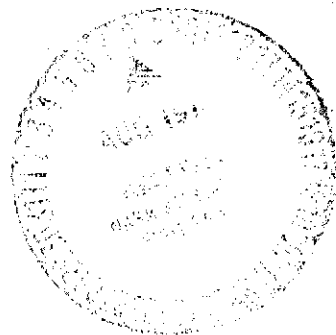/39043

AIR TRAFFIC CONTROL SURVEILLANCE ACCURACY AND UPDATE RATE STUDY

FINAL REPORT

J. H. CRAIGIE, D. D. MORRISON, I. ZIPPER, ET AL.

MARCH 1973

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER

TRW
SYSTEMS GROUP

ONE SPACE PARK • REDONDO BEACH, CALIF. • 90278

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ABSTRACT

This report describes the results of the Air Traffic Control
Surveillance Accuracy and Update Rate Study which TRW performed
for the National Aeronautics and Space Administration, Goddard
Space Flight Center. The objective of the ATC Surveillance
Accuracy and Update Rate Study was to establish quantitative
relationships between the surveillance accuracies, update rates,
and the communications load associated with the tactical control
of aircraft for conflict resolution. These relationships are
established for typical types of aircraft, phases of flight, and
types of airspace. The Surveillance Accuracy/Update Rate (SAUR)
computer program developed to determine these relationships "flies"
two aircraft towards one another under various realistic circum-
stances, and analyzes the interaction between the two pilots and
the controller. Ten specific cases are analyzed to determine
relationships between the surveillance accuracies and update
rates which will be required in order to prevent these aircraft
from getting too close to each other.

For these ten specific cases, involving a broad range of aircraft
types, phases of flight, and types of airspace, the demands on
surveillance system accuracies span a wide range, i.e., from less
than 100 feet to greater than 10 miles. The necessary surveil-
lance system update interval does not cover such a large range.
With one exception it varies from 1 to 10 seconds. In the case
study evaluation, a short update interval was needed in more
cases than was a high accuracy surveillance system. Therefore,
it appears that reducing total system reaction time is more
important than increasing surveillance accuracy.

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ACKNOWLEDGEMENT

CONTENTS

CONTENTS (Continued)

CONTENTS (Continued)

ILLUSTRATIONS

ILLUSTRATIONS (Continued)

Page

ix

ILLUSTRATIONS (Continued)

PAGE

TABLES

PAGE

1. PREFACE

1.1 INTRODUCTION

This document contains a description of the essential elements of the Air Traffic Control Surveillance Accuracy and Update Rate Study which TRW performed for the National Aeronautics and Space Administration, Goddard Space Flight Center, under Contract NAS-5-21603. This study was initiated at a time when NASA had responsibility for the development of the technology associated with application satellites such as those associated with air traffic control. NASA had, under a previous contract, developed the technology of satellite-based air traffic control (ATC) to the point where the feasibility of a very high performance satellite-based ATC surveillance system was established. However, the need for this technology was not clear at that time. Thus, NASA initiated this study in order to establish a substantial basis for the performance capability that an air traffic control satellite would be required to provide.

Subsequent to this time other studies have indicated the desirability of satellites for domestic ATC application. The contents of this report, however, can be applied to the question of ATC surveillance accuracy and update rate requirements, independent of the particular surveillance technique generating the data.

1.2 BACKGROUND

The Department of Transportation Air Traffic Control Advisory Committee in December of 1969 stated that the three critical problems which urgently required solutions were:

1) The shortage of terminal capacity

2) The need for new means of assuring separation

3) The limited capacity and increasing cost of ATC.

This Air Traffic Control Surveillance Accuracy Update Rate Study addresses the second of those three critical problems. The Air Traffic Control Advisory Committee went on to state that "measures beyond the present use of 'see-and-avoid' in portions of 'mixed airspace' will become mandatory by 1980." The committee report (Reference 1) went on to recommend a process

1

called Intermittent Positive Control (IPC) which involved automating and making more precise the air traffic advisory service. Although this study addresses all levels of control and types of airspace, an evaluation of the IPC concept and the problems associated with mixed airspace is the essence of the study.

1.3 APPROACH

The objective of the ATC Surveillance Accuracy and Update Rate Study was to establish quantitative relationships between the surveillance accuracies, update rates, and the communications load associated with the tactical control of aircraft for conflict resolution. These relationships are established for typical types of aircraft, phases of flight, and types of airspace. The Surveillance Accuracy/Update Rate (SAUR) computer program developed to determine these relationships "flies" two aircraft towards one another under various realistic circumstances, and analyzes the inter-action between the two pilots and the controller. For a given pair of collision avoidance commands to the two aircraft, the program searches through possible maneuvers by the two aircraft and finds the minimum dis-tance of closest approach. The program then searches through all possible command pairs to find the maximum of the minimum distances of closest approach. Ten specific cases are analyzed in this way. Application of this program determines relationships between the surveillance accuracies and update rates which will be required in order to prevent these aircraft from getting too close to each other. In addition, the communication rate associated with traffic vectoring under various conditions of traffic load-ing is determined. The resulting combination of information allows one to render sound judgements in the selection of two key surveillance system design parameters.

1.4 CONTENTS

Section 2 of this report contains a development of the surveillance accuracy/update rate relationships as they pertain to the pilot/controller interaction in the separation assurance process. Section 3 contains an analysis of the key input parameters in the Surveillance Accuracy/Update Rate and Communication Rate formulations. Sections 4 and 5 contain descrip-tions of the Surveillance Accuracy/Update Rate and Collision Warning Commu-nication Rate formulations. Section 6 contains the results of ten case

2

studies and a discussion concerning the implications of these results. Section 7 contains a report on the filter mechanization analysis, which was an investigation to determine the accuracy with which the present position and velocity state of various aircraft could be estimated, and how that accuracy depends on the surveillance system, aircraft dynamics, and data processing (filtering) procedure. Section 8 contains two special studies, the first a brief analysis of some 55 actual mid-air collisions, and the second an analysis of air traffic control communications in remote areas.

1.5 CONCLUSIONS AND RECOMMENDATIONS

This section summarizes the major results of the Air Traffic Control Surveillance Accuracy/Update Rate Study.

1.5.1 <u>Conclusions</u>

The following conclusions are drawn from the analyses described in this report:

1) <u>Surveillance Accuracy and Update Rate</u>

For the ten specific cases involving a broad range of aircraft types, phases of flight, and types of airspace, the demands on surveillance system accuracies span a wide range i.e., from less than 100 feet to greater than 10 miles. The necessary surveillance system update interval does not cover such a large range. With one exception it varies from 1 to 10 seconds.

The need for high performance surveillance in terminal areas is clearly indicated. Accuracy demands drop off rapidly, so that in the transition and enroute areas present radar accuracy capabilities appear sufficient. The update rates indicated do not drop off so rapidly; and, in general, both existing terminal and enroute radar update intervals are longer than desirable.

2) <u>System Reaction Time</u>

In the case study evaluation, a short update interval was needed in more cases than was a high accuracy surveillance system. Therefore, it appears that reducing total system reaction time is more important than increasing surveillance accuracy.

The present concept, involving voice communications and acknowledgement, as well as the reaction times by both pilot and controller, results in total system reaction

3

times that can cause significant increases in communication loads and false alarm rates.

3) Single Aircraft Response

In the event that the decision is made to assume that only one aircraft will respond to a collision avoidance command, the surveillance accuracy and update rate requirements could become significantly more stringent than has been indicated in this study. When only one aircraft performed a commanded maneuver, the results were quite variable. In a high performance aircraft/low performance aircraft encounter situation the results did not change if the low performance aircraft was not commanded. On the other hand, if the two aircraft were of similar performance capabilities the surveillance accuracy update demands usually become much more stringent.

4) Collision Avoidance Maneuvers

Commanded horizontal maneuvers were shown to be much more effective than vertical maneuvers in collision avoidance. If the ATC commands were limited to changes in altitude, the separation that a controller could guarantee would be markedly reduced, resulting in more stringent surveillance requirements.

5) Filter Mechanization

Sophisticated filters for accuracy or prediction did not help the conflict prediction and resolution process. The accuracy with which the position and velocity of aircraft can be specified depends strongly on the characteristics of the surveillance system, aircraft, and trajectories. However, the accuracy was shown to be relatively independent of the complexity of the filter used.

The existing ATCRBS system with 10 seconds surveillance intervals is incapable of tracking many types of currently operational aircraft (executing maneuvers within their allowed dynamic constraints) with an error of less than 2500 feet irrespective of the degree of sophistication of the data processing used. This is mainly due to the length of time between surveillance updates (not measurement inaccuracy) — an aircraft can be perfectly on course at one surveillance time, then execute a 2 or 3 g maneuver and be 3000-5000 feet off its original flight plan 10 seconds later just before the next measurement occurs.

4

The performance of a high quality multilateration system with a 1 second surveillance interval is totally determined by and directly proportional to the bias errors in the range measurements. Nominal values for these bias errors in a typical* satellite system produce a maximum error of 400 feet.

6) Remote Area Communications

Remote area air traffic control communications do not appear to impose critical or state-of-the art requirements on a satellite-based ATC system. Based on a brief message traffic analysis the remote area communications needs (exclusive of collision avoidance) can be met through 1995 by some eight 1200-bit per second data link channels.

1.5.2 Observations

In arriving at the findings described above it was possible to make a number of observations regarding the system implications of the foregoing results and also on the study methodology itself.

1.5.2.1 System Implications

High accuracy multilateration concepts, which are readily implemented with satellites, provide their greatest usefulness around airports (Cases 4, 6A, and 6B). Terminal and enroute radar from an accuracy point of view are usually adequate to do the conflict prediction and resolution tasks; but the update interval of the enroute radar, and, to some extent, the terminal radar, is usually too long. Reduction of total system reaction time is of such importance that detailed investigation of the use of data link is clearly called for. The future of satellites for domestic air traffic control does not appear to hinge on the high accuracy capability of the satellite systems alone, because the high accuracy requirements occur in limited areas where ground-based systems could be competitive. It has been shown in previous studies that satellites providing communication, navigation, and/or surveillance functions in remote areas are cost competitive. Thus, the decision to use satellites for domestic ATC may well be made on the basis of a combination of the foregoing performance, coverage, and cost advantages, rather than on any one alone.

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*Not optimized for accuracy. See Reference 2

1.5.2.2 Study Methodology

The SAUR/CR methodology appears valid and useful. The relationships between surveillance accuracy and update rate obtained in this study appear valid for preliminary indications of the major system design parameters; but the study was not of sufficient scope and the data is not of sufficient fidelity on which to base final implementation or design conclusions. Application of this methodology to obtain additional, more detailed, and higher fidelity data appears desirable for advanced air traffic management system analyses and for design tradeoff studies.

1.5.3 Recommendations

Several near-term efforts to support advanced air traffic management system planning and design efforts are recommended. They include:

- Additional use of the Surveillance and Command Communications formulations developed here to provide higher resolution data, tailored for specific cases as part of Advanced Air Traffic Management System analysis efforts.

- More detailed investigation of the multiple aircraft encounter situation. Although the command communications formulation used here is an extension of previous analyses, the problem merits even further extension in order to verify the important design parameters of the vital IPC concept.

- Additional analysis and test efforts relating to reduction of total system reaction time, especially with regard to implementation of data link.

- Analyses of navigation/surveillance interaction using smaller pilot-initiated maneuvers that are related to probable navigation errors.

These individual efforts should be performed as a prelude to or as part of a complete IPC Concept synthesis, based on the surveillance formulation, for real-time collision avoidance.

Longer term efforts should include design tradeoff studies involving cost analyses of both the ground and airborne elements of the communications, navigation, surveillance and data management functions of the total Advanced Air Traffic Management System.

2. SURVEILLANCE ACCURACY/UPDATE RATE RELATIONSHIP

To illustrate important facets of pilot/controller interaction, consider the aircraft situation as shown in Figure 1. Consider a hypothetical controller,* faced with the task of preventing a collision between the two aircraft. (The reasoning of the hypothetical controller is presumed to be that of the program.) A Cessna 172 is currently in a level turn at 5000 feet through a roughly northwest heading. A Boeing 737 is currently descending through 5500 feet and going roughly southeast. The controller knows the position and velocity of both aircraft from the surveillance system.

If both aircraft continue their present flight paths, there will be a collision 60 seconds from now. If either one changes its flight path, they will miss each other. If both change, they will very probably miss each other, but could possible still collide at a different point.

Although the controller is concerned about the situation, it is not obvious, without further calculations, that any commands are necessary. The controller might find that he can wait and it might happen that one or both of the two aircraft will change course and create non-collision situation. The controller must decide whether he can afford to wait and; if he decides he can't wait, he must determine what commands to send to the aircraft.

To make the decision, the controller first asks what would happen if he gave commands now. There are a large number of choices available to him for commands, so he must analyze sets of commands.

For example, he might command the 737 (now descending) to climb, and the 172 (now flying level) to descend. Even if he gives the commands now, however, they will not be acted on until they have been sent out and the pilot has reacted to them. The controller might assume that this will

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*The "hypothetical controller" could be a human controller operating with today's manual system, a human controller aided by various levels of automation, or a completely automated system, where the computer could be either on the ground (centralized control) or in the aircraft (distribution control). The discussion here is more from the point of view of centralized control.

B737, DESCENDING THROUGH 5500 FEET AT 500 FT/MIN, 250 KNOTS, MAINTAINING A TRUE COURSE OF 154°

NORTH

NON-COLLISION COURSE

ORIGINAL COLLISION COURSES

WEST

EAST

COLLISION POINTS

C172, FLYING AT A CONSTANT ALTITUDE OF 5000 FEET, MAINTINAING 100 KNOTS, IN A 1°/SEC RIGHT TURN, PASSING THROUGH A TRUE COURSE OF 300°

SOUTH
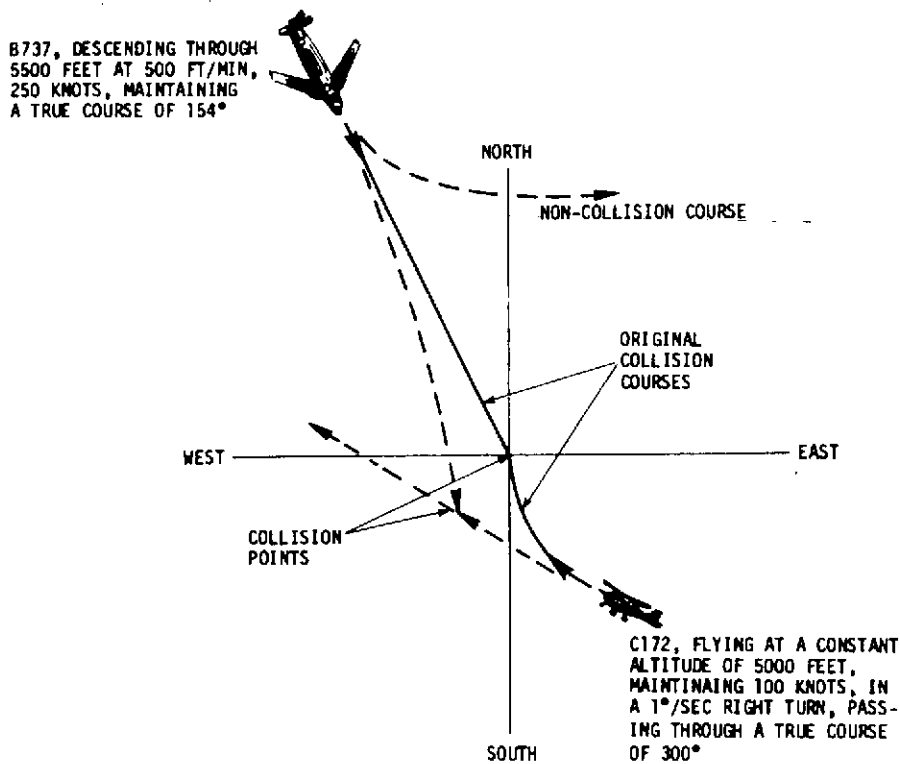
Figure 1. Collision Situation Geometry

take 10 seconds — i.e., if the controller decides to give a command now, the two pilots will start moving the controls to follow the command in 10 seconds.

During this first 10 seconds after the commands are given the controller has no control. He therefore assumes the worst. The 172, now flying level, might suddenly start climbing — this will partly negate the effect of the descend command the controller is thinking about sending. The 737, now descending, might continue to descend. (Actually, the controller might want to assume that the 737 will start descending even faster and the program allows for this. In the sample problem we assumed that it was already descending at its maximum rate.)

Assuming these worst case conditions about any pilot-initiated maneuvers, it is possible to calculate for the particular pairs of commands which were given to the two aircraft how close the aircraft will get. Using a set of assumed values of climb rates, etc., the program computes

8

a miss distance of about 8000 feet, about (not necessarily exactly), 60 seconds from now. If the controller's requirements are to guarantee a separation of 1000 feet under all circumstances, this now suggests that it is not necessarily to give a command yet. This would be the correct answer with a surveillance system which is providing perfect, continuous data.

However, suppose each aircraft's position is only known to within an uncertainty of 3500 feet (the surveillance accuracy). Then the two aircraft might be twice that distance, or 7000 feet closer together than the controller thinks they are, and thus they might well be 7000 feet closer together in 60 seconds. This would make the predicted worst case distance of closest approach (DCA) equal to 1000 feet.[*] It would therefore be necessary to give a command now. If the surveillance system gives better than 3500 feet accuracy, it would be necessary to give a command now with a surveillance system which provides data continuously.

Suppose that the above test shows that it is not necessary to give a command now. However, suppose also that the surveillance system only gives data every 10 seconds. (This is the present update interval for an enroute radar.) The next chance to make a decision will be 10 seconds from now. If no command is given now, the controller must assume that the aircraft will be doing the "worst" maneuvers for the next 20 seconds — the 10 second surveillance data interval plus the 10 second system reaction time. He should therefore perform the previous calculations with an effective system reaction time of 20 seconds.[**] The results show the aircraft getting to within 2000 feet of each other. In this case if the surveillance system uncertainty is worse than 500 feet he cannot guarantee 1000 feet separation unless he gives the commands now. Note that the increased update interval results in an increased accuracy requirement.

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[*] From this reasoning one can infer the relationship between the surveillance accuracy required, the distance of closest approach, and guaranteed separation:

$$SAR = 1/2 \ (DCA - GS)$$

[**] From this it may be seen that system reaction time and update interval are identical in their effect on surveillance accuracy requirements. Or, put another way, the update interval is part of the total effective reaction time in the system.

Figure 2 shows curves of DCA and surveillance accuracy requirements versus total reaction time, T (which includes controller reaction time; communication transmission time, including delays; pilot reaction time; and surveillance update interval) for various decision times. Decision time (τ) is defined as the length of time prior to a potential mid-air collision that the controller makes a decision to intervene or not to intervene. If the controller goes through the thought process described above at an earlier point in time (a larger τ), he can allow a larger total reaction time and/or a larger surveillance position determination uncertainty. It can be seen from this figure that if the total effective reaction time exceeds about 30 seconds, a τ of 60 seconds does not allow sufficient maneuver time to provide the desired 1000 feet "guaranteed separation," regardless of surveillance accuracy. It might by pointed out here that a shortened update interval will reduce the required accuracy (i.e. increase the position uncertainty), but at the same time it will increase the achievable accuracy (reduce the position uncertainty) for a particular surveillance system concept.
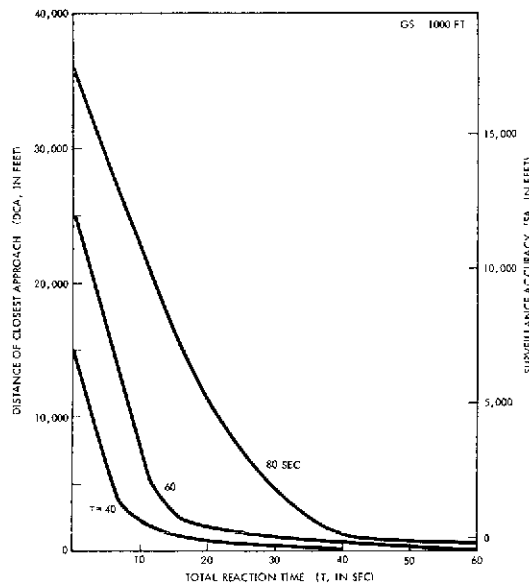
Figure 2.   Surveillance Accuracy Versus Reaction
Time and Decision Time

10

In the complete analysis, the controller would also analyze many other pairs of commands. He might choose those commands which would maximize the distance of closest approach, or he might choose the most expedient commands that yielded some desirable separation standard.

The SAUR Program works by following the above line of reasoning. For a given pair of commands and a given set of uncommanded maneuvers it computes a distance of closest approach (DCA). One can also specify the commands and the program will search through a number of possible uncommanded maneuvers to find the maneuvers which give the smallest DCA. The program will also search through a number of command pairs to determine which command pair gives the largest of these smallest DCA's, i.e., the max-min DCA. It is this last, most general case that provides the results such as are shown in Figure 2. Section 4 presents a description of the detailed operation of the SAUR Program. Section 5 describes a collision warning communication rate analysis which was performed in conjunction with the SAUR analysis in order to shed more light on the preferred value of the surveillance parameters.

The significant difference between the SAUR formulation and many similar analyses performed in the past is that whereas many of the latter dealt with probable aircraft flight paths and the probability of collision the SAUR formulation deals with the envelope of possible aircraft flight paths. The air traffic controller must deal with all possible maneuvers if he is to guarantee separation between aircraft; and in the event that either aircraft is free to alter his flight path at will (VFR or IPC conditions), or in the presence of unintentional, unplanned and/or unforseen flight path changes for aircraft under instrument flight rules, that is precisely his responsibility. It should be pointed out that no claim is made for the "superiority" of the SAUR formulation. Both types of analyses — probabilistic and deterministic — can be valuable tools in air traffic control analysis. For example, the heanling of IFR anomalies will probably not be a sufficiently strong case for the SAUR formulation and the use of probable navigation and surveillance errors is useful; but dealing with VFR/IPC aircraft in mixed airspace is clearly of sufficient importance to warrant this type of analysis.

11

3. ANALYSIS OF SYSTEM VARIABLES

This section contains a brief discussion of the key major elements of the problem and the key input parameters to the SAUR and Communication Rate Analyses. The selection of an appropriate value or range of values for the system variables used will, of course, have a significant effect on the end results. Thus, this section might be thought of as the quantification of the justification for the assumptions made in the analysis.

3.1 SELECTION OF CASES FOR ANALYSIS

The cases selected for study are listed in Table 1. Although it is clear that these cases represent only a small fraction of the great number of collision encounter situations possible, they are considered to be representative. The cases are also bounding in the sense that some place high demands on surveillance system performance while others will allow very relaxed accuracies and update rates.

In order to come up with a representative set of cases, it seemed reasonable to examine the characteristics of mid-air collisions which occurred over a number of years (References 3 through 22). The results of this examination are reported in Section 8 of this report. It was found that from 1959 through 1968 there were 223 mid-air collisions involving U.S.-registered aircraft. About half of these accidents were fatal, resulting in 528 fatalities. Ninety-eight percent of these collisions involved general aviation aircraft. Although air carrier aircraft were involved in only 6.7 percent of the accidents, the occupants of these aircraft accounted for 66 percent of the fatalities.

Accordingly 7 of the 10 cases involve general aviation aircraft because of their predominant involvement on a percentage basis. Since more loss of life is involved with midair collisions involving air carrier aircraft, they too are involved in 7 of the 10 cases. Military aircraft are involved in four of the cases.

Cases 4, 5, and 6 are intended to represent bounding cases on surveillance accuracy and update rate. Case 5, for high altitude enroute situations, makes very relaxed demands. Case 6 involves very tight requirements associated with independently operated instrument runways with only

13

Table 1. SAUR Cases

Case No.	Description	Representative Aircraft Types #1	#2	Representative Aircraft Type(s)
1.	Air carrier descending toward encounters light general aviation aircraft in a level turn	172	737	Mixed* or positive control
2.	A number of air carrier and light general aviation encounters	172	737	Mixed or positive control
3.	Military very high performance jet in a steep climb from airport encounters a general aviation business jet in level flight	F-104	Citation	Mixed or positive control
4.	Two light general aviation aircraft in VFR landing pattern encounter	172	Volksplane	Uncontrolled, mixed or positive control
5.	a) Very high performance (supersonic military jet) encounters air carrier in level flight	747	YF-12	Positive control
	b) A high altitude encounter between two supersonic aircraft	YF-12	YF-12	Positive control
6.	Two aircraft in parallel track encounters			Mixed or positive control
	a) Parallel runway (5000 ft separation)	737	737	
	b) Parallel runway (2500 ft separation)	737	737	
	c) Airway	737	Citation	
7.	An all-V/STOL encounter situation	Twin Otter	UH-1	Mixed or positive control
8.	The March 1967 mid-air collision between a Beechcraft B-55 and a McDonnell Douglas DC-9 near Urbana, Ohio	DC-9	B-55	Mixed or positive control
9.	The 4 December 1965 mid-air collision between a Boeing 707 and a Lockheed constellation near Carmel, New York	707	Constellation	Mixed or positive control
10.	The June 1971 mid-air collision between a Marine F-4 and a McDonnell Douglas DC-9 near Durate, California	F-4	DC-9	Mixed or positive control

*Also called "Controlled" airspace

14

a 2500 foot separation distance (wherein one aircraft initiates a turn toward the other runway). Case 4, involving light general aviation aircraft in a traffic pattern, also represents tight requirements.

From Reference 22 it was found that the problem of mixed airspace is also critical, as indicated by the following:

> "The Board has noted, from studies of recent near mid-air collision reports and its findings in the investigation of several catastrophic mid-air collision accidents of the past several years, that conflict between the "known" and "unknown" traffic (VFR/IFR mix) was a factor. The problem stems from traffic cleared to operate under instrument flight rules but operating in VFR conditions. Such operation does not relieve the IFR operator from the responsibility to see and avoid, even though the cockpit duties for instrument flight are more numerous than those for VFR operations. This conflict is now beginning to reveal the true magnitude of the impact on our Air Traffic System created by the ever-growing number of VFR and IFR operations."

In recognition of the seriousness of this problem, the study cases have been selected so that in 8 of the cases the results are directly applicable to mixed airspace encounters. For a ninth case (Case 6), the results could also apply to VFR traffic approaching an aircraft making an instrument approach.

3.2 CONTROL PHILOSOPHY

As pointed out in Reference 2, there are a number of competing control philosophies. Indeed, they are reflected in the competing airspace organizations listed in Table 2. The following listing is representative of the various types of control exercised over aircraft with the addition of one new classification which evolved from the Surveillance Accuracy/Update Rate Study. Various levels of control, listed somewhat in order of increasing control requirements are:

1) Procedural

   Rules of the Air, e.g., quadrantal altitude separation, speed limits, and radio-out procedures.

2) Flight Following

   No surveillance data. Delayed position reports allow ground control only a rough knowledge of aircraft position. Therefore, tight tactical or strategic control is not possible.

Table 2. Airspace Categories as Designated by Various Sources

1967 — Radio Technical Commission for Aeronautics (Ref. 23)

1. Controlled
2. Uncontrolled
3. Special use

1969 — DOT/ATC Advisory Committee (Ref. 1)

1. High density airspace
2. Positive controlled airspace
3. Mixed airspace (with IPC)
4. Uncontrolled airspace

1969 — General Aviation (Ref. 24)

1. Controlled
2. Mixed airspace
3. Uncontrolled

1969 — FAA (Ref. 25)

1. PCA
2. TPCA
3. Control zones
4. High density terminal area airspace (initially mixed airspace, going TPCA)

1969 — Air Transport Association (Ref. 26)

1. Controlled:
   - Type (1): Positive control of aircraft
   - Type (2): IFR/IFR control
     IFR/VFR control
     VFR/VFR advisory
   - Type (3): IFR/VFR — no service
     IFR/IFR control
     IFR/VFR control
2. Uncontrolled: No service

1972 — FAA (Ref. 27)

1. Positive control area
2. Control area (mixed airspace)
3. Terminal control area (positive control)
4. Control zone
5. Low altitude routes and airways (mixed airspace)
6. High altitude routes (positive control)
7. Special use airspace
8. Uncontrolled airspace

1972 — Boeing (Ref. 28)

1. High density positive control
2. Medium and low density positive control
3. Mixed
4. Uncontrolled

1972 — Lincoln Laboratories (Ref. 28)

1. High density positive control airspace
2. High density controlled (mixed) airspace
3. Low density positive control airspace
4. Low density controlled (mixed) airspace

1972 — Autonetics (Ref. 28)

1. Positive control
   1.1 High density
   1.2 Medium density
   1.3 Low density

2. Intermediate (mixed)
   2.1 High density
   2.2 Low and medium density

3. Uncontrolled

16

3) <u>Strategic Control</u>

   Control wherein the aircraft is under surveillance and
   is required to conform to his flight plan but the flight
   plan itself, having been determined to be conflict-free,
   remains unchanged.

4) <u>Tactical/Vectoring</u>

   Tactical control means control of aircraft involving
   changes to their flight plan and conformance to the new
   flight plan. Tactical/vectoring control is defined here
   to mean tactical control wherein the flight plan is
   changed early enough and greatly enough to avoid the
   possibility of an encounter situation. The closed-loop
   dynamics of the aircraft/control system are not
   important.

5) <u>Tactical/Encounter</u>

   Control is delayed long enough to allow a non-critical
   encounter situation to develop, exercising control only
   when deemed necessary to avoid a collision or close
   passage between two aircraft. In this case the closed-
   loop dynamics of the aircraft, aircrew, surveillance
   system, communications system, controller, and control
   laws must be taken into account. It is this situation
   that the IPC concept addresses and the SAUR formulation
   simulates.

These various levels of control each carry with them responsibilities
on the part of the controlling agency as well as on the part of the pilots.
Services which can be rendered to the pilot (again listed in order of in-
creasing in-flight requirements) include:

1) <u>Milestone checking</u>

   Milestone checking, e.g., cross-checking between sectors
   along an aircraft flight path, with possible initiation
   of search and rescue in the event of excessively late
   arrival at the destination or enroute reporting point.

2) <u>In-flight services</u>

   In-flight services which require general knowledge of
   aircraft position, e.g., notification of severe weather
   phenomena.

3) <u>Traffic advisories</u>

   Traffic advisories which involve determination of
   position of the individual aircraft plus other aircraft
   in the area.

17

4) Traffic control

   Traffic control, which is an attempt to provide
   separation of aircraft in a given area to the degree
   possible with the equipment available. Under circum-
   stances, however, aircraft may be close enough together
   to allow one aircraft to veer into another before ATC
   can effect an escape maneuver on the part of one or
   both aircraft.

5) Guaranteed separation

   Guaranteed separation — a service which, in the absence
   of failures or deliberate violations, will allow the
   air traffic controller to guarantee that participating
   aircraft are not involved in a mid-air collision.

Table 3 shows a postulated set of relationships between the flight
plan filed by the pilot, the equipment onboard the aircraft in question,
the services provided by the air traffic control system, and the degree
of control required in order to provide those services. Note that the
term "Intermittent Positive Control" does not appear in either the flight
plan or the control column. The pilot, when he flies a flight plan, as
pointed out earlier, signs a contract with the government. He either
agrees to be under control or he doesn't. Controlled conformal means that
the pilot wishes to conform to his approved flight plan and is willing to
accept control commands from the ground in order to meet safety require-
ments even if this involves a change to a new flight plan (to which he
also agrees to conform, once he accepts the new amended clearance). Con-
trolled non-conformal flight plans reflect what was in the minds of the
authors of the Intermittent Positive Control concept, i.e., the pilot
prefers to fly a non-conformal flight, not necessarily adhering to any
specific predetermined trajectory. On the other hand, he is willing to
accept control commands for safety of flight purposes. Therefore, he is
on a controlled flight plan. ATC may not actually exercise control over
him, just as they may never have to control a controlled conformal flight
which is able to adhere closely to plan; but he has agreed to accept con-
trol. The point is that there can be nothing intermittent about the
authority to exercise or the ability to follow positive control.

A pilot may file an uncontrolled cooperative flight plan wherein he
does not accept control from the ground (nor is he necessarily provided

Table 3. Conceptual Flight Plan, Control, Equipment, and Service Chart

FLIGHT PLAN	CONTROL	COMM		NAV. ACCURACY			SURV.	SEPARATION SERVICE PROVIDED (VS. OTHER FLIGHT PLAN)			
		DATA	VOICE	HI	MED	LO		1	2	3	4
1a CONTROLLED CONFORMAL	S, TV or TE	X	X	X			X	GS	GS	GS	N
1b CONTROLLED CONFORMAL	S or TV		X	X			X	GS	GS	GS	N
1c CONTROLLED CONFORMAL	S or TV	X	X	X				GS	GS	GS	N
1d CONTROLLED CONFORMAL	TV, TE or S	X	X				X	GS	GS	GS	N
2a CONTROLLED NON-CONFORMAL	TV or TE	X	X		X		X	GS	GS	TC	N
2b CONTROLLED NON-CONFORMAL	TV	X	X		X		X	GS	GS	TC	N
2c CONTROLLED NON-CONFORMAL	TV	X	X		X			GS	GS	TC	N
2d CONTROLLED NON-CONFORMAL	TV or TE	X	X				X	GS	GS	TC	N
3a UNCONTROLLED COOPERATIVE	P or FF	X	X			X	X	TA	TA	TA	N
3b UNCONTROLLED COOPERATIVE	P or FF		X			X	X	TA	TA	TA	N
4 UNCONTROLLED/ NON-COOPERATIVE	P		X			X		N	N	N	N

NOTE: TE — TACTICAL/ENCOUNTER
       TV — TACTICAL/VECTORING
       S — STRATEGIC
       FF — FLIGHT FOLLOWING
       P — PROCEDURE

NOTE: X = EQUIPMENT OPERATING

NOTE: GS — GUARANTEED SEPARATION
       TC — TRAFFIC CONTROL
       TA — TRAFFIC ADVISORIES
       N — NONE

19

separation by ATC), but where he is willing to make himself visible to the system by carrying a cooperative surveillance device. Thus, other aircraft can be warned of his presence and/or vectored around him. Mixed airspace as it presently exists would no longer be allowed. Aircraft which are both invisible and unknown to the system would no longer be allowed to operate in controlled airspace. Note, the distinction between invisible and unknown here is important. The air traffic control system could handle an invisible aircraft (e.g., one whose transponder has failed in flight or one flown by a pilot who has filed a special "no transponder" flight plan in advance) by following its position using verbal position reports and providing larger separation standards around that aircraft. It cannot, of course, handle unknown aircraft. Should such an aircraft be discovered the responsible person(s) should be subject to legal prosecution.

A pilot who is neither willing to accept commands nor carry cooperative equipment would file an underlined uncontrolled non-cooperative flight plan (undoubtedly some euphemism would be developed for this flight plan). Flight plans 1b through 1d and 2b through 2d are listed to show that aircraft can operate in the system with equipment failures. For example, an aircraft who files a 1a flight plan might become a 1b if he loses his data light or a 1c if he loses his surveillance device. Certain types of equipment failures could possibly prevent an aircraft from filing a flight plan for a given area but, of course, the system must be able to continue to provide the contracted-for service even if the pilot loses part of his equipment in flight. In actual practice, more subcases than are shown here would have to be accounted for. The Surveillance Accuracy/Update Rate Analysis is relevant to all encounters involving flight plan 1 aircraft; all encounters involving flight plan 2 aircraft; and those encounters involving flight plan 3 aircraft which also involve a flight plan 1 or flight plan 2 aircraft. Thus, any time guaranteed separation involving tactical/ encounter control is provided as indicated in Table 3, the SAUR formulation and results are relevant. It might also be pointed out that the control philosophy is independent of time of day and weather conditions.

3.3 SURVEILLANCE ACCURACY AND UPDATE RATE

The surveillance position determination uncertainties of interest range from several miles, as in the case of some of the poorer radar

situations, to uncertainties on the order of 100 feet or less, representative of the LIT concept and several other multilateration techniques. Update intervals associated with present surveillance systems are from four to twenty seconds. For LIT and other satellite systems the update intervals are considered to run from a fraction of a second to about four seconds, but can certainly be longer if desired. The update interval as pointed out in Section 2 is simply a linear term in the total system reaction time.

3.4 TOTAL SYSTEM REACTION TIME

This parameter is defined as the length of time from a collision avoidance command to when the aircraft collision avoidance maneuver begins. It takes into account a number of variables (including update interval) each of which deserves discussion. Table 4 has reaction time budgets for both a manual system and a semi-automatic system wherein a human controller looks at a radar scope to determine relative position of various aircraft, interprets the scope, decides on a course of action, reacts, executes a communication message of some specific length; this is then interpreted by a pilot who decides on his proper conformity action and, following some reaction time on his own part, initiates a maneuver.

In the semi-automatic system it is assumed that the data on the two aircraft locations are fed into a computer which decides that there is a conflict, determines what the appropriate resolution of that conflict should be, and provides a human controller with a concise presentation concerning the existence of a conflict, the aircraft involved, and the action required by the controller. This action is presumed to consist of pushing a button which releases a formatted message via data link to the aircraft through a wide bandwidth circuit involving virtually no communication delays. The situation in the aircraft changes very little. The pilot reaction time might be expected to be less, but interpretation of his collision warning message could actually take him longer if the technique providing him with this information is not well designed. For the SAUR study, however, we have assumed a simple unequivocal display, accompanied by an aural signal to get the pilot's attention, producing the same time budget for the pilot as in the manual system. Whereas a total system reaction time of six or seven seconds is quite possible, it would not be prudent to design the system at that figure unless absolutely necessary and unless appropriate procedures were invoked.

21

Table 4.   Total System Reaction Time

	Manual System		Semi-automatic System	
	Full Range Seconds	Likely Range Seconds	Full Range Seconds	Likely Range Seconds
CONTROLLER				
Interpretation	1-5	2	1-3	1
Determination of Necessary Action	1-10	3-4	1-3	0-2
Human Response	0-1	0	0-1	0
	2-16	5-6	2-7	1-3
COMMUNICATIONS				
Delay/Execution	0-10	0-2	0-1	0
Message Length	2-5	4	0-1	0
	2-15	4-6	0-2	0
PILOT				
Interpretation	1-5	2	1-5	0
Determination of Necessary Action	1-20	2-4	1-20	2-4
Human Response	0-1	0	0-1	0
Initiate Maneuver	1-4	2	1-4	2
	3-30	6-8	3-30	6-8
SURVEILLANCE				
Update Interval				
Standard	10-20	5-10	10-20	5-10
High Performance	1-4	1-2	1-4	1-2
TOTAL Standard	17-81	20-30 (25)	15-59	14-23 (19)
High Performance	8-65	17-22 (19)	6-43	10-15 (12)

The full range reaction time budget for the manual system indicates that it is quite possible for over a minute to elapse from the moment when a collision situation develops to the time that one or both pilots initiates a collision avoidance maneuver. This is not unrealistic. For example, if one aircraft banked sharply, blanking out the ATC RBS antenna for two pulses, thirty seconds could elapse between updates. If this put the aircraft in an unexpected position, or his reappearing on the scope caused some confusion for the controller, it is possible that his interpretation and decision time could involve another 10 or 15 seconds. Next, the controller's warning command might at first be blocked by another transmission causing a 10 second delay, forcing the communications time up near the 15 second mark. Finally, if the pilot was busy and under stress, it could easily take him another 10 or 15 seconds to react. Clearly, this is not a likely occurrence; but it is not an impossible one either. The values in parentheses at the bottom of the likely range columns for both manual and semi-automatic systems are the ones used in the analysis.

It is important to note that additional work in the area is needed to establish a sound basis for appropriate design levels of reaction time as a function of the various surveillance, communications, and control concepts which could be implemented.

3.5 DECISION TIME

The decision time (τ) is used in the SAUR Program as the length of time prior to the instant of closest approach at which the controller makes a decision to intervene or not to intervene in the case of two aircraft in potential conflict. This conflict is considered in the tactical sense, i.e., the aircraft are flying in reasonably close proximity such that it is possible that some action will be required by a controller, who can then observe the results of his action in real-time, to ascertain that a collision will be avoided. The SAUR analysis iteslf sheds no light on a desirable value of decision time. (See Section 5.) Consequently, the range of values over which τ should be examined was determined as follows:

- For very large values of τ — say, 3 to 5 minutes — the SAUR analysis does not apply. The aircraft are so far apart that the situation is not tactical. The con-troller is not concerned with what one or both pilots

might do in the seconds prior to his giving them a
vector command. If large values are used (e.g., a
decision time of 8 minutes and a total effective
reaction time of 4 minutes), then one or both air-
craft could conduct maneuvers such as full 360° turns
and the mathematical model itself becomes less relevant.
A decision time of much over 90 seconds would involve
more strategic control.

● If the decision time is very small, say, ten seconds,
then it is very likely too late to avoid a collision.
The decision time must exceed the reaction time by at
least the length of time to perform an escape maneuver
which will provide the desired guaranteed separation
distance. Experience and other analyses (e.g., those
related to airborne CAS) have indicated that it is
difficult to guarantee safe separation after some
25 or 30 seconds before collision

● The range of values for decision time considered most
relevant in this study, then, is that related to tactical
encounter control, i.e., from about 30 to 90 seconds.

3.6 GUARANTEED SEPARATION

In the derivation of the required surveillance accuracy from the
distance of closest approach, as determined by the SAUR model, the assumption
is made that uncorrelated errors in the positions of the two aircraft are pre-
cisely co-linear but opposite in sign. To do less (e.g., RSSing the errors)
would mean that the controller would be providing probable, not guaranteed
separation. The question arises, "What value of guaranteed separation should
be used?" Remembering that the chances of two aircraft actually getting that
close as a result of surveillance errors is quite remote, an arbitrary selec-
tion of 1,000 feet was made in the SAUR formulation, except in the case of
two supersonic aircraft in a head-on collision situation where 10,000 feet
was used. A discussion of the effects on the output results of varying
guaranteed separation is given in Section 6.2.

3.7 AIRCRAFT DYNAMICS

Of major importance in the SAUR analysis is the nature of aircraft
maneuvers — both those commanded by the controller and those initiated by
the pilot on his own volition. The maneuvering capability of each aircraft
was calculated using basic data from Reference 29, performance techniques
from References 30 and 31, and airplane stability and control techniques

from References 30 and 32. The pilot-initiated maneuvers were assumed to be relatively close to the limits of the maneuvering capability of the aircraft, modified as appropriate by the particular case (e.g., limitations in maneuvers close to the ground). The controller-initiated maneuvers were somewhat smaller in that it could not be certain that a controller could count on the pilot being able to get the full range of performance out of the aircraft in responding to a collision warning command. For example, some pilots could be willing and able to execute a very steep turn in a certain type situation, whereas the second pilot might, for reasons of passenger comfort, safety, or his own ability, execute a collision avoidance command with a less pronounced maneuver. Thus, the controller-initiated maneuvers were usually some fraction (e.g., 1/2 to 2/3) of the pilot-initiated maneuvers.

3.8 ATC SYSTEM LOAD AND AIRCRAFT DENSITY

Although the SAUR Program is strictly a two-body problem, the communication rate analysis does take into account the local aircraft population in order to determine the communication rates associated with solving all of the individual two-body problems. Using instantaneous airborne count figures which were estimated using data from References 1, 2, and 3, the following population densities were calculated for the ten cases studied:

- United States average — 0.01 aircraft per square mile

- Cases 1, 2, 6, and 8 — 1.0 aircraft per square mile

- Case 3 — 0.1 aircraft per square mile

- Cases 4 and 7 — 0.5 aircraft per square mile

- Case 5 — 0.0001 aircraft per square mile

- Case 9 — 0.5 aircraft per square mile

- Case 10 — 0.01 aircraft per square mile

These figures were arrived at by dividing the peak number of aircraft estimated to be airborne in a given region (around 1990) by the area of that region and multiplying by the fraction of aircraft that could be expected to occupy the altitude band of interest.

4. THE SURVEILLANCE ACCURACY/UPDATE RATE (SAUR) PROGRAM

The development of the SAUR Program began with an examination of some of the existing models for theoretical ATC studies. These models had certain ideas in common, and these ideas were abstracted and generalized in order to develop a more universal approach to the problem. The existing models used very simplified assumptions about control laws and aircraft performance. The SAUR approach uses more realistic assumptions and hence more complex computations.

4.1 GENERAL MODEL

The model considers the interaction of two aircraft and a controller. The basic inputs are the initial positions, speeds, and headings of the two aircraft; the aircraft and pilot performance parameters; and the parameters of the control law.

Consider two aircraft which are under surveillance. The current estimates of position and/or velocity are known to a controller. (The controller may be a man or a computer.) The measurements of position and velocity are subject to error.

Consider what happens if the controller sends a separation maneuver to one or both aircraft. Before the message is received by the aircraft, the aircraft will fly a path which is not completely predictable. For example, the turn rate of the aircraft is not known, but it is known (or assumed) to satisfy certain limiting conditions. The limits may reflect aircraft performance capability or, in the case of IFR traffic, procedural rules. After the command is received at the aircraft and the pilot reacts, there is a certain additional time required for the aircraft to react.

After the aircraft reacts, the turn rate of the aircraft will be assumed to be the commanded turn rate.

The motion of each aircraft is described in three stages:

1) The time period required to perform the computations, format and send the message, display it to the pilot and for the pilot to react;

2) The time period for the aircraft to react; and

3) The time period during which the command is being obeyed.

27

The motion of the aircraft would be completely predictable if all of the parameters of flight were known. These parameters are divided into three classes: 1) Constants; 2) Uncontrolled parameters (there are parameters which the controller can do nothing about, e.g., data errors, heading before pilot reaction time); and 3) Controlled parameters (e. g., heading after aircraft reaction time). The values of the constants are assumed known and the values of the controlled and uncontrolled parameters are assumed to fall within known limits.

Let D be the distance of closest approach between the two aircraft. Let $\vec{q}$ be a vector whose components are all of the uncontrolled parameters. Let $\vec{p}$ be a vector whose components are all of the controlled parameters. Then D is a function of $\vec{p}$ and $\vec{q}$: $D = D(\vec{p},\vec{q})$.

If the controller knew the values of $\vec{q}$, he would select $\vec{p}$ to maximize $D(\vec{p}, \vec{q})$. Unfortunately, he doesn't know $\vec{q}$. Hence, consider what would happen if he gives a particular command $\vec{p}$. The worst thing that can happen is that $\vec{q}$ happens to be such that $D(\vec{p}, \vec{q})$ is minimum. Let this minimum be

$$H(\vec{p}) = \min_{\vec{q}} D(\vec{p}, \vec{q}).$$

The controller can now choose $\vec{p}$ to make $H(\vec{p})$ as large as possible. The basic computation is, therefore

$$D^* = \max_{\vec{p}} \min_{\vec{q}} D(\vec{p}, \vec{q}).$$

The reasoning behind the basic formula is that one assumes the worst about the uncontrolled parameters and the best about the controlled parameters. The controller gives commands (i. e., selects controlled parameters) which will work even in the worst assumed set of circumstances - the minimum over $\vec{q}$. The controller is assumed to be rational and able to give commands to assume maximum separation - the maximum over $\vec{p}$.

The formula is used in the following way. Let D_0 be the minimum acceptable distance between the two aircraft (or between one aircraft and a straight line). Let D* be the solution of the max-min problem above. The value of D* is a function of total pilot reaction time T, or D* = D* (T). D* (T) is a decreasing function of T. If D* (T) > D_0, a command given now would assure sufficient separation of the two aircraft. To find whether it is necessary to give a command now, consider what would happen if the controller waits one surveillance cycle. This is done by computing D* (T + Δt), where Δt is the data interval. If D* (T + Δt) $\leq D_0$, it is not wise to wait until the next cycle; it will be too late to provide the required separation. The rule is: if D*(T+Δt) $\leq D_0$, give a command now—otherwise wait.

The SAUR program is designed to compute D*, i.e., to solve the max-min problem.

4.2 GENERAL PROGRAM STRUCTURE[1]

The program starts with the initial position and velocity of the two aircraft and a number of other parameters which are described in Section 3.9.

The basic assumptions about the two aircraft trajectories are as follows:

1) Speed

The initial speed is input. The speed increases or decreases, with constant acceleration, until it reaches a pilot selected value which is one of the "uncontrolled parameters" described earlier. If speed is a controlled parameter, i. e., if the controller gives a speed command, then, at time T (the total reaction time for that aircraft), the speed begins changing, with constant acceleration, until it reaches the controller selected value.

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[1] A detailed description of the program equations and program input listing is presented in Appendix A. Instructions for running the program are included.

2) Heading

The initial heading is input. The heading increases or
decreases, with constant turn rate, until it reaches a
pilot selected value, which is one of the "uncontrolled
parameters" described earlier. If heading is a control-
led parameter, i. e., if the controller gives a heading
command, then, at time T (the total reaction time for
that aircraft), the heading begins changing, with con-
stant turn rate, until it reaches the controller
selected value.

3) Altitude

The initial altitude is input. The altitude increases
or decreases, with constant climb (descent) rate until
it reaches a pilot selected value, which is one of the
"uncontrolled parameters" described earlier. If altitude
is a controlled parameter, i. e., if the controller given
an altitude command, then, at time T (the total reaction
time for that aircraft), the altitude begins changing,
with constant acceleration, until it reaches the control-
ler selected value.

The control law assumption is that the controller will give one com-
mand to each aircraft and the aircraft will start to respond to that com-
mand at the end of the system reaction time. The two commands may represent
different trajectory parameters; for example, the controller may tell one
aircraft to increase speed to 300 knots and tell the other aircraft to
descend to 2000 feet.

The program solves the max-min problem by trial and error, according
to instruction from the program user. For example, if the initial heading
of aircraft #1 is 100°, the user may tell the program to try pilot selected
headings of 90°, 95°, 100°, 105°, and 110°. The program will fly the two
aircraft along each of the possible trajectories and choose the maximum
(for each controlled parameter) of the minimum (for each pilot parameter)
of the distance between the two aircraft.

4.3 ALTITUDE CALCULATIONS

The program calculations for altitude (Z) are handled differently
from the calculations for horizontal (X,Y) motion in that the altitude
part of the problem is solved directly and only the horizontal part of the
max-min problem is solved by trial and error.

The various commands considered are divided into seven sets:

Set	Aircraft 1	Aircraft 2
1	Climb	Descend
2	Descend	Climb
3	Climb	Other
4	Descend	Other
5	Other	Climb
6	Other	Descend
7	Other	Other

In this table "other" means other than an altitude type of command (i.e., heading or speed). For each of the above seven sets, the altitude program (routine ZHIST) computes upper and lower limits on the magnitude of the altitude difference between the two aircraft.

Consider the first aircraft with a measured initial altitude $Z_0^{(1)}$. In Figure 3 . The measured altitude is subject to an altitude error. This error is assumed to be less than E_Z. Hence the true altitude of the aircraft is between $Z_0^{(1)} - E_Z$ and $Z_0^{(1)} + E_Z$.
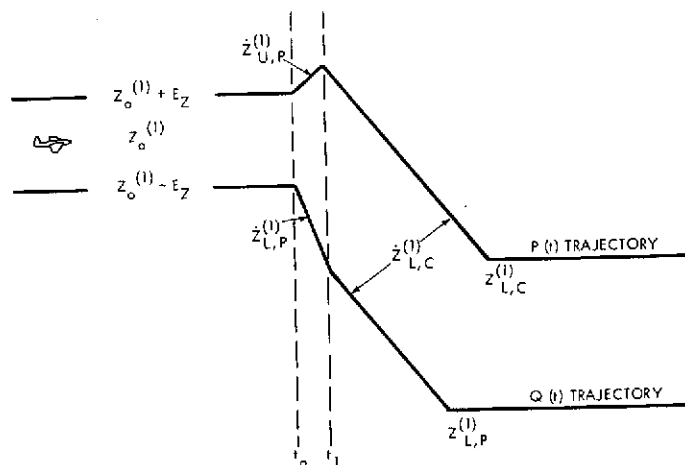
FOR AIRCRAFT 1	$Q(t) \leq Z^{(1)}(t) \leq P(t)$
SIMILARLY FOR AIRCRAFT 2	$S(t) \leq Z^{(2)}(t) \leq R(t)$
$D_Z(t) = (Z^{(1)}(t) - Z^{(2)}(t))$ MIN	

Figure 3. Altitude Program

31

Suppose for example the first aircraft is told to descend (case 2 or 4 above). Before time T_1 (the system reaction time for aircraft 1) the altitude history of the aircraft is not completely predictable. However, the highest it can be at any time can be calculated by assuming that it starts at the highest possible altitude $Z_0^{(1)}+E_Z$, and it climbs at its maximum (uncontrolled parameter) rate $\dot{Z}_{U,P}^{(1)}$ until it reaches its maximum (uncontrolled parameter) altitude $Z_{U,P}^{(1)}$. The superscript 1 refers to the aircraft, the U refers to the upper limit and the P refers to the fact that these are limits on pilot-initiated maneuvers. After time T_1, the command to descend takes effect. This means that the aircraft then descends at a rate $\dot{Z}_{L,c}^{(1)}$ until it reaches its minimum altitude $Z_{L,c}^{(1)}$. The subscript c refers to limits on controller-initiated maneuvers. This altitude trajectory (start as high as possible, climb as fast as possible to the maximum altitude, then descends at the given rate to the commanded altitude) defines the highest possible altitude. It is called $P(t)$ in the program.

The maximum altitude function $P(t)$, computed by a general routine "WROUT," is a piecewise linear function. The values of the function $P(t)$ at each of the "break points" and the times of these points are output by routine WROUT.

The minimum altitude $Q(t)$ function for aircraft 1 is computed similarly. This function is determined by considering an aircraft which starts at $Z_0^{(1)}-E_Z$, descends as quickly as allowed—i.e., at a rate $\dot{Z}_{L,p}^{(1)}$ to a minimum allowed altitude $Z_{L,p}^{(1)}$. At time T_1 it continues to descend at a rate $\dot{Z}_{L,c}^{(1)}$ until it reaches $Z_{L,c}^{(1)}$. The subscript L means "lower" limit. Again the routine WROUT is used to compute $Q(t)$.

Aircraft 1 is then known to be between $P(t)$ and $Q(t)$ at any time t:

$$Q(t) \le Z^{(1)}(t) \le P(t),$$

where $Z^{(1)}(t)$ is the true (unknown) altitude of aircraft 1 at time t.

Using the same methods, the upper and lower limits or altitude functions for aircraft 2 are determined. They are called R(t) and S(t):

$$S(t) \leq Z^{(2)}(t) \leq R(t)$$

The program uses WROUT four times to compute P(t), Q(t), R(t), and S(t). $D_Z(t)$ is then computed by routine ZHIST. The main property of $D_Z(t)$ is that the altitude difference between the two aircraft at time t is not less than $D_Z(t)$.

The function $D_Z(t)$ is computed for set 1 and then for set 2 of Table 1. Denote the two functions by $D_Z^{(1)}(t)$ and $D_Z^{(2)}(t)$. Routine MAX DZ computes and outputs the maximum of these two functions:

$$D_Z^{(M)}(t) = \max \left[D_Z^{(1)}(t), D_Z^{(2)}(t) \right]$$

The meaning of $D_Z^{(M)}(t)$ is as follows. At any time $t_{(M)}$, there is a pair of altitude commands such that an altitude separation of $D_Z^{(M)}(t)$ can be guaranteed.

4.4 HORIZONTAL MANEUVERS AND THE CALCULATION OF DISTANCE OF CLOSEST APPROACH

Next the program considers horizontal maneuvers. The details of this part of the program are described in Appendix A. The program tries a large number of possible horizontal maneuvers on the part of both aircraft. For example, one combination might involve aircraft one turning right 10° and speeding up to 350 knots, while aircraft 2 turns left 30° and slows down to 250 knots. For each combination of maneuvers it calculates the distance of closest approach (DCA) of the two aircraft, using the previous calculated vertical separation. The program takes the minimum of all of these DCA's. This minimum DCA is the separation which can be assured by using an altitude maneuver. Denote this distance by D*.

The program searches over a number of specific new headings for each aircraft, but the number of new headings is finite. Further, the program does not search over all possible turn rates that could place an aircraft on a new final heading. A single turn rate is used. As a result the loci of possible positions of the two aircraft may be thought of as a continuum

33

and the loci of calculated positions of the two aircraft as sets of paths forming a screen or grid covering this continuum. This approximation introduces noise into the calculations of DCA's and it usually prevents a zero DCA from being calculated when actually one could occur; but in a detailed or preliminary design phase, however, a more complex version of the program should be used in order to provide higher fidelity data.

The program next considers set 3. The maximum and minimum altitudes for aircraft 1 [$P(t)$ and $Q(t)$] are the same as for set 1. The maximum and minimum altitudes for aircraft 2 [$R(t)$ and $S(t)$] are computed without any altitude command being given. The maximum is computed by considering an aircraft, initially at the nominal altitude plus the surveillance uncertainty in altitude, which climbs at the pilot-initiated rate to the maximum altitude and then stays at that altitude.

Once the functions $P(t)$, $Q(t)$, $R(t)$, and $S(t)$ are computed for set 3 the function $D_Z(t)$ is computed as before. The corresponding function for set 4 is also computed. The maximum of these two functions is computed as before. This is the altitude separation which can be guaranteed by giving an altitude command to aircraft 1 while aircraft 2 is not given an altitude command.

After computing $D_Z^{(M)}(t)$, the program considers horizontal maneuvers again. This time the horizontal calculations are more complex because a horizontal command is given to aircraft 2.

The program first considers a particular horizontal command to aircraft 2; for example, increase speed by 20 knots. Given this command, the speed of aircraft 2 after T_2 is determined, but there are a number of other variables which are still undetermined — the speed of aircraft 2 before time T_2, the heading of aircraft 2, the speed of aircraft 2, and the heading of aircraft 1. The program tries all possible combinations of these undetermined parameters, computes a DCA for each combination, assuming that the altitude separation is $D_Z^{(M)}(t)$, and computes the minimum of these DCA.

This distance is the separation which can be assumed with an altitude command to aircraft 1 and a command to aircraft 2 to increase speed by 20 knots. This distance is compared with D* and the maximum of the two replaces D*.

The program next considers another command to aircraft 2 (e.g., slow down 20 knots). It again tries all possible combinations for the other parameters, gets a DCA with each combination, and takes the minimum of these DCAs. Again, this distance is compared with D* and the maximum of these numbers replaces D*.

The process is repeated until all possible horizontal commands have been considered for aircraft 2. After this is done, D* represents the distance which can be guaranteed for all commands considered so far, i. e., for a vertical command to aircraft 1 and a horizontal or vertical command to aircraft 2.

Next the program returns to sets 5 and 6 and computes $D_Z^{(M)}(t)$ as before. It then repeats the computations as in sets 3 and 4, but this time considering horizontal commands for aircraft 1. When this is done, D* represents the guaranteed separation for all commands considered so far, i. e., for cases in which at least one aircraft is given an altitude command.

The program then considers set 7. In this case, the functions P, Q, R, and S are computed without any vertical maneuvers. The function $D_Z(t)$ is computed as before. It is not necessary to use MAX DZ this time and the program simply sets $D_Z^{(M)}(t)=D_Z(t)$.

Next, all possible pairs of horizontal commands are tried, e.g., aircraft 1 is told to turn right 90° and aircraft 2 is told to slow down 20 knots. For each pair of commands the program tries all possible combinations of the free parameters and computes the minimum DCA for that pair of commands. As each pair of commands is computed, the minimum DCA is compared with D* and the maximum replaces D*.

Finally, at the end of the program, D*(t) is the separation which can be guaranteed using the best of the commands which were tried.

5. COLLISION WARNING COMMAND RATE ANALYSIS

5.1 INTRODUCTION

It became apparent early in the SAUR Program that in establishing the relationship between surveillance accuracy and update rate requirements we would arrive at a relationship such as that indicated in Figure 2. Inspection of this figure reveals that a surveillance system designer would not be able to prescribe a unique accuracy or update interval based on that data unless he knew what the appropriate decision time (τ) should be. For this reason an attempt was made to develop quantitative relationships between communications load or false alarm rate, decision time and system reaction time to determine a preferred value of accuracy and update interval.

Whereas the SAUR Program investigates the interaction of two aircraft in specific circumstances, some control problems require the consideration of whole aircraft populations. Problems related to estimation of communications loads are of this type. This section describes a method for the application of a max/min control model to an aircraft population. The purpose is to analyze conflicts, collision warning, command loads and false alarm rates as a function of the quality of the surveillance system.

The qualitative relationship between surveillance accuracy, update rate, collision warning, communications load and false alarm rate are fairly well understood. For example, if everything is held constant except that surveillance accuracy is degraded, the collision warning command communications load will go up. Aircraft that are actually a safe distance apart must be considered to be closer together than they actually are (due to the increased uncertainty in their position), thus precipitating collision warning commands when none may be necessary. This situation can be considered a false alarm situation. A similar situation exists if the surveillance update interval is increased, i.e., the alarm rate and false alarm rate are both increased since the uncertainty in aircraft position becomes greater and commands must be generated sooner.

To quantitatively study this problem and estimate command loads for particular aircraft population, it is necessary to assume an aircraft population model and a controller model. The aircraft population model is the set of assumptions about the density and uncontrolled motion of the aircraft. Examples of such assumptions are:

1) Aircraft fly along known input flight paths with known trajectories.

2) Aircraft fly along known paths with initial time of entry into the paths being random.

3) Aircraft fly at random, i.e., at any time the location and direction of any aircraft is random, but the aircraft fly in straight lines (gas model).

4) Aircraft fly in random paths at random times and not generally in straight lines.

The gas model has been used in previous studies to get an estimate of conflicts and command loads and this model will be used here.

The controller model is the set of assumptions used to determine whether commands have to be given to avoid a conflict between aircraft. Controller models include:

1) The controller gives a command pair to any two aircraft which get within a fixed distance of each other.

2) The controller gives a command pair to any two aircraft when a conflict is possible within a fixed warning time.

3) The controller assumes that the aircraft can maneuver, and gives the best command pair to assure separation when and only when necessary. This is the max/min model considered in the SAUR Program.

In summary, the models used in this study are a gas model population model and a simplified max/min controller model.

5.2 THE METHOD

Consider a region R which contains a large number of aircraft with random positions and headings. Choose one of the aircraft at random. The first problem is to determine the number of times it is necessary to give a command pair to avoid a conflict between the chosen aircraft and one of the aircraft in R.

38

Assuming that all aircraft fly in straight lines, it is fairly easy to calculate the expected number of "conflicts" or times that the chosen aircraft will come within a fixed distance of one of the aircraft in R. This is done by first looking at the subset of aircraft in R which are moving at a particular heading and counting the number of conflicts for the aircraft in this subset. This is repeated for other subsets with other headings and the results are integrated over all headings to get the total number of conflicts with the chosen aircraft. This is the technique used in a number of previous studies. The technique is valid (for the given assumptions) for estimating the number of conflicts. The same technique has been used to estimate the number of commands by making the approximation that two aircraft must be given commands if they fly within a certain distance of each other. This latter approximation is not valid, and the present study will not use it.

Consider a particular aircraft in the region R, and consider a subset of aircraft in R which all have the same headings. The question is which of these aircraft will require commands. In principle, the question can be answered by repeatedly running the SAUR Program.

Consider an aircraft at a particular point in R with a particular heading at the initial time. By running the SAUR Program it is possible to determine whether a command is necessary for that aircraft and an aircraft in the subset at that time. If the answer is "no", the time is incremented and the question is asked again. By testing all the times between the times of entering and leaving region R, it is possible to find out whether there is any time at which a command is necessary. By repeating the process for other points in the region it would be possible to find the set of points which result in a command being necessary. For each of these points it is possible to determine whether the command pair was a false alarm. A false alarm results when a command pair is given but the straight line extrapolation of the two aircraft paths resulted in a distance of closest approach greater than the conflict distance.

For the particular heading it is then possible to average over all positions to get the expected number of command pairs, and the expected number of false alarms. The process is repeated and results are averaged over all headings.

39

The above program is not practical in the simple "brute force" form described above. The number of runs of the SAUR Program would be excessive. It is therefore necessary to use special techniques to obtain the boundary of the subregion in R which contains points which result in command pairs.

In Figure 4 the region R contains aircraft at random position, all with the same headings (indicated by the velocity vectors in the figure). Select one particular aircraft and one particular time, say, t=0. Consider another aircraft at an arbitrary position P inside the region R. Suppose that all of the parameters required for the SAUR 2 Program have been defined. It is possible to run the program to determine whether, for the selected aircraft and the aircraft now at P, either a) it is not necessary to give a command pair now to avoid a conflict, b) if a command pair is given now the two aircraft will just barely avoid a conflict, or c) it is too late to give a command pair to avoid a conflict. These three situations define a region S. Points satisfying condition a), b), or c) are respectively outside, on the boundary, and inside of region S.
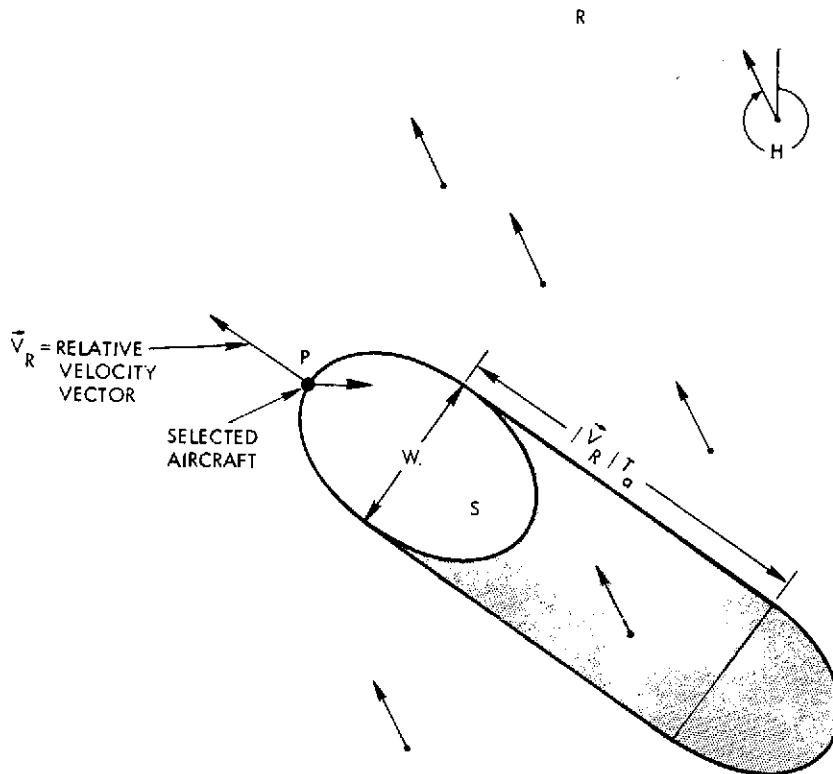
Figure 4. Command Load Calculations

40

Next consider the question of whether it is necessary to give a command in the future. Consider an arbitrary time T_a. Consider the selected aircraft as being fixed, and the other aircraft to be moving at relative velocity $\vec{V}_R$. The region S is fixed to the selected aircraft. For any other aircraft which flies into the region S a command pair must be given. If W is the width of the region S in the direction perpendicular to $\vec{V}_R$, the figure shows that the aircraft which result in command pairs being given before time T_a are those aircraft in the shaded region. The area of the shaded region is equal to the area of a rectangle of width W and length $|\vec{V}_R|T_a$. The expected number of such aircraft is the density of aircraft times and area of the rectangle, or $\rho W|\vec{V}_R|T_a$, where ρ is the density of aircraft with the given heading H.

Once the expected number of command pairs is computed for an entering aircraft, the process is repeated for other values of heading and the results are summed over all headings to get the total number of commands for this entering aircraft. The result is multiplied by the number of aircraft to get the total command load for the situation being studied. The major task is to calculate the width W of the critical command region. The following paragraphs outline a method for computing W.

Consider an aircraft which is heading, say, due East (Figure 5). The system reaction time for the aircraft is T, the maximum pilot initiated turn rate is ω_p, and the controller initiated turn rate is ω_c.

As in the SAUR Program, we don't know what the pilot will do before time T, and we therefore must consider a whole family of possibilities. Suppose, for example, the pilot turns to a heading H and maintains that heading until time T, at which time he responds to a left turn command from the controller. The trajectory will appear as in Figure 5. Since, however, the value of H is not known, it is necessary to consider a whole family of such trajectories, as in Figure 6. The outer envelope of these trajectories is a curve C_L. (The L subscript refers to left turn commands.) The significance of the curve C_L is that the controller can guarantee, by giving a left turn command, that the aircraft will stay inside of the shaded region S_L bounded by C_L.
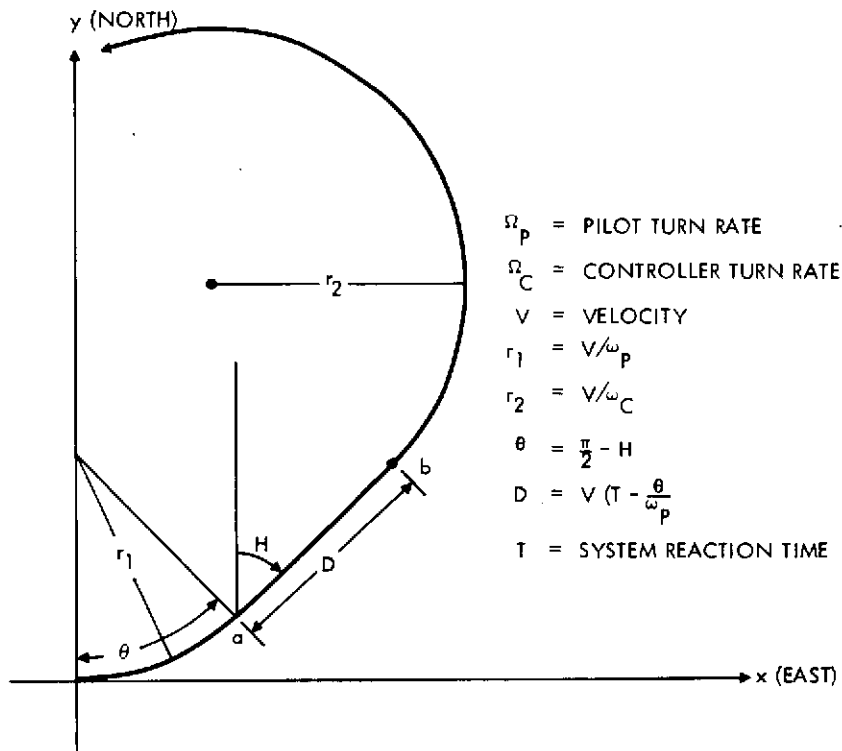
Figure 5. Single Aircraft Turn Geometry

In the figure legend:

Ω_P = PILOT TURN RATE

Ω_C = CONTROLLER TURN RATE

V = VELOCITY

r_1 = V/ω_P

r_2 = V/ω_C

θ = $\frac{\pi}{2} - H$

D = $V \left(T - \frac{\theta}{\omega_P}\right)$

T = SYSTEM REACTION TIME
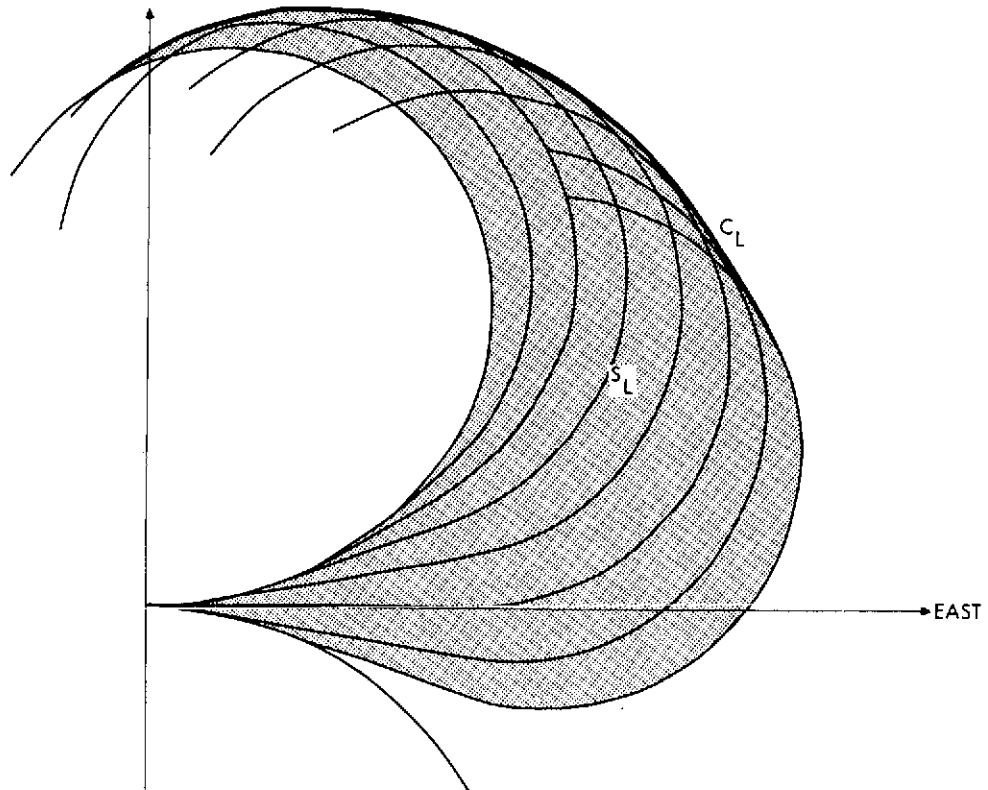
Figure 6. Envelope of Left-Turn Command Trajectories

42

By considering right turn commands, another curve C_R and region S_R, are defined. Now consider the intersection (the common set of points) S of the two regions S_R and S_L. The significance of the region S is that the controller can guarantee, by a heading command, that the aircraft will stay with the region S. (See Figure 7.)

Next consider an approaching aircraft with an initial heading H_A. The controller will give commands, if necessary, in order to ensure that the aircraft will stay a distance DCA=GS+2SA apart. The distance GS is the required "protected distance" and a safety factor is added, where the maximum surveillance position error is SA. This is the same relationship derived in Section 2.

Consider the selected aircraft to be fixed and the approaching aircraft to be flying by with velocity vector $\vec{V}_R$, the relative velocity vector. (See Figure 8.) Consider a region S fixed to the selected aircraft and another region S_A attached to the approaching aircraft. In this case the two aircraft are presumed to be identical so the region S_A is exactly the same size and shape as S. The lateral distance d_1 and d_2 are set such that
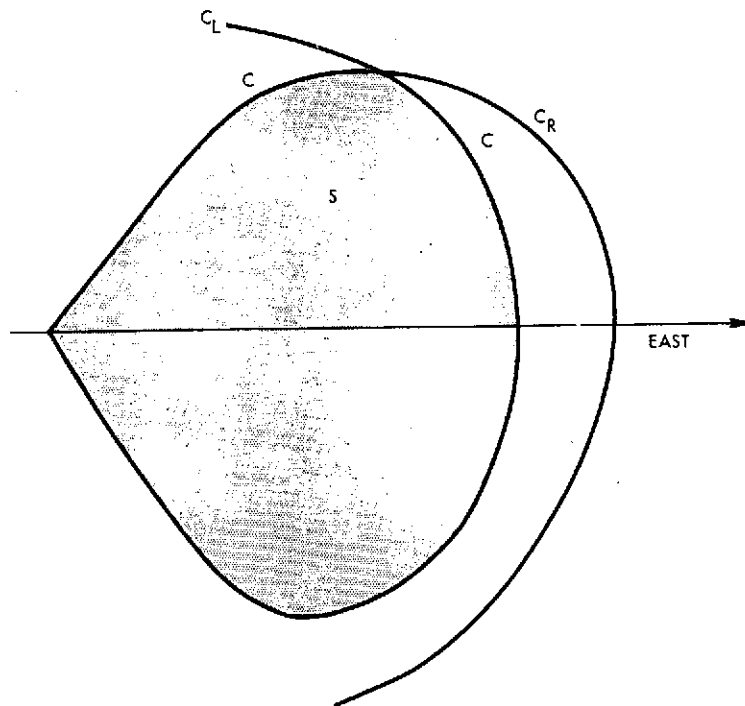
Figure 7. Intersection of C_L and C_R Envelopes

43
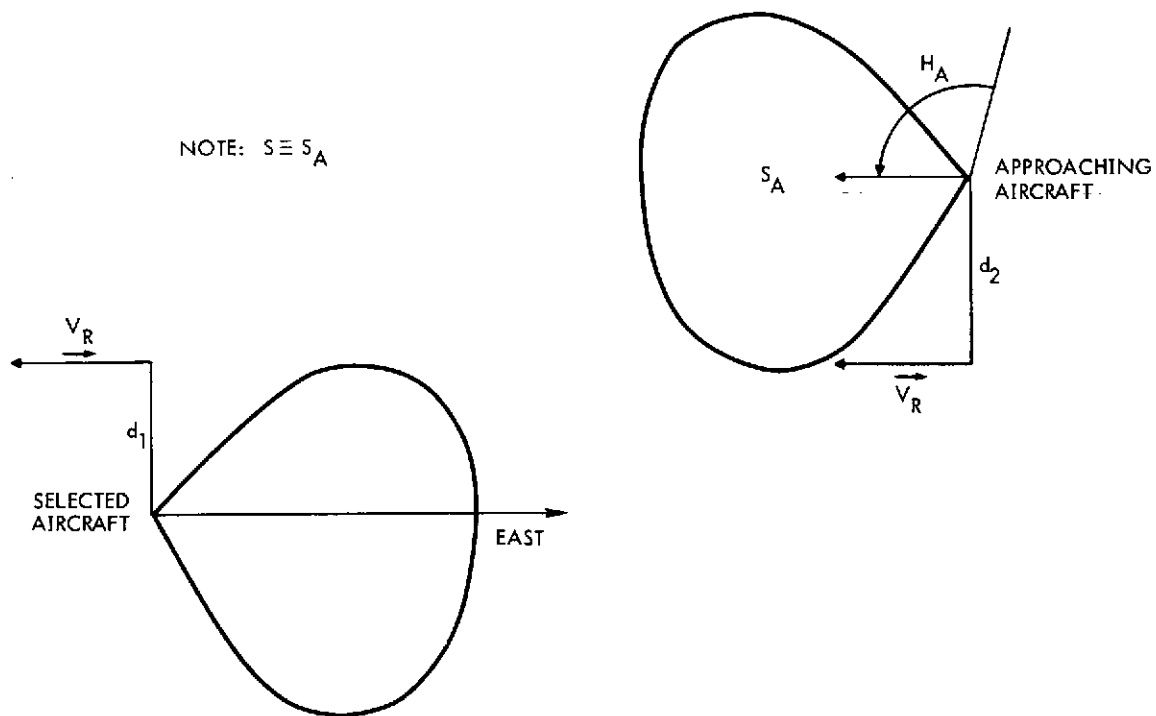
Figure 8. Geometry of Passing Regions

the two regions will just graze. If the region S_A "flies by" at a distance greater than DCA, it will not be necessary to give commands to the two aircraft to ensure separation. To determine whether the regions will remain separated by DCA, it is necessary to compute the distances d_1 and d_2, which depend on the approach heading H_A.

Summarizing, first the envelopes C_R and C_L are computed and the intersection envelope C is computed from these two curves. Next, the approach heading is varied over all possible approach headings between -180° and 180°. For each of these approach headings, the distances d_1 and d_2 are computed. The width W is computed from $W=2(d_1+d_2)$. The results are averaged over all approach headings to get the average width. The result is multiplied by $\rho|V_R|\tau$ to get the total number of commands per hour.

The mathematics of these calculations are described in more detail in Appendix A. A description of the inputs and the results of the calculations for the ten cases are in the following subsection.

44

5.3 COMMAND RATE CALCULATIONS

The command rate calculations were made in the manner just described. Inputs to the calculations, using values appropriate to the individual case, are shown in Table 5. The CR relationships for Cases 1, 2, 6, and 8 are shown in Figure 9. These following results are fairly typical:

- The rate of change of DCA or SA with a change in T is large and negative for each value of CR. The CR lines are essentially straight and parallel.

- The value of CR increases roughly linearly with T.

The actual values of CR are very scenario-dependent and the large uncertainty in the values of some of the input parameters (e.g., aircraft density) make it difficult to determine the absolute value of the command rate even for any particular scenario. In spite of this, the relationships established here are considered to be very useful and it does appear that significant conclusions can be drawn that will help determine the accuracy and update rate requirements. The method of determining this will be described in the next section.
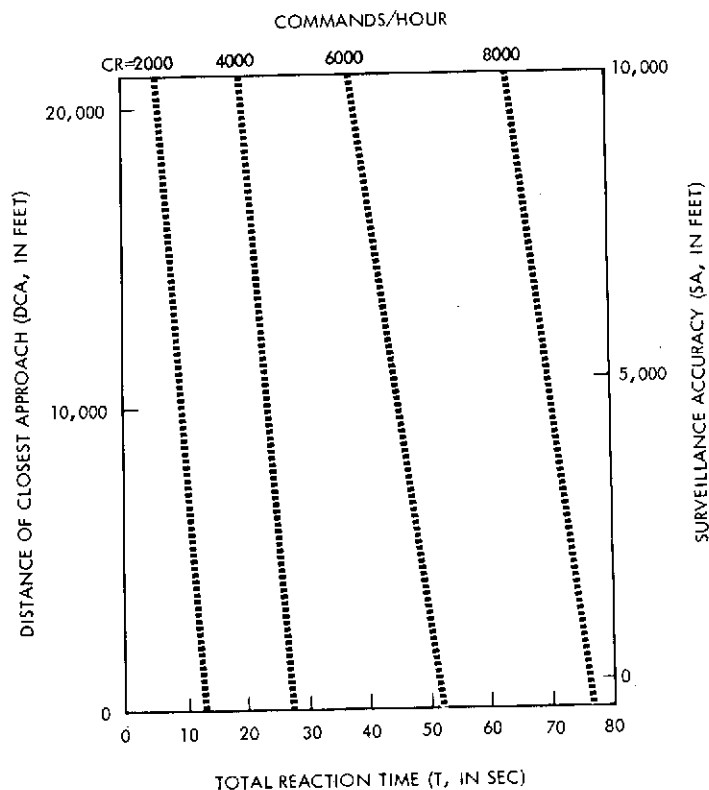
Figure 9. Command Rate (Cases 1, 2, 6, and 8)

45

Table 5. Inputs to Collision Warning Command Rate Analysis

	Case 1,2,6,8	Case 4,7	Case 5	Case 3	Case 9	Case 10
Surveillance Accuracy in Feet	100-20,000	100-10,000	100-100,000	100-20,000	100-20,000	10-20,000
Guaranteed Separation in Feet	1,000	1,000	10,000	1,000	1,000	1,000
Density in Aircraft per Cubic Mile	1.0	0.5	0.0001	0.1	0.5	0.01
Total System Reaction Time in Seconds	0-90	0-90	0-90	0-90	0-90	0-90
Average Speed in Feet Per Second	420	200	2,000	845	700	845
Pilot Initiated Turn Rate in Degrees Per Second	6	6	3	4.5	6	4.5
Controller Initiated Turn Rate in Degrees Per Second	3	3	0.5	3	3	3

Finally, it should be pointed out that "command rate" and "communications load" are closely related but not synonymous. For each "command" at least two communications would probably take place (at least one message to each aircraft). Also, the relationship between positive and negative commands is not established here. Each command here is a positive command (e.g., "turn left").

6. ANALYSIS OF CASE STUDIES

The primary output of the Surveillance Accuracy Update Rate Study is embodied in the SAUR/Communication Rate (SAUR/CR) analyses of the ten specific cases selected for this purpose and described in Section 3.1. From these case studies it is possible to establish quantitative relationships between the surveillance accuracies, the update rates, and the communications loads associated with the tactical control of aircraft for conflict resolution for a wide range of circumstances covering various types of aircraft, phases of flight, and types of airspace. The explicit results of the ten cases are reported in Section 6.1. Special cases are discussed in Section 6.2, and comparisons with other analyses are made in Section 6.3.

6.1 INDIVIDUAL CASE STUDY RESULTS

Each of the cases listed in Section 3.6 was examined in detail, using the SAUR and CR analysis tools described in the previous sections. Cases 1 through 10 are discussed in detail in Section 6.1.1 and Cases 2 through 10 are discussed briefly in Sections 6.1.2 through 6.1.10. Section 6.1.11 summarizes the results of the ten case studies and contains a discussion of the implications of these results.

6.1.1 Case 1 Analysis

This case represents a possible collision situation between an air carrier descending toward an airport, which encounters a light general aviation aircraft in a level turn (Figure 10). Assuming that this situation would probably occur in relatively busy airspace (e. g., an approach into Friendship Airport, Baltimore, Maryland), the CR analysis was based on a high aircraft density, resulting in a high communications loading. The SAUR and CR curves are shown overlaid in Figure 11, together with identification of six specific points labeled A through F:

- Point A is representative of an ATC system today wherein radar provides accuracies on the order of a mile and the ATC system as indicated in Section 3 involves a typical reaction time of 25 seconds. This point will be a starting point in the Surveillance Accuracy/Update Interval selection process; and various design change strategies will be employed to determine a recommended air traffic control system capability.

47

B737, DESCENDING THROUGH
5500 FEET AT 500 FT/MIN,
250 KNOTS, MAINTAINING
A TRUE COURSE OF 154°

NORTH

NON-COLLISION COURSE

ORIGINAL
COLLISION
COURSES

WEST ——— EAST

COLLISION
POINTS

C172, FLYING AT A CONSTANT
ALTITUDE OF 5000 FEET,
MAINTINAING 100 KNOTS, IN
A 1°/SEC RIGHT TURN, PASS-
ING THROUGH A TRUE COURSE
OF 300°

SOUTH
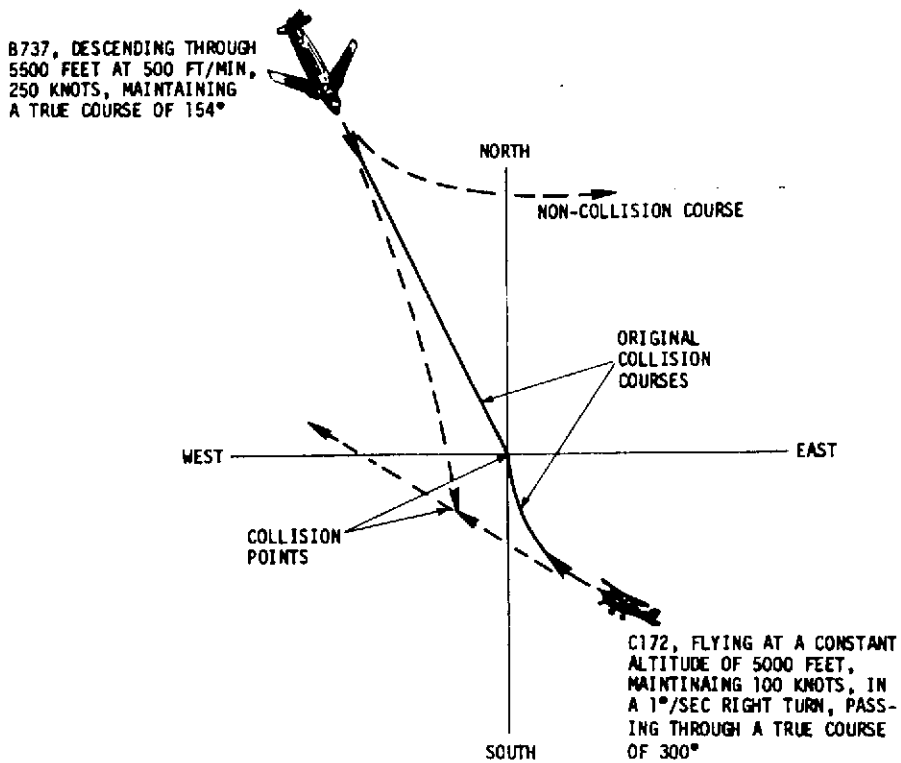
Figure 10.   Case 1 Encounter
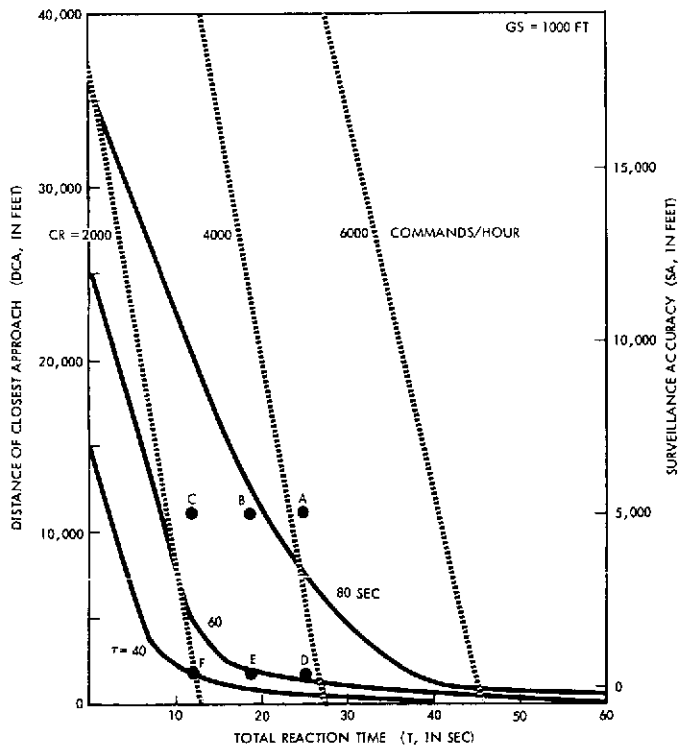
Figure 11.   Case 1 SAUR/CR Curves

48

- Point B represents a decrease in total system reaction time which could be obtained by decreasing the update interval from 10 to 4 seconds or by decreasing pilot, controller, and communication delays a similar amount.

- Point C represents a larger decrease in total system reaction time, implying both shorter update interval and reduced pilot, controller, and/or communications delays.

- Points D, E, and F represent accuracies typical of those which could be provided by a multilateration system such as LIT and with the same total system reaction times as discussed in Points A, B, and C.

These points are shown as examples of surveillance accuracy/reaction time combinations. The selection process did not limit the choices to one of these six points, but they were found to be helpful in comparing design alternatives of strategies.

The SAUR reasoning and selection process is outlined in the first two columns of Table 6. From Figure 11 the appropriate values of decision time and the communications rate for Point A are extracted and logged in Block 1 for visibility. The minimum strategy (Block 2) is a reflection of the tactical aspects of the SAUR problem. In this case, since tau is already below 90 seconds, this block is not applicable. The first potential strategy (Block 3a) is investigated in order to determine the impact of reducing T on the communications load. In this case, if the reaction time is reduced to 19 or 12 seconds, the command rate drops from 4200 to 3200 or 2400 commands per hour. These CR drops were considered significant. Clearly, the decision as to what CR reduction is "significant" is a subjective one. In those cases where a reduction in T (or in SA) could reduce the communications rate by both a large percentage and an absolute value of over a thousand commands per hour, the CR drop was considered significant. If the CR drops was, say, 10 or 15%, or was small in absolute terms (e. g., from 22 commands per hour to 11 commands per hour), then this was not considered a significant CR drop. Ultimately, of course, one would prefer to do a direct cost trade of the communications and surveillance subsystems, a necessary step in the preliminary design stage. The present approach, however, does provide an indication of those cases which possess the

Table 6. Surveillance Accuracy/Update Interval Selection

RULE	REASONING	APPLICATION TO CASE 1
1. Starting Point (Point A) SAR=5000 ft & T = 25 sec	This is representative of the enroute ATC (radar/voice) systems in use today.	τ = 87 CR = 4200
2. Minimum Strategy If τ at the starting point is greater than 90 sec, it will be necessary to use one or more of the strategies below to get down to 90 sec or below. Pick lowest τ if all are above 90 sec.	If τ is very large, the case in point requires better performance than radar provided in order to provide tactical control for collision avoidance ("Tactical/Encounter Control")	N/A
3. Potential Strategies a) Go to semi-automation and/or reduced update interval (Points B, C) only if 1) CR drops significantly; 2) it is necessary to bring τ down to 90 sec.	Data link and/or reduced update interval will only be more cost effective than voice under high CR conditions or unless it is necessary for tactical/encounter control.	Going to Point B (or C) drops CR to 3200 (or 2400). The CR change is probably significant.
b) Drop down from starting point to lower SAR(e. g., Point D) only if 1) CR drops significantly; or 2) it is necessary to bring τ down to 90 sec.	Presumably more accuracy will require a more expensive surveillance system. CR must drop substantially in order to save communications system costs	Dropping from Point A to Point D causes CR to drop to 3,800. The CR change is probably not significant.
c) Allow larger values of SAR if possible, subject to τ equal to or less than 90 sec.	Same as 3b	N/A
d) A combination of strategies is allowed (e. g., Points E and F).	The logic behind the moves is not mutually exclusive.	At Point F CR = 2,000. The CR change is probably significant but not much more than at Point C.
4. Product: A preferred or indicated set of values of SAR/T/τ, which is equivalent to SAR/UI the desired product.		SAR = 5,000 ft T = 12 sec UI = 1 sec τ = 66 sec CR = 2,400 comm/hr

greatest potential for payoff, and will provide the communications rate versus accuracy/update interval relationships required when the quantitative cost comparisons are made.

The potential strategies of Blocks 3b, 3c, and 3d likewise are measuring the impact of more stringent or more relaxed SU/UI/T design parameters or any combination thereof. In this case it is seen that the additional CR reduction which could be achieved by going to Points D, E, and F, are much smaller than the initial drop obtained by reducing T. Block 4 records the values of SAR, T, UI, tau, and CR which result from this thought process; and Block 5 shows the indicated ATC system implications. Summarizing this case, it appears that the communication load (and also the false alarm rate) can be dropped significantly by reduction in total system reaction time from 25 to 12 seconds. On the other hand, a reduction in position determination uncertainty at any particular value of T does not appear to appreciably reduce the communications load or false alarm rate. Thus, the indicated surveillance accuracy requirement is 5000 feet; the update interval is 1 second; and the resulting communications load is 2400 commands per hour.

6.1.2 Case 2

It was not clear at the outset of the study whether or not minor variations in a particular scenario would cause fairly large changes in the surveillance accuracy, update rate, and communications requirements. Therefore, Case 2 represents a fairly mild departure from Case 1 and involves a number of similar sub-cases.

6.1.2.1 Case 2A

Description

This case concerns a general aviation piston aircraft (Cessna 172) in a collision situation with a commercial jet (Boeing 737). The situation, as indicated below, is very similar to Case 1 except that both aircraft are' in stright and level flight.

AC#2 (Boeing 737)
      154°
      5,000 ft
      250 Kt
      Level

AC#1 (Cessna 172)
      334°
      5,000 ft
      100 kt
      Level
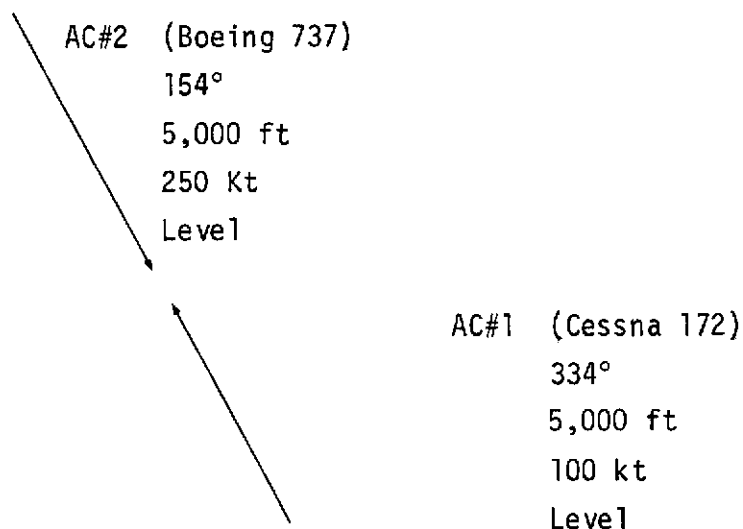
Figure 12. Case 2A Encounter

Discussion

The results of Case 2A are virtually identical to Case 1. The command rate curves do not change at all since only minor changes in the aircraft flight paths are made. The rationale for selection of Point C was again the same, namely a significant reduction in communication load with a reduction in system reaction time, but not so much improvement by going to a high accuracy surveillance system.

COMMANDS/HOUR
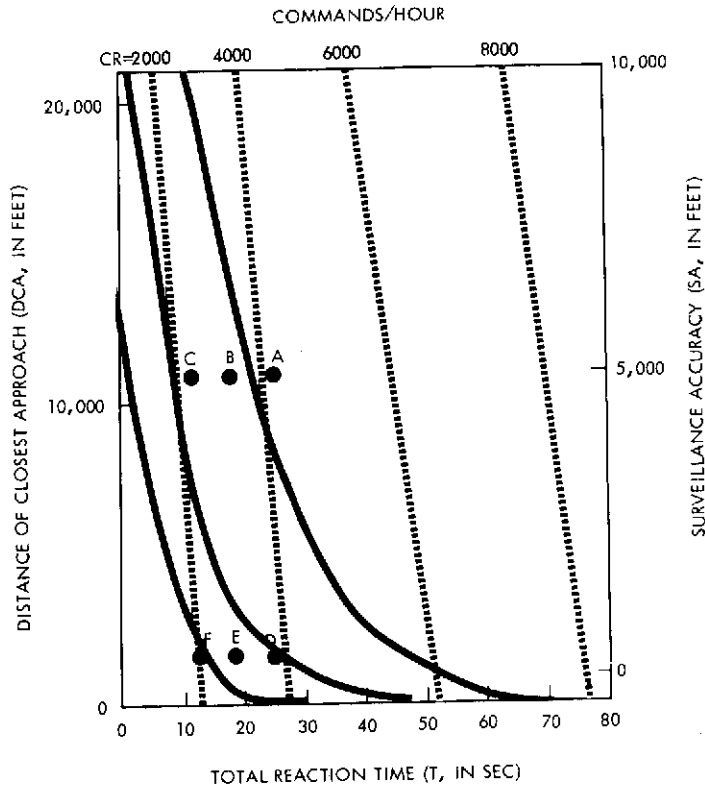
Figure 13. Case 2A SAUR/CR Curves

Table 7. Case 2A SAUR Selection

RULE	APPLICATION
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ = 90 sec CR = 4,200 comm/hr
2. Minimum Strategy Reduce τ below 90 seconds	N/A
3. Potential Strategies a) Semiautomation and/or reduced update interval (Points B, C)	Going to Point B (or C) drops CR to 3200 (or 2400). The CR change is probably significant.
b) Better accuracy	Going to Point D drops CR to 3800. CR change is probably no significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e. g., Points E and F)	At Point F CR=2000. The CR change is probably significant, but not much more than at Point C.
4. Product: A preferred or indicated set of design	SAR=5000 ft) Point C $\quad \tau$ = 65 sec T =12 sec) $\quad\quad$ CR = 2,400 comm/hr UI =1 sec

53

Case 6.1.2.2 <u>Case 2B</u>

<u>Description</u>

This case is identical to Case 2A except that the two aircraft flight paths meet at right angles.

AC#2 (Boeing 737)

154°

5,000 ft

250 kt

Level

AC#1 (Cessna 172)

064°

5,000 ft.

100 kt

Level
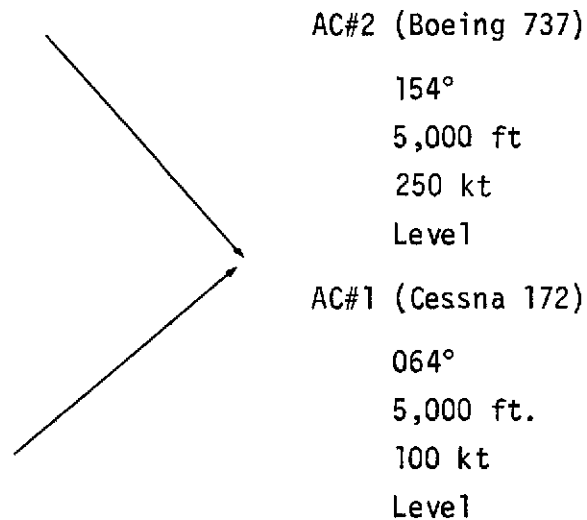
Figure 14.   Case 2B Encounter

<u>Discussion</u>

The right angle case is slightly more difficult than the head-on case from the air traffic controller's point of view. At any given point in time the two aircraft are simply closer together. This causes the tau curves to "droop" somewhat. Nevertheless, the preferred design point is again Point C for the same reasons as before.
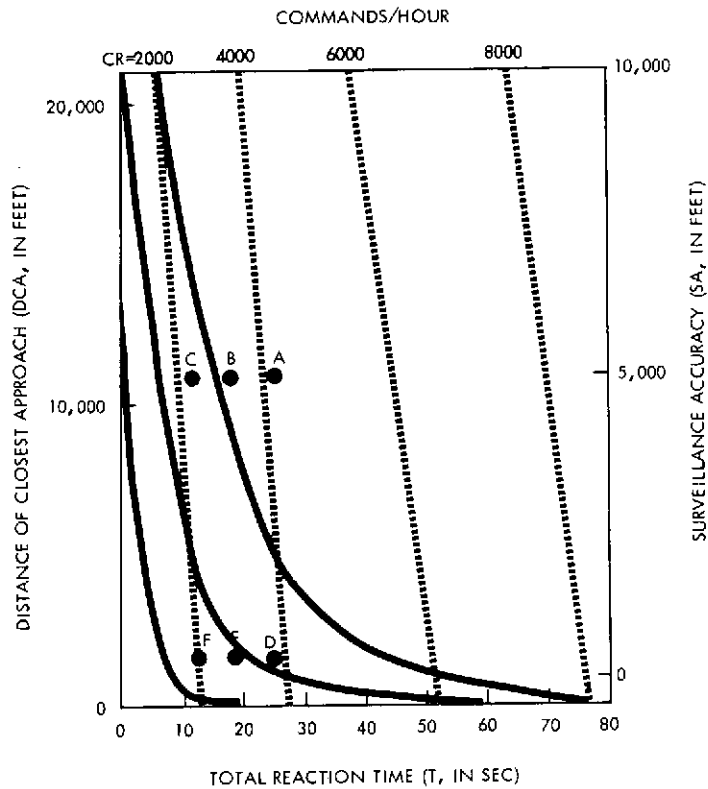
Figure 15. Case 2B SAUR/CR Curves

Table 8. Case 2B SAUR Selection

RULE	APPLICATION
1. <u>Starting Point</u> (Point A) <u>SAR=5000 ft and T=25 sec</u>	τ = > 100 sec CR = 4,200 comm/hr
2. <u>Minimum Strategy:</u> <u>Reduce</u> τ below 90 sec.	Must reduce SA to 2600 to T to 19 sec or an equivalent combination.
3. <u>Potential Strategies:</u> a) Semiautomation and/or reduced update interval (Points B,C)	Going to Point B (or C) drops CR to 3200 (or 2400). The CR change is probably significant.
b) Better accuracy	At Point D CR drops to 3800. The CR change is probably not significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e. g., Points E and F).	At Point F CR=2000. The CR change is probably significant but not much more than Point C.
4. <u>Product:</u> A preferred or indicated <u>set of design values.</u>	SAR=5000 ft) T =12 sec)Point C τ =72 sec UI = 1 sec) CR =2,400 comm/ hr

6.1.2.3 Case 2C

Description

This case is identical to Case 2A except that the two aircraft are in a tail-chase.

AC#2 (Boeing 737)

    154°

    5,000 ft.

    250 kt

    Level

AC#1 (Cessna 172)

    154°

    5,000 ft.

    100 kt

    Level
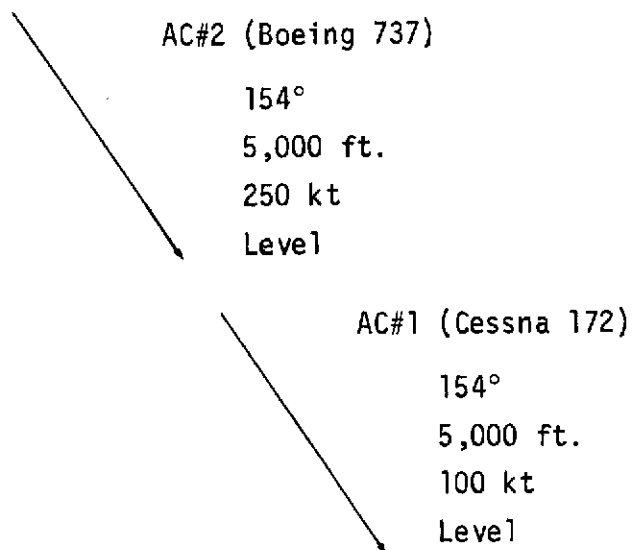
Figure 16. Case 2C Encounter

Discussion

Here for the first time the minimum strategy rule had to be invoked. Point G brings tau down to about 85 seconds and it cuts CR in half at 2100. Points H and F, however, do not improve CR significantly; so Point G was selected as the preferred design point.
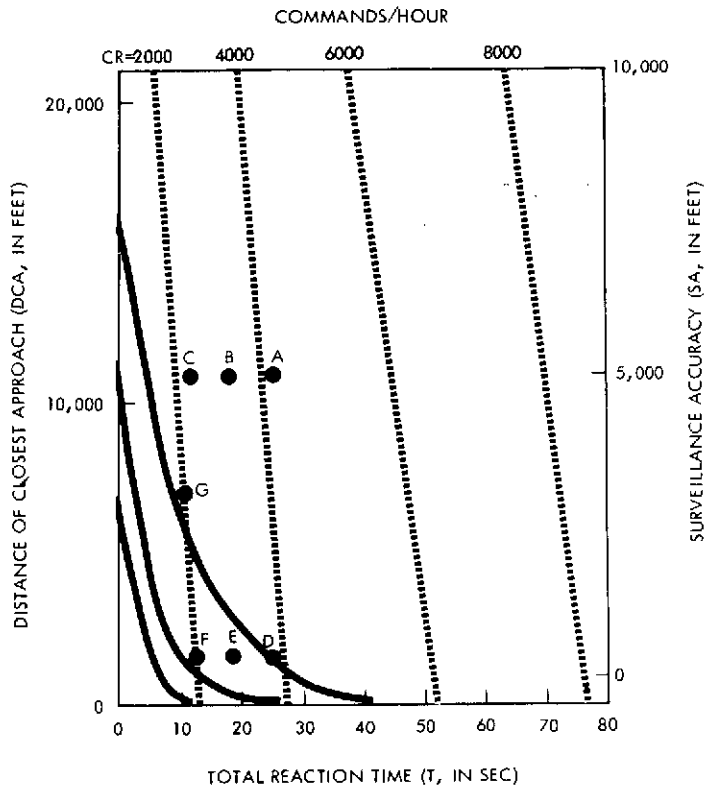
Figure 17.   Case 2C SAUR/CR Curves

Table 9.   Case 2C SAUR Selection

RULE	APPLICATION
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ = > 100 sec CR = 4,200 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	Must reduce SA to 3000 and T to 15 sec, or SA to 2000 and T to 22 sec, or an equivalent combination.
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B, C)	At SA=3000, if T drops to 12 (Point G), drops to 85 and CR drops to 2100. The CR cahgne is probably significant. At SA-1000 and T=12 (Point H), CR=2000. The change is probably significant, but not a significant improvement over Point G.
b) Better accuracy	Dropping from Point G to Point D causes significant CR increase.
c) Reduced accuracy	N/A
d) Combination of strategies Points E and F).	At Point F CR=2000. The CR change is probably significant but not more than Point G.
4. Product: A preferred or indi-cated set of design values.	SAR=3000 ft) T =12 sec)Point C τ =85 sec UI =1 sec CR=2,100 comm/hr

6.1.2.4 Case 2D

Description

This case is identical to Case 2A except that both aircraft are Boeing 737's.

AC#2 (Boeing 737)

154°

5,000 ft.

250 kt

Level

AC#1 (Boeing 737)

334°

5,000 ft.

250 kt

Level
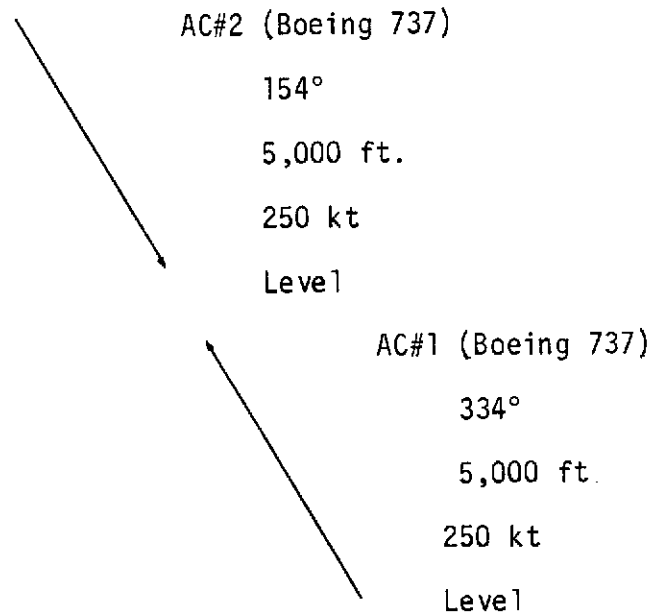
Figure 18. Case 2D Encounter

Discussion

The rationale for selection here is very similar to previous sub-cases, and Point C was again selected. While it is not the intent of this study to stress hardware implementation, it is worth pointing out that, in terms of implementation, Point G is representative of a terminal radar/voice system used today. Although the CR reduction going from Point A to Point G is only half that associated with going to Point C, it might be preferable from a total cost point of view to retain terminal radar in those areas where it already provides an adequate capability.

Figure 19.   Case 2D SAUR/CR Curves

Table 10.   Case 2D SAUR Selection

RULE	APPLICATION
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ = 74 sec CR = 4,200 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	N/A
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B,C)	Going to Point B (or C) drops CR to 3200 (or 2400). The CR change is probably significant
b) Better accuracy	Going to Point D drops CR to 3800. The CR change is probably not significant.
c) Reduced accuracy	At T=25, if τ is increased to 90, SA is increased to 10,300, and CR goes up to 4,600.
d) Combination of strategies (e.g., Points E and F).	At Point F CR-1900. The CR change is probably significant but not much more than Point C.
4. Product: A preferred or indicated set of design values.	SAR=5000 ft) Point C τ =58 sec T =12 sec) UI =1 sec CR=2,400 comm/hr

59

6.1.2.5 **Case 2E**

Description

This case is identical to Case 2A except that both aircraft are Cessna 172's.

AC#2 (Cessna 172)

154°

5,000 ft.

100 kt

Level

AC#1 (Cessna 172)

334°

5,000 ft.

100 kt

Level
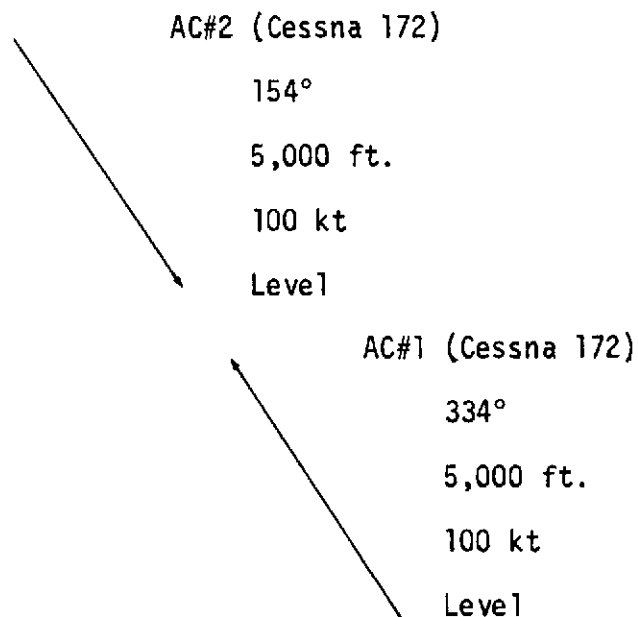
Figure 20. Case 2E Encounter

Discussion

The results here remain essentially identical to the previous cases. Since the aircraft in this case are flying much slower, the selected design Point C involves a significantly longer decision time.

Figure 21. Case 2E SAUR/CR Curves

Table 11. Case 2E SAUR Selection

RULE	APPLICATION
1. <u>Starting Point</u> (Point A) <u>SAR=5000 ft and T=25 sec</u>	τ = 100 sec CR = 4,200 comm/hr
2. <u>Minimum Strategy</u>: Reduce τ below <u>90 sec.</u>	Must reduce SA to 5000 and T to 12 sec or SA to 2500 and T to 25 sec, or an equivalent combination.
3. <u>Potential Strategies:</u> a) <u>Semiautomation</u> and/or reduced update interval (Points B,C)	Going to Point B (or C) drops CR to 3200 (or 2400). The CR change is probably significant.
b) Better accuracy	Going to Point D drops CR to 3800. The CR change is probably not significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e. g., Points E and F)	At Point F CR=2400. The CR change is probably significant but not much more than Point C.
4. <u>Product</u>: A preferred or indi- <u>cated</u> set of design values.	SAR=5000 ft $\big)$ T =12 sec $\big)$ Point C $\quad\tau$ =90 sec UI =1 sec $\qquad$ CR=2,400 comm/hr

61

6.1.2.6 Case 2F

Description

This case is identical to Case 2A except that the Cessna 172 is displaced 1000 feet north of a collision course.

AC#2 (Boeing 737)

154°

5,000 feet

250 knots

Level

AC#1 (Cessna 172)

334°

5,000 feet

100 knots

Level
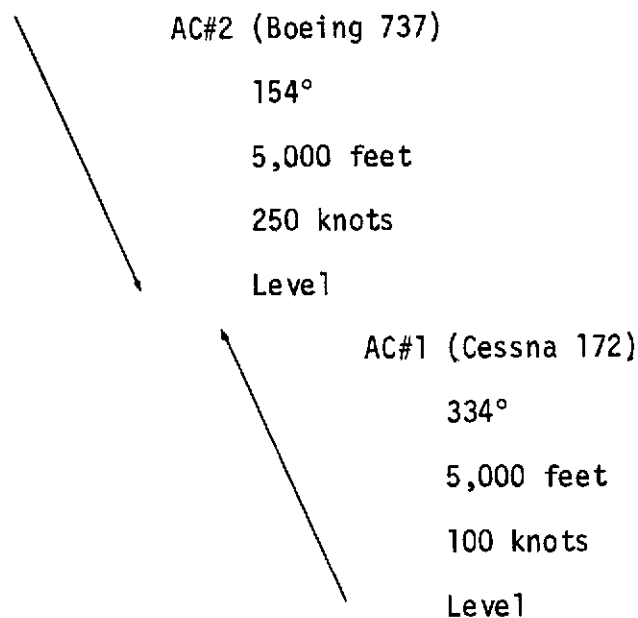
Figure 22. Case 2F Encounter

Discussion

This small perturbation on Case 2A caused virtually no change from the previous results. See Section 6.2.5, however, for a more complete discussion of the topic of offset flight paths as related to possible heading errors. The results here are considered to be slightly non-conservative.

COMMANDS/HOUR

DISTANCE OF CLOSEST APPROACH (DCA, IN FEET)

SURVEILLANCE ACCURACY (SA, IN FEET)

TOTAL REACTION TIME (T, IN SEC)
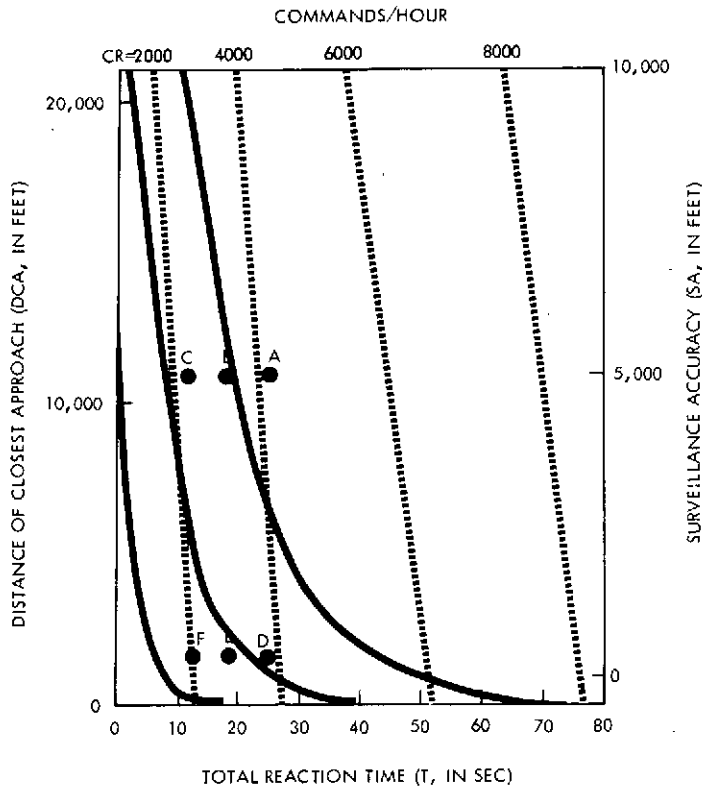
Figure 23. Case 2F SAUR/CR Curves

Table 12. Case 2F SAUR Selection

RULE	APPLICATION
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ = 82 sec CR=4200 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	N/A
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B,C)	Going to Point B (or C) drops CR to 3200 (or 2400). The CR change is probably significant.
b) Better accuracy	Going to Point D drops CR to 3800. The CR change is probably not significant.
c) Reduced accuracy	Starting point near τ =90. No significant change.
d) Combination of strategies (e. g., Points E and F).	At Point F CR drops to 1900. CR change probably significant but not much more than Point C.
4. Product: A preferred or indicated set of design values.	SAR=5000 ft T =12 sec UI =1 sec } Point C τ =75 sec CR=2,400 comm/hr

63

6.1.2.7 Case 2G

Description

This case is identical to Case 2A except that the Cessna 172 is in level flight at an altitude of 4,000 feet, which is 1000 feet below a collision course.

AC#2 (Boeing 737)

154°

5,000 feet

250 knots

Level

AC#1 (Cessna 172)

334°

4,000 feet

100 knots

Level
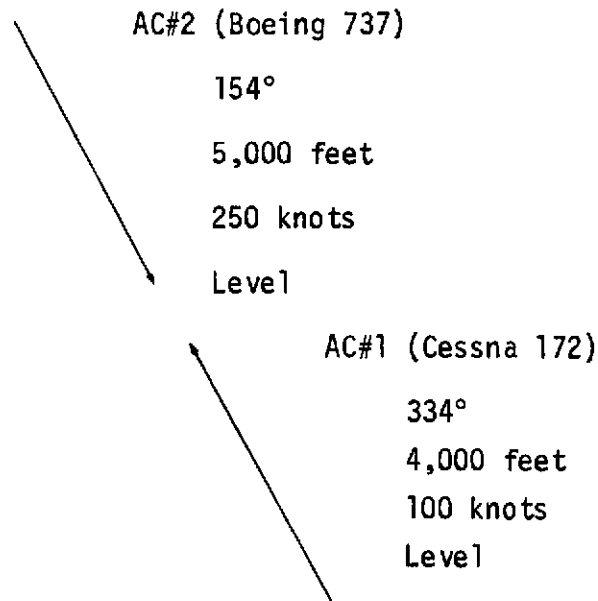
Figure 24. Case 2G Encounter

Discussion

This small perturbation on Case 2A did not change the SAUR/CR results at all.[*] Obviously, if these two aircraft were flying under instrument flight rules at these assigned altitudes, no intervention would be required at all. The presumptions would be made that 1) the instrumentation in both aircraft would be operating correctly; 2) the pilots would be flying at or very close to their assigned altitudes; and 3) that neither would initiate a vertical maneuver to cause a potential collision situation. However, in the event that either aircraft were on a VFR/IPC clearance (or if the controller had any other reason to believe that one of the foregoing conditions might not be met), he would have to make a decision to intervene just about a full minute prior to their point of close approach and would require a short update interval and short system reaction time.

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[*] See Section 6.2.5.

64

COMMANDS/HOUR
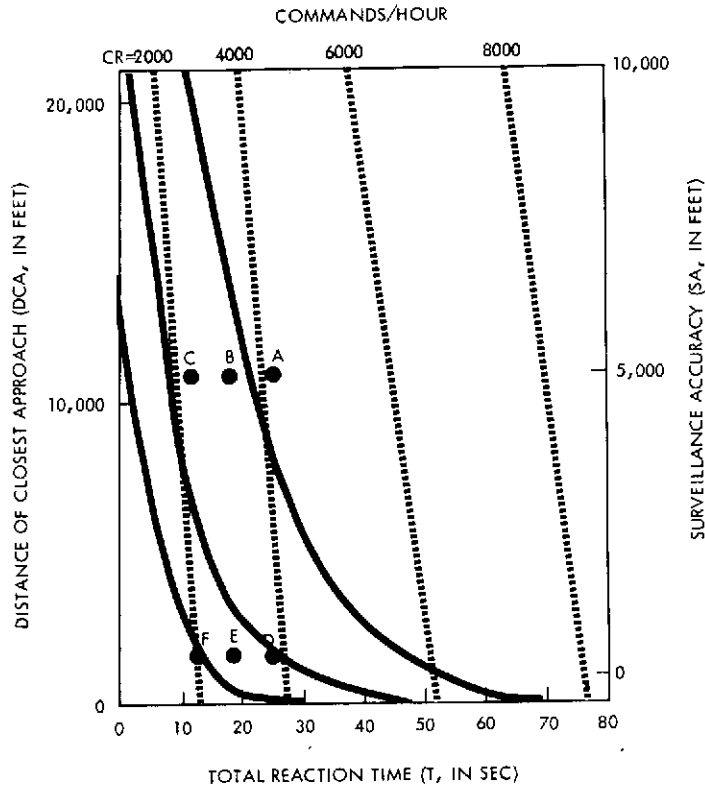
Figure 25. Case 2G SAUR/CR Curves

Table 13. Case 2G SAUR Selection

RULE	APPLICATION
1. <u>Starting Point</u> (Point A) SAR=5000 ft and T=25 sec	τ = 84 SEC CR= 4200 comm/hr
2. <u>Minimum Strategy</u>: Reduce τ below 90 sec.	N/A
3. <u>Potential Strategies:</u> a) Semiautomation and/or reduced update interval (Points B,C)	Point to Point B (or C) drops CR to 3200 (or 2200). The CR change is probably significant.
b) Better accuracy	At Point D CR drops to 3800. The CR change is probably not significant.
c) Reduced accuracy	Starting point near τ =90. No significant change.
d) Combination of strategies (e. g., Points E and F).	At Point F CR drops to 2000. CR change is probably significant but not more than Point C.
4. <u>Product</u>: A preferred or <u>indicated set of design values.</u>	SAR=5000 ft) Point C τ = 63 sec T =12 sec) CR= 2,200 comm/hr UI = 1 sec

6.1.3 Case 3

Description

This case concerns an executive jet (Cessna Citation) in a collision situation with a supersonic military jet (F-104). The Citation is in level cruise on a flight from Los Angeles to Las Vegas. The F-104 has just taken off from Palmdale and climbing at 24,000 feet/min toward the northwest.

```
                                060° HEADING
                                30,000 ft. ALT
                                340 kt
                                Level FLIGHT


AC#2   ──────────────────────►

(Cessna Citation)
                                    300° HEADING
                                    5,000 ft. ALT
                                    500 kt
                                    24,000 FPM CLIMB
                                    ── AC#1

                                      (F-104)
```
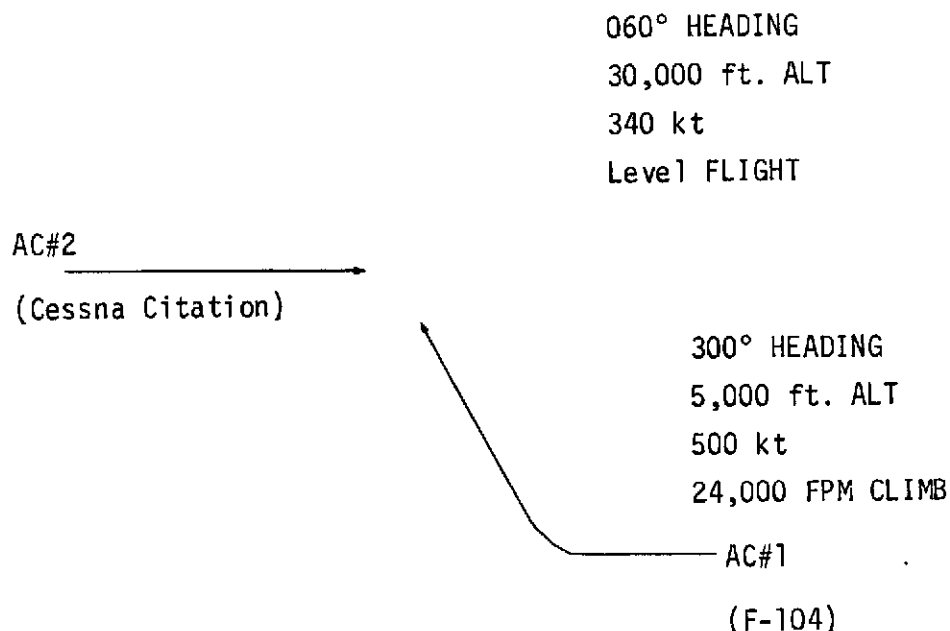
Figure 26. Case 3 Encounter

Discussion

The SAUR CR curves are noticably different in this case because the traffic density has been reduced. (The CR values are down by a factor of 3). Furthermore, the DCA's which the controller can generate are significantly larger. Thus Points A through F appear to have dropped lower with respect to the tau curves. In this case, there is again about a 50 percent CR drop in going from Point A to Point C, but the absolute values are smaller. Thus, it becomes difficult to pick a preferred point between A, B and C. As before, however, there is little to be gained in terms of CR drop by going from Point A to Point D. Conversely, there is not a great deal to lose in going from Point A to Point G. Point H is of interest because it represents today's terminal radar capability. Finally, Point B was selected as a design point since the drop in going from B to C was potentially less than that obtained in going from A to B and since the drop from B to H produced a negligible CR drop.
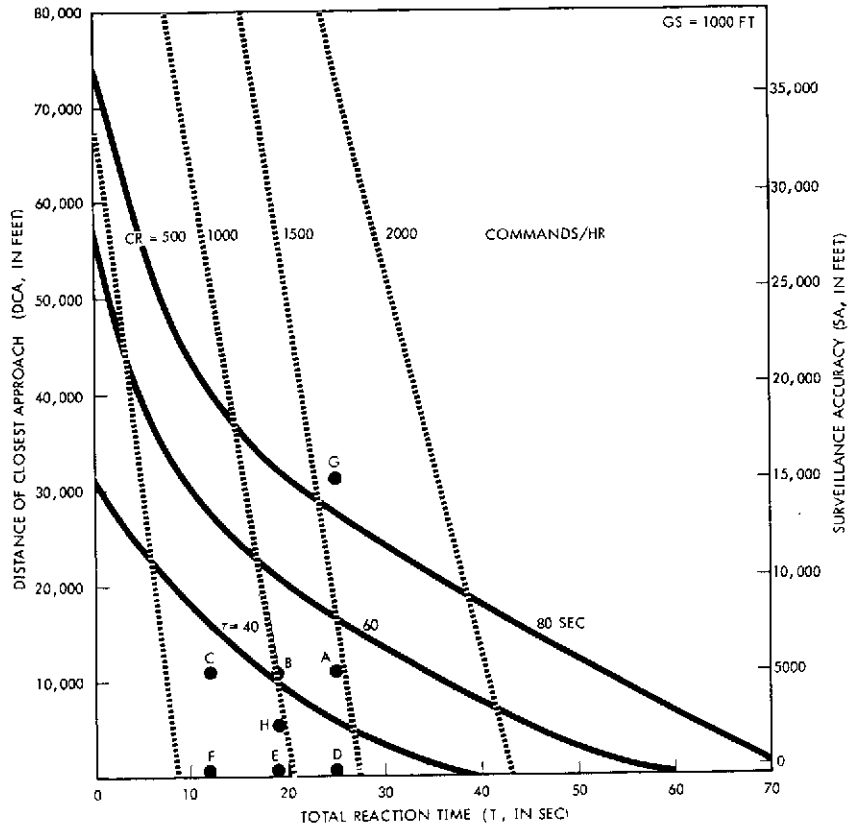
Figure 27. Case 3 SAUR/CR Curves

Table 14. Case 3 SAUR Selection

RULE	APPLICATION
1. <u>Starting Point</u> (Point A) SAR=5000 ft and T=25 sec	τ = 49 sec CR = 1,450 comm/hr
2. <u>Minimum Strategy</u>: Reduce τ below 90 sec.	N/A
3. <u>Potential Strategies:</u> a) Semiautomation and/or reduced update interval (Points B,C)	Going to Point B (or C) drops CR to 1000 (or 700). CR change may be significant.
b) Better accuracy	Going to Point D drops CR to 1,350. CR change not significant.
c) Reduced accuracy	If τ is increased to 90 (Point G), SA increases to 15,000, and CR goes up to 1,600.
d) Combination of strategies (e. g., Points E and F).	Going to Point F drops CR to 605. CR drop may be significant but not much more than Point C.
4. Product: A preferred or indicated set of design values.	SAR=5,000 ft) T =19 sec)Point B τ =34 sec UI =4 sec CR=1,000 comm/hr

67

6.1.4 Case 4

Description

This concerns two light general aviation aircraft (Cessna 172 and Evans Volksplane) in a VFR landing pattern encounter. The Cessna 172 is proceeding on the downwind leg and the Evans Volksplane is entering downwind at an angle of 45°.

AC#1 (Cessna 172)
090°
1000 ft.
70 kt
Level

AC#2 (Evans Volksplane)
045°
1000 ft.
70 kt
Level
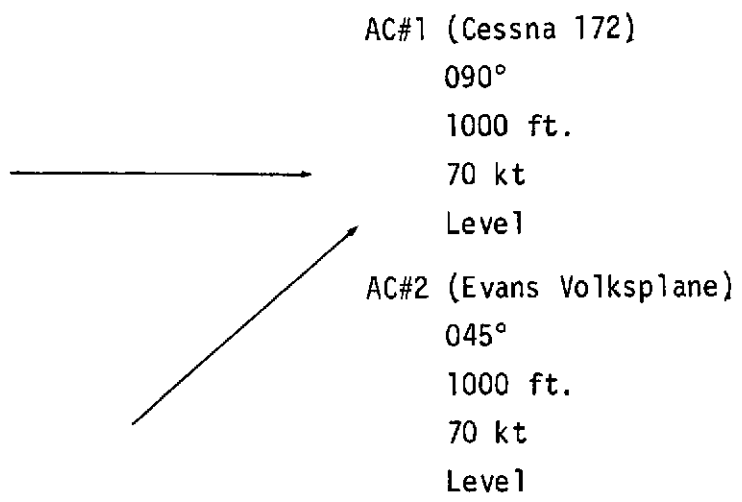
Figure 28. Case 4 Encounter

Discussion

Figure 29 reveals immediately that separation cannot be guaranteed according to the SAUR ground rules. Reducing the guaranteed separation distance to 500 feet changes the required surveillance accuracies as indicated in the lower right-hand corner of Figure 29. (See Section 6.2.4 for an explanation.) Even with this reduced guaranteed separation, a surveillance accuracy of 100 feet allows a total system reaction time of only 7 seconds. The implications seem clear. If traffic patterns are to be controlled in such a way as to guarantee the prevention of mid-air collisions, then the airspace must be much more organized and all users operating in a highly disciplined mode. Under these circumstances, shorter total system reaction times might be considered realistic. (It would be very beneficial to re-run case 4 with more restricted pilot-initiated maneuvers as inputs consistent with a postulated set of traffic pattern guide rules that would make the control at least partially strategic.) In addition to improved safety of flight, it also appears that several hundred commands per hour could be saved.
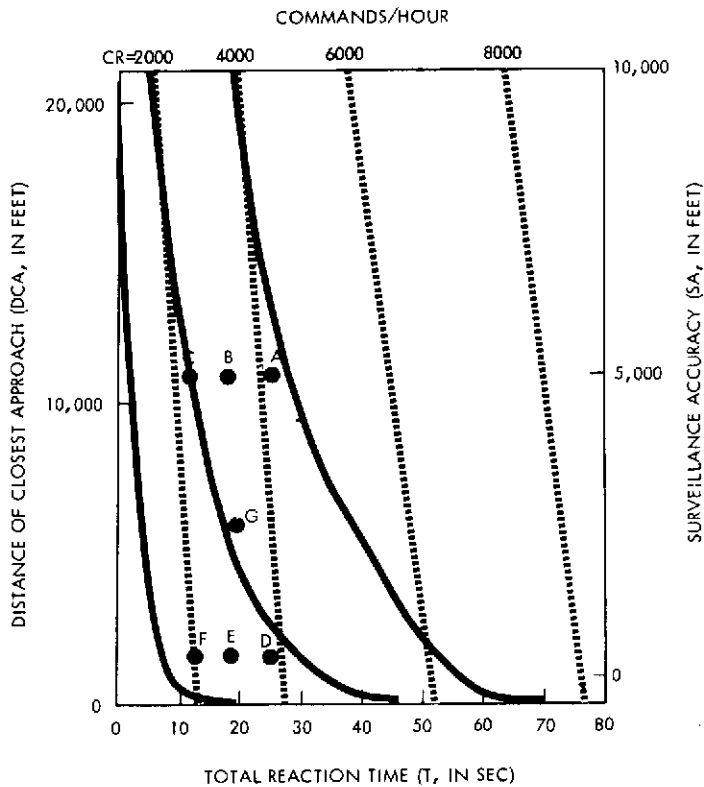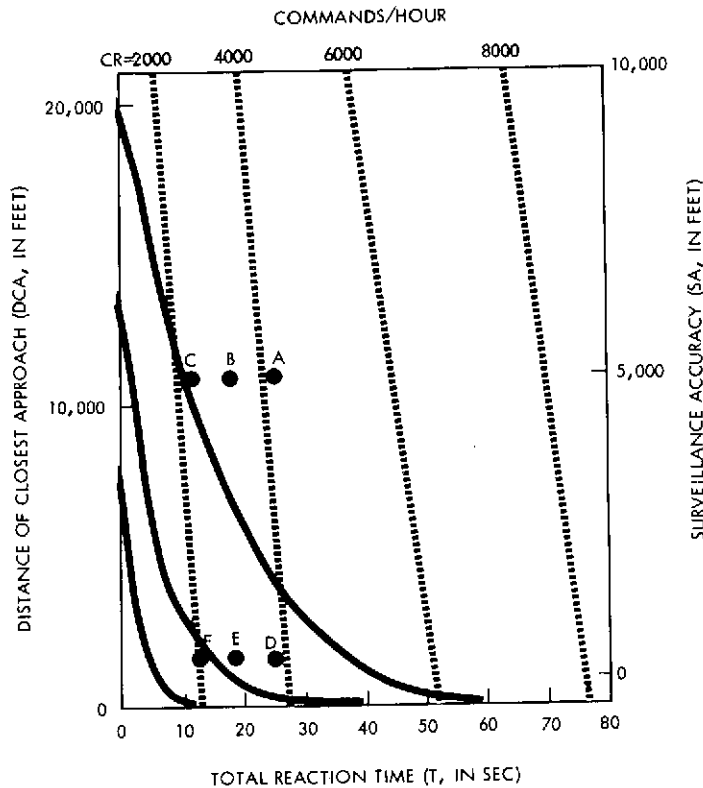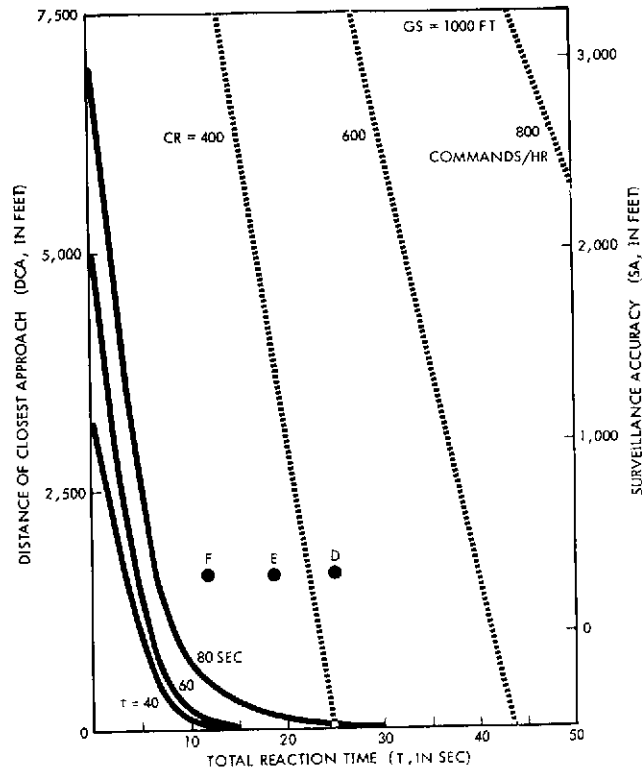
Figure 29. Case 4 SAUR/CR Curves

Table 15. Case 4 SAUR Selection

RULE	APPLICATION
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ = >>100 sec CR = 540 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	SA and T must be reduced to very small values in order to get τ = 90.
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B,C)	SA and T must be reduced to very small values in order to get τ = 90.
b) Better accuracy	N/A
c) Reduced accuracy	N/A
d) Combination of strategies (e.g., Points E and F).	Separation cannot be guaranteed according to SAUR ground rules.
4. Product: A preferred or indicated set of design values.	For GS=500 ft, SA=100 ft Will allow T=7 (see discussion)

69

6.1.5 Case 5

Cases 5A and 5B deal with possible problems associated with control of supersonic aircraft.

6.1.5.1 Case 5A

This case concerns a very high performance military jet (YF-12) on a collision course with a commercial jet (Boeing 747)

AC#2 (Boeing 747)

225°

43,000 ft

500 kt

Level

AC#1 (YF-12)

270°

43,000 ft

1200 kt

Level
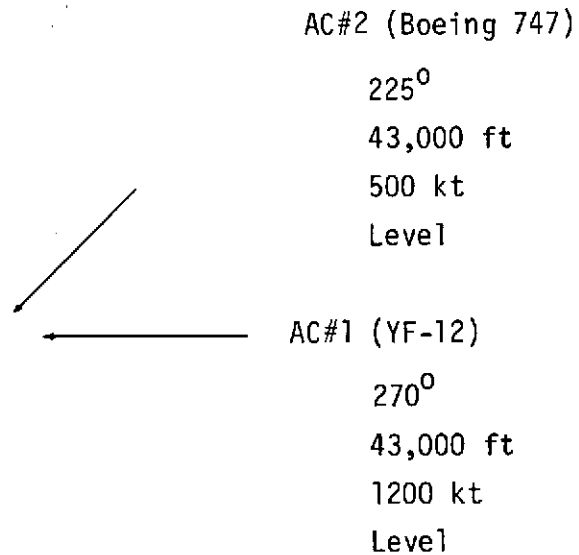
Figure 30. Case 5 Encounter

Discussion

The most significant change during this and previous cases is the very low command rate associated with this encounter at high altitudes and low aircraft densities. Also, despite the fact that this is a quartering tail chase, the controller can generate fairly large DCA's and therefore points A, B and C once again fall within the SAUR curves. Because of the low command rate, however, no combination of strategies will provide a significantreduction in command rate. Therefore, Point A is a preferred design point.
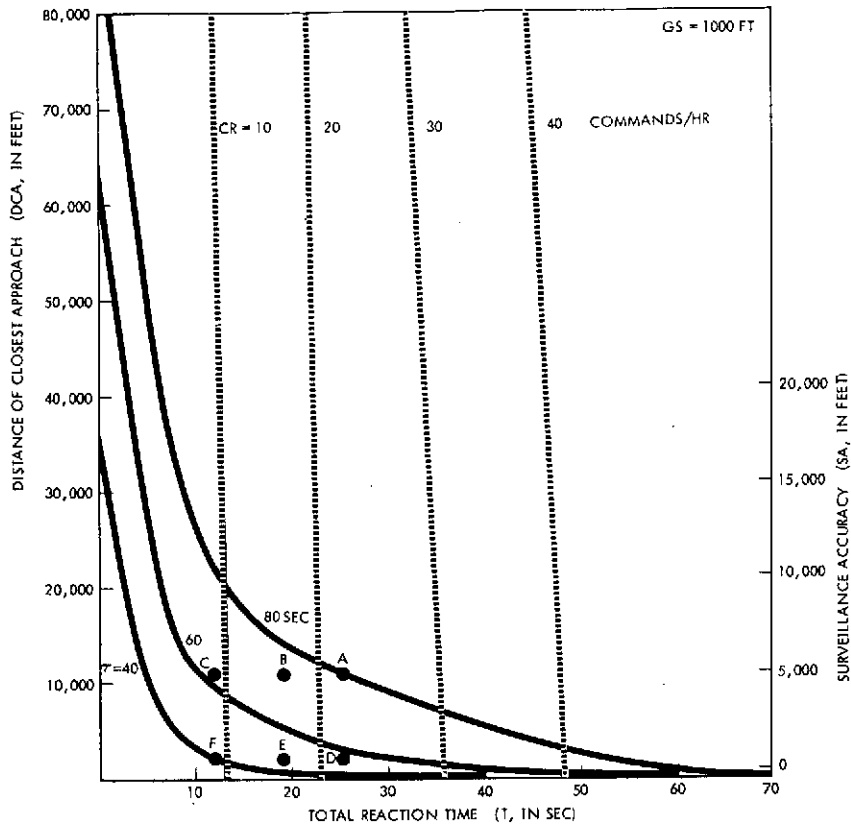
Figure 31. Case 5 SAUR/CR Curves

Table 16. Case 5A SAUR Selection

Rule	Application
1. **Starting Point** (Point A) SAR=5000 ft and T=25 sec	τ =80 sec CR=22 comm/hr
2. **Minimum Strategy**: Reduce τ below 90 sec.	N/A
3. **Potential Strategies**: a) Semiautomation and/or reduced update interval (Points B, C)	Going to Point C drops CR to 9. CR drop not significant.
b) Better accuracy	Going to Point D drops CR less than 1 comm/hr. CR change not significant.
c) Reduced accuracy	Starting point near τ = 90. No significant change.
d) Combination of strategies (e.g., Points E and F).	No combination of strategies will provide a significant change in command rate.
4. **Product**: A preferred or indicated set of design values.	SAR=5,000 ft) T =25 sec) Point A τ =80 sec UI =10 sec CR=22 comm/hr

71

6.1.5.2 Case 5B

Description

This case concerns two supersonic military aircraft (both YF-12's) in a collision situation with each other.

AC#2 (YF-12)
180°
50,000 ft.
1200 kt
Level

AC#1 (YF-12)
000°
50,000 ft.
1200 kt
Level
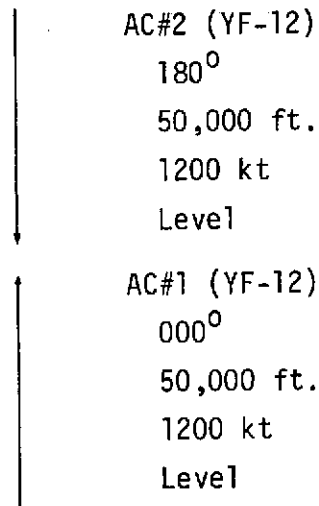
Figure 32. Case 5B Encounter

Discussion

At first glance it might seem that two aircraft approaching each other in a head-on collision situation at supersonic speeds might pose a difficult collision avoidance problem. But precisely the opposite results evolve from the SAUR formulation. With a decision time of 60 seconds, assuming a reaction time of 25 seconds, an air traffic controller can generate a distance of closest approach of about 30 miles. Because of the very low density of aircraft at the assumed altitude, even the very high speeds involved produce very modest command rates. Although not representative of any particular surveillance concept, Point G was arbitrarily selected as a design point to show that very relaxed accuracies up-date intervals in system reaction times are possible for this case. Finally, because of the very low density at that altitude and the very high speeds, the guaranteed separation was opened up to 10,000 feet, although the res-lts for required surveillance accuracy were virtually unaffected by this change.
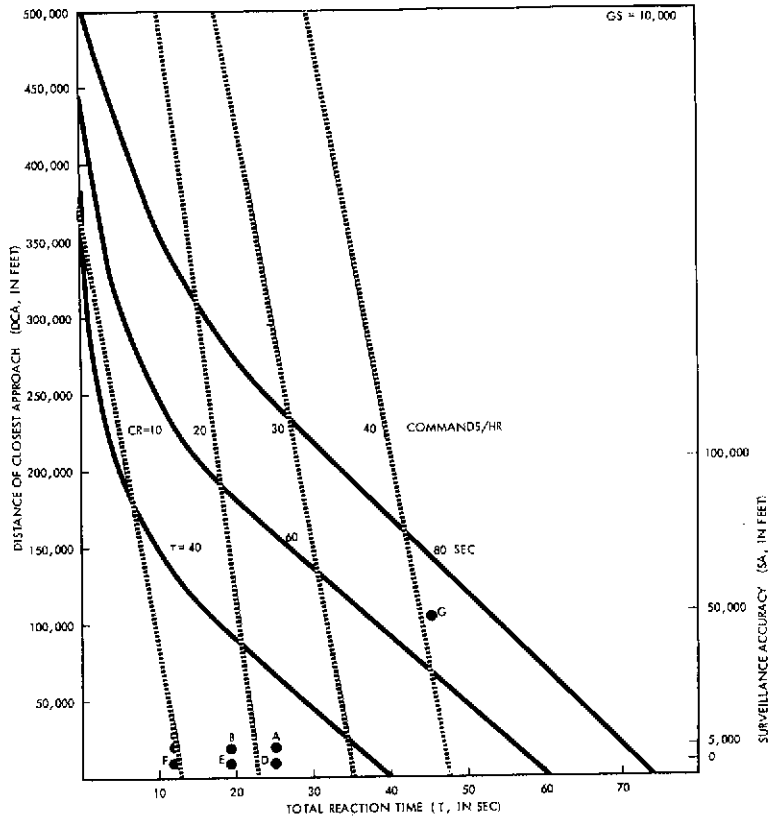
Figure 33. Case 5B SAUR/CR Curves

Table 17. Case 5B SAUR Selection

Rule	Application
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ =31 sec CR=23 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	N/A
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B, C)	Going from Point A to Point C drops CR to 10. CR change not significant.
b) Better accuracy	Going from Point A to Point drops CR to 22. CR change not significant.
c) Reduced accuracy	If allow SA to go to 10 miles, this allows τ to go to 70 sec and CR goes to 41. CR change not significant.
d) Combination of strategies (e.g., Points E and F).	No significant changes in CR for any combinations.
4. Product: A preferred or indicated set of design values.	SAR=50,000 ft) T =45 sec) Point G τ =70 sec UI =30 sec CR=41 comm/hr

6.1.6 Case 6

Case 6 involves three parallel track situations: the first two at a major airport and the third in controlled airways.

6.1.6.1 Cases 6A and 6B

Description

These cases deal with a parallel ILS approach situation involving two air carrier jets (both Boeing 737's). The separation distance between the ILS runways is 5,000 feet for Case 6A and 2,500 feet for Case 6B.

AC#2 (Boeing 737)
  270°
  2,000 ft.
  150 kt
  500 fpm

5,000 ft (Case 6A), 2,500 ft (Case 6B)

AC#1 (Boeing 737)
  270°
  2,000 ft.
  150 kt
  500 fpm
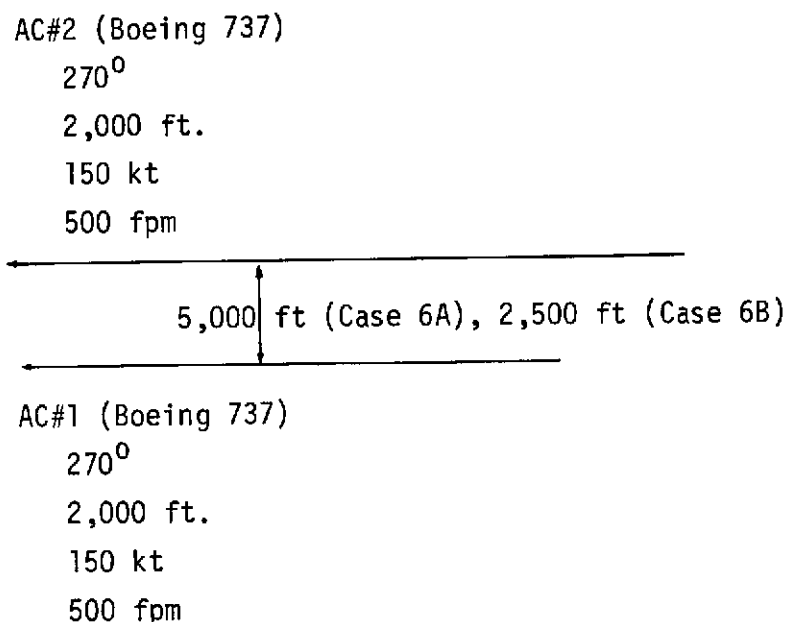
Figure 34. Case 6 Encounter

Discussion

As in Case 4, separation cannot be guaranteed according to the SAUR formulation. A surveillance accuracy of 100 feet with an update interval of one second and total system reaction time of 10.5 seconds (Point G) almost makes the SAUR formulation. And considering the high quality of pilots and controllers involved in all-weather instrument approaches to major airport runways, the reduction of T from 12 to 10.5 seconds can probably be tolerated. For Case 6B, however, the guaranteed separation must be reduced to 500 feet and the system reaction time to 6 seconds. This is probably a questionable design point. However, the SAUR analysis should probably be rerun with reduced or restricted pilot-initiated maneuvers indicating more strategic control.

74
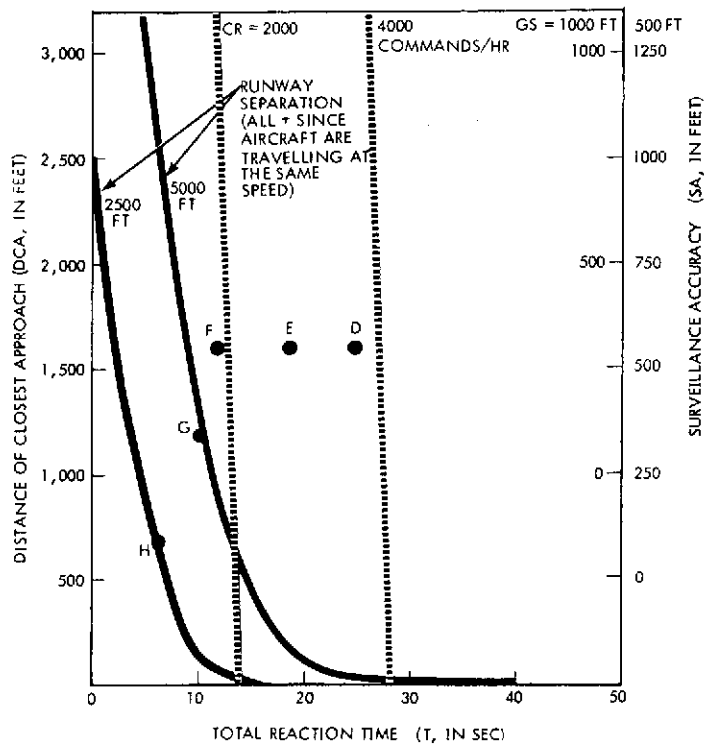
Figure 35. Case 6 SAUR/CR Curves

Table 18. Cases 6A and 6B SAUR Selection

Rule	Application
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ =N/A (see discussion) CR=Approx. 4,000 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	N/A
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B, C)	Using a SAR of 300 ft allows T of only 2 & 9 sec (cases A & B). Reduce T (and update interval) not practical. CR changes with T are fairly large but do not appear critical.
b) Better accuracy	Going to a SAR of 100 ft allows T of 4 & 10.5 sec (cases A and B).
c) Reduced accuracy	N/A
d) Combination of strategies (e.g., Points E and F).	Allow GS to go to 500 ft. Allow a T of 6 sec. Then SAR must be 100 ft.
4. Product: A preferred or indicated set of design values.	SAR=100 ft) T =10.5 sec) Point G UI =1 sec τ =N/A CASE 6A CR=1,000 comm/hr GS=1,000 ft
	SAR=100 ft) T =6 sec) Point H UI =1 sec τ =N/A CASE 6B CR=1,700 comm/hr GS=500 ft

75

6.1.6.3 Case 6C

Description

This case deals with two aircraft flying parallel tracks in any airway. Their lateral separation is 5 miles. AC #1 (Boeing 737) is overtaking the Citation.

AC#1 (Boeing 737)

090°

30,000 ft

500 kt

Level

5 n mi

AC#2 (Cessna Citation)

090°

30,000 ft

300 kt

Level
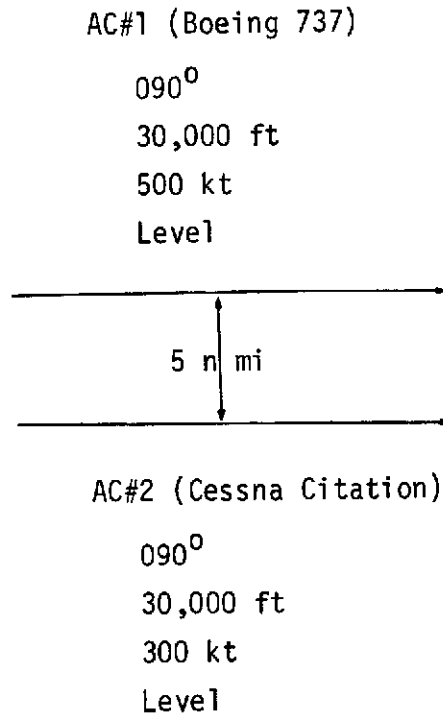
Figure 36. Case 6C Encounter

Discussion

The rationale and results of this parallel airway situation are very similar to those in Case 1. The one-mile accuracy and one-second update interval are again selected because shortening the reaction time greatly reduces the communication load and improving the accuracy has a small effect.
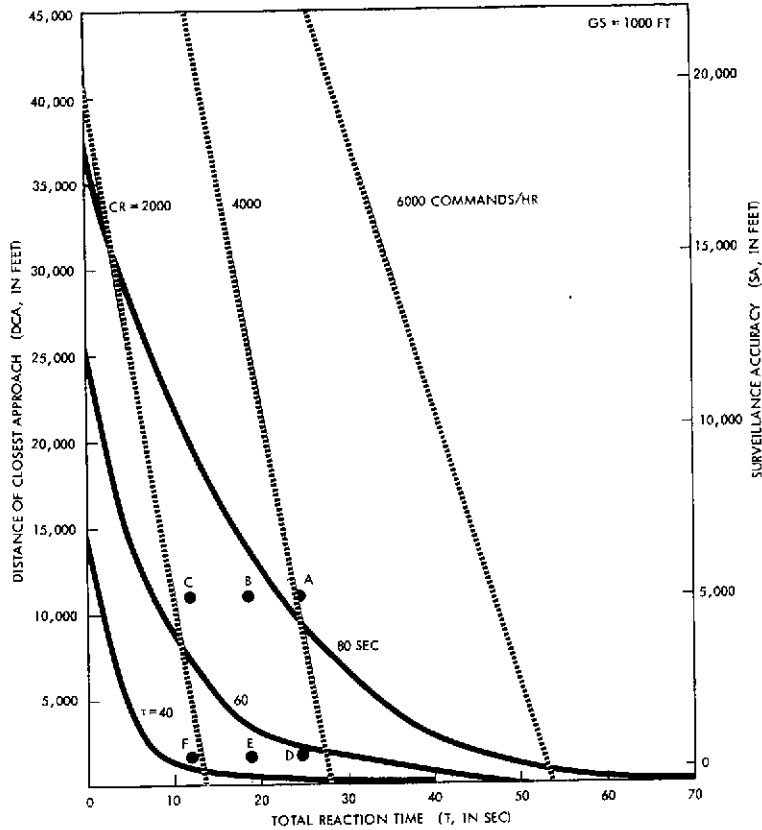
Figure 37. SAUR/CR Curves

Table 19. Case 6C SAUR Selection

Rule	Application
1. <u>Starting Point</u> (Point A) <u>SAR=5000</u> ft and T=25 sec	τ =25 sec CR=4,200 comm/hr
2. <u>Minimum Strategy</u>: Reduce τ below 90 sec	N/A
3. <u>Potential Strategies</u>: a) Semiautomation and/or reduced update interval (Points B, C)	Going from Point A to Point C drops CR to 2,300. Change in CR is significant.
b) Better accuracy	Going from Point A drops CR to 3,700. CR change not significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e.g., Points E and F).	Going to Point F, CR goes to 2,000. Change in CR over (3a) not significant.
4. <u>Product</u>: A preferred or indicated set of design values.	SAR=5,000 ft) T =12 sec) Point A τ =44 sec UI =1 sec CR=2,300 comm/hr

77

6.1.7 Case 7

Description

This case concerns a V-STOL aircraft (Twin Otter) and a helicopter (Bell UH-1) on a collision course in a terminal area.

AC#2 (Bell UH-1)

225°

3,000 ft

90 kt

Level

AC#1 (Twin Otter)

0000°

3,000 ft

150 kt

Level
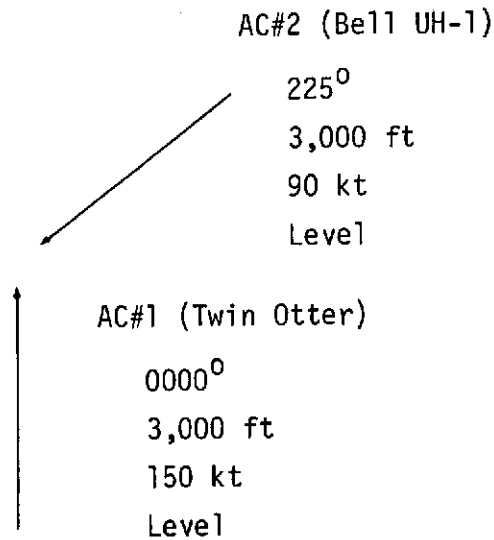
Figure 38. Case 7 Encounter

Discussion

This case is somewhat different in that the slope of the CR curves is less steep. The reduction in CR in going from Point A to Point D is still less than going from Point A to Point C, but not by as large a factor as in the earlier cases. Point G was selected because its CR penalty over Point C was not very large and in recognition of the fact that today's terminal radars do possess this capability.

78
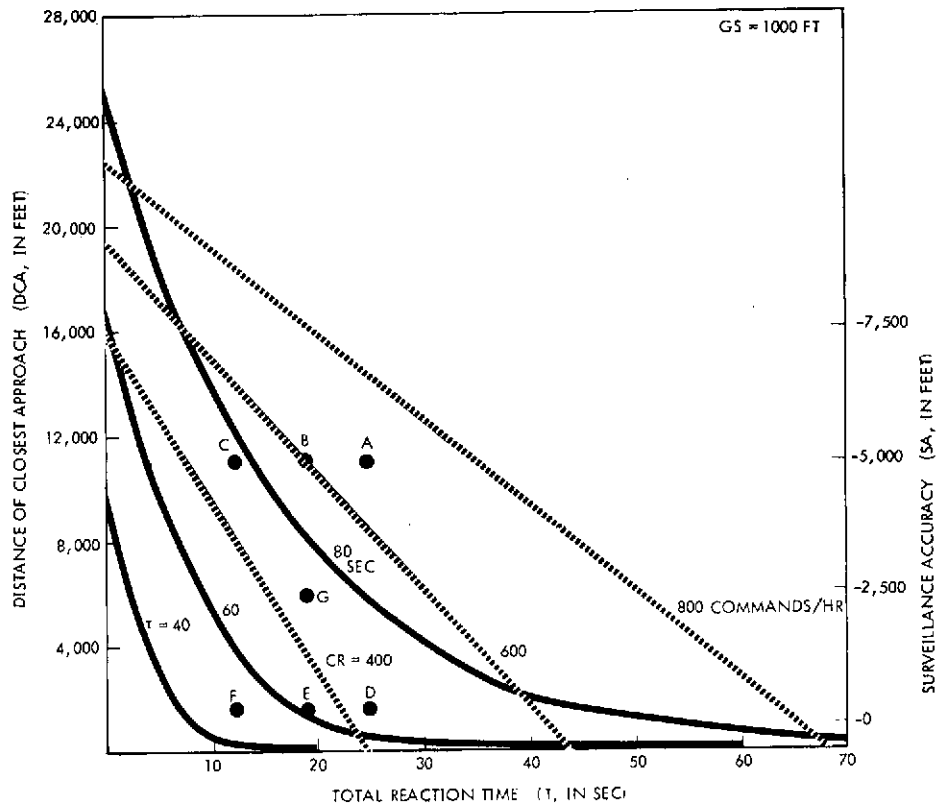
Figure 39. SAUR/CR Curves

Table 20. Case 7 SAUR Selection

Rule	Application
1. <u>Starting Point</u> (Point A) <u>SAR=5000 ft</u> and T=25 sec	τ =90 sec CR=530 comm/hr
2. <u>Minimum Strategy</u>: Reduce τ below <u>90 sec</u>.	Going from Point A to Points B, C, D, or E reduces τ to less than 90 sec.
3. <u>Potential Strategies</u>: a) <u>Semiautomation and/or reduced update interval</u> (Points B, C)	Going from Point A to Point D, CR drops to 350. CR change not significant.
b) Better accuracy	Going from Point A to Point D, CR drops to 420. CR change not significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e.g., Points E and F).	Going to T=19 sec, and SAR=2,500 ft drops CR to 400. CR improvement between 3a & 3b will less SAR & UI penalty.
4. <u>Product</u>: A preferred or indi-<u>cates set</u> of design values.	SAR=2,500 ft) T =19 sec) Point G τ =73 sec UI =4 sec CR=400 comm/hr

6.1.8 Case 8

Description

This case considers an actual mid-air collision involving a Douglas DC-9 and a Beechcraft Baron that took place near Urbana, Ohio on March 9, 1967. Details are available in the National Transportation Safety Board Report (Reference 10).

AC#1 (DC-9)

232°

4,500 ft

323 kt

3,500 fpm descent

AC#2 (Beechcraft Baron)

195°

4,500 ft

170 kt

Level
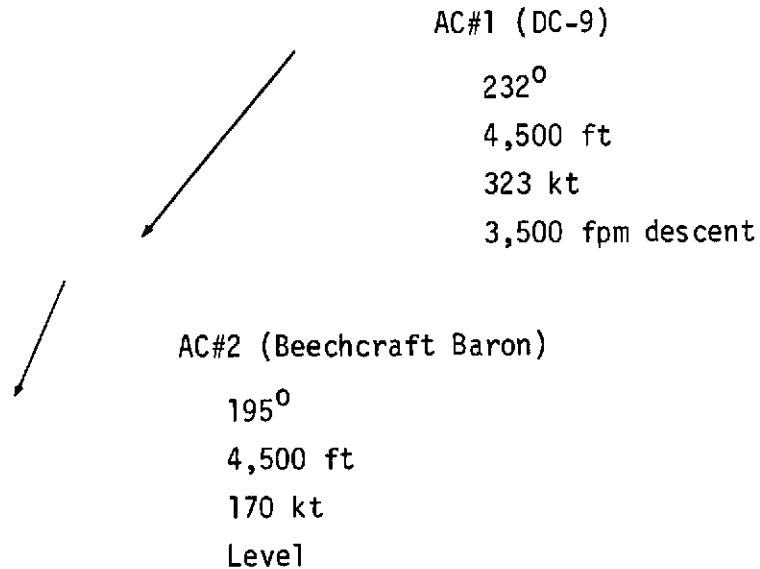
Figure 40. Case Encounter

Discussion

Although this actual mid-air collision involved a tail case rather than a near head-on case, the results are very similar. The rationale for selection and the selected design point (5000-foot accuracy, one-second update interval) are the same.
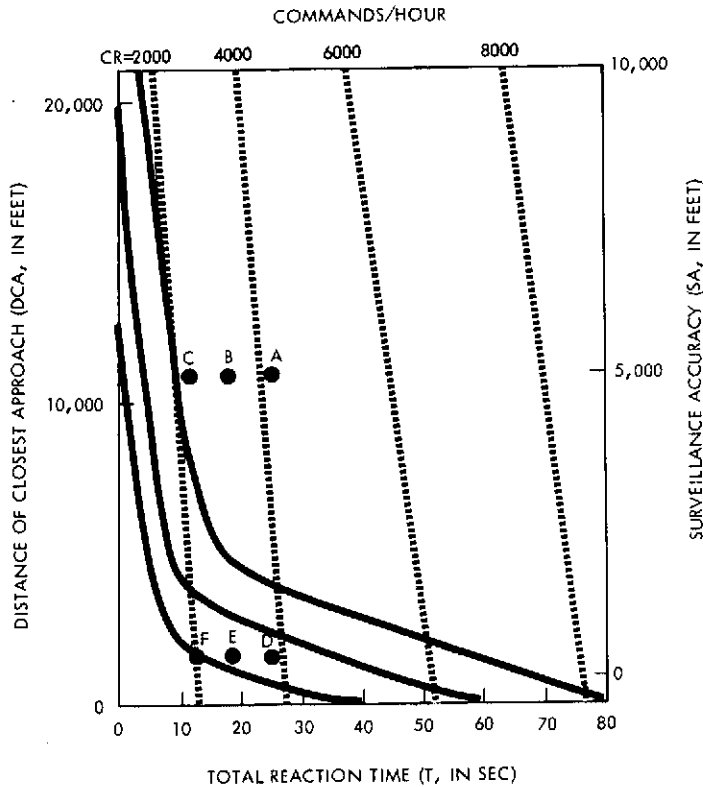
Figure 41. SAUR/CR Curves

Table 21. Case 8 SAUR Selection

Rule	Application
1. Starting Point (Point A) SAR=5000 ft and T=25 sec	τ =100 sec CR=4,300 comm/hr
2. Minimum Strategy: Reduce τ below 90 sec.	It is necessary to reduce T to 12 sec, reduce SA below 2,000 ft, or some suitable combination.
3. Potential Strategies: a) Semiautomation and/or reduced update interval (Points B, C)	Going from Point A to Point C drops CR to 2,400. CR change is significant. τ = 90.
b) Better accuracy	Going from Point A to Point D drops CR to 3,800. CR change not significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e.g., Points E and F).	Going to SAR of 2,500 ft and T=12 sec, CR drops to 2,000. CR change over 3a is not significant.
4. Product: A preferred or indicated set of design values.	SAR=5,000 ft) Point C T =12 sec) τ =90 sec UI =1 sec CR=2,400 comm/hr

81

6.1.9 Case 9

This case concerns an actual collision over Carmel, New York on December 4, 1965 between Eastern Airlines Flight 843, a Lockheed Constellation, and Trans World Airlines Flight 42, a Boeing 707. This case is especially interesting in that the Lockheed Constellation was 1,000 feet below the 707 but the pilot thought he was at the same altitude and pulled up into the 707. Both aircraft were on IFR flight plans. Details are available in the National Transportation Safety Board Report (Reference 8).

AC#1 (Boeing 707)	AC#2 (Lockheed Constellation)
105°	210°
11,000 ft	10,000 ft (until pull-up)
450 kt	210 kt
Level	Level (until pull-up)
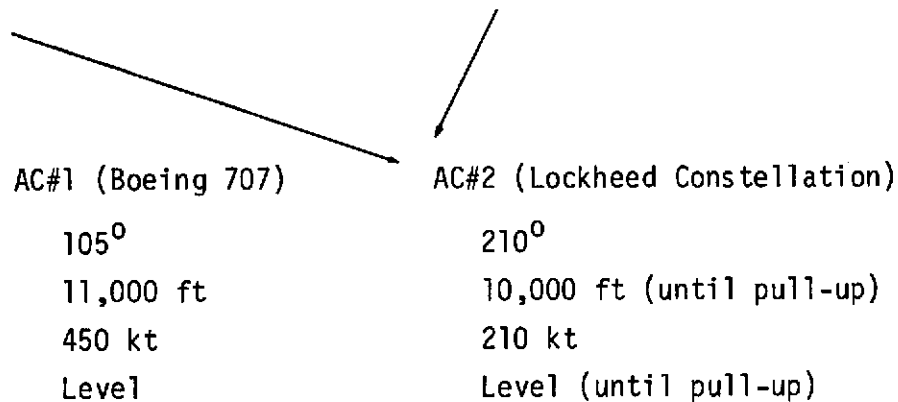
Figure 42. Case Encounter

Discussion

The selection of the surveillance accuracy and update interval rationale in Case 9 is very similar to Cases 1, 2 and 8; and the recommended design point is also 5000 ft accuracy and one-second update interval. It should be noted, however, in this particular case that the decision time of 61 seconds is substantially larger than the amount of time the controller would have had in order to avoid a collision in this case. The Constellation was not required to have a flight recorder and was not carrying one. Therefore, the precise time at which the Constellation pilot initiated his pullup is not clear. Examination of the nominal Transportation Safety Board Report leads one to believe that the Constellation initiated its collision causing maneuver probably less than 30 seconds to impact. The SAUR parameters can be inferred from the tau = 30 curve in the lower left-hand corner of Figure 43. The unavoidable conclusion as applied here is that if two aircraft are allowed to get fairly close together because it is assumed that they are going to continue on their present flight paths, the pilots can then confound the controller with surprise maneuver(s).

82
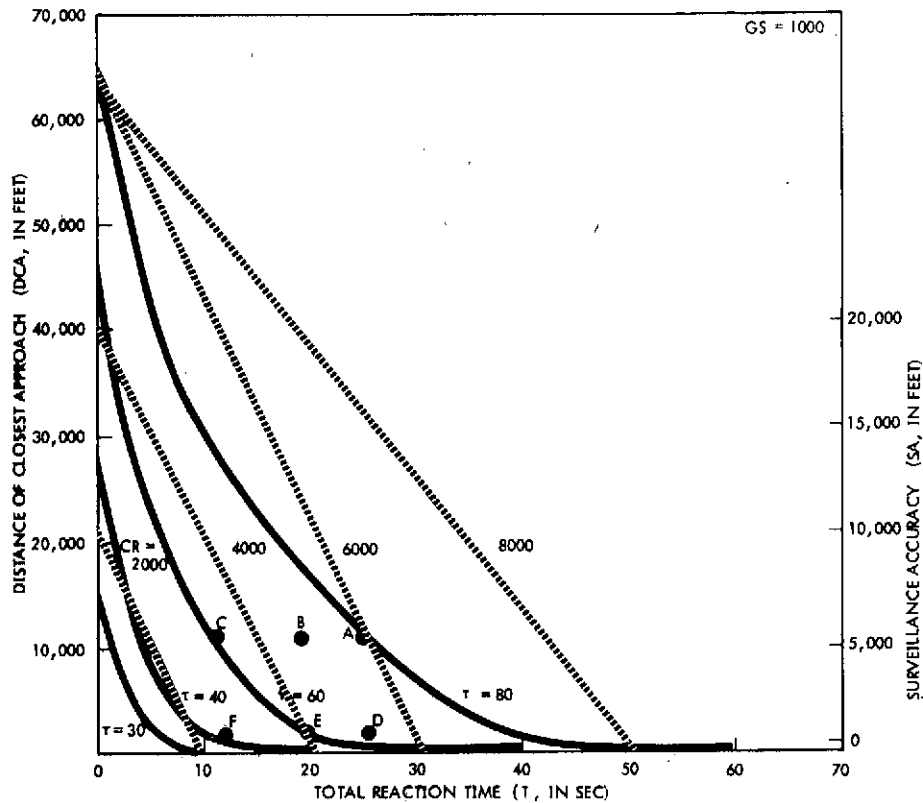
Figure 43. SAUR/CR Curves

Table 22. Case 9 SAUR Selection

Rule	Application
1. **Starting Point** (Point A) <u>SAR=5000 ft and T=25 sec</u>	τ =79 sec CR=5,900 comm/hr
2. **Minimum Strategy:** Reduce τ below <u>90 sec.</u>	N/A
3. **Potential Strategies:** a) Semiautomation and/or reduced update interval (Points B, C)	Going from Point A to C drops CR to 3,200. CR change is significant.
b) Better accuracy	Going from Point A to Point D drops CR to 5,200. CR change not significant.
c) Reduced accuracy	Going to higher SAR, CR change is not significant.
d) Combination of strategies (e.g., Points E and F).	Going to SAR of 2,500 ft and T=12 sec, CR drops to 3,000. CR change over Point C not significant.
4. **Product:** A preferred or indi-<u>cated</u> set of design values.	SAR=5,000 ft) T =12 sec) Point C UI =1 sec τ =61 sec CR=3,200 comm/hr

83

6.1.10 Case 10

Description

This case is an actual mid-air collision between a Hughes Air West DC-9 and a Marine F-4 over Duarte, California on June 6, 1971. The collision occurred at an altitude of 15,150 feet. The DC-9 was departing from Los Angeles International Airport and the F-4 was descending to land at El Toro MCAS. The F-4 transponder was inoperative. The National Transportation Safety Board report on this accident has not been released as of the time of this study. Data was taken primarily from the public media.

AC#2 (F-4)

150°

15,150 ft

420 kt

1,500 fpm (descent)

AC#1 (DC-9)

041°

15,150 ft

330 kt

2,000 fpm (climb)
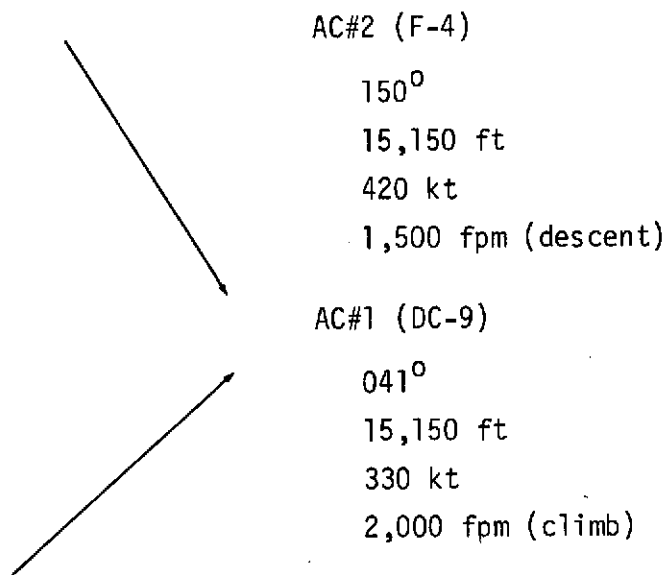
Figure 44. Case Encounter

Discussion

Because this mid-air collision took place at an altitude of 15,000 feet, the assumed traffic density was fairly small and the resulting command communications load was fairly small. As a result, there was no strong case for adopting Point C rather than Point A as the design point. However, if this collision had occurred at, say, 9,000 feet near the Los Angeles basin, the magnitude of the CR values would have been increased by a factor of about 20. In this case, Point C would have been selected as the design point, rather than Point A.
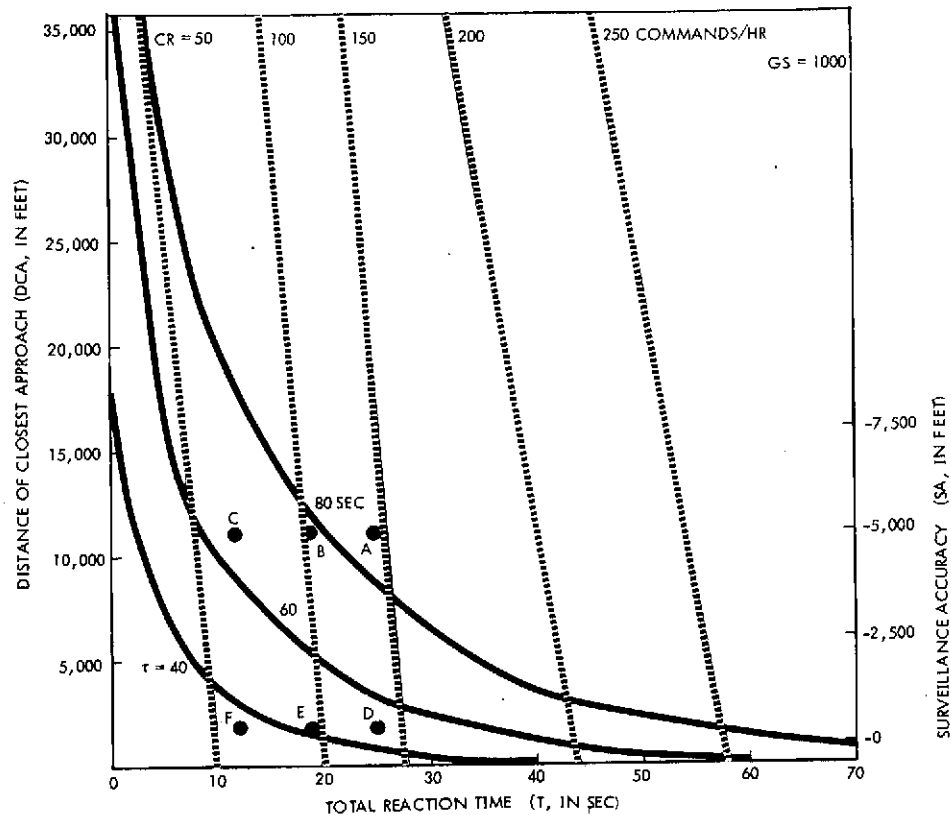
Figure 45. SAUR/CR Curves

Table 23. Case 10 SAUR Selection

Rule	Application
1. **Starting Point** (Point A) SAR=5000 ft and T=25 sec	τ =88 sec CR=150 comm/hr
2. **Minimum Strategy**: Reduce τ below 90 sec.	N/A
3. **Potential Strategies**: a) Semiautomation and/or reduced update interval (Points B, C)	Going from Point A to Point C, CR drops to 66. CR change not significant.
b) Better accuracy	Going from Point A to Point D drops CR to 52. CR change not significant.
c) Reduced accuracy	N/A
d) Combination of strategies (e.g., Points E and F).	CR change is not significant for all strategies.
4. **Product**: A preferred or indicated set of design values.	SAR=5,000 ft) T =25 sec) Point A τ =88 sec UI =10 sec CR=150 comm/hr

85

6.1.11 Case Study Summary

The results of the ten case studies are summarized in Table 24. Examination of this table brings out the following points:

1. The cases span a broad range of aircraft types, phases of flight, and types of airspace.

2. The indicated surveillance system accuracies span a wide range, i.e., from less than 100 feet to greater than 10 miles.

3. The indicated surveillance system update interval does not span such a large range; with one exception it varies from 1 to 10 seconds.

4. A short update interval was indicated more often than was high accuracy. Thus, methods of reducing total system reaction time appear to be more important than increasing surveillance accuracy.

6.2 ADDITIONAL ANALYSES

6.2.1 Single Aircraft Response

There are several special cases that must be considered in the SAUR analysis which involve limitations on the maneuvers of one or both aircraft. The first important variation on the standard SAUR analysis is one in which only one of the two aircraft maneuvers in response to a collision warning command. This single aircraft response situation could result from the fact that one pilot does not receive the message sent to him or is simply unable to respond. It could also be the case if he were simply not equipped to receive collision warning commands. The first case is obviously relevant in a system failure mode analysis, whereas the second is relevant when dealing with many aircraft types which may possess varying levels of capability and equipment. To investigate the effects of these single aircraft response cases, the SAUR program is exercised using a reaction time greater than the decision time for one of the two aircraft. Allowing the reaction time of the other aircraft to vary as in the earlier cases. This had the effect of not allowing the first aircraft to follow a commanded maneuver at all. The results of this analysis for Cases 1, 2D, 3, and 10 are shown in Figures 46 through 49. For simplicity, only the "tau equals 60" curves (wich are representative of all tau curves) are plotted. Examination of Figure 46 reveals that for the Boeing 737 air carrier descending toward a

Table 24. Summary of Case Study Results

CASE NUMBER	DESCRIPTION	REPRESENTATIVE AIRCRAFT TYPES		REPRESENTATIVE AIRSPACE TYPE(S) AND PHASE(S) OF FLIGHT	INDICATED SURVEILLANCE SYSTEM PARAMETERS	
		NO. 1	NO. 2		ACCURACY, FT.	UPDATE INTERVAL, SEC.
1.	AIR CARRIER DESCENDING TOWARD AIR-PORT ENCOUNTERS LIGHT GENERAL AVIATION AIRCRAFT IN A LEVEL TURN	CESSNA 172	BOEING 737	MIXED OR CONTROLLED (SEE NOTE A) 172=CRUISE 737=DESCENT/APPROACH	5,000	1
2.	A NUMBER OF AIR CARRIER AND LIGHT GENERAL AVIATION ENCOUNTERS INCLUDING CESNA 172/BOEING 737 ENCOUNTERS FOR VARIOUS RELATIVE BEARINGS; 737/737 AND 172/172 HEAD-ON ENCOUNTERS; AND EN-COUNTERS WHEREIN THE AIRCRAFT ARE NOT INITIALLY ON A COLLISION COURSE	CESSNA a) 172 b) 172 c) 172 d) 737 e) 172 f) 172 g) 172	BOEING 737 737 737 737 172 737 737	MIXED OR CONTROLLED 737-APPROACH OR DEPARTURE 172-CRUISE	5,000 5,000 3,000 5,000 5,000 5,000 5,000	1 1 1 1 1 1 1
3.	MILITARY VERY HIGH PERFORMANCE JET IN A STEEP CLIMB FROM AIR-PORT ENCOUNTERS A GENERAL AVIATION BUSINESS JET IN LEVEL FLIGHT	LOCKHEED F-104	CESSNA CITATION	MIXED OR CONTROLLED F-104=DEPARTURE CITATION=CRUISE	5,000	4
4.	TWO LIGHT GENERAL AVIATION AIR-CRAFT IN VFR AIRPORT TRAFFIC PATTERN ENCOUNTER	CESNA 172	EVANS VOLKS-PLANE	UNCONTROLLED, MIXED, OR CONTROLLED LANDING/TAKEOFF	100 (SEE NOTE B)	1
5.a)	VERY HIGH PERFORMANCE (SUPER-SONIC) MILITARY JET ENCOUNTERS AIR CARRIER IN LEVEL FLIGHT	BOEING 747	LOCKHEED YF-12	CONTROLLED CRUISE	5,000	10
b)	A HIGH ALTITUDE ENCOUNTER BETWEEN TWO SUPERSONIC AIRCRAFT	BOEING 747	LOCKHEED YF-12	CONTROLLED CRUISE	5,000	30
6.	TWO AIRCRAFT IN PARALLEL TRACK ENCOUNTER a) PARALLEL RUNWAY (2500 FT. SEP.) b) PARALLEL RUNWAY (5000 FT. SEP.) c) PARALLEL AIRWAY (5 MILE SEP.)	BOEING a) 737 b) 737 c) 737	BOEING 737 737 CITATION	CONTROLLED (RESULTS VALID FOR MIXED) a) APPROACH/DEPARTURE b) APPROACH/DEPARTURE c) CRUISE	5,000 a) 100(SEE NOTE C) b) 100(SEE NOTE D) c) 5000	1 1 1
7.	AN ENCOUNTER SITUATION SIMILAR TO CASE 2, BUT INVOLVING A TWIN OTTER AND A UH-1 BOTH MOVING AT SLOW SPEEDS AND IN RELATIVELY CLOSE PROXIMITY	DEHAVIL-LAND TWIN OTTER	BELL UH-1	MIXED OR CONTROLLED CRUISE, APPROACH, DEPARTURE ON TRAFFIC PATTERN	2,500	4
8.	THE MARCH 1967 MID-AIR COLLISION BETWEEN A BEECH-CRAFT B-55 AND A McDONNELL DOUGLAS DC-9 NEAR URBANA OHIO	McDON-NELL DOUGLAS DC-9	BEECH B-55	MIXED OR CONTROLLED DC-9: ASCENT/APPROACH B-55: CRUISE	5,000	1
9.	THE 4 DECEMBER 1965 MID-AIR COLLISION BETWEEN A BOEING 707 AND A LOCKHEED CONSTEL-LATION NEAR CARMEL, NEW YORK	BOEING 707	LOCKHEED CONSTEL-LATION	MIXED OR CONTROLLED CRUISE	5,000	1
10.	THE JUNE 1971 MID-AIR COLLISION BETWEEN A MARINE F-4 AND A MCDONNELL DOUGLAS DC-9 NEAR DUARTE, CALIFORNIA	McDON-NELL DOUGLAS F-4	McDON-NELL DOUGLAS DC-9	MIXED OR CONTROLLED F-4: CRUISE (OR APPROACH/DEPARTURE) DC-9: DEPARTURE (OR APPROACH OR CRUISE)	5,000	10

NOTE A: "MIXED" AIRSPACE HERE IS EQUIVALENT TO THE FAA's "CONTROLLED" AIRSPACE. "CONTROLLED" AIRSPACE HERE IS EQUIVALENT TO THE FAA's "POSITIVE CONTROL AIRSPACE (PCA)

NOTE B: SEPARATION CANNOT BE GUARANTEED ACCORDING TO SAUR GROUND RULES. FOR GS-500 FEET, SA=100 FEET WILL ALLOW T=7 SEC.

NOTE C: SEPARATION CANNOT BE GUARANTEED ACCORDING TO SAUR GROUND RULES. FOR GS=1000 FEET, SA=100 FEET WILL ALLOW T=10.5 SEC. FOR ALL τ.

NOTE D: SEPARATION CANNOT BE GUARANTEED ACCORDING TO SAUR GROUND RULES. FOR GS=500 FEET, SA=100 FEET WILL ALLOW T=6 SEC. FOR ALL τ.
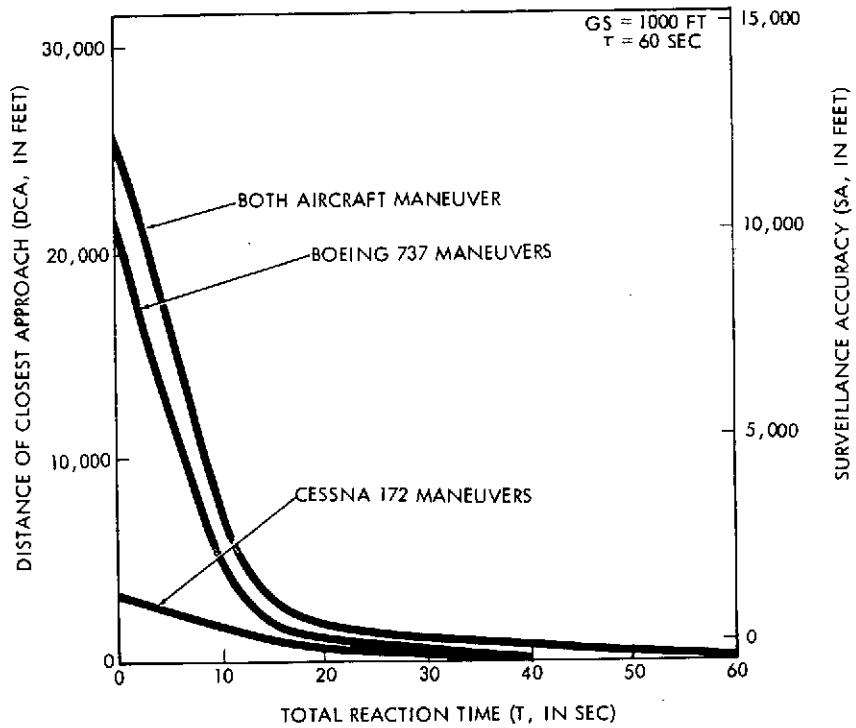
Figure 46. Case 1 Single Aircraft Maneuvers
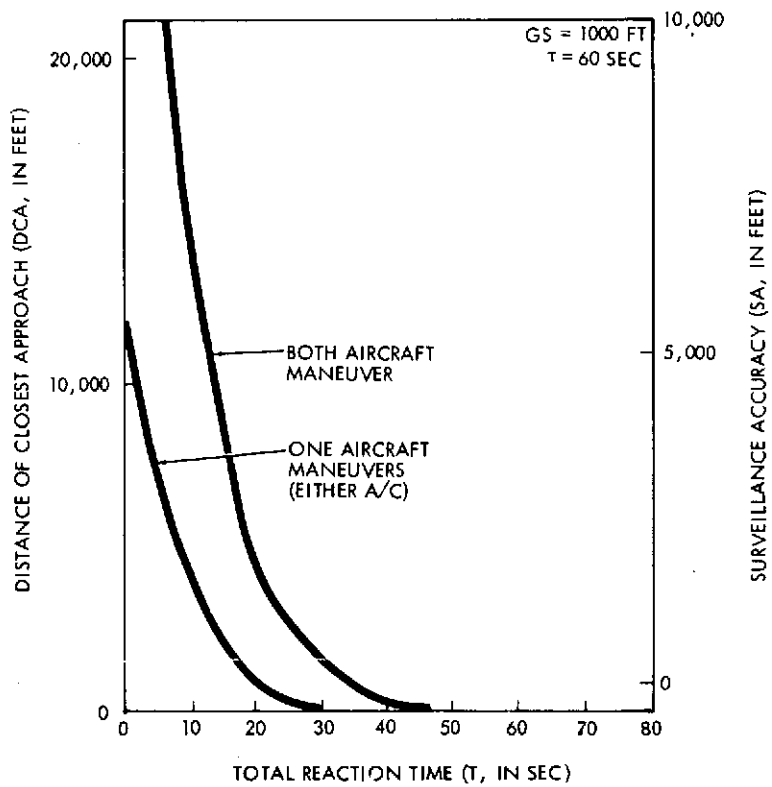
Figure 47. Case 2D Single Aircraft Maneuvers
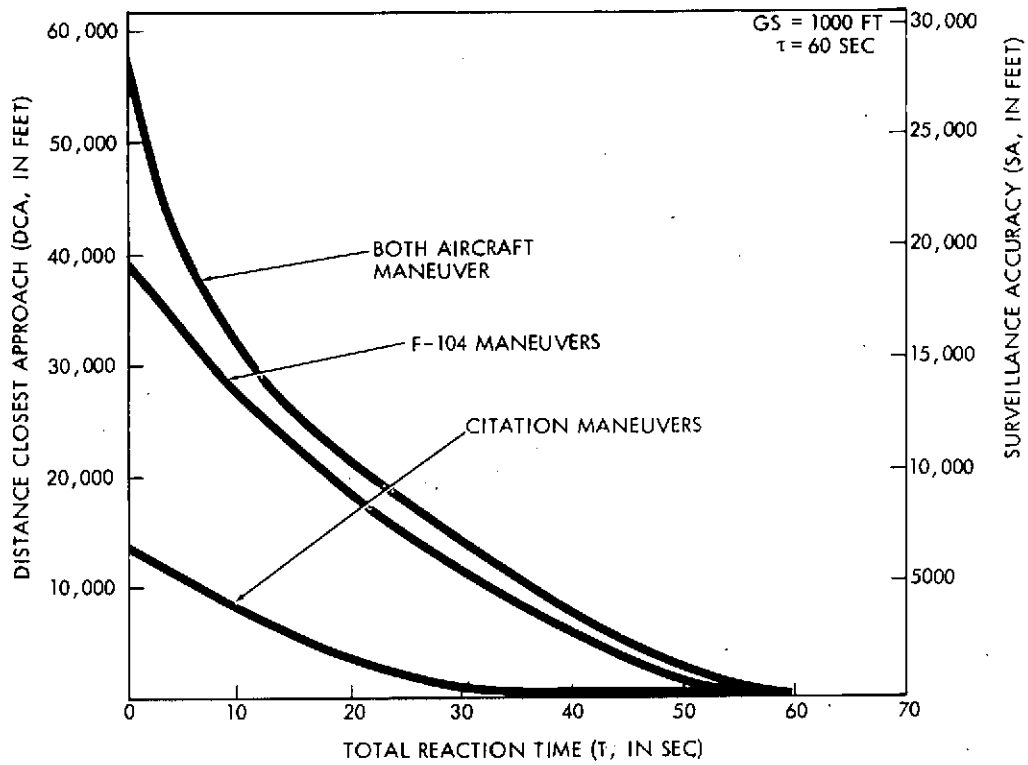
Figure 48. Case 3 Single Aircraft Maneuvers
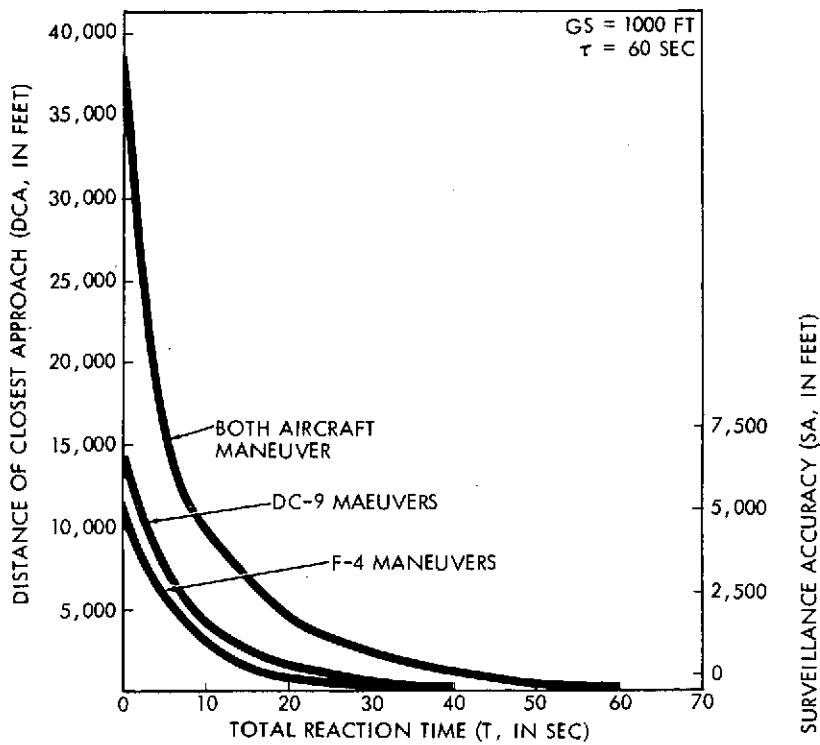
Figure 49. Case 10 Single Aircraft Maneuvers

89

general aviation Cessna 172, if the air traffic controller can get the 737 to respond, the DCA he can provide is almost as large as when both aircraft respond to his command. On the other hand, it does virtually no good to have the 172 to maneuver if the 737 is still free to maneuver throughout its much wider maneuvering envelope. This point arose consistently in similar cases and agrees with one's intuition that the air traffic controller has more control over the situation if he can get to the highest performance aircraft and command that aircraft to maneuver. Since from the standpoint of the aircraft ability to carry reliable and possibly redundant equipment, the higher performance aircraft are more likely to be responsive to commands than the low performance aircraft. Thus, the SAUR/CR results will probably be valid even if an IPC concept is implemented involving a mix of fully and partially equipped aircraft; the latter would possess only that equipment required to make them visible to the system and not the equipment which would allow them to receive collision warning commands.

Case 2D, shown in Figure 47 involves two high performance aircraft. In fact, these are identical (Boeing 737) aircraft flying in a direct head-on collision situation. In this case, because of the geometry of the situation, it is almost mandatory that the controller be able to vector both aircraft. The reason for this is that because of the symmetry of the situation, if one aircraft is given a particular commanded maneuver in an attempt by the controller to avoid a collision, it is possible for the other aircraft to, in effect, fly the mirror image trajectory and still result in a mid-air collision. Thus, the uncontrolled aircraft can "do enough mischief," even with a single set of maneuvers, such that the maneuvers which can be performed by the controlled aircraft during the escape time are not sufficient to provide very large DCA's. Case 3, in Figure 48, is again a situation involving two high performance aircraft. The performance difference between the F-104 and the Citation is large enough however, such that the F-104 is clearly the preferred aircraft to command. As indicated in Figure 49, the results for Case 10 are similar to those of Case 2D. Both aircraft are high performance aircraft so that it doesn't matter which of the two aircraft are controlled; but a substantial drop in DCA's occurs in the event that either aircraft is not controllable.

The foregoing results bring out three points:

- In the event that one aircraft is high performance and the other is low performance, it is much more important to be able to command the high performance aircraft. In fact, from the standpoint of the SAUR formulation little is lost if the low performance aircraft cannot be commanded.

- If both aircraft are high performance then the situation is geometry-dependent. It may be necessary to be able to command both aircraft or it may be sufficient to command either. In an operational situation, if the controller (or control algorithm in the case of automatic control) were to make the conservative assumption that only one aircraft would respond to a command, then the situation such as the one indicated in Case 2D could not be allowed to develop.

- If, in the course of more detailed analysis or in the preliminary design phase of an advanced air traffic control system, the decision is made to assume a single aircraft response situation, then the surveillance accuracy and update rate requirements could become more stringent. It would appear that higher performance radar or LIT might have to replace enroute radar unless control procedures were altered. The question certainly warrants more analysis.

6.2.2 Different Reaction Times

The above case can be considered a special case of a more general one wherein the two aircraft simply have different reaction times. Indeed, it would be a coincidence in the real world situation if the two aircraft did begin collision avoidance maneuvers at the same instant. The SAUR Program was exercised in all ten cases allowing the two aircraft different reaction times. The results may be summarized by comparing three hypothetical cases:

- Case A: $T_1 = T_2 = 30$ seconds
- Case B: $T_1 = 40$ seconds; $T_2 = 20$ seconds
- Case C: $T_1 = 20$ seconds; $T_2 = 40$ seconds
- Aircraft 1 is high performance
- Aircraft 2 is low performance

In cases such as this it was observed that the DCA's obtained in Case B were smaller than for Case A since it took a longer time for the higher performing aircraft to react, whereas Case C gave larger DCA's than Case A since the higher performing aircraft had more time to maneuver. These results were, of course, consistent with the case where one of the two aircraft failed to maneuver at all.

6.2.3 Altitude-Only Maneuvers

Finally, as pointed out in Section 4 the SAUR Program in all cases also computed DCA's where only altitude commands were allowed. The results are shown in Table 25 and indicate quite clearly that much larger DCA's are possible when horizontal commands can be used. Part b) of the list shows as the reacton time becomes larger the difference between the two sets of DCA's becomes smaller. Finally, as T approaches tau the altitude maneuvers become the preferred ones. In other words, if only a few seconds are available for an escape maneuver the vertical maneuvers will provide a larger DCA.

Clearly it would not be necessary to produce large DCA's in an operational case. Sufficiency of DCA's and guaranteed separation associated with a particular pair of commands is of most interest operationally. Thus, it is recognized that there will be times when, for example, a vertical maneuver will be preferable to a horizontal maneuver even though the latter could guarantee a larger separation. The central point here, however, is that increased DCA's and therefore relaxed surveillance accuracy requirements, larger update intervals, and larger total effective reaction times can be allowed if horizontal as well vertical maneuvers are employed.

6.2.4 Effect of Variations in Guaranteed Separation Distance

In Section 2 it was pointed out that the assumed values of separation distance used in deriving required surveillance accuracy from the distance of closest approach were arbitrary. It is necessary at this point then to evaluate the effect of variations in this input parameter. Recalling the relationship:

$$SAR = \frac{1}{2} (DCA - GS)$$

Table 25. Vertical Maneuvers

(a) All Cases ($\tau=60$, $T_1=T_2=0$		
Case	Horizontal Maneuvers	Vertical Maneuvers Only
1	26,708	2.070
2A	25,280	1,540
3	57,650	29,950
4	5,010	10
5A	63,440	10,890
5B	447,300	1,830
6C	33,050	3,700
7	17,400	1,180
8	19,520	4,540
9	45,170	2,600
10	37,910	14,480

(b) Case 1 (Various τ's, T's)				
τ	C172 T_1	B737 T_2	Horizontal Maneuvers	Vertical Maneuvers Only
40	0	0	15,425	1,383
	20	20	1,363	728
	30	30	621	394
	35	35		
	40	40	412	344
60	0	0	26,708	2,070
	10	10	6,948	1,404
	20	20	2,598	786
	30	30	1,708	465
	60	60	465	465
80	0	0	37,443	3,772
	20	20	13,318	2,885
	30	30	8,138	2,609
	40	40	2,743	2,266
	60	60	2,069	976
	80	80	912	524

Various sets of SAR scales (consistent with various assumed values of GS) can be shown on the DCA vs T vs τ curves. Figure 50 shows the effect of varying the guaranteed separation parameter for Case 2A. The right side of Figure 50 shows four different scales of surveillance accuracy corresponding to input values of GS of 0, 1,000 feet, 5,000 feet, and 10,000 feet. The 0 and 1,000 foot cases essentially look alike. At first glance it would appear that as GS is increased from 1,000 to 5,000 and then to 10,000 feet, since the resulting SAR is thereby decreased from 5,000 to 2,600 and then to 500 feet, that the surveillance system choice might go from enroute radar to terminal radar to LIT on that basis. Surprisingly, this does not turn out to be the case. As GS is increased from 1,000 to 10,000 feet, point C moves up to C prime and the other five points move up in the same fashion. If one then goes through the same thought process as before, C prime is preferable to A prime or F prime for the same reason that C was preferable to A and F before, i.e., a significant decrease in CR for the introduction of data link and an insignificant decrease in CR as a result of going to higher accuracies. Although figures for the other nine cases are not shown, this result is fairly typical. Cases 4 and 6 are obvious exceptions, since separation could not be guaranteed under the SAUR ground rules in these cases. An increase in GS will usually not change the recommended surveillance/communication implementation. In Case 10, it does appear that a change of recommended implementation from enroute radar to terminal radar and/or data link would take place if GS were increased to 10,000 feet.

In summary, it does not appear that the effect of varying GS within the ranges indicated is a major factor; but it probably should be given some attention in the preliminary design phase.

6.2.5 Effect of Surveillance Errors

The SAUR Program defines what the surveillance system overall position accuracy must be in order to guarantee a given separation distance. It is conservative in the sense that the position errors for the two aircraft (whether or not they are equal) are considered to be in the direction of the position vectors from each aircraft to the other.
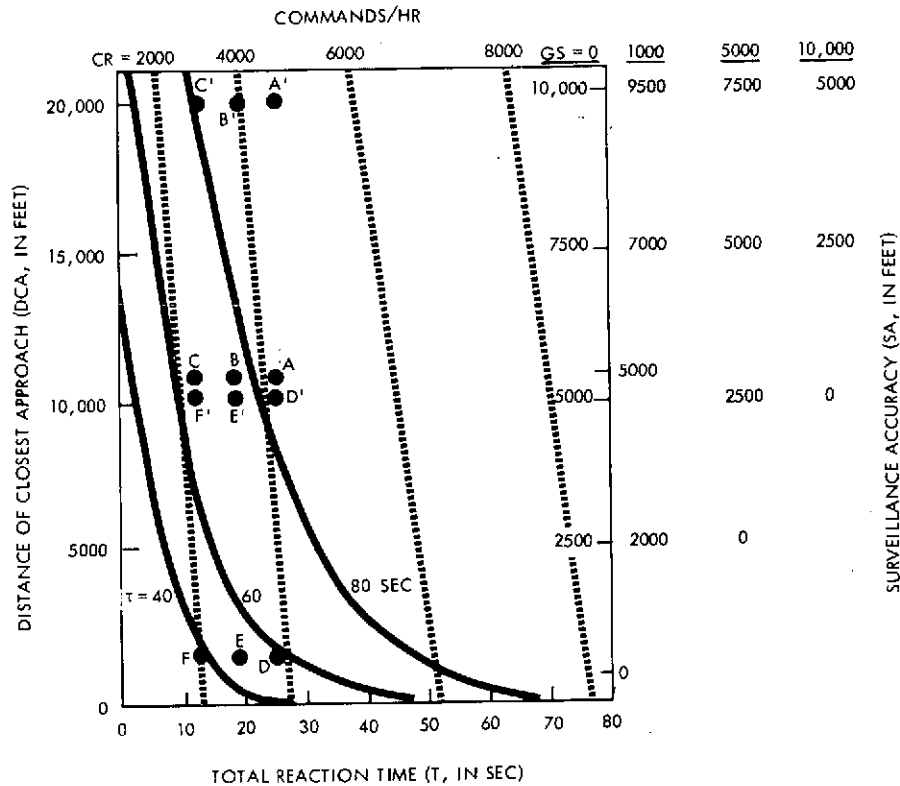
Figure 50. Effects of Varying Guaranteed Separation

On the other hand, the SAUR Program does not explicitly take into account the velocity and acceleration errors which exist at the τ epoch. The question is whether these errors for a given set of parameters (i.e., surveillance system type, filter type and pre-τ aircraft dynamics) as propagated throughout the decision time are significant with respect to range of values in the uncontrolled and controlled parameters used in SAUR.

Section 7 describes what the surveillance errors are likely to be for four different aircraft types on four types of trajectories. The results for two different surveillance systems using varying levels of filter sophistication are shown. The errors listed are the RSS of orthogonal components carried in the filter program.

Regarding the nature and the effect of these errors the following points are relevant:

1. The surveillance accuracy in general will be different for the two aircraft.

2. The position uncertainty for the two aircraft will increase with time following the decision epoch.

3. The relative magnitude of the components of initial surveillance error can influence the determination of which maneuvers result in the estimated[*] DCA.

If the results of runs[**] 6 and 23 of Section 7 (shown in Table 25) are applied to the nominal positions and velocities at the τ epoch, it is possible to determine how position uncertainty increases with time from the decision epoch. Only the horizontal components of the average position and velocity from these cases errors are shown in Table 25, but the vertical case will be considered here. If the speed or velocity and acceleration uncertainties of Table 25 were allowed to propagate over 60 seconds accord-into the formula

$$P + Vt + 1/2 \ At^2$$

the resulting position determination uncertainty would be over 20,000 feet for both runs 23 and 6. Fortunately, the actual aircraft dynamics will almost invariably prevent such very large propagated position errors. Each of the surveillance error types will now be discussed. It will be seen that with one exception the SAUR Program already assumes the worst situation that is physically possible, and is therefore conservative.

Speed or Linear Velocity Uncertainty

In this case the SAUR Program is slightly non-conservative. For example, a Cessna 172 presumably traveling at 170 feet per second is made by the SAUR Program to accelerate to its maximum speed at its maximum acceleration capability. Given the velocity uncertainty shown on Line 2,

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[*] Estimated DCA is used here to include the effect of propagated surveillance errors.

[**] These runs are roughly comparable to the aircraft types and trajectories for the SAUR Case 1 exemplified in Section 6.1.1. Both runs reflect a radar type surveillance system using 6-state variable Kalman filter.

		Run 23	Run 6
1.	Position Uncertainty at τ.(ft)	$\sqrt{(493)^2+(468)^2}=679$	$\sqrt{(527)^2+(2836)^2}=2884$
2.	Velocity Uncertainty at τ.(ft/sec)	$\sqrt{(56)^2+(76)^2}=94$	$\sqrt{(68)^2 \quad (144)^2}=159$
3.	Acceleration Uncertainty at τ.(ft/sec^2)	$\sqrt{(7.4)^2+(7.4)^2}=10.5$	$\sqrt{(8.5)^2 \quad (8.2)^2}=11.8$

it actually could have been at V max already. Comparing the linear distances traveled in 60 seconds:

- Nominal: $170(60) = 10,200$ feet

- SAUR: $1/2 (170+220) (25) + 220(35) = 12,575$ feet

- Possible: $220(60) = 13,200$ feet

It is seen that the SAUR Program in allowing for aircraft maneuvers, also very nearly allows for velocity uncertainty. In the case of high performance aircraft the ratio of velocity uncertainty to velocity should probably be smaller producing an even more favorable situation. A SAUR run was made using Case 1, which verifies this analysis.

Acceleration Uncertainty

The SAUR Program already allows an instantaneous change to the max acceleration capability of the aircraft. Larger acceleration uncertainties which might be obtained from the filter mechanization analysis would not be physically possible.

Heading Errors

Here the SAUR Program again assumes the worst situation but in a different way. Consider the two aircraft shown in Figure 51. The air traffic controller assumes that the aircraft have velocity vectors V_1 and V_2 respectively. If they have heading (or velocity) uncertainty as represented by $\varepsilon 1$ and $\varepsilon 2$ (or U 1 and U2), they could actually be on a direct collision course, with velocity vectors W1 and W2. But the collision situation is precisely what the SAUR Program is already set up to

investigate. From an operational standpoint the controller would tend to treat the V_1/V_2 case as though it were the head-on W1/W2 case, which makes an assessment of velocity and acceleration errors for any surveillance system vital to the system design process. In this case, however, the W1/W2 case was the one which was visually examined. The W_1/W_2 case can be non-conservative. For example, suppose the controller assumed on the basis of his surveillance data that the W_1/W_2 case existed and commanded aircraft #1 to turn left, and aircraft #2 to turn right. If they were actually on flight paths V_1/V_2 they would not be turning toward each other, thus reducing the DCA and SAR. This phenomenon was investigated briefly in Cases 2F and 2G and changes appeared to be insignificant. In retrospect, however, it is felt that the subject needs to be quantitatively investigated further in order to establish adequate confidence in these results, because they do lie on the non-conservative side of the truth.
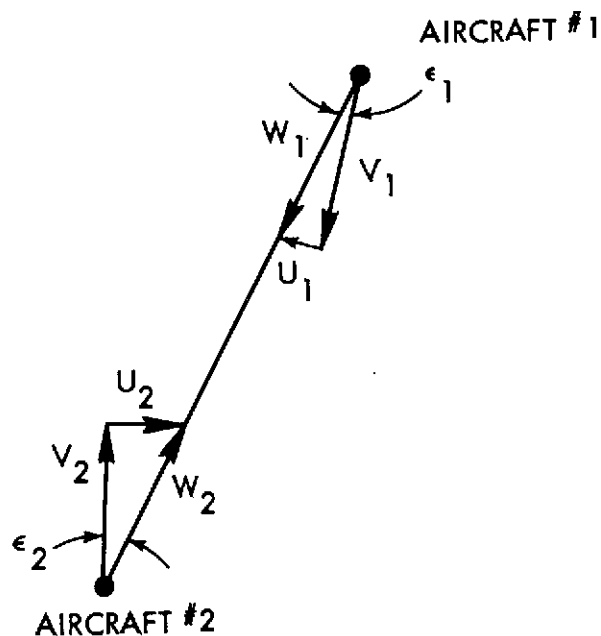
Figure 51. Effect of Heading Errors

Vertical Errors

The SAUR altitude program likewise allows the maximum climb rate and descent rate physically possible. Therefore, any uncertainties in vertical speed which are physically realizable can be analyzed by the SAUR Program.

6.3 COMPARISON OF SAUR WITH OTHER ATC ANALYSES

It is of interest to compare SAUR with other ATC studies dealing with the subject of aircraft separation hazards and conflict resolution. Three such studies are: "Separation Hazard Criteria," by Holt and Marner (Volume II of Reference 1), "Data Acquisition System Design Considerations," by Blake and Smith, (Volume II of Reference 1) and "Navigation/Traffic Control Satellite Mission Study," by Craigie, et al (Reference 33). The results of these studies are compared with SAUR. Although each study contains unique constraints or assumptions, quantitative differences still provide an insight into the effect of these assumptions on particular aircraft situations.

6.3.1 "Separation Hazard Criteria" -- Holt and Marner

This analysis is similar to SAUR in several respects. It seeks to define a horizontal and vertical hazard region $R^*(t) = R_2(t) - R_1(t)$ and $z^*(t) = z_2(t) - z_1(t)$ and to predict the hazard region (under certain assumptions over a period of time, called the total escape time, t_e^{**}. If the region satisfies given safe passage criteria, i.e., $|R(t)| \geq \rho$ and $|z(t)| \geq h$, (where ρ and h are selected constants), then intervention can be deferred until the next decision time.

The predicted hazard region takes into account:

1. Intended flight path of the aircraft $[R_{(o)}, \dot{R}_{(o)}t, \ddot{R}_{(o)} \frac{t^2}{2}]$.

2. Surveillance position and velocity measurement errors $(\Delta R, \Delta \dot{R})$.

3. Deviation from intended flight path, including flight technical errors and "freedom of choice afforded to the pilot," which can include position, velocity and acceleration terms $[F(t)]$.

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[*] The notation of the reference is used here where capital letters denote vectors and small letters scalars.

[**] t_e is equivalent to τ used in this report.

For the situation where there are no intended accelerations, it is sufficient to show for safe passage that

$$R_{(o)} + \dot{R}_{(o)}t \geq at^2 + bt + c \text{ over the predicted interval where}$$

a = acceleration included in item 3, above;

b = the velocity term in 3. above + the surveillance error in velocity;

c = the position term in 3. above + surveillance position error + ρ.

Numerical examples for several different ground-based surveillance systems are given using this approach. Within these examples, one type represents a fairly well defined flight corridor situation (A + 0.1g); another type defines a "considerable freedom" case (a = 0.5g). The latter is chosen for comparison with SAUR.

First, however, the expected surveillance accuracy for the enroute ATCRBS is compared from the reference and from Section 7 of the report. The reference gives, over the prediction interval, a position error of $|\Delta R + \Delta R t_e|$ total 32,700 feet (3σ). This includes smoothed position and derived velocity errors using an α,β filter and the lag errors associated with 0.5g accelerations. From Section 7, Run Number 3 (737 aircraft, same surveillance system, α-β filter, and moderate turn trajectory), a position error of 31,500 feet over the same time interval is indicated.

Comparing hazard results, it is seen that for a delay time of 18 seconds and a total escape time of 37 seconds, the reference shows a half-width hazard or alarm region of 47,300 feet, which includes the above surveillance error, 13,900 feet for 0.5g freedom and 600 feet minimum passing distance. SAUR (Figure 11) indicates that it would be barely possible to guarantee 600 feet separation for a delay time of 18 seconds and a τ of 40 seconds even with a perfectly accurate surveillance system giving data at 10 second intervals. Another way of regarding Figure 11 is that for a (roughly) 2500 foot surveillance it would require a τ of about 70 seconds to guarantee the separation for the same delay time. The difference in results would have to be attributed to what the definition of "considerable freedom" is in the two methods.

6.3.2 "Data Acquisition System Design Considerations" -- Neal A. Blake and Edward E. Smith

This approach is a deterministic model which treats the particular problems of required runway or airway separation distances in the face of navigation or flight technical error, data acquisition error, and update rate. The models are based on the concept of normal operating zones (NOZ) or lane width, and a buffer zone between lanes. The relationships are developed under the constraint that an aircraft will be given a command at some point which will prevent him from entering the buffer zone. Figure 52 was reproduced from the reference to aid the reader in visualizing the situation.

By definition, the above buffer zone width is essentially equivalent to the GS of this study. For one set of results the reference used the following parameters:

Aircraft Velocity	150 knots
Maximum turn-away rate	1.5°/second
Response time	5 seconds
Recovery maneuver rate	3°/second
Width of buffer zone	500 feet
Update interval	4 seconds

SAUR was run with similar parameters except for the maximum turnaway (or pilot-initiated maneuver), which was 5°/sec in SAUR. To get a better comparison, the Blake-Smith method was used to compute NOZ for 2500 feet runway separation and a turnaway rate of 5°/sec. The results indicate that to achieve the runway separation of 2500 feet, a NOZ of approximately 100 feet would be required and the equivalent delay* would be abut 10 seconds. Figure (Case 6A) indicates that to achieve runway separation and guaranteed separation of 500 feet at the given surveillance accuracy and update rate, the reaction time would have to be about 7 seconds. These results are reasonably close considering some of the subtleties involved. For example, SAUR starts the aircraft 100 feet closer together; also,

---

*Equivalent delay was computed from the 5-second response time, 4-second update interval and the time required to cross the DAS error -- to obtain equivalence with SAUR delay time, T.
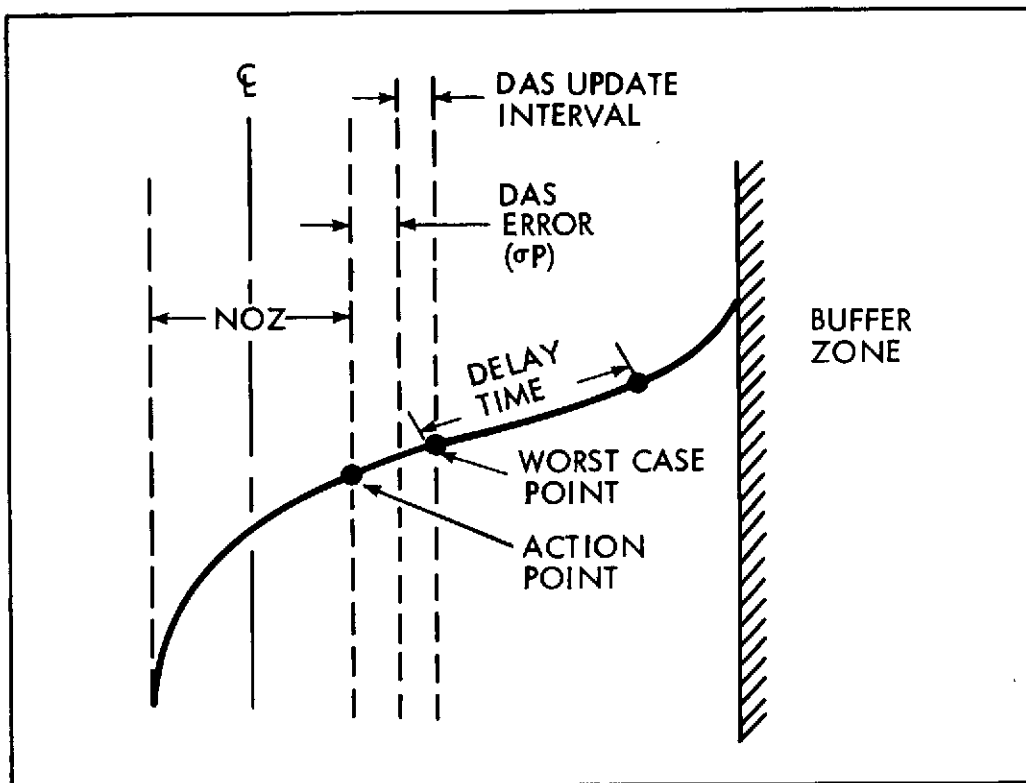
Figure 52. Turnoff and Recovery Geometry — Position Only
Case (Taken from Reference 1)

since it was found in this example that it takes only 3 seconds to traverse the NOZ, it is probable (see Figure 2) that a DA sample for action would show the aircraft already outside the NOZ. An interesting observation is that the reference uses a NOZ to determine the initiation of the action; SAUR essentially creates an equivalent NOZ depending on the circumstances.

At 5000 feet runway separation and a realistic (commensurate with navigation accuracy) NOZ of 600 feet, the reference indicates a required surveillance accuracy of about 720 feet and an equivalent delay of 9.7 seconds. (Figure (Case 6B) shows good agreement with these parameters, showing a required delay time of about 10 seconds for a similar data acquisition accurancy.

6.3.3 "Navigation Traffic Control Satellite Mission Study" — Craigie, et al

The mission study also used a deterministic model (similar to Blake and Smith) designed to analyze relationships between surveillance accuracy, update interval and separation standards for IFR traffic such as the North

Atlantic routes. Using the mission study surveillance model and the appropriate data from Case 6C in the SAUR analysis yields update interval requirements of 0.2 to 1.0 sec-nds for the Boeing 737 and 3 to 15 seconds for the Citation, as compared with the one second figure obtained here. This assumes a surveillance accuracy of 300 feet and heading errors of 30°, which is the case most comparable to the SAUR analysis. Different update intervals were obtained for the two aircraft since, in the Reference 27 formulation, each aircraft is controlled individually in the event it deviates from its assigned flight path. The difference between the results of the two studies can be attributed directly to the difference in the assumed perturbation maneuvers by the two aircraft.

6.3.4 Comparison with Probabalistic Analyses

By its deterministic nature, SAUR is difficult to compare directly with techniques which employ pro-abilistic estimates of collision risk, maneuver dynamics, navigation accuracy, and other parameters, as, for example, in References 34 and 35. This is not to say that comparisons are not ultimately possible since certainly probabilities can be assigned to the parameters used in SAUR. As pointed out by Dr. Koenke in Reference 34, "....prediction of potential threats could be quite tedious and time-consuming....if maneuvers are limited to standard maneuvers within a control region, then calculations for potential threat evaluation are very straightforward.... ." The need for answers encompassing large aircraft populations is recognized. Nevertheless, under circumstances where the number of aircraft is limited, such as in parallel approach and landing, it is found that comparable results occur. A final example, in Reference 35, the relationship between surveillance accuracy, update rate and runway separation shows that 5000 feet runway separation and an update interval of 4 seconds require a surveillance accuracy of about 700 feet.

Dr. Koenke also brings out a point that is quite relevant in evaluating the validity of the SAUR/CR results. This point has to do with the application of a 2-body formulation such as SAUR to the n-body problem which is the "real world". The present aircraft-pair capability may appear as a serious limitation. It would be desirable to increase the capability to include large numbers of aircraft but this would be prohibitive in computation time. It is not unreasonable to increase the

103

capacity to three or perhaps four aircraft. However, as pointed out in Reference 34 the probability of simultaneous threats is small. Koenke's analysis shows that the probability of encountering one intruder and then encountering a second intruder while the system is resolving the first conflict is 1 part in 10^6 for the New York terminal area, using 1968 statistics, a terminal flight time of 20 minutes and a system warning time of 30 seconds. In future efforts this type of analysis should be applied to the given scenarios to determine if the surveillance capability must be increased beyond that indicated by the SAUR Program.

7. FILTER MECHANIZATION STUDY

The objective of the filter mechanization portion of the SAUR study was to determine the accuracy with which the state (i.e., the position and velocity of the various aircraft) can be estimated and how this accuracy depends on the surveillance system, aircraft dynamics, and data processing (filtering) procedure.

7.1 METHODOLOGY

The performance of the separation assurance function is dependent on the accuracy with which the state (i.e., the position and velocity) of the various aircraft can be estimated. The accuracy of the state estimate depends on three major factors — surveillance accuracy and update rate, aircraft dynamics, and the data processing (filtering) procedure. The problem is complicated by the fact that there are several types of surveillance systems, a wide variety of aircraft with differing dynamics, and many filtering techniques. Just as in the accuracy/update rate analysis an adequate cross-section of the various possibilities must be examined in sufficient detail to allow meaningful conclusions to be reached.

This is accomplished in the following manner. The two types of aircraft surveillance systems which seem most representative of systems likely to be used in ATC applications over the next two decades are the presently existing ATCRBS (Air Traffic Control Radar Beacon System) and a proposed multi-lateration system similar to the LIT concept developed in Reference 2. Since they also essentially span the performance spectrum likely in this time span, both are considered in this study. First, the wide range of aircraft that will use domestic airspace are broken down into five classes. In this analysis, aircraft considered to be representative of this spectrum are used — although only four cases will be required. Four different filters of varying levels of complexity are considered and the tradeoffs between filter performance and filter complexity are examined. In this manner, the various relevant possibilities are considered and their performance compared.

7.2 AIRCRAFT SURVEILLANCE SYSTEMS

Two substantially different types of aircraft surveillance systems have been proposed for application to the air traffic control problem.

The first is an extension of the presently existing Air Traffic Control Radar Beacon System (ATCRBS); the second is a multilateration system using either satellites or ground stations. Since the characteristics of these systems differ considerably, both are considered in this study.

The ATCRBS concept, commonly referred to as secondary radar, was established as the National Standard in 1961 and its mandatory use in positive control airspace was required. As of January 1968, the system was being used in 89 civil ground stations in enroute areas and by 109 stations in terminal areas. The FAA has established the ATCRBS system as the primary source of identity, altitude, and position information in the presently evolving semi-automated ATC system, scheduled to be completed in 1973.

Three separate measurements are made by the ATCRBS system: range, azimuth, and altitude. The range and azimuth measurements are made by the radar itself, while the altitude measurements is made by the aircraft altimeter and is transmitted to the ground station by the aircraft transponder as part of the radar ranging signal.

The National Standard (Reference 36) has specified that the ATCRBS systems must have a range accuracy of ± 1000 feet and an azimuth accuracy of $\pm 1.0°$ (at the display). However, observations of existing operational ground stations indicate a somewhat better performance than that dictated by the National Standard. The average observed accuracy of the ATCRBS (Reference 37) was found to be:

Range Bias: 380 feet (1 sigma)

Azimuth Bias: 0.25 degrees (1 sigma)

The ATCRBS ground stations utilize a directional antenna having a typical beamwidth of 4°. The sweep period (time to rotate 360°) is 10 seconds. While in the beam, each aircraft can be interrogated at a rate of 1200 times per second. The average time duration in the beam (per sweep) is approximately 0.11 seconds; hence, 132 samples can be obtained. The noise on each range measurement (due to transponder reply jitter and variations in pulse rise times) is approximately 110 feet. Since changes in the aircraft-radar geometry will be small during each 0.11 second interrogation interval, we shall assume that all of the 132 measurements will be averaged and only this average used for updating the filter. Then, the equivalent

106

data rate provided by the ATCRBS system is 1 sample per 10 seconds and the equivalent measurement noise is 10 feet (i.e., $10 \simeq 110/132$).

The accuracy of the altitude measurement will depend on the aircraft instrumentation, installation error (which includes effects of aircraft airspeed, altitude, mach number, and configuration), and flight technical error. Munnikhuysen, in Reference 1, indicates that typical figures for general aviation and newer types of transports should exhibit 3σ errors of 665 feet and 420 feet respectively. He also projects a "possible" error of 260 to 285 feet (3σ). Since the horizontal errors will normally be much larger, for simplicity this analysis assumes an altitude bias error of 100 feet (1σ) and noise intensity of 25 feet (1σ).

A rough indication of the level of accuracy which can be obtained from the ATCRBS system is provided by the following analysis. The error in measured aircraft position, ΔP, due to the range, azimuth, and altitude biases is given by the expression

$$\Delta P = \left[\left(\frac{.25\ r}{57.3}\right)^2 + (380.)^2 + (100.)^2\right]^{1/2}$$

where r denotes the aircraft range. The position error, ΔP varies with range as indicated below:

Range	Position Error
0. n.miles	393. feet
25. n.miles	763. feet
100. n.miles	2647. feet

Thus, while the accuracy of the ATCRBS system is acceptable close in, it degrades rapidly with increasing range.

The ATCRBS surveillance model is summarized in the table below.

ATCRBS Surveillance Model

Measurement	Bias (1σ)	Noise (1σ)
Altitude	380. feet	10. feet
Azimuth	.25 degree	.10 degree
Altitude	100. feet	25. feet

Surveillance Data Rate: 1 measurement every 10 seconds

A number of multilateration techniques have been proposed for ATC applications. One such system has been studied extensively (Reference 2) for NASA. This system would provide position updates about every 1.0 to 1.3 seconds. The accuracy of the surveillance system varies somewhat (+25%) with position; hence, average values for the continental United States will be used. The multilateration surveillance model is summarized in the table below.

Multilateration Surveillance Model

Measurement	Bias (1σ)	Noise (1σ)
Lateral Position	180. feet	15. feet
Longitudinal Position	180. feet	15. feet
Vertical Position	300. feet	15. feet

Surveillance Data Rate: 1 measurement per second

7.3 FLIGHT ERROR MECHANISMS

The three major factors which cause an aircraft to deviate from its intended trajectory are 1) navigation errors, 2) flight technical errors, and 3) environmental effects. Navigation errors; which might be simply the result of limitations in the design of the Navigation System or caused by pilot error, are of such long term that they may be neglected in the filter mechanization evaluation. Flight technical errors are those deviations which result from errors by either the pilot or aircraft (including its instruments, etc.). Environmental effects include all errors which arise from sources external to the aircraft (e.g., atmospheric turbulence) and are considered in this study.

The flight technical errors were modeled as deviations from the intended nominal in the speed, altitude, heading, and rate-of-climb (or descent) of the aircraft. Each of these errors was represented as a zero mean gaussian random variable which was added directly to the aircraft dynamics as perturbing state noise. These error sources were modeled as being mutually uncorrelated. It was assumed that during nominally level flight the pilot would attempt to hold a constant altitude and that during ascending or descending flight he would attempt to hold some specified rate of descent. Therefore, altitude variations were modeled by using the

altitude error source during nominally level flight and by using the rate of ascent (descent) error source during ascending or descending portions of flight. The standard deviations for each of these gaussian variables was obtained from the data in the track-keeping portion of the aircraft performance characteristics.

The problem of accurately characterizing the environmental effects (turbulence, wind gusts, etc.) which perturb the motion of aircraft is difficult because of the wide variety of possible atmospheric conditions. Nevertheless, a considerable amount of research has been conducted on the subject in recent years. The work which seems most relevant to this study is contained in References 38, 39, and 40.

Four significant facts are evident.

1) Significant air turbulence occurs rather infrequently. The probability that turbulence of "moderate" or "severe" intensity (as evaluated by pilots) will be encountered during a 100-mile segment of a flight is less than 5%.

2) When turbulence occurs, the correlation between the vertical, longitudinal, and lateral components of gust velocity is high. Measurements show correlations varying from a low of .793 to a high of .971.

3) Turbulence tends to increase with altitude up to the tropopause.

4) Although the distribution of gust velocities is probably not truly gaussian (probability densities from experimental data are somewhat broader than that of a gaussian density), a gaussian distribution will serve as an adequate model.

The turbulence model which was adopted for this study was derived from that developed in Reference 39. The turbulence is modeled as a highly correlated random gaussian vector which exerts velocity increments to the vertical, longitudinal, and lateral components of the aircraft velocity. This is equivalent to state noise on the velocity components of the state vector. The values which were used for the standard deviations of this gaussian vector — 2.3 ft/sec vertical, and 3.0 ft/sec lateral and longitudinal — were taken directly from Table XIII of Reference 39.

These values were derived (in Reference 39) from a vast quantity of experimental data taken in the 30,000 to 70,000 foot altitude range. They represent the "average" amount of turbulence in this region. As noted previously, however, the intensity of turbulence may vary widely and the "average" value may be completely unrepresentative of a "worst case" situation. Therefore, simulations will be performed with the above values increased by a factor of five to simulate "worst case" turbulence conditions.

7.4 AIRCRAFT TRAJECTORIES

The set of aircraft trajectories which will be used to evaluate the performance of the various filters will now be defined. The basic requirement on this set is that it should represent a reasonable range of aircraft maneuvers which the ATC surveillance system is likely to encounter. Five trajectories appear to be sufficient to accomplish this:

Trajectory A: Straight and Level Flight

T = 0 to T = 120 seconds:	Straight and level flight

Trajectory B: Level Turn (Standard to Moderate Rate)

T = 0 to T = 30 seconds:	Straight and level flight
T = 30 to T = 90 seconds:	Level Turn (standard to moderate rate)
T = 90 to T = 120 seconds:	Straight and level flight

Trajectory C: Level Turn (High Rate)

T = 0 to T = 30 seconds:	Straight and level flight
T = 30 to T = 90:	Level Turn (high rate)
T = 90 to T = 120 seconds:	Straight and level flight

Trajectory D: Descent with Turn (Standard to Moderate Rate)

T = 0 to T = 30 seconds:	Straight and level flight
T = 30 to T = 90 seconds:	Steep Descent with Turn (standard to moderate rate)
T = 90 to T = 120 seconds:	Straight and level flight

Trajectory E: Stop and Hover (Helicopter only)

T = 0 to T = 30 seconds:	Straight and level flight
T = 30 to T = 90 seconds:	Stop and hover
T = 90 to T = 120 seconds:	Straight and level flight

The specific values of the various parameters used in the trajectory description will depend on the particular aircraft involved. A table listing the values of these parameters for each of the classes of aircraft under consideration is presented in Table 26.

Table 26. Aircraft Parameters used in Filter Mechanization Trajectories

Parameter	Class I YF-12	Class II B737	Class III Citation	Class IV UH-1
Airspeed (ft/sec)	2,533.3	760.0	422.2	168.9
Low-Moderate Turn Rate (deg/sec)	1.5	1.5	3.0	6.0
Rapid Turn Rate (deg/sec)	2.83	2.42	4.48	10.91
Descent Rate (ft/sec)	83.33	16.66	16.66	40.00
Altitude (ft)	60,000.	40,000.	25,000.	10,000.
Range to ATCRBS Radar (ft)	608,000.	608,000.	152,000.	60,800.

At T = 0 the aircraft is located at the specified distance from the ATCRBS radar and is flying directly toward it at the indicated airspeed. Then, the appropriate maneuver described in the trajectory scenario is executed.

7.5 DESCRIPTION OF FILTERS

One of the principal objectives of this study is to examine the tradeoff between filter performance and filter complexity. This type of analysis is useful from two standpoints. First, it will provide qualitative insight into the relative importance of various aspects of the aircraft dynamics (for example, it would answer questions such as: "How much is the filter performance degraded by assuming that the aircraft's velocity is constant between updates?") Second, it would provide a quantitative basis for making a detailed optimal tradeoff in the allocation of available computer space between the filter and the separation assurance algorithm.

The above-stated objective will be accomplished by considering a sequence of four filters arranged in an ascending order of complexity. The first is an extremely simple constant coefficient filter. The second is a substantially more complex six state variable (position, velocity)

111

Kalman filter; its computer requirements (memory and computational time) are approximately five times those of the first filter. The third and fourth are even more complex nine state variable [(position, velocity, acceleration) and (position, velocity, measurement bias)] Kalman filters whose computational requirements are approximately ten times those of the first filter. Each will be described in detail in subsequent portions of this section.

The following formulation will provide a convenient representation for defining the filters. The aircraft dynamics will be expressed in state variable form as

$$\underline{x}_{i+1} = \underline{\Phi}_i \, \underline{x}_i + \underline{w}_i$$

where

$\underline{x}_i$: denotes the system state vector at t_i

$\underline{\Phi}_i$: denotes the state transition matrix from t_i to t_{i+1}

$\underline{w}_i$: denotes the random gaussian vector of state noise representing the flight technical errors and environmental effects

The relationship between the surveillance measurements and the state vector is

$$\underline{y}_i = \underline{M}_i \, \underline{x}_i + \underline{v}_i$$

where

$\underline{y}_i$: denotes the vector of surveillance measurements at t_i

$\underline{M}_i$: denotes the measurement matrix

$\underline{v}_i$: denotes the random gaussian vector representing the noise on each of the surveillance measurements

The following notation will be adopted and used in the subsequent filter descriptions.

E : denotes the expectation operator

$$\underline{Q}_i = E \left[\underline{w}_i \, \underline{w}_i^T \right]$$

$$\underline{R}_i = E\left[\underline{v}_i \ \underline{v}_i^T\right]$$

$$\overline{\underline{w}}_i = E\left[\underline{w}_i\right]$$

$\hat{\underline{x}}_i$: denotes the state estimate at t_i prior to update

$\overline{\underline{x}}_i$: denotes the state estimate at t_i after update

$$\hat{\underline{C}}_i = e\left[\left(\underline{x}_i - \hat{\underline{x}}_i\right)\left(\underline{x}_i - \hat{\underline{x}}_i\right)^T\right]$$

$$\overline{\underline{C}}_i = E\left[\left(\underline{x}_i - \overline{\underline{x}}_i\right)\left(\underline{x}_i - \overline{\underline{x}}_i\right)^T\right]$$

7.5.1 Simple α-β Constant Coefficient Filter

This filter is the simplest of the four filters and has the least demanding computational requirements. In its basic form, it consists of two prespecified gains which are used to update the state. It does not require the computation of any covariance matrices.

This filter uses a 6-element state vector. The first three components are the position coordinates; the last three are the velocity components. The filter equations are given below

$$\hat{\underline{x}}_{i+1} = \underline{\Phi}_i \ \overline{\underline{x}}_i + \overline{\underline{w}}_i$$

$$\overline{\underline{x}}_i = \hat{\underline{x}}_i + \underline{K}_i \left(\underline{y}_i - \underline{M}_i \ \hat{\underline{x}}_i\right)$$

where

$$\underline{\Phi}_i = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

with T being the sample interval.

Although this filter can be constructed in an arbitrary coordinate system, the complexity of the expression for the gain matrix $\underline{K}_i$ is

113

significantly reduced if the coordinates of the filter and surveillance system are coincident. We shall assume this to be the case. Then, the expression for the gain matrix $\underline{K}_i$ is simply

$$
\underline{K}_i = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \\ \beta/T & 0 & 0 \\ 0 & \beta/T & 0 \\ 0 & 0 & \beta/T \end{bmatrix}
$$

for the cases of interest in this study where the measurement vector consists of a position measurement along each of the three coordinates. Note that $\underline{K}_i$ does not change with time and that it is completely specified by the constants α and β along with the sample interval T.

If α and β are appropriately selected, the performance of this filter in steady state situations can be extremely good. However, its transient behavior is less impressive.

7.5.2 Six State Kalman Filter

This filter is a classical 6 state variable Kalman filter which produces a minimum variance state estimate. It has an intermediate degree of complexity. As with the α-β filter, the first three components of the state vector are the position coordinates; the last three are the velocity components. The filter equations are given below

$$
\hat{\underline{C}}_{i+1} = \underline{\Phi}_i \, \overline{\underline{C}}_i \, \underline{\Phi}_i^T + \underline{Q}_i
$$

$$
\underline{K}_i = \left(\underline{M}_i \, \hat{\underline{C}}_i \right)^T \left[\underline{M}_i \, \hat{\underline{C}}_i \, \underline{M}_i^T + \underline{R}_i \right]^{-1}
$$

$$
\overline{\underline{C}} = \hat{\underline{C}}_i - \underline{K}_i \left(\underline{M}_i \, \hat{\underline{C}}_i \right)
$$

$$
\hat{\underline{x}}_{i+1} = \underline{\Phi}_i \, \overline{\underline{X}}_i + \overline{\underline{w}}_i
$$

$$
\overline{\underline{x}}_i = \hat{\underline{x}}_i + \underline{K}_i \left(\underline{y}_i - \underline{M}_i \, \hat{\underline{x}}_i \right)
$$

114

where the transition matrix $\underline{\Phi}_i$ is identical to that used with the α-β filter.

The performance of this filter should show a moderate improvement over that of the α-β filter under steady state conditions and a substantial improvement in transient situations. The two major limitations on its performance which result from the limited size of the state vector are

1) The acceleration vector is not estimated and is assumed to be zero.

2) Biases in the surveillance system are not estimated and likewise assumed to be zero.

These defects should not produce a significant degradation in performance when used with the satellite-based multilateration system due to its high accuracy and rapid data rate. However, their effect should be considerably more pronounced when used with the ATCRBS system.

7.5.3 Nine State Kalman Filter (Pos., Vel., and Acc.)

This filter, the most complex of those under consideration, is an extension of the preceeding filter; it was formed by adding the three components of acceleration to the state vector. The filter equations are identical to those given for the six state variable Kalman filter. Of course, the vectors and matrices have dimension nine instead of six. For this case the state transition matrix becomes

$$\Phi_i = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 & T^2/2 & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 & 0 & T^2/2 & 0 \\
0 & 0 & 1 & 0 & 0 & T & 0 & 0 & T^2/2 \\
0 & 0 & 0 & 1 & 0 & 0 & T & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & T & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & T \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

The addition of the three acceleration components to the state vector should produce an improvement in the performance during transient situations involving large accelerations (rapid turns, etc.) if the update rate is

115

sufficiently rapid so that the acceleration does not change significantly between updates. If this condition is violated, the performance of this filter may be worse than that of the simpler filters considered previously. For "normal" low acceleration flight, all three filters should produce essentially the same results.

7.5.4 Nine State Kalman Filter (Pos., Vel., and Meas. Biases)

This filter has the same level of complexity as the previous one; in fact, it was formed by merely replacing the three components of acceleration in the state vector with the three measurement biases. For this case the state transition matrix becomes.

$$\Phi_i = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The motivation for considering this type of filter was the belief that the biases in the surveillance systems would be a dominant factor limiting system performance. If these biases could be accurately estimated, the performance should be substantially improved.

7.6 SIMULATION DESCRIPTION

In evaluating the performance of a filter, one must consider both the "real world" and the "filter world." The "real world" is a model which represents all of the significant error sources which influence the performance of the system; it represents reality. The "filter world" consists of those error sources which the filter models and represents. Thus, the "filter world" is the filter's simplified model of the "real world" and is determined by the filter design. If one were to evaluate a given filter in its own "filter world" (i.e., consider only those error sources which the filter modeled), one would obtain an overly optimistic

116

prediction of the filter's actual performance because many of the actual error sources would be omitted. To obtain an accurate indication of the operational performance of a given filter, one must evaluate this filter in a "real world" environment. For the purpose of this study the "real world" error model was defined to consist of the following four error sources.

1) Flight technical errors

2) Environmental effects

3) Noise on the surveillance measurements

4) Biases on the surveillance measurements

A 15 state filter was constructed which correctly modeled all of the error terms in the "real world" model.

Surveillance noise was the only one of the "real world" error sources which was correctly modeled by all four filters under evaluation. Surveillance bias was correctly modeled by the fourth filter only. State noise was used by all four to approximate error sources one and two, but the modeling was inexact.

A block diagram of the computer program is presented in Figure 53. The input parameters control the selection of which trajectory, aircraft, filter, and surveillance system will be used as well as specifying the numerical values of the various system parameters (noise level, biases, intensity of atmospheric turbulence, etc.). The operation of the program can be described as follows. First, the trajectory generator writes the entire trajectory profile (position, velocity, and acceleration) on tape. Then, the desired filter is selected and the flight of the aircraft along the given trajectory is simulated using the selected filter to update the state. The value of the filter gain matrix $\underline{K}_f$ is stored on tape. Then, the flight of the aircraft is repeated along the same trajectory. This time the "real world" filter model is used. However, the general update equation valid for both optimal and suboptimal updates

$$\underline{\overline{C}} = (\underline{I} - \underline{K}\,\underline{M})\,\underline{\hat{C}}\,(\underline{I} - \underline{K}\,\underline{M})^T + \underline{K}\,\underline{R}\,\underline{K}^T$$
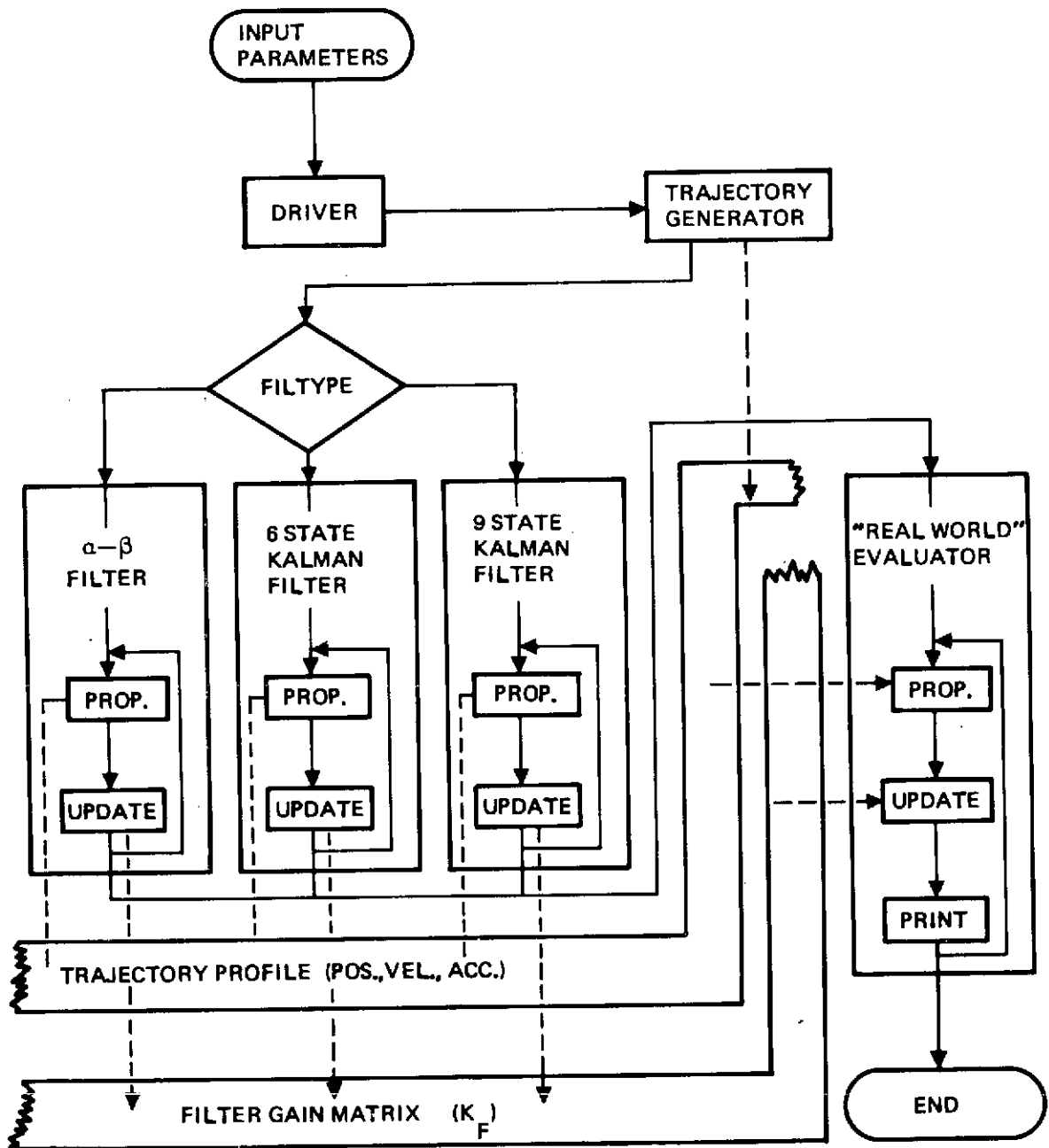
Figure 53. Block Diagram of Computer Program

is used with $\underline{K}$ set equal to the previously computed filter gain matrix $\underline{K}_f$. In this manner one can evaluate the performance of any given filter in a "real world" environment. The relevant statistics are printed out at each time point along the trajectory. The program automatically terminates when the end of the trajectory is reached.

7.7 SIMULATION RESULTS

The various simulations which were performed are described in this section and their results presented and compared. Altogether, a total of 34 simulations were made.

7.7.1 State Noise

Prior to performing these simulations, it was necessary to specify the state noise matrix (required by the six and nine state variable Kalman filters) as well as the two coefficients of the α-β filter. The performance of the filters will depend in a significant manner on the values selected for these quantities since they determine the relative importance which the filter will assign to the surveillance measurements with respect to the projected state estimate. Therefore, care must be exercised to select appropriate values for these quantities.

This problem is complicated by the fact that the best or "optimal" set of values for these parameters depends on the specific application (i.e., aircraft characteristics and trajectory). For the purposes of this study, the average position error was selected as the appropriate criterion for evaluating filter performance; hence, all references of "best" or "optimal" are with respect to this criterion. In an actual operational situation neither the specific aircraft characteristics nor its future trajectory will be known; therefore, the values selected for the state noise and the coefficients α and β must produce acceptable performance over the entire spectrum of possible situations.

The specification of these parameters was accomplished in the following manner. Initially, a variety of computer runs was made using various parameter values, aircraft types, and trajectories. The results indicated that small values of these parameters (i.e., $\alpha \simeq \beta \simeq 0.3$ or 0.4) were optimal in low dynamic situations (i.e., straight and level trajectories with low performance aircraft). Large values ($\alpha \simeq \beta \simeq 0.85$ or 0.90) were optimal in

high dynamic situations (i.e., turns with high performance aircraft). However, the most significant result provided by these runs was that the large parameter values also provided excellent performance (approximately 5% suboptimal) in low dynamic situations while the performance of the low parameter values in high dynamic situations was unsatisfactory — producing errors which were 200% to 300% above optimal. Thus, the obvious conclusion is that the parameters should be selected to produce optimal performance in high dynamic situations. This not only guarantees optimal performance in the most critical situations, but also guarantees satisfactory performance in all situations.

Based on these results, the scenario involving aircraft type II and trajectory D was utilized to determine the state noise matrix and the coefficients α and β. These values were used in all of the simulations. The optimal values of α and β for this scenario using the ATCRBS Surveillance system are:

$$\alpha = 0.85$$

$$\beta = 0.85$$

The optimal state noise matrix for the 6-state variable Kalman filter and the 9-state variable Kalman filter modeling measurement biases was:

$$\underline{Q} = \begin{bmatrix} 2000. & 0. & 0. & 2000. & 0. & 0. \\ 0. & 4000. & 0. & 0. & 4000. & 0. \\ 0. & 0. & 200. & 0. & 0. & 200. \\ 2000. & 0. & 0. & 2000. & 0. & 0. \\ 0. & 4000. & 0. & 0. & 4000. & 0. \\ 0. & 0. & 200. & 0. & 0. & 200. \end{bmatrix}$$

The optimal state noise matrix for the 9-state variable Kalman filter modeling acceleration components was:

$$\underline{Q} = \begin{bmatrix} 5. & 0. & 0. & 5. & 0. & 0. & 0. & 0 & 0 \\ 0. & 5. & 0. & 0. & 5. & 0. & 0. & 0. & 0. \\ 0. & 0. & 5. & 0. & 0. & 5. & 0 & 0. & 0. \\ 5. & 0. & 0. & 5. & 0. & 0. & 0. & 0. & 0. \\ 0. & 5. & 0. & 0. & 5. & 0. & 0. & 0. & 0. \\ 0. & 0. & 5. & 0. & 0. & 5. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

The large values of α and β and the large values appearing in the state noise matrix for the 6-state variable filter both indicate that the filter will weight the measurement very heavily with respect to the propagated state estimate. The small values appearing in the state noise matrix for the 9-state variable filter were initially surprising because of the six state variable noise matrix. However, a careful analysis indicates that this difference is more apparent than real. The explanation is that when the three acceleration components are included in the state vector, the acceleration errors propagate into position and velocity errors and have essentially the same effect as state noise; hence, the much smaller values in the optimal 9-state variable state noise matrix.

The performance of the filters with the multilateration system was found to be relatively insensitive to the state noise matrix, or the coefficients α and β, as long as they were reasonably large. This behavior is due to the rapid update available with this system. Hence, the values used for the ATCRBS system were also used for the multilateration system.

The parameters used in each simulation, the purpose of each simulation, and the corresponding results are tabulated in Table 27; the average and worst-case values of the errors in the position, velocity, and acceleration are tabulated.

Table 27. Simulation Description and Results

Run	Traj	Air-craft Class	Filter type	Sur-veil-lance type	Position Error Av.	Position Error W.C.	Velocity Error Av.	Velocity Error W.C.	Acceleration Error Av.	Acceleration Error W.C.	Purpose
1	A	II	1	1	2750.	3120.	120.	135.	0.1	0.1	
2	C	II	1	1	3225.	4640.	265.	491.	21.4	32.1	
3	D	II	1	1	2970.	3803.	198.	334.	13.4	26.0	
4	A	II	2	1	2727.	3040.	104.	114.	0.1	0.1	Comparison of Filters
5	C	II	2	1	3082.	4539.	238.	482.	21.4	32.1	
6	D	II	2	1	2910.	3809.	181.	341.	13.4	26.0	
7	A	II	3,4	1	2751.	3129.	129.	176.	4.4	5.0	
8	C	II	3,4	1	3178.	5077.	259.	577.	24.4	40.8	
9	D	II	3,4	1	2927.	3715.	193.	341.	16.7	25.5	
10	A	III	2	1	747.	915.	54.3	71.9	0.1	0.1	
11	C	III	2	1	1622.	4810.	242.7	550.2	22.0	33.0	
12	D	III	2	1	1291.	2747.	179.	355.	14.8	27.7	
13	A	III	2	2	394.	394.	17.8	22.3	0.1	0.1	Comparison of Surveillance Systems
14	C	III	2	2	395.	395.	38.4	52.0	22.0	33.0	
15	D	III	2	2	394.	395.	31.9	40.8	14.8	27.7	
16	A	I	2	1	2987.	3694.	108.	133.	0.1	0.1	
17	B	I	2	1	3303.	7650.	438.	1013.	44.6	83.3	
18	C	I	2	1	4901.	12167.	799.	1792.	83.4	125.	
19	D	I	2	1	3312.	7666.	422.	1017.	45.1	107.	
20	B	II	2	1	2902.	2775.	175.	333.	13.3	19.9	Comparison of Aircraft and Trajectories
21	B	III	2	1	1038.	1995.	143.	297.	14.8	22.1	
22	A	IV	2	1	532.	749.	54.1	79.7	0.1	0.1	
23	B	IV	2	1	748.	1693.	122.	248.	12.1	40.0	
24	C	IV	2	1	947.	2537.	183.	361.	21.7	40.0	
25	D	IV	2	1	823.	1914.	54.0	284.	12.4	43.7	
26	E	IV	2	1	545.	749.	54.0	80.0	0.45	40.0	
27	D	III	2	1	1912.	4241.	383.	610.	14.8	27.7	Pilot Error X5
28	D	III	2	1	1230.	2815.	187.	365.	14.8	27.7	Environ. Error X5
29	D	III	2	1	1851.	3013.	180.	355.	14.8	27.7	Bias and Noise X2
30	D	III	2	2	788.	788.	33.9	42.2	14.8	27.7	Bias and Noise X3
31	D	III	2	1	909.	1401.	117.	246.	14.8	27.7	Update Rate =4 sec
32	D	III	2	2	399.	400.	32.0	41.0	14.9	27.7	Pilot Error X5
33	D	III	2	2	395.	396.	31.9	40.8	14.9	27.7	Environ. Error X5
34	X	III	2	2	416	480.	65.7	122.5	14.8	27.7	Update Rate = 4

Sensitivity Analysis (applies to runs 27–34)

The units of the above quantities are feet, feet/sec, and feet/sec^2.

Nomenclature:

Trajectory Types	A : Straight and Level
	B : Level Turn (Standard to Moderate)
	C : Level Turn (High Rate)
	D : Descending Turn (Standard to Moderate)
	E : Stop and Hover (Class IV Only)

Aircraft Classes	I : YF-12
	II : B-737
	III: Citation
	IV : UH-1

Filter Types	1 : Constant Coefficient
	2 : 6-State Kalman (Positive, Velocity)
	3 : 9-State Kalman (Positive, Velocity, Acceleration)
	4 : 9-State Kalman (Positive, Velocity, Measurement Bias)

Surveillance Types	1 : ATCRBS
	2 : LIT

7.7.2 Results

A comparison of the performance (i.e., the average position errors) of the four filters was conducted for both surveillance systems. The results showed that the performance of the multilateration system was essentially independent of the filter used. The variation in performance among the four filters was less than 1%. This result is due to the combined effect of the rapid update rate and the unmodeled measurement biases of this surveillance system. This filter must weigh each measurement very heavily relative to its projected state estimate. Hence, the sophistication of the more complex filters is of little utility with the multilateration surveillance systems.

A similar comparison was conducted with the ATCRBS surveillance system and the results are presented in Table 28; a Class II aircraft was used in all runs.

Table 28. Comparison of Filters

Trajectory	α-β Filter	6-State Filter	(Pos., Vel., and Acc.)
A. Straight and level	2750. ft	2727. ft	2751. ft
C. High rate turn	3225. ft	3082. ft	3178. ft
D. Descending turn	2970. ft	2910. ft	2927. ft

Again, the major conclusion to be drawn from the above table is that the performance of all three filters is essentially equivalent. The 6-state variable filter does produce slightly better results (particularly in high dynamic situations), but is maximum improvement is less than 5% over that of the α-β filter. This small improvement in performance hardly seems to justify its greater complexity.

The performance of the 9-state variable Kalman filter which estimates measurement biases (Runs 7, 8, 9,) is precisely identical to the 6-state variable Kalman filter and it was unable to improve on the a priori estimates of the measurement biases. This was due to two factors:

1. Since the aircraft's initial position was completely unknown, the filter was unable to differentiate between the aircraft's position and the measurement biases.

123

2. The large amount of state noise required to force the filter to track the aircraft acceptably during higher rate maneuvers "washed out" the past history of the aircraft trajectory and precludes the possibility of varying geometry helping the estimation process.

Therefore, the procedure of estimating the measurement biases with a 9-state variable Kalman filter does not appear useful and should not be pursued.

A comparison of the performance (i. e., the average position errors) of the two surveillance systems is presented in Table 29. A Class III aircraft and the 6-state variable Kalman filter were used.

Table 29. Comparison of Surveillance Systems

Trajectory	ATCRBS	Multilateration
A	747. ft	394. ft
C	1622. ft	395. ft
D	1291. ft	394. ft

The performance of the multilateration system was significantly better than that of the ATCRBS over all trajectories. Furthermore, the multilateration error was consistently in the range of 390-400 feet; it was almost completely independent of the aircraft and trajectory. In contradistinction, the error produced by the ATCRBS system was very sensitive to the aircraft type and trajectory.

The variation in the performance (i. e., the average position error) of the ATCRBS system with trajectory and aircraft type is shown in Table 30.

Table 30. Comparison of Trajectories and Aircraft Type

| Trajectory | Aircraft Class | | | |
	I	II	III	IV
A	2987. ft	2727. ft	747. ft	532. ft
B	3303. ft	2902. ft	1038. ft	748. ft
C	4901. ft	3082. ft	1622. ft	947. ft
D	3312. ft	2910. ft	1291. ft	823. ft
E	--	--	--	545. ft

The sensitivity of the performance (i. e., the average position error) to certain factors is presented in Table 31. Class III aircraft, trajectory D, and filter II were used.

Table 31. Sensitivity Analysis

Simulation Condition	ATCRBS	Multilateration
Nominal	1291. ft	394. ft
Pilot Error X5	1912. ft	399. ft
Environ. Eff. X5	1320. ft	395. ft
Surveillance Bias X2	1851. ft	788. ft
Sample period = 4 Sec	909. ft	416. ft

These results show that pilot error, surveillance errors (biases, etc.), and the surveillance update rate have a critical effect on the ATCRBS system performance.

On the other hand, only the biases in the surveillance system have an appreciable effect on the performance of the multilateration system. In particular, note that decreasing the surveillance time from 1 second to 4 seconds degrades performance only 5%. The effects of the environment (atmospheric turbulence, etc.) were completely negligible.

For a system with serially uncorrelated (white) noise and no biases, the estimation of position and velocity would be expected to improve by a factor proportional to the square root of the ratio of sample intervals. With bias, and sample intervals close to the correlation time constants, the improvement would be less. As the sample interval become very short compared to the time constant, the error in state estimate begins to look like bias and the measurement bias becomes more significant.

For the ATCRBS, although there appears to be an appreciable improvement in going from one sample per 10 seconds to one sample per 4 seconds, a similar improvement with shorter sample intervals would not be expected since the biases would become more prominent.

For the multilateration system, going from sample intervals of 4 seconds to 1 second, the small improvement in state estimate achieved would indicate that the measurement biases are already dominating and even smaller improvements would be achieved by a further decrease in sample interval.

7.8 SUMMARY DISCUSSION

The objective of this portion of the study was to determine the accuracy with which the state (i. e., the position and velocity of the various aircraft) can be estimated and how this accuracy depends on the surveillance system, aircraft dynamics, and data processing (filtering) procedure. The results of this analysis have led to the following observations:

1. The accuracy with which the position and velocity of the aircraft can be specified depends strongly on the characteristics of the surveillance system, aircraft, and trajectory; however, the accuracy is relatively independent of the type of filter used. This indicates that in this application (tracking and anomaly detection) sophisticated data processing is unable to compensate for poor data (i. e., noisy data containing large biases generated at a slow data rate). Hence, it should prove more profitable to concentrate on improving the surveillance system rather than the data processing scheme.

2. The existing ATCRBS system with 10 seconds surveillance intervals is incapable of tracking many types of currently operational aircraft (executing maneuvers within their allowed dynamic constraints) with an error of less than 2500 feet irrespective of the degree of sophistication of the data processing used. This is mainly due to the length of time between surveillance updates (not measurement inaccuracy) — an aircraft can be perfectly on course at one surveillance time, then execute a 1.5 or 3 g maneuver and be 2000-3000 feet off its origional 10 seconds later just before the next measurement occurs.

3. The performance of the multilateration system with 1 second surveillance intervals is totally determined by and directly proportional to the bias errors in the range measurements. Nominal values for these bias errors in a typical* satellite system produce a maximum error of 400 feet.

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*Not optimized for accuracy. See Reference 22.

4. There are two possibilities for improving an ATCRBS-based system; first, decrease the measurement, biases; and second, either increase the surveillance rate or incorporate on-board data (i. e., the measured acceleration and orientation of the aircraft) into the data processing scheme. The former will reduce the steady-state errors while the latter will improve the detection and tracking of transient maneuvers.

5. The same two possibilities exist for improving a multilateration system. The measurement bias could be reduced by establishing several precisely located calibration stations which would continuously estimate the bias at their location. TRW has performed an extensive analysis (Reference 9) on the navigation accuracy obtainable by combining multilateration surveillance with low-cost on-board inertial instruments; the general result is that the aircraft's position can be continuously determined to within 50-75 feet.

6. One of the questions posed at the outset of the study was "If studies indicate that, for a particular case, a position accuracy of, say, 1000 feet is required, could this accuracy be obtained with a filtering technique applied to a system with raw surveillance accuracy of, say, 5000 feet?" As stated previously, the accuracy with which the position and velocity of the aircraft can be specified depends on the characteristics of the surveillance system, aircraft, and trajectory. If the aircraft flies a straight line trajectory with a constant velocity, the filter may reduce the position uncertainty by a factor of from 5 to 10 below that of the raw surveillance data; however, in high dynamic situations the improvement is minimal.

7. Another question posed in the study was "For cases in which the control law uses velocity data, how accurately can one obtain this velocity through the application of an "optimal" filter process?" Again, this depends strongly on the particular surveillance system, aircraft, and trajectory under consideration as shown in Table II. The average error in the velocity estimate varies from a low of 17.8 ft/sec to a high of 38.4 ft/sec for the multilateration system and from a low of 54.0 ft/sec to a high of 799. ft/sec for the ATCRBS system. The lower errors occur in constant velocity trajectories while the large errors result from high dynamic situations.

8. The surveillance data rate is a very sensitive parameter in the ATCRBS system, but rather insignificant in the multilateration system. An increase in the ATCRBS surveillance data rate from 1 measurement every 10 seconds to 1 measurement every 4 seconds produced a 37% reduction in the position estimation error. A decrease in the multilateration surveillance data rate from 1 measurement every second to 1 measurement every 4 seconds only produced a 5% increase in the position estimation error.

8. SPECIAL STUDIES

This Section contains two special studies performed as part of the Surveillance Accuracy/Update Rate analysis. The first is an investigation of a number of actual mid-air collisions and the second is a brief analysis of Remote Area ATC communications.

8.1 MID-AIR COLLISION STUDY

8.1.1 Background

The mid-air collision in September 1969 between an Allegheny DC-9 and a corporate PA-28 (Reference 18) prompted the National Transportation Safety Board to review the entire collision problem to determine its magnitude, what actions were being taken to solve the problem, additional research required, and state-of-the-art in collision avoidance systems. The report of those proceedings (Reference 22) contains the following indictment of the see and be seen concept:

> "For many years it has become increasingly apparent that conditions other than weather conditions are being encountered which directly affect aircraft separation and of which account must be taken in the continued development of the air traffice rules. For instance, it appears that under certain circumstances the rate of closure of very high-speed aircraft is such that the total time in which an aircraft may be visible to a pilot of another aircraft is so short that pilots cannot be expected to insure separation between aircraft irrespective of the weather conditions in which they are flying. It is also apparent that the density of air traffic, particularly in the vicinity of certain major air terminals, has approahced or is approaching serious proportions. Obviously, the greater the number of aircraft movements within a given airspace the more difficult it is for a pilot to separate himself adequately from other aircraft regardless of the vigilance exercised."

It is even more tragic to realize that that statement was made by the deputy director of the Bureau of Safety Regulation of the Civil Aeronautics Board — not in 1969, but in 1956!

Another insidious aspect of visual flight rules is the concept of mixed airspace. Ironically, it is formally called "controlled airspace." Aircraft flying in mixed airspace may be on instrument flight rules,

perhaps going in and out of clouds such that the pilot on board is flying primarily on instruments.[*] He is in communication with an air traffic controller who is giving him advisories on other traffic in the area. Thus, he naturally and inevitably develops a frame of mind wherein he feels "protected." Another aircraft can be flying quite legally under visual flight rules in this same area. Because flying VFR he does not need to have an ATC transponder, or, if he has one, he is not required to turn it on. Thus, the VFR pilot may be exercising proper vigilance and the IFR pilot may also be performing his normal IFR flight taks diligently, but may be concentrating more on his flight instruments than looking outside, especially if he is flying in and out of clouds. A dangerous situation could develop even though the VFR pilot, the IFR pilot and the air traffic controller are all performing their individual tasks competently and diligently. The probability of a mid-air collision could actually be increased by giving pilots on instrument clearances periodic advisories on the VFR traffic. Another aspect of the problem is speed itself. Two turbo-jet aircraft in a head-on or right-angle encounter close on each other so fast that a threat aircraft can loom from a tiny, almost invisible speck on the windscreen to a real threat in a matter of seconds. Furthermore, even after one pilot spots the other aircraft approaching him at a relative speed of 800 to 1000 knots, it is often difficult to guage what the correct maneuver should be in time to perform it.

Of course, the question of prevention of mid-air collision goes beyond the deficiencies of mixed airspace and the see-and-avoid concept. The preponderance of loss of life associated with mid-air collisions involving air air carrier aircraft indicates the importance of an examination of mid-air collisions involving air carriers in all types of airspace. Because of the similar characteristics of air carrier and military aircraft, and because of the importance of the military/civil interface itself, a large number of mid-air collisions involving both air carrier/general aviation and military/civil encounters were examined. Furthermore, since the numberical preponderance of mid-air collisions occurs between general aviation aircraft at or near an airport, this area, too, was examined carefully.

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[*]See the narrative of MAC #10 in Table 32.

Fifty-five major accidents spanning the years from 1949 to 1971 were evaluated, first to determine if general patterns or trends existed, as part of the control philosophy analysis; second, to aid in the selection of the cases used in the SAUR analysis. The first thirty-five cases treat mid-air collisions involving military/civil and air carrier/general aviation mid-air collisions. The next twenty cases treat mid-air collisions that occurred in airport traffic patterns during 1968.

8.1.2 Results

The results for the first thirty-five are tabulated by individual case in Table 32, and the next twenty cases are shown in Figure 54. Although it is not evident from Table 32, the amount of data available on the various accidents (References 3-21) varied a great deal. Those accidents of major importance (e. g., MAC's #20, 22, 26, 31, and 35) were documented extensively. Others are documented only in annual National Transportation Safety Board (formerly Civil Aeronautics Board) briefs. Prior to 1960 the briefs were quite short and the "indicated cause" was inferred herein. In addition, even for the more recent and more complete reports this author occasionally added "indicated causes" to the "probable cause" determined by the National Transportation Safety Board (e. g., transponder item, MAC #22).

8.1.3 Discussion

Examination of a large set of mid-air collision reports makes evident the point that the "see-and-avoid" concept requires:

- The _undivided_ attention of all pilots,

- Favorable weather, lighting, and geometry, _and_

- Low aircraft density.

It is quite clear that all of the above conditions simply will not prevail all of the time. When they do not it is a certainty that a number of mid-air collisions will take place.

Pilots cannot be expected to give their undivided attention to looking for other aircraft when they are preoccupied with tasks such as:

- Instrument flying (regardless of weather conditions),

- Taking off, landing (including approach/departure transition),

- Participating in aircrew instruction.

131

Table 32. Mid-Air Collision Evaluation

MAC Number	Date and Location	Aircraft Involved	Brief Narrative	Indicated Cause
1.	7/30/49 Chesterfield New Jersey	F-6F-5, DC-3	The DC-3 was in cruising flight under VFR conditions. The F-6 was engaged in illegal high-speed acrobatics on a civil airway.	• Violation of procedures. • High speed differential
2.	10/24/56 Midland, Texas	T-33, Cessna 170	The T-33 was in a climb over-taking the Cessna from the rear. The T-33 was observed to roll just prior to impact.	• T-33 pilot FTSA[*] • High-speed differential
3.	5/29/57 Baltimore, Maryland	C-45, Cessna 182	Under VFR conditions, both air-craft were practicing IFR ap-proaches. Mid-air collision occurred just after they passed the radio range.	• Both pilots FTSA
4.	9/9/57 Fullerton, California	S-2F, PA-22	The PA-22 was on a simulated instrument approach to the air-port. Both aircraft made sharp diving turns just prior to impact.	• Both pilots FTSA • Probable wrong evasive maneuver
5.	10/17/57 Barrington, Illinois	F-86, PA-23	A mid-air collision occurred at night at 2000 feet 10 miles from an airport.	• Both pilots FTSA
6.	11/14/57 Sioux City, S. Dakota	F-84, Beech G-35	The F-84 was landing at one air-port and the Beech had just taken off from another	• Both pilot FTSA
7.	4/5/58 Huntington Beach, Calif.	Navy T-34, Cessna 170	The T-34 rapidly overtook the Cessna 170. No evasive maneu-vers by either aircraft.	• T-34 pilot FTSA • High speed differential
8.	4/21/58 Las Vegas, Nevada	F-100F, DC-8	The F-100F was on an instrument training flight from Nellis AFB. The mid-air collision occurred at a very high rate of closure with with aircraft approaching at approximately right angles. The F-100 pilot rolled just prior to impact.	• High rate of closure • Human and clock limitations • USAF and CAA procedural defficiencies • Probable wrong escape maneu-ver by F-100 pilot
9.	5/20/58 Brunswick, Maryland	T-33 Viscount	Under VFR conditions the T-33 on a local flight, overtook the Viscount (290 knots versus 235 knots). No evasive maneuvers.	T-33 pilot FTSA
10.	8/24/59 Morehead City N. Carolina	Two A-4D's, PA-18	The PA-18 was on a commercial fish spotting flight flying in a restricted area. The flight of two A-4's while on a radar approach came out of clouds and hit the PA-18 10 seconds after leaving the clouds.	• Regulatory deficiencies • Possible violation of procedures by the PA-18
11.	11/7/59 Mansfield, Ohio	Four F-84F's, PA-22	A flight of four F-84F's were making a pass at Mansfield Airport when the No. 4 air-craft in the flight struck the PA-22. The PA-22 was flying in the airport control zone without having contacted the tower. The CAB placed about equal blame on the F-84 lead pilot, the PA-22 pilot and the tower operator. All were considered to have been deficient in not spotting all aircraft involved.	• F-84 lead pilot and PA-22 pilot — FTSA* • High speed differential • Possible violation of procedures by the PA-22 pilot

[*]FTSA — Failed to See and Avoid

Table 32. Mid-Air Collision Evaluation (Continued)

MAC Number	Date and Location	Aircraft Involved	Brief Narrative	Indicated Cause
12.	Cheyenne, Wyoming	Two F-86L's Beech C-35	The Beech was flying VFR in an airport control zone. The F-86 flight overtook the Beech at a high rate of closure and the No. 2 aircraft struck the Beech.	• F-86 flight lead — FTSA • High rate of closure
13.	1-27-60 El Cajon, California	AD-5, Cessna 182	The Cessna 182 was on an instrument training flight and was in a turn when the AD-5 who was starting a turn collided with the 182.	• Both pilots — FTSA
14.	4/20/60 Hickory, N. Carolina	Piedmont Cessna 310	The Cessna 310 overtook the F-27 on final approach to the Hickory Airport. The Cessna 310 was on the flight service station frequency while the F-27 was on the airline company frequency. There was no tower at the airport.	Cessna pilot — FTSA
15.	5/27/60 Pt. Mugu, California	F-4D, Cessna 172	The Cessna 172 was on an instrument navigation flight (dual). The F-4D accelerating after the GCA collided with the 172 on climb-out. The F-4D pilot tried to nose-down just prior to collision.	Both pilots — FTSA*
16.	11/17/60 Denver, Colorado	UAL DC-6, Beechcraft	This mid-air collision occurred during heavy traffic conditions at Stapleton Field, Denver, Colorado. Many aircraft and very heavy voice traffic existed.	• DC-6 pilot — FTSA due to sun glare • Beechcraft pilot failed to follow procedures • Heavy traffic
17.	12/16/60 Staten Island, New York	TWA L-1049, UAL DC-8	Both aircraft were approaching New York airports under instrument flight rule conditions. The DC-8 exceeded its clearance limits, causing the mid-air collision which apparently took place in the clouds.	Violation of procedures by DC-8 pilots
18.	5/16/64 Westminister, California	A-4, Ercoupe	The Ercoupe was in cruising flight, the A-4 was descending to land when the collision took place under IFR conditions.	Both pilots — FTSA
19.	10/12/65 Montgomery, Alabama	T-33, PA-28	The T-33 was on an instructional flight, the PA-28 was in normal cruise, when the collision occurred.	T-33 pilot — FTSA
20.	12/4/65 Carmel, New York	Lockheed 1049, Boeing 707	The two aircraft were flying under VFR conditions. As the Lockheed 1049 approached a radio fix, the first officer saw the 747 at his 2 o'clock position. Because he believed the jet was at his altitude and on a collision course, he called "look out!" and grasped the control wheel to assist the captain in a pull-up. At about the same time the captain of the other aircraft saw the 1049 at his 10 o'clock position. He rolled into a right turn and pulled back on the yoke.	• Human error • No ATC assistance or advisories

*FTSA — Failed to See and Avoid

133

Table 32. Mid-Air Collision Evaluation (Continued)

MAC Number	Date and Location	Aircraft Involved	Brief Narrative	Indicated Cause
20.			(Continued) He then decided this maneuver would not clear the other aircraft and, assisted by his first officer, attempted to reverse and turn by rolling to the left and pushing on the yoke. The aircraft collided at an altitude of about 11,000 feet. The CAB found that mis-judgement of altitude separation by the 1049 PLM because of an optical illusion created by the upslope effect of cloud tops led to this collision.	
21.	11/6/66 Merced, California	C-141, Cessna 150	The C-141 was holding in a turn when it struck the Cessna which was in cruising flight.	C-141 pilot — FTSA*
22.	3/9/67 Urbana, Ohio	DC-9, Beech B-55	The DC-9 was under positive control by an Air Force radar approach control facility. 18 seconds prior to the collision the RAPCON issued a traffic advisory to the DC-9. The Beechcraft was operating under VFR and was not in contact with any FAA facility, nor was its transponder operating.	• DC-9 pilot — FTSA • B-55 operating with with transponder turned off
23.	4/28/67 Andersonville, Tenn.	RF-101 Beech B-55	Both aircraft were cruising under VFR conditions.	Both pilots — FTSA
24.	5/30/67 Phoenix, Arizona	T-33, Cessna 182	Both aircraft were in crusing flight under VFR conditions.	T-33 pilot — FTSA
25.	6/22/67 Saigon, Vietnam	RF-4C, Lockheed 1049	This was a night mid-air collision involving aircraft operating without running lights due to a combat situation.	Special conditions
26.	7/19/67 Hendersonville, N. Carolina	Boeing 727 Cessna 310	This mid-air collision occurred shortly after the Boeing 727 lifted off from runway 16 at the Ashville Municipal Airport. The 727 was proceeding according to clearance when it collided with the Cessna 310. Apparently the pilot of the 310 had become confused and was making the wrong approach to the Ashville Airport.	• Cessna 310 pilot deviated from IFR instructions
27.	6/20/68 Indian Springs, Nevada	F-105, Mooney M-20A	This collision occurred under VFR conditions. Both aircraft in normal cruise.	F-105 pilot — FTSA*
28.	3/5/69 Julian, California	F-8J, PA-28	Both aircraft were under normal cruise, but were not under radar control. The F-8 apparently attempted some unknown evasive action just prior to the collision.	F-8 pilot — FTSA
29.	4/20/69 El Paso, Texas	T-37, PA-23	The T-37 was descending and the PA-23 in cruise flight when the collision occurred. The PA-23 attempted some evasive action. There was no radar service not ATC advisories.	• T-37 pilot — FTSA • PA-23 pilot inadequate planning and failed to follow procedures.**
30.	8/3/69 Ft. Worth, Texas	Boeing 707, Cessna 172	Both aircraft were reported to be in normal cruise, although the 707 was under approach control and in radar contact. The radar operator did not see the 172. Neither aircraft had warning and no evasive action was taken.	Both pilots — FTSA

*FTSA — Failed to see and Avoid
**Not explained in accident brief.

Table 32. Mid-Air Collision Evaluation (Continued)

MAC Number	Date and Location	Aircraft Involved	Brief Narrative	Indicated Cause
31.	9/9/69 Fairland, Indiana	DC-9, PA-29	The PA-29 was on a solo VFR cross-counter. The aircraft did not have a transponder. The DC-9 was in and out of clouds on a radar approach.	ATC deficiencies, i. e., mixed airspace
32.	3/18/70 Lakeland, Florida	A-7B, PA-18	Both aircraft were in normal cruise under VFR conditions and not under control of ATC. The A-7 tried evasive action.	High rate of closure prevented successful evasive action
33.	5/25/70 LaPorte,	F-102, PA-28	The F-102 was on a scramble climb under radar control and did not see the PA-28. The accident brief notes that the PA-28 executed improper evasive maneuvers but did not explain. The brief also points out that the radar controller didn't observe the return from the PA-28.	F-102 pilot — FTSA*
34.	6/19/70 Myrtle Beach, S. Carolina	T-33, PA-24	The T-33 was operating on tower frequency with an operative transponder which was turned off. The PA-24 was operating under sender frequency, was under radar surveillance but the T-33 was not visible to the radar.	• Both pilots — FTSA* • T-33 operative transponder turns off
35.	6/6/71 Duarte, California	F-4 DC-9	A hughes Air West DC-9 which had departed from Los Angeles International Airport on a flight to Salt Lake City was reported on course and climbing at 12,000 feet when the collision occurred. The Marine F-4 was enroute from Fallon Naval Air Station, east of Reno, to El Toro Marine Corps station in Orange County, Calif. and was in a descent, apparently, at the time of the collision. The National Transportation Safety Board's findings have not been published. Most of the data available on this accident was taken from the public media.	• Both pilots — FTSA • Very high rate of closure • F-4 transponder inoperative • Regulatory deficiencies • Probable wrong evasive maneuver by the F-4 pilot.

*FTSA — Failed to See and Avoid.

135
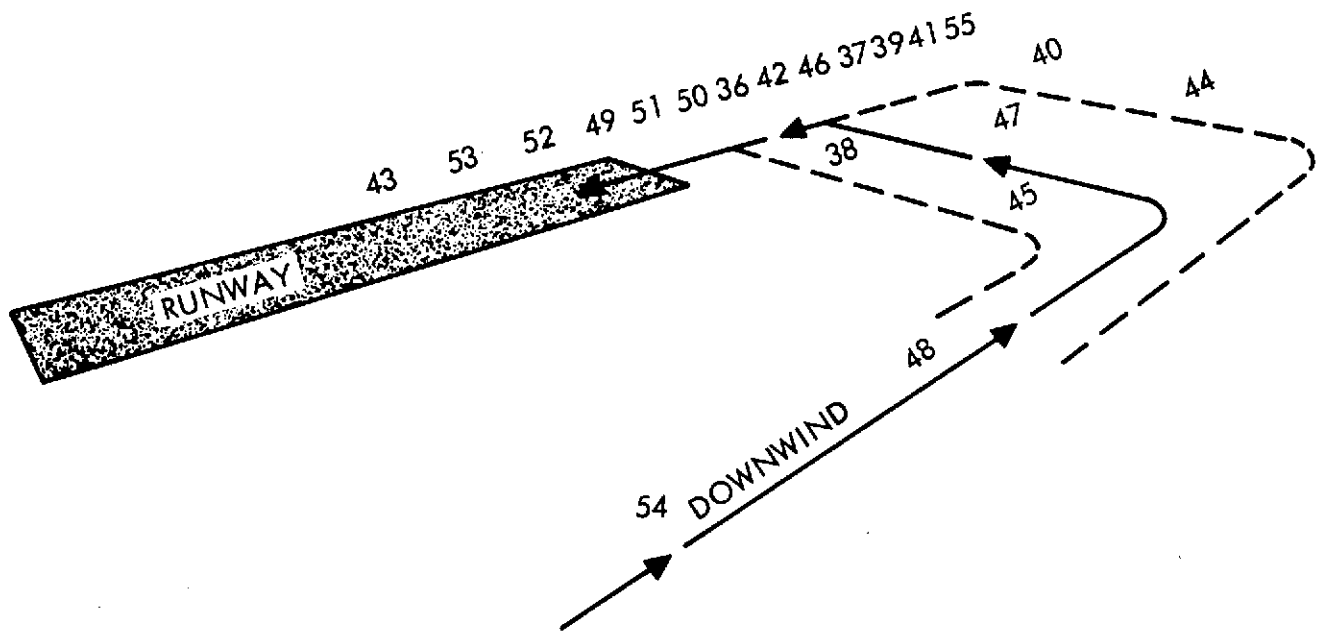
Indicated / Probable Cause

36.	One Pilot FTSA, violation	46.	Two Pilots FTSA, violation
37.	One Pilot FTSA, possible violation	47.	Two Polots FTSA
38.	One Pilot FTSA	48.	Two Pilots FTSA
39.	Two Pilots FTSA, violation	49.	One pilot FTSA, traffic controller improper performance
40.	One Pilot FTSA, violation		
41.	Two Pilots FTSA, weather factor	50.	One Pilot FTSA
42.	Two pilots FTSA	51.	One Pilot FTSA
43.	One Pilot FTSA, weather	52.	Not determined
44.	Two Pilots FTSA	53.	Not determined
45.	One Pilot FTSA, possible violation	54.	Not determined
		55.	Not determined

Figure 54. Airport Traffic Pattern Mid-Air
Collisions, 1968 (Cases 36-55)

136

It seems logic then to "protect" pilots engaged in these activities by placing them under some form of active air traffic control. Similarly, it seems logical to control aircraft flying in poor flight conditions and in high density regions. The weather problem has, of course, been addressed by instrument flight rules for decades. The high density regions are now beginning proposed or controlled areas. It now seems prudent to address the item of pilot activity and preoccupation.

From the tabulation of indicated causes (Table 33) the most obvious point is that in the majority of cases the indicated cause was simply that one or both pilots failed to see and avoid (noted as "FTSA" in the table) the other aircraft. It is likely that in most cases the "FTSA," "high rate of closure," and "equipment not activated" causes can be equated with regulatory deficiencies. For example, Cases 3, 8, and 15 indicate that regulations might be altered to insure that aircraft practicing instrument flying in fairly dense regions operate either under positive control or some form of observation and advisory (e. g., Intermittent Positive Control) conditions. Regulations have already been changed to alleviate the high rate of closure problem which was noted in 8 of the 35 mid-air collisions.

Table 33. Tabulation of Indicated Causes

		MAC #1-35	MAC #36-55	TOTAL
A.	Failed to See and Avoid Other Aircraft (FTSA)	26	16	42
B.	High Rate of Closure	8	0	8
C.	Regulatory Deficiencies	3	0	3
D.	In-Flight Equipment Failures	0	0	0
E.	Equipment not Activated	3	0	3
F.	Violations or Failures to Follow Procedures (including possible violations)	8	6	14
G.	Wrong Evasive Maneuver	4	0	4
H.	Other Human Error	2	1	3
I.	Other/Not Determined	4 / 58	6 / 29	10 / 87

Note: Sum of causes exceeds the number of MAC's because some accidents involved multiple causes.

137

Of course, it is not reasonable to expect that mid-air collisions will cease to exist. The violations and human errors (Causes F, G, and H) simply reflect the human element in the air transportation equation. The negative effects of this element can be reduced with sufficient effort, however. Civil and military flying safety programs stressing the human element have been shown to be beneficial. For example, pilots could be made aware of an observation concerning wrong evasive maneuvers on the part of the pilot (Item G. of Table 33). Specifically, there are a number of documented cases where a pilot would see another aircraft just prior to a mid-air collision and then roll his aircraft in order to avoid a collision. This is precisely the wrong maneuver. Almost invariably the optimum maneuver would be a pull-up. Assuming that the two aircraft trajectories would result in a collision, the pilot must change his flight path as much and as fast as possible without doing structural damage to the aircraft. A rolling maneuver — for the first few seconds — normally changes the flight path little if at all. The reason is that in executing a turn a pilot first rolls or banks the aircraft and then pulls back on the control column to cause the aircraft to turn. The banking maneuver takes up that valuable 1 to 4 seconds that it is not until the elevator is raised, starting a turn, that the flight path changes at all. Simply put then, the quickest way to apply the most acceleration and therefore change flight path by the greatest number of feet in a given amount of time is to pull back on the elevator. Assuming that most of the time only one pilot will see the other aircraft and attempt a maneuver (Reference 41), this would increase the probability of avoiding a collision. It could, for example, have prevented the loss of 50 lives in the Duarte collision (Case 35). Attacking the human element is not central to this study, but understanding its relationship to terminology factors is important.

During the period in which this SAUR study was performed for NASA, the MITRE Corporation conducted an extensive Civil Aviation Collisions Analysis for the Office of Systems Engineering Management, Federal Aviation Administration (Reference 42). An overview of the highlights and conclusions of the MITRE study became available at the conclusion of this SAUR STUDY. The MITRE effort involved determining the statistics of actual mid-air collisions; assessing the effectiveness of the present ATC system in preventing collisions; and comparing proposed solutions with actual

mid-air collisions data. Although it was not possible to evaluate Reference 42 in detail, it does appear that the results of the two efforts (in those areas where a particular topic was addressed in both studies) are in agreement. For example, the two services agreed on the implications of the mid-air collisions that occurred at Carmel, New York October 19, 1965. We also agree with the MITRE concerning the advisability of pursuing cost-effective delivery of traffic advisory and control services, e.g., IPC.

In conclusion it is significant — and ironic — to note that none of the mid-air collisions studied here had as even a secondary cause in-flight equipment failures. In the final analysis, however, over half the indicated causes are "fail to see and avoid" and it is that cause which must be attacked if the nation is serious about resolving the mid-air collision problem. The answer appears to lie in a higher degree of control, recognizing that the task must be one that is manageable and the cost must be acceptable to participating parties. The SAUR study is designed to examine some of the key parameters of this control function.

8.2   AIR TRAFFIC CONTROL COMMUNICATIONS IN REMOTE AREAS

8.2.1   Introduction

As pointed out in Section 1.1, NASA initiated this study in order to establish a substantial basis for the performance capability than an air traffic control satellite system would be required to provide. Although the primary emphasis on the study has been on the surveillance aspects of the problem, it was also recognized at the outset that the possibility existed that the communications function, especially as it related to remote area communications could be a pacing factor. The FAA has pointed out (Reference 43) that there are towers at less than one-half the airports. As a result, service to the second and third level carriers is less than satisfactory. FAA further stated that they need relatively inexpensive surveillance and communications in these areas. The average communications load would not be great but the need for traffic control to notify the aircraft involved is immediate when a collision situation developes. Accordingly, TRW did a rough order of magnitude communications load estimate for collision warning communications as part of Reference 2. The

remainder of the remote area ATC communications was not investigated, however, and it was recognized that if this load were quite heavy it would greatly influence the design and even the viability of a satellite-based ATC system. Accordingly, this brief remote area communications load analysis was undertaken. The results of this analysis indicate that if data link is used, the communications load which might be imposed on a satellite-based ATC system (about nine 1200 bit-per-second data channels in 1980 and fifteen such data channels in 1995) poses no difficult design or technology problems. Attempting to handle the same communications load using satellite-relayed voice communications, however, appears to be out of the question.

The problem of ATC communications to and from remote areas can be separated from the general problem of ATC communications. In this context, the term "remote area" shall refer to a civil airport which fulfills the following two conditions: is non-controlled (i. e., has no FAA tower), and is not in a hub area. Since the remote area traffic is much less than the airway or terminal traffic, its communication load is much lighter. For this reason, the remote area problem can be handled as a special case.

The importance of making this distinction arises since the use of satellites is not considered practical for the general case. It is still possible that satellite communication could be used for remote area communication at a lower price than establishing remote area communication facilities; and the purpose of this analysis is to establish the associated communications load.

Both general aviation and air carrier traffic is considered, but not military since military flights seldom go into civil remote airports. (Military Air Bases may be in a remote area but they have their own towers, and, in any case, they represent only a small portion of traffic in the remote areas.) Although general aviation air carriers operate under different regulations, they are handled the same by the ATC system.

The discussion covers the present situation, the future situation, and the conclusion. The present situation is discussed from the viewpoint of how remote area ATC communications function now, whereas the future situration is discussed from the viewpoint of the requirements imposed

upon the remote area ATC communications. The results of both the present and future situation sections are discussed along with the communication load based on mission analysis and traffic densities.

8.2.2 Current Scenario

8.2.2.1 Mission Analysis

A mission analysis is done for both to and from a remote area for both a typical VFR flight and an IFR flight. The origion (on flights to a remote area) or destination (on flights from a remote area) is not important so the flight is analyzed only between the airways and the remote area. The scenario concerns the flight of Cessna 60615 between Los Angeles, and Bishop, California.

8.2.2.1.1 VFR Missions

The basic function performed by ATC for VFR aircraft which file flight plans is flight following. If requested and possible, traffic reporting is also done. The mission analyses consider only VFR aircraft on a flight plan and do not consider traffic advising.

On a flight to a remote area there is no contact with ATC until the flight is complete or irtually complete, at which time the aircraft closes its flight plan and receives the local weather from a Flight Service Station. A typical conversation is shown in Table 34.

On a flight from a remote area the only contact with ATC is filing and opening the flight plan. The filing is done on the ground in person or by telephone with a flight service station. The flight plan is opened by radio once under way. T typical message sequence is given in Table 1.

Using Table 1, and averaging the to and from trips, results in an average communication load of 80 words per flight.

8.2.2.1.2 IFR Missions

The flight is assumed to be in controlled airspace but it is not assumed that there is an approach/departure control, tower, radar coverage or even ARTCC the entire way. The FSS relays communications to and from ARTCC through a remote facility. The functions of approach/departure

Table 34. VFR Communications

To Remote Area

FSS	AIRCRAFT
	"Bishop Radio, this is Cessna 60615, on 123.6; over."
"Cessna 60615, go ahead."	
	"Would like to close VFR flight plan from Los Angeles to Bishop and also get the Bishop weather; over."
"Cessna 60615, your flight plan from Los Angeles to Bishop is closed. The weather at Bishop is scattered clouds at 7000, winds out of 330 at 20 knots, and the altimeter setting is 29.98; over."	
	"Cessna 60615; thank you."

Total communication requriements of 91 words.

From Remote Area

FSS	AIRCRAFT
	"Bishop Radio, this is Cessna 60615 on 123.6; over."
"Cessna 60615, go ahead."	
	"Would like to open VFR flight plan from Bishop to Los Angeles. Time off 1:25; over."
"Cessna 60615, your flight plan from Bishop to Los Angeles is opened. Close with Los Angeles Radio; over."	
	"Cessna 60615; roger thank you."

Total communication requirement of 76 words.

control and ARTCC are performed through this facility. In general, the following services would be performed by ATC.

Departure Control — originates departure clearance to provide separation between departing and arriving IFR flights.

Approach Control — formulates and issues approach clearances and instruction to provide separation between arriving IFR aircraft.

ARTCC

1. Control of aircraft operating under IFR in controlled airspace.

2. Air traffic advisories to aircraft concerning potential hazards to flight, anticipated delays, and any other data of importance to the pilot for the safe conduct of the flight.

3. Navigation assistance by radar vectors for detouring thunderstorm and expediting routing.

4. Transmission of pilot reports and weather advisories to enroute aircraft.

5. Flight assistance to aircraft in distress.

The pilot is required to report to ATC at the following times:

ARTCC (Continued)

1. On request of ATC

2. Compulsory reporting points

3. Malfunction of required equipment

4. Time and altitude/FL reaching a holding fix or point to which cleared

5. When vacating any previously assigned altitude/FL for a newly assigned altitude/FL

6. When leaving any assigned holding fix or point

7. When leaving final approach fix inbound on final approach

8. When an approach has been missed (request clearance for specific action, i. e., to alternate airport, another approach, etc.).

9. A corrected estimate any time it becomes apparent that a previously submitted estimate to a reporting point will be in error in excess of three minutes.

10. That an altitude change will be made if operating on a clearance specifying "VFR conditions on top."

IFR position reports should include the following:

1. Identification

2. Position

3. Time

4. Altitude

5. ETA over next reporting point

6. Name of next reporting point

7. Remarks if necessary

The aircraft considered is a hypothetical 200K commuter-type aircraft and the flight distance will be 300 nautical miles resulting in an approximate duration of 1.5 hours.

Typical to/from missions can now be constructed using the above information. Tables 35 and 36 contain the text for each type of message chosen.

To Remote Area Mission

The scenario for the flight to a remote area consists of the following:

1. One transmission of weather report

2. Position report at each of two reporting points

3. One altitude change request to climb clear of clouds

4. Time and altitude reaching a holding fix

5. Leaving above holding fix

6. Leaving final approach fix inbound on final

7. Closing flight plan

144

Table 35. IFR Communications to Remote Area

ATC	AIRCRAFT
	"Los Angeles Center, this is Cessna 60615. What is the Bishop weather/ Over."
"Cessna 60615, the Bishop weather is scattered 6000 feet with visibility 15 miles. The surface winds are 10 knots out of 330. The altimeter setting is 29.99; over."	
	"Thank you."
	"Los Angeles Center, this is Cessna 60615 at_____intersection. Time is 9:45; altitude is 8000 feet. Expect_____intersection at 10:24; over."
"Thank you, Cessna 60615. Report _____intersection."	
	"Los Angeles Center, this is Cessna 60615. Over."
"Go ahead, Cessna 60615."	
	"Request 10,000 feet to avoid clouds. Over."
"Cessna 60615, climb and maintain 10,000 feet; over."	
	"Climb 10,000 Cessna 60615."
	"Los Angeles Center, this is Cessna 60615 at_____intersection. Time is 10:25. Altitude is 16,000 feet. Expect_____intersection at 10:25. Over."
"Thank you, Cessna 60615."	
"Cessna 60615, hold south at _____intersection, right turn. Over."	
	"Cessna 60615, hold south_____ intersection, right turn."
	"Bishop Radio, this is Cessna 60615, holding at_____intersection at 13,000 feet. Time 11:00."
"Thank you, Cessna 60615."	
	"Cessna 60615 leaving_____intersection."
	"Cessna 60615, inbound from_____ intersection."
	"Bishop Radio, this is Cessna 60615. Wish to close flight plan. Over."
"Flight plan closed, Cessna 60615."	

Total communication requirement of 287 words.

Table 36. IFR Communications from Remote Area

ATC	AIRCRAFT
	"Bishop Radio, this is Cessna 60615 on_____. Over."
"This is Bishop Radio; go ahead. Over."	
	"Opening flight plan, Bishop to Los Angeles. Time off 8:22. Over."
"Cessna 60615, your flight plan is opened." Report_____ intersection. Over."	
	"Report_____intersection. Roger."
	"Bishop Radio, this is Cessna 60615. What is the Los Angeles weather? Over."
"Cessna 60615, Los Angeles weather is broken 2000, haze and smoke, visiblity 5 miles. The surface winds are 270 at 10 knots. The altimeter setting is 30.01. Over."	
	"Thank you."
	"Los Angeles Center, this is Cessan 60615 at_____intersection. Time is 8:45. Altitude is 9000 feet. Exepct _____intersection at 9:21. Over."
"Thank you, Cessna 60615. Report_____intersection."	
	"Los Angeles Center, this is Cessna 60615. Over."
"Cessna 60615, go ahead. Over."	
	"Request 11,000 feet to clear clouds ahead. Over."
"Cessna 60615, climb and maintain 11,000 feet. Over."	
	"Leaving 9000 for 11000."
	"Los Angeles Center, this is Cessna 60615 at_____intersection. Time is 9:23. Altitude is 11,000. Expect _____intersection at 10:01. Over."
"Thank you, Cessna 60615."	

Total communication requirement is 233 words.

The required verbal communications appear in Table 35. Summing yields a total of 287 words for the mission.

From Remote Area Mission

The scenario for the flight from a remote area consists of the following:

1. File flight plan by telephone (mandatory if IFR conditions)

2. Open flight plan by radio once airborne

3. One transmission of weather report

4. Position report at each of two reporting points

5. An altitude change request to climb to altitude clear of clouds

The required verbal communications appear in Table 36. Summing yields a total of 239 words for the mission.

Averaging the to and from mission yields an average communication load of 263 words for the 1-1/2 hour flight.

8.2.2.2 Traffic Density

In 1968 there were an estimated 12,800 aircraft airborne over the United States at a peak instant. About two-fifths of all operations were conducted at non-tower airports in 1968. It is calculated that 3/4 of all flights into uncontrolled airports are in remote areas yields a peak figure of 3,840 aircraft airborne in remote areas. Assuming that one-fourth are IFR yields 960 IFR and 2,880 VFR (not all necessarily on a VFR flight plan).

8.2.2.3 Communication Load

VFR

Using the mission communications loads from Subsection 8.2.2.1 and the peak traffic density from Subsection 8.2.2.2 yields:

$$\text{communication load} = \frac{(80 \text{ words})}{(90 \text{ min})} (2880)$$

$$= 2560 \frac{\text{words}}{\text{min}} \text{ peak load}$$

147

IFR

As for VFR:

$$\text{communication load} = \frac{263}{90} \ (960)$$

$$= 2800 \ \frac{\text{words}}{\text{min}} \ \text{peak load}$$

Total Communication Load in Units if Messages

Communication load = 5360 words min

The average length of the messages in Tables 1-3 is:

$$\left. \begin{array}{l} 30 \ \text{messages} \\ 684 \ \text{total words} \end{array} \right\} -- \ 22.3 \ \text{words/message}$$

Using this figure with the above communications yields:

communication load = 235 msg/min

8.2.3 Future Scenario

8.2.3.1 Mission Analysis

It will be assumed that the basic ATC procedures will be the same as now (as in Subsection 8.2.2.1). The main difference considered is that no position reporting will be required as this study presupposes a satellite surveillance system.

The communications are then basically the same with only the IFR position reports deleted. The figures are below.

VFR Mission

To remote area -- 91 words

From remote area--68 words   average=80 words

IFR Mission

To remote area -- 211 words

From remote area --163 words   average = 187 words

8.2.3.2 Traffic Density[*]

In order to extrapolate traffic into the future the change in three parameters must be determined: fraction of flights that are IFR, peak total aircraft airborne, and fraction of total flights that are into uncontrolled airports.

The fraction of flights that are IFR is extrapolated by using the extrapolation of IFR iterant flights. This yields:

Fraction of flights that are IFR: 1980--.37

1995--.42

The peak total aircraft airborne will be as follows:

1980--22,220

1995--54,400

The fraction of flights into non-controlled airports will be approximately:

1980--1/4

1995--1/6

Combining these parameters yields a peak aircraft airborne in remote areas count of (based on the assumption that 3/4 of all flights into uncontrolled airports are into remote airports):

	1980	1995
VFR	2620	3960
IFR	1540	2910

8.2.3.3 Communication Load

Using the mission communications load from Subsection 8.2.3.1 and the traffic density from Subsection 8.2.3.2 yields:

VFR

1980   communication load = $\frac{80 \text{ words}}{90 \text{ min}}$ (2620)

$= 2330 \frac{\text{words}}{\text{min}}$ peak load

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[*]From Reference 1

<u>VFR</u> (Continued)

1995   communication load $= \dfrac{80 \text{ words}}{90 \text{ min}}$ (3960)

$$= 3520 \ \frac{\text{words}}{\text{min}} \ \text{peak load}$$

<u>IFR</u>

1980   communication load $= \dfrac{187 \text{ words}}{90 \text{ min}}$ (1540)

$$= 3200 \ \frac{\text{words}}{\text{min}} \ \text{peak load}$$

1995   communication load $= \dfrac{187 \text{ words}}{90 \text{ min}}$ (2910)

$$= 6040 \ \frac{\text{words}}{\text{min}} \ \text{peak load}$$

Total Communication Load in Units of Messages

communication load-- 1980--5530 words/min

1995--9560 words/min

The average message length of the messages in Tables 33-35 is 22.8 words/min.

Using this figure with above communication load yields:

communication load--1980--242 msg/min

1995--420 msg/min

8.2.3.4 <u>Queueing Analysis</u>

This analysis (based on a method used in Reference 33) first considers voice channel solutions to the problem of remote communication. In this analysis the following factors are of interest:

- Probability of communication saturation

- Utilization rate

- Expected number of messages in the system

- Expected message time.

The following assumptions are made in the following analysis:

- Messages arrive at a satellite according to a Poisson distribution with an average arrival rate of λ message per minute. Since the Poisson distribution corresponds to random arrivals of messages at the satellite for a small time interval, this assumptions appears to be quite reasonable for this situation.

- The distribution of the length of aircraft messages in remote areas has not been investigated in this analysis. It is assumed that the length of aircraft and marine messages will follow an expoential distribution with an average length of $1/\mu$ words per message.

- The queue discipline assumed in this model is a first-in-first-out discipline.

- The system has N voice channels with a transmission rate of C words per minute.

Each aircraft will communicate on an assigned channel. The problem then is determining how many channels (N) are required and what their waiting times are. The system utilization rate, ρ_S, is defined as:

$$\rho_S = \frac{\lambda \; 1/\mu}{C}$$

Dividing the message traffic by the number of channels yields the channel utilization rate, ρ_C, of each channel:

$$\rho_C = \frac{\lambda \; 1/\mu}{NC}$$

or,

$$N = \frac{\lambda \; 1/\mu}{\rho_C} \frac{1}{C}$$

From Reference 33, a utilization factor of 50% is chosen. A transmission rate of 100 words/minute is assumed. This yields channel requirement of:

1980 - 110 channels
1995 - 191 channels

The voice channel requirement is prohibitive, but using a ratio of 13 voice channels for 1 - 1200 bit data channel (from Reference 33), this yields a data channel of:

1980	5 data channels
1995	8 data channels

This does appear to be readily achievable from a satellite design point of view. Although it should be pointed out that the cost of data link equipment may be too high to warrant its use in many aircraft.

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10. NEW TECHNOLOGY

After a diligent review of the work performed under this contract, it was determined that no new innovation, discovery, improvement or invention was made.

APPENDIX A

SAUR PROGRAM DESCRIPTION

This appendix contains a description of the detailed equations of the SAUR Program together with a program listing and instructions for its use.

Altitude History (Routine WROUT)

This routine is used to calculate the upper and lower bounds on the altitude history of each aircraft. It is also used to compute the heading and speed history of both aircraft.

The output of this routine is a piecewise linear function $w(t)$. Depending on which part of the program is called the routine, w may represent altitude, heading, or speed.

There are two options of the program, depending on whether the particular function is subject to a command. The commanded option will be described first.

The input to the routine consists of:

w_o = initial value of the function

T = system reaction time

$\dot{w}_{u,p}$ = upper bound on the value of $\dot{w}$ before time T. (Must be positive.)

$\dot{w}_{l,p}$ = lower bound on the value of $\dot{w}$ before time T. (Must be negative.)

$\dot{w}_{u,c}$ = upper bound on the value of $\dot{w}$ after time T (positive)

$\dot{w}_{l,c}$ = lower bound on the value of $\dot{w}$ after time T (negative)

$w^{(1)}$ = the value of w to ards which w is to move (and perhaps reach) before time T

$w^{(2)}$ = the value of w towards which w is to move after time T.

The computations are as follows:

The routine sets $w_1=w_0$ and $t_1 = 0$.

If $w^{(1)} \geq w_0$, it sets $\dot{w} = w_{u,p}$.

If $w^{(1)} < w_0$, it sets $\dot{w} = w_{1,p}$.

Next the program computes the time at which the function will reach $w^{(1)}$. This time is $\tau = (w^{(1)}-w_0)/\dot{w}$.

If this time is earlier than T, the program sets

$w_2=w_3=w^{(1)}$ and $t_2=\tau$, $t_3=T$.

In this case w reaches the value $w^{(1)}$ and stays there until T. Next $w^{(2)}$ is compared with $w^{(1)}$.

If $w^{(2)} \leq w^{(1)}$, $\dot{w} = \dot{w}_{1,c}$.

If $w^{(2)}$ J $w^{(1)}$, $\dot{w} = \dot{w}_{1,c}$.

The time $\overline{\tau}$ at which w reaches $w^{(2)}$ is calculated. This time is

$$\overline{\tau} = T + (w^{(2)}-w^{(1)})/\overline{\tau}.$$

The program sets $w_4 = {}^{(2)}$ and $t_4= \overline{\tau}$. There are four "break points" in this function.

The calculations are similar in the case where $\tau>T$ - - i.e., if the value $w^{(1)}$ is not reached before time T. In this case only three break points result.

If the variable being considered is not being commanded, part of the above computations are left out. In this case the program sets $w_1 = w_0$, $t_1 = 0$, and then:

If $w^{(1)} > w_0$, then the program sets $\dot{w} = \dot{w}_{u,p}^{(1)}$.

If $w^{(1)} < w_0$, then $\dot{w} = w_{1,p}^{(1)}$.

In either case t_2 and w_2 are calculated from

$$t_2 = (w^{(1)} - w_0)/\dot{w} \quad \text{and} \quad w_2 = w^{(1)}.$$

In this case there are only two break points in the function.

160

The routine outputs the number of break points and the value w_i, t_i at each of these break points. The times are given by t_i and the value of the function $w(t)$ at those times are given by w_i.

Trajectory Calculations

Because the program must calculate the positions of both aircraft a very large number of times, it is important to calculate these positions as rapidly as possible. This section describes the methods for performing these calculations. Since the relative altitudes of the two aircraft are considered in another part of the program, this section describes only the calculations for the (x,y) motion of the aircraft.

In the next few paragraphs, consider one of the two aircraft. The first step is to compute the piecewise linear functions which describe the velocity (v) and heading (H) history of the aircraft. These functions are computed with the routine WROUT. The history of heading is given by pairs of numbers (H_1, t_1^H), $(H_2 t_2^H)$, etc. The history of velocity is given by pairs of numbers (v_1, t_1^V), (v_2, t_2^V), etc.

The next step is to merge the two time lists t_1^V, t_2^V, . . . and t_1^H, t_2^H, . . . and obtain a common list of times and calculate the values of H and v on the merged list. For example, if t_2^V is not equal to any of the times t_1^H, t_2^H, etc. the value of H at the time t_2^V is calculated by linear interpolation. The result of this merging is a single list of times $t_1^{(c)}$, $t_2^{(c)}$, . . . and corresponding lists $H_1^{(c)}$, $H_2^{(c)}$, . . . and $v_1^{(c)}$, $v_2^{(c)}$, . . . of values of H and v at these times. The superscript c means a "combined" list has been formed. In the program these quantities are called CT, CH, and CV. The superscript c will be dropped in the discussion below.

Turning Aircraft

The motion of an aircraft which is turning at a rate ω and which has a linear acceleration a is described by the following equations:

$$t = \text{current time (arbitrary)}$$

$$t_o = \text{some fixed time}$$

$$H_o = \text{heading at time } t_o$$

v_0 = velocity at time t_0

$H = H_0 + w(t-t_0)$ = current heading

$v = v_0 + a(t-t_0)$ = current velocity

$x = A - \frac{v}{w} \cos H + \frac{a}{w^2} \sin H$

$y = B + \frac{v}{w} \sin H + \frac{a}{w^2} \cos H$

A = constant = $x_0 + \frac{v_0}{w} \cos H_0 - \frac{a}{w^2} \sin H_0$

B = constant = $y_0 - \frac{v_0}{w} \sin H_0 + \frac{a}{w^2} \cos H_0$

x_0 = x at time t_0

y_0 = y at time t_0

To show that the formulas for x and y are correct, note that $x(t_0) = x_0$ and $y(t_0) = y_0$. Also, differentiating x and y with respect to t gives

$\dot{x} = \frac{v}{w}w \sin H - \frac{a}{w} \cos H + \frac{a}{w} \cos H = v \sin H$

$\dot{y} = \frac{v}{w} w \cos H + \frac{a}{w} \sin H - \frac{a}{\omega} \sin H = v \cos H.$

The formulas $\dot{x}$= v sin H and $\dot{y}$= v cos H are consistent with the definitions of v and H. The heading H is the angle measured clockwise from the y axis.

The calculation of x and y requires the sine and cosine of the current heading angle. These calculations are very expensive in terms of computing time. To avoid trigonometric calculations, the program is constructed to take advantage of the fact that the calculations are performed at equally spaced times. That is, x and y are calculated at times t_0, t_0 +h, t_0 + 2h, etc. If the sine and cosine are known at a time t, the next value of sin H is calculated from

$\sin H (t+h) = \sin [H(t) + wh]$
$= \sin H(t) \cos wh + \cos H(t) \sin wh$
$\cos H(t+h) = \cos H(t) \cos wh - \sin w(t) \sin wh$

162

The value of $\cos w_h$ and $\sin w_h$ are precomputed by the program at the start. The values of $\sin H$ and $\cos H$ at the starting time are also pre-computed. Then successive values of $\sin H$ and $\cos H$ are computed by the above formulas for successive values of time. Whenever the value of changes (e.g., when the aircraft stops a 3°/sec turn and starts a 2°/sec turn) the formulas are reinitialized by calculation of the sine and cosine of the heading at the first time point past the change.

The calculations of position in the case of a turning aircraft are summarized as follows. Let

h = time increment

$\bar{c}$ = $\cos wh$

$\bar{s}$ = $\sin wh$

t_k = most recent break point

t = current time

$$t_k \leq t < t_{k+1}$$

x_k, y_k, H_k, etc. = values of x, y, H, etc. at time t_k.

$$A_k = x_k + \frac{v_k}{\dot{H}_k} \cos H_k - \frac{\dot{v}_k}{\dot{H}_k} \sin H_k$$

$$B_k = y_k - \frac{v_k}{\dot{H}_k} \sin H_k - \frac{\dot{v}_k}{\dot{H}_k} \cos H_k$$

$$C_k = \frac{v_k - \dot{v}_k t_k}{\dot{H}_k}$$

$$D_k = \frac{v_k}{\dot{H}_k}$$

$$E_k = \frac{\dot{v}_k}{\dot{H}_k^2}$$

Suppose that the trajectory has just been computed at time t. This means that the values of cos H and sin H are available. Call these values $C^{(OLD)}$ and $S^{(OLD)}$. The time is incremented by an amount h and the corresponding values of cos H and sin H are called C and S. The computations are as follows:

$$C = \overline{C}\ C^{(OLD)} - \overline{S}\ S^{(OLD)}$$

$$S = \overline{C}\ S^{(OLD)} + \overline{C}\ C^{(OLD)}$$

$$\psi = C_k + D_k t$$

$$x = A_k - \psi C + E_k S$$

$$y = B_k + \psi C + E_k S$$

Note that the current aircraft position (x,y) is computed with only nine multiplication and seven additions.

Non-Turning Aircraft

Unfortunately, it is not possible to set the turn rate to zero in the above equations to get the motion of an aircraft travelling in a straight line. The turn rate appears in the denominator. It is possible to rewrite the equations in such a form that it is possible to set the turn rate to zero. However, these equations are more complicated than those above In order to keep the equations as simple as possible, it was decided to make a special case of non-turhing aircraft.

The equations in this case are as follows:

t_o = some fixed time

v_o = speed at time t_o

a = acceleration

v = current velocity = $v_o + a(t - t_o)$

H_o = heading at time t_o

(x,y) = current aircraft position

(x_0, y_0) = aircraft position at time t_0

x = $A + \psi \sin H_0$

y = $B + \psi \cos H_0$

ψ = $v_0 (t - t_0) + \frac{a}{2} (t - t_0)^2$

A = x_0

B = y_0

The equations can be checked by noting that $\dot{x} = v \sin H_0$, $\dot{y} = v \cos H_0$, $x(t_0) = x_0$, and $y(t_0) = y_0$.

In the trajectory calculation in the program the equations are used in the following form:

t = current time

t_k = most recent break point

$t_k \leq t < t_{k+1}$

x_k, y_k, v_k, etc. = values of x, y, v, etc. at time t_k

$A_k = x_k$

$B_k = y_k$

$C_k = \sin H_k$

$D_k = \cos H_k$

$\Delta t = t - t_k$

$\psi = (v_k + \dot{v}_k \Delta t / 2) \Delta t$

$x = A_k + \psi C_k$.

$y = B_k + \psi D_k$.

Note that the constants A_k, B_k, C_k, D_k, mean something different than they did in the case of a turning aircraft.

To compute the trajectory of an aircraft the constants A_k, B_k, etc. are computed first according to the above formulas for $k=1, 2, 3, \ldots \ldots$ If the aircraft is turning ($\dot{H}_k \neq 0$), the turning equations are used. If the aircraft is not turning ($\dot{H}_k = 0$), the non-turning equations are used.

After the constants are computed, the values of x and y at any time t can be computed. First, it is necessary to determine which time interval contains the time t, i.e., determine the value of k such that $t_k \leq t_{k+1}$. Then H_k is examined. If $\dot{H}_k$ is not zero, the turning formulas are used to compute x and y. If $\dot{H}_k$ is zero, the non-turning formulas are used.

DCA Calculation

The calculations of the DCA of two aircraft are described below.

a) Compute the heading history of aircraft 1.

b) Compute the velocity history of aircraft 1.

c) Combine the above histories.

d) Compute the values of A_k, B_k, C_k, D_k, E_k at each of the break points.

e) Save the values of A_k, B_k, etc. by setting $A_k^{(1)} = A_k$, $B_k^{(1)} = B_k$, etc.

f) Repeat a) - e) for aircraft 2.

g) Start with an initial time $t = \min(T_1, T_2)$.

h) Determine the (x,y) portions of both aircraft at time t, using the trajectory formulas above. Denote these by (x_1, y_1) and (x_2, y_2). Determine the value of $\Delta Z (=$ altitude separation at time t) from the table of values DZM. Compute the distance at the current time by

$$D^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (\Delta Z)^2$$

i) Increment the time: t+h t and repeat step h). In general one will see the distances decrease and then start to increase. As soon as the increase is noted, the program assumes that the current time is an approximate DCA.

j) Refine the time of closest approach by passing a parabola through the last three points and computing the time at which the parabola is minimized.

k) Using this refined time, recompute the distance. This is taken to be the DCA between the two aircraft.

Program Inputs

Table 36 is the standard input sheet for the program. There are two options for initial conditions. The initial conditions (XO(1), YO(1), XO(2), etc.) may be specified, or they may be determined by the "flyback" option. In the latter case, the initial conditions are determined by starting at a collision point, flying the aircraft back for some time τ (TAUB), then turning the aircraft around (by adding 180° to the heading) to get the initial conditions. The second option is assumed if TAUB $\neq$ 0.

Table 37. SAUR Input Sheet

	Initial Conditions	
	x	XO(1) =
	y	YO(1) =
A/C #1	z	ZO1 =
	heading	HNØT1 =
	velocity	VNØT1 =
		XO(2) =
		YO(2) =
A/C #2		ZO2 =
		HNØT2 =
		VNØT2 =

	Errors (altitude)	
	A/C #1	EZ1 =
	A/C #2	EZ2 =
	Reaction Times	
	A/C #1	TØNE =
	A/C #2	TTWØ =
	Print Control	
	Case Selector* for Trajectory Print	FIP =
		FJP =
	Increment for Trajectory Print	DTP =
	Final Time for Trajectory Print	TE =
	Increment for DCA Calculation	DT =

Table 37. SAUR Input Sheet (Continued)

Performance Parameters

A/C #1	altitude	pilot	upper bound	ZUP1 =
			lower bound	ZLP1 =
		controller	upper bound	ZUC1 =
			lower bound	ZLC1 =
	ż	pilot	upper bound	ZDUP1 =
			lower bound	ZDLP =
		controller	upper bound	ZDUC1 =
			lower bound	ZDLC1 =
	turn rate	pilot	upper bound	HDUP1 =
			lower bound	HDLP1 =
		controller	upper bound	HDUC1 =
			lower bound	HDLC1 =
	Acceleration	pilot	upper bound	VDUP1 =
			lower bound	VDLP1 =
		controller	upper bound	VDUC1 =
			lower bound	VDLC1 =

A/C #2

ZUP2 =
ZLP2 =
ZUC2 =
ZLC2 =
ZDUP2 =
ZDLP2 =
ZDUC2 =
ZDLC2 =

HDUP2 =
HDLP2 =
HDUC2 =
HDLC2 =

VDUP2 =
VDLP2 =
VDUC2 =
VDLC2 =

Search Parameters

pilot A/C #1	heading	smallest	HMP1 =
		delta	DMP1 =
		number -1	FIM(1)=
	velocity	smallest	VMP1 =
		delta	DVP1 =
		number -1	FIM(2)=
controller A/C #1	heading	smallest	HMC1 =
		delta	DHC1 =
		number -1	FJM(1)=
	velocity	smallest	VMC1 =
		delta	DVC1 =
		number -1	FJM(2)=

HMP2 =
DHP2 =
FIM(3)=
VMP2 =
DVP2 =
FIM(4)=

A/C #2

HMC2 =
DHC2 =
FJM(3)=
VMC2 =
DVC2 =
FJM(4)=

Flyback Option

TAUB =

A/C #1	initial heading	HBR1 =
	initial heading	VBR1 =
	initial z	ZDBR1 =
	initial _	WBR1 =
A/C #2		HBR2 =
		VBR2 =
		ZDBR2 =
		WBR2 =

initial x   EXB =
initial y   EYB =
initial z   EZB =

The input list contains brief descriptions of all the parameters except for the "print control" and "miscellaneous" parameters which are used for special debug and test runs.

The program can be run from the time share terminal. Because of the large number of inputs required to generate a single case, however, it is not convenient to input all of the data from the time share terminal. Therefore, the standard mode of operation is to key punch the basic data for a case on cards, submit the cards to create an input file, and then call the file from the time share terminal. At the terminal it is possible to change any of the inputs in order to modify the basic case.

The input and output units will be consistent, e.g., if the input is feet, second and feed per second, the output will be in feet. If the input uses meters, the output will be in meters. All of the inputs must be in consistent units (i.e., one cannot input speed in knots and altitude in feet). All angles are measured in degrees and heading is measured from north (the Y axis).

SAUR Program Operation

Figure 55 gives a listing of the inputs for the case. The initial conditions for the case are found by starting with the two aircraft at the same point, flying them for TAUB=40 seconds, and turning them around (i.e., increasing heading by 180°). Aircraft 1 is flown back with a heading of HBRI=154°, a velocity VBRI=169 ft/sec, and a zero climb rate (ZDBRI=0). Aircraft 2 has parameters HBR2=334°, VBR=422 ft/sec, ZDBR2=8.33 ft/sec.

Figure 56 shows the first output of the program. It consists of a listing of the trajectory trials which the program will attempt. For example, for aircraft 1, the program will try trajectories with pilot initiated maneuvers to headings of 244°, 289°, 334°, 379°, and 424°. Also for aircraft 1, the program will try controller commanded headings of 244° and 424°. The trajectories to be tried are controlled by the inputs — for example the pilot initiated headings were input by the parameters HMP1 = 244, DPH1 = 45°, and FIM(1) = 4. The determined that the heading values to be tried are 244°, 244° + 45°, 244° +2(45°), 244° + 3(45°), 244° + 4(45°).

```
MAXD=6000.                          DVP1=40
EZ1=300                             FIM(2)=2
EZ2=300                             HMC1=244
TONE=20                             DHC1=180
TTWO=20                             FJM(1)=1
TAUB=40                             VMC1=139
HBR1=180                            DVC1=60
VBR1=169                            FJM(2)=1
ZDBR1=0                             ZUP1=20000
WBR1=-1                             ZLP2=0
HBR2=334                            ZUC2=20000
VBR2=422                            ZLC2=0
ZDBR2=8.33                          ZDUP2=50
WBR2=0                              ZDLP2=-16.6667
EXB=0, EYB=0, EZB=5000,             ZDUC2=33.3
ZUP1=20000                          ZDLC2=-8.3333
ZLP1=0                              HDUP2=5
ZLP1=0                              HDLP2=-5
ZUC1=20000                          HDUC2=3
ZLC1=0                              HDLC2=-3
ZDUP1=11                            VDUP2=4
ZDLP1=-16.6667                      VDLP2=-1
ZDUC1=8.3333                        VDUC2=3
ZDLC1=-8.3333                       VDLC2=-0.6
HDUP1=8                             HMP2=94
HDLP1=-8                            DHP2=30
HDUC1=3                             FIM(3)=4
HDLC1=-3                            VMP2=342
VDUP1=2                             DVP2=80
VDLP1=-2                            FIM(4)=2
VDUC1=2                             HMC2=64
VDLC1=-2                            DHC2=180
HMP1=244                            FJM(3)=1
DHP1=45                             VMC2=362
FIM(1)=4                            DVC2=120
VMP1=129                            FJM(4)=1
```

Figure 55. Input Listing (On Line)

```
1     PILOT MANEUVERS              CONTROLLER COMMANDS
0     A/C 1        I1  HEADING      A/C  1       J1  HEADING
                   0     244.00                  0     244.00
                   1     289.00                  1     424.00
                   2     334.00                  2       0.00
                   3     379.00                  3       0.00
                   4     424.00                  4       0.00
0     A/C 1        I2  VELOCITY     A/C  1       J2  VELOCITY
                   0     129.00                  0     139.00
                   1     169.00                  1     199.00
                   2     209.00                  2       0.00
0     A/C 2        I3  HEADING      A/C  2       J3  HEADING
                   0      94.00                  0      64.00
                   1     124.00                  1     244.00
                   2     154.00                  2       0.00
                   3     184.00                  3       0.00
                   4     214.00                  4       0.00
0     A/C 2        I4  VELOCITY     A/C  2       J4  VELOCITY
                   0     342.00                  0     362.00
                   1     422.00                  1     482.00
                   2     502.00                  2       0.00
```

Figure 56. Trajectory Trial Listing (On Line)

170

Figure 57 is an off-line program output which gives the results of the trajectories flown. The first two columns of this output can be ignored. For the remaining columns a typical output consist of the numbers

$$0, 1, 1, 2, 1, 99, 99, 1, 1.7011162E+08$$

Each such line represents a trajectory pair which the program ran to calculate a distance of closest approach. The first set of integers identifies the trajectory by referring back to Figure 56. The example trajectory pair is given by:

0: pilot heading 244°, aircraft 1

1: pilot velocity 169 ft/sec, aircraft 1

1: pilot heading 124°, aircraft 2

2: pilot velocity 502 ft/sec, aircraft 2

1: commanded heading 424°, aircraft 1

99: no velocity command given for aircraft 1

99: no heading command given for aircraft 2

1: commanded velocity 482 ft/sec, aircraft 2.

Note that "99" is a special code for "no command" for that particular component.

The last number 1.7011162E+08 is the square of the distance of closest approach for that particular trajectory.

The program first considers altitude commands to both aircraft. For the first lines of the output the heading and velocity command columns are 99, i.e., no heading or velocity commands are given to either aircraft.

For each command set, note that the program tries all the possible combinations of pilot maneuvers which the input specified should be tried. It determines which of these trajectories give the smallest distance. The minimum for the altitude only commands is for the trajectory 4,2,1,1,99,99, 99,99.

This result is also printed on line, as shown in Figure 58. (It would be too expensive and slow to print all of the intermediate results on line, so only the winning trajectory data is printed on line.) The on line

171

		PILOT PHASE1		CONTROLLER PHASE2					
IMPCT	DSTR	H	V	H	V	H	V	H	V	DCA
*****									
1.0000000E+20	0.	0	0	0	0	99	99	99	99	3.1821723E+08
3.1821723E+08	0.	0	0	0	1	99	99	99	99	3.1632218E+08
3.1632218E+08	0.	0	0	0	2	99	99	99	99	3.0940588E+08
3.0940588E+08	0.	0	0	1	0	99	99	99	99	1.8352643E+08
1.8352643E+08	0.	0	0	1	1	99	99	99	99	1.7917834E+08
1.7917834E+08	0.	0	0	1	2	99	99	99	99	1.6789075E+08
1.6789075E+08	0.	0	0	2	0	99	99	99	99	2.7753142E+07
2.7753142E+07	0.	0	0	2	1	99	99	99	99	2.3815788E+07
2.3815788E+07	0.	0	0	2	2	99	99	99	99	1.5374026E+07
1.5374026E+07	0.	0	0	3	0	99	99	99	99	3.3064987E+07
1.5374026E+07	0.	0	0	3	1	99	99	99	99	3.9707405E+07
1.5374026E+07	0.	0	0	3	2	99	99	99	99	5.1696945E+07
1.5374026E+07	0.	0	0	4	0	99	99	99	99	2.4006493E+08
1.5374026E+07	0.	0	0	4	1	99	99	99	99	2.4668295E+08
1.5374026E+07	0.	0	0	4	2	99	99	99	99	2.5724243E+08
1.5374026E+07	0.	0	1	0	0	99	99	99	99	3.2400495E+08
1.5374026E+07	0.	0	1	0	1	99	99	99	99	3.2205353E+08
1.5374026E+07	0.	0	1	0	2	99	99	99	99	3.1482778E+08
1.5374026E+07	0.	0	1	1	0	99	99	99	99	2.0328874E+08
1.5374026E+07	0.	0	1	1	1	99	99	99	99	1.9853789E+08
1.5374026E+07	0.	0	1	1	2	99	99	99	99	1.8527235E+08
1.5374026E+07	0.	0	1	2	0	99	99	99	99	4.5413088E+07
1.5374026E+07	0.	0	1	2	1	99	99	99	99	3.9372528E+07
1.5374026E+07	0.	0	1	2	2	99	99	99	99	2.7267897E+07
1.3230184E+07	0.	0	1	3	0	99	99	99	99	1.3230184E+07
1.3230184E+07	0.	0	1	3	1	99	99	99	99	1.9436922E+07
1.3230184E+07	0.	0	1	3	2	99	99	99	99	3.1898849E+07
1.3230184E+07	0.	0	1	4	0	99	99	99	99	2.0041997E+08
1.3230184E+07	0.	0	1	4	1	99	99	99	99	2.1441043E+08
1.3230184E+07	0.	0	1	4	2	99	99	99	99	2.3372242E+08
1.3230184E+07	0.	0	2	0	0	99	99	99	99	3.2865179E+08
1.3230184E+07	0.	0	2	0	1	99	99	99	99	3.2668816E+08
1.3230184E+07	0.	0	2	0	2	99	99	99	99	3.1933772E+08
1.3230184E+07	0.	0	2	1	0	99	99	99	99	2.2050308E+08
1.3230184E+07	0.	0	2	1	1	99	99	99	99	2.1560544E+08
1.3230184E+07	0.	0	2	1	2	99	99	99	99	2.0099958E+08
1.3230184E+07	0.	0	2	2	0	99	99	99	99	6.5213223E+07

Figure 57. "DCA-Squared for Each Trajectory Pair (Off Line)"

DNOT	DSTR
1.3230184E+07	0.
1.3230184E+07	0.
1.3230184E+07	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.7088048E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.

PILOT PHASE1				CONTROLLER PHASE 2				DCA
H	V	H	V	H	V	H	V	
0	2	2	1	99	99	99	99	5.7431776E+07
0	2	2	2	99	99	99	99	4.1420004E+07
0	2	3	0	99	99	99	99	2.7088048E+06
0	2	3	1	99	99	99	99	6.0992830E+06
0	2	3	2	99	99	99	99	1.6134061E+07
0	2	4	0	99	99	99	99	1.4798303E+08
0	2	4	1	99	99	99	99	1.7450986E+08
0	2	4	2	99	99	99	99	2.0505656E+08
1	0	0	0	99	99	99	99	2.3919756E+08
1	0	0	1	99	99	99	99	2.3919583E+08
1	0	0	2	99	99	99	99	2.3848481E+08
1	0	1	0	99	99	99	99	1.0701217E+08
1	0	1	1	99	99	99	99	1.0602467E+08
1	0	1	2	99	99	99	99	1.0341206E+08
1	0	2	0	99	99	99	99	5.8036107E+06
1	0	2	1	99	99	99	99	5.1614416E+06
1	0	2	2	99	99	99	99	2.6460335E+06
1	0	3	0	99	99	99	99	4.5724407E+07
1	0	3	1	99	99	99	99	4.9832917E+07
1	0	3	2	99	99	99	99	5.9015538E+07
1	0	4	0	99	99	99	99	1.9610927E+08
1	0	4	1	99	99	99	99	2.0262475E+08
1	0	4	2	99	99	99	99	2.1612892E+08
1	1	0	0	99	99	99	99	2.3506971E+08
1	1	0	1	99	99	99	99	2.3517392E+08
1	1	0	2	99	99	99	99	2.3485503E+08
1	1	1	0	99	99	99	99	1.1127175E+08
1	1	1	1	99	99	99	99	1.1018867E+08
1	1	1	2	99	99	99	99	1.0719041E+08
1	1	2	0	99	99	99	99	9.4234693E+06
1	1	2	1	99	99	99	99	8.3645272E+06
1	1	2	2	99	99	99	99	5.2213683E+06
1	1	3	0	99	99	99	99	3.0319417E+07
1	1	3	1	99	99	99	99	3.4424066E+07
1	1	3	2	99	99	99	99	4.4159881E+07
1	1	4	0	99	99	99	99	1.6086219E+08
1	1	4	1	99	99	99	99	1.7016866E+08
1	1	4	2	99	99	99	99	1.8932858E+08
1	2	0	0	99	99	99	99	2.3143680E+08
1	2	0	1	99	99	99	99	2.3161401E+08
1	2	0	2	99	99	99	99	2.3159155E+08
1	2	1	0	99	99	99	99	1.1500107E+08

Figure 57. "DCA-Squared for Each Trajectory Pair (Off Line)" (Continued)

DNQT	DSTR	PILOT PHASE1				CONTROLLER PHASE 2				DCA
		H	V	H	V	H	V	H	V	
2.6460335E+06	0.	1	2	1	1	99	99	99	99	1.1386182E+08
2.6460335E+06	0.	1	2	1	2	99	99	99	99	1.1057749E+08
2.6460335E+06	0.	1	2	2	0	99	99	99	99	1.3514834E+07
2.6460335E+06	0.	1	2	2	1	99	99	99	99	1.2117368E+07
2.6460335E+06	0.	1	2	2	2	99	99	99	99	8.2671786E+06
2.6460335E+06	0.	1	2	3	0	99	99	99	99	1.8901919E+07
2.6460335E+06	0.	1	2	3	1	99	99	99	99	2.2593534E+07
2.6460335E+06	0.	1	2	3	2	99	99	99	99	3.2024388E+07
2.6460335E+06	0.	1	2	4	0	99	99	99	99	1.2712842E+08
2.6460335E+06	0.	1	2	4	1	99	99	99	99	1.3854881E+08
2.6460335E+06	0.	1	2	4	2	99	99	99	99	1.6228023E+08
2.6460335E+06	0.	2	0	0	0	99	99	99	99	1.5985130E+08
2.6460335E+06	0.	2	0	0	1	99	99	99	99	1.6293263E+08
2.6460335E+06	0.	2	0	0	2	99	99	99	99	1.7006966E+08
2.6460335E+06	0.	2	0	1	0	99	99	99	99	4.0309066E+07
2.6460335E+06	0.	2	0	1	1	99	99	99	99	4.1624827E+07
2.6460335E+06	0.	2	0	1	2	99	99	99	99	4.4798739E+07
2.6460335E+06	0.	2	0	2	0	99	99	99	99	4.6953106E+06
2.6460335E+06	0.	2	0	2	1	99	99	99	99	4.8071602E+06
2.6460335E+06	0.	2	0	2	2	99	99	99	99	3.9517171E+06
2.6460335E+06	0.	2	0	3	0	99	99	99	99	1.0039991E+08
2.6460335E+06	0.	2	0	3	1	99	99	99	99	1.0218000E+08
2.6460335E+06	0.	2	0	3	2	99	99	99	99	1.0661049E+08
2.6460335E+06	0.	2	0	4	0	99	99	99	99	2.3707777E+08
2.6460335E+06	0.	2	0	4	1	99	99	99	99	2.3940067E+08
2.6460335E+06	0.	2	0	4	2	99	99	99	99	2.4515987E+08
2.6460335E+06	0.	2	1	0	0	99	99	99	99	1.4188731E+08
2.6460335E+06	0.	2	1	0	1	99	99	99	99	1.4561516E+08
2.6460335E+06	0.	2	1	0	2	99	99	99	99	1.5468167E+08
2.6460335E+06	0.	2	1	1	0	99	99	99	99	3.4707401E+07
2.6460335E+06	0.	2	1	1	1	99	99	99	99	3.6087403E+07
2.6460335E+06	0.	2	1	1	2	99	99	99	99	3.9584497E+07
2.6460335E+06	0.	2	1	2	0	99	99	99	99	4.5214458E+06
2.6460335E+06	0.	2	1	2	1	99	99	99	99	4.6156137E+06
2.6460335E+06	0.	2	1	2	2	99	99	99	99	3.9291763E+06
2.6460335E+06	0.	2	1	3	0	99	99	99	99	9.2046240E+07
2.6460335E+06	0.	2	1	3	1	99	99	99	99	9.3988151E+07
2.6460335E+06	0.	2	1	3	2	99	99	99	99	9.9048275E+07
2.6460335E+06	0.	2	1	4	0	99	99	99	99	2.1963723E+08
2.6460335E+06	0.	2	1	4	1	99	99	99	99	2.2274098E+08
2.6460335E+06	0.	2	1	4	2	99	99	99	99	2.3078520E+08
2.6460335E+06	0.	2	2	0	0	99	99	99	99	1.2608830E+08

Figure 57. "DCA-Squared for Each Trajectory Pair (Off Line)" (Continued)

DNOT	DSTR
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.
2.6460335E+06	0.

PILOT PHASE1		CONTROLLER PHASE2						DCA
H	V	H	V	H	V	H	V	
2	2	0	1	99	99	99	99	1.3021044E+08
2	2	0	2	99	99	99	99	1.4065712E+08
2	2	1	0	99	99	99	99	3.0144681E+07
2	2	1	1	99	99	99	99	3.1516578E+07
2	2	1	2	99	99	99	99	3.5168932E+07
2	2	2	0	99	99	99	99	4.3940217E+06
2	2	2	1	99	99	99	99	4.4640518E+06
2	2	2	2	99	99	99	99	3.9066999E+06
2	2	3	0	99	99	99	99	8.4935016E+07
2	2	3	1	99	99	99	99	8.6949120E+07
2	2	3	2	99	99	99	99	9.2416635E+07
2	2	4	0	99	99	99	99	2.0346556E+08
2	2	4	1	99	99	99	99	2.0716214E+08
2	2	4	2	99	99	99	99	2.1705823E+08
3	0	0	0	99	99	99	99	1.2544815E+08
3	0	0	1	99	99	99	99	1.3227484E+08
3	0	0	2	99	99	99	99	1.4573759E+08
3	0	1	0	99	99	99	99	1.1570917E+07
3	0	1	1	99	99	99	99	1.3736898E+07
3	0	1	2	99	99	99	99	1.8882600E+07
3	0	2	0	99	99	99	99	3.0642147E+07
3	0	2	1	99	99	99	99	2.8936108E+07
3	0	2	2	99	99	99	99	2.4211301E+07
3	0	3	0	99	99	99	99	1.6824041E+08
3	0	3	1	99	99	99	99	1.6724200E+08
3	0	3	2	99	99	99	99	1.6438294E+08
3	0	4	0	99	99	99	99	2.9103920E+08
3	0	4	1	99	99	99	99	2.9073479E+08
3	0	4	2	99	99	99	99	2.8915304E+08
3	1	0	0	99	99	99	99	9.4388040E+07
3	1	0	1	99	99	99	99	1.0314285E+08
3	1	0	2	99	99	99	99	1.2067348E+08
3	1	1	0	99	99	99	99	4.7130381E+06
3	1	1	1	99	99	99	99	6.3991311E+06
3	1	1	2	99	99	99	99	1.0998796E+07
3	1	2	0	99	99	99	99	3.7942557E+07
3	1	2	1	99	99	99	99	3.5825377E+07
3	1	2	2	99	99	99	99	3.0168951E+07
3	1	3	0	99	99	99	99	1.7139366E+08
3	1	3	1	99	99	99	99	1.7030996E+08
3	1	3	2	99	99	99	99	1.6707770E+08
3	1	4	0	99	99	99	99	2.8634951E+08

Figure 57. "DCA-Squared for Each Trajectory Pair (Off Line)" (Continued)

DNOT	DSTR	PILOT PHASE1				CONTROLLER PHASE2				DCA
		H	V	H	V	H	V	H	V	
2.6460335E+06	0.	3	1	4	1	99	99	99	99	2.8613788E+08
2.6460335E+06	0.	3	1	4	2	99	99	99	99	2.8488971E+08
2.6460335E+06	0.	3	2	0	0	99	99	99	99	6.7146134E+07
2.6460335E+06	0.	3	2	0	1	99	99	99	99	7.6842142E+07
2.6460335E+06	0.	3	2	0	2	99	99	99	99	9.6972904E+07
2.6460325E+06	0.	3	2	1	0	99	99	99	99	1.1328339E+06
1.1328339E+06	0.	3	2	1	1	99	99	99	99	2.1087525E+06
1.1328339E+06	0.	3	2	1	2	99	99	99	99	5.5499185E+06
1.1328339E+06	0.	3	2	2	0	99	99	99	99	4.4859662E+07
1.1328339E+06	0.	3	2	2	1	99	99	99	99	4.2439615E+07
1.1328339E+06	0.	3	2	2	2	99	99	99	99	3.5944271E+07
1.1328339E+06	0.	3	2	3	0	99	99	99	99	1.7392651E+08
1.1328339E+06	0.	3	2	3	1	99	99	99	99	1.7279576E+08
1.1328339E+06	0.	3	2	3	2	99	99	99	99	1.6930028E+08
1.1328339E+06	0.	3	2	4	0	99	99	99	99	2.8198844E+08
1.1328339E+06	0.	3	2	4	1	99	99	99	99	2.8184751E+08
1.1328339E+06	0.	3	2	4	2	99	99	99	99	2.8086364E+08
1.1328339E+06	0.	4	0	0	0	99	99	99	99	1.5495415E+08
1.1328339E+06	0.	4	0	0	1	99	99	99	99	1.6134779E+08
1.1328339E+06	0.	4	0	0	2	99	99	99	99	1.7122296E+08
1.1328339E+06	0.	4	0	1	0	99	99	99	99	7.2150804E+06
1.1328339E+06	0.	4	0	1	1	99	99	99	99	9.9586555E+06
1.1328339E+06	0.	4	0	1	2	99	99	99	99	1.5895509E+07
1.1328339E+06	0.	4	0	2	0	99	99	99	99	5.6133676E+07
1.1328339E+06	0.	4	0	2	1	99	99	99	99	5.1377679E+07
1.1328339E+06	0.	4	0	2	2	99	99	99	99	4.1027597E+07
1.1328339E+06	0.	4	0	3	0	99	99	99	99	2.1559156E+08
1.1328339E+06	0.	4	0	3	1	99	99	99	99	2.1232862E+08
1.1328339E+06	0.	4	0	3	2	99	99	99	99	2.0267756E+08
1.1328339E+06	0.	4	0	4	0	99	99	99	99	3.1246723E+08
1.1328339E+06	0.	4	0	4	1	99	99	99	99	3.1109180E+08
1.1328339E+06	0.	4	0	4	2	99	99	99	99	3.0581977E+08
1.1328339E+06	0.	4	1	0	0	99	99	99	99	1.2056916E+08
1.1328339E+06	0.	4	1	0	1	99	99	99	99	1.3251662E+08
1.1328339E+06	0.	4	1	0	2	99	99	99	99	1.4901315E+08
1.1328339E+06	0.	4	1	1	0	99	99	99	99	1.1507156E+06
1.1328339E+06	0.	4	1	1	1	99	99	99	99	2.2599387E+06
1.1328339E+06	0.	4	1	1	2	99	99	99	99	6.6325676E+06
1.1328339E+06	0.	4	1	2	0	99	99	99	99	7.4782152E+07
1.1328339E+06	0.	4	1	2	1	99	99	99	99	6.8630012E+07
1.1328339E+06	0.	4	1	2	2	99	99	99	99	5.4988168E+07
1.1328339E+06	0.	4	1	3	0	99	99	99	99	2.2654073E+08

Figure 57. "DCA-Squared for Each Trajectory Pair (Off Line)" (Continued)

			PILOT PHASE1				CONTROLLER PHASE2				
DNOT		DSTR	H	V	H	V	H	V	H	V	DCA
1.1328339E+06	0.		4	1	3	1	99	99	99	99	2.2314425E+08
1.1328339E+06	0.		4	1	3	2	99	99	99	99	2.1238468E+08
1.1328339E+06	0.		4	1	4	0	99	99	99	99	3.0956085E+08
1.1328339E+06	0.		4	1	4	1	99	99	99	99	3.0818758E+08
1.1328339E+06	0.		4	1	4	2	99	99	99	99	3.0296917E+08
1.1328339E+06	0.		4	2	0	0	99	99	99	99	8.0586061E+07
1.1328339E+06	0.		4	2	0	1	99	99	99	99	1.0011286E+08
1.1328339E+06	0.		4	2	0	2	99	99	99	99	1.2417735E+08
1.1328339E+06	0.		4	2	1	0	99	99	99	99	2.6442505E+06
1.1328339E+06	0.		4	2	1	1	99	99	99	99	7.0474835E+05
7.0474835E+05	0.		4	2	1	2	99	99	99	99	1.3255015E+06
7.0474835E+05	0.		4	2	2	0	99	99	99	99	9.2918667E+07
7.0474835E+05	0.		4	2	2	1	99	99	99	99	8.5845987E+07
7.0474835E+05	0.		4	2	2	2	99	99	99	99	6.9201169E+07
7.0474835E+05	0.		4	2	3	0	99	99	99	99	2.3456117E+08
7.0474835E+05	0.		4	2	3	1	99	99	99	99	2.3119966E+08
7.0474835E+05	0.		4	2	3	2	99	99	99	99	2.1992132E+08
7.0474835E+05	0.		4	2	4	0	99	99	99	99	3.0643939E+08
7.0474835E+05	0.		4	2	4	1	99	99	99	99	3.0510981E+08
7.0474835E+05	0.		4	2	4	2	99	99	99	99	3.0003535E+08
7.0474835E+05		4	2	1	1	99	99	99	99		
7.0474835E+05	4.0000000E+00	2	1	1	99	99	99	99			

Figure 57. "DCA-Squared for Each Trajectory Pair (Off Line)" (Continued)

```
7.0474835E+05  8.3949291E+02    4    2    1    1    99    99    99    99
7.0474835E+05  8.3949291E+02    4    2    1    1    99    99    99    99
7.0474835E+05  8.3949291E+02    4    2    1    1    99    99    99    99
1.2822390E+06  1.1323599E+03    4    2    1    0    99    0    99    1
       MAXIMUM POSSIBLE TRAJECTORIES        5625
       ACTUAL TRAJECTORIES                  2494
```

Figure 58. Winning Trajectory Data Print (On Line)

print contains the distance squared, the distance, and the parameters of the winning trajectory. The winning trajectory has a distance of closest approach of 839 feet. This means that with altitude commands a separation of 839 feet can be guaranteed (subject, of course, to the assumption that enough pilot maneuvers have been considered by the program user).

The program next considers altitude commands for aircraft 1 and other (heading and speed) commands for aircraft 2. First it considers the command set (99,99,99,0) — i.e., aircraft 2 is given a heading command of 244°. For this command set the program starts to consider all possible pilot maneuvers. Note, however, that it does not need to complete all the possible maneuvers. For example, when it gets to case (4,1,1,0,99,99,99,0) it finds that the distance is less than the distance which can be guaranteed by altitude maneuvers only. Consequently, there is no need to consider this command set further, and the program goes on to the next command set (99,99,99,1).

For all of the command sets in which aircraft 1 is given an altitude maneuver, the program finds the same thing — the guaranteed separation cannot be improved beyond the separation already guaranteed by the altitude separation. At this point, the program prints out the results so far. Since a better command pair has not been found, the print is a repeat of the previous line.

Next the program considers the situation when heading or speed commands are given to both aircraft. It considers all possible such command sets. For the command set (99,0,99,0) it runs through all possible pilot maneuvers until it finds a distance which is less than the separation already guaranteed. For the next command set (99,0,99,1) it runs all the way through the pilot maneuvers and finds that the minimum of the distances is greater than the separation already guaranteed. The critical trajectory is (4,1,1,0,99,0,99,1) and the corresponding distance (squared) is 1.28×10^6 which is greater than the previous value 7.05×10^5.

178

The program continues through all the other command pairs and never finds a better guaranteed separation. The final result is printed on line. (The fourth line of Figure 58).

The program also prints out the number of trajectories which it ran and the number which it "considered," i.e., those it would have run if the command pair cases were not terminated early as explained above. (As a matter of interest, the program on Case 1 considered 67,500 and ran 24,354 trajectory pairs in 380 seconds CPU time. Thus the portion of Case 1 shown in Figure 58 took about 30 seconds CPU time to run. This indicates that this particular SAUR formulation and this ground computer mechanization (CDC 6500) would not allow this problem to be worked in real time for operational use. More analysis would be needed in order to evaluate the operational computer speed and capacity requirements).

Upon request, the program prints out the details of any trajectory. An example of such a printout is shown in Figure 59. The first part of printout contains a set of constants which are used to compute the trajectories. The major part of the printout contains a listing of the speed, heading, and (x,y) positions of both aircraft, the guaranteed vertical separation "DELTAZ," and the distance between the two aircraft.

TIME	HEADING	TURN RATE	SPEED	ACCEL	X	Y	A	B	C	D	E
AIRCRAFT 1											
0.00	294.00	0.00	169.000	2.000	4764.59	-4601.11	5.351E+03	-3.537E+03	1.210E+03	1.432E+01	1.026E+02
16.25	424.00	0.00	201.500	2.000	4810.18	-2195.05	4.810E+03	-2.195E+03	8.988E-01	4.384E-01	0.
23.00	424.00	0.00	209.000	-2.000	5501.97	-1857.64	5.502E+03	-1.858E+03	8.988E-01	4.384E-01	
55.00	424.00	0.00	139.000	0.000	10975.63	812.04	1.098E+04	8.120E+02	8.988E-01	4.384E-01	
AIRCRAFT 2											
0.00	154.00	-5.00	422.000	-1.000	-7399.70	15171.64	-2.996E+03	1.717E+04	-4.836E+03	1.146E+01	-1.313E+02
6.00	124.00	0.00	416.000	-1.000	-5770.33	13294.88	-5.770E+03	1.329E+04	8.290E-01	-5.592E-01	0.
20.00	124.00	0.00	402.000	3.000	-1023.26	10092.94	-1.023E+03	1.009E+04	8.290E-01	-5.592E-01	
46.67	124.00	0.00	482.000	0.000	3748.33	3501.92	8.748E+03	3.502E+03	8.290E-01	-5.592E-01	

TIME	K	DIST	HEADING	SPEED	X1	Y1	HEADING	SPEED	X2	Y2	DELTA X	DELTA Y	DELTA Z
0.00	1	23214.90	294.00	169.00	4764.59	-4601.11	154.00	422.00	-7399.70	15171.64	-12164.29	19772.75	0.00
1.00	1	22647.72	302.00	171.00	4614.62	-4521.34	149.00	421.00	-7198.65	14801.34	-11813.27	19322.68	0.00
2.00	1	22069.05	310.00	173.00	4475.59	-4420.30	144.00	420.00	-6966.64	14450.79	-11442.24	18871.10	0.00
3.00	1	21485.16	318.00	175.00	4350.55	-4299.52	139.00	419.00	-6705.59	14122.59	-11056.13	18422.11	0.00
4.00	1	20902.52	326.00	177.00	4242.30	-4160.92	134.00	418.00	-6417.61	13819.11	-10659.90	17980.04	0.00
5.00	1	20327.76	334.00	179.00	4153.39	-4006.89	129.00	417.00	-6105.02	13542.55	-10258.41	17549.44	0.00
6.00	1	19767.55	342.00	181.00	4086.03	-3840.12	124.00	416.00	-5770.33	13294.88	-9856.36	17135.00	0.00
7.00	1	19219.97	350.00	183.00	4042.06	-3663.66	124.00	415.00	-5425.86	13062.53	-9467.92	16726.20	0.00
8.00	1	18680.74	358.00	185.00	4022.87	-3480.82	124.00	414.00	-5082.22	12830.75	-9105.09	16311.57	0.00
9.00	1	18152.97	366.00	187.00	4029.38	-3295.08	124.00	413.00	-4739.42	12599.52	-8768.80	15894.61	0.00
10.00	1	17639.74	374.00	189.00	4062.02	-3110.09	124.00	412.00	-4397.46	12368.86	-8459.46	15478.95	0.00
11.00	1	17144.00	382.00	191.00	4120.71	-2929.55	124.00	411.00	-4056.29	12138.75	-8177.00	15068.29	0.00
12.00	1	16658.54	390.00	193.00	4204.83	-2757.13	124.00	410.00	-3715.97	11909.20	-7920.80	14666.33	0.00
13.00	1	16215.87	398.00	195.00	4313.24	-2596.44	124.00	409.00	-3376.48	11680.21	-7689.72	14276.65	0.00
14.00	1	15788.19	406.00	197.00	4444.30	-2450.92	124.00	408.00	-3037.82	11451.78	-7482.12	13902.70	0.00
15.00	1	15347.30	414.00	199.00	4595.87	-2323.76	124.00	407.00	-2699.99	11223.91	-7295.86	13547.67	0.00
16.00	1	15014.52	422.00	201.00	4765.36	-2217.89	124.00	406.00	-2362.98	10996.60	-7128.34	13214.48	0.00
17.00	2	14662.74	424.00	203.00	4946.52	-2128.55	124.00	405.00	-2026.81	10769.84	-6973.33	12898.40	0.00
18.00	2	14312.82	424.00	205.00	5129.87	-2039.12	124.00	404.00	-1691.46	10543.65	-6821.33	12582.77	0.00
19.00	2	13963.90	424.00	207.00	5315.02	-1948.82	124.00	403.00	-1356.94	10318.02	-6671.97	12266.84	0.00
20.00	3	13615.98	424.00	209.00	5501.97	-1857.64	124.00	402.00	-1023.26	10092.94	-6525.23	11950.58	0.00
21.00	3	13267.22	424.00	207.00	5688.92	-1766.46	124.00	405.00	-688.74	9867.31	-6377.66	11633.76	0.00
22.00	3	12915.72	424.00	205.00	5874.07	-1676.15	124.00	408.00	-351.74	9639.99	-6225.81	11316.15	0.00
23.00	3	12561.49	424.00	203.00	6057.43	-1586.72	124.00	411.00	-12.25	9411.01	-6069.67	10997.73	0.00
24.00	3	12204.50	424.00	201.00	6238.98	-1498.17	124.00	414.00	329.73	9180.34	-5909.25	10678.51	0.00
25.00	3	11844.75	424.00	199.00	6418.74	-1410.50	124.00	417.00	674.20	8947.99	-5744.55	10358.49	0.00
26.00	3	11482.23	424.00	197.00	6596.70	-1323.70	124.00	420.00	1021.15	8713.97	-5575.55	10037.67	0.00

Figure 59. Trajectory Print (Off Line)

TIME	K	DIST	HEADING	SPEED	X1	Y1	HEADING	SPEED	X2	Y2	DELTA X	DELTA Y	DELTA Z
27.00	3	11116.94	424.00	195.00	6772.87	-1237.78	124.00	423.00	1370.59	8478.27	-5402.28	9716.05	0.00
28.00	3	10743.86	424.00	193.00	6947.23	-1152.74	124.00	426.00	1722.52	8240.89	-5224.72	9393.63	0.00
29.00	3	10378.00	424.00	191.00	7119.80	-1068.57	124.00	429.00	2076.93	8001.84	-5042.87	9070.41	0.00
30.00	3	10004.36	424.00	189.00	7290.57	-985.28	124.00	432.00	2433.83	7761.11	-4856.74	8746.39	0.00
31.00	3	9627.95	424.00	187.00	7459.55	-902.87	124.00	435.00	2793.22	7518.70	-4666.33	8421.56	0.00
32.00	3	9248.77	424.00	185.00	7626.72	-821.33	124.00	438.00	3155.09	7274.61	-4471.63	8095.94	0.00
33.00	3	8866.84	424.00	183.00	7792.10	-740.67	124.00	441.00	3519.45	7028.84	-4272.65	7769.51	0.00
34.00	3	8482.18	424.00	181.00	7955.68	-660.88	124.00	444.00	3886.30	6781.40	-4069.38	7442.29	0.00
35.00	3	8094.83	424.00	179.00	8117.46	-581.98	124.00	447.00	4255.64	6532.28	-3861.82	7114.26	0.00
36.00	3	7704.83	424.00	177.00	8277.45	-503.95	124.00	450.00	4627.46	6281.48	-3649.99	6785.43	0.00
37.00	3	7312.24	424.00	175.00	8435.64	-426.79	124.00	453.00	5001.77	6029.01	-3433.86	6455.80	0.00
38.00	3	6917.11	424.00	173.00	8592.03	-350.52	124.00	456.00	5378.57	5774.85	-3213.46	6125.37	0.00
39.00	3	6519.57	424.00	171.00	8746.62	-275.12	124.00	459.00	5757.86	5519.02	-2988.76	5794.14	0.00
40.00	3	6119.73	424.00	169.00	8899.41	-200.60	124.00	462.00	6139.63	5261.51	-2759.79	5462.11	0.00
41.00	3	5717.76	424.00	167.00	9050.41	-126.95	124.00	465.00	6523.89	5002.33	-2526.53	5129.28	0.00
42.00	3	5313.91	424.00	165.00	9199.61	-54.18	124.00	468.00	6910.63	4741.46	-2288.98	4795.64	0.00
43.00	3	4908.48	424.00	163.00	9347.01	17.71	124.00	471.00	7299.87	4478.92	-2047.15	4461.21	0.00
44.00	3	4501.93	424.00	161.00	9492.62	88.73	124.00	474.00	7691.59	4214.71	-1801.03	4125.98	0.00
45.00	3	4094.89	424.00	159.00	9636.43	158.87	124.00	477.00	8085.79	3948.81	-1550.63	3789.94	0.00
46.00	3	3688.28	424.00	157.00	9778.44	228.13	124.00	480.00	8482.49	3681.24	-1295.95	3453.10	0.00
47.00	3	3283.64	424.00	155.00	9918.65	296.52	124.00	482.00	8881.53	3412.08	-1037.12	3115.56	0.00
48.00	3	2884.83	424.00	153.00	10057.06	364.03	124.00	482.00	9281.13	3142.55	-775.93	2778.52	0.00
49.00	3	2495.64	424.00	151.00	10193.68	430.66	124.00	482.00	9680.72	2873.02	-512.95	2442.36	0.00
50.00	3	2121.63	424.00	149.00	10328.50	496.42	124.00	482.00	10080.32	2603.48	-248.18	2107.07	0.00
51.00	3	1772.75	424.00	147.00	10461.52	561.29	124.00	482.00	10479.92	2333.95	18.40	1772.66	0.00
52.00	3	1467.42	424.00	145.00	10592.74	625.30	124.00	482.00	10879.51	2064.42	286.77	1439.13	0.00
53.00	3	1238.73	424.00	143.00	10722.17	688.42	124.00	482.00	11279.11	1794.89	556.94	1106.47	0.00
54.00	3	1134.56	424.00	141.00	10849.80	750.67	124.00	482.00	11678.70	1525.36	828.91	774.69	0.00
55.00	4	1198.63	424.00	139.00	10975.63	812.04	124.00	482.00	12078.30	1255.83	1102.67	443.79	0.00
56.00	4	1381.99	424.00	139.00	11100.56	872.98	124.00	482.00	12477.90	986.30	1377.34	113.32	0.00
57.00	4	1666.21	424.00	139.00	11225.49	933.91	124.00	482.00	12877.49	716.77	1652.00	-217.14	0.00
58.00	4	2002.97	424.00	139.00	11350.43	994.34	124.00	482.00	13277.09	447.24	1926.66	-547.61	0.00
59.00	4	2369.99	424.00	139.00	11475.36	1055.78	124.00	482.00	13676.68	177.71	2201.33	-878.07	0.00
60.00	4	2755.19	424.00	139.00	11600.29	1116.71	124.00	482.00	14076.28	-91.83	2475.99	-1208.54	0.00

Figure 59. Trajectory Print (Off Line) (Continued)

A more detailed description of the method of calculating the total number of commands per unit time can be obtained by referring to Figure 60. The X, Y coordinates of the aircraft on the path Γ are computed as follows:

ω_p = pilot turn rate

ω_c = controller turn rate

V = velocity

$r_1 = V/\omega_p$

$r_2 = V/\omega_c$

$\Theta = \frac{\pi}{2} - H$
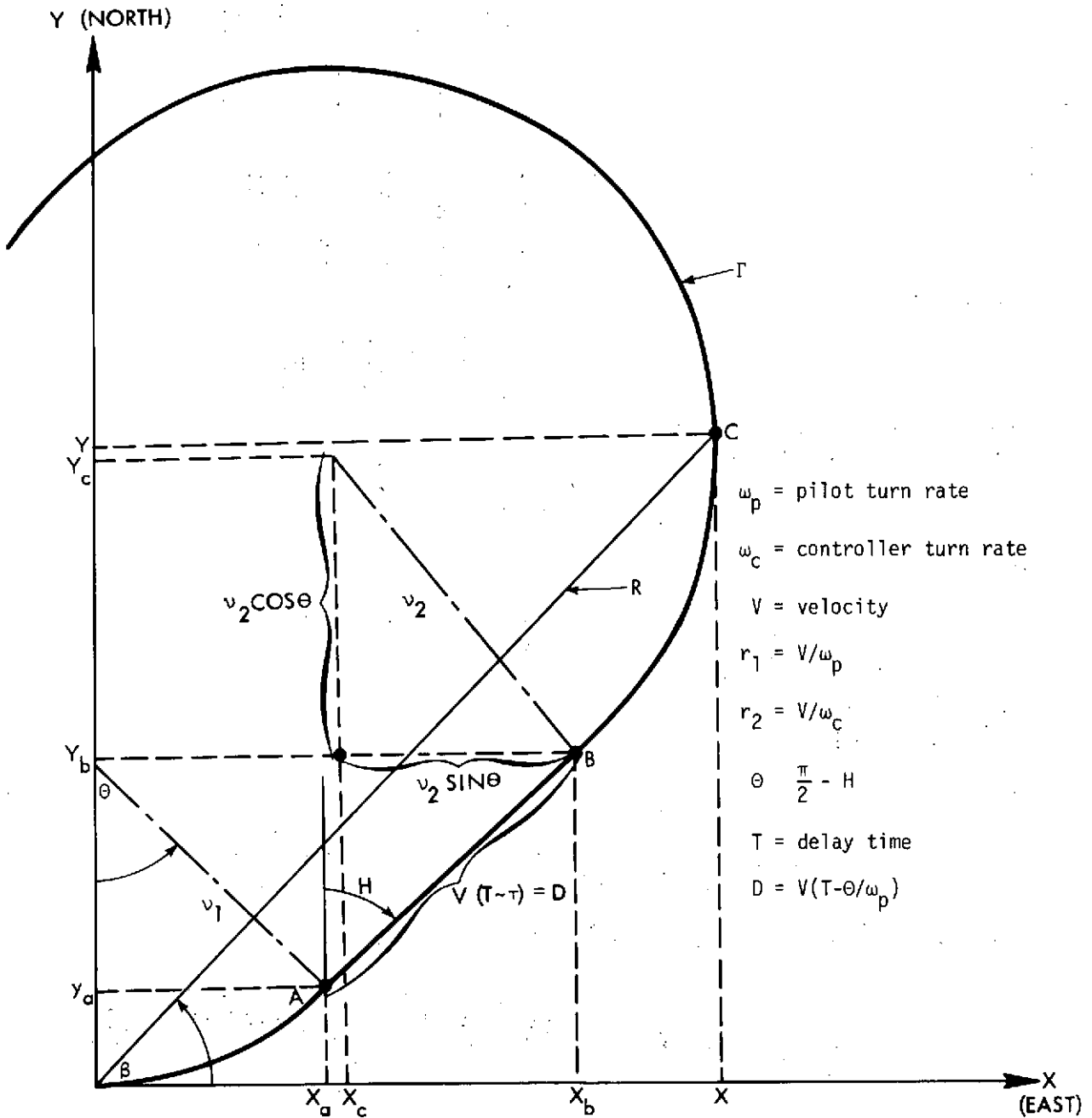
T = delay time

$D = V(T - \Theta/\omega_p)$

Figure 60. Turning Geometry

From the diagram:

$$X_a = r_1 \sin \Theta$$

$$Y_a = r_1 (1-\cos \Theta)$$

Also:

$$\tau = |\Theta|/\omega_p$$

Then:

$$C_b = X_a + V \cos \Theta (T-\tau)$$

$$Y_b = Y_a + V \sin \Theta (T-\tau)$$

So:

$$X_c = X_b - r_2 \sin \Theta S_T$$

$$Y_c = Y_b + r_2 \cos \Theta S_T$$

Where:

$$S_T = \begin{cases} +1 & \text{left turn} \\ -1 & \text{right turn} \end{cases}$$

The circle Γ is defined by:

$$(X-X_c)^2 + (Y-Y_c)^2 = r_2^2$$

Or:

$$(X^2+Y^2) - 2(XX_c + YY_c) + (X_2+Y_c^2-r_2^2) = 0$$

Let: $X = R \cos \beta$
      $Y = R \sin \beta$

Then:

$$R^2 - 2R (X_c \cos \beta + Y_c \sin \beta) + (X_c^2+Y_c^2-r_2^2) = 0$$

So: the solution for the larger R is given by:

$$R = (X_c \cos \beta + Y_c \sin \beta) + \sqrt{(X_c \cos \beta + Y_c \sin \beta)^2 - (X_c^2+Y_c^2-r_2^2)}$$

184

An aircraft approaching at heading H_A will potentially interfere with the aircraft at the origin if its containment contour intersects the region defined by:

$$\omega = \max(d_1) + \max(d_2)$$

If the two aircraft are identical (i.e., have identical containment contours) the interference distance W is doubled.

$$W = 2\omega = 2[\max(d_1) + \max(d_2)]$$

Thus the program computes the value of W for each heading H_A and forms the average

$$C_R = \rho \frac{1}{N} \sum_{i=1}^{N} W_i V_{Ri}$$

where: ρ is the aircraft density, V_{ri} is the relative velocity for heading H_{Ai}, and N is the number of approach headings considered. The result is the command rate, C_R. Simultaneously, the program computes

$$C_{R \ reg} = \rho \frac{1}{N} \sum_{i=1}^{N} P_D V_{Ri} = \rho P_D \frac{1}{N} \sum_{i=1}^{N} V_{Ri}$$

where P_D is a (constant) minimum separation distance. The quantity $C_{R \ reg}$ represents the required minimum communication rate. By comparing C_R with $C_{R \ reg}$ it is possible to observe the increase in communication rate caused by maneuvers as a function of the parameters of the maneuver.

The basic procedure now is to find the maximum value of R (and the associated X and Y) for all permissible values of Θ in a left turn. This set of points, parameterized by β, represents the aircraft containment contour for left turns. Call this set of points $R_{max\ L}(\beta)$, $X_{max\ L}(\beta)$, $Y_{max\ L}(\beta)$. Repeat the process for right turns to obtain the set $R_{maxR}(\beta)$, $X_{maxR}(\beta)$, $Y_{maxR}(\beta)$. Now the desired contour is the set of points which, for a given value of β, represent the smaller value of $R_{max\substack{L \\ R}}$. This contour represents that area in which the controller can guarantee the presence of the aircraft regardless of the pilot-initiated turn. Define the points on this contour (Figure 61) as $E_x(\beta)$, $E_y(\beta)$, then:

$$E_x(\beta) = \begin{cases} X_{max\ R}(\beta) & \text{if } R_{max\ R}(\beta) < R_{max\ L}(\beta) \\[6pt] X_{max\ L}(\beta) & \text{if } R_{max\ L}(\beta) < R_{max\ R}(\beta) \\[6pt] Y_{max\ R}(\beta) & \text{if } R_{max\ R}(\beta) < R_{max\ L}(\beta) \\[6pt] Y_{max\ L}(\beta) & \text{if } R_{max\ L}(\beta)\ R_{max\ R}(\beta) \end{cases}$$
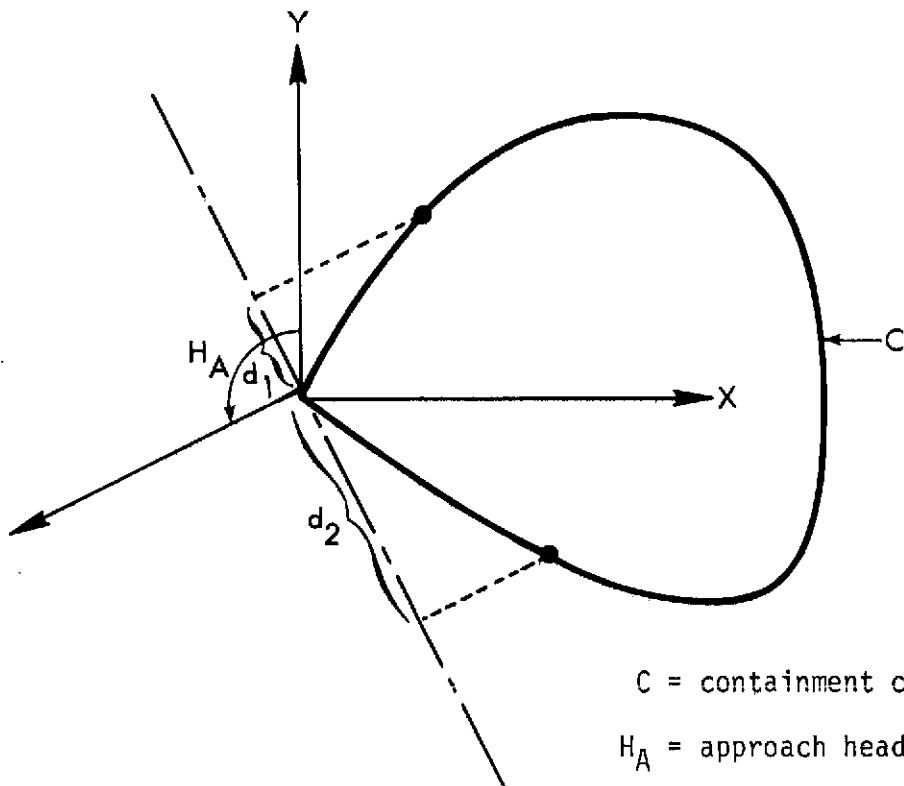

C = containment contour

H_A = approach heading of second A/C

Figure 61. Containment Contour