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A STUDY OF TWO STATISTICAL METHODS AS APPLIED TO SHUTTLE SOLID ROCKET BOOSTER EXPENDITURES

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Prepared for

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812

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FOREWORD

The work described in this report was conducted by Northrop Services, Inc., Huntsville, Alabama, for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Space Sciences Laboratory, under contract no. NAS8-21810, Appendix A schedule order number 13. Dr. George H. Fichtl was the Technical Coordinator for this task.

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Section I

INTRODUCTION

In launching the Space Shuttle, it is expected that two recoverable Solid Rocket Boosters (SRB) will be used. These boosters will detach from the main vehicle and drop to the ocean, from which they are to be recovered. In some cases, a component, or possibly a whole booster, will not be recoverable and will have to be replaced.

It is important to know the probable number of booster components needed to successfully accomplish the Space Shuttle Mission of 440 flights, so that advance planning on the optimum number of SRB components to be built can be carried out. Several situations are analyzed, namely, these cases where:

(1) The booster is used continuously until not recoverable;
(2) The booster is used for 20 launches unless not recoverable;
(3) The booster is made up of identical, independent, replaceable components.

Two independent techniques have been used in this analysis. The first method, to be called the State Probability Method, consists of defining an appropriate state space for the outcome of the random trials. Then the probability of reaching each state is evaluated. The second method is called the Monte Carlo Technique, or the Model Simulation Method. Detailed discussion of the Monte Carlo Method can be found in references 1 and 2. The Monte Carlo Method is simpler to formulate than the State Probability Method, but requires longer computer running time and can have statistical uncertainty due to sampling size. Both methods are compared to check the results.

Finally, a simple analysis of expected cost is carried out. This includes the case where the boosters can be built initially at some initial cost and the remainder built as needed. The resulting optimum number to build initially and the minimum expected cost are evaluated.

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1. *Graves, M. E. and Perlmutter, M. "Statistical Analysis of Flight Times for Space Shuttle Ferry Flights", NASA CR-129021, Jan 1974.*
2. *Shreider, Y. U. A., "The Monte Carlo Method", Pergamon Press, 1966.*

Section II

STATE PROBABILITY AND MONTE CARLO METHODS

The State Probability Method consists of defining the statistical problem to be solved in terms of a state space of pertinent possible outcomes of the random trials. The probability of obtaining each of these outcomes is then calculated. The method will be illustrated in the solution to the present problem.

The Monte Carlo Method simulates the problem, using a random number generator on a computer. A sufficient number of trials are simulated to obtain the statistical behavior of the system.

The probability density of the number of one-component boosters needed for a mission of 440 launches is calculated in subsections 2.1 and 2.2. This is found for a given attrition rate of losing the booster in a launch. The booster is used continuously until nonusable.

Subsection 2.1 gives the State Probability Method of solution, while subsection 2.2 gives the Monte Carlo solution.

Subsection 2.3 and 2.4 consider the case where the booster can only be used for a maximum of 20 launches. Subsection 2.3 considers the State Probabilty solution while subsection 2.4 uses the Monte Carlo solution.

Subsections 2.5 and 2.6 extend the results to the multicomponent rocket booster. In this case, the booster is made of identical replaceable components. Each component has the same attrition rate, q. The probability distribution of the number of components needed for 440 missions is calculated. In subsection 2.5 the State Probability Method is used, while in subsection 2.6 the Monte Carlo analysis is used.

2.1 STATE PROBABILITY METHOD – CONTINUOUS USE, ONE-COMPONENT BOOSTER

To evaluate the probability density of the number of one-component boosters needed to achieve a mission of 440 launches for a given attrition rate, q, per launch where the booster is used continually until lost, then replaced, the following steps are taken. First, a state $S_{r,k}$ is defined as shown in Figure 2-1. The term $S_{r,k}$ refers to the state of k launches with r booster replacements. The state probability for the booster replacement in 1 launch is given by

$$p(S_{1,k}) = p_k \qquad (2\text{-}1)$$

where p_k is the probability of replacing one booster in k launches.

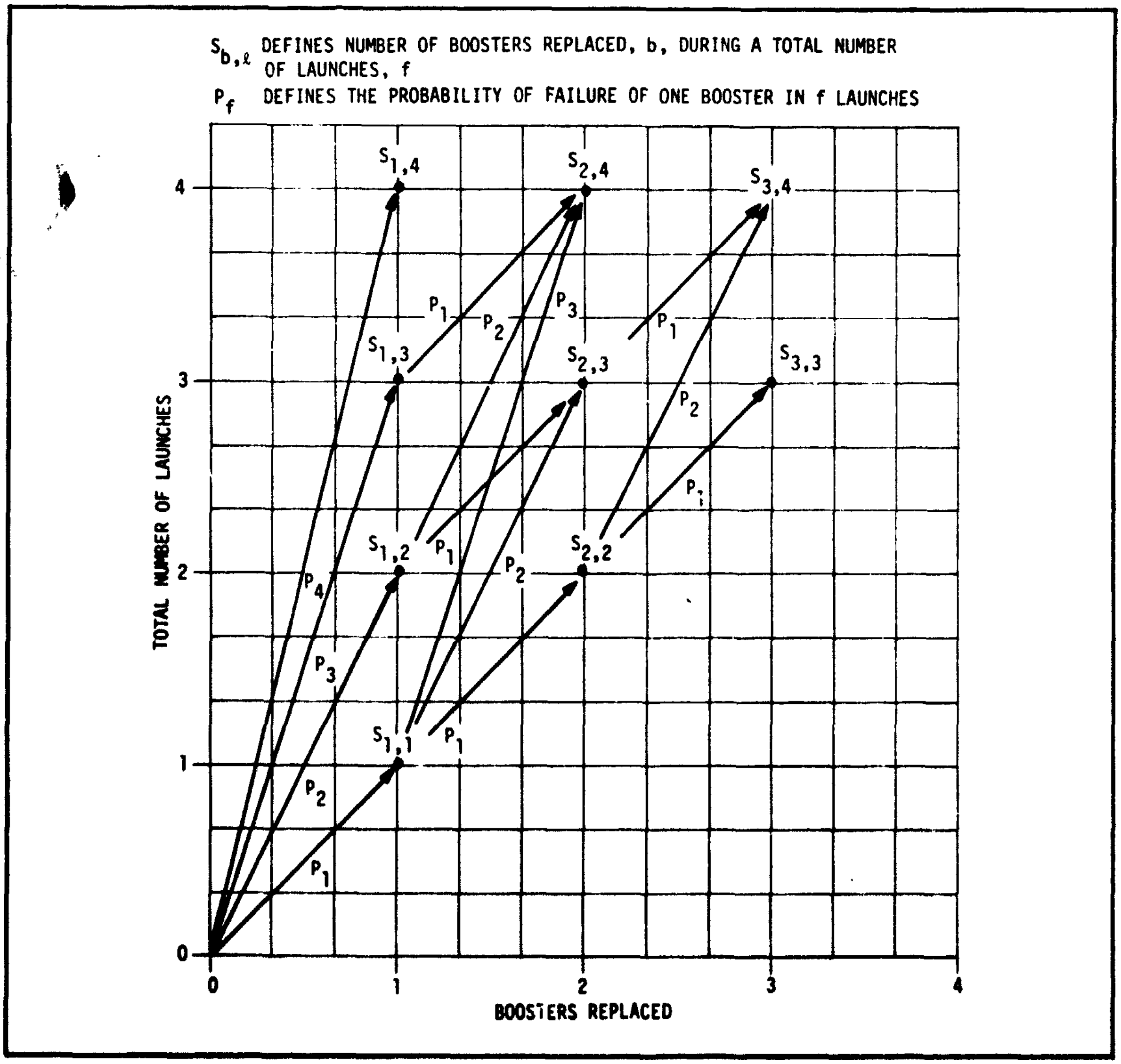

Figure 2-1. STATE SPACE DEFINING NUMBER OF BOOSTERS REPLACED FOR TOTAL NUMBER OF LAUNCHES

The attrition rate, or probability of booster loss per launch, is q, so letting p=1-q, then

$$p_1 = q \qquad p_2 = qp \qquad p_k = qp^{k-1} \tag{2-2}$$

Therefore,

$$\sum_{k=1}^{\infty} p(S_{1,k}) = q\ [1 + p + p^2 + \ldots\ ] = \frac{q}{1-p} = 1 \quad . \tag{2-3}$$

The probability of 0 booster replacements in the mission of 440 flights is given by

$$P_{1,0} = \sum_{j=440}^{\infty} p(S_{1,j}) = 1-q \sum_{j=1}^{439} p^{j-1} = p^{439} \quad . \tag{2-4}$$

This is the same result given by the binomial distribution for 0 replacements in 439 launches.

$$P_{1,0} = B\ (k = 439,\ r = 0) \tag{2-5}$$

where, $$B(k,r) = \frac{k!}{r!(k-r)!}\ p^{k-r}\ q^r \tag{2-6}$$

This process is continued to the state $S_{2,k}$, which refers to replacing two boosters in k total launches. The probability of reaching state $S_{2,k}$ as can be seen from Figure 2-1 is given by

$$\begin{aligned} p\ (S_{2,j}) &= p\ (S_{1,j-1})\ p_1 + p\ (S_{1,j-2})\ p_2 + \ldots + p\ (S_{1,1})\ p_{j-1} \\ &= 2p_{j-1}\ p_1 + 2p_{j-2}\ p_2 + \ldots + 2p_{\frac{j+1}{2}}\ p_{\frac{j-1}{2}} \quad j \text{ odd} \\ &= 2p_{j-1}\ p_1 + 2p_{j-2}\ p_2 + \ldots + p_{\frac{j}{2}}\ p_{\frac{j}{2}} \quad . \quad j \text{ even} \end{aligned} \tag{2-7}$$

This can be seen to be equal to

$$p(S_{2,j}) = (j-1)\ q^2\ p^{j-2} \tag{2-8}$$

This can be checked as before

$$\sum_{j=2}^{\infty} p(S_{2,j}) = \sum_{j=2}^{\infty} (j-1)\ q^2\ p^{j-2} = q^2\ [1 + 2p + 3p^2 + \ldots\ ] \tag{2-9}$$

$$= \frac{q^2}{(1-p)^2} = 1 \quad .$$

The probability of using one or fewer booster replacements in the mission is then given by

$$P_{1,1} = \sum_{j=440}^{\infty} p(S_{2,j}) = 1 - \sum_{j=2}^{439} p(S_{2,j}) = 1 - \sum_{j=2}^{439} (j-1)\ q^2\ p^{j-2} \quad . \tag{2-10}$$

It can be seen that (ref. 3, Section III)

$$\sum_{j=2}^{439} (j-1)\ p^{j-2} = 1 + 2p^1 + 3p^2 + \ldots + 438p^{437}$$

$$= \frac{1}{q} + p\ \frac{(1-p^{437})}{q^2} - \frac{[1 + 437]\ p^{438}}{q} \tag{2-11}$$

$$= \frac{1}{q^2} - p^{438}\ \left(\frac{1}{q^2} + \frac{438}{q}\right) \quad .$$

Then,

$$P_{1,1} = p^{438}\ (q + p) + 438q\ p^{438} = p^{439} + 439q\ p^{438} \tag{2-12}$$

This is equivalent to the cumulative binomial distribution with 1 or less replacements in 439 launches (see eq. 2-6)

$$P_{1,1} = B(k = 439,\ r = 0) + B\ (k = 439,\ r = 1) \tag{2-13}$$

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3. *Jolley, L. B. W., "Summation of Series", Dover Publicatins 1961.*

This process can be continued to give

$$P_{1,2} = B(k = 439, r = 0) + B(k = 439, r = 1) + B\ (k = 439, r = 2), \tag{2-14}$$

etc.

Besides the cumulative distribution $P_{1,r}$ that is, the probability of using r replacements or less in the mission, $p_{1,r}$ can be written, the distribution which gives the probability of using exactly r replacements for the mission. Thus,

$$\begin{aligned} p_{1,0} &= B\ (k = 439, r = 0) \\ p_{1,1} &= B\ (k = 439, r = 1) \\ p_{1,2} &= B\ (k = 439, r = 2) \end{aligned} \tag{2-15}$$

The probability distribution $p_{1,n}$ shown in Figure 2-2 gives the probability of using n one-component boosters in a mission of 440 launches.

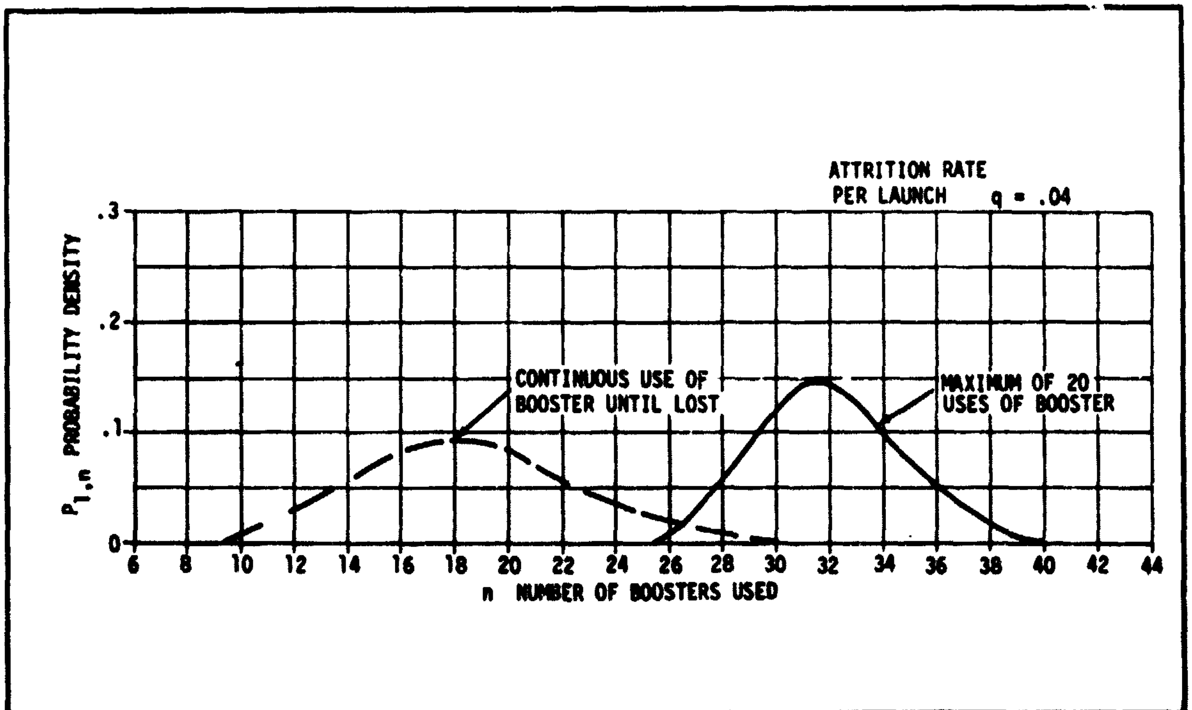

Figure 2-2. PROBABILITY DENSITY OF NUMBER OF SINGLE COMPONENT BOOSTERS USED IN 440 LAUNCHES

The cumulative distribution, $P_{1,n}$ which gives the probability of using n or less one-component boosters in 440 launches, is shown in Figure 2-3. As has been shown previously, the probability density in Figure [illegible] is the binomial distribution for 439 launches or trials where the probability of failure or attrition is q and the number of failures, or boosters replaced, is r The mean and variance of the binomial is given by

$$\mu = 439q \text{ and } \sigma^2 = 439q\,(1-q). \qquad (2\text{-}16)$$

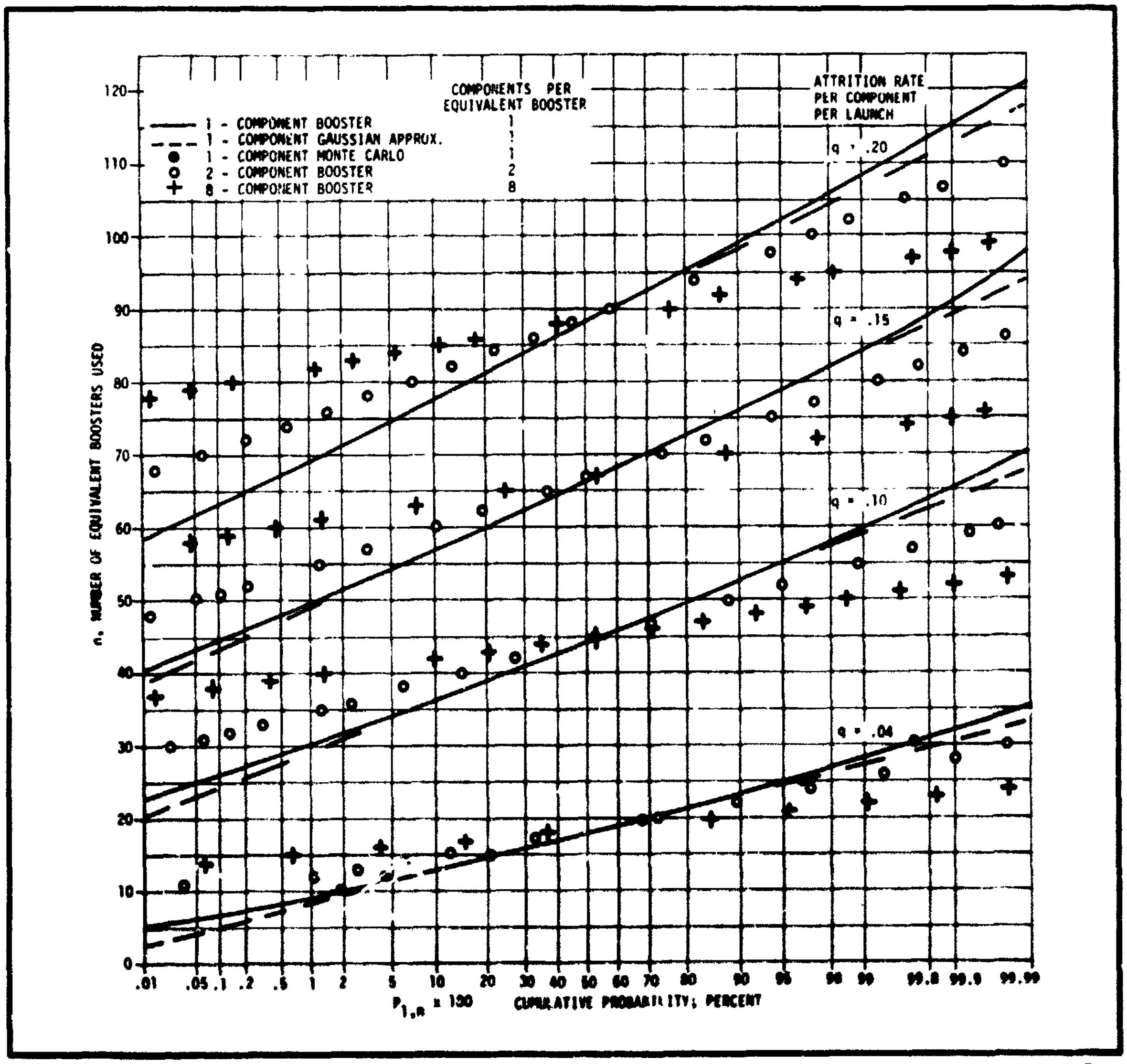

Figure 2-3. CUMULATIVE PROBABILITY OF USING n OR LESS COMPONENTS IN 1, 2 OR 8 COMPONENT BOOSTERS IN 440 LAUNCH MISSIONS; BOOSTERS REPLACED AT FAILURE

For a large number of launches, the binomial distribution can be approximated by a Gaussian distribution with the same mean and variance as the binomial. The 95 percent probability of using r replacements of the booster in 439 launch missions is given by the Gaussian distribution as

$$1.654 = \frac{r - \mu}{\sigma} \qquad (2\text{-}17)$$

The 50 percent ($P_{1,n} = .5$) and 95 percent ($P_{1,n} = .95$) cumulative probability for number of single-component boosters needed for a mission of 440 launches is plotted in Figure 2-4.

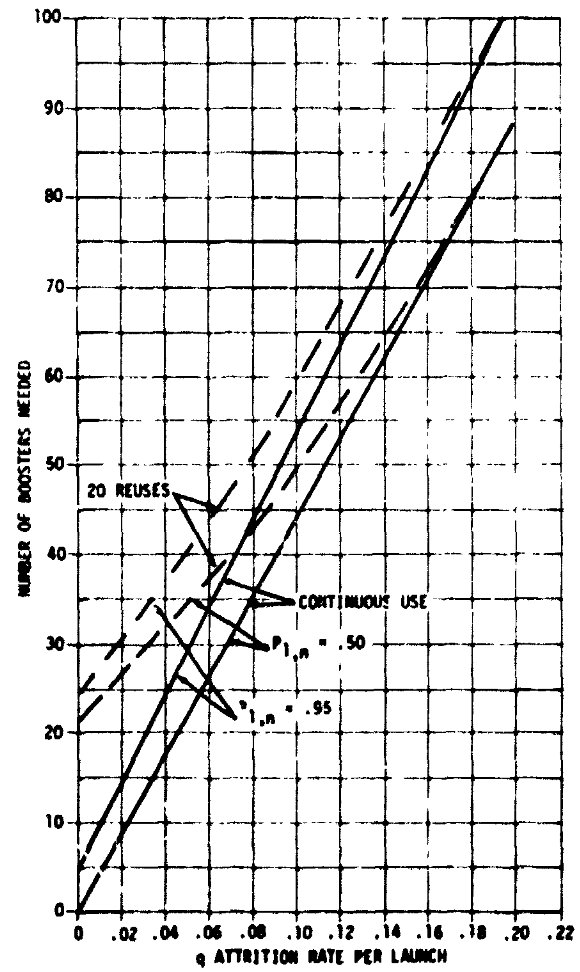

Figure 2-4. THE 50 PERCENT ($P_{1,n} = .50$) AND 95 PERCENT ($P_{1,n} = .95$) CUMULATIVE PROBABILITY FOR NUMBER OF SINGLE COMPONENT BOOSTERS NEEDED FOR A 440 LAUNCH MISSION

2.2 MONTE CARLO METHOD – CONTINUOUS USAGE, ONE-COMPONENT BOOSTER

For the same conditions discussed in the previous subsection a Monte Carlo solution can also be obtained. A set of pseudo random numbers which are uniformly distributed between 0 and 1 is generated. Sequences of these numbers are examined and recovery of SRB is assumed if the values are less than 1-q and non-recovery of SRB is assumed for any number exceeding 1-q. The probability of 440 successful missions is calculated for a given number of starting SRB. These results are plotted in Figure 2-3 and are in good agreement with the results of the State Probability Method.

2.3 STATE PROBABILITY METHOD – 20 USES ONE-COMPONENT BOOSTER

A calculation similar to that of the continuous usage case can be carried out if the booster is replaced after 20 uses or failure. For this case, instead of (eq. 2-2), the probability of replacing one booster in k launches is obtained,

$$p_k = q\, p^{k-1},\ 1 \leq \ell < 20 \tag{2-18}$$

$$p_{20} = (1)p^{19}$$

$$p_k = 0\ ;\quad \ell > 20 \quad .$$

As before, (eq. 2-1) gives

$$p(S_{1,k}) = p_k \quad . \tag{2-19}$$

Now it can be seen that

$$\sum_{k=1}^{\infty} p(S_{1,k}) = \sum_{j=1}^{\infty} p_j = q[1 + p + p^2 + \ldots + p^{18}] + p^{19}$$
$$= q[1 + p + p^2 + p^{17}] + p^{18} = 1 \quad . \tag{2-20}$$

Then as before,

$$P_{1,0} = \sum_{k=440}^{\infty} p(S_{1,k}) = 1 - \sum_{k=1}^{439} p(S_{1,k}) \quad . \tag{2-21}$$

This process is continued as before to yield

$$P_{1,1} = 1 - \sum_{j=2}^{439} p(S_{2,j}) \quad , \qquad (2\text{-}22)$$

where $p(S_{2,j})$ is given by equation 2-7. This procedure is then continued to obtain $P_{1,2}$, $P_{1,3}$ ... .

The probability distribution for the number of single-component boosters for 20 uses in a 440 launch mission is shown in Figure 2-2, and the cumulative distribution which gives the probability that n or less booster components will be used for the mission is shown in Figure 2-5. The result can be seen to be close to Gaussian, since the cumulative distribution is close to a straight line on the probability graph in Figure 2-5. The 50 percent ($p_{1,n}$=0.5) and 95 percent ($p_{1,n}$=0.95) cumulative probability for the number of boosters needed are also plotted in Figure 2-4, along with the case for continuous usage of the boosters. It can be seen that for high attrition rate, q = 0.2. The results for these two cases converge.

2.4 MONTE CARLO METHOD – 20 USES, ONE-COMPONENT BOOSTER

The Monte Carlo procedure in the present case of a maximum 20 uses of the booster is similar to the Monte Carlo solution for the continuous use case, except in the simulation the booster is replaced after 20 consecutive launches. Again the probability of the number of boosters needed to achieve 440 launches is calculated and plotted in Figures 2-2 and 2-5.

2.5 STATE PROBABILITY METHOD – MULTI-COMPONENT BOOSTER

The SRB can be assumed to be built of replaceable components, each component having an independent attrition rate. Interest is in the probable total number of components needed for a mission of 440 flights if each component has the same attrition rate, q. Again, a state space $c_{i,r}$ is drawn as shown in Figure 2-6. The state $c_{i,r}$ represents i independent components having a total of r replacements during a mission of 440 launches.

Considering a one-component rocket:

$$p(c_{1,r}) = p_{1,r} \; ; \; r = 0, \ldots, 439 \qquad (2\text{-}23)$$

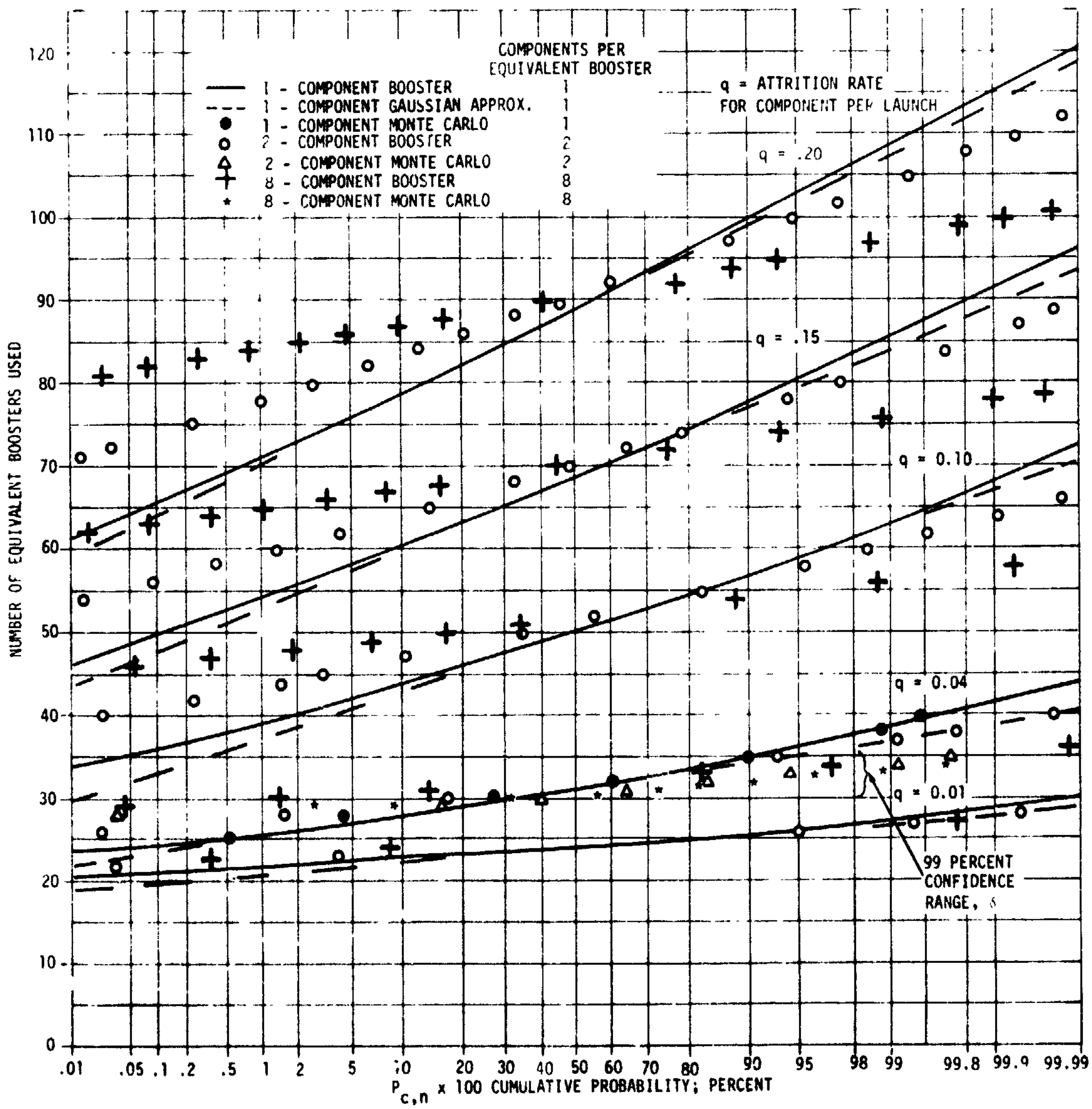

Figure 2-5. CUMULATIVE PROBABILITY OF USING η OR LESS COMPONENTS IN 1, 2 OR 8 COMPONENT BOOSTER IN 440 LAUNCHES: BOOSTER REPLACED AFTER 20 USES OR FAILURE

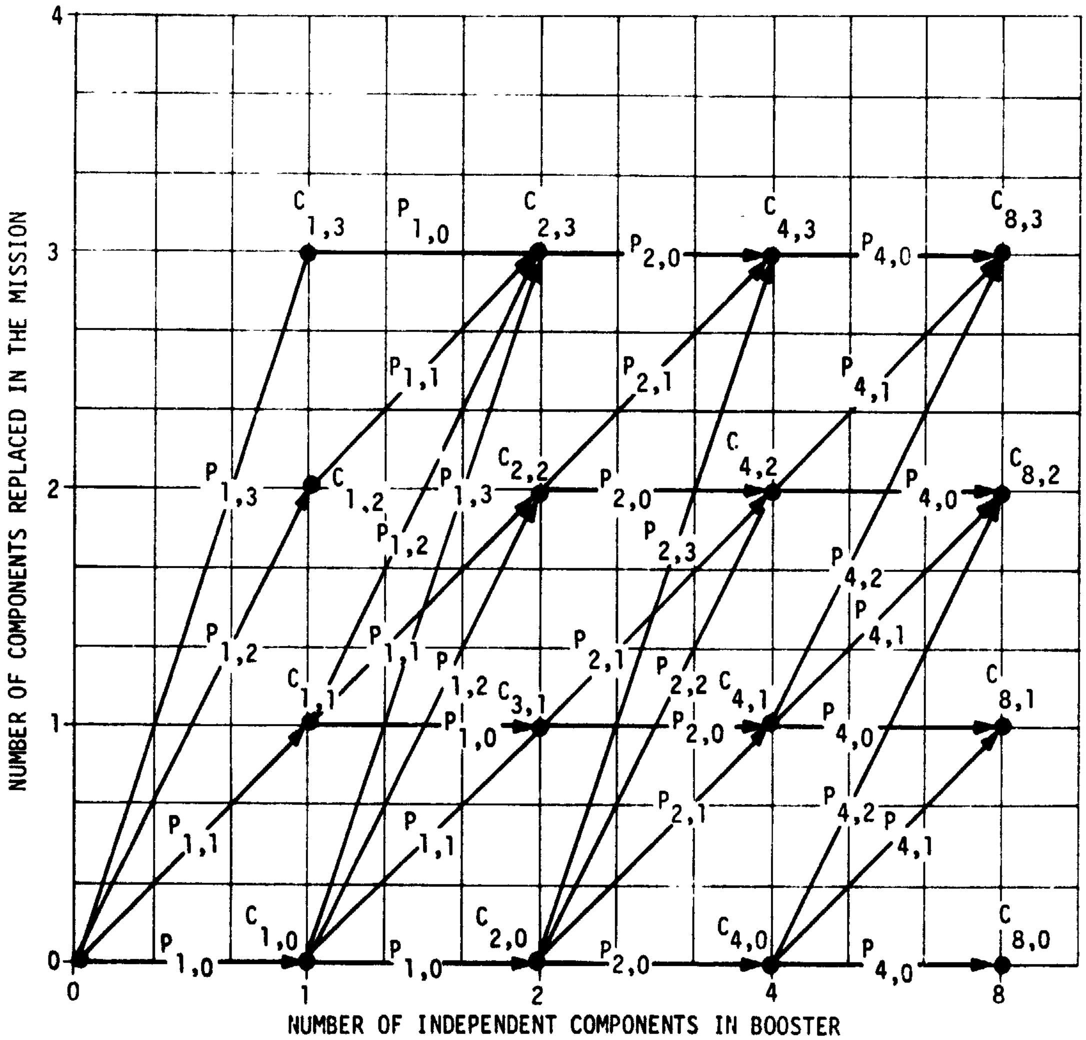

Figure 2-6. STATE SPACE DEFINING THE TOTAL NUMBER OF COMPONENTS REPLACED IN THE MISSION FOR A ROCKET BOOSTER BUILT WITH A GIVEN NUMBER OF REPLACEABLE COMPONENTS; $C_{c,r}$ DEFINES NUMBER OF COMPONENTS REPLACED, r DURING A SUCCESSFUL MISSION USING ROCKET BOOSTER CONTAINING c REPLACEABLE COMPONENTS; $P_{c,r}$ IS THE PROBABILITY OF STATE $C_{c,r}$ OCCURRING

where $p_{1,r}$ is the probability of r replacements of the one-component during a successful mission. This result was obtained previously in equation 2-15, etc., and is shown in Figure 2-2.

Obviously,

$$\sum_{r=0}^{\infty} p_{1,r} = 1 . \tag{2-24}$$

As seen in Figure 2-6, state $C_{2,r}$, which represents the state of a two-component booster with a total of r component replacements in the mission can be found as follows. Consider the state $c_{2,0}$, that is a two-component booster with zero component replacements in the mission. The probability of obtaining this state is given by

$$p(c_{2,0}) = p_{2,0} = p(c_{1,0})\, p_{1,0} = p_{1,0}\, p_{1,0} . \tag{2-25}$$

This process can be continued to give

$$p(c_{2,1}) = p_{2,1} = p(c_{1,0})\, p_{1,1} + p(c_{1,1})\, p_{1,0} = 2p_{1,1}\, p_{1,0} \tag{2-26}$$

or for an odd number replacements, r odd,

$$p_{2,r} = p(c_{2,r}) = 2p_{1,r}\, p_{1,0} + 2p_{1,r-1}, p_{1,1} + \dots + 2p_{1,\frac{r+1}{2}}\, p_{1,\frac{r-1}{2}} , \tag{2-27}$$

while for r even,

$$p_{2,r} = 2p_{1,r}\, p_{1,0} + \dots + p_{1,\frac{r}{2}}\, p_{1,\frac{r}{2}} . \tag{2-28}$$

This can be extended to the state $c_{4,r}$, which represents a 4-component booster with a total of r components replaced in the mission as shown in Figure 2-6. Continuing as in the previous case, the probability of achieving state $c_{4,0}$, that is, a 4-component booster with 0 component replacements in the mission, can be written as

$$p_{4,0} = p(c_{4,0}) = p(c_{2,0})\, p_{2,0} = p_{2,0}\, p_{2,0} \quad . \tag{2-29}$$

Then continuing as before,

$$p_{4,1} = p(c_{4,1}) = p(c_{2,0})\, p_{2,1} + p(c_{2,1})\, p_{2,0} = 2p_{2,0}\, p_{2,1} \tag{2-30}$$

$$p_{4,2} = 2p_{2,2}\, p_{2,0} + p_{2,1}\, p_{2,1} \quad . \tag{2-31}$$

This can be extended for r odd to

$$p_{4,r} = p(c_{4,r}) = 2p_{2,r}\, p_{2,0} + 2p_{2,r-1}\, p_{2,1} + \cdots + 2p_{2,\frac{r+1}{2}}\, p_{2,\frac{r-1}{2}} \tag{2-32}$$

and for r even,

$$p_{4,r} = 2p_{2,r}\, p_{2,0} + \cdots + p_{2,\frac{r}{2}}\, p_{2,\frac{r}{2}} \quad . \tag{2-33}$$

This procedure can be applied to 8 similar components in a similar manner, giving

$$p_{8,0} = p(c_{8,0}) = p(c_{4,0})\, p_{4,0} = p_{4,0}\, p_{4,0} \tag{2-34}$$

$$p_{8,1} = p(c_{8,1}) = p(c_{4,1})\, p_{4,0} + p(c_{4,0})\, p_{4,1} = 2p_{4,1}\, p_{4,0} \tag{2-35}$$

Again, for a total of r replacements of an 8 independent component booster for r odd,

$$p_{8,r} = 2p_{4,r}\, p_{4,0} + 2p_{4,r-1}\, p_{4,1} + \cdots + 2p_{4,\frac{r+1}{2}}\, p_{4,\frac{r-1}{2}} \tag{2-36}$$

while for r even,

$$p_{8,r} = 2p_{4,r}\, p_{4,0} + \cdots + p_{4,\frac{r}{2}}\, p_{4,\frac{r}{2}} \tag{2-37}$$

These results are shown in Figures 2-3 and 2-5 as cumulative distributions,

$$P_{c,r} = \sum_{d=0}^{r} p_{c,d} \quad . \tag{2-38}$$

2.6 MONTE CARLO METHOD - MULTI-COMPONENT BOOSTER

The simulation for the multi-component SRB is similar to the Monte Carlo analyses for the single-component SRB. Two sets of pseudo random numbers are used for the 2-component booster, while 8 sets of pseudo random numbers are used for the 8-component SRB. The same method as used in the single-component SRB is then used. The results of these calculations are shown in Figure 2-5. The probability of having a specified number of booster components remaining after 440 missions, given the number of booster components initially available, is also shown in Figure 2-7.

The confidence range of the present Monte Carlo calculation can be found following the procedure shown in reference 1. Assume M_s is the number of 440 launch missions using n or less boosters in N Monte Carlo trials. Then the probability of this occurring $p_{c,n}$, is given as

$$\frac{M_s}{N} \simeq p_{c,n} \quad . \qquad (2\text{-}39)$$

The 99 percent confidence ranges, (see ref. 1) on $\frac{M_s}{N}$ is given by

$$\left|\frac{M_s}{N} - p_{c,n}\right| = \delta \leq 2.57 \sqrt{p_{c,n}(1-p_{c,n})/N} \qquad (2\text{-}40)$$

If, considering $p_{c,n} = 0.98$ one may obtain for N=350 trails

$$\delta \leq 2.57 \left[\frac{0.98(.02)}{350}\right] = 0.019 \quad . \qquad (2\text{-}41)$$

This result gives an error range in $p_{c,n}$ of ± 0.019. This would correspond, using the two-component State Probability results, to an equivalent error range of ± 6 equivalent boosters or ± 12 components. The Monte Carlo results fall well within this range, as can be seen in Figure 2-5.

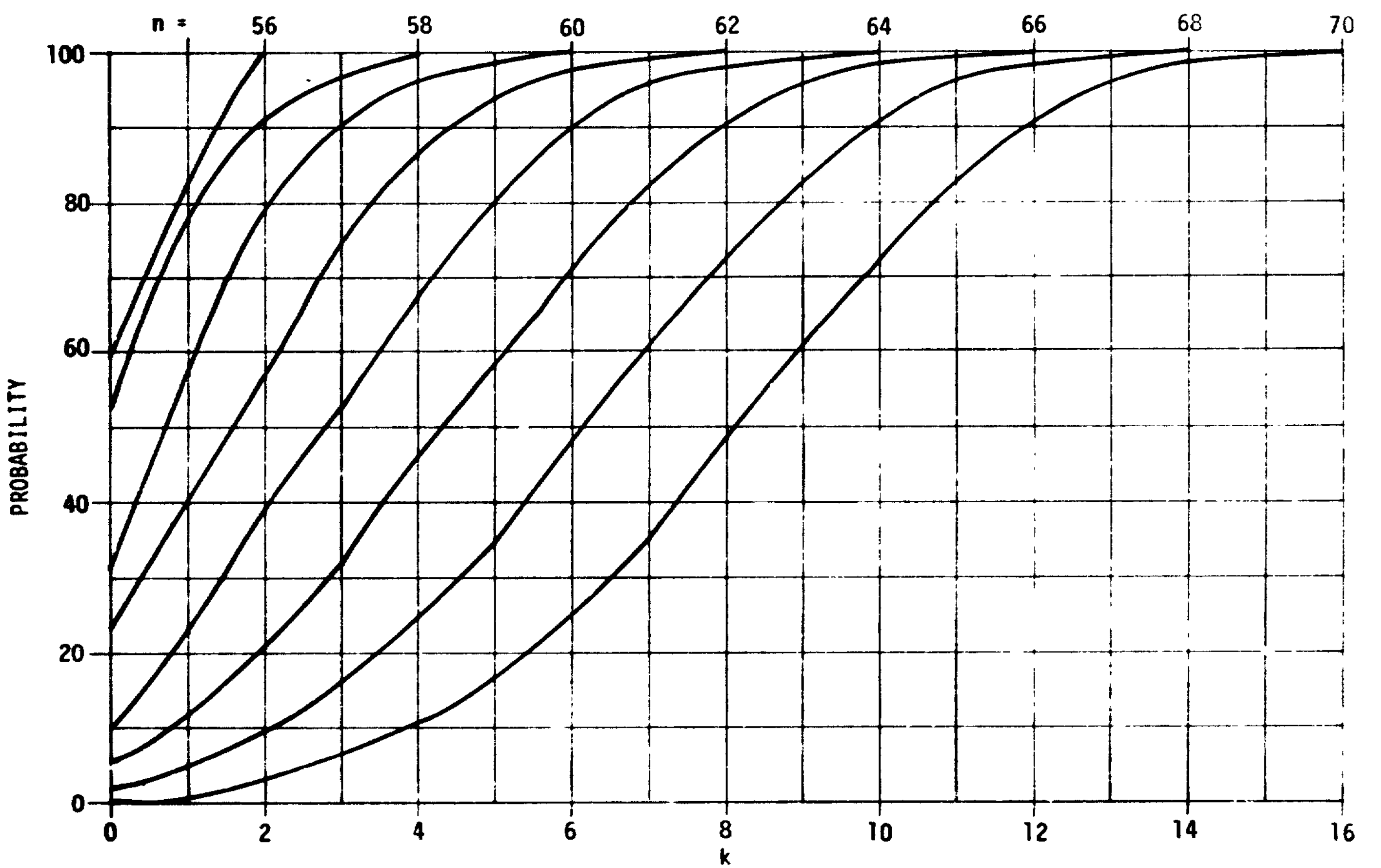

Figure 2-7. PROBABILITY OF k OR LESS BOOSTER COMPONENTS REMAINING AFTER 440 MISSIONS GIVEN n INITIAL BOOSTER COMPONENTS WITH THE ATTRITION RATE q OF 0.04 (LIMIT 20 REUSES OF EACH COMPONENT); TWO COMPONENT BOOSTER

Section III

APPLICATION OF RESULTS TO SHUTTLE SRB COST ANALYSES

The present results can indicate expected costs of the SRB and aid in the decision-making process to achieve minimum costs.

If the cost of building a booster is assumed as c, then the expected cost of building the boosters for the 440 launch mission is given by

$$E.C. = C \sum_{n=1}^{\infty} np_{c,n} = c\ \mu_n \qquad (3\text{-}1)$$

This shows that the expected cost of the booster for the mission is equal to the average of the distribution of number of boosters used in the mission times the cost per booster. Figure 2-4 indicates that the average number of boosters for the mission, which is approximately equal to $P_{1,n} = 0.5$, and also therefore the cost, will rise with the attrition rate and is higher for the 20 reuses case than for the continuous usage case. If we assume the multi-component booster cost is equal to the cost of a one-component booster, then, since the mean for the multicomponent booster is close to the value for the mean of a one-component booster (see Figures 2-3, 2-5), the cost for the multi-component boosters should be close to the cost for the single-component booster. However, the components are likely to have a significantly lower attrition rate which would give significantly lower cost for the multi-component booster compared to the single-component booster.

The decision process can be extended to the case where an initial group of boosters can be built at one cost and the remainder built as needed at a different cost. An objective is to find the optimum number to build initially so as to obtain a minimum cost. This can be done as follows. Assume an initial cost of building booster components of C_i. Then if additional booster components are needed to complete the mission, they can be built at cost c_f. A decision table can be constructed as shown in Table 3-1.

Table 3-1. DECISION TABLE FOR BOOSTER COMPONENTS
COMPONENTS BUILT INITIALLY

NUMBER OF COMPONENTS BUILT INITIALLY

NUMBER OF BOOSTER COMPONENTS NEEDED FOR MISSION

n \ b=	1	2	3	...
1	$N_{1,1}$ $C(N_{1,1})=C_i$ $EC(N_{1,1})=C(N_{1,1})p_{c,1}$	$N_{1,2}$ $C(N_{1,2})=2C_i$ $EC(N_{1,2})=C(N_{1,2})p_{c,1}$	$N_{1,3}$ $C(N_{1,3})=3C_i$ $EC(N_{1,3})=C(N_{1,3})p_{c,1}$	...
2	$N_{2,1}$ $C(N_{2,1})=C_i+C_f$ $EC(N_{2,1})=C(N_{2,1})p_{c,2}$	$N_{2,2}$ $C(N_{2,2})=2C_i$ $EC(N_{2,2})=C(N_{2,2})p_{c,2}$	$N_{2,3}$ $C(N_{2,3})=3C_i$ $EC(N_{2,3})=C(N_{2,3})p_{c,2}$	...
3	$N_{3,1}$ $C(N_{3,1})=C_i+2C_f$ $EC(N_{3,1})=C(N_{3,1})p_{c,3}$	$N_{3,2}$ $C(N_{3,2})=2C_i+C_f$ $EC(N_{3,2})=C(N_{3,2})p_{c,3}$	$N_{3,3}$ $C(N_{3,3})=3C_i$ $EC(N_{3,3})=C(N_{3,3})p_{c,3}$	...
	⋮	⋮	⋮	...
	$EC(1)=\sum_{n=1}^{\infty} C(N_{n,1})p_{c,n}$	$EC(2)=\sum_{n=1}^{\infty} C(N_{n,2})p_{c,n}$	$EC(3)=\sum_{n=1}^{\infty} C(N_{n,3})p_{c,n}$	

The possible decision on the number of components to build initially is along the top of the table and the number needed for the mission along the side. The state $N_{n,b}$ is defined, where b refers to the number of components built initially and n refers to the number needed. The cost of each state is given by

$$C(N_{n,b}) = bC_i + (n-b)C_f \quad \text{for } n > b \tag{3-2}$$
$$= bC_i \quad \text{for } n \le b$$

This can be normalized,

$$c(N_{n,b}) = \frac{C(N_{n,b})}{C_i} = b + (n-b)(C_f/C_i) \quad \text{for } n > b$$
$$= b \qquad\qquad \text{for } n \leq b \tag{3-3}$$

The probability of each state occurring can be written as

$$p(N_{n,b}) = p_{c,n} \tag{3-4}$$

where the subscript c refers to the number of components per booster and n refers to the number of components used per mission. Then the expected cost of each state is given by $EC(N_{n,b}) = c(N_{n,b})p_{c,n}$. The expected cost of each decision can then be found to be given by

$$EC(b) = \sum_{n=1}^{\infty} c(N_{n,b})p_{c,n} \quad . \tag{3-5}$$

The optimum decision of the number of booster components to be built initially, b_n, is the value that causes the expected cost to be a minimum EC_m so that

$$EC(b_n) = EC_m \quad . \tag{3-6}$$

These results have been evaluated which are shown in Figures 3-1 and 3-2 as a function of the cost ratio C_f/C_i.

The results in Figure 3-1 are plotted in percent cumulative probability of the number of booster components used in the mission. The results indicate that if the ratio of final cost per booster to initial cost C_f/C_i is less then some minimum value, the optimum procedure is to build one booster initially, then build the rest as needed. For values of C_f/C_i greater then this minimum, build initially most of the components needed for the mission so that one is 80 percent or more confident that sufficient boosters are built to complete the mission, even though some may be left over.

In Figure 3-1 the normalized cost of the optimum decision is shown. Much more detailed calculations than those shown here are necessary for a complete cost analysis.

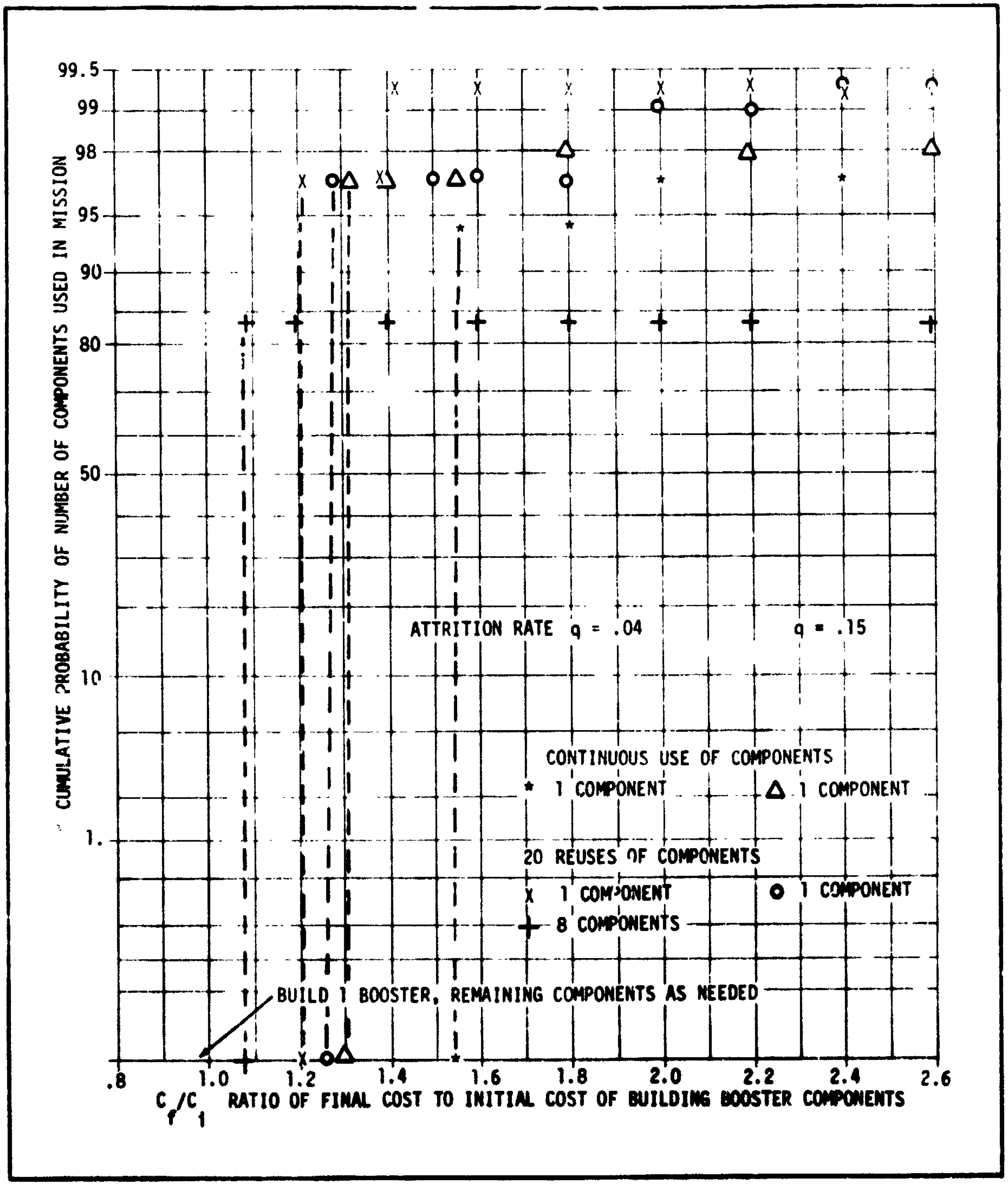

Figure 3-1. PERCENT CUMULATIVE PROBABILITY OF BOOSTER COMPONENTS USED IN MISSION BUILT INITIALLY FOR MINIMUM COST

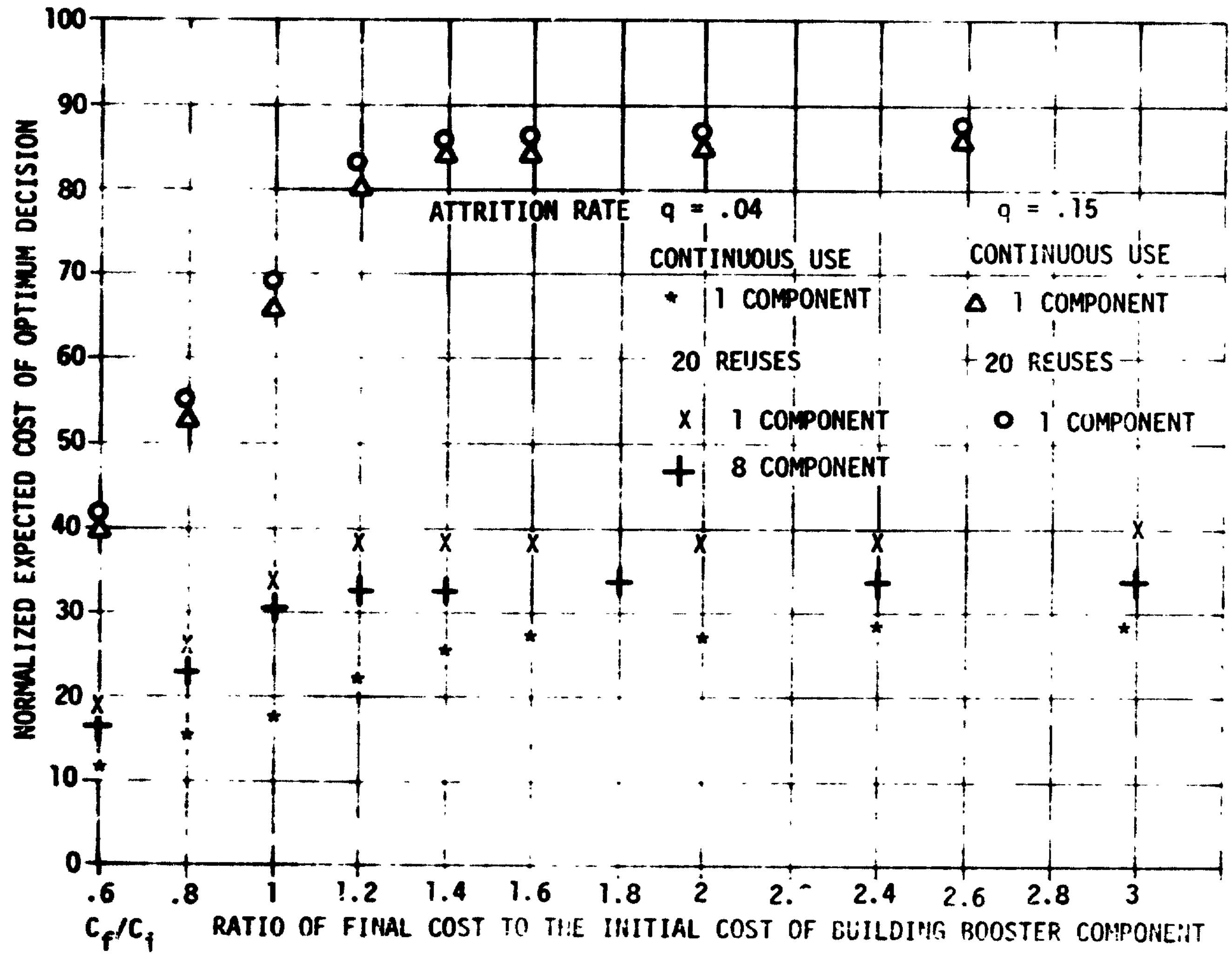

Figure 3-2. NORMALIZED COST FOR OPTIMUM DECISION $EC(b_m)/C_i$

Section IV

CONCLUSIONS

The Space Shuttle will use recoverable solid rocket boosters. However, at times due to damage or wear the rocket booster will not be recoverable and must be replaced. It is of importance to know the probable number of solid rocket boosters that will [illegible] be replaced in the nominal mission of 440 Space Shuttle launches. This information is necessary to the planning of the mission so that the solid rocket booster replacements will be available as needed at a minimum cost.

The present analysis calculated the probability distribution of solid rocket boosters needed for a mission of 440 launches for a given attrition rate or probable loss of the solid rocket booster per launch. Two methods of calculating the probability distributions were used. These are the State Probability Method and the Monte Carlo Method. The results from both methods agreed. It was found that the Monte Carlo Method was easier to formulate, while the State Probability Method required less computing time and was more accurate.

The results of the calculation showed that the probability distribution of booster replacements for the 440-launch Shuttle mission was close to gaussian with a mean value that rose linearly with the attrition rate. Thus, as can be seen in Figure 2-4, for an attrition rate of 0.04 the mean or expected value of boosters needed for the 440-launch mission, which in this case is equivalent to the 50 percent cumulative probability, is 18. This number goes up to 44 boosters for an attrition rate of 0.1 and increases further to 88 boosters for an attrition rate of 0.2.

Also shown in Figure 2-4 is the number of boosters for the 95 percent cumulative probability value. The probability is 0.95 that this number of boosters or less will be needed to successfully complete the mission of 440 launches. As shown in Figure 2-4, at an attrition rate of 0.04 the 95 percent cumulative probability number is 25 boosters. This number rises to 54 boosters for an attrition rate of 0.1, and to 103 boosters at an attrition rate of 0.2.

The effect of wear on the solid rocket boosters was included in the analysis by replacing the booster after 20 launches. This caused the expected number of rocket boosters needed to complete the mission to increase. As can be seen from Figure 2-4, the mean or expected values of solid rocket boosters needed for a 440-launch mission changed from 18 to 33 for an attrition rate of q = 0.04 if the rocket booster was replaced after 20 launches instead of being used continuously. At an attrition rate of 0.1 the number of boosters needed went from 44 to 49 in going from continuous use to 20 uses, while at an attrition rate of 0.2 the number of boosters needed was the same for the case of 20 uses as for the continuous use case.

The boosters can be built up in replaceable components. If damaged, these components can be replaced without replacing the entire booster. An analysis was carried out to calculate the probability distribution of the number of replaceable components needed for a 440-launch mission for a given component attrition rate. This result is shown in Figure 2-3 for the continuous booster use case. As can be seen from the figure the expected number of components needed for the mission is given in terms of equivalent boosters. An equivalent booster is equal to the number of components in a booster, so that 10 equivalent boosters for an eight component booster consists of eighty components. Thus for a component attrition rate of 0.04 it can be seen that the expected number of equivalent boosters for a booster made up of eight replaceable components is nearly the same as that for a booster with one component, and is 18. This would make the expected number of components for the eight component booster equal to 144.

For the case where the components are replaced after 20 uses Figure 2-5 is used. If the results are examined for a component attrition rate of 0.1 we see that the expected number of equivalent boosters for the eight component booster is larger (52) than the expected number for the one component booster (50). This would make the expected number of components needed for the 440-launch mission with an eight component booster with an attrition rate per component per launch of 0.1 equal to 416.

A simple cost analysis was carried out, based on the probability distribution obtained. The results indicate that the expected cost should be proportional to the expected number of boosters needed. Thus the cost would rise as the attrition rate rises and should be higher for the 20 reuse case than for the continuous use case; see Figure 2-4. Assuming the cost of the multicomponent equivalent booster is close to the cost of a single component booster, and since the expected values are close for both cases for the same attrition rate, the costs should be similar for the two cases. The attrition rate should be lower, however, for the multicomponent booster. This is because if one component in an eight component booster is damaged only one component needs to be replaced whereas for a one component rocket the entire rocket must be replaced. This would give an attrition rate for the eight component booster 1/8 that for the one component booster. This would result in an eight fold cost cut for the multicomponent booster, compared to the one component booster.

The cost analysis was extended to the case where an initial group of boosters could be built at some initial cost and the remainder built as needed at a final cost. The objective was to find the optimum number to build initially as a function of the ratio of initial to final cost so as to minimize the overall cost. In Figure 3-1 the results show that if the ratio of final cost to initial cost is below a certain value the boosters are built as needed. However, if they are above this value, then enough boosters should be built initially so that one is fairly certain of having sufficient boosters to complete the 440-launch mission. Thus from Figure 3-1 for a one component booster in continuous use with an attrition rate of 0.04, if the ratio of final to initial cost is below 1.54, build the boosters as needed; if this ratio is higher than 1.54, then build enough boosters so that the probability is 0.94 that there are sufficient boosters to complete the mission.

The analysis can readily be extended to other values of attrition rate, number of launches per mission, and number of components per rocket.