NB
241*      IF(I.EQ.J) GO TO 37
242*      PS(I,J,1,1)=-TM*(DYC(I,J)*DYC(J,I)+DZC(I,J)*DZC(J,I))
243*      PS(I,J,1,2)=TM*DXC(J,I)*DYC(I,J)
244*      PS(I,J,1,3)=TM*DXC(J,I)*DZC(I,J)
245*      PS(I,J,2,1)=TM*DYC(J,I)*DXC(I,J)
246*      PS(I,J,2,2)=-TM*(DXC(I,J)*DXC(J,I)+DZC(I,J)*DZC(J,I))
247*      PS(I,J,2,3)=TM*DYC(J,I)*DZC(I,J)
248*      PS(I,J,3,1)=TM*DZC(J,I)*DXC(I,J)
249*      PS(I,J,3,2)=TM*DZC(J,I)*DYC(I,J)
250*      PS(I,J,3,3)=-TM*(DXC(I,J)*DXC(J,I)+DYC(I,J)*DYC(J,I))
251*      37  CONTINUE
252*      DC 751 M=1,3
253*      DO 751 N=1,3
254*      751  PS(1,1,M,N)=PH(1,M,N)
255*      C
256*      C      VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
257*      C      (QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
258*      C      VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
259*      C
260*      DC 230 N=1,NB
261*      CPX=0.
262*      CPY=0.
263*      CPZ=0.
264*      DC 2301 L=1,NB
265*      IF(N.EQ.L) GO TO 2301
266*      WEX=WYO(L)*DZO(L,N)-WZO(L)*DYO(L,N)
267*      WDY=WZO(L)*DXO(L,N)-WXO(L)*DZO(L,N)
268*      WEZ=WXO(L)*DYO(L,N)-WYO(L)*DXO(L,N)
269*      WWDX=TM*(WYO(L)*WDZ-WZO(L)*WDY)+FEYC(L)
270*      WWDY=TM*(WZO(L)*WDX-WXO(L)*WDZ)+FEYC(L)
271*      WWDZ=TM*(WXO(L)*WDY-WYO(L)*WDX)+FEZC(L)
272*      DWWDX=DYO(N,L)*WWDX-DZO(N,L)*WWDY
273*      DWWDY=DZO(N,L)*WWDX-DXO(N,L)*WWDZ
274*      DWWDZ=DXO(N,L)*WWDY-DYO(N,L)*WWDX
275*      CFX=CPX+DWWDX
276*      CPY=CPY+DWWDY
277*      CFZ=CPZ+DWWDZ
278*      2301 CONTINUE
279*      OFX=DYC(N,K)*FEZO(N,J)-DZO(N,N)*FEYC(N)

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280*      DFY=DZO(N,N)*FEXO(N)-DXO(N,N)*FEZO(N)
281*      DFZ=DXO(N,N)*FEYO(N)-DYO(N,N)*FEXO(N)
282*      HX=PS(N,N,1,1)*WXO(N)+PS(N,N,1,2)*WYO(N)+PS(N,N,1,3)*WZO(N)
283*      HY=PS(N,N,2,1)*WXO(N)+PS(N,N,2,2)*WYO(N)+PS(N,N,2,3)*WZO(N)
284*      HZ=PS(N,N,3,1)*WXO(N)+PS(N,N,3,2)*WYO(N)+PS(N,N,3,3)*WZO(N)
285*      K = 3*(N-1)
286*      E(K+1,1)=HY*WZO(N)-HZ*WYO(N)+TXO(N)+CPX+DFX
287*      E(K+2,1)=HZ*WXO(N)-HX*WZO(N)+TYO(N)+CPY+DFY
288*      E(K+3,1)=HX*WYO(N)-HY*WXO(N)+TZO(N)+CPZ+DFZ
289*      C(NTINUE)
290*      C
291*      C      ACC MATRIX ELEMENT COMPUTATION (3X3)
292*      C
293*      D( 3001 I=1,3
294*      DO 3001 J=1,3
295*      3001  ACC(I,J)=D.
296*      DO 3 I=1,NB
297*      D( 3 J=1,NB
298*      ACC(1,1)=ACC(1,1)+PS(I,J,1,1)
299*      ACC(1,2)=ACC(1,2)+PS(I,J,1,2)
300*      ACC(1,3)=ACC(1,3)+PS(I,J,1,3)
301*      ACC(2,2)=ACC(2,2)+PS(I,J,2,2)
302*      ACC(2,3)=ACC(2,3)+PS(I,J,2,3)
303*      ACC(3,3)=ACC(3,3)+PS(I,J,3,3)
304*      3      CONTINUE
305*      ACC(2,1)=ACC(1,2)
306*      ACC(3,1)=ACC(1,3)
307*      ACC(3,2)=ACC(2,3)
308*      C
309*      C      ACK VECTOR ELEMENT COMPUTATION (3X1)
310*      C
311*      DC 7 K=1,NH
312*      AV(K,1)=0.
313*      AV(K,2)=0.
314*      AV(K,3)=0.
315*      DC 7 I=1,NB
316*      DO 7 J=1,NB
317*      IF(EPS(K,J).EQ.0) GO TO 7
318*      DO 10 M=1,3
319*      DC 10 L=1,3
320*      10  AV(K,M)=AV(K,M)+PS(I,J,M,L)*GO(K,L)
321*      7      C(NTINUE)
322*      C
323*      C      APM SCALAR ELEMENT COMPUTATION
324*      C
325*      DC 14 K=1,NH
326*      DO 14 M=1,NH
327*      IF(K.GT.M) GO TO 14
328*      AIS(1)=0.
329*      AIS(2)=0.
330*      AIS(3)=0.
331*      DC 15 J=1,NB
332*      DO 15 I=1,NB
333*      IF((EPS(K,J).EQ.0).OR.(EPS(M,I).EQ.0)) GO TO 15
334*      DO 18 N=1,3
335*      DC 18 L=1,3
336*      18  AIS(N)=AIS(N)+PS(J,I,N,L)*GO(M,L)
337*      15  C(NTINUE)
338*      AIS(K,M)=GO(K,1)+AIS(1)+GO(K,2)+AIS(2)+GO(K,3)+AIS(3)
339*      14  C(NTINUE)
340*      C
341*      C      C(MPUTE CORRECTION ELEMENTS FOR (E) VECTOR
342*      C
343*      DC 40 I=1,NB
344*      EA(1)=0.
345*      EA(2)=0.
346*      EA(3)=0.
347*      DC 41 J=1,NB
348*      DO 411 M=1,3
349*      411  CA(M)=0.
350*      DO 42 K=1,NH

```

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351*      IF(EPS(K,J).EQ.0) GO TO 42
352*      KB=KK(K)
353*      CW(1)=CW(1)+GMD(K)*(WYO(KB)*GO(K,3)-WZC(KB)*GO(K,2))
354*      CW(2)=CW(2)+GMD(K)*(WZO(KB)*GO(K,1)-WXO(KB)*GO(K,3))
355*      CW(3)=CW(3)+GMD(K)*(WXO(KB)*GO(K,2)-WYC(KB)*GO(K,1))
356*      42  CONTINUE
357*      DC 45 M=1,3
358*      DO 45 L=1,3
359*      45  E(M)=EA(M)+PS(I,J,M,L)*CW(L)
360*      41  CONTINUE
361*      K1=3*(I-1)
362*      E(K1+1,1)=E(K1+1,1)-EA(1)
363*      E(K1+2,1)=E(K1+2,1)-EA(2)
364*      E(K1+3,1)=E(K1+3,1)-EA(3)
365*      40  CONTINUE
366*      DO 55 MI=1,3
367*      55  EC(MI)=E(MI,1)
368*      DO 52 J=2,NB
369*      DC 52 M=1,3
370*      K1=3*(J-1)*M
371*      52  E(M)=EC(M)+E(K1,1)
372*      I=0
373*      DC 60 K=1,NH
374*      IF(PI(K).NE.0) GO TO 60
375*      I=I+1
376*      EC(I+3)=0.
377*      DC 601 M=1,3
378*      601  CE(M)=0.
379*      DC 61 J=1,NB
380*      IF(EPK(K,J).EQ.0) GO TO 61
381*      DC 65 M=1,3
382*      J1=3*(J-1)*M
383*      65  CE(M)=CE(M)+E(J1,1)
384*      61  CONTINUE
385*      DO 66 L=1,3
386*      66  E(I+3)=EC(I+3)+GO(K,L)*CE(L)
387*      EC(I+3)=EC(I+3)+TH(K)
388*      60  CONTINUE
389*      DO 610 I=1,3
390*      DC 610 J=1,NH
391*      IF(PI(IJ).EQ.0) GO TO 610
392*      EC(I)=EC(I)-AV(J,I)*GMDD(IJ)
393*      610  CONTINUE
394*      K=0
395*      DO 612 I=1,NH
396*      IF(PI(I).NE.0) GO TO 612
397*      K=K+1
398*      IV=K+3
399*      DO 611 J=1,NH
400*      IF(PI(IJ).EQ.0) GO TO 611
401*      IF(I.GT.J) AS(I,J)=AS(J,I)
402*      EC(K+3)=EC(K+3)-AS(I,J)*GMDD(IJ)
403*      611  CONTINUE
404*      612  CONTINUE
405*      C
406*      C      LOAD SYSTEM MATRIX (A) WITH A00,A0K,AKM ELEMENTS
407*      C
408*      DC 23 I=1,3
409*      DO 23 J=1,3
410*      23  A(I,J)=A00(I,J)
411*      DO 24 I=1,3
412*      K=0
413*      DO 24 J=1,NH
414*      IF(PI(IJ).NE.0) GO TO 24
415*      K=K+1
416*      A(K+3,I)=AV(IJ,I)
417*      A(I,K+3)=AV(IJ,I)
418*      24  CONTINUE
419*      K=0
420*      DC 250 I=1,NH
421*      IF(PI(I).NE.0) GO TO 250

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422*      K=K+1
423*      L=0
424*      DO 25 J=1,NH
425*      IF(PI(J).NE.0) GO TO 25
426*      L=L+1
427*      IF(K.GT.L) GO TO 26
428*      A(K+3,L+3)=A(1+J)
429*      GO TO 25
430*      2E  A(K+3,L+3)=A(L+3,K+3)
431*      25  CONTINUE
432*      250 CONTINUE
433*      C
434*      C      SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
435*      C              (RELATIVE) ROTATIONAL ACCELERATIONS
436*      C
437*      CALL SOD(A,V,IV,EC,V,1,$92,WRK)
438*      DO 910 J=V+4
439*      K=J-3
440*      IF(PI(K).NE.0) GO TO 911
441*      EC(J)=EC(IV)
442*      IV=IV-1
443*      GO TO 910
444*      911  EC(J)=GMDD(K)
445*      910  CONTINUE
446*      92   CONTINUE
447*      RETURN
448*      END

```

## Appendix B

### Summary of MBDYTL, MBDYL User Requirements

#### *Subroutine Entry Statements*

CALL MBDYTL(NH,H,MB,MA,PB,PA,G,PI)  
or CALL MBDYL(NH,H,MB,MA,FB,PA,G,PI)

CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,WO,WDOT)

#### *Input/Output Variable Type and Storage Specifications—Fortran V*

INTEGER NH, H( $n$ ), PI( $n$ )

REAL MB(7), MA( $n,7$ ), PB( $n,3$ ), PA( $n,n,3$ ), G( $n,3$ ), TH( $n$ ),  
TB(3), TA( $n,3$ ), FB(3), FA( $n,3$ ), GM( $n$ ), GMD( $n$ ),  
GMDD( $n$ ), WO(3)

DOUBLE PRECISION WDOT( $n + 3$ )(where  $n$  = number of system hinges)

#### *External Subroutines Called*

AINVD — double precision matrix inversion subroutine; user-supplied (see line 224 in MBDYTL listing, line 223 in MBDYL listing)

This subroutine inverts the matrix  $A$ , where  $A$  is of order  $IV \times IV$ ,  $IV \geq 3$ . The result is returned in  $A$ .

#### *Subroutine Setup*

Insert the Fortran V statement (see listings)

PARAMETER Q =  $n$

where  $n$  = number of system hinges

#### *Data Restrictions*

$n \geq 1$

#### *Core Storage Required*

Code: 1500 words

Data: 500 words (minimum; increases with  $n$ )

#### *Listings*

```

1*      SUBROUTINE MBDYTLINH,H,MB,MA,PB,PA,G,PI)
2*      C
3*      C      ADJUSTABLE DIMENSIONS
4*      C
5*      INTEGER HIJ,PI(1)
6*      REAL MB(1),MAINH(7),PBINH(3),PAINH,NH(3),GINH(3)
7*      PARAMETER Q=4
8*      PARAMETER S=Q+1,V=Q+3,U=4+V,S3=3*S
9*      C
10*     C      ADDITIONAL DIMENSIONED VARIABLES
11*     C
12*     DOUBLE PRECISION A(V,V),WRK(V4),BMASS(S)
13*     DIMENSION PHIS(3,3),DX(S,S),DY(S,S),DZ(S,S),DXO(S,S),DYO(S,S),
14*     $DZO(S,S),G0(Q,3),T(1,S,3,3),LX(S,S),LY(S,S),LZ(S,S),FEXO(S),
15*     $PEYO(S),PEZO(S),IXO(S),IYO(S),IZO(S),AB(3,3),
16*     $CE(3),AIS(3),PS(S,S,3,3),AS(Q,Q),AV(Q,3),          AGC(3,3)
17*     INTEGER KK(Q),EPSIG(S)
18*     REAL LX,LY,LZ,IXX(S),IYY(S),IZZ(S),IXY(S),IXZ(S),IYZ(S)
19*     EQUIVALENCE (PS,A),(LX,DXO),(LY,DYO),(LZ,DZO),(PH,T)
20*     NB=NH+1
21*     C
22*     C      INITIALIZE EPSIK,J WITH ZEROS AND DEFINE KKIK)
23*     C
24*     DO 85 K=1,NH
25*     KK(K)=K+1
26*     DO 85 J=1,NB
27*     85 EPSIK,J)=0
28*     C
29*     C      DEFINE EPSIK,J USING HIJ)
30*     C
31*     DO 86 K=1,NH
32*     DO 86 J=2,NB
33*     IF(K.EQ.(J-1)) EPSIK,J)=1
34*     IF(K.LT.(J-1)) GO TO 87
35*     GO TO 86
36*     87 CONTINUE
37*     JQ=K+1
38*     JJ=J-1
39*     DO 89 L=JQ,JI
40*     IF(K.GT.(L-1)) GO TO 89
41*     IF((EPSIK,_) .EQ. 1) .AND. (HIJ-1).EQ.(L-1)) EPSIK,J)=1
42*     89 CONTINUE
43*     86 CONTINUE
44*     C
45*     C      INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.G.S
46*     C      AND HINGE POINTS
47*     C
48*     IXX(1)=MB(1)
49*     IYY(1)=MB(2)
50*     IZZ(1)=MB(3)
51*     IXY(1)=MB(4)
52*     IXZ(1)=MB(5)
53*     IYZ(1)=M0(6)
54*     BMASS(1)=MB(7)
55*     TM=BMASS(1)
56*     DO 35 J=2,NB
57*     IXX(J)=MA(J-1,1)
58*     IYY(J)=MA(J-1,2)
59*     IZZ(J)=MA(J-1,3)
60*     IXY(J)=MA(J-1,4)
61*     IXZ(J)=MA(J-1,5)
62*     IYZ(J)=MA(J-1,6)
63*     BMASS(J)=MA(J-1,7)
64*     35 TM=TM+BMASS(J)
65*     DO 149 I=1,NB

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66*      I1=I-1
67*      DO 149 J=1,NB
68*      J1=J-1
69*      IF(I.EQ.J) GO TO 163
70*      IF(I.GT.J) GO TO 70
71*      IF(I.EQ.1) GO TO 80
72*      IF(EPS(I1,J).EQ.1) GO TO 400
73*      70  LX(I,J)=PA(I1,I1,1)
74*      LY(I,J)=PA(I1,I1,2)
75*      LZ(I,J)=PA(I1,I1,3)
76*      GO TO 149
77*      400  CONTINUE
78*      DO 600 K=I,J1
79*      IF(EPS(K,J).EQ.1) GO TO 500
80*      600  CONTINUE
81*      GO TO 149
82*      500  LX(I,J)=PA(I1,K,1)
83*      LY(I,J)=PA(I1,K,2)
84*      LZ(I,J)=PA(I1,K,3)
85*      GO TO 149
86*      80  DO 90 L=1,J1
87*      IF(EPS(L,J).EQ.1) GO TO 101
88*      90  CONTINUE
89*      GO TO 149
90*      101  LX(I,J)=PB(L,1)
91*      LY(I,J)=PB(L,2)
92*      LZ(I,J)=PB(L,3)
93*      GO TO 149
94*      163  LX(I,J)=0.
95*      LY(I,J)=0.
96*      LZ(I,J)=0.
97*      149  CONTINUE
98*      DO 13 N=1,NB
99*      DO 13 J=1,NB
100*     DX(N,J)=LX(N,J)
101*     DY(N,J)=LY(N,J)
102*     DZ(N,J)=LZ(N,J)
103*     DO 13 K=1,NB
104*     DX(N,J)=DX(N,J)-(BMASS(K)/TM)*LX(N,K)
105*     DY(N,J)=DY(N,J)-(BMASS(K)/TM)*LY(N,K)
106*     DZ(N,J)=DZ(N,J)-(BMASS(K)/TM)*LZ(N,K)
107*     C
108*     C   CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
109*     C
110*     DO 31 N=1,NB
111*     PH(N,1,1)=IXX(N)
112*     PH(N,1,2)=-IXY(N)
113*     PH(N,1,3)=-IXZ(N)
114*     PH(N,2,2)=IYY(N)
115*     PH(N,2,3)=-IYZ(N)
116*     PH(N,3,3)=IZZ(N)
117*     DO 30 J=1,NB
118*     PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
119*     PH(N,1,2)=PH(N,1,2)-BMASS(J)*DX(N,J)*DY(N,J)
120*     PH(N,1,3)=PH(N,1,3)-BMASS(J)*DX(N,J)*DZ(N,J)
121*     PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
122*     PH(N,2,3)=PH(N,2,3)-BMASS(J)*DY(N,J)*DZ(N,J)
123*     30  PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)
124*     PH(N,2,1)=PH(N,1,2)
125*     PH(N,3,1)=PH(N,1,3)
126*     31  PH(N,3,2)=PH(N,2,3)
127*     C
128*     C   ADDITIONAL AUGMENTED INERTIA DYADICS (IN BODY I FRAME)
129*     C
130*     DO 751 J=1,NB
131*     DO 751 M=1,3
132*     DO 751 N=1,3
133*     751  PS(J,J,M,N)=PH(J,M,N)
134*     DO 37 I=1,NB
135*     DO 37 J=1,NB
136*     IF(I.EQ.J) GO TO 37

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137*      PS(I,J,1,1)=-TM*(DY(I,J)*DY(J,I)+DZ(I,J)+CZ(J,I))
138*      PS(I,J,1,2)=TM*DX(J,I)*DY(I,J)
139*      PS(I,J,1,3)=TM*DX(J,I)*DZ(I,J)
140*      PS(I,J,2,1)=TM*DY(J,I)*DX(I,J)
141*      PS(I,J,2,2)=-TM*(DX(I,J)*DX(J,I)+DZ(I,J)+DZ(J,I))
142*      PS(I,J,2,3)=TM*DY(J,I)*DZ(I,J)
143*      PS(I,J,3,1)=TM*DZ(J,I)*CX(I,J)
144*      PS(I,J,3,2)=TM*DZ(J,I)*DY(I,J)
145*      PS(I,J,3,3)=-TM*(DX(I,J)*DX(J,I)+DY(I,J)+DY(J,I))
146*      . 37 CONTINUE
147*      C
148*      C      ADD MATRIX ELEMENT COMPUTATION (3X3)
149*      C
150*      DO 3 I=1,NB
151*      DO 3 J=1,NE
152*      ADD(1,1)=ADD(1,1)+PS(I,J,1,1)
153*      ADD(1,2)=ADD(1,2)+PS(I,J,1,2)
154*      ADD(1,3)=ADD(1,3)+PS(I,J,1,3)
155*      ADD(2,2)=ADD(2,2)+PS(I,J,2,2)
156*      ADD(2,3)=ADD(2,3)+PS(I,J,2,3)
157*      ADD(3,3)=ADD(3,3)+PS(I,J,3,3)
158*      3 CONTINUE
159*      ADD(2,1)=ADD(1,2)
160*      ADD(3,1)=ADD(1,3)
161*      ADD(3,2)=ADD(2,3)
162*      C
163*      C      ADK VECTOR ELEMENT COMPUTATION (3X1)
164*      C
165*      DO 7 K=1,NH
166*      AV(K,1)=0.
167*      AV(K,2)=0.
168*      AV(K,3)=0.
169*      DO 7 I=1,NB
170*      DO 7 J=1,NB
171*      IF(EPS(K,J).EQ.0) GO TO 7
172*      DO 10 M=1,3
173*      DO 10 L=1,3
174*      10 AV(K,M)=AV(K,M)+PS(I,J,M,L)*G(K,L)
175*      7 CONTINUE
176*      C
177*      C      AKM SCALAR ELEMENT COMPUTATION
178*      C
179*      DO 14 K=1,NH
180*      DO 14 M=1,NH
181*      IF(K.GT.M) GO TO 14
182*      AIS(1)=0.
183*      AIS(2)=0.
184*      AIS(3)=0.
185*      DO 15 J=1,NB
186*      DO 15 I=1,NB
187*      IF((EPS(K,J).EQ.0).OR.(EPS(M,I).EQ.0)) GO TO 15
188*      DO 18 N=1,3
189*      DO 18 L=1,3
190*      18 AIS(N)=AIS(N)+PS(J,I,N,L)*G(M,L)
191*      15 CONTINUE
192*      AS(K,M)=G(K,1)*AIS(1)+G(K,2)*AIS(2)+G(K,3)*AIS(3)
193*      14 CONTINUE
194*      C
195*      C      LOAD SYSTEM MATRIX (A) WITH ADD,ADK,AKM ELEMENTS
196*      C
197*      DO 23 I=1,3
198*      DO 23 J=1,3
199*      23 A(I,J)=ADD(I,J)
200*      DO 24 I=1,3
201*      K=C
202*      IV=3
203*      DO 24 J=1,NH
204*      IF(PI(J).NE.0) GO TO 24
205*      K=K+1
206*      A(K+3,I)=AV(J,I)
207*      A(I,K+3)=AV(J,I)

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208*      24      CONTINUE
209*      K=0
210*      DO 250 I=1,NH
211*      IF(PI(I).NE.0) GO TO 250
212*      K=K+1
213*      IV=IV+1
214*      L=0
215*      DO 25 J=1,NH
216*      IF(PI(J).NE.0) GO TO 25
217*      L=L+1
218*      IF(K.GT.L) GO TO 26
219*      A(K+3,L+3)=AS(I,J)
220*      GO TO 25
221*      26      A(K+3,L+3)=A(L+3,K+3)
222*      25      CONTINUE
223*      250     CONTINUE
224*      CALL AINVD(A,V,IV,81099,WRK)
225*      1099    CONTINUE
226*      RETURN
227*      ENTRY KRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMD,WE,EQ)
228*      DIMENSION TB(3),TA(NH,3),FB(3),FA(NH,3),GM(1),GMD(1),GMD(1),
229*      *TH(NH),WC(3),E(S3,1)
230*      DOUBLE PRECISION EC(V),EQ(V)
231*      C
232*      C      BODY-TO-BODY COORDINATE TRANSFORMATIONS
233*      C
234*      DO 335 J=1,NH
235*      K=H(J)
236*      AB(1,1)=1.
237*      AB(1,2)=GM(J)*G(J,3)
238*      AB(1,3)=-GM(J)*G(J,2)
239*      AB(2,1)=-AB(1,2)
240*      AB(2,2)=1.
241*      AB(2,3)=GM(J)*G(J,1)
242*      AB(3,1)=-AB(1,3)
243*      AB(3,2)=-AB(2,3)
244*      AB(3,3)=1.
245*      IF(K.EQ.0) GO TO 335C
246*      DO 334 L=1,3
247*      DO 334 M=1,3
248*      T(1,J+1,L,M)=0.
249*      DO 334 I=1,3
250*      334     T(1,J+1,L,M)=T(1,J+1,L,M)+AB(L,I)*T(1,K+1,I,M)
251*      GO TO 335
252*      335C    CONTINUE
253*      DO 335I L=1,3
254*      DO 335I M=1,3
255*      335I    T(1,J+1,L,M)=AB(L,M)
256*      335     CONTINUE
257*      C
258*      C      COORD. TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
259*      C
260*      DO 362 I=1,NH
261*      DO 362 J=1,3
262*      G0(I,J)=0.
263*      DO 362 K=1,3
264*      G0(I,J)=G0(I,J)+T(1,I+1,K,J)*G(I,K)
265*      362     CONTINUE
266*      C
267*      C      EXTERNAL FORCES, EXTERNAL TORQUES
268*      C
269*      FEX0(I)=FB(1)
270*      FEY0(I)=FB(2)
271*      FEZ0(I)=FB(3)
272*      TX0(I)=TB(1)
273*      TY0(I)=TB(2)
274*      TZ0(I)=TB(3)
275*      DO 17 I=2,NB
276*      K=I-1
277*      FEX0(I)=T(1,I,1,1)*FA(K,1)+T(1,I,2,1)*FA(K,2)+T(1,I,3,1)*FA(K,3)
278*      FEY0(I)=T(1,I,1,2)*FA(K,1)+T(1,I,2,2)*FA(K,2)+T(1,I,3,2)*FA(K,3)

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279*      FEZO(I)=T(1,I,1,3)*FA(K,1)+T(1,I,2,3)*FA(K,2)+T(1,I,3,3)*FA(K,3)
280*      TXO(I) =T(1,I,1,1)*TA(K,1)+T(1,I,2,1)*TA(K,2)+T(1,I,3,1)*TA(K,3)
281*      TYO(I)= T(1,I,1,2)*TA(K,1)+T(1,I,2,2)*TA(K,2)+T(1,I,3,2)*TA(K,3)
282*      TZO(I)= T(1,I,1,3)*TA(K,1)+T(1,I,2,3)*TA(K,2)+T(1,I,3,3)*TA(K,3)
283*      DO 17 J=1,NB
284*      DXO(I,J)=Y(1,I,1,1)*DX(I,J)+Y(1,I,2,1)*DY(I,J)+Y(1,I,3,1)*DZ(I,J)
285*      DYO(I,J)=Y(1,I,1,2)*DX(I,J)+Y(1,I,2,2)*DY(I,J)+Y(1,I,3,2)*DZ(I,J)
286*      DZO(I,J)=Y(1,I,1,3)*DX(I,J)+Y(1,I,2,3)*DY(I,J)+Y(1,I,3,3)*DZ(I,J)
287*      17 CONTINUE
288*      DO 367 I=1,NB
289*      DXO(1,I)=DX(1,I)
290*      DYO(1,I)=DY(1,I)
291*      367 DZO(1,I)=DZ(1,I)
292*      C
293*      C VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
294*      C (QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
295*      C VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
296*      C
297*      DO 230 N=1,NB
298*      CPX=0.
299*      CPY=0.
300*      CPZ=0.
301*      DO 2301 L=1,NB
302*      CPX=CPX+DYO(N,L)*FEZO(L)-DZO(N,L)*FEYO(L)
303*      CPY=CPY+DZO(N,L)*FEXO(L)-DXO(N,L)*FEZO(L)
304*      CPZ=CPZ+DXO(N,L)*FEYO(L)-DYO(N,L)*FEXO(L)
305*      2301 CONTINUE
306*      K = 3*(N-1)
307*      E(K+1,1)=TXO(N)+CPX
308*      E(K+2,1)=TYO(N)+CPY
309*      E(K+3,1)=TZO(N)+CPZ
310*      230 CONTINUE
311*      C
312*      C COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR
313*      C
314*      DO 55 MI=1,3
315*      55 EC(MI)=E(MI,1)
316*      DO 52 J=2,NB
317*      DO 52 M=1,3
318*      K1=3*(J-1)+M
319*      52 EC(M)=EC(M)+E(K1,1)
320*      I=0
321*      DO 60 K=1,NH
322*      IF(PI(K).NE.0) GO TO 60
323*      I=I+1
324*      EC(I+3)=0.
325*      DO 601 M=1,3
326*      601 CE(M)=0.
327*      DO 61 J=1,NB
328*      IF(EPS(K,J).EQ.0) GO TO 61
329*      DO 65 M=1,3
330*      J1=3*(J-1)+M
331*      65 CE(M)=CE(M)+E(J1,1)
332*      61 CONTINUE
333*      DO 66 L=1,3
334*      66 EC(I+3)=EC(I+3)+GO(K,L)*CE(L)
335*      EC(I+3)=EC(I+3)+TH(K)
336*      60 CONTINUE
337*      DO 610 I=1,3
338*      DO 610 J=1,NH
339*      IF(PI(J).EQ.0) GO TO 610
340*      EC(I)=EC(I)-AV(J,I)*GMDD(J)
341*      610 CONTINUE
342*      K=0
343*      IV=3
344*      DO 612 I=1,NH
345*      IF(PI(I).NE.0) GO TO 612
346*      K=K+1
347*      IV=IV+1
348*      DO 611 J=1,NH
349*      IF(PI(J).EQ.0) GO TO 611

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350*          IF(I.GT.J) AS(I,J)=AS(J,I)
351*          EC(K+3)=EC(K+3)-AS(I,J)*GMDC(J)
352*          611 CONTINUE
353*          612 CONTINUE
354*          C
355*          C SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND FINGE
356*          C (RELATIVE) ROTATIONAL ACCELERATIONS
357*          C
358*          DO 671 I=1,IV
359*          EQ(I)=0.
360*          DO 671 J=1,IV
361*          EQ(I)=EQ(I)+A(I,J)*EC(J)
362*          671 CONTINUE
363*          DO 910 J=V,4
364*          K=J-3
365*          IF(PI(K).NE.D) GO TO 911
366*          EQ(J)=EQ(IV)
367*          IV=IV-1
368*          GO TO 910
369*          911 EQ(J)=GMDC(K)
370*          910 CONTINUE
371*          92 CONTINUE
372*          RETURN
373*          END

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1*      SUBROUTINE MBOYL(NH,H,MB,MA,PB,PA,G,PI)
2*      C
3*      C      ADJUSTABLE DIMENSIONS
4*      C
5*      INTEGER HI(1),PI(1)
6*      REAL MB(1),MAINH(7),PBINH(3),PAINH,NH(3),G(NH,3)
7*      PARAMETER Q=4
8*      PARAMETER S=Q+1,V=Q+3,V4=4*V,S3=3*S
9*      C
10*     C      ADDITIONAL DIMENSIONED VARIABLES
11*     C
12*     DOUBLE PRECISION A(V,V),WRK(14),BMASS(5)
13*     DIMENSION PHIS(3,3),DX(S,S),DY(S,S),DZ(S,S),
14*     LX(S,S),LY(S,S),LZ(S,S),FEX(S),FEY(S),FEZ(S),TX(S),TY(S),TZ(S),
15*     $CE(3),AIS(3), PS(S,3,3),AS(0,0),AV(0,3),      A0(3,3)
16*     INTEGER KK(3),EPS(0,S)
17*     REAL LX,LY,LZ,IXX(S),IYY(S),IZZ(S),IXY(S),IXZ(S),IYZ(S)
18*     EQUIVALENCE (PS,LX,A)
19*     NB=NH+1
20*     C
21*     C      INITIALIZE EPS(K,J) WITH ZEROS AND DEFINE KK(K)
22*     C
23*     DO 85 K=1,NH
24*     KK(K)=K+1
25*     DO 85 J=1,NB
26*     EPS(K,J)=0
27*     C
28*     C      DEFINE EPS(K,J) USING HI(J)
29*     C
30*     DO 86 K=1,NH
31*     DO 86 J=2,NB
32*     IF(K.EQ.(J-1)) EPS(K,J)=1
33*     IF(K.LT.(J-1)) GO TO 87
34*     GO TO 86
35*     87 CONTINUE
36*     JD=K+1
37*     JI=J-1
38*     DO 89 L=JD,JI
39*     IF(K.GT.(L-1)) GO TO 89
40*     IF((EPS(K,L).EQ.1).AND.(H(J-1).EQ.(L-1))) EPS(K,J)=1
41*     89 CONTINUE
42*     85 CONTINUE
43*     C
44*     C      INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.G.S
45*     C      AND HINGE POINTS
46*     C
47*     IXX(1)=MB(1)
48*     IYY(1)=MB(2)
49*     IZZ(1)=MB(3)
50*     IXY(1)=MB(4)
51*     IXZ(1)=MB(5)
52*     IYZ(1)=MB(6)
53*     BMASS(1)=MB(7)
54*     TM=BMASS(1)
55*     DO 35 J=2,NB
56*     IXX(J)=MA(J-1,1)
57*     IYY(J)=MA(J-1,2)
58*     IZZ(J)=MA(J-1,3)
59*     IXY(J)=MA(J-1,4)
60*     IXZ(J)=MA(J-1,5)
61*     IYZ(J)=MA(J-1,6)
62*     BMASS(J)=MA(J-1,7)
63*     35 TM=TM+BMASS(J)
64*     DO 149 I=1,NB

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65*      I1=I-1
66*      DO 149 J=1,NB
67*      JJ=J-1
68*      IF(I.EQ.J) GO TO 163
69*      IF(I.GT.J) GO TO 70
70*      IF(I.EQ.1) GO TO 80
71*      IF(EPS(I1,J).EQ.1) GO TO 400
72*      70  LX(I,J)=PA(I1,I1,1)
73*      LY(I,J)=PA(I1,I1,2)
74*      LZ(I,J)=PA(I1,I1,3)
75*      GO TO 149
76*      400  CONTINUE
77*      DO 600 K=I,J1
78*      IF(EPS(K,J).EQ.1) GO TO 500
79*      600  CONTINUE
80*      GO TO 149
81*      500  LX(I,J)=PA(I1,K,1)
82*      LY(I,J)=PA(I1,K,2)
83*      LZ(I,J)=PA(I1,K,3)
84*      GO TO 149
85*      80  DO 90 L=1,J1
86*      IF(EPS(L,J).EQ.1) GO TO 101
87*      90  CONTINUE
88*      GO TO 149
89*      101  LX(I,J)=PB(L,1)
90*      LY(I,J)=PB(L,2)
91*      LZ(I,J)=PB(L,3)
92*      GO TO 149
93*      163  LX(I,J)=0.
94*      LY(I,J)=0.
95*      LZ(I,J)=0.
96*      149  CONTINUE
97*      DO 13 N=1,NB
98*      DO 13 J=1,NB
99*      DX(N,J)=LX(N,J)
100*      DY(N,J)=LY(N,J)
101*      DZ(N,J)=LZ(N,J)
102*      DO 13 K=1,NB
103*      DX(N,J)=DX(N,J)-(BMASS(K)/TM)*LX(N,K)
104*      DY(N,J)=DY(N,J)-(BMASS(K)/TM)*LY(N,K)
105*      DZ(N,J)=DZ(N,J)-(BMASS(K)/TM)*LZ(N,K)
106*      13  C
107*      C      CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
108*      C
109*      DO 31 N=1,NB
110*      PH(N,1,1)=IXX(N)
111*      PH(N,1,2)=-IXY(N)
112*      PH(N,1,3)=-IXZ(N)
113*      PH(N,2,2)=IYY(N)
114*      PH(N,2,3)=-IYZ(N)
115*      PH(N,3,3)=IZZ(N)
116*      DO 30 J=1,NB
117*      PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
118*      PH(N,1,2)=PH(N,1,2)-BMASS(J)*DX(N,J)*DY(N,J)
119*      PH(N,1,3)=PH(N,1,3)-BMASS(J)*DX(N,J)*DZ(N,J)
120*      PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
121*      PH(N,2,3)=PH(N,2,3)+BMASS(J)*DY(N,J)*DZ(N,J)
122*      30  PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)
123*      PH(N,2,1)=PH(N,1,2)
124*      PH(N,3,1)=PH(N,1,3)
125*      31  PH(N,3,2)=PH(N,2,3)
126*      C
127*      C      ADDITIONAL AUGMENTED INERTIA DYADICS (IN BODY I FRAME)
128*      C
129*      DO 751 J=1,NB
130*      DO 751 M=1,3
131*      DO 751 N=1,3
132*      751  PS(J,J,M,N)=PH(J,M,N)
133*      DO 37 I=1,NB
134*      DO 37 J=1,NB
135*      IF(I.EQ.J) GO TO 37

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136*      PS(I,J,1,1)=-TM*(DY(I,J)+DY(J,I))+DZ(I,J)+DZ(J,I)
137*      PS(I,J,1,2)=TM*DX(J,I)+DY(I,J)
138*      PS(I,J,1,3)=TM*DX(J,I)+DZ(I,J)
139*      PS(I,J,2,1)=TM*DY(J,I)+DX(I,J)
140*      PS(I,J,2,2)=-TM*(DX(I,J)+DX(J,I))+DZ(I,J)+DZ(J,I)
141*      PS(I,J,2,3)=TM*DY(J,I)+DZ(I,J)
142*      PS(I,J,3,1)=TM*DZ(J,I)+DX(I,J)
143*      PS(I,J,3,2)=TM*DZ(J,I)+DY(I,J)
144*      PS(I,J,3,3)=-TM*(DX(I,J)+DX(J,I)+DY(I,J)+DY(J,I))
145*      3? CONTINUE
146*      C
147*      C   ADD MATRIX ELEMENT COMPUTATION (3X3)
148*      C
149*      DO 3 I=1,NB
150*      DO 3 J=1,NB
151*      AGG(1,1)=AGG(1,1)+PS(I,J,1,1)
152*      AGG(1,2)=AGG(1,2)+PS(I,J,1,2)
153*      AGG(1,3)=AGG(1,3)+PS(I,J,1,3)
154*      AGG(2,2)=AGG(2,2)+PS(I,J,2,2)
155*      AGG(2,3)=AGG(2,3)+PS(I,J,2,3)
156*      AGG(3,3)=AGG(3,3)+PS(I,J,3,3)
157*      3 CONTINUE
158*      AGG(2,1)=AGG(1,2)
159*      AGG(3,1)=AGG(1,3)
160*      AGG(3,2)=AGG(2,3)
161*      C
162*      C   AGK VECTOR ELEMENT COMPUTATION (3X1)
163*      C
164*      DO 7 K=1,NH
165*      AV(K,1)=G.
166*      AV(K,2)=0.
167*      AV(K,3)=0.
168*      DO 7 I=1,NB
169*      DO 7 J=1,NB
170*      IF(EPK(I,J).EQ.0) GO TO 7
171*      DO 10 M=1,3
172*      DO 10 L=1,3
173*      10 AV(K,M)=AV(K,M)+PS(I,J,M,L)*G(K,L)
174*      7 CONTINUE
175*      C
176*      C   AKM SCALAR ELEMENT COMPUTATION
177*      C
178*      DO 14 K=1,NH
179*      DO 14 M=1,NH
180*      IF(K.GT.M) GO TO 14
181*      AIS(1)=0.
182*      AIS(2)=0.
183*      AIS(3)=0.
184*      DO 15 J=1,NB
185*      DO 15 I=1,NB
186*      IF((EPK(K,J).EQ.0).OR.(EPS(I,I).EQ.0)) GO TO 15
187*      DO 18 N=1,3
188*      DO 18 L=1,3
189*      18 AIS(N)=AIS(N)+PS(I,J,N,L)*G(M,L)
190*      15 CONTINUE
191*      AIS(K,M)=G(K,1)*AIS(1)+G(K,2)*AIS(2)+G(K,3)*AIS(3)
192*      14 CONTINUE
193*      C
194*      C   LOAD SYSTEM MATRIX (A) WITH AGG,AGK,AKM ELEMENTS
195*      C
196*      DO 23 I=1,3
197*      DO 23 J=1,3
198*      23 A(I,J)=AGG(I,J)
199*      DO 24 I=1,3
200*      K=0
201*      IV=3
202*      DO 24 J=1,NH
203*      IF(PI(I,J).NE.0) GO TO 24
204*      K=K+1
205*      A(K+3,I)=AV(I,J)
206*      A(I,K+3)=AV(J,I)

```

```

207*      24  CONTINUE
208*          K=0
209*          DO 250 I=1,NH
210*             IF(FI(I).NE.0) GO TO 250
211*             K=K+1
212*             IV=IV+1
213*             L=0
214*             DO 25 J=1,NH
215*                IF(PI(J).NE.0) GO TO 25
216*                L=L+1
217*                IF(K.GT.L) GO TO 26
218*                A(K+3,L+3)=A5(I,J)
219*                GO TO 25
220*      26  A(K+3,L+3)=A(L+3,K+3)
221*      25  CONTINUE
222*      250 CONTINUE
223*          CALL AINVD(A,V,IV,$1099,WRK)
224*      1099 CONTINUE
225*          RETURN
226*          ENTRY MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMD0,W0,EQ)
227*          DIMENSION TB(3),TAINH(3),F9(3),FA(NH,3),GM(1),GMD(1),GMD0(1),
228*          $TH(NH),W0(3),E(53,1)
229*          DOUBLE PRECISION EC(IV),EQ(IV)
230*          C
231*          C   EXTERNAL FORCES,EXTERNAL TORQUES,HINGE TORQUES
232*          C
233*          FEX(1)=FB(1)
234*          FEY(1)=FB(2)
235*          FEZ(1)=FB(3)
236*          TX(1)=TB(1)
237*          TY(1)=TB(2)
238*          TZ(1)=TB(3)
239*          DO 33 J=2,NB
240*             TX(J)=TA(J-1,1)
241*             TY(J)=TA(J-1,2)
242*             TZ(J)=TA(J-1,3)
243*             FEX(J)=FA(J-1,1)
244*             FEY(J)=FA(J-1,2)
245*      33  FEZ(J)=FA(J-1,3)
246*          C
247*          C   VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
248*          C   QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
249*          C   VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
250*          C
251*          DO 230 N=1,NB
252*             CPX=0.
253*             CPY=0.
254*             CPZ=0.
255*             DO 2301 L=1,NB
256*                CPX=CPX+DY(N,L)*FEZ(L)-DZ(N,L)*FEY(L)
257*                CPY=CPY+DZ(N,L)*FEX(L)-DX(N,L)*FEZ(L)
258*                CPZ=CPZ+DX(N,L)*FEY(L)-DY(N,L)*FEX(L)
259*      2301 CONTINUE
260*             K = 3*(N-1)
261*             E(K+1,1)=TX(N)+CPX
262*             E(K+2,1)=TY(N)+CPY
263*             E(K+3,1)=TZ(N)+CPZ
264*      230 CONTINUE
265*          C
266*          C   COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR
267*          C
268*          DO 55 MI=1,3
269*      55  EC(MI)=E(MI,1)
270*          DO 52 J=2,NB
271*             DO 52 M=1,3
272*                KI=3*(J-1)+M
273*      52  EC(M)=EC(M)+E(KI,1)
274*          I=0
275*          DO 60 K=1,NH
276*             IF(PI(K).NE.0) GO TO 60
277*             I=I+1

```



```

278*      EC(I+3)=G.
279*      DO 601 M=1,3
280*      501  CE(M)=G.
281*      DO 61 J=1,NB
282*      IF(EP(S(K,J),EG,C) GO TO 61
283*      DO 65 M=1,3
284*      J1=3+(J-1)*M
285*      65  CE(M)=CE(M)+E(I,J1,1)
286*      61  CONTINUE
287*      DO 66 L=1,3
288*      66  EC(I+3)=EC(I+3)+G(K,L)*CE(L)
289*      EC(I+3)=EC(I+3)+TH(K)
290*      60  CONTINUE
291*      DO 610 I=1,3
292*      DO 510 J=1,NH
293*      IF(PI(I,J),EQ,0) GO TO 610
294*      EC(I)=EC(I)-AV(J,I)*GMDD(I,J)
295*      610  CONTINUE
296*      K=0
297*      IV=3
298*      DO 512 I=1,NH
299*      IF(PI(I),NE,0) GO TO 612
300*      K=K+1
301*      IV=IV+1
302*      DO 511 J=1,NH
303*      IF(PI(I,J),EQ,0) GO TO 611
304*      IF(I,GT,J) AS(I,J)=AS(J,I)
305*      EC(K+3)=EC(K+3)-AS(I,J)*GMDD(I,J)
306*      611  CONTINUE
307*      612  CONTINUE
308*      C
309*      C   SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
310*      C   (RELATIVE) ROTATIONAL ACCELERATIONS
311*      C
312*      DO 671 I=1,IV
313*      EQ(I)=G.
314*      DO 571 J=1,IV
315*      EQ(I)=EQ(I)+A(I,J)*EC(J)
316*      671  CONTINUE
317*      DO 910 J=V,4
318*      K=J-3
319*      IF(PI(K),NE,0) GO TO 911
320*      EQ(J)=EQ(IV)
321*      IV=IV-1
322*      GO TO 910
323*      911  EQ(J)=GMDD(K)
324*      910  CONTINUE
325*      92  CONTINUE
326*      RETURN
327*      END

```

## Appendix C

### Summary of MBDYPL User Requirements

#### *Subroutine Entry Statements*

CALL MBDYPL(NH,H,MB,MA,PB,PA,G,PI,LI,TI)

CALL MRATE(NH,TH,TB,TA,FB,FA,GM,GMD,GMDD,WO,WDOT)

#### *Input/Output Variable Type and Storage Specifications—Fortran V*

INTEGER NH, H( $n$ ), PI( $n$ ), LI( $n$ ), TI( $n$ )

REAL MB(7), MA( $n$ ,7), PB( $n$ ,3), PA( $n$ , $n$ ,3), G( $n$ ,3), TH( $n$ ),  
TB(3), TA( $n$ ,3), FB(3), FA( $n$ ,3), GM( $n$ ), GMD( $n$ ),  
GMDD( $n$ ), WO(3)

DOUBLE PRECISION WDOT( $n + 3$ )(where  $n$  = number of system hinges)

#### *External Subroutines Called*

1. AINVD—double precision matrix inversion subroutine; user-supplied (see line 272 in MBDYPL listing)

This subroutine inverts the matrix  $AT$ , where  $AT$  is of order  $NMT \times NMT$ ,  $NMT \geq 1$ . The result is returned in  $AT$ .

2. SOD—double precision matrix equation solving subroutine; user-supplied (see line 666 in MBDYPL listing)

This subroutine solves the matrix equation  $A\ddot{x} = EE$ , where  $A$  is of order  $NL \times NL$ ,  $NL \geq 3$ .

#### *Subroutine Setup*

Insert the Fortran V statement (see listing)

PARAMETER Q =  $n$

where  $n$  = number of system hinges.

#### *Data Restrictions*

$n \geq 1$

#### *Core Storage Required*

Code: 3000 words

Data: 1000 words (minimum; increases with  $n$ )

#### *Listing*

```

1*      SUBROUTINE MBDYPL(NH,H,MB,MA,PB,PA,G,PI,LI,TI)
2*      C
3*      C      ADJUSTABLE DIMENSIONS
4*      C
5*      INTEGER H(1),PI(1),LI(1),TI(1)
6*      REAL MB(1),MA(NH,7),PB(NH,3),PA(NH,NH,3),G(NH,3)
7*      PARAMETER G=4
8*      PARAMETER U=G+1,V=G+3,U4=4*V,S3=3*U
9*      C
10*     C      ADDITIONAL DIMENSIONED VARIABLES
11*     C
12*     DOUBLE PRECISION A(V,V),WRK(1V4),BMASS(U)
13*     DOUBLE PRECISION A1(Q,Q),AE(V,V),EE(V),AL(V,V)
14*     DIMENSION PS(U,U,3,3),DX(U,U),DY(U,U),DZ(U,U),
15*     $      LX(U,U),LY(U,U),LZ(U,U),PH(U,3,3)
16*     DIMENSION AB(3,3),CE(3),CW(3),AIS(3),EA(3),GO(3,3),ADD(3,3),AV(3,3
17*     $),AS(3,3),T(1,3,3),BA(3,3),ASB(3,3),DXO(U,U),DYO(U,U),DZO(U,U),
18*     $TXO(U),TYO(U),TZO(U),FEXO(U),FEYO(U),FEZO(U)
19*     DIMENSION G1S(Q),G2S(Q),G3S(Q),G12(Q),G13(Q),G23(Q)
20*     INTEGER KK(Q),EPS(Q,U),R,S,TAM(Q),TAMF(Q),NUMF(U),TAB
21*     REAL LX,LY,LZ,IXX(U),IYY(U),IZZ(U),IXY(U),IXZ(U),IYZ(U)
22*     EQUIVALENCE (A,PS),(LX,DXO,AS),(LY,DYO),(LZ,DZO)
23*     NB=NH+1
24*     INT=V
25*     DO 506 K=1,NH
26*     LISUM=LISUM+LI(K)
27*     C
28*     C      INITIALIZE EPS(K,J) WITH ZEROS AND DEFINE KK(K)
29*     C
30*     DO 85 K=1,NH
31*     KK(K)=K+1
32*     DO 85 J=1,NB
33*     85 EPS(K,J)=0
34*     C
35*     C      DEFINE EPS(K,J) USING H(J)
36*     C
37*     DO 86 K=1,NH
38*     DO 86 J=2,NB
39*     IF(K.EQ.(J-1)) EPS(K,J)=1
40*     IF(K.LT.(J-1)) GO TO 87
41*     GO TO 86
42*     87 CONTINUE
43*     JO=K+1
44*     JI=J-1
45*     DO 89 L=JO,JI
46*     IF(K.GT.(L-1)) GO TO 89
47*     IF((EPS(K,L).EQ.1).AND.(H(J-1).EQ.(L-1))) EPS(K,J)=1
48*     89 CONTINUE
49*     86 CONTINUE
50*     C
51*     C      LOCATE POTENTIAL TERMINAL APPENDAGE BODIES
52*     C
53*     NN=0
54*     M=0
55*     LL=0
56*     805 CONTINUE
57*     DO 809 J=1,NH
58*     IF(H(J).EQ.M) GO TO 808
59*     GO TO 809
60*     808 LL=LL+1
61*     TAM(LL)=J
62*     809 CONTINUE
63*     DO 810 L=1,NH
64*     K=TAM(L)

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65*      IF(TAM(L),EQ,0) GO TO 810
66*      NPI=0
67*      NEP=0
68*      DO 807 J=2,NB
69*      IF(EPS(K,J),EQ,1) NEP=NEP+1
70*      IF((LI(J-1),EQ,1).AND.(EPS(K,J),EQ,1)) NPI=NPI+1
71* 807   CONTINUE
72*      IF(NPI,EQ,0) GO TO 810
73*      IF(NPI,EQ,NEP) GO TO 820
74*      M=TAM(L)
75*      TAM(L)=0
76*      GO TO 805
77* 820   CONTINUE
78*      TAM(L)=0
79*      DO 827 J=2,NB
80*      IF(EPS(K,J),EQ,1) GO TO 828
81*      GO TO 827
82* 828   NN=NN+1
83*      TAMF(NN)=J-1
84* 827   CONTINUE
85* 810   CONTINUE
86*      C
87*      C   LOCATE TERM. APPENDAGE BASE BODIES AND NO. OF T.A. BODIES ON EACH
88*      C
89*      DO 836 L=1,NB
90* 836   NUMFIL=0
91*      DO 835 L=1,NH
92*      DO 834 K=1,NH
93*      IF(TAMFIL,NE,K) GO TO 834
94*      N=H(K)+1
95*      DO 833 J=2,NB
96*      IF(EPS(K,J),EQ,0) GO TO 833
97*      NUMF(N)=NUMF(N)+1
98* 833   CONTINUE
99* 834   CONTINUE
100* 835   CONTINUE
101*      C
102*      C   FIND BASE BODY WITH THE MOST TERMINAL APPENDAGE BODIES ON IT
103*      C
104*      NMX=0
105*      DO 837 L=1,NB
106*      IF(NUMFIL,GE,NMX) GO TO 837
107*      GO TO 837
108* 838   TAB=L-1
109*      NMX=NUMFIL)
110* 837   CONTINUE
111*      C
112*      C   IDENTIFY MEMBERS OF THE TERMINAL APPENDAGE SET
113*      C
114*      DO 898 L=1,NH
115* 898   TI(L)=0
116*      IF(TAB,EQ,NH) GO TO 840
117*      DO 839 J=1,NH
118*      K=TAMF(J)
119*      IF(H(K),EQ,TAB) GO TO 829
120*      GO TO 839
121* 829   CONTINUE
122*      DO 888 L=2,NB
123*      IF(EPS(K,L),EQ,1) TI(L-1)=1
124* 888   CONTINUE
125* 839   CONTINUE
126* 840   CONTINUE
127*      C
128*      C   INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.G.S
129*      C   AND HINGE POINTS
130*      C
131*      IXX(1)=MB(1)
132*      IYY(1)=MB(2)
133*      IZZ(1)=MB(3)
134*      IXY(1)=MB(4)
135*      IXZ(1)=MB(5)

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136*      IYZ(I)=MB(I)
137*      BMASS(I)=MS(I)
138*      TM=BMASS(I)
139*      DO 35 J=2,NB
140*      IXX(J)=MA(J-1,1)
141*      IYY(J)=MA(J-1,2)
142*      IZZ(J)=MA(J-1,3)
143*      IXY(J)=MA(J-1,4)
144*      IXZ(J)=MA(J-1,5)
145*      IYZ(J)=MA(J-1,6)
146*      BMASS(J)=MA(J-1,7)
147*      35  TM=TM+BMASS(J)
148*      DO 149 I=1,NB
149*      IJ=I-1
150*      DO 149 J=1,NB
151*      JJ=J-1
152*      IF(I,EQ,J) GO TO 163
153*      IF(I,GT,J) GO TO 70
154*      IF(I,EQ,1) GO TO 80
155*      I=(EPS(I1,J),EQ,1) GO TO 400
156*      70  LX(I,J)=PA(I1,I,1)
157*      LY(I,J)=PA(I1,I,2)
158*      LZ(I,J)=PA(I1,I,3)
159*      GO TO 149
160*      400  CONTINUE
161*      DO 600 K=I,J1
162*      IF(EPS(K,J),EQ,1) GO TO 500
163*      600  CONTINUE
164*      GO TO 149
165*      500  LX(I,J)=PA(I1,K,1)
166*      LY(I,J)=PA(I1,K,2)
167*      LZ(I,J)=PA(I1,K,3)
168*      GO TO 149
169*      80  DO 90 L=1,J1
170*      IF(EPS(L,J),EQ,1) GO TO 100
171*      90  CONTINUE
172*      GO TO 149
173*      101  LX(I,J)=PB(L,1)
174*      LY(I,J)=PB(L,2)
175*      LZ(I,J)=PB(L,3)
176*      GO TO 149
177*      163  LX(I,J)=0.
178*      LY(I,J)=0.
179*      LZ(I,J)=0.
180*      149  CONTINUE
181*      DO 13 N=1,NB
182*      DO 13 J=1,NB
183*      DX(N,J)=LX(N,J)
184*      DY(N,J)=LY(N,J)
185*      DZ(N,J)=LZ(N,J)
186*      DO 13 K=1,NB
187*      DX(N,J)=DX(N,J)-(BMASS(K)/TM)*LX(N,K)
188*      DY(N,J)=DY(N,J)-(BMASS(K)/TM)*LY(N,K)
189*      DZ(N,J)=DZ(N,J)-(BMASS(K)/TM)*LZ(N,K)
190*      13  C
191*      C      CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
192*      C
193*      DO 31 N=1,NB
194*      PH(N,1,1)=IXX(N)
195*      PH(N,1,2)=-IXY(N)
196*      PH(N,1,3)=-IXZ(N)
197*      PH(N,2,2)=IYY(N)
198*      PH(N,2,3)=-IYZ(N)
199*      PH(N,3,3)=IZZ(N)
200*      DO 30 J=1,NB
201*      PH(N,1,1)=PH(N,1,1)+BMASS(J)*(DY(N,J)**2+DZ(N,J)**2)
202*      PH(N,1,2)=PH(N,1,2)-BMASS(J)*DX(N,J)*DY(N,J)
203*      PH(N,1,3)=PH(N,1,3)-BMASS(J)*DX(N,J)*DZ(N,J)
204*      PH(N,2,2)=PH(N,2,2)+BMASS(J)*(DX(N,J)**2+DZ(N,J)**2)
205*      PH(N,2,3)=PH(N,2,3)-BMASS(J)*DY(N,J)*DZ(N,J)
206*      30  PH(N,3,3)=PH(N,3,3)+BMASS(J)*(DX(N,J)**2+DY(N,J)**2)

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```

207*      PH(N,2,1)=PH(N,1,2)
208*      PH(N,3,1)=PH(N,1,3)
209*      31  PH(N,3,2)=PH(N,2,3)
210*      DO 571 R=1,NB
211*      DO 571 J=1,3
212*      DO 571 K=1,3
213*      571  PS(R,R,J,K)=PH(R,J,K)
214*      DO 763 J=1,NH
215*      G1S(J)=G1(J,1)**2
216*      G2S(J)=G1(J,2)**2
217*      G3S(J)=G1(J,3)**2
218*      G12(J)=G1(J,1)*G1(J,2)
219*      G13(J)=G1(J,1)*G1(J,3)
220*      763  G23(J)=G1(J,2)*G1(J,3)
221*      DO 3700 R=2,NB
222*      DO 3700 S=2,NB
223*      IF(R.EQ.S) GO TO 3700
224*      IF((I1(R-1).NE.1).OR.(I1(S-1).NE.1)) GO TO 3700
225*      DX2=DX(R,S)*DX(S,R)*TM
226*      DY2=DY(R,S)*DY(S,R)*TM
227*      DZ2=DZ(R,S)*DZ(S,R)*TM
228*      PS(R,S,1,1)=-DY2-DZ2
229*      PS(R,S,1,2)=DX(S,R)*DY(R,S)*TM
230*      PS(R,S,1,3)=DX(S,R)*DZ(R,S)*TM
231*      PS(R,S,2,1)=DY(S,R)*DX(R,S)*TM
232*      PS(R,S,2,2)=-DX2-DZ2
233*      PS(R,S,2,3)=DY(S,R)*DZ(R,S)*TM
234*      PS(R,S,3,1)=DZ(S,R)*DX(R,S)*TM
235*      PS(R,S,3,2)=DZ(S,R)*DY(R,S)*TM
236*      PS(R,S,3,3)=-DX2-DY2
237*      3700 CONTINUE
238*      IT=0
239*      DO 1401 I=1,NH
240*      IF(I1(I).NE.1) GO TO 1401
241*      IT=IT+1
242*      IF(I1(I).EQ.1) IT=IT-1
243*      KT=0
244*      DO 1400 K=1,NH
245*      IF(I1(K).NE.1) GO TO 1400
246*      KT=KT+1
247*      IF(I1(K).EQ.1) KT=KT-1
248*      IF(I.G1,K) GO TO 1400
249*      DO 1410 J=1,3
250*      1410  CW(J)=0.
251*      DO 1800 R=2,NB
252*      DO 1800 S=2,NB
253*      IF((EPS(K,S).EQ.0).OR.(EPS(I,R).EQ.0)) GO TO 1800
254*      DO 1802 J=1,3
255*      CE(J)=0.
256*      DO 1802 L=1,3
257*      1802  CE(J)=CE(J)+PS(R,S,J,L)*G1(K,L)
258*      DO 1430 J=1,3
259*      1430  CW(J)=CW(J)+CE(J)
260*      1800  CONTINUE
261*      ASB(I,K)= G1(I,1)*CW(1)+G1(I,2)*CW(2)+G1(I,3)*CW(3)
262*      IF((I1(I).EQ.1).OR.(I1(K).EQ.1)) GO TO 1400
263*      AT(I,K)=ASB(I,K)
264*      AT(K,I)=AT(I,K)
265*      1400  CONTINUE
266*      1401  CONTINUE
267*      NMT=KT
268*      C
269*      C  INVERT THE TERMINAL APPENDAGE MATRIX OF CONSTANTS
270*      C
271*      IF(NMT.EQ.0) GO TO 1422
272*      CALL AINV0(AT,Q,NMT,$1422,WRK)
273*      1422  CONTINUE
274*      RETURN
275*      ENTRY MRATE(NH,TH,IB,TA,FB,FA,GM,GMD,GMD0,W0,EC)
276*      DIMENSION IB(3),TA(NH,3),FB(3),FA(NH,3),GM(1),GMD(1),GMD0(1),
277*      W0(3),W0(4),W0(4),W0(4),E(5,1),TH(NH)

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278*      DOUBLE PRECISION EC(IV)
279*      C
280*      C      BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES
281*      C
282*      DO 335 J=1,NH
283*      K=H(J)
284*      IF(LI(J).EQ.1) GO TO 5021
285*      SGM=SIN(GM(J))
286*      CGM=SQRT(1.-SGM**2)-
287*      IF(ABS(GM(J)).LT.(1.570796327)) GO TO 5020
288*      IF(ABS(GM(J)).GT.(4.712388981)) GO TO 5019
289*      CGM=-CGM
290*      GO TO 5020
291*      5019 IF(ABS(GM(J)).GT.(7.85398)) CGM=COS(GM(J))
292*      5020 CONTINUE
293*      CGM1=1.-CGM
294*      G12CG=G12(J)+CGM1
295*      G13CG=G13(J)+CGM1
296*      G23CG=G23(J)+CGM1
297*      G1SG=G1(J,1)*SGM
298*      G2SG=G1(J,2)*SGM
299*      G3SG=G1(J,3)*SGM
300*      BA(1,1)=CGM+G1S(J)+CGM1
301*      BA(1,2)=G3SG+G12CG
302*      BA(1,3)=-G2SG+G13CG
303*      BA(2,1)=-G3SG+G12CG
304*      BA(2,2)=CGM+G2S(J)+CGM1
305*      BA(2,3)=G1SG+G23CG
306*      BA(3,1)=G2SG+G13CG
307*      BA(3,2)=-G1SG+G23CG
308*      BA(3,3)=CGM+G3S(J)+CGM1
309*      GO TO 5022
310*      5021 CONTINUE
311*      BA(1,1)=1.
312*      BA(1,2)=G1(J,3)+GM(J)
313*      BA(1,3)=-G1(J,2)+GM1(J)
314*      BA(2,1)=-BA(1,2)
315*      BA(2,2)=1.
316*      BA(2,3)=G1(J,1)+GM1(J)
317*      BA(3,1)=-BA(1,3)
318*      BA(3,2)=-BA(2,3)
319*      BA(3,3)=1.
320*      5022 CONTINUE
321*      IF(L.EQ.0) GO TO 3350
322*      DO 334 L=1,3
323*      DO 334 M=1,3
324*      T(1,J+1,L,M)=0.
325*      DO 334 I=1,3
326*      334 T(1,J+1,L,M)=T(1,J+1,L,M)+BA(L,I)*T(I,K+1,I,M)
327*      GO TO 335
328*      3350 CONTINUE
329*      DO 3351 L=1,3
330*      DO 3351 M=1,3
331*      3351 T(1,J+1,L,M)=BA(L,M)
332*      335 CONTINUE
333*      C
334*      C      COORD. TRANSFORMATION OF 6 VECTORS ITO REF. BODY FRAME)
335*      C
336*      DO 362 I=1,NH
337*      DO 362 J=1,3
338*      G0(I,J)=0.
339*      DO 362 K=1,3
340*      G0(I,J)=G0(I,J)+T(1,I+1,K,J)*G(I,K)
341*      362 CONTINUE
342*      C
343*      C      ANG. VELOCITY COMPONENTS OF EACH BODY (IN REF. BODY FRAME)
344*      C
345*      DO 361 J=1,NB
346*      WX0(J)=W0(1)
347*      WY0(J)=W0(2)
348*      WZ0(J)=W0(3)
349*      DO 36 K=1,NH

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350*      IF (EPS(K,J),EQ,G) GO TO 36
351*      WXO(J)=WXO(J)+GMD(K)*GO(K,1)
352*      WYO(J)=WYO(J)+GMD(K)*GO(K,2)
353*      WZO(J)=WZO(J)+GMD(K)*GO(K,3)
354*      36
355*      361 CONTINUE
356*      C
357*      C TRANSFORM AUGMENTED BODY INERTIA DYADICS TO REF. BODY FRAME
358*      C
359*      DO 363 I=2,NB
360*      DO 364 J=1,3
361*      DO 364 K=1,3
362*      AB(J,K)=0.
363*      DO 364 L=1,3
364*      AB(J,K)=AB(J,K)+PH(I,J,L)*T(1,I,L,K)
365*      364 CONTINUE
366*      DO 365 J=1,3
367*      DO 365 K=1,3
368*      PS(I,I,J,K)=0.
369*      DO 365 L=1,3
370*      PS(I,I,J,K)=PS(I,I,J,K)+T(1,I,L,J)*AB(L,K)
371*      365 CONTINUE
372*      363 CONTINUE
373*      DO 471 J=1,3
374*      DO 471 K=1,3
375*      471 PS(I,I,J,K)=PH(1,J,K)
376*      FEXO(1)=FB(1)
377*      FEYC(1)=FB(2)
378*      FEZC(1)=FB(3)
379*      TXO(1)=TB(1)
380*      TYO(1)=TB(2)
381*      TZO(1)=TB(3)
382*      DO 17 I=2,NB
383*      K=I-1
384*      FEXO(I)=T(1,I,1,1)*FA(K,1)+T(1,I,2,1)*FA(K,2)+T(1,I,3,1)*FA(K,3)
385*      FEYC(I)=T(1,I,1,2)*FA(K,1)+T(1,I,2,2)*FA(K,2)+T(1,I,3,2)*FA(K,3)
386*      FEZC(I)=T(1,I,1,3)*FA(K,1)+T(1,I,2,3)*FA(K,2)+T(1,I,3,3)*FA(K,3)
387*      TXO(I)=T(1,I,1,1)*TA(K,1)+T(1,I,2,1)*TA(K,2)+T(1,I,3,1)*TA(K,3)
388*      TYO(I)=T(1,I,1,2)*TA(K,1)+T(1,I,2,2)*TA(K,2)+T(1,I,3,2)*TA(K,3)
389*      TZO(I)=T(1,I,1,3)*TA(K,1)+T(1,I,2,3)*TA(K,2)+T(1,I,3,3)*TA(K,3)
390*      DO 17 J=1,NB
391*      DXO(I,J)=T(1,I,1,1)*DX(I,J)+T(1,I,2,1)*DY(I,J)+T(1,I,3,1)*DZ(I,J)
392*      DYO(I,J)=T(1,I,1,2)*DX(I,J)+T(1,I,2,2)*DY(I,J)+T(1,I,3,2)*DZ(I,J)
393*      DZO(I,J)=T(1,I,1,3)*DX(I,J)+T(1,I,2,3)*DY(I,J)+T(1,I,3,3)*DZ(I,J)
394*      17 CONTINUE
395*      DO 367 I=1,NB
396*      DXO(1,I)=DX(1,I)
397*      DYO(1,I)=DY(1,I)
398*      367 DZO(1,I)=DZ(1,I)
399*      C
400*      C ADDITIONAL AUGMENTED INERTIA DYADICS (IN REF. BODY FRAME)
401*      C
402*      DO 37 I=1,NB
403*      DO 37 J=1,NB
404*      IF (I,EQ,J) GO TO 37
405*      PS(I,J,1,1)=-TM*(DYO(I,J)*DYO(J,I)+DZO(I,J)*DZO(J,I))
406*      PS(I,J,1,2)=TM*(DXO(J,I)*DYO(I,J)
407*      PS(I,J,1,3)=TM*(DXO(J,I)*DZO(I,J)
408*      PS(I,J,2,1)=TM*(DYO(J,I)*DXO(I,J)
409*      PS(I,J,2,2)=-TM*(DXO(I,J)*DXO(J,I)+DZO(I,J)*DZO(J,I))
410*      PS(I,J,2,3)=TM*(DYO(J,I)*DZO(I,J)
411*      PS(I,J,3,1)=TM*(DZO(J,I)*DXO(I,J)
412*      PS(I,J,3,2)=TM*(DZO(J,I)*DYO(I,J)
413*      PS(I,J,3,3)=-TM*(DXO(I,J)*DXO(J,I)+DYO(I,J)*DYO(J,I))
414*      37 CONTINUE
415*      C
416*      C VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.
417*      C (QUADRATIC TERMS INVOLVING THE CONNECTING BODY ANGULAR
418*      C VELOCITIES AND THE MUTUAL BARYCENTER-HINGE VECTORS)
419*      C
420*      DO 230 N=1,NB

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421*      CPX=0.
422*      CPY=0.
423*      CPZ=0.
424*      DO 2301 L=1,NB
425*      IF(N.EQ.L) GO TO 2301
426*      WDX=WYO(L)*DZO(L,N)-WZO(L)*DYO(L,N)
427*      WDY=WZO(L)*DXO(L,N)-WXO(L)*DZO(L,N)
428*      Wdz=WXO(L)*DYO(L,N)-WYO(L)*DXO(L,N)
429*      WWDX=TM*(WYO(L)*Wdz-WZO(L)*WDY)+FEXO(L)
430*      WWDY=TM*(WZO(L)*WDX-WXO(L)*Wdz)+FEYO(L)
431*      WWdz=TM*(WXO(L)*WDY-WYO(L)*WDX)+FEZO(L)
432*      DWWDX=DYO(L)*WWdz-DZO(L)*WWDY
433*      DWWDY=DZO(L)*WWDX-DXO(L)*WWdz
434*      DWWdz=DXO(L)*WWDY-DYO(L)*WWDX
435*      CPX=CPX+DWWDX
436*      CPY=CPY+DWWDY
437*      CPZ=CPZ+DWWdz
438* 2301 CONTINUE
439*      DFX=DYO(N,N)+FEZO(N)-DZO(N,N)+FEYO(N)
440*      DFY=DZO(N,N)+FEXO(N)-DXO(N,N)+FEZO(N)
441*      DFZ=DXO(N,N)+FEYO(N)-DYO(N,N)+FEXO(N)
442*      HX=PS(N,N,1,1)*WXO(N)+PS(N,N,1,2)*WYO(N)+PS(N,N,1,3)*WZO(N)
443*      HY=PS(N,N,2,1)*WXO(N)+PS(N,N,2,2)*WYO(N)+PS(N,N,2,3)*WZO(N)
444*      HZ=PS(N,N,3,1)*WXO(N)+PS(N,N,3,2)*WYO(N)+PS(N,N,3,3)*WZO(N)
445*      K=3*N-1
446*      E(K+1,1)=HY*WZO(N)-HZ*WYO(N)+TXO(N)+CPX+DFX
447*      E(K+2,1)=HZ*WXO(N)-HX*WZO(N)+TYO(N)+CPY+DFY
448*      E(K+3,1)=HX*WYO(N)-HY*WXO(N)+TZO(N)+CPZ+DFZ
449* 230 CONTINUE
450* C
451* C      ADD MATRIX ELEMENT COMPUTATION (3X3)
452* C
453*      DO 3001 I=1,3
454*      DO 3001 J=1,3
455* 3001 ADD(I,J)=0.
456*      DO 3 I=1,NB
457*      DO 3 J=1,NB
458*      ADD(1,1)=ADD(1,1)+PS(I,J,1,1)
459*      ADD(1,2)=ADD(1,2)+PS(I,J,1,2)
460*      ADD(1,3)=ADD(1,3)+PS(I,J,1,3)
461*      ADD(2,2)=ADD(2,2)+PS(I,J,2,2)
462*      ADD(2,3)=ADD(2,3)+PS(I,J,2,3)
463*      ADD(3,3)=ADD(3,3)+PS(I,J,3,3)
464* 3 CONTINUE
465*      ADD(2,1)=ADD(1,2)
466*      ADD(3,1)=ADD(1,3)
467*      ADD(3,2)=ADD(2,3)
468* C
469* C      ADD VECTOR ELEMENT COMPUTATION (3X1)
470* C
471*      DO 7 K=1,NH
472*      AV(K,1)=0.
473*      AV(K,2)=0.
474*      AV(K,3)=0.
475*      DO 7 I=1,NB
476*      DO 7 J=1,NB
477*      IF(EPS(K,J).EQ.0) GO TO 7
478*      DO 10 M=1,3
479*      DO 10 L=1,3
480* 10 AV(K,M)=AV(K,M)+PS(I,J,M,L)*GO(K,L)
481* 7 CONTINUE
482* C
483* C      AKM SCALAR ELEMENT COMPUTATION
484* C
485*      DO 14 K=1,NH
486*      DO 14 M=1,NH
487*      IF(K.GT.M) GO TO 14
488*      IF((I(K).EQ.1).AND.(I(M).EQ.1)) GO TO 1399
489*      AIS(1)=0.
490*      AIS(2)=0.
491*      AIS(3)=0.

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492*      DO 15 J=1,NB
493*      DO 15 I=1,NB
494*      IF((EPS(K,J).EQ.0).OR.(EPS(M,I).EQ.0)) GO TO 15
495*      DO 18 N=1,3
496*      DO 18 L=1,3
497*      18 AIS(N)=AIS(N)+PS(J,I,N,L)*GO(M,L)
498*      15 CONTINUE
499*      AS(K,M)=GO(K,1)*AIS(1)+GO(K,2)*AIS(2)+GO(K,3)*AIS(3)
500*      GO TO 14
501*      1399 AS(K,M)=AS3(K,M)
502*      14 CONTINUE
503*      C
504*      C COMPUTE CORRECTION ELEMENTS FOR IEJ VECTOR
505*      C
506*      DO 40 I=1,NB
507*      EA(1)=0.
508*      EA(2)=0.
509*      EA(3)=0.
510*      DO 41 J=1,NB
511*      DO 411 M=1,3
512*      411 CW(M)=0.
513*      DO 42 K=1,NH
514*      IF(EPS(K,J).EQ.0) GO TO 42
515*      KR=KK(K)
516*      CW(1)=CW(1)+GMD(K)*IWY0(KB)*GO(K,3)-WZ0(KB)*GO(K,2)
517*      CW(2)=CW(2)+GMD(K)*IWZ0(KB)*GO(K,1)-WX0(KB)*GO(K,3)
518*      CW(3)=CW(3)+GMD(K)*IWX0(KB)*GO(K,2)-WY0(KB)*GO(K,1)
519*      42 CONTINUE
520*      DO 45 M=1,3
521*      DO 45 L=1,3
522*      45 EA(M)=EA(M)+PS(I,J,M,L)*CW(L)
523*      41 CONTINUE
524*      KI=3*(I-1)
525*      E(KI+1,1)=E(KI+1,1)-EA(1)
526*      E(KI+2,1)=E(KI+2,1)-EA(2)
527*      E(KI+3,1)=E(KI+3,1)-EA(3)
528*      40 CONTINUE
529*      DO 55 MI=1,3
530*      55 EC(MI)=E(MI,1)
531*      DO 52 J=2,NB
532*      DO 52 M=1,3
533*      KI=3*(J-1)+M
534*      52 EC(M)=EC(M)+E(KI,1)
535*      I=0
536*      DO 60 K=1,NH
537*      IF(PI(K).NE.0) GO TO 60
538*      I=I+1
539*      EC(I+3)=0.
540*      DO 601 M=1,3
541*      601 CE(M)=0.
542*      DO 61 J=1,NB
543*      IF(EPS(K,J).EQ.0) GO TO 61
544*      DO 65 M=1,3
545*      JI=3*(J-1)+M
546*      65 CE(M)=CE(M)+E(JI,1)
547*      61 CONTINUE
548*      DO 66 L=1,3
549*      66 EC(I+3)=EC(I+3)+GO(K,L)*CE(L)
550*      EC(I+3)=EC(I+3)+TH(K)
551*      60 CONTINUE
552*      DO 610 I=1,3
553*      DO 610 J=1,NH
554*      IF(PI(J).EQ.0) GO TO 610
555*      EC(I)=EC(I)-AV(J,I)*GMDD(J)
556*      610 CONTINUE
557*      K=0
558*      IV=3
559*      DO 612 I=1,NH
560*      IF(PI(I).NE.0) GO TO 612
561*      K=K+1
562*      IV=IV+1
563*      DO 511 J=1,NH

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564*      IF(PI(I,J).EQ.0) GO TO 611
565*      IF(I.GT.J) AS(I,J)=AS(J,I)
566*      EC(K+3)=EC(K+3)-AS(I,J)*BMDD(I,J)
567*      611 CONTINUE
568*      612 CONTINUE
569*      C
570*      C LOAD SYSTEM MATRIX (A) WITH AGG, AOK, AKH ELEMENTS
571*      C
572*      DO 23 I=1,3
573*      DO 23 J=1,3
574*      23 A(I,J)=AGG(I,J)
575*      DO 24 I=1,3
576*      K=0
577*      DO 24 J=1,NH
578*      IF(PI(I,J).NE.0) GO TO 24
579*      K=K+1
580*      A(K+3,I)=AV(J,I)
581*      A(I,K+3)=AV(J,I)
582*      24 CONTINUE
583*      K=0
584*      DO 250 I=1,NH
585*      IF(PI(I,I).NE.0) GO TO 250
586*      K=K+1
587*      L=0
588*      DO 25 J=1,NH
589*      IF(PI(I,J).NE.0) GO TO 25
590*      L=L+1
591*      IF(L.GT.L) GO TO 26
592*      A(K+3,L+3)=AS(I,J)
593*      GO TO 25
594*      26 A(K+3,L+3)=A(L+3,K+3)
595*      25 CONTINUE
596*      250 CONTINUE
597*      C
598*      C SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
599*      C (RELATIVE) ROTATIONAL ACCELERATIONS
600*      C
601*      C EXCHANGE ROWS IN A-MATRIX AND RT.HAND SIDE VECTOR
602*      C
603*      DO 619 K=1,3
604*      EE(K)=EC(K)
605*      DO 619 I=1,IV
606*      619 AE(K,I)=A(K,I)
607*      L=0
608*      DO 620 K=1,NH
609*      IF(TI(K).EQ.1) GO TO 620
610*      L=L+1
611*      EE(L+3)=EC(K+3)
612*      DO 621 I=1,IV
613*      621 4E(L+3,I)=A(K+3,I)
614*      620 CONTINUE
615*      DO 630 K=1,NH
616*      IF(TI(K).EQ.0) GO TO 630
617*      L=L+1
618*      EE(L+3)=EC(K+3)
619*      DO 631 I=1,IV
620*      631 AE(L+3,I)=A(K+3,I)
621*      630 CONTINUE
622*      C
623*      C EXCHANGE COLUMNS IN A-MATRIX
624*      C
625*      DO 639 K=1,3
626*      DO 639 I=1,IV
627*      639 A(I,K)=AE(I,K)
628*      L=0
629*      DO 640 K=1,NH
630*      IF(TI(K).EQ.1) GO TO 640
631*      L=L+1
632*      DO 641 I=1,IV
633*      641 A(I,L+3)=AE(I,K+3)
634*      640 CONTINUE
635*      DO 650 K=1,NH

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636*          IF (T(I,K).EQ.0) GO TO 650
637*          L=L+1
638*          DO 651 I=1,IV
639*          651  A(I,L+3)=AE(I,K+3)
640*          650  CONTINUE
641*          C
642*          C      SOLVE FOR HINGE-ANGLE ACCELERATIONS (NON-TERM. APPENDAGE)
643*          C
644*          NL=IV-NM1
645*          IF (NM1.EQ.0) GO TO 6561
646*          DO 652 I=1,NL
647*          DO 652 K=1,NM1
648*          AL(I,K)=0.
649*          DO 652 L=1,NM1
650*          LL=NL+L
651*          652  AL(I,K)=AL(I,K)+A(I,LL)*AT(L,K)
652*          DO 654 I=1,NL
653*          DO 654 K=1,NL
654*          AE(I,K)=0.
655*          DO 653 L=1,NM1
656*          LL=NL+L
657*          653  AE(I,K)=AE(I,K)+AL(I,LL)*A(LL,K)
658*          654  A(I,K)=A(I,K)-AE(I,K)
659*          DO 656 I=1,NL
660*          EC(I)=0.
661*          DO 655 L=1,NM1
662*          LL=NL+L
663*          655  EC(I)=EC(I)+AL(I,LL)*EE(LL)
664*          656  EE(I)=EE(I)-EC(I)
665*          6561  CONTINUE
666*          CALL SOD(A,V,NL,EE,V,I,$92,WRK)
667*          C
668*          C      SOLVE FOR HINGE-ANGLE ACCELERATIONS (TERM. APPENDAGE)
669*          C
670*          IF (NM1.EQ.0) GO TO 6581
671*          DO 657 I=1,NM1
672*          II=NL+I
673*          EC(II)=EE(II)
674*          DO 657 L=1,NL
675*          657  EC(II)=EC(II)-A(II,L)*EE(L)
676*          DO 658 I=1,NM1
677*          II=NL+I
678*          EE(II)=0.
679*          DO 658 L=1,NM1
680*          658  EE(II)=EE(II)+A(II,L)*EC(L)
681*          6581  CONTINUE
682*          C
683*          C      RECONSTRUCT SOLUTION VECTOR IN ORIGINAL ORDER
684*          C
685*          EC(1)=EE(1)
686*          EC(2)=EE(2)
687*          EC(3)=EE(3)
688*          L=3
689*          DO 659 K=1,NH
690*          IF ((T(I,K).EQ.1).OR.(P(I,K).EQ.1)) GO TO 659
691*          L=L+1
692*          EC(K+3)=EE(L)
693*          659  CONTINUE
694*          L=NL
695*          DO 661 K=1,NH
696*          IF ((T(I,K).NE.1).OR.(P(I,K).EQ.1)) GO TO 661
697*          L=L+1
698*          EC(K+3)=EE(L)
699*          661  CONTINUE
700*          DO 662 K=1,NH
701*          IF (P(I,K).NE.1) GO TO 662
702*          EC(K+3)=GMDO(I,K)
703*          662  CONTINUE
704*          92  CONTINUE
705*          RETURN
706*          END

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