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EFFECT OF MOTION FREQUENCY SPECTRUM ON SUBJECTIVE COMFORT RESPONSE

STOL Program Memorandum Report 403212

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EFFECT OF NOTION FREQUENCY SPECTRUM ON SUBJECTIVE COMFORT RESPONSE -35 p HC \$4.75 CSCL 05E (Virginia Univ.) 36

N74-28630 unclas 16057 G3/05

November 1973

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INTRODUCTION

In order to model passenger reaction to present and future aircraft environments, it is necessary to obtain data in several ways. First, of course, is the gathering of environmental and passenger reaction data on commercial aircraft flights. Although these commercial passengers are the group the model is intended for, it is virtually impossible to obtain from commercial flights the complete range of environmental variables and their interactions required for the development of a versatile model. In addition, detailed analyses of particular aspects of human reaction to the environment are best studied in a controllable experimental situation. Thus the use of simulators, both flight and ground based, is suggested.

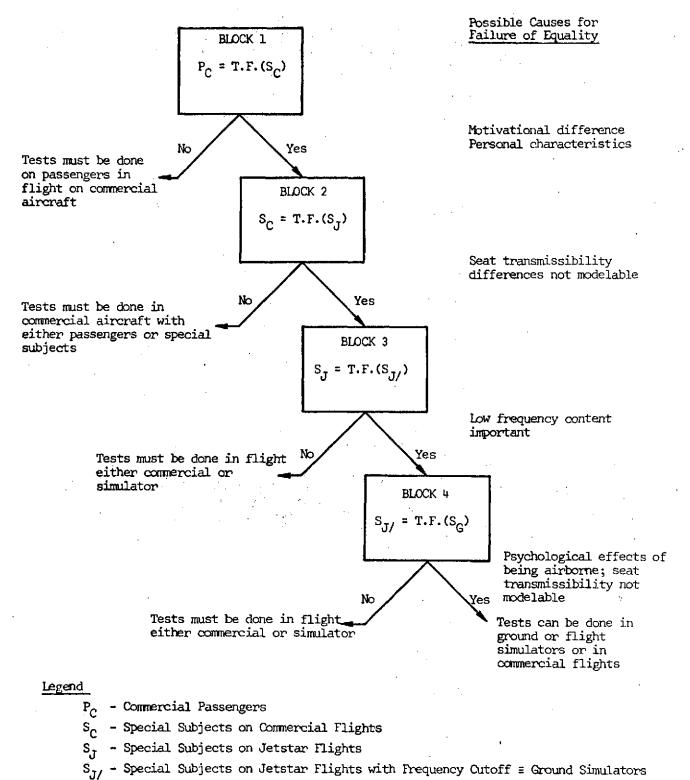
The applicability of any results from simulators for use in predicting human response in a commercial environment hinges on determining:

- a) the usability of test subjects in place of passengers;
- b) the psychological differences between reaction to a simulator vs. a commercial flight environment; and
- c) the fidelity of the simulation.

A scheme for evaluating each of these is illustrated in Figure 1, where

Block 1: tests the correlation between commercial passengers and special test subjects, (a above)

Block 2: the differences between commercial vs. non-commercial flights, (b above)



S_G - Special Subjects in Ground Simulated Flights

T.F.- Transfer Function

FIGURE 1. SIMULATOR VALIDATION STRATEGY

- Block 3: analyzes the effects of low frequency content (this is the portion of the motion spectrum over which groundbased simulators have poor fidelity) (c above)
- Block 4: compares ground-based simulator responses vs. flight data (b above).

Block 1 (subjects vs. passengers) is evaluated in detail in reference 1 with the results indicating that one may expect good correlation between passenger and test subjects. This memorandum describes an experiment conducted to explore the relationships shown in Blocks 2 and 3 of Figure 1. Block 4 can best be answered by a series of flight and ground based tests.

DESCRIPTION OF EXPERIMENT

Twelve ten-minute flight segments, two per flight, two flights per day, using four test subjects were conducted on the NASA, Flight Research Center, Jetstar aircraft with the GPAS (General Purpose Airborne Simulator) system. The test protocol for each two-hour flight consisted of the following:

- Brief two test subjects on purpose of program and use of the 5-point rating scale;
- 2. Airplane boarding;
- 3. Taxi, take off and climb to test altitude;
- Maintain cruise conditions and engage GPAS system;
- 5. Record comfort responses and environment for ten one-minute acceleration segments, each with a randomly chosen vertical acceleration level;
- 6. Disengage GPAS, make 180[°] turn and establish straight and level cruise;
- 7. Engage GPAS system;
- 8. Same as (5) above for second set of programmed motion conditions;

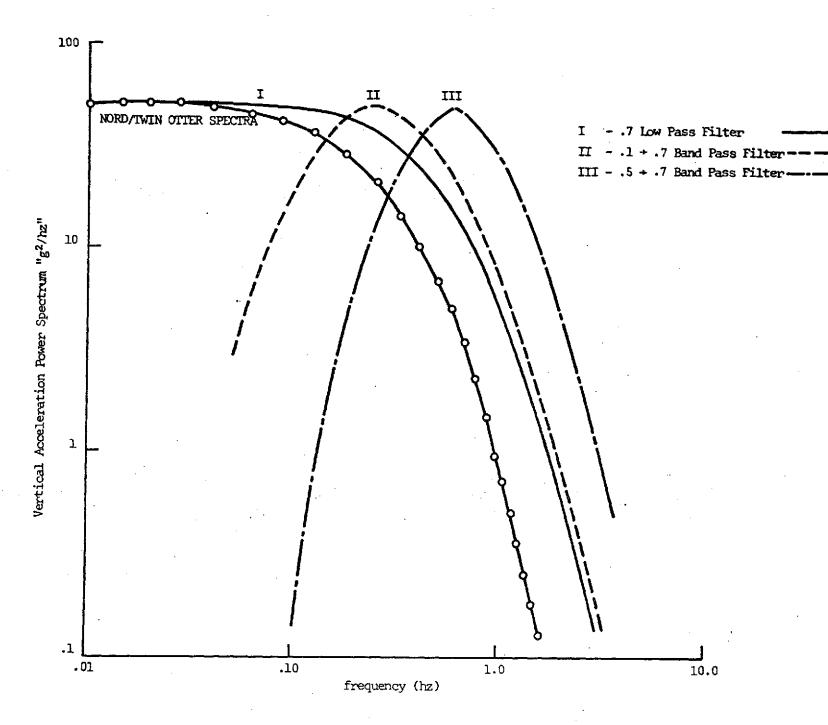
9. Disengage GPAS, return to boarding gate; and

10. Disembark subjects.

This was repeated for the second set of two subjects on the same day. Each succeeding day the protocol for different test conditions was repeated.

The flight conditions are summarized in Table 1, with the spectra (I, II, III) shown in Figure 2. The raw data is given in Appendix I along with the test subject comfort ratings which were taken at the end of each one-minute segment of constant acceleration. Each acceleration level represents the area under the power spectrum curve.

FIGURE 2. VERTICAL POWER SPECTRA



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TABLE I

Flight	Vertical Acceleration (rms)	Lateral Acceleration (rms)	6n onterror
			Spectrum
1	Randomly Varied	0	I
2	Randomly Varied	0	II
3	Randomly Varied	0	III
4	Randomly Varied	.02	I
5	Randomly Varied	.02	II
6	Randomly Varied	.02	III
		indue hal bei in the second	····

JETSTAR TEST CONDITIONS

As can be seen, the motion profiles utilized three different spectra: I - approximating the actual environment; II - .1-.7 hz band-pass spectrum which reduced the amount of very low frequency content in the motion; and, III - a .5-.7 hz band-pass-filtered spectrum which further reduced the low frequency content. The comfort scale used was basically the same as that used by the University of Virginia in its commercial flight tests--a five point scale with 1 signifying very comfortable and 5, very comfortable. The equipment and manner in which the data was recorded was identical to tests conducted on commercial flights. These are described in reference 2.

RESULTS

First, the matter of the importance of the low frequency components of motion will be considered. Figures 3 through 8 indicate the acceleration for each of the six test conditions along with subjective comfort ratings of the subjects exposed. Here, one can see that although equivalent acceleration levels were desired for each flight, this did not occur. These data have been examined using two techniques. The first is a gross analysis where levels of vertical acceleration are identified which will elicit prescribed responses. With the note that there is variability in subject responses, this type of analysis gives use to the following subjective response acceleration bands. (Table II; also shown in Figures 3-8.)

As can be seen from these figures and Table II, there does not appear to be significant differences between the regions predicted for any of the power spectra. This implies that the "low" frequency content of power spectrum I may not be necessary in determining ride quality criteria in motion.

The second type of analysis is somewhat more rigorous. Here we assume that the data is drawn from a normally distributed population, that the responses are independent, and that there is homogeneity of variance. A well-known technique of data analysis, the t test, is applied and is extremely insensitive to violations of the normal distribution and homogeneity of variance assumptions. This means that these assumptions can be violated without affecting the inference derived from the test (reference 3, 4). The t test is used to test the hypothesis that for a given acceleration level the mean response for the atmospheric spectrum (I)

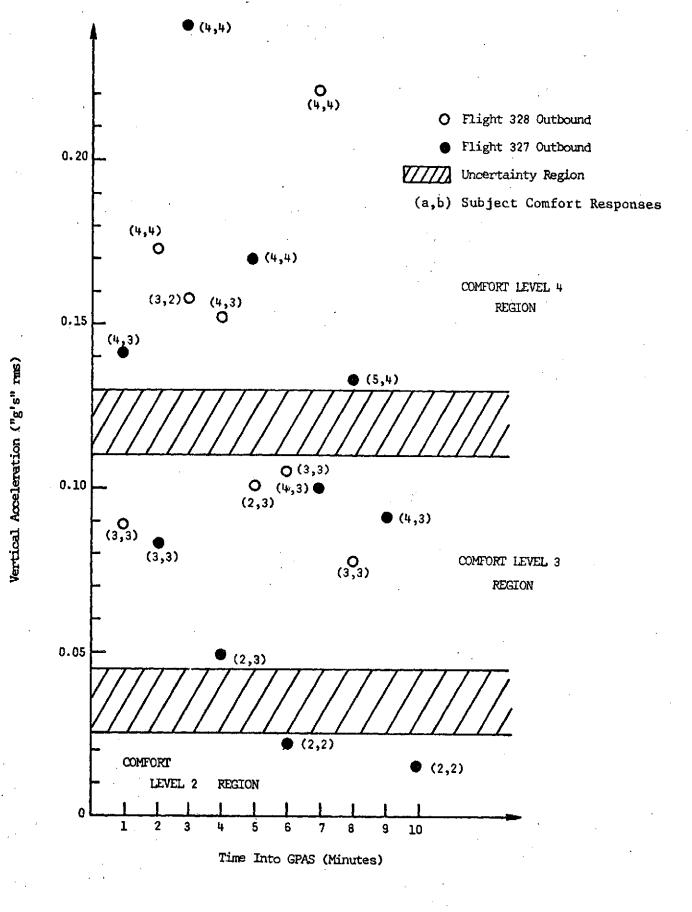


FIGURE 3. COMFORT RESPONSES, SPECTRUM I (0.7 BAND PASS), NO LATERAL ACCELERATION

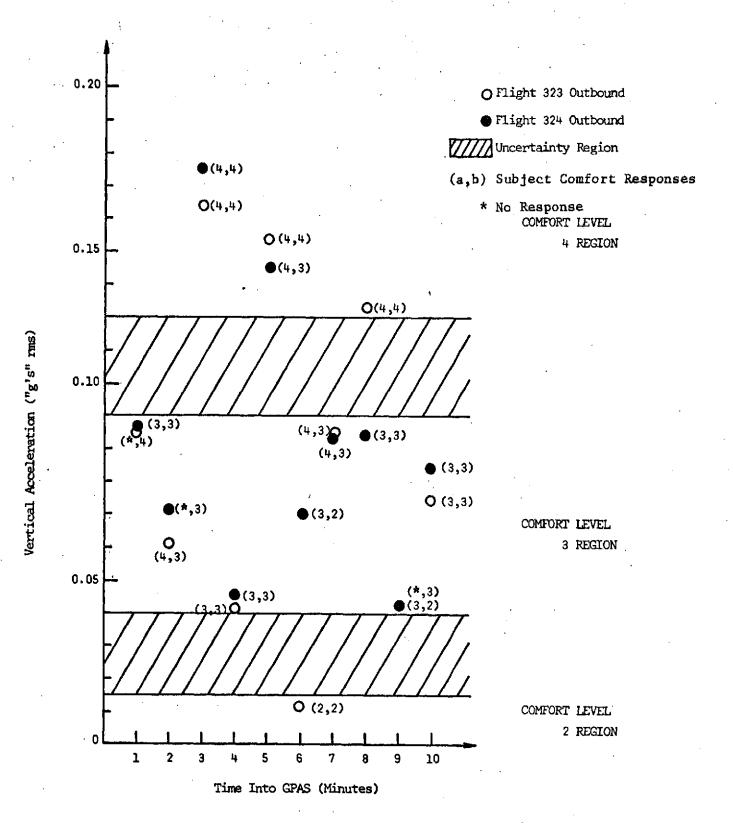


FIGURE 4. COMFORT RESPONSES, SPECTRUM II (0.1 - 0.7 BAND PASS), NO LATERAL ACCELERATION

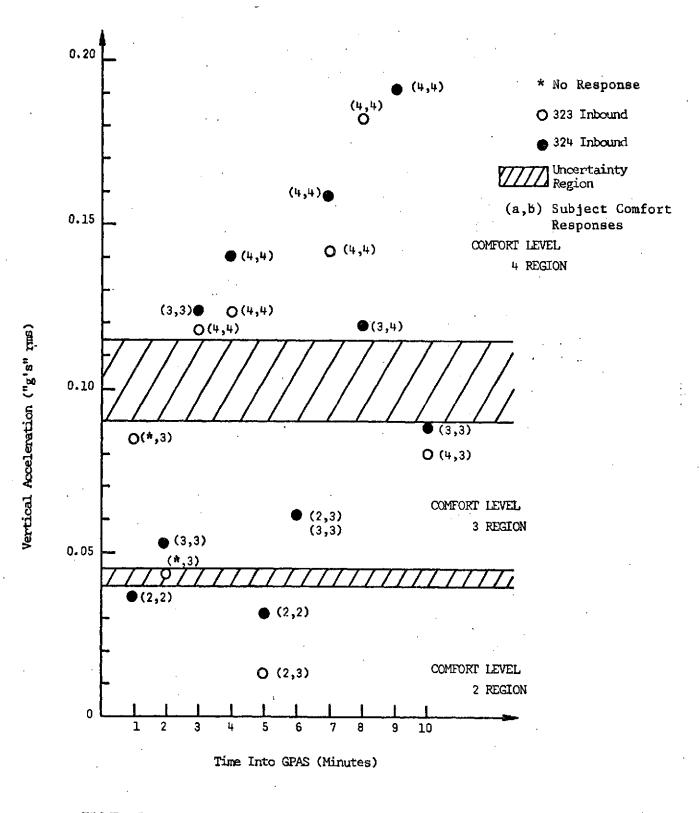


FIGURE 5. COMFORT RESPONSES, SPECTRUM III (0.5 - 0.7 BAND PASS), NO LATERAL ACCELERATION

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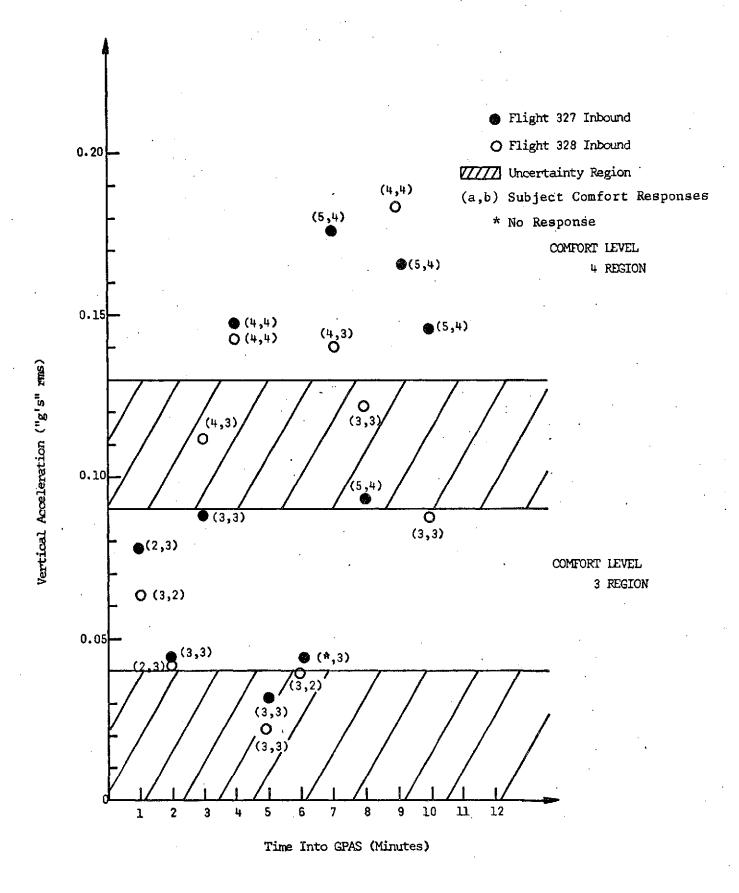


FIGURE 6. COMFORT RESPONSES, SPECTRUM I (0.7 BAND PASS), CONSTANT LATERAL ACCELERATION

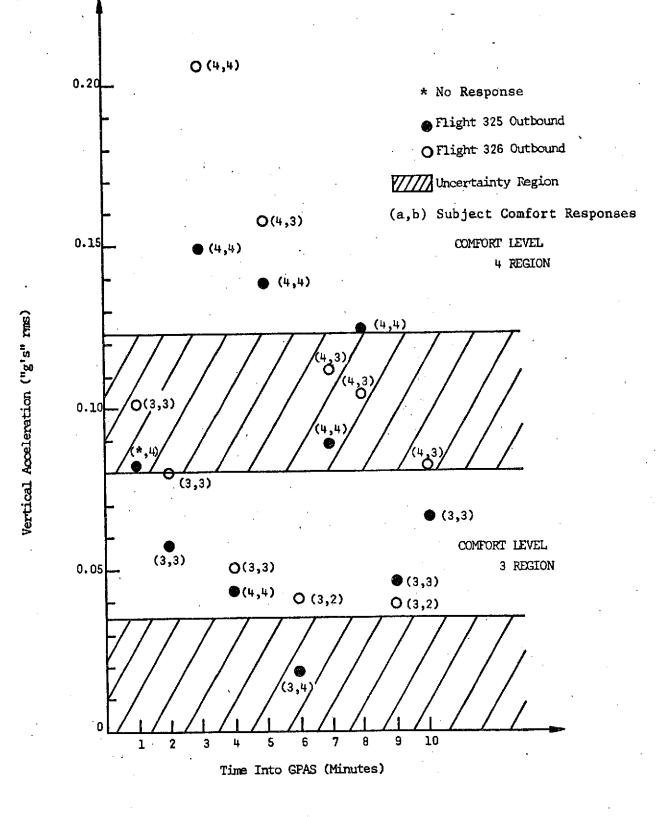


FIGURE 7. COMFORT RESPONSES, SPECTRUM II (0.1 - 0.7 BAND PASS), CONSTANT LATERAL ACCELERATION

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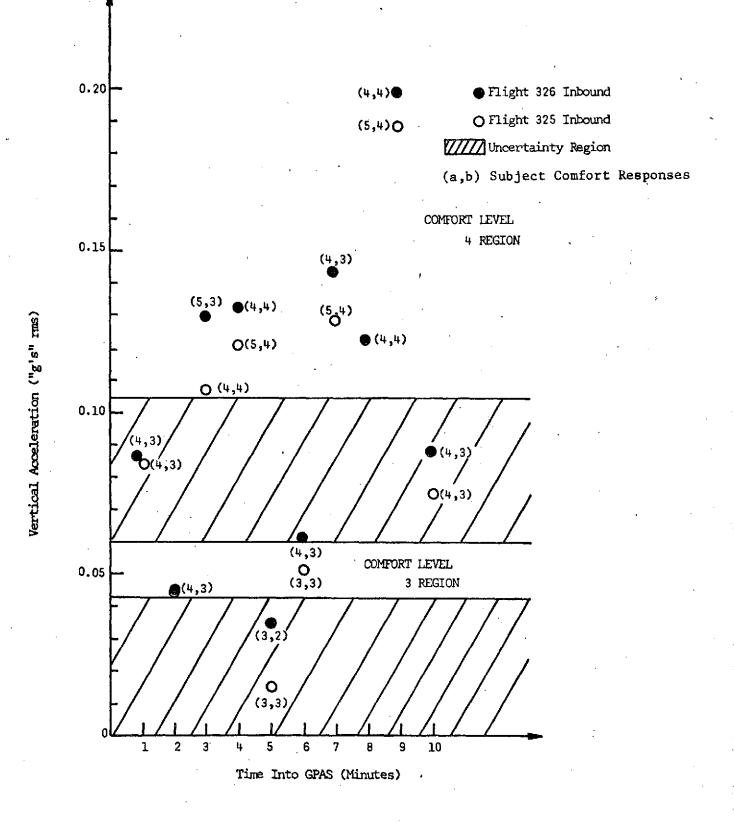


FIGURE 8. COMFORT RESPONSES, SPECTRUM III (0.5 - 0.7 BAND PASS), CONSTANT LATERAL ACCELERATION

TABLE II

ACCELERATION LEVELS

(all values in "g" rms)

Res	ponse Cate	gory		Frequency Spectra	
		· · · · · · · · · · · · · · · · · · ·	<u>I</u>	<u> </u>	III
·					
	<u><</u> 2	(Comfortable)	<.025	<.015	<.04
No	2-3	(Uncertainty Region)	.025→.045	.015+.04	.04 →.045
Lateral	3	(Neutral)	.045→.11	.04 →.10	.045→.09
Acceleration	3-4	(Uncertainty Region)	.11 → . 13	.10 →.13	.09 →.115
	4	(Uncomfortable)	>.13	>.13	>.115
	2	(Comfortable)			
Constant	2-3	(Uncertainty Region)	<.045	<.045	<.05
Latera1	3	(Neutral)	.045+.095	.045→.085	.05 →.065
Acceleration	3-4	(Uncertainty Region)	.095→.13	.085+.125	.065→.105
	4	(Uncomfortable)	>.13	>.125	>.12

is the same as that for the other spectra (II, III). (Details of hypothesis testing are described in Appendix II, along with the analysis of the present flight data.) The actual hypothesis used is:

> H_1 : At a given acceleration level, the mean response on flights using spectrum I is .5 greater than the mean response on flights using spectra II or III. (This implies that spectrum I is at least .5 less comfortable than spectra II or III. The consequence of H_1 being true is the necessity of doing tests with an atmospheric spectrum.)

The test is arranged in this way in order to make the most costly error (using spectra II or III for our tests, when in fact they are not suitable) a Type I error (rejecting a true hypothesis, see Appendix II).

The results show that in all but three cases the hypothesis H_1 can be rejected at the .1 significance level or lower. This means that there is a 10% chance of H_1 being true. Consequently, we can be 90% confident that the hypothesis is false or that there is not a significant difference in the responses for any of the three spectra.

The second aspect investigated through the use of these flight data is the equivalence of test subject responses on commercial vs. noncommercial flights. To examine this equivalence, the model generated from commercial flights^{*} is used to predict the responses on the Jetstar.

^{*}The interested reader may wish to refer to reference(s) (1) and (2) for a better insight into the development of this model.

Figure 9 shows a comparison between the observed subjective response and the predicted comfort model value given by $C = 2 = 11.9 a_{vert} + 7.5 a_{trans} + 0.12 a_{vert}^{a} t_{trans} + a_{long}^{a} + 0.1a_{yaw}^{a} + 0.1a_{roll}^{c}$. Overall, the model predicts comfort response within ± 0.5 in 82% of the cases. The majority of points where the model failed occurred in two segments, Flight 325 inbound (spectrum III and 0.3 lateral acceleration) and Flight 326 outbound (spectrum II and 0.3 lateral acceleration).

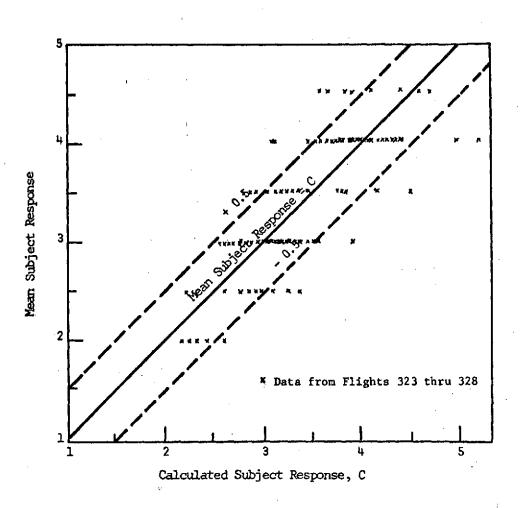


FIGURE 9. COMPARISON BETWEEN OBSERVED SUBJECTIVE RESPONSE AND PREDICTED COMFORT MODEL

CONCLUSIONS AND RECOMMENDATIONS

It has been shown that there is a reasonably high probability that the low frequency end of the spectrum will not be necessary for simulation purposes. That is, the fidelity of any simulation which omits the very low frequency content (such as spectra II or III) will not yield results which differ significantly from the "real" environment. In addition, there does not appear to be significant differences between the responses obtained in the airborne simulator environment (Jetstar) versus those obtained on commercial flights.

These results should not, however, be considered conclusive; only promising. Since the matter is of great importance, additional tests are planned as soon as time becomes available on a flight simulator (estimated to be late spring or summer, 1974). In the meantime, at least order of magnitude guidance on these issues is available to those wishing to use laboratory simulators for the many problems to which they are ideally suited. The need to proceed with a study of the importance of psychological effects (e.g., anxiety, apprehension, etc.) associated only with being airborne is immediate.

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- Baker, Bela O., Bardyck, Curtis F., and Peterinovich, Lewis F. Weak Measurements vs. Strong Statistics: An Empirical Critique of S.S. Stevens' Proscriptions on Statistics. <u>Educational and Psychological Measurement</u>, Vol. XXVI, No. 2, 1966.

APPENDICES

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DATE: JUNE 18, 1973

Outbound Segment

Inbound Segment

,

APPENDIX I

Spectrum II (0.1 - 0.7 Band Pass) No Lateral Acceleration

Spectrum III (0.5 - 0.7 Band Pass) No Lateral Acceleration

Serial Number	Subject 5 Response	Subject 2 Response		σ _{lat}	σlong	o yaw	^σ roll	^o pitch	Serial Number		Subject 2 Response		σlat	^σ long	σ yaw	σ _{rol1}	σpitch
			-				•								- <u>-</u>		
787	*	. 4	.0941	.0014	.0028	.0598	.4344	.3478	799	. *	3	.0848	.0023	.0012	.0739	.4196	2885
788	4	3	.0614	.0014	.0017	.0827	.3754	.2800	800	*	3			.0022			
789	4	4	.1641	.0015	.0081	.0632	.4270	.9028	801	4	4			.0034			
790	3	3	.0413	.0009	.0014	.0503	.3129	.2244	802	4	4			.0055			
791	4	4					.5561		803	2	3						-
792	2	2					.1228		804	3	- -			.0014			
793	4	3					.4696			_	3		- i -	.0017			
	•	•							805	4	.4	.1419	.0046	.0052	.0742	.5585	.7981
794	4	4	.1332	.0011	.0041	.0495	.5457	.5328	806	4 .	4	.1815	.0043	.0095	.0666	5788	1.2091
795	*	3	.0437	.0011	.0018	.0512	.3595	.2072								12700	1.2071
796	3	3	.0747	.0016	.0025	.0661	.4540	.3636	808	4	3	.0800	.0039	.0020	. 0599	.2958	.2651

* = Subject did not respond

21

 σ = rms acceleration (mean biased out)

Outbound Segment

Spectrum II (0.1 - 0.7 Band Pass) No Lateral Acceleration

DATE: JUNE 18, 1973

Inbound Segment

Spectrum III (0.5 -0.7 Band Pass) No Lateral Acceleration

Seríal Number	Subject 8 Response	Subject 3 Response		^o lat	⁰ long	d yaw	σroll	σ _{pitch}	Serial Number		8 Subject 3 Response		σ _{lat}	σ long	o yaw	σ _{roll}	^o pitch
819	3	3	.0945	.0011	.0048	.0580	.2894	.4289	831	2	2	0373	0017	0011	0060		
820	*	3					.3387		832	3	3			.0011			
821	4	4					.5653		833	3. 3	3			.0011			
822	3	3					.1713		834	4	4			.0046			
823	4	3					.4400		835	2	+ 2			.0041			
824	3	2					.3312		836	2	2			.0009			
826	4	3					.4483		837	4	4			.0008			
827	3	3	.0961						838	3	4			.0054			
828	3		.0446						839	4				.0031			
830	3		.0845						840	4 3				.0072			1.1098 .3357

* = Subject did not respond

 σ = rms acceleration (mean biased out)

· 22

Outbound Segment

Inbound Segment

DATE: JUNE 19, 1973

Spectrum II (0.1 - 0.7 Band Pass) Constant Lateral Acceleration Spectrum III (0.5 - 0.7 Band Pass) Constant Lateral Acceleration

Serial Number	Subject 5 Response			σ _{lat}	^σ long	o yaw	^o roll	^o pitch	Serial Number		Subject 3 Response	σ vert	σ _{lat}	dlong	σ yaw	σ _{roll}	^o pitch
									•								
987	*	4	.0820	.0277	.0041	.8895	3.1000	.3827	200	4	3	.0835	.0213	.0027	.2995	1.5324	.2655
988	3	3	.0567	.0223	.0035	. 7405	2.4031	. 3539	201	4	3	.0451	.0235	.0016	.4180	2.0154	.1751
989	4	4	.1487	.0211	.0076	.7422	1.8119	.9162	202	.4	4	.1069	.0237	.0042	.4091	1.9637	. 3778
990	4	4	.0434	.0270	.0027	1.0071	2.8954	.2170	203	5	4					2.2489	
991	4	4	.1386	.0234	.0055	.6585	2.6920	.5706	204	3	3					1.8479	
992	3	4	.0181	.0257	.0027	.7600	2.9895	.0701	205	3	3					1.6176	
993	4	4	.0886	.0230	.0037	.7622	2.4342	.4210	206	5	4			_		1.6680	
994	4	4	.1242	.0202	.0055	.6483	2.2886	.4481	207	*	*	*	*	*	*	*	*
995	3	3	.0440	.0220	.0033	.5646	2.3937	.7280	208	5	4	.1885	.0234	.0093	.4060	2.1692	
996	3	3					2.4460		209	4	3					1.7838	

- * = Subject did not respond
- σ = rms acceleration (mean biased out)

DATE: JUNE 19, 1973

Outbound Segment

Inbound Segment

Spectrum II (0.1 - 0.7 Band Pass) Constant Lateral Acceleration Spectrum I

and Pass)	Constant Lateral	Acceleration	Spectrum III	(.5 – (0.7 Band	Pass) (Constant	Lateral	Acceleration	

Serial Number	Subject 8 Response	Subject 2 Response	o vert	^o lat	Jong	σ yaw	^o roll	^σ pitch	Serial Number	Subject 8 Response	Subject 2 Response	σvert	σlat	σlong	σ yaw	^o roll	^σ pitch
	_	• •															
221	3	3	.1016	.0332	.0037	1,1235	2.7397	.4587	233	4	3	.0839	.0231	.0012	.4322	.0208	2.0034
222	3	3	.0795	.0275	.0036	.7151	2.6652	.4007	234	4	2	.0446	.0261	.0010	.5246	2.2935	-, 1828
223	4	4	.2057	.0344	.0101	1.0903	3.4686	.9251	235	5	3	.1295	.0231	.0031	.3303	1.7760	. 3099
224	3	3	.0504	.0341	.0015	.9365	3.0138	.2115	236	4	4	.1326	.0246	.0036	.4716	1,9094	.5706
225	4	3	.1578	.0306	.0061	.9557	2:9379	.4895	237	3	2 ·	.0342	.0246	.0014	.5465	2.0942	.1306
227	3	2	.0412	.0377	.0025	1,3609	2.7818	.2843	238	4	3	.0617	.0296	.0012	.6144	2.5524	.2123
228	4	3	.1118	.0738	.0031	.7572	1.9669	.4357	239	4	3	.1428	.0246	.0049	.3808	2,0430	.4797
229	4	3	.1036	.0243	.0022	.7328	2.1726	.3600	241	4	· 4	.1219	.0299	.0040	.5642	2.5256	. 3848
230	3	2	.0391	.0296	.0010	.7351	2.7416	.1786	242	4	4	.1992	.0288	.0096	.6417	2.4487	1.1677
231	4	3	.0823	.0225	.0014	,6021	2,2165	.4446	243	4	3	.0876	.0273	.0021	.4539	2.2138	.3254

* = Subject did not respond

24

 σ = rms acceleration (mean biased out)

Outbound Segment

Spectrum I (0.7 Low Band Pass) No Lateral Acceleration

DATE: JUNE 20, 1973

Inbound Segment

Spectrum I (0.7 Low Band Pass) Constant Lateral Acceleration

Serial Number	Subject 5 Response		σ vert	⁰ lat	^d long	o yaw.	σ _{roll}	^o pitch	Serial Number	Subject 5 Response	Subject 3 Response	vert	o lat	σlong	o yaw	σ _{roll}	^o pitch
		· ·															
347	4	3	.1411	.0008	.0064	.0734	.4244	.6293	360	2	3	.0775	.0185	.0024	.6209	1.9969	.3045
348	3	3	.0818	.0008	.0022	.0591	.2352	.4209	361	2	3						7 .1858
349	4	4	.2408	.0008	.0184	.0670	.4214	1.5819	362	3	3						.4114
350	3	3	.0486	.0006	.0014	.0484	.1508	.1801	363	4	4	· -					, .7061
351	4	4	.1698	.0013	.0106	.0777	.3247	1.1701	364	3	3						3.1837
352	2	2			.0021				365	* *	:						5 .2208
353	4	3			.0031				366	*	*						.2518
354	5	4			.0066			-	367	*	3						. 1825
355	4	3			.0034				368	5	4						
356	2	2			.0051				370	5.	4	-					.8192
								11240	371	-							.3802
										5	4	.1048	.0210	.0098	.6958	2.1906	.8673
									372	5	4	.1454	.0165	.0110	.4626	1.9698	8 1.0143

* = Subject did not respond

 σ = rms acceleration (mean biased out)

Outbound Segment

Spectrum I (0.7 Low Band Pass) No Lateral Acceleration

DATE: JUNE 20, 1973

Inbound Segment

Spectrum I (0.7 Low Band Pass) Constant Lateral Acceleration

	Subject 8 Response	Subject 2 Response	σ vert	^o lat	σlong	σ yaw	^σ roll	σpitch	Serial Number	Subject 8 Response	-	d vert	Jat	^o long	o yaw	^o roll	^o pitch
			1						394	3	2	.0634	.0165	.0014	.4283	1.5229	9.3298
384	3	3	0890	.0009	.0018	.0725	.2334	. 3960	395	3	3						7.2279
385	4	4	.1727	.0014	.0064	₀0626	.4376	. 7854	396	4	3						4.3893
386	3	2	.1578	.0009	.0061	.0721	.2758	.8228	397	4	4						5.5404
387	4	3	·1521	.0027	.0073	.0646	,4184	.6711	398	3	3			.0011			
388	2	2	.1014	.0015	.0075	.0567	.1823	.3243	399	3	2			.001			
389	3	3	.1058	.0014	.0066	.0543	.2762	.4905	400	4	3	.139	.0232	.0043	.7735	1.977	5.6437
390	4	. 4	.2203	.0014	.0135	.0614	.5063	1.2674	401	3	3						.5254
391	3	3	.0782	.0010	.0042	.0572	. 2968	.4059	402	4	4			.0111			
		·							403	3	3	.0868	.0237	.0066	.5917	1.774	. 34:59

* = Subject did not respond

σ = rms acceleration (mean biased out)

•••

APPENDIX II

General Background

Hypothesis testing involves making assumptions about the distribution function of a random variable and deciding if those assumptions are consistent with the observed data. In testing hypotheses about differences of means, it is common to use the "t" statistic:

$$t_{obs} = \frac{(\overline{x}_a - \overline{x}_b) - (\mu_a - \mu_b)}{\sqrt{S_p^2 \left(\frac{1}{n_a} + \frac{1}{n_b}\right)}}$$

where

 $\overline{X}_{a}, \overline{X}_{b}$ are the observed means of the experimental data sets a and b, respectively μ_{a}, μ_{b} are the true means of the sets from which the experimental data was taken $S_{p}^{2} = \frac{(n_{a} - 1) S_{a}^{2} + (n_{b} - 1) S_{b}^{2}}{(n_{a} - 1) + (n_{b} - 1)}$

where

 S_a , S_b are the observed standard deviations of the experimental data n_a , n_b are the number of data points in each experimental data set.

This statistic is a random variable having a "Student's t" distribution with $v = n_a + n_b - 2$ degrees of freedom. Our assumptions about this distribution will concern the value of the difference between the true means of the two sets from which the data is taken. The hypothesis being tested is referred to as H₁, the alternate hypothesis H₂. Let's say that H₁ is is $\mu_a - \mu_b = 0$ (i.e., there is no difference in the means of the two sets from which the experimental data was drawn). For v degrees of freedom (which depends on the number of data points), the distribution function of t is shown in Figure II-1.

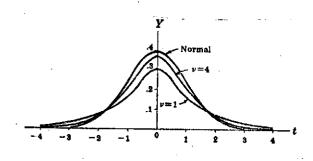


Figure II-1. Student's t Distributions for various values of v.

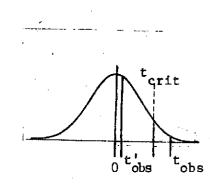


Figure II-2.

Student's t Distribution with Two Observed Values of t and a t crit

We must now decide on a criterion based on the observed data for accepting or rejecting H₁. If our observed difference of means has a low probability of occurrence when H₁ is true, then we should be suspicious of our hypothesis. In Figure II-2 the probability that $t \geq t_{obs}$ is the area under the curve to the right of t_{obs} . Since this area is small, the probability of observing a t this extreme will be small, and we should consider the possibility that our H₁ does not represent the true difference of means. Similarly, the probability of observing t'_{obs} is relatively high (area to the right of t'_{obs}) and thus poor evidence for rejecting H₁.

Let us choose as our criterion variable, the probability, when H_1 is true, of the occurrence of the observed difference of means. If our observed data have a lower probability than some arbitrarily selected lower limit, called α , then we will be sufficiently suspicious of H_1 to reject it. This corresponds to picking a t value, say t_{crit} , such that the probability that $t_{obs} \geq t_{crit}$ is less than α . The limiting probability, denoted by α is called the level of significance of the test and is the probability of rejecting a true hypotehsis. In Figure II-2 t_{obs} would cause H_1 to be rejected at whatever α level t_{crit} represents, while t'_{obs} would cause H_1 to be accepted.

We are not out of the woods yet. We must consider the probability of our data falling in H₁'s acceptance region (in Figure II-2 to the left of t_{crit}) when H₁ is not true. If H₁ is false, i.e., $\mu_a - \mu_b \neq 0$, then some alternate hypothesis H₂ is true. Lets say that our H₂ is $\mu_a - \mu_b = \delta > 0$ graphically:

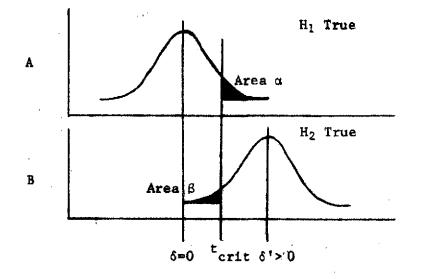


Figure II-3 Effects of an Hypothesis Shift on the Power and Significance Level of Test.

From Figure II-3 it is obvious that when H_1 is not true, it is possible for the observed data to fall in the acceptance region of H_1 . The probability of this event, denoted β , equals the area to the left of t in Figure II-3B. A quantity called the power of the test, equal to 1- β , is the probability of rejecting a false hypothesis. It should be clear that the power of the test depends on α and δ , and decreases as α decreases.

It is common in hypothesis testing to speak of Type 1 and Type 2 errors. They are respectively rejecting a true hypothesis and accepting a false hypothesis. The probabilities of making these errors are α and β , respectively.

	True Sit	tuation
Action	H _l True	H ₂ True
Accept H_1	No error	Type 2 error prob = β
Reject H _l	Type 1 error prob = α	No error

The best test would have both α and β small. Since α is directly under the experimenter's control, the test is usually arranged so that the Type 1 error is the most expensive, and its probability can be made arbitrarily low. For a given significance level, the power of the test depends on δ ; moving δ

to the right in Figure II-3B (i.e., H_2 becomes $\mu_a - \mu_b = \delta' > \delta$) would decrease β , increasing the power. If the Type 2 error is relatively expensive, it may be necessary to choose an α somewhat larger than desired in order to obtain a reasonable power. If increasing α is not desirable due to the expense of a Type 1 error, the only solution lies in increasing the sample size, since the distribution becomes more sharply peaked as the sample size increases, i.e., $\nu = n_a + n_b - 2$ increases (see Figure II-1).

Test Data

Our hypotheses for these tests are as follows:

Hypothesis

H₁: At a given acceleration level, the mean response on flights using spectrum I is .5 greater than the mean response on flights using spectra II or III. (This implies that the atmospheric spectrum is at least .5 less comfortable. The consequence of H₁ being true is the necessity of doing tests with an atmospheric spectrum.)

Alternate Hypothesis

 H_2 : At a given acceleration level, the mean response on flights using spectrum I is less than .5 greater than the mean response on flights using spectra II or III. (This implies that the atmospheric spectrum is less than .5 less comfortable than spectra II or III. The consequence of H_2 being true is the ability to use modified spectra with less low frequency content.)

Putting our hypotheses in symbolic terms:

$$H_1: \mu_a - \mu_b \ge .5$$

 $H_2: \mu_a - \mu_b < .5$

where

 $\boldsymbol{\mu}_{_{\boldsymbol{\mathcal{B}}}}$ is the true mean response to spectrum I

 $\boldsymbol{\mu}_{\underline{b}}$ is the true mean response to spectra II or III.

To calculate the t-statistic, t obs, for a given acceleration level, we proceed as follows:

Let

 \overline{X}_{A} = observed mean response to spectrum I \overline{X}_{B} = observed mean response to spectra II or III S_{a} = observed standard deviation for spectrum I S_{b} = observed standard deviation for spectra II or III P_{a} = the number of mean response to spectra I or III

 $n_a, n_b =$ the number of responses to spectrum I and spectra II or III

Then

$$t_{obs} = \frac{\overline{x}_{A} - \overline{x}_{B} - (\mu_{a} - \mu_{b})}{\sqrt{s_{p}^{2} (\frac{1}{n_{a}} + \frac{1}{n_{b}})}}$$

where
$$S_p^2 = \frac{(n_a - 1) S_a^2 + (n_b - 1) S_b^2}{n_a - 1 + n_b - 1}$$

The acceleration levels are broken into six bands. An example of the calculations is shown for band 3 of vertical acceleration, 0 lateral acceleration, spectrum I compared with spectrum II. Here the values for the needed variables are:

 $\overline{X}_{A} = 3.0$ $S_{a} = .756$ $n_{a} = 8$ $\overline{X}_{b} = 3.33$ $S_{b} = .5$ $n_{b} = 9$

This yields a t value given by:

Filter 1

$$t_{obs} = \frac{3.0 - 3.33 - .5}{.399 \times .236} = \frac{-.83}{.307} = -2.7$$

This compares with a t value for 15 degrees of freedom of -2.6.

Thus we can reject H_1 at the 90% level of significance. In other words, if we reject H_1 on the basis of this evidence, the probability that we are rejecting a true hypothesis is less than 10%.

Performing a similar calculation for the other g levels, we get the following results:

O Lateral Acceleration

Filter 2

		tobs	reject at α=	tobs	reject at α=	
Band	1	-4.12	.005	-3.39	.005	
	2	-1.06	.2	-1.29	.2	
	3	-2.32	.025	-3.87	.005	
	4	-2.70	.01	-3.06	.005	
	5 .	-2.04	.05	-2.26	.025	
	6	_ ∞	0	 ∞	0	

.3 Lateral Acceleration

	<u>F1</u>	<u>Filter 1</u>		<u>Filter 2</u>	
	tobs	reject at α=	t _{obs}	reject at α=	
1	-1.74	.1	-1.46	.1	
2	-3.61	.005	326	.005	
3	-4,38	.005	-4.46	.005	
4	-1.74	.1	-3.76	.005	
5	-4.18	.005	-1.48	.1	
6	-1.08	.2	-3.24	.01	
	2 3 4 5	$ \frac{t_{obs}}{1} -1.74 $ 2 -3.61 3 -4.38 4 -1.74 5 -4.18	$\frac{t_{obs}}{1} = \frac{reject \text{ at } \alpha}{1}$ $\frac{1}{2} = -3.61 = .005$ $3 = -4.38 = .005$ $4 = -1.74 = .1$ $5 = -4.18 = .005$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	

Of the 24 cases tested, only 3 could not be rejected at $\alpha = .1$ or lower. This means that in 21 of the 24 cases, if we rejected H₁, the probability that we were rejecting a true hypothesis is less than 10%. Of the remaining three cases, we can reject H₁at $\alpha = .2$.