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**COMPARISON OF SEVERAL METHODS  
FOR PREDICTING SEPARATION  
IN A COMPRESSIBLE  
TURBULENT BOUNDARY LAYER**

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# COMPARISON OF SEVERAL METHODS FOR PREDICTING SEPARATION IN A COMPRESSIBLE TURBULENT BOUNDARY LAYER

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## SUMMARY

Several methods for predicting the separation point for a compressible turbulent boundary layer were applied to the flow over a bump on a wind-tunnel wall. Measured pressure distributions were used as input. Two integral boundary-layer methods, three finite-difference boundary-layer methods, and three "simple" methods were applied at five free-stream Mach numbers ranging from 0.354 to 0.7325.

Each of the boundary-layer methods failed to explicitly predict separation. However, by relaxing the theoretical separation criteria, several boundary-layer methods were made to yield reasonable separation predictions, but none of the methods accurately predicted the important boundary-layer parameters at separation.

Only one of the "simple" methods consistently predicted separation with reasonable accuracy in a manner consistent with the theory. The other methods either indicated several possible separation locations or only sometimes predicted separation.

## INTRODUCTION

A calculation procedure for analyzing the viscous flow around bodies could consist of an inner viscous-flow analysis coupled with an outer inviscid-flow analysis. The capabilities of the viscous-flow analysis would have to include attached and separated flows. The objective of this investigation is to determine whether a boundary-layer computation method could be used for the attached-flow portion of the viscous layer analysis up to, and including the prediction of, the separation point. It should be noted that if a separate method is used for the attached-flow region, then this method must also provide accurate values of the important boundary-layer parameters in the neighborhood of the separation point so that a separated-flow analysis can be started.

In many practical problems, the boundary layer ahead of separation is turbulent. Recently, Cebeci, Mosinskis, and Smith (ref. 1) compared several methods for predicting turbulent-boundary-layer separation for incompressible flows. The methods considered in reference 1 were an integral boundary-layer analysis, a finite-difference boundary-layer analysis, and two "simple" methods. Of the four methods considered, only one of the "simple" methods did not agree well with experimental results.

The present investigation is concerned with predicting the pressure-gradient-induced separation point of the compressible turbulent boundary layer as well as determining the boundary-layer properties at separation. Alber, Bacon, Masson, and Collins (ref. 2) have recently made extensive measurements of the flow over a two-dimensional, modified circular-arc profile which included both pressure gradient and shock-induced separation. Several earlier studies (refs. 3 to 5) of compressible-boundary-layer separation were concerned only with separation caused by interactions between the shock and the boundary layer.

In this study, the measured pressure distributions of reference 2 were used as input for eight methods of predicting the separation point. Of the eight methods, two were integral boundary-layer methods, three were finite-difference boundary-layer methods, and three were "simple" methods. The calculations were performed for five free-stream Mach numbers ranging from 0.354 to 0.7325, with peak Mach numbers on the body between 0.45 and 1.31.

## SYMBOLS

a	speed of sound
$C_f$	skin-friction coefficient
$C_p$	pressure coefficient $\left( C_p = \frac{p - p_m}{\frac{1}{2} \rho u_m^2} \right)$
c	chord length
H	boundary-layer shape factor ( $H = \delta^*/\theta$ )
M	Mach number
P	total pressure
p	local surface static pressure
$Re_x$	Reynolds number based on surface distance x

S	compressible Stratford separation parameter
$S_i$	incompressible Stratford separation parameter
T	absolute temperature
$\bar{T}$	reference temperature
u	time mean velocity
x	coordinate along body surface
z	coordinate along body axis measured from point of maximum thickness
$\alpha$	parameter defined by eq. (6)
$\beta_p$	Lees-Alber separation parameter $\left( \beta_p = \frac{\theta}{\rho_e u_e^2} \frac{dp}{dx} \right)$
$\gamma$	ratio of specific heats
$\delta^*$	boundary-layer displacement thickness
$\theta$	boundary-layer momentum thickness
$\kappa$	von Kármán constant
$\nu$	kinematic viscosity
$\rho$	density

Subscripts:

e	edge of boundary layer
i	transformed "incompressible"
m	minimum pressure point
sep	at separation
0	free-stream stagnation
$\infty$	free-stream (far upstream) conditions

## DISCUSSION OF METHODS

### Integral Boundary-Layer Methods

Integral boundary-layer methods are characterized by the fact that the boundary-layer equations are satisfied in some mean sense instead of at each point in the boundary layer. Such methods usually require a significant amount of empirical input, the mini-

mum being the selection of a velocity profile family and some law relating skin friction to other parameters of the flow.

In the following, all boundary-layer prediction methods will be identified by the name(s) of the author(s) who published the method.

Sasman-Cresci method (ref. 6). - The Sasman-Cresci method uses the integral momentum and moment of momentum equations to predict boundary-layer behavior. A Stewartson type transformation is used to reduce the equations to an "incompressible" form. The method uses the Ludwig-Tillman (ref. 7) correlation and a reference temperature for the skin friction and assumes the incompressible velocity profiles are represented by power laws.

The use of the Ludwig-Tillman correlation precludes the possibility of predicting separation as the point where  $C_f = 0$ . For many integral methods, separation is predicted from the "incompressible" shape factor  $H_1$ . No strict value for  $H_1$  at separation can be given; values from 1.8 to 4.05 have appeared in the literature (ref. 8). For the present purposes, if  $H_1$  is between 1.8 and 2.8 and increasing rapidly, separation is assumed to occur.

White-Christoph method (ref. 9). - The heart of this method lies in the twin assumptions that the law of the wall can be extended to compressible flow and that the law of the wall velocity profile is representative of the entire boundary layer. A "compressible" law of the wall velocity profile is derived and substituted into the boundary-layer partial differential equations, which are then integrated across the boundary layer. The result is a single, first-order, ordinary differential equation which is solved for the skin-friction coefficient.

Parameters appearing in the equation include the first and second derivatives of the free-stream velocity and a "stretched" Reynolds number. In principle, separation is predicted when  $C_f = 0$ ; in practice, separation is predicted by comparing the calculated skin-friction coefficient with a separation value which is correlated to the local "stretched" Reynolds number.

### Finite-Difference Boundary-Layer Methods

In this class of methods, the full partial differential equations of the boundary layer are attacked. Derivatives are replaced with finite differences, and the resulting set of algebraic equations is solved numerically. In all such methods, some method of obtaining the apparent turbulent shearing stress must be included; it is here that empiricism must be introduced.

Herring-Mellor method (ref. 10). - In this method, the turbulent shear stresses are derived from an eddy-viscosity formulation. A two-layer model of the turbulent boundary layer is hypothesized; the eddy viscosity is evaluated by different formulations in

each of the layers. A central feature of the model is the existence of an overlap region, in which both eddy-viscosity models apply. The location and extent of the overlap region is a function of local Reynolds number. Separation is predicted in the method when  $C_f = 0$ .

McDonald-Fish method (ref. 11). - The turbulence model uses an eddy viscosity which is evaluated from an extended mixing length approach. As reported in reference 11, the method solves the turbulence kinetic energy equation simultaneously with the momentum equation to obtain a streamwise variation of the outer-region mixing length; however, this feature of the analysis is not publicly available, and the method as presently employed uses a constant value for the ratio of outer-region mixing length to boundary-layer thickness. Separation is predicted by the failure of the method to converge as  $C_f$  approaches zero.

Bradshaw-Ferris method (ref. 12). - This method represents a significantly different concept of turbulence modeling. The turbulent shear stress is not related to the local mean velocity gradient; instead, it is assumed that the shear stress is uniquely related to the turbulence kinetic energy. The turbulence kinetic energy equation is included in the system with the momentum and continuity equations to form a hyperbolic set which is solved by the method of characteristics. Separation is predicted when  $C_f = 0$ .

### "Simple" Methods for Predicting Boundary-Layer Separation

In this class of methods, it is not necessary to predict the development of the entire boundary layer; instead, separation is predicted by considering only the pressure gradient or pressure level. Most criteria of this type were originally derived for incompressible flow; it is assumed that these methods can be extended to compressible flows, at least for moderate free-stream Mach numbers, by using a suitable transformation.

Goldschmied method (ref. 13). - In deriving a separation criterion for low-speed flows, Goldschmied assumed that the turbulent boundary layer can be divided into inner and outer regions. By considering the inner region of the layer, he derived the following criterion for the pressure coefficient at separation:

$$C_{p, \text{sep}} = 200C_{f, m} \quad (1)$$

where the subscript  $m$  is identified with the beginning of the adverse pressure gradient. By using the Stewartson type transformation used by Sasman and Cresci (ref. 6), this may be solved for the Mach number at separation in a compressible flow:

$$M_{sep} = M_m \sqrt{1 - 200 \frac{\bar{T}_m}{T_0} C_{f,m}} \quad (2)$$

where  $\bar{T}$  is a reference temperature.

Stratford method (ref. 14). - The Stratford approach, originally derived for incompressible flow, is likewise based on the idea of dividing the boundary layer into inner and outer regions. With the presence of fully turbulent flow and an adverse pressure gradient starting from the leading edge, the parameter

$$S_i(x) = C_p \left( x \frac{dC_p}{dx} \right)^{1/2} \left( 10^{-6} Re_x \right)^{-0.1} \quad (3)$$

is calculated. Strictly speaking, separation is predicted at the point where  $S_i(x) = 1.25 \kappa$  ( $\kappa =$  von Kármán constant). Stratford suggests that separation occurs if  $S_i(x) \geq 0.4$ . Based on application of the method to incompressible separating flows with the use of measured pressure distributions, Cebeci et al. (ref. 1) suggest the following modifications:

(1) If  $S_i$  is greater than 0.5, the flow separates at  $S_i = 0.5$ .

(2) If a maximum value of  $S_i$  occurs between 0.3 and 0.5, the flow separates at the maximum value.

(3) If  $S_i$  is less than 0.3, the flow does not separate.

If the flow is initially laminar or has a region of favorable pressure gradient preceding the pressure rise, the parameter  $x'$ , defined as

$$x' = x - x_m + x'' \quad (4)$$

is used in place of  $x$  in computing  $S_i$ . In equation (4),  $x_m$  is the location of the minimum pressure point, and  $x''$  is the length of run of a flat-plate turbulent boundary layer required to match the momentum thickness at the minimum pressure point.

To extend Stratford's method to compressible flow, the transformation used by Sasman and Cresci is introduced. The resulting expression for  $S(x)$  is



$$S(x) = \left[ 1 - \left( \frac{M_e}{M_m} \right)^2 \right] \left\{ \left[ \int_{x_m}^x \alpha dx + 94.4 \theta_{i,m} \left( \frac{a_0 M_m \theta_{i,m}}{\nu_0} \right)^{1/5} \right] \left( - \frac{2M_e}{\alpha M_m^2} \frac{dM_e}{dx} \right) \right\}^{1/2} \\ \times \left\{ 10^{-6} \left[ \int_{x_m}^x \alpha dx + 94.4 \theta_{i,m} \left( \frac{a_0 M_m \theta_{i,m}}{\nu_0} \right)^{1/5} \right] \frac{a_0 M_m}{\nu_0} \right\}^{-0.1} \quad (5)$$

where  $\theta_i$  is the "incompressible" momentum thickness, and

$$\alpha = \frac{T_0}{T} \left( \frac{T_e}{T_0} \right)^{(\gamma+1)/2(\gamma-1)} \quad (6)$$

Lees-Alber method (refs. 15 and 16). - Lees and Alber have discussed a low speed separation parameter of the form

$$\beta_p = \frac{\theta}{\rho_e u_e^2} \frac{dp}{dx} \quad (7)$$

Separation is presumed to be imminent if  $\beta_p > 0.004$ . It should be noted that this is not really a "simple" method, since the distribution of  $\theta$  along the body must be known.

Alber et al. (ref. 2) have suggested that the criterion may be applied without modification to compressible flows, at least up to Mach numbers of the order of 1.0.

In applying the method, the momentum thicknesses as calculated by the Sasman-Cresci boundary-layer method were used, since no significant difference was present between these values and those predicted by other methods.

## EXPERIMENT

The ultimate test of any computation scheme is comparison with proper experimental data. Cebeci et al. (ref. 1) present a large amount of flow-separation data for incompressible flow; however, data in which compressibility effects are present are not as abundant. In addition, most of the compressible-flow data are concerned with the separation resulting from the interaction of the shock and the boundary layer.

Recently, Alber et al. (ref. 2) have presented the results of a careful and extensive investigation of boundary-layer separation in a compressible flow. The geometry considered was a two-dimensional, modified circular-arc bump on a wind-tunnel wall. The flow speed ranged from essentially incompressible ( $M_\infty \approx 0.35$ ) to the initiation of shock-interaction type separation ( $M_\infty \approx 0.73$ ). For all cases in which separation was pressure-gradient induced, the wall pressure distributions are presented in figure 1. For the highest Mach number case shown ( $M_\infty = 0.7325$ ), extensive surveys of velocity profiles were made. In addition, integral thicknesses and skin-friction coefficients were calculated. In the present study, the extensive data available for  $M_\infty = 0.7325$  provide a method of checking the overall predictive abilities of the boundary-layer computation methods.

It should be pointed out that the skin-friction measurements reported are somewhat indirect, since they were obtained by fitting a law of the wall and law of the wake velocity profile to the measured points. One result of this procedure is that a positive skin-friction coefficient is indicated downstream of the separation point indicated by oil flow. Closer investigation of the measured velocity profiles indicates much better agreement between the separation points indicated by probes and by oil flow. It has been noted previously (refs. 8, 17, and 18) that the proper view of turbulent separation is a spectrum of states representing a transition from attached to separated flow and occurring over a region of finite length. The separation point indicated by oil flow corresponds to the farthest forward penetration of reverse flow, while the location indicated by total-pressure probes, which is a time-averaged location of zero shear stress at the wall, is somewhat downstream. It can be seen that defining a separation point experimentally or theoretically is a difficult problem.

It is this set of data (ref. 2) which was used for evaluation of the separation prediction methods. It was realized that using one set of data would not give a complete test of the various methods; however, this selection was made for the following reasons: (a) apparently, no other comparable set of compressible-flow separation data is available; (b) the data are not exhaustive, by any means, but they do contain information for several different Mach numbers; (c) care had been taken in the experiment to ensure two-dimensionality of flow, etc.; and (d) apparently, this set of data is accepted as a valid test case by researchers in the field of separated flows (refs. 19 and 20).

## DISCUSSION OF RESULTS

All of the separation prediction methods outlined have been applied to all cases present in the data, with the exception of the highest Mach number case, since separation in that case was shock induced. Due to the availability of experimental velocity profiles for the  $M_\infty = 0.7325$  case, calculations of boundary-layer development were begun at

the bump peak for this case. For all other cases, calculations were begun 1/4 chord length upstream of the bump, where the boundary layer is approximately in equilibrium.

### Performance of Boundary-Layer Methods

Each of the boundary-layer prediction methods requires essentially different information to begin calculations. The method of White and Christoph requires only an initial value of the skin friction, while the Sasman-Cresci method requires initial values of momentum thickness and shape factor. The finite-difference methods all can accept a detailed initial velocity profile as starting information. The method of Bradshaw and Ferris also requires an initial shear-stress profile. All of the finite-difference methods are also capable of generating an "equilibrium" start. For the equilibrium start, the Herring-Mellor and McDonald-Fish methods require initial displacement thickness and pressure gradient, while the Bradshaw-Ferris method requires initial momentum thickness and skin friction.

Since detailed velocity-profile data were available for only one case and since data on initial shear-stress distribution were absent for all cases, all calculations were started by letting each finite-difference program generate its own equilibrium start.

Simultaneous comparison of all of the boundary-layer prediction methods can only be in terms of the skin-friction coefficient, since this is the only parameter common to all. Actually, this is ideal for the present purposes, since most of the methods use  $C_f = 0$  as a separation prediction.

The theoretical skin-friction distributions for all methods for the  $M_\infty = 0.52$  case are shown in figure 2(a). Only the adverse-pressure-gradient region is shown. The skin-friction predictions of the finite-difference calculations are similar and exhibit approximately the same maximums and minimums. The integral method of White and Christoph predicts an appreciably higher skin friction than do the finite-difference methods. The skin friction predicted by the integral method of Sasman and Cresci falls between the predictions of the finite-difference methods and the prediction of the White-Christoph method. It should be noted that no method predicts zero skin friction; thus, no separation is predicted. This behavior is typical of the other low-speed cases ( $M_\infty \leq 0.70$ ).

Figure 2(b) shows skin friction predictions for the  $M_\infty = 0.7325$  case, as well as the experimental data points. Again, only the downstream side of the bump is shown. In this case, all of the prediction methods yield significantly different results. The Herring-Mellor method provides reasonable agreement with the experimental data for part of the bump; while the Bradshaw-Ferris method agrees reasonably well for another part. The reason for the strange dip in the Bradshaw-Ferris calculations is not clear; however, it may be a consequence of the equilibrium start used in this method. The

severe drop in skin friction predicted by the McDonald-Fish method may be caused by sensitivity to the oblique shock which occurs in the free stream but is not of sufficient strength to cause shock-induced separation. Once again it is noted that no method predicts  $C_f = 0$ . The failure of the Sasman-Cresci method to predict  $C_f = 0$  is not a source of great concern, because separation predictions using this method are based on the "incompressible" shape factor  $H_1$ . Plots of this parameter for the downstream side of the bump for  $M_\infty = 0.52$  and  $M_\infty = 0.7325$  are presented in figure 3. It should be noted that in terms of the criterion of  $H_1$  lying between 1.8 and 2.8 and increasing rapidly, no separation is predicted.

### Performance of Simple Methods

Separation predictions by the methods of Goldschmeid (ref. 13), Stratford (ref. 14), and Lees and Alber (refs. 15 and 16) were incorporated in the Sasman-Cresci method (ref. 6) computer program.

The axial distribution of the Stratford parameter on the downstream side of the bump for  $M_\infty = 0.52$  and  $M_\infty = 0.7325$  is shown in figure 4. The behavior of  $S$  at  $M = 0.52$  is typical of the cases with  $M_\infty \leq 0.70$ . The abnormally large value of  $S$  at  $(2z/c) = 0.25$  for  $M_\infty = 0.7325$  may be attributed to the presence of an oblique shock in the free stream. Since the maximum value of  $S$  is greater than 0.5 for this case, separation is taken to occur where  $S = 0.5$ . For the  $M_\infty = 0.52$  case, the maximum value of  $S$  is between 0.3 and 0.5, and separation is taken to occur at the location of the maximum value of  $S$ . It should be noted that for the remainder of the low-speed cases, the Stratford method predicts boundary-layer separation.

The distribution of the Lees-Alber separation parameter  $\beta_p$  for  $M_\infty = 0.52$  and  $M_\infty = 0.7325$  is shown in figure 5. The curve for  $M_\infty = 0.52$  is typical of the other low-speed cases; the severe peak in the curve for  $M_\infty = 0.7325$  is apparently due to the presence of the oblique shock in the free stream. As noted previously, separation is predicted if  $\beta_p > 0.004$ . Examination of the curves shows that there are several regions where this criterion is satisfied. It is unclear which point should be properly identified as separation.

The Goldschmeid separation criterion does not lend itself to presentation in a figure, since it is concerned only with locating a specific point in the flow; no "parameter" is computed at each streamwise location. It was found that the Goldschmeid criterion predicted separation in two cases,  $M_\infty = 0.700$  and  $0.7325$  (table I).

## Interpretation of Results

In closely examining the results of the boundary-layer computations, it is evident that none of the methods explicitly predicts separation. It is also observed that with the exception of the method of White and Christoph, all of the methods behave in a rather strange manner near separation. Such behavior was noticed and discussed in some detail by Cebeci et al. (ref. 1). The root of the trouble lies in the attempt to calculate a separating boundary layer with a measured pressure distribution. In the vicinity of separation, the viscous layer interacts strongly with the free stream and causes a relaxation in pressure gradient. As pointed out in reference 1, using a measured pressure distribution is equivalent to being told the position of separation.

It seems obvious that a complete viscous-layer computation scheme should be capable of predicting separation with a measured pressure distribution, provided that separation does occur. Consider a boundary-layer computation as one element of a viscous-layer computation in an iteration scheme to compute the pressure distribution and viscous-layer characteristics for an arbitrary body. If such a scheme were convergent to the correct answer, then presumably the viscous-layer method should converge to the correct answer (i. e., prediction of separation under the action of the converged (correct) pressure gradient).

Viewed from another point, use of the actual pressure gradient confronts the viscous-layer method with actual physical information. In order to be satisfactory, the method should be capable of correctly predicting the actual physical flow. Any other behavior would not be consistent.

With the above comments in mind, it is concluded that all of the methods of boundary-layer prediction considered herein fail to explicitly predict separation. It follows that it is not feasible to use the boundary-layer method near the region of separation; a viscous-flow analysis which more realistically models a region of incipient separation would have to be matched to a boundary-layer method somewhere upstream of the separation region.

It is of some interest to determine whether a separation-point location can be inferred from the results already obtained. In some cases it is possible to locate a separation point with reasonable accuracy provided that it is first known that separation does indeed exist. Examination of the skin-friction predictions of the three finite-difference methods (fig. 2) shows that after a drastic drop in skin-friction coefficient, the values suddenly level off at some value of the order of 0.001. This leveling off occurs in a region of adverse pressure gradient, where this type of behavior is not expected. If the calculations are carried through the entire field, it is found that a similar behavior occurs near reattachment (fig. 2(a)). The first occurrence of such behavior is taken as an indication of separation, and the point of minimum skin friction in this region is taken as the separation point.

Turning to the integral methods, the skin-friction predictions of the White-Christoph method (fig. 2) do not exhibit a pronounced drop and leveling off behavior and, taken by themselves, would not lead one to suspect separation, since the minimum  $C_f$  never drops below approximately 0.002. If it is decided by some other means that separation does occur, then the minimum  $C_f$  can be taken to indicate separation.

Examination of the behavior of  $H_i$  from the Sasman-Cresci method (fig. 3) shows that  $H_i$  increases rapidly to a value of approximately 1.8, reaches a maximum, and drops off sharply. This occurs in a region of adverse pressure gradient, where such behavior is not to be expected. The separation criterion is therefore adjusted as follows: if  $H_i > 1.8$ , separation is assumed to occur at the maximum value of  $H_i$ .

These adjustments to the separation criteria were also used by Cebeci et al. (ref. 1) to predict separation in incompressible flow. They also proposed calculating the boundary layer on the basis of an extrapolated pressure distribution in the vicinity of separation. Use of the extrapolated pressure distribution generally caused the skin friction to approach zero or caused  $H_i$  to increase rapidly, which rigorously predicted separation. Separation points thus predicted were in good agreement with measured separation points and with separation points predicted by identifying the minimum  $C_f$  or maximum  $H_i$  calculated with the actual pressure distribution as the separation point.

For the present investigation, figure 1 shows that there is no obvious pressure extrapolation suggested by the curves. For the  $M_\infty = 0.7325$  case, two equally reasonable extrapolations were tried; one led to a strict separation prediction, the other did not. It is concluded that extrapolation of the measured pressure distribution is not feasible in all cases.

Turning now to the simple methods, it was found that the Stratford method is the only one of all the methods considered that consistently predicts separation in a rigorous manner. No special interpretation of the results of this method is necessary. Considering Goldschmeid's method, no special interpretation is possible, since it either does or does not strictly predict separation.

Some interpretation of the results of applying the Lees-Alber criterion is necessary. It is indeed true that  $\beta_p > 0.004$  occurs in a separating flow; however, it occurs several times. Without any specific guidance as to which  $\beta_p > 0.004$  point to identify as separation, it is assumed that separation occurs at the first occurrence of  $\beta_p > 0.004$ .

When all of these adjustments of the separation criteria were used it was found that separation was predicted in a majority of the cases considered (table I). The results show that, for the most part, the Sasman-Cresci method predicts separation in the vicinity of reattachment and misses the actual separation point. This also occurs in one case each with the Herring-Mellor, McDonald-Fish, and Goldschmeid methods. The predictions of the Herring-Mellor method are satisfactory in all cases but one; those of the McDonald-Fish method follow the Herring-Mellor predictions closely, with the exception of  $M_\infty = 0.7325$ , in which case the shock apparently has some effect on the calculation.

The Bradshaw-Ferris method probably yields the best predictions of the finite-difference methods. It should be noted that the minimum  $C_f$  predicted by this method is lower in all cases than any other predicted  $C_f$ . From table I it would appear that the White-Christoph method gives reasonable predictions; however, it should be recalled that the minimum  $C_f$  predicted by this method is in all cases greater than 0.002 and probably should not be interpreted as separation at all. The Sasman-Cresci method is accurate in only one case. The Lees-Alber method strictly predicts separation in all cases but not very accurately. The accuracy of this method could be improved by taking the maximum value of  $\beta_p$  or the second point where  $\beta_p = 0.004$  as the separation point; however, there is no theoretical justification for this. The Stratford method is the only one which predicts separation rigorously, uniquely, and reasonably accurately in the three cases for which the flow is completely subsonic. This method predicts early separation in all cases.

As mentioned previously, it is desirable that a separation prediction method also provide accurate predictions of important boundary-layer parameters (e. g. ,  $\theta$ ,  $H$ , and  $C_f$ ) at separation. The "simple" separation prediction methods do not calculate such parameters, this being the essence of their simplicity. Since it was found that the criteria for separation had to be modified in all cases for the boundary-layer prediction methods, it is clear that none of these methods was able to accurately calculate  $C_f$  or  $H$  near separation. Calculation of  $\theta$  was generally satisfactory for all methods in which it was calculated.

## SUMMARY OF RESULTS

Several methods for predicting the separation point for a compressible turbulent boundary layer were applied to the flow over a bump on a wind-tunnel wall with measured pressure distributions used as input. Two integral boundary-layer methods, three finite-difference boundary-layer methods, and three simple methods were applied at five free-stream Mach numbers ranging from 0.354 to 0.7325.

Each of the boundary-layer methods failed to explicitly predict separation. However, by relaxing the theoretical separation criteria, several boundary-layer methods were made to yield reasonable separation predictions, but none of the methods accurately predicted the important boundary-layer parameters at separation.

Only one of the simple methods consistently predicted separation with reasonable accuracy in a manner consistent with the theory. The other methods indicated either several possible separation locations or only sometimes predicted separation.

Lewis Research Center,  
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TABLE I. - PREDICTED AND MEASURED

SEPARATION LOCATIONS

Method	Mach number				
	0.354	0.520	0.663	0.700	0.7235
	Separation location, $2z/c$				
Sasman-Cresci (ref. 6)	-----	<sup>a</sup> 1.0	<sup>a</sup> 1.08	<sup>a</sup> 1.16	<sup>a</sup> 0.54
White-Christoph (ref. 9)	<sup>a</sup> 0.63	<sup>a</sup> .58	<sup>a</sup> .50	<sup>a</sup> .46	<sup>a</sup> .44
Herring-Mellor (ref. 10)	<sup>a</sup> .67	<sup>a</sup> .96	<sup>a</sup> .54	<sup>a</sup> .56	<sup>a</sup> .50
McDonald-Fish (ref. 11)	<sup>a</sup> .67	<sup>a</sup> .96	<sup>a</sup> .58	<sup>a</sup> .62	<sup>a</sup> .21
Bradshaw-Ferris (ref. 12)	<sup>a</sup> .64	<sup>a</sup> .67	<sup>a</sup> .54	<sup>a</sup> .56	<sup>a</sup> .52
Goldschmeid (ref. 13)	-----	-----	-----	1.08	.49
Stratford (ref. 14)	.63	.58	.50	.42	.20
Lees-Alber (refs. 15 and 16)	<sup>b</sup> .47	<sup>b</sup> .44	<sup>b</sup> .37	<sup>b</sup> .35	<sup>b</sup> .20
Experiment (ref. 2)	.71	.63	.54	.525	.46

<sup>a</sup>Separation not strictly predicted but predicted as discussed in Interpretation of Results section.

<sup>b</sup>Multiple locations predicted.

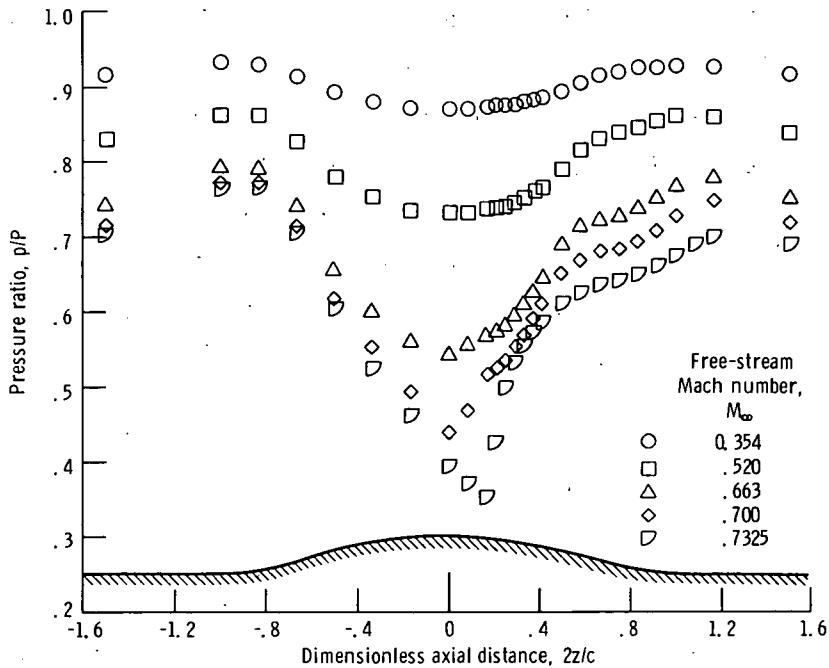


Figure 1. - Geometry and pressure distributions (from ref. 2) used with the methods for predicting separation.

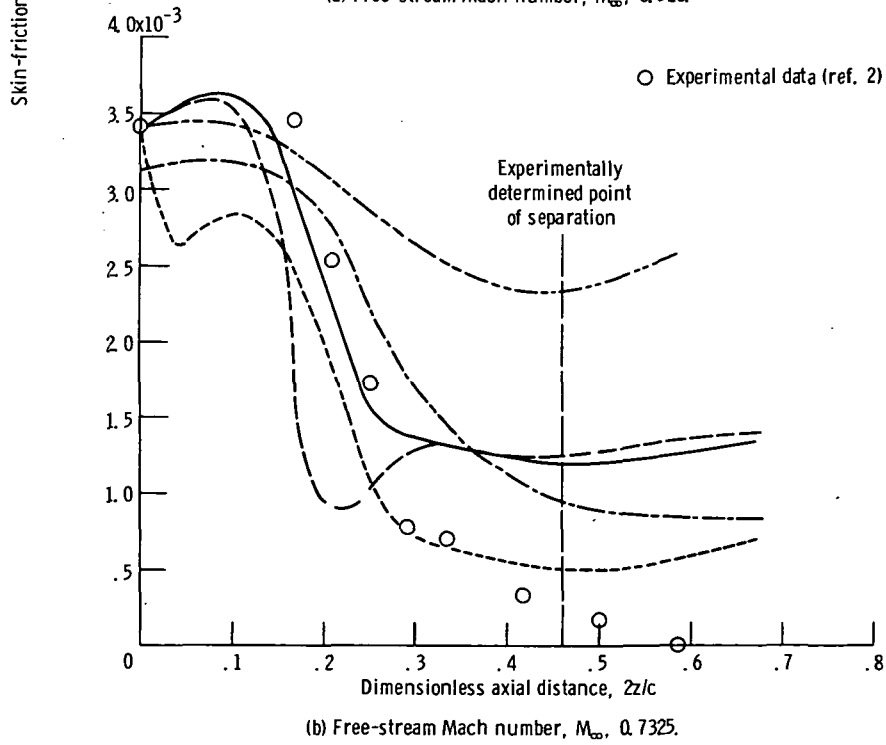
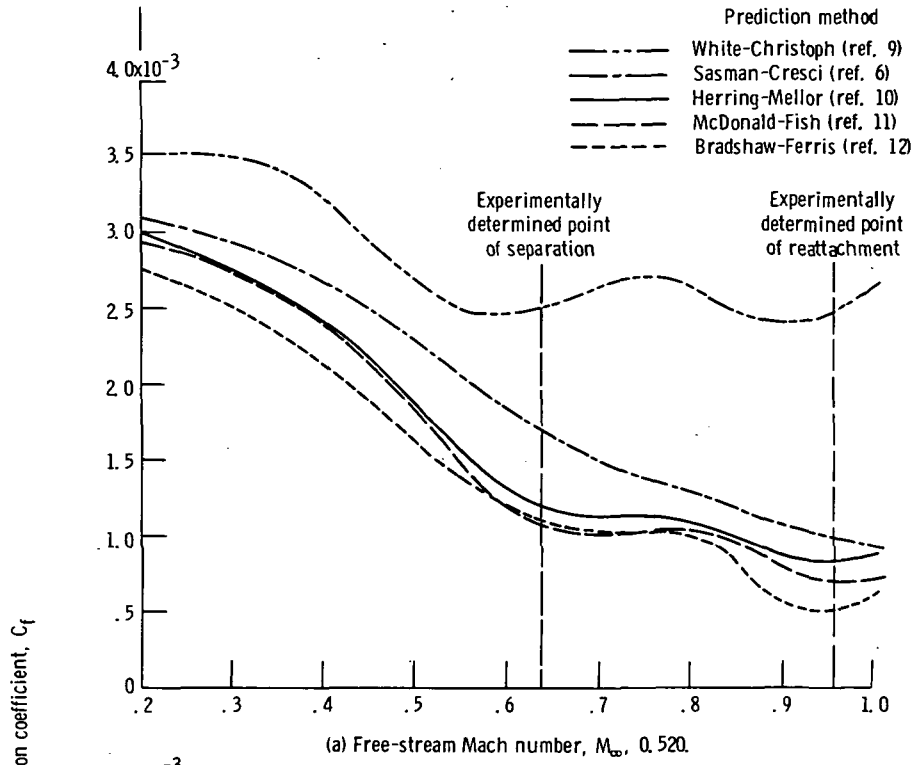


Figure 2 - Calculated skin-friction coefficients.

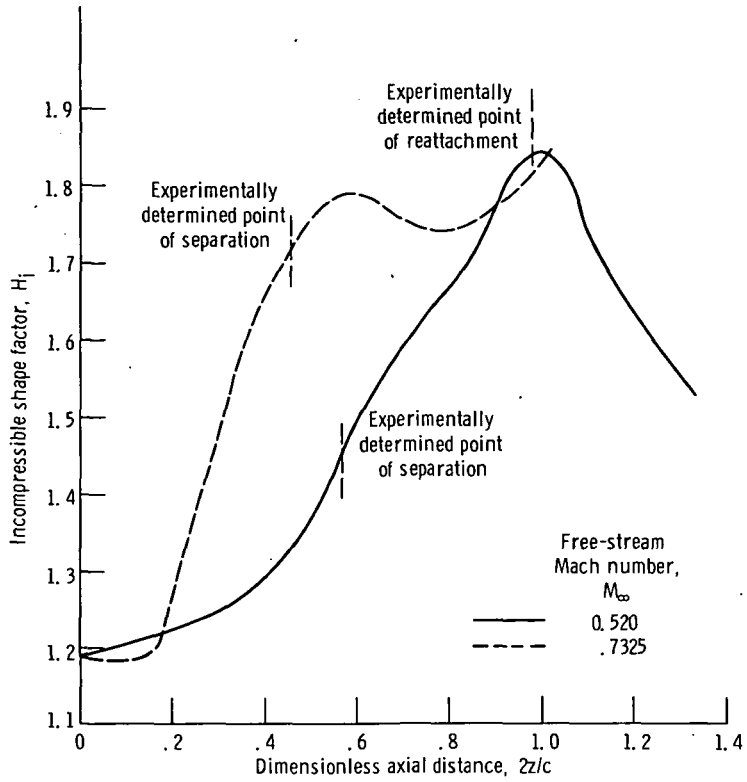


Figure 3. - Incompressible shape-factor distribution calculated by Sasman-Cresci method (ref. 6).

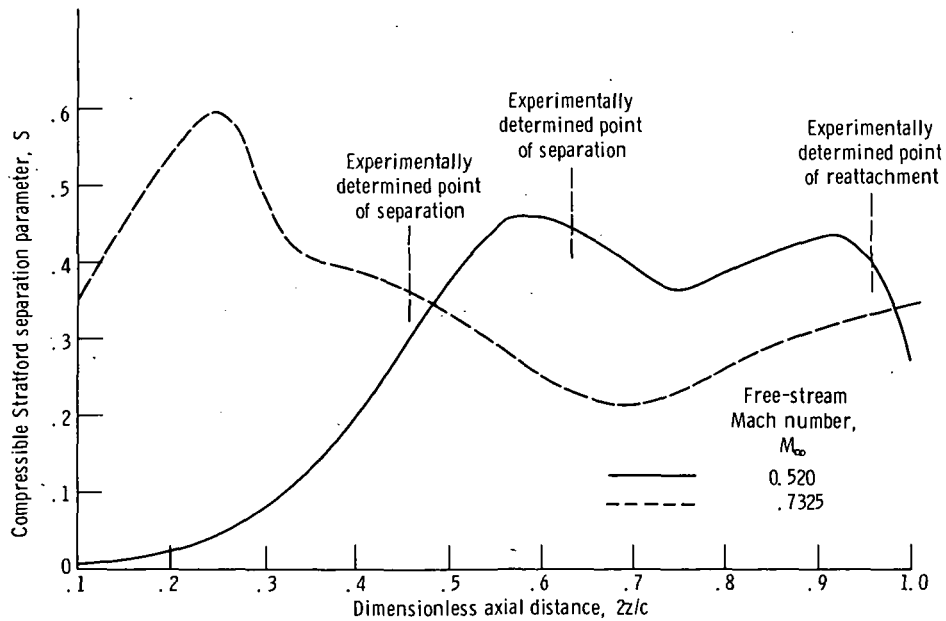


Figure 4. - Axial variation of compressible Stratford separation parameter.

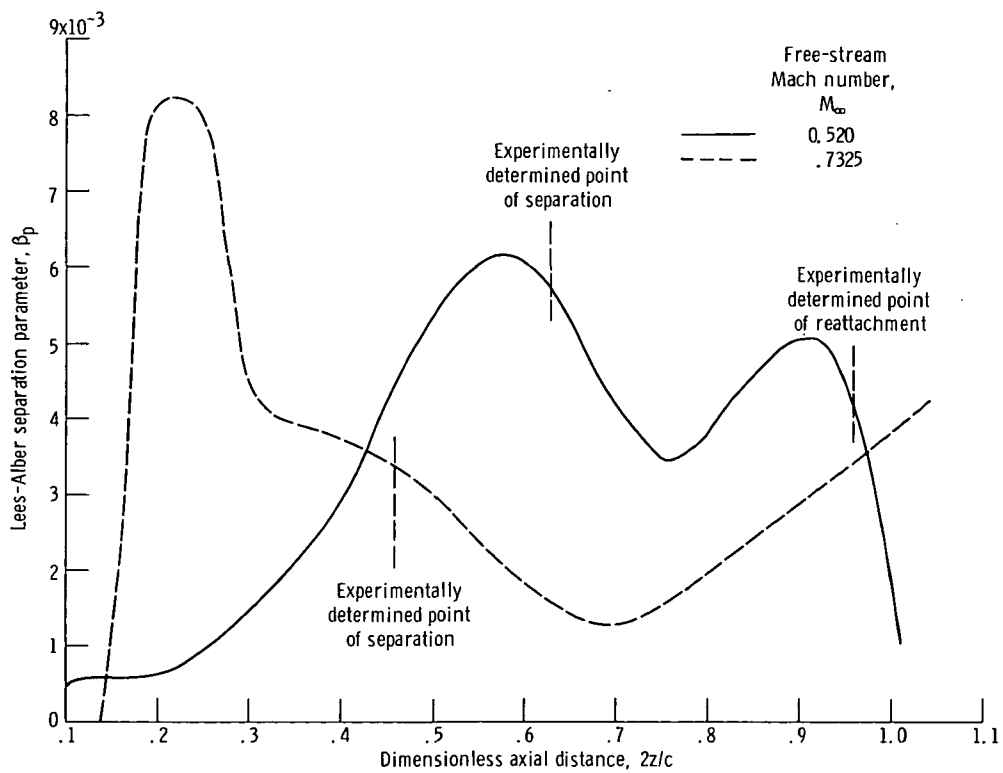


Figure 5. - Axial variation of Lees-Alber separation parameter.



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