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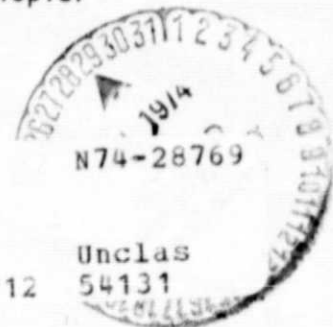
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CONSTANT TEMPERATURE HOT WIRE
ANEMOMETRY DATA REDUCTION PROCEDURE

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ABSTRACT

The theory and data reduction procedure for constant temperature hot wire anemometry are presented. The procedure is valid for all Mach and Prandtl numbers, but limited to Reynolds numbers based on wire diameter between 0.1 and 300. The fluids are limited to gases which approximate ideal gas behavior. Losses due to radiation, free convection and conduction are included.

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NOMENCLATURE

Most of the nomenclature is listed; those excepted are the numerous factors in the correlation equations which are obvious in the context.

a	non-dimensional temperature
A	$a(\xi) - a^*$
C_N	Nusselt number end loss correction factor
C_R	recovery ratio end loss correction factor
d	wire diameter (in.)
g	gravitational acceleration, 32.17 ft/sec^2
Gr	Grasshof number
h	convective heat transfer (BTU/hr-ft^2-°R)
I	current (amps)
k	thermal conductivity (BTU/hr-ft-°R)
K	unit length resistance (ohms/in.)
Kn	Knudsen number
ℓ	effective wire length (in.)
M	Mach number
Nu	Nusselt number
p	pressure (lbf/in^2)
Pr	Prandtl number
$\dot{q}$	heat transfer rate (BTU/hr)
q_m	measured hot wire power dissipation (watts/sec)
r	wire resistance (ohms)
R	gas constant (ft-lbf/lbm °R)
S	end loss correction parameter

T	temperature $(^{\circ}R)$
U	velocity (ft/sec)
x	length coordinate (in.)
z	non-dimensional length coordinate
α_1	temperature coefficient of resistance, $(^{\circ}R^{-1})$
α_2	second temperature coefficient of resistance $(^{\circ}R^{-2})$
β	isothermal expansivity coefficient $(^{\circ}R^{-1})$
γ	specific heat ratio
ε	emissivity
ζ	temperature difference, $T(x) - T_e$
η	temperature recovery ratio, T_e/T_s
$\overline{\eta}$	normalized recovery ratio
η_c	recovery ratio at continuum limit
η_f	recovery ratio at free molecule limit
θ	flow direction angle
μ	viscosity (1bm/ft-hr)
ξ	non-dimensional length coordinate
ρ	density $(1bm/ft^3)$
σ	Stefan-Boltzmann constant
ω	recovery ratio end loss correction parameter

Subscripts

$(\ )_a$	air
$(\ )_c$	conduction
$(\ )_d$	duct or tunnel walls
$(\ )_e$	equilibrium
$(\ )_f$	fluid
$(\ )_m$	measured

$(\ )_r$ radiation

$(\ )_s$ wire support

$(\ )_t$ total or stagnation

$(\ )_{wi}$ wire

$(\ )_{0,1,2}$ reference values except on α

$(\ )_\infty$ local free stream if on M, Re, or θ; infinite wire if on Nu

$(\overline{\ \ })$ average

1.0 INTRODUCTION

The theory and data reduction procedure for hot wire anemometry are presented in this paper. The basis of hot wire anemometry is that the heat transfer rate is a function of the flow properties of the flow to be measured. By heating the wire above the equilibrium temperature and measuring the current fluctuations to maintain the wire at a constant temperature, the heat loss and hence the flow speed is determined via a series of Mach, Reynolds, and Nusselt (heat transfer coefficient) correlation curves.

The hot wire loses heat by all three heat transfer modes: convection, conduction, and radiation. The effect of each one is considered, since the correlation curves are valid only for the forced convective heat loss. Due to the finite length of the wire, the conduction end losses are considerable, which affects both the heat transfer (convective) and the cold wire equilibrium temperature, which again affects the heat transfer. An analysis of all these "end" losses, free convective and radiative losses is given.

A complete data reduction procedure is given. The computer program and its data input requirements are given. The appendix also presents the non-linear effects of temperature on the wire resistance and a limited uncertainly analysis of the Mach number due to uncertainties in the wire diameter.

The technique for obtaining the data is not presented here. The program listed is quite general, being valid for all types of gases at all Mach numbers for Reynolds numbers based on wire diameter between 0.1 and 300.

2.0 HOT WIRE ANEMOMETRY THEORY

The hot wire probe is a device for measuring the local mass flux of a flowing fluid. The probe consists of a very fine wire electrically heated and immersed in a fluid. By measuring the heat transfer rate off the wire, the mass flow can be deduced. The heat transfer rate is easily obtained by measuring the electrical resistance and the voltage drop of the hot wire.

Two principles are the basis of hot wire anemometry: 1) the electrical resistivity of the wire is temperature dependent, preferably linearly, and 2) the heat transfer rate is a function of the flow properties of the flow. The first principle is necessary so that the temperature of the wire needed for the heat transfer determination can be accurately measured. The need for the second principle is obvious.

The hot wire has two modes of operation: constant current and constant temperature. In the constant current mode, the electrical current through the wire is held constant by an electronic circuit. A variation in the mass flow changes the heat transfer, temperature and, thus, resistance of the wire. The instantaneous resistance is measured to obtain the heat transfer rate and the mass flow. In the second mode the current is modulated with the varying flow to maintain it at constant temperature and resistance. The instantaneous current through the wire is measured to obtain the heat loss. The constant temperature mode has several advantages over the constant current mode, among which are simpler and more straightforward data acquisition and reduction procedures. These and other reasons have made the constant temperature method more useful and will be used in this report.

The theoretical analysis of heat transfer from circular cylinders

will not be given here. As yet no complete analysis with all the compli-
cations exists. Partial theories have been developed for the free molec-
ular, high Reynolds number, and low Reynolds number flow regimes. For
the transition zones empirical data only are available. Extensive empirical
data have been published on the heat transfer rates in terms of Nusselts
number of infinitely long circular cylinders as a function of the flow
properties, namely Mach and Reynolds numbers. These data are reduced into
a set of "universal" correlation curves, Fig. 3. The universal correlation
curves allow the direct determination of the Mach and Reynolds (based on
wire diameter) from the measured heat transfer corrected for the effects
of finite wire length.

The finite wire differs from the infinitely long wire in that heat
is conducted to the supports of the finite wire in addition to the con-
vected and radiated heat loss. This conduction end loss causes a nonuniform
temperature distribution. The infinite wire loses heat only by convection
and radiation and therefore has a uniform temperature distribution. The
end loss of the finite wire also affects the adiabatic recovery temperature
of the wire which also affects the heat transfer rate of the finite wire
as compared to the infinite wire.

The end loss effects of the finite wire must, therefore, be carefully
taken into account before the correlation curves of Fig. 3 for infinitely
long cylinders are applicable. Since the end effects are so important
(the end conduction heat loss can be as great as 50% of the total heat loss
for wire aspect ratios of 100), the theoretical analyses of the end losses
are given in detail below.

2.1 Derivation

The system to be analyzed is shown in Fig. 1a and b. Figure 1a

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shows the complete wire probe and 1b, the wire and coordinate system. The steady state case will be considered only.

The wire segment of length Δx loses heat by four mechanisms, which are

<u>forced convection</u>

$$\dot{q}_h = h \, \pi \, d \, \Delta x \, (T(k)-T_e)$$

where h = coefficient of convective heat transfer

$T(x)$ = temperature of wire

T_e = equilibrium (adiabatic wall) temperature of wire,

<u>free convection</u>

$$\dot{q}_{f.c.} = h_{f.c.} \, \pi \, d \, \Delta x (T(k)-T_e)$$

where $h_{f.c.}$ = average free convective heat transfer coefficient

$$\approx 1.16 \, \frac{k_f}{d} \, (Gr \, Pr)^{1/10} \qquad \text{(Ref. 21)}$$

$$\text{for } 10^{-5} < Gr < 10^3$$

and Gr = Grashoff's number = $\dfrac{\rho^2 g\beta(T(k)-T_e)d^3}{\mu_f^2}$,

<u>radiation</u>

$$\dot{q}_r = \sigma \, \epsilon \, \pi \, d \, \Delta x (T(k)^4 - T_d^4)$$

where T_d = duct temperature.

and

<u>conduction</u>

$$\dot{q}_h = -\frac{\pi \, d^2}{4} \, k_{wi} \, \frac{d^2 T(x)}{dx^2} \qquad .$$

The heat losses are balanced by the ohmic heating of the wire

$$\dot{q}_{generated} = r(x) \, I^2 \, \Delta x$$

    where I = electrical current through the wire

    and

        $r(x)$ = resistance per unit length of the wire at

            temperature $T(x)$

            = $r_0[1 + \alpha_1(T(x)-T_0) + \alpha_2(T(x)-T_0)^2]$

        T_0 = reference temperature

        r_0 = reference resistance at T_0

        α_1 = first temperature coefficient of resistivity

        α_2 = second temperature coefficient. The influence

            of α_2 is negligible for small overheat ratios

            $[(r(x)-r_e)/r_e]$, as is shown in Appendix B.

Defining $\zeta(x) = T(x) - T$ and assuming that $T_d \sim T_e$ and
$|[T(x)-T_d)]/T_d]| < 1$ or

$$T(x)^4 - T_d^4 \approx 4 \, T(x)^3 [T(x)-T_e]$$

and simplifying yields the energy equation

$$h \, \zeta(x) + 4 \, \sigma \, \epsilon \, T(x)^3 \, \zeta(x) + 1.16 \, \frac{k_f}{d} \, (Gr \, Pr)^{1/10} \, \zeta(x)$$

$$-\frac{k_{wi} \, d}{4} \, \frac{d^2\zeta(x)}{dx^2} = \frac{I^2 \, r_e}{\pi \, d} \, (1 + \alpha_1 \, \zeta(x)) \quad .$$

The boundary conditions are

$$\zeta(\pm \tfrac{\ell}{2}) = T_s - T_e \quad \text{and} \quad \frac{d\zeta(o)}{dx} = 0 \quad .$$

Introducting the nondimensional variables

$$\xi = \frac{2x}{\ell} \qquad \text{and} \qquad a(\xi) = \alpha_1 \zeta(x)$$

simplifies the differential equation to

$$\left[\frac{hd}{k_f} + \frac{4 \sigma \varepsilon T^3(\xi) d}{k_f} + 1.16(Gr\ Pr)^{1/10} - \frac{\alpha_1 I^2 r_e}{\pi k_f} \right] a(\xi)$$

$$= \frac{k_{wi}}{k_f} \frac{d^2}{\ell^2} \frac{d^2 a(\xi)}{d\xi^2} + \frac{\alpha_1 I^2 r_e}{\pi k_f}$$

The first and last terms within the brackets (forced convection and ohmic heating) are of $O(1)$. The other two terms within the brackets (radiation and free convection) are of smaller order as shown and hence are negligible. Typical values for air flows at $M = 2.5$ with overheat ratios of 1 and $d = 0.0004''$ are $T(\xi) \sim 1000° R$, $T_e \sim 500° R$, $k_f \sim 0.02$ BTU/hr-ft-°R, $C_p = 0.25$ BTU/lbm-°R, $\mu_f = 0.05$ lbm/hr-ft, $\rho_f = 0.006$ lbm/ft^3, $g = 32$ ft/sec^2 and $\beta = 0.002/°R$. The radiation term

$$\frac{4 \sigma \varepsilon T^3(\xi) d}{k_f} \approx 10^{-2}$$

and the free convection term

$$1.16(Gr\ Pr)^{1/10} \approx 0.2.$$

Therefore radiation and free convection are negligible provided that the overheat ratios are less than unity and wire diameters do not exceed $0.0004''$. If they are larger the free convection term must be considered. A theoretical discussion of free and forced convection of fine hot wires is given by Woods (22).

The governing differential equation reduces to

$$\frac{hd}{k_f} a(\xi) - \frac{\alpha_1 I^2 r_e}{\pi k_f} (1 + a(\xi)) = \frac{k_{wi}}{k_f} \frac{d^2}{\ell^2} \frac{d^2 a(\xi)}{d\xi^2}$$

with the boundary conditions

$$\frac{d\, a(o)}{d\xi} = 0 \qquad \text{and} \qquad a(\pm 1) = \alpha_1 (T_s - T_e)$$

2.2 Nusselt Number Correction

The relation between the Nusselt number (hd/k_f) for an infinitely long wire and a finite wire must be determined. Denoting the infinite wire by $(\ )^*$

$$a^*(\xi) = a^* = \text{constant, i.e., no end losses.}$$

Therefore the equation becomes

$$h = \frac{\alpha I^2 r_e}{\pi d} \frac{1 + a^*}{a^*}$$

The average convective coefficient of a finite wire is

$$\bar{h} = \frac{\alpha I^2 r_e}{\pi d} \frac{1 + \bar{a}}{\bar{a}}$$

where $\bar{a}$ = average non-dimensional temperature of the finite wire

and also

$$\bar{a} = \alpha_1 (T_w - T_e) = \frac{r_w - r_e}{r_e} \qquad \text{(overheat ratio)}$$

Therefore

$$\frac{Nu_\infty}{Nu_m} = \frac{h}{\bar{h}} = \frac{1 + a^*}{1 + \bar{a}} \frac{\bar{a}}{a^*}$$

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which gives the Nusselt number of an infinite wire in terms of the measured (average) Nusselt number and $\bar{a}/a^*$. To determine $\bar{a}/a^*$, the differential equation is solved for $a(\xi)$ and averaged over the wire length.

On substituting for h and $\alpha_1 I^2 r_e/\pi d$, in terms of $\bar{h}$, $\bar{a}$, and a^*, and rearranging, the differential equation becomes

$$\frac{\bar{h} \, d}{k_f} \frac{\bar{a}}{1 + \bar{a}} \frac{\bar{a}}{a^*} (a(\xi) - a^*) = \frac{k_{wi}}{k_f} \frac{d^2}{\ell^2} \frac{d^2 \, a(\xi)}{d\xi^2}.$$

The measured Nusselt number $Nu_m = \bar{h}d/k_f$ and letting
$S = d/\ell \; [(1+\bar{a}/Nu_m)(k_{wi}/k_f)]^{1/2}$ obtains

$$\frac{a^*}{\bar{a}} S^2 \frac{d^2 \, a(\xi)}{d\xi^2} - a(\xi) + a^* = 0$$

which becomes with one more change of variables

$$z = \xi \frac{1}{S} (\bar{a}/a^*)^{1/2} \quad \text{and} \quad A(z) = a(\xi) - a^*$$

$$\frac{d^2 \, A(z)}{dz} - A(z) = 0$$

with the boundary conditions

$$\frac{d \, A(o)}{dz} = 0 \quad \text{and} \quad A[\pm (\bar{a}/a^*)^{1/2} (1/S)] = \alpha_1 (T_s - T_e) - a^*$$

The solution is

$$A(z) = \frac{\alpha_1 (T_s - T_e) - a^*}{\cosh[(1/S)(\bar{a}/a^*)^{1/2}]} \cosh (z)$$

or

$$a(\xi) = a^* + \frac{\cosh[(1/S)(\overline{a}/a^*)^{1/2}\xi]}{\cosh[(1/S)(\overline{a}/a^*)^{1/2}]} \; [\alpha_1(T_s-T_e) - a^*]$$

Integrating with respect to ξ and averaging yields

$$\frac{\overline{a}}{a^*} = 1 - \frac{[1 - \frac{\alpha_1}{a^*}(T_s-T_e)]}{\frac{1}{S}(\overline{a}/a^*)^{1/2}} \; \tanh \; [\; \frac{1}{S}(\overline{a}/a^*)^{1/2} \;]$$

Kovasznay (1) assumed that the wire support temperature T_s equals the cold wire equilibrium temperature T_e and also that T_e is unaffected by the end losses. With this assumption, the end loss correction factor for the Nusselt number is

$$\frac{\overline{a}}{a^*} = 1 - \frac{\tanh[\frac{1}{S}(\overline{a}/a^*)^{1/2}]}{\frac{1}{S}(\overline{a}/a^*)^{1/2}}$$

The graphical solution of this relation is shown in Fig. 2a.

Dewey (10) included the effects of different wire support temperatures. These effects are also shown in Fig. 2a and are seen to be secondary so they can be neglected.

The equilibrium (adiabatic wall) temperature of finite wire as measured differs from that of an infinite wire due to the end heat conduction losses. This difference, ignored by Kovasznay (1), has been considered by Dewey (10). The equilibrium temperature ratio or recovery ratio end loss correction as a function of S is shown in Fig. 2b.

The recovery temperature correction requires knowledge of the flow properties of the flow which are as yet unknown. Therefore in this paper the two corrections are considered separately. The Nusselt number corrections are given by Kovasznay in Fig. 2a and the recovery temperature

corrections by Dewey in Fig. 2b. Applying the Nusselt number corrections with the <u>measured</u> recovery temperature, a first approximation of the flow properties, Mach and Reynolds numbers, is obtained from the heat transfer correlation curves described below. These then can be used to correct for the recovery temperature end loss effect. The corrected recovery temperature is then used in place of the measured recovery temperature and the entire procedure is repeated. The procedure for correcting the recovery temperature will be described after the discussion of the heat transfer correlation curves.

2.3 <u>Nusselt -Mach-Reynolds Numbers Correlation</u>

Theoretical analyses for predicting the heat transfer to infinitely long cylinders in the continuum flow and free molecule flow are available, but the transition between the two regimes must be obtained empirically. Since extensive empirical data have been published, no theoretical analyses will be presented here. The experimental data have been summarized and correlated by Dewey (10) in terms of Nusselt number of infinitely long cylinders as functions of the local free stream Mach and Reynolds numbers. These correlations are presented graphically and analytically.

The best correlations were found to exist if all the fluid properties were evaluated at the total free stream temperature, Laufer and McClellan (9). The correlation curves are shown in Fig. 3. The correlation equations are given in two parts, as shown,

$$Nu_\infty(Re_\infty, M_\infty) = Nu_\infty(Re_\infty, \infty) \; \phi(Re_\infty, M_\infty)$$

where $Nu(Re_\infty, \infty)$ is the Nusselt number of an infinite cylinder at $M_\infty = \infty$. (Subscript ∞ on Nu refers to infinite cylinder and on

Re and M refers to the <u>local</u> free stream conditions. Re is based on wire diameter.)

$$Nu_\infty(Re_\infty, \infty) = Re_\infty^{\;n}(0.14 + N_1 + N_2)$$

$$n = 1 - \frac{Re_\infty^{\;0.6713}}{5.142 + 2\,Re_\infty^{\;0.6713}}$$

$$N_1 = \frac{0.2302\,Re_\infty^{\;0.7114}}{15.44 + Re_\infty^{\;0.7114}}$$

$$N_2 = \left(\frac{0.01569}{0.3077 + Re_\infty^{\;0.7378}}\right)\left(\frac{15}{15 + Re_\infty^{\;3}}\right)$$

and $\phi(Re_\infty, M_\infty)$ is the deviation of the Nu at M_∞ from the Nu at $M_\infty = \infty$.

$$\phi(Re_\infty, M_\infty) = 1 + \phi_1 \cdot \phi_2 \cdot \phi_3$$

$$\phi_1 = \frac{0.06309}{M_\infty} + 0.5701\left[\left(\frac{M_\infty^{\;1.222}}{1 + M_\infty^{\;1.222}}\right)^{1.569} - 1\right]$$

$$\phi_2 = 1.834 - 1.634\left(\frac{Re_\infty^{\;1.109}}{2.765 + Re_\infty^{\;1.109}}\right)$$

$$\phi_3 = 1 + \left[0.300 - \frac{0.065}{M_\infty^{\;1.67}}\left(\frac{Re_\infty}{4 + Re_\infty}\right)\right]$$

These correlation equations are valid only in the range

$$0.4 \le M_\infty < \infty \quad\text{and}\quad 0.01 \le Re_\infty \le 1000.$$

For boundary layer investigations, correlations for $M_\infty < 0.4$ are required, hence the following correlation equations were derived for

$$0 \le M_\infty < 0.4 \qquad .$$

The $M_\infty = 0$ correlation, as obtained from the Oseen solution of Cole and Roshko (16) and the data of Hilpert (17) is

$$Nu_\infty(Re_\infty, 0) = 0.19 + 0.66 \; Re_\infty^{0.45} \qquad .$$

In the Mach range $0 < M_\infty < 0.4$, a weighted logarithmic average between the $M_\infty = 0$ correlation above and the Dewey correlation at $M_\infty = 0.4$ is used.

$$Nu_\infty = 10^{\{F \; log[Nu_\infty(Re_\infty, 0)] + (1-F) \; log[Nu_\infty(Re_\infty, 0.4)]\}}$$

where

$$F = [0.618 + (2.5 \; M_\infty)^{0.9}]^{-1} - 0.618 \qquad .$$

The extended correlation is valid only for the range

$$0 \leq M_\infty < 0.4 \qquad and \qquad 0.1 < Re_\infty < 300 \qquad .$$

Some of the original data, Dewey's correlation, and the new correlation are shown in Fig. 3.

Other correlations have been used, specifically the Yanta correlation (18) as used by Sfeir (3). Yanta modified the Dewey equations to extend M_∞ down to 0.1. However, the correlation equations as reported by Yanta and Sfeir produced no correlation at all for $M_\infty \leq 0.3$. This difficulty and the desire to extend the $Nu_\infty - M_\infty - Re_\infty$ correlations to all Mach numbers required the addition to the Dewey equations for $M_\infty < 0.4$.

Measuring the local free stream static pressure, in addition to the hot wire measurements, the Reynolds number is obtained as a unique function of the Mach number

$$Re_\infty = \frac{p_\infty d}{\mu_t (R \; T_t)^{1/2}} \; M_\infty [(1 + \frac{\gamma-1}{2} M_\infty^2) \gamma]^{1/2} \qquad .$$

It can be seen then that the Mach number must be determined iteratively from the correlation equations.

2.4 Recovery Temperature Correction

To obtain the first approximation of the Mach and Reynolds numbers, the measured recovery temperature, T_{em}, for the finite wire, was used in the procedure of the previous section. The correlation data are, however, valid only for infinite wires, hence the recovery ratio or temperature as measured must also be corrected for the end loss effects.

The procedure for correcting for the recovery ratio end loss as outlined by Dewey (10) is as follows:

The end loss recovery correction factor is

$$C_R = \frac{T_e}{T_{em}^*} = \frac{\eta}{\eta_m^*} = [1 - \omega(\frac{T_s}{T_{em}})](1-\omega)^{-1}$$

$$= [1 - \omega(\frac{\eta_s}{\eta_{em}})](1-\omega)^{-1}$$

where

T_{em}^* = cold wire recovery temperature of the infinite wire

ω = a parameter that depends primarily on S and is given in Fig. 2b,

and

$\eta = T/T_t$, the temperature ratio or recovery ratio depending on T.

Since it is not feasible to experimentally determine the wire support temperature, a good approximation is

$$T_s = T_t \frac{1 + Pr^{1/2} \cdot \frac{\gamma-1}{2} M_\infty^2}{1 + \frac{\gamma-1}{2} M_\infty^2}$$

A normalized recovery ratio $\bar{\eta}$ of a conducting cylinder is defined in terms of the free molecular and high Reynolds number continuum limits recovery ratio, η_f and η_c, respectively,

$$\bar{\eta} = (\eta - \eta_c)/(\eta_f - \eta_c) \quad .$$

The continuum limit and free molecular recovery ratios have been correlated by Morkovin (7) as

$$\eta_c = 1 - 0.050 \left(\frac{M_\infty^{3.4}}{1.175 + M_\infty^{3.5}} \right)$$

and

$$\eta_f - \eta_c = 0.2167 \left(\frac{M_\infty^{2.80}}{0.8521 + M_\infty^{2.80}} \right)$$

The normalized recovery ratio is empirically correlated in terms of the local free stream Knudsen number in Fig. 4. The equation of the universal curve shown in the figure is

$$\bar{\eta} = \left(\frac{Kn_\infty^{1.193}}{0.4930 + Kn_\infty^{1.193}} \right)$$

and

$$Kn_\infty = [(\pi\gamma/2)(M_\infty/Re_\infty)]^{1/2}$$

Sufficient information exists to determine η_m^* ($= T_{em}^*/T_t$) given the M_∞ and Re_∞ from the first Nu correction and the $Nu - M - Re$ correlation. The final Nu_∞, M_∞, Re_∞ and T_{em}^* are determined by repeating the Nu correction with the computed η_m^* (or T_{em}^*) instead of the measured η_m (or T_{em}). Usually less than five iterations are required to obtain 0.01% accuracy in η_m^*. Sfeir (3) claims the same accuracy with only three iterations.

2.5 Local Flow Direction

The Mach number obtained in the preceding section is the actual Mach number only if the hot wire is perpendicular to the flow. If the wire is not oriented perpendicularly to the flow then the Mach number obtained is only the component of the flow normal to the wire. By obtaining the Mach number components at two different wire orientations, the resultant Mach number and direction are obtained as follows:

The resultant local flow direction relative to some reference line (e.g., a wind tunnel centerline as in Fig. 5) is given by

$$\theta_\infty = \tan^{-1} \left(\frac{M_1 \sin \theta_2 - M_2 \sin \theta_1}{M_1 \cos \theta_2 - M_2 \cos \theta_1} \right)$$

and the resultant Mach number is

$$M_\infty = M_1/\cos\left(\frac{\pi}{2} - \theta_1 + \theta \right)$$

$$= M_2/\cos\left(\frac{\pi}{2} - \theta_2 + \theta \right)$$

With the resultant local free stream Mach number, the remaining flow properties may be obtained

$$T_\infty = T_t/(1 + \frac{\gamma-1}{2} M_\infty^2)$$

$$\rho_\infty = P_\infty/RT_\infty$$

and

$$U_\infty = M_\infty(\gamma RT_\infty)^{1/2} \quad .$$

This completes the basic hot wire anemometry theory. A more complete analysis required for the data reduction procedure would have to include an uncertainty analysis. No attempt was made to determine the uncertainty

except that due to the wire diameter. The wire diameters were very diffi-
cult to measure accurately since they are on the order of 0.0004 to 0.0001
inches. The manufacturer's nominal values are used and are ±3% uncertain.
The derivation of the uncertainty due to the wire diameter is given in the
appendix.

3.0 COMPUTATIONAL PROCEDURE

In this section, the step by step computation procedure for reducing the hot wire data is outlined. The computer program (the listings are in the appendix) follows this procedure exactly. The procedure for obtaining the data will not be discussed.

1. Calibrate the wire to obtain the first temperature resistivity coefficient, α_1,

$$\alpha_1 = \frac{(r_2 - r_1)}{r_1 (T_2 - T_1)} \quad .$$

Handbook values are used for the second temperature coefficient of resistivity, α_2. The temperature of the wire is

$$T = T_1 + \frac{1}{\alpha_1} \frac{r - r_1}{r_1} - \frac{\alpha_2}{\alpha_1^3} \left(\frac{r - r_1}{r_1} \right)^2 \quad .$$

2. Determine effective wire length, ℓ, from the manufacturer's unit length resistance at a reference temperature T_k and the actual measured resistance at T_k

$$\ell = \frac{r}{k} \quad .$$

3. Also determine the wire diameter or use the manufacturer's nominal specification (see step 25 for the wire calibration).

4. Start the run and determine the cold wire measured equilibrium temperature, the heated wire temperature, and the total temperature, T_t.

$$T_{em} = T_1 + \frac{1}{\alpha_1} \frac{r_{em}-r_1}{r_1} - \frac{\alpha_2}{\alpha_1^3} \left(\frac{r_{em}-r_1}{r_1} \right)^2$$

$$T_{wm} = T_{em} + \frac{1}{\alpha_1} \frac{r_w-r_{em}}{r_{em}} - \frac{\alpha_2}{\alpha_1^3} \left(\frac{r_w-r_{em}}{r_{em}} \right)^2$$

5. Determine the viscosity and conductivity of air at the total temperature. Sutherland's viscosity equation (Ref. 12)

$$\mu_t \times 10^7 = 7.323 \, T^{3/2}/(T + 198.72) \quad \left(\frac{1bm}{ft\text{-}sec} \right)$$

and conductivity

$$k_a = \frac{1.1408 \times 10^{-3} \, T^{3/2}}{T + (441.72/10^{21.6/T})} \quad \left(\frac{BTU}{hr\text{-}ft\text{-}^\circ R} \right)$$

6. Determine the thermal conductivity of the wire as a function of temperature (Ref. 14)

$$k_{wi} = k_{wi,1} + (T-T_1)\left(\frac{k_{wi,1} - k_{wi,2}}{T_1 - T_2} \right)$$

and compute k_{wi} at $T = T_{wm}$.

7. Measure the power dissipation in the wire

$$\dot{q}_m = I^2 \, r_{wm} = \frac{v^2}{r_{wm}}$$

where v = voltage drop across the hot wire and is given in terms of the bridge voltage, V, by

$$v = V \frac{r_{wm}}{(r_{bridge} + r_{lead} + r_{wm})} \quad .$$

8. Assume an equilibrium recovery factor or use the measured value $(\eta_m = T_{em}/T_t)$, and compute

$$T_{em} = \eta_m T_t$$

9. Compute the air conductivity from step 5 with $T = T_{em}$

$$k_{am} = f(T_{em})$$

10. Compute the measured Nusselt number

$$Nu_m = \frac{4.097 \ q_m}{\pi(T_{wm} - T_{em})\ell \ k_{am}}$$

11. Calculate the end loss parameter, S,

$$S = \frac{d}{\ell} \left(\frac{k_{wi}}{k_{am}} \ \frac{1}{Nu_m} \ \frac{r_{wm}}{r_{em}} \right)^{1/2}$$

and the overheat ratio

$$\bar{a}_w = \frac{r_{wm} - r_{em}}{r_{em}}$$

12. Read off the value for

$$\frac{\bar{a}_w}{a_w^*} = f(S)$$

from Fig. 2a or use the cubic polynomial

$$\frac{\bar{a}_w}{a_w^*} = 1 - 1.100 \ S + 0.0999 \ S^2 - 250 \ S^3$$

hence $a^* = \bar{a}_w/(\bar{a}_w/a_w^*)$

and the Nusselt number correction factor for the end loss

$$C_N = \frac{\overline{a}_W}{a_W^*} \left(\frac{1 + a_W^*}{1 + \overline{a}_W} \right)$$

Thus the corrected Nusselt number for an infinitely long wire is

$$Nu_\infty = C_N \, Nu_m$$

13. Assume M_∞ and with the measured p_∞, compute Re_∞

$$Re_\infty = 68.067 \, \frac{p_\infty d}{\mu_t (RT_t)^{1/2}} M_\infty [\gamma(1 + \frac{\gamma-1}{2} M_\infty^2)]^{1/2}$$

14. With these values for M_∞ and Re_∞, determine Nu_∞ from the correlation Nu - Ma - Re correlation curves as given in Fig. 2 and Section 2.3.

$$Nu_\infty = f(Re_\infty, M_\infty)$$

15. Compare this Nu_∞ with the Nu_∞ obtained in Step 12. Readjust M_∞ in Step 13 and recompute until

$$Nu_\infty(\text{step } 12) \equiv Nu_\infty(\text{step } 14).$$

16. Knowing the M_∞ and Re_∞, compute the Knudsen number

$$Kn_\infty = \left(\frac{\pi \gamma}{2} \right) \frac{M_\infty}{Re_\infty} \quad ,$$

the continuum recovery ratio

$$\eta_c = 1.0 - 0.05 \, \frac{M_\infty^{3.5}}{(1.175 + M_\infty^{3.5})} \quad ,$$

the free-molecule flow recovery ratio

$$\eta_f = \eta_c + 0.2167 \left(\frac{M_\infty^{2.80}}{0.8521 + M_\infty^{2.80}} \right) \qquad ,$$

and the universal recovery factor

$$\bar{\eta} = \frac{Kn_\infty^{1.193}}{0.4930 + Kn_\infty^{1.193}} \qquad .$$

17. The recovery factor for the infinite wire is then

$$\eta = \bar{\eta}(\eta_f - \eta_c) + \eta_c$$

18. Compute the adiabatic wire support temperatures from M

$$T_s = T_t(1 + (Pr)^{1/2} \frac{\gamma-1}{2} M_\infty^2)/(1 + \frac{\gamma-1}{2} M_\infty^2)$$

and the support temperature ratio

$$\eta_s = T_s/T_t$$

19. Read the ratio $\omega/1-\omega$ as a function of S from Fig. 2b or compute from

$$\frac{\omega}{1-\omega} = 0.700 \, S + 1.125 \, S^2 + 4.250 \, S^3$$

and solve for ω

$$\omega = \frac{[\omega/(1-\omega)]}{1 + [\omega/(1-\omega)]}$$

20. The end loss recovery factor correction ratio is

$$C_R = \frac{[1 - \omega(\eta_s/\eta_m)]}{1 - \omega}$$

21. The "measured" wire recovery ratio is now computed

$$\eta_m' = \eta/C_R$$

22. Compare with the measured or assumed value of η_m of Step 8. If the two differ, then go back to Step 8 and repeat the entire procedure with the computed "measured" wire recovery ratio η_m'. Usually no more than five iterations are required to obtain $\pm 0.01\%$ precision in M_∞.

23. The normal M_∞ component has now been determined for one wire orientation. Repeating the entire procedure obtains another normal M_∞ component at a different wire orientation at the same point in the flow field. With the given wire orientation angles θ_1 and θ_2 and the computed normal M_∞ components, the resulting M_∞ direction and magnitude are

$$\theta = \tan^{-1} \left(\frac{M_1 \sin \theta_2 - M_2 \sin \theta_1}{M_1 \cos \theta_2 - M_2 \cos \theta_1} \right)$$

and

$$M_\infty = M_1/\cos\left(\frac{\pi}{2} - \theta_1 + \theta \right)$$

24. The remaining flow properties are now computed from the resultant M_∞,

$$T_\infty = T_t/\left(1 + \frac{\gamma-1}{2} M_\infty^2\right) \qquad ,$$

$$\rho_\infty = \frac{144\ p_\infty}{R\ T_\infty}$$

and

$$U_\infty = M_\infty (\gamma R T_\infty)^{1/2} \quad .$$

25. If the wire diameter used in Step 3 is of doubtful accuracy, the wire diameter is calibrated by measuring a very accurately known M_∞ and adjusting the wire diameter until the computed M_∞ equals the known M_∞.

This completes the procedure for reducing the hot wire data at one particular point in the flow field.

A final word of warning is necessary. The whole point of using the Nu - Ma - Re correlation curves is to eliminate the need for calibration curves. This avoids the danger of calibrating bad data into "good" data. Calibrating the wire diameter as in Step 25 reintroduces that danger, and any adjustments of wire diameter greater than the manufacturer's tolerances should be considered suspect. In other words, the source of any discrepancy lies elsewhere.

The following section outlines the input data required for the data reduction program listed in the appendix.

4.0 DATA INPUT

The data input required for the data reduction program can be divided into three types: program control data, general tunnel and wire conditions, and the specific data output of the hot wire anemometer. All data are initialized either at zero or a default value, as shown below. The data read-in is controlled by Subroutine DATA (IEOF), shown in Appendix A.

Each group of data within each of the three types is identified by a preceding card containing an integer 1 through 20 in the first two columns, right justified. Thus there are 20 data groups. The only exception to this rule is the data group labeled by the integer 1. This card must initially contain another integer in Columns 3 to 5, right justified. The other 19 identifiers contain one integer only; Columns 3 to 5 must be blank, otherwise the program stops.

Decimal data following the identifier card must be formatted in six words per card of 10 columns per word and must be right justified if in exponential form. Again the exception to this rule is the data labeled by Integer 1. Here the format is 8 columns for the first word, and 10 columns for each of the remaining 5 words. The reason for this exception is required by the peculiarities of the remote teletype terminal used to record the data. There is another difference in the Integer 1 data. The card immediately following the label card must either be blank or contain alpha-numeric characters in Columns 1 - 72. The same characters are printed in the output as a title. Omitting this card will cause a program termination.

Integer data are only 2 words per card and must be right justified in Columns 1 - 2, and 3 to 5. There are only two groups requiring integer data, namely, the data labeled by integers 14 and 15. The data sets

labeled by integers 19 and 20 are blank and have been provided for future expansion of the program.

The three types of data required are described below, along with their default or initial values.

Program Control Data

This type of data is used to control the accuracy required, maximum number of iterations allowed, number of data sets to be reduced, and special printout statements for diagnosis and reminders. In each of the group descriptions below the first line contains the data label integer and the second the variable names.

10

DNUIC, DETAM

where

> DNUIC - fractional allowable error on the Nusselt number
>
> calculation in Subroutine MARENU. Default value
>
> = 0.002.
>
> DETAM - fractional allowable error on the recovery factor
>
> calculation in Program HOTWIRE. Default value = 0.002.

11

EBLT — an estimation of the boundary layer thickness. Default value = 0.10. This variable has no effect on any computations, all it does is control the distance over which a certain number (KK, see below) of data points are read; printout is "estimated boundary layer thickness = EBLT."

12

DETAIL   - a parameter to be set to non-zero if a detailed
                  printout is required for diagnosis. Default
                  value = 0.0 for no detailed printout.

14

NPROFL   - the number of profiles or data sets to be reduced.
                  used only if the flow direction is to be computed.
                  No default value.

15

ITMARN, ITETAM

    where

        ITMARN - the maximum number of iterations allowed for sub-
               routine MARENU to determine the Nusselt number
               from the correlation curves. Default number = 10.
        ITETAM - the maximum number of iterations allowed for program
               HOTWIR to compute the recovery ratio. Default
               value = 5.

Note that there are two controls on the required accuracy of Mach
number and Nusselt number. If the maximum numbers of iterations
are exceeded a printout "Solution not accurate" will occur along
with the inaccurate solution.

Tunnel and Wire Data

These data specify the fluid, wire, and anemometer constants. The
default values are for the fluid being air, and the wire a 0.0004
inch platinum - 10% rhodium wire. The bridge resistance of the
hotwire anemometer is 50 ohms.

4

GAM, R, PR, CMU

    where

        GAM - specific heat ratio. Default value = 1.4.

          R - gas constant. Default value = 53.61.

         PR - Prandtl's number. Default value = 0.71.

        CMU - fluid viscosity. No default. Computed internally

              using Sutherland's viscosity formula for air. If a

              different fluid is used, the Sutherland equation

              must be bypassed.

5

WD   - wire diameter. Default value = 0.0004 inches.

7

RESIST   - hotwire anemometer bridge resistance. Default

              = 50 ohms.

8

CALCON   - parameter near 1 to calibrate wire diameter if

              necessary. See computational procedure step 25.

              Default value = 1.0.

9

WK, TK

    where

        WK - unit length resistivity of the wire at the given

              temperature. Default value = 57.25 ohms per inch.

        TK - temperature at which the above resistivity is

              specified. Default = 527.7°R.

16

WIK1, TW1, WIK2, TW2

> where

>> WIK1, WIK2 - thermal conductivity of the hot wire at two
>> different temperatures. Default values = 43.35,
>> 42.47 BTU/hr-ft-°R.

>> TW1, TW2   - the corresponding temperatures. Default
>> = 450, 720 °R.

17

ALFA, BETA

> where

>> ALFA - temperature coefficient of resistance. Default
>> = 0.96E-3 °R^{-1}.

>> BETA - second temperature coefficient of resistance.
>> Default = - 0.462E-7 °R^{-2}.

18

>> TT   - total temperature. Default   535 °R.

Specific Test Data

These data include output voltages of the anemometer, temperature-resistivity calibration, static pressure, total temperature, overheat and wire lead resistances, and the wire orientation angles.

1   K

(Alpha-numeric title or blank)

VOLTS(1), VOLTS(2), VOLTS(3), ... , ... , VOLTS(6)

VOLTS(7), ... , VOLTS(I), ... , VOLTS(K)

where

K - number of output voltages to be read for this
particular data run, $K \leq 200$.

(title) - any alpha-numeric or blank statement to be printed
as a title for this test run. If omitted, program
terminates.

VOLTS(I) - the bridge voltage of the anemometer. Default = 0.0
and program termination.

2

PINF   - static pressure. Must be obtained separately as it is not
part of the hot wire anemometry instrumentation. Default
value = 0.495 psi.

3

T1, WR1, T2, WR2

where

WR1, WR2 - the wire resistances as measured by the anemometer
bridge at the corresponding temperatures. Default
values = 2.6, 3.047 ohms.

T1, T2 - the corresponding temperatures. Default values = 492,
672 °R.

6

WRWM, RESISL

where

WRWM   - overheat resistance. No default. Omission causes
program termination.

RESISL - hot wire lead resistance as measured by the hot wire
anemometer bridge. No default.

13

THETAI, THETA1, THETA2

where

> THETAI - parameter to be set to 1 or 2 if the program is to
> determine local flow direction. If THETAI = 1., then
> data following is for THETA1, and if THETAI = 2., data
> following for THETA2. Default = 0.0. If omitted, no
> angular calculation.

> THETA1 - wire orientation or angle for the first Mach number
> component as shown in Figure 5. Default = 90°.

> THETA2 - wire angle for the second Mach number. Default = 45°.

The execution starts after reading in VOLTS(K). Hence the data sets
2, 3, 6, 13 must precede set 1. If THETAI is non-zero then two sets
of data must be read in each with the same K. PINF need not be same
but should not differ by more than 10%. The two data sets for the
angles need not be together. A maximum number of profiles or data
sets can be read in at one angle and then the profiles at the second
angle may be read in. The only restriction is the number of profiles
at each angle must be the same and the sequence of profiles must be
in the same order for each angle.

5.0 CONCLUSIONS

The data reduction procedure is limited by the following restrictions:

1) fluid can be gases only, which approximate the ideal gas law,

2) overheat ratio ≤ 0.5, ·

3) Reynolds number based on wire diameter $0.1 \leq Re_\infty \leq 300$, and

4) wire diameter should be calibrated.

Other than these restrictions, the program is quite flexible. There are no restrictions on the Mach or Prandtl number.

The default values in the computer program for the fluid and wire properties are those for air and a 0.0004 inch diameter platinum-10% rhodium wire. These values may be changed if other gases and wires are used. The data input requirements are given in the previous section. The program listing is in Appendix A.

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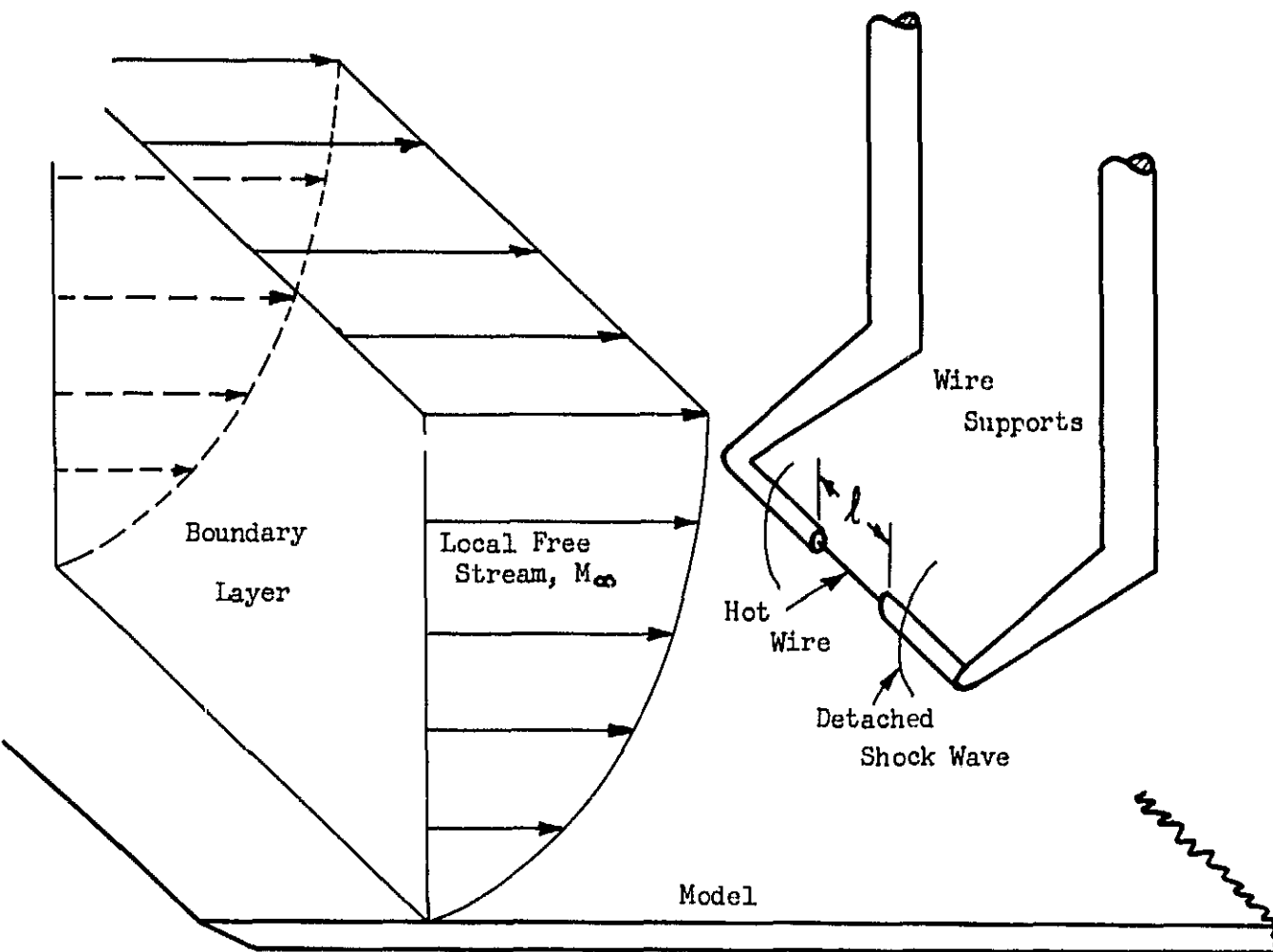
FIGURE 1a. Schematic of Hot Wire in the Boundary Layer of the Model.
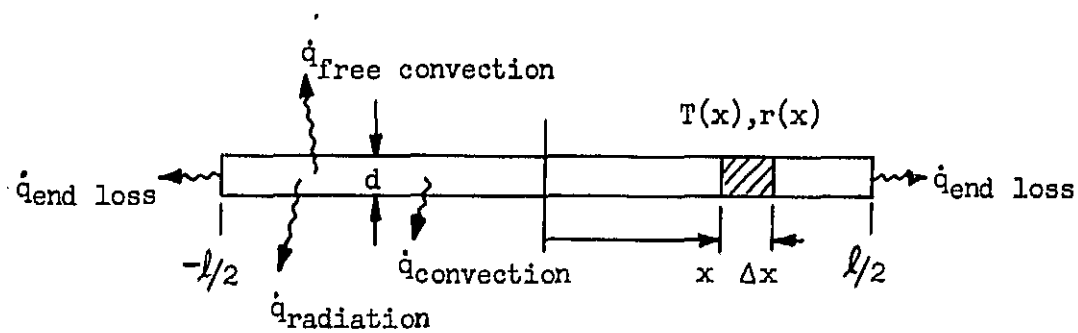

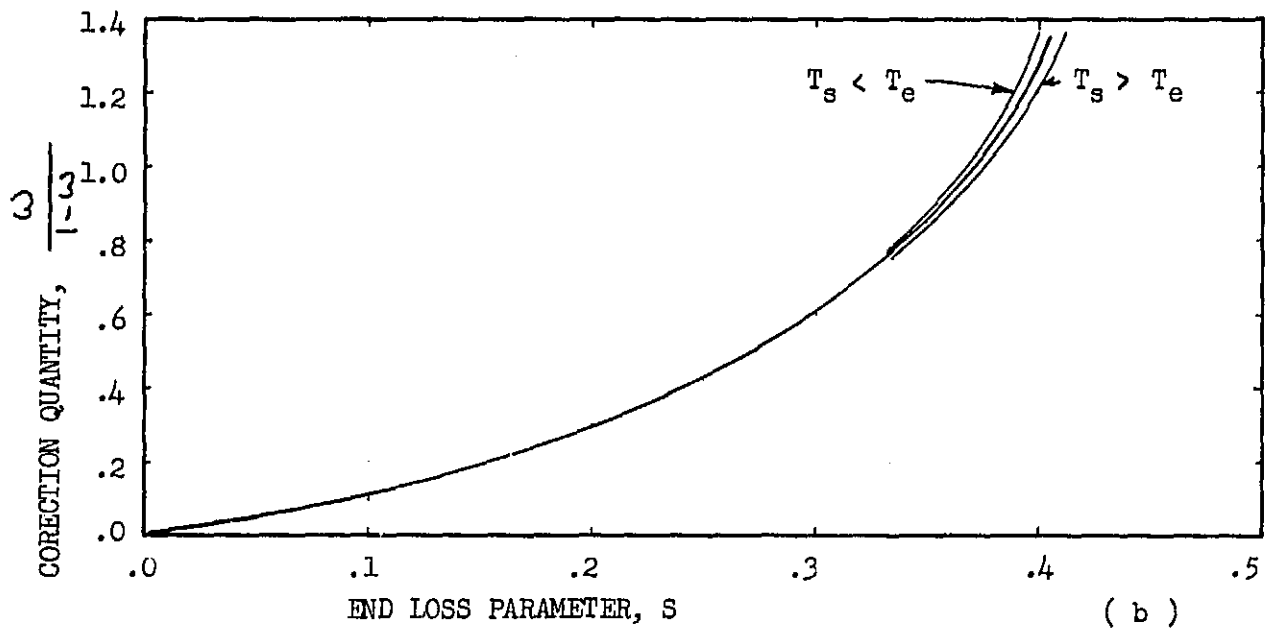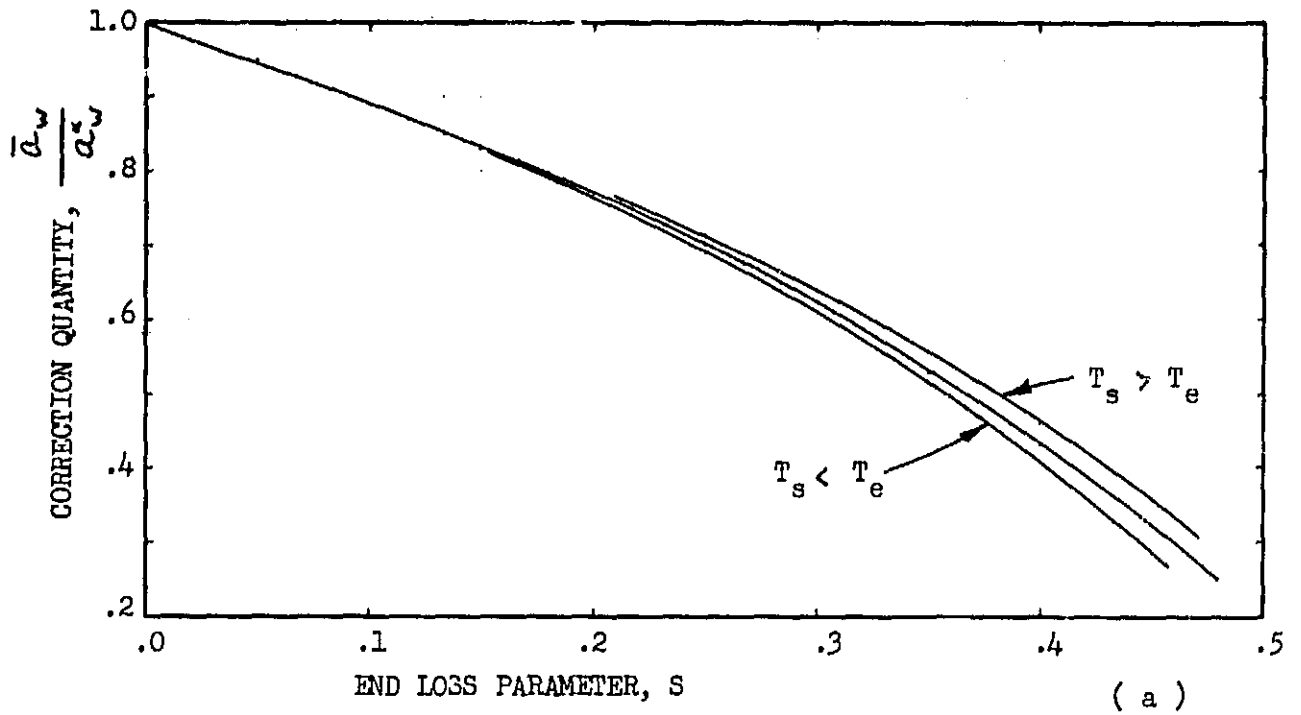FIGURE 1b. Coordinate System and Heat Losses of the Hot Wire.

FIGURE 2. End loss correction for the Nusselt number (a) and
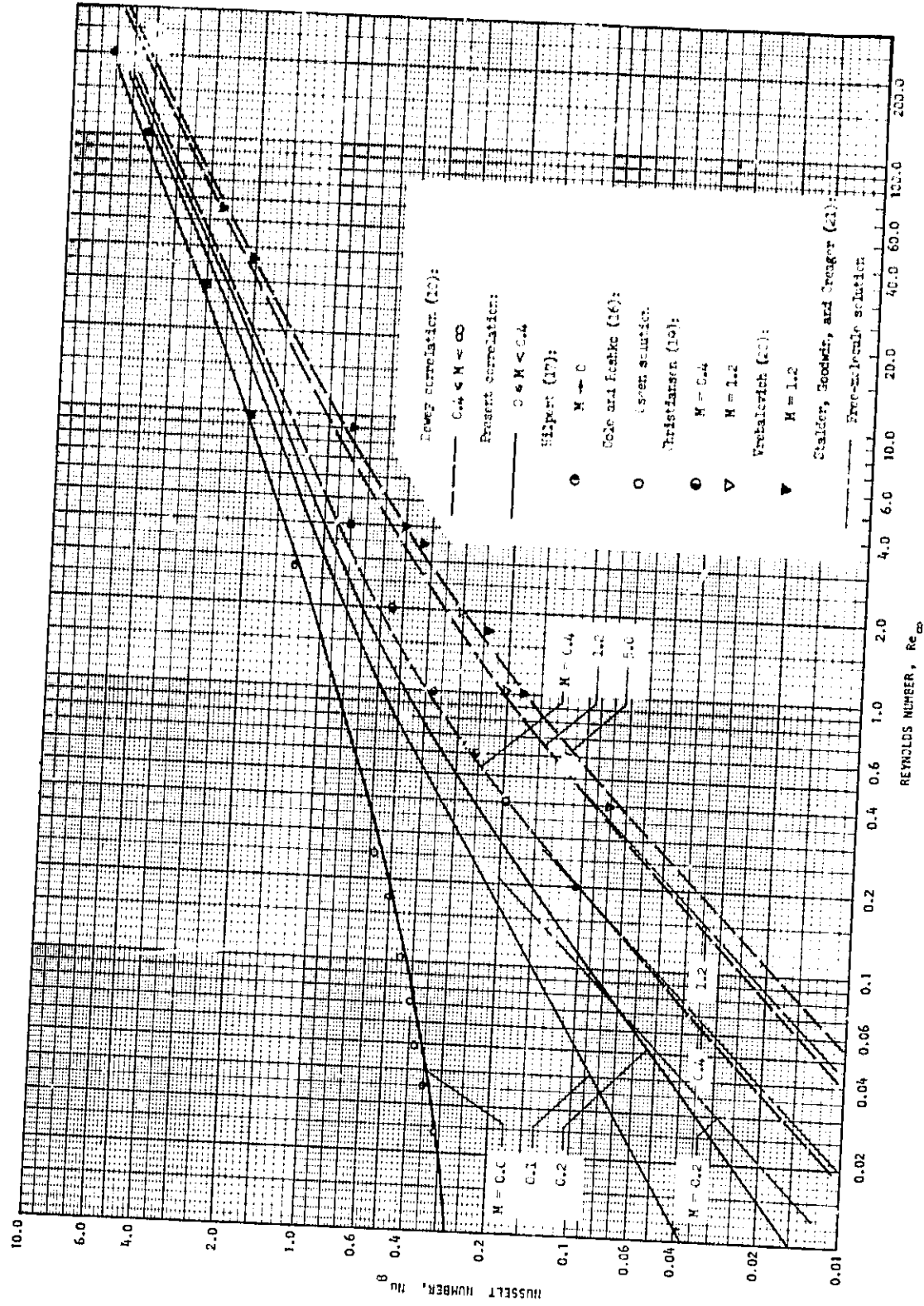recovery temperature (b), Reference 10.

FIGURE 3. Empirical convective heat transfer correlation for cylinders
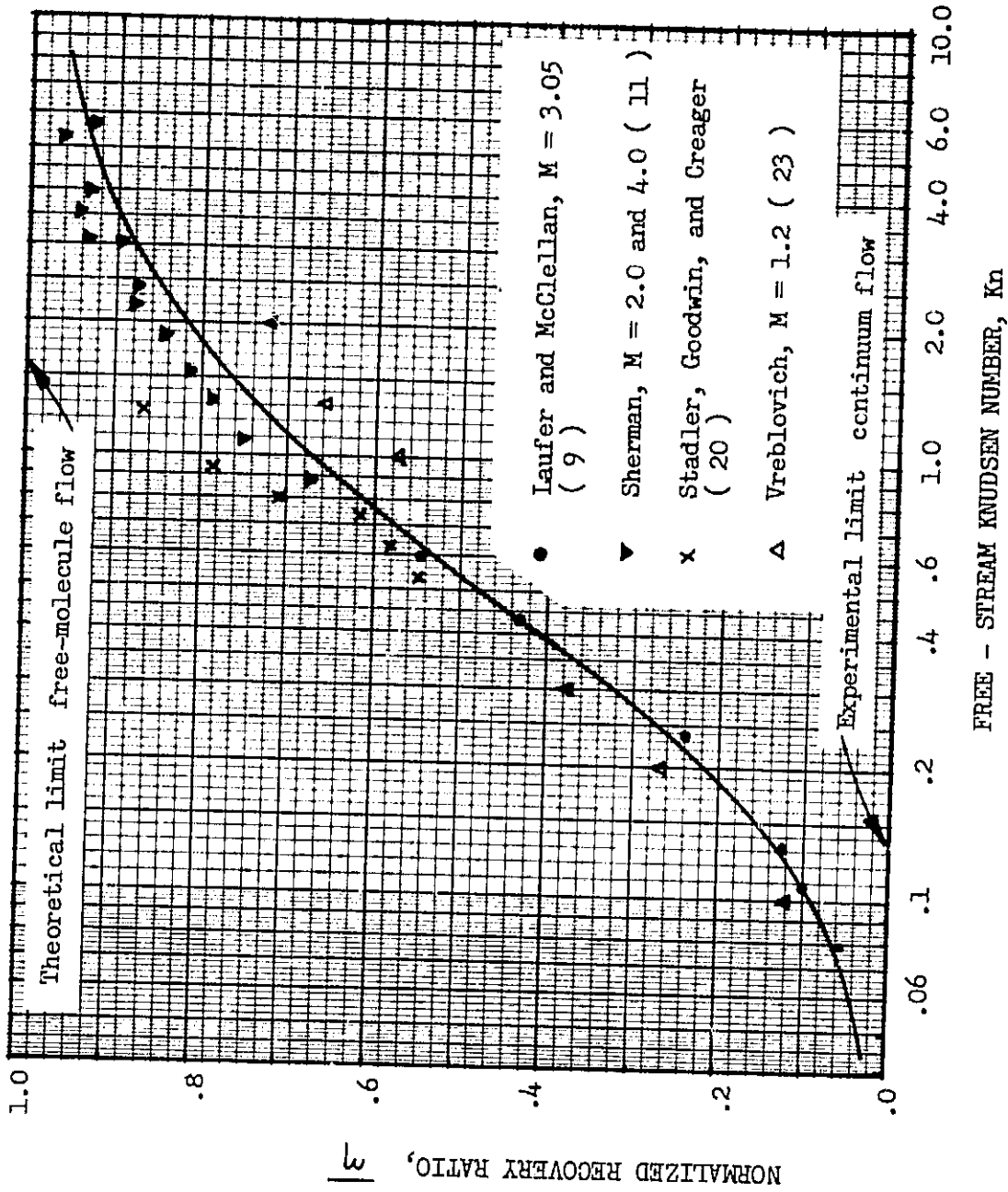
FIGURE 4. Free-Stream Knudsen Number Correlation Curve, (reference 10).
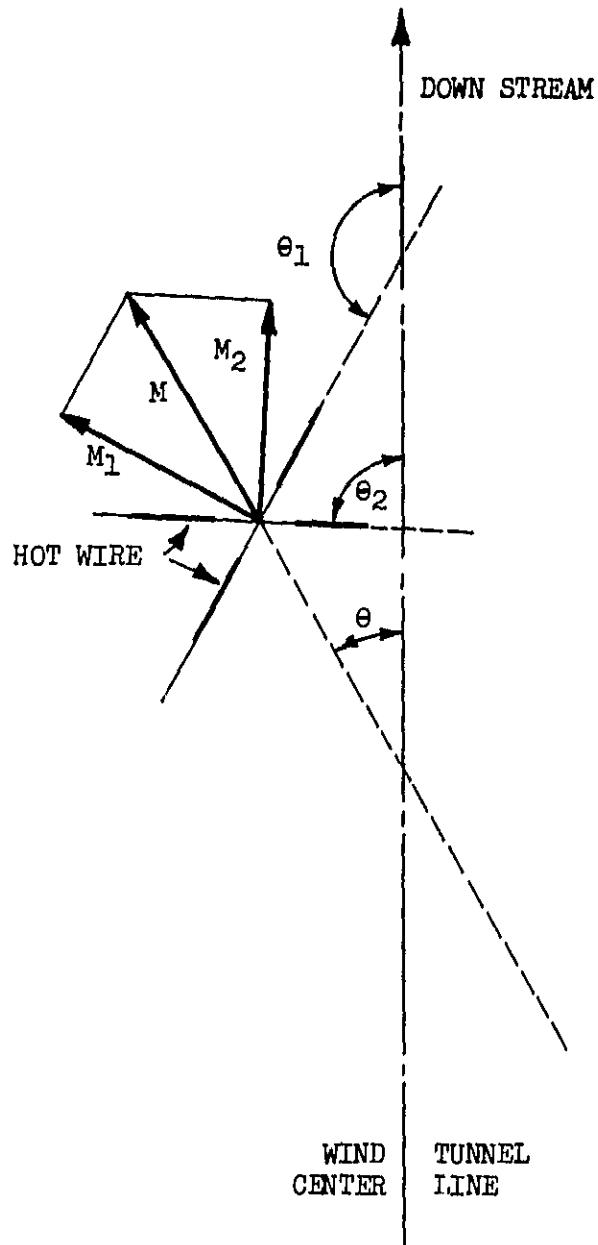
FIGURE 5. Coordinate System for the Local Flow Direction

APPENDIX A

PROGRAM LISTINGS

```
      PROGRAM HOTWIR(INPUT,OUTPUT,TAPE50 = INPUT)
C
C   PROGRAM HOT WIRE *** COMPUTES MACH NUMBERS AND VELOCITIES
C
      LOGICAL IEOF

      DIMENSION XMTH1(20,200),XMTH2(20,200),PINF1(20),PINF2(20)
      COMMON/RESULT /XM,REI,CNUI,CKNI,CNUM,CNUIC
      COMMON/ETA/ ETA,ETAC,ETAF,ETAS,ETABAR,ETAM,ETAM1,ETADEL
      COMMON/CORCOE/ CN,CR
      COMMON/ENDLOS/ S,AWBAR,AWSTAR,AWRA,OMEGA,OMGRA
      COMMON/TUNNEL/PINF,TT,TINF
      COMMON/WIRCAL/TW1,WIK1,TW2,WIK2,T1,WR1,T2,WR2,WK,TK,CALCON
      COMMON/WIRE/WD,WL,ALFA,BETA,WIK,WREM,WRWM,WRM,TFM,TWM,DELT
      COMMON/GAS/GAM,P,PR,CMU,RGAM
      COMMON/VOLTS/VOLTS(200)
      COMMON/RESIST/RESIST,RESISL
      COMMON/KONST/K
      COMMON/INDEX/IJI
      COMMON/INDEKS/KJI,JJJ,LLL,ITMARN,ITETAM
      COMMON/PRECIS/DNUIC,DETAM
      COMMON/EBLT/EBLT
      COMMON/DETAIL/DETAIL
      COMMON/THETAI/THETAI,THETA1,THETA2
      COMMON/NPROFL/NPPOFL

      DATA (VOLTS(I),I=1,200)/200*0.0/
      DATA ALFA,BETA/.96E-3,-.462E-7/,TK,WK/527.7,57.25/
      DATA WIK1,TW1/43.35,450./,WIK2,TW2/42.47,720./
      DATA R,GAM,PR /53.61,1.4,0.710/
      DATA T1,WR1,T2,WR2 /491.7,2.6,671.7,3.047/
      DATA PINF,TINF,TT/.495 ,450.,535./
      DATA WD,WL/0.0004,.0446/
      DATA RESIST/50./
      DATA RESISL/.24/
      DATA DNUIC,DETAM/.002,.002/
      DATA CALCON/1./
      DATA EBLT/0.10/
      DATA DETAIL/0.0/
      DATA (PINF1(I),I=1,20)/20 *0.0/
      DATA (PINF2(I),I=1,20)/20 *0.0/
      DATA THETAI,THETA1,THETA2/ 0.0,90.,45./
      DATA ITMARN,ITETAM/10,5/
```

```
      NPTH1   = 0
      NPTH2   = 0

    1 CONTINUE
      CALL DATA(IEOF)
      IF( IEOF ) GO TO 5000
      IF ( THETAI .EQ. 1.) GO TO 15
      IF ( THETAI .EQ. 2.) GO TO 16
      GO TO 4000
   15 CONTINUE
      NPTH1   = NPTH1 + 1
      PINF1(NPTH1)   = PINF
      GO TO 4000
   16 CONTINUE
      NPTH2   = NPTH2 + 1
      PINF2(NPTH2)   = PINF
 4000 CONTINUE
      PGAM    = (GAM - 1.)/2.
      SGAMPI  = SQRT(GAM*1.5708)
      GAMRC   = 32.174*R*GAM
C
C     DETERMINE AIR THERMAL CONDUCTIVITY (BTU/HR-FT-R)    AND
C       VISCOSITY ( LBM/FT-SEC)
C

      T       = TT
      T15     = T**1.5
      AKM     = .0011408*T15/(T + 441.72/(10.**(21.6/T)))
      CMU     =  (7.323E-7)*T15/(T + 198.72)
C
C     CALIBRATE
C
      R1      = WR1
      R2      = WR2
      ALFA    = (R2-R1)/(R1*(T2-T1))
      WR      = R1*(1.+ALFA*(TK-T1))
      WL      = WR/WK
      PRINT 2000
C
C     DETERMINE MEASURED HEATED WIRE TEMPERATURE
C
      TWM     =  T1+((WRWM-WR1)/(ALFA*WR1))*(1.-BETA*(WRWM-WR1)/
     1  (WR1*ALFA*ALFA))
      WIK     = WIK1+(TWM-TW1)*(WIK1-WIK2)/(TW1-TW2)

      XM      = .05
C     ASSUME VALUE FOR THE ETAM
   50 ETAM    = 1.
```

```
      DO 1000 I =1,K
      JJJ       = C
   75 TEM       = ETAM*TT
      JJJ       = JJJ + 1
      DELT      = TWM-TEM
      WREM      =  R1*(1.+(TEM-T1)*(ALFA + BETA*(TEM-T1)))
      WRM       = WREM
      EWR       = WREM
      VOLT      = VOLTS(I)*WRWM/(WRWM+RESIST+RESISL)
      OM        =(VOLT*VOLT/WRWM)
      CNUM      =  13.042 * OM/(AKM*WL*DELT)
      S         =  (WD/WL)*SQRT(WIK*WRM/(AKM*EWR*CNUM))*CALCON
      AWBAR     = (WRM-EWR)/EWR
      AWRA      = 1. + S*(-1.1+S*(.0999+S*(-2.25)))
  250 AWSTAR    = AWBAR/AWRA
      CN        = AWRA*(1.+AWSTAR)/(1.+AWBAR)
      CNUI      = CN*CNUM
      IJI       = I
C
C    SOLVE FOR MACH NUMBER VIA ITERATION
C
      KJI       = 0
      CALL MARFNU
      IF ( DETAIL  .EQ. 0.0)  GO TO 13
      IF ( IJI .NE. 1  .AND. IJI .NE. K)  GO TO 13
C   *****************************************************
      PRINT 10
   10 FORMAT (*   ALFA,WL,WIK,WRM,EWR,TT,TEM /DELT,ETAM,VOLT,QM,CNUM,S,
     1 AWBAR/AWRA,AWSTAR,CN,CNUI,CNUIC,REI,XM    *)
      PRINT 12, ALFA,WL,WIK,WRM,EWR,TT,TEM
      PRINT 12, DELT,ETAM,VOLT,QM,CNUM,S,AWBAR
      PRINT 12, AWRA,AWSTAR,CN,CNUI,CNUIC,REI,XM
   12 FORMAT (7F14.4)
C   **************************R**************************************
   13 CONTINUE
      CKNI      = (XM/REI)*SGAMPI
      RM35      = XM**3.5
      ETAC      = 1.-.05*RM35/(1.175+RM35)
      RM28      = XM**2.8
      ETADEL    = .2167*RM28/(.8521+RM28)
      RK11      = CKNI**1.193
      ETABAR    = RK11/(.493+RK11)
      ETA       = ETABAR*ETADEL+ETAC
      RM2GAM    = PGAM*XM*XM
C
C     COMPUTE SUPPORT RECOVERY RATIO  = ETAS = TS/TT
C
      ETAS      = (1.+SQRT(PR)*RM2GAM)/(1.+RM2GAM)
      OMGRA     = S*(.7+S*(+1.125+S*11.25   ))
  900 CONTINUE
      OMEGA     = OMGRA/(1.+OMGRA)
```

```
C
C         SOLVE FOR END LOSS RECOVERY FACTOR CORRECTION RATIO
C
          CR        = (1.-OMEGA*(ETAS/ETAM))/(1.-OMEGA)
C
C         COMPUTE FOR ETAM
C
          ETAM1    = ETA/CR
C
C         COMPARE COMPUTED AND ASSUMED ETA
C
          IF(ABS((ETAM-ETAM1)/ETAM) .LE. DETAM)   GO TO 950
  920 CONTINUE
          IF(JJJ.GT. ITETAM) GO TO 951
          ETAM     = ETAM1 -(ETAM1-ETAM)/2.
          GO TO 75
  951 CONTINUE
          PRINT 718
  718 FORMAT (*   SOLUTION NOT ACCURATE- MORE ETAM ITERATION NEEDED    *)
  950 CONTINUE
          IF(LLL.GT.ITMARN) PRINT 717
  717 FORMAT (*   SOLUTION NOT ACCURATE - MORE MARENU ITERATION NECSY *)
C
C         COMPUTE LOCAL FREE STREAM TEMPERATURE,DENSITY,MASS FLOW, AND VELOCITY
C
          TINF     =   TT/(1.+RM2GAM)
          RHO      =   144.*PINF/(R*TINF)
          V        = XM*SQRT(GAMRC*TINF)
          IF ( DETAIL .EQ. 0.0)   GO TO 14
          IF ( IJI .NE. 1 .AND. IJI .NE. K ) GO TO 14
C *******************************
          PRINT 7001
 7001 FORMAT (*   CN,CNUI,CKNI,ETAC,ETADEL,ETABAR,ETA/ETAS,OMGRA,OMEGA,
     1  CR,ETAM1,TINF,PINF/WD,REI,R ,RHO,T15 ,V,RM2GAM    *)
          PRINT 5001, CN,CNUI,CKNI,ETAC,ETADEL,ETABAR,ETA
          PRINT 5001, ETAS,OMGRA,OMEGA,CR,ETAM,TINF,PINF
          PRINT 5001, WD,REI,R, RHO,T15 ,V,RM2GAM
 5001 FORMAT (7F14.4)
          PRINT 6002
 6002 FORMAT (*   TWM,DELT,WIK,T,CNUIC,REI,XM    *)
          PRINT 5001 , TWM,DELT,WIK,T,CNUIC,REI,XM
C *******************************
   14 CONTINUE
          AI       = I
          PRINT 980, AI,XM,V,RHO,TINF
  980 FORMAT(1X,F3.0,E20.4,10X,F20.4,10X,E20.4,10X,E20.4)
 2000 FORMAT (1H ,10X,19H LOCAL MACH NUMBER ,10X,20H  LOCAL VELOCITY,F/S
     1   ,10X,20H  LOCAL DENSITY,L/CF,3X,32H LOCAL FREE STREAM TEMPERATU
     2RE,R    )
          IF ( THETAI .EQ. 1.) GO TO 1015
          IF ( THETAI .EQ. 2.) GO TO 1016
          GO TO 1020
```

```
1015 CONTINUE
     XMTH1(NPTH1,I)  = XM
     GO TO 1020
1016 CONTINUE
     XMTH2(NPTH2,I)  = XM
1020 CONTINUE
     IF((NPTH1+NPTH2).EQ.(2*NPROFL) .AND. I.EQ.K) GO TO 3000
1111 CONTINUE
1000 CONTINUE
     GO TO 1
3000 CONTINUE
     IF ( THETA1 .NE. 1. .AND.THETA1 .NE. 2.) GO TO 5000
     RTET1   = THETA1/57.296
     RTET2   = THETA2/57.296
     DO 8100 NP = 1,NPROFL
     IF ( PINF1(NP) .NE. PINF2(NP))GO TO 1017
     PINF    = PINF1(NP)
     GO TO 1018
1017 CONTINUE
     PINF    = (PINF1(NP) + PINF2(NP))/2.
1018 CONTINUE
     ANP     = NP
     PRINT 8000, ANP
8000 FORMAT (1H1,F3.0,* PROFILE * )
     PRINT 8001
8001 FORMAT(8X,32H LOCAL MACH NO.       ANGLE, DEG  ,24H LOCAL VELOCITY,
    1 F/S     , 2X,21H LOCAL DENSITY, L/CF ,2X,35H LOCAL FREE STREAM TEMP
    2ERATURE,R    )
     DO 8200 J = 1,K
     AJ      = J
     RTETA   = ATAN((XMTH1(NP,J)*SIN(RTET2)- XMTH2(NP,J)*SIN(RTET1))/
    1  (XMTH1(NP,J)*COS(RTET2) - XMTH2(NP,J)*COS(RTET1)))
     THETA   = RTETA*57.296
     XM      = XMTH1(NP,J)/COS(1.5708 - RTET1 + RTETA)
     RM2GAM  = RGAM*XM*XM
     TINF    = TT/(1. + RM2GAM)
     RHO     = 144.*PINF/(R*TINF)
     V       = XM*SQRT(GAMRC*TINF)
     PRINT 8002, AJ,XM,THETA,V,RHO,TINF
8002 FORMAT(1X,F3.0,E15.4,12X,F5.1,3X,E15.4,7X,E20.4,10X,E20.4)
8200 CONTINUE
8100 CONTINUE
     NPTH1   = 0
     NPTH2   = 0
     THETA1  = 0.0
     THETA2  = 0.0
     GO TO 1
5000 CONTINUE
     STOP
     END
```

```
      SUBROUTINE MAPENU


      COMMON/RESULT /XM,REI,CNUI,CKNI,CNUM,CNUIC
      COMMON/ETA/ ETA,ETAC,ETAE,ETAS,ETABAR,ETAM,ETAM1,ETADEL
      COMMON/CORCOE/ CN,CR
      COMMON/ENDLOS/ S,AWBAR,AWSTAR,AWRA,OMEGA,OMGRA
      COMMON/TUNNEL/PINF,TT,TINF
      COMMON/WIRCAL/TW1,WIK1,TW2,WIK2,T1,WR1,T2,WR2,WK,TK,CALCON
      COMMON/WIRE/WD,WL,ALFA,BETA,WIK,WREM,WRWM,WRM,TEM,TWM,DELT
      COMMON/GAS/GAM,R,PR,CMU,RGAM
      COMMON/VOLTS/VOLTS(200)
      COMMON/KONST/K
      COMMON/INDEX/IJI
      COMMON/RESIST/RESIST,RESISL
      COMMON/INDEKS/KJI,JJJ,LLL,ITMAPN,ITETAM
      COMMON/PRECIS/DNUIC,DETAM
      COMMON/ERLT/ERLT


      LLL       = 0
      Y0        = ALOG10(CNUI)
      DLX       = 0.1
      X1        = ALOG10(XM)
      KKK       = 1
      GO TO 100
    1 CONTINUE
      IF(ABS((CNUIC-CNUI)/CNUIC) .LE. DNUIC)  GO TO 600
      Y1        = ALOG10(CNUIC)
 1000 CONTINUE
      X2        = X1 + DLX
      KKK       = 2
      XM        = 10.**X2
      GO TO 100
    2 CONTINUE
      LLL       = LLL + 1
      IF( LLL .GT. ITMARN ) GO TO 600
C     COMPARE CNUIC AND CNUI AND RETURN IF GOOD
      IF(ABS((CNUIC-CNUI)/CNUIC) .LE. DNUIC)  GO TO 600
      Y2        = ALOG10(CNUIC)
      DLX       = (Y0 - Y2)*(X2 - X1 )/(Y2 -Y1 )
      X1        = X2
      Y1        = Y2
      GO TO 1000
```

```
  100 CONTINUE

      REI     = 68.06655*(XM*PINF*WD/CMU)*CALCON*
     1 SQRT(GAM*(1. + RGAM*XM*XM)/(R*TT))
C   IF XM LESS THAN .4   THEN AN AVERAGE IS TAKEN OF THE VALUES OF CNUIC
C   AT XM = 0.0 AND XM = .4

      IF(XM-.4)  400,400,450
  400 CONTINUE
      R6      = REI**0.6713
      SN      = 1.-(R6)/(5.142+2.*R6)
      R7      = REI**0.7114
      CN1     = 0.2302*R7/(15.44+R7)
      CN2     = (.01569/(.3077+REI**.7378))*(15./(15.+REI**3.))

      PHI1    = 1.00283
      R11     = REI**1.109
      PHI2    = 1.834 - 1.634*(R11/(2.765+R11))
      PHI3    = 1. - .000242*(REI/(4. + REI))
      PHI     = 1. + PHI1*PHI2*PHI3
      CNUIM4  = (.14+CN1+CN2)*(REI**SN)*PHI
      CNUIMO  =  .19 + .66*REI**.45
      FF      = -.618034 + 1./((XM/.4)**.9 + .618034)
      CNUIC   =  10.**(FF*ALOG10(CNUIMO) + (1. - FF)*ALOG10(CNUIM4))
      GO TO 500

  450 CONTINUE
      R6      = REI**0.6713
      SN      = 1.-(R6)/(5.142+2.*R6)
      R7      = REI**0.7114
      CN1     = 0.2302*R7/(15.44+R7)
      CN2     = (.01569/(.3077+REI**.7378))*(15./(15.+REI**3.))
      RM12    = XM**1.222

      PHI1    = .6039/XM + .5701*((RM12/(1. + RM12))**1.569 - 1.)

      R11     = REI**1.109
      PHI2    = 1.834 - 1.634*(R11/(2.765+R11))
      PHI3    = 1.+(.3-.065/(XM**1.67))*(REI/(4.+REI))
      PHI     = 1. + PHI1*PHI2*PHI3
      CNUIC   = (.14+CN1+CN2)*(REI**SN)*PHI
  500 CONTINUE
      GO TO (1,2),KKK

  600 RETURN
      END
```

```
      SUBROUTINE DATA(IEOF)


      LOGICAL IEOF

      COMMON/RESULT /XM,REI,CNUI,CKNI,CNUM,CNUIC
      COMMON/ETA/ ETA,ETAC,ETAF,ETAS,ETABAR,ETAM,ETAMI,ETADEL
      COMMON/COPCOE/ CN,CR
      COMMON/ENDLOS/ S,AWBAR,AWSTAR,AWRA,OMEGA,OMGRA
      COMMON/TUNNEL/PINF,TT,TINF
      COMMON/WIRCAL/TW1,WIK1,TW2,WIK2,TI,WR1,T2,WR2,WK,TK,CALCON
      COMMON/WIRE/WD,WL,ALFA,BETA,WIK,WPEM,WRWM,WRM,TEM,TWM,DELT
      COMMON/GAS/GAM,R,PP,CMU,RGAM
      COMMON/VOLTS/VOLTS(200)
      COMMON/KONST/K
      COMMON/RESIST/RESIST,RESISL
      COMMON/INDEKS/KJI,JJJ,LLL,ITMARN,ITETAM
      COMMON/PRECIS/DNUIC,DETAM
      COMMON/EBLT/EBLT
      COMMON/DETAIL/DETAIL
      COMMON/THETAI/THETAI,THETA1,THETA2
      COMMON/NPROFL/NPROFL



 1000 CONTINUE
      READ 100,II,KK
  100 FORMAT ( I2,I5)
      IF ( EOF,50 ) 2000,101
  101 CONTINUE
      IEOF   = .FALSE.
      IF (KK.GT. 0)  K = KK
C
C     KK SHOULD BE READ IN ONLY IF II = 1 AND IF NUMBER OF
C     VOLTAGE READINGS PER RUN CHANGES
C
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20),II
C
C     NOTE   ALL UNITS ARE AS FOLLOWS
C     RESISTANCES   OHMS
C     TEMPERATURES  DEGREES RANKINE
C     LENGTHS   INCHES
C     VISCOSITY  LBM/FT-SEC
C     GAS CONSTANT   FT-LBF/LBM-R
C     CONDUCTIVITY   BTU/HR-FT-R
C
C     A CARD MUST PRECEDE EACH DATA BLOCK TO LABEL THE FOLLOWING DATA
C     THE CARD PRECEDING THE VOLTAGE DATA CONTAINS TWO INPUTS, II AND K
C     THE LABEL CARD FOR ALL OTHER DATA BLOCKS CONTAINS ONLY ONE INPUT, II
C
```

```
C
C     II = 1   SHOWS   THE VOLTAGES OF ANEMOMETER READINGS
C      K = NUMBER OF VOLTAGE READINGS TAKEN FOR THAT RUN
C
C     II = 2   LOCAL FREE STREAM STATIC PRESSURE, PINF
C     II = 3   WIRE CALIBRATIONS T1,WR1,T2,WR2
C     II = 4   GAS DATA  GAM,R,PR,CMU
C     II = 5   WIRE DIAMETER, WD
C     II = 6   HEATED WIRE RESISTANCE,WRWM , AND LEAD RESISTANCE,RESISL
C     II = 7   BRIDGE RESISTANCE R3,RESIST
C     II = 8   CALIBRATION CONSTANT, CALCON
C     II = 9   WIRE UNIT LENGTH RESISTIVITY(OHMS/IN.),WK, AT TEMP., TK
C     II = 10  PRECISION ON NUSSELTS NO. AND EQ. TEMP. RATIO,DNUIC,DETAM
C     II = 11  ESTIMATED B.L. THICKNESS, EBLT, (IN.)
C     II = 12  PARAMETER TO BE SET TO NON-ZERO IF A DETAILED PRINTOUT IS
C              REQUIRED, DETAIL.   THE P.O. WILL BE FOR THE FIRST AND LAST
C              RUNS ONLY.
C     II = 13  PARAMETER TO BE SET TO 1. OR 2. IF ANGLE DEPENDENCE IS REQD
C              THETAI = 1. FOR 1ST ANGLE
C              THETAI = 2. FOR 2ND ANGLE
C              AND THE VALUES OF THOSE TWO ANGLES, THETA1, THETA2
C     II = 14  NUMBER OF PROFILES TO BE READ IN AT EACH ANGLE,NPROFL
C     II = 15  NUMBER OF ITERATIONS TO COMPUTE NU,ITMARN, AND COMPUTED
C              RECOVERY RATIO,ITETAM
C     II = 16  THERMAL CONDUCTIVITIES OF WIRE AT TWO TEMPERATURES,
C              WIK1,TW1,WIK2,TW2
C     II = 17  FIRST AND SECOND TEMPERATURE COEFFICIENT OF RESISTIVITY,
C              ALFA AND BETA
C     II = 18  TOTAL TEMPERATURE,TT
C
C     ALL DATA MUST BE RIGHT JUSTIFIED
C
C

    1  READ 200
  200  FORMAT(70H
     1                                              )
       IF ( EOF,50 ) 2000,201
  201  CONTINUE
       IEOF   = .FALSE.
       PRINT 300
  300  FORMAT (1H1)
       PRINT 200
       PRINT 350
  350  FORMAT (1H )
       READ 401,(VOLTS(I),I=1,K)
  401  FORMAT (F8.4,5F10.4)
  400  FORMAT (6F10.4)
       RETURN
    2  READ 400, PINF
       GO TO 1000
    3  READ 400, T1,WR1,T2,WR2
       GO TO 1000
    4  READ 400, GAM,R,PR,CMU
       GO TO 1000
```

```
 5   READ 400, WD
     GO TO 1000
 6   READ 400, WRWM,RESISL
     GO TO 1000
 7   READ 400, RESIST
     GO TO 1000
 8   READ 400, CALCON
     GO TO 1000
 9   READ 400, WK,TK
     GO TO 1000
10   READ 400, DNUIC,DETAM
     GO TO 1000
11   READ 400, FBLT
     PRINT 982, FBLT
982  FORMAT(*    ESTIMATED BOUNDARY LAYER THICKNESS =   *,F6.3, *INCHES*)
     GO TO 1000
12   READ 400, DETAIL
     GO TO 1000
13   READ 400, THETAI,THETA1,THETA2
     GO TO 1000
14   READ 100, NPROFL
     GO TO 1000
15   READ 100, ITMARN,ITETAM
     GO TO 1000
16   READ 400, WIK1,TW1,WIK2,TW2
     GO TO 1000
17   READ 400, ALFA,BETA
     GO TO 1000
18   READ 400, TT
     GO TO 1000
19   CONTINUE
     GO TO 1000
20   CONTINUE
     GO TO 1000
2000 CONTINUE
     IEOF     = .TRUE.
     RETURN
     END
```

APPENDIX B

NON-LINEAR RESISTIVITY EFFECTS

The effects of neglecting the second temperature coefficient of resistivity are considered in this appendix.

The resistance of the wire at temperature T_w is

$$r_w = r_0[1 + \alpha_1(T_w - T_0) + \alpha_2(T_w - T_0)^2]$$

where

r_0 = resistance at temperature T_0

α_1 = first temperature resistivity coefficient (linear)

α_2 = second temperature resistivity coefficient.

Rewriting in terms of overheat ratio

$$\frac{r_w - r_0}{r_0} = \alpha_1(T_w - T_0) + \alpha_2(T_w - T_0)^2$$

and solving for $(T_w - T_0)$

$$T_w - T_0 = \frac{1}{\alpha_1} \frac{r_w - r_0}{r_0} \qquad \text{if} \quad \alpha_2 = 0$$

$$T_w - T_0 = \frac{1}{2} \frac{\alpha_1}{\alpha_2} [(1 + \frac{4\alpha_2}{\alpha_1^2} \frac{r_w - r_0}{r_0})^{1/2} - 1] \qquad \text{if} \quad \alpha_2 \neq 0$$

expanding the bracketed term with

$$\left| \frac{4\alpha_2}{\alpha_1^2} (\frac{r_w - r_0}{r_0}) \right| < 1$$

yields

$$T_W - T_0 = \frac{1}{\alpha_1} \frac{r_W - r_0}{r_0} - \frac{1}{4} \frac{\alpha_2}{\alpha_1^3} \left(\frac{r_W - r_0}{r_0} \right)^2 + \frac{1}{32} \frac{\alpha_2^2}{\alpha_1^5} \left(\frac{r_W - r_0}{r_0} \right)^3 + \ldots$$

therefore the error in the temperature due to neglecting the non-linear effect α_2 is

$$\frac{\alpha_1 r_0 (T_W - T_0)}{r_W - r_0} - 1 = \frac{1}{4} \frac{\alpha_2}{\alpha_1^2} \left(\frac{r_W - r_0}{r_0} \right) - \frac{1}{32} \frac{\alpha_2^2}{\alpha_1^4} \left(\frac{r_W - r_0}{r_0} \right)^2 + \ldots$$

The temperature error for typical values of $|\alpha_2| \sim 10^{-7}$ and $\alpha_1 \sim 10^{-3}$ (Ref. 2) is 1% for an overheat ratio of 0.4 and 2.5% for an overheat of 1.0.

The non-linear temperature coefficient of resistivity may be safely neglected if the overheat ratio does not exceed 0.4.

APPENDIX C

UNCERTAINTY ANALYSIS OF THE WIRE DIAMETER

The effects of the wire diameter uncertainties are considered here. The uncertainties of all the required input data should be considered in determining the uncertainty of the output of the computational procedure of Section 3.0, i.e., Mach number. However, the diameter of the wire is the most uncertain of all the data inputs, and furthermore, the diameter is not readily measurable, either directly or indirectly. Usually the manufacturer's nominal diameter is used (accuracy ±3%). The remaining data can be determined rather accurately, hence their uncertainties will not be considered.

The uncertainty of a function, $\Delta f/f$ (e.g., Mach number), is defined by Kline and McClintock [13] as

$$\frac{\Delta f}{f} = \left[\sum_{i=1}^{N} \left(\frac{x_i}{f} \frac{\partial f}{\partial x_i} \right)^2 \left(\frac{\Delta x_i}{x_i} \right)^2 \right]^{1/2}$$

$$= \left[\sum_{i=1}^{N} \left(x_i \frac{\partial \ln f}{\partial x_i} \right)^2 \left(\frac{\Delta x_i}{x_i} \right)^2 \right]^{1/2}$$

where $f = f(x_i)$, and the x_i's and $(\Delta x_i/x_i)$'s are the independent data inputs (and their respective uncertainties) required to determine f.

The uncertainty of mass flow due to uncertainty of wire diameter is determined as follows

$$Nu = A (Re)^{1/2} + B$$

where for

incompressible flow   A = 0.690   B = 0.318
    (King's Eq., Ref. 1)

and

supersonic flow $\qquad$ A = 0.580 $\qquad$ B = -0.795

$$Nu = \frac{hd}{k} \quad ; \quad Re = \frac{\rho U d}{\mu} \quad .$$

From the defining uncertainty equation

$$\rho U = (\frac{hd}{kA} - \frac{B}{A})^2 \frac{\mu}{d}$$

$$\therefore \quad \ell n \, \rho U = \ell n \, \mu - \ell n \, d + 2 \ell n (\frac{hd}{kA} - \frac{B}{A})$$

$$\mu \frac{\partial \, \ell n \, \rho U}{\partial \mu} = 1$$

$$d \frac{\partial \, \ell n \, \rho U}{\partial d} = \frac{hd + Bk}{hd - Bk} = \frac{Nu + B}{Nu - B}$$

$$\therefore \quad \frac{\Delta \rho U}{\rho U} = [(\frac{\Delta \mu}{\mu})^2 + (\frac{Nu + B}{Nu - B})^2 (\frac{\Delta d}{d})^2]^{1/2} \quad .$$

For typical values in the subsonic range

$$(\frac{\Delta \rho U}{\rho U})_{M \ll 1} = \pm [(0.01)^2 + (\frac{2 + 0.318}{2 - 0.318})^2 (0.10)^2]^{1/2}$$

$$\approx \pm 14\% \quad \text{if} \quad \frac{\Delta d}{d} = \pm 10\%$$

$$\text{and} \quad \frac{\Delta \mu}{\mu} = \pm 1\%$$

$$\text{and} \quad Nu = 2 \quad .$$

For typical values in the supersonic range

$$\left(\frac{\Delta\rho U}{\rho U} \right)_{M>1} = \pm \left[(0.01)^2 + \left(\frac{8 - 0.795}{8 + 0.795} \right)^2 (0.10)^2 \right]$$

$$\approx \pm 8\% \quad \text{if} \quad \frac{\Delta d}{d} = \pm 10\%$$

$$\text{and} \quad \frac{\Delta\mu}{\mu} = \pm 1\%$$

$$\text{and} \quad Nu \approx 8$$

Therefore the uncertainty in the mass flow is of the same order as the uncertainty in the wire diameter. Since the mass flow is directly proportional to the Mach number, the uncertainties in Mach number have the same values. Hence an effort must be made to correct for uncertain wire diameters.