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CALIBRATION OF A CYLINDRICAL RF CAPACITANCE PROBE

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Antenna capacitance probe measurements provide a technique of determining the local electron density in the ionosphere and magnetosphere. Several comparisons of capacitance probe measurements with the results of other density measurement techniques have been made in the laboratory and at low altitudes in the ionosphere [see, for example, Heikkila *et al.*, 1968; and Smith, 1972]. This report briefly considers the calibration of an RF antenna capacitance probe carried aboard the RAE-1 spacecraft and the correction of the probe for external effects, believed to be primarily due to local positive ion sheaths and/or photoelectron sheaths surrounding the antenna.

The RAE-1 spacecraft was launched in July 1968 into a 5850-km. circular orbit of 121-degree inclination and carried several antenna and radiometer systems covering a frequency range of 0.2 to 9.2 MHz for radio astronomical studies. The RF capacitance probe measurements discussed in this report utilized a 37-meter electric dipole antenna formed by two monopoles made of silver-coated beryllium-copper alloy tapes formed into hollow cylindrical tubes 1.3 cm in diameter. [For a detailed description of the RAE-1 spacecraft, see Weber *et al.*, 1971].

The capacitance probe measures the dipole antenna capacitance in four frequency ranges centered at approximately 0.25, 0.70, 1.0, and 2.2 MHz. It finds the resonant frequency for a tuned LC circuit consisting of the dipole and one of four inductors available in the probe. After accounting for uncertainties due to oscillator stability, the resonant frequency can be measured to 0.1 percent, corresponding to sensitivity to capacitance changes of less than one percent.

Considering the case where the plasma frequency is much less than the observing frequency and the impedance of the short dipole is predominantly a capacitive reactance, the ratio of the antenna capacitance C in the plasma to the free-space capacitance C_o can be written as [see Stone *et al.*, 1966].

$$\frac{C}{C_o} = S \left(\cos^2\theta + \frac{P}{S}\sin^2\theta\right)^{1/2} , \qquad (1)$$

where θ is the angle between the antenna and the static magnetic field and, following the notation of Stix [1962] for a collisionless plasma,

$$S = 1 - \frac{X}{1-Y^2} ,$$

$$P = 1 - X ,$$

$$X = f_p{}^2/f^2$$

and

$$Y = f_g/f.$$

Here f_p, f_g, and f are the electron plasma, gyro, and observing frequencies, respectively. For magnetic field values and orientations encountered near the equator at L values near 2 by RAE-1, we may assume Y to be zero with resulting errors of less than two percent. With the observing frequency chosen so that X is much less than unity, equation (1) reduces to

$$\frac{C}{C_o} \cong 1 - X. \qquad (2)$$

Solving equation (2) for X yields the plasma frequency of the medium and hence the electron density in the absence of a positive ion sheath or a photoelectron sheath about the antenna.

It is well-known that a body placed in a plasma will generally acquire a potential different from that of the plasma, due to the fact that the thermal velocity of the electrons is very much higher than that of the ions, and also since a body in sunlight will be charged by photo-emission. The potential of the body will then affect the electron density in the near vicinity of the body so that the measured density is different from that of the ambient density in the absence of the body. Moreover, the density as a function of radial distance from the antenna is not known and theoretical treatment of the problem is quite difficult.

Consider a conducting cylinder with charge Q and whose radius R_o is small compared to its length. At the center of the cylinder, the electrostatic field varies as 1/r in the radial coordinate until, at some radial distance R_T from the cylinder, the field begins to change to a $1/r^2$ radial dependence due to fringing of the field from the ends of the cylinder. The potential of the cylinder can be written (in rationalized MKS units) as

$$V = \int_{R_o}^{\infty} E \cdot dr = \frac{Q}{2\pi\epsilon_o}\left[\int_{R_o}^{R_T} \frac{dr}{\epsilon r} + \int_{R_T}^{\infty} \frac{F(r)}{\epsilon}\, dr\right], \quad (3)$$

where F(r) is the radial field dependence beyond the transition distance R_T and

$$\epsilon = 1 - 81N/f^2$$

is the relative dielectric constant as given by cold plasma theory, where

$$N = \text{electron concentration}$$

and

$$f = \text{operating frequency.}$$

For the RAE-1 dipole antenna and plasma conditions, R_T is much greater than a Debye length in the plasma and the contribution to the potential in equation (3) beyond R_T can be neglected. The capacitance of the antenna can then be written as

$$C = \frac{2\pi \epsilon_o}{\int_{R_o}^{R_T} \frac{dr}{\epsilon r}} , \tag{4}$$

where end effects due to fringing of the electrostatic field have been neglected, as well as collisions and magnetic field, and the relative dielectric constant ϵ is assumed to be a function only of the radial coordinate r. For free space, $\epsilon = 1$ and equation (4) becomes

$$C = C_o = \frac{2\pi \epsilon_o}{\ln R_T/R_o} ,$$

Following a similar calculation done by Heikkila *et al.*, [1968], the relative change in antenna capacitance caused by the plasma is then

$$\frac{\Delta C}{C_o} = \frac{C-C_o}{C_o} = \frac{\left(\int_{R_o}^{R_T} \frac{dr}{\epsilon r}\right)^{-1} - (\ln R_T/R_o)^{-1}}{(\ln R_T/R_o)^{-1}}$$

which can be written as

$$\frac{\Delta C}{C_o} = \frac{-\int_{R_o}^{R_T} \frac{dr}{r\epsilon}(1-\epsilon)}{\int_{R_o}^{R_T} \frac{dr}{r\epsilon}}$$

For the case of plasma frequency much less than observing frequency, $\epsilon(r)$ is only slightly smaller than unity and, using small number approximations, the above equation can be written as

$$\frac{\Delta C}{C_o} \cong \frac{1}{\ln R_T/R_o} \int_{R_o}^{R_T} \frac{dr}{r}(1-\epsilon). \tag{5}$$

This is basically the same result obtained by Heikkila et al., [1968] for a spherical probe geometry and, as pointed out in that paper, the absence of an ion sheath would result in a capacitance decrease of

$$1-\epsilon = 81N/f^2,$$

or linearly proportional to electron density. However, for an actual ion sheath or photoelectron sheath, ϵ is some unknown function of r and must remain in the integrand. Heikkila *et al*. avoided the complex and difficult evaluation of equation (5) by showing experimental results which indicated that, for a spherical probe geometry, the net effect of the sheath is to reduce the sensitivity of the capacitance probe measurement by an amount which depends only weakly upon the plasma parameters and obtained the result of $\Delta C \cong$ (constant) N.

The results which will be presented in this report, however, indicate that the same approximation cannot be used for the cylindrical probe geometry and that ΔC is not linearly proportional to N but rather varies as root N,

$$\Delta C \propto (\text{constant})\ N^{1/2},$$

indicating a much more complex behavior of ϵ in the integrand of equation (5) than was suggested by Heikkila *et al*. [1968] for a spherical probe geometry.

In order to obtain an absolute calibration of the RAE-1 capacitance probe, the electron plasma frequency was determined from both the capacitance probe measurement and from observations of the local upper-hybrid resonance (UHR) noise bands [for a discussion of this technique, see Mosier *et al.*, 1973]. The difference between these two determinations of plasma frequency is then plotted as a function of plasma frequency. Figure 1 illustrates the results of 48 such calculations for values of plasma frequency between 150 kHz and 470 kHz. The upper limit of this range was chosen so that the small-number approximations used in deriving equation (5) would result in an error of less than ten percent. For the range of plasma frequency in Figure 1, ε varies from 0.98 to 0.74. A positive capacitance probe error in Figure 1 indicates that the capacitance probe determination of plasma frequency is too high. Also shown in Figure 1 is the result of a least-squares analytical fit to the data. Of the many analytical curves which were attempted, only the straight line yielded a good fit. Thus, ΔC varies linearly as plasma frequency or, in terms of density,

$$\Delta C = -2.88\ N^{1/2} + 127.$$

In summary, the correction factor for the RAE-1 capacitance probe has been accurately determined from comparison with resonance techniques, the correction being linearly proportional to plasma frequency. Since the antenna potential is unknown, we are unable to calculate the function $\varepsilon(r)$ in equation (5) through the sheath region. In principle, however, the measurement of the probe potential would permit such a calculation since boundary conditions could then be fit to the numerical studies performed by several investigators. As Heikkila et al. [1968] point out, a particularly serious feature of the capacitance probe correction is its dependence on probe potential. It thus appears that the capacitance probe is a more valuable tool in regions where the probe potential is likely to be slowly varying as in the equatorial magnetosphere. The difference between the probe correction obtained by Heikkila and the correction described in this report is not understood but is probably due in part to the differences in probe geometry and the resulting changes in boundary conditions.

REFERENCES

Heikkila, W.J., N. Eaker, J.A. Fejer, K.R. Tipple, J. Hugill, D.E. Schneible, and W. Calvert, "Comparison of several probe techniques for ionospheric electron concentration measurements," J. Geophys. Res., 73, 3511, 1968.

Mosier, S.R., M.L. Kaiser, and L.W. Brown, "Observations of noise bands associated with the upper hybrid resonance by the IMP 6 radio astronomy experiment," J. Geophys. Res., 78, 1673, 1973.

Smith, J., "Comparison of Lanqmuir probe and rf capacitance probe electron density measurements in low-density plasmas," J. Appl. Phys., 43, 3031, 1972.

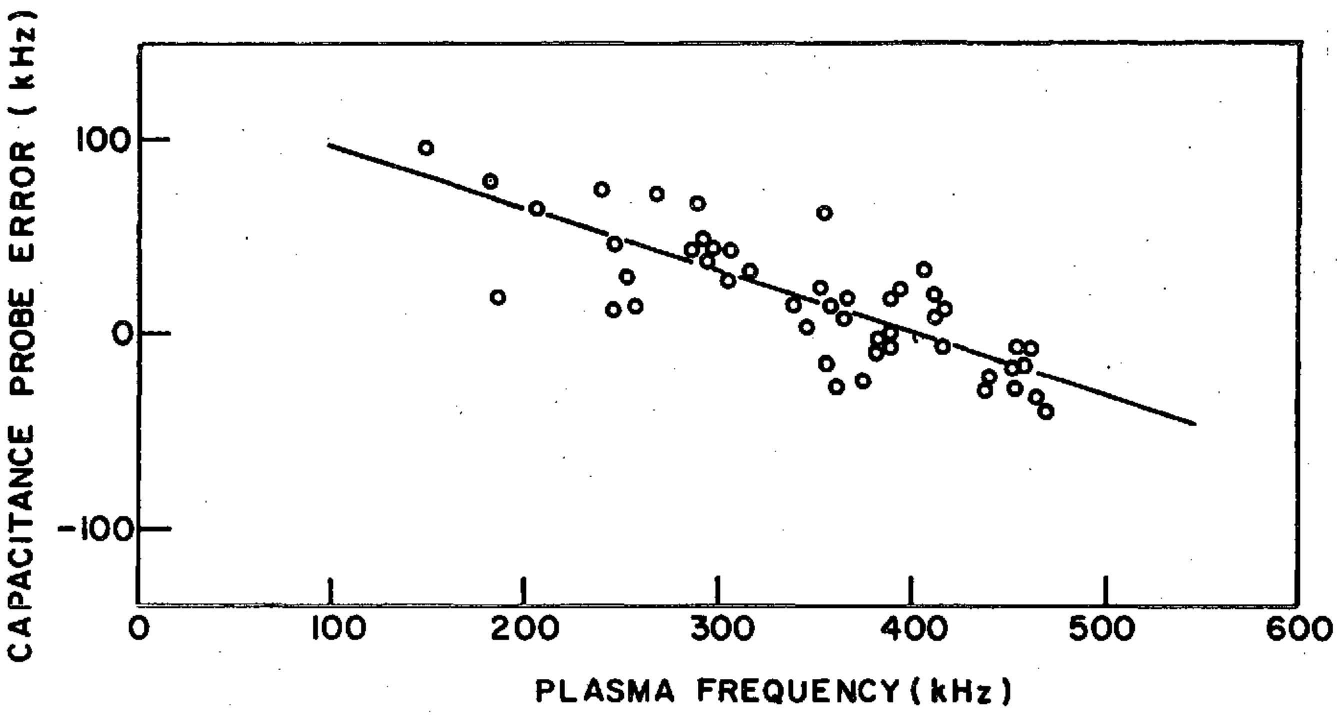
Stix, T.H., The Theory of Plasma Waves, McGraw-Hill, New York, 1962.

Stone, R.G., J.K. Alexander, and R.R. Weber, "Magnetic field effects on antenna reactance measurements at frequencies well above the plasma frequency," Planet. Space Sci., 14, 1227, 1966.

Weber, R.R., J.K. Alexander, and R.G. Stone, "The Radio Astronomy Explorer Satellite, a low-frequency observatory," Radio Sci., 6, 1085, 1971.

Figure Captions

Figure 1 - Capacitance probe error as a function of plasma frequency.



CAPACITANCE PROBE ERROR (kHz)
100
0
-100
0
100
200
300
400
500
600
PLASMA FREQUENCY (kHz)

Figure 1