MUTUAL IMPEDANCE OF NONPLANAR-SKEW SINUSOIDAL DIPOLES

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ABSTRACT

The mutual impedance of nonplanar-skew sinusoidal dipoles is presented rigorously as a summation of several exponential integrals with complex arguments.

TABLE OF CONTENTS

I. INTRODUCTION

In 1957, H. E. King [1] published the mutual impedance expressions for parallel dipoles in terms of sine-integrals and cosine-integrals. This paper gives the analogous expressions for non-parallel dipoles.

Previous communications consider the mutual impedance of coplanar-skew sinusoidal dipoles [2], planar and nonplanar-skew dipoles [3] and coplanar V dipoles [4]. A computer program is available [5] for thin-wire antennas and scatterers with sinusoidal bases and Galerkin's method. In this program, subroutine GGMM evaluates the mutual impedance between filamentary monopoles with sinusoidal current distributions. The mutual impedance of two V dipoles is the sum of four monopole-monopole impedances. This paper presents the expressions programmed in GGMM. For thin-wire structures, the speed and accuracy [6] of the sinusoidal-Galerkin program is attributed largely to the use of these expressions.

When the monopoles are relatively far apart, the impedance can be calculated faster with numerical integration (as in subroutine GGS). The expressions given here are most useful when the monopoles are close together.

II. NEAR-ZONE FIELD OF THE SINUSOIDAL MONOPOLE

Many books [7] and papers [8] give the near-zone fields of a sinusoidal line source in free space. This section presents the fields of a time-harmonic electric line source (monopole) in a homogeneous conducting medium. The filamentary source is located on the z axis with endpoints z_1 and z_2 and length d as in Figure 1. A convenient expression for the current distribution is

$$(1) \qquad I(z) = \frac{I_1 \sinh(z_2 - z) + I_2 \sinh(z - z_1)}{\sinh d}$$

$$(2) \qquad \gamma = s\sqrt{\mu\varepsilon}$$

where the complex constants I_1 and I_2 denote the endpoint currents. The complex frequency is $s = j\omega$, and ω may be real or complex. In Eq. (1) and the subsequent equations, quantities of the form γx are denoted simply by x. This convention applies only when x represents a linear dimension (such as d) or a metric coordinate such as z.

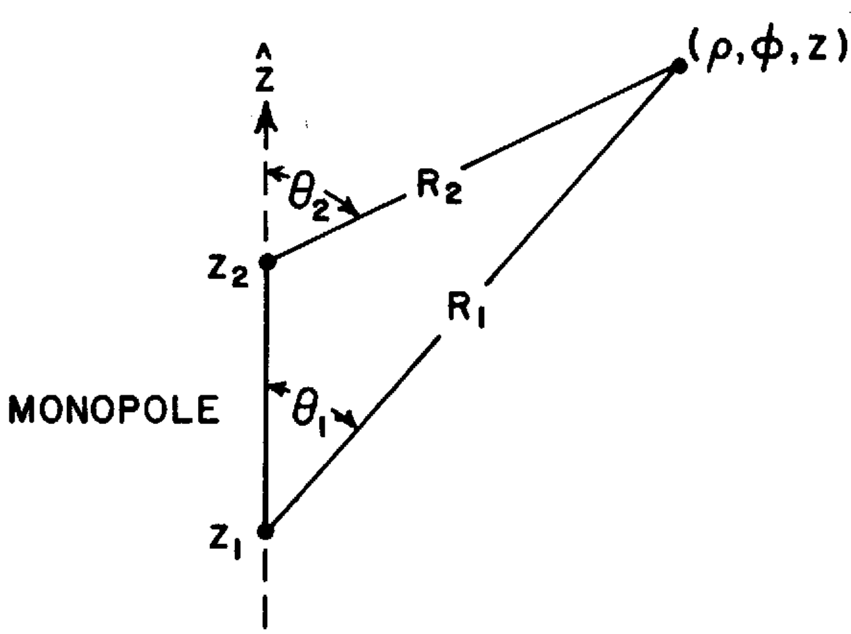

Fig. 1--Electric monopole and the coordinate system.

When the sinusoidal monopole radiates in a homogeneous conducting medium with complex permeability μ and complex permittivity ε, the field is given by

(3) $$E_\rho = \eta \frac{A}{\rho} \Big[(I_1 e^{-R_1} - I_2 e^{-R_2}) \sinh d + (I_1 \cosh d - I_2) e^{-R_1} \cos\theta_1 + (I_2 \cosh d - I_1) e^{-R_2} \cos\theta_2 \Big]$$

(4) $$E_z = \eta A \left[(I_1 - I_2 \cosh d) \frac{e^{-R_2}}{R_2} + (I_2 - I_1 \cosh d) \frac{e^{-R_1}}{R_1} \right]$$

(5) $$H_\phi = \frac{A}{\rho} \Big[(I_1 \sinh d \cos\theta_1 + I_1 \cosh d - I_2) e^{-R_1} - (I_2 \sinh d \cos\theta_2 - I_2 \cosh d + I_1) e^{-R_2} \Big]$$

$$(6) \qquad A = \frac{\gamma}{(4\pi \ \sinh d)}$$

$$(7) \qquad \eta = \sqrt{\mu/\varepsilon}$$

where (ρ, ϕ, z) denote the cylindrical coordinates. (R_i, θ_i, ϕ) are the coordinates in a spherical system with origin at the endpoint z_i. The sinusoidal monopole has point charges at the endpoints, but they will disappear when another monopole is connected to form a dipole. For this reason, the contributions from the point charges are omitted in the above field expressions.

III. MUTUAL IMPEDANCE OF SINUSOIDAL MONOPOLES

In the induced emf formulation, the mutual impedance of coupled dipoles is

$$(8) \qquad Z = -\int \underline{I}_2(t) \cdot \underline{E}_1(t)\, dt$$

where $I_2(t)$ denotes the current distribution (normalized to unit terminal current) on dipole 2, and $\underline{E}_1(t)$ is the field of dipole 1 when it transmits with unit terminal current. Distance along the axis of dipole 2 is denoted by the coordinate t. In Eq. (8), $\underline{E}_1$ may be expressed as the sum of the fields from each of the monopoles comprising dipole 1. Furthermore, the integral in Eq. (8) is the sum of the integrations over each of the monopoles comprising dipole 2. Thus, the dipole-dipole mutual impedance may be expressed as the sum of four monopole-monopole impedances.

With no loss of generality, we again locate the source monopole on the z axis with endpoints at z_1 and z_2 and length $d_1 = z_2 - z_1$. As shown in Figure 2, the receiving monopole is located on the t axis with endpoints at t_1 and t_2 and length $d_2 = t_2 - t_1$. Monopole 2 lies in the plane $y = d$. The coordinate origin for (x,y,z) is located at the apparent intersection of the t axis and the z axis in Figure 2. The origin for the t coordinate is at $(x,y,z) = (0,d,0)$. Monopole 1 is a segment of dipole 1, and monopole 2 is a segment of dipole 2. The monopole-monopole impedance will not necessarily satisfy the reciprocity theorem. The reference directions for current are $\hat{z}$ and $\hat{t}$. The mutual impedance is defined by

$$(9) \qquad Z = -\int_{t_1}^{t_2} \bar{I}(t)\, E(t)\, dt$$

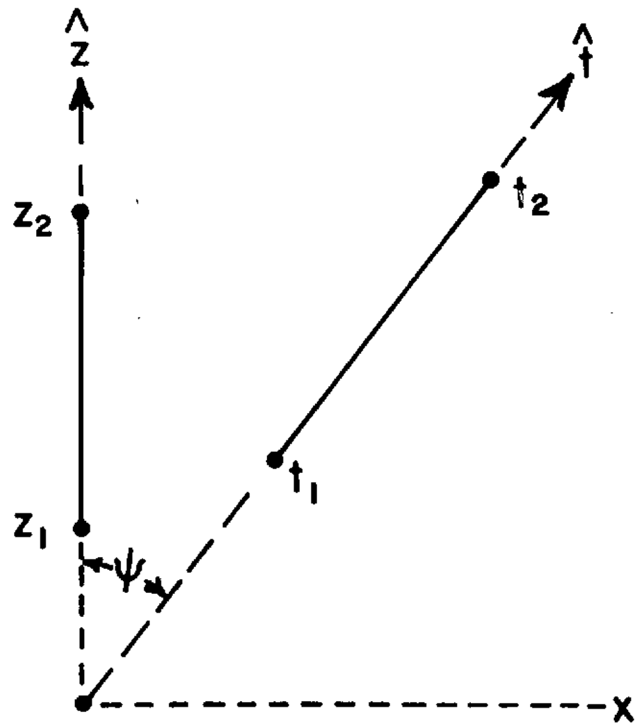

Fig. 2--Monopole 1 lies on the z axis and monopole 2 on the t axis which is parallel with the xz plane and displaced a distance d from it.

(10) $E(t) = E_\rho \cos\phi \sin\psi + E_z \cos\psi$

(11) $\cos\phi = x/\rho$

In Eqs. (8) and (9) the integration variable is t and not γt. Let the current distribution on monopole 1 be given by Eq. (1), and the current on monopole 2 is

(12) $\bar{I}(t) = [\,\bar{I}_1 \sinh(t_2 - t) + \bar{I}_2 \sinh(t - t_1)] / \sinh d_2$

where $\bar{I}_1$ and $\bar{I}_2$ are the endpoint currents.

To be consistent with Eqs. (8) and (9), the currents $I(z)$ and $\overline{I}(t)$ must have unit value at one end of the monopole and vanish at the other end. Thus a double-subscript notation Z_{ij} is convenient for the mutual impedance of two monopoles. The first subscript i has value 1 if $\underline{I}_1 = 1$ and value 2 if $\underline{I}_2 = 1$. The second subscript j has value 1 if $\overline{I}_1 = 1$ and value 2 if $\overline{I}_2 = 1$. (The monopole has unit current at the endpoint corresponding to the dipole terminals.) The first subscript is associated with the "source monopole" whose field $E(t)$ appears in Eq. (9). The second subscript is associated with the "receiving monopole" with current $\overline{I}(t)$. As illustrated in Figure 3, the source monopole is always a segment of the source dipole.

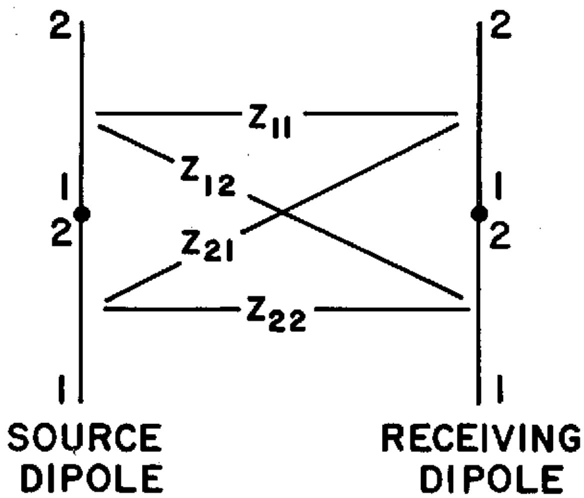

Fig. 3--The mutual impedance of coupled dipoles may be expressed as the sum of four monopole-monopole impedances: $Z = Z_{11} + Z_{12} + Z_{21} + Z_{22}$.

Omitting the lengthy derivations, the mutual impedance Z_{ij} for skew monopoles is

$$Z_{ij} = (-1)^{i+j} B\left[e^{t_n} \left(F_{i1} - e^{-z_m} G_{12} + e^{z_m} G_{22}\right) - e^{-t_n}\left(F_{i2} - e^{-z_m} G_{11} + e^{z_m} G_{21}\right)\right] \tag{13}$$

(14) $$B = \frac{\eta}{(16 \pi \sinh d_1 \sinh d_2)}$$

where m = 2/i and n = 2/j. The functions F_{ik} are defined by:

(15) $$F_{ik} = 2 \sinh d_1 \, e^{\ell z_i \cos\psi} \, E(R_i + \ell z_i \cos\psi - \ell t)$$

where

$\ell = (-1)^k$. The functions $G_{k\ell}$ are defined as follows:

(16) $$G_{k\ell} = E(R_2 + mz_2 + nt - j\beta) + E(R_2 + mz_2 + nt + j\beta) - E(R_1 + mz_1 + nt - j\beta) - E(R_1 + mz_1 + nt + j\beta)$$

where $m = (-1)^k$, $n = (-1)^\ell$ and $\beta = mb + nc$.

In Eq. (16) and all the subsequent equations, $j = \sqrt{-1}$. Let

(17) $$b = c \cos\psi$$

(18) $$c = d/\sin\psi$$

In Eqs. (15) and (16), t denotes the position of an observation point somewhere on monopole 2. R_1 and R_2 are distances from the endpoints of monopole 1 to this observation point. Finally, the E functions are defined as follows:

(19) $$E(\alpha + j\beta) = e^{j\gamma\beta} \int_{\alpha_1+j\beta}^{\alpha_2+j\beta} \frac{e^{-\gamma w} \, dw}{w}$$

where α and β are real quantities with dimensions of length, α is a function of t, $\alpha_1 = \alpha(t_1)$, $\alpha_2 = \alpha(t_2)$ and γ is given by Eq. (2). In the right-hand side of Eq. (19), we abandon the convention of letting x represent γx.

The integral in Eq. (19) may be expressed in terms of exponential integrals as follows:

(20) $$S = \int_{v_1}^{v_2} \frac{e^{-v}\,dv}{v} = E_1(v_1) - E_1(v_2) + j\,2n\pi$$

The integration path is a horizontal line in the w plane or an inclined straight line from v_1 to v_2 in the v plane. The integer n is zero unless this path intersects the negative real v axis at a point between v_1 and v_2. When there is such an intersection,

a) $n = 1$ if v_1 lies above the real axis and v_2 is below, or

b) $n = -1$ if v_1 lies below the real axis and v_2 is above.

A subroutine EXPJ is available for the integral in Eq. (19). This subroutine is used with GGMM for skew monopoles and dipoles. Although the expressions given here fail for parallel monopoles, the subroutines do not.

IV. CONCLUSIONS

Rigorous expressions are presented for the mutual impedance of nonplanar-skew sinusoidal monopoles in a homogeneous conducting medium in the complex frequency domain.

APPENDIX

Two basic integrals are required to derive the results presented herein. These "reaction integrals" were evaluated by N.H. Geary in October 1968 and April 1969 with substitution of variables and partial fraction expansions. The integrals are defined as follows:

$$(22) \qquad P = 2 \int_{t_1}^{t_2} e^{\pm\gamma t}\, e^{-\gamma R} \left[\frac{\cos\psi}{R} - \frac{\cos\phi \cos\theta \sin\psi}{\rho} \right] dt$$

$$(23) \qquad Q = 2 \sin\psi \int_{t_1}^{t_2} e^{\pm\gamma t}\, e^{-\gamma R}\, \frac{\cos\phi}{\rho}\, dt$$

In this appendix, a subscript i is understood on the quantities R, θ and z. The subscript may take on values 1 and 2. In the integrands, all quantities are functions of t except γ and ψ. In terms of the E function in Eq. (19), P and Q are given rigorously as follows:

$$(24) \qquad P = e^{-\gamma z} E(x_1) - e^{\gamma z} E(x_2) + e^{-\gamma z} E(x_3) - e^{\gamma z} E(x_4)$$

$$(25) \qquad Q = e^{-\gamma z} E(x_1) + e^{\gamma z} E(x_2) + e^{-\gamma z} E(x_3) + e^{\gamma z} E(x_4) - 2\, e^{\pm\gamma z \cos\psi} E(x_5)$$

$$(26) \qquad x_1 = R \mp t - z + jb \pm jc$$

$$(27) \qquad x_2 = R \mp t + z - jb \pm jc$$

$$(28) \qquad x_3 = R \mp t - z - jb \mp jc$$

$$(29) \qquad x_4 = R \mp t + z + jb \mp jc$$

$$(30) \qquad x_5 = R \mp t \pm z \cos\psi$$

All quantities not defined here are defined in the main text.

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