FLIGHT CONTROL SYSTEMS RESEARCH

RE-89

Annual Report
(period 9-1-72 - 9-1-73)
for

NASA Grant NGL-22-009-548

by

H.P. Whitaker
Y. Baram
Y. Cheng

November, 1973

(NASA-CR-139595) FLIGHT CONTROL SYSTEMS RESEARCH Annual Report, 1 Sep. 1972 - 1 Sep. 1973 (Massachusetts Inst. of Tech.) 50 p HC CSCL 01C
N74-31427
Unclas
G3/02 16531

ACKNOWLEDGEMENT

This report was prepared under OSP Project 73598 sponsored by the NASA-Edwards Flight Research Center, through National Aeronautics and Space Administration grant number NGL-22-009-548.

The publication of this report does not constitute approval by the NASA of the findings or conclusions contained herein. It is published only for the exchange and stimulation of ideas.

TABLE OF CONTENTS

NOTATION

A,B,C,D	system coefficient matrices
C^*	aircraft handling qualities quantity; a linear combination of pitch rate and load factor (or specific force) expressed in g's.
$G_{()}(p)$	transfer function of component ( )
I	identity matrix
J	alternate notation for performance index
(PI)	alternate notation for performance index
$S_{( )}[q_i,q_0)$	static sensitivity of component ( ); the steady-state ratio of the incremental change in the output quantity, q_0, for a incremental change in input quantity, q_i; where there is only one input and one output, the bracketed subscripts can be omitted.
T	sampling period in seconds
$( )^T$	transpose of quantity ( )
X	see equation 2.49
a	coefficient of denominator polynomial of the system transfer function.
a_z	specific force component along the aircraft z-body axis.
b	coefficient of numerator polynomial of the system transfer function.
e	error signal
g	acceleration of gravity
i(t)	system input quantity as a function of time

p	Laplace operator
q	general notation for a signal quantity
t	time
u	input signal to the continuous section of the control system.
x	state variable
Δx	difference between model and system states
y	output signal of the continuous section of the control system
z	Z-transform variable, e^{pT}
α	coefficient of denominator polynomial of the model transfer function
β	coefficient of the numerator polynomial of the model transfer function
δ_e	elevator deflection, radians, positive trailing edge down.
$\underline{\Gamma}$	convolution integral in state variable form, a column vector
γ	element of the convolution column vector
Φ	system state transition matrix
$\Delta\Phi$	difference between system and model state transition matrices
θ	airplane pitch angle; rotation about the airplane Y-axis; positive nose up.
___	underline denotes a vector quantity

CHAPTER 1

Summary

The purpose of this report is to summarize the work performed under this grant for the period 1 September 1972 to 1 September 1973. Chapter 2 describes the theoretical development of the parameter optimization design technique needed for digital flight control system design. Chapter 3 presents the results of an example case study applying the previously developed optimization technique for continuous systems to an F-8 aircraft C* control system. It illustrates the concept of evolving the simplest system configuration that is capable of meeting a specified set of performance requirements.

CHAPTER 2

Theoretical Development of a Parameter Optimization Design Technique for Digital Flight Control Systems.

The automatic control system design techniques that are being investigated are applicable to systems whose configuration is suggested generally by the block diagram of Figure 2.1. That diagram omits the detailed arrangement corresponding to any specific system, and although there is no restriction intended thereby, the consideration of such a configuration is motivated by interest in flight control systems for aircraft, spacecraft, or missiles. In these systems there is a physical body whose linear and angular motions are to be controlled by applying appropriate control forces and moments. This body is usually the vehicle, and on the diagram it is termed the controlled member. The control forces are generated by various control surface displacements or other devices which may be actuated in turn by any of the many different types of actuators, servo systems, and drive mechanisms. The controlled member motions are continuous, and in most manned vehicle applications the control forces are continuous although they may be actuated in a discontinuous manner. The control force generating system is actuated by signals transmitted by an information processing system. The latter combines and modifies signal inputs to the system in various ways as well as the available information as to the output motions of the vehicle indicated by whatever sensor system is used.

For the design technique to be useful in the many different possible design situations, no conceptual restrictions are made upon the system configuration either in the number of devices, feedback paths, inner loop paths, nor interconnection of the various signals and devices. There is the restriction that the system be linear however. In practice computer programming considerations also impose restrictions in the form of available storage, computer time, and numerical accuracy.

The design procedure involves definition of performance specifications, interpretation of the specifications in terms of the response of a model of the system, establishing a fixed configuration for the system and specifying thereby those parameters considered to be variables under the designer's control, and then performing a parameter optimization to select parameter values which cause the system to meet its performance requirements.

Previous work has developed such a procedure for continuous systems.[1,2] The present work extends that to the case for which part of the information processing is digital in nature. In Figure 2.1 therefore the information processing system which connects the sensor system to the control force generating system includes both analog and digital processing sections. To suggest further the functional nature of the various devices, Figure 2.2 provides a somewhat more detailed elaboration of Figure 2.1. Various filters are indicated to denote signal processing and summation which can occur in either the analog or the digital sections although there may not be the physical separation of equipment shown by the diagram.

From a mathematical modelling standpoint the system is described by the relationship of various continuous state variables and various discrete state variables. Symbolically the mathematical block diagram of Figure 2.3 suggests this, although the correlation with the physical devices is thereby obscured. The continuous system elements are represented by an n_c-dimension state vector $\underline{x}_c$, and the discrete system elements by an n_d-dimension state vector $\underline{x}_d$. The input to the system enters the digital section, and the input to the continuous element section is considered to be the output of zero-order hold devices at the digital to analog interface. The continuous section is described by the differential equation

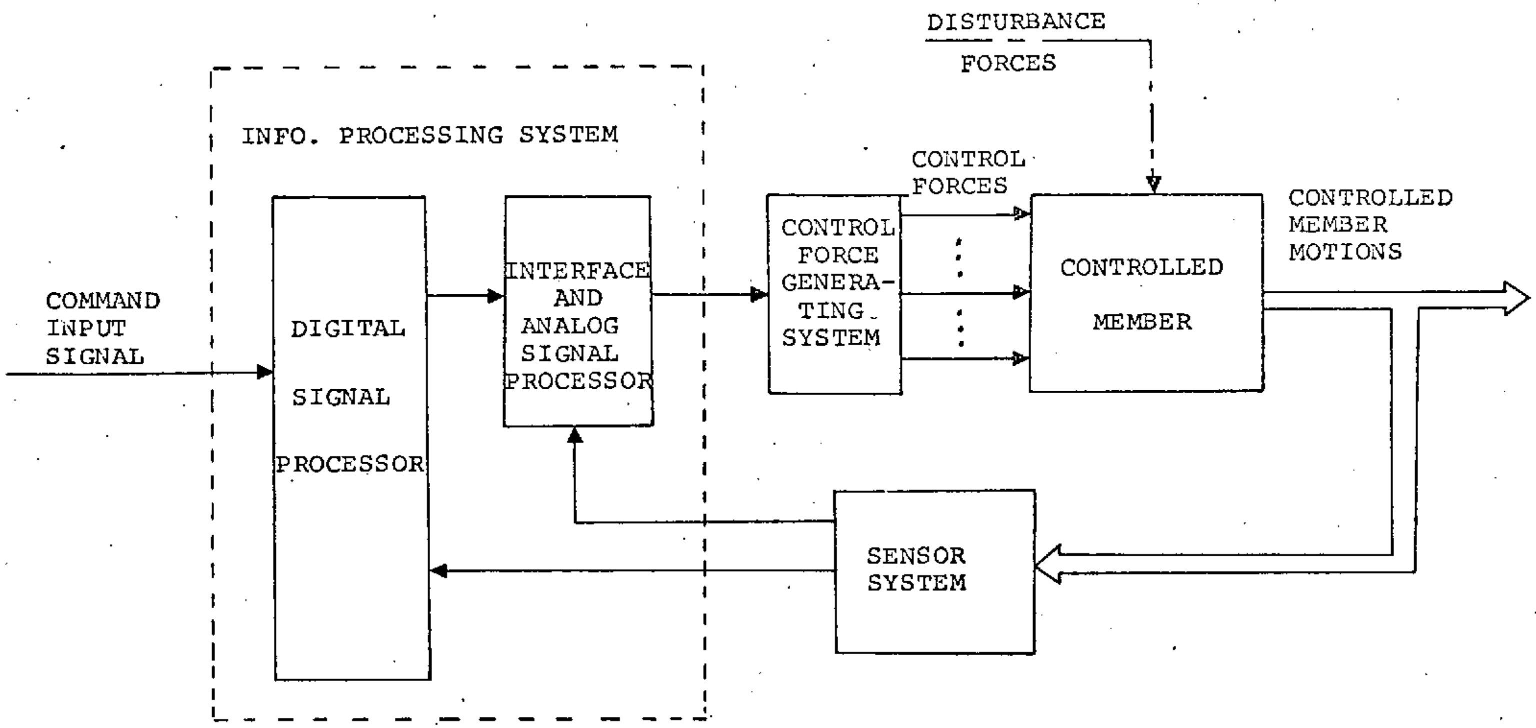


FIG. 2.1 GENERAL CONTROL SYSTEM

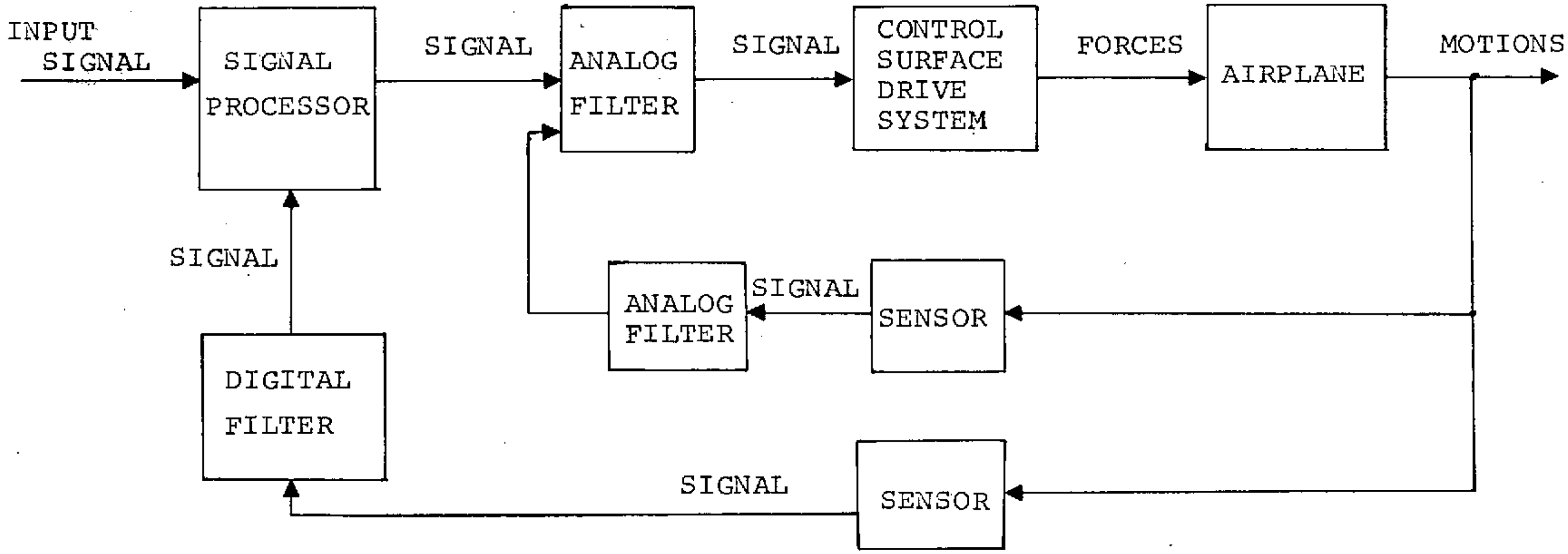
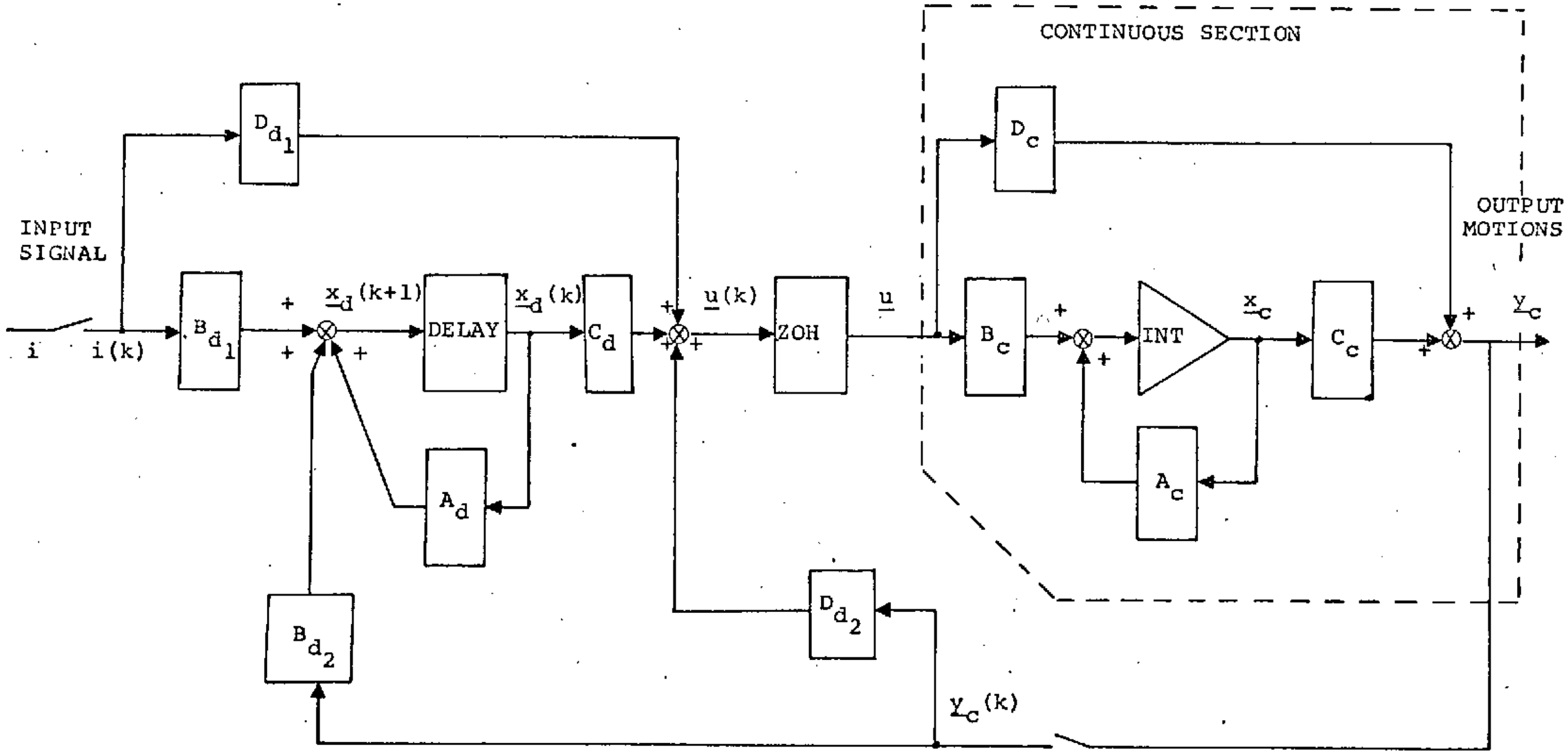


FIG. 2.2 TYPICAL DIGITAL FLIGHT CONTROL SYSTEM

8



Discrete State Variables $\equiv \underline{x}_d$. Continuous State Variables $\equiv \underline{x}_c$

FIG. 2.3 MATHEMATICAL BLOCK DIAGRAM OF GENERAL DIGITAL FLIGHT SYSTEM

$$\left(\frac{d\underline{x}_c}{dt}\right) = A_c\underline{x}_c + B_c\underline{u} \qquad (2\text{-}1)$$

with initial condition $\underline{x}_c(0) = \underline{x}_{c_0}$

$\underline{u}$ = rth - dimension vector input to the continuous section.

$\underline{x}_c = n_c$th - dimension vector of state variables of the continuous section.

$A_c = n_c \times n_c$ - matrix of coefficients

$B_c = n_c \times r$ - matrix of coefficients

Since $\underline{u}$ is the output of the zero order holds,

$$\underline{u}(t) = \underline{u}(k), \text{ for } kT \leq t < (k+1)T \qquad (2\text{-}2)$$

where T is the sampling period of the digital section and the simplification in notation, $u(kT) \equiv u(k)$, has been used. The various signal paths are given by

$$\underline{y}_c = C_c\underline{x}_c + D_c\underline{u} \qquad (2\text{-}3)$$

where the matrices are appropriately dimensioned depending upon the number of y signals that are of interest. The digital section is described by the difference equations

$$\underline{x}_d(k+1) = A_d\underline{x}_d(k) + B_{d_1} i(k) + B_{d_2}\underline{y}_c(k) \qquad (2\text{-}4)$$

$$\underline{u}(k) = C_d\underline{x}_d(k) + D_{d_1} i(k) + D_{d_2}\underline{y}_c(k) \qquad (2\text{-}5)$$

where $\underline{x}_d = n_d$-th dimension vector of state variables of the digital section

$\underline{u}(k)$ = output signal vector of the digital section which is the input to the zero order holds at the digital-analog interface

The D_{d_2} matrix represents those output feedback paths which are processed in the digital information section and modified only in magnitude but not in phase. Those which also receive dynamic compensation are represented in the B_{d_2} matrix. The

The input signal, i(t), is chosen to be a step function so that

$$i(t) = \begin{cases} 0 & , \quad t<0 \\ 1.0 & , \quad t>0 \end{cases} \tag{2-6}$$

It is convenient in analyzing this overall system to obtain a discrete representation of the continuous section giving the state variable values at the sampling time of the digital section. Making use of the state transition matrix.

$$\underline{x}_c(k+1) = \Phi_c \underline{x}_c(k) + \Gamma_c \underline{u}(k) \tag{2-7}$$

where

$$\Phi_c = e^{A_c T} \tag{2-8}$$

$$\Gamma_c = \int_0^T e^{A_c(T-t)} B_c \cdot dt \tag{2-9}$$

Letting $\underline{x}$ denote the combined state vector

$$\underline{x} = \begin{bmatrix} \underline{x}_c \\ \underline{x}_d \end{bmatrix} \tag{2-10}$$

the complete system can be described as

$$\underline{x}(k+1) = \begin{bmatrix} \underline{x}_c(k+1) \\ \underline{x}_d(k+1) \end{bmatrix} = \begin{bmatrix} (\Phi_c + \Gamma_c D_{d_2}) & , \Gamma_c D_d \\ B_{d_2} & , A_d \end{bmatrix} \begin{bmatrix} \underline{x}_c(k) \\ \underline{x}_d(k) \end{bmatrix} + \begin{bmatrix} \Gamma_c D_{d_1} \\ B_{d_1} \end{bmatrix} i(k) \tag{2-11}$$

or in shortened notation

$$\underline{x}(k+1) = \Phi\underline{x}(k) + \underline{\Gamma}\, i(k)$$
$$\underline{y}(k) = C\underline{x}(k) + D\, i(k) \qquad (2\text{-}12)$$

The state variables, $\underline{x}$, and the signal quantities, $\underline{y}$, of equation (2-12) may or may not be directly observable physical quantities. This depends upon the particular mathematical modelling which is employed, and that is often chosen on the basis of expediting the mathematical analysis. The system output quantities of interest to the designer therefore may be some of the y_i signal points or may be combinations of them. In any event the output quantities will be linear combinations of the state variables and of the input to the system.

The Performance Index

The designer desires to select values of those parameters which have not been specified by other design requirements. The available parameters may be located in either the discrete or the continuous sections of the system. The technique described here is very similar to that developed for continuous systems in references 1 and 2 . A system configuration is specified, the design parameters are designated, and a parameter optimization is performed.

The assumption is made that one can interpret the system operational requirements in terms of a desired time response of the system to a step function input. It is further assumed that the time response is equivalent to the step function response of a model whose transfer function can be selected by the designer from his knowledge of the desired operating characteristics. Following the work of Rediess and Palsson, this can lead one to the definition of a performance index which is a measure of the degree to which the system response can be made to approximate the response of the model and accordingly to meet the operational specifications. For the case of digital control systems the development is very similar except for a discrete representation of the system performance.

One method of examining the behavior of the complete system is to make use of the Z-transform

$$\frac{q_1(z)}{i(z)} = \frac{b_m z^m + b_{m-1} z^{m-1} + \ . \ . \ . + b_0}{z^n + a_{n-1} z^{n-1} + \ . \ . \ . + a_0} \ , \ m<n \qquad (2\text{-}13)$$

This is equivalent to the time domain representation

$$\underline{q}(k+1) = \Phi \underline{q}(k) + \underline{\Gamma}\, i(k), \ \underline{q}(0) = 0 \qquad (2\text{-}14)$$

where

$\underline{q}$ = n - dimension vector of state variables

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0, & -a_1, & -a_2, & - \cdots & -a_{n-1} \end{bmatrix} \qquad (2\text{-}15)$$

$$\underline{\Gamma} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \cdot \\ \cdot \\ \cdot \\ \gamma_n \end{bmatrix} \equiv \text{ an n - dimensioned input coefficient vector, } n>m \qquad (2\text{-}16)$$

and

$$\gamma_i = \begin{cases} 0 \text{ if } (n-m)>1 \text{ and } 1 \leq i < (n-m) \\ b_{n-i} - \sum_{j=1}^{i-1} a_{n-j}\, \gamma_{i-j} \quad \text{if } (n-m) \leq i \leq n \end{cases} \tag{2-17}$$

$i(k) =$ scaler input which in this case is taken to be a step function sampled at the times $t=kT$

The restriction, $n>m$, states that the input does not directly change the output until after at least one sampling delay.

In this choice of state variables, the successive q_i represent the past values of the output at the sampling instants.

$$\underline{q} = \begin{bmatrix} q(k) \\ q(k-1) \\ \cdot \\ \cdot \\ \cdot \\ q(k-n+1) \end{bmatrix} \tag{2-18}$$

Equation (2-14) is a set of difference equations which could be simulated using ideal unit delay elements as in Figure 2.4. The systems of interest are such that the output q_1 reaches a predictable steady state level, $q_{1_{ss}}$ as time becomes larger. The static sensitivity of the system is the ratio of the output to the input in steady state. For a unit step input the final value theorem applied to equation 2-13 yields

$$q_{1_{ss}} = \frac{\sum_{i=0}^{m} b_i}{1 + \sum_{i=0}^{n-1} a_i} = S_{cs} \tag{2-19}$$

If one subtracts the steady state value from the state vector one can consider the system's transient time response to a unit step input to be equivalent to that of an autonomous system in response to an appropriate set of initial conditions.

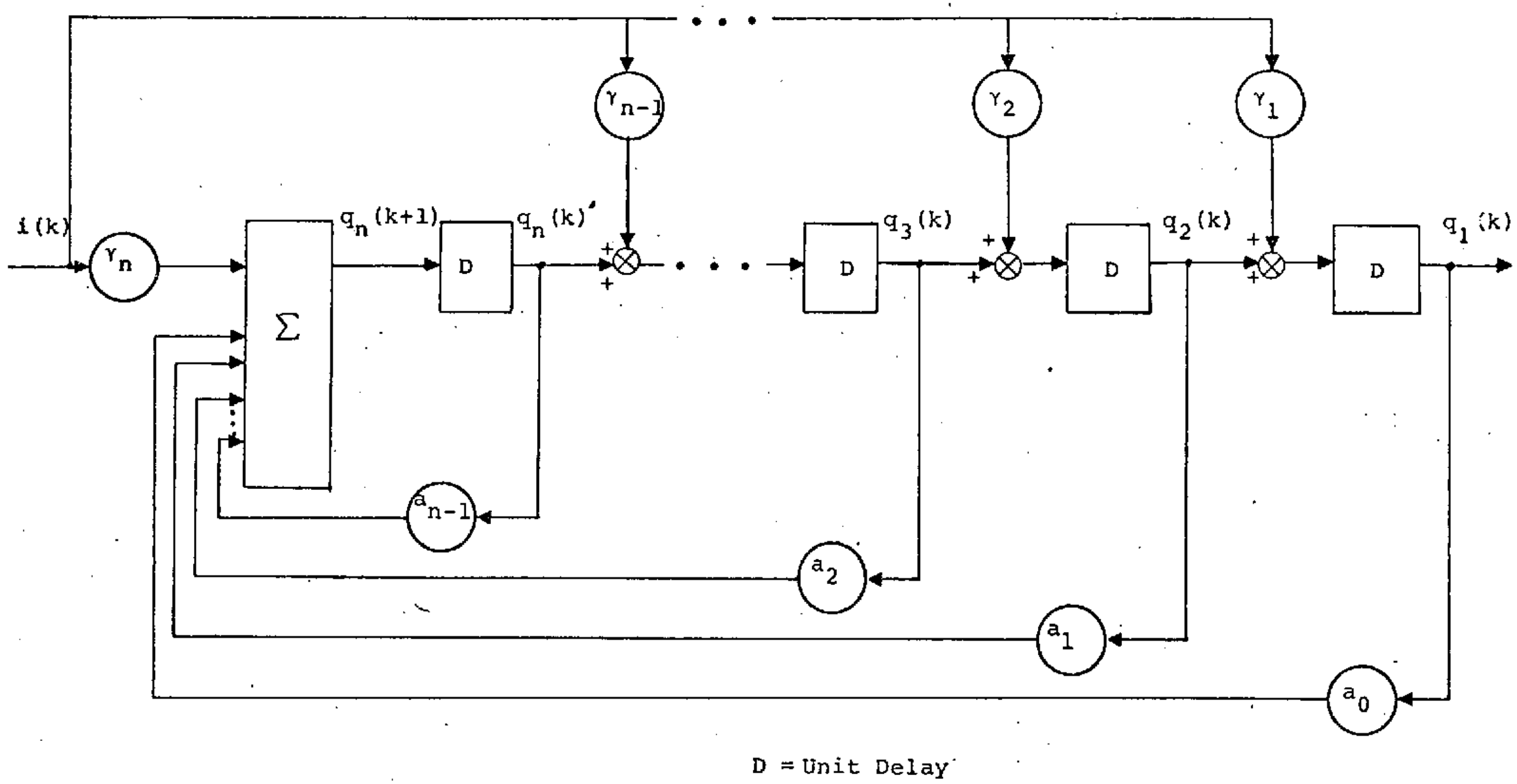


FIG. 2.4 SIMULATION OF CONTROL SYSTEM DIFFERENCE EQUATION

The steady state value for the state vector then is obtained from equation (2-14) by letting k become large. Then for a unit step input

$$\underline{q}_{ss} = \Phi \underline{q}_{ss} + \underline{\Gamma} \tag{2-20}$$

since in steady state no change in the state occurs from one sample time to another. Hence

$$\underline{q}_{ss} = (I-\Phi)^{-1}\underline{\Gamma} \tag{2-21}$$

where I is the identity matrix.

$$\underline{q}_{ss} = \frac{1}{1+\sum_{i=0}^{n-1} a_i} \begin{bmatrix} 1+\sum_{i=1}^{n-1} a_i, & 1+\sum_{i=2}^{n-1} a_i, & 1+\sum_{i=3}^{n-1} a_i & \cdots & 1 \\ -a_0 & 1+\sum_{i=2}^{n-1} a_i, & 1+\sum_{i=3}^{n-1} a_i & \cdots & 1 \\ -a_0 & -a_0-a_1 & 1+\sum_{i=3}^{n-1} a_i & \cdots & 1 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ -a_0 & -a_0-a_1 & -a_0-a_1-a_2 & \cdots & 1 \\ -a_0 & -a_0-a_1 & -a_0-a_1-a_2 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \cdot \\ \cdot \\ \cdot \\ \\ \gamma_n \end{bmatrix} \tag{2-22}$$

Equation (2-17) shows that (n-m-1) of the first γ_i are zero. In steady state the output state variable q_1 will remain at its steady state value, $q_{1_{ss}}$, for a time greater than n delay times. Figure 2.4 thus shows that the outputs of (n-m) of the delay units on the output side of the system will eventually all be equal to $q_{1_{ss}}$. Note that for a typical flight control system the state variables associated with these lower numbered delay units correspond to the difference equation representation of continuous state variables of the controlled member. Since these relate to the output motion of the controlled member and the time derivatives of that motion, these state variables will not be equal. This is equivalent to noting that for these control systems (n-m) can be expected to be 1. The Z-transform of equation 2-13, which is analogous to the transfer function of the continuous systems, can be expected to have zeroes as well as poles, and these will differ from those of the model. This is an added complexity of the digital control system in comparison with the continuous control system.

It is convenient to develop the performance index considering first the case for a model whose Z transform has no zeroes. Using the symbol ^ for model quantities, the Z transforms then are

$$\text{Model: } \quad \hat{G}(z) = \frac{\beta_0}{z^r + \alpha_{r-1} z^{-1} + \ldots + \alpha_1 z + \alpha_0} = \frac{\hat{q}_1(z)}{i(z)} \qquad (2\text{-}23)$$

$$\text{System: } \quad G(z) = \frac{b_m z^m + b_{m-1} z^{m-1} + \ldots + b_1 z + b_0}{z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0} = \frac{q_1(z)}{i(z)} \qquad (2\text{-}24)$$

The transient responses for a unit step input are obtained by subtracting the steady state value. Let

$$\hat{x}(z) = \hat{q}_1(k) - \hat{q}_{1_{ss}}$$
$$x(z) = q_1(k) - q_{1_{ss}} \qquad (2\text{-}25)$$

Both the system and the model are initially at rest with zero initial conditions.

The equivalent time domain representation of the model's transient response then is

$$\hat{\underline{q}}(k+1) = \hat{\Phi}\, \hat{\underline{q}}(k) + \hat{\underline{\Gamma}} i(k) \qquad (2\text{-}26)$$

where

$$\hat{\underline{\Gamma}} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \beta_0 \end{bmatrix} \text{ since } m=0$$

$$\hat{\underline{q}}(0) = \underline{0}$$

In steady-state all of the q_i are equal since there is no change in the output from sample to sample. The corresponding transient response then is

$$\hat{\underline{x}}(k) = \hat{\underline{q}}(k) - \hat{\underline{q}}_{ss}. \qquad (2\text{-}27)$$

The model transient response can be represented by

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \\ \vdots \\ \hat{x}_r(k+1) \\ \vdots \\ \hat{x}_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & & 0 & 0 & & 0 \\ 0 & 0 & 1 & & 0 & 0 & & 0 \\ \vdots & & & & & & & \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & & & \\ \vdots & & & & \vdots & & & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \\ \vdots \\ \hat{x}_r(k) \\ \vdots \\ \hat{x}_n(k) \end{bmatrix} \qquad (2\text{-}28)$$

with a set of initial conditions

$$\hat{\underline{x}}(0)$$

The transient response to the initial condition will be the same as the transient response obtained with the step input if

$$\begin{bmatrix} \hat{x}_1(0) \\ \hat{x}_2(0) \\ \vdots \\ \hat{x}_r(0) \\ \vdots \\ \hat{x}_n(0) \end{bmatrix} = \begin{bmatrix} -\hat{q}_{1_{ss}} \\ -\hat{q}_{1_{ss}} \\ \vdots \\ -\hat{q}_{1_{ss}} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{where } \hat{q}_{1_{ss}} = \frac{\beta_0}{1 + \sum_{i=0}^{r-1} \alpha_i} \qquad (2\text{-}29)$$

The corresponding state-space representation of the system is

$$\underline{x}(k+1) = \Phi \underline{x}(k) \tag{2-30}$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \cdot \\ \cdot \\ \cdot \\ x_r(k+1) \\ x_{r+1}(k+1) \\ \cdot \\ \cdot \\ \cdot \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & & 0 & 0 & & 0 \\ 0 & 0 & 1 & & 0 & 0 & & 0 \\ \cdot & & & & & & & \\ \cdot & & & & & & & \\ \cdot & & & & & & & \\ 0 & 0 & 0 & & 1 & 0 & & 0 \\ 0 & 0 & 0 & & 0 & 1 & & 0 \\ \cdot & & & & & & & \\ \cdot & & & & & & & \\ \cdot & & & & & & & \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{r-1} & -a_r & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \cdot \\ \cdot \\ \cdot \\ x_r(k) \\ x_{r+1}(k) \\ \cdot \\ \cdot \\ \cdot \\ x_n(k) \end{bmatrix} \tag{2-31}$$

$$\underline{x}(0) = \underline{q}(0) - \underline{q}_{ss} = - \underline{q}_{ss}$$

The steady-state values are given by equation 2-22
By examining this equation one notes that if $(n-m) \geq r$, the first r initial condition for the system response are the same which is shown in the following manner. From the definition of $\underline{x}$,

$$x_1(0) = - q_{1_{ss}} \tag{2-32}$$

$$x_2(0) = - q_{2_{ss}}$$

If one examines the difference between $x_1(0)$ and $x_2(0)$, equation (2-22)shows that

$$x_2(0) - x_1(0) = q_{1_{ss}} - q_{2_{ss}}$$

$$= - \frac{1 + \sum_{i=0}^{n-1} a_i}{1 + \sum_{i=0}^{n-1} a_i} \gamma_1 = -\gamma_1 \tag{2-33}$$

If $(n-m)>1$, $\gamma_1 = 0$ and hence $x_1(0) = x_2(0)$. Similarly if $(n-m)>2$, $\gamma_2 = 0$ and

$$x_3(0) - x_2(0) = -\gamma_2 = 0$$

Continuing if $(n-m)> r-1$ or $(n-m)\geq r$

$$x_r(0) - x_{r-1}(0) = -\gamma_{r-1} = 0$$

Consequently

$$\underline{x}(0) = \begin{bmatrix} -q_{1_{ss}} \\ -q_{1_{ss}} \\ \cdot \\ \cdot \\ \cdot \\ -q_{1_{ss}} \\ -q_{1_{ss}} + \gamma_{n-m} \\ -q_{1_{ss}} + \sum_{i=n-m}^{n-m+1} \gamma_i \\ \cdot \\ \cdot \\ \cdot \\ -q_{1_{ss}} + \sum_{i=n-m}^{n-1} \gamma_i \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \\ r\text{th component} \\ \\ \\ \\ \\ \\ \\ \end{matrix} \qquad (2\text{-}34)$$

Define the error state to be the difference between the system and the model states

$$\Delta\underline{x}(k) = \underline{x}(k) - \hat{\underline{x}}(k) \qquad (2\text{-}35)$$

and

$$(n-m) \geq r$$

then

$$\Delta\underline{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{r-1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \Delta\underline{x}(k)$$

$$+ \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & & & & & \\ \alpha_0 & \alpha_1 & \alpha_2 & \cdots & \alpha_{r-1} & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{r-1} & -a_r & \cdots & -a_{n-1} \end{bmatrix} \underline{x}(k) \qquad (2\text{-}36)$$

$$\underline{x}(0)^T = [0, 0, .. 0, x_{r+1}(0), \ldots x_n(0)] \qquad (2\text{-}37)$$

where the system and model static sensitivities have been set equal to each other. Equation (2-36) has the form

$$\Delta\underline{x}(k+1) = \hat{\Phi}\Delta\underline{x}(k) + \Delta\Phi\underline{x}(k).$$

It can be modelled by the diagram of Figure 2.5. If there were no input excitation of the error model, $\Delta\underline{x}$ would be zero, and the system would match the model. It can be seen from the homogeneous part of equation (2-36) that the higher order states do not affect the lower order error response states. The only excitation of the first r error states is the scalar input

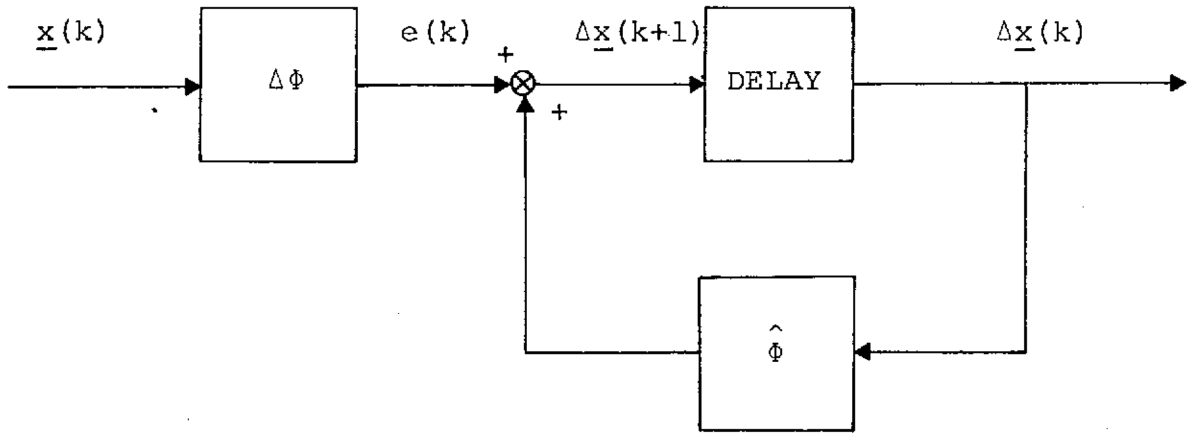


FIG. 2.5 MATHEMATICAL BLOCK DIAGRAM OF THE MODEL SYSTEM ERROR EQUATION

$$e(k) = \sum_{i=0}^{r-1} \alpha_i \; x_{i+1}(k) + x_{r+1}(k) \tag{2-38}$$

$$= \underline{\tilde{\alpha}}^T \underline{x} \tag{2-39}$$

if $\underline{\tilde{\alpha}}^T$ is defined as the n-dimension vector

$$\underline{\tilde{\alpha}}^T = [\alpha_0, \; \ldots \; \alpha_{r-1}, \; 1, \; 0, \; \ldots 0] \tag{2-40}$$

Minimizing the square of the error excitation leads to a performance index

$$(PI) = \sum_{k=0}^{\infty} e^2(k) \tag{2-41}$$

$$= \sum_{k=0}^{\infty} \underline{x}^T(k) \; Q \; \underline{x}(k) \qquad \text{where } Q = \underline{\tilde{\alpha}} \; \underline{\tilde{\alpha}}^T \tag{2-42}$$

Consider then the case for which the model has zeros.

$$\hat{G}(z) = \frac{\beta_\ell z^\ell + \ldots + \beta_0}{z^r + \ldots + \alpha_0} = \frac{\hat{q}_1(z)}{i(z)} \tag{2-43}$$

The error response can be defined as

$$\Delta q_1(z) = q_1(z) - \hat{q}_1(z)$$

$$= (x_1(z) + q_{1_{ss}}) - (\hat{x}_1(z) + \hat{q}_{1_{ss}})$$

$$= \Delta x_1(z)$$

since $\quad q_{ss} = \hat{q}_{ss}$

A diagram for the error response is shown in Figure 2.6a. If the model zeroes are cascaded as the poles of an added component, the diagram of Figure 2.6b results with the error output $\Delta x'$.

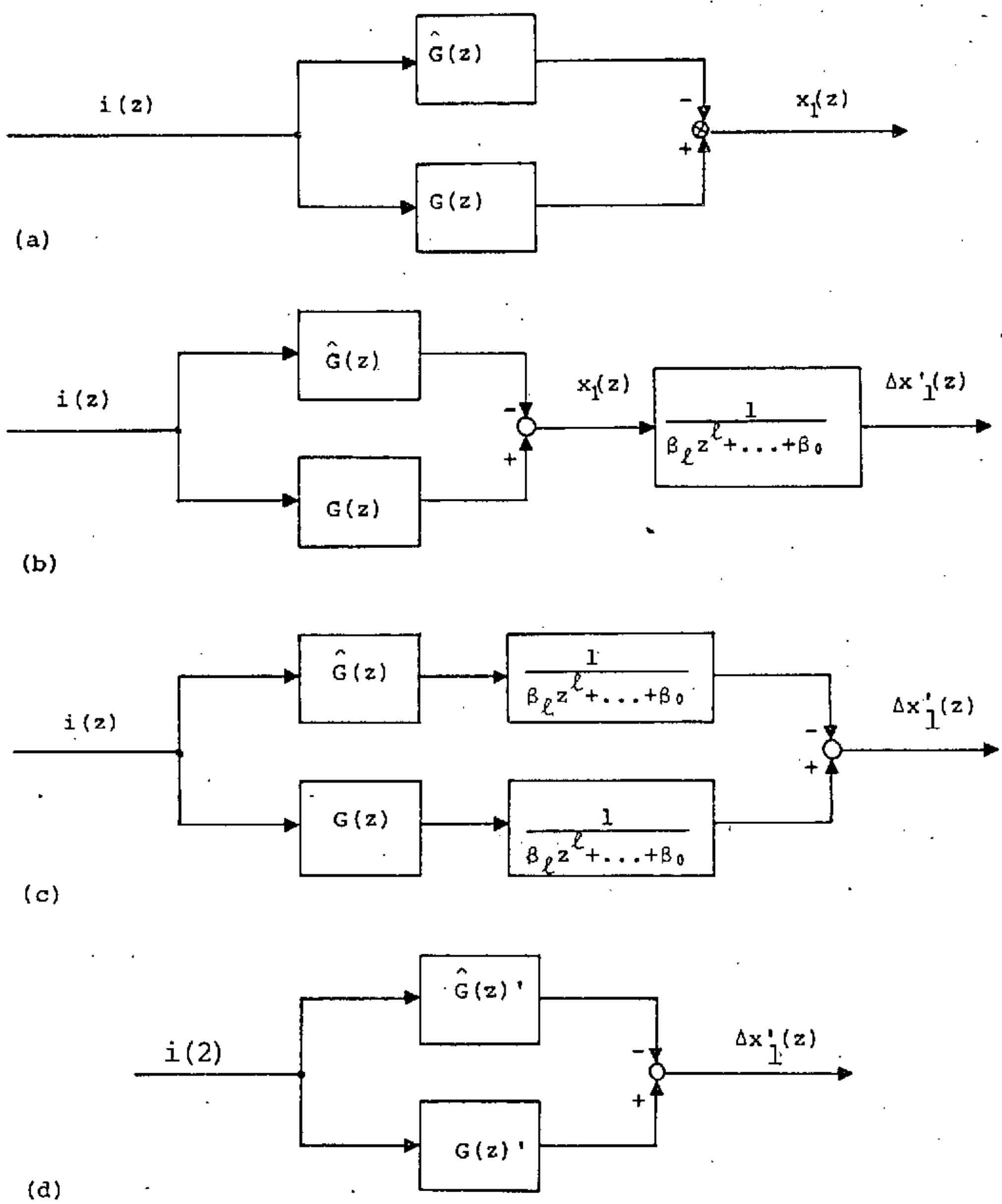


FIG. 2.6 EQUIVILANT REPRESENTATIONS FOR THE CASE OF MODEL ZEROS

By block diagram algebra one obtains Figures 2.6c and 2.6d. The final figure has a new system, $G(z)'$, of order $(n+\ell)$ since the model zeroes have been cascaded as system poles. The new model $\hat{G}(z)'$ has no zeroes. Following the same development as before, one arrives at a performance index

$$(PI) = \sum_{k=0}^{\infty} \underline{x}^T(k)\ Q\ \underline{x}(k) \tag{2-44}$$

where $\underline{x}$ is a state vector of order $(n+\ell)$

$$Q = \underline{\tilde{\alpha}}\ \underline{\tilde{\alpha}}^T \tag{2-45}$$

$$\underline{\tilde{\alpha}}^T = [\alpha_0,\ \alpha_1,\ \ldots\ \alpha_{r-1},\ 1,\ 0,\ \ldots 0] \tag{2-46}$$

and

$$n+\ell - m \geq r$$

or

$$(n-m) \geq (r-\ell)$$

The latter restriction states that the transport lag of the model can not be greater than that of the system for this representation to be valid.

Equation (2-44) then is the expression for the performance index to be used for the digital flight control system optimization. Instead of an integration there is a summation of state variable values at the sampling instants. For purposes of discussing the evaluation of the performance index, let

$$J \triangleq \sum_{k=0}^{\infty} \underline{x}^T(k)\ Q\ \underline{x}(k) \tag{2-47}$$

$$= tr(QX) \tag{2-48}$$

where $$X = \sum_{k=0}^{\infty} \underline{x}(k)\ \underline{x}(k)^T \tag{2-49}$$

and tr denotes trace

The difference equation for the state variables is

$$\underline{x}(k+1) = \Phi \underline{x}(k) \quad , \quad \underline{x}(0) = \underline{x}_0 \tag{2-50}$$

If one premultiplies equation (2-51) by Φ , postmultiplies by Φ^T and makes use of equation (2-50),

$$\Phi X \Phi^T = \sum_{k=0}^{\infty} \Phi \; \underline{x}(k) \; \underline{x}^T(k) \Phi^T \tag{2-51}$$

$$= \sum_{k=0}^{\infty} \underline{x}(k+1) \; \underline{x}^T(k+1) \tag{2-52}$$

The right hand side of this equation is equal to X except for the very first term. Thus

$$\Phi X \Phi^T = X - \underline{x}_0 \; \underline{x}^T{}_0 \tag{2-53}$$

or

$$\Phi X \Phi^T - X = - \underline{x}_0 \; \underline{x}_0^T \tag{2-54}$$

which can be solved for X and the performance index obtained from equation (2-48). The numerical method of solving equation (2-54) can be that used for a similar equation for the continuous systems described in reference 2. As noted in the discussion of the example control system design of Chapter 3 of this report, the numerical accuracy of that technique is under investigation. Further description will therefore be postponed until the outcome of that effort is available.

Equation (2-54) involves the computation of the state transition matrix over the sampling period. A computer program has been written for calculating Φ by a series expansion. Additional subroutines permit one to specify the input data for the digital section in a manner similar to that of reference 3 for continuous systems. The continuous section and the discrete section can be described in terms of elemental sub-units with suitable signal summation points. These will be more fully described in future reports.

CHAPTER 3

Design Example

The parameter optimization techniques developed under this grant have been applied to the design of a pitch control system for a F-8 fighter airplane. The control system is a fly-by-wire system, and the purpose of the system is to provide a controlled response of the quantity, C^*, for pilot stick inputs. C^* is a linear mixture of the load factor and the pitch angular velocity responses of the airplane. It was first proposed by the Boeing Company[4] as a criterion for insuring acceptable handling qualities. A block diagram of the system is presented in Figure 3.1. In this figure the feedback configuration has been left undefined at this point since the objective of the design procedure is to specify an acceptable feedback structure and the values of the design variables. The input to the system is a C^* command signal obtained from a suitable pilot hand controller in the cockpit. The C^* mixture used for this analysis was

$$C^* = -0.031\ a_z + 12.43\ \dot{\theta} \qquad (3\text{-}1)$$

where units of C^* are g's.

a_z = component of the resultant nonfield (specific) force along the aircraft body z axis in ft./sec^2

$\dot{\theta}$ = component of aircraft angular velocity with respect to inertial space along the aircraft body Y axis in radian/second.

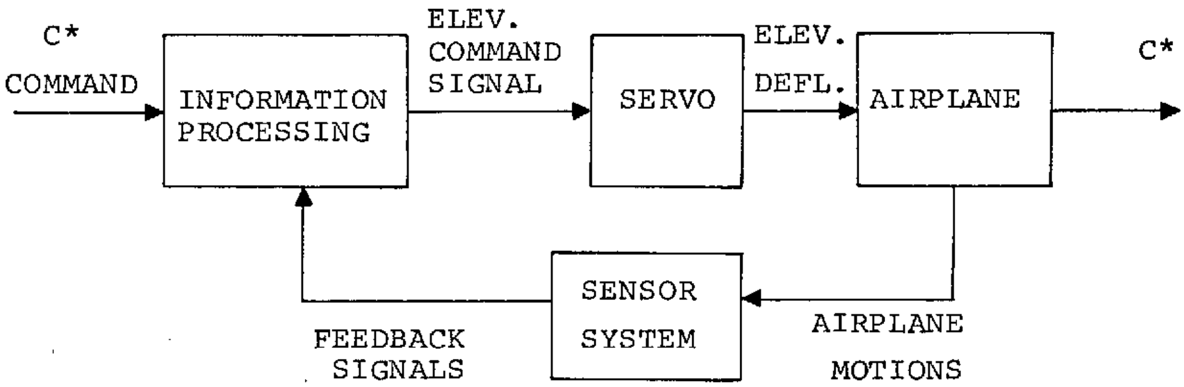


FIG. 3.1 PRELIMINARY FUCTIONAL BLOCK DIAGRAM

Airplane data is summarized in Appendix A. Flight condition 4 corresponding to Mach 1.1 at 45000 feet (13,700 meters) was used as a reference condition for preliminary design. The performance specification used was that the C^* response of the airplane for a step function C^* command should be approximately the same as the step function response of the C^* model within the tolerance boundaries given in Figure 3.2. The transfer function of the C^* model was

$$G_m(p) = \frac{(1+p/2.9)(1+p/61.1)}{\left(1+\frac{2(0.94)}{5.4}p+\frac{p^2}{(5.4)^2}\right)(1+p/10)} \qquad (3-2)$$

The elevator servo actuator was modelled as a first order component with a pole at -12 radian/sec. The input to the system is a signal proportional to the angular movement of the pilot's control stick. Considering C^* to be the primary system output, inspection of the aircraft transfer functions reveals that there is no integration in the aircraft C^* response. Therefore the closed-loop static sensitivity relating C^* to pilot input of such a control system would vary with flight condition. Since a force feel system is assumed to be present, the units of this sensitivity can be expressed in g's per pound of stick force. An additional performance specification was imposed by requiring a constant stick force per g characteristic at all flight conditions. This required the addition of integral compensation for the final design configuration.

Design procedure

The design technique being utilized assumes that a reasonable preliminary system configuration can be specified on the basis of knowledge of the operational requirements, availability of practical sensors, and limiations upon system complexity. The parameter optimization computer program then is used to select the design parameters which permit the closest approximation of the system reponse to the model response in the sense

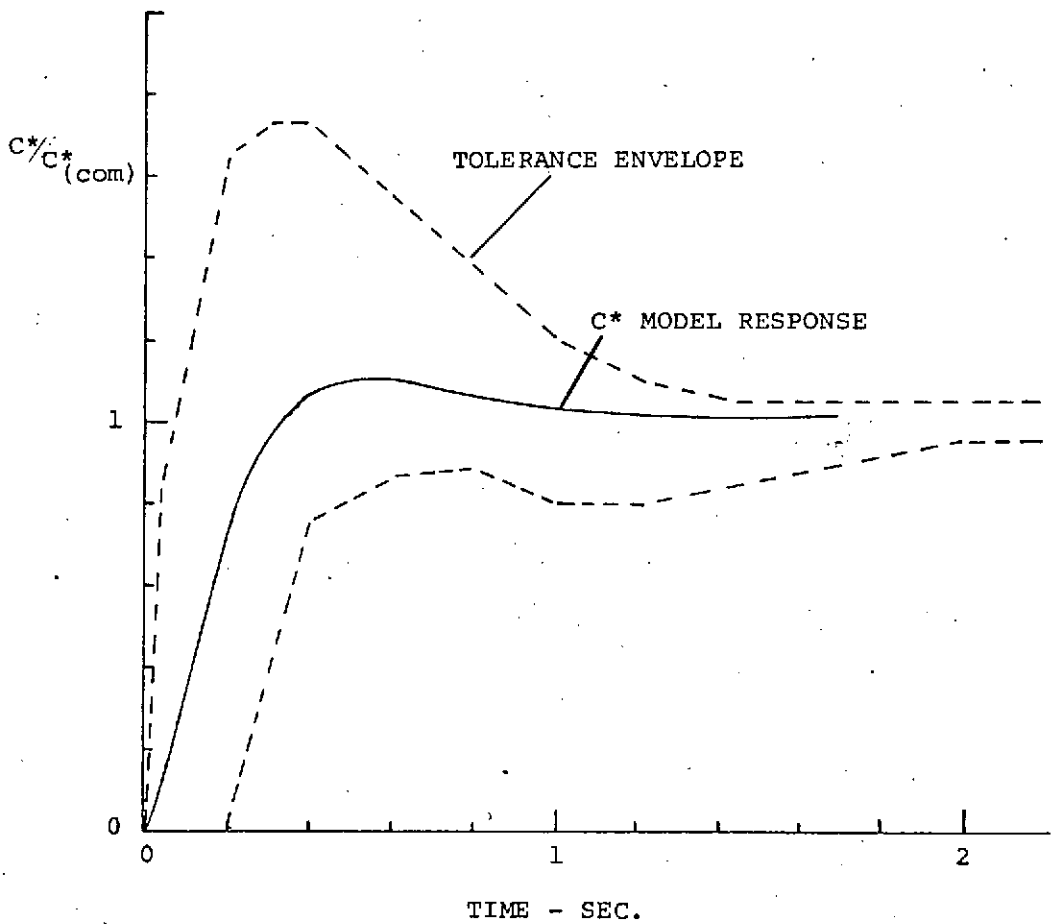


FIG. 3.2 C* MODEL RESPONSE TO STEP INPUT

proposed in reference 1 by minimizing the Model Performance Index. If the resultant design fails to satisfy the specifications, added compensation must be investigated at the price of additional complexity. The intent is to proceed from the simplest design to the simplest acceptable design letting the design process reveal the need for the additional complexity. Note that in all cases the model used is one for C^* output, although it is not necessary to feed back C^*.

The Model Performance Index is a quadratic function of the phase variables representing the transient response of the system to a step input. The weighting factors used for the various variables are specified by the reference model chosen. Geometrically the minimization process is one of minimizing the projection distance of the system trajectory onto the model's characteristic hyperplane. One can thus interpret the magnitude of the index as a fraction of the initial output error, which is in turn equal to the input step magnitude. If only one state variable were used, the index could be interpreted as the average (RMS) value of error between model and system. The higher order state variables add to this index, but it is useful to express the index value as a percentage of the steady state level for purposes of interpreting the meaning of a given minimum value that results from the optimization search procedure. Thus if the minimum index were 10%, one would expect the output of the system to deviate from the model's response on the average less than 10% of the steady-state value. For specifying the time average, 1.6 seconds was used since the tolerance envelope permits variation of the response over approximately that time interval.

In all of the cases described then, the data presented are those which the computer program selected as corresponding to the approximate minimum value of the performance index. The program permits one to specify which parameters are fixed and which can be used for optimization. Similarly design limits can be imposed upon the parameters if desired.

Simple pitch rate loop

Since it is desirable to design the simplest possible control system, it is informative to examine an idealized system first and to add complexity as it is required. The airplane's longitudinal short period mode is rather poorly damped, $\zeta = 0.074$. If a single sensor feedback loop were to be considered, the airplane transfer functions show that a_z feedback would not improve damping. Accordingly a pitch rate feedback offers the possibility of the simplest system.

To obtain first a feel for the basic control problem, the system was further idealized by neglecting the servo dynamics and the need for integral compensation. The result of the parameter optimization was a value of feedback static sensitivity

$$(\delta_e/\dot{\theta}) = 0.52 \text{ sec.} \qquad (3\text{-}3)$$

and the resultant performance index value was 83 per cent of the steady-state value of C^*. The step function response is presented as curve A in Figure 3.3. The C^* response lies just barely within the tolerance boundary and the damping ratio of the short period mode has been increased to 0.7 with a natural frequency of 5.0 rad/sec. So while a close model match is not possible, the idealized simple pitch rate control system does neet the C^* envelope specification.

Including the servo actuator dynamics, the optimization selected

$$(\delta_e/\dot{\theta}) = 0.93 \text{ sec.} \qquad (3\text{-}4)$$

The short period mode characteristics which then resulted were a higher natural frequency, 11,8 rad/sec, to overcome the increased lag associated with the additonal closed-loop real mode (pole at -2.33 rad/sec), and the damping ratio decreased to 0.46. The response shown as curve B of Figure 3.3 lies further within the tolerance boundary, although the velocity

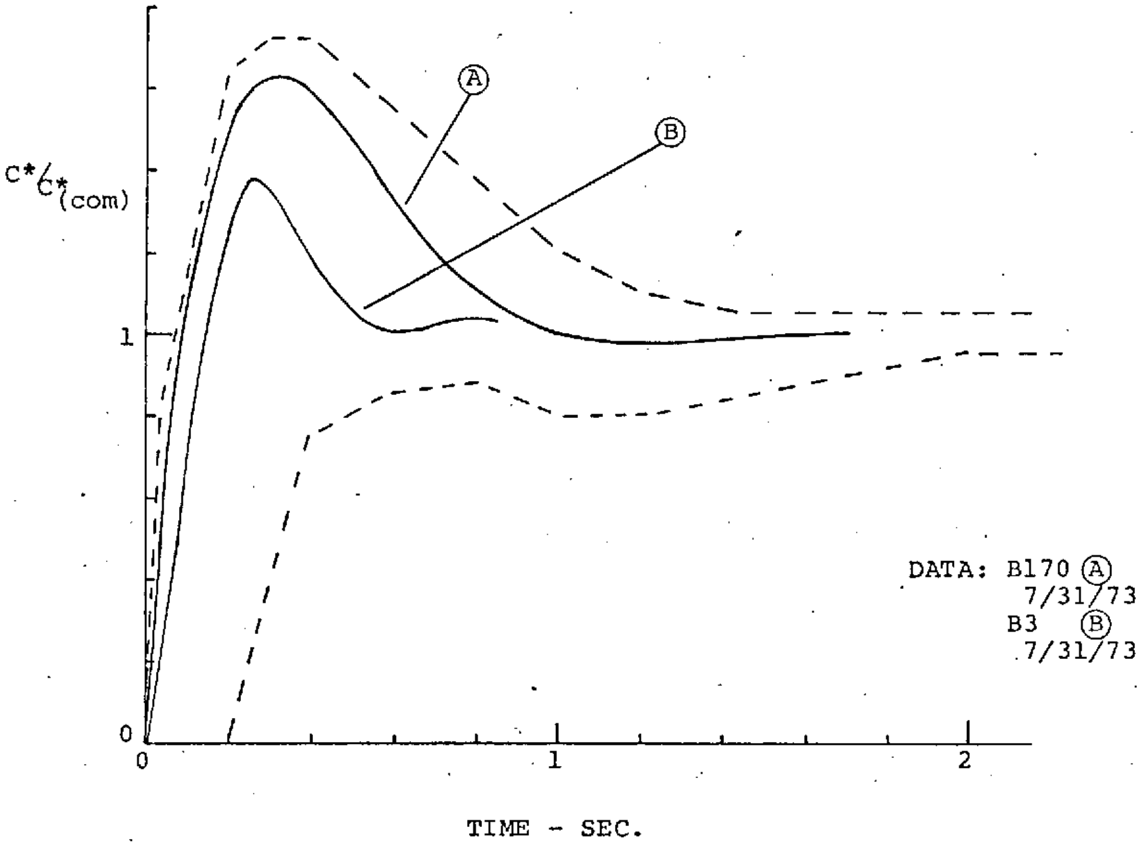


FIG. 3.3 RESPONSE OF A SIMPLE PITCH RATE SYSTEM
A: IDEALIZED SERVO
B: SERVO POLE AT -12 RAD/SEC.

response may be more objectionable due to the lowered damping ratio. In any event the pitch rate system presents difficulties from the standpoint of meeting the requirement for a constant steady-state stick force per g characteristic. Since normal acceleration and pitch rate are interrelated by a factor of true airspeed, one would need to vary the pilot input command sensitivity with airspeed.

State variable feedback

Since the aircraft is second order, complete state variable feedback is achieved by using two sensors, the pitch rate gyro and a normal accelerometer giving the system of Figure 3.4. The parameter optimization for the idealized case selected the following sensitivities.

$$S_{c[\dot{\theta},\delta_e]} = (\delta_e/\dot{\theta}) = 0.23 \text{ sec.} \qquad (3-5)$$

$$S_{c[a_3,\delta_e]} = (\delta_e/a_z) = 4.7 \text{ deg./g} \qquad (3-6)$$

The resultant C^* response is shown in Figure 3.5. Here the performance index has a value of 17.9 per cent. The initial response is well within the tolerance envelope, but the tail of the response approaches the boundary. The positive value for the accelerometer feedback indicates that a better model match is achieved by using this feedback to reduce M_α of the aircraft so as to reduce the natural frequency of the short period mode. The latter mode ends up with a natural frequency of 2.72 rad/sec and damping ratio of 0.8. The closed-loop poles do not coincide with the model poles even though state variable feedback is used because the aircraft zeroes differ from the model zeros.

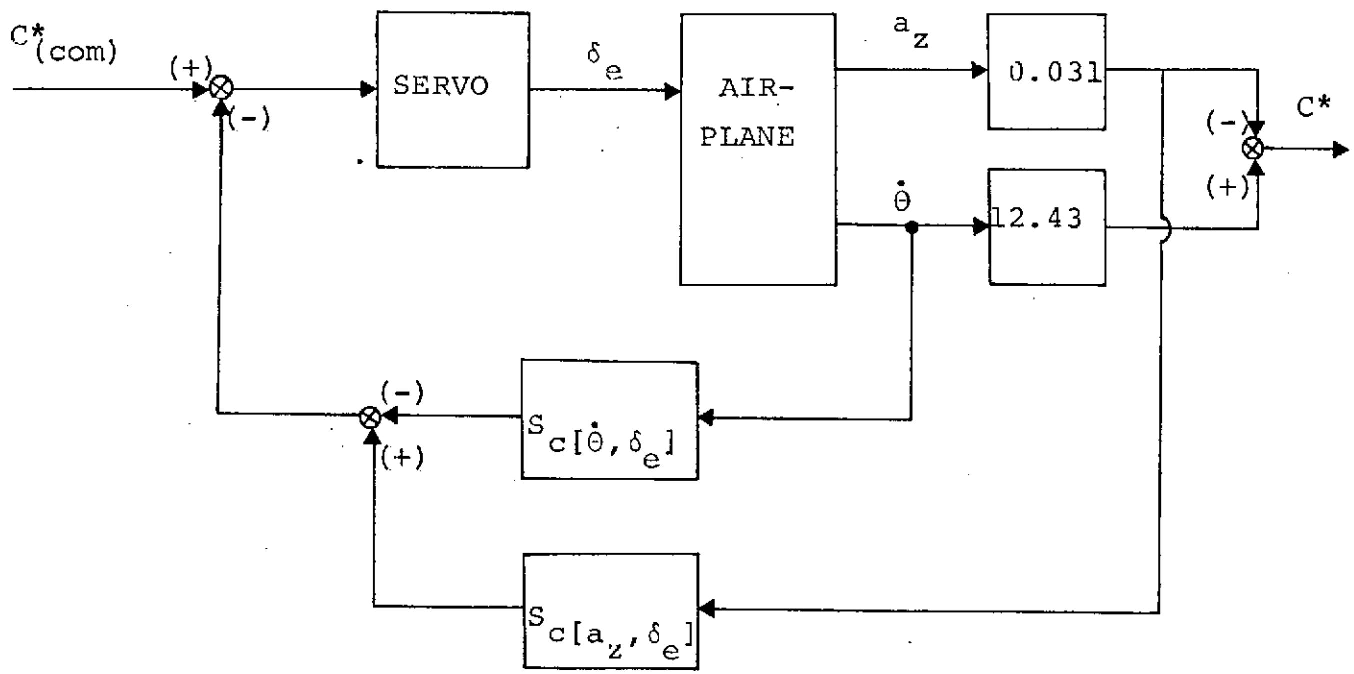


FIG. 3.4 SIMPLIFIED STATE VARIABLE FEEDBACK SYSTEM
MATHEMATICAL BLOCK DIAGRAM

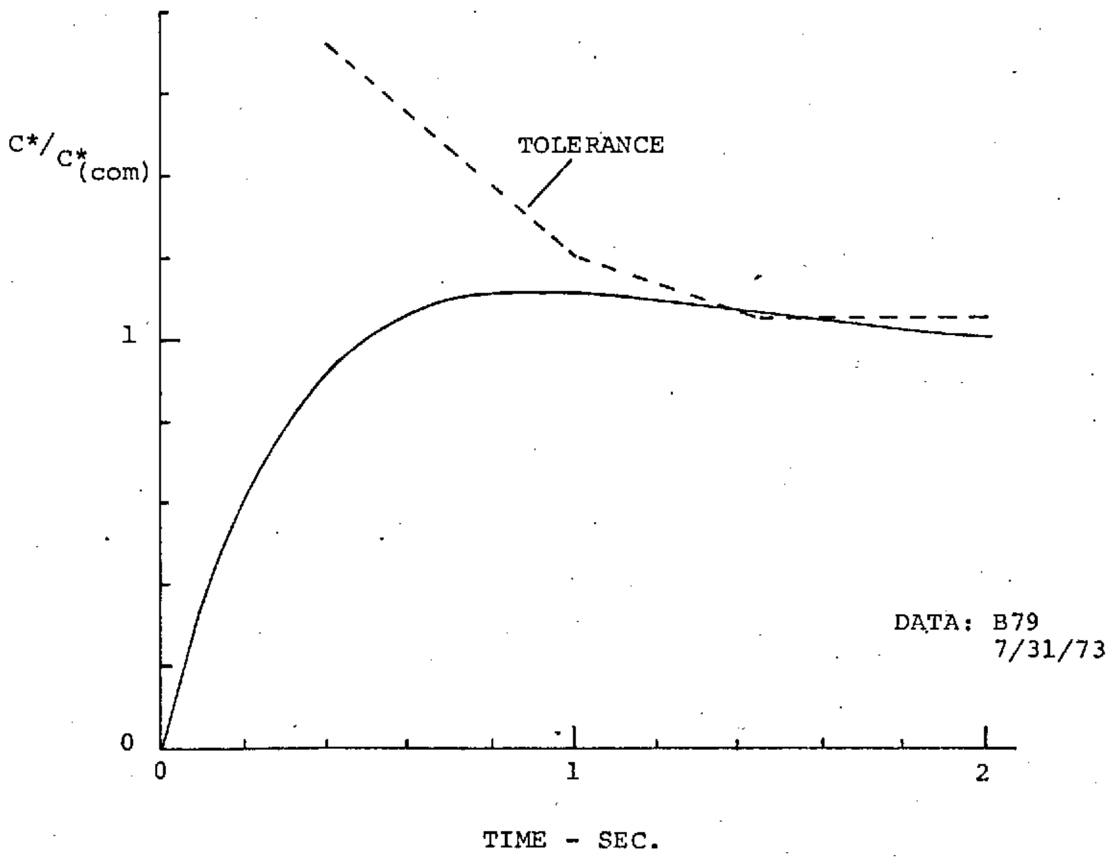


FIG. 3.5 RESPONSE OF AN IDEAL STATE VARIABLE FEEDBACK SYSTEM

C^* feedback

The performance specification calls for a desired C^* step function response. Since C^* is a linear combination of pitch rate and normal acceleration, a control system which used C^* as the feedback signal to be summed with a C^* command input is equivalent to feeding back pitch rate and normal acceleration in a specified ratio of the feedback gains. However the previous results indicate that the desired C^* ratio of acceleration to rate signals does not provide an optimum feedback signal at this flight condition, and one would not expect to obtain as good performance. For completeness however an idealized C^* feedback loop was investigated of the form of Figure 3.6. The optimization resulted in the static sensitivity

$$S_{c[C^*,\delta]} = (\delta_e/C^*) = 1.5 \text{ degree/g} \qquad (3\text{-}7)$$

The response exhibits excessive overshoot and fails to meet the specifications as shown by the response of Figure 3.7.

The previous results indicate that one should not attempt to add compensation to improve the response of the C^* loop since that would have to be more complicated than just using a different ratio of the acceleration and rate information in the feedback structure.

Final design

In the block diagram of Figure 3.4 the input was labelled a C^* command signal even though there is no C^* comparision point in the system. The reason for this is that in steady-state C^* is proportional to the input command so that the input could be calibrated in terms of units of desired C^* response. However, the figure also shows that the input could just as easily be termed either a pitch rate command or an acceleration command. Since there is no integration in the open loop, these calibrations would vary with flight condition. In order to meet the added requirement of a constant stick

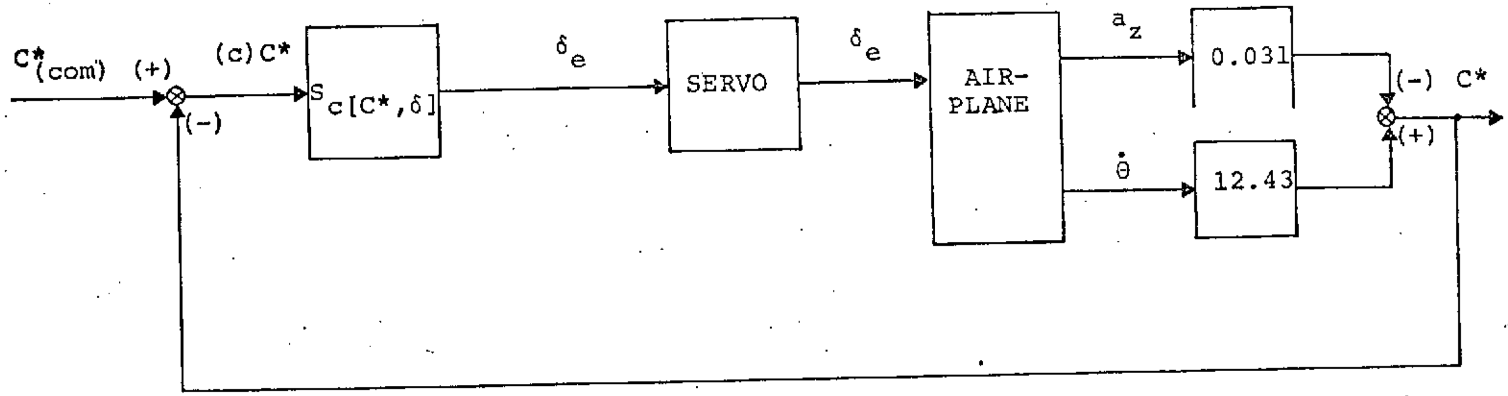


FIG. 3.6 MATHEMATICAL BLOCK DIAGRAM, SIMPLE C* LOOP

force per g characteristic, additional complication is required.

Since C^* is the output whose dynamic response is modelled by the specification, comparing C^* with a C^* command is a straightforward proposal for a control loop configuration. Since Figure 3.7 indicates that a simple C^* feedback would not meet the specification envelope, one could postulate a C^* feedback loop closed around the inner acceleration and pitch rate loops of Figure 3.4 in order to provide a C^* comparison point. For the reasons given in the next paragraph, this configuration was not proposed, but it motivates the discussion of the final system configuration.

Since the steady state pitch rate is proportional to normal acceleration and inversely proportional to true airspeed, a constant C^* per unit command is not a constant acceleration per unit command over the flight envelope of the airplane. Therefore if one considers integral compensation to provide the open-loop integration required to give a constant closed-loop static sensitivity, such compensation cannot be used following a C^* comparison point. Rather it must be compensation applied to an acceleration error signal. However the result of the previously discussed studies show that the optimization resulted in an acceleration feedback which was regenerative in order to decrease the aircraft's inherent static stability margin. Therefore integral compensation would not be usable on the acceleration error signal.

To deal with this added complication, one could postulate a C^* feedback in which the individual acceleration and rate signals are fed back separately to two summation points as shown by the block diagram of Figure 3.8. The total feedback portion of the second summation point is then C^*. This configuration however permits one to place integral compensation following the acceleration error summation point so as to maintain the desired constant closed-loop static sensitivty.

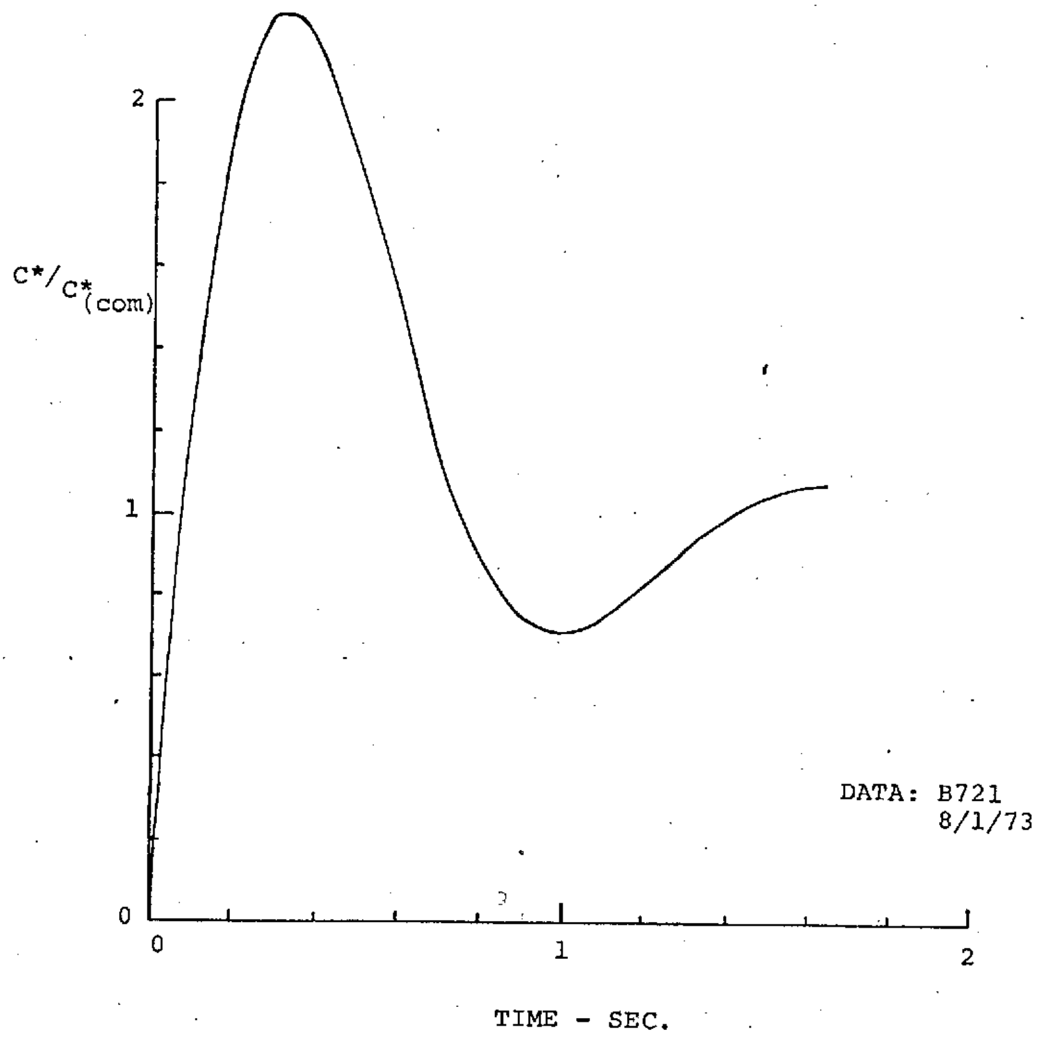


FIG. 3.7 RESPONSE OF SIMPLE C* LOOP

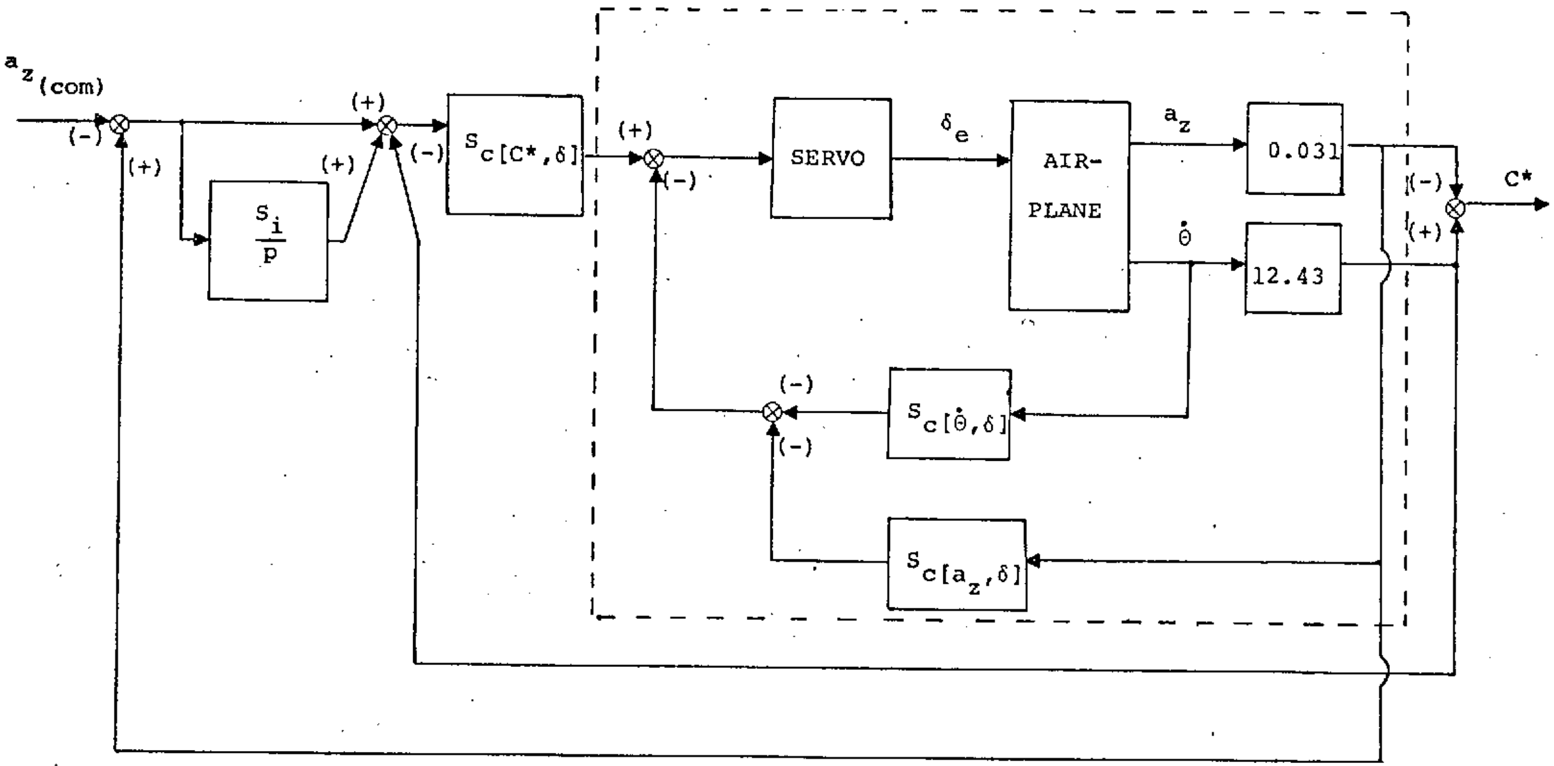


FIG. 3.8 MATHEMATICAL BLOCK DIAGRAM CONSTANT STICK FORCE PER g SYSTEM

The gain of the integration path could be selected as a design variable also. When this configuration was optimized, the following design parameters were selected:

$$(\delta_e/\dot{\theta})_{total} = 0.44 \text{ sec.} \tag{3-8}$$

$$\left.\begin{aligned}(\delta e/a_z)_{inner} &= 4.9 \text{ deg./g} \\ (\delta_e/C^*) &= -3.0 \text{ deg/g}\end{aligned}\right\} \text{Effective gain of 1.9 deg/g} \tag{3-9}$$

$$S_i = 0.95 \text{ sec}^{-1}$$

From noise considerations, there was interest in a further restriction upon the allowable integral compensation static sensitivity of $S_{i\ (max)} = 0.75 \text{ sec}^{-1}$. The optimization including this restriction gave a system defined by

$$(\delta_e/\dot{\theta})_{total} = 0.73 \text{ sec} \tag{3-10}$$

$$\left.\begin{aligned}(\delta_e/a_z)_{inner} &= 5.4 \text{ deg/g} \\ (\delta_e/C^*) &= -3.4 \text{ deg/g}\end{aligned}\right\} \text{Effective gain of 2.0 deg/g} \tag{3-11}$$

The time responses of these two systems are presented in Figure 3.9. Both responses lie within the tolerance envelope. The unrestricted integral gain gives a better model match with an average performance index value of (PI) = 5.3%. The short period mode has an undamped natural frequency of 8.1 rad/sec and damping ratio of 0.54. Restricting the integral gain yielded an average index of 12%, a natural frequency of 10.6 rad/sec, with a damping ratio of 0.46. The value of the unrestricted integral compensation gain was only 26 per cent higher than the restricted one, and the short period mode characteristics which resulted may be preferable.

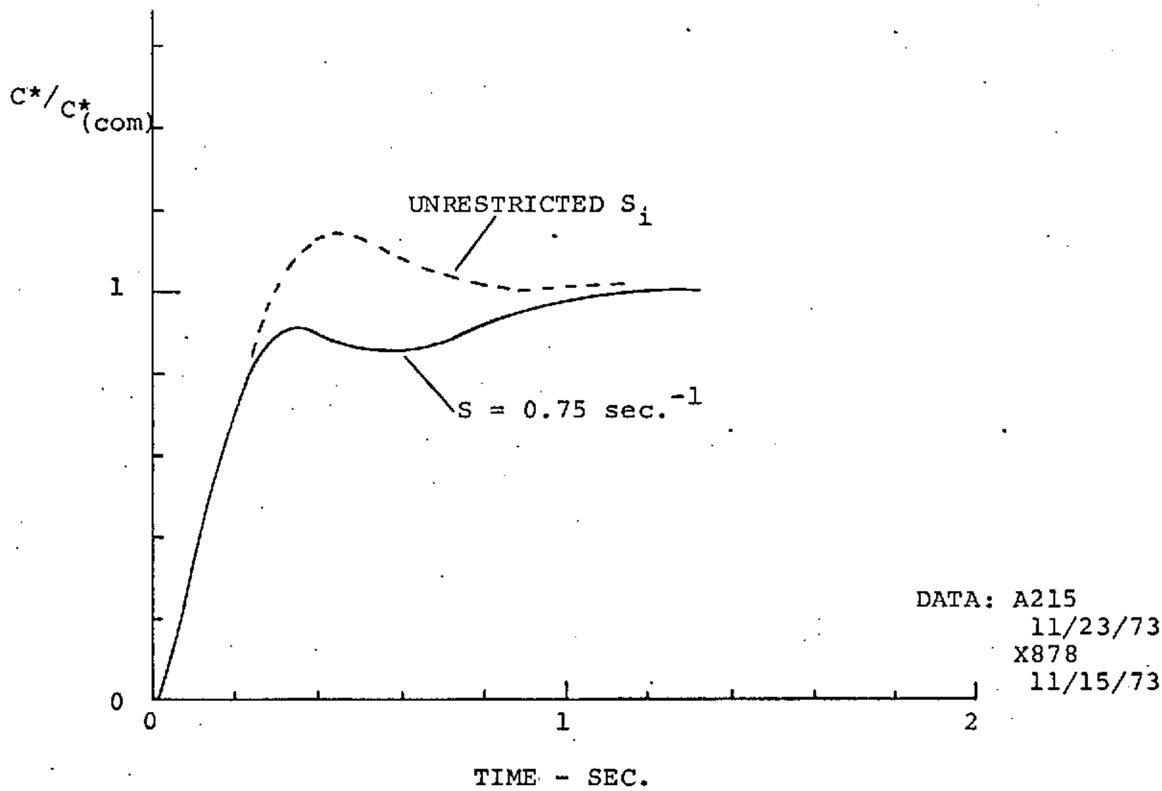


FIG. 3.9 CONSTANT STICK FORCE /g SYSTEM RESPONSE

The configuratin of Figure 3.8 obviously shows two pitch rate feedback paths. By block diagram rearrangement these can be reduced to a single pitch rate feedback path with a different sensitivity definition.

Continuation of the design procedure

At this period a possible control system configuration has been established. The next step would be to investigate the configuration for other flight conditions and investigate design parameter sensitivty. This was not accomplished in this reporting period due to encountering numerical accuracy problems as noted subsequently. It is interesting that the parameter optimization design method automatically suggested the regenerative acceleration feedback path. This is a design possibility that one probably would have overlooked if one had used more conventional design iterations. (The total acceleration feedback, that is, including the component of the effective C^* feedback,is less regenerative.) This is undoubtedly the simplest configuration. If one were concerned however that a possible failure mode existed in which the outer acceleration feedback signal was opened, one could impose a restraint upon the sign of the inner loop and proceed to add compensation as needed to achieve as good performance as this system exhibited. Some of the cases studied, but not included in the above summary, indicated that much higher values of the integrator sensitivity would be required, and that that would be objectionable for practical considerations. Proper redundancy design should provide an acceptable fail-safe configuration. On the other hand, the high integrator sensitivity systems were less sensitive to flight condition change than a preliminary examination of this system seems to indicate. Thus the design procedure has brought the engineer to a design point at which he can evaluate his alternatives on the basis of other operational requirements than performance and can do so knowing the performance capability of his present design.

Numerical accuracy of the computations

In the investigation of the previously described design example, numerical accuracy difficulties were encountered from time to time. These are serious enough to make the technique of limited usefulness unless they can be overcome. Accordingly the design example was interrupted, and effort was expended to understand the source of the trouble. The general cause is the numerical rounding-off due to a finite word length of the digital computer in the matrix operations being employed. It shows up partly as errors in a matrix inversion that is required, but that alone may not be the complete source of the difficulty. A Master of Science thesis is attempting to delineate these troubles more clearly and to explore ways to overcoming them.

The indication of inaccuracy is the generation of inaccurate or even impossible performance index values for some combinations of design parameter values. An independent check of the performance index by numerical integration was performed for each of the cases reported above so that confidence is high that these results are valid. Unfortunately such an evaluation is more expensive in computer costs, and hence it is not an attractive alternative as a permanent fix.

APPENDIX A

Airplane Data

The longitudinal aerodynamic stability derivative data for the F-8 airplane are tabulated in Table A-1. This table also serves to define the flight conditions to be investigated. The mass and dimensional data are as follows:

Tail length:	15.7 feet (4.8m)
Chord:	11.8 feet (3.59m)
Wing area:	374.9 $(\text{ft.})^2$ (34.87 m^2)
Mass :	648.8 slug (9993.7 kg)
Pitch moment of inertia:	87492 slug-ft^2(118640 kg-m^2)

The results presented in this report were all for flight Condition 4. The corresponding transfer function for the airplane are presented below. These relate the pitch angular velocity, $\dot{\theta}$, the Z-axis component of specific force, a_z, and the quantity, C^*, to the elevator motion, δ_e.

$$\frac{\dot{\theta}}{\delta_e} = G_A[\delta,\dot{\theta}] = \frac{-0.2973(1+p/0.5187)}{D(p)} \quad \sec^{-1} \qquad \text{(A-1)}$$

$$\frac{a_z}{\delta_e} = G_A[\delta,\ a_z] = \frac{+\ 316.7\ (1-p/z_1)(1-p/\bar{z}_1)}{D(p)} \quad \frac{(\text{ft./sec.}^2)}{\text{radian}}$$

$$z_1 = -\ 0.4813 + 8.244j$$

$$\frac{C^*}{\delta_e} = G_{A[\delta,C^*]} = \frac{-13.51\ (1+p/1.935)(1+p/48.51)}{D(p)} \quad \frac{(g)}{(\text{radian})}$$

$$D(p) = 1+\frac{2(0.0736)}{4.50}p + \frac{p^2}{(4.50)^2}$$

The matrix equations of the airplane are: (F.C.4)

$$\begin{bmatrix} \dot{w} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -0.653, & 1065. \\ -0.019, & -0.010 \end{bmatrix} \begin{bmatrix} w \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} -82.0 \\ 11.6 \end{bmatrix} \delta_e \qquad \text{(A-2)}$$

TABLE A-1 Aerodynamic Data for the F-8 Airplane, Longitudinal Axis

F8 NON DIMENSIONAL LONGITUDINAL STABILITY DERIVATIVES

F.C.	1	2	3	4	5	6	7	8	9
C_D	0.031	0.0415	0.0430	0.0480	0.0410	0.0195	0.0230	0.0175	0.0210
C_{D_M}	0.115	0.116	0.116	0.116	-0.0167	-0.0175	-0.018	0.115	0.115
$\partial T/\partial u$	0.649	-0.1	1.47	0.649	0.50	-0.67	0.3	3.68	1.47
C_L	0.284	0.0775	0.143	0.224	0.452	0.11	0.240	0.11	0.23
C_{L_M}	0.120	0.117	0.116	0.116	0.144	0.109	0.109	0.08	0.123
C_{D_α}	0.328	0.106	0.217	0.335	0.29	0.29	0.29	0.072	0.210
C_{L_α}	4.864	4.866	4.866	4.866	3.80	3.80	3.81	4.30	4.30
C_{m_M}	-0.145	0.068	-0.002	-0.144	-0.077	-0.07	-0.002	0.01	-0.145
C_{m_α}	-0.745	-1.55	-1.55	-1.55	-0.715	-0.492	-0.477	-0.525	-0.59
$C_{L_{\delta_e}}$	0.573	0.58	0.58	0.58	0.55	0.55	0.550	0.604	0.532
$C_{m_{\delta_e}}$	-0.967	-0.882	-0.84	-0.89	-0.895	-0.88	-0.87	-0.96	-0.964
C_m	1.16	2.7	2.7	2.7	-0.283	-0.233	-0.200	-0.070	-0.070
C_{m_q}	-3.45	-2.85	-2.85	-2.85	-3.50	-3.47	-3.45	-4.1	-4.1
Altitude (1000 ft)	45	17	35	45	25	4	20	20	35
Mach No.	0.94	1.1	1.1	1.1	0.5	0.56	0.6	0.85	0.85
α_0(deg)	4.7	1.5	2.4	3.7	7.7	2.7	4.6	2.3	3.9
δ_{e_0}	-4.5	-1.26	-2.9	-5.0	-4.68	-2.22	-2.6	-1.93	-2.6
V(fps)	910	1154.	1070	1065	508	617	622	881	827
q(lb/ft^2)	101	932	423	262	137	401	245	492	252

REFERENCES

1. Rediess, H.A. and Whitaker, H.P., "A New Model Performance Index for Engineering Design of Flight Control Systems", AIAA Journal of Aircraft, Vol. 7, Nov. 1970.

2. Palsson, T. and Whitaker, H.P., "Parameter Uncertainties in Control System Design", Proceedings of the 1972 Joint Automatic Control Conference of the American Automatic Control Council, p. 248, August, 1972.

3. Whitaker, H. Philip, "The System Description Program", Measurement Systems Laboratory Report RE-79, Revision 1, M.I.T., Sept. 1972.

4. Malcom, L.G. and Tobie, H.N., "New Short Period Handling Qalities Criterion for Fighter Aircraft", Boeing Co. Document D6-17841 T/N, Seattle, Washington, Sept. 1965.