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By S. M. Seltzer Systems Dynamics Laboratory

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#### TECHNICAL MEMORANDUM X-64867

## ENHANCING SIMULATION EFFICIENCY WITH ANALYTICAL TOOLS\*

### INTRODUCTION

The relatively recent development of computers — particularly digital machines — has strongly influenced engineering design. Modern engineers and designers place heavy reliance on digital, analogue, and hybrid computers to simulate models of systems being designed. This is particularly true in the area of dynamics and control, where computers are used extensively to simulate the dynamic behavior of such models. In addition, digital computers often are used to solve the equations of motion describing the model of the dynamics of a particular system for the associated eigenvalues.

An older (but continually updated) approach to system design is to perform mathematical analysis of the representative equations of motion. This usually leads to closed form solutions of these equations.

Recently NASA has placed particular emphasis on the development of low cost aerospace systems. The area of computer simulation has been identified as one of the many areas where cost reduction should be investigated. The purpose of this paper is to indicate some means of combining both computer simulation and analytical techniques in order to mutually enhance their efficiency as design tools and to motivate those involved in engineering design to consider using such combinations. While the idea is not new, heavy reliance on computers often seems to overshadow the potential utility of analytical tools. Although the example used in this paper is drawn from the area of dynamics and control, the principles espoused are applicable to other fields.

### APPROACH

Computers provide a relatively rapid means of solving the equations of motion of models of physical systems. They can handle large order systems of

<sup>\*</sup>Portions of this paper were presented at the Eighth Annual Allerton Conference on Circuit and System Theory at Monticello, Ill., Oct. 7-9, 1970, and at the First Annual Research and Technology Review, George C. Marshall Space Flight Center, Alabama, Feb. 22-23, 1973.

equations of a nature and dimension that historically have troubled analysts. These outstanding and well known traits often lead to exclusive reliance on computer simulation techniques by contemporary engineers. This exclusive reliance can lead to technical pitfalls in the form of erroneous results because of programing errors or to improper attention being devoted to implicit computer characteristics such as accuracy (round off errors) and clock time. Furthermore, suitable numerical values for parameters used in the simulation often are determined by "cut-and-try" methods, which can be time consuming and inefficient.

Mathematical analysis, on the other hand, usually requires relatively low order systems with the attendant simplifications leading from the original, more complex (and more accurate) system model. Further, the user of present analytical techniques often encounters difficulty in dealing with nonlinear systems or time varying parameters. However, when a relatively simple system model can be obtained, analytical techniques exist that can predict the dynamic behavior of that model, often as a closed form solution.

From the foregoing, it is seen that analysis techniques and computer simulations might be used to mutually enhance their strengths and decrease their weaknesses. Obvious examples would be to use analytical techniques to predict the dynamic behavior of the computer simulation and thereby help "debug' the program. Another use is to apply analytical techniques to determine numerical values for system parameters that are needed for the computer simulation, thereby reducing or eliminating "cut-and-try" efforts. It is apparent that the use of simplified models might lead to erroneous results. This danger is reduced by comparing the dynamics of the simplified model used for analysis with the dynamics of the computer simulation. Disagreement indicates an error in the choice of the simplified model, a mistake in the associated analysis, or a programing error. While such disagreements are inconvenient to resolve, once an agreement is achieved the simulation results may be used with a high degree of confidence. The major portion of the remainder of this paper will be devoted to showing how an analysis technique may be used efficiently to select numerical values for a large scale computer simulation. This was actually performed in the design and development of the Skylab control system.

### SKYLAB EXAMPLE

In late 1970 a hybrid simulation of the Skylab control system, simulating the rigid body dynamics, was under development for use in the design of the

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control system being developed for NASA's first manned orbital space station, Skylab (Fig. 1)[1]. It was found that the system response was unstable unless the sampling period was decreased to an unacceptably small value in terms of computer utilization. Hence, a simple digital filter was introduced into the system. The effect of the numerical values chosen for the basic control system parameters and filter parameters on the system dynamics, and the sampling period, was analyzed and is described below.



#### Figure 1. Skylab.

### **Definition of Problem**

The Skylab control system is shown in block diagram form in Figure 2. If it is assumed that the reaction moment  $(\underline{M}_R)$  is kept equal to the commanded moment  $(\underline{M}_C)$  and only rotation about a single axis is considered, the block diagram may be simplified as shown in Figure 3. If at first no digital filter is present  $(K_2 = 0)$ , the problem is to choose values for control gains  $K_0$  and  $K_1$ , and sampling period (T) such that desirable transient characteristics accrue. However, as will be shown subsequently, a simple digital filter (indicated on Fig. 1) must be added to the system to achieve acceptable sampling periods. Now, the problem becomes one of choosing values for the control gains  $K_0$  and  $K_1$ , and the filter element  $K_2$  so that an acceptable value of T will provide desirable transient characteristics.



Figure 2. Block diagram of Skylab control system.

## Description of Parameter Plane Method

The parameter plane technique for analysis and synthesis of linear and nonlinear control systems is amply described in Siljak's recent monograph on the subject [2]. Reference 3 describes the application of the technique to the analysis and synthesis of linear sampled-data control systems.



Figure 3. Simplified control system block diagram.

Once the system characteristic equation has been obtained, the parameter plane method enables the designer to evaluate graphically the roots of the equation. Hence, he may design the control system in terms of the chosen performance criteria; e.g., absolute stability, damping ratio, and settling time. He is able to see the effect on the characteristic equation roots of changing two adjustable parameters. Siljak further simplified the design procedure by introducing Chebyshev functions into the equations, thereby

putting them in a form particularly amenable to their solution by a digital computer. The method has been extended to portray the effect of varying the sampling period [4]. Thus one can see the effect of the choice of values assigned to the sampling period on absolute and relative stability. Also, the recursive formulas shown herein are simpler in form than the Chebyshev functions [3]. Although the brevity of this paper precludes an explicit demonstration, the resulting formulation is deliberately cast in a form that makes it particularly amenable to solution by a digital computer or a desk calculator, again emphasizing the interplay between analysis and computing machines. Because a calculator is usually readily available to the design engineer, the example portrayed here was solved using a Hewlett-Packard desk calculator.

The technique requires that the control system be described by a characteristic equation which is transformed into the z-domain. Two adjustable parameters  $(k_0, k_1)$  are selected, and the characteristic equation (CE) is recast in terms of them; i.e.,

$$CE = \sum_{j=0}^{n} \gamma_{j} z^{j} = 0 , \qquad (1)$$
  
$$\gamma_{j} = a_{j} k_{0} + b_{j} k_{1} + c_{j} , \qquad (2)$$
  
$$z = \epsilon^{TS} = r \epsilon^{i\theta} , \qquad (3)$$

$$r = r(\zeta, \omega_n, T) = \epsilon^{-\zeta \omega_n T},$$
 (4)

and

$$\beta = \beta(\zeta, \omega_n, T) = \cos \theta = \cos (\omega_n T \sqrt{1-\zeta^2}) ,$$
 (5)

where  $\zeta, \omega_n$ , and T represent the damping ratio, natural frequency, and sampling period, respectively. To transform the characteristic equation from a differential equation into an algebraic equation,  $z^j$  may be defined as

$$z^{j} = X_{j} + iY_{j}\sqrt{1-\beta^{2}}$$
, (6)

(7)

where  $X_0 = 1$ ,  $Y_0 = 0$ ,  $X_1 = r\beta$ , and  $Y_1 = r$ . The following recursion formulas are found to enable one to solve for other values of  $X_j$  and  $Y_j$ :

$$X_{j+1} - 2r\beta X_j + r^2 X_{j-1} = 0$$

and

$$Y_{j+1} - 2r\beta Y_{j} + r^2 Y_{j-1} = 0$$

If equation (6) is substituted into equation (1) and the real and imaginary parts of the resulting equation are separated, two simultaneous algebraic equations are obtained. They may be solved for the adjustable gains  $k_0$  and  $k_1$ :

$$k_{0} = \frac{B_{1}C_{2} - B_{2}C_{1}}{J}$$

$$k_{1} = \frac{A_{2}C_{1} - A_{1}C_{2}}{J}$$

$$J = A_{1}B_{2} - A_{2}B_{1}$$

$$A_{1} = \sum_{j=0}^{n} a_{j}X_{j}$$

$$B_{1} = \sum_{j=0}^{n} b_{j}X_{j}$$

$$C_{1} = \sum_{j=0}^{n} c_{j}X_{j}$$

$$A_{2} = \sum_{j=0}^{n} a_{j}Y_{j}$$

$$B_{2} = \sum_{j=0}^{n} b_{j}Y_{j}$$

$$C_{2} = \sum_{j=0}^{n} c_{j}Y_{j}$$

$$C_{3} = \sum_{j=0}^{n} c_{j}Y_{j}$$

$$C_{4} = \sum_{j=0}^{n} c_{j}Y_{j}$$

$$C_{5} = \sum_{j=0}^{n} c_{j}Y_{j}$$

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$$C_{5} = \sum_{j=0}^{n} c_{j}Y_{j}$$

$$C_{5} = \sum_{j=0}^{n} c_{j}Y_{j}$$

Recalling that  $X_j$  and  $Y_j$  are functions of  $\zeta, \omega_n$ , and T and observing that  $a_j$ ,  $b_j$ , and  $c_j$  may be functions of T, we see that contours for specified values of  $\zeta$  may be plotted as functions of  $\omega_n$  and T in the  $k_0 - k_1$  parameter plane. The locations of these contours in the parameter plane represent values of the complex conjugate roots of the characteristic equation. If one chooses numerical values for  $\omega_n$  and  $\zeta$ , values for parameters  $k_0$  and  $k_1$  and the sampling period T are established. If the coefficients  $a_j$ ,  $b_j$ , and  $c_j$  contain exponential terms with T appearing in the exponents, then each exponent must be replaced by its power series and truncated according to the accuracy that is desired.

The real root locations corresponding to values of z when  $\theta$  equals 0 degrees and 180 degrees may also be plotted on the parameter plane by setting z equal to a positive or negative real constant, substituting that value into equation (1), and solving for  $k_1$  as a function of  $k_0$ . Each resulting contour corresponds to a location of a real root in the z-domain. When z = +a,

$$\sum_{j=0}^{n} \gamma_{j} a^{j} = 0 , \qquad (11)$$

and when z = -a,

$$\sum_{j=0}^{n/2} \gamma_{2j} a^{2j} - \sum_{j=0}^{(n-2)/2} \gamma_{(2j+1)} a^{(2j+1)}, \text{ n even }, \qquad (12a)$$

$$\sum_{\substack{j=0\\j=0}}^{(n-1)/2} \left[ \gamma_{2j} a^{2j} - \gamma_{(2j+1)} a^{(2j+1)} \right], n \text{ odd } ,$$
 (12b)

where a is a positive real number.

For a linear sampled data control system to be stable, it is necessary that all roots of the characteristic equation lie within the unit circle on the z-plane. It is assumed that the system being designed possesses low-pass filter characteristics so that only the primary strip (corresponding to  $0 \le \theta \le \pi$ in the z-plane) need be considered. The stable region is bounded by the semicircle defined by the upper half of the unit circle. This region may be defined by mapping three contours from the z-plane onto the parameter plane. The

and the second

 $\mathbf{7}$ 

first contour is defined by setting  $\zeta = 0$  in equations (8). The resulting complex root stability boundary is seen to be a function of  $\omega_n$  and T. The second and

third contours correspond to real root locations at z = +1 and z = -1 and are found by setting "a" equal to unity in equations (11) and (12), respectively. The stable region is determined by applying a shading criterion or using a test point [3]. If the Jacobian, defined by equation (9), is greater than zero, then the stable region (if it exists) lies to the left of the  $\zeta = 0$  contour as  $\omega_{p}T$ 

increases; the left side of the line is double crosshatched to indicate a boundary associated with double, or complex conjugate, roots. (If the Jacobian is less than zero, the stable region lies to the right.) Single crosshatching is used on the two contours associated with the real roots. The side of the contour on which to place the crosshatching is determined by the requirement that crosshatching be continuous, or on the same side, of the contours as the intersections corresponding to z = +1 and z = -1 are approached along either the complex root or real root stability boundary.

Once the stable region, if it exists, has been delineated in the parameter plane, the transient characteristics of the system can be determined in terms of the locations of the roots of the characteristic equation. For the complex conjugate roots, these locations are defined in terms of damping ratio and natural frequency. Contours of constant  $\zeta$  are determined as functions of  $\omega_n$  and T from equation (8) and plotted on the parameter plane.

Similarly, values of "a" corresponding to real root locations may be substituted into equations (11) and (12) to plot contours corresponding to these locations. Now the effect of any chosen design point on the parameter plane may be found in terms of  $\zeta$ ,  $\omega_{p}$ , a, and T.

The analytical technique developed permits the designer to observe the effect of simultaneously changing three control parameters and the sampling period. Most existing conventional techniques permit the observation of the effect of changing only one control parameter and do not show the effect of various sampling periods.

Sometimes it is specified that the settling time of the system be less than a prescribed value. This corresponds to requiring that the real part of the roots of the characteristic equation be less than a prescribed negative real constant. A boundary corresponding to this requirement can be drawn on the parameter plane by mapping a circle of constant radius (for a chosen value of  $\omega_{n}\zeta$  and T) from the z-plane onto the parameter plane [3]. Relations exist for estimating the maximum overshoot and peak time of transient

response when it is valid to assume a second order system [5]. However, a simple estimate can sometimes be made by merely looking at the difference equation representing the system response, estimating when the overshoot will occur, and plotting a corresponding line on the parameter plane. This procedure is brought out in the example. Steady state response may be found from the open loop transfer function and the assumed forcing functions in the conventional manner [5].

## Application to Skylab Design Problems

If at first no digital filter is present, the problem is to choose values for control gains  $k_0$  and  $k_1$  and sampling period T such that desirable transient characteristics accrue. The open loop transfer may be written in the z-domain:

$$G(z) = \mathcal{Z}\left\{ \left(\frac{K_0}{s} + K_1\right) \left(\frac{1 - \epsilon^{-Ts}}{s}\right) \left(\epsilon^{-T} d^s\right) \left(\frac{1}{Is^2}\right) \right\}$$
(13)

If the computation delay  $T_d$  is made equal to the sampling period T, then equation (13) may be rewritten as

$$G(z) = \frac{(k_0 + k_i) z + (k_0 - k_i)}{z(z - 1)^2}$$

where

$$\mathbf{k}_0 = \frac{\mathbf{K}_0 \mathbf{T}^2}{2\mathbf{I}}$$

and

$$\mathbf{k}_1 = \frac{\mathbf{K}_1 \mathbf{T}}{\mathbf{I}}$$

(15)

9

(14)

The corresponding characteristic equation is equation (1), where n = 3. Table 1 defines the numerical values of the coefficients  $a_j$ ,  $b_j$ , and  $c_j$  for this case if  $K_2$  is set equal to zero.

j	a <sub>j</sub>	bj	c <sub>j</sub>
0	1	-1	K <sub>2</sub>
1	1	1	$1 - 2K_2$
2	0	0	K <sub>2</sub> - 2
3	0	0	1

TABLE 1. COEFFICIENTS OF CHARACTERISTIC EQUATION

Using Table 1 values and setting  $\zeta = 0$  and a = 1 for equations (4), (5), (7), and (8) through (12) provides J<0 and the three stability boundaries shown in Figure 4 (again setting  $K_2 = 0$ ). The number of stable roots in each region is indicated in parentheses. It is seen that the stable region lies in the first quadrant. Contours corresponding to values of  $\zeta > 0$  may now be plotted as functions of  $\omega_n T$ . Several are shown on Figure 5 with equal values of  $\omega_n T$  connected by dashed lines.



Figure 4. Stability contours with filter.





If a value of  $\zeta = 0.7$  is desired,  $\omega_n$  T must be less than 0.8. The predicted system natural frequency is 0.  $1\pi$ , yielding a maximum possible sampling time of approximately 2.4 seconds. To obtain acceptable transient characteristics, the actual value of T used would have to be considerably less than 2.4 seconds. With the goal of utilizing less computer time, it is desired to add a simple digital filter to increase the sampling period. A filter of the form,

$$F(z) = \frac{z}{z + K_2}$$

is introduced (Fig. 3). The qualitative effect of the filter may be seen by referring to Figure 4. For positive values of  $K_2$ , the z = -1 stability boundary is displaced upward. The effect is to move the  $\zeta = 0$  curve so that it bulges more to the right and upward, enclosing a larger region and increasing the maximum value of  $\omega_n T$  for stability. An example is shown for  $K_2 = 1$  (Fig. 6).

(16)



Figure 6. Relative stability contours ( $K_2 = 1$ ).

However, if the value of  $K_2$  is set too large, the z = -1 boundary will cut off part (or all) of the stability boundary, as indicated in Figure 7. As  $K_2$  assumes negative values, the z = -1 boundary is displaced downward and the  $\zeta = 0$  curve moves to the left until, for values of  $K_2 < -1$ , no stability region remains. The steady state error is zero for step displacement and velocity inputs. For acceleration inputs, it is a constant value which is proportional to  $K_2$  and inversely proportional to  $k_0$ .

First a value of  $K_2$  should be chosen. It is seen that as  $K_2$  increases positively, larger values of  $\omega_n T$  will lie in the stable region for corresponding values of  $\zeta$ . Although the purpose of the filter is to increase the value of T,  $K_2$  must not be increased to the value where the stable region disappears or to a value greater than the computer clock time. The clock time is 2.5 milliseconds, and the simulation runs are performed at 100 times real time, so the sampling period T must not exceed 2.5 seconds ( $\omega_n T < 0.79$ ). Looking at the difference equation associated with the system response  $\phi$ , one sees that in general large values of  $k_0$  and  $k_1$  will result in large overshoots of the response:



Figure 7. Relative stability contours ( $K_2 = 2$ ).

 $\phi(nT) = (k_0 + k_1) \phi_r [(n - 2)T] + (k_0 - k_1) \phi_r [(n - 3)T]$   $-(K_2 - 2) \phi [(n - 1)T] - (k_0 + k_1 + 1 - 2K_2) \phi [(n - 2)T]$   $-(k_0 - k_1 + K_2) \phi [(n - 3)T] \qquad (17)$ 

As a design compromise, a value of  $K_2 = 1$  was chosen. If a value of 0.707 is used for  $\zeta$ , the maximum value of  $\omega_n^T$  that will remain within the stable region is 2.1, which is greater than the clock-time constraint. Choosing values for  $\omega_n^T$  on a particular  $\zeta$ -contour establishes the location of the real root.

If both the difference equation (17) and the parameter plane contours are examined simultaneously, one usually can quickly deduce the sampling period in which the maximum overshoot will occur. For example, if it is assumed that the unit step response will have a maximum overshoot in the vicinity of the third sampling instant, contours corresponding to selected magnitudes of the response at the third sampling instant may be mapped onto the parameter plane, using equation (17). Contours corresponding to unit step response values at the third sampling instant of 1.2, 1.4, and 1.6 are indicated on Figure 6. If it is desired that the overshoot not exceed 60 percent, then it is estimated that  $1.5 < \omega_{\rm T} < 0.7$ . Because of the clock-time constraint,  $\omega_{\rm T}$  is set at the lower value. The resulting step input response is shown on Figure 8.





## OTHER EXAMPLES

Other examples are available which further amplify the prudence of not accepting computer results without checking them by analysis. Schiehlen [6] found that the results reported in Reference 7 are incorrect — probably due to round-off errors in the digital computation. Similar pitfalls exist with analogue computation: the difficulty in obtaining the well known stability boundaries of the Mathiew equation is usually a frustrating exercise in futility [6].

## CONCLUSION

The main body of this paper describes an analytical technique that was used to predict desirable numerical values for simulation parameters. Numerical values were rapidly found and used successfully in the hybrid simulation. This combination of an analytical technique with the simulation technique obviated the need for "cut-and-try" methods to choose the numerical values, thereby saving both time and computer utilization.

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## APPROVAL

## ENHANCING SIMULATION EFFICIENCY WITH ANALYTICAL TOOLS

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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