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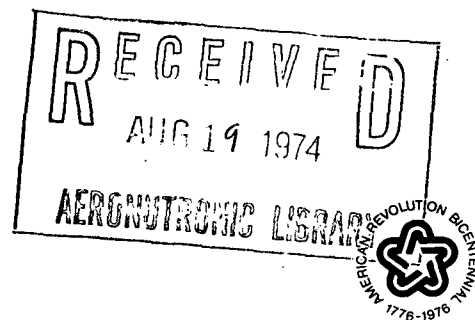
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**CURVE FITS OF PREDICTED INVISCID  
STAGNATION-POINT RADIATIVE  
HEATING RATES, COOLING FACTORS,  
AND SHOCK STANDOFF DISTANCES  
FOR HYPERBOLIC EARTH ENTRY**

*by John T. Suttles, Edward M. Sullivan,  
and Stephen B. Margolis*

*Langley Research Center  
Hampton, Va. 23665*



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SUMMARY

Curve-fit formulas are presented for the stagnation-point radiative heating rate, cooling factor, and shock standoff distance for inviscid flow over blunt bodies at conditions corresponding to high-speed earth entry. The data which were curve fitted were calculated by using a technique which utilizes a one-strip integral method and a detailed nongray radiation model to generate a radiatively coupled flow-field solution for air in chemical and local thermodynamic equilibrium. The range of free-stream parameters considered were altitudes from about 55 to 70 km and velocities from about 11 to 16 km/sec. Spherical bodies with nose radii from 30 to 450 cm and elliptical bodies with major-to-minor axis ratios of 2, 4, and 6 were treated.

Power-law formulas are proposed and a least-squares logarithmic fit is used to evaluate the constants. It is shown that the data can be described in this manner with an average deviation of about 3 percent (or less) and a maximum deviation of about 10 percent (or less). The curve-fit formulas provide an effective and economic means for making preliminary design studies for situations involving high-speed earth entry.

INTRODUCTION

One of the problems encountered by hypervelocity heat-shield designers and trajectory analysts is that of calculating the radiative flux to an entry body. For economic reasons, it is desirable to conduct studies and preliminary design work by using simplified approaches and correlation equations such as those in reference 1. The radiative heating-rate correlation used in reference 1 was derived from early heating-rate predictions which were based on a transparent, constant property shock layer. More recent studies have shown the early work to be grossly in error for high-speed entry problems. The survey given in reference 2 shows that for high-speed entries a heating-rate analysis must include the combined effects of shock-layer radiative cooling (which results in

properties varying across the shock layer), nongray self-absorption, continuum radiation, and atomic line radiation to be valid.

Numerous analyses are now available (e.g., refs. 3 to 8) which include these important effects for inviscid, stagnation region flow. Several of these techniques (refs. 4 to 6) have used approximate absorption-coefficient models for the radiation calculations in the interest of making engineering simplifications to this complex problem. However, these approaches are not in a convenient form for designers and analysts who desire estimates of shock-layer radiative heat transfer without resorting to lengthy computer solutions. Based on the success of the graphical correlation in reference 9, it is reasonable to expect that a simple analytic expression for radiative heating rates, such as that used in reference 1, can be derived by curve fitting the results of detailed computer solutions. Therefore, such an effort was undertaken.

In addition to heating rates an effort was made to curve fit the cooling factor since it is believed that the results presented in this form will be of more lasting value than the heating-rate results. The cooling factor is simply the ratio of the heating rate for a non-adiabatic shock layer to the heating rate for an adiabatic shock layer with the heating rates based on the same radiation model, body size, and flight conditions. (It is well known that for hypervelocity flows nonadiabatic effects significantly reduce the radiative heating compared to the heating calculated by assuming adiabatic conditions.)

Olstad (refs. 4 and 9) has demonstrated that the cooling factor can be correlated as a function of the heating rate for an adiabatic shock layer normalized by the free-stream kinetic-energy flux ( $\frac{1}{2} \rho_{\infty} V_{\infty}^3$ ). The twofold usefulness of the cooling factor was also suggested by Olstad (ref. 9). First, the use of the cooling factor minimizes the effects of the radiation model used in the calculation; heating rates which have significantly different values as a consequence of the use of different radiation models show good agreement when compared on the cooling-factor basis. This rationale is used for the comparison of results in the present report. Second, because the cooling factor has this property of bringing radiative heating rates calculated with diverse radiation models to a common base, it can be used in conjunction with an adiabatic heating rate calculated with the best available radiation model to calculate the nonadiabatic heating rate. With this approach it is not necessary to use time-consuming coupled flow-field solutions to obtain radiative heating rates, nor is it necessary to recalculate the cooling factors as improvements are made in radiation models.

Since inviscid shock-standoff-distance values are obtained when heating rates and cooling factors are calculated, a curve fit for this quantity was derived also. The shock standoff distance is of interest to heat-shield designers, for example, in evaluating the effects of a probe extending from the body into the shock layer. Also, flow-field analysts

find that the shock standoff distance has numerous uses, two of which are for comparing various results and for initiating iterative solutions.

The present report presents the results of curve fitting information on calculated heating rate, cooling factor, and shock standoff distance with simple expressions involving free-stream conditions and vehicle geometry parameters. The calculations were made by using a radiatively coupled flow-field computer program (ref. 3) which includes a detailed nongray radiation model (refs. 10 and 11). The range of free-stream parameters considered were altitudes from about 55 to 70 km and velocities from about 11 to 16 km/sec. Spherical bodies with nose radii from 30 to 450 cm and elliptical bodies with major-to-minor axis ratios of 2, 4, and 6 were treated. The simple form used for the curve fits is suitable for hand calculation with a slide rule or electronic desk calculator and can be readily included in a parametric analysis.

### SYMBOLS

$a_{ji}$	coefficient matrix of equation (5)
$A_i$	constants used in heating-rate curve fit (see eq. (1))
$b_{ji}$	coefficient matrix of equation (8)
$B_i$	constants used in cooling-factor curve fit (see eq. (7))
$c_{ji}$	coefficient matrix of equation (10)
$C_i$	constants used in shock-standoff-distance curve fit (see eq. (9))
$D_j$	constant vector of equation (5)
$e$	base for natural logarithms ( $e = 2.71828 \dots$ )
$E$	total-error measure (see eq. (4))
$F_C$	cooling factor, ratio of nonadiabatic-to-adiabatic heating rates
$G_j$	constant vector of equation (8)
$H_j$	constant vector of equation (10)

i	index used for constants in curve-fit equations
j	index used for equations resulting from minimization procedure
K	constant in functional form assumed for curve-fit equations
n	index used to identify individual data points which are to be curve fitted (case number)
N	total number of data points for a curve fit
$q_R$	radiative heating rate, W/cm <sup>2</sup>
r	axis ratio of elliptical bodies, free-stream velocity always oriented normal to major axis
$R_B$	body nose radius, cm
$V_\infty$	free-stream velocity, km/sec
X	denotes any of quantities to be curve fitted
$\delta$	stagnation-point shock standoff distance, cm
$\epsilon_n$	error of curve fit at the nth data point
$\rho_\infty$	free-stream density (obtained from 1962 standard atmosphere for a given altitude), g/cm <sup>3</sup>

Subscripts:

C	curve fit
D	data
r	axis ratio
$\delta$	shock standoff distance

## Superscripts:

a,b,c            exponents in the functional form assumed for curve-fit equations

## METHOD

The data to be curve fitted were obtained from solutions of the inviscid radiating flow field in the stagnation region of a blunt body (fig. 1), which is entering the Earth's atmosphere. These solutions were obtained by using the one-strip integral method of reference 3 which incorporates the nongray radiation model of reference 10 as a subroutine. The radiation model ("RATRAP") is described in further detail and compared with other models in reference 11. The computational method is for the inviscid flow of air in chemical and local thermodynamic equilibrium. The radiation calculations assume a plane slab shock-layer geometry and neglect absorption by the air upstream of the shock layer and byproducts of ablation at the body surface. Discussion of the importance of these effects can be found in reference 2.

The data on which the curve fits are based are given in table I. The free-stream conditions, velocity and density (density determined from 1962 standard atmosphere for a given altitude), were picked to cover the range of interest for high-speed entry cases where radiative heating will be important. Figure 2 illustrates the conditions chosen: a typical manned planetary-return lifting-entry trajectory (ref. 12), a range of high-velocity ballistic-entry trajectories (from an unpublished study), and a matrix of cases which span the trajectories. The body sizes were selected from results of an unpublished study and cover a range of interest from small unmanned probes of preliminary flight-test vehicles to large manned spacecraft. Thus, the curve fits should be of value for a wide range of applications.

The effort was undertaken on the premise that if sufficient data were available it would be possible to write a curve-fit equation of the form

$$X = K\rho_{\infty}^a R_B^b V_{\infty}^c$$

where X represents either the heating rate, cooling factor, or the standoff distance and a, b, and c may be allowed to vary with the free-stream conditions and/or body size. This form has been used to correlate convective heating rates (see, e.g., ref. 13), and it is well suited for calculations made with either a slide rule or electronic desk calculator. It can also be readily programed for repetitive calculations on a digital computer.

## CURVE-FIT EQUATIONS

### Heating Rates

Spherical bodies.- A curve fit of the stagnation-point radiative heating rate  $q_R$  with the free-stream velocity  $V_\infty$ , free-stream density  $\rho_\infty$ , and the body radius  $R_B$  was obtained by fitting a multidimensional curve through the spherical body data (cases 1 to 91) in table I. Based on a preliminary analysis, the functional form

$$q_{R,C} = e^{A_1} \rho_\infty^{A_2+A_3 V_\infty} R_B^{A_4+A_5 V_\infty+A_6 V_\infty^2} V_\infty^{A_7+A_8 V_\infty+A_9 V_\infty^2} \quad (1)$$

was assumed and the constants  $A_1, A_2, \dots, A_9$  were found by a least-squares analysis. To facilitate application of the least-squares method, equation (1) was first linearized with respect to the  $A_i$  values by taking logarithms as follows:

$$\begin{aligned} \ln q_{R,C} = & A_1 + A_2 \ln \rho_\infty + A_3 V_\infty \ln \rho_\infty + A_4 \ln R_B + A_5 V_\infty \ln R_B \\ & + A_6 V_\infty^2 \ln R_B + A_7 \ln V_\infty + A_8 V_\infty \ln V_\infty + A_9 V_\infty^2 \ln V_\infty \end{aligned} \quad (2)$$

The error function for the least-squares method is, therefore,

$$\epsilon_n = \ln q_{R,D}^{(n)} - \ln q_{R,C}^{(n)} \quad (3)$$

where  $q_{R,D}^{(n)}$  is the value of  $q_R$  corresponding to the  $n$ th data point  $[\rho_\infty^{(n)}, V_\infty^{(n)}, \text{ and } R_B^{(n)}]$  from table I, and  $q_{R,C}^{(n)}$  is the  $q_R$  value obtained from the curve fit (eq. (1)) for the  $n$ th point. The total-error measure in the method is given by

$$E = \sum_{n=1}^N \epsilon_n^2 \quad (4)$$

and is to be minimized with respect to each of the  $A_i$  values. The minimization leads to a set of linear simultaneous equations for the  $A_i$  values whose matrix form is as follows:

$$a_{ji} A_i = D_j \quad (i, j = 1, 2, \dots, 9) \quad (5)$$

where the  $A_i$  values have previously been defined and the coefficient matrix  $a_{ji}$  and  $D_j$  values are given in table II. The elements of the coefficient matrix involve summations of functions involving  $\rho_\infty^{(n)}, R_B^{(n)}, \text{ and } V_\infty^{(n)}$ ; and the  $D_j$  values involve summa-



tions of functions involving  $\rho_\infty(n)$ ,  $R_B(n)$ ,  $V_\infty(n)$ , and  $q_{R,D}(n)$ . In each case, the summations are over the  $N$  data points. Equation (5) was solved by inverting the coefficient matrix  $a_{ji}$ . The results for the  $A_i$  values are given in the appendix, and the results using equation (1) with the  $A_i$  values are given in table III(a) for the 91 points used. A comparison of  $q_{R,D}$  with  $q_{R,C}$  in table III(a) shows an average deviation of approximately 3 percent. Also, of the 91 data points only 3 have deviations larger than 7 percent, and these are for small bodies at high altitudes - conditions for which the calculated radiative heat-transfer rates tend to be very low and the inaccuracy may be large.

Elliptical bodies.- The curve fits presented were based on the data for spherical bodies given in table I. Data are given in table I (cases 102 to 150) for elliptical bodies with axis ratios of 2, 4, and 6. For the elliptical shapes the free-stream velocity is normal to the semimajor axis and  $R_B$  is the stagnation-point radius of curvature. When the curve fit for the spherical-body data was applied to the nonspherical-body data, a systematic error appeared which depended on the axis ratio. This error was attributed to the fact that the axis ratio influences the tangential velocity gradient and the shock standoff distance. It was found possible to compensate for this effect by use of a correction factor (called the shape factor) which is a linear function of the axis ratio. The shape factor was obtained by calculating  $q_{R,C}$  for each of the nonspherical-body cases by using equation (1) and forming the ratio of  $q_{R,C}$  to  $q_{R,D}$ . These ratios were collected into groups of data corresponding to common axis ratios, and simple averages for each group were determined. The value of  $\frac{q_{R,C}}{q_{R,D}}$  for the spherical bodies (unit axis ratio) was taken to be 1.0. Thus, values for the shape factor were generated for axis ratios of 1, 2, 4, and 6. A linear function was assumed for the variation of the shape factor with axis ratio, and a least-squares curve-fit method was used to find the slope and intercept. Figure 3 presents the data used in the shape-factor calculations and the line obtained from the least-squares fit. With the resulting shape factor, the expression for  $q_{R,C}$  for nonspherical bodies is

$$q_{R,C,r} = \frac{q_{R,C}}{(0.955859 + 0.03645r)} \quad (6)$$

which is used for  $1 < r \leq 6$ . It is noted that the shape factor is not precisely 1.0 at unit axis ratio since this condition was used merely as a data point rather than as a constraint for the curve fit.

The  $q_{R,C,r}$  values are compared with the corresponding  $q_{R,D}$  values in table III(b). The results are excellent with an average deviation of 3 percent and a maximum of 8 percent for the cases where  $R_B$ ,  $\rho_\infty$ , and  $V_\infty$  are in the range for which the  $A_i$  values were originally calculated.

## Cooling Factors

A curve fit for the cooling factor  $F_C$  was derived from the 53 cases in table I for which  $F_C$  values were given. The curve-fit formula for  $F_C$  as a function of  $\rho_\infty$ ,  $V_\infty$ , and  $R_B$  was obtained in the same manner as that for  $q_R$ . A function of the form

$$F_{C,C} = e^{B_1 \rho_\infty^{B_2+B_3 R_B+B_4 R_B^2+B_5 R_B^3}} R_B^{B_6} V_\infty^{B_7} \quad (7)$$

was assumed based on a preliminary data analysis. Taking the logarithm of equation (7) and applying the least-squares method resulted in a set of equations for the  $B_i$  values whose matrix form is as follows:

$$b_{ji} B_i = G_j \quad (i, j = 1, 2, \dots, 7) \quad (8)$$

where the  $B_i$  values are the constants appearing in equation (7) and the coefficient matrix  $b_{ji}$  and  $G_j$  values are given in table IV. The elements of the coefficient matrix involve summations over the  $N$  data points of functions involving  $\rho_\infty(n)$ ,  $V_\infty(n)$ , and  $R_B(n)$ , whereas the  $G_j$  values involve summations of functions involving  $\rho_\infty(n)$ ,  $V_\infty(n)$ ,  $R_B(n)$ , and  $F_{C,D}(n)$ . Equation (8) was solved by inverting the coefficient matrix  $b_{ji}$ . The results for the  $B_i$  values are presented in the appendix, and the results using equation (7) with the  $B_i$  values to compute  $F_{C,C}$  are given in table V. A comparison of  $F_{C,C}$  with  $F_{C,D}$  is also given. These results for the cooling factor have an average deviation of approximately 3 percent and a maximum deviation of about 10 percent.

## Shock Standoff Distances

Spherical bodies.- A curve-fit formula for the shock standoff distance at the stagnation streamline as a function of  $\rho_\infty$ ,  $V_\infty$ , and  $R_B$  was obtained in the same manner as those for  $q_R$  and  $F_C$ . Ninety-one spherical-body cases (cases 1 to 91) were used. A function of the form

$$\delta_C = e^{C_1 \rho_\infty^{C_2+C_3 V_\infty R_B^{C_4+C_5 V_\infty V_\infty}} V_\infty^{C_6+C_7 V_\infty+C_8 V_\infty^2}} \quad (9)$$

was assumed based on a preliminary analysis of the available standoff-distance data. Again, by taking the logarithm of the assumed function and applying the least-squares method, a set of equations for the  $C_i$  values was obtained. The matrix form of these equations is as follows:

$$c_{ji} C_i = H_j \quad (i, j = 1, 2, \dots, 8) \quad (10)$$

where the  $C_i$  values are the constants appearing in equation (9) and the coefficient matrix  $c_{ji}$  and  $H_j$  values are given in table VI. The elements of the coefficient matrix involve summations over the  $N$  data points of functions involving  $\rho_\infty(n)$ ,  $R_B(n)$ , and  $V_\infty(n)$ ; whereas the  $H_j$  values involve summations of functions involving  $\rho_\infty(n)$ ,  $R_B(n)$ ,  $V_\infty(n)$ , and  $\delta_D(n)$ . Equation (10) was solved by inverting the matrix  $c_{ji}$ , and the  $C_i$  values are given in the appendix. Table VII(a) gives a comparison of the original standoff-distance data  $\delta_D$  with  $\delta_C$  values calculated by using equation (9) and the derived  $C_i$  values. The results show that when compared to  $\delta_D$  the corresponding  $\delta_C$  values have an average deviation of about 1 percent with a maximum deviation of about 5 percent.

Elliptical bodies.- A shape factor for the standoff distance for the nonspherical-body cases was derived in the same manner as that for the heating-rate correlation. The resultant expression for the shock standoff distance for the elliptical bodies is

$$\delta_{C,r} = \frac{\delta_C}{(0.825784 + 0.14816r)} \quad (11)$$

which is applied for  $1 < r \leq 6$ . The data from which the shape factor for equation (11) was derived are shown in figure 4. The comparison of  $\delta_{C,r}$  with  $\delta_{D,r}$  given in table VII(b) shows that the curve-fit values have an average deviation of 4 percent compared to the original data.

## DISCUSSION

As shown in tables III, V, and VII the curve-fit equations give good representations of the basic data (table I) in the ranges specified in the appendix. In this section the basic data and/or the curve fits will be compared with other available data. In addition, the cooling-factor curve fit was studied extensively in order to establish its sensitivity to out-of-range conditions. The results of this study are also presented.

### Heating Rates

Radiative heat-transfer calculations are known to be significantly influenced by both the flow-field calculation technique and the radiation transfer model. The basic data (table I) for heating rate and standoff distance were calculated by using the one-strip integral technique of reference 3, which includes the radiation model of reference 10. Results from calculations using these techniques have been compared with several other methods and these comparisons have been published (e.g., refs. 3, 7, and 8). The general finding is that the one-strip integral technique produces results which compare very well

with other techniques, provided that the same radiation model is used. Thus, it can be concluded that the fluid mechanics of the method are adequate.

The effects resulting from the use of different radiation models have also been explored (refs. 3 and 11). It has been found that the calculated heating rates for a given set of conditions can differ by 50 percent or more. This result is not surprising when one considers the diversity in the amount of detail programed into various radiation models. In view of the lack of experimental data, at the conditions of interest here, it is difficult to assess which of the available radiation models gives the most accurate results and which provides the best compromise between accuracy and required computer time. For these reasons the present heating-rate results are not compared with previously published values. Instead, such comparisons will be made on a cooling-factor basis in the next section.

### Cooling Factors

As indicated in the "Introduction," Olstad (ref. 9) has shown that when the cooling factor  $F_C$  is taken as a basis, good agreement is obtained between results calculated by using different radiation models. Therefore, the  $F_C$  curve fit of the present report will be compared to the charts of  $F_C$  presented by Olstad in reference 9. It is emphasized that to derive heating rates from  $F_C$  values a companion calculation of the adiabatic radiative heating rate is required. That is,

$$(q_R)_{\text{nonadiabatic}} = F_C (q_R)_{\text{adiabatic}} \quad (12)$$

and  $(q_R)_{\text{adiabatic}}$  does not require a time-consuming coupled flow-field solution. Since  $F_C$  is relatively insensitive to the radiation model (ref. 9), this allows the user to select the model most appropriate to his purpose to determine the adiabatic value. If the user desires to accept the radiation model applied in the calculation of the basic data of this report, he can simply use the heating-rate curve fit presented herein.

The basic data have been curve fitted wherever possible (the 53 cases having  $F_C$  values in table I) in terms of  $F_C$ ; the resulting curve-fit equation is given in the appendix. It can be seen (table V) that the agreement between the basic data and the curve fit is excellent with a maximum error of approximately 10 percent. As an independent check, a large number (180) of cooling-factor values were calculated by using the curve-fit equation (eq. (7)) and compared with the curves given in reference 9. Figures 5, 6, and 7 show the results of these calculations. Note that some calculations shown in the figures are out of the range of the correlation. This was done to show the sensitivity of the curve fit. (The calculations also included cases with  $R_B = 800$  cm; however, they produced an unreasonably small  $F_C$  and have not been plotted.) In figure 5 it is seen that the values

from the curve fit for those cases where all three parameters are within the ranges specified in the appendix are in excellent agreement with the results of reference 9. The maximum difference is approximately 17 percent for one case of a 30-cm body at an altitude near the upper limit of the altitude range. The values from figures 5, 6, and 7 for cases where one or more of the parameters are out of range show that the curve fit is most sensitive to body size and least sensitive to velocity.

In view of the results shown in figures 5, 6, and 7, the authors believe that the cooling-factor curve fit is accurate provided the ranges specified in the appendix are not exceeded.

### Shock Standoff Distances

Figure 8 presents a comparison of the shock standoff distances from the basic data (table I), the curve-fit equation (the appendix), and the results of Callis (ref. 6) for  $\rho_\infty = 1.225 \times 10^{-7}$  g/cm<sup>3</sup> and for  $V_\infty = 15.24$  km/sec and 12.19 km/sec. The basic data do not contain identically corresponding results, and, therefore, data for the closest conditions ( $\rho_\infty = 1.2959 \times 10^{-7}$  g/cm<sup>3</sup> and  $V_\infty = 15$  km/sec and 12 km/sec) are given. It is seen from figure 8 that these slight variations in  $V_\infty$  and  $\rho_\infty$  have little effect on the comparison. The comparison shows that the curve-fit equation gives an excellent representation of the basic data and, although the trends are different, the basic data and the results of Callis agree within 10 percent or less. A more detailed comparison between the two methods (i.e., Suttles (ref. 3) and Callis (ref. 6)) is given in reference 3, where it is shown that the differences between radiation models can account for a significant difference in the results. Therefore, the variation in the trends of the results in figure 8 are most probably due to the different radiation models used.

Some comparisons were made (but not shown here) between the results of the present work and the results of Callis for bodies smaller ( $R_B = 1$  to 30 cm) and for bodies larger ( $R_B = 450$  to 1000 cm) than those considered in the curve fits. This comparison indicated that for bodies in the range of  $R_B$  from 10 to 30 cm, the results remained in good agreement (10 percent or less); but for smaller bodies and bodies larger than 450 cm, the results began to deviate significantly. It is concluded that, as in the case of the cooling-factor curve fit, the user should be very hesitant in applying the shock-standoff-distance curve fit outside the range of the basic data.

### CONCLUDING REMARKS

Curve-fit formulas are presented for the stagnation-point radiative heating rate, cooling factor, and shock standoff distance for inviscid flow over blunt bodies at conditions corresponding to high-speed earth entry. The data on which the curve fits are based were calculated by using a technique which utilizes a one-strip integral method

and a detailed nongray radiation model to generate a radiatively coupled flow-field solution for air in chemical and local thermodynamic equilibrium. The range of free-stream parameters considered were altitudes from about 55 to 70 km and velocities from about 11 to 16 km/sec. Spherical bodies with nose radii of from 30 to 450 cm and elliptical bodies with major-to-minor axis ratios of 2, 4, and 6 were treated.

Power-law formulas are proposed and a least-squares logarithmic fit is used to evaluate the constants. It is shown that the data can be described in this manner with an average deviation of about 3 percent (or less) and a maximum deviation of about 10 percent. A study of the sensitivity of the formulas indicates that they should be used only within the range of the free stream and vehicle geometry parameters of the data. These curve-fit formulas provide an effective and economic means for making preliminary design studies for situations involving high-speed earth entry.

Langley Research Center,  
National Aeronautics and Space Administration,  
Hampton, Va., April 30, 1974.

## APPENDIX

### CURVE-FIT FORMULAS AND RELATED PARAMETERS

For the convenience of the reader, the curve-fit formulas, the constants derived for the formulas, the ranges of applicability of the formulas, and the average deviation of the formulas from the basic data are listed in this appendix.

#### Heating Rates

Formula.- The formula for  $q_{R,C}$  in  $W/cm^2$  is

$$q_{R,C} = K_r e^{A_1 \rho_\infty^{A_2 + A_3 V_\infty}} R_B^{A_4 + A_5 V_\infty + A_6 V_\infty^2} V_\infty^{A_7 + A_8 V_\infty + A_9 V_\infty^2}$$

Derived constants.-

$$K_r = \begin{cases} \frac{1}{(0.955859 + 0.03645r)} & (r \neq 1) \\ 1 & (r = 1) \end{cases}$$

$$A_1 = -69.099$$

$$A_6 = 0.005381$$

$$A_2 = 1.320$$

$$A_7 = 51.89$$

$$A_3 = -0.01223$$

$$A_8 = -1.558$$

$$A_4 = 1.688$$

$$A_9 = 0.02659$$

$$A_5 = -0.1796$$

Ranges of applicability.-

$$\rho_\infty = 1.078 \times 10^{-7} \text{ to } 6.53 \times 10^{-7} \text{ g/cm}^3$$

$$\text{Altitude} = 53.75 \text{ to } 68.4 \text{ km}$$

$$R_B = 30 \text{ to } 450 \text{ cm}$$

$$V_\infty = 11 \text{ to } 16 \text{ km/sec}$$

$$r = 1 \text{ to } 6$$

## APPENDIX

Average deviation.- 3 percent

### Cooling Factors

Formula.- The cooling factor  $F_{C,C}$  is dimensionless and is given by

$$F_{C,C} = e^{B_1 \rho_\infty^{B_2 + B_3 R_B + B_4 R_B^2 + B_5 R_B^3} R_B^{B_6} V_\infty^{B_7}}$$

Derived constants.-

$$\begin{aligned} B_1 &= -3.679 & B_5 &= 0.00000005567 \\ B_2 &= -0.3598 & B_6 &= 1.059 \\ B_3 &= 0.002024 & B_7 &= -1.931 \\ B_4 &= -0.000005583 \end{aligned}$$

Ranges of applicability.-

$$\rho_\infty = 5.61 \times 10^{-7} \text{ to } 8.75 \times 10^{-8} \text{ g/cm}^3$$

$$\text{Altitude} = 55 \text{ to } 70 \text{ km}$$

$$R_B = 30 \text{ to } 450 \text{ cm}$$

$$V_\infty = 11 \text{ to } 16 \text{ km/sec}$$

Average deviation.- 3 percent

### Shock Standoff Distances

Formula.- The formula for shock standoff distance in cm is

$$\delta_C = K_{r,\delta} e^{C_1 \rho_\infty^{C_2 + C_3 V_\infty R_B^{C_4 + C_5 V_\infty} R_B^{C_6 + C_7 V_\infty} + C_8 V_\infty^2}}$$

Derived constants.-

$$K_{r,\delta} = \begin{cases} \frac{1}{(0.825784 + 0.14816r)} & (r \neq 1) \\ 1 & (r = 1) \end{cases}$$



## APPENDIX

$$C_1 = -3.697$$

$$C_5 = -0.01248$$

$$C_2 = 0.07375$$

$$C_6 = 0.7718$$

$$C_3 = -0.003338$$

$$C_7 = -0.02572$$

$$C_4 = 1.134$$

$$C_8 = 0.00009347$$

### Ranges of applicability.-

$$\rho_\infty = 1.078 \times 10^{-7} \text{ to } 6.53 \times 10^{-7} \text{ g/cm}^3$$

$$\text{Altitude} = 53.75 \text{ to } 68.4 \text{ km}$$

$$R_B = 30 \text{ to } 450 \text{ cm}$$

$$V_\infty = 11 \text{ to } 16 \text{ km/sec}$$

$$r = 1 \text{ to } 6$$

### Average deviation.- 2 percent

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TABLE I.- BASIC DATA

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$q_{R,D}$ , W/cm <sup>2</sup>	$F_{C,D}$
1	11.000	58.0	3.9072E-07	30.00	1.0	1.421	306.0	-
2	11.000	58.0	3.9072E-07	150.00	1.0	7.036	544.0	-
3	11.000	58.0	3.9072E-07	300.00	1.0	13.974	661.0	-
4	11.000	58.0	3.9072E-07	450.00	1.0	20.844	752.0	-
5	13.000	58.0	3.9072E-07	30.00	1.0	1.368	1607.0	.582
6	13.000	58.0	3.9072E-07	150.00	1.0	6.680	2263.0	-
7	13.000	58.0	3.9072E-07	300.00	1.0	12.887	2699.0	-
8	13.000	58.0	3.9072E-07	450.00	1.0	18.971	3080.0	-
9	14.000	58.0	3.9072E-07	30.00	1.0	1.339	2578.0	.508
10	14.000	58.0	3.9072E-07	150.00	1.0	6.374	3537.0	-
11	14.000	58.0	3.9072E-07	300.00	1.0	12.268	4289.0	.307
12	14.000	58.0	3.9072E-07	450.00	1.0	17.943	4816.0	.285
13	15.000	58.0	3.9072E-07	30.00	1.0	1.308	3709.0	.452
14	15.000	58.0	3.9072E-07	150.00	1.0	6.093	5124.0	.298
15	15.000	58.0	3.9072E-07	300.00	1.0	11.611	6108.0	.268
16	15.000	58.0	3.9072E-07	450.00	1.0	16.850	6917.0	.252
17	16.000	58.0	3.9072E-07	30.00	1.0	1.277	5462.0	.401
18	16.000	58.0	3.9072E-07	150.00	1.0	5.830	7039.0	.261
19	16.000	58.0	3.9072E-07	300.00	1.0	11.011	8176.0	.231
20	16.000	58.0	3.9072E-07	450.00	1.0	15.873	9382.0	.224
21	11.000	61.0	2.7018E-07	30.00	1.0	1.397	194.0	.812
22	11.000	61.0	2.7018E-07	150.00	1.0	6.921	365.0	.626
23	11.000	61.0	2.7018E-07	300.00	1.0	13.725	447.0	.541
24	11.000	61.0	2.7018E-07	450.00	1.0	20.464	509.0	.509
25	12.000	61.0	2.7018E-07	30.00	1.0	1.378	520.0	.710
26	12.000	61.0	2.7018E-07	150.00	1.0	6.760	843.0	.497
27	12.000	61.0	2.7018E-07	300.00	1.0	13.390	958.0	.412
28	12.000	61.0	2.7018E-07	450.00	1.0	19.844	1087.0	.383
29	13.000	61.0	2.7018E-07	30.00	1.0	1.348	1027.0	.630
30	13.000	61.0	2.7018E-07	150.00	1.0	6.640	1545.0	.429
31	13.000	61.0	2.7018E-07	300.00	1.0	12.710	1786.0	.361
32	13.000	61.0	2.7018E-07	450.00	1.0	18.800	1951.0	.327
33	14.000	61.0	2.7018E-07	30.00	1.0	1.319	1737.0	.566
34	14.000	61.0	2.7018E-07	150.00	1.0	6.280	2353.0	.356
35	14.000	61.0	2.7018E-07	300.00	1.0	12.090	2839.0	.318
36	14.000	61.0	2.7018E-07	450.00	1.0	17.660	3243.0	.303
37	15.000	61.0	2.7018E-07	30.00	1.0	1.287	2656.0	.514
38	15.000	61.0	2.7018E-07	150.00	1.0	5.995	3471.0	.314
39	15.000	61.0	2.7018E-07	300.00	1.0	11.440	4154.0	.282
40	15.000	61.0	2.7018E-07	450.00	1.0	16.850	4690.0	.268

TABLE I.- BASIC DATA - Continued

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$q_{R,D}$ , W/cm <sup>2</sup>	$F_{C,D}$
41	16.000	61.0	2.7018E-07	30.00	1.0	1.256	3778.0	.466
42	16.000	61.0	2.7018E-07	150.00	1.0	5.713	4771.0	.272
43	16.000	61.0	2.7018E-07	300.00	1.0	10.920	5362.0	.233
44	16.000	61.0	2.7018E-07	450.00	1.0	15.550	6174.0	.227
45	11.000	67.0	1.2959E-07	30.00	1.0	1.360	70.0	-
46	11.000	67.0	1.2959E-07	150.00	1.0	6.754	149.0	-
47	11.000	67.0	1.2959E-07	300.00	1.0	13.445	190.0	-
48	11.000	67.0	1.2959E-07	450.00	1.0	20.019	215.0	-
49	12.000	67.0	1.2959E-07	30.00	1.0	1.342	194.0	-
50	12.000	67.0	1.2959E-07	150.00	1.0	6.597	363.0	-
51	12.000	67.0	1.2959E-07	300.00	1.0	13.045	442.0	-
52	12.000	67.0	1.2959E-07	450.00	1.0	19.392	490.0	-
53	13.000	67.0	1.2959E-07	30.00	1.0	1.317	396.0	-
54	13.000	67.0	1.2959E-07	150.00	1.0	6.392	674.0	-
55	13.000	67.0	1.2959E-07	300.00	1.0	12.525	796.0	-
56	13.000	67.0	1.2959E-07	450.00	1.0	18.498	860.0	-
57	14.000	67.0	1.2959E-07	30.00	1.0	1.292	688.0	-
58	14.000	67.0	1.2959E-07	150.00	1.0	6.194	1079.0	-
59	14.000	67.0	1.2959E-07	300.00	1.0	12.022	1218.0	-
60	14.000	67.0	1.2959E-07	450.00	1.0	17.636	1347.0	-
61	15.000	67.0	1.2959E-07	30.00	1.0	1.268	1064.0	.620
62	15.000	67.0	1.2959E-07	150.00	1.0	5.980	1576.0	.362
63	15.000	67.0	1.2959E-07	300.00	1.0	11.470	1750.0	.291
64	15.000	67.0	1.2959E-07	450.00	1.0	16.710	1913.0	.269
65	16.000	67.0	1.2959E-07	30.00	1.0	1.242	1531.0	-
66	16.000	67.0	1.2959E-07	150.00	1.0	5.756	2150.0	-
67	16.000	67.0	1.2959E-07	300.00	1.0	10.920	2360.0	-
68	16.000	67.0	1.2959E-07	450.00	1.0	14.850	2450.0	-
69	13.000	55.0	5.6080E-07	30.00	1.0	1.394	2388.0	-
70	13.000	55.0	5.6080E-07	150.00	1.0	6.725	3411.0	-
71	13.000	55.0	5.6080E-07	300.00	1.0	13.064	4156.0	-
72	13.000	55.0	5.6080E-07	450.00	1.0	19.194	4752.0	-
73	15.000	55.0	5.6080E-07	30.00	1.0	1.332	5851.0	-
74	15.000	55.0	5.6080E-07	150.00	1.0	6.209	7632.0	.291
75	15.000	55.0	5.6080E-07	300.00	1.0	11.770	9381.0	.268
76	15.000	55.0	5.6080E-07	450.00	1.0	17.040	10877.0	.255
77	16.000	55.0	5.6080E-07	30.00	1.0	1.298	8247.0	-
78	16.000	55.0	5.6080E-07	150.00	1.0	5.895	10294.0	-
79	16.000	55.0	5.6080E-07	300.00	1.0	11.070	12360.0	-
80	16.000	55.0	5.6080E-07	450.00	1.0	15.918	14314.0	-

TABLE I.- BASIC DATA - Continued

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$q_{R,D}$ , W/cm <sup>2</sup>	$F_{C,D}$
81	15.025	68.3	1.0780E-07	342.70	1.0	12.978	1542.0	-
82	14.897	66.3	1.4270E-07	342.70	1.0	13.040	2036.0	-
83	14.737	64.7	1.7580E-07	342.70	1.0	13.243	2348.0	-
84	14.549	63.4	2.0170E-07	342.70	1.0	13.383	2616.0	-
85	14.346	62.9	2.1600E-07	342.70	1.0	13.535	2618.0	-
86	14.161	62.8	2.1680E-07	342.70	1.0	13.669	2435.0	-
87	13.560	62.8	2.1680E-07	342.70	1.0	14.068	1892.0	-
88	13.650	63.5	2.0259E-07	60.96	1.0	2.634	1210.0	-
89	13.169	59.3	3.3412E-07	60.96	1.0	2.716	1732.0	-
90	12.845	57.4	4.2147E-07	60.96	1.0	2.766	1883.0	-
91	12.009	53.7	6.5255E-07	60.96	1.0	2.904	1771.0	-
92	15.000	59.5	3.2535E-07	150.00	1.0	6.079	4187.0	.304
93	15.000	59.5	3.2535E-07	300.00	1.0	11.608	4936.0	.268
94	15.000	59.5	3.2535E-07	450.00	1.0	16.870	5531.0	.252
95	15.000	64.0	1.8837E-07	150.00	1.0	6.015	2311.0	.328
96	15.000	64.0	1.8837E-07	300.00	1.0	11.530	2582.0	.272
97	15.000	64.0	1.8837E-07	450.00	1.0	16.625	2854.0	.253
98	15.000	70.0	8.7535E-08	30.00	1.0	1.258	635.0	.692
99	15.000	70.0	8.7535E-08	150.00	1.0	5.944	1031.0	.413
100	15.000	70.0	8.7535E-08	300.00	1.0	11.468	1169.0	.324
101	15.000	70.0	8.7535E-08	450.00	1.0	16.599	1252.0	.289
102	13.650	63.5	2.0259E-07	121.92	2.0	4.688	1514.0	-
103	13.169	59.3	3.3412E-07	121.92	2.0	4.843	2090.0	-
104	12.845	57.4	4.2147E-07	121.92	2.0	4.934	2230.0	-
105	12.458	55.5	5.2997E-07	121.92	2.0	5.044	2236.0	-
106	12.009	53.7	6.5255E-07	121.92	2.0	5.173	2092.0	-
107	11.499	52.0	8.0413E-07	121.92	2.0	5.290	1723.0	-
108	13.650	63.5	2.0259E-07	243.84	4.0	7.268	1639.0	-
109	13.169	59.3	3.3412E-07	243.84	4.0	7.501	2210.0	-
110	12.845	57.4	4.2147E-07	243.84	4.0	7.625	2419.0	-
111	12.458	55.5	5.2997E-07	243.84	4.0	7.783	2480.0	-
112	12.009	53.7	6.5255E-07	243.84	4.0	7.978	2325.0	-
113	11.499	52.0	8.0413E-07	243.84	4.0	8.242	1966.0	-
114	13.650	63.5	2.0259E-07	365.76	6.0	8.728	1720.0	-
115	13.169	59.3	3.3412E-07	365.76	6.0	8.963	2331.0	-
116	12.845	57.4	4.2147E-07	365.76	6.0	9.178	2553.0	-
117	12.458	55.5	5.2997E-07	365.76	6.0	9.352	2628.0	-
118	12.009	53.7	6.5255E-07	365.76	6.0	9.575	2512.0	-
119	11.499	52.0	8.0413E-07	365.76	6.0	9.800	2263.0	-
120	13.580	64.2	1.8447E-07	243.70	4.0	6.994	1375.0	-

TABLE I.- BASIC DATA - Concluded

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$q_{R,D}$ , W/cm <sup>2</sup>	$F_{C,D}$
121	13.232	60.1	3.0232E-07	243.70	4.0	7.195	2041.0	-
122	12.701	56.2	4.8433E-07	243.70	4.0	7.423	2570.0	-
123	11.949	52.6	7.4351E-07	243.70	4.0	7.791	2565.0	-
124	10.963	49.3	1.1199E-06	243.70	4.0	8.134	1612.0	-
125	12.700	59.2	3.3958E-07	179.83	4.0	5.644	1650.0	-
126	12.254	55.4	5.3436E-07	179.83	4.0	5.817	2049.0	-
127	11.611	51.9	8.1349E-07	179.83	4.0	6.063	1994.0	-
128	13.161	61.0	2.6849E-07	231.64	4.0	7.094	1800.0	-
129	12.767	57.1	4.3343E-07	231.64	4.0	7.370	2365.0	-
130	12.190	53.4	6.7347E-07	231.64	4.0	7.545	2703.0	-
131	11.398	50.0	1.0247E-06	231.64	4.0	7.947	2339.0	-
132	13.467	64.2	1.8330E-07	262.12	4.0	7.840	1327.0	-
133	12.909	59.2	3.3693E-07	262.12	4.0	8.073	1964.0	-
134	12.220	55.5	5.2983E-07	262.12	4.0	8.408	2168.0	-
135	11.295	52.0	7.9770E-07	262.12	4.0	8.847	1635.0	-
136	13.050	61.1	2.6667E-07	252.98	4.0	7.724	1652.0	-
137	12.592	57.2	4.2847E-07	252.98	4.0	8.220	2169.0	-
138	11.932	53.6	6.6135E-07	252.98	4.0	8.336	2255.0	-
139	11.052	50.3	9.9419E-07	252.98	4.0	8.695	1607.0	-
140	12.700	59.2	3.3938E-07	249.94	4.0	7.788	1763.0	-
141	12.254	55.4	5.3436E-07	249.94	4.0	8.013	2219.0	-
142	11.611	51.9	8.1349E-07	249.94	4.0	8.396	2206.0	-
143	13.016	62.0	2.3774E-07	231.64	4.0	7.075	1374.0	-
144	12.723	58.1	3.8424E-07	231.64	4.0	7.276	2009.0	-
145	12.281	54.4	6.0208E-07	231.64	4.0	7.484	2534.0	-
146	11.656	50.9	9.2060E-07	231.64	4.0	7.845	2589.0	-
147	10.819	47.6	1.3803E-06	231.64	4.0	8.238	1719.0	-
148	12.909	59.2	3.3693E-07	179.83	4.0	5.594	1829.0	-
149	12.220	55.5	5.2983E-07	179.83	4.0	5.820	1976.0	-
150	11.931	56.6	4.6395E-07	219.46	4.0	7.114	1444.0	-

TABLE II.- MATRIX ELEMENTS AND CONSTANT VECTOR FOR HEATING-RATE CURVE FIT

(a) Matrix elements  $a_{ji}$

$j \backslash i$	1	2	3	4	5	6	7	8	9
1	$\sum \ln \rho_\infty$	$\sum V_\infty \ln \rho_\infty$	$\sum \ln R_B$	$\sum V_\infty \ln R_B$	$\sum V_\infty^2 \ln R_B$	$\sum \ln V_\infty$	$\sum V_\infty \ln V_\infty$	$\sum V_\infty^2 \ln V_\infty$	
2	$\sum (\ln \rho_\infty)^2$	$\sum V_\infty (\ln \rho_\infty)^2$	$\sum \ln \rho_\infty \ln R_B$	$\sum V_\infty \ln \rho_\infty \ln R_B$	$\sum V_\infty^2 \ln \rho_\infty \ln R_B$	$\sum \ln \rho_\infty \ln V_\infty$	$\sum V_\infty \ln \rho_\infty \ln V_\infty$	$\sum V_\infty^2 \ln \rho_\infty \ln V_\infty$	
3		$\sum (V_\infty \ln \rho_\infty)^2$	$\sum V_\infty \ln \rho_\infty \ln R_B$	$\sum V_\infty^2 \ln \rho_\infty \ln R_B$	$\sum V_\infty^3 \ln \rho_\infty \ln R_B$	$\sum V_\infty \ln \rho_\infty \ln V_\infty$	$\sum V_\infty^2 \ln \rho_\infty \ln V_\infty$	$\sum V_\infty^3 \ln \rho_\infty \ln V_\infty$	
4			$\sum (\ln R_B)^2$	$\sum V_\infty (\ln R_B)^2$	$\sum (V_\infty \ln R_B)^2$	$\sum \ln V_\infty \ln R_B$	$\sum V_\infty \ln V_\infty \ln R_B$	$\sum V_\infty^2 \ln V_\infty \ln R_B$	
5				$\sum (V_\infty \ln R_B)^2$	$\sum V_\infty^3 (\ln R_B)^2$	$\sum V_\infty \ln V_\infty \ln R_B$	$\sum V_\infty^2 \ln V_\infty \ln R_B$	$\sum V_\infty^3 \ln V_\infty \ln R_B$	
6					$\sum V_\infty^4 (\ln R_B)^2$	$\sum V_\infty^2 \ln V_\infty \ln R_B$	$\sum V_\infty^3 \ln V_\infty \ln R_B$	$\sum V_\infty^4 \ln V_\infty \ln R_B$	
7						$\sum (\ln V_\infty)^2$	$\sum V_\infty (\ln V_\infty)^2$	$\sum (V_\infty \ln V_\infty)^2$	
8							$\sum (V_\infty \ln V_\infty)^2$	$\sum V_\infty^3 (\ln V_\infty)^2$	
9								$\sum V_\infty^4 (\ln V_\infty)^2$	

SYMMETRIC  $a_{ij} = a_{ji}$

\*The symbol N represents the total number of data points, and all summations are from 1 to N; that is,  $\sum_{n=1}^N$ .

(b) Constant vector  $D_j$

$$D_j = \left\{ \begin{array}{l} \sum \ln q_{R,D} \\ \sum (\ln q_{R,D} \ln \rho_\infty) \\ \sum (V_\infty \ln q_{R,D} \ln \rho_\infty) \\ \sum (\ln q_{R,D} \ln R_B) \\ \sum (V_\infty \ln q_{R,D} \ln R_B) \\ \sum (V_\infty^2 \ln q_{R,D} \ln R_B) \\ \sum (\ln q_{R,D} \ln V_\infty) \\ \sum (V_\infty \ln q_{R,D} \ln V_\infty) \\ \sum (V_\infty^2 \ln q_{R,D} \ln V_\infty) \end{array} \right\}$$

TABLE III.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR HEATING RATE

(a) Spherical bodies

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$q_{R,D}$ , W/cm <sup>2</sup>	$q_{R,C}$ , W/cm <sup>2</sup>	Error, percent
1	11.000	58.0	3.9072E-07	30.00	1.0	306.0	296.5	3.107
2	11.000	58.0	3.9072E-07	150.00	1.0	544.0	532.2	2.166
3	11.000	58.0	3.9072E-07	300.00	1.0	661.0	684.7	-3.589
4	11.000	58.0	3.9072E-07	450.00	1.0	752.0	793.5	-5.513
5	13.000	58.0	3.9072E-07	30.00	1.0	1607.0	1531.5	4.701
6	13.000	58.0	3.9072E-07	150.00	1.0	2263.0	2336.9	-3.267
7	13.000	58.0	3.9072E-07	300.00	1.0	2699.0	2803.4	-3.870
8	13.000	58.0	3.9072E-07	450.00	1.0	3080.0	3118.4	-1.247
9	14.000	58.0	3.9072E-07	30.00	1.0	2578.0	2586.9	-0.347
10	14.000	58.0	3.9072E-07	150.00	1.0	3537.0	3735.5	-5.611
11	14.000	58.0	3.9072E-07	300.00	1.0	4289.0	4375.8	-2.025
12	14.000	58.0	3.9072E-07	450.00	1.0	4816.0	4800.2	0.328
13	15.000	58.0	3.9072E-07	30.00	1.0	3709.0	3875.3	-4.484
14	15.000	58.0	3.9072E-07	150.00	1.0	5124.0	5387.7	-5.146
15	15.000	58.0	3.9072E-07	300.00	1.0	6108.0	6209.1	-1.656
16	15.000	58.0	3.9072E-07	450.00	1.0	6917.0	6746.6	2.464
17	16.000	58.0	3.9072E-07	30.00	1.0	5462.0	5387.4	1.365
18	16.000	58.0	3.9072E-07	150.00	1.0	7039.0	7337.3	-4.238
19	16.000	58.0	3.9072E-07	300.00	1.0	8176.0	8381.4	-2.512
20	16.000	58.0	3.9072E-07	450.00	1.0	9382.0	9059.7	3.435
21	11.000	61.0	2.7018E-07	30.00	1.0	194.0	191.5	1.307
22	11.000	61.0	2.7018E-07	150.00	1.0	365.0	343.7	5.839
23	11.000	61.0	2.7018E-07	300.00	1.0	447.0	442.2	1.081
24	11.000	61.0	2.7018E-07	450.00	1.0	509.0	512.4	-0.665
25	12.000	61.0	2.7018E-07	30.00	1.0	520.0	493.6	5.076
26	12.000	61.0	2.7018E-07	150.00	1.0	843.0	809.9	3.927
27	12.000	61.0	2.7018E-07	300.00	1.0	958.0	1002.4	-4.635
28	12.000	61.0	2.7018E-07	450.00	1.0	1087.0	1135.6	-4.470
29	13.000	61.0	2.7018E-07	30.00	1.0	1027.0	997.9	2.832



TABLE III.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR HEATING RATE - Continued

(a) Spherical bodies - Continued

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$q_{R,D}$ , W/cm <sup>2</sup>	$q_{R,C}$ , W/cm <sup>2</sup>	Error, percent
30	13.000	61.0	2.7018E-07	150.00	1.0	1545.0	1522.8	1.438
31	13.000	61.0	2.7018E-07	300.00	1.0	1786.0	1826.8	-2.283
32	13.000	61.0	2.7018E-07	450.00	1.0	1951.0	2032.0	-4.152
33	14.000	61.0	2.7018E-07	30.00	1.0	1737.0	1693.3	2.515
34	14.000	61.0	2.7018E-07	150.00	1.0	2353.0	2445.1	-3.914
35	14.000	61.0	2.7018E-07	300.00	1.0	2839.0	2864.3	-.890
36	14.000	61.0	2.7018E-07	450.00	1.0	3243.0	3142.0	3.113
37	15.000	61.0	2.7018E-07	30.00	1.0	2656.0	2548.1	4.062
38	15.000	61.0	2.7018E-07	150.00	1.0	3471.0	3542.5	-2.061
39	15.000	61.0	2.7018E-07	300.00	1.0	4154.0	4082.7	1.718
40	15.000	61.0	2.7018E-07	450.00	1.0	4690.0	4436.0	5.416
41	16.000	61.0	2.7018E-07	30.00	1.0	3778.0	3558.4	5.814
42	16.000	61.0	2.7018E-07	150.00	1.0	4771.0	4846.3	-1.578
43	16.000	61.0	2.7018E-07	300.00	1.0	5362.0	5535.9	-3.243
44	16.000	61.0	2.7018E-07	450.00	1.0	6174.0	5983.9	3.079
45	11.000	67.0	1.2959E-07	30.00	1.0	70.0	80.1	-14.480
46	11.000	67.0	1.2959E-07	150.00	1.0	149.0	143.8	3.458
47	11.000	67.0	1.2959E-07	300.00	1.0	190.0	185.1	2.597
48	11.000	67.0	1.2959E-07	450.00	1.0	215.0	214.5	.253
49	12.000	67.0	1.2959E-07	30.00	1.0	194.0	208.5	-7.453
50	12.000	67.0	1.2959E-07	150.00	1.0	363.0	342.0	5.776
51	12.000	67.0	1.2959E-07	300.00	1.0	442.0	423.3	4.223
52	12.000	67.0	1.2959E-07	450.00	1.0	490.0	479.6	2.127
53	13.000	67.0	1.2959E-07	30.00	1.0	396.0	425.2	-7.385
54	13.000	67.0	1.2959E-07	150.00	1.0	674.0	648.9	3.724
55	13.000	67.0	1.2959E-07	300.00	1.0	796.0	778.4	2.206
56	13.000	67.0	1.2959E-07	450.00	1.0	860.0	865.9	-.686
57	14.000	67.0	1.2959E-07	30.00	1.0	688.0	728.1	-5.826
58	14.000	67.0	1.2959E-07	150.00	1.0	1079.0	1051.3	2.564
59	14.000	67.0	1.2959E-07	300.00	1.0	1218.0	1231.6	-1.114

TABLE III.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR HEATING RATE - Continued

(a) Spherical bodies - Concluded

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$q_{R,D}$ , W/cm <sup>2</sup>	$q_{R,C}$ , W/cm <sup>2</sup>	Error, percent
60	14.000	67.0	1.2959E-07	450.00	1.0	1347.0	1351.0	-0.297
61	15.000	67.0	1.2959E-07	30.00	1.0	1064.0	1105.5	-3.902
62	15.000	67.0	1.2959E-07	150.00	1.0	1576.0	1537.0	2.477
63	15.000	67.0	1.2959E-07	300.00	1.0	1750.0	1771.3	-1.217
64	15.000	67.0	1.2959E-07	450.00	1.0	1913.0	1924.6	-0.606
65	16.000	67.0	1.2959E-07	30.00	1.0	1531.0	1557.8	-1.748
66	16.000	67.0	1.2959E-07	150.00	1.0	2150.0	2121.6	1.322
67	16.000	67.0	1.2959E-07	300.00	1.0	2360.0	2423.5	-2.689
68	16.000	67.0	1.2959E-07	450.00	1.0	2450.0	2619.6	-6.922
69	13.000	55.0	5.6080E-07	30.00	1.0	2388.0	2329.8	2.438
70	13.000	55.0	5.6080E-07	150.00	1.0	3411.0	3555.1	-4.226
71	13.000	55.0	5.6080E-07	300.00	1.0	4156.0	4264.9	-2.619
72	13.000	55.0	5.6080E-07	450.00	1.0	4752.0	4744.0	0.169
73	15.000	55.0	5.6080E-07	30.00	1.0	5851.0	5843.6	0.127
74	15.000	55.0	5.6080E-07	150.00	1.0	7632.0	8124.1	-6.448
75	15.000	55.0	5.6080E-07	300.00	1.0	9381.0	9362.8	0.194
76	15.000	55.0	5.6080E-07	450.00	1.0	10877.0	10173.1	6.471
77	16.000	55.0	5.6080E-07	30.00	1.0	8247.0	8087.9	1.930
78	16.000	55.0	5.6080E-07	150.00	1.0	10294.0	11015.2	-7.006
79	16.000	55.0	5.6080E-07	300.00	1.0	12360.0	12582.6	-1.801
80	16.000	55.0	5.6080E-07	450.00	1.0	14314.0	13600.9	4.982
81	15.025	68.3	1.0780E-07	342.70	1.0	1542.0	1488.8	3.449
82	14.897	66.3	1.4270E-07	342.70	1.0	2036.0	1962.9	3.589
83	14.737	64.7	1.7580E-07	342.70	1.0	2348.0	2358.9	-0.465
84	14.549	63.4	2.0170E-07	342.70	1.0	2616.0	2586.4	1.133
85	14.346	62.9	2.1600E-07	342.70	1.0	2618.0	2601.8	0.617
86	14.161	62.8	2.1680E-07	342.70	1.0	2435.0	2439.7	-0.193
87	13.560	62.8	2.1680E-07	342.70	1.0	1892.0	1910.4	-0.974
88	13.650	63.5	2.0259E-07	60.96	1.0	1210.0	1215.8	-0.479
89	13.169	59.3	3.3412E-07	60.96	1.0	1732.0	1691.5	2.336
90	12.845	57.4	4.2147E-07	60.96	1.0	1883.0	1839.6	2.304
91	12.009	53.7	6.5255E-07	60.96	1.0	1771.0	1739.5	1.781

TABLE III.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR HEATING RATE - Continued

(b) Nonspherical bodies

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$q_{R,D}$ , W/cm <sup>2</sup>	$q_{R,C}$ , W/cm <sup>2</sup>	Error, percent
102	13.650	63.5	2.0259E-07	121.92	2.0	1514.0	1394.8	7.874
103	13.169	59.3	3.3412E-07	121.92	2.0	2090.0	1963.6	6.050
104	12.845	57.4	4.2147E-07	121.92	2.0	2230.0	2154.5	3.385
105	12.458	55.5	5.2997E-07	121.92	2.0	2236.0	2227.8	.367
106	12.009	53.7	6.5255E-07	121.92	2.0	2092.0	2092.1	-.004
107	11.499	52.0	8.0413E-07	121.92	2.0	1723.0	1788.6	-3.810
108	13.650	63.5	2.0259E-07	243.84	4.0	1639.0	1537.2	6.209
109	13.169	59.3	3.3412E-07	243.84	4.0	2210.0	2189.7	.919
110	12.845	57.4	4.2147E-07	243.84	4.0	2419.0	2424.1	-.212
111	12.458	55.5	5.2997E-07	243.84	4.0	2480.0	2536.0	-2.256
112	12.009	53.7	6.5255E-07	243.84	4.0	2325.0	2417.3	-3.968
113	11.499	52.0	8.0413E-07	243.84	4.0	1966.0	2105.8	-7.112
114	13.650	63.5	2.0259E-07	365.76	6.0	1720.0	1588.6	7.640
115	13.169	59.3	3.3412E-07	365.76	6.0	2331.0	2278.5	2.254
116	12.845	57.4	4.2147E-07	365.76	6.0	2553.0	2535.6	.683
117	12.458	55.5	5.2997E-07	365.76	6.0	2628.0	2670.7	-1.624
118	12.009	53.7	6.5255E-07	365.76	6.0	2512.0	2568.0	-2.229
119	11.499	52.0	8.0413E-07	365.76	6.0	2263.0	2261.8	.052
120	13.580	64.2	1.8447E-07	243.70	4.0	1375.0	1337.3	2.745
121	13.232	60.1	3.0232E-07	243.70	4.0	2041.0	2011.2	1.460
122	12.701	56.2	4.8433E-07	243.70	4.0	2570.0	2631.3	-2.387
123	11.949	52.6	7.4351E-07	243.70	4.0	2565.0	2700.9	-5.298
124	10.963	49.3	1.1199E-06	243.70	4.0	1612.0	1938.4	-20.250
125	12.700	59.2	3.3958E-07	179.83	4.0	1650.0	1599.7	3.047

TABLE III.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR HEATING RATE - Concluded

(b) Nonspherical bodies - Concluded

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$q_{R,D}$ , W/cm <sup>2</sup>	$q_{R,C}$ , W/cm <sup>2</sup>	Error, percent
126	12.254	55.4	5.3436E-07	179.83	4.0	2049.0	2060.0	-0.536
127	11.611	51.9	8.1349E-07	179.83	4.0	1994.0	2112.6	-5.949
128	13.161	61.0	2.6849E-07	231.64	4.0	1800.0	1670.6	7.190
129	12.767	57.1	4.3343E-07	231.64	4.0	2365.0	2366.3	-0.053
130	12.190	53.4	6.7347E-07	231.64	4.0	2703.0	2791.2	-3.262
131	11.398	50.0	1.0247E-06	231.64	4.0	2339.0	2534.5	-8.356
132	13.467	64.2	1.8330E-07	262.12	4.0	1327.0	1283.5	3.281
133	12.909	59.2	3.3693E-07	262.12	4.0	1964.0	1971.7	-0.393
134	12.220	55.5	5.2983E-07	262.12	4.0	2168.0	2230.0	-2.860
135	11.295	52.0	7.9770E-07	262.12	4.0	1635.0	1801.8	-10.200
136	13.050	61.1	2.6667E-07	252.98	4.0	1652.0	1602.7	2.984
137	12.592	57.2	4.2847E-07	252.98	4.0	2169.0	2165.3	-0.171
138	11.932	53.6	6.6135E-07	252.98	4.0	2255.0	2353.0	-4.345
139	11.052	50.3	9.9419E-07	252.98	4.0	1607.0	1856.3	-15.511
140	12.700	59.2	3.3938E-07	249.94	4.0	1763.0	1750.1	.732
141	12.254	55.4	5.3436E-07	249.94	4.0	2219.0	2270.2	-2.308
142	11.611	51.9	8.1349E-07	249.94	4.0	2206.0	2353.6	-6.691
143	13.016	62.0	2.3774E-07	231.64	4.0	1374.0	1346.8	1.980
144	12.723	58.1	3.8424E-07	231.64	4.0	2009.0	2006.5	.124
145	12.281	54.4	6.0208E-07	231.64	4.0	2534.0	2597.2	-2.494
146	11.656	50.9	9.2060E-07	231.64	4.0	2589.0	2750.6	-6.243
147	10.819	47.6	1.3803E-06	231.64	4.0	1719.0	2115.9	-23.090
148	12.909	59.2	3.3693E-07	179.83	4.0	1829.0	1783.5	2.487
149	12.220	55.5	5.2983E-07	179.83	4.0	1976.0	1994.0	-0.913
150	11.931	56.6	4.6395E-07	219.46	4.0	1444.0	1483.7	-2.749

TABLE IV.- MATRIX ELEMENTS AND CONSTANT VECTOR FOR COOLING-FACTOR CURVE FIT.

(a) Matrix elements  $b_{ji}$

$j \backslash i$	1	2	3	4	5	6	7
1	*N	$\sum \ln \rho_\infty$	$\sum (R_B \ln \rho_\infty)$	$\sum (R_B^2 \ln \rho_\infty)$	$\sum (R_B^3 \ln \rho_\infty)$	$\sum \ln R_B$	$\sum \ln V_\infty$
2		$\sum (\ln \rho_\infty)^2$	$\sum [R_B (\ln \rho_\infty)^2]$	$\sum (R_B \ln \rho_\infty)^2$	$\sum [R_B^3 (\ln \rho_\infty)^2]$	$\sum (\ln \rho_\infty \ln R_B)$	$\sum (\ln \rho_\infty \ln V_\infty)$
3			$\sum (R_B \ln \rho_\infty)^2$	$\sum [R_B^3 (\ln \rho_\infty)^2]$	$\sum [R_B^4 (\ln \rho_\infty)^2]$	$\sum (R_B \ln \rho_\infty \ln R_B)$	$\sum (R_B \ln \rho_\infty \ln V_\infty)$
4				$\sum [R_B^4 (\ln \rho_\infty)^2]$	$\sum [R_B^5 (\ln \rho_\infty)^2]$	$\sum (R_B^2 \ln \rho_\infty \ln R_B)$	$\sum (R_B^2 \ln \rho_\infty \ln V_\infty)$
5					$\sum [R_B^6 (\ln \rho_\infty)^2]$	$\sum (R_B^3 \ln \rho_\infty \ln R_B)$	$\sum (R_B^3 \ln \rho_\infty \ln V_\infty)$
6						$\sum (\ln R_B)^2$	$\sum (\ln V_\infty \ln R_B)$
7							$\sum (\ln V_\infty)^2$

SYMMETRIC  $b_{ij} = b_{ji}$

\*The symbol N represents the total number of data points, and all summations are from 1 to N; that is,  $\sum_{n=1}^N$ .

(b) Constant vector  $G_j$

$$G_j = \left\{ \begin{array}{l} \sum \ln F_{C,D} \\ \sum (\ln F_{C,D} \ln \rho_\infty) \\ \sum (R_B \ln F_{C,D} \ln \rho_\infty) \\ \sum (R_B^2 \ln F_{C,D} \ln \rho_\infty) \\ \sum (R_B^3 \ln F_{C,D} \ln \rho_\infty) \\ \sum (\ln F_{C,D} \ln R_B) \\ \sum (\ln F_{C,D} \ln V_\infty) \end{array} \right\}$$

TABLE V.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR COOLING FACTOR

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	F <sub>C,D</sub>	F <sub>C,C</sub>	Error, percent
5	13.000	58.0	3.9072E-07	30.00	1.0	.582	.580	.383
9	14.000	58.0	3.9072E-07	30.00	1.0	.508	.502	1.089
11	14.000	58.0	3.9072E-07	300.00	1.0	.307	.305	.800
12	14.000	58.0	3.9072E-07	450.00	1.0	.285	.290	-1.636
13	15.000	58.0	3.9072E-07	30.00	1.0	.452	.440	2.700
14	15.000	58.0	3.9072E-07	150.00	1.0	.298	.302	-1.432
15	15.000	58.0	3.9072E-07	300.00	1.0	.268	.267	.538
16	15.000	58.0	3.9072E-07	450.00	1.0	.252	.254	-.608
17	16.000	58.0	3.9072E-07	30.00	1.0	.401	.388	3.176
18	16.000	58.0	3.9072E-07	150.00	1.0	.261	.267	-2.241
19	16.000	58.0	3.9072E-07	300.00	1.0	.231	.235	-1.872
20	16.000	58.0	3.9072E-07	450.00	1.0	.224	.224	-.078
21	11.000	61.0	2.7018E-07	30.00	1.0	.812	.895	-10.279
22	11.000	61.0	2.7018E-07	150.00	1.0	.626	.584	6.666
23	11.000	61.0	2.7018E-07	300.00	1.0	.541	.504	6.786
24	11.000	61.0	2.7018E-07	450.00	1.0	.509	.474	6.890
25	12.000	61.0	2.7018E-07	30.00	1.0	.710	.757	-6.616
26	12.000	61.0	2.7018E-07	150.00	1.0	.497	.494	.623
27	12.000	61.0	2.7018E-07	300.00	1.0	.412	.426	-3.470
28	12.000	61.0	2.7018E-07	450.00	1.0	.383	.401	-4.603
29	13.000	61.0	2.7018E-07	30.00	1.0	.630	.649	-2.947
30	13.000	61.0	2.7018E-07	150.00	1.0	.429	.423	1.359
31	13.000	61.0	2.7018E-07	300.00	1.0	.361	.365	-1.176
32	13.000	61.0	2.7018E-07	450.00	1.0	.327	.343	-4.971
33	14.000	61.0	2.7018E-07	30.00	1.0	.566	.562	.691
34	14.000	61.0	2.7018E-07	150.00	1.0	.356	.367	-3.019

TABLE V.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS FOR COOLING FACTOR - Concluded

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$F_{C,D}$	$F_{C,C}$	Error, percent
35	14.000	61.0	2.7018E-07	300.00	1.0	.318	.317	.458
36	14.000	61.0	2.7018E-07	450.00	1.0	.303	.297	1.819
37	15.000	61.0	2.7018E-07	30.00	1.0	.514	.492	4.284
38	15.000	61.0	2.7018E-07	150.00	1.0	.314	.321	-2.230
39	15.000	61.0	2.7018E-07	300.00	1.0	.282	.277	1.751
40	15.000	61.0	2.7018E-07	450.00	1.0	.268	.260	2.843
41	16.000	61.0	2.7018E-07	30.00	1.0	.466	.434	6.795
42	16.000	61.0	2.7018E-07	150.00	1.0	.272	.283	-4.188
43	16.000	61.0	2.7018E-07	300.00	1.0	.233	.245	-4.978
44	16.000	61.0	2.7018E-07	450.00	1.0	.227	.230	-1.265
61	15.000	67.0	1.2959E-07	30.00	1.0	.620	.615	.793
62	15.000	67.0	1.2959E-07	150.00	1.0	.362	.362	.041
63	15.000	67.0	1.2959E-07	300.00	1.0	.291	.299	-2.828
64	15.000	67.0	1.2959E-07	450.00	1.0	.269	.275	-2.074
74	15.000	55.0	5.6080E-07	150.00	1.0	.291	.285	2.071
75	15.000	55.0	5.6080E-07	300.00	1.0	.268	.257	4.233
76	15.000	55.0	5.6080E-07	450.00	1.0	.255	.247	3.139
92	15.000	59.5	3.2535E-07	150.00	1.0	.304	.311	-2.442
93	15.000	59.5	3.2535E-07	300.00	1.0	.268	.272	-1.388
94	15.000	59.5	3.2535E-07	450.00	1.0	.252	.257	-1.948
95	15.000	64.0	1.8837E-07	150.00	1.0	.328	.340	-3.794
96	15.000	64.0	1.8837E-07	300.00	1.0	.272	.288	-5.783
97	15.000	64.0	1.8837E-07	450.00	1.0	.253	.267	-5.635
98	15.000	70.0	8.7535E-08	30.00	1.0	.692	.693	-1.143
99	15.000	70.0	8.7535E-08	150.00	1.0	.413	.386	6.598
100	15.000	70.0	8.7535E-08	300.00	1.0	.324	.312	3.770
101	15.000	70.0	8.7535E-08	450.00	1.0	.289	.282	2.258

TABLE VI.- MATRIX ELEMENTS AND CONSTANT VECTOR FOR STANDOFF-DISTANCE CURVE FIT

(a) Matrix elements  $c_{ji}$

$\begin{matrix} i \\ j \end{matrix}$	1	2	3	4	5	6	7	8
1	*N	$\sum \ln \rho_\infty$	$\sum (V_\infty \ln \rho_\infty)$	$\sum \ln R_B$	$\sum (V_\infty \ln R_B)$	$\sum \ln V_\infty$	$\sum (V_\infty \ln V_\infty)$	$\sum (V_\infty^2 \ln V_\infty)$
2		$\sum (\ln \rho_\infty)^2$	$\sum [V_\infty (\ln \rho_\infty)^2]$	$\sum (\ln \rho_\infty \ln R_B)$	$\sum (V_\infty \ln \rho_\infty \ln R_B)$	$\sum (\ln \rho_\infty \ln V_\infty)$	$\sum (V_\infty \ln \rho_\infty \ln V_\infty)$	$\sum (V_\infty^2 \ln \rho_\infty \ln V_\infty)$
3			$\sum [V_\infty^2 (\ln \rho_\infty)^2]$	$\sum (V_\infty \ln \rho_\infty \ln R_B)$	$\sum (V_\infty^2 \ln \rho_\infty \ln R_B)$	$\sum (V_\infty \ln \rho_\infty \ln V_\infty)$	$\sum (V_\infty^2 \ln \rho_\infty \ln V_\infty)$	$\sum (V_\infty^3 \ln \rho_\infty \ln V_\infty)$
4				$\sum (\ln R_B)^2$	$\sum [V_\infty (\ln R_B)^2]$	$\sum (\ln V_\infty \ln R_B)$	$\sum (V_\infty \ln V_\infty \ln R_B)$	$\sum (V_\infty^2 \ln V_\infty \ln R_B)$
5					$\sum [V_\infty^2 (\ln R_B)^2]$	$\sum (V_\infty \ln V_\infty \ln R_B)$	$\sum (V_\infty^2 \ln V_\infty \ln R_B)$	$\sum (V_\infty^3 \ln V_\infty \ln R_B)$
6						$\sum (\ln V_\infty)^2$	$\sum [V_\infty (\ln V_\infty)^2]$	$\sum [V_\infty^2 (\ln V_\infty)^2]$
7							$\sum [V_\infty^2 (\ln V_\infty)^2]$	$\sum [V_\infty^3 (\ln V_\infty)^2]$
8								$\sum [V_\infty^4 (\ln V_\infty)^2]$

SYMMETRIC  $c_{ij} = c_{ji}$

\*The symbol N represents the total number of data points, and all summations are from 1 to N; that is,  $\sum_{n=1}^N$

(b) Constant vector  $H_j$

$$H_j = \left\{ \begin{array}{l} \sum \ln \delta_D \\ \sum (\ln \delta_D \ln \rho_\infty) \\ \sum (V_\infty \ln \delta_D \ln \rho_\infty) \\ \sum (\ln \delta_D \ln R_B) \\ \sum (V_\infty \ln \delta_D \ln R_B) \\ \sum (\ln \delta_D \ln V_\infty) \\ \sum (V_\infty \ln \delta_D \ln V_\infty) \\ \sum (V_\infty^2 \ln \delta_D \ln V_\infty) \end{array} \right\}$$



TABLE VII.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS  
FOR SHOCK STANDOFF DISTANCE

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	(a) Spherical bodies			$\delta_D$ , cm	$\delta_C$ , cm	Error, percent
				$R_B$ , cm	Axis ratio				
1	11.000	58.0	3.9072E-07	30.00	1.0	1.421	1.413	.532	
2	11.000	58.0	3.9072E-07	150.00	1.0	7.036	7.030	.085	
3	11.000	58.0	3.9072E-07	300.00	1.0	13.974	14.028	-0.387	
4	11.000	58.0	3.9072E-07	450.00	1.0	20.844	21.014	-0.816	
5	13.000	58.0	3.9072E-07	30.00	1.0	1.368	1.381	-0.946	
6	13.000	58.0	3.9072E-07	150.00	1.0	6.680	6.598	1.229	
7	13.000	58.0	3.9072E-07	300.00	1.0	12.887	12.940	-0.411	
8	13.000	58.0	3.9072E-07	450.00	1.0	18.971	19.189	-1.149	
9	14.000	58.0	3.9072E-07	30.00	1.0	1.339	1.352	-1.007	
10	14.000	58.0	3.9072E-07	150.00	1.0	6.374	6.333	.636	
11	14.000	58.0	3.9072E-07	300.00	1.0	12.268	12.314	-0.378	
12	14.000	58.0	3.9072E-07	450.00	1.0	17.943	18.169	-1.260	
13	15.000	58.0	3.9072E-07	30.00	1.0	1.308	1.318	-0.764	
14	15.000	58.0	3.9072E-07	150.00	1.0	6.093	6.049	.719	
15	15.000	58.0	3.9072E-07	300.00	1.0	11.611	11.660	-0.425	
16	15.000	58.0	3.9072E-07	450.00	1.0	16.850	17.117	-1.586	
17	16.000	58.0	3.9072E-07	30.00	1.0	1.277	1.279	-0.139	
18	16.000	58.0	3.9072E-07	150.00	1.0	5.830	5.753	1.329	
19	16.000	58.0	3.9072E-07	300.00	1.0	11.011	10.993	.164	
20	16.000	58.0	3.9072E-07	450.00	1.0	15.873	16.056	-1.154	
21	11.000	61.0	2.7018E-07	30.00	1.0	1.397	1.394	.196	
22	11.000	61.0	2.7018E-07	150.00	1.0	6.921	6.935	-0.197	
23	11.000	61.0	2.7018E-07	300.00	1.0	13.725	13.838	-0.821	
24	11.000	61.0	2.7018E-07	450.00	1.0	20.464	20.729	-1.295	
25	12.000	61.0	2.7018E-07	30.00	1.0	1.378	1.385	-0.473	
26	12.000	61.0	2.7018E-07	150.00	1.0	6.760	6.749	.159	
27	12.000	61.0	2.7018E-07	300.00	1.0	13.390	13.352	.285	
28	12.000	61.0	2.7018E-07	450.00	1.0	19.844	19.900	-0.283	
29	13.000	61.0	2.7018E-07	30.00	1.0	1.348	1.366	-1.303	

TABLE VII.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS

FOR SHOCK STANDOFF DISTANCE - Continued

(a) Spherical bodies - Continued

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$\delta_C$ , cm	Error, percent
30	13.000	61.0	2.7018E-07	150.00	1.0	6.640	6.524	1.740
31	13.000	61.0	2.7018E-07	300.00	1.0	12.710	12.796	-.676
32	13.000	61.0	2.7018E-07	450.00	1.0	18.800	18.975	-.933
33	14.000	61.0	2.7018E-07	30.00	1.0	1.319	1.339	-1.522
34	14.000	61.0	2.7018E-07	150.00	1.0	6.280	6.271	.149
35	14.000	61.0	2.7018E-07	300.00	1.0	12.090	12.192	-.846
36	14.000	61.0	2.7018E-07	450.00	1.0	17.660	17.989	-1.862
37	15.000	61.0	2.7018E-07	30.00	1.0	1.287	1.307	-1.517
38	15.000	61.0	2.7018E-07	150.00	1.0	5.995	5.997	-.026
39	15.000	61.0	2.7018E-07	300.00	1.0	11.440	11.559	-1.040
40	15.000	61.0	2.7018E-07	450.00	1.0	16.850	16.968	-.703
41	16.000	61.0	2.7018E-07	30.00	1.0	1.256	1.269	-1.053
42	16.000	61.0	2.7018E-07	150.00	1.0	5.713	5.710	.061
43	16.000	61.0	2.7018E-07	300.00	1.0	10.920	10.911	.084
44	16.000	61.0	2.7018E-07	450.00	1.0	15.550	15.936	-2.483
45	11.000	67.0	1.2959E-07	30.00	1.0	1.360	1.357	.232
46	11.000	67.0	1.2959E-07	150.00	1.0	6.754	6.748	.082
47	11.000	67.0	1.2959E-07	300.00	1.0	13.445	13.466	-.158
48	11.000	67.0	1.2959E-07	450.00	1.0	20.019	20.173	-.767
49	12.000	67.0	1.2959E-07	30.00	1.0	1.342	1.351	-.646
50	12.000	67.0	1.2959E-07	150.00	1.0	6.597	6.584	.194
51	12.000	67.0	1.2959E-07	300.00	1.0	13.045	13.025	.151
52	12.000	67.0	1.2959E-07	450.00	1.0	19.392	19.414	-.111
53	13.000	67.0	1.2959E-07	30.00	1.0	1.317	1.335	-1.400
54	13.000	67.0	1.2959E-07	150.00	1.0	6.392	6.381	.179
55	13.000	67.0	1.2959E-07	300.00	1.0	12.525	12.514	.090
56	13.000	67.0	1.2959E-07	450.00	1.0	18.498	18.557	-.318
57	14.000	67.0	1.2959E-07	30.00	1.0	1.292	1.313	-1.606
58	14.000	67.0	1.2959E-07	150.00	1.0	6.194	6.147	.753
59	14.000	67.0	1.2959E-07	300.00	1.0	12.022	11.953	.577

TABLE VII.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS

FOR SHOCK STANDOFF DISTANCE - Continued

(a) Spherical bodies - Concluded

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$\delta_C$ , cm	Error, percent
60	14.000	67.0	1.2959E-07	450.00	1.0	17.636	17.635	.004
61	15.000	67.0	1.2959E-07	30.00	1.0	1.268	1.284	-1.261
62	15.000	67.0	1.2959E-07	150.00	1.0	5.980	5.893	1.452
63	15.000	67.0	1.2959E-07	300.00	1.0	11.470	11.360	.963
64	15.000	67.0	1.2959E-07	450.00	1.0	16.710	16.676	.205
65	16.000	67.0	1.2959E-07	30.00	1.0	1.242	1.250	-.676
66	16.000	67.0	1.2959E-07	150.00	1.0	5.756	5.625	2.279
67	16.000	67.0	1.2959E-07	300.00	1.0	10.920	10.749	1.566
68	16.000	67.0	1.2959E-07	450.00	1.0	14.850	15.700	-5.722
69	13.000	55.0	5.6080E-07	30.00	1.0	1.394	1.396	-.156
70	13.000	55.0	5.6080E-07	150.00	1.0	6.725	6.671	.808
71	13.000	55.0	5.6080E-07	300.00	1.0	13.064	13.083	-.144
72	13.000	55.0	5.6080E-07	450.00	1.0	19.194	19.401	-1.077
73	15.000	55.0	5.6080E-07	30.00	1.0	1.332	1.329	.201
74	15.000	55.0	5.6080E-07	150.00	1.0	6.209	6.101	1.737
75	15.000	55.0	5.6080E-07	300.00	1.0	11.770	11.761	.080
76	15.000	55.0	5.6080E-07	450.00	1.0	17.040	17.264	-1.317
77	16.000	55.0	5.6080E-07	30.00	1.0	1.298	1.288	.754
78	16.000	55.0	5.6080E-07	150.00	1.0	5.895	5.795	1.697
79	16.000	55.0	5.6080E-07	300.00	1.0	11.070	11.074	-.037
80	16.000	55.0	5.6080E-07	450.00	1.0	15.918	16.175	-1.612
81	15.025	68.3	1.0780E-07	342.70	1.0	12.978	12.811	1.283
82	14.897	66.3	1.4270E-07	342.70	1.0	13.040	12.988	.402
83	14.737	64.7	1.7580E-07	342.70	1.0	13.243	13.168	.564
84	14.549	63.4	2.0170E-07	342.70	1.0	13.383	13.349	.251
85	14.346	62.9	2.1600E-07	342.70	1.0	13.535	13.520	.111
86	14.161	62.8	2.1680E-07	342.70	1.0	13.669	13.655	.104
87	13.560	62.8	2.1680E-07	342.70	1.0	14.068	14.081	-.094
88	13.650	63.5	2.0259E-07	60.96	1.0	2.634	2.650	-.609
89	13.169	59.3	3.3412E-07	60.96	1.0	2.716	2.725	-.332
90	12.845	57.4	4.2147E-07	60.96	1.0	2.766	2.768	-.080
91	12.009	53.7	6.5255E-07	60.96	1.0	2.904	2.865	1.329

TABLE VII.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS  
FOR SHOCK STANDOFF DISTANCE -- Continued

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$\delta_C$ , cm	Error, percent
102	13.650	63.5	2.0259E-07	121.92	2.0	4.688	4.606	1.753
103	13.169	59.3	3.3412E-07	121.92	2.0	4.843	4.756	1.800
104	12.845	57.4	4.2147E-07	121.92	2.0	4.934	4.845	1.807
105	12.458	55.5	5.2997E-07	121.92	2.0	5.044	4.945	1.967
106	12.009	53.7	6.5255E-07	121.92	2.0	5.173	5.051	2.352
107	11.499	52.0	8.0413E-07	121.92	2.0	5.290	5.165	2.355
108	13.650	63.5	2.0259E-07	243.84	4.0	7.268	7.106	2.230
109	13.169	59.3	3.3412E-07	243.84	4.0	7.501	7.368	1.773
110	12.845	57.4	4.2147E-07	243.84	4.0	7.625	7.527	1.286
111	12.458	55.5	5.2997E-07	243.84	4.0	7.783	7.708	.964
112	12.009	53.7	6.5255E-07	243.84	4.0	7.978	7.905	.919
113	11.499	52.0	8.0413E-07	243.84	4.0	8.242	8.119	1.492
114	13.650	63.5	2.0259E-07	365.76	6.0	8.728	8.688	.459
115	13.169	59.3	3.3412E-07	365.76	6.0	8.963	9.030	-.751
116	12.845	57.4	4.2147E-07	365.76	6.0	9.178	9.240	-.678
117	12.458	55.5	5.2997E-07	365.76	6.0	9.352	9.481	-1.380
118	12.009	53.7	6.5255E-07	365.76	6.0	9.575	9.745	-1.777
119	11.499	52.0	8.0413E-07	365.76	6.0	9.800	10.035	-2.400
120	13.580	64.2	1.8447E-07	243.70	4.0	6.994	7.106	-1.596
121	13.232	60.1	3.0232E-07	243.70	4.0	7.195	7.321	-1.757
122	12.701	56.2	4.8433E-07	243.70	4.0	7.423	7.603	-2.420
123	11.949	52.6	7.4351E-07	243.70	4.0	7.791	7.954	-2.093
124	10.963	49.3	1.1199E-06	243.70	4.0	8.134	8.370	-2.896
125	12.700	59.2	3.3958E-07	179.83	4.0	5.644	5.590	.962

(b) Nonspherical bodies

TABLE VII.- COMPARISON OF BASIC DATA AND CURVE-FIT RESULTS

FOR SHOCK STANDOFF DISTANCE - Concluded

(b) Nonspherical bodies - Concluded

Case	$V_{\infty}$ , km/sec	Alt., km	$\rho_{\infty}$ , g/cm <sup>3</sup>	$R_B$ , cm	Axis ratio	$\delta_D$ , cm	$\delta_C$ , cm	Error, percent
126	12.254	55.4	5.3436E-07	179.83	4.0	5.817	5.767	.853
127	11.611	51.9	8.1349E-07	179.83	4.0	6.063	5.985	1.286
128	13.161	61.0	2.6849E-07	231.64	4.0	7.094	6.967	1.788
129	12.767	57.1	4.3343E-07	231.64	4.0	7.370	7.190	2.441
130	12.190	53.4	6.7347E-07	231.64	4.0	7.545	7.469	1.003
131	11.398	50.0	1.0247E-06	231.64	4.0	7.947	7.812	1.701
132	13.467	64.2	1.8330E-07	262.12	4.0	7.840	7.661	2.285
133	12.909	59.2	3.3693E-07	262.12	4.0	8.073	7.998	.933
134	12.220	55.5	5.2983E-07	262.12	4.0	8.408	8.356	.616
135	11.295	52.0	7.9770E-07	262.12	4.0	8.847	8.785	.699
136	13.050	61.1	2.6667E-07	252.98	4.0	7.724	7.624	1.290
137	12.592	57.2	4.2847E-07	252.98	4.0	8.220	7.891	3.997
138	11.932	53.6	6.6135E-07	252.98	4.0	8.336	8.225	1.329
139	11.052	50.3	9.9419E-07	252.98	4.0	8.695	8.623	.832
140	12.700	59.2	3.3938E-07	249.94	4.0	7.788	7.706	1.048
141	12.254	55.4	5.3436E-07	249.94	4.0	8.013	7.966	.585
142	11.611	51.9	8.1349E-07	249.94	4.0	8.396	8.289	1.279
143	13.016	62.0	2.3774E-07	231.64	4.0	7.075	6.985	1.275
144	12.723	58.1	3.8424E-07	231.64	4.0	7.276	7.176	1.368
145	12.281	54.4	6.0208E-07	231.64	4.0	7.484	7.414	.930
146	11.656	50.9	9.2060E-07	231.64	4.0	7.845	7.708	1.742
147	10.819	47.6	1.3803E-06	231.64	4.0	8.238	8.057	2.196
148	12.909	59.2	3.3693E-07	179.83	4.0	5.594	5.543	.908
149	12.220	55.5	5.2983E-07	179.83	4.0	5.820	5.773	.808
150	11.931	56.6	4.6395E-07	219.46	4.0	7.114	7.065	.685

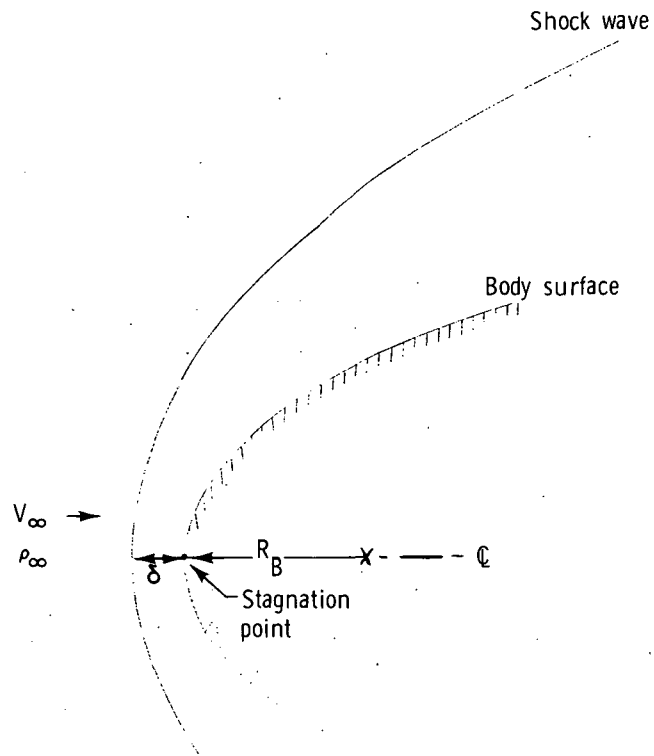


Figure 1.- Parameters of inviscid radiating flow-field solutions for stagnation region of a blunt body.

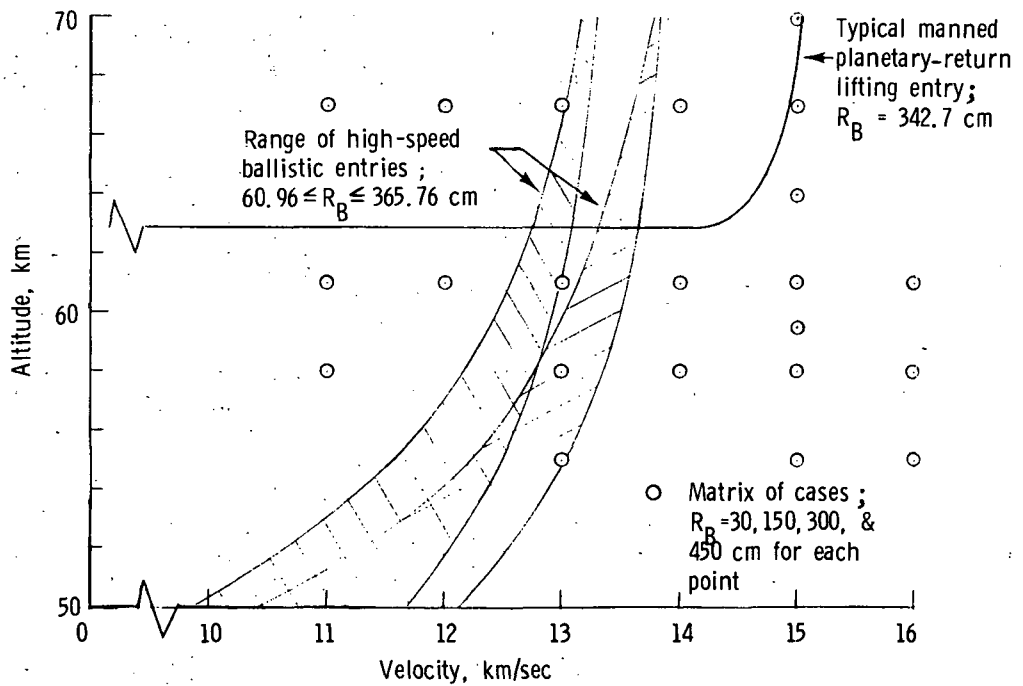


Figure 2.- Free-stream conditions and body sizes used in generating data to be curve fitted.

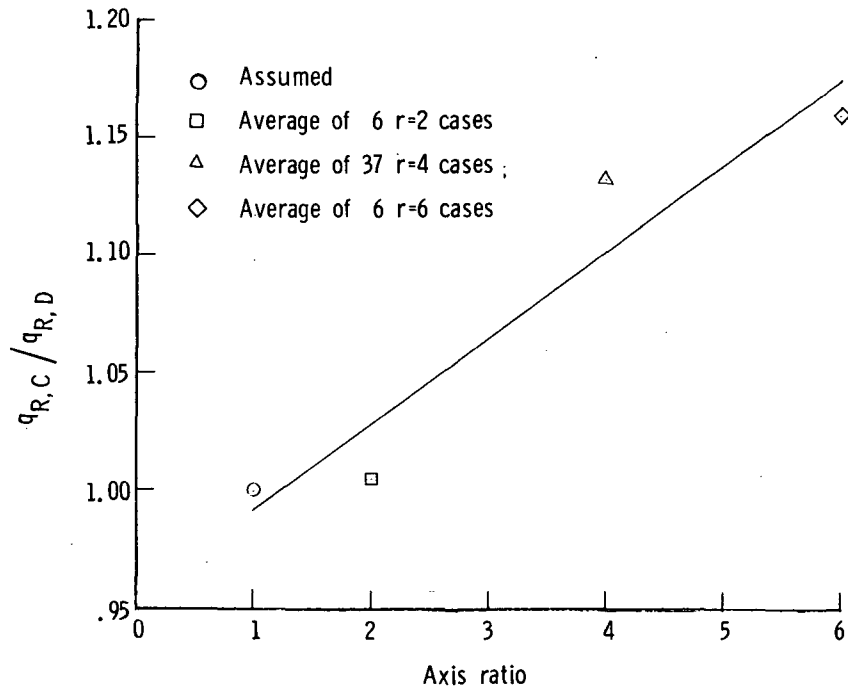


Figure 3.- Shape-factor correction for heating rate of elliptical bodies (free stream aligned with semiminor axis).

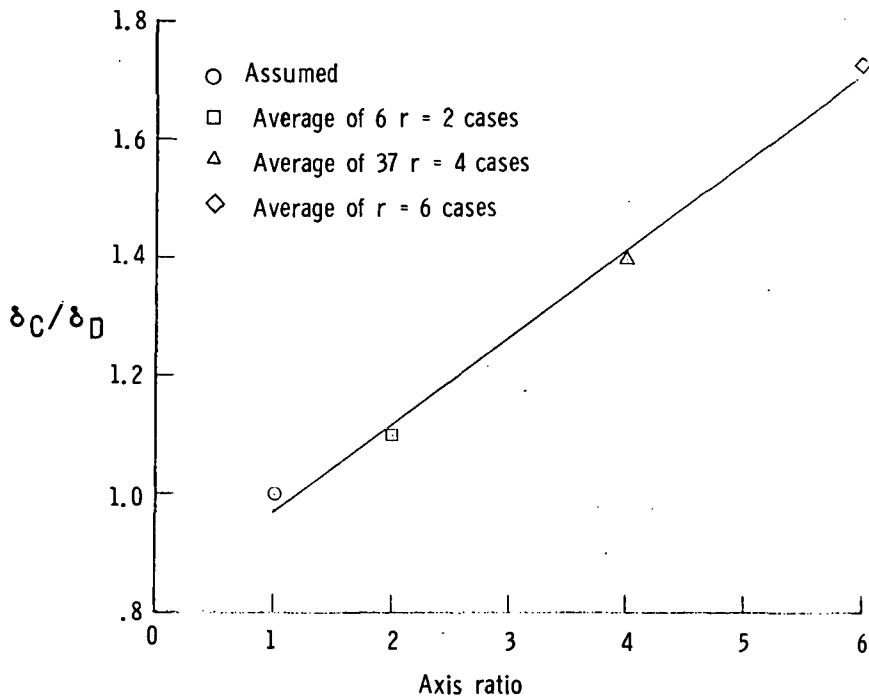


Figure 4.- Shape-factor correction for stagnation-point shock standoff distance of elliptical bodies (free stream aligned with semiminor axis).

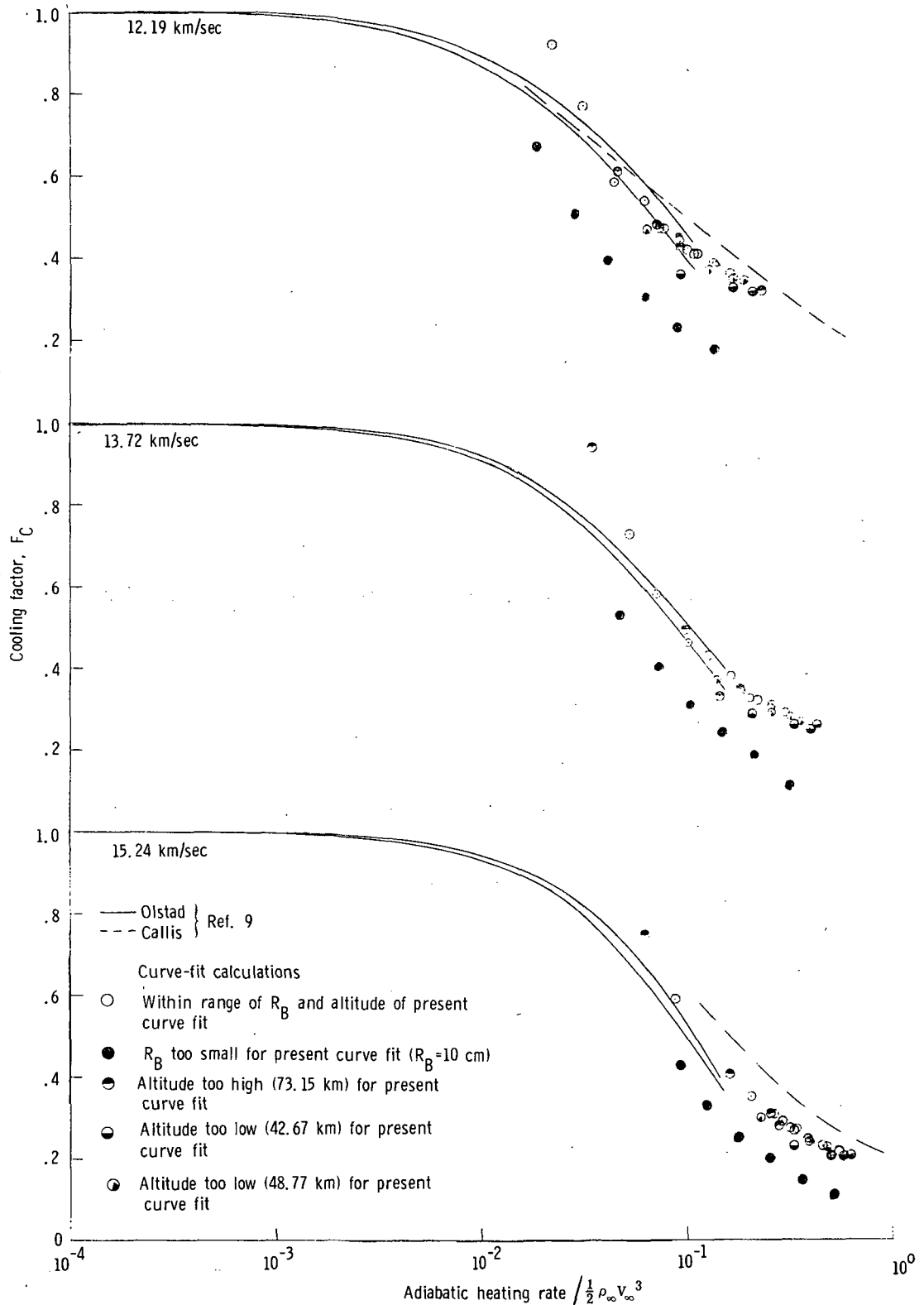


Figure 5.- Comparison of cooling-factor correlation curves from reference 9 with results of present curve fit for three velocities within the range of the present curve fit.



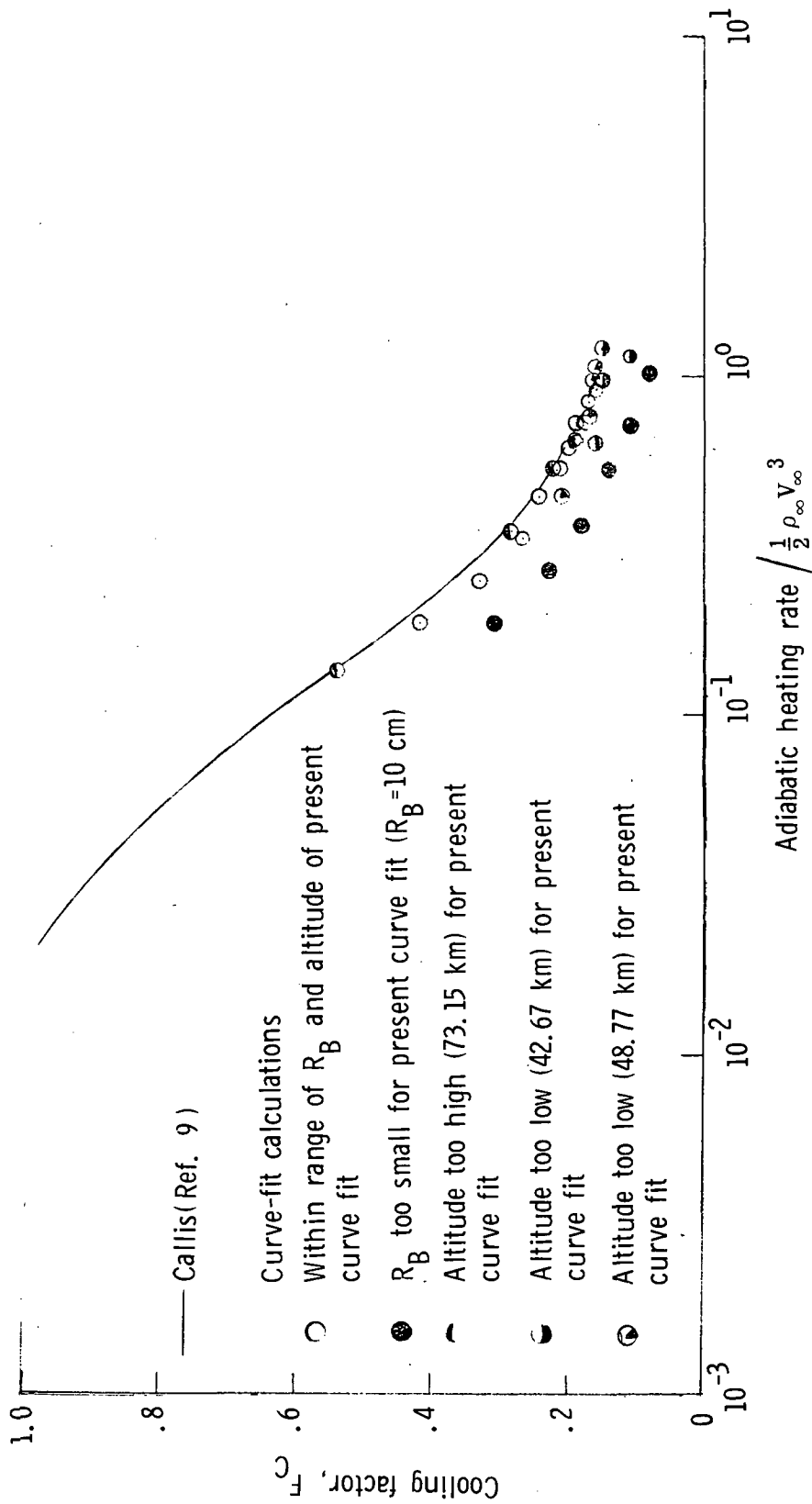


Figure 6.- Comparison of cooling-factor correlation curves from reference 9 with results of present curve fit for a velocity ( $V_{\infty} = 18.28$  km/sec) higher than those of the present curve fit.

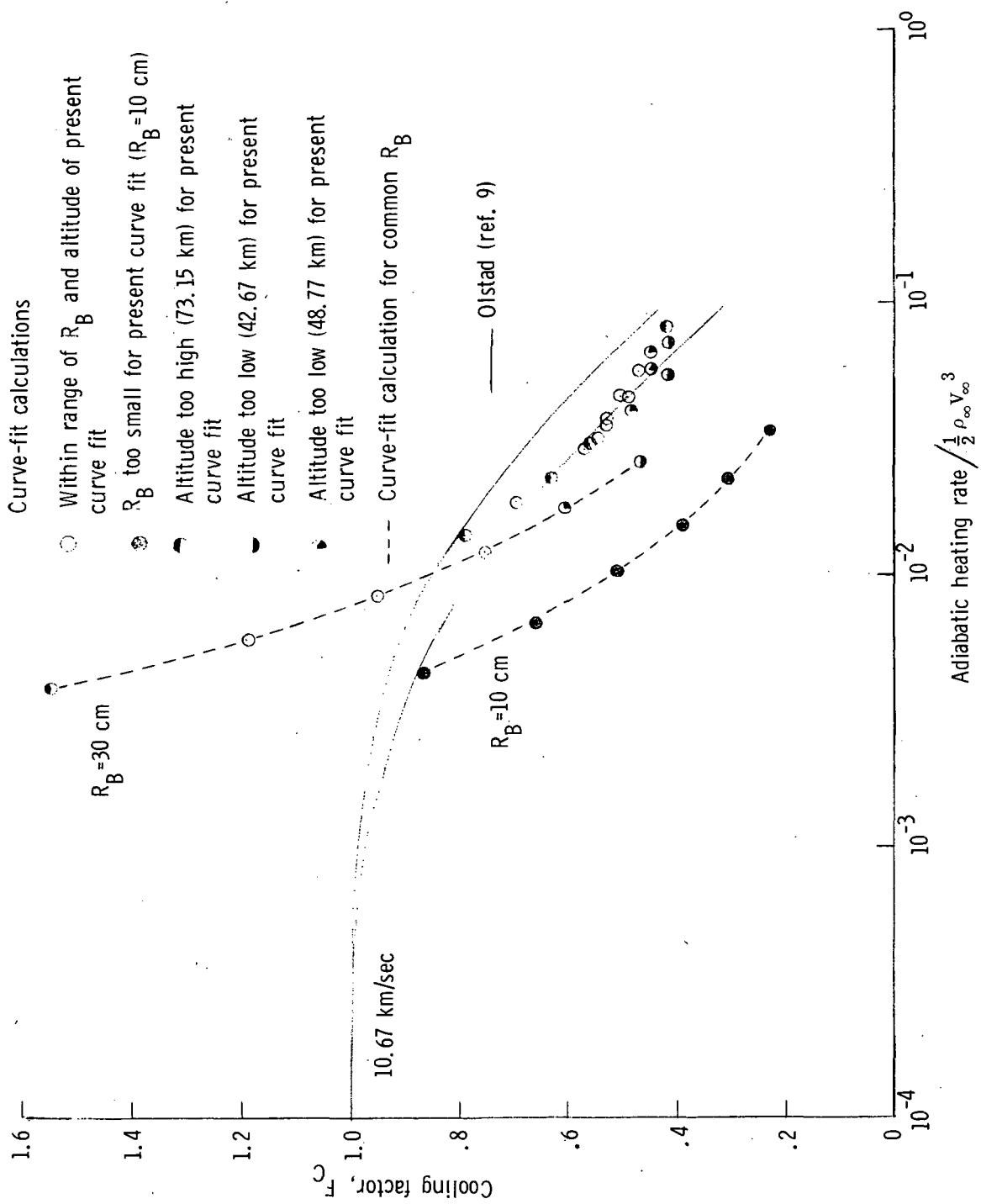
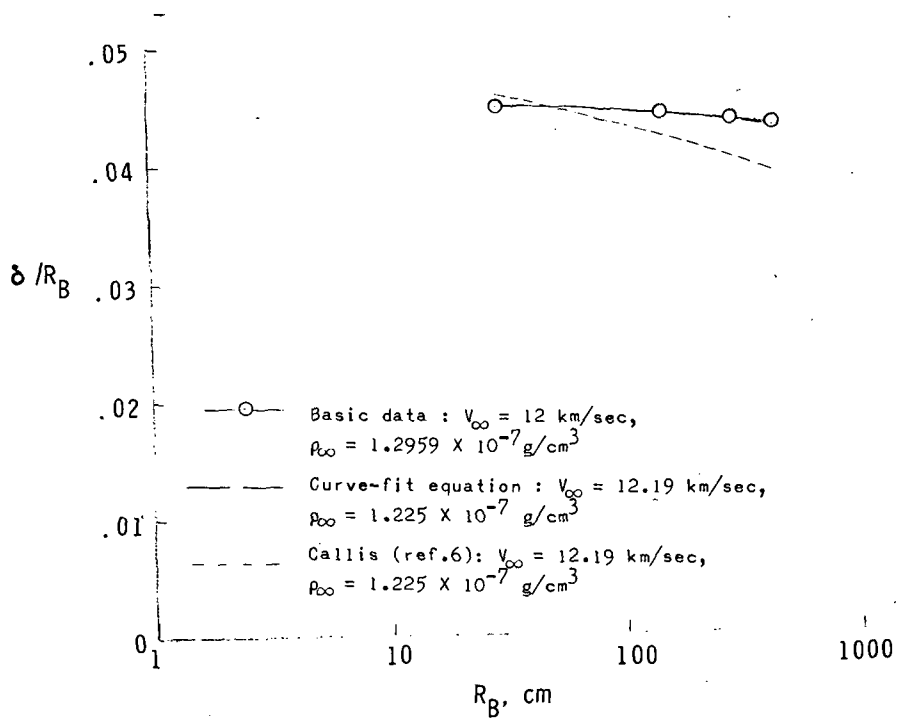
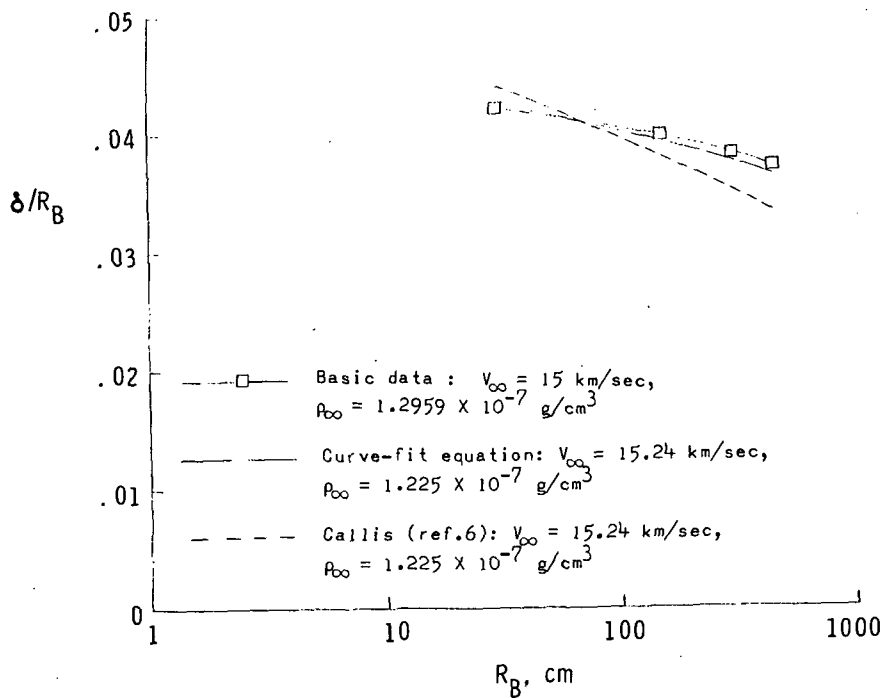


Figure 7.- Comparison of cooling-factor correlation curves from reference 9 with results of present curve fit for a velocity ( $V_\infty = 10.67$  km/sec) lower than those of the present curve fit.



(a) Low velocity.



(b) High velocity.

Figure 8.- Comparison of shock standoff distances for basic data, curve-fit equation, and results of Callis (ref. 6).

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