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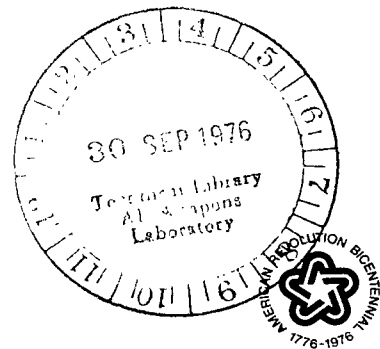
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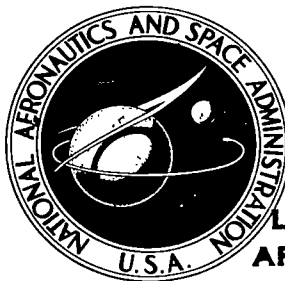
RELIABILITY-BASED ECONOMETRICS OF AEROSPACE STRUCTURAL SYSTEMS: DESIGN CRITERIA AND TEST OPTIONS

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16. ABSTRACT

   Several new areas in aerospace structural reliability were investigated. The more significant original contributions were:
   1. Development of a decision methodology for selecting a combination of structural tests and structural design factors for aerospace systems on the basis of optimized expected cost. The decision method involves the use of Bayesian statistics and statistical decision theory as exemplified by the work of Pratt, Raiffa, and Schlaifer of the Harvard School of Business Administration.
   2. Development of procedures for obtaining and updating data-based probabilistic strength distributions for aerospace structures when test information is available and for obtaining subjective distributions when data are not available. Techniques used in developing these distributions included:
      a. The use of subjective expert opinion obained through mail questionnaires. Such subjective data were required because observed data were not available for one particular type of testing considered in the optimization procedure. Probabilistic models were derived for analyzing the subjective opinions.
      b. Incorporation of new information with prior data through the use of Bayesian statistics. Laboratory test results were used to determine a prior distribution, and flight data were used as new information for revising the prior distribution.
   The methodology developed was applied to several typical aerospace structures to illustrate the effect of system characteristics such as value of weight, cost of failure, and cost of testing on the optimum decision.

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SUMMARY

Several new areas in aerospace structural reliability were investigated. The investigations were directed toward the development of a methodology for selecting a combination of structural tests and structural design factors for aerospace systems on the basis of optimized expected cost. The optimization method involves the use of Bayesian statistics and statistical decision theory as exemplified by the work of Pratt, Raiffa, and Schlaifer of the Harvard School of Business Administration.

From the basic theory, a specific model applicable to the problem of selecting structural tests and design factors was developed. This was accomplished by establishing the decision to be made as a selection of a test method from four alternatives or options and the selection of a design factor from a continuous spectrum. The strength of the structure and the loads applied to the structure were assumed to be random variables that influence the decision. The decision criterion was minimum expected cost, where probabilistic as well as deterministic costs were considered.

Probabilistic strength distributions needed in the decision model required new developments. Subjective expert opinion, obtained through mail questionnaires, was used to obtain a strength distribution for a test option where observed data were not available. The method of acquiring opinions and the probabilistic models used in the analysis of the opinions were first verified

by a test questionnaire which gave results that could be compared with observed data.

A technique employing Bayesian statistics was used to incorporate new information with prior data in obtaining another strength distribution. Laboratory test results were used to obtain a prior distribution, and flight data were used as new information for revising the prior distribution.

The methodology developed was applied to several typical aerospace structures to illustrate the effect of system characteristics such as value of weight, cost of failure, and cost of testing on the optimum decision. From the example problem results, it was concluded that the method provides a reasonable means of selecting tests and design factors for aerospace structures.

CHAPTER I

INTRODUCTION

1.1 Organization of Material

The research will be introduced by a brief discussion of some background concepts required for understanding the remainder of the work. Following this, since the research falls into the broad category of aerospace structural reliability, a review of the literature in this area is given.

In light of this review of previous work, a statement of the research problem and the various topics composing the problem is presented in Chapter III, followed by an overview of the method of solution.

The three major original contributions of the research and several other contributing innovations are presented in Chapters IV through VII. The analysis method developed is then applied to several structural assemblies of NASA space vehicles in Chapter VIII to illustrate the technique and to show how the system characteristics of the various assemblies affect the outcome.

Some results and conclusions from the research are discussed in Chapter IX. Recommendations, particularly in the area of possible further research, are also given in that chapter. Nomenclature for the entire dissertation is given in Appendix A.

1.2 Background

On the following pages certain concepts which are essential to further development of the research are briefly outlined. In most cases, the concepts discussed here have developed into common usage and only typical references, usually not to the original contributor, are cited. The references mentioned generally give a coherent discussion of the idea under consideration, as applicable to the present research.

1.2.1 Reliability

Tribus [1] credits the National Aeronautics and Space Administration with the following definition of reliability: the probability of a device performing adequately for the period of time intended under the operating conditions encountered. Some key words and phrases in this definition require careful interpretation in the light of each problem.

Benjamin and Cornell [2] discuss probability as the relative frequency of occurrence of an event in repeated trials or, more liberally, as an individual's measure of the relative likelihoods of the possible outcomes of an experiment. Many have argued, including Tribus [1] and McGee [3], that the individual's belief about the relative likelihood or plausibility of an event is the appropriate concept of probability and that the relative frequency interpretation is only a special case of this more liberal interpretation. For example, if there is to be only one launch of a particular type of spacecraft, there is no such thing as a long-run relative frequency in connection with the probability of success. The single mission will either succeed or fail. Given information about the

spacecraft, one might assign a number between 0 and 1 as a plausibility (probability) measure that the mission will be a success. Note that such a plausibility measure can include cases where relative frequency is meaningful. It is this concept of probability as a plausibility measure that will be used in this research.

In the reliability context there are two possible events or outcomes — success and failure. Thus, reliability becomes the decision-maker's assessment of the probability or plausibility of success. Since success and failure are prescribed to be the only possible outcomes, the following relationship results from an axiom of probability as given by Benjamin and Cornell [2]:

$$P_F + L = 1 \qquad\qquad (1.1)$$

or

$$P_F = 1 - L \qquad ,$$

where P_F is the probability of failure and L is the reliability or probability of success. In this research, probability of failure will be a frequently used idea with the understanding that the corresponding reliability is readily obtainable from equation (1.1).

Device is the next word in the definition to be discussed and is the identification of the article, component, or system for which the reliability is being discussed. The device could range from a single simple tension member, to a complete airframe, to an entire fleet of airplanes.

The period of time intended and the operating conditions encountered must also be clearly stated. These conditions of operation could range from withstanding a single prescribed application of load to survival of all expected conditions in a complete airplane service life.

A precise description of adequate performance is equally important; otherwise, reliability is an ambiguous term. In some situations, the appearence of any defect requiring repair would be termed inadequate performance. In other instances, only a catastrophic loss of the system is of interest.

It should be noted that these assertions of the device under consideration, the overall conditions of operation, and the description of adequate performance should be specified in deterministic form as a prelude to calculating the probabilistic quantity, reliability.

A misunderstanding of any of these assertions can cause a difference in reliability of several orders of magnitude. Of course, more detailed probabilistic requirements are derived from the specified deterministic requirements in calculating reliability.

1.2.2 Resistance and Load

Two generally accepted and understood ideas of structural reliability discussions are the concepts of resistance and load, under various names. Names often used for resistance are strength, capacity, and capability. Terms used for load are stress, strain, and demand. Generally speaking, the load is a measure of the environment acting on the system, and the resistance is a measure of the ability of the system to withstand the environment.

Frequently, though not always, the load represents effects not under the immediate control of the designer, and resistance is visualized as a quality to be determined by the designer; that is, the designer is to select a design which has a resistance sufficient to withstand the load. For example, the designer may be required to select a structural design with sufficient static strength to withstand a given static or quasi-static load. As another example, a design may be required to have fatigue capability (resistance) to withstand a particular stress for a given number of cycles. Very simple relationships can be used to denote a satisfactory design, as

$$R \geq S \quad , \quad \text{or} \quad R - S \geq 0 \quad , \quad \text{or} \quad \frac{R}{S} \geq 1 \quad , \tag{1.2}$$

where R is resistance and S is load. Such relationships are discussed in numerous publications. For example, see Asplund [4], Brewer [5], Brown [6], Chilver [7], Disney, Lipson, and Sheth [8], Freudenthal [9], or Su [10].

Complexities arise in the satisfaction of the inequalities of equation (1.2) because of uncertainties in the resistance and load. These uncertainties stem from two basic sources. First, there is a natural or intrinsic uncertainty because loads and resistances are probabilistic phenomena. Second, the engineer cannot precisely describe the phenomena. Uncertainties can also be classified as objective (those that can be expressed in terms of measured data) and subjective (those that cannot be or have not been measured and depend on personal assessment). Uncertainty dictates that a completely rational description of the design adequacy cannot be expressed by equations (1.2) but requires

probability statements about the inequalities in (1.2). Given sufficient information about R and S, the engineer may make the following statements,

$$P[R \geq S] = \alpha \quad , \text{ or } P[R - S \geq 0] = \alpha \quad , \text{ or } P[\frac{R}{S} \geq 1] = \alpha \quad , \quad (1.3)$$

where the first statement, for example, is read, "the probability that R is greater than or equal to S is α." If R and S are defined appropriately, then $\alpha = L$, the reliability.

1.2.3 Safety Measures

In this discussion of safety measures, resistance R and load S are assumed to be described in terms of statistical parameters, $\overline{R}$ (mean resistance), $\overline{S}$ (mean load), σ_R (standard deviation of resistance), and σ_S (standard deviation of load).

The term "safety measure" as used in the heading of this section means any parameter devised to provide a safety increment between load and resistance. Safety factors, safety margins, and safety indices are safety measures to be discussed in this section. A number of commonly used safety measures are discussed by Kececioglu and Haugen [11]. Any safety measure can be regarded as either a prescribed constant which controls or specifies the design or as a variable which describes the status of the design. These are actually two distinct concepts with subtle differences, and this has caused a considerable degree of difficulty in communication. The two concepts will be discussed in the following paragraphs in terms of the safety measure known as the safety factor. The safety factor can be expressed as follows:

$$\nu = \frac{R}{S} \quad , \tag{1.4}$$

where ν is the safety factor. In the first concept, this factor is perceived as a design requirement or design specification and the interpretation would be,"It is required that the resistance-to-load ratio be ν." It is evident that ν is a constant in this case, and the equation is ambiguous unless particular values of R and S such as $\overline{R}$ and $\overline{S}$ are used in the equation. In this case, ν is called the mean or central safety factor and could be appropriately labeled as $\overset{\circ}{\nu}$. Other values of R and S could be used in the specification. For example,

$$\overset{\wedge}{\nu} = \frac{R - p\,\sigma_R}{S + q\,\sigma_S} = \frac{R_p}{S_q} \quad , \tag{1.5}$$

where p and q are constants, has frequently been used. In this dissertation a superscript " $\circ$ ", " $\wedge$ ", or other identification will always be used to distinguish the specified safety factor, and the term, design safety factor, will be used in the text.

In the second concept, the safety factor is a property of the design. Equation (1.4) would be interpreted as "The ratio of resistance to load is ν." Thus, ν is a variable for the second concept, which describes the status of the design. If R and S are random variables, ν is also a random variable, and equation (1.4) can be substituted into (1.3) to yield,

$$P[\nu \geq 1] = \alpha = L \quad , \tag{1.6}$$

which directly relates the safety factor to the reliability.

Another frequently used safety measure is safety margin. Two different definitions of safety margin are as follows:

$$SM = R - S \qquad (1.7)$$

and

$$SM = \frac{R_p}{\hat{\nu} S_q} - 1 \qquad . \qquad (1.8)$$

Through equation (1.3), the reliability can be calculated directly from definition (1.7) as

$$L = P[R - S \geq 0] = P[SM \geq 0] \qquad . \qquad (1.9)$$

Equation (1.8) is used extensively in the aerospace industry and gives the fraction by which the design resistance exceeds the requirement.

The safety index provides another means of specifying or calculating structural reliability. Its definition is

$$\beta = \frac{\overline{R} - \overline{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad . \qquad (1.10)$$

The safety index will be used extensively in this research. It is a relatively new concept which was mentioned by Corso in a discussion of Freudenthal's paper [12]. The concept has been used by Blake [13], and Mau and Sexsmith [14].

Cornell [15] suggested its use in a structural code.

It should be mentioned that in any given situation it would be possible to apply any one of the preceding safety measures. The primary factors to be considered in selecting the safety measure to be used are convenience and communication. The convenience is usually computational. For example, if resistance and strength are normally distributed random variables, then R - S is also normally distributed. This could well lead one to utilize the safety margin or safety index for normally distributed R and S. Communication is a tremendously important aspect which is frequently overlooked. It is unlikely that a new research idea will gain widespread acceptance unless the results are eventually cast in some form easily recognizable to practitioners in the field. This could lead to use of a well-known safety measure or at least to a demonstration of how the safety measure used relates to a familiar measure such as the safety factor recognized by most structural engineers.

1.2.4 Structural Testing

A brief explanation of structural testing will be given here to place it in proper context with respect to this dissertation. Small-scale static and fatigue tests on specially designed specimens to determine basic material properties could properly be called structural tests. However, the kind of testing considered here is on a much larger scale. The concern here is with the testing of large structural assemblies which make up an airframe, or even the testing of the complete airframe itself. This is the type of testing discussed

by Lowndes and Cavanagh [16], Thomas [17], and Rosenfield [18], which will be described as the "baseline" testing.

In most airplanes or aerospace systems, all structural assemblies of the airframe have been tested in the laboratory, either as separate units or as a complete assembly. Both methods have been successful. If the assemblies are tested separately, care must be taken to simulate the boundary conditions where the tested structure joins the remainder of the airframe. Because of various practical considerations, testing by assemblies has the advantage of providing for more refined application of loads and more densely distributed instrumentation. If the complete airframe is tested, larger facilities are required, loading is less refined, and instrumentation is more sparse. However, the important boundary condition problem is eliminated.

Ideally, the testing discussed above is accomplished on an airframe built from the same specifications and under the same conditions as the actual flight articles. Often, there are compromises in this respect.

Studies have been made to determine the need for such testing. These studies have revealed a failure rate ranging from 32 percent to 65 percent in this type of testing; that is, approximately one-third to two-thirds of the test articles experience some type of failure at less than predicted failure load, as shown by Bouton and Trent [19], Jablecki [20], and Thomas [17]. There can be no doubt that full-scale structural testing has been an effective "error discloser" as discussed by Bouton and Trent [19].

Nevertheless, the increasing size of airplane and aerospace structural systems and the complexity of environments such as temperatures applied to the systems have encouraged a search for other means of assuring structural reliability. Lowndes and Cavanagh [16] discuss facility and operational costs in testing large high-temperature systems and show that costs could be an order of magnitude higher than previous costs for lower temperatures.

Other methods of testing have been used to some extent, usually as a supplement to the testing discussed above. These include model tests (a scaling down of the test article size), component tests (the testing of individual small subassemblies which make up the major assembly), and proof tests (the pretesting of each actual flight assembly). These alternative types of tests will be expanded in scope and considered as options which could be used in place of the "baseline" type of testing previously discussed.

1.2.5 Relationship of Safety Measures, Testing, and Cost

Alternatives to the "baseline" type of testing appear more plausible if studied in conjunction with safety measures and cost. To gain an understanding of the relationships involved, consider some extreme examples. As a first example, assume that the design safety measure has been set at a very high value so that even if errors exist in the analysis, failure is unlikely. Then there is no need for testing, and the test cost has been saved with perhaps no decrease in reliability and no increase in maintenance cost. However, the large safety measure is certain to cause weight increases which could perhaps be tolerated in some systems but not in others.

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As a second example, assume that testing of not one but several complete airframes identical to the flight article is planned and that design changes will be made on all flight articles to correct any deficiencies observed during testing. This gives a high assurance that fewer errors will remain in the design or manufacturing after the test program has been completed. Then the design safety measure used in the initial design could be much lower, again, with no decrease in reliability and no increase in maintenance cost, but with the attendant lower weight and increased performance. It is quite possible that this increased performance could be well worth the increased test cost in some systems.

Although the baseline method of testing has usually produced acceptably reliable systems, there is certainly no proof that it is the best method for all systems. Alternative methods deserve careful study.

1.2.6 Decision Theory

Statistical decision theory, as exemplified by Pratt, Raiffa, and Schlaifer [21], provides an excellent means of studying the complex interrelationships discussed above in a quantitative manner. A lengthy discussion would be required to present even a rudimentary development of decision theory. Instead of embarking on such a development, we will devote this section to describing what can be done with the theory.

Statistical decision theory is one technique which provides a mathematical model for making decisions in the face of uncertainty. For example, the fact that uncertainties exist in loads and resistance has already been

discussed, yet a decision may be required on what design safety factor to specify.

The decision-making process is formulated as the process of choosing an action, a, from among the available alternative actions, a_n, the members of the action space, A. For each possible action the true state of nature will yield a certain value, a measure of the consequences of the action. For example, the action could be the choice of design safety factor, the state of nature could be the reliability and system weight achieved, and the value could be the total tonnage of goods deliverable by a fleet of airplanes.

If the true state of nature were known with certainty, one would readily calculate the value of each alternative action and select the action with highest value. Statistical decision theory provides a method of determining the action with the highest expected value when the true state of nature is not known with certainty, but probabilistic information is available about the state of nature or its constituents.

Further, if Bayesian statistical methods are used, the probabilistic information can be in the form of observed data or of the engineer's subjective degree of belief. One distinct advantage of the Bayesian approach is that experimental data can be combined with the prior information available. It is required only that the resulting probabilities represent the relative likelihoods upon which the individual is prepared to base his decision.

The decision methodology also provides for selecting an experiment (or no experiment) from among several candidates which achieves the best

balance between experiment cost and reduced risk in the action choice. Through

Bayes' theorem a means is provided for incorporating new data with existing

data to update probability estimates.

CHAPTER II

REVIEW OF THE LITERATURE

2.1 Scope of Literature Review

The scope of the present research extends into several areas, such as probability, statistics, decision theory, and testing. However, the major motivation for the work and the primary thrust of the research is directed toward the reliability of aerospace structures. Since it would not be feasible to attempt a reasonably complete literature review in all of the areas mentioned above, it seems appropriate to concentrate on the area of primary emphasis, aerospace structural reliability.

In some respects, a broad view of the topic of structural reliability is included. For example, literature in the field of structural criteria such as safety measures is included, whether or not the safety measure is related by the author to structural reliability. Also, a number of publications outside of the aerospace field, particularly in the area of civil engineering structures where a great deal of closely related work has been done, are included if the techniques presented can be applied directly to aerospace structures. In other respects, the review is restricted to a narrower viewpoint. For example, although several important articles on fatigue are included, the review in this area is by no means complete, because the present research was restricted

to the static strength of structures. For the same reason, material on structural loads is included only as required to make the material on structural strength more meaningful.

The review is conducted in essentially a chronological order. Exceptions to this are made when a grouping of several articles in a closely related area improves the continuity of the presentation. The primary purpose of the review is to simply relate what was done in each publication, but evaluative remarks are sometimes made in relating the publication to the present research.

2.2 Historical Observations

According to Bouton and Trent [19], the concept of the structural safety factor for aircraft dates back to the Wright Flyer, when the Wright brothers established as their objective a structural system that would sustain five times the weight of the vehicle. It would appear that such a goal may have been somewhat arbitrary and that some rational, quantitative basis for selecting safety factors is desirable, but to this date no widely accepted quantitative means of selecting structural safety factors has appeared in the aerospace industry. In spite of attempts by some engineers to introduce rational means of safety factor selection, usually on the basis of reliability analysis, safety factors are still arbitrarily selected in many cases, and the particular values are based on precedent and tradition.

In general, such an approach has produced acceptably reliable

structural systems in the past; hence, there has been a great reluctance to change the approach. Nevertheless, it has certainly not been proved that no better method of selecting safety factors can be devised. It is highly probable that some alternate method could produce equally reliable structures with better performance and at lower cost, or possibly, more reliable structures with equal performance and cost.

The phrase safety factor could refer to any parameter used to control the strength of a structure relative to the applied load. However, since the term "safety factor" has acquired a more specific meaning, the term "safety measure" will be used to refer to such parameters as discussed in Chapter I.

2.3 Contents of the Literature

Tye [22] suggested in 1944 that aircraft performance envelopes, rather than arbitrary factors, be used to establish yield strength and ultimate strength requirements. For example, the ultimate condition could be specified as one which occurs once in 100,000 hours and the yield condition one which occurs once in 100 hours. It was argued that arbitrary factors sometimes resulted in physically impossible load conditions. The paper contains examples of typical safety factors in use in the 1940's. Evidently, this was a transition period in which the yield factor was changing from 1.5 or 1.25 to 1.125 and the ultimate factor from 2.0 to 1.5.

In 1947 Freudenthal published one of his classic papers [23] containing many fundamental concepts of reliability analysis. The safety factor

is defined as the ratio of the random variable resistance to the random variable load, rather than the ratio of a "minimum" resistance to a "maximum" load. Several topics such as superposition of influences, types of loads, temperature effects, wind forces, boundary conditions, material properties, and uncertainty in dimensions were discussed in analytical form.

Chilver [7] showed that the frequency of structural failure could be calculated by integrating either over increments in the strength of the structure, considering the load fixed, or over increments in load considering the strength fixed in each increment. These relationships were shown as sums over discrete increments and in integral form. Pugsley [24] discussed several items affecting structural safety, including fatigue, random loading, and corrosion effects. The probabilistic effects of strength and loading were considered, and the effect of safety factor on the probability of failure was shown graphically. A trend toward reducing the reliability problem to one of economics was discussed, and a simple cost minimization model, in which initial cost and cost of failure were considered, was presented.

In a 1955 doctoral thesis [20] Jablecki presented the results of static load testing of a large number of aircraft major components. Roughly one-half of the components failed at less than design ultimate load. The report seems to substantiate the fact that static testing is essential for aircraft structures designed with small safety factors. The link of safety factors to the amount of testing needed has received little attention, in general. The publication is a valuable source of information on structural failures, and in

many cases, the location and cause of failure were given. Unfortunately,
strength values of structures sustaining greater than design ultimate load were
not given, and a strength distribution cannot be developed from the data. It is
noteworthy that, regardless of the component, almost all failures occurred in
some type of joint, fitting, cutouts, or some other type of load transition area.

A general review of the role of safety factors in aircraft structural
safety was given by Williams [25]. The importance of the ultimate strength
test in demonstrating that an aircraft can survive greater than limit load was
stressed. Williams visualized the ultimate safety factor as an assurance that
the few aircraft in a fleet which might encounter excessive loads will survive.
The fatigue or life factor assures that each individual aircraft in the fleet will
sustain the required load repetitions.

Freudenthal published a handbook-type exposition on structural
safety [12] in 1956. The relationship of economics to reliability was intro-
duced. Analytical relationships among several safety measures, statistical
distributions, and the probability of failure were given. In discussions of the
paper, the safety index (unnamed at the time) and additional ways to compute
the probability of failure were introduced. Asplund [4] exploited the idea that
some risk is unavoidable in design and related this risk to mortality and the
efficient use of resources.

A progress report of an ASCE committee on factors of safety [26]
was written by Chairman O.G. Julian in 1957. The committee recommended
the use of two separate factors: a factor of safety and a factor of serviceability.

The factor of safety was defined as the ratio of mean collapse resistance to the mean load and was to assure a sufficiently high probability that the structure would not collapse. The factor of serviceability is a similar ratio, but with respect to serviceability rather than collapse. Nomograms and tables were given for determining probability of failure versus safety factor for normal and log-normal load and strength distributions.

In Part I of a United States Air Force Report [27], Landes, Wagner, and Kriegshauser reviewed the structural requirements and characteristics of several missile systems with the goal of developing design criteria on a reliability basis. Because of the lack of data to implement such criteria, they recommended, on an interim basis, that separate factors be applied to speed, quality, and maneuverability instead of applying a single safety factor to loads. In Part II of the report [28], Ready developed a framework for establishing design requirements to achieve a given level of reliability. The methodology deals almost entirely with load variability with little attention to strength variability. Because of an attempt to make the method all-inclusive, it appears very difficult to understand and to use. The report also contains some questionable assumptions in the area of probability and statistics. Nevertheless, it is the first attempt to develop structural criteria adaptable to very complex environmental conditions.

Su [10], Brown [6], and Svensson [29] discussed means of combining the variability of several effects such as strength, loads, dimensions, and environments to obtain a single safety factor. For example, Brown combined

a factor on loads, a factor on strength, and a factor on social consequences into a single safety factor to be multiplied by a characteristic load value to establish a characteristic strength value.

Although a few authors had mentioned the possibility of optimizing reliability toward some desired goal such as minimum costs, it appears that Hilton and Feigen contributed the first analytical structural optimization problem in reliability [30]. Their problem was to minimize the weight of a structure for a preassigned probability of failure. To accomplish this, Gaussian distributions for loads and strength were assumed. The weight of the structure and the probability of failure were both expressed as functions of the member areas, and the problem was formulated as a minimization of the member weights with the prescribed probability of failure as a constraint. It was shown that the heavier structural members should be assigned relatively higher individual failure probabilities to achieve minimum weight.

Freudenthal [9] published a method for calculating reliability when the combined effects of fatigue and ultimate failure are considered. The reliability was expressed as a function of the life of the structure in terms of the number of load repetitions for both fatigue and ultimate failure. By assuming independence of the two failure modes, the combined probability of failure was obtained by adding the two contributions. The probability of ultimate failure was considered constant over the life, but the probability of fatigue failure increased with the number of load repetitions. Freudenthal recognized that the assumption of independence of the failure modes may not

hold in many cases, since a fatigue-damaged structure is more susceptible to ultimate load failure. However, if the dependence of the failure modes is recognized in the analysis, ultimate strength data on fatigue-damaged structures are required in the analysis, and these data are not normally available.

Structural reliability as a probabilistic phenomenon was discussed by Bouton [31], who pointed out that the probability of survival is an appropriate quantitative measure of reliability. The conflict of influences tending to increase reliability (economics of failure of the system) and influences tending to decrease reliability (system weight) were recognized. The author indicated that the selection of an appropriate reliability is primarily economic in nature but considered the analysis required to perform such a cost optimization impractical at that time. The effect of variability in loads and strength on reliability was discussed, and the results of some example problems were shown, but the method of calculation was not given. A flow chart showing the various factors that influence component strength and the environment of the component, and hence determine the reliability, was developed.

A unique method of characterizing loading was given by Leve [32] In this method, the loading is characterized by a set of deterministic life histories, and the reliability over an individual life history is the minimum value attained along the history. The total reliability is obtained by summing the products of the individual reliabilities and their probability of occurrence. It was also shown in this paper that an assumption of independence of failure

of individual members is invalid if the loading in the members is not independent. Techniques for apportioning reliability were also discussed, and a modification of the Hilton-Feigen model [30] was suggested, which accounts for dependence of the failure modes.

The reliability analysis and allocation methods advocated by Leve hinge on the idea that there are only a few predominant failure modes in a complex structure and that other failure modes, less likely to occur, are associated with these predominant modes. Only the predominant modes are assumed independent. The result of such an approach is that the overall reliability goal for a structure can be achieved with lower individual member reliabilities, yielding a lower structural weight than an assumption of independence of all failure modes. The problem with such an approach appears to be that the predominant failure modes must be known a priori, whereas in many instances oversights or errors by the designer is a major contributor to unreliability as discussed by Bouton and Trent [19].

Possible alternatives in testing large airframes were discussed by Lowndes and Cavanagh [16]. The rapidly escalating cost of full-scale testing, primarily due to more severe thermal environments, was discussed. The continued need for full-scale testing was substantiated by data which showed that failures occurred in many static test programs of aircraft between 1940 and 1963. The authors pointed out that testing is effective in determining manufacturing effects, determining analytical deficiencies, and disclosing human errors.

Four alternative methods of testing were reviewed with regard to their effectiveness in performing the above functions. These are (1) loading the entire vehicle but heating only local areas, (2) separating into components and heating and testing each separately, (3) testing to higher than flight loads with lower than flight temperatures, and (4) model testing. No conclusion was reached as to the best alternative. It should be mentioned that component testing was successfully used by the National Aeronautics and Space Administration in the Saturn program as discussed by Thomas [17]. The problem of boundary conditions where the test component joins the rest of the airframe was usually overcome by building a test fixture with stiffness to simulate the adjoining structure or actually using a part of the adjoining structure as part of the test fixture.

A linear-perturbation method of reliability estimation and weight optimization was published by Broding, Diederich, and Parker [33]. Reliability was related to safety factor by assuming that the safety factor is a normally distributed random variable. In the linear perturbation method, weight and safety factor are expressed as linear functions of the design variables through a Taylor series expansion about some reference values in the range of interest. The weight is then minimized with respect to each design variable such as strength or thickness, with a probability of failure for each mode of failure as constraints. The authors stated that the approach could be extended to the case of a prescribed overall reliability. The method appears to be useful when the interest is in optimizing on several design

variables and safety factor and weight are approximately linear functions of the design variables in the range of interest. In an example problem, temperature, allowable strain, coefficient of thermal expansion, and elastic modulus are treated as design variables.

Switzky [34] produced an analysis procedure for minimizing weight of components with a constraint on the overall probability of failure. This was accomplished by assuming that the ratio of any given component weight to the total weight is independent of the probability of failure for small probabilities of failure. Normal distributions of load and strength were used, and the probability of failure was expressed as the inverse of the cumulative distribution of the safety index. The mean safety factor was shown explicitly as a function of the probability of failure and the load and strength coefficients of variation. The weight minimization was accomplished using a Lagrange multiplier technique for the constrained minimization problem. A technical note by Ghista [35] also addressed the problem of weight minimization with a probability of failure constraint. The suggested trial-and-error technique for solution yielded satisfactory results for a two-member example problem.

A survey paper by Coutinho [36] reviewed basic statistical and reliability concepts and discussed some government reliability specifications and their applicability. An interesting breakdown of types of failure and the responsibility for each was given: initial failures are usually traceable to manufacturing and quality control, wear-out failures to maintenance, and random failures to design. The philosophy and operational aspects of

reliability assurance engineering were discussed. Coutinho listed the four

major causes of unreliability as:

1. Lack of reliable methods for predicting operating environments.

2. Unit-to-unit variations in resistance (within specification).

3. Design errors.

4. Manufacturing errors.

A qualitative comparison of the relative reliability of different types of aero-

space systems was given. Airframes are credited with the highest reliability

and electronics the poorest. However, it should be mentioned that alternate

or redundant electronic systems can usually be provided at much less cost

and weight increase than structural systems.

Moon, Shinn, and Hyler [37] discussed the determination of design

allowables for Military Handbook 5 [38], the primary source of materials

strength data in the aerospace industry. Statistical techniques for determining

allowable strengths on a probability basis, the use of specification values and

typical values, the derivation of design allowables for a given property from

closely related properties, and the determination of allowables at other than

room temperature were discussed.

A final report of the Task Committee on Factors of Safety, American

Society of Civil Engineers, was written by Freudenthal, Garrelts, and

Shinozuka [39]. This is an extremely useful reference which defined

commonly used terms in reliability and gave concise derivations of

important structural reliability relationships for numerous cases. As would

be expected for this type of report, much of the material had been developed and published earlier, but this publication brought the information together in an easy-to-use handbook style. The report will be frequently referenced in this research, especially where it concisely states some commonly accepted principle that has been gradually developed by a number of contributors.

The task committee defined the safety factor as the ratio of resistance to load, and since resistance and load are random variables, the safety factor is also a random variable. The report showed how to treat multiple member structures under multiple load cycles. This was done for the case that resistance and load are independent of the number of load applications and for the case that one or both depends on the number of previous load applications. Consideration was given to loads applied at specified times (life is measured in number of load applications) and to loads governed by a Poisson law (life is measured in time). Fatigue was treated by the residual strength concept, where resistance is a decreasing function with number of load applications.

Evidently, the first attempt at recommending specific safety factors for aerospace structures that vary depending on the desired reliability and the nature of the design was made by Bouton, Trent, and Chenoweth [40]. These authors developed, based on statistical analyses, a matrix of safety factors to be used under various conditions. To select the appropriate safety factor, one of three overall reliability objectives is chosen: 0.99 for high-risk vehicles, 0.9999 for standard vehicles, and 0.999999 for low-risk vehicles.

Beyond this, the safety factor selected depends upon the expected strength scatter in the structure under consideration (four classes are listed) and whether the loads and strength are verified by test. The recommended safety factor ranges from 1.15 for a high-risk vehicle, with small scatter in strength and loads and strength verified by test, to 4.0 for all structures not verified by strength test and all structures with large strength scatter and no test verification of loads. The approach seems to be a very reasonable step beyond the usual practice of arbitrary selection of safety factors, and it is surprising that the approach or some variation thereof has not come into general use in the aerospace industry. Perhaps the reluctance to use the method stems from the a priori judgments which must be made: selection of a reliability goal and determination of strength variability. The present research contributes to a rational means of resolving this problem through the use of decision theory.

Statistical analysis of fail-safe designs was considered by Shinozuka and Itagaki [41]. It was shown in an example problem, which could idealistically represent a three-spar wing, that the probability of survival after failure of one member is undesirably low (in the range of 0.20 to 0.33), even for a ductile material. The failure condition considered was yielding of the structure, and the authors indicated that different conclusions could be expected for a fatigue failure mode.

An article by Turkstra [42] discussed the choice of design from a set of designs with equal functional value based on minimization of expected loss. The basic idea offered was that, within a reasonable approximation

under certain conditions, only bounds on reliability of the candidate designs are

needed, and the design selected should be the one with minimum cost which

has a reliability within a prescribed value; that is, all designs with reliability

falling within the prescribed reliability are equally acceptable, regardless of

the exact reliability. This can considerably simplify cost optimization calcula-

tions. The costs considered were initial costs and the expected cost of failure.

Initial cost was assumed to be a linear function of the mean safety factor.

Turkstra's method would be very limited in aerospace applications, since the

assumption of equal functional value for all structures would rarely apply.

For example, the functional value of aerospace structures depends greatly

on the weight of the structure, which is a function of the safety factor. Thus,

designs with different safety factors have distinctly different functional values,

because increased weight in the structure usually results in decreased payload

weight.

A paper by Moses and Kinser [43] gave computational methods for

multiple member structures under multiple load conditions. The authors

emphasized the bounds on reliability and the relationship of the bounds to the

degree of dependence among the failure modes. Computations of the reliability

were made, using the known dependence of failure modes, for comparison with

the bounds when complete independence, then complete dependence, of the

failure modes were assumed. It was concluded that the essential relationship

which controls the dependence, assuming that loads on all members arise from

a common source, is the load variability compared with the strength variability.

A small relative variability of load leads to statistical independence, and a large variability of load relative to strength variability leads to statistical dependence.

It was clearly demonstrated in the paper that an assumption of independence of failure modes, when dependence actually exists, can cause a significant underprediction of reliability. However, the particular example problem chosen seems to amplify the effects of such an assumption because of the low reliabilities involved. For more typical, higher reliability, the difference in predictions using assumptions of dependence and independence would not differ so much.

Weight optimization with a constraint on the probability of failure, which was examined by Hilton and Feigen [30] and Switzky [34], was treated by Moses and Kinser [44] with dependence of failure modes considered. They showed that additional reduction in weight can be achieved if the assumption of the independence of failure modes is discarded in favor of a more exact treatment.

A book by Pugsley [45] treated the subject of structural safety in an easy-to-read fashion. Several chapters dealt with elementary structural concepts and analysis, followed by a discussion of the variability in strengths and loads and the risk of failure. The safety rules (or design codes) used to obtain safe structures were then discussed. An interesting chapter on "Some Famous Accidents" illustrated some of the conditions which can result in unsafe structures.

An excellent treatise of the bounds on structural reliability was given by Cornell [46]. The bounds depend upon the degree of dependence among loads and among failure modes. Expressions for upper and lower bounds on reliability were first given for a fundamental problem with time-independent, equally distributed resistance subjected to a sequence of random loads. A generalization was then made to time-dependent loads, time-dependent resistance, and unequally distributed resistance.

Blake [13] discussed the direct application of statistical decision theory to structural reliability problems. He emphasized the importance of the cost effectiveness of a design as opposed to the desirability of the same fixed safety factor or the same fixed value of reliability for every component in a structural design. Relationships were developed for sizing a structural design to obtain a reliability such that system effectiveness is maximized. If the loads on the system and the strength of the system are normally distributed, the reliability was shown to be related to the safety index, and the optimum safety index was tabulated against a measure of the change in system effectiveness with safety index.

Blake continued his development by considering the possible loss of system effectiveness for nonoptimum safety index. Since exact knowledge of effectiveness, safety index, and reliability is not usually available, the approach adopted was to minimize the loss in effectiveness for nonoptimum choice of the safety index. This leads to a nonoptimum design size with a conservative bias. The foregoing preceded a discussion as to whether more

information, which would permit selection of a design size nearer the optimum, would be worth the cost of such information, and led to a realization that, in addition to the intrinsic random nature of design variables affecting reliability, there is also uncertainty associated with a lack of complete information. Both of these uncertainties should be considered in cost optimization. The author also pointed out that the Bayesian approach permits the incorporation of subjective judgments with objective data in reaching a design decision.

Benjamin [47] described the advantages of probabilistic design as compared with deterministic procedures. The author envisioned deterministic approaches as lacking in informational content, modeling of reality, refinement of analysis and design, and decision-making. Illustrations showed that the designer is not usually fully aware of the true state of affairs when deterministic approaches are used. A simplified Bayesian-type decision tree was presented, illustrating the choice of an optimum building design for earthquake loading.

Bouton and Trent [19] elaborated on the ideas initially set forth by Bouton, Trent, and Chenoweth [40]. They described in great detail how a variable safety factor requirement could be incorporated into the technical and contractual management of aerospace systems and made an extensive evaluation of previous structural criteria proposals to show that none of these could fulfill what they considered the essential requirements of structural criteria. Most of the other proposed criteria could not satisfy a requirement, considered essential by the authors, that a means of proof of compliance must

exist for the criteria. For example, if the criterion is that the structure must have a certain reliability, then proof of compliance is virtually impossible before the structure is placed in service. Thus, structural criteria which specify reliability would not be acceptable to the authors.

A proposal for separating loads into "limit" and "omega" conditions was presented. The limit condition is a load at which the structure is expected to survive, and the omega condition is a load, possibly caused by malfunction of other systems, beyond which the structure is not expected to survive.

The primary thrust of the work by Bouton and Trent was an advocation of changes in the present safety factor system for airframes. They showed how structural reliability fits into the total system approach, and emphasized the need for establishing exactly what is expected of the structural system and design and test procedures for obtaining and measuring compliance with what is expected. The system created by them utilizes deterministic safety factors for analysis and testing which have been established by probabilistic techniques. The approach retains the advantage of having designers and analysts working with a given safety factor but also has the advantage that the safety factor has been determined in a rational manner. The basic premise for setting the safety factor is a reliability goal, but the safety factor also depends upon the scatter in strength of like structures and upon the number of tests contemplated.

There can be little doubt that such a comprehensive evaluation and thorough proposal was a result of much study and deliberation. It appears that

the proposed criteria would be workable and would be a significant improvement over present practices. It does seem that the authors may have overlooked some plausible alternatives because of a preoccuptation with the legalistic aspects of the problem. For example, the proof of compliance previously mentioned and a requirement that blame for any failure must be traceable may be desirable goals but probably should not be permitted to dominate or mask other important objectives.

Kececioglu and Haugen [11] summarized various definitions of safety factors, safety margins, and measures of reliability which have been used and illustrated the use of some of them. A curve showing reliability as a function of safety index for normal distribution of loads and strength was given. Disney, Lipson, and Sheth [8] published formulas for determination of the probability of failure for several commonly used distributions of loads and strength.

Some simplifications to the Hilton-Feigen model for minimum weight [30] were made by Murthy and Subramanian [48]. Some approximations were introduced that resulted in a set of equations which, unlike the Hilton-Fiegen model, could be solved without iteration. Some results in an example problem using the approximate solution had a close correspondence to the exact solution.

Shinozuka presented a model for weight-optimized design subject to a constraint on expected cost [49]. The expected cost consisted of the expected cost of failure in service and in proof testing. The proof test was considered

to truncate the strength distribution at the proof-test level, and the reliability in service was thus enhanced. The optimization was performed with respect to the proof-test load level; that is, a proof-test level which minimized expected cost or weight was the result of the analysis. The procedure was developed for both determinant and indeterminant structures.

The use of statistical decision theory as a tool in design is discussed by Cornell [50]. The Bayesian method of incorporating all physical, parameter, and model uncertainties into a single probability law was emphasized, since such a law can be used directly in design decisions and reliability estimates.

A qualitative description of the residual strength concept of fatigue was given by Bouton and Trent [51]. The basis of the concept is that fatigue failures are, in reality, ultimate strength failures of fatigue-damaged structures. The analysis of such structures had previously been considered by Freudenthal, Garrelts, and Shinozuka [39]. The relationship of the concept to the amount and type of testing required was discussed.

Shah [52] advocated a design code in which resistance and load are recognized as probabilistic quantities. Some of the fundamental aspects of reliability and statistics were reviewed. A factor to account for differences in laboratory and field strengths was considered. Sexsmith and Nelson [53] discussed the limitations in the application of probabilistic concepts to structural design. Each limitation mentioned was accompanied by a discussion of how it might be overcome. Some of the limitations examined were problems in selecting a model, consideration of subjective information, use of

information in making a decision, and confidence limits on reliability.

A generalization of the classical probability of failure concept as a structural criterion was proposed by Ang and Amin [54]. A concept of the "probability of unsafety" was introduced such that the event $(R/S < \nu)$, $\nu \geq 1$, constitutes a state of unsafety, where R is structural strength or resistance, S is the applied load, and ν is a factor for uncertainty. For $\nu = 1$, the concept reduced to a conventional reliability analysis. It was proposed that the known probabilistic information be incorporated into R and S and that ν account for any remaining dispersions or unknowns. The structural requirement would then be expressed by the condition $P[R/S < \nu] \leq \alpha$, where α is the allowable risk. It was further shown that a reasonable and conservative choice for α is pq where p is the probability of having a structure with resistance less than a characteristic value, R_p, and q is the probability of occurrence of a load greater than a characteristic value, S_q.

The principal advantages of the approach are that it separates the uncertainty into subjective and objective parts and it makes the design much less sensitive to the assumed distributions of R and S if α is chosen as pq. The disadvantage of the approach seems to be that it involves several somewhat arbitrary choices (ν, p, and q) instead of one choice, for instance, a level of reliability. It also appears that Bayesian statistics may be a more consistent way to introduce subjective uncertainties than the proposed method.

A structural design code was suggested by Cornell [15] in which the safety factor selected depends only upon the means and coefficients of variations

of resistance and load, but not upon their specific probability distributions. To apply the method, however, a value of the safety index β must be selected, evidently somewhat arbitrarily. The author suggested that β be specified in the code and that its value depend upon the consequences of failure.

Yao and Yeh [55] concentrated upon developing a procedure for systematically counting the failure paths in a redundant structure in the formulation of a reliability analysis.

Thomas [17] summarized the test results for large structural assemblies in the Saturn V launch vehicle system and gave a rationale for determining safety factors for structures which would not be tested. The Saturn V test data confirmed the trend of the Jablecki data [20] in showing that structural failures are to be expected frequently in static testing of large assemblies. Such failures were seen as error disclosures by Bouton and Trent [19] and are usually attributed to analysis inaccuracies or errors. The Saturn V data showed a trend toward more accurate analysis as compared with the Jablecki data which were compiled from tests in the 1940's.

An analysis of the Jablecki data was performed by Chenoweth [56] to determine the safety factor which would be required for untested structures. The analysis showed that large safety factors (in a range from 4 to 11) would be required to assure a reliability of 0.9999 in sustaining limit load. Although the conclusions of the paper are generally acceptable, the definition of reliability as the probability of sustaining limit load is inadequate for quantitative decision methods. In a more refined analysis, the probabilistic aspects

of the loading should be considered, and reliability should be defined in terms of the probability of failure in service. This would undoubtedly result in lower required factors of safety than those shown in Chenoweth's paper, since limit load is usually considerably larger than the expected load.

Stevenson and Moses [57] presented an analysis method for determining the probability of failure in structures for which the failure mechanism is expressed as a linear combination of several resistances and loads. The method was applied to the limit design of frames in which the resistances were the plastic hinge moment capability of joints and the loads were applied to various points of the frame. It was shown that variance in strength and load has a large effect on reliability, that the particular form of statistical distribution assumed has a small effect for high failure probabilities and a large effect for low failure probabilities, and that the dependence between failure modes is not very significant if the dependence is weak.

A paper by Shah and Tang [58] developed a procedure for determining load factors. One criterion used was cost, based on a simple model consisting of construction cost and expected cost of failure. A summary of the views of several European committees studying structural safety was given by Rowe [59]. The basic approach was deterministic, but the deterministic factor may be developed from several sources of uncertainty which could be represented probabilistically.

Optimum design with respect to weight with a constraint on the probability of failure was the subject of a paper by Moses and Stevenson [60]. The

application was to both elastic design and limit design of frame structures. The problem was similar to that addressed by Hilton and Feigen [30], Switzky [34], and Moses and Kinser [44] except that limit design failures were considered.

Ghare [61] discussed the perogative of the designer in trading off the safety factor used in design with the quality factor associated with production to obtain a required reliability within prescribed resources.

An ultimate strength analysis of more than 300 data points of different types of aircraft structures and components was presented in terms of test strength compared with analytical strength [62] by Freudenthal and Wang. A comparison with the Jablecki data [20] showed some improvement in analytical prediction. The data were used in two reliability analyses of aircraft by disregarding the fact, recognized as questionable by the authors, that strength deficiencies discovered in tests would be rectified.

Blume [63] discussed the philosophy of public safety in civil structures and the consideration of economic and safety trade-offs. The paper was a discussion only, with no analysis given. In the same volume, Freudenthal [64] discussed similar questions and suggested a model for selecting reliability based on minimizing the total expected loss. The expression for loss accounted for initial costs (which may be a function of reliability) and expected cost of failure, the product of cost of failure and probability of failure.

Carnahan published an investigation of the reliability growth in orbital launches from 1958 to 1968 [65]. Data in this article showed that the overall

reliability has grown from about 50 percent in 1958 to about 95 percent in 1966-1968 and has leveled off at about 95 percent. The causes of failure were examined and found to be almost entirely hardware related, as opposed to personnel or procedural. Ninety percent of the failures were mechanical, and of these 48 percent were in nonmoving parts. The author defended component testing as the least expensive means of increasing launch reliability.

An excellent review of statistical methods in fatigue was given by Wirsching and Yao [66]. Emphasis was on various ways of presenting fatigue data and the use of the Weibull distribution for representing fatigue life. An estimation of parameters for the two-parameter and the three-parameter distribution was discussed and illustrated. The graphical method, the method of moments, and the maximum likelihood method were presented.

Ang [67] further developed the concept of unsafety introduced by Ang and Amin [54]. The concept is that separate devices are used to account for subjective and objective uncertainties. Subjective uncertainties are taken into account by the conventional technique of requiring that some characteristic value of resistance is equal to the product of a safety factor and a character-istic value of the applied load. Objective uncertainty is considered by requiring that the probability that the desired subjective safety factor exists be a predetermined value. This value is called the probability of unsafety. Ang recommended that a log-normal distribution always be used for the subjective safety factor, based on physical grounds. When this is done, the objective probability of unsafety becomes much less sensitive to the assumed

statistical distribution of resistance and load than would exist if all uncertainty is incorporated into the probability of unsafety. It should be noted that two separate deterministic selections must be made to apply the method: a safety factor and a desired probability of unsafety.

An extension of the Shinozuka model [49] was presented by Heer and Yang [68]. The area of extension was in the application to designs susceptible to brittle fracture in which the growth of flaws due to cyclic and sustained loading was considered. The effect of the flaw growth is to lower the structural resistance with use of the structure. In the optimization procedure, weight was optimized with a constraint on the total expected cost. The prediction of the reliability of structures subjected to stationary random excitations was considered by the same authors [69]. The degradation of resistance by the loading history was considered in the reliability prediction. The application of the random loading was to a single-degree-of-freedom system, along with other quite restrictive assumptions.

In a philosophical paper [70] in 1971, Lind formalized the art of safety factor selection and showed that the rationale of several proposed design codes can be derived from various combinations of a small set of postulates. The postulates common to all of the proposed codes were first enumerated. For example, the consideration of strength as a random variable is one of these postulates. Next, the postulates peculiar to the individual code proposals were stated. For example, Cornell [15] postulated that the means and coefficients of variations of loads and resistances should be considered as a

sufficient statistical description.

Hardrath [71] presented an excellent capsulation of the fundamentals of fatigue and fracture mechanics and included a bibliography of approximately 200 sources. The Task Committee on Structural Safety of the ASCE Structural Division published a literature review of structural safety [72] which contains a useful bibliography of approximately 400 references.

A paper by Tang and Yao [73] extended the Miner fatigue damage criterion such that the fatigue lives (in cycles) at each stress level are treated as random variables instead of constants. In this case, the cumulative damage itself then becomes a random variable.

Campion, et al. [74] evaluated the criteria developed by Bouton and Trent [19]. The method was reviewed to determine the data required and the availability, the implications of such an approach in the structural design process, the methods by which the implementation could be achieved without discontinuity, and handbooks. A trial application of the procedure was made to the C-141 transport aircraft wing. The evaluators concluded that total application of the procedure is premature, essentially because of a lack of data for implementation. Additional trial applications to existing systems were recommended, along with efforts to collect and develop the required data.

A development with an optimization objective closely paralleling that of the present research was presented by Mau and Sexsmith [14]. This was a problem of optimizing expected cost as an unconstrained minimization

problem. The result was a Bayesian-type decision process, illustrated by the authors in a simple decision tree. The entire model was extremely simplified to minimize computations. For example, the initial cost of the structure was related only to the material costs, failure modes were assumed independent, costs consisted only of initial costs plus the expected cost of failure, and load and resistance were assumed normally distributed. Permitting different types of testing in the design alternatives was not considered.

Lind [75] proposed distribution functions for load and resistance composed of a normal central portion and an exponential tail portion for reliability calculations. Lind believed that the incorporation of new data with prior data by simply recalculating the required statistics was simpler than a Bayesian procedure for the proposed distributions.

Several probabilistic models were reviewed by Heller [76] with respect to their representation of component life. The viewpoint was a comparison of the physical phenomena to be represented with the properties of the models (normal, log-normal, exponential, or Weibull). It was shown that the Weibull model provides a convenient representation of life-type phenomena. Graphical means of determining model parameters were discussed and illustrated with a fatigue example. This was done for both complete samples and for extreme observations.

A review of statistical methods applicable to structural reliability was

given by Lemon [77]. The concepts of conceptual models, distributions, and parameters were reviewed, including the basic terminology used to describe distributions. A very readable explanation of order statistics was given. Point and interval estimation of parameters was explained and "within" and "between" batch variability was discussed. Chi-square and Kolmogorov-Smirnov goodness-of-fit tests were explained. A fairly thorough treatment of the Weibull distribution was presented, including illustration of the effects of parameters, parameter estimation, distribution of certain functions of the parameters, and confidence bounds on the parameters.

Forney [78] discussed the recent improvements in reliability methods for computing fatigu. · ,es of aircraft structures, particularly the development of models for estimating the expected time to the first failures in a fleet. A multiphase program of the Air Force Materials Laboratory for reducing structural reliability analysis methods to practice was described. One phase of the program was concerned with cost and weight trade-offs as affected by reliability; this is closely related to the present research.

Some of the problem areas in structural reliability computation and communication were pointed out by Bouton [79]. Among these are the effect of analytical errors and the problem of giving a precise definition to reliability. For example, the importance of defining whether the reliability pertains to the entire fleet or to a single aircraft was pointed out, as well as the need for carefully defining what constitutes a failure.

Buntin [80] discussed a proof-test program currently in use on the

F-111 aircraft. The main purpose of the proof-test program is to detect flaws in high-strength steel parts in the aircraft which could cause failure during service. The program was instituted because an early flight failure was attributed to a flaw in a wing-pivot fitting. Basic fracture mechanics and proof test concepts were described, including a method of allowing for a yield zone at the crack tip. A flow diagram for calculating the probability of surviving a given number of flight hours after proof test was shown, but analytical details were not given. From a total of 325 operational aircraft which were proof tested, two failures occurred during testing, neither related to flaws. This demonstrated the ability of proof testing to detect anomalies other than flaws.

A discussion of the latest developments in the U.S. Air Force structural integrity program was presented by Haviland and Tiffany [81]. The discussion revealed a trend toward more attention to fatigue and fracture toughness problems, a result of encountering such problems on the F-111 and C-5A airplanes. Rosenfield [18] discussed the static test philosophy of the U.S. Navy on major aircraft structural assemblies. A review of [16], [17], [18], [19], [20], [62], and [81] reveals a marked similarity in static structural test methods used by the Air Force, Navy, and the National Aeronautics and Space Administration.

There can be little doubt that some important contributions in the field of structural reliability have not been included in this literature review. However, it is hoped that the review does include a sufficient number and variety

of contributions to provide the reader a beginning point in the pursuit of some

particular aspect of this many-faceted and interesting field of research.

CHAPTER III

STATEMENT OF THE PROBLEM, THE SETTING, AND THE METHOD OF SOLUTION

3.1 Statement of the Problem

In this section a concise statement of the research problem, a discussion of the related topics to be considered, the problem setting, and an overview of the method of solution will be outlined.

3.1.1 Concise Statement of the Research Problem

The need for structural testing of aerospace structural systems has been shown separately by Freudenthal and Wang [62], Jablecki [20], and Thomas [17]. Their results show that one-third to two-thirds of the tested structures fail at a load lower than that predicted by the analysts. The terms "analyst's predicted failure load" or "design ultimate load" have been used to designate the actual load predicted by the analysts at which failure is predicted to take place. No safety factors are included in this load. Aerospace structural testing is expensive, and the sample sizes are usually small. In addition to the influence of structural testing on the cost of the structural system, a prescribed safety measure also influences the cost through its effect on reliability and performance. Safety measure has been used as a general term meaning the safety factor, reliability measure, safety index, or other variable used to control the strength of the design.

The research program discussed in this dissertation is to develop a

methodology for selecting a suitable test option and an appropriate safety measure based on optimized expected cost. In the course of the development two additional problems have been considered, the solutions to which were required to provide information needed as input to the cost optimization model.

1. In preliminary design and in making design decisions on the test options and safety measures based on optimized expected cost, it is necessary to estimate the probabilistic strength or failure distribution before testing has actually been carried out and the observed data become available. In this dissertation a methodology will be developed to systematically obtain the subjective expert opinion by using mail questionnaires and to use the answers to these questionnaires to represent the probabilistic strength distribution analytically. These investigations and developments are discussed in Chapter V.

2. In developing the methodology to select the test option and safety measure, it is necessary to update the probabilistic strength distributions as new data become available in the form of test data or flight experience. Furthermore, any procedure for updating should be adapted to the structural testing procedure used in the aerospace industry, which will be discussed in Chapter IV. This has been accomplished using Bayesian techniques and is discussed in Chapter VI.

3.1.2 Related Topics

Several individual, closely related topics must be investigated and placed in a proper relationship to each other in the research program. Each

topic is underlined in the following text.

The first topic to be discussed is "structural testing." The topic can perhaps best be introduced by examining an almost universally used structural test procedure which may be regarded as a "standard" procedure. An examination of Thomas [17], Rosenfeld [18], and Bonton and Trent [19] shows that such a procedure is used by the United States Air Force, the Navy, and the National Aeronautics and Space Administration in testing airframes. This standard procedure consists of testing one full-scale specimen of each of the major structural assemblies which compose an airframe. The testing is accomplished by applying static loads to the assembly with hydraulic cylinders, weights, airbags, etc. Ideally, the test article is constructed from the same design drawings as the flight articles. Often, there are compromises in this respect. If a failure occurs at less than the predicted failure load, a design change is made, and a retest is accomplished. This procedure is iterated until one design "passes" the test, and the successful design is then used in production airframes. To permit a lucid description, the above procedure has been highly idealized.

In some aerospace programs, records have been kept which give some insight into the levels at which failure can be expected in such testing. Freudenthal and Wang [62], Jablecki [20], and Thomas [17] present such records. The failure levels in certain groups of structures are illustrated by the cumulative distribution functions (CDF's) plotted in Figure 1. The CDF's for the recent aircraft data and the aircraft data from the 1940's are incomplete
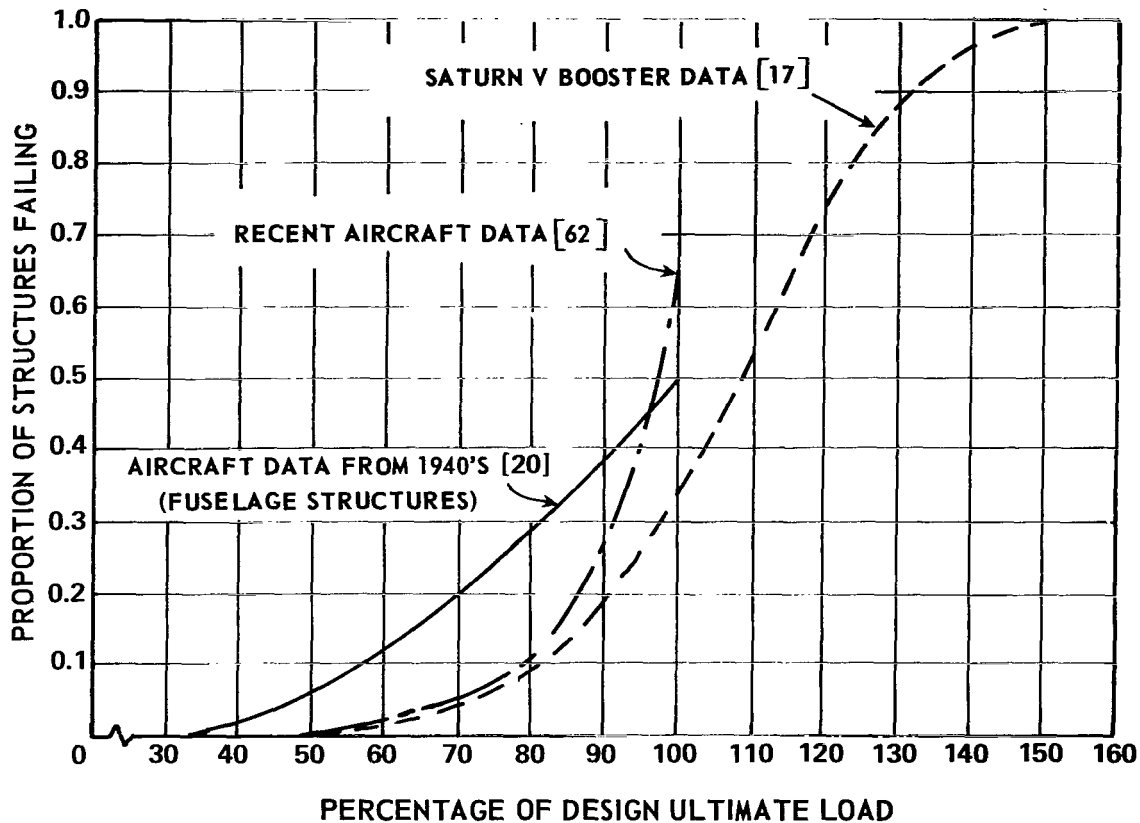
Figure 1. Failure Data for Structural Assemblies.

because the data were preserved only for assemblies which failed at less than

100 percent of design load.

Although the difference between the 1940 data and the more recent

results indicates a significant improvement in analytical accuracy, the role of

the standard static test as an error discloser is still very evident. It is

important to realize that the distributions in Figure 1 do not represent the final

product, since the standard procedure requires a design change and a retest of

any structure failing to achieve design ultimate load. Although some uses have

been made of test and flight data, no procedure has been developed in the

aerospace industry to analytically update the strength distribution after redesign,

retest, and flight experience. Bayesian techniques will be used to develop such

a distribution in the present research.

The standard test procedure has been effective in producing acceptably

reliable structural systems. However, as pointed out in Chapter I, there is no

evidence that it is the most cost-effective procedure for every system,

especially if the prospect of varying the design safety measure is admitted.

Alternatives to the standard procedure will be introduced and discussed in

Chapter IV.

To use statistical decision theory in selecting a particular test option

from various available test options, it is necessary to know the probabilistic

representation of the structural strength distribution for each proposed test

option. For example, one of the CDF's in Figure 1 could be taken to represent

the strengths before any redesigns are applied to the understrength assemblies.

Beyond this simplest case, no observed data have been located to calculate

strength distributions for other test options. However, Bayesian procedures

[2], for example, permit the use of any available information — observed or

subjective — in obtaining strength distributions or in other elements of the

decision process.

Furthermore, even for test procedures where no observed data have been

accumulated, the subjective belief of the decision-maker can be used to establish

a distribution for the purpose of making a decision. Collective expert opinion

has been used to establish probability data for oil spills [82], by Nair, Shah, Smith, and Shah. A similar technique will be developed in the present research program on structural reliability. The accuracy of such a procedure will be established by direct comparision of some subjective distributions obtained from expert opinions with the available observed data, such as shown in Figure 1.

Once particular distributions of strength have been established for the various test options, a second design variable under the control of the designer can be exercised. This is the design safety measure as discussed in Chapter I. The safety measure commonly used in aerospace structures is the safety factor, which establishes some characteristic value of the structural strength or resistance relative to a characteristic value of the load. The design safety measure to be exploited in the present research is the safety index, since it permits the designer a greater flexibility by allowing him to control variability as well as a characteristic value.

The safety index has been shown to provide computational convenience if both load and resistance are normally distributed. For example, see Kececioglu and Haugen [11] or Mau and Sexsmith [14]. Cornell [15] proposed the use of the safety index in a design code and pointed out that it can be used as a safety measure when only the means and variances of load and resistance are known, and no other information is available about the actual distributions. The role of the safety measure in controlling the resistance is illustrated in Figure 2. For simplicity, the safety measure used for illustration in this
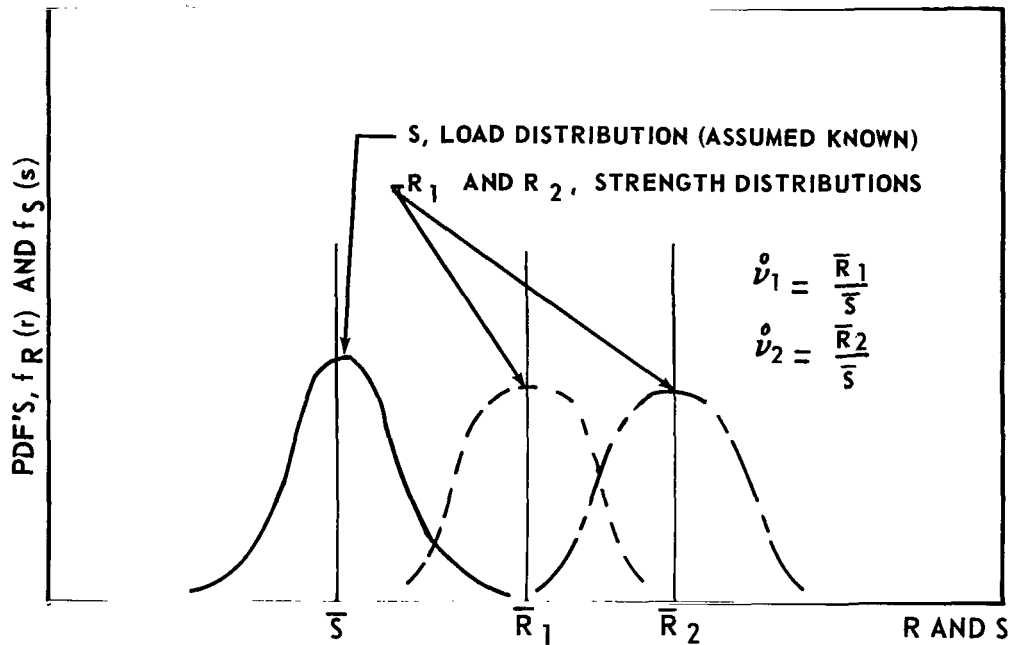
Figure 2. Effect of Safety Factor.

figure is the mean safety factor, $\overset{o}{\nu}$.

In Figure 2, the load distribution is assumed to be known, and the figure is assumed to be drawn for a particular test procedure, which determines the shape of R. The shape is the same for R_1 and R_2. The designer can thus control the strength of his design by selecting a design mean safety factor, $\overset{o}{\nu}_1$ or $\overset{o}{\nu}_2$ (or any other value). A similar argument holds for any other design safety measure.

The role of the test procedure and the design safety measure in controlling the design strength has been discussed. The designer is confronted with the decisions of what test procedure and what value of the design safety measure to use for his design. Since the load and strength are not deterministic

values but are represented by probabilit. distributions, this is a decision in the face of uncertainty. Provided that the designer can quantify in a prescribed manner his preferences among the possible outcomes that will result from each of his alternative decisions and an assessment of the probability of each possible outcome, statistical decision theory provides a formal framework for making such a decision.

In this particular application, the preference for outcomes will be expressed in terms of the cost of each outcome. The cost model is expected to include all of the significant costs relevant to the decision. These include development costs, maintenance costs, cost of failure, and cost of loss in performance, each of which may be further subdivided.

3.2 The Problem Setting

It is important to delineate the setting in which the decisions of what type of testing to do and what safety measure to use are most conveniently made. Of course, such decisions could be made at almost any time during the design process. However, the decision process developed in this research would appear to be more appropriately applied during one particular phase of the typical design cycle.

To apply the decision methodology developed here, certain information peculiar to the particular structural system in question must be available. This includes such information as the change in weight as a function of the design safety measure, the economic value of changes in the structural system

weight, and the anticipated cost of various methods of testing. It seems evident that a preliminary design must be available to develop this type of information. Of course, it might be possible to extend the present method to use probabilistic estimates of this information before design has begun.

On the other hand, if a detailed design has already been developed, fabrication drawings have been released, and test hardware and facilities committed, a decision to change the design safety measure or the method of testing would be costly.

From the preceding discussion, it can be inferred that the appropriate time for applying the decision methodology to select a test program and a design safety measure is after preliminary design is completed but before detailed design begins. For example, the methodology is not intended to provide a means of selecting between a skin-stringer design and a honeycomb sandwich design, for, say, an interstage structure. However, given that the design is skin-stringer, the methodology provides a means of selecting a safety measure which will, in turn, determine the required skin thickness and stringer cross sectional properties. Likewise, the methodology would dictate face sheet thickness and core properties, given a honeycomb sandwich design. This is not to say that the method would not be useful in selecting between these two design concepts. However, to influence the selection, the method must be applied to each design separately and the final results compared.

Equally as important as the time phasing is an appreciation of the level of assembly to which the methodology applies. The method could be adapted to

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any level of assembly, provided that data are available to determine statistical strength distributions for the particular level under consideration. However, for several reasons the present research is directed toward large structural assemblies such as complete wings, thrust structures, propellant tanks, or interstages. First, the available data for developing statistical strength distributions are applicable to this level of assembly. Second, structural tests are usually conducted on these large assemblies. Next, it would be very inconvenient for the designer to attempt to apply different safety measures to the various subassemblies within the large assembly.

The method is expected to be most applicable to structures under overall compressive and bending loads. Again, the data from which the strength distributions are developed are for these types of structures. However, the compressive or bending loads in the test sample were often applied in a complex manner such as concentrated engine thrust applied to a large longeron which, in turn, shears the load into the fuselage structure. Thus, the results are expected to be valid for such structures. Extension to other types of structures would involve only a development of strength distributions for the structure of interest.

3.3 Method of Solution

In the first section of this chapter on the statement of the problem, some preview information on the method of solution was discussed in describing the problem and its ramifications. In this section, the method of solution is to be

made more specific. Although it is intended to give a complete outline of the proposed solution, extended discussions will be deferred to later chapters, where various subjects are developed in detail. It is intended to provide in this section a general outline of how the developments in Chapters IV through VI will be used in Chapter VII.

An <u>action</u> or decision <u>space</u> is created. This consists of a discrete portion (the selection of one test procedure from several discrete possibilities) and a continuous portion (the selection of a design safety index from a continuous spectrum). A complete action must consist of a selection of both a test procedure and a safety index. Selection of the test option determines the shape of the probabilistic strength distribution relative to the analysts' predicted failure load. This strength distribution is considered to be a part of the <u>state of nature</u>. Selection of the safety index determines the location of the strength distribution relative to the probabilistic distribution of the applied load, also considered to be a part of the state of nature. Every combination of course of action and state of nature results in a known <u>value</u> to the decision-maker. In particular, the negative of value in this problem is the cost associated with each combination of action and state of nature. The cost may include the development cost, cost of manufacturing, maintenance cost, cost of failure in operation and testing, and cost of loss in performance.

The <u>expected cost</u> for each course of action is obtained from the definition of expectation. The action is selected (i. e. , the decision is made to use the test procedure and safety index) which results in the minimum expected cost.

The formulation just described is illustrated graphically by the <u>decision tree</u> in Figure 3. The fan-shaped displays for β, R_i, and S indicate that these are continuous variables. No formal treatment of decision analysis or decision trees is given here. Raiffa [83] gave a very readable introduction to the subject. Pratt, Raiffa, and Schlaifer [21] gave a virtually self-contained development, and Benjamin and Cornell [2] gave a development oriented to engineering decisions. Savage [84] presented an exposition on the relationship of the Bayesian approach to other schools of thought in probability and statistics. Tribus [1] related decision analysis to a broader spectrum of ideas. After the problem has been cast in the form of a decision tree, the decision process will be based on the procedures presented by Pratt, Raiffa, and Schlaifer [21]. To analyze the decision tree, the expected cost for each test option, $E[U/\beta, T_i]$, is determined. This expected cost includes the direct cost C_{D_i}. The direct cost is that portion of the cost not affected by the state of nature. Figure 4 shows graphically how the minimum cost test procedure and corresponding safety index are selected. The optimum decision for the case illustrated in the figure is to select test option T_0 and the corresponding safety index, β_0.

The decision problem has been placed in the context of statistical decision theory to take advantage of the formal developments in this field, which establish the conceptual and mathematical rigor of the procedure used to optimize expected cost. Chapter VII will be devoted to a more complete development of the decision model.
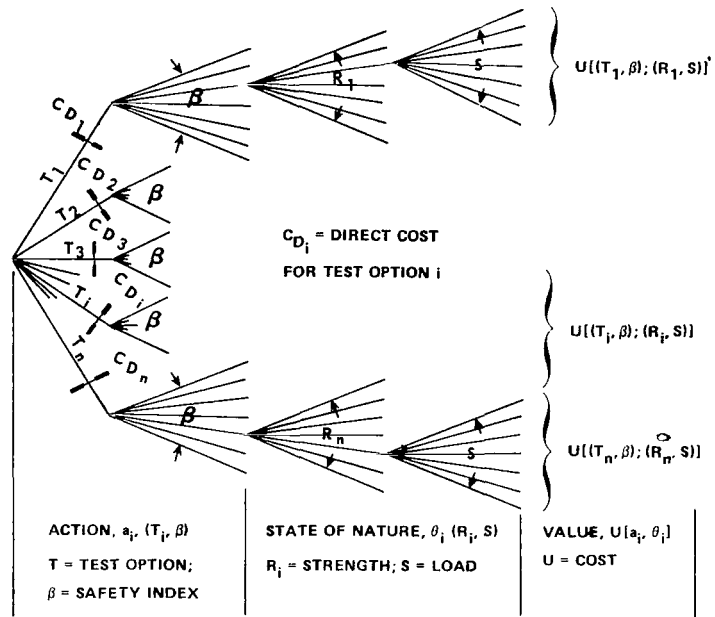
Figure 3. Decision Tree for an Assembly.
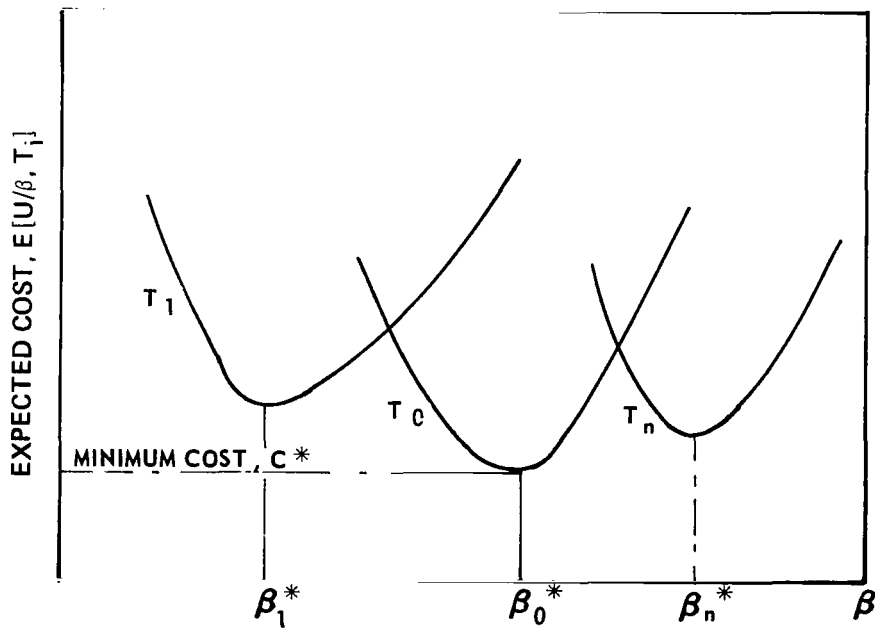
Figure 4. Optimum Decision.

CHAPTER IV

TEST OPTIONS, STRENGTH DISTRIBUTIONS, AND SAFETY MEASURES

Statistical decision theory provides the basis for the research problem. However, it is instructive to discuss the problem in some of the terminology of optimal structural design to clarify the role of some of the variables involved. An introduction to this terminology has been presented in a review article by Sheu and Prager [85].

Geometric design constraints restrict the choices of structural type and shape open to the designer. In this dissertation, all overall structural dimensions, structural member arrangements, and the type of construction are considered fixed. The only choices open to the designer are local dimensions such as thicknesses and cross-sectional areas.

Behavioral design constraints set bounds on quantities that characterize the behavior of the structure under conditions of service. In the present research, the assumed behavioral constraint is that the structure shall not exhibit any behavior which indicates an inability to sustain the applied loads.

The design objective for the structures considered here is the minimization of the total expected cost of the structure, including the probabilistic as well as the deterministic costs. It will be assumed that the test options and the design safety measures are the two design variables that can be used by the

designer to control the strength of a structural assembly. The test option can be used to control the strength by disclosing rectifiable design errors or by eliminating random understrength designs. The test option can also be used to eliminate some uncertainties regarding the engineer's knowledge of the design strength, thereby improving the accuracy in estimating the design performance. The safety measure can be used as a design variable to control the design strength by establishing a characteristic design strength relative to a characteristic load, or more generally, by controlling both a characteristic value and the variance of strength.

4.1 Test Options

The concept of a test option is introduced in this section to explain a procedure by which the method of testing can be used as a design variable. Unlike the classical experiment of statistical decision theory, the test option includes a predetermined (deterministic) response by the designer to each possible test outcome. The testing and predetermined actions by the designer together are used to <u>control</u> the state of nature. The classical experiment in statistical decision processes is used to <u>observe</u> the state of nature. From a probability standpoint, a predetermined action means that for each possible outcome of the test, one particular subsequent course of action will be taken with probability 1, and there is zero probability that any other course of action will be followed. The standard test procedure discussed in Chapter I provides a ready example. For the outcome that the structural assembly fails to pass the test, the

predetermined action is to redesign and retest. For the outcome that the

assembly passes the test, the predetermined action is to make no design change

and to discontinue testing, even if the test reveals an overdesign.

Four test options are included in this research. An unlimited number of

test options could conceivably exist, and each could be treated in a manner

similar to that presented here. It would be possible to create new test options

by various combinations of the test options considered here. A description of

each of the four test options is given as follows.

4.1.1 No-Test Option, T_0

Under the No-Test Option procedure no structural testing is done. Analyt-

ical predictions alone are used to establish the design and predict the strength.

This is much like what is done on most civil engineering structures where

structural testing is not accomplished on each individual design. It would be

inprudent, of course, to use this option if some radically new design concept is

to be used.

4.1.2 Standard Test Option, T_1

The Standard Test Option procedure has been introduced in Chapters I

and III. This is the normal procedure used in the aerospace industry to qualify

the airframe for flight use. It consists of testing one each of the large assem-

blies such as a complete wing, a fuel tank, or an interstage structure which

make up the airframe. More than one assembly may be tested in a single test

setup. It is not unusual to test a complete airframe. A design change is

required to be made on any structure failing to sustain design ultimate load.

Failure does not necessarily mean collapse or breakup; it could mean that some measured parameter such as displacement or strain falls outside a prescribed range. Testing and redesign are to be iterated until a design passes the test. Testing is to be done on assemblies built to the same specifications as the flight articles.

4.1.3 Proof Test Option, T_2

In the Proof Test Option procedure, every production flight article is pretested in the laboratory to some prescribed load level less than design ultimate load. Articles which fail to pass the proof test are discarded from the population or repaired and retested until they pass the proof test. This effectively prohibits the use of random understrength structures as flight articles and establishes a lower bound for the strength of those structures that are used. It is possible that proof testing could alter or damage the structure in a way that would invalidate its original characteristics. Before the proof test option is used, sufficient studies must be made to assure that the proof test will not be damaging to the structure. The proof-test procedure has been used extensively for pressure vessels, including the large propellant tanks of launch vehicles. Little use has been made of it for other types of structures and loadings. In the present research, test option T_2 includes the use of proof testing for all types of structures and loadings. For example, bending of wings and combined compression and bending of fuselage structures are considered appropriate for proof testing.

4.1.4 Model Test Option, T_3

The Model Test Option procedure is to perform all testing on either scale models of the actual structure or subassemblies (components) instead of on complete assemblies. Such a procedure would not require large, complex test facilities. The evaluation approach would be an iteration of design, analysis, and testing until the analysis and test results agree. The same design would then be used for the flight hardware, and the analysis procedure which agreed with the model test results would be used to analyze the flight design.

To accomplish the above procedure, it would be necessary to identify and numerically define the requirements of the flight article and to measure certain parameters identified with these requirements in the model testing. An analytical prediction of the same parameters must be made on both the test and flight articles, and the analytical predictions must agree with each other and with the test results before the flight article design is placed in service. This is the method by which model test results are projected to full-scale design.

It would be desirable that the descriptions of test options be completely free of any ambiguity. However, the testing of complex aerospace hardware contains so many compromises that it is doubtful if this could ever be achieved. Hopefully, the preceding descriptions are specific enough to permit mutually exclusive classifications of the available test options within reasonable tolerances. It is not the objective to define a test option so rigorously that it would be difficult to plan a program which falls within a given test option.

4.2 Strength Distributions

4.2.1 Qualitative Comparison

The test options described above are deterministic in nature in that an a priori disposition of every test outcome has been established. Because of this, it is reasonable to postulate that a probabilistic strength distribution exists for the population of structural assemblies developed by each test option. Each of these distributions can be referenced to the strength value predicted by the analyst. This value is the 100-percent value shown in Figure 5, where a qualitative comparison of the strength distributions that might result from the various test options is shown.

The general shape of the No-Test Option probability density function (PDF) is inferred from the Saturn Booster Data [17] for which the CDF is shown in Figure 1 of Chapter III. Note that other available data in Figure 1 tend to confirm the general shape of the left side of Figure 5(a). For the standard test option, all test articles which fail to sustain 100-percent load are redesigned and retested. However, this does not completely assure that all flight articles have strengths greater than 100 percent, since there would be some scatter among nominally identical structures. A distribution of the general shape of Figure 5(b) results for this test option. In Figure 5(c), r_0 is the proof test level. Thus, any structures from the original population shown in Figure 5(a) with strengths less than r_0 are discarded from the population. This would result in a distribution similar to that shown in Figure 5(c). A model test program would eliminate some of the unknowns causing the scatter
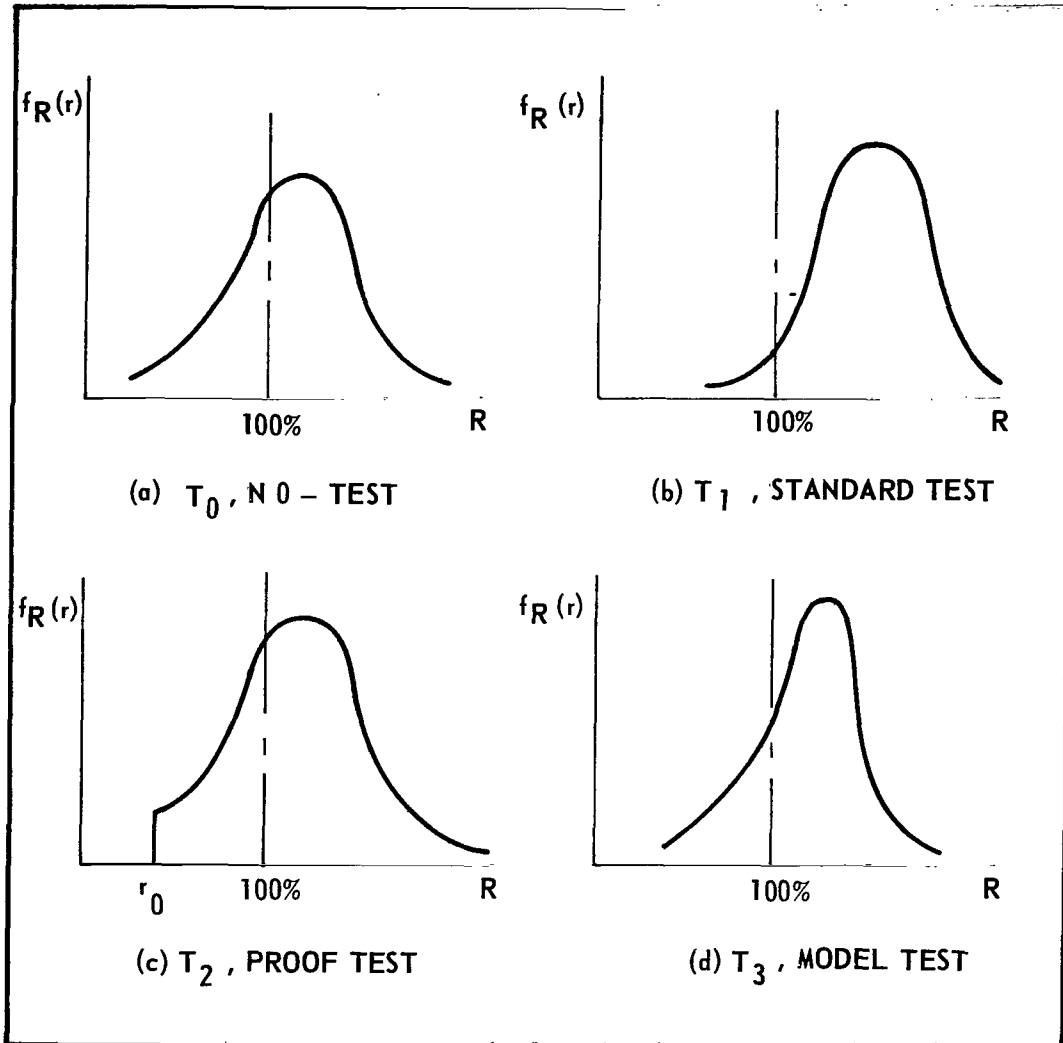
Figure 5. Qualitative Comparison of Strength Distributions.

in Figure 5(a), resulting in a more peaked distribution as shown in Figure 5(d).

The scarcity of applicable data for calculating the preceding distributions

is unfortunate, considering the amount of testing that has been done on large

structural assemblies. The lack of published data can almost certainly be

attributed to the fact that testing of such large assemblies has been done as a

part of specific system developments rather than as scientific research. In development programs the need for recording and compiling data in the form required to determine PDF's has generally not been recognized.

Four sources of data on large structural assemblies have been located, but for various reasons only one, the Saturn V Booster data [17], appears usable at this time. The Jablecki data [20] represents the state-of-the-art in the 1940's and is useful for showing the historical trend of analysis improvement. However, data are given only for assemblies which failed at less than limit load so that a complete PDF cannot be determined. Freudenthal and Wang's data on recent aircraft [62] also give the results on large assemblies, but only for premature failures. In addition, some panel, beam, and frame data (structures which should not really be described as large structural assemblies) are included. Martin and Bouton [86] presented the so-called Lustig data representing aircraft from 1950 to 1970. Again, only premature failures were listed, and in this case not even the total number of tests is known. This makes it impossible to construct even a portion of the CDF, as was done for the other data in Figure 1, Chapter III.

The two following sections in this chapter will be devoted to determining strength distributions that can be realized by test options T_0 and T_2, using reasonably well-established statistical analysis methods. Strength distributions that can be realized by test options T_1 and T_3 require special treatment and will be the subject of Chapters V and VI.

4.2.2 Strength Distribution for No-Test Option, T_0

4.2.2.1 Saturn V Test Data. The test data of Thomas [17] are directly

applicable to the No-Test Option, since these data represent a sample of

structures before any redesign or retesting was accomplished. As such, the

data can be interpreted as a sample from the population of structures which has

not been previously tested. Table 1 is a listing of the failure loads for Saturn V

structural assemblies as a percentage of the design ultimate load, assumed to be

the analyst's predicted failure load. Thus, each data point is the strength or

resistance, r_i, of the particular structure tested.

The data represent all of the major structural qualification tests on the

Saturn V launch vehicle, including several assemblies from each of the three

stages, interstages, and the instrument unit. In some cases, where more than

one load condition was applied to a structure and the load conditions were

distinctly different from each other, more than one data point was taken on a

single structure. Note that there are a number of censored data points (tests

which were terminated before failure occurred) in the sample. Tests were

terminated for various reasons such as economy of testing or a desire to save

the assembly for some other use.

It would be impossible to give a general physical description of the

structure or a general description of the types of loading that would characterize

the sample of Saturn V structures. Construction ranged from riveted skin-

stringer-frame design to honeycomb sandwich. Loading was usually

Table 1. Saturn V Test Data

Structure Number (Order Number)	Failure Load, r_i (Percentage of Design Ultimate Load)	Structure Number (Order Number)	Failure Load, r_i (Percentage of Design Ultimate Load)
1	71.5	26	99.3
2	77.2	27	100.0
3	80.0	28	100.0
4	82.2	29	100.0+*
5	83.7	30	100.0+*
6	85.7	31	100.0+*
7	87.2	32	100.0+*
8	87.2	33	100.0+*
9	89.3	34	100.0+*
10	89.3	35	100.0+*
11	89.3+*	36	100.7+*
12	90.8	37	100.7
13	91.5	38	101.3
14	92.2+*	39	103.6+*
15	93.0	40	107.2
16	93.0+*	41	107.2
17	93.0+*	42	110.8
18	93.0+*	43	112.1+*
19	93.0+*	44	113.0
20	93.0+*	45	115.0+*
21	93.0+*	46	115.7
22	93.0+*	47	118.0
23	93.0+*	48	125.0
24	93.0+*	49	131.3
25	94.3	50	151.3

*Censored sample

compressive or bending, but loads were often concentrated loads applied to large longerons or to beam or truss structures attached to the fuselage shell.

It is important to note that throughout this research all large structural assemblies are assumed to belong to a single population for purposes of statistical analysis. This is done irrespective of the mode of failure or the type of structure. Freudenthal and Wang [62] made an identical assumption, justified by the following statement:

> This assumption is unavoidable because replications of ultimate load tests of large structures and structural parts are and will always be severely limited by technical and economic considerations. Without it, reliability analysis of aircraft structures becomes obviously impossible since the individual small samples are useless for this purpose.

4.2.2.2 Computer Program for Two-Parameter Weibull Distribution. A computer program developed by Hanagud [87] for estimating parameters of the two-parameter Weibull distribution for complete and censored samples was available at the Georgia Institute of Technology, School of Aerospace Engineering. The computer program is based on the maximum likelihood method for estimating parameters as presented by Cohen [88]. The cumulative distribution function of the Weibull distribution is given by

$$F_R(r) = 1 - e^{-\left(\frac{r}{v}\right)^k} \quad , \tag{4.1}$$

and the probability density function is

$$\ldots i) = \frac{k}{v} \left(\frac{r}{v} \right)^{k-1} e^{-\left(\frac{r}{v} \right)^{k}} \quad , \qquad (4.2)$$

where R is the random variable, v is the scale parameter, and k is the shape parameter. The method of maximum likelihood is based on determination of the values of parameters k and v which maximize the joint probability of observing all of the data points in a particular random sample. For the present problem, this means that the method of Cohen can be used to determine values of k and v that maximize the likelihood of the joint occurrence of the data points in Table 1, including censored values.

More specifically, the maximum likelihood estimator of the vector of parameters $[k, v]$ is the value of $[k, v]$ which causes the likelihood function to be a maximum. The likelihood function is proportional to the joint probability distribution of the random sample, assuming independent observations. Assume that $(R_1, R_2 \ldots R_\ell)$ are observed failure loads and that $(R_{\ell+1}, R_{\ell+2} \ldots R_n)$ are observations at which no failure occurred (censored values). The joint probability of $(R_1, R_2 \ldots R_\ell)$ for known $[k, v]$ is

$$f_{R_1, R_2 \ldots R_\ell} (r_1, r_2 \ldots r_\ell \mid k, v) = \qquad (4.3)$$

$$f_{R_1} (r_1 \mid k, v) \, f_{R_2} (r_2 \mid k, v) \ldots f_{R_\ell} (r_\ell \mid k, v) = \prod_{i=1}^{\ell} f_{R_i} (r_i \mid k, v) \quad .$$

Similarly, the joint probability of the observed censored values is

$$P[R_{\ell+1} > r_{\ell+1}] \ldots P[R_n > r_n] = \prod_{i=\ell+1}^{n} [1 - F_{R_i}(r_i|k, v)] \qquad (4.4)$$

Regarding equations (4.3) and (4.4) as functions of unknown parameters [k, v], the likelihood function L gives the relative likelihood of observing·the sample $(R_1 \ldots R_\ell, R_{\ell+1} \ldots R_n)$ as a function of [k, v].

$$L(k, v|R_1 = r_1 \ldots R_\ell = r_\ell; R_{\ell+1} > r_{\ell+1}, \ldots R_n > r_n) \qquad (4.5)$$

$$= \prod_{i=1}^{\ell} f_{R_i}(r_i|k, v) \prod_{j=\ell+1}^{n} [1 - F_{R_j}(r_j|k, v)]$$

Explicitly,

$$L = \prod_{i=1}^{\ell} \left(\frac{k}{v}\right)\left(\frac{r_i}{v}\right)^{k-1} \exp\left[\left(-\frac{r_i}{v}\right)^k\right] \prod_{j=\ell+1}^{n} \exp\left[\left(-\frac{r_j}{v}\right)^k\right]. \qquad (4.6)$$

Since there is a monotonic, one-to-one relationship between the likelihood function and its logarithm, they have a maximum at the same values $[\hat{k}, \hat{v}]$. Thus,

$$\ell n\, L = \sum_{i=1}^{\ell} \left[\ell n\left(\frac{k}{v}\right) + (k-1)\,\ell n\left(\frac{r_i}{v}\right) - \left(\frac{r_i}{v}\right)^k\right] - \sum_{j=\ell+1}^{n} \left(\frac{r_j}{v}\right)^k$$

$$(4.7)$$

Maximizing ℓn L, the following equations are obtained for k and v:

$$\frac{\ell}{k} + \sum_{i=1}^{\ell} \ell n\left(\frac{r_i}{v}\right) - \sum_{i=1}^{n} \left(\frac{r_i}{v}\right)^k \ell n\left(\frac{r_i}{v}\right) = 0 \quad , \qquad (4.8)$$

$$- \frac{\ell k}{v} + \frac{k}{v} \sum_{i=1}^{n} \left(\frac{r_i}{v}\right)^k = 0 \quad . \qquad (4.9)$$

These equations can be simplified by eliminating v and obtaining an equation

for k to be solved by Newton's iteration technique.

$$\frac{1}{k} + \frac{1}{\ell} \sum_{i=1}^{\ell} \left[\ell n\, r_i - k\, \ell n\left(\frac{1}{\ell} \sum_{i=1}^{n} r_i^{\,k}\right) \right] \qquad (4.10)$$

$$= \sum_{i=1}^{n} \left[\frac{r_i^{\,k}}{\sum\limits_{i=1}^{n} r_i^{\,k}} \, k \left\{ \ell n\, r_i - k\, \ell n\left(\frac{1}{\ell} \sum_{i=1}^{\ell} r_i^{\,k}\right) \right\} \right]$$

The value, $\hat{k}$, satisfying equation (4.10) is the maximum likelihood estimator

for k and can be substituted into equation (4.9) to solve for the maximum

likelihood estimator $\hat{v}$. A computer program for obtaining these estimators

is given by Hanagud [87].

When the method was applied to the data in Table 1, maximum likelihood

estimators of the parameters were determined as $\hat{k}$ = 6.43 and $\hat{v}$ = 115.5. A

plot of the probability density function $f_R(r)$ that can be realized with test

option T_0 is given in Figure 6. A Chi-square goodness-of-fit test, in which

$$f_R(r) \times 10^2$$

$$T_0 : \bar{R} = 107.53$$
$$\gamma_R = 0.1829$$

R, STRENGTH – PERCENT OF ANALYST'S PREDICTION
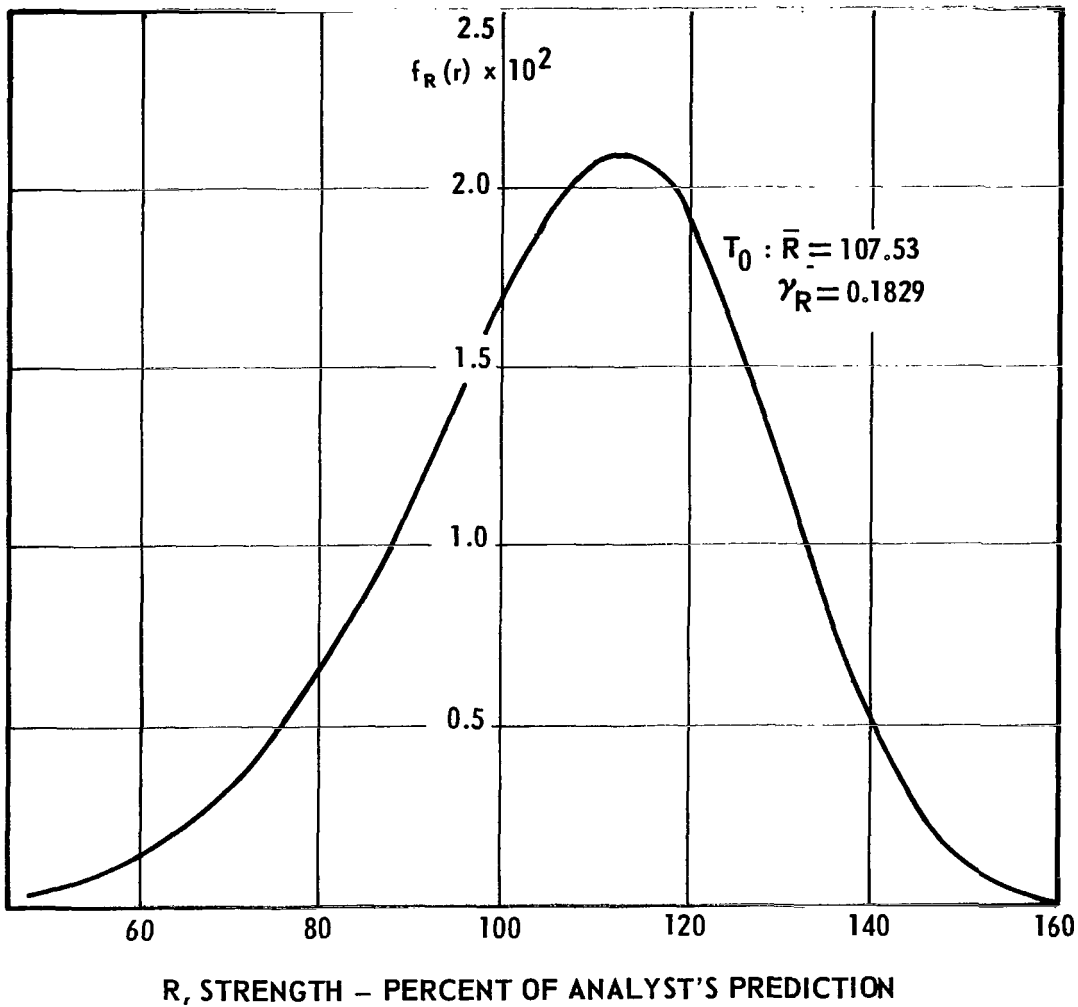
Figure 6. Strength Distribution for No–Test Option.

only the uncensored data points could be used, showed that the model could be

accepted at the 11-percent significance level. Six intervals of equal probability

were used in the test. A computer program from Hanagud [87] was used for

the Chi-square test.

4.2.3 Strength Distribution for Proof-Test Option, T_2

A very common method of assuring the reliability of pressure vessels

and tanks in both the aerospace and civil engineering fields is the proof test. In this method of testing, before each production pressure vessel or tank is placed in service, it is pressure tested in the laboratory to a pressure greater than that expected to be encountered in service. The technique has seldom been used for structures other than containers, but there are notable exceptions. Buntin [80] described a proof-test program for the F-111 aircraft. The main purpose of the program was to detect flaws or crack-like defects in certain steel parts in the aircraft, which could cause failure in service. Although no such flaws were detected in proof testing of 325 operational aircraft, two premature failures due to other causes did occur. This indicates the ability of the proof test to detect and eliminate defective structures of all types from the operational population of structures.

An idealized representation of the effect of proof testing on the strength distribution is a straightforward application of truncated distributions as discussed by Benjamin and Cornell [2]. If equation (4.2) is the strength distribution of the population of structures that have not been tested, then the distribution for proof-tested structures is assumed to be

$$f_R(r) = 0 \quad , \quad r < r_0 \qquad\qquad (4.11)$$

$$f_R(r) = \frac{1}{1 - F_R(r_0)} \frac{k}{v} \left(\frac{r}{v}\right)^{k-1} e^{-\left(\frac{r}{v}\right)^k} \quad , \quad r \geq r_0 \quad ,$$

where $F_R(r_0)$ is the cumulative distribution of equation (4.1) evaluated at

the proof-test level r_0. The resulting distribution was illustrated qualitatively
in Figure 5(c). The distribution is plotted for 60-percent and 80-percent
proof test levels in Figure 7.
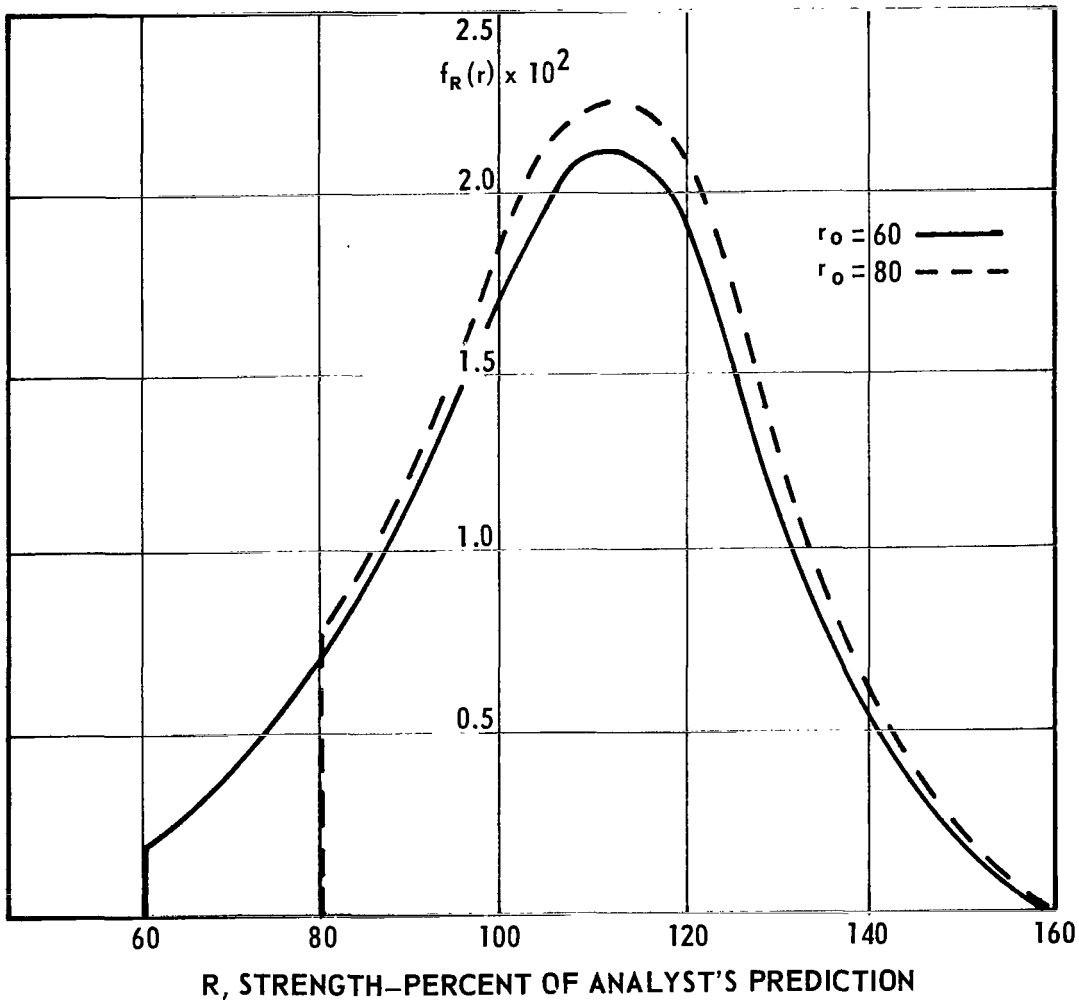
Figure 7. Strength Distributions for Proof-Test Options.

4.3 Safety Measures

Safety measures were defined and discussed in Chapter I. The effect of

the mean safety factor in controlling resistance relative to load was illustrated in Chapter III. In this section attention will be focused on the relationship between the safety index and safety factor, and a particular safety factor will be carefully defined.

In equation (1.5) a characteristic safety factor was defined as

$$\hat{\nu} = \frac{\overline{R} - p\,\sigma_R}{\overline{S} + q\,\sigma_S} \tag{4.12}$$

In equation (1.10), the safety index was expressed as

$$\beta = \frac{\overline{R} - \overline{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{4.13}$$

Suppose that $\overline{S}$, σ_R, and σ_S are known quantities, and a design safety index $\hat{\beta}$ has been prescribed. Then the mean resistance required in the design can be established from equation (4.13) as

$$\overline{R} = \hat{\beta}\sqrt{\sigma_R^2 + \sigma_S^2} + \overline{S} \tag{4.14}$$

If it is now desired to determine the design safety factor $\hat{\nu}$ consistent with the prescribed $\hat{\beta}$, equation (4.14) can be substituted into equation (4.12) to give

$$\hat{\nu} = \frac{\overline{S} + \hat{\beta}\sqrt{\sigma_R^2 + \sigma_S^2} - p\,\sigma_R}{\overline{S} + q\,\sigma_S} \tag{4.15}$$

Equations (4.12) and (4.13) can be used to derive a number of relationships between $\hat{\nu}$ and $\hat{\beta}$ similar to equation (4.15), depending on which parameters are assumed to be known and which are unknown.

4.3.1 A Special Safety Factor Definition

For the subsequent work in this research and for convenient application of the results, it is desirable to use a particular definition of design safety factor. Although the definition to be used here has not previously been explicitly stated, its use has been implied in many situations in civil and aerospace engineering. In fact, in deterministic design approaches the implied use of such a definition is probably the rule rather than the exception in the design of most structures. This definition is

$$\tilde{\nu} = \frac{\tilde{R}}{\tilde{S}} \qquad , \tag{4.16}$$

where $\tilde{R}$ is the strength of the structure calculated by the analyst, $\tilde{S}$ is a so-called limit load, and $\tilde{\nu}$ is the design safety factor. Very often in the past, $\tilde{S}$ has been statistically determined as

$$\tilde{S} = \bar{S} + q\,\sigma_S \qquad , \tag{4.17}$$

where $\bar{S}$ is the mean load, σ_S is the standard deviation, and q is a constant. Much less frequently, $\tilde{R}$ has been statistically defined, and in many cases where this has been done, consideration of factors affecting strength variability has been limited. For example, in many analyses the variance in R has been

assumed to be due only to scatter in material properties. As will be seen in subsequent chapters, scatter in material properties is a relatively minor contributor to the variance in R. A key assumption in this research is that $\tilde{R}$ is **not** defined as $\overline{R} - p\,\sigma_R$. $\tilde{R}$ is defined as the strength predicted by the analyst. The relationship of the deterministic quantity $\tilde{R}$ to the mean value will depend upon the type of testing (test option) contemplated. Consequently, the probability that the strength will be less than $\tilde{R}$ is different for each test option. This is illustrated in Figure 5 where $\tilde{R}$, the analyst's predicted strength, is 100 percent for each test option.

Furthermore, it will often be convenient to use the coefficient of variation of load rather than the standard deviation:

$$\gamma_S = \frac{\sigma_S}{\overline{S}} \quad , \tag{4.18}$$

where γ_S is the coefficient of variation. Equation (4.17) becomes

$$\tilde{S} = \overline{S} + q\,\overline{S}\,\gamma_S = \overline{S}(1 + q\,\gamma_S) \quad . \tag{4.19}$$

Equation (4.16) can then be written as

$$\tilde{\nu} = \frac{\tilde{R}}{\overline{S}(1 + q\,\gamma_S)} = \frac{100}{\overline{S}(1 + q\,\gamma_S)} \quad , \tag{4.20}$$

where $\overline{S}$ is expressed as a percentage of the design ultimate load. Equation (4.20) is the objective of the entire development in this section and will be used

extensively in the computations in subsequent chapters. The design safety factor $\tilde{\nu}$ is illustrated graphically in Figure 8. As has been previously mentioned, the design safety factor establishes the location of R relative to S , and hence in effect, controls the design for a known load distribution. The opposite view-point can be taken. The factor $\tilde{\nu}$ can be considered to establish the permissible loading which can be applied to a given design with strength R (characterized by $\tilde{R}$).

DESIGN SAFETY FACTOR □ $\tilde{\nu} = \dfrac{100}{\bar{S}(1+q\gamma_s)}$

$\bar{S}$, LOAD DISTRIBUTION

R, STRENGTH DISTRIBUTION

$\bar{S}$ $\bar{S}(1+q\gamma_s)$ $\tilde{R}=100$ R AND S
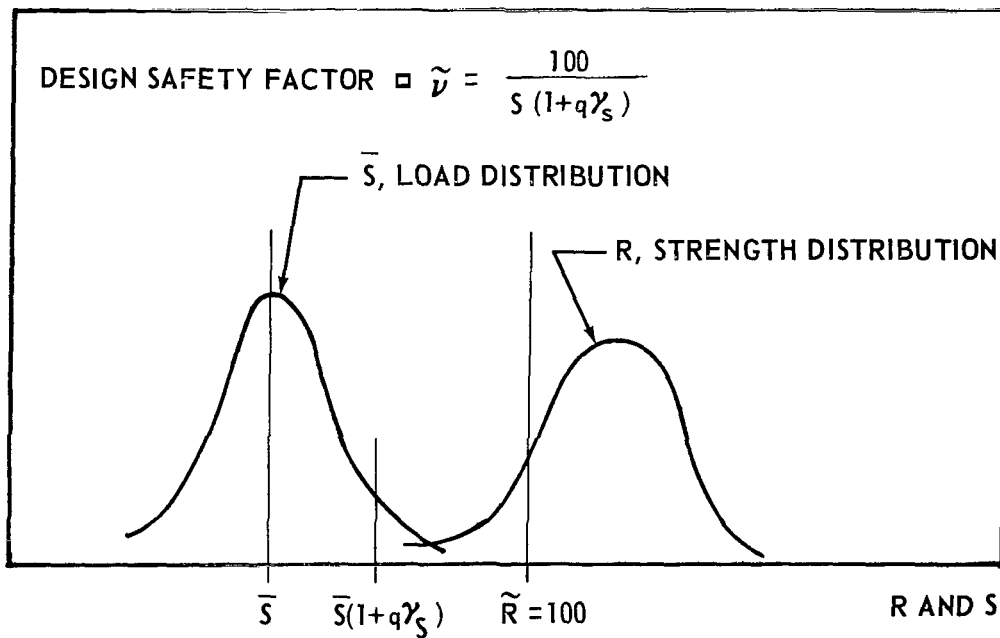
Figure 8. Special Design Safety Factor.

CHAPTER V

ANALYSIS OF SUBJECTIVE EXPERT OPINIONS

Helmer [89] made the following observation:

> While model-building is an extremely systematic expedient to pro-
> mote the understanding and control of our environment, reliance on the use
> of expert judgment, though often unsystematic, is more than an expedient:
> it is an absolute necessity. Expert opinion must be called on whenever it
> becomes necessary to choose among several alternative courses of action
> in the absence of an accepted body of the theoretical knowledge that would
> clearly single out one course as the preferred alternative.

The use of subjective expert opinion is as old as civilization, but systematic

methods have largely been restricted to application with observed data. Helmer's

purpose was the systematic collection of expert opinions through a method called

the Delphi technique. The distinguishing feature of this technique is that a

sequence of questionnaires is sent to the experts, and each questionnaire in the

sequence is based on information from the previous questionnaires. The

respondents are thus made aware of the opinions of other experts and are en-

couraged to make use of these opinions in formulating new responses or revising

previous responses.

Since each questionnaire in the Delphi technique is based on previous

responses, the model for analyzing results must necessarily be loosely defined.

A more rigorous model for analyzing subjective information with respect to pre-

dicting the future was presented by Gordon and Hayward [90] and Enzer [91].

This model, the cross-impact matrix method of forecasting, permits the probability of each event in a forecasted set to be adjusted in view of judgments relating to potential interactions with the other forecasted events. This is accomplished by determining the probability of each event, conditional upon the occurrence of each of the other events. The effect of the events on each other is called cross impact and is often ignored in less rigorous methods.

The foregoing developments were adapted and applied to the social sciences. Nair, Shah, Smith, and Shah [82] used a probabilistic model to study the technical problem of oil spills in the seas. They used expert opinions, gathered through mail questionnaires and personal interviews, as input to the model. The investigators were able to construct a probabilistic model of spills with respect to size, cause, and location.

5.1 Bayesian Viewpoint of Subjective Information

An overall appraisal of the Bayesian viewpoint in statistics was eloquently presented by Savage [84]. McGee [3], Papoulis [92], and Zellner [93] gave excellent presentations on the adaptation of these concepts to probability analysis. Tribus [1] related the Bayesian viewpoint to a broader spectrum of ideas, including the rational description of problems and use in design. A complete decision methodology based on the Bayesian viewpoint was constructed by Pratt, Raiffa, and Schlaifer [21]. An orientation of Bayesian statistics to decisions in engineering problems was given by Benjamin and Cornell [2].

The Bayesian viewpoint originates in the concept of probability as an

individual's degree of belief rather than the traditional concept that probability must be interpreted as a relative frequency of occurrence. Thus, probability is a result of the encoding of the decision-maker's personal belief into a plausibility (probability) measure. This is accomplished by assigning a real number between zero and unity to the event in question, which is then taken as a measure of the relative likelihood that the event will occur. An assignment of zero represents no likelihood of occurrence, and unity represents certainty of occurrence. An assignment of 0.5 represents an equal likelihood of occurrence and nonoccurrence. Note that if the individual has no information available to him other than a measured relative frequency of occurrence in an experiment, it would be unreasonable for him to assign a probability other than the measured relative frequency. In this sense the "frequentist" concept of probability is a special case of the Bayesian concept.

The Bayesian concept of probability leads to the conclusion that all available information should be used in making probability estimates. This information may be objective, subjective, or both. Bayesian statistical methods have been devised [1], [3], [93] which permit the consolidation of both types of information into probability estimates. An application of such methods is used in Chapter VI.

The real strength of the Bayesian viewpoint lies in its applicability to decision methodology. Since a probability assignment represents the individual's degree of belief based on all information available to him, the implication is that he should be willing to base his decisions on such probability assignments. On

the strength of this implication, a formal decision procedure which permits the
use of Bayesian statistical methods in statistical decision theory has been devel-
oped [21], which makes consistent use of the decision-maker's probability
assignments. The method is discussed in Chapters III and VII.

5.2 Mail Questionnaires

Given that subjective information is appropriate for use in decision
methodology, the questions arise as to how such information is best obtained
and formulated in a manner suitable for use in decisions. In the present research
there is a need to determine a strength distribution for test option T_3 for which
no observed data are available. The existing state-of-the-art in obtaining such
a distribution would be to guess or to consult with experts and then guess. The
objective in this chapter is to develop a methodology to systematically obtain
unbiased subjective expert opinions, statistically analyze these opinions, and
obtain probability distributions for strength to be used in applications in later
chapters.

Two questionnaires (Appendix B) were devised to elicit the opinions of
structural analysis and testing experts about strength distributions. Question-
naire A was sent to approximately 55 supervisory and technical management
personnel in industry and government, known by the author to have a background
pertinent to the subject matter. This questionnaire was used to test the method
of acquiring opinions, the probabilistic models, and the ability of experts to
provide the type of information needed. It contained questions leading to strength

distributions of aerospace structures in a typical development program before
test results are available. The experts' opinions were compared with some
available observed data to determine the validity of results.

After preliminary analysis of Questionnaire A results, Questionnaire B
was sent to approximately 30 respondents of Questionnaire A. To assure
validity of results, Questionnaire B was identical in format to Questionnaire A
but contained questions leading to a strength distribution for structural assemblies
developed through an extensive model and small component development test
program, test option T_3. No observed data are available for this case, and the
subjective expert opinions are the only source of information for obtaining the
needed strength distribution.

An attempt was made in sending out the questionnaires to follow techniques
which had proved to elicit a high percentage of responses. Such techniques are
discussed by Bauer and Meissner [94], Campbell and Katona [95], Jeanne and
Gullahorn [96], and Parten [97]. Hints provided by these authors include careful
and precise construction of questions, minimization of writing required of
respondents, appearance and neatness of the questionnaire, transmittal by
personal letter on prestigious letterhead, enclosure of self-addressed stamped
envelope, and pretesting of the questionnaire. Pretesting was accomplished by
asking several faculty members to answer and comment on the questionnaire
and then making changes based on the comments.

Considering that the questionnaires were lengthy and that no tangible
reward was offered for completing them, the response can be considered very

good. Thirty-nine of 55 copies of Questionnaire A were completed, and 19 of 30

copies of Questionnaire B were completed.

5.2.1 Verification of Method — Questionnaire A

Questionnaire A was so devised that two different approaches to calcu-

lating a strength distribution from the responses could be used. For the situation

described in the questionnaire, observed data were available from the Saturn V

launch vehicle structures for direct comparison with the questionnaire results.

A favorable comparison with the observed data would indicate that the expert

opinions and the approach used were valid.

The first approach utilizes only the response in Table 1 of the question-

naire, which gives the experts' overall views of the scatter in test failure loads as

compared with the analysts' predicted failure load. In the second approach, data

in Tables 2 through 10 of the questionnaire are used to construct a counterpart to

Table 1, which gives the same information as Table 1, but is constructed by use

of a probabilistic model from the various causes which contribute to the scatter.

The approach yielding results which compare more favorably with observed data

can then be used to determine strength distributions for other circumstances

when observed data are not available.

Similar to the construction of the counterpart to Table 1, a counterpart

to Table 3 could be constructed from the data in Tables 11 through 19 of the

questionnaire to check consistency.

Certain events related to the various entries were labeled in accomplishing

the questionnaire analysis. Let I_i denote the event that a structural failure

occurs in load increment i, where $i = 1, 2 \ldots 10$; that is, I_1 is the event that a structural failure occurs at less than 60 percent of the analysts' predicted failure load, I_2 is the event that a structural failure occurs between 60 and 70 percent of the analysts' predicted failure load, $\ldots I_{10}$ is the event that a structural failure occurs at greater than 140 percent of the analysts' predicted failure load.

$I_1 \sim\ <$ 60 percent

$I_6 \sim$ 100 percent – 110 percent

$I_2 \sim$ 60 percent – 70 percent

$I_7 \sim$ 110 percent – 120 percent

$I_3 \sim$ 70 percent – 80 percent

$I_8 \sim$ 120 percent – 130 percent

$I_4 \sim$ 80 percent – 90 percent

$I_9 \sim$ 130 percent – 140 percent

$I_5 \sim$ 90 percent – 100 percent

$I_{10} \sim\ >$ 140 percent.

Since failure of a given structure occurs in one and only one of these load increments, the I_i are always assumed to be a set of mutually exclusive and collectively exhaustive events.

The event that a particular cause is the reason for a difference between predicted and actual strengths is denoted C_j, where $j = 1, 2, \ldots 8$. For example, C_1 is the event that the analysts' mathematical model is a poor representation of the actual structure. All of the causes are listed in Table 2 of Questionnaires A and B, Appendix B.

D_k denotes the event that a particular discrepancy between the analysts' mathematical model and the actual structure is the reason for failure, where $k = 1, 2 \ldots 8$. The discrepancies are listed in Table 11 of the questionnaires, Appendix B. Note that the discrepancies are actually special cases or subcases

of cause number 1 in Table 2; that is, it is assumed that the eight discrepancies are the different reasons that the analysts' mathematical model may be a poor representation of the actual structure.

The first step in analysis of the results consists of combining the entries of the n respondents. This is accomplished by simply determining the mean response for each entry in each table. Let E represent any one of the table entries. Let E_p represent the entry of respondent p $(p = 1, 2 \ldots n)$. The value of the entry to be used in all further analysis is

$$\overline{E} = \frac{E_1 + E_2 + \ldots + E_n}{n} = \frac{\sum_{p=1}^{n} E_p}{n} , \qquad (5.1)$$

where $\overline{E}$ is the mean response.

5.2.1.1 Probabilistic Models. Table 1 in the questionnaire represents the experts' overall assessment of the strength distribution of the population of typical aerospace structural assemblies which have not been structurally tested. This distribution can be compared directly with the distribution determined for untested structures from Saturn V data in Figure 6 of Chapter IV. This comparison will be made later in this chapter along with a comparison with results from probabilistic models.

Basic probability concepts as discussed in standard textbooks such as Benjamin and Cornell [2] and Hines and Montgomery [98] are used in constructing probabilistic models for analyzing questionnaire results. Three counterparts

to Table 1 of the questionnaire are constructed from the data in Tables 2 through 10 through the use of conditional probabilities. In this context I_i, C_j, and D_k are interpreted as events. The symbol $P[A]$ stands for the probability that the event A will occur. The symbol $P[A|B]$ stands for the probability that A will occur given that B has occurred. $A \cap B$ (the intersection of A and B) is the event that both events A and B occur, and $A \cup B$ (the union of A and B) is the event that either event A or B occurs.

$P[A|B]$ is called the conditional probability of A given B and is defined [2] as

$$P[A|B] = \frac{P[A \cap B]}{P[B]} \tag{5.2}$$

This can be rewritten to give

$$P[A \cap B] = P[A|B] \, P[B] \tag{5.2a}$$

Note that questionnaire A is devised so that an entry in Table 2 can be interpreted as $P[C_j]$ since frequency of occurrence can be interpreted as a probability. Further, an entry in Tables 3 through 10 can be interpreted as $P[I_i|C_j]$ since each entry represents the probability of a test failure in load increment i conditional upon a particular cause j.

Then from equation (5.2),

$$P[I_i \cap C_j] = P[I_i|C_j] \, P[C_j] \quad ; \tag{5.3}$$

that is, the unconditional probability that a failure will occur in a particular load increment due to a particular cause is given by (5.3). An interpretation of this is that $P[I_i \cap C_j]$ represents the probability that a failure will occur in increment i and that C_j will be the reason for any difference between the actual and predicted failure loads.

Three separate probabilistic models will be developed for determining the strength distribution by various methods of combining the eight causes of differences between actual and predicted failure loads. These models will be called X, Y, and Z, and the counterparts to Table 1 of the Questionnaire A constructed from these models will be shown in Tables 2, 3, and 4 in this chapter.

5.2.1.2 <u>Model X.</u> In Model X the causes of differences between actual and predicted failure loads C_j are assumed to be a set of mutually exclusive and collectively exhaustive events. This implies that any given difference between actual and predicted failure loads has only one cause, and by an axiom of probability,

$$P[C_1] + P[C_2] + \ldots + P[C_8] = \sum_{j\,1}^{8} P[C_j] = 1 \quad . \quad\quad (5.4)$$

Since the mean responses to Questionnaire A, Table 2, do not conform to this axiom, the responses must be normalized. It should be noted that respondents were asked to answer based on the premise that causes could occur together, and this normalization violates that premise. It cannot be proved that the respondents would not have answered differently if they had been told that the causes were

mutually exclusive. Normalization is accomplished by taking

$$P[C_j] = \frac{P[C_{jr}]}{\sum\limits_{j=1}^{8} P[C_{jr}]} \quad , \tag{5.5}$$

where $P[C_{jr}]$ is the mean response opposite cause j (Table 2 in Questionnaire A). This normalization is illustrated by the Venn diagrams of Figure 9.

Since no two causes occur together in general, they do not occur together within a given failure load increment i. Therefore, the event $I_i \cap C_j$ excludes any other event $I_i \cap C_\ell$, $\ell \neq j$. This means that the events $I_i \cap C_j$ and $I_i \cap C_\ell$ are mutually exclusive and by an axiom of probability [2],

$$P[(I_i \cap C_1) \cup (I_i \cap C_2) \cup \ldots \cup (I_i \cap C_8)] = \sum\limits_{j=1}^{8} P[I_i \cap C_j] = P[I_i]_{1X} \tag{5.6}$$

where $P[I_i]_{1X}$ denotes the probability of failure in increment i for Model X. In words, the probability of the occurrence of failure in a particular increment, i, due to all causes is the sum of the intersection of I_i with each of the causes [as calculated from equation (5.3)]. But the probability of occurrence of I_i due to all causes is precisely what was requested of respondents in Table 1 of the questionnaire. Thus, it is clear that Table 2 can be constructed through equations (5.3), (5.5) and (5.6), which should compare favorably with Table 1 of the questionnaire and the observed data to validate the model and the experts'
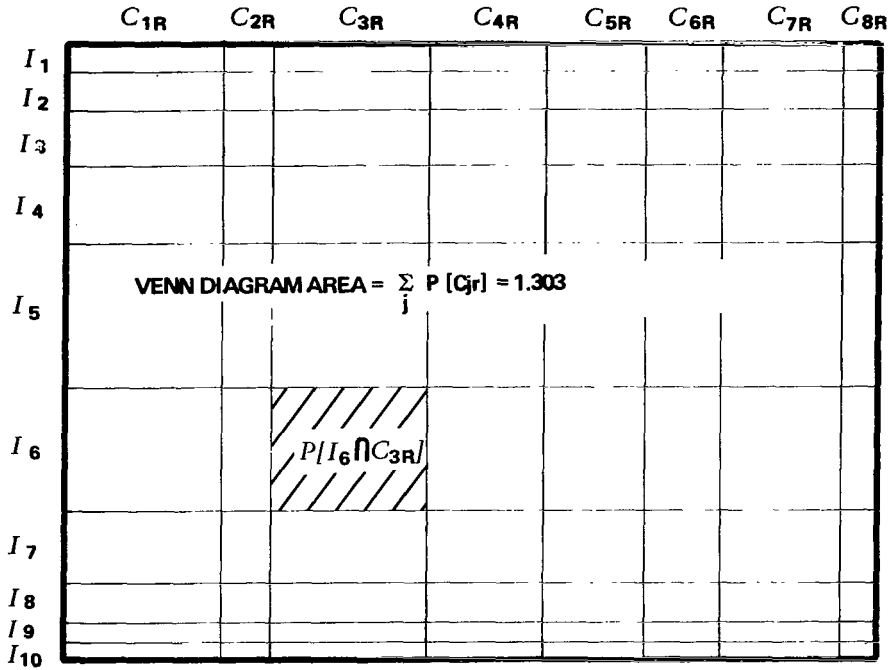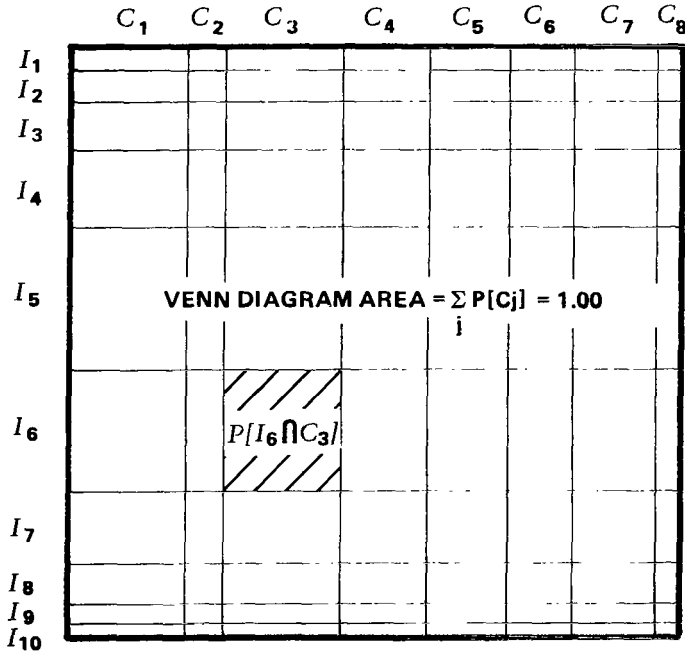
Figure 9. Normalization of Venn Diagram for Model X.

responses in Tables 2 through 10, Questionnaire A.

An example may be instructive. Suppose it is desired to calculate the

entry for column (6) in Table 2 to compare with Table 1 in Questionnaire A.

The entries in column (6) $(P[I_6 | C_j])$ of each of the Tables 3 through 10 in the

questionnaire would be multiplied by the corresponding frequency of occurrence

$(P[C_j])$ from normalized entries in Table 2 of the questionnaire according to

equation (5.3). The resulting products are then summed according to equation

(5.6) to obtain the entry, $P[I_6]_{1X}$, in column (6) for Table 2.

These computations have been carried out for each load increment i and

the results are shown in Table 2.

Table 2. Failure Probabilities for Model X

	(1)	(2)	(3)	(4)	(5)	I_i (6)	(7)	(8)	(9)	(10)
	<60	60– 70	70– 80	80– 90	90– 100	100– 110	110– 120	120– 130	130– 140	>140
$P[I_i]_{1X}$	0.040	0.054	0.084	0.134	0.242	0.206	0.120	0.068	0.034	0.018

These data will be compared with the Saturn V observed data along with the other

models later in this chapter.

5.2.1.3 Model Y. In model X the causes were assumed to be mutually

exclusive and collectively exhaustive. It may be possible that more than one

cause could be a contributor to a given difference in predicted and actual failure

load. This eventuality was considered in constructing the revised Table 2 in

Questionnaire A by permitting the responses to total 100 percent or more. If

two causes j and k are not mutually exclusive, then they may have a joint

occurrence $C_j \cap C_k$. In Model Y the probability of this joint occurrence is considered in each of the failure load increments i. A Venn diagram depicting this condition is shown in Figure 10, where only two causes are shown for clarity. The intersections of these causes with the mutually exclusive and collectively exhaustive events I_i are also shown. Now consider the relationships existing within a given load increment i. This is illustrated in Figure 11, where only two causes are shown for clarity. In the following derivation it is assumed that all causes and the intersection of any two causes may exist but that intersections of three or more causes are negligible.

From the Venn diagram, Figure 11, and generalization to eight causes,

$$P[I_i] = \sum_{j=1}^{8} P[C_j \cap I_i] - \frac{1}{2} \sum_{\substack{j=1 \\ j \neq k}}^{8} \sum_{k=1}^{8} P[C_j \cap C_k \cap I_i] \qquad (5.7)$$

If the causes are assumed to be independent within i,

$$P[C_j \cap C_k \cap I_i] = P[C_j \cap I_i] \, P[C_k \cap I_i] \qquad (5.8)$$

From the definition of conditional probability [2],

$$P[C_j \cap I_i] = P[I_i | C_j] \, P[C_j] \qquad (5.9)$$

From (5.8) and (5.9),

$$P[C_j \cap C_k \cap I_i] = P[I_i | C_j] \, P[C_j] \, P[I_i | C_k] \, P[C_k] \qquad (5.10)$$

Substituting (5.9) and (5.10) into (5.7),

$$P[I_i] = \sum_{j=1}^{8} P[I_i|C_j]P[C_j] - \frac{1}{2} \sum_{\substack{j=1 \\ j \neq k}}^{8} \sum_{k=1}^{8} P[I_i|C_j] \, P[C_i] \, P[I_i|C_k] \, P[C_k] \quad .$$

$$(5.11)$$

Since I_i are mutually exclusive and collectively exhaustive events,

$$\sum_{i=1}^{10} P[I_i] = 1 \quad .$$

$$(5.12)$$

Now the quantities $P[C_j]$ are precisely the quantities represented by entries in Table 2 of Questionnaire A, and the quantities $P[I_i|C_j]$ are represented by entries in questionnaire Tables 3 through 10. If the mean responses to Questionnaire A are substituted into equation (5.11) and the summation of $P[I_i]$ is taken as indicated by equation (5.12), then

$$\sum_{i=1}^{10} P[I_i] = 1.207 \neq 1 \quad .$$

$$(5.13)$$
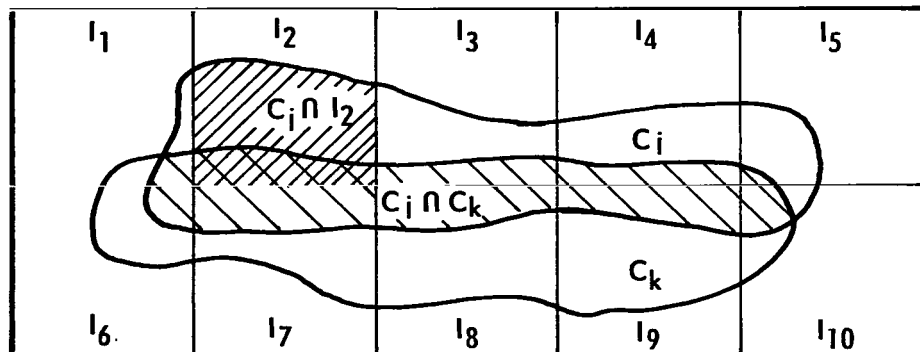
Figure 10. Venn Diagram for Intersecting Causes.
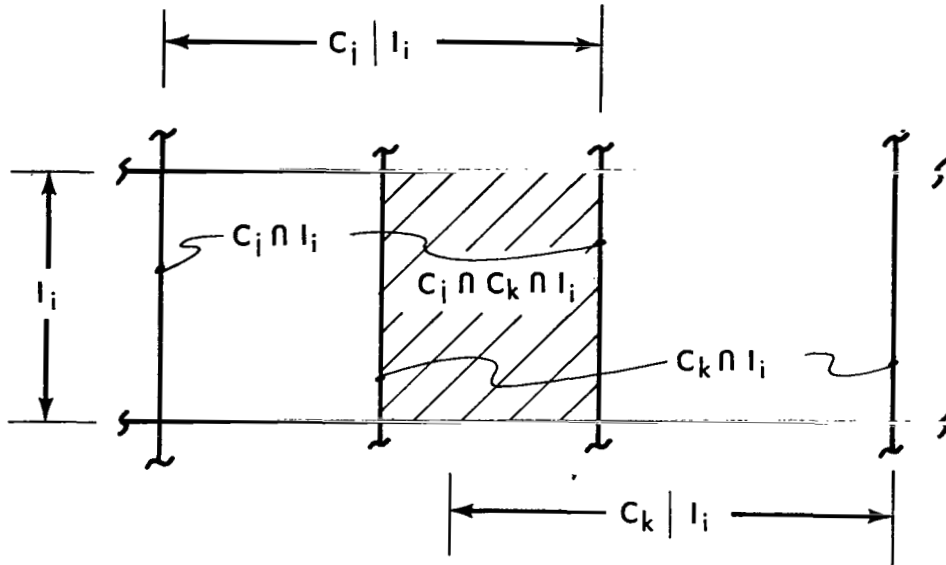
Figure 11. Intersection of Causes with Load Increment.

This result indicates that the assumption of independence of causes is not compatible with the questionnaire responses. Note that the assumption of independence was involved in writing equation (5.8). Assume that a factor α exists which accounts for the dependence among causes such that

$$P[C_j \cap C_k \cap I_i] = \alpha\, P[C_j \cap I_i]\, P[C_k \cap I_i] \quad . \tag{5.14}$$

Then, in place of equation (5.11) there results

$$P[I_i] = \sum_{j=1}^{8} P[I_i|C_j]\, P[C_j] - \frac{\alpha}{2} \sum_{\substack{j=1 \\ j\neq k}}^{8} \sum_{k=1}^{8} P[I_i|C_j]\, P[C_j]\, P[I_i|C_k]\, P[C_k] \quad . \tag{5.15}$$

Substituting this result into equation (5.12)

$$\sum_{i=1}^{10} \sum_{j=1}^{8} P[I_i|C_j]P[C_j] - \frac{\alpha}{2} \sum_{i=1}^{10} \cdot \sum_{j=1}^{8} \sum_{k=1}^{8} P[I_i|C_j]P[C_j]P[I_i|C_k]P[C_k] = 1 \ ,$$

$$(5.16)$$

from which

$$\alpha = \cfrac{\displaystyle\sum_{i=1}^{10} \sum_{j=1}^{8} P[I_i|C_j] \, P[C_j] - 1}{\frac{1}{2} \displaystyle\sum_{i=1}^{10} \sum_{\substack{j=1 \\ j \ne k}}^{8} \sum_{k=1}^{8} P[I_i|C_j]P[C_j]P[I_i|C_k]P[C_k]} \qquad (5.17)$$

Once α has been determined from equation (5.17) the desired results $P[I_i]$ for model Y can be determined from equation (5.15). These computations have been carried out for each load increment i, using mean responses from Questionnaire A to obtain the results shown in Table 3.

Table 3. Failure Probabilities for Model Y

					I_i					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	<60	60– 70	70– 80	80– 90	90– 100	100– 110	110– 120	120– 130	130– 140	>140
$P[I_i]_{1Y}$	0.049	0.065	0.095	0.140	0.183	0.201	0.125	0.079	0.042	0.023

These data will be compared with Saturn V objective data later in this chapter.

5.2.1.4 Model Z. A third probabilistic model was constructed, for which the development and discussion is identical to that for Model Y through equation (5.13). At that point instead of assuming a factor α to account for dependence of causes, the sample space is normalized by scaling both the causes

and intersections by a constant factor λ. Thus, in model X intersections were assumed to be zero, and probabilities of causes only were scaled; in model Y, probabilities of intersections only were scaled, and in model Z, probabilities of both causes and their intersections were scaled.

If model Z is applied to equation (5.11), there results

$$P[I_i] = \lambda \left(\sum_{j=1}^{8} P[I_i|C_j]P[C_j] - \frac{1}{2} \sum_{\substack{j=1 \\ j \neq k}}^{8} \sum_{k=1}^{8} P[I_i|C_j]P[C_j]P[I_i|C_k]P[C_k] \right)$$

(5.18)

$$\equiv \lambda \left(A_i - B_i \right) \quad .$$

Substituting this result into (5.12),

$$\sum_{i=1}^{10} P[I_i] = \lambda \left(\sum_{i=1}^{10} A_i - \sum_{i=1}^{10} B_i \right) \equiv \lambda (A - B) = 1 \quad . \qquad (5.19)$$

Solving equation (5.19) for the value of λ required to normalize the sample space,

$$\lambda = \frac{1}{A - B} \quad . \qquad (5.20)$$

Once λ has been obtained, $P[I_i]$ can be calculated from equation (5.18). These computations have been carried out for Questionnaire A, and the results shown in Table 4 were obtained.

5.2.1.5 <u>Comparison with Observed Data.</u> Three probabilistic models

Table 4. Failure Probabilities for Model Z

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	<60	60– 70	70– 80	80– 90	90– 100	100– 110	110– 120	.120– 130	130– 140	140– 150
$P[I_i]_{1Z}$	0.042	0.057	0.087	0.136	0.226	0.204	0.121	0.071	0.036	0.019

I_i

of the strength distribution for untested structures have been constructed from the experts' responses to Questionnaire A in Tables 2 through 10. In addition, the experts' overall assessment of what the strength distribution should be is given directly by the response in Table 1 of Questionnaire A. In this section, the resulting four strength distributions will be compared with the strength distribution from the Saturn V observed data shown in Figure 6, Chapter IV. This will accomplished by a Chi-square test comparison of each of the four strength distributions with the observed data and a visual comparison of CDF's.

A summary of how the Chi-square test is being used here and in Chapter IV may be helpful. First, in Chapter IV, a two-parameter Weibull model is hypothesized to represent the Saturn V observed data, and the parameters of the distribution are calculated by the maximum likelihood method. A Chi-square test is then made by comparing the Saturn V observed data with the hypothesized model. Next, in this chapter, the same Weibull model is hypothesized to represent the data from one of the models of subjective expert opinions, and the hypothesis is checked by the Chi-square test.

For the Chi-square test, six load increments with a minimum of five expected failures in each increment are considered. The test will give a

measure of the difference in the number of structures expected to fail in each

load increment from the distribution determined by observed data versus the

number expected to fail in each increment from the subjective data. Let S_i be

the number of structures from the total of 50 structures expected to fail in

increment i from the subjective data and O_i be the corresponding number from

the distribution calculated from the observed data. The D_1 statistic is defined

[2] as

$$D_1 = \sum_{i=1}^{k} \frac{(S_i - O_i)^2}{O_i} \quad , \quad k = 6 \quad . \tag{5.21}$$

This statistic is compared with tabulated values of the Chi-square distribution

[2] to determine the significance level at which the subjective and observed

data agree; that is,

$$P[D_1 \geq \chi^2_{\alpha, k-1}] = \alpha \quad , \tag{5.22}$$

where $\chi^2_{\alpha, k-1}$ is the value of the Chi-square random variable with k-1 degrees

of freedom and cumulative distribution α. The value of α is known as the

significance level and is a measure of the agreement between the hypothesized

model and subjective data.

Calculation of the D_1 statistic for each of the four subjective models and

determination of the significance level from standard tables (five degrees of

freedom) yielded the results shown in Table 5.

Table 5. Results of Chi-Square Test

	Overall Assessment	Model X	Model Y	Model Z
d_1 Statistic	4.9	14.1	12.8	13.4
Significance Level	42%	1.5%	2.5%	2.0%

This indicates a very good fit for the overall assessment and a mediocre fit for the models.

Cumulative distribution functions are shown in Figure 12, where the Weibull model of observed data from Chapter IV is shown by the solid line, the experts' overall assessment by the dotted line, and an envelope of the three probabilistic models of experts' assessment from causes by the cross-hatched area.

It is evident from these results that the experts' overall assessment is far superior to the distribution determined from probabilistic models based on experts' opinions of causes. To establish the reason for this would require additional questionnaires and analysis of the results. With the information now available it cannot be clearly established whether there is an intrinsic ability of experts to provide a better overall assessment or whether the questionnaire and probabilistic models could be improved to a point where the model results would be as good as the overall results. However, it is known that all of the experts consulted have had occasion to directly observe data applicable to the overall assessment. It is not known how many of these experts have directly observed data applicable to the assessment from causes.
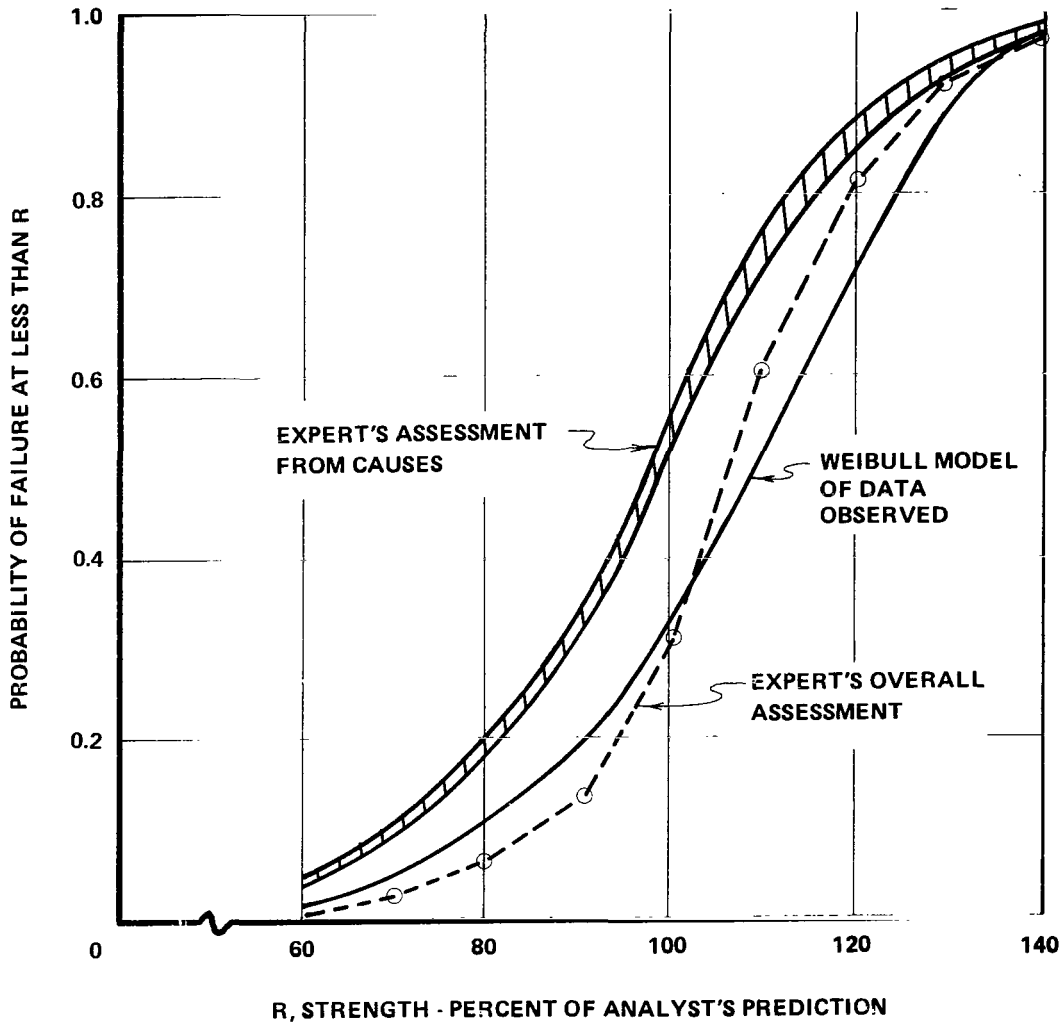
Figure 12. Comparison of Observed and Subjective CDF's.

The simplicity of the overall assessment compared with the assessment

from causes could also have been a factor. The latter required considerable

time and deliberation and possibly a reordering of information from a form

available to the respondent to that required to answer the questionnaire. Some

respondents may have been unwilling to spend sufficient time to do this carefully.

It should be mentioned that, although the model results are not nearly as

good as the overall assessment results, the model result significance level is in a range sometimes accepted by statisticians in the absence of better information. A significance level sometimes accepted is 1 percent, and the model results fall within that criterion.

5.2.2 Strength Distribution for T_3 – Questionnaire B

The primary objective of Questionnaires A and B was to determine a strength distribution for test option T_3, the model test option. The analysis of Questionnaire A results in the previous section demonstrated the ability of experts to predict strength distributions and established that the overall assessment is likely to give better results than the probabilistic models of causes. Therefore, the experts' overall assessment, Table 1 in Questionnaire B, was used to determine a strength distribution for test option T_3. This was accomplished by fitting a two–parameter Weibull model to the data (mean responses) in Table 1, Questionnaire B.

Each percentage point in the mean response was interpreted as one data point, and the resulting data were input into the computer program by Hanagud [87]. Maximum likelihood estimates $\hat{v} = 109.4$ and $\hat{k} = 8.8$ were determined for the parameters of the Weibull distribution,

$$f_R(r) = \frac{k}{v} \left(\frac{r}{v}\right)^{k-1} e^{-\left(\frac{r}{v}\right)^k} \quad . \tag{5.25}$$

A significance level of 60 percent was obtained in a Chi–square goodness–of–fit test between model and data, using 10 increments of equal probability. A

comparison of CDF's of the model and the subjective data is shown in Figure 13.
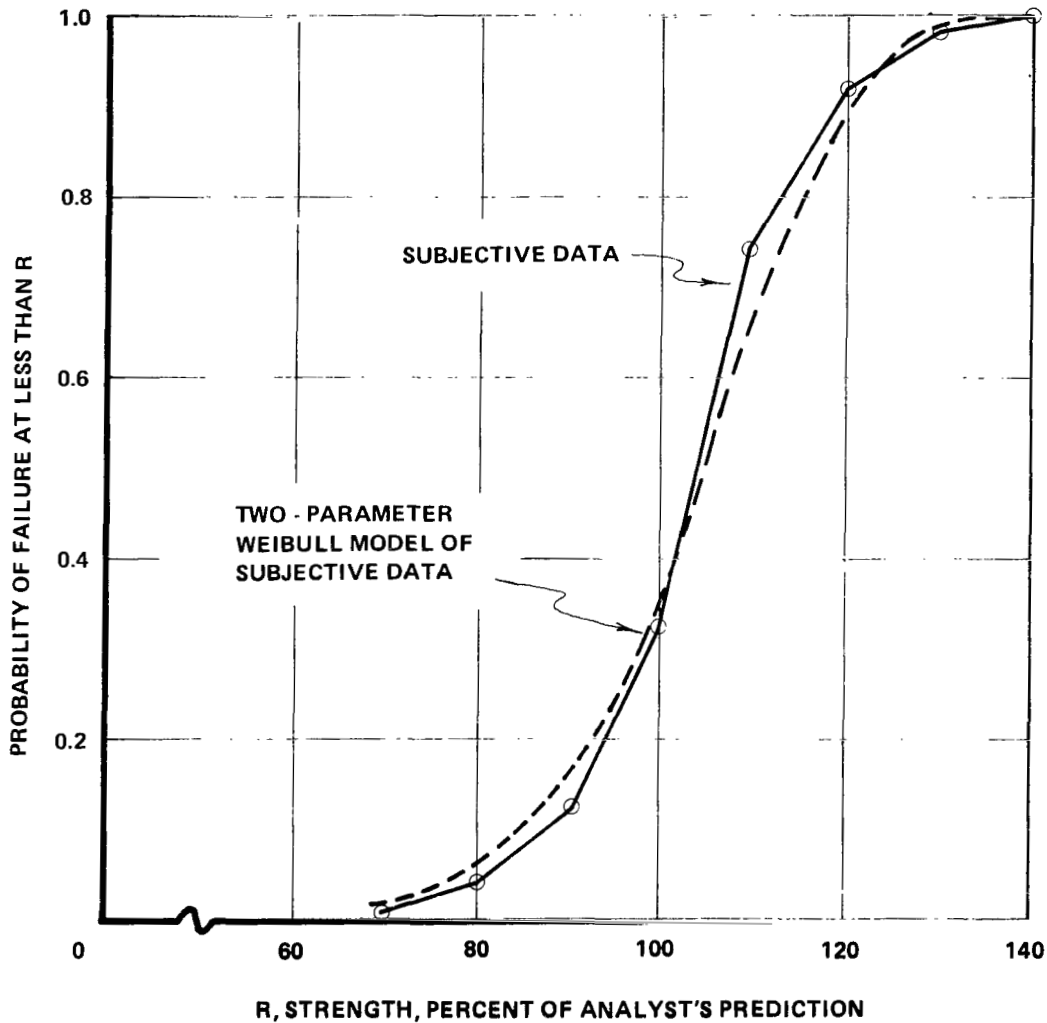
Figure 13. Comparison of Subjective Data and Weibull Model.

In Figure 14 the strength distribution that can be realized by test option T_3 is compared with the distributions that can be realized by T_0 and T_1. The comparison is intuitively appealing. For the Model Test Option, T_3, the frequencies of both grossly overstrength and grossly understrength structures are reduced as compared with No-Test Option, T_0, resulting in a distribution sharply
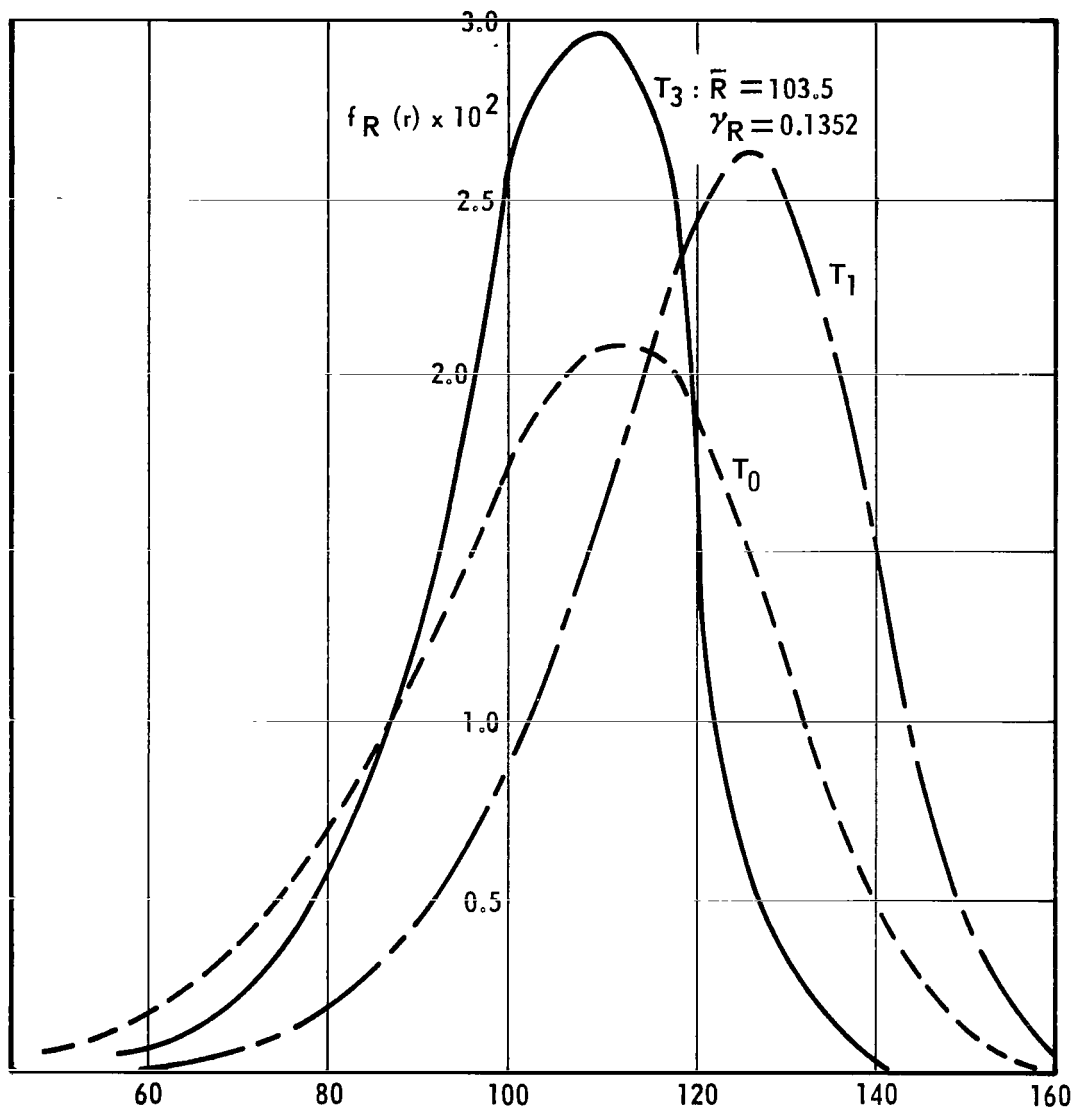
Figure 14. Comparison of Strength Distributions.

peaked near the analyst's predicted value. Note that this is in contrast to the

Standard Test Option, T_1, for which it was assumed that design changes are made

only in understrength structures, and overstrength structures remain a part of

the population. The strength distribution given by equation (5.23) and plotted in Figure 14 will be used for T_3 in all further analysis.

CHAPTER VI

A BAYESIAN APPROACH TO UPDATING A STRENGTH DISTRIBUTION

In this chapter, Bayesian statistics will be used to determine a strength distribution for test option T_1, the standard test procedure. Bayesian statistics provides a means of updating statistical distributions as new information becomes available. For the present analysis, test data accumulated during the laboratory structural testing of Saturn V structural assemblies are regarded as prior information, and the data obtained through flight experience are regarded as new information used to update the distribution determined from the prior information.

The Bayesian approach recognizes the validity of subjective information for use in probability analysis, and such information is used in determining the strength distribution for test option T_1. However, the effect of the subjective data on the distribution is relatively minor, and the resulting distribution can be regarded as primarily based on observed data.

The method to be used will be developed through discussions of Bayes' theorem, the use of Bayes' theorem to combine new data with prior information, and the updating of the scale parameter of the Weibull distribution. Next, the method is applied to the Saturn V test and flight data to determine a two-parameter Weibull distribution for test option T_1. The final distribution which can be realized by T_1 is obtained from the Weibull distribution by using a

Bayesian analysis to account for uncertainty in the scale parameter.

6.1 Bayes' Theorem

A conditional probability of the event A given that the event B_j has occurred is defined as the ratio of the probability of the intersection of events A and B_j to the probability of event B_j and is denoted by $P[A|B_j]$.

$$P[A|B_j] = \frac{P[A \cap B_j]}{P[B_j]} \qquad , \qquad (6.1)$$

from which

$$P[A \cap B_j] = P[A|B_j] \, P[B_j] \qquad . \qquad (6.2)$$

Similarly,

$$P[B_j \cap A] = P[B_j|A] \, P[A] \qquad \circ \qquad (6.3)$$

Since $P[A \cap B_j] \equiv P[B_j \cap A]$,

$$P[A|B_j] \, P[B_j] = P[B_j|A] \, P[A] \qquad . \qquad (6.4)$$

Now following Hines and Montgomery [98], if B_1, B_2 ... B_n represent a set of collectively exhaustive and mutually exclusive events, then an event A can be written as (Fig. 15)

$$A = (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n) \qquad . \qquad (6.5)$$
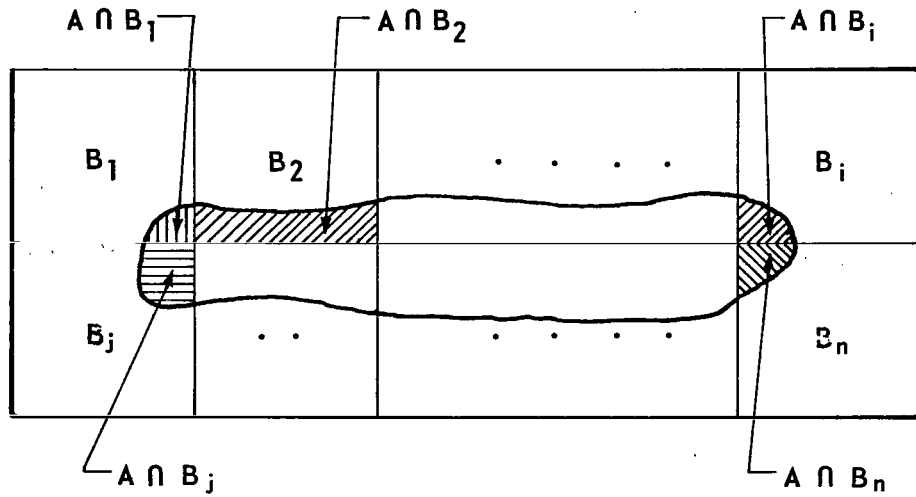
Figure 15. Event A in Terms of Events B_i.

$$P[A] = P[A \cap B_1] + P[A \cap B_2] + \ldots + P[A \cap B_n] \qquad (6.5a)$$

$$= \sum_j P[A \cap B_j] \quad .$$

Substituting equation (6.5) into equation (6.4),

$$P[B_j|A] = \frac{P[A|B_j]P[B_j]}{\sum_k P[A \cap B_k]} \qquad (6.6)$$

Then from equations (6.2) and (6.6),

$$P[B_j|A] = \frac{P[A|B_j] \ P[B_j]}{\sum_k P[A|B_k] \ P[B_k]} \qquad (6.7)$$

Equation (6.7) is known as Bayes' theorem and provides the means of incor-

porating new information with prior probability estimates to yield posterior

probabilities.

6.2 Combining of New Data with Prior Information

A continuous version (following Papoulis [92]) of equation (6.7) can be

derived. Returning to equation (6.1), let the event B_j be the event

$\{x < X \leq x + \Delta x\}$ where X is a random variable, and x is a specific value of

X. Then (6.1) becomes

$$P[A | x < X \leq x + \Delta x] \; = \; \frac{P[A \cap x < X \leq x + \Delta x]}{P[x < X \leq x + \Delta x]} \qquad . \qquad (6.8)$$

Now by definition of the CDF,

$$P[x < X \leq x + \Delta x] \; = \; F_X(x + \Delta x) \; - \; F_X(x) \qquad . \qquad (6.9)$$

By definition of the conditional CDF,

$$F_X(x|A) \; = \; \frac{P[X \leq x \cap A]}{P[A]} \qquad\qquad (6.10)$$

and

$$F_X(x_2|A) \; - \; F_X(x_1|A) \; = \; P[x_1 < X \leq x_2 | A] \qquad (6.11)$$

$$= \; \frac{P[A \cap x_1 < X \leq x_2]}{P[A]}$$

Substituting equations (6.9) and (6.11) into equation (6.8),

$$P[A | x < X \leq x + \Delta x] = \frac{[F_X(x + \Delta x | A) - F_X(x | A)] \, P[A]}{F_X(x + \Delta x) - F_X(x)} \qquad (6.12)$$

Now

$$\lim_{\Delta x \to 0} \frac{[F_X(x + \Delta x) - F_X(x)]}{\Delta x} = f_X(x) \qquad (6.13)$$

and

$$\lim_{\Delta x \to 0} \frac{F_X(x + \Delta x | A) - F_X(x | A)}{\Delta x} = f_X(x | A) \qquad (6.14)$$

Now define,

$$P[A | X = x] = \lim_{\Delta x \to 0} P[A | x < X \leq x + \Delta x] \qquad (6.15)$$

Applying equations (6.13), (6.14) and (6.15) to equation (6.12),

$$P[A | X = x] = \frac{f_X(x | A) \, P[A]}{f_X(x)} \quad , \qquad (6.16)$$

or

$$f_X(x | A) = \frac{P[A | X = x] \, f_X(x)}{P[A]} \qquad (6.17)$$

From equation (6.16) we can write,

$$\int_{-\infty}^{\infty} P[A \mid X = x] \, f_X(x) \, dx = \int_{-\infty}^{\infty} f_X(x \mid A) \, P[A] \, dx \equiv P[A] \qquad (6.18)$$

Substituting for P[A] in equation (6.17)

$$f_X(x \mid A) = \frac{P[A \mid X = x] \, f_X(x)}{\int_{-\infty}^{\infty} P[A \mid X = x] \, f_X(x) \, dx} \qquad (6.19)$$

Equation (6.19) is the continuous version of Bayes' theorem. In this expression

let X represent a parameter of the distribution used to model the random

variable of interest and A be a sequence of independent observations of the

random variable. Thus, the theorem states that the distribution of the parameter

X, given the observation A, is proportional to the product of the likelihood

function P[A | X = x], sometimes denoted L[X | A], and the distribution of X

$f_X(x)$ before the observations. The distribution $f_X(x \mid A)$, sometimes denoted

$f_X''(x)$, is called the posterior distribution of X, and $f_X(x)$, sometimes denoted

$f_X'(x)$, is called the prior distribution of X.

Incorporated in the above viewpoint is the idea that the parameter of a

distribution is itself a random variable, a central concept in the Bayesian

approach. This leads to the recognition that the distribution of a random variable

such as a strength distribution, wherein the parameters are considered constant,

does not fully represent the uncertainty involved with the random variable. A

complete representation of the uncertainty must allow for the recognition that the parameters of the distribution are not known with certainty. A method often employed to accomplish this is to use the parameter distribution given by equation (6.19) as a weighting function on the PDF of the random variable and to integrate over the domain of the parameter to obtain a so-called predictive or Bayesian PDF of the random variable. Thus,

$$\tilde{f}_Y(y) = \int_X f_Y(y) \, f_X(x|A) \, dx \qquad , \qquad (6.20)$$

where $f_Y(y)$ is the distribution of the random variable Y, with parameters assumed given, and $\tilde{f}_Y(y)$ is the Bayesian or predictive distribution of Y. The Bayesian distribution can be used to make inferences about as-yet-unobserved data and will include the uncertainty in the parameters as well as the uncertainty represented by $f_Y(y)$.

6.3 Updating the Weibull Distribution

Assume that the strength distribution for test option T_1 is given by the two-parameter Weibull distribution,

$$f_R(r) = \frac{k}{v} \left(\frac{r}{v}\right)^{k-1} e^{-\left(\frac{r}{v}\right)^k} \qquad (6.21)$$

In the case of this strength distribution, the shape of parameter k will be considered constant (a condition which has been observed to be approximately

true for some physical phenomena), and the scale parameter will be recognized

as a random variable. For convenience, in this portion of the analysis the

parameter λ will be used in place of v^{-k}.

The Bayesian distribution of the strength distribution (6.21) will now be

obtained. From Soland [99] (also, see Section 4.2.2.2), the likelihood function

of n observations of r in which $(R_1, R_2 \ldots R_\ell)$ represent observations of

failures and $(R_{\ell+1}, \ldots R_n)$ represent observations of load levels in which no

failure occurred (a censored sample) is

$$L[\lambda | k, R_1, \ldots R_n] = k^\ell \lambda^\ell (R_1 R_2 \ldots R_\ell)^{k-1} \exp\left(-\lambda \sum_{i=1}^{n} R_i^k\right) .$$

$$(6.22)$$

For the present problem, equation (6.19) takes the form,

$$f_\Lambda''(\lambda) = \frac{L(\lambda | k, R_1 \ldots R_n) \, f_\Lambda'(\lambda)}{\int_\lambda L \, f_\Lambda'(\lambda) \, d\lambda} , \qquad (6.23)$$

where f' indicates a prior distribution, and f'' indicates a posterior distribu-

tion. The foregoing discussion emphasizes the fact that a Bayesian updating of

a probability distribution is accomplished through an updating of the parameter(s)

of the distribution. Note that the posterior distribution of the parameter is

proportional to the prior distribution and to the likelihood function. The integral

in the denominator can be interpreted as a normalizing constant.

Under certain restrictive conditions the mathematics indicated in equation (6.23) need not be performed explicitly. It is mathematically convenient to use a distribution for the parameter λ such that the operation indicated by (6.23) results in a posterior distribution of λ of the same form as the prior distribution. Such a distribution of the parameter is called a natural conjugate of the likelihood function.

Such a distribution for λ (a gamma distribution) was displayed by Soland [99]:

$$f_{\Lambda}(\lambda) = \frac{y^{\ell} \lambda^{\ell-1} e^{-\lambda y}}{\Gamma(\ell)} \quad , \quad 0 \leq \lambda < \infty \quad ; \quad 0 < \ell, \text{ and } y < \infty , \quad (6.24)$$

where ℓ and y are the parameters of the distribution, and Γ is the gamma function. Soland applied a Bayesian statistical analysis and decision methods to a problem of sampling electronic components for quality control. In the present research the application is to an updating of strength distributions for aerospace structural assemblies.

If the parameters of the prior distribution $f_{\Lambda}'(\lambda)$ are y' and ℓ', and a sample of the Weibull process yields the likelihood function (6.22), it was shown by Soland [99] that the posterior distribution $f_{\Lambda}''(\lambda)$ has the parameters:

$$y'' = y' + y \quad , \tag{6.25}$$

$$\ell'' = \ell' + \ell \quad ,$$

where

$$y = \sum_{i=1}^{n} R_i^{\,k} \qquad\qquad (6.26)$$

Through the use of the natural conjugate, the mechanics of updating are reduced to the reevaluation of parameters indicated in (6.25) rather than the mathematical manipulations indicated in (6.23).

The Bayesian strength distribution, using (6.20), is

$$\tilde{f}_R(r) = \int_{\lambda=0}^{\infty} f_R(r)\, f_\Lambda(\lambda)\, d\lambda \qquad\qquad (6.27)$$

$$= \int_0^{\infty} \lambda k\, r^{k-1}\, e^{-\lambda r^k}\ \frac{(y)^\ell\, \lambda^{\ell-1}\, e^{-\lambda y}}{\Gamma(\ell)}\ \ d\lambda$$

$$= \int_0^{\infty} \frac{k r^{k-1}\, (y)^\ell}{\Gamma(\ell)}\ \lambda^\ell\, e^{-\lambda(r^k + y)}\ d\lambda$$

Integrating equation (6.27),

$$\tilde{f}_R(r) = \frac{k\, r^{k-1}\, (y)^\ell}{\Gamma(\ell)}\ \frac{\Gamma(\ell+1)}{(r^k + y)^{\ell+1}} = \frac{\ell\, k\, r^{k-1}\, (y)^\ell}{(r^k + y)^{\ell+1}} \qquad\qquad (6.28)$$

This is the Bayesian distribution of strength to be used in the decision process.

In equation (6.28), r is the random variable, k is the shape parameter (assumed constant), and y and ℓ are either prior or posterior parameters of the distribution of λ. Thus, the problem of selecting a strength distribution reduces to obtaining a value for k and determining prior values y' and ℓ' and data-based values y and ℓ from which y'' and ℓ'' can be calculated from equation (6.25) and used in equation (6.28).

6.4 Strength Distribution for T_1

The following step-by-step approach is deemed a reasonable procedure for using Saturn V test and flight data to determine values for k, ℓ'', and y'' for the Standard Test Option T_1.

1. Determine the shape parameter k and prior estimates of y' and ℓ' from static structural test data, with some subjective interpretations of the applicability of certain portions of the data.

2. Determine data-based values of y and ℓ from the flight successes of the Saturn V launch vehicle, using an approximation of the loads experienced during flight.

3. Calculate posterior parameters y'' and ℓ'' from equations (6.25).

6.4.1 Saturn V Test Data as Prior Information

An understanding of how the test option T_1 test procedure affects the test data presented in Table 1, Chapter IV, is required in describing step 1. Let the data in Table 1 be divided into two groups, I and II. Group I consists of structures 27 through 50 which failed or for which the test was terminated at

greater than 100 percent of design ultimate load. Group II consists of structures 1 through 26 which failed to achieve design ultimate load. The Standard Test Option T_1 requires that no changes be made in the structures in group I and that all structures in group II be modified and retested to greater than design ultimate load. The modified and retested group (not shown in Table 1) is designated group III. The flight articles are built from the same specifications as the structures in groups I and III.

Note now that all samples nominally identical to the flight articles have been tested to at least 100 percent of design ultimate load. Nevertheless, it is probable that some of the production flight articles would fail at less than design ultimate load if all flight articles could be tested, possibly because of the variation in material properties, variation (within tolerance) of dimensions, or an occasional manufacturing error not evident in the test article. Questionnaire C, Appendix B, was used to obtain expert opinions on the probable percentage of redesigned structures which would fail in each of three load increments less than design ultimate load. The percentages predicted by the experts are given in Table 1, Questionnaire C, Appendix B. If these percentages are multiplied by the 26 structures in group II and rounded to the nearest integer number of structures, no structures are predicted to fail in increments 70- to 80- and 80- to 90-percent, and two structures are predicted to fail in increment 90- to 100-percent of design ultimate load. These two data points will be as- sumed at the midpoint (95 percent) of the increment and will be designated as

group IV. Note that group IV could be regarded as a remnant of the original group II.

The data base for the prior distribution for test option T_1 will be taken as groups I, III, and IV. The data are tabulated in Table 6. In this chapter decimal fractions rather than percentages are used to avoid computations with extremely large numbers. The computer program by Hanagud [87] was used to estimate parameters of a two-parameter Weibull distribution model for the data in Table 6. The maximum likelihood parameter estimates were $\hat{k} = 9.27$ and $\hat{v} = 1.265$ [see equation (6.21)]. Then

$$\hat{\lambda} = \hat{v}^{-k} = 1.265^{-9.27} = 0.113 \tag{6.29}$$

It will now be shown that this is also the expected value of λ from equation (6.24) using the same data. From equation (6.26),

$$y' = \sum_{i=1}^{n} r_i^{9.27} = 132.3 \quad , \tag{6.30}$$

where the r_i are data in Table 6. Also from Table 6, $\ell' = 15$ is the number of failures. The prior distribution of λ from equation (6.24) is then

$$f_\Lambda'(\lambda) = \frac{(132.3)^{15} (\lambda)^{14} e^{-132.3\lambda}}{\Gamma(15)} = 0.7632 \times 10^{21} \lambda^{14} e^{-132.3\lambda} \tag{6.31}$$

From Benjamin and Cornell [2], the expected value of λ is

Table 6. Saturn V Test Data for Redesigned Structures

Group Number	Structure Number	Failure Load, r_i (Fraction of Design Ultimate Load)	Group Number	Structure Number	Failure Load, r_i (Fraction of Design Ultimate Load)
III	1	1.000+[a]	I	27	1.000
	2	1.000+[a]		28	1.000
	3	1.000+[a]		29	1.000+[a]
	4	1.000+[a]		30	1.000+[a]
	5	1.000+[a]		31	1.000+[a]
	6	1.000+[a]		32	1.000+[a]
	7	1.000+[a]		33	1.000+[a]
	8	1.000+[a]		34	1.000+[a]
	9	1.000+[a]		35	1.000+[a]
	10	1.000+[a]		36	1.007+[a]
	11	1.000+[a]		37	1.007
	12	1.000+[a]		38	1.013
	13	1.000+[a]		39	1.036+[a]
	14	1.000+[a]		40	1.072
	15	1.000+[a]		41	1.072
	16	1.000+[a]		42	1.108
	17	1.000+[a]		42	1.121+[a]
	18	1.000+[a]		44	1.130
	19	1.000+[a]		45	1.150+[a]
	20	1.000+[a]		46	1.157
	21	1.000+[a]		47	1.180
	22	1.000+[a]		48	1.250
	23	1.000+[a]		49	1.313
	24	1.000+[a]		50	1.513
	25	1.000+[a]			
	26	1.000+[a]	IV[b]	51	0.950
				52	0.950

a. Censored sample
b. Subjective data, Questionnaire C

$$E(\lambda) \;=\; \frac{\ell'}{y'} \;=\; \frac{15}{132.3} \;=\; 0.113 \tag{6.32}$$

As expected, this is the same as the maximum likelihood estimate, equation (6.29).

Summarizing, the prior distribution of resistance R is the Weibull distribution, equation (6.11) with parameters $k = 9.27$ (assumed constant) and $v = \lambda^{-1/k} = 1.265$.

$$f_R'(r) \;=\; \frac{9.27}{(1.265)^{9.27}} \, r^{8.27} \, e^{-\left(\frac{r}{1.265}\right)^{9.27}} \tag{6.33}$$

$$= 1.0488 \, r^{8.27} \, e^{-0.113 \, r^{9.27}}$$

The prior distribution of λ is given by equation (6.31). As indicated by equation (6.20), the uncertainty in λ for purposes of decision analysis can be taken into account by determining the Bayesian distribution of r. The prior estimates of k, ℓ, and y are substituted into equation (6.28) to obtain the prior Bayesian distribution,

$$\tilde{f}_R'(r) \;=\; \frac{\ell' k \, r^{k-1} \, (y')^{\ell'}}{(r^k + y')^{\ell'+1}} \;=\; 9.252 \times 10^{33} \; \frac{r^{8.27}}{(r^{9.27} + 132.3)^{16}} \quad . \tag{6.34}$$

6.4.2 Saturn V Flight Success as New Data

Since the method by Soland [98] (to be used to update the distribution) permits censored samples, the Saturn V flight data, which include no failures,

can be used as new information for updating the prior strength distribution which was established from laboratory test data only. There have been 13 successful Saturn V flights, with each flight having 50 structural assemblies. This is a total of 650 assemblies that have been flown without failure. The opinion of experts on the loads experienced by these structures is given in Table 2 of Questionnaire C, Appendix B. The data in this table will be approximated as shown in Table 7 for purposes of analysis.

Table 7. Saturn V Flight Loads

	Flight Load — Percentage of Design Ultimate Load (R_i)						
	<30	35	45	52.5	57.5	62.5	68
Percentage of Structures Experiencing Load	15	7.22	9.22	12.44	20.22	13.78	22.11
Number of Structures Experiencing Load	98	47	60	81	131	90	144

Using these data as new information and noting that the contribution of structures loaded to less than 30 percent will be negligible, the parameters of the posterior distribution of λ can be calculated from equations (6.25) and (6.26):

$$y = \sum_{i=1}^{n} R_i^k = 47(0.35)^{9.27} + 60(0.45)^{9.27} + \dots \qquad (6.35)$$

$$+ 144(0.68)^{9.27} = 6.2 \qquad ,$$

$y' = 132.3$ from equation (6.30),

$\ell = 0$ (no flight failures),

$\ell' = 15$ (15 test failures),

$y'' = y + y' = 6.2 + 132.3 = 138.5$,

$\ell'' = \ell + \ell' = 15$.

Then from equation (6.24),

$$f_\Lambda''(\lambda) = \frac{(138.5)^{15} \lambda^{14} e^{-138.5\lambda}}{\Gamma(15)} = 1.519 \times 10^{21} \lambda^{14} e^{-138.5\lambda} \quad . \tag{6.36}$$

The expected posterior value of λ is

$$E''(\lambda) = \frac{\ell''}{y''} = \frac{15}{138.5} = 0.1083 \quad , \tag{6.37}$$

and the expected value of the scale parameter v is

$$E''(v) = \left[E''(\lambda)\right]^{-\frac{1}{k}} = (0.1083)^{-\frac{1}{9.27}} = 1.271 \tag{6.38}$$

The posterior distribution of r is then given by the Weibull distribution, equation (6.21), with parameters $k = 9.27$, $v = 1.271$:

$$f_R(r) = \frac{9.27}{(1.271)^{9.27}} r^{8.27} e^{-\left(\frac{r}{1.271}\right)^{9.27}} = 1.0038 r^{8.27} e^{-0.1083 r^{9.27}} \quad . \tag{6.39}$$

The posterior Bayesian distribution is, from equations (6.28) and (6.35),

$$\tilde{f}_{R}''(r) = \frac{\ell'' \, k \, r^{k-1} (y'')^{\ell''}}{(r^{k} + y'')^{\ell''+1}} = 1.841 \times 10^{34} \frac{r^{8.27}}{(r^{9.27} + 138.5)^{16}} \quad .$$

$$(6.40)$$

This is tne final strength distribution for test option T_1 to be used in the decision analysis. Differences between the prior distribution, equation (6.33); the Bayesian prior distribution, equation (6.34); the posterior distribution, equation (6.39), and the Bayesian posterior distribution, equation (6.40), are quite small. Some values of the distributions are shown in Table 8 to illustrate this. Plots of the distributions would all appear as shown by the T_1 test option in Figure 16.

The small differences between the Bayesian and non-Bayesian distributions indicate that there is sufficient data to make a reasonably accurate determination of the scale parameter. Even though there were a comparatively large number of flight structures and no failure occurred, the flight data caused only a small change between the prior and posterior distributions because of the low load levels in flight compared with the test loads.

The improvement in strength that can be gained by testing according to test option T_1 is illustrated in Figure 16. The dotted line represents the strength distribution before the test and redesign cycle (derived in Chapter IV), and the solid line is the distribution from equation (6.40) after the test, redesign, and retest cycle.

Table 8. Comparison of Strength Distributions

$r \times 10^2$	$f_R{}'(r)$	$\tilde{f}_R{}'(r)$	$f_R{}''(r)$	$\tilde{f}_R{}''(r)$
20	1.666×10^{-6}	1.743×10^{-6}	1.664×10^{-6}	1.665×10^{-6}
40	5.367×10^{-4}	5.378×10^{-4}	5.136×10^{-4}	5.138×10^{-4}
60	0.01533	0.01536	0.01467	0.01467
80	0.1633	0.1635	0.1564	0.1563
100	0.9366	0.9318	0.9008	0.8949
120	2.5657	2.4973	2.5209	2.4538
140	1.3105	1.3584	1.3993	1.4414
160	0.007504	0.03580	0.01048	0.03473

Prior Distribution: $\quad f_R{}'(r) = 1.0488 \, r^{8.27} \, e^{-0.113 \, r^{9.27}}$

Prior Bayesian Distribution: $\quad \tilde{f}_R{}'(r) = 9.252 \times 10^{33} \dfrac{r^{8.27}}{(r^{9.27} + 132.3)^{16}}$

Posterior Distribution: $\quad f_R{}''(r) = 1.0038 \, r^{8.27} \, e^{-0.1083 \, r^{9.27}}$

Posterior Bayesian Distribution: $\quad \tilde{f}_R{}''(r) = 1.841 \times 10^{34} \dfrac{r^{8.27}}{(r^{9.27} + 138.5)^{16}}$
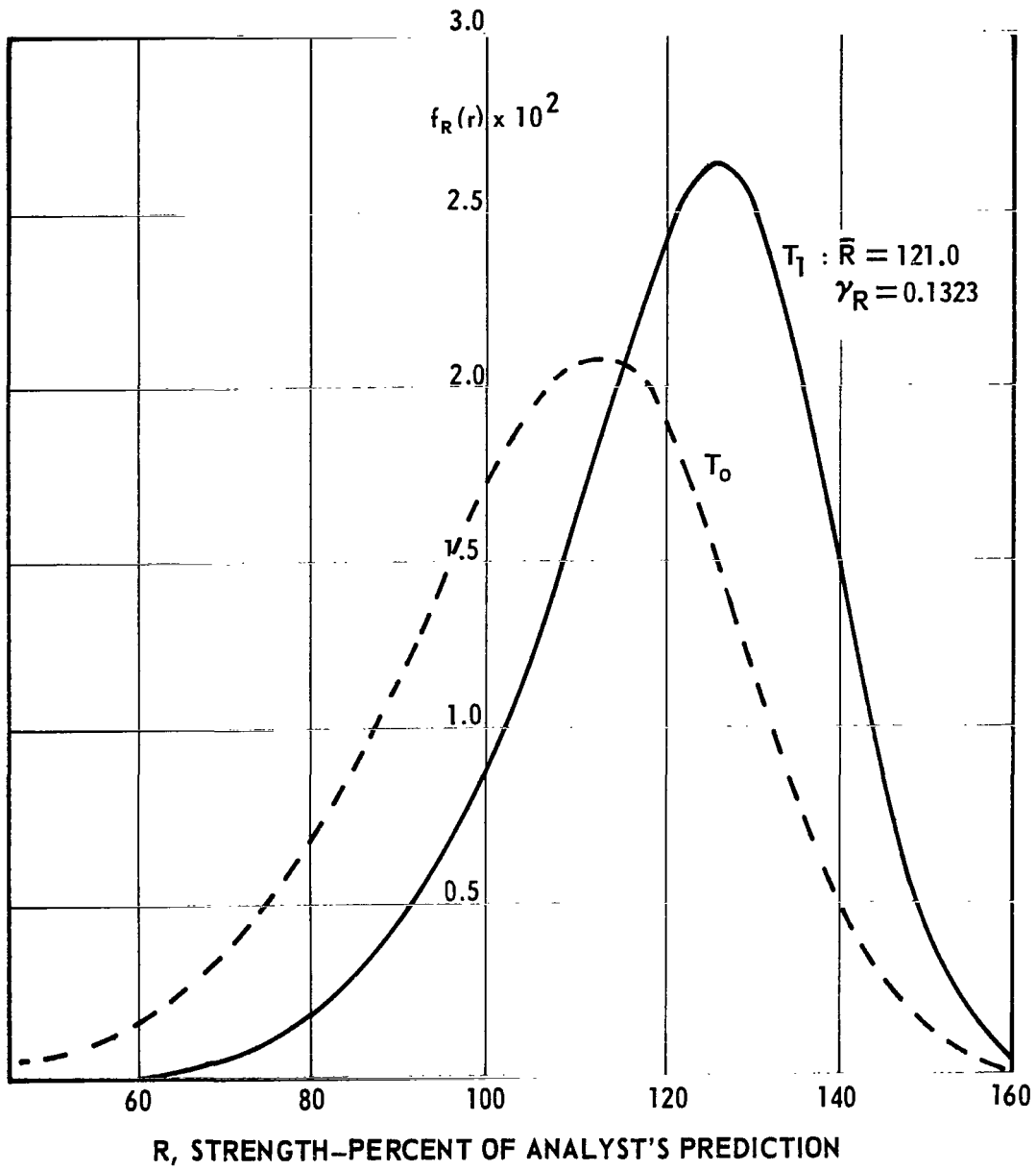
Figure 16. Comparison of Strength Distributions for
No-Test and Standard Test Options.

CHAPTER VII

THE DECISION MODEL

In Chapter III, it was shown how the decision to be made falls within the purview of Bayesian statistical decision theory. The fact that this is true negates the requirement for formal proof here that the decision procedure is valid. The validity has already been established by others in development of the theory. Entire volumes are devoted to the subject by Pratt, Raiffa, and Schlaifer [21], Raiffa [83], Morgan [100], Raiffa and Schlaifer [101], and Schlaifer [102]. A treatment sufficient for the present problem is included in Benjamin and Cornell [2]. An essential feature of the theory is that expected costs of uncertain events, based on probability analysis, are treated in conjunction with known or certain costs in determining a cost optimum decision.

The structuring of the decision tree, accomplished in Chapter III, will not be repeated in this chapter. Instead, a flow-chart description of the process, which may help to demonstrate some points that could not be shown on the decision tree, will be developed. Each element of the flow chart will then be examined separately in more detail. Care has been taken to assure that all assumptions and procedures in the flow chart are consistent with statistical decision theory. This chart should give a good idea of the interrelationships of the problem variables and should help to distinguish what information is

treated as an integral part of the decision model as opposed to information needed as input to the model.

Figure 17 illustrates the decision process. Consistent with statistical decision theory, the process is divided into three major elements: actions, state of nature, and values (or utilities). Generally, actions represent alternatives available to the decision-maker, the state of nature represents factors (usually not known with certainty) which affect the consequences of the actions, and the values are numerical measures of the consequences of action-state combinations.

7.1 Action — The Decision to be Made

The alternative actions or decisions are illustrated in the left portion of Figure 17. In the present problem, an action consists of two elements: (1) selection of a test option or method of testing from among four discrete alternatives and (2) selection of a design safety index from a continuous spectrum encompassing (theoretically) all real numbers. The action is denoted by the symbol $A_j (\beta)$, where j represents the test option, and β represents the safety index. For example, $A_3 (1.5)$ denotes the decision to use test option 3 and a safety index of 1.5.

7.1.1 Selection of Test Option

Clearly, one of the means available to the designer to control structural reliability is the selection of a method of testing. Testing tends to decrease uncertainties and design errors, thereby permitting the designer to more
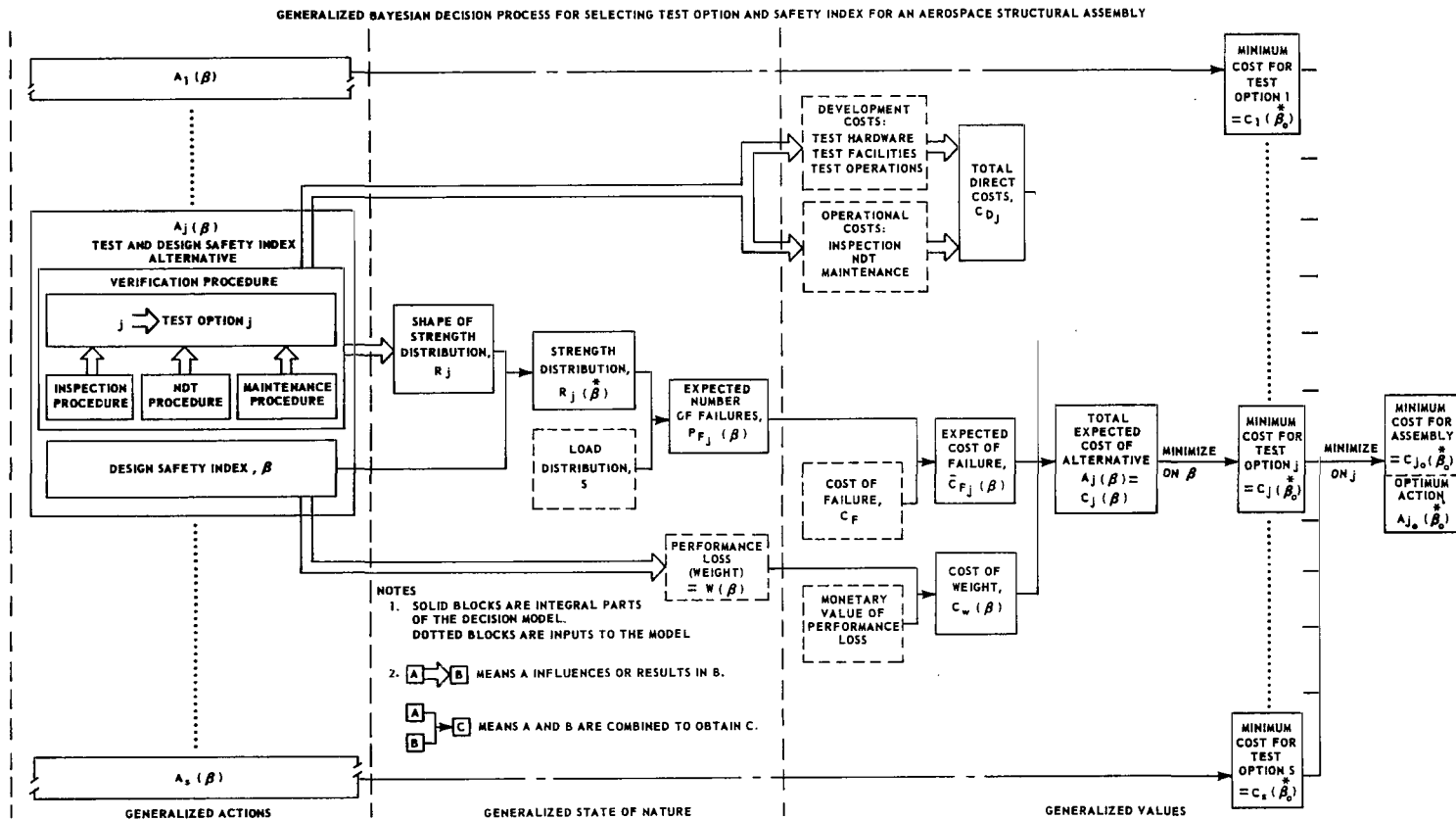
Figure 17. Flow Diagram for Decision Process.

accurately estimated the strength of the design. This affects the probabilistic

strength distribution, which in turn influences the structural reliability.

In Chapter IV, four possible methods of testing were described. In the

present research, it is assumed that the designer is to make a selection of one

of these discrete methods of testing (test options) and that the test option

selected controls the shape of the probabilistic strength distribution of the design

as shown in previous chapters. Since the strength distributions were basically

derived from experience on aerospace structures built in the past, there is also

an implication that the inspection procedures, nondestructive test procedures,

and maintenance procedures for a given test option should be consistent with

those on the past projects from which the data were obtained. Such procedures

are usually designed to disclose errors and other anomalies which result in

defective parts. It should then be expected that these procedures affect the

strength distribution of the population of structural assemblies placed in service,

since any defects discovered would be rectified. If the procedures in a new de-

sign are not consistent with those in past projects from which the strength distri-

butions were derived, there is reason to question the applicability to the new de-

sign. This is emphasized on the flow chart by showing these procedures as an

integral part of the test option.

Test option T_2, the proof test option, requires elaboration here. If T_2

is selected as the test method, the choice of a proof test load level is also

necessary. Five discrete levels of proof testing will be considered as a part

of the decision space; that is, if the proof test option is selected, a proof test

level must also be selected to complete the decision. The proof test level

will be expressed as a percentage of the design ultimate load. Figure 18 shows

the matrix of test options.

T_0, N 0 – TEST OPTION

T_1, STANDARD TEST OPTION

T_2, PROOF TEST OPTION

50% PROOF TEST

60% PROOF TEST

70% PROOF TEST

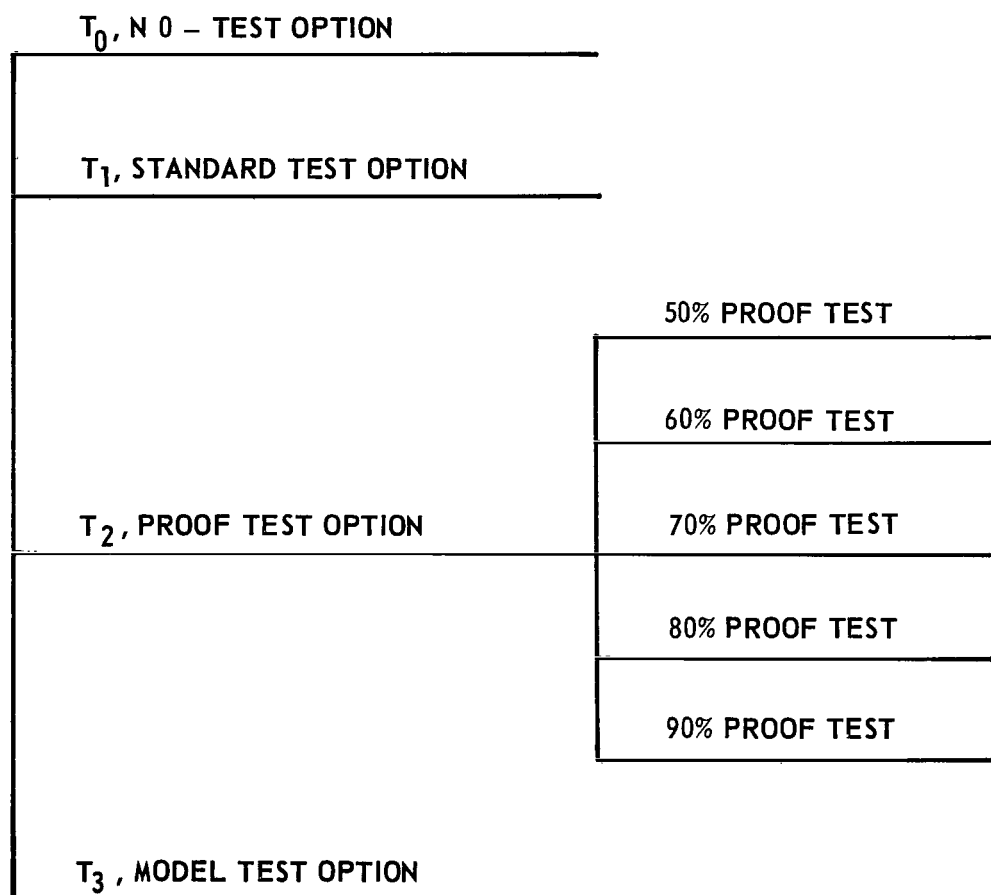80% PROOF TEST

90% PROOF TEST

T_3, MODEL TEST OPTION

Figure 18. Matrix of Test Options.

7.1.2 Selection of Safety Index

A second means available to the designer to control structural reliability

is the selection of a safety index. Once the safety index has been obtained, it is

simple to convert the result to the safety factor in any given problem. In the

applications in Chapter VIII, both the safety index and the safety factor will be considered.

Thus, the second element of the decision to be made is the selection of a design safety index, β. The domain of β is all real numbers. Negative values are physically and mathematically possible, but it seems unlikely that negative values would ever be useful in a real situation, since this implies that the expected strength is less than the expected load.

7.2 State of Nature

7.2.1 Load and Strength Distributions

The state of nature is illustrated in the middle portion of the flow diagram, Figure 17. Basically, there are two ingredients which define the state of nature: (1) the structural assembly itself and (2) the environmental conditions to which the structural assembly is subjected. It is convenient to identify specifically the important characteristics of the structural assembly and the environmental conditions which are pertinent to the decision at hand.

The important characteristics of the structural assembly are its strength and weight. As indicated in the flow diagram and discussed in previous chapters, the shape of the strength distribution R_j depends upon the test option. Further, the final strength distribution $R_j(\beta)$ depends on both the test option and the design safety index, since the design safety index is used to control the predicted strength of the design. Within a given test option and under the assumption that the basic configuration of the structural assembly is fixed, an

increase in the design safety index to increase the strength will result in an increase in the weight. The dependence of weight on the safety index is indicated in the flow diagram.

The environmental condition of importance is the load distribution applied to the structure. As indicated in the flow diagram, the load distribution is assumed to be known and is considered as an input to the decision model.

7.2.2 The Probability of Failure

An important quantity which characterizes a given combination of strength and load distributions is the probability of failure. In this section, the procedure for calculating the probability of failure of a given structural assembly, subjected to n applications of identically distributed random loads, will be developed. The n applications of loads could represent n separate flights. The development here is consistent with that of Freudenthal, Garrelts, and Shinozuka [39].

Let $F_R(x)$ be the cumulative distribution function (CDF) of strength of a structural assembly and $f_S(x)$ be the probability density function (PDF) of the load to be applied to the assembly. By definition of the CDF and PDF,

$$F_R(x) = P[R \le x] \tag{7.1}$$

and

$$f_S(x) \, dx = P[x \le S \le x + dx] \tag{7.2}$$

Thus, the probability that R will be less than x and that S will be in the

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increment dx is

$$P[R \leq x \text{ and } x \leq S \leq x + dx] = P[E\{R \leq x\} \cap E\{x \leq S \leq x + dx\}] \quad , \tag{7.3}$$

where $E\{\}$ represents the event that the statement in the brackets is true.

If R and S are assumed independent of each other,

$$P[E\{R \leq x\} \cap E\{x \leq S \leq x + dx\}] = F_R(x) f_S(x) dx \tag{7.4}$$

by the well-known product rule. Equation (7.4) represents the probability that

R will be less than S when S is in a general increment dx. If the probability

that R is less than S is desired for all possible load values, an integration

over all values of x must be performed:

$$P[R < S] = \int_0^\infty F_R(x) f_S(x) dx \tag{7.5}$$

Note that $P[R < S]$ represents the probability that a structural assembly with

CDF $F_R(x)$ will fail because of a single application of load from the distribu-

tion $f_S(x)$. Thus, the probability of failure on the kth application of load is

$$p_f(k) \equiv P[R < S] = \int_0^\infty F_R(x) f_S(x) dx \quad . \tag{7.6}$$

There are assumed to be only two possible outcomes on the kth load

application, failure or survival. Since the probability of failure is given by

equation (7.6), the probability of survival, from an axiom of probability, is

$$\bar{p}_f (k) = 1 - p_f (k) \quad .$$
(7.7)

The probability that the assembly will survive n load applications is

$$L_N (n) = P \left[E_1 \cap E_2 \cap \ldots \cap E_k \cap \ldots \cap E_n \right] \quad ,$$
(7.8)

where E_k denotes the event that the assembly survives the kth load application, and the intersection $E_1 \cap \ldots \cap E_n$ denotes the occurrence of all events $E_1 \ldots E_n$. If the survival of each of n load cycles is independent of the survival of all other load cycles, the probability of $E_1 \cap \ldots \cap E_n$, and hence the probability that the assembly will survive n load applications, is

$$L_N (n) = \bar{p}_f (1) \, \bar{p}_f (2) \ldots \bar{p}_f (k) \ldots \bar{p}_f (n)$$

$$= \left[1 - p_f (1) \right] \left[1 - p_f (2) \right] \ldots \left[1 - p_f (n) \right]$$

$$= \prod_{k=1}^{n} \left[1 - p_f (k) \right] \quad .$$
(7.9)

Since the n applied loads are identically distributed, $p_f (1) = p_f (2) = \ldots = p_f (n)$, and

$$L_N (n) = \left[1 - p_f (k) \right]^n$$
(7.10)

The probability of failure of the structural assembly during the n load applications is

$$F_N (n) = 1 - L_N (n) = 1 - \left[1 - p_f (k) \right]^n = 1 - \left[\bar{p}_f (k) \right]^n$$
(7.11)

Similar results were obtained by Freudenthal, Garrelts, and Shinozuka [39].

A special case of equation (7.11) arises for the consideration of the application of a single deterministic load such as a proof test load. Let the magnitude of the deterministic load be r_0. Then, in place of equations (7.4) and (7.5),

$$P\left[R < S\right] = P\left[E\left\{R \le r_0\right\} \cap E\left\{S = r_0\right\}\right] = F_R\left(r_0\right) \qquad , \qquad (7.12)$$

where $P\left[E\left\{S = r_0\right\}\right] \equiv 1$ and $P\left[E\left\{R \le r_0\right\}\right] = F_R\left(r_0\right)$ from equation (7.1). Then the probability of failure during proof testing is

$$p_f\left(p\right) = P\left[R < S\right] = F_R\left(r_0\right) \qquad . \qquad (7.13)$$

If m identical structural assemblies are to be subjected to the proof test, the probability that one or more of these articles will fail the proof test is given by

$$F_M\left(m\right) = 1 - \left[1 - F_R\left(r_0\right)\right]^m \qquad . \qquad (7.14)$$

7.3 The Cost Model: Utilities

In Bayesian decision theory, utilities or values are, in general, a measure of the consequences of various action/state-of-nature combinations. The proper expression and use of the utility concept has absorbed a considerable portion of the development of the theory. The present problem requires the use of only the very elementary concepts of utility, and these have a very high degree of intuitive appeal. The utility concepts to be used are described below and are consistent with Bayesian decision theory.

The measure of utility is very direct — the expected cost in dollars of action/state-of-nature combinations is used as the measure of desirability of the combinations. The lower the expected cost, the higher the desirability. Expected costs include not only direct monetary outlays but also the monetary equivalent of other consequences such as performance loss due to weight of the structural assembly (assumed to be deterministic) and possible monetary losses due to structural failure (assumed probabilistic). As would be intuitively expected, any costs common to all possible actions may be omitted from consideration. This is equivalent to saying that the choice of reference point does not affect the decision. As indicated in Figure 3 of Chapter III, direct costs (costs not affected by the state of nature) may be considered directly and need not be included in the utility function. However, these costs could be included in the utility function, and the final results would not change.

The total expected cost to be minimized to obtain the optimum decision consists of three basic parts: direct costs C_D, cost of weight C_w, and expected cost of failure $\overline{C}_F$:

$$C = C_D + C_w + \overline{C}_F .$$

$$(7.15)$$

Each of these cost constituents will be discussed separately.

7.3.1 Direct Costs

The direct costs do not depend upon the state of nature but may depend upon the action. As shown in the flow diagram of Figure 17, these costs include such items as development costs C_d and operational costs C_0.

$$C_D = C_{D_j} = C_d + C_0 \qquad . \qquad (7.16)$$

The notation C_{D_j} is used in the flow diagram to indicate that these costs primarily depend upon the test option selected. The development costs include such items as test hardware, test facilities, and test operations, including an allowance for correcting deficiencies which may be discovered in testing. Operational costs include items such as inspection, nondestructive tests, and maintenance. In the area of direct costs, it is particularly important to keep in mind that costs common to all actions need not be considered; this can significantly reduce the amount of information required as input to the decision model. For example, if all courses of action under consideration require the use of the same test facility, the cost of the facility need not be included in the cost model.

7.3.2 <u>Cost of Weight</u>

Weight is usually of prime importance in aerospace systems. In fact, this is the basic reason that aerospace systems are usually designed with much smaller safety factors than civil structures. However, weight does not have equal importance in all aerospace systems. It is not unusual for the monetary value of weight to vary by an order of magnitude between the first stage and the upper stages of a launch vehicle. This important fact has been largely disregarded in the past in establishing structural criteria such as safety factors. The value of weight can easily be included in the present decision model.

Within a given design concept, the weight of a structural assembly largely depends upon the design safety index. Let the weight relative to some reference value be a known function of the design safety index:

$$W = W(\beta) \tag{7.17}$$

Let the known monetary value of one unit of weight in the assembly under consideration be c_w. Then the cost of weight as a function of β appropriate for use in equation (7.14) is

$$C_w = C_w(\beta) = c_w W(\beta) \quad . \tag{7.18}$$

7.3.3 Expected Cost of Failure

In Benjamin and Cornell [2], it is shown that the expected cost of an uncertain event is given by the product of the actual cost of the event and the probability that the event will occur. In the present context, the uncertain event is the failure of the structural assembly under consideration. Note that the probability of failure derived in equation (7.11) is the probability of having one or more failures. The possible events that can occur during the application of n load cycles to a fleet of structural assemblies are no failures, one failure, two failures ... or n failures. According to statistical decision theory [2], the expected cost of these mutually exclusive events is

$$C_F = C_0 P_0 + C_1 P_1 + C_2 P_2 + \ldots + C_i P_i + \ldots + C_n P_n , \tag{7.19}$$

where C_i is the cost of i failures, and P_i is the probability of having exactly

i failures.

From equation (7.6) and (7.7), the probability of having a failure on the kth load cycle is $p_f(k)$, and the probability of no failure is $\bar{p}_f(k)$. The probability of no failures in n load cycles, $P_0 \equiv L_N(n)$, was given by equation (7.10). Assuming independence of the events of failure, the probability of surviving the 1st, 2nd ... $(i-1)^{th}$, $(i+1)^{th}$ ... and n^{th} cycles and having a failure on the ith cycle is given by

$$p_i = \bar{p}_f(1)\bar{p}_f(2) \ldots \bar{p}_f(i-1)p_f(i)\bar{p}_f(i+1) \ldots \bar{p}_f(n) \qquad . \qquad (7.20)$$

Similarly, the probability of failing on the jth cycle but surviving all others is

$$p_j = \bar{p}_f(1)\bar{p}_f(2) \ldots \bar{p}_f(j-1)p_f(j)\bar{p}_f(j+1) \ldots \bar{p}_f(n) \qquad . \qquad (7.21)$$

Note that the failure events, $E\{$failure on the 1st cycle, survival of all other cycles$\}$, $E\{$failure on the 2nd cycle, survival of all other cycles$\}$ ... $E\{$failure on the nth cycle, survival of all other cycles$\}$ are mutually exclusive. Then, by an axiom of probability [2], the probability of any one of these events occurring is

$$P_1 = p_1 + p_2 + \ldots + p_i + \ldots + p_n = \sum_{i=1}^{n} p_i \qquad , \qquad (7.22)$$

where P_1 is the probability of having exactly one failure. Substituting tion (7.20) into equation (7.22) and assuming $p_f(1) = p_f(2) = \ldots p_f(k) \ldots = p_f(n)$,

$$P_1 = [\overline{p}_f(k)]^{n-1} p_f(1) + [\overline{p}_f(k)]^{n-1} p_f(2) + \ldots + [\overline{p}_f(k)]^{n-1} p_f(i) \quad (7.23)$$

$$+ \ldots + [\overline{p}_f(k)]^{n-1} p_f(n) = n \, p_f(k) [\overline{p}_f(k)]^{n-1} \quad .$$

Similarly, the probability of having failures on the ith and r^{th} cycles and surviving all other cycles is

$$p_{i,r} = \overline{p}_f(1) \overline{p}_f(2) \ldots \overline{p}_f(i-1) p_f(i) \overline{p}_f(i+1) \ldots \overline{p}_f(r-1) \, p_f(r) \overline{p}_f(r+1) \ldots \overline{p}_f(n) .$$

$$(7.24)$$

If all possible combinations of i and r are considered, the probability of having exactly two failures is

$$P_2 = [\overline{p}_f(k)]^{n-2} p_f(1) p_f(2) + [\overline{p}_f(k)]^{n-2} p_f(1) p_f(3) + \ldots \quad (7.25)$$

$$+ [\overline{p}_f(k)]^{n-2} p_f(i) p_f(r) \ldots + [\overline{p}_f(k)]^{n-2} p_f(n-1) p_f(n)$$

$$= \frac{n \, (n-1)}{2 \, !} \, [p_f(k)]^2 [\overline{p}_f(k)]^{n-2} \quad .$$

Generalizing equation (7.23) and (7.25),

$$P_q = \frac{n \, !}{q \, ! \, (n-q) \, !} \, [p_f(k)]^q \, [\overline{p}_f(k)]^{n-q} \quad . \quad (7.26)$$

Equation (7.26) is recognized as the well-known binomial distribution [2], which has an expected value $n \, p_f(k)$.

If equation (7.26) is substituted into equation (7.19), and $C_0 = 0$ is assumed,

$$C_F = C_1 \, n \, p_f(k) \, [\bar{p}_f(k)]^{n-1} + \ldots + C_q \frac{n!}{q!(n-q)!} \, [p_f(k)]^q [\bar{p}_f(k)]^{n-q}$$

$$(7.27)$$

$$+ \ldots + C_n [p_f(k)]^n [\bar{p}_f(k)] \quad .$$

Now assuming that $C_q = q \, C_1$, that is, q failures cost q times as much as one failure,

$$\bar{C}_F = C_1 \{ n p_f(k) [\bar{p}_f(k)]^{n-1} + \ldots + q \frac{n!}{q!(n-q)!} [p_f(k)]^q [\bar{p}_f(k)]^{n-q}$$

$$(7.28)$$

$$+ \ldots + n \, [p_f(k)]^n [\bar{p}_f(k)] \} \quad .$$

The term in the braces in equation (7.28) is recognized as the definition of the expected value of the binomial distribution. Therefore,

$$\bar{C}_F = C_F \, n \, p_f(k) \quad , \tag{7.29}$$

where $C_F \equiv C_1$ is the cost of flight failure. A physical interpretation is that $n \, p_f(k)$ is the expected number of failures. $P_{F_j}(\beta)$ has been used to represent $n \, p_f(k)$ in the flow diagram of Figure 17. This emphasizes the dependence of the expected number of failures on the test option j and the safety index, β. The expected cost of failure is then given by

$$\bar{C}_F = \bar{C}_{F_j}(\beta) = C_F \, P_{F_j}(\beta) = C_F \, n \, p_f(k) \quad . \tag{7.30}$$

An identical development to equations (7.19) through (7.29) can be constructed, using $p_f(p) = F_R(r_0)$ from equation (7.13) in place of $p_f(k)$, the number of

proof tests m in place of the number of load cycles n, and the cost of proof test failure C_{F_p} in place of the cost of flight failure C_F. When this is done, equation (7.30) is generalized for the proof test option to yield

$$\overline{C}_F = \overline{C}_{F_2}(\beta) = C_F \, n \, p_f(k) + C_{F_p} \, m \, p_f(p) \qquad . \tag{7.31}$$

7.4 Cost Optimization

The cost constituents from equations (7.16), (7.18), and (7.30) or (7.31) are now substituted into equation (7.15):

$$C_j(\beta) = C_{D_j} + C_w(\beta) + \overline{C}_{F_j}(\beta) \qquad . \tag{7.32}$$

The notation here emphasizes the dependence of costs on the test option j and the safety index β. The cost optimization problem now reduces to the minimization of $C_j(\beta)$ with respect to the discrete variable j and the continuous variable β. Consideration of the discrete variable j leads to separate equations for each test option:

$$C_0(\beta) = C_{D_0} + C_w(\beta) + \overline{C}_{F_0}(\beta) \tag{7.33}$$
$$\vdots$$
$$C_3(\beta) = C_{D_3} + C_w(\beta) + \overline{C}_{F_3}(\beta) \qquad .$$

A necessary condition for a relative minimum cost within a given test option is

$$\frac{dC_j(\beta)}{d\beta} = 0 \qquad , \tag{7.34}$$

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provided that $C_j(\beta)$ is continuous throughout the domain of β, and $dC_j(\beta) \mid d\beta$ exists at the minimum point. This relative minimum is the absolute minimum provided that $C_j(a)$ and $C_j(b)$ are greater than $C_j(c)$ where a and b are the end points of the domain of β under consideration and c is the value of β at the relative minimum given by equation (7.34). Applying equation (7.34) to (7.33),

$$\frac{d\, C_0(\beta)}{d\beta} = 0 \tag{7.35}$$

$$\frac{d\, C_3(\beta)}{d\beta} = 0 \qquad .$$

Let the solutions of equations (7.35) be denoted by $\beta = \beta_j^*$. The minimum costs for each of the four test options are obtained by substituting these solutions into equation (7.33).

$$\overset{*}{C}_0(\beta) = C_{D_0} + C_w(\overset{*}{\beta}_0) + C_{F_0}(\overset{*}{\beta}_0) \tag{7.36}$$

$$\overset{*}{C}_3(\beta) = C_{D_3} + C_w(\overset{*}{\beta}_3) + C_{F_3}(\overset{*}{\beta}_3) \qquad .$$

The minimum cost among all test options is selected by

$$\overset{*}{C} = \min\{\overset{*}{C}_0, \overset{*}{C}_1, \overset{*}{C}_2, \overset{*}{C}_3\} \qquad . \tag{7.37}$$

Thus, the minimum cost test option and safety index combination is that one corresponding to $\overset{*}{C}$. The minimization procedure just described was illustrated

by Figure 4 in Chapter III. Because of the complexity of the expressions for the expected cost of failure, the minimization procedure must be done numerically or graphically. A graphical method will be illustrated by the applications in Chapter VIII.

Throughout Chapter VII there is an implicit assumption that the minimum cost for the particular structural assembly under consideration is not affected by the other assemblies in the structural system. This assumption is not believed to be particularly restrictive for the typical aerospace system. The high reliabilities expected in aerospace systems tends to reduce the dependence of the probabilities of failure of the various assemblies on each other. Moses and Kinser [43] showed that an assumption of independence, when dependence actually exists, can cause significant errors in reliability prediction for structures with low reliabilities.

CHAPTER VIII

APPLICATIONS

8.1 Discussion

The purpose of this chapter is to demonstrate the application of the concepts developed in previous chapters to typical aerospace structural assemblies. These concepts culminate in the cost optimization presented in section 7.4. The applications given in this chapter are a direct application of the cost optimization procedure, where the costs for each test option, as given by equation (7.33), are calculated for several values of β. Then the minimum cost decision is obtained by plotting the results to obtain the solutions of equations (7.35) and (7.37) graphically.

Some parametric results useful in applications have been obtained and included as Appendix C. These results provide graphical means of determining $p_f(k)$ needed in equation (7.31) Graphs are also presented in Appendix C for converting between the safety index and design safety factor.

8.2 Input Data

To conserve space and to provide for easy comparison of input and results, the applications examples are presented together in parallel instead of sequentially. Example structures are representative of structural assemblies in NASA's Saturn V launch vehicle and Space Shuttle systems. The information

presented regarding these systems should not be taken as official NASA data but should be considered as typical of such systems. The Saturn V system is illustrated in Figure 19. The S-IVB stage aft skirt and the S-II forward skirt are the structural assemblies from this system selected as examples. A sketch of the Space Shuttle system is shown in Figure 20. The External Tank (ET) intertank, Orbiter aft fuselage, Orbiter wings, and Solid Rocket Booster aft skirt are selected as examples from the Space Shuttle.

Data required as input to the decision model for the example problems are shown in Table 9. The table headings indicate where each input is used in the equations of Chapter VII. Weight versus safety index is also needed in the decision model. This is most readily obtained in the form of weight versus safety factor, which is given for the example problems in Figures 21 through 26. To plot weight versus safety index would require a family of curves. One way of plotting would be to plot a curve for each test option, which would hold for only one particular coefficient of variation of load. The weight versus safety factor curve holds for any test option and any coefficient of variation of load. Since all of the example problem structures except the S-II stage are designed for a safety factor of 1.4, the reference value of weight mentioned in section 7.3.2 should be the value from the figures corresponding to a 1.4 safety factor (1.3 for the S-II).

8.3 Results

Results of the application of the decision model to the input data of section 8.2 are shown in Figures 27 through 32. For each example problem,
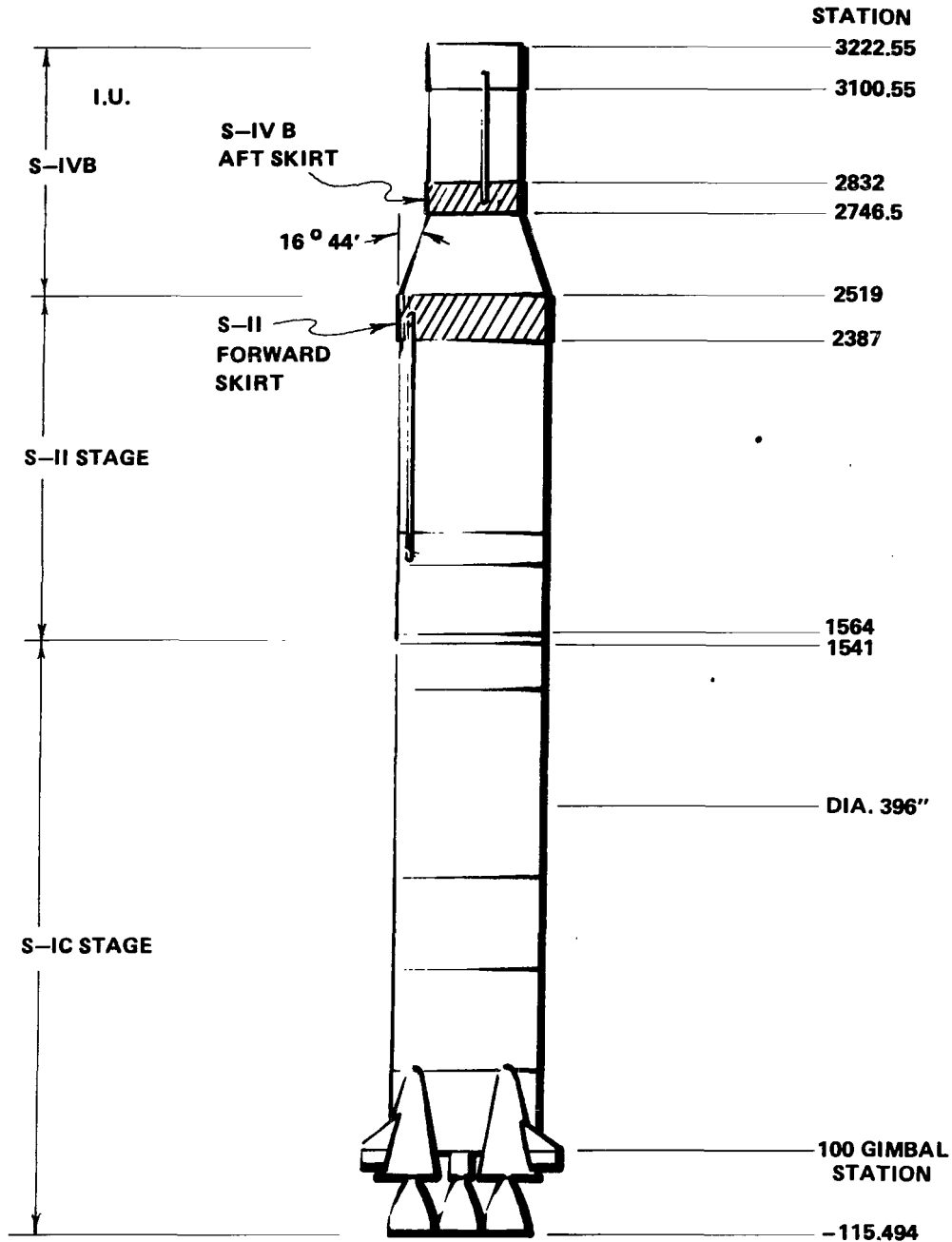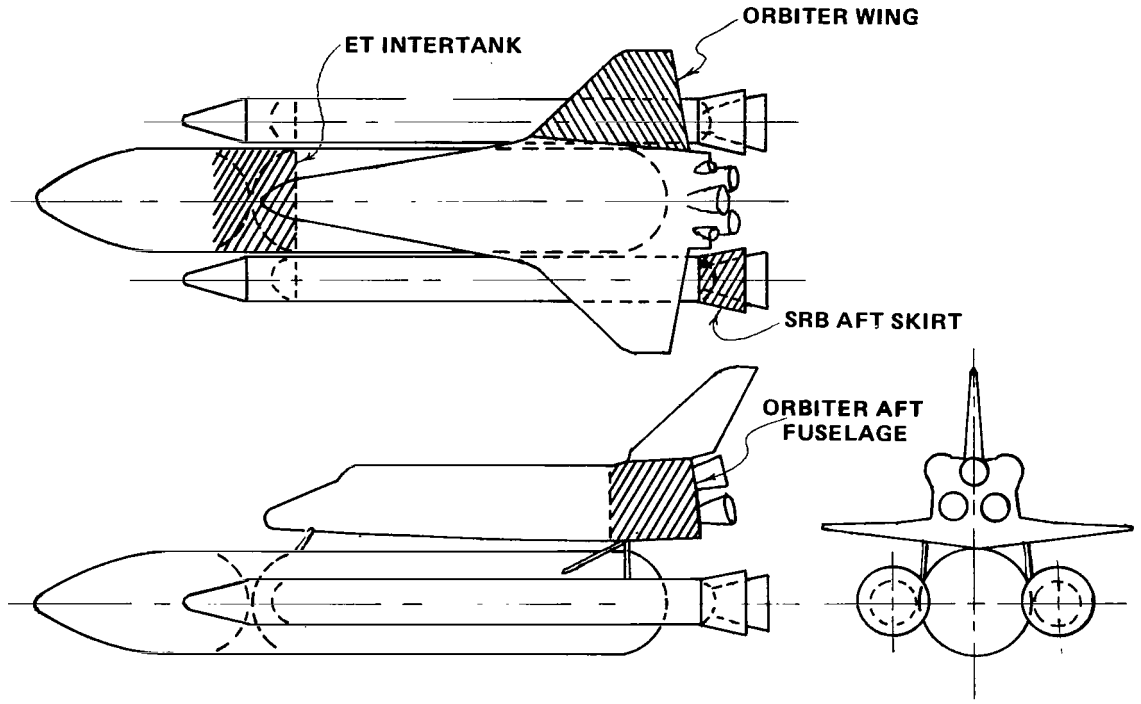
Figure 19. Saturn V System.

Figure 20. Space Shuttle System.

Table 9. Input Data for Example Problems

Example Problem	Value of Weight (dollars per pound) c_W Equation (7.18)	Cost of Flight Failure (dollars) c_F Equation (7.30)	Cost of Proof Test Failure (dollars) c_{F_p} Equation (7.31)	Cost of Test Option T_1 (dollars) c_{D_1} Equation (7.16)	Cost of Test Option T_2 (dollars) c_{D_2} Equation (7.16)	Cost of Test Option T_3 (dollars) c_{D_3} Equation (7.16)	Coefficient of Variation of Load γ_S Equation (C-4)	Number of Flight Load Applications n Equation (7.30)	Number of Proof Tests m Equation (7.31)
1. S-IVB Aft Skirt	11,500	250×10^6	0.72×10^6	2.0×10^6	3.28×10^6	1.0×10^6	0.12916	15	15
2. S-II Forward Skirt	4,230	250×10^6	1.23×10^6	5.3×10^6	8.69×10^6	2.65×10^6	0.12916	15	15
3. ET Intertank	13,706	125×10^6	0.625×10^6	2.0×10^6	22.25×10^6	1.0×10^6	0.05787	445	445
4. Orbiter Aft Fuselage	18,832	250×10^6	2.0×10^6	5.3×10^6	3.3×10^6	2.65×10^6	0.05787	445	5
5. Orbiter Wings	18,832	250×10^6	2.0×10^6	5.3×10^6	6.6×10^6	2.65×10^6	0.37	890	10
6. SRB Aft Skirt	1,624	14.8×10^6	0.35×10^6	2.0×10^6	8.25×10^6	1.0×10^6	0.37	890	50

the expected cost versus safety index and versus safety factor is plotted. Two sample calculations are shown in Appendix D, and the results are plotted as points A and B in Figure 27 and as points C and D in Figure 29. For clarity, the proof test option T_2 has been plotted separately, since there is a separate curve for each proof test level. In Figure 27(a), the results for a simpler cost model in which $P_{F_j}(\beta)$ was assumed equal to $F_N(n)$ (equation 7.11) are plotted as dotted lines. This model applies only if the probabilities of failure are very small.

A summary of the final results is given in Table 10 for all example problems and all test options. The results were obtained by selecting minimum points on the curves in Figures 27 through 32.

8.4 Trends Shown by Results

The optimum decision for each of these example problems is the result of a complex interaction of several factors which contribute to the expected cost. Each of the inputs in Table 9 contributes to the expected cost.

A higher value of weight in a structural assembly tends to produce lower optimum safety factors. A higher cost of flight failure tends to produce higher optimum safety factors. A high cost of proof test failure tends to discourage use of the Proof Test Option T_2 and to reduce the optimum level of the proof test load. The optimum proof test level represents the most cost effective balance between expected costs of proof test failure and flight failure. The number of flight load applications has a pronounced influence on the probability

of flight failure, resulting in a tendancy toward higher safety factors for larger numbers of flights.

The Proof Test Option, T_2, which has seldom been used in aerospace structures of this type, appears remarkably effective, even if the maximum proof test level is restricted to 70 percent, as was done in Table 10. This effectiveness appears to arise because of the ability of this test option to produce more reliable structures with smaller safety factors than the other test options. This trend can be observed by comparing Figure 36(a) with Figure 37(a) in Appendix C, for example. The advantage of test option T_2 would be even more pronounced for one-of-a-kind structures, since only one item instead of several would have to be tested $(m = 1)$, resulting in an even lower cost.

The SRB aft skirt optimum safety factors are generally lower than those of the other structures. This is very likely because of the lower cost of flight failure for this structure, resulting in a lower required reliability as compared with the other structures.

An encouraging aspect of these results is the compatibility of the test option T_1 optimum factors with past experience. The S-IVB and S-II example problems were included to establish this compatibility. The design safety factor for these structures was 1.40, which compares closely with the 1.47 optimum value for T_1 in Table 10. A lack of compatibility with previous experience would have made the validity of the model developed here questionable.

Table 10. Summary of Cost Optimum Decisions

Cost Optimum Decisions	S-IVB Aft Skirt	S-II Forward Skirt	ET Intertank	Orbiter Aft Fuselage	Orbiter Wing	SRB Aft Skirt
Overall Option	T_1	T_3	T_2	T_2	T_2	T_1
$\overset{*}{\beta}$	4.12	4.15	2.25	2.32	3.15	4.13
$\overset{\sim}{\nu}$	1.47	1.77	1.37	1.42	1.43	1.33
Reliability	0.995	0.995	0.955	0.993	0.951	0.984
Option T_0						
$\overset{*}{\beta}$	3.62	3.70	3.80	4.07	4.10	3.75
$\overset{\sim}{\nu}$	2.10	2.25	2.65	3.25	2.25	1.80
Reliability	0.988	0.989	0.870	0.950	0.900	0.768
Option T_1						
$\overset{*}{\beta}$	4.18	4.18	4.10	4.65	4.45	4.13
$\overset{\sim}{\nu}$	1.47	1.47	1.55	1.87	1.50	1.33
Reliability	0.995	0.996	0.805	0.953	0.949	0.984
Option T_2[†]						
$\overset{*}{\beta}$	2.6	2.6	2.25	2.32	3.15	3.08
$\overset{\sim}{\nu}$	1.35	1.35	1.37	1.42	1.43	1.37
Reliability	0.998	0.998	0.955	0.993	0.951	0.933
Option T_3						
$\overset{*}{\beta}$	4.10	4.15	4.20	4.65	4.45	4.00
$\overset{\sim}{\nu}$	1.70	1.77	1.95	2.30	1.80	1.53
Reliability	0.994	0.995	0.850	0.956	0.940	0.818

† The maximum proof test level considered feasible for these structures is 70 percent. Higher levels could damage the structures by yielding.
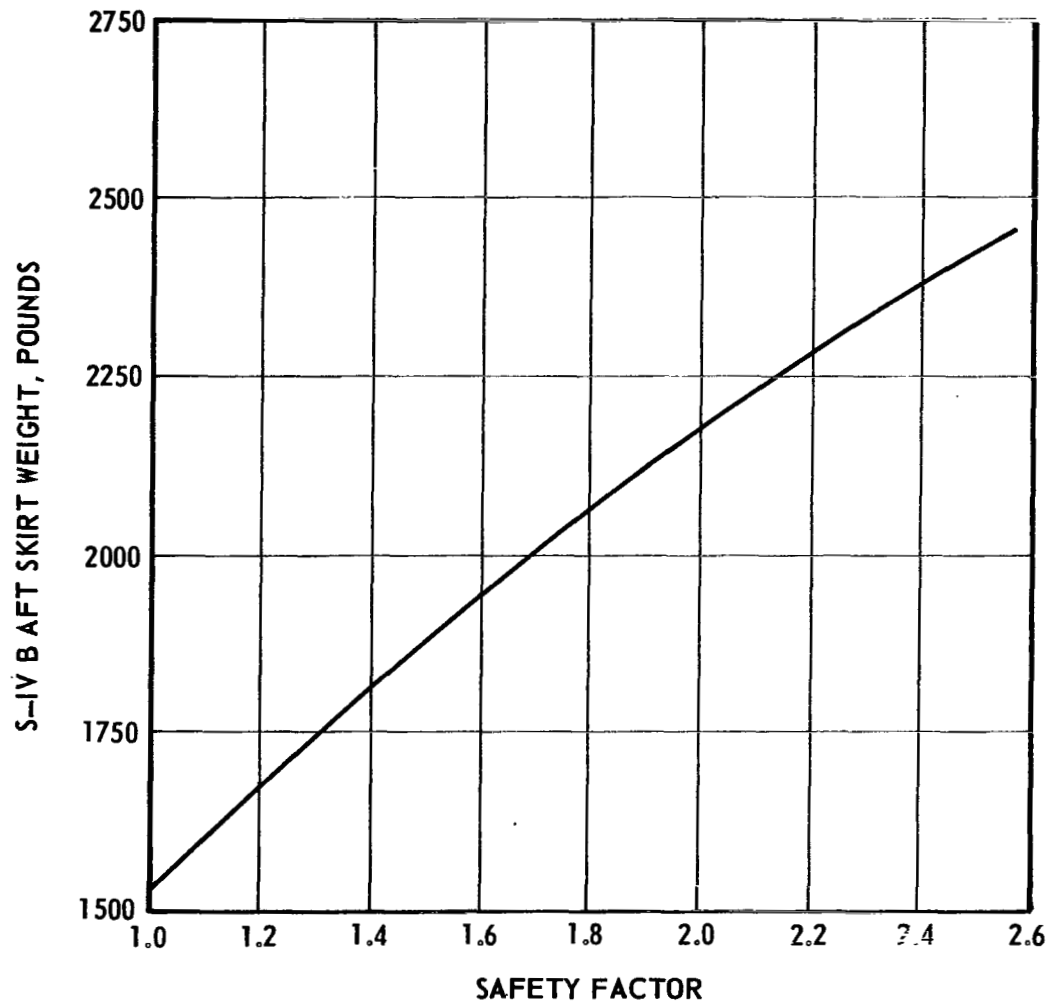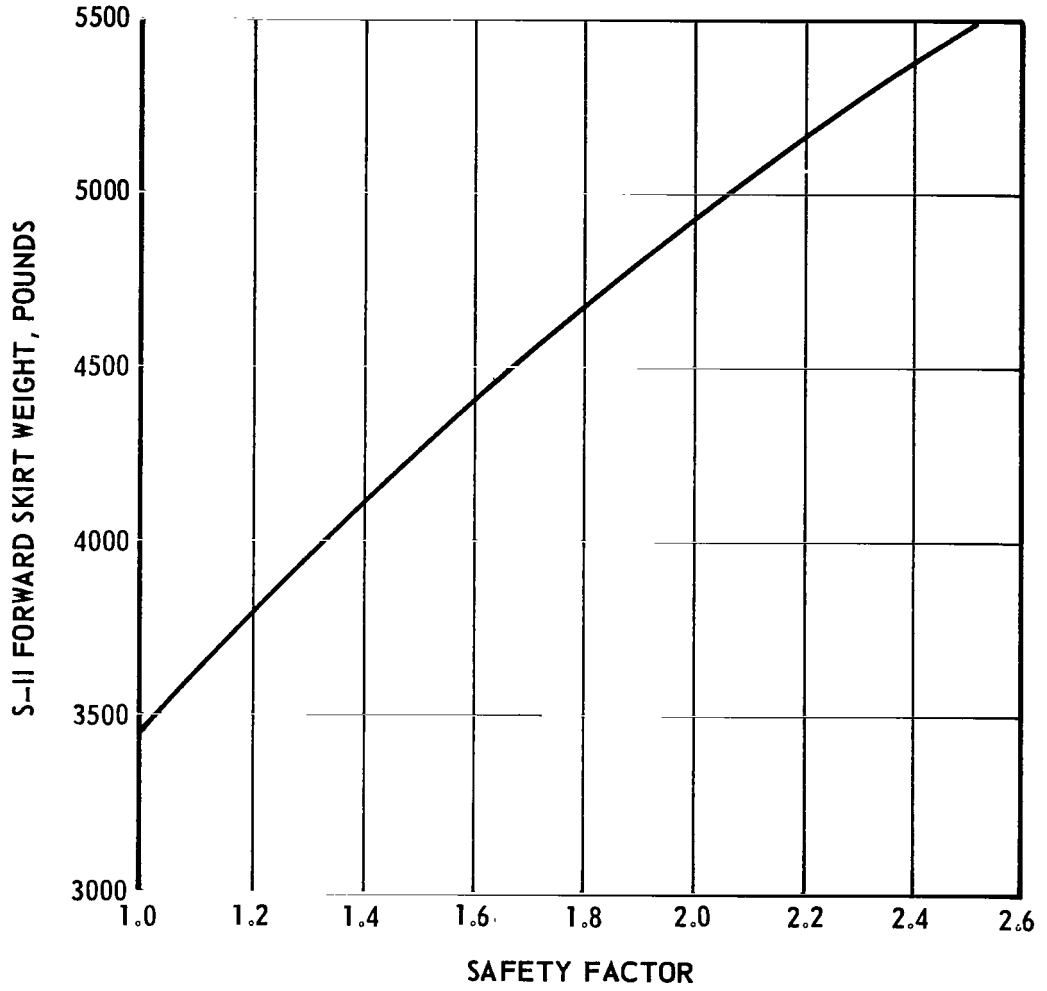
Figure 21.   S–IVB Aft Skirt Weight.
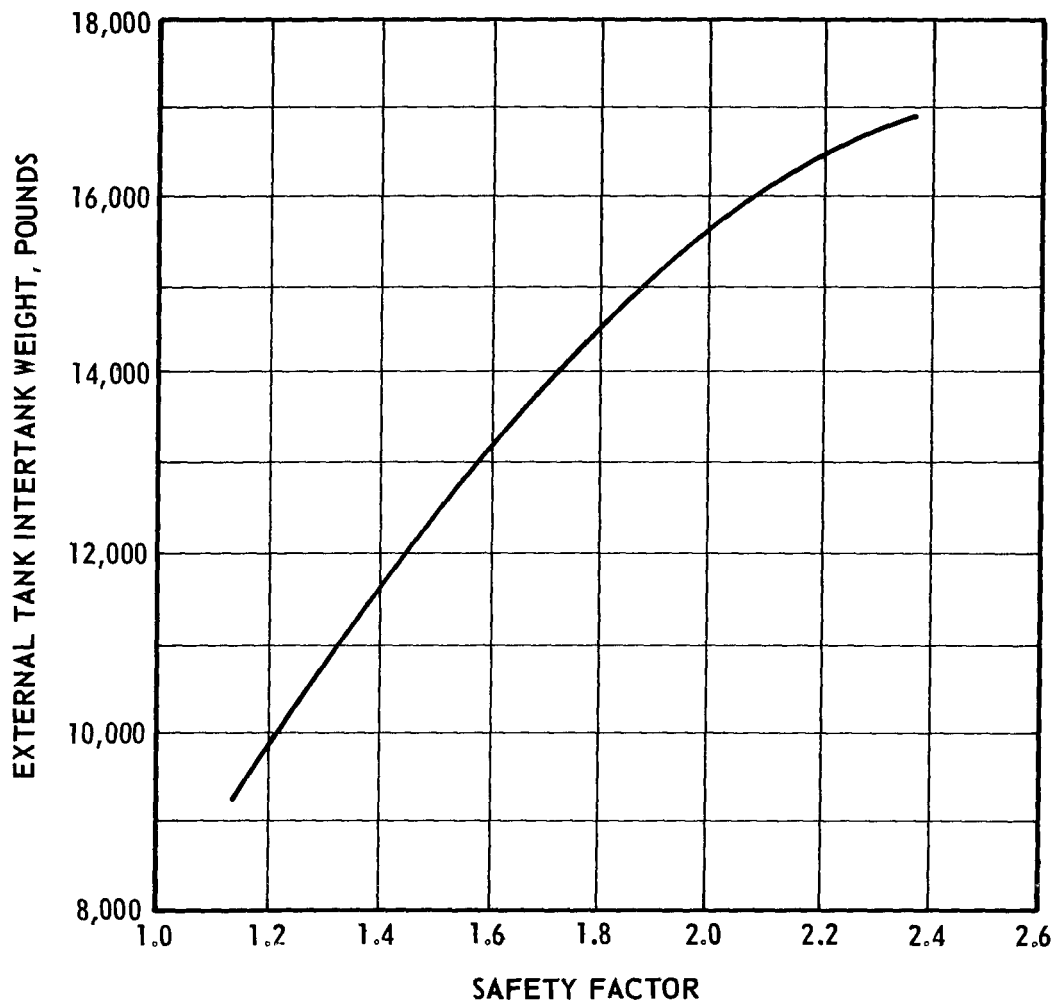
Figure 22. S-II Forward Skirt Weight.
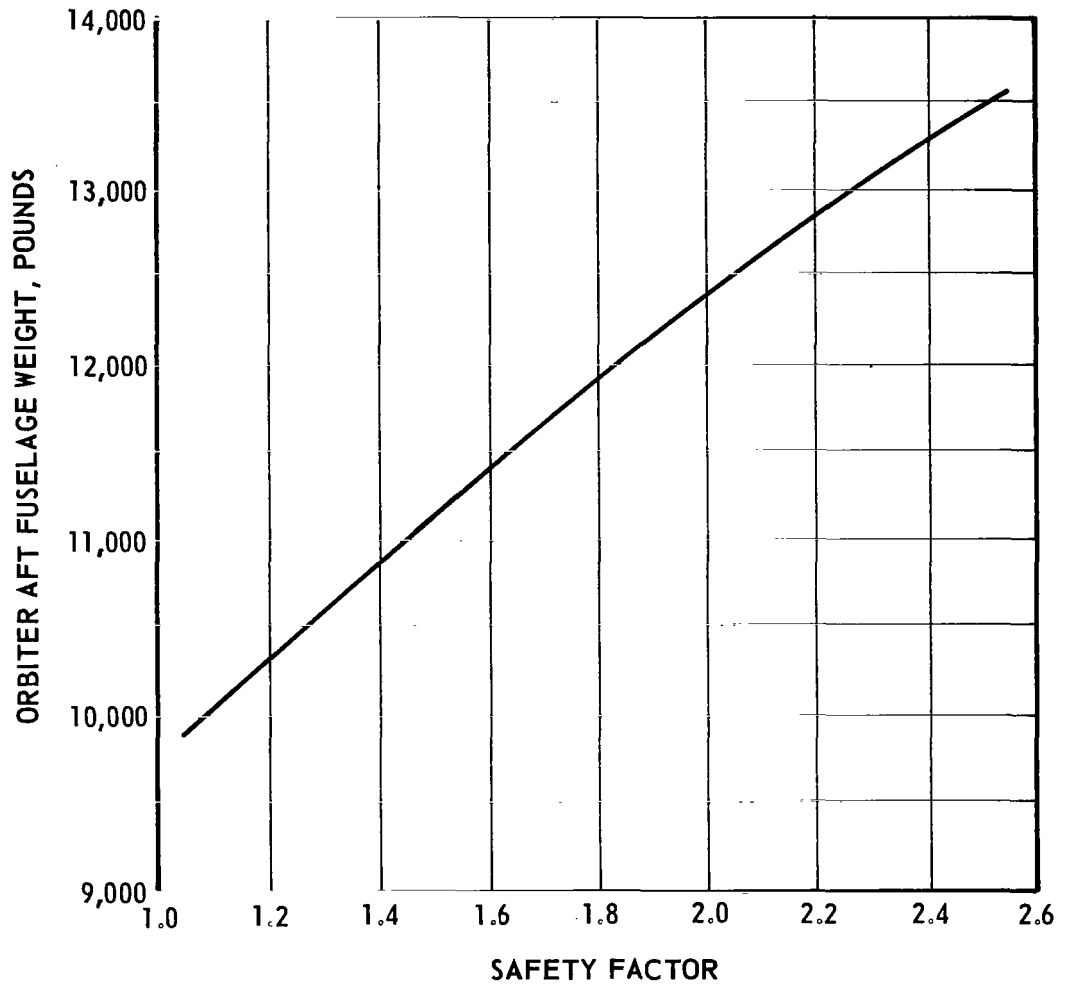
Figure 23. External Tank Intertank Weight.

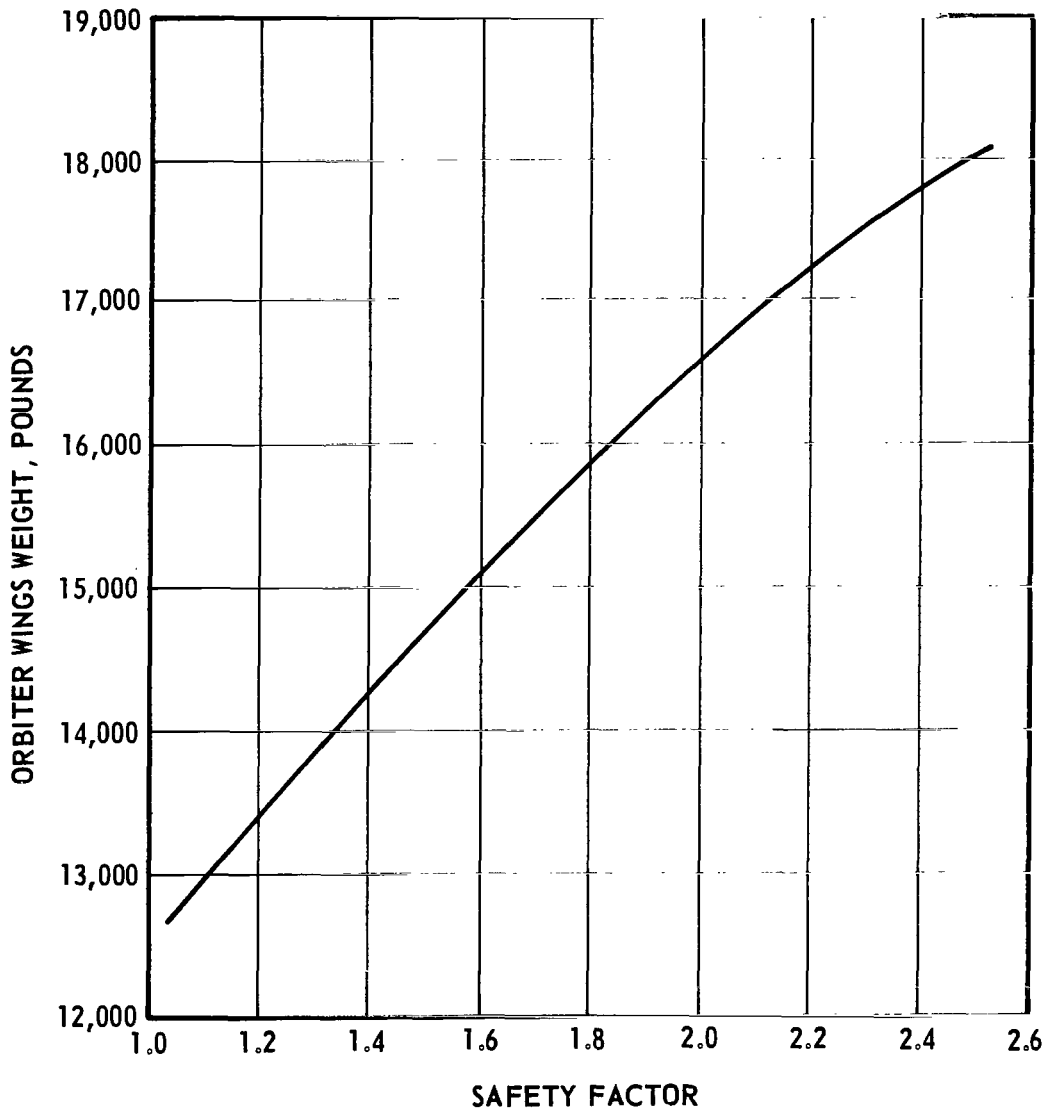Figure 24.   Orbiter Aft Fuselage Weight.
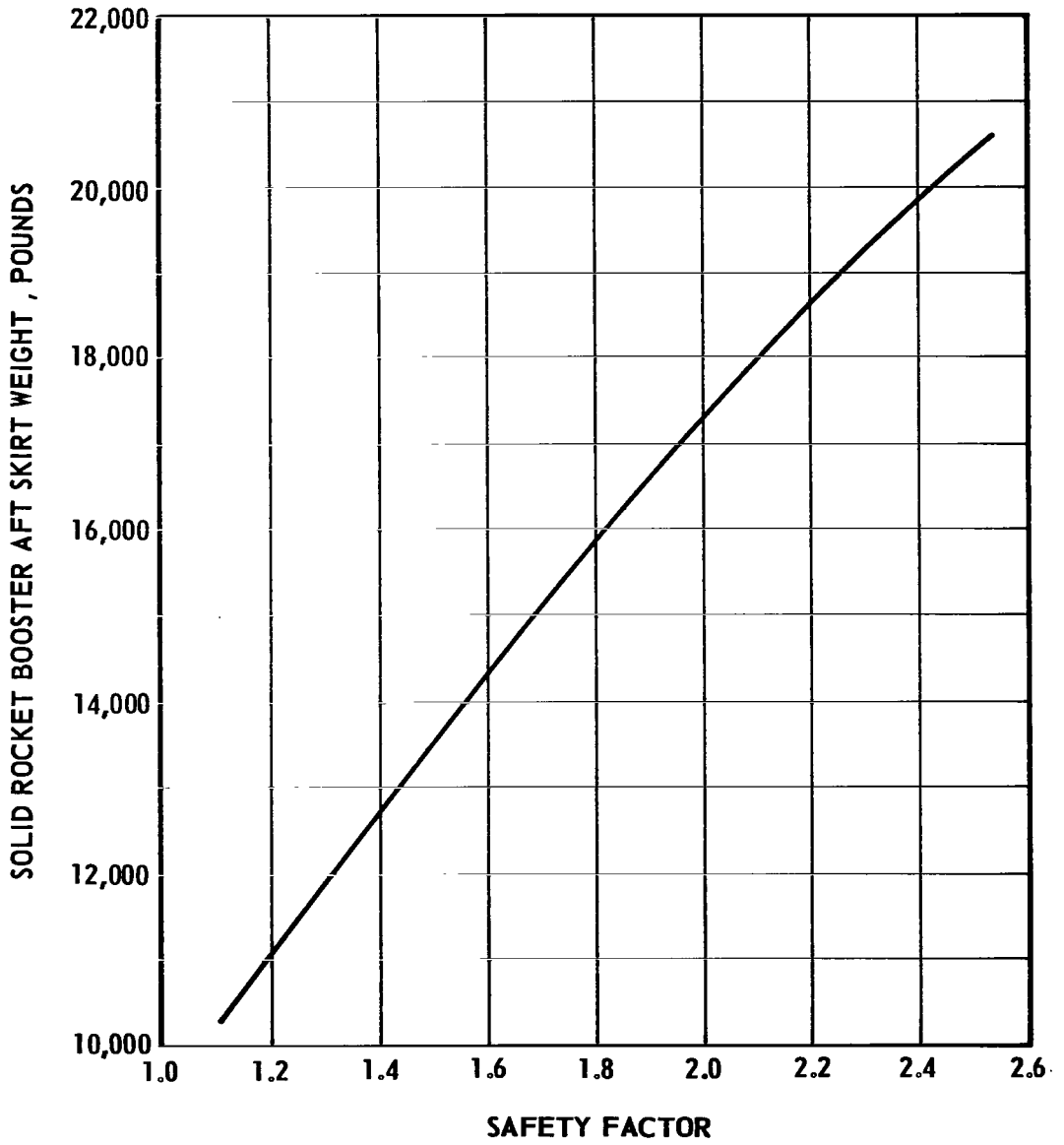
Figure 25. Orbiter Wings Weight.

Figure 26. Solid Rocket Booster Aft Skirt Weight.
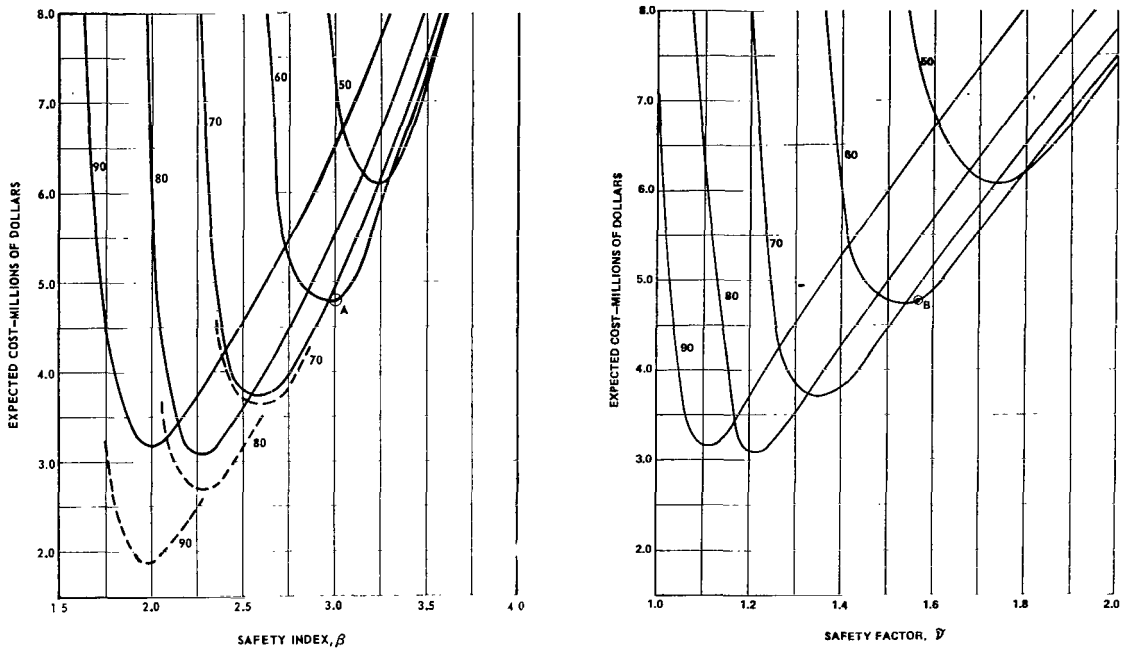
a. Test Option T_2.
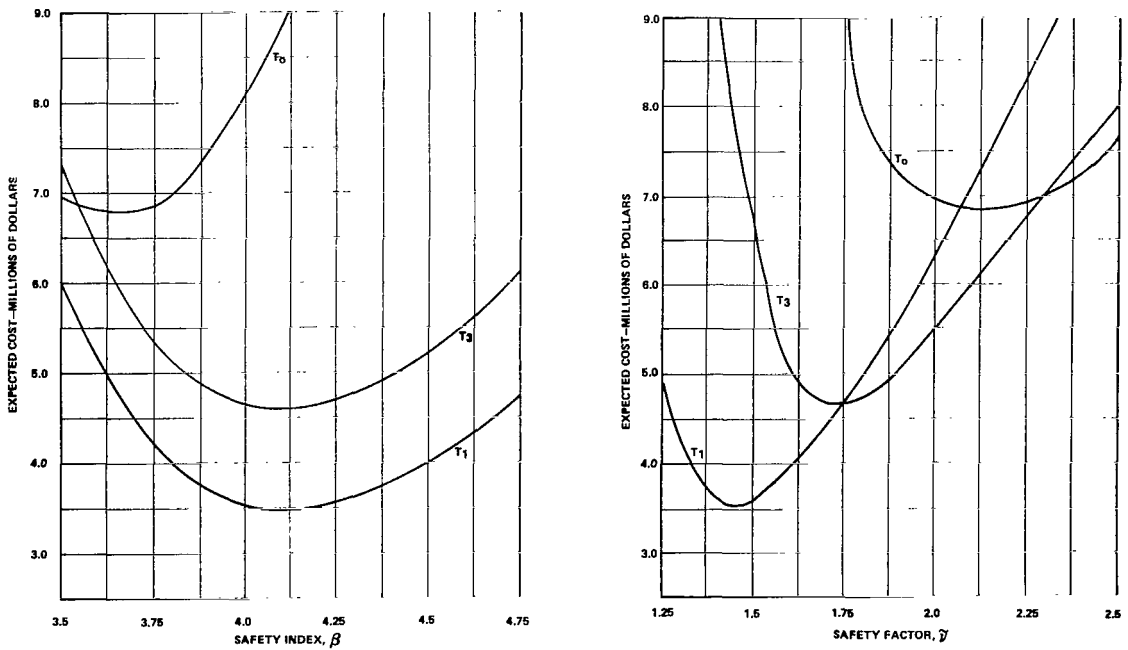
b. Test Options T_0, T_1, and T_3.

Figure 27. Expected Costs for S-IVB Aft Skirt.

c. Safety Factor versus Safety Index.

Figure 27. (Concluded).

a.   Test Option T_2.

b.   Test Options T_0, T_1, and T_3.

Figure 28.   Expected Costs for S-II Forward Skirt.
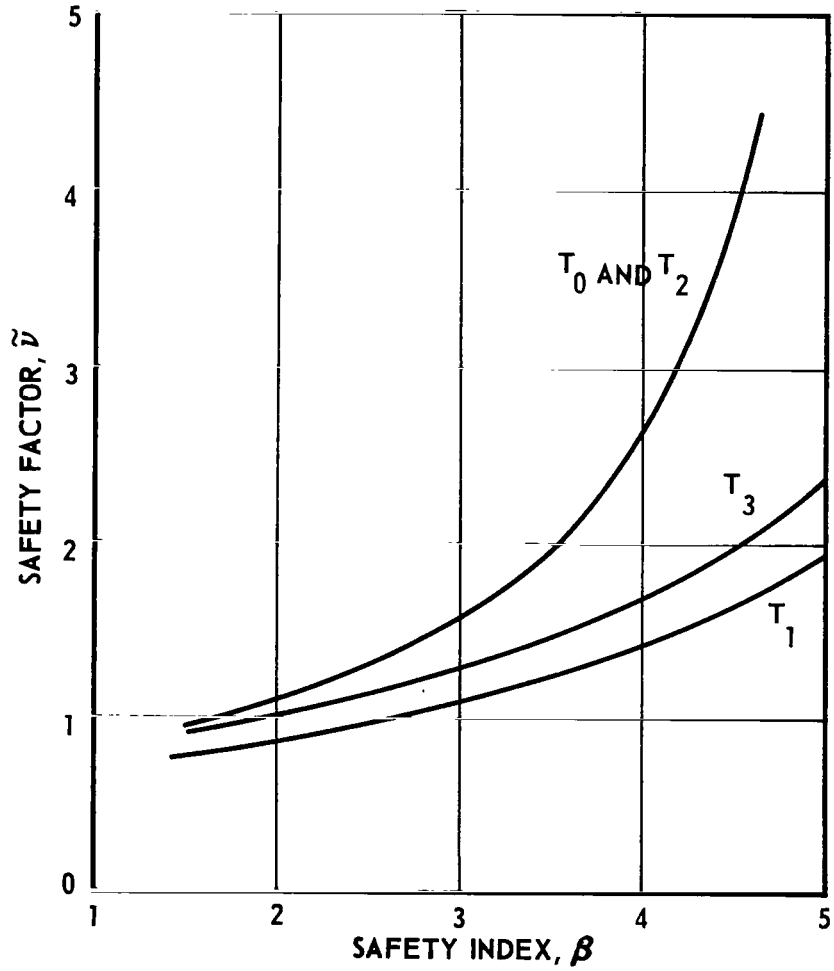
c. Safety Factor versus Safety Index.

Figure 28. (Concluded).
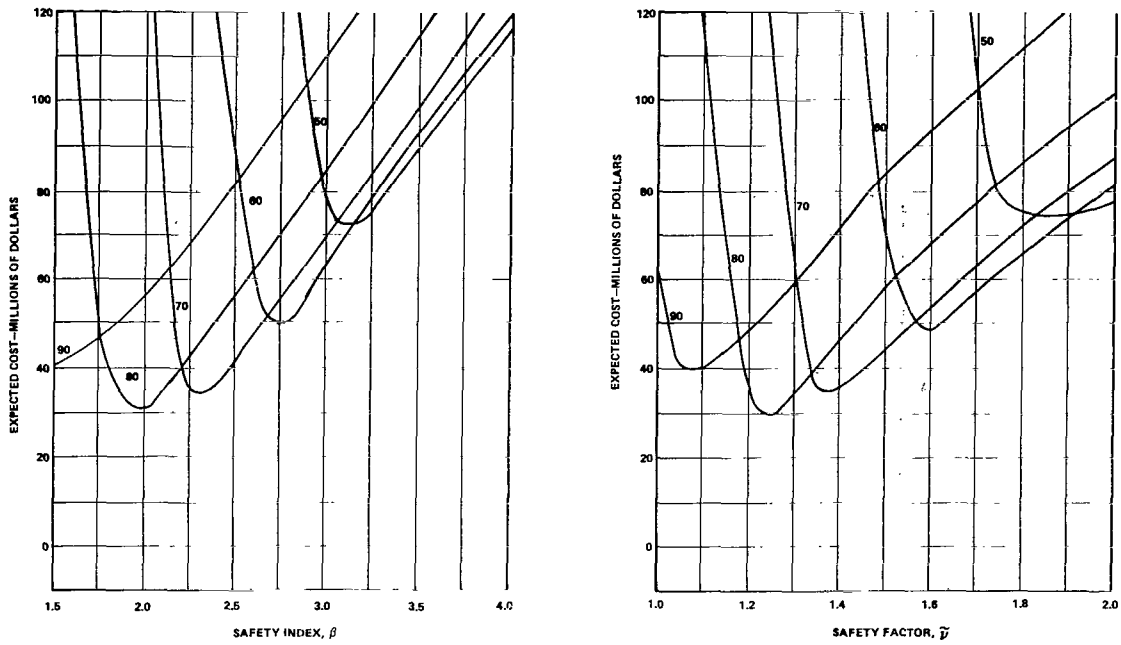
a. Test Option T_2.
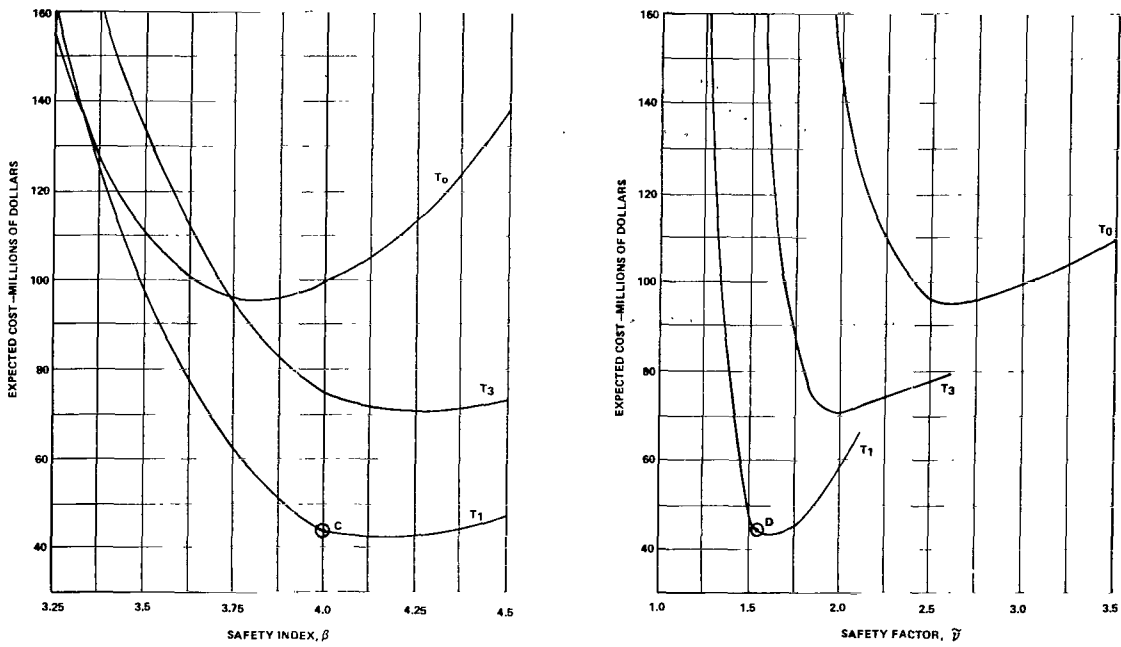
b. Test Options T_0, T_1, and T_3.

Figure 29. Expected Costs for ET Intertank.
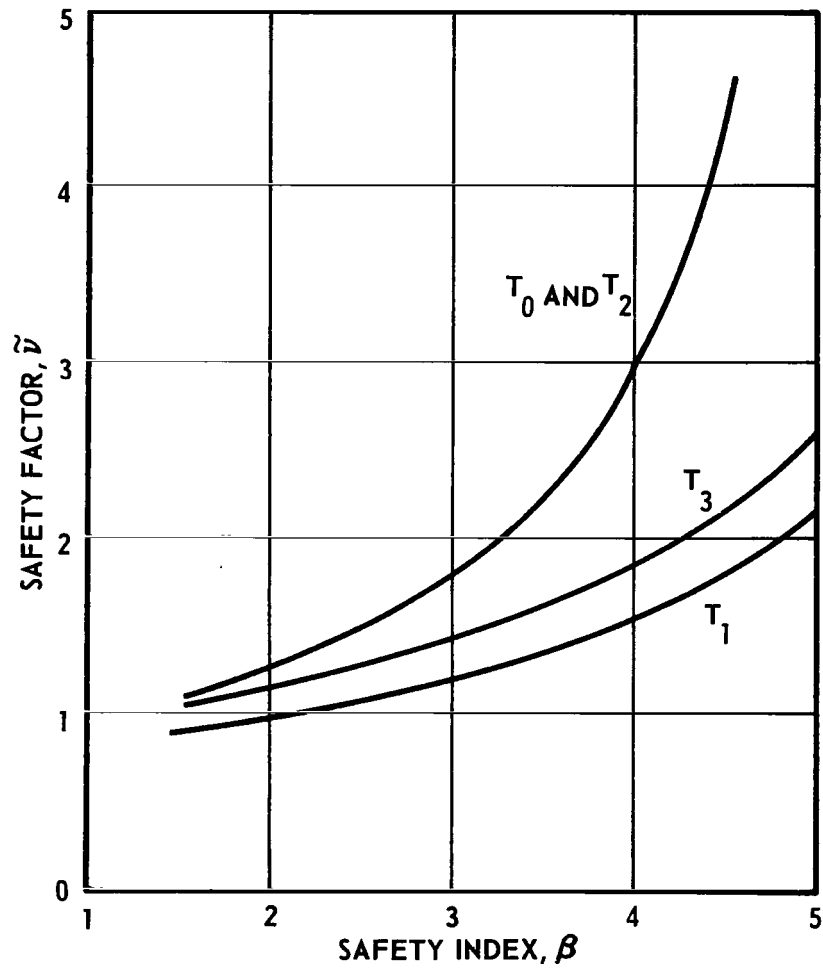
c. Safety Factor versus Safety Index.

Figure 29. (Concluded).
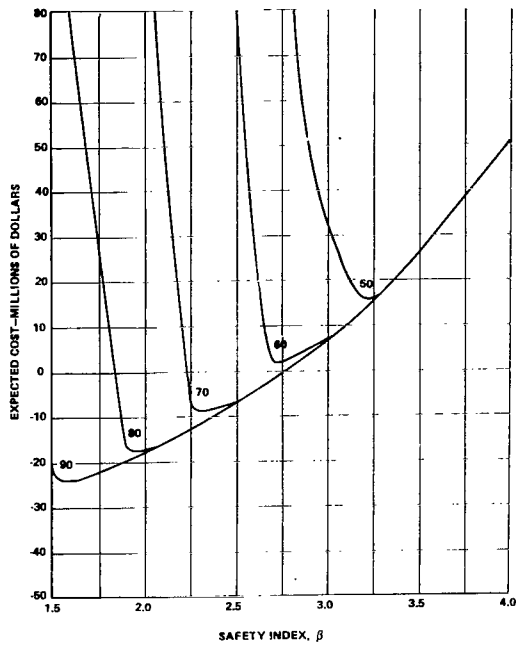
a. Test Option T_2.
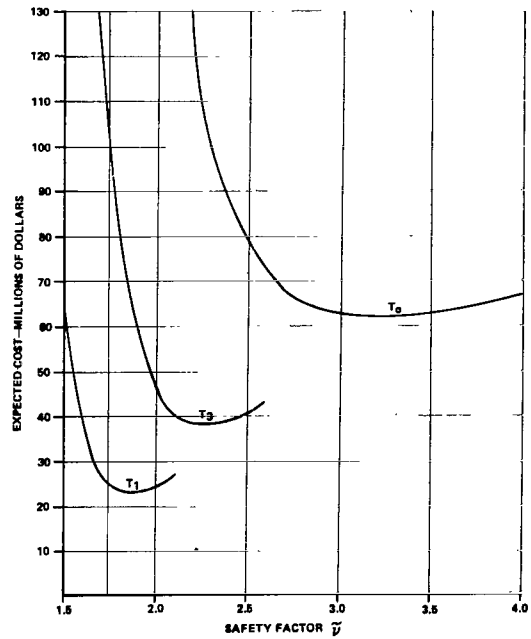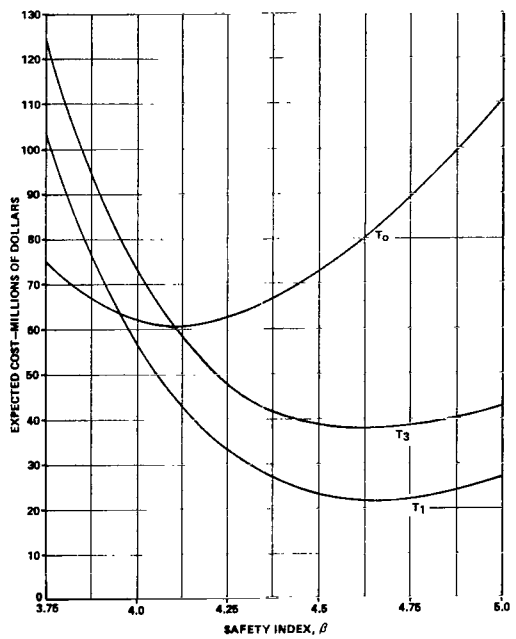
b. Test Options T_0, T_1, and T_3.

Figure 30. Expected Costs for Orbiter Aft Fuselage.
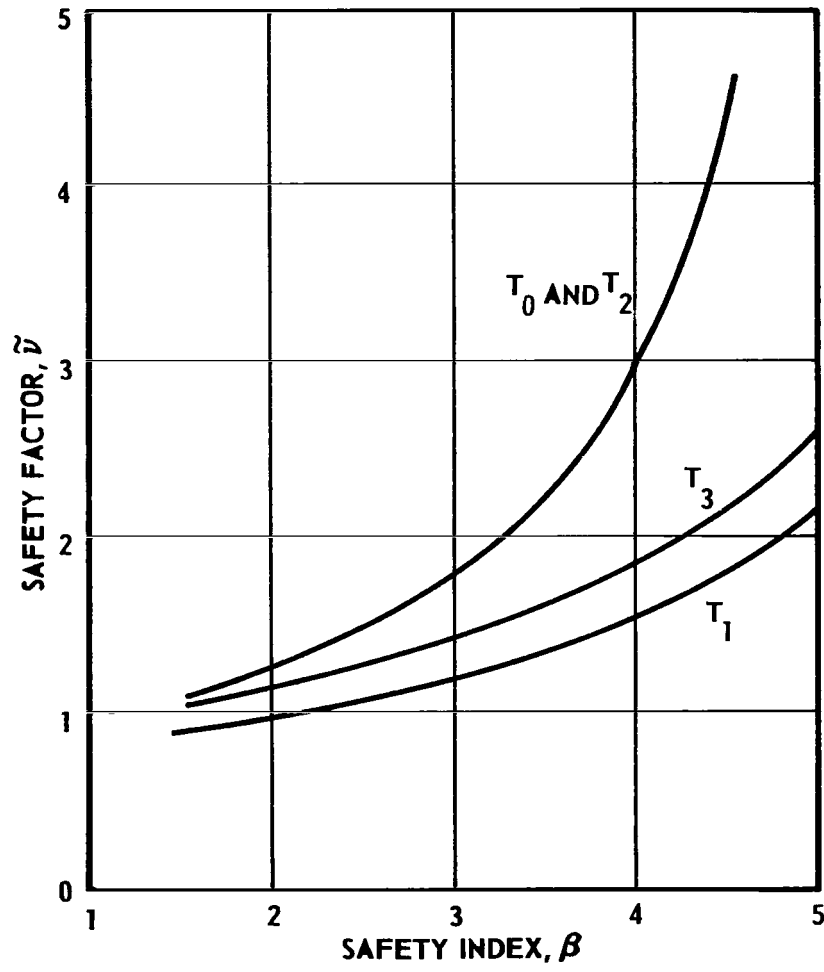
c. Safety Factor versus Safety Index.

Figure 30. (Concluded).
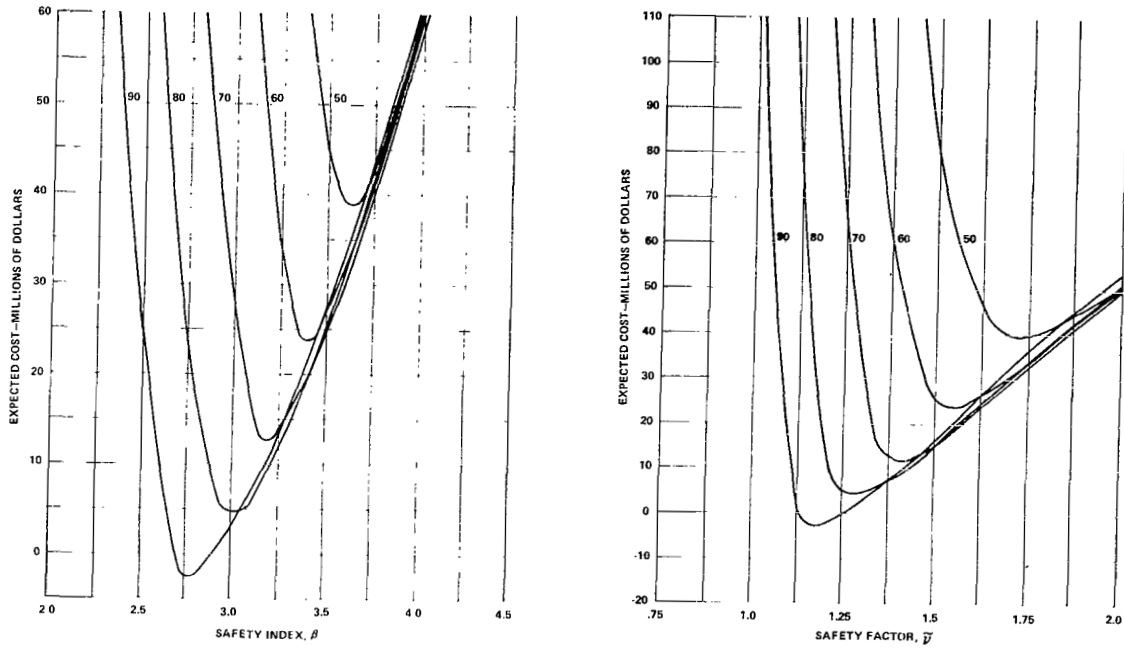
a. Test Option T_2.
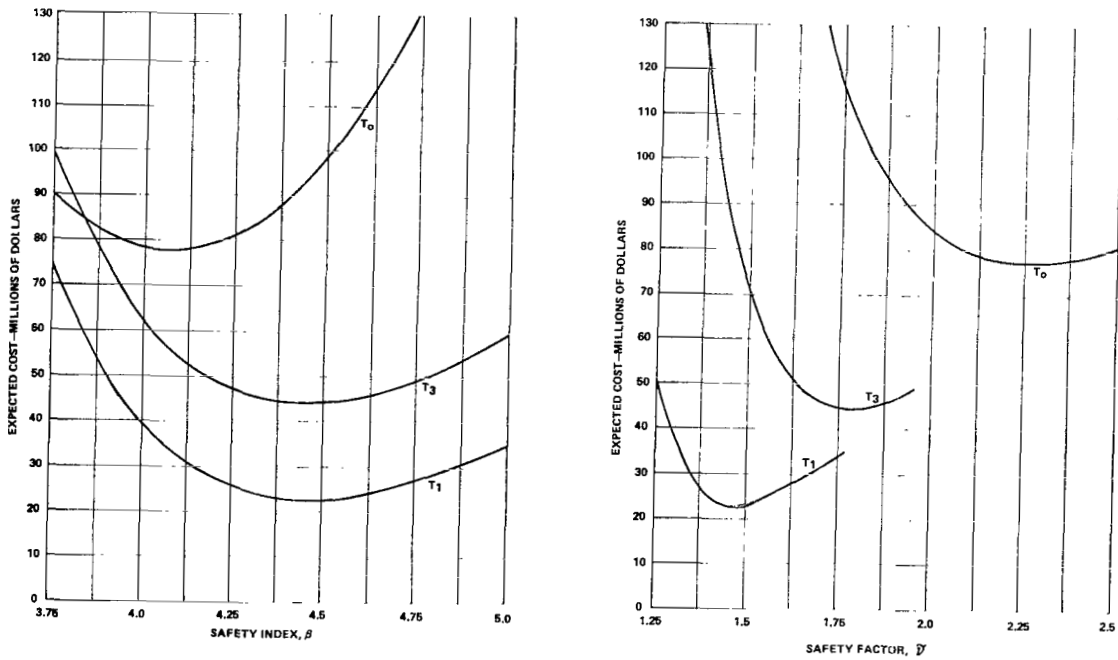
b. Test Options T_0, T_1, and T_3.

Figure 31. Expected Costs for Orbiter Wing.
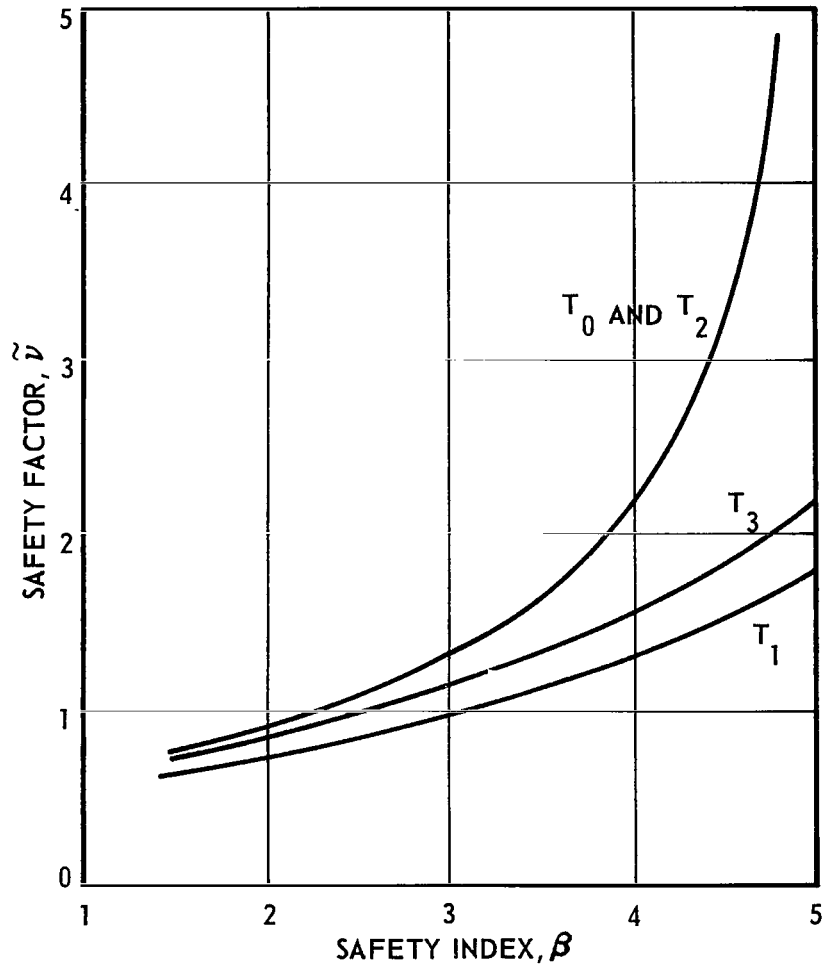
c. Safety Factor versus Safety Index.
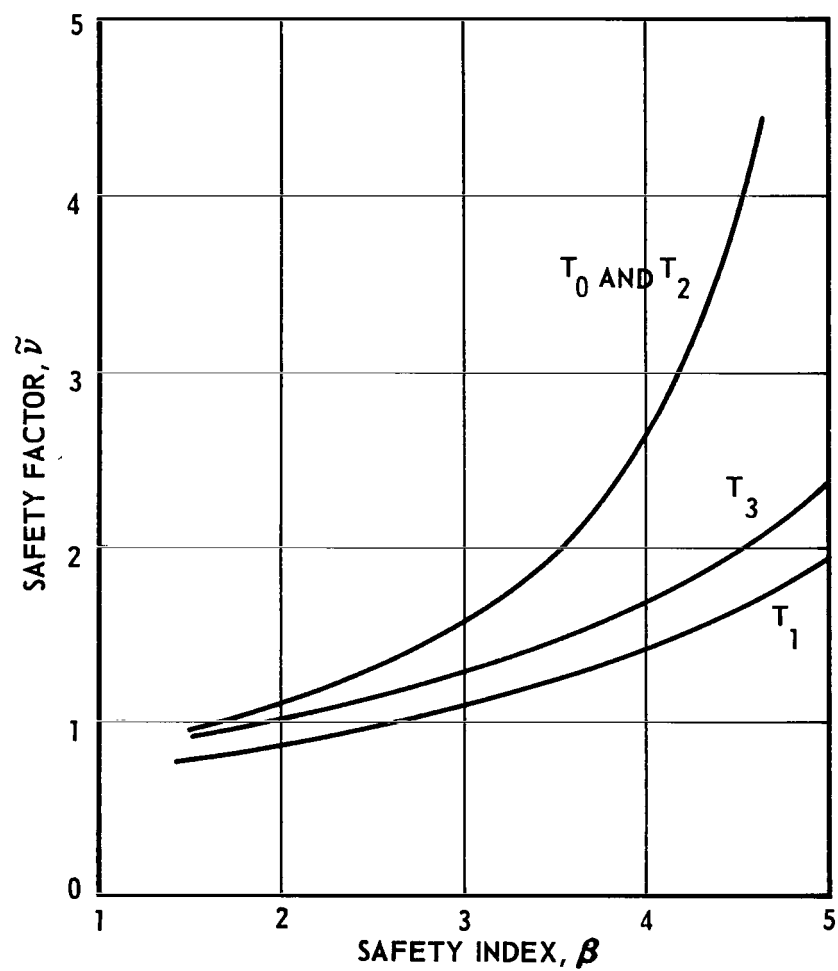
Figure 31. (Concluded).
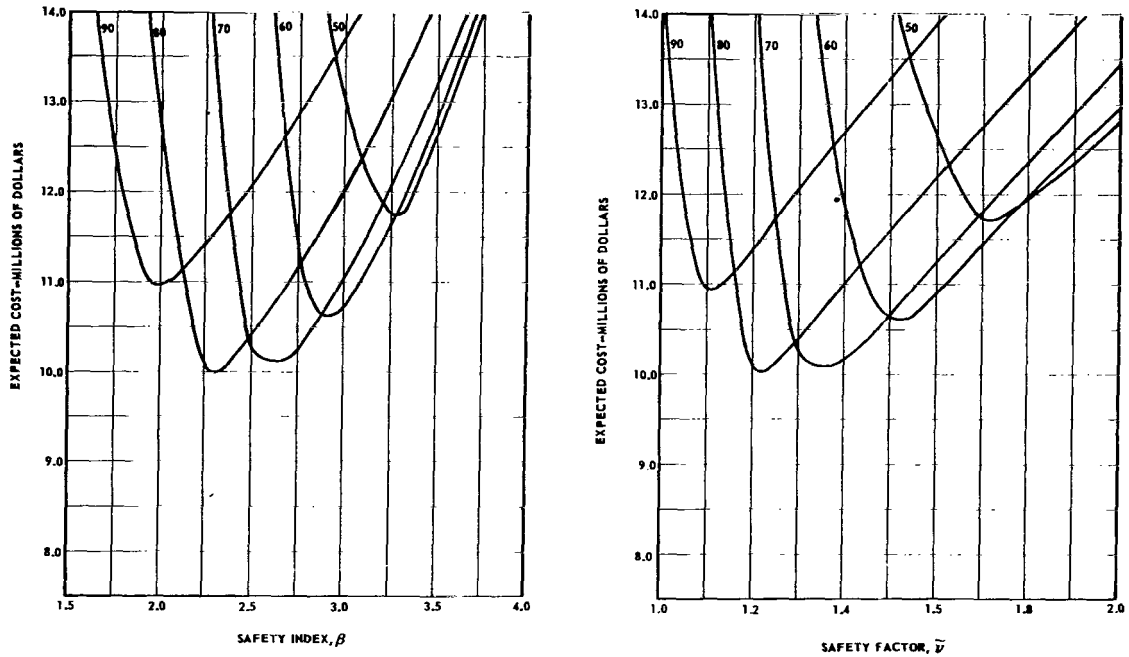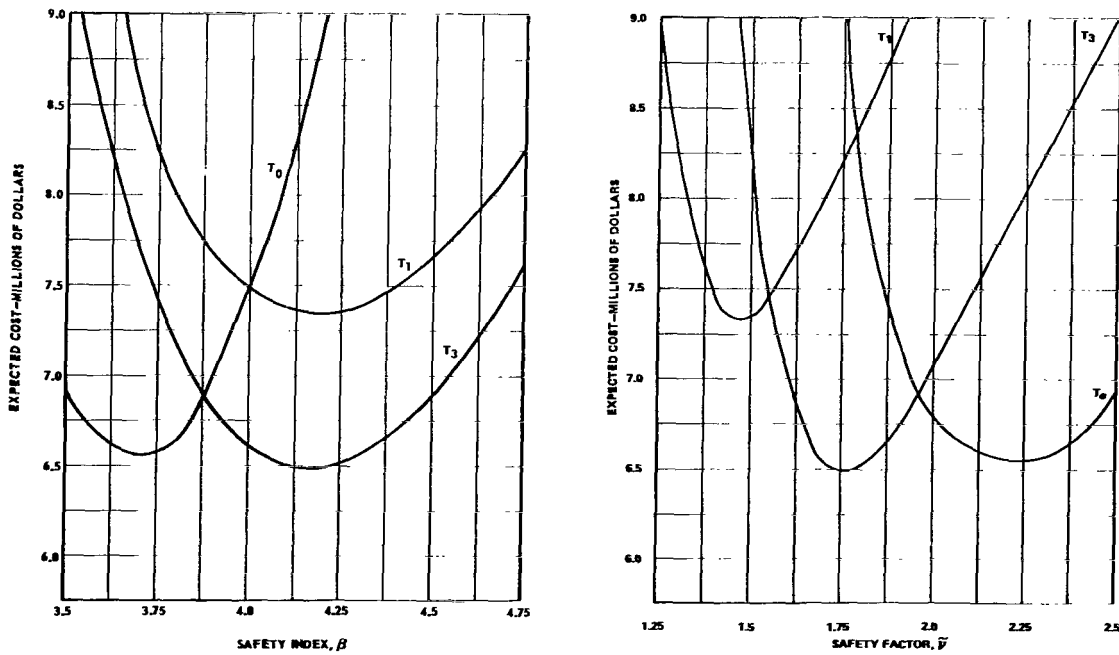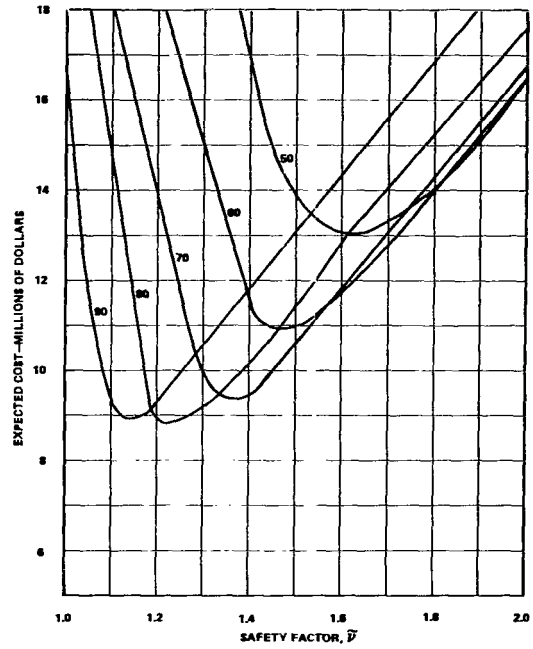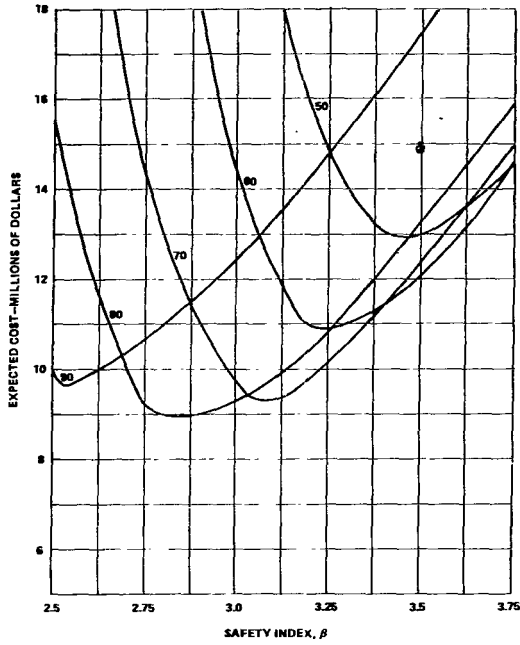
a. Test Option T_2.
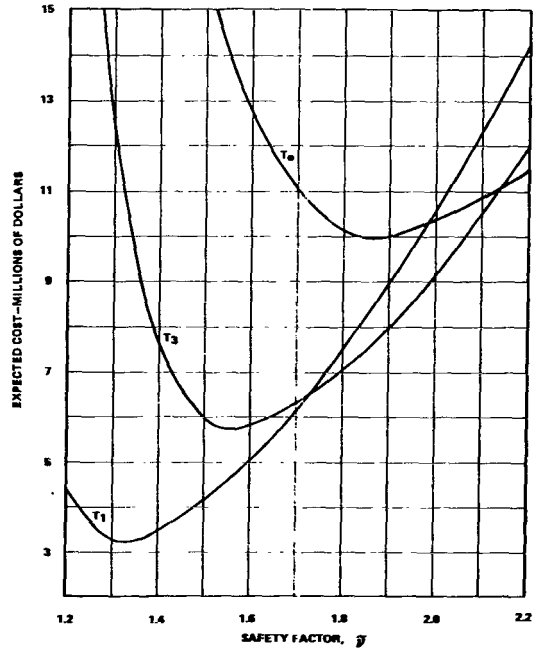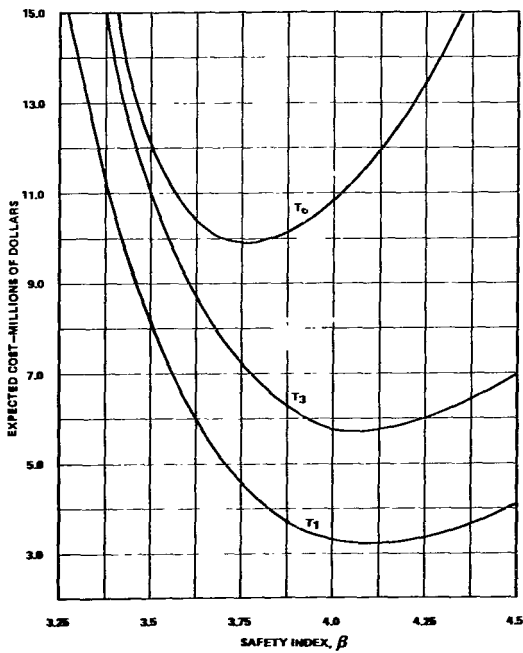
b. Test Options T_0, T_1, and T_3.

Figure 32.   Expected Costs for SRB Aft Skirt.
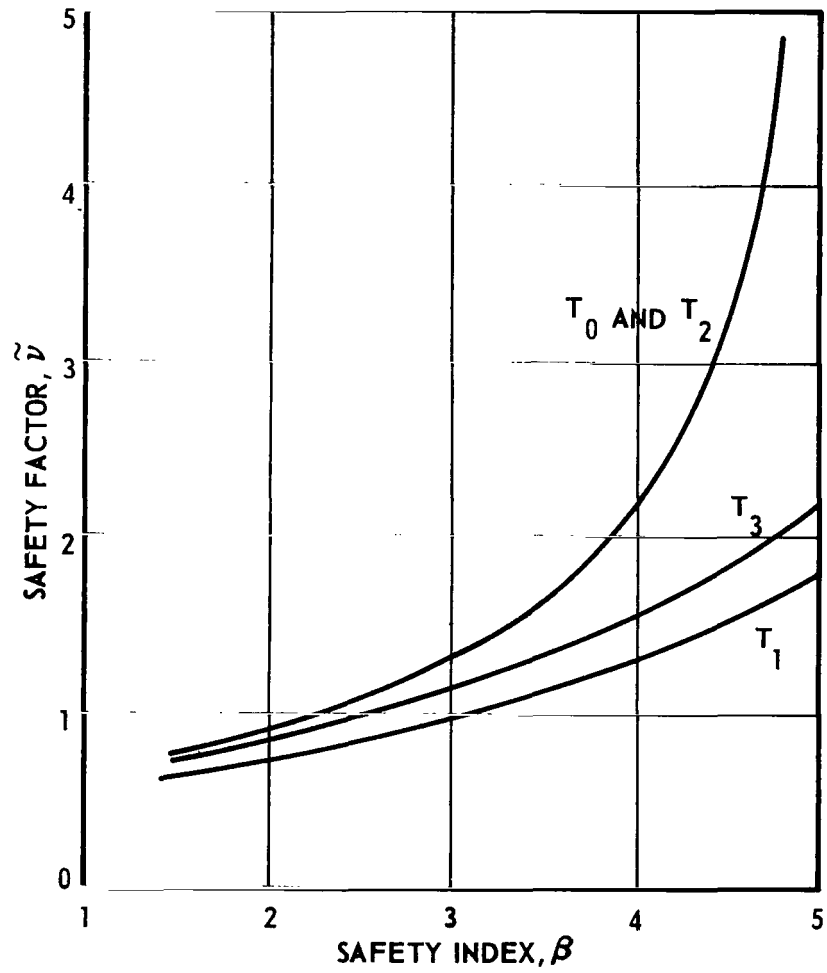
c. Safety Factor versus Safety Index.

Figure 32. (Concluded).

CHAPTER IX

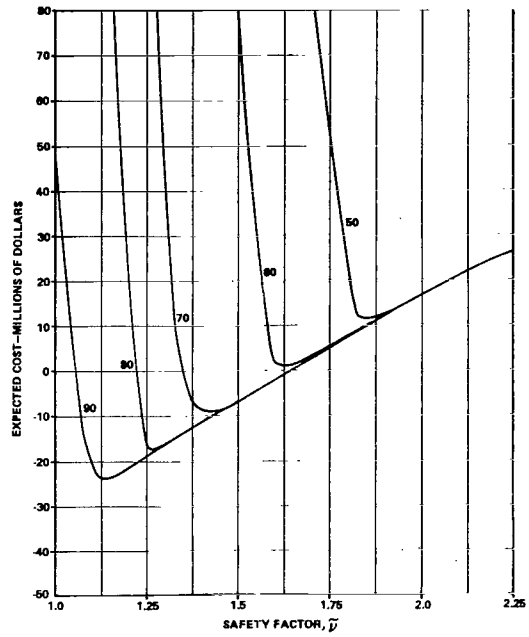RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

9.1 Results

A review of the background material in structural reliability was pre-
sented in Chapter I. This background was based on generally accepted principles
that had evolved from the literature on the subject, which was reviewed in
Chapter II. A problem was posed, and a method of solving the problem, based
on statistical decision theory was presented in Chapter III. Some concepts to be
used in the problem solution were carefully explained in Chapter IV. The test
option concept and a special safety factor definition were new ideas introduced
in Chapter IV. The test option formalizes certain structural test procedures in
a manner suitable for use as one of the decision elements in statistical decision
theory. The special safety factor definition will make the methods developed in
this dissertation much easier for the typical user to employ than would alterna-
tive safety factor definitions.

The use of subjective expert opinion in decision methods was exploited in
Chapter V. Questionnaires to obtain experts' opinions on structural reliability
and probabilistic models for analyzing the opinions were developed. The validity
of the questionnaire and modeling techniques was tested by using a trial ques-
tionnaire concerning a problem for which observed data were available for
comparison with the experts' opinions. This comparison revealed that one

particular model, which utilized the experts' overall assessment of a particular strength distribution, agreed remarkably well with the observed data. This model and questioning technique was subsequently used to develop a strength distribution for a test option for which no observed data were available.

In Chapter VI, Bayesian statistical methods were used for combining prior information from laboratory tests with new information from flight experience to develop a strength distribution for a particular test option. This was accomplished by fitting a two-parameter Weibull model to the laboratory data by the maximum likelihood method, then updating the scale parameter of this distribution with flight data, using Bayesian statistical methods.

A decision model for selecting the test option and design safety index which results in minimum expected cost was developed in Chapter VII and applied to six typical aerospace structural assemblies in Chapter VIII. Results which appeared reasonable were obtained for each of the six example problems.

9.2 Conclusions

1. The decision techniques developed in this research can readily and confidently be used for selecting design factors and test options for aerospace structures, if minimum expected cost is the desired objective.

2. Expert opinions are a valuable source of quantitative information in structural problems, provided that adequate care and deliberation are used in acquiring the opinions and analyzing the results.

3. Bayesian statistical techniques provide a viable method of combining prior information with new data in structural reliability problems.

9.3 Recommendations

As is the case with most research, the completion of this endeavor provides several new problem areas in which further research could be initiated. Some of these will be discussed briefly.

One group of such problems could come under the heading of an expansion of the decision space. The introduction of failure modes other than static strength failures would constitute such an expansion. For example, inclusion of fatigue failures would require that the decision space be expanded to include the selection of a fatigue test procedure and a fatigue safety measure, such as a design scatter factor. Another example would be the consideration of fracture mechanics where the decision would again involve selection of a test procedure and a safety measure appropriate to the fracture failure mode. Another consideration which would expand the decision space would be the selection of an optimal sample size (e.g. the optimum number of tests to run) which would minimize cost.

A second group of related problems would involve a coupling of the decision for a given assembly with the decision for all other assemblies in the system. Although all the ramifications of such a coupling have not been fully explored, it is quite certain that it would lead to a very sophisticated optimization procedure, possibly far beyond any that has been developed. This could be an area of research in its own right. One cause of such a coupling would be an assumption that the reliability of one assembly depends upon the reliability of another. The following conditions tend to reduce such a dependence: (1) a small

variance in load compared with a variance in strength, (2) loads in the various assemblies arising from different sources, and (3) high reliability in all assemblies. Another cause of assembly coupling would be the application of a constraint on the total reliability of a system made up of several assemblies. Such a constraint could possibly be dictated by management or contractual requirement and would cause the expected cost to be higher than the attainable minimum.

Other features which would increase the quality of a decision would be the consideration of multiple load conditions and of the lack of symmetry of the cost with respect to an error in the decision. The first feature would simply require that all load conditions be included in calculating the probability of failure. A special case of this would be the omega condition proposed by Bouton and Trent [19]. The omega condition is one of infrequent occurrence, such as a load that might be caused by failure of a nonstructural system. Such a loading could be treated as a load distribution conditional upon the probability of failure of the nonstructural system.

The unsymmetrical cost feature would be so devised that the decision-maker would not make the decision which absolutely minimizes cost but would bias his decision in the direction and by an amount that would minimize his risk when the asymmetry of cost with respect to his decision is considered.

In the previous discussion of test options in Chapter IV, it was implied that a test option is deterministic in nature, and this is the intention in the present research. However, a more refined approach would not prescribe a

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completely deterministic course of action but would recognize the probabilistic

aspects of testing, such as noise in the test results. To be consistent with the

Bayesian approach, it would not be sufficient to make inductive statistical

inferences about the probabilities involved, but a means would need to be

devised to cause such probabilities to be properly weighted as to their effect on

the decision at hand. This could be a challenging and rewarding area.

APPENDIX A

NOMENCLATURE

Symbol	Definition
A	A general event.
A, $A_j(\beta)$	Action or decision space.
A|B	Event A conditional on event B.
A ∩ B	Intersection of events A and B.
A ∪ B	Union of events A and B.
a	A particular action or decision.
B	A general event.
$\dot{C}$	Cause of a difference between actual and predicted strengths.
C, $C_j(\beta)$	Total expected cost.
$\overset{*}{C}$	Optimum cost.
C_D	Direct costs.
C_d	Development costs.
C_F, C_1	Cost of one failure.
$\overline{C}_F$, $\overline{C}_{F_j}(\beta)$	Expected cost of failure.
C_i	Cost of i failures.
C_w, $C_w(\beta)$	Cost of weight.
CDF	Cumulative distribution function.

NOMENCLATURE (Continued)

c_w	Unit cost of weight.
D	Discrepancy between mathematical model and actual structure.
D_1	A statistic measuring the difference between data and model.
d_1	A particular value of D_1.
E	An entry in response to questionnaire.
$\overline{E}$	Mean value of repondents' questionnaire entries.
$E(\lambda)$	Expected value of λ.
$E\{A\}$	The event that the statement A is true.
e	A constant, 2.71828
$F_X(x)$	Cumulative distribution function of the random variable X.
$f_X(x)$	Probability density function of the random variable X.
$f_X'(x)$	Prior density function.
$f_X''(x)$	Posterior density function.
$\tilde{f}_X(x)$	Bayesian density function.
I_i	Event of failure in load increment i.
i, j, k, l, m, n	Enumeration indices.
k	Shape parameter of the Weibull distribution.
$\hat{k}$	Maximum likelihood estimate of k.
L	Reliability; likelihood function.
$L_N(n)$	Probability that a structural assembly will survive n load applications.
ℓ	Number of observed failures in a sample.

NOMENCLATURE (Continued)

ℓ, ℓ', ℓ''	General, prior, and posterior values of a parameter of the gamma distribution.
m	Number of structural assemblies to be proof tested.
n	Number of observations in a sample; number of load cycles experienced by a structure; number of alternative actions.
O_i	Number of structures expected to fail in load increment i from observed data.
$P[A]$	Probability that event A will occur.
$P_F, P_{F_j}(\beta)$	Expected number of failures.
P_i	Probability of having exactly i failures.
PDF	Probability density function.
p	A constant; proof test cycle.
$p_f(k)$	Probability of failure on the kth application of load.
$\bar{p}_f(k)$	Probability of no failure on the kth application of load.
p_i	Probability of failure on the ith application of load and surviving all others.
$p_{i,r}$	Probability of failure on the ith and r^{th} applications of load and surviving all others.
q	A constant.
$R, R_j(\beta)$	Resistance or strength of a structural assembly.
R_p	A characteristic value of the resistance.
$\bar{R}$	Mean or expected value of the resistance.
r	A particular value of R.
r_0	Proof test load level.

NOMENCLATURE (Continued)

S	Load applied to a structural assembly.
S_q	A characteristic value of the load.
$\bar{S}$	Mean or expected value of the load.
SM	Safety margin.
$\overline{SM}$	Mean or expected value of the safety margin.
T	Test option.
U	Utility function.
v	Scale parameter of the Weibull distribution.
$\hat{v}$	Maximum likelihood estimate of v.
W, W(β)	Weight of a structural assembly.
X, Y, Z	Particular probabilistic models of experts' opinions.
y, y', y''	General, prior, and posterior values of a parameter of the gamma distribution.
α	A constant; an allowable risk; a scaling factor; significance level.
β	Safety index.
$\overset{*}{\beta}$	Optimum safety index.
Γ	The gamma function.
γ_R, γ_S	Coefficients of variation of resistance and load, respectively.
Θ	Vector of parameters of a probabilistic model; the state of nature.
θ	A particular value of Θ.
Λ	A parameter of the Weibull distribution.

NOMENCLATURE (Concluded)

λ A particular value of Λ.

ν Safety factor; factor for uncertainty.

$\hat{\nu}$ A safety factor in terms of characteristic values of resistance and load.

$\overset{\circ}{\nu}$ Safety factor in terms of mean resistance and mean load.

$\tilde{\nu}$ Safety factor in terms of analysts' predicted resistance and a characteristics value of load.

$\prod$ Indicates the product of following terms.

$\sum$ Indicates a summation of following terms.

$\sigma_R,\ \sigma_S,\ \sigma_{SM}$ Standard deviations of resistance, load, and safety margin, respectively.

APPENDIX B

QUESTIONNAIRES

This appendix contains the questionnaires, as well as the introductory letters, used to elicit the opinions of experts in structural analysis and testing. The opinions obtained were used in calculating strength distributions for some of the test options used in this dissertation.

Questionnaire A was used to test the method of acquiring opinions, the probabilistic models, and the ability of experts to provide the type of information needed. The results are analyzed in Chapter V. Questionnaire B was used for obtaining information to develop a strength distribution for test option T_3, the model test option. The required distribution is calculated from the Questionnaire B results in Chapter V. The Questionnaire C results yielded subjective information which was combined with observed data to obtain a strength distribution for test option T_1, the standard test option in Chapter VI.

The questionnaires are included here exactly as they were sent to the experts. To conserve space, the mean responses to the questionnaires [see equation (5.1)] are entered in boldface type in the spaces provided for the experts' responses.

GEORGIA INSTITUTE OF TECHNOLOGY

ATLANTA, GEORGIA 30332

SCHOOL OF
AEROSPACE ENGINEERING

404-894-3000

DANIEL GUGGENHEIM SCHOOL
OF AERONAUTICS

Dear _____ :

      I am engaged in research on structural reliability with Dr. S. V. Hanagud at Georgia Tech in aerospace structural engineering. The end product of this research will be a methodology for selecting a combination of design safety factors (or equivalent parameters) and structural testing options which yields a minimum expected system cost.

      The particular decision theory to be used in this process permits subjective information as well as objective information to be employed in reaching a decision. One type of subjective information is expert opinion. In certain test cases a remarkable correlation between collective expert opinion and independent objective data has been observed.

      The enclosed questionnaire is being sent to a few experts, such as yourself, in the area of structural analysis and testing. It is the first of a series of two similar questionnaires which I hope you can find time to answer.

      There is no reason for me to believe that your opinions in this area or your employment affiliation should be sensitive information. Nevertheless, your reply will be treated as sensitive, and no names or employers will be disclosed in the publication of results. In fact, the replies will be sanitized (identification removed) upon receipt, and the publication will be a conglomerate of replies in which no individual reply appears separately. It may be interesting for you to compare your own opinions with the collective opinions of other experts when the results are compiled. If you wish to receive a copy of the summary results, please check the box on the first page of the questionnaire.

      As you may know, I am an employee of the National Aeronautics and Space Administration. However, I am presently assigned to full-time graduate study and am asking this as a personal favor in the interest of research, and not as an employee of NASA. To answer the questionnaire will require some deliberation and at least one hour of your time. However, we believe this effort will be amply rewarded by a greater insight into the factors affecting structural reliability.

The questionnaire has three parts, arranged in order of importance. If time does not permit you to answer the entire questionnaire, perhaps you could complete Part I, which is especially simple. If you intend to reply, please do so by July 15, 1973. A mail-back envelope is enclosed for your reply to

>Dr. S. Hanagud
>School of Aerospace Engineering
>Georgia Institute of Technology
>Atlanta, Georgia 30332

If you are unable to complete the questionnaire for any reason, please mail it back anyway so we can keep track of our returns. Detach this letter for your records if you wish.

Thank you very much for your time.

>Sincerely,

>Jerrell M. Thomas

JMT/jct

Serial Number __**Mean Response**__
Questionnaire A

A STUDY OF FACTORS AFFECTING
THE ACCURACY OF AEROSPACE STRUCTURAL ANALYSIS

S. V. Hanagud and J. M. Thomas
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

( ) Please send summary of study results.

INTRODUCTION

EXPLANATION OF SERIES OF QUESTIONNAIRES

The series of questionnaires consists of two separate questionnaires, A and B. Questionnaire A is enclosed. Questionnaire B will be sent to respondents of A, after the results of A have been analyzed.

The purpose of A is to determine the validity of the expert opinion approach in this particular application. Some objective data are available for comparison with the collective expert opinion to be obtained from A.

If there is good correlation between the objective data and expert opinion from A, B will be sent out. B will relate to a situation where objective data are not available. Thus, the collective expert opinion will be the only source of data available for the situation in B.

EXPLANATION OF QUESTIONNAIRE A

The questionnaire seeks to determine the amount and source of errors which can be expected in aerospace static structural analysis before the analyst has obtained qualification test results. This is done by using the analyst's predicted failure load as a baseline value and attempting to determine the scatter about this baseline. The analyst's prediction contains no safety factor. That is, the 100% value is not limit load or limit load times the required safety factor, but is the load at which the analyst predicts the structure will fail. It is assumed that there is always some difference (sometimes very small) between the analyst's prediction and the actual failure load.

Innumerable questions would have to be answered to remove all ambiguity from the questionnaire. For example, one could legitimately ask: was the analysis done by the superior methods of my own company or by the obsolete methods of Company X? How much development testing or prior experience is associated with the design in question? What is the type of construction, etc? You are asked to answer these types of questions for yourself (and to yourself) by drawing on your own experience as to what constitutes a typical development program of a typical aerospace system.

It is perfectly acceptable to consult with your co-workers or to review any data which you may have in hand in answering the questionnaire. On the other hand, it is also appropriate to give completely "off-the-cuff" responses.

QUESTIONNAIRE A

PART I. HOW ACCURATE ARE ANALYSTS' PREDICTIONS?

Suppose that during a typical development program of a typical major aerospace system, the point in time has been reached for structural static qualification testing to begin. The test program is to be accomplished by testing one each of a number of large structural assemblies such as a complete wing, a thrust structure, a large fuselage section, an interstage structure, etc. For each assembly, the responsible stress analysts have given predictions of the ultimate failure loads under static conditions (no fatigue involved). We are interested in your judgement of how accurate we can expect these predictions to be.

In the table below are ten test failure load increments, expressed as a percentage of analysts' predictions. Please indicate the approximate percentage of test failures you would expect to fall in each increment for static structural tests of this sort. For example, if you believe 20% of the assemblies tested will fail between 80% and 90% of failure load predicted by the analysts, you would enter 20% in column (4).

Table 1. Accuracy of Analysts' Predictions

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.9	2.2	3.3	7.1	17.2	29.3	20.6	11.3	4.5	3.5

Check: Do your estimates of percent failures total 100%.

PART II(A). WHAT CAUSES ANALYSTS' ERRORS?

We are interested in your opinion of what typically causes predicted failure loads to be different from actual test failure loads in tests of large structural assemblies. Listed below are several possible causes for these differences, with space for you to add any other causes which might account for the differences.

In the table below, you are asked to indicate your judgement of the approximate percentage of all errors which are primarily caused by each of the causes listed (including any you have added). For example, if you feel that 5 out of every 100 prediction errors are due to "Scatter in material properties", please enter "5" opposite item 4 in the table.

Table 2. Causes of Errors

*Responses not shown. See letter introducing Questionnaire B.

Approximate
Percentage of
Errors from this
Cause

Primary Cause of Prediction Error

1. Analysts' mathematical model is a poor representation of the actual structure.   ____*_ %

2. Analyst made a mathematical error.   ____*_ %

3. Analyst overlooked (or failed to analyze) some critical area or failure mode.   ____*_ %

4. Scatter in material properties.   ____*_ %

5. Variation (within tolerance) of dimensions.   ____*_ %

6. Undetected manufacturing error.   ____*_ %

7. Test load condition did not represent flight condition used by analyst for prediction. (e.g., aerodynamic loads and boundary conditions are difficult to duplicate).   ____*_ %

8. Other causes (Please list).

_____

_____

_____   ____*_ %

Check: Do your percentages sum to 100%?   TOTAL 100%

PART II(B). SIZE OF ERRORS FROM DIFFERENT CAUSES

On the previous page you gave an indication of what you believe are the causes for differences in predicted and test strengths of structural assemblies. In the tables below you are asked to indicate the _amounts_ by which you believe these causes can affect the difference. The causes are to be considered one at a time. You are asked to assume that a given cause has been determined to be the reason for differences between test and predicted strengths in a number of assemblies.

Please enter in the tables below your opinion of the percentages of assemblies which you believe would fail in each load increment due to the given cause. The sum of entries in each table should total 100%.

Table 3. Analysts' Mathematical Model Is A Poor
Representation of the Actual Structure

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130 140	(10) >140
Your Estimate of Percentage of Failures	3.9	5.3	7.4	12.0	17.4	17.8	15.8	10.9	5.0	4.5

Table 4. Analyst Made a Mathematical Error

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	8.7	7.4	9.4	14.6	15.6	16.1	12.5	6.6	4.3	4.8

PART II(B). SIZE OF ERRORS FROM DIFFERENT CAUSES (Continued)

Table 5. Analyst Overlooked (or Failed to Analyze)
Some Critical Area or Failure Mode

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	7.6	11.2	16.2	21.2	26.6	7.3	5.3	3.2	1.1	0.4

Table 6. Scatter in Material Properties

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.3	0.5	1.2	5.8	25.4	35.0	17.5	8.9	4.4	1.0

Table 7. Variation (Within Tolerance) of Dimensions

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.2	0.4	2.3	7.0	32.4	36.7	12.3	5.1	3.1	0.5

PART II(B). SIZE OF ERRORS FROM DIFFERENT CAUSES (Continued)

Table 8. Undetected Manufacturing Error

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	8.6	10.9	13.6	16.9	24.3	14.4	5.5	3.7	1.5	0.7

Table 9. Test Load Condition Did Not Represent Flight Condition Used by Analyst for Prediction

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	1.3	3.0	7.9	16.1	25.7	20.8	13.9	7.0	3.6	0.7

Table 10. Other Causes

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	3.1	5.1	8.9	10.1	24.2	15.4	11.5	10.2	6.3	5.2

Check: Do your entries in each table total 100%?

PART III(A). DISCREPANCIES IN MATHEMATICAL MODELS

The purpose of Part III is to further quantify the errors in mathematical models (Cause Number 1). That is, the individual discrepancies which cause the analysts' mathematical model to be a poor representation of the actual structure will be examined.

In the table below, please indicate in the Frequency of Occurrence column the percentage of occasions you believe the discrepancy in the left column will be the primary reason that the analysts' mathematical model is in error. The Frequency of Occurrence column should total 100%.

Table 11. Frequency of Discrepancies

Discrepancy Between Analysts' Mathematical Model and Actual Structure	Frequency of Occurrence
1. A stress distribution is assumed instead of calculated. Example: Linear stress distribution assumed over beam cross section near end of beam.	12.4 %
2. A linear analysis is used where a nonlinear analysis is closer to reality. Examples: Elastic analysis used in plastic area. Small deformation theory used where deformations are large.	18.2 %
3. Empirical data is extrapolated too far. Example: Lightly stiffened shell buckling coefficients used for heavily stiffened shell.	13.1 %
4. Incorrect boundary conditions are used. Example: Fixed edge used where component joins another elastic structure.	17.3 %
5. Finite element model is too coarse. Example: Rapid stress changes at discontinuity or cutout not predicted by model.	16.6 %
6. Geometric idealization is incorrect. Example: Sheet-stringer combination treated as isotropic plate.	12.1 %
7. Anisotropic or inhomogeneous materials treated as isotropic or homogeneous. Example: Composite material analyzed by methods applicable only to homogeneous, isotropic materials.	6.1 %
8. Other discrepancies (Please list).	
	4.2 %
	Total: 100%

PART III(B). SIZE OF ERRORS FROM DISCREPANCIES IN MATHEMATICAL MODELS

In this part you are asked to indicate the amounts by which you believe the discrepancies listed on the previous page can affect the difference between predicted and test strengths. The discrepancies are to be considered one at a time. You are asked to assume that a given discrepancy is known to be the reason for differences between predicted and test strengths.

Please enter in the tables below your opinion of the percentages of assemblies which you believe would fail in each load increment due to the given discrepancy. The sum of entries in each table should total 100%.

Table 12. A Stress Distribution Is Assumed Instead of Calculated

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	3.5	3.3	5.6	15.7	23.3	18.8	14.9	7.9	5.0	2.0

Table 13. A Linear Analysis Is Used Where a
Nonlinear Analysis Is Closer to Reality

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	5.0	4.6	6.2	15.5	19.0	17.3	10.7	10.2	8.3	3.3

PART III(B). SIZE OF ERRORS FROM DISCREPANCIES IN MATHEMATICAL MODELS
(Continued)

Table 14. Empirical Data Is Extrapolated Too Far

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	3.2	5.6	10.0	15.7	24.1	19.5	10.3	6.7	3.7	1.2

Table 15. Incorrect Boundary Conditions Are Used

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	2.2	3.9	7.9	14.5	23.4	17.8	14.8	7.9	5.3	2.2

Table 16. Finite Element Model Is Too Coarse

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	2.5	4.3	11.0	24.1	24.5	15.0	9.8	4.7	3.1	1.0

PART III(B). SIZE OF ERRORS FROM DISCREPANCIES IN MATHEMATICAL MODELS
(Continued)

Table 17. Geometric Idealization Is Incorrect

	Test Failure Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	3.9	4.0	7.6	14.2	24.0	23.1	11.7	5.6	4.4	1.6

Table 18. Anisotropic or Inhomogeneous Materials
Treated As Isotropic or Homogeneous

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	8.3	7.0	10.2	16.5	22.3	16.1	10.2	5.0	3.2	1.3

Table 19. Other Discrepancies

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	7.2	3.3	7.8	15.6	18.9	22.2	12.2	7.2	3.9	1.7

Check: Do your entries in each table total 100%?

PART IV

Comments:

End of Questionnaire A

Please enclose in envelope provided and mail to:

Dr. S. Hanagud
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

Name: _____

Employer: _____

GEORGIA INSTITUTE OF TECHNOLOGY

ATLANTA, GEORGIA 30332

SCHOOL OF
AEROSPACE ENGINEERING

DANIEL GUGGENHEIM SCHOOL
OF AERONAUTICS

Dear _____:

Your response to our first questionnaire on factors affecting the accuracy of aerospace structural analysis was certainly appreciated. Some meaningful preliminary results have already been obtained from Questionnaire A, and a summary of those results is enclosed. Analysis of Questionnaire A is continuing along the line of searching for better data reduction techniques.

As mentioned in our first letter an enclosed second questionnaire has been prepared, which is similar to the first questionnaire, but deals with the increased analytical accuracy obtained through an expanded component and model development test program.

It has been determined that Part II(A) in Questionnaire A was probably poorly constructed (see enclosed Preliminary Results from Questionnaire A). Therefore, a revision of Part II(A) is included in this mailing. Please complete the revised Table 2 and return it with Questionnaire B.

Your reply to Questionnaire B is especially important to us, since it is being sent only to respondents of Questionnaire A.

If you intend to reply, please do so by September 17, 1973. Thank you again for spending part of your valuable time in helping with this reaearch.

Sincerely,

Jerrell M. Thomas

JMT/jt
Enclosure

Serial Number _____ **Mean Response** _____
Questionnaire B

A STUDY OF THE EFFECTS OF
MODEL AND COMPONENT TESTING IN INCREASING
THE ACCURACY OF AEROSPACE STRUCTURAL ANALYSIS

S. V. Hanagud and J. M. Thomas
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

( ) Please send summary of study results

INTRODUCTION

This questionnaire seeks to determine the amount and source of errors in aerospace static structural analysis, assuming that the analyst has available to him the results of an extensive component and model development test program. This is done by using the analyst's predicted value as a baseline value and attempting to determine the scatter about this baseline. The analyst's prediction contains no safety factor. That is, the 100% value is the load at which the analyst predicts the structure will fail. It is assumed that there is always some difference (sometimes very small) between the analyst's prediction and the actual failure load.

In Questionnaire A you were asked to assume that the analyst's predictions were based on a typical development program of a typical aerospace system. The point in time assumed was before any static qualification testing had been accomplished. The only change in these assumptions for Questionnaire B is that much more than a typical development program is assumed to have been accomplished in the area of component and model testing. The expanded component and model test program to be assumed is described qualitatively in the following paragraph.

A component is defined as a reasonably small structural subassembly such as a ring segment, a thrust post (longeron), or a major fitting. A model is a scaled (in the engineering sense) replica of a large structural assembly. Development testing is accomplished on such components and models through the use of an iterative process. The iterative process involves basing the test conditions on expected service conditions, prediction of test results by the best available analysis methods, direct and detailed comparison of test results with analytical predictions, and repetition of design, analysis, and testing until analysis and test results agree. Design of every subassembly of the flight article is then based on these components and models, and the flight article is analyzed by the same techniques which agreed with the model and component test results.

All parts of Questionnaire B pertain to the accuracy in analysis of the flight article after such an extensive development program but before any full scale structural qualification testing.

Even if you have no data or observations in such a situation, we are interested in your opinion about the expected accuracy.

QUESTIONNAIRE B ·

PART I. HOW ACCURATE ARE ANALYSTS' PREDICTIONS?

Suppose that during development of a typical aerospace system, a number of large structural assemblies have been developed through the model and component test procedure described in the introduction. For each assembly, the responsible stress analysts have given predictions of the ultimate failure loads under static conditions (no fatigue involved). We are interested in your judgement of how accurate we can expect these predictions to be.

In the table below are ten test failure load increments, expressed as a percentage of analysts' predictions. Please indicate the approximate percentage of test failures you would expect to fall in each increment if each assembly developed by the above procedure is subjected to a static test. For example, if you believe 20% of the assemblies tested will fail between 80% and 90% of failure load predicted by the analysts, you would enter 20% in column (4).

Table 1. Accuracy of Analysts' Predictions

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.11	0.94	3.33	8.00	19.72	41.40	17.23	7.36	1.72	0.17

Check: Do your estimates of percent failures total 100%?

PART II(A). WHAT CAUSES DIFFERENCES BETWEEN PREDICTED AND ACTUAL FAILURE LOADS?

Assume that differences of various magnitudes have been observed between analysts' predicted failure load and actual test failure load in each of 100 structural assemblies consisting of components developed according to the expanded component and model test program described in the introduction. In the table below is a list of possible causes of the observed differences.

Assume that in the sample of 100 structural assemblies you will always find one of the causes present, and that occasionally you may find more than one cause present. For example, scatter in material properties and variation of dimensions could be present in addition to an analyst's poor mathematical model.

In what percentage of the tests do you believe that each of the causes was a contributor to the observed difference, regardless of the magnitude of tne difference? Your entries in Table 2 can also be interpreted to represent how _often_ or how _frequently_ you think the given cause contributes to differences in predicted and actual strengths. Because the sample is concerned with all structures that differed from the analyst's prediction, without regard for the magnitude of the differences, any one or more of these causes (e.g. scatter in material properties or variation of dimensions* or any one of the other 8 causes in the list) may be responsible (Hypothetically) for only small magnitudes of differences, but could be present in large numbers in the sample of 100. Causes 4 and 5 are to be interpreted as normal or natural occurrences and the remaining causes are intended to represent errors or anomalies. Since one or more causes are assumed to always be present, your entries should total 100% or more.

Table 2. Causes of Differences

Cause of Difference	Frequency of Occurrence
1. Analysts' mathematical model is a poor representation of the actual structure.	**21.33** %
2. Analyst made a mathematical error.	**6.72** %
3. Analyst overlooked (or failed to analyze) some critical area or failure mode.	**24.44** %

*These causes are given as examples only and should not be permitted to influence your response.

Table 2. Causes of Differences (continued)

Cause of Difference	Frequency of Occurrence
4. Scatter in material properties.	18.00 %
5. Variation (within tolerance) of dimensions.	15.78 %
6. Undetected manufacturing error.	14.94 %
7. Test load condition did not represent flight condition used by analyst for prediction. (e.g., aerodynamic loads and boundary conditions are difficult to duplicate).	17.28 %
8. Other causes (Please list).	
	3.72 %
Total:	**122.21 %**

PART II(B). SIZE OF ERRORS FROM DIFFERENT CAUSES

On the previous page you gave an indication of what you believe are the causes for differences in predicted and test strengths of structural assemblies. In the tables below you are asked to indicate the amounts by which you believe these causes can affect the difference. The causes are to be considered one at a time. You are asked to assume that a given cause has been determined to be the reason for differences between test and predicted strengths in a number of assemblies.

Please enter in the tables below your opinion of the percentages of assemblies which you believe would fail in each load increment due to the given cause. The sum of entries in each table should total 100%.

Table 3. Analysts' Mathematical Model Is A Poor
Representation of the Actual Structure

| | Test Failure Load Increment - Percent of Analysts' Prediction | | | | | | | | | |
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	1.89	2.22	8.17	15.11	22.94	23.33	15.28	6.11	3.50	1.44

Table 4. Analyst Made a Mathematical Error

| | Test Failure Load Increment - Percent of Analysts' Prediction | | | | | | | | | |
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	1.69	3.88	8.56	18.44	23.88	21.81	11.88	5.81	2.81	1.25

PART II(B). SIZE OF ERRORS FROM DIFFERENT CAUSES (Continued)

Table 5. Analyst Overlooked (or Failed to Analyze)
Some Critical Area or Failure Mode

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	3.83	8.00	13.28	23.89	27.22	12.78	7.22	2.72	1.00	0.06

Table 6. Scatter in Material Properties

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.06	0.13	1.81	9.81	28.25	34.63	19.91	4.81	0.50	0.06

Table 7. Variation (Within Tolerance) of Dimensions

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.06	0.50	2.25	10.00	31.56	33.75	15.81	4.38	1.63	0.06

PART II(B). SIZE OF ERRORS FROM DIFFERENT CAUSES (Continued)

Table 8. Undetected Manufacturing Error

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	4.00	7.11	11.78	23.83	28.33	16.39	6.44	1.28	0.67	0.17

Table 9. Test Load Condition Did Not Represent Flight Condition Used by Analyst for Prediction

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.13	1.56	8.44	16.38	28.88	22.19	13.94	5.81	2.19	0.50

Table 10. Other Causes

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	2.50	11.00	10.50	16.00	24.50	15.50	9.00	6.50	3.00	1.50

Check: Do your entries in each table total 100%?

PART III(A). DISCREPANCIES IN MATHEMATICAL MODELS

The purpose of Part III is to further quantify the errors in mathematical models (Cause Number 1). That is, the individual discrepancies which cause the analysts' mathematical model to be a poor representation of the actual structure will be examined.

In the table below, please indicate in the Frequency of Occurrence column the percentage of occasions you believe the discrepancy in the left column will be the primary reason that the analysts' mathematical model is in error. The Frequency of Occurrence column should total 100%.

Table 11. Frequency of Discrepancies

Discrepancy Between Analysts' Mathematical Model and Actual Structure	Frequency of Occurrence
1. A stress distribution is assumed instead of calculated. Example: Linear stress distribution assumed over beam cross section near end of beam.	11.72 %
2. A linear analysis is used where a nonlinear analysis is closer to reality. Examples: Elastic analysis used in plastic area. Small deformation theory used where deformations are large.	17.65 %
3. Empirical data is extrapolated too far. Example: Lightly stiffened shell buckling coefficients used for heavily stiffened shell.	12.94 %
4. Incorrect boundary conditions are used. Example: Fixed edge used where component joins another elastic structure.	18.94 %
5. Finite element model is too coarse. Example: Rapid stress changes at discontinuity or cutout not predicted by model.	20.00 %
6. Geometric idealization is incorrect. Example: Sheet-stringer combination treated as isotropic plate.	13.59 %
7. Anisotropic or inhomogeneous materials treated as isotropic or homogeneous. Example: Composite material analyzed by methods applicable only to homogeneous, isotropic materials.	3.76 %
8. Other discrepancies (Please list).	2.00 %
	Total: 100%

PART III(B). SIZE OF ERRORS FROM DISCREPANCIES IN MATHEMATICAL MODELS

In this part you are asked to indicate the amounts by which you believe the discrepancies listed on the previous page can affect the difference between predicted and test strengths. The discrepancies are to be considered one at a time. You are asked to assume that a given discrepancy is known to be the reason for differences between predicted and test strengths.

Please enter in the tables below your opinion of the percentages of assemblies which you believe would fail in each load increment due to the given discrepancy. The sum of entries in each table should total 100%.

Table 12. A Stress Distribution Is Assumed Instead of Calculated

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.33	2.67	10.67	15.73	25.33	24.13	12.40	5.33	3.13	0.27

Table 13. A Linear Analysis Is Used Where a
Nonlinear Analysis Is Closer to Reality

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	1.69	2.13	8.63	18.69	23.75	22.81	12.88	5.75	2.69	1.00

PART III(B). SIZE OF ERRORS FROM DISCREPANCIES IN MATHEMATICAL MODELS (Continued)

Table 14. Empirical Data Is Extrapolated Too Far

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.47	4.59	13.06	22.24	25.12	16.53	10.88	4.82	2.12	0.18

Table 15. Incorrect Boundary Conditions Are Used

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.53	3.06	5.82	16.12	32.71	19.53	10.18	7.82	3.53	0.71

Table 16. Finite Element Model Is Too Coarse

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1) <60	(2) 60-70	(3) 70-80	(4) 80-90	(5) 90-100	(6) 100-110	(7) 110-120	(8) 120-130	(9) 130-140	(10) >140
Your Estimate of Percentage of Failures	0.71	4.59	13.00	22.76	30.76	15.59	7.53	3.76	1.18	0.12

PART III(B). SIZE OF ERRORS FROM DISCREPANCIES IN MATHEMATICAL MODELS
             (Continued)

Table 17. Geometric Idealization Is Incorrect

	Test Failure Increment - Percent of Analysts' Prediction									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	<60	60-70	70-80	80-90	90-100	100-110	110-120	120-130	130-140	>140
Your Estimate of Percentage of Failures	0.41	2.59	11.41	19.82	29.71	19.59	10.41	4.47	1.47	0.12

Table 18. Anisotropic or Inhomogeneous Materials
Treated As Isotropic or Homogeneous

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	<60	60-70	70-80	80-90	90-100	100-110	110-120	120-130	130-140	>140
Your Estimate of Percentage of Failures	2.73	5.67	12.00	17.27	25.33	17.33	10.93	5.00	3.33	0.40

Table 19. Other Discrepancies

	Test Failure Load Increment - Percent of Analysts' Prediction									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	<60	60-70	70-80	80-90	90-100	100-110	110-120	120-130	130-140	>140
Your Estimate of Percentage of Failures	1.67	3.83	7.50	22.00	21.67	26.33	9.17	4.17	2.00	1.67

Check: Do your entries in each table total 100%?

PART IV. COST OF DEVELOPMENT TEST PROGRAM

We are interested in the relative cost of the model and component development test program considered in this questionnaire. All costs related to the testing are of interest except construction of facilities. Some pertinent costs are test hardware, instrumentation, fixtures, and test operations.

What would you expect such a model and component test program to cost, expressed as a percentage of a typical static structural qualification test program?

50 %

PART V. COMMENTS

End of Questionnaire B

Please enclose in envelope provided and mail to:

Dr. S. Hanagud
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

Name: _____

Employer: _____

Revision to Part II(A) of Questionnaire A

Please complete and return with Questionnaire B

PART II(A). WHAT CAUSES DIFFERENCES BETWEEN PREDICTED AND ACTUAL FAILURE
LOADS?

Assume that differences of various magnitudes have been observed
between analyst's predicted failure load and actual test failure load in
each of 100 structural assemblies developed according to what you would
consider typical aerospace structural development program. In the table
below is a list of possible causes of the observed differences.

Assume that in the sample of 100 structural assemblies you will
always find one of the causes present, and that occasionally you may find
more than one cause present. For example, scatter in material properties
and variation of dimensions could be present in addition to an analyst's
poor mathematical model.

In what percentage of the tests do you believe that each of the
causes was a contributor to the observed difference, regardless of the
magnitude of the difference? Your entries in Table 2 can also be inter-
preted to represent how <u>often</u> or how <u>frequently</u> you think the given cause
contributes to differences in predicted and actual strengths. Because the
sample is concerned with all structures that differed from the analyst's
prediction, without regard for the magnitude of the differences, any one
or more of these causes (e.g. scatter in material properties or variation
of dimensions* or any one of the other 8 causes in the list) may be re-
sponsible (Hypothetically) for only small magnitudes of differences, but
could be present in large numbers in the sample of 100. Causes 4 and 5
are to be interpreted as normal or natural occurrences and the remaining
causes are intended to represent errors or anomalies. Since one or more
causes are assumed to always be present, your entries should total <u>100%
or more</u>.

Table 2. Causes of Differences

Cause of Difference	Frequency of Occurrence
1. Analysts' mathematical model is a poor representation of the actual structure.	**25.00** %
2. Analyst made a mathematical error.	**8.38** %
3. Analyst overlooked (or failed to analyze) some critical area or failure mode.	**24.69** %

---

* These causes are given as examples only and should not be permitted
to influence your response.

Table 2. Causes of Differences (continued)

Cause of Difference	Frequency of Occurrence
4. Scatter in material properties.	19.13 %
5. Variation (within tolerance) of dimension.	16.00 %
6. Undetected manufacturing error.	12.81 %
7. Test load condition did not represent flight condition used by analyst for prediction. (e.g., aerodynamic loads and boundary conditions are difficult to duplicate).	19.63 %
8. Other causes (Please list).	
	4.69 %
Total:	130.33 %

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AEROSPACE STRUCTURES

STRENGTH IMPROVEMENT BY DESIGN MODIFICATIONS

AND

LOADS EXPERIENCED BY SATURN V FLIGHT STRUCTURES

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QUESTIONNAIRE C

Questionnaire C consists of two separate areas of interest: improvement
in strength of structural assemblies gained by design modifications made
in the structure after test failures and loads typically experienced by
structures in flight. These areas will be considered separately in
Parts I and II.

Part I. IMPROVEMENT IN STRENGTH BY DESIGN MODIFICATIONS

Suppose that a large number of large aerospace structural assemblies,
designed and analyzed by typical methods, have been static tested. We
are interested in the portion of these structures that failed at less
than the analysts' predicted failure load, which is designated as 100%
design ultimate load (DUL). Suppose that of those structures failing,
8% failed between 70% and 80%, 35% failed between 80% and 90% , and
57% failed between 90% and 100% of the analysts' predicted failure load.
This is shown in line 1 of Table 1, below.

Now assume that failure analyses of these structures are conducted,
design changes are made and incorporated in the test assemblies, and
the assemblies are retested and all sustain at least 100% load.

Suppose that a new group of these assemblies is now built, and the
same design changes incorporated in the test assemblies are incorporated
in the new build. If this new build of assemblies should now be tested,
what percentage would you expect to fail in each load increment given
in Table 1? Enter your estimates in line 2. Your estimates should
total 100%.

Table 1. Strength Improvement

| Line | Description | Test Failure Load Increment - Percent of Analysts' Prediction (DUL) | | | |
		(1) 70 - 80	(2) 80-90	(3) 90-100	(4) > 100
1	Percent of Structures Failing First Test in Each Increment	8	35	57	-
2	Your Estimate of Percent of Redesigned Structures Failing in Each Increment	0.11	1.22	6.78	91.89

Part II. LOADS EXPERIENCED BY SATURN V FLIGHT STRUCTURES

We are interested in loads experienced by large structural assemblies
of the Saturn V system in actual operation. Many structural assemblies
have more than one critical design condition. In all that follows,
consider the maximum load experienced in each mission for each design
condition as one "loading" of the structure. Of all loadings experienced
by all assemblies of the Saturn V launch vehicle in all missions to date,
what percentage of the loadings do you believe fell in each load increment
shown in Table 2, below? The Table is in terms of design ultimate load.
Your entries should total 100%.

Table 2. Loads on Saturn V Structures in Operation

	Flight Load Increment - Percent of Design Ultimate Load						
	(1) <30	(2) 30- 40	(3) 40- 50	(4) 50- 55	(5) 55- 60	(6) 60- 65	(7) 65- 71*
Your Estimate of Percent of Structures Experiencing Loading in Each Increment	15.00	7.22	9.22	12.44	20.22	13.78	22.11

* 71% of design ultimate load is approximately limit load. If you are
aware of any Saturn structures experiencing greater than limit load,
please discuss briefly in the space below.

APPENDIX C

PARAMETRIC ANALYSIS

This appendix gives the results of a parametric analysis which was done to establish the sensitivity of the probability of failure of a structural assembly to certain variables: test option T_i, safety index β, safety factor $\tilde{\nu}$, and coefficient of variation of load γ_S. The figures presented here are very useful for determining the probability of failure for a single application of load, $p_f(k)$, as given by equation (7.6), Chapter VII. The $p_f(k)$ determined from these figures can then be entered into the cost model through equation (7.30), Chapter VII, or used for any other purpose for which $p_f(k)$ is required.

The figures, in conjunction with the nomenclature in Appendix A, are essentially self-contained, and no detailed explanation of each figure is required.

An outline of the method used to calculate a typical point for plotting on the $p_f(k)$ versus β curves follows. From the definition of the safety index,

$$\beta = \frac{\overline{R} - \overline{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad , \qquad \text{(C-1)}$$

where $\sigma_R = \gamma_R \overline{R}$ and $\sigma_S = \gamma_S \overline{S}$. Then

$$\beta = \frac{\overline{R} - \overline{S}}{\sqrt{\gamma_R^2 \overline{R}^2 + \gamma_S^2 \overline{S}^2}} \qquad \text{(C-2)}$$

Note that for positive values of β, $\overline{R} > \overline{S}$. If both sides of equation (C-2) are squared, and the resulting equation is solved for $\overline{S}$,

$$\overline{S} = \overline{R} \; \frac{1 \pm \sqrt{1 - (1 - \gamma_S^2 \beta^2)(1 - \gamma_R^2 \beta^2)}}{1 - \gamma_S^2 \beta^2} \qquad \text{(C-3)}$$

Interest is usually in positive values of β, and $\overline{S}$ and $\overline{R}$ are defined as positive real quantities. Note that the denominator of equation (C-3) is always less than unity. Therefore, if $\overline{R}$ is to be greater than $\overline{S}$ as must be the case for positive β, the numerator in brackets must also be less than unity. This implies that the minus sign must always be chosen in front of the radical.

$$\overline{S} = \overline{R} \; \frac{1 - \sqrt{1 - (1 - \gamma_S^2 \beta^2)(1 - \gamma_R^2 \beta^2)}}{1 - \gamma_S^2 \beta^2} \qquad \text{(C-4)}$$

Now, beginning with an assumed test option (which uniquely determines $\overline{R}$ and γ_R), an assumed value of γ_S, and an assumed positive value of β, $\overline{S}$ can be calculated from equation (C-4). If a normal distribution of the applied load is assumed, then sufficient information is now available to evaluate the integral equation (7.6) of Chapter VII and plot one point on the $p_f(k)$ versus β curves. Next, additional values of β are assumed with the other parameters held fixed, and several points are plotted to establish one curve. These computations are easily carried out on a hand calculator or a small computer.

Once $\overline{S}$ has been determined from equation (C-4), the safety factor $\tilde{\nu}$ consistent with the assumed value for β can be determined from equation (4.20)

of Chapter IV. To do this, a value of q must be selected, and a value of 3.0

was selected for q for all numerical computations in this dissertation. The

results of the parametric calculations are shown in Figures 33 through 38.
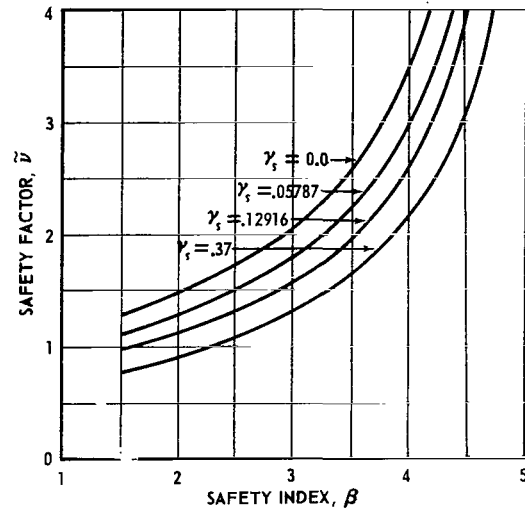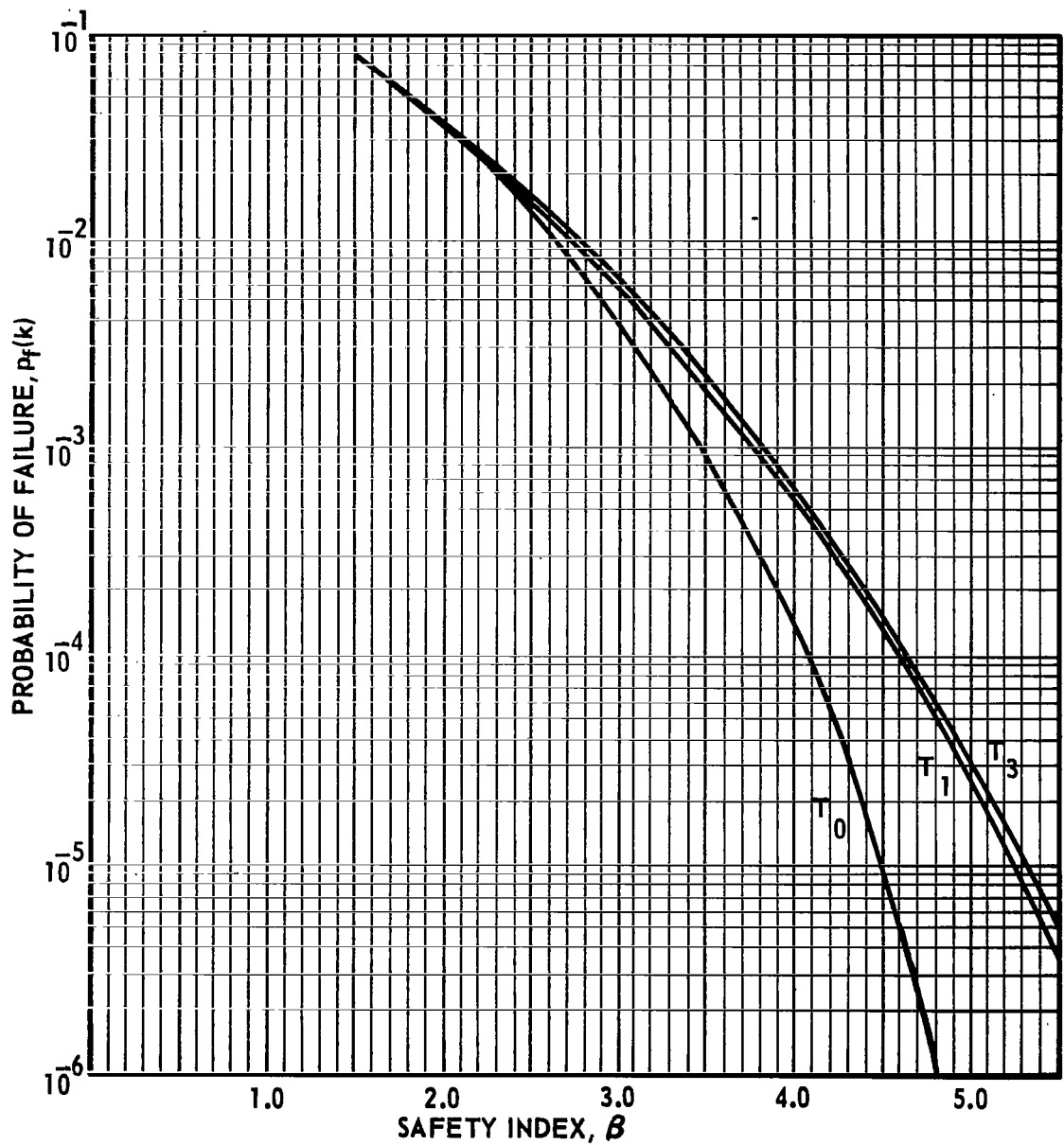
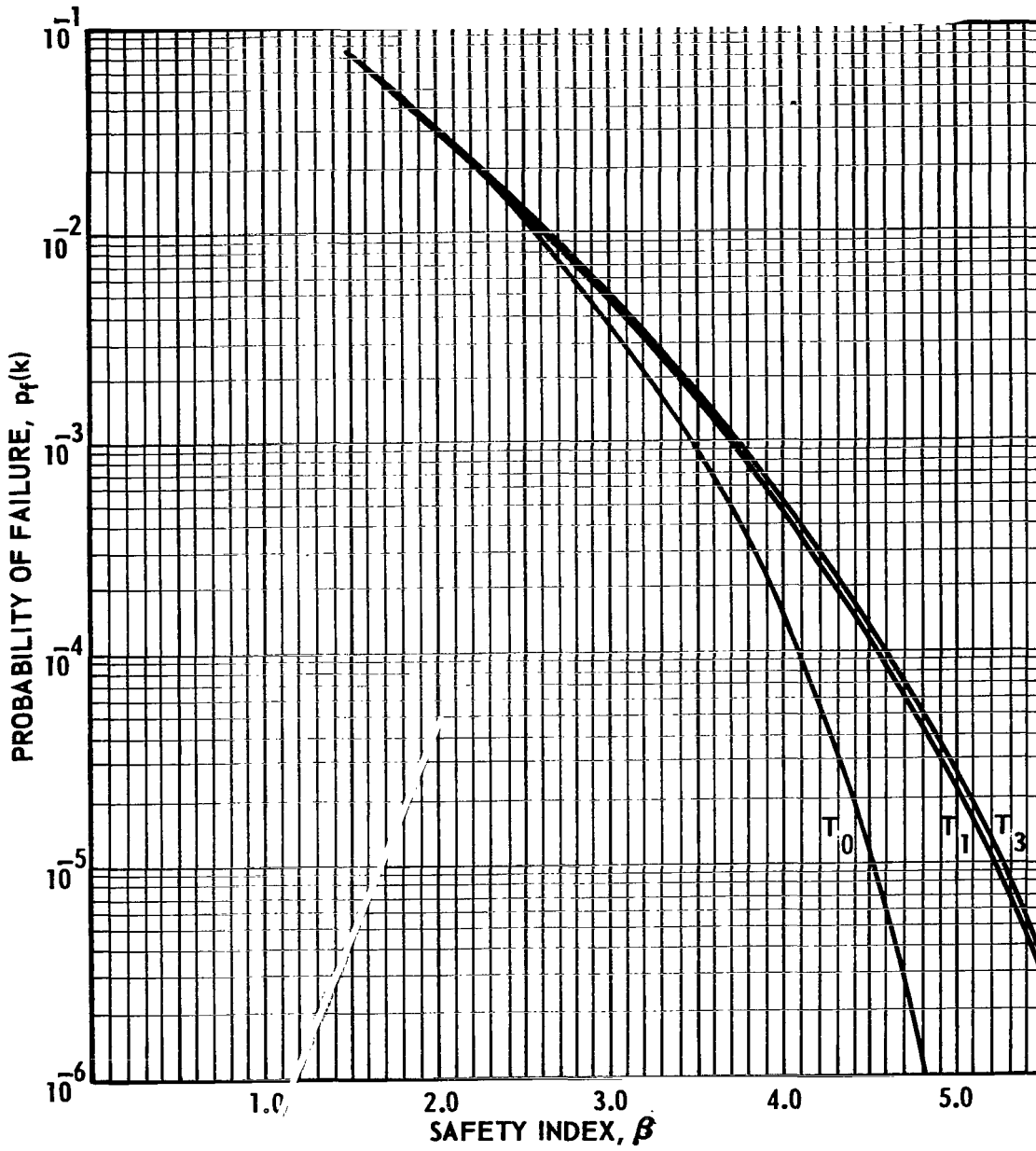a. Test Options T_0 and T_2.
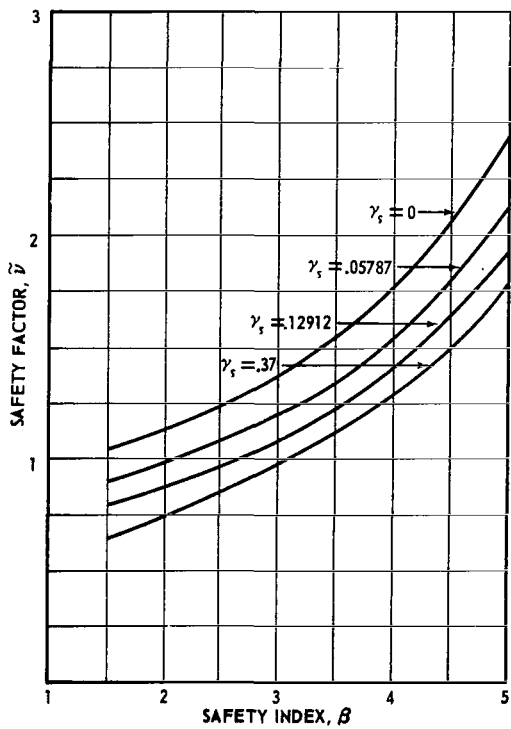
b. Test Option T_1.
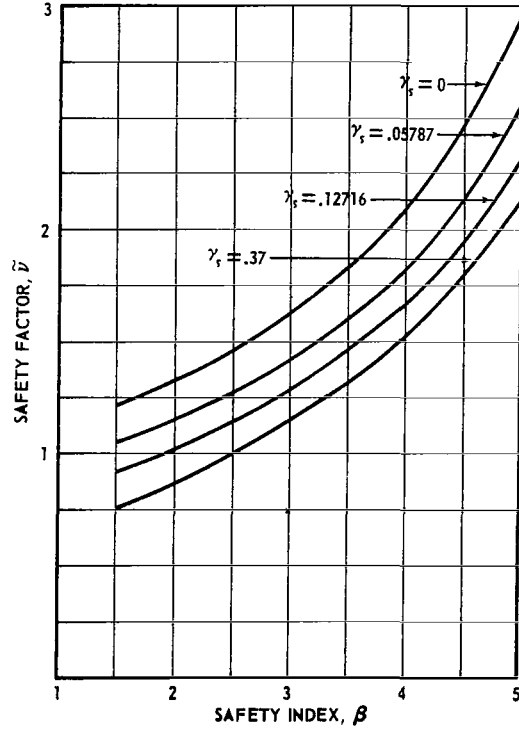
c. Test Option T_3.

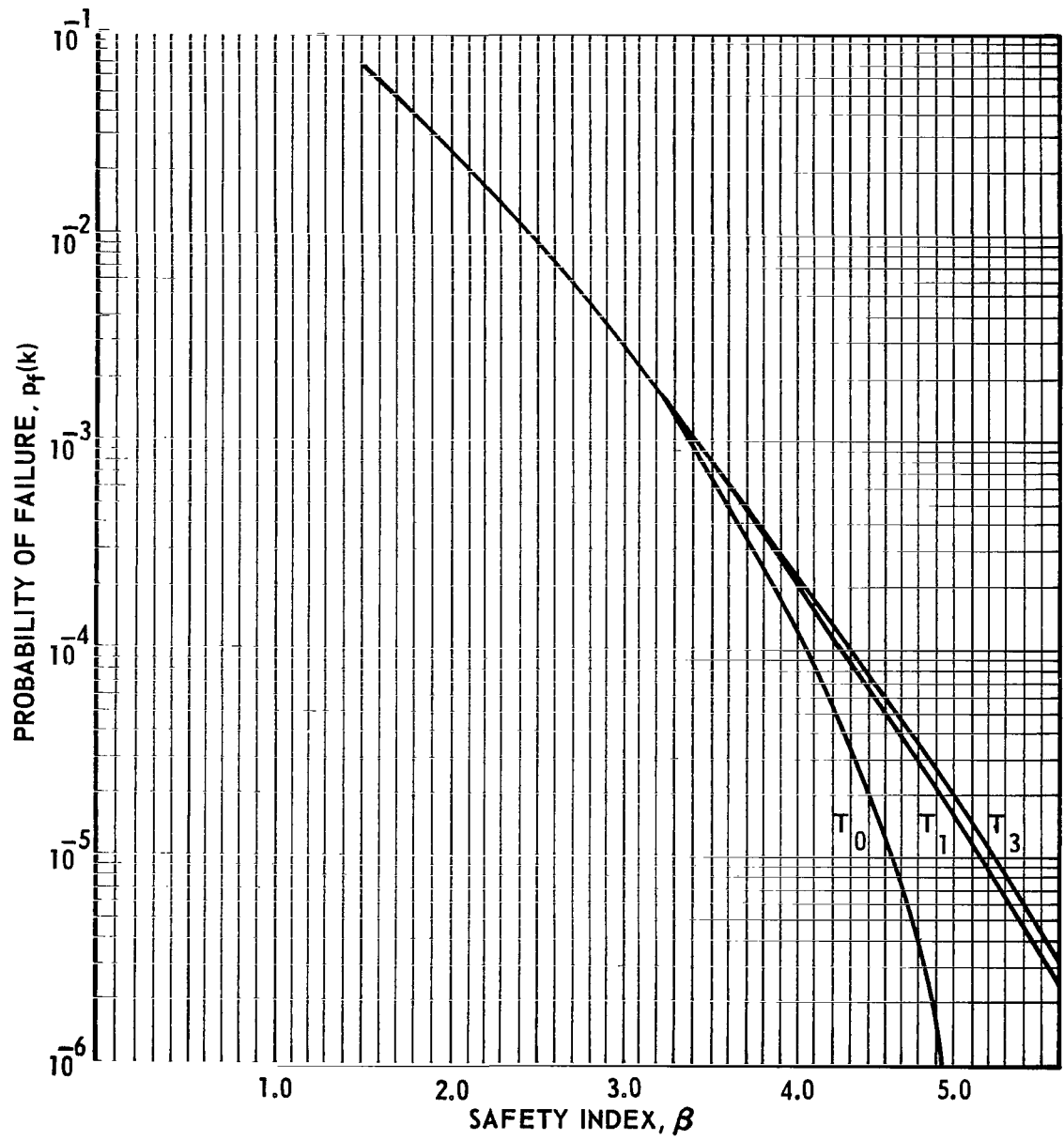Figure 33. Relationship of Safety Factor and Safety Index.

a. $\gamma_S = 0.05787.$

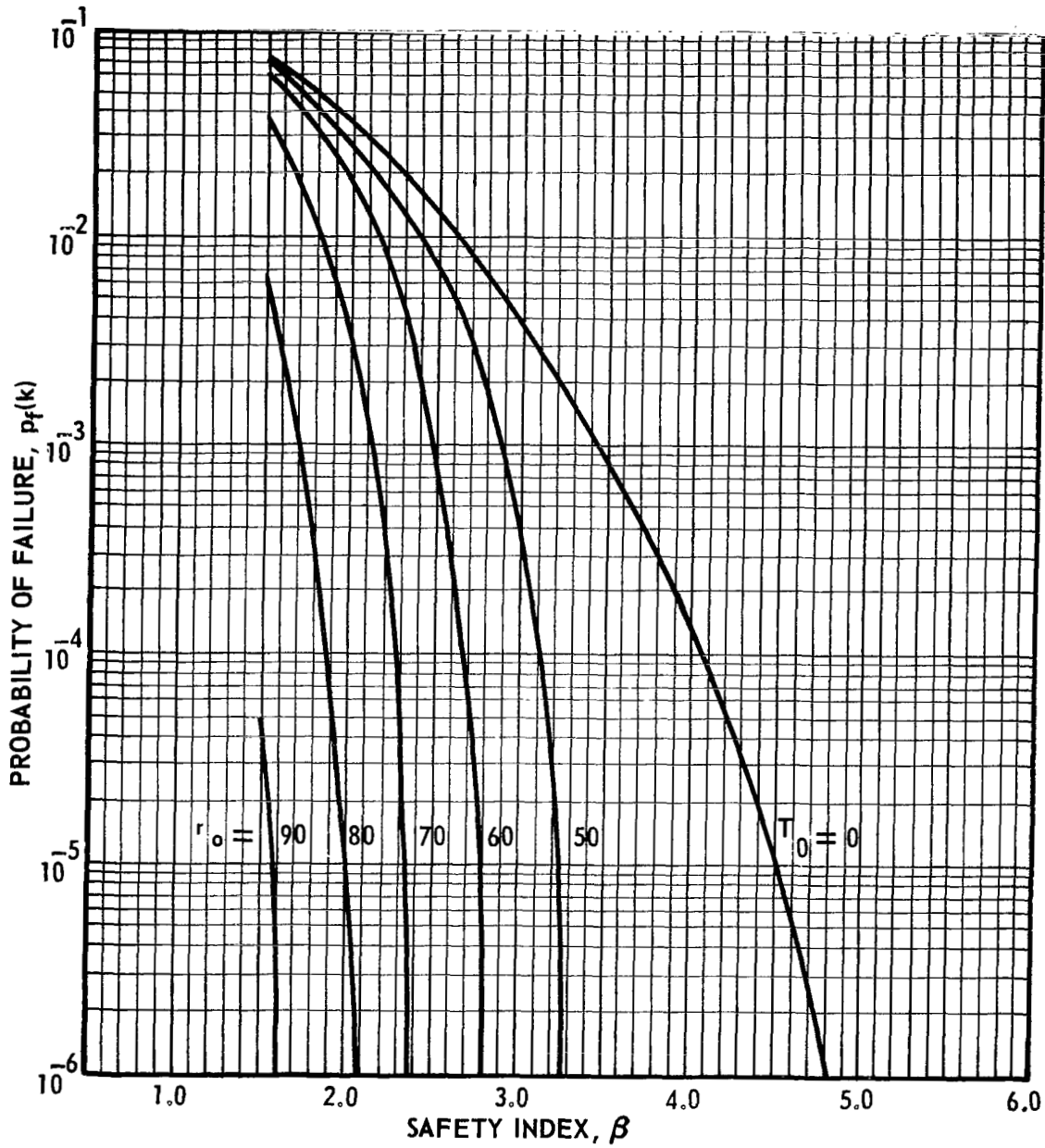Figure 34. Probability of Failure versus Safety Index for
Test Options 0, 1, and 3.
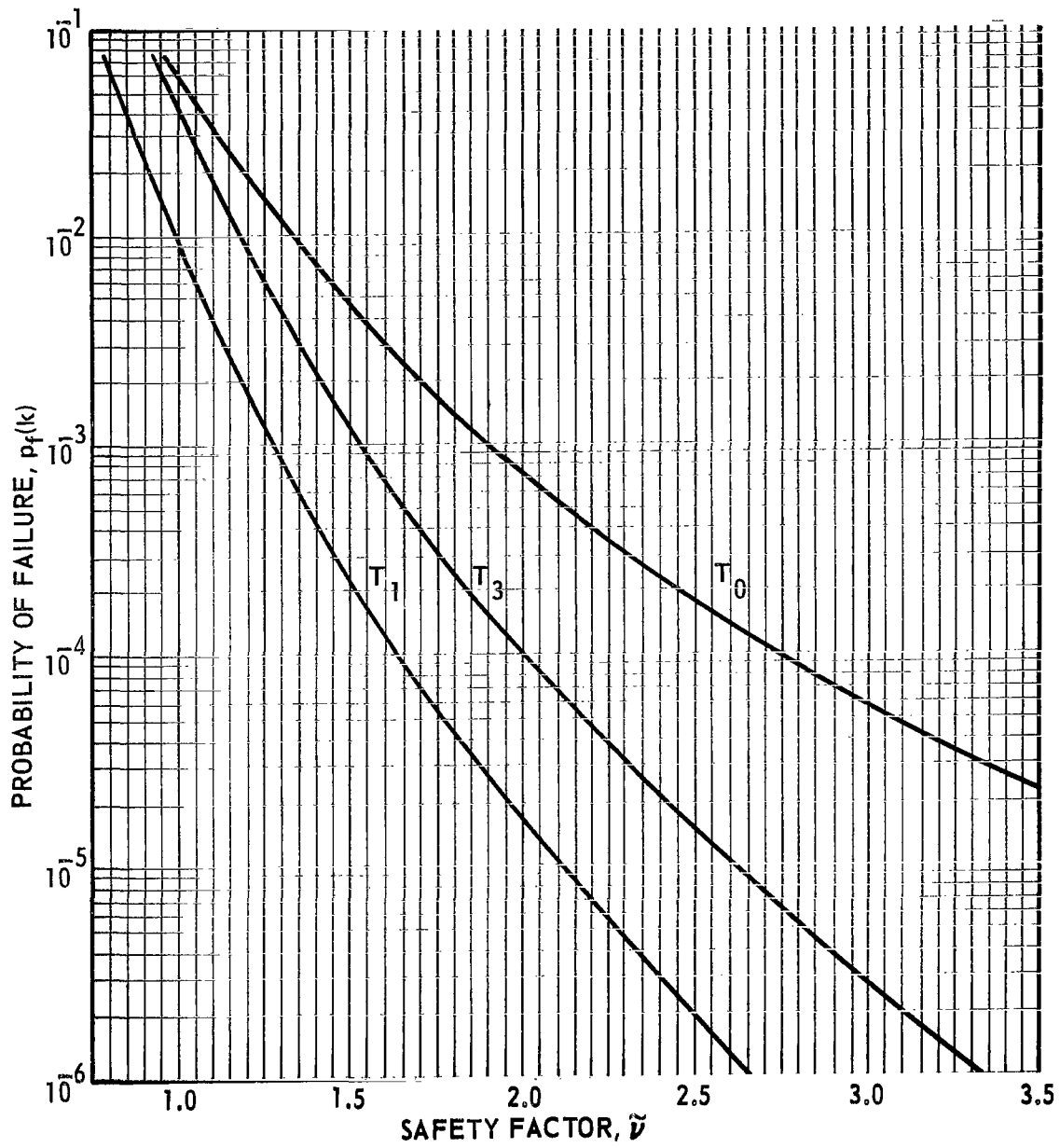
b. $\gamma_S = 0.12916$.

Figure 34. (Continued).

c. $\gamma_S = 0.37$.

Figure 34. (Concluded).

a. $\gamma_S = 0.05787.$

Figure 35. Probability of Failure versus Safety Index for
Test Option 2.
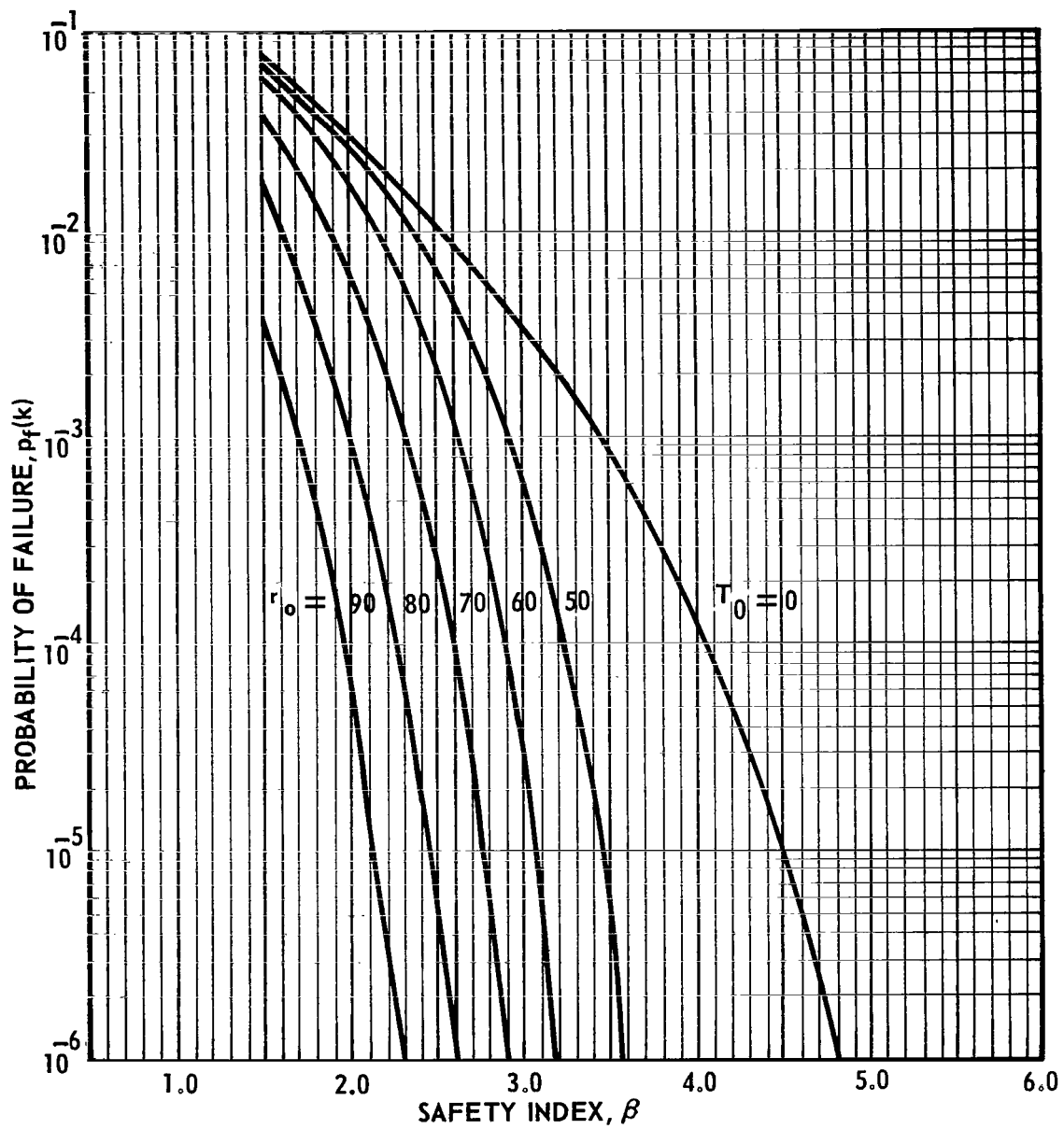
b. $\gamma_S = 0.12916$.

Figure 35. (Continued).
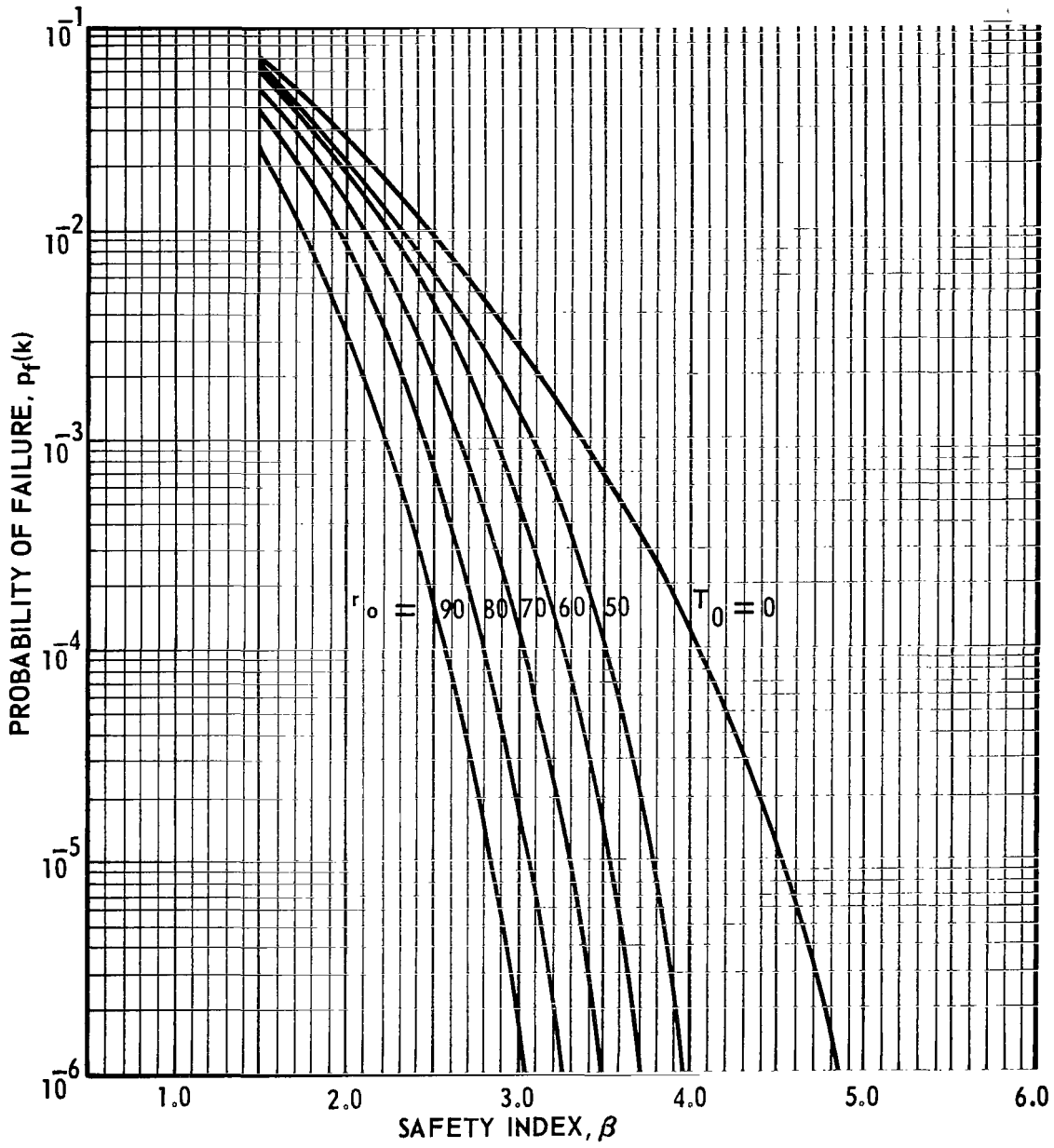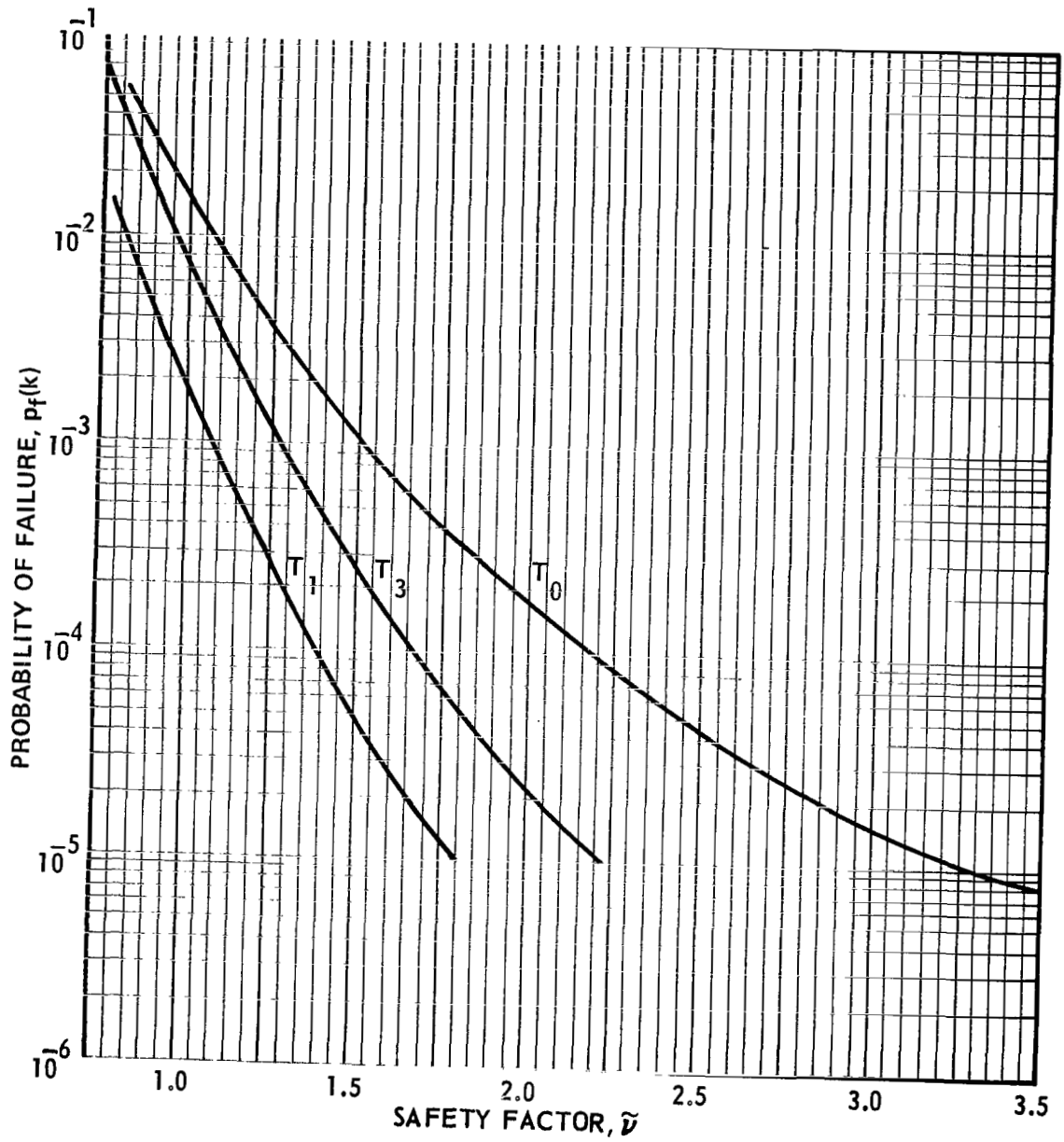
c. $\gamma_S = 0.37$.

Figure 35. (Concluded).
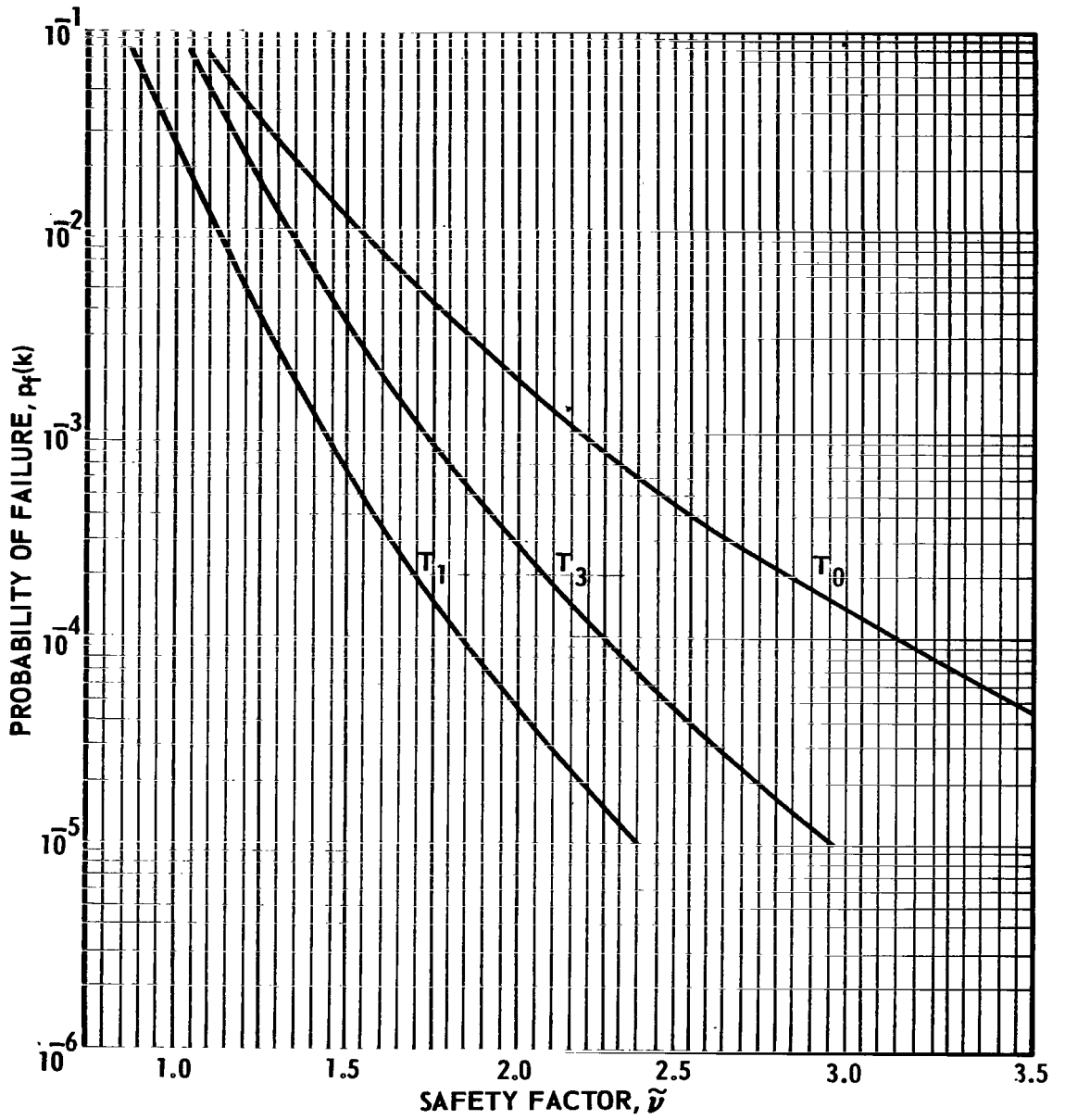
a. $\gamma_S = 0.05787$.

Figure 36. Probability of Failure versus Safety Factor for
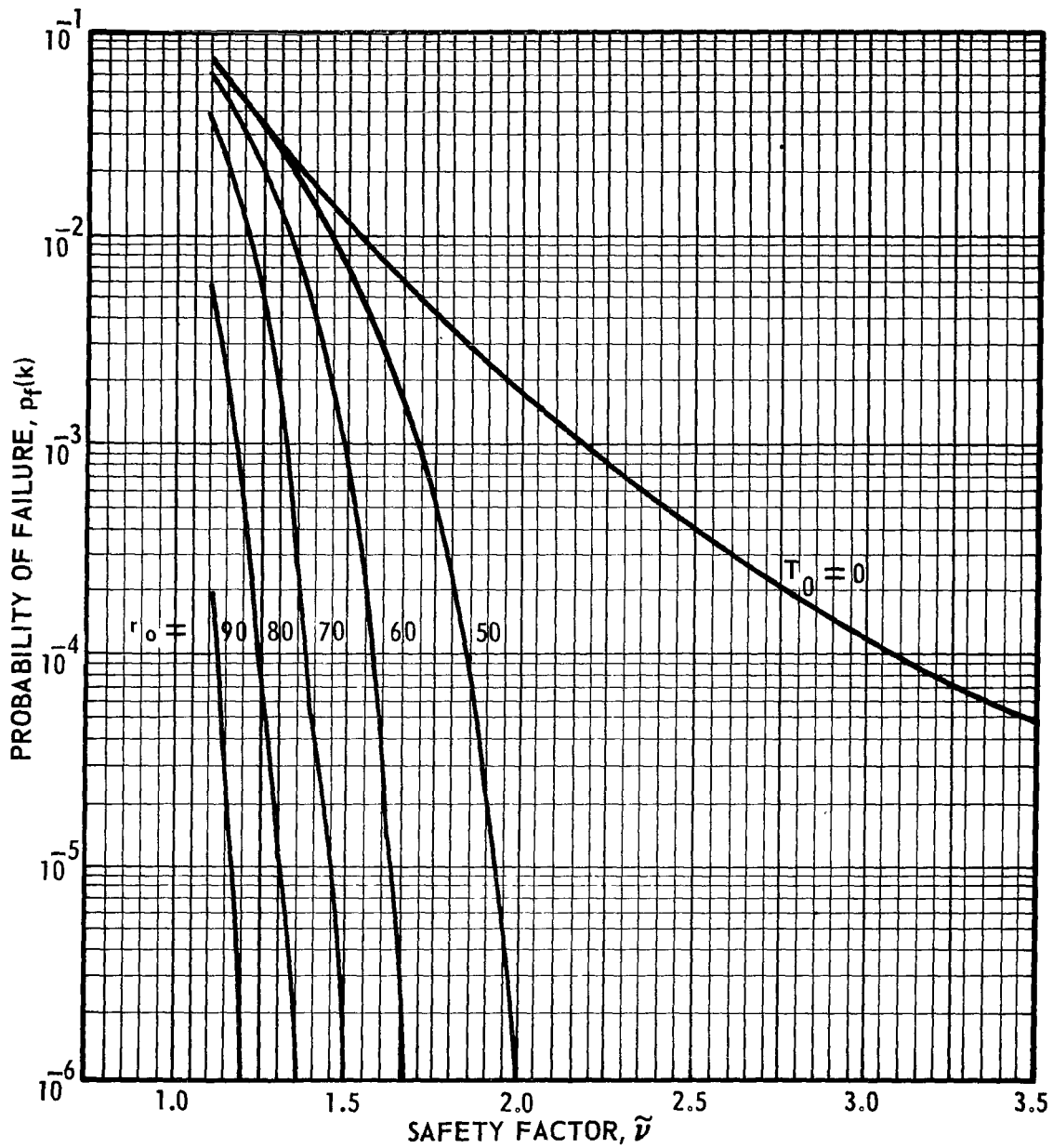Test Options 0, 1, and 3.

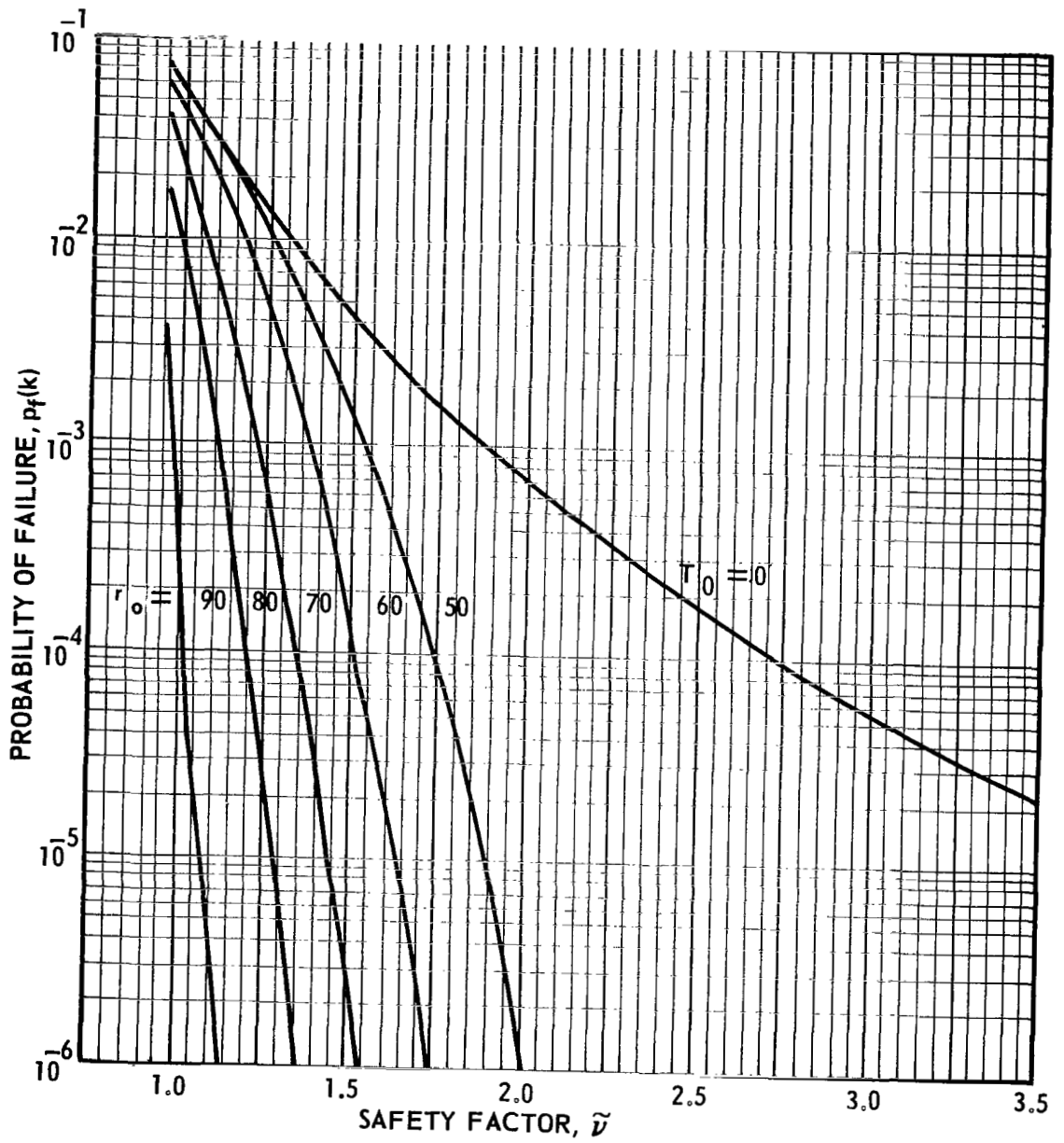b. $\gamma_S = 0.12916.$

Figure 36. (Continued).

c. $\gamma_S = 0.37$.
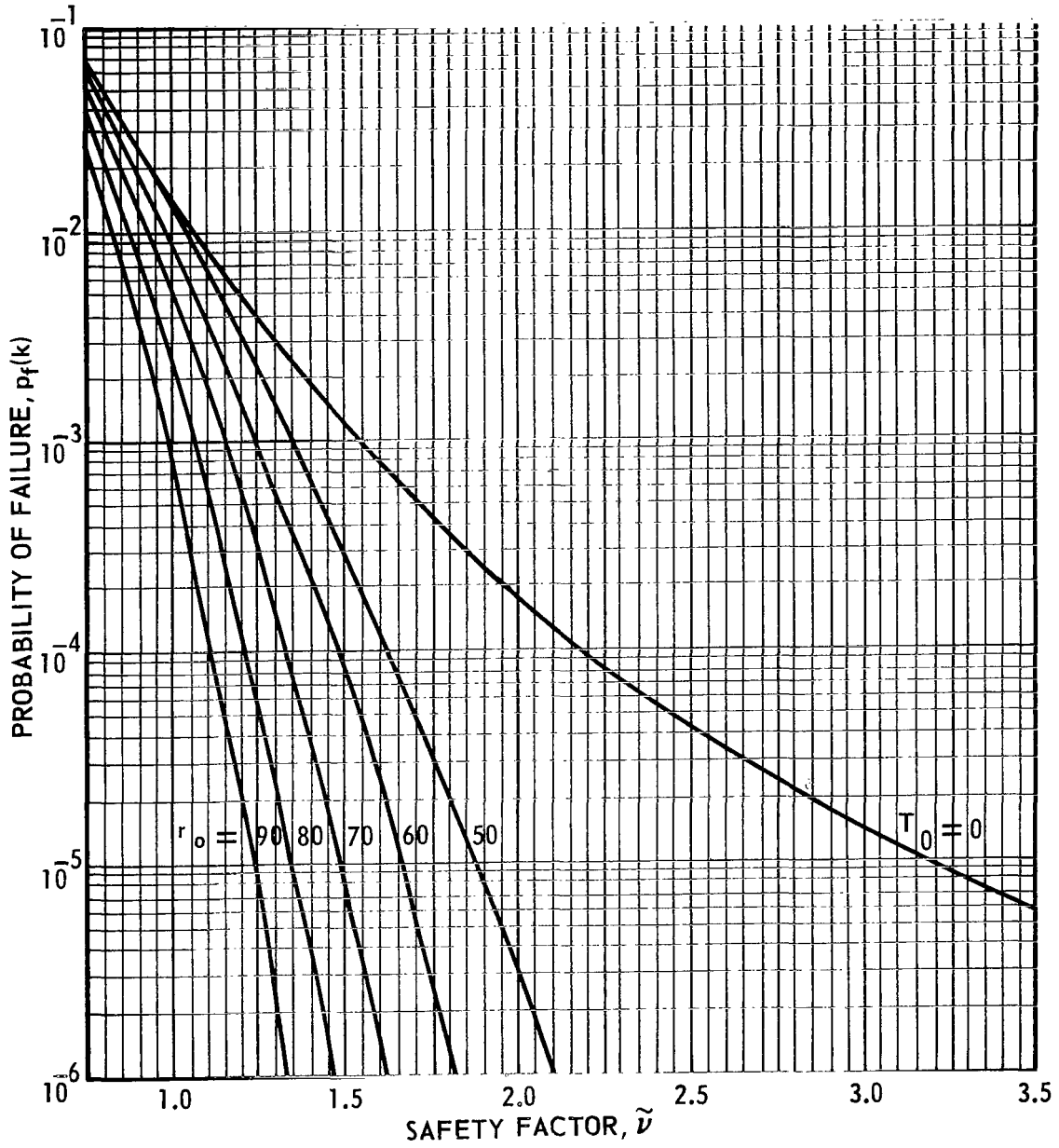
Figure 36. (Concluded).

a. $\gamma_S = 0.05787$.

Figure 37. Probability of Failure versus Safety Factor for Test Option 2.

b. $\gamma_S = 0.12916.$

Figure 37. (Continued).

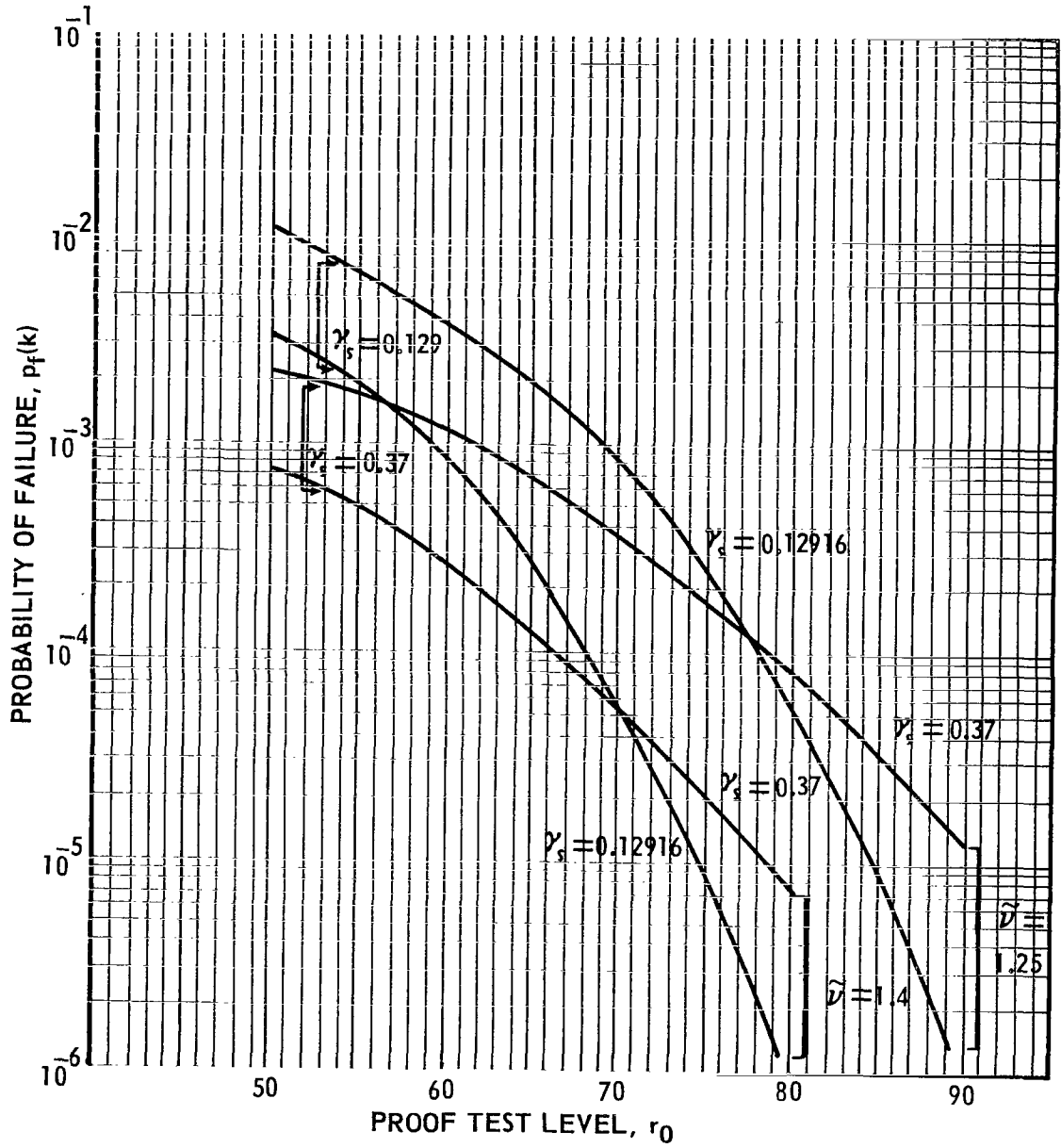c. $\gamma_S = 0.37$.

Figure 37. (Concluded).

Figure 38. Probability of Failure versus Proof Test Level.

APPENDIX D

SAMPLE CALCULATIONS

To illustrate the method used for plotting the cost optimization curves,

Figures 27 through 32 in Chapter VIII, sample calculations will be performed in

this appendix for two of the sample problems. This is accomplished by applying

the equations of Chapter VII to data for the example problems in Table 9.

D.1 Procedure

A step-by-step procedure for calculating one point in the cost optimiza-

tion curves will be given.

1. List the input data from Table 9: c_w, C_F, C_{F_p}, C_{D_j}, γ_S, n, m.

2. Assume a value of β.

3. From Figure 33, read the corresponding value of $\tilde{\nu}$ for the appro-

priate test option and γ_S.

4. From the appropriate Figures 21 through 26, read the weight value

corresponding to $\tilde{\nu}$ from step 3.

5. Determine $W(\beta)$ by subtracting the baseline weight from the weight

determined in step 4. The baseline weight may be selected arbitrarily and the

expected cost will then be relative to this arbitrary baseline. In the examples

in this dissertation, baseline weights corresponding to $\tilde{\nu} = 1.4$ were used

except for the S-II forward skirt, where $\tilde{\nu} = 1.3$ was used.

6. Calculate the cost of weight, $C_w = c_w W(\beta)$, equation (7.18). $W(\beta)$ is from step 5, and c_w is from the input data.

7. Read $p_f(k)$ from the appropriate Figures 34 through 38.

8. If the calculation is for the proof test option, determine

$$p_f(p) = F_R(r_0) = 1 - e^{\left(\frac{r_0}{115.5}\right)^{6.43}} \quad \text{from equation (4.1)}.$$

9. Calculate the expected cost of failure, $\overline{C}_F = C_F \, n \, p_f(k) + C_{F_p} \, m \, p_f(p)$ from equation (7.31). C_F and C_{F_p} are input data, and $p_f(k)$ and $p_f(p)$ are from steps 7 and 8.

10. Calculate the total expected cost for this example problem from equation (7.32), using C_{D_j} (direct costs) from the input data, C_w from step 6, and $\overline{C}_F$ from step 9: $C_j(\beta) = C_{D_j} + C_W + \overline{C}_F$.

11. Plot a point on the expected cost versus β curve using the assumed value of β from step 2 and $C_j(\beta)$ from step 10. Plot a point on the expected cost versus $\tilde{\nu}$ curve using the value of $\tilde{\nu}$ from step 3 and $C_j(\beta)$ from step 10.

D.2 Sample Calculation for S-IVB Aft Skirt

A sample calculation for test option T_2 with $r_0 = 60$ will be made for the S-IVB aft skirt using the procedure outlined in section D.1.

1. Input data from Table 9: $c_w = 11,500$ dollars per pound, $C_F = 250 \times 10^6$ dollars, $C_{F_p} = 0.72 \times 10^6$ dollars, $C_{D_2} = 3.28 \times 10^6$ dollars, $\gamma_S = 0.12916$, n = 15, and m = 15.

2. Assume that $\beta = 3.00$.

3. Read $\tilde{\nu} = 1.57$ from Figure 33a for $\beta = 3.00$ and $\gamma_S = 0.12916$.

4. From Figure 21, the S-IVB aft skirt weight for $\tilde{\nu} = 1.57$ is

$$W(\tilde{\nu} = 1.57) = 1920 \text{ lb.}$$

5. $W(\beta) = W(\tilde{\nu} = 1.57) - W(\tilde{\nu} = 1.4) = 1920 - 1815 = 105 \text{ lb.}$

6. $C_W = c_w W(\beta) = 11,500 \, (105) = 1,208,000 \text{ dollars.}$

7. From Figure 35b, $p_f(k) = 3.2 \times 10^{-5}$.

8. $p_f(p) = F_R(60) = 1 - e^{\left(\frac{60}{115.5}\right)^{6.43}} = 0.01472.$

9. $C_F = C_F \, n \, p_f(k) + C_{F_p} \, m \, p_f(p) = (250 \times 10^6) \, (15) \, (3.2 \times 10^{-5})$

$$+ \, (0.72 \times 10^6) \, (15) \, (0.01472) = 279,000 \text{ dollars.}$$

10. $C_2(3.00) = C_{D_j} + C_W = C_F = 3,280,000 + 1,208,000 + 279,000$

$$= 4,767,000 \text{ dollars.}$$

11. The result from step 10 is plotted in Figure 27a as points A and B.

D.3. Sample Calculation for ET Intertank

A sample calculation for test option T_1 will be made for the ET intertank.

1. Input data from Table 9: $c_w = 13,706$ dollars per pound, $C_F = 125 \times 10^6$ dollars, $C_{D_1} = 2.0 \times 10^6$ dollars, $\gamma_S = 0.05787$, n = 445, and m = 445.

2. Assume that $\beta = 4.00$.

3. From Figure 33, $\tilde{\nu} = 1.53$ for $\beta = 4.00$, and $\gamma_S = 0.05787$.

4. From Figure 23, $W(\tilde{\nu} = 1.53) = 12,600$ lb.

5. $\dot{W}(\beta) = W(\tilde{\nu} = 1.53) - W(\tilde{\nu} = 1.4) = 12,600 - 11,600 = 1,000$ lb.

6. $C_w = c_w W(\beta) = 13,706 \, (1,000) = 13,706,000$ dollars.

7. From Figure 34, $p_f(k) = 5.2 \times 10^{-4}$ for $\beta = 4.00$, test option T_1.

8. Not applicable for test option T_1.

9. $\overline{C}_F = C_F \, n \, p_f(k) = 125 \times 10^6 \, (445) \, (5.2 \times 10^{-4}) = 28.925 \times 10^6$

dollars.

10. $C_1(4.00) = C_{D_j} + C_w + C_F = 2,000,000 + 13,706,000 + 28,925,000$

$= 44,631,000$ dollars.

11. The result from step 10 is plotted as points C and D in Figure 29b.

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