# TURBULENT TRANSPORT OF HEAT AND MOMENTUM IN A BOUNDARY HAYER SUBJECT TO DECELERATION, SUCTION ANI VARHABLE WALL TEMPERATURE 

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The National Aeronamics and Space Administrat


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By

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#### Abstract

The objective of this work was the study of the relationship between the turbulent transport of heat and momentum in an adverse pressure gradient boundary layer. An experimental study was conducted of turbulent boundary layers subject to strong adverse pressure gradients with suction. Near-equilibrium flows were attained, evidenced by outerregion similarity in terms of defect temperature and defect velocity profiles.

The relationship between Stanton number and enthalpy thickness was shown to be the same as for a flat plate flow both for constant wall temperature boundary conditions and for steps in wall temperature. The superposition principle used with the step-wall-temperature experimental result was shown to accurately predict the Stanton number variation for two cases of arbitrarily varying wall temperature.

The Reynolds stress tensor components were measured for strong adverse pressure gradient conditions and different suction rates. Two peaks of turbulence intensity were found: one in the inner and one in the outer regions. The outer peak is shown to be displaced outward by an adverse pressure gradient and suppressed by suction.

The correlation coefficient and the ratio between the turbulent shear stress and the turbulent kinetic energy are shown to be constant and independent of pressure gradient and transpiration rate in the outer region of the boundary layer under nearly equilibrium conditions.

Temperature fluctuation and turbulent heat transfer were measured directly using the same probe used for the velocity fluctuations. The turbulent Prandtl number was then calculated directly from the turbulence and the mean profile measurements without the usual position error sensitivity. High values at the wall and low values in the outer region were measured, in accordance therefore, with some accepted studies.

A new procedure was developed to estimate the turbulent Prandtl number at the wall based on extrapolations of data near the wall. An adverse pressure gradient is shown to increase the turbulent Prandtl number at the wall while suction, on the other hand, is shown to decrease it.


## Table of Contents

Page
Acknowledgments ..... 1ii
Abstract ..... v
Table of Contents ..... vi
List of Figures ..... ix
List of Tables ..... xili
Nomenclature ..... xiv
CHAPTER

1. INTRODUCTION AND OBJECTIVES ..... 1
2. EXPERIMENTAL BOUNDARY CONDITIONS ..... 5
2.1. The Concept of Equilibrium Flows ..... 6
2.2. Boundary Conditions on $U$ and $F$ for the Structural Studies ..... 9
2.3. Boundary Conditions on $T_{\text {wall }}$ and $F$ for the Step Wall Temperature Studies ..... 10
2.4. Identification of the Boundary Condition ..... 11
3. EXPERIMENTAL APPARATUS, INSTRUMENTATION AND QUALIFI- CATION TESTS ..... 12
3.1. Main Air Flow ..... 12
3.2. Test Section
15
3.3. Transpiration Air System
3.4. Pressure Measurement ..... 15
3.5. Procedure for Setting Up a New Run ..... 16
3.6. Sequential Measurement of Mean Velocity and Temperature ..... 17
3.7. Hot Wire Instrumentation -- Choice of the Probe for Mean Temperature and Velocity Measurement ..... 18
3.8. Hot Wire Instrumentation -- The Measurement of Turbulent Quantities ..... 21
3.9. Calibration for Mean and RMS Temperature Measurement ..... 28
3.10. The Use of a Linearizing Circuit for Processing theOutput from the Anemometer30
3.11. Calibration of the Horizontal Wire for the Measurement of Mean and RMS Quantities ..... 31
3.12. Calibration of the Slant Wire for the Measurement of Turbulence Quantities ..... 34
3.13. Qualification of the Apparatus ..... 39
3.14. Qualification of the Measurement Procedure for Mean Temperature and Velocity Profiles ..... 40
3.15. Qualification of the Procedure for Turbulence Measurements ..... 40

## Table of Contents (cont.)

CHAPTER Page
4. ANALYSIS OF STANTON NUMBER DATA ..... 52
4.1. Stanton Numbers for Constant Wall Temperature Conditions ..... 53
4.2. Stanton Numbers for Variable Wall Temperature Conditions ..... 54
4.3. Conclusions Regarding the Stanton Number Behavior ..... 56
5. ANALYSIS OF MEAN TEMPERATURE AND VELOCITY PROFILES ..... 62
5.1. Behavior of $\overline{u^{\prime} v^{\prime}}$ and Its Derivative in the Region Very Close to the Wall ..... 62
5.2. Behavior of $v^{\prime} t^{\prime}$ and Its Derivatives in the Region Very Near the Wall ..... 63
5.3. Validity of the Couette Flow Assumption in the Region Very Near the Wall ..... 64
5.4. The Location of the First Data Point with Respect to the Wall ..... 65
5.5. Mean Velocity Profiles ..... 66
5.6. Mean Temperature Profiles ..... 68
5.7. Determination of the Friction Coefficients ..... 68
6. ANALYSIS OF THE TURBULENCE MEASUREMENTS ..... 82
6.1. Hydrodynamics ..... 82
6.1.1. Similarity variables for turbulence measurements ..... 82
6.1.2. Comments on the measurement of turbulence quantities ..... 83
6.1.3. The zero pressure gradient flow ..... 84
6.1.4. Adverse pressure gradient flow ..... 85
6.2. Temperature ..... 87
6.2.1. Similarity variables for temperature and heat flux measurements ..... 88
6.2.2. The zero pressure gradient flow ..... 89
6.2.3. Adverse pressure gradient flow ..... 90
7. TURBULENT TRANSPORT OF HEAT AND MOMENTUM ..... 108
7.1. Previous Theoretical and Experimental Studies ..... 108
7.2. Behavior of the Turbulent Prandtl Number Close to the Wall ..... 114
7.3. A New Measurement Procedure for Estimating Pr at the Wall ..... 115
7.4. Analysis of the Turbulent Prandtl Number Data ..... 118
7.5. Uncertainty Intervals ..... 120
8. SUMMARY AND CONCLUSIONS ..... 130
REFERENCES ..... 133
Table of Contents (cont.)Page
Appendix A. Analysis of a Resistance Thermometer Response to Mean and Fluctuating Temperature ..... 139
Appendix B. The Measurement of Turbulence Quantities ..... 149
Appendix C. The Measurement of Shear Stress in Completely Developed Rectangular Channel Flow ..... 155
Appendix D. The Direct Measurement of the Turbulent Kinetic
Energy and the Shear Stress by Means of a Single Slanted, Rotatable Hot Wire Probe . . . . 157
Appendix E. Tabulations of Experimental Data ..... 160
List of Figures
Figure Chapter 33.1 Schematic of the test apparatus42
3.2 Photograph of the test section with a traversing mechanism in position ..... 43
3.3 Spacing of the plate thermocouples ..... 43
3.4 A longitudinal cross section of the tunnel test section ..... 44
3.5 Cross sectional view of a typical compartment ..... 44
3.6 The horizontal hot-wire probe ..... 45
3.7 The rotatable hot-wire probe ..... 46
3.8 The horizontal hot-wire calibration for different wire temperatures ..... 47
3.9 The hot-wire sensitivity to velocity ..... 48
3.10 Hot-wire measurements of velocity on a flat plate: cold wall vs. hot wall ..... 49
3.11 Temperature measurements on a flat plate: thermocouple vs. resistance thermometer ..... 50
3.12 Channel flow shear stress measurements: checking the hot-wire system ..... 51
Chapter 4
4.1 Stanton number vs. enthalpy thickness Reynolds number -- strong adverse pressure gradient and flat plate values ..... 58
4.2 The ratio between the actual Stanton number and its value with no transpiration at the same enthalpy thickness Reynolds number ..... 59
4.3 Wall-temperature distributions for the variable tempera- ture test cases $(-0.15,0)$ and $(-0.275,-0.004)$ ..... 60
4.4 Comparison of measured and predicted Stanton numbers for the variable wall temperature situations ..... 61
Chapter 5
5.1 Defect velocity profiles at different x-stations$(-0.275,0)$71
5.2 Defect temperature profiles at different $x$-stations $(-0.275,0)$ ..... 72
5.3 Defect velocity profiles at different x-stations ( $-0.275,-0.001$ ) ..... 73
List of Figures (cont.)
5.4 Defect temperature profiles at different x-stations (-0.275, -0.001) ..... 74
5.5 Defect velocity profiles at different x-stations (-0.275, -0.002) ..... 75
5.6 Defect temperature profiles at different x-stations (-0.275, -0.002) ..... 76
5.7 Defect velocity profiles at different x-stations (-0.275, -0.004) ..... 77
5.8 Defect temperature profiles at different x-stations (-0.275, -0.004) ..... 78
5.9 Clauser shape factors for velocity as a function of $x$ for different suction rates ..... 79
5.10 Temperature shape factors as a function of $x$ for different suction rates ..... 80
5.11 Friction factor as a function of momentum thickness Rey- nolds number for strong adverse pressure gradient with suction ..... 81
Chapter 6
6.1 Axial velocity fluctuation profiles -- comparison with the data of Klebanoff ..... 93
6.2 Axial velocity fluctuation proftles for strong adverse pressure gradient flows with different suction rates ..... 94
6.3 Normal velocity fluctuation profiles for strong adverse pressure gradient flows with different suction rates ..... 95
6.4 Transverse velocity fluctuation profiles for strong adverse pressure gradients with different suction rates ..... 96
6.5 Turbulent shear stress profiles for strong adverse pressure gradient with different suction rates ..... 97
6.6 Correlation coefficients between the longitudinal and normal velocities -- mild adverse pressure gradient ..... 98
6.7 Correlation coefficients between the longitudinal and normal velocities -- strong adverse pressure gradient ..... 99
6.8 The ratio between turbulent shear stress and turbulent kinetic energy -- mild adverse pressure gradient flows ..... 100
6.9 The ratio between turbulent shear stress and turbulent kinetic energy -- strong adverse pressure gradient flows ..... 101

|  | List of Figures (cont.) |  |
| :---: | :---: | :---: |
|  |  | Page |
| 6.10 | Flat plate measurements of temperature fluctuations -comparison with the data of Fulachier and Dumas . . . | 102 |
| 6.11 | The temperature fluctuation profile compared with the longitudinal velocity fluctuation profile -flat plate data | 103 |
| 6.12 | Measured $\overline{v^{\top} t^{\top}}$ profiles normalized on $u_{\tau} T^{--}$flat plate | 104 |
| 6.13 | The temperature fluctuation profiles in a strong adverse pressure gradient with different suction rates | 105 |
| 6.14 | The temperature-normal velocity correlation for a strong adverse pressure gradient with different suction rates | 106 |
| 6.15 | The temperature-normal velocity correlation coefficient for a strong adverse pressure gradient with different suction rates | 107 |
|  | Chapter 7 |  |
| 7.1 | The turbulent Prandtl number distribution in a flat plate boundary layer ( 0,0 ) | 122 |
| 7.2 | The turbulent Prandtl number distribution in an adverse pressure gradient $(-0.275,0)$ | 123 |
| 7.3 | The turbulent Prandtl number distribution in an adverse pressure gradient with mild suction ( $-0.275,-0.001$ ) | 124 |
| 7.4 | The turbulent Prandtl number distribution in an adverse pressure gradient with suction ( $-0.275,-0.002$ ) . . | 125 |
| 7.5 | The turbulent Prandtl number distribution in an adverse pressure gradient with strong suction (-0.275, -0.004) | 126 |
| 7.6 | The ratio $\partial T / \partial U$ within boundary layers subject to strong adverse pressure gradient with different suction rates | 127 |
| 7.7 | Turbulent Prandtl number as a function of $\mathrm{y}^{+}$; strong adverse pressure gradients with suction | 128 |
| 7.8 | Turbulent Prandtl number as a function of the turbulent Peclet number; strong adverse pressure gradients with suction | 129 |
|  | Appendix A |  |
| A. 1 | Gold-plated tungsten wire $\left(\mathrm{D}_{\mathrm{G}} / \mathrm{O}_{\mathrm{T}}=6\right)$. . . . . . . . . . | 139 |
| A. 2 | Analysis of the wire . . . . . . . . . . . . . . . . . | 141 |
| A. 3 | Flat plate temperature fluctuations: comparison between <br> the $1 \mu \mathrm{~m}$ and the $5 \mu \mathrm{~m}$ wires | 148 |

List of Figures (cont.)
Appendix B Page
B. 1 The geometry and coordinates of the slant wire ..... 150
Appendix C
C. 1 Channel flow and the system of coordinates ..... 155
Table Page
2.2.1 Summary of the Boundary Conditions Used ..... 9
2.3.1 Nominal Pressure Gradient and Transpiration Rates Studied with a Step in Wall Temperature Applied with a Step in Wall Temperature Applied at P1ate 10 (38 inches) ..... 10
A-1 Values of $v$ as a Function of Velocity for the DISA 55F02 $5 \mu \mathrm{~m}$ Tungsten Probe and DISA 55F01 $1 \mu \mathrm{~m}$ Platinum Probe ..... 145
$\mathrm{E}-1$ Friction Factor Data ..... 160
E-2 Experimental Stanton Number at a Constant Wall Temper- ature Condition ..... 162
E-3 Experimental Stanton Numbers: Step in Wall Temperature Condition ..... 165
E-4 Mean Temperature and Velocity Profiles ..... 170
E-5 Reynolds Stress Tensor Components (Isothermal) ..... 195
E-6 Velocity and Temperature Fluctuation Profiles ..... 199
E-7 Turbulent Prandt1 Numbers ..... 203

NOMENCLATURE
A Van Driest length scale.
$A^{+} \quad$ Van Driest length scale (dimensionless) $=A u_{\tau} / V$
$A_{G} \quad$ Cross section area of gold plated portion of wire.
$A_{T} \quad$ Cross section area of tungsten wire.
B Heat transfer length scale
$\mathrm{B}^{+} \quad$ Heat transfer length scale $=\mathrm{Bu}_{\tau} / \nu$.
$B_{f} \quad$ Blowing parameter $=F /\left(C_{f} / 2\right)=\rho_{\infty} v_{o} U_{\infty} / \tau_{o}$.
$B_{h} \quad$ Heat transfer blowing parameter $=F / S t$
$C_{p} \quad$ Specific heat at constant pressure
$C_{f} / 2$ Friction coefficient $\tau_{0} / \rho_{\infty} U_{\infty}^{2}$.
D Diameter of wire.
$D_{G} \quad$ Diameter of the gold plated portion of the wire
$\mathrm{D}_{\mathrm{T}} \quad$ Diameter of the tungsten wire.
E Output from anemometer.
$e^{\prime} \quad$ Fluctuating value, output from anemometer.
$\mathrm{F} \quad$ Blowing fraction $=\mathrm{v}_{\mathrm{o}} / \mathrm{U}_{\infty}$.
G Clauser shape factor, Eq. 2.4.
$\mathrm{G}_{\mathrm{h}} \quad \Delta_{4} / \Delta_{3}$.
$h$ Heat transfer coefficient.
$K$ Karman constant.
$\mathrm{K} \quad$ Acceleration parameter $=\left(v / \mathrm{U}_{\infty}^{2}\right) \times\left(\mathrm{d} \mathrm{U}_{\infty} / \mathrm{dx}\right)$.
$\mathrm{K}_{\mathrm{T}} \quad$ Thermal conductivity of tungsten wire.
$K_{G} \quad$ Thermal conductivity of gold.
L Length of wire
\& Unheated length; also mixing-length.
P Static pressure.
Pr Molecular Prandtl number.
$P_{o} \quad$ Free stream static pressure
$P^{+} \quad v / \rho_{\infty} U_{\tau}^{3} \times \mathrm{d} \rho / \mathrm{dx}$.
$\mathrm{Pr}_{\mathrm{t}} \quad$ Turbulent Prandt1 number, $\varepsilon_{\mathrm{M}} / \varepsilon_{\mathrm{H}}$.
$\dot{q}_{0}^{\prime \prime} \quad$ Heat flux at the wall.
$\overline{q^{2}} \quad$ Turbulent kinetic energy $=\overline{u^{\prime 2}}+\overline{v^{\prime 2}}+\overline{w^{\prime 2}}$.
$\operatorname{Re}_{\Delta_{\mathrm{T}}} \quad$ Enthalpy thickness Reynolds number $=U_{\infty} \Delta_{\mathrm{T}} / v$.

| R | Wire resistance. |
| :---: | :---: |
| $\mathrm{R}_{\mathrm{G}}$ | Resistance of gold plated part of the wire. |
| $\mathrm{R}_{T}$ | Resistance of tungsten wire. |
| $\mathrm{R}_{\mathrm{W}}$ | Average resistance of wire. |
| St | Stanton number $=\dot{q}_{0}^{\prime \prime} / \rho_{\infty} \mathrm{U}_{\infty}\left(\mathrm{T}_{W}-\mathrm{T}_{\infty}\right) c_{p}$. |
| $S t_{0}$ | Stanton number, no transpiration. |
| St ${ }_{\text {T }}$ | Stanton number, isothermal plate. |
| $\mathrm{St}_{\mathrm{o}_{\mathrm{T}}}$ | Stanton number, isothermal plate, no transpiration. |
| $\mathrm{T}_{\mathrm{W}}$ | Wire temperature, wall temperature. |
| T | Mean local temperature. |
| $\mathrm{T}_{\infty}$ | Free stream temperature. |
| $\mathrm{T}^{+}$ | $\overline{\mathrm{T}} \sqrt{\mathrm{C}_{\mathrm{f}} / 2} / \mathrm{st}$ |
| T | $\left(\mathrm{T}-\mathrm{T}_{\mathrm{W}}\right) /\left(\mathrm{T}_{\infty}-\mathrm{T}_{W}\right)$. |
| $\mathrm{T}_{\tau}$ | $\left(\mathrm{T}_{\mathrm{o}}-\mathrm{T}_{\infty}\right) \mathrm{St} / \sqrt{\mathrm{C}_{\mathrm{f}} / 2}$. |
| $t^{\prime}$ | F1uctuating temperature. |
| $t_{\infty}^{\prime}$ | Fluctuating free stream temperature. |
| $t_{m}^{\prime}$ | F1uctuating average wire temperature. |
| $\mathrm{T}_{\mathrm{m}}$ | Average wire temperature. |
| t | Time. |
| u | Local velocity, instantaneous value. |
| $\overline{\mathrm{u}}$ | Local velocity, average value. |
| $\mathrm{U}_{\infty}$ | Free stream velocity. |
| ${ }_{\text {u }}$ | Friction velocity $=\mathrm{U}_{\infty} \sqrt{\mathrm{C}_{\mathrm{f}} / 2}$. |
| $u^{\prime}$ | Fluctuating longitudinal velocity. |
| $\overline{u^{\prime} v^{\prime}}$ | Turbulent shear stress. |
| $\begin{aligned} & U_{\text {eff }} \\ & u^{+} \end{aligned}$ | Effective velocity (the velocity the wire "sees"). $\mathrm{u} / \mathrm{u}_{\tau}$. |
| $\overline{u^{\prime} t^{\top}}$ | Longitudinal velocity-temperature correlation. |
| $\overline{u^{\top} w^{\top}}$ | Longitudinal-tangential velocity correlation. |
| $U^{\prime}$ eff | Fluctuating effective velocity. |


| $v$ | Normal velocity, instantaneous value. |
| :---: | :---: |
| v | Normal velocity, average value. |
| $v_{0}$ | Normal velocity at the wall. |
| $\mathrm{v}^{\prime}$ | Fluctuating normal velocity. |
| $\overline{v^{\prime} t^{\prime}}$ | Normal velocity-temperature correlation. |
| $\mathrm{v}^{+}$ | $\mathrm{v}_{0} / \mathrm{u}_{\tau}$. |
| $\overline{v^{\prime} w^{\prime}}$ | Normal-tangential velocity correlation. |
| x | Stream wise coordinate. |
| $\mathrm{x}_{0}$ | Virtual origin of momentum boundary layer. |
| w | Tangential velocity, instantaneous value. |
| w | Tangential velocity, average value. |
| $w^{\prime}$ | Fluctuating tangential velocity. |
| $\overline{w^{\prime} t^{\prime}}$ | Tangential velocity-temperature correlation. |
| y | Normal coordinate. |
| $\mathrm{y}^{+}$ | $\mathrm{yu}_{\tau} / \nu$. |
| $z$ | Transverse coordinate. |
| Greek |  |
| $\alpha$ | Thermal diffusivity $=k / \rho c_{p}$ |
| $\beta$ | Eq. 2.5 . |
| $\Delta$ | Clauser boundary layer thickness, Eq. 2.2. |
| $\Delta_{1}$ | $\text { Enthalpy thickness }=\int_{0}^{\infty} \frac{\rho \bar{u}}{\rho_{\infty} U_{\infty}} \frac{T-T_{\infty}}{T_{W}-T_{\infty}} d y .$ |
| $\Delta_{3}$ | Eq. 2.8 . |
| $\Delta_{4}$ | Eq. 2.8 |
| $\delta$ | Momentum boundary layer thickness $\overline{\mathbf{u}}(\delta)=0.99 \mathrm{U}_{\infty}$. |
| $\delta_{T}$ | Thermal boundary layer thickness $\left(T-T_{w}\right) /\left(\mathrm{T}_{\infty}-\mathrm{T}_{\mathrm{w}}\right)=0.99$. |
| $\delta_{1}$ | $\text { Displacement thickness }=\int_{0}^{\infty}\left(1-\frac{\overline{\mathrm{u}}}{\mathrm{U}_{\infty}}\right) \mathrm{dy} .$ |
| $\delta_{2}$ | Momentum thickness $=\int_{0}^{\infty} \frac{\rho \bar{u}}{\rho_{\infty} U_{\infty}}\left(1-\frac{\bar{u}}{U_{\infty}}\right) d y$. |
| $\varepsilon_{M}$ | Eddy diffusivity for momentum $=-\overline{u^{\prime} v^{\top}} /(\partial \bar{u} / \partial y)$ |
| $\varepsilon_{\text {H }}$ | Eddy diffusivity for heat $=-v^{\prime} t^{\prime} /(\partial \mathrm{T} / \partial \mathrm{y})$. |

$u \quad$ Dynamic viscosity.
$v$. Kinematic viscosity $=\mu / \rho$.
$\rho \quad$ Local density.
$\rho_{\infty} \quad$ Free stream density.

## INTRODUCTION AND OBJECTIVES

It is presently not possible to predict, with the desired accuracy, boundary layers subjected to arbitrary variations of free stream velocity, wa11 temperature and transpiration. Much progress has been made in the past few years, with the increasing use of finite difference computer programs capable of predicting the velocity and temperature distributions inside the boundary layer. These programs require, for closure, a structural model of the turbulent mixing process and some function relating the transport of energy to the transport of momentum. One currently successful approach uses a mixing length model for turbulence and a turbulent Prandtl number function. Others use the turbulent kinetic energy to predict the mixing, still others deal directly with the effective "stresses" induced by the turbulence. These models can most easily be developed by examination of large bodies of coherent data: data taken by following a "rational path" through the wide field of possible combinations of velocity, transpiration and wall temperature. Popular paths include "constant velocity flows", "equilibrium decelerations", and "asymptotic accelerations". Transpiration is adjusted to provide either "uniform blowing parameter" or "uniform blowing velocity" along the surface. From such data sets are drawn the values of the constants used in the predictive programs. The test of the validity of the programs comes from attempting to predict results not included within the data sets which were used in evaluating the constants: runs with steps in wall temperature, non-equilibrium decelerations, and arbitrary variations in transpiration.

The Heat and Mass Transfer Group at Stanford has been engaged for some time in studies of the momentum and heat transfer behavior of turbulent boundary layers, aimed at generating data in both classes: predictive and "test cases". Moffat [1] built the experimental facility to allow measurements in two-dimensional boundary layers and obtained measurements of Stanton numbers for zero pressure gradient flows with
both blowing and suction within the limits of boundary layer behavior. Simpson [2] studied the hydrodynamic behavior of zero pressure gradient flows and Whitten [3] the heat transfer behavior for variable transpiration and wall temperature conditions. Julien [4] and Thielbahr [6] investigated moderate accelerations (asymptotic boundary layer flows) and Loyd [5] and Kearney [7] studied strong accelerations. Most recently, attention was turned to decelerating flows with Andersen [8] and Blackwell [9] dealing with mild adverse pressure gradients. From all of this work has come a wealth of data for developing operational models for the turbulent transport of heat and momentum. Incorporated into a finite difference computer program based on the Spalding-Patankar method, these models have been refined by challenges from other data from pipe flows, and from rocket nozzle data, mostly from accelerating or, at most, mildy decelerating flows.

The area of adverse pressure gradients with transpiration has not been investigated previously except for one study by McLean [57]. McLean investigated strong adverse pressure gradients with blowing, with primary emphasis on the onset of separation. The free stream velocity in these experiments was decreased linearly in the streamwise direction. Velocity profiles were measured with pitot probes and skin friction determined using Stevenson's [58] law of the wall. Stanton numbers were reported, but no temperature profiles. Such a data set is useful as a check on the ability of a model to predict Stanton numbers but the absence of temperature profiles makes it difficult to use the data to refine the model. Stanton number data alone are not sufficient to resolve problems with most models. Since finite difference prediction schemes predict mean velocity and temperature profiles, such data are needed to confirm these predictions. If prediction schemes using more than simple mixing length theory for closure are to be developed, then additional information about the turbulent structure of the boundary layer is required to develop the new models.

The present study has three main objectives related to the general problem of predicting turbulent boundary layer heat transfer: (1) extension of the body of hydrodynamic and heat transfer data into the area of strong decelerations, (2) development of a model for predicting
the effect of the adverse pressure gradient on the turbulent Prandtl number in the near wall region, and (3) development of an improved experimental technique for studying temperature and velocity fluctuations In the boundary layer.

Particular emphasis was placed upon the development of new experimental techniques for evaluating the turbulent transports of momentum and energy. It appears likely that the "next generation" of turbulence models will lean heavily upon the stationary statistical properties of the turbulence and such data are difficult to acquire. The "present generation" has great need of more rellable data regarding turbulent Prandtl number in the near wall region.

The intermediate steps for carrying out this program are as follows:
(1) Provide Stanton number data for constant wall and step in wall temperature conditions, for a nearly equilibrium flow behavior, that would be used as a reliable source of comparisons for turbulent boundary layer prediction methods.
(2) Develop a model that would allow one to predict the Stanton number under different free stream adverse pressure gradient and transpiration rate conditions, for an arbitrary variation of the wall temperature.
(3) Develop the technique to allow the simultaneous measurement of mean temperature and velocity, with just one probe.
(4) Provide data on mean temperature and velocity for flows under strong free stream adverse pressure gradient and transpiration rate conditions.
(5) Provide data and the analysis of the hydrodynamic turbulence structure of nearly equilibrium decelerating flows.
(6) Provide data and the analysis of the direct measurement of the turbulent heat transfer and temperature fluctuation for nearly equilibrium flow conditions.
(7) Develop a measurement procedure to allow the turbulent Prandt1 number at the wall to be estimated and investigate its behavior in the neighborhood of the wall.
(8) Provide the turbulent Prandtl number data from direct measurement of the turbulent transport of momentum and heat.

CHAPTER 2

## EXPERIMENTAL BOUNDARY CONDITIONS

The objective of the experimental part of this investigation was to examine the mechanisms of the transport of momentum and energy in a transpired turbulent boundary layer under strong adverse pressure gradient conditions, including some effects of an arbitrary wall temperature distribution.

The experimental program was carried out on the Stanford Heat and Mass Transfer apparatus, which has been modified by Andersen [8] and Blackwell [9] to permit the establishment and accurate control of an adverse pressure gradient. The wall temperature control, however, is the same as used by Moffat [1].

This investigation was limited to low-speed, nearly constant property flows, with the transpiration fluid the same as the free stream (air). Hydrodynamic boundary conditions leading to strong deviations from "equilibrium" (for example, steps in the pressure gradient or in the transpiration rate) were not considered, nor were adverse pressure gradient conditions strong enough to cause separation. The energy transport portion of the problem was studied, in part, by imposing strong departures from "thermal equilibrium": steps in wall temperature were applied and used to generate the kernel function for superposition solutions to the arbitrary wall temperature distribution problem. Only heat transfer coefficients were measured for this non-equilibrium condition -no profiles. Extensive profile measurements, both mean and fluctuating, were taken for the constant wall temperature conditions. The wall-to-free-stream temperature difference used in the measurements was small $\left(20-30^{\circ} \mathrm{F}\right)$ and, therefore, nearly constant property flows were obtained. The maximum free-stream-to-wall density ratio was about 1.04.

### 2.1 The Concept of Equilibrium Flows

Because the temperature field depends on the velocity field, it is natural to analyze the momentum boundary layer first when attention is given to the estab1ishment of an equilibrium momentum and thermal boundary layer.

The outer $90 \%$ region of the turbulent boundary layer is known to react more slowly than the inner wall region to changes in the boundary conditions. In fact, the wall region may be considered to be always in equilibrium, in the sense that only local values of the pressure gradient and transpiration rate are important. The outer region, on the other hand, shows a pronounced history effect. Information from upstream stations is important in determining the hydrodynamic behavior in the outer region.

The constant pressure layer has been known, for a long time, to possess both inner and outer region similarity, respectively known as the "law of the wall" and the "velocity defect law".

The velocity defect, $-\left(\mathrm{U}_{\infty}-\overline{\mathrm{u}}\right) / \mathrm{u}_{\tau}$, is a similarity variable for the outer region of the constant pressure boundary layer if plotted against $y / \delta$. The outer $\simeq 90 \%$ of the boundary layer has a unique shape (independent of the Reynolds number) when plotted this way. Thus,

$$
\begin{equation*}
-\frac{u_{\infty}-\bar{u}}{u_{\tau}}=F\left(\frac{y}{\delta}\right) \tag{2.1}
\end{equation*}
$$

represents a defect law for the zero pressure gradient boundary layer ( $\delta$ is the boundary layer thickness).

Clauser [27], in 1954, considered the problem of similarity in turbulent boundary layers. His idea was to extend the concept of similarity to include turbulent boundary layers in adverse pressure gradient. He succeeded in experimentally creating adverse pressure gradient boundary layers with a defect similarity just as in the case of the zero pressure gradient boundary layer -- "equilibrium boundary layers".

Clauser defined a new boundary layer thickness, $\Delta$, such that

$$
\begin{equation*}
\Delta=\delta \int_{0}^{1} \frac{\mathrm{U}_{\infty}-\bar{u}}{u_{\tau}} d\left(\frac{y}{\delta}\right) \tag{2.2}
\end{equation*}
$$

When the boundary layer has outer region similarity, the Clauser thickness $\Delta$ is a constant factor times $\delta$, the boundary layer thickness, which is difficult to measure. Because of the fact that $\Delta$ can be more precisely determined from experimental data than $\delta$, the defect law will be written as

$$
\begin{equation*}
-\frac{\mathrm{U}_{\infty}-\overline{\mathrm{u}}}{\mathrm{u}_{\tau}}=F\left(\frac{y}{\Delta}\right) \tag{2.3}
\end{equation*}
$$

Clauser then defined a shape factor for equilibrium velocity profiles as

$$
\begin{equation*}
G=\frac{\delta}{\Delta} \int_{0}^{I}\left[\frac{U_{\infty}-\bar{u}}{u_{\tau}}\right]^{2} d\left(\frac{y}{\delta}\right) \tag{2.4}
\end{equation*}
$$

and reasoned that in such flows the ratio of the wall shear force and the pressure force acting on the boundary layer is constant. This condition implies that

$$
\begin{equation*}
\beta=\frac{\delta_{1}}{\tau_{0}} \frac{d p}{d x}=\text { constant } \tag{2.5}
\end{equation*}
$$

Bradshaw [28] found that such an adverse equilibrium pressure gradient flow corresponds, experimentally, to a decreasing free-stream velocity in which $U_{\infty} \propto x^{m} \quad(m<0)$.

Anderson [8] extended the concept of equilibrium flows to include transpiration based on the momentum integral equation, written in Bradshaw's form as

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{dx}}\left(\delta_{2} \mathrm{U}_{\infty}^{2}\right)=\frac{\tau_{0}}{\rho}\left(1+\beta+\mathrm{B}_{\mathrm{f}}\right) \tag{2.6}
\end{equation*}
$$

The conditions to give equilibrium flows are constant $\beta$ and constant $B_{f}$, where $B_{f}=2 F / C_{f}$ (the Blowing Parameter).

The constant $\beta$ condition is satisfied by setting the free-stream velocity so that $U_{\infty} \propto x x^{m}$. The constant $B_{f}$ condition is satisfied by setting $F \propto x^{m_{f}}$, where $F=\rho_{o} v_{o} / \rho_{\infty} U_{\infty}$ and $m_{f}$ is dependent on $m$. Boundary layers in flows adjusted in this way have been experimentally shown to display similarity (Anderson [8]).

Blackwell [9] approached the problem of equilibrium thermal boundary layers by defining a defect enthalpy law which can be written, for a constant properties flow, as:

$$
\begin{equation*}
T_{\infty}^{+}-T^{+}=f_{2}\left(y / \delta_{T}\right), \tag{2.7}
\end{equation*}
$$

where $\delta_{T}$ is the thermal boundary layer thickness. Following the same line as C1auser [27], the thickness parameters $\Delta_{3}$ and $\Delta_{4}$ were defined

$$
\begin{align*}
& \Delta_{3}=\delta_{T} \int_{0}^{1}\left(\mathrm{~T}_{\infty}^{+}-\mathrm{T}^{+}\right) \mathrm{d}\left(\mathrm{y} / \delta_{\mathrm{T}}\right) \\
& \Delta_{4}=\delta_{\mathrm{T}} \int_{0}^{1}\left(\mathrm{~T}_{\infty}^{+}-\mathrm{T}^{+}\right)^{2} \mathrm{~d}\left(\mathrm{y} / \delta_{\mathrm{T}}\right) \tag{2.8}
\end{align*}
$$

When the thermal boundary layer has an outer region similarity, the thickness $\Delta_{3}$ is a constant times $\delta_{T}$, which is as hard to measure as was $\delta$. As in the case of momentum boundary layers, $\Delta_{3}$ can be experimentally determined more accurately, and the defect enthalpy law will be written as

$$
\begin{equation*}
\mathrm{T}_{\infty}^{+}-\mathrm{T}^{+}=\mathrm{F}_{2}\left(\mathrm{y} / \Delta_{3}\right) \tag{2.9}
\end{equation*}
$$

The defect enthalpy profile shape factor is then defined as

$$
G_{h}=\frac{\Delta_{4}}{\Delta_{3}}
$$

If outer-1ayer similarity exists, then $G_{h}$ is approximately constant.
Blackwell [9] also investigated conditions leading to equilibrium thermal layers. The energy integral equation for two-dimensional incompressible flows can be written in terms of the enthalpy thickness, $\Delta_{\mathrm{T}}$, as:

$$
\begin{equation*}
\frac{d}{d x}\left(\rho_{\infty} U_{\infty} c_{p}\left(T_{w}-T_{\infty}\right) \Delta_{T}\right)=\dot{q}_{o}^{\prime \prime}\left(1+B_{h}\right), \tag{2.10}
\end{equation*}
$$

where $B_{h}=F / S t$, the heat transfer parameter.

A necessary condition for thermal equilibrium is a constant $B_{h}$, which can be achieved, as shown by Blackwell [9], by setting $U_{\infty} \propto x^{m}$ and $F \propto x^{m_{f}}$.
2.2 Boundary Conditions on $U_{\infty}$ and $F$ for the Structural Studies Two different velocity distributions and four different transpiration rates were used in the studies of the turbulence structure. Taking the free-stream velocity variations to be given as

$$
\begin{equation*}
\mathrm{U}_{\infty}=\bar{u}_{1}\left(\frac{\mathrm{x}-\mathrm{x}_{\mathrm{o}}}{\mathrm{x}_{1}-\mathrm{x}_{\mathrm{o}}}\right)^{m} \tag{2.11}
\end{equation*}
$$

and

$$
\begin{equation*}
F=F_{1}\left(\frac{x-x_{0}}{x_{1}-x_{0}}\right)^{m_{F}} \tag{2.12}
\end{equation*}
$$

the flows can be summarized as in Table 2.2.1.

Table 2.2.1
Summary of the Boundary Conditions Used

| m | $\bar{u}_{1} \mathrm{ft} / \mathrm{sec}$ | $\mathrm{F}_{1}$ | $\mathrm{~m}_{\mathrm{F}}$ | $\mathrm{x}_{\mathrm{o}}(\mathrm{in})$ | $\mathrm{x}_{1}(\mathrm{in})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.000 | 31.5 | 0.00 | 0.00 | - | - |
| -0.275 | 30.1 | 0.000 | 0.000 | -3 | -4 |
|  |  | -0.001 | 0.000 |  |  |
|  |  | -0.002 | 0.000 |  |  |
|  |  | -0.004 | 0.000 |  |  |

Only two flows ( $m=0, F=0$ and $m=-0.275, F=0$ ) are truly in hydrodynamic equilibrium. Constant $F$ flows such as these are not truly equilibrium flows but tend to behave very well, since ${ }_{B_{f}}$ changes so slowly along the surface that a "local equilibrium" seems to prevail (Whitten [ 3]).
2.3 Boundary Conditions on $T$ wall and $F$ for the Step Wall Temperature

Studies
Mean temperature, velocity, and turbulent profiles were taken for constant wall temperature conditions for each of the previously tabulated sets of conditions. A separate series of conditions was used for the step wall temperature tests in which the step in wall temperature was fixed at plate 10 ( 38 inches downstream).

Several cases were examined, as indicated by Table 2.3.1.

Table 2.3.1
Nominal Pressure Gradient and Transpiration Rates Studied with a Step in Wall Temperature Applied at Plate 10
(38 inches)

| m | $\mathrm{F}_{1}$ | $\mathrm{~m}_{\mathrm{f}}$ |
| :---: | ---: | :---: |
| 0.00 | 0.000 | - |
|  | 0.000 |  |
| -0.15 | +0.001 | 0.00 |
|  | +0.002 |  |
|  | +0.004 |  |
|  | 0.000 |  |
| -0.275 | -0.001 | 0.000 |
|  | -0.002 |  |
|  | -0.004 |  |

Two additional cases ( $m=-0.15, F_{1}=0 ; m=-0.275, F_{1}=0.004$ ) were run with an arbitrary wall temperature distribution to test predictive capability, using the information obtained from the step wall temperature cases.

### 2.4 Identification of the Boundary Conditions

From the previous sections, it is obvious that the three parameters ( $m, F_{1}, m_{F}$ ) nominally describe the pressure gradient and transpiration boundary condition. Therefore, each run will be identified by these three parameters. The designation of $m_{F}$, however, will be omitted in these runs because no constant $B_{F}$ flows were examined. The run designation $(-0.275,-0.004)$ will indicate $U_{\infty} \sim x^{-0.275}, F_{1}=-0.004$.

## EXPERIMENTAL APPARATUS, INSTRUMENTATION AND QUALIFICATION TESTS

The basic apparatus used in this study was an open loop wind tunnel first described by Moffat [1]. It was originally designed for zero pressure gradient studies of the transpired turbulent boundary layer, and the description of subsequent modifications can be found in References [2] - [9]. The most relevant modification consisted of a new design for the upper surface of the test channel for the study of decelerating flows, as described by Andersen [8] and B1ackwell [9]. Only a brief description will be given below, references being made to Fig. 3.1.

### 3.1 Main Air Flow

The main air flow enters the filter box, made of a 0.7 micron retention felt-type material. The main air blower is of the centrifugal type and has a 2000 scfm capacity at 30 inches of water. The flow turning header was designed to provide a uniform velocity across the face of the heat exchanger. Honeycomb is placed both upstream and downstream of the heat exchanger which is supplied with cooling water from the Stanford University water system. Since this heat exchanger has a very high effectiveness, any fluctuations in cooling water temperature would cause fluctuations in the test section air temperature. It was found that during evenings and weekends, the drift in mainstream temperature was less than $0.1^{\circ} \mathrm{F}$ per hour. For the measurement of turbulent quantities, a four hour period was usually required and a drift of $0.3^{\circ} \mathrm{F}$ at most was observed, although some measurements were taken with only a $0.1^{\circ} \mathrm{F}$ drift.

After leaving the heat exchanger, the main flow air passes through a 1.5 in. thick honeycomb with $3 / 16$ in. cell size. Following the honeycomb and located in a constant area section $23 \times 23$ inches there are six $32 \times 32$ mesh stainless steel screens. The function of this set of screens is to reduce spanwise non-uniformities in dynamic pressure. It is well known that even a slight crease in a screen can considerably affect the
uniformity of the flow through the screen, so these screens received careful attention.

Following the screen pack, the flow enters a 4:1 contraction (over 26 inches) nozzle which provides an almost two dimensional contraction to the $6 \times 20$ inch outlet. The nozzle is symmetric about both the vertical and the horizontal planes, with the contraction taking place primarily in the vertical dimension. Andersen [8] and Blackwell [9] found a separation bubble in the inlet of the nozzle, and the nozzle shape was modified to provide an initial contracting angle of approximately $5^{\circ}$ downstream of the last screen. This eliminated the separation.

At the exit of the nozzle, a $3 / 16$ inch wide slot was cut through the bottom and the side walls of the nozzle. Since the static pressure in the tunnel is slightly above atmospheric, this acts as a suction slot and removes the boundary layer that develops in the nozzle. To insure a fully turbulent profile at the first test plate, a $1 / 32$ inch high by $1 / 4$ inch wide boundary layer trip was located upstream of the first test plate. There were no trips on the side or top walls.

### 3.2 Test Section

The test section consists of a $6 \times 20$ inch rectangular cross section duct 8 feet in length. The side walls are $1 / 2$ inch plexiglass, the top is $5 / 16$ inch aluminum tool plate, and the bottom wall (the actual test surface) is porous sintered bronze.

One of the side walls contains static pressure taps, which are used in conjunction with Kiel probes for free stream velocity determination. The side wall static pressure taps are 0.040 inch diameter with sharp edges and are spaced two inches apart in the streamwise direction. The distance above the bottom wall test surface is 1 inch. Every 12 inches in the streamwise direction four additional pressure taps are provided at $2,3,4$, and 5 inches above the test surface, on each side wall. The function of these vertical columns of pressure taps is to check the static pressure uniformity in the vertical and transverse directions. The side-to-side static pressure variation was found to be less than 0.002 inches of water. The pressure variation in the vertical direction
was less than 0.001 inches of water, which is also the accuracy of the pressure measuring system. The static pressure taps are evident in Fig. 3.2.

The bottom wall of the test section consists of 24 individual porous plates mounted in four separate aluminum base castings. Each plate is thermally isolated from the base casting and its neighboring plates. The physical characteristics of the plates are as follows:

Material - sintered bronze
Dimensions - $18.0 \times 3.975 \times 0.25$ inches
Particles - spherical, varying in diameter in the range $0.002-0.007$ inches

Porosity - approximately $40 \%$, uniform within $\pm 6 \%$ in center 6 -inch section

Roughness - maximum of 200 microinches (RMS) measured with a stylus of 0.0005 inch radius

Thermal Conductivity - $6.5 \mathrm{Btu} / \mathrm{hr}-\mathrm{ft}-{ }^{\circ} \mathrm{F}$, minimum
Surface Emittance - 0.37 average

Plate temperatures are monitored by five iron/constantan thermocouples, each located 0.040 inches below the surface. The spacing of the plate thermocouples is shown in Fig. 3.3. Each plate is electrically heated by nichrome wires located in grooves on the underside of the plate. The power supplied to each plate is individually controlled by a rheostat. This allows one to vary the power to each plate individually to maintain a uniform surface temperature.

The upper wall of the test section is used to control the pressure gradient. It consists of a series of 24 aluminum plates, each $5 / 6$ inch thick by 3.5 inches wide. These plates are arranged on top of the side walls such that there is a $1 / 2$ inch space (in the flow direction) between adjacent plates. Each of the 23 resulting $1 / 2$ inch wide slots is partially covered by two $1 / 2$ inch thick aluminum bars. One of these bars is permanently fixed to the 3.5 inch plate, while the other bar is allowed to move relative to the fixed bar. The result is to form a slot whose
width can be varied between 0 and 0.4 inches. A side view of the tunnel with the slot arrangement is shown in Fig. 3.4. The slot width can be set to the desired width within 0.001 inches. Wing nuts are used to keep the movable bar in place once the slot width has been set.

The test section is extended 14 inches past the last heated plate by a projecting shelf which plays no role in the experiment other than to present "clean" flow on the last plate. A movable gate slides up and down vertically across this extension piece. This gate, plus the throttle valve at the main stream air blower are used to control the main stream air flow rate. The function of this extension is to insure that the influence of the sliding gate valve is not appreciably felt at the last test station.

### 3.3 Transpiration Air System

The transpiration air system is quite similar to the main air system. The main difference is that after leaving the transpiration blower the air goes to a header and then to 24 individually calibrated pairs of rotameters. The air supplied to a given plate can be routed to one of two rotameters, depending on the magnitude of the flow rate. By using the two rotameters in parallel, flow rates in the range of 0.5 to 18 scfm can be measured. Each of these 48 rotameters was individually calibrated by Kearney [7]. A typical transpiration compartment and plate assembly is shown in Fig. 3.5 .

### 3.4 Pressure Measurement

All pressure measurements were made with a Statham PM-97 unbonded strain gauge differential pressure transducer for pressures in the range $0-1.4$ inches of water. The Wheatstone bridge was excited by a stable Hewlett-Packard model 6213 DC power supply. The output voltage from the pressure transducer was read by an integrating digital voltmeter, Hew-lett-Packard Model 2401C. An external quartz oscillator time base generator was used to give an integration time of 10 seconds. The transducer was calibrated at regular intervals against a MERIAM micromanometer model 34FB2. The calibration curve was found to be linear and stable to $\pm 0.001$ inches of water.

### 3.5 Procedure for Setting Up a New Run

The setting-up of a new run to a specified free-stream velocity, transpiration boundary condition and wall temperature is an iterative procedure. For the first trial the flow field is left isothermal, that is, the plates are not heated. From a statement of the desired velocity distribution (given in terms of the values of free stream pressure gradient and the velocity at the beginning of the test section) a computer program, Slot [8], supplies the desired dynamic pressure distribution. The total and static pressure at $x=2$ inches are held constant during the course of the iterations. The rotameter readings (in case transpiration is used) and the slot widths are reset at each iterative cycle, again with the help of the above mentioned computer program. On the basis of the measured distribution of dynamic pressures, the total pressure and the rotameter settings, a correction is computed so that resetting would result in attainment of the desired boundary conditions. As a practical matter the iterations were discontinued when slot predicted changes in slot width of 0.001 inches or less. The acceleration parameter $K$, could be set at a desired value within about $3 \%$ by comparing the measured value to the desired value and iterating.

After having set up the rig for hydrodynamic conditions, the plates were heated and the power to them iteratively adjusted to obtain a constant plate temperature within $\pm 0.25^{\circ} \mathrm{F}$. Due to the small wall-to-freestream temperature difference ( $20-30^{\circ} \mathrm{F}$ ), no significant deviation from the originally set up hydrodynamic conditions was observed within the uncertainty of the measurements. The hydrodynamic and thermal conditions were reset at the beginning of each data taking procedure, to compensate for any ambient condition drift.

All free stream velocities were measured with a pitot probe, and the free stream temperature was measured with a calibrated iron-constantan thermocouple. However, both velocity and temperature inside the boundary layer were measured with a resistance thermometry approach, as indicated in the following section.

### 3.6 Sequential Measurement of Mean Velocity and Temperature

Prior studies in this series have relied mainly upon isothermal hydrodynamic data used with temperature profile data from a nominally identical run, except for the heated wall. This has introduced problems of two sorts into the structural studies of turbulent heat transfer: first, the position uncertainty inherent in overlaying the two profiles and, second, the possibility of changes in the shapes of the profiles caused by variable properties effects. Most of the prior work in this series has used a 20 to $30^{\circ} \mathrm{F}$ difference between wall and free stream temperature. This corresponds typically to a maximum density ratio, $\bar{p}_{\infty} / \bar{\rho}_{o}$, of about 1.04 hence has required considerably care in combining the isothermal hydrodynamic data with the hot wall temperature profiles. Thielbahr [6] investigated both experimentally and numerically the question of which one of the following candidates would be most nearly preserved: $\overline{\mathrm{u}} / \mathrm{U}_{\infty}, \overline{\rho \mathrm{u}} / \rho_{\infty} U_{\infty}$, or $\overline{\rho u^{2}} / \rho_{\infty} \mathrm{U}_{\infty}^{2}$. He found that the minimum error in integral parameters was achieved by assuming the preservation of $\bar{u} / U_{\infty}$. Thus the variable properties effect is under reasonable control. The position uncertainty arises because of the need to interpolate in one of the two data sets ( $T$ or $\bar{u}$ ) to obtain corresponding data. The uncertainty in locating either probe can give appreciable errors, close to the wall. If the first data point is 0.005 inches from the wall, then an error of only 0.0005 inches yields a $10 \%$ position error. When $y$ derivatives are to be compared, such errors are important.

These problems were solved in the present program by measuring the velocity and temperature sequentially with the same hot wire probe: first, the velocity was measured using a constant temperature anemometer, then the temperature was measured by switching to a constant current anemometer using the same probe as a resistance thermometer.

This sequential measurement of velocity and temperature eliminated the need for overlaying an isothermal velocity profile onto the temperature profile.

The proposal seems straightforward and initially it was believed that the DISA 55DO1 anemometer would be able to perform both measurements using one chassis since it can be operated in either the constant temper-
ature mode (for velocity measurements) or the constant current mode (for temperature measurements). The amplifier drift in the constant current mode, however, limits the accuracy of mean temperature measurement, even though this does not constitute a problem for temperature fluctuation measurement. Equally important, however, is the fact that both modes of operation use the same Wheatstone bridge and different currents are used (40-60 ma for constant temperature, 3 ma for constant current). Because of the change in bridge current the re-stabilizing of the system after switching from one mode to the other, requires a long waiting time. This problem is particularly important when taking a $30-40$ point profile: the waiting time can be prohibitive.

The solution to this problem is the use of two anemometers. A 55D01 system was used as a constant temperature anemometer with a 55 M system operated in the constant current mode. This new 55M unit has a temperature controlled input transistor in the first stage of amplification, and both the noise level and the amplifier drift are greatly reduced. Very high gains can be used, increasing greatly the sensitivity for temperature measurements.

Very low probe currents ( 2 ma with a 5 micron tungsten wire; 0.1 ma with a 1 micron platinum wire) can be used and the velocity contamination in the anemometer response for mean temperature measurement can be virtually eliminated.

The final stage of the measuring system is the coupling between the two anemometers. The DISA 55D65 Probe selector with very low contact resistance can be used. This unit was initially designed to allow several probes to be used with one anemometer; an inversion of the connections had to be made to use two anemometers and just one probe.

## 3. 7 Hot Wire Instrumentation - Choice of the Probe for Mean Temperature and Velocity Measurement

Ideally an infinitely long wire would be the indicated one for mean velocity and mean temperature measurements in a 2-dimensional flow. The wire diameter, however, must be small if a good frequency response is to be obtained. This limits the wire length for strength reasons. For
velocity measurements, a short length of wire has the disadvantage that it responds to the aerodynamic interference of its supports [11]. In the temperature measuring mode, deviation from the calibration curve can be caused by a non-unfform temperature distribution along the wire, the deviation being a function of the fluid velocity and the wire current [10].

The DISA 55F04 5 micron boundary layer probe meets the requirement of minimum aerodynamic interference. The wire is 3 mm long, gold plated to a diameter of 6 times its sensitive portion (about 1.2 mm 1 ong ) for strengthening reasons. Although this was not the original purpose, it can be used for temperature measurement. According to Maye [10], this larger portion has practically no influence on the measured resistance and therefore on the probe sensitivity. Being located on the same isotherm as the sensing wire and having a diameter smaller than the prong diameter it can contribute effectively towards lowering the conduction between the sensing wire and the prongs. Appendix A describes a conduction error analysis when the wire is operated at very low currents, and used as a resistance thermometer.

The sensitivity of the wire is another important factor for temperature measurement. The highest possible wire current must be chosen, but it cannot be large enough so that the heat loss from the wire becomes appreciable. With a high amplifier gain (3500) and a very low current (2 ma), a good signal was obtained with the 55 M system with a very high signal-to-noise ratio and very low drift.

Another factor to be taken into consideration is the position of the probe prongs with respect to the mean flow direction. Thinh [12] reports that the inclination giving the correct values (minimum aerodynamic interference) corresponds to a support placed parallel to the direction of the mean flow. Maye [10] reports that the prongs must be long and pointed, and located in the isothermal plane passing through the wire, if minimum conduction errors are desired when measuring temperature.

The shape of the 5 micron boundary layer type probe used in this investigation permits measurements to be taken very close to the wall ( 0.005 in ) and meets the above requirements.

Mean velocity and mean temperature measurements were performed using the system described in Section 3.6. The output from the two anemometers was read by a VIDAR 5206 D-DAS Data Acquisition System employing a D.E.C. PDP 8/L computer. It uses a VIDAR digital voltmeter with a very high input impedance (larger than 10 megohms). The sampling period was 0.167 seconds and the data taking was programmed with a variable integration time to provide an average value of the signal from the anemometer which was steady to within $\pm 1$ mv out of a 3 volt output. This corresponds, for velocity, to less than $0.1 \mathrm{ft} / \mathrm{sec}$ at the lowest sensitivity. For temperature, this corresponds to $0.02^{\circ} \mathrm{F}$. Very close to the wall an integration time higher than 10 seconds was required; in the outer region only a few seconds were needed.

The hot wire probe and its support are sketched in Fig. 3.6. The only difference between it and Andersen's probe [8] is the wire element. The probe has a "wall stop" that effectively prevents the wire from accidentally being damaged by the wall. The wire distance from the wall, when the wall stop touches it, was measured by an optical comparator and set nominally to 0.005 inches. As observed by Andersen [8], the probe stem position may deviate up to $\approx 0.001$ inches. It appears, however, that for a given $x$ position (i.e. a given access hole) the distance from the wall to the wire is reproducible to a greater accuracy; this conclusion, according to Andersen [8] is based on repeated evaluation of the wall shear stress using the first data point for the computation of the velocity gradient near the wall.

In making boundary layer traverses, the wall is located in the following way. The probe is lowered down until, visually, the wall stop touches the wall. This is observed by lighting the region from the back of the probe, as viewed from the observer, and adjusting until no light passes between the wall stop and the wall. Then the probe is advanced 0.002 in. to compensate for the micrometer backlash. The micrometer is rotated in the opposite direction until the probe starts being displaced upwards. Readings are taken for both mean temperature and velocity every 0.001 in . in the direction perpendicular to the wall for a set of 5 to 10 points, and a procedure based on the constancy of the temperature
difference between a point and its neighbor (approximately $1^{\circ} \mathrm{F}$ ) is followed to complete the profile. After having reduced the data, the profile is visually inspected and the first points (usually 2 or 3 ) are discarded. The criterion is to eliminate points having the same or only slightly different values of velocity and temperature. This would indicate that the probe had not been displaced yet from its position, accounting for micrometer backlash. Also some spring effects of the trasversing mechanism were observed when pressing the wall stop against the wall. The above procedure also takes into account this factor. The first data point is then assigned a value of 0.005 in. , as measured by the optical comparator. The uncertainty is estimated to be $\pm 0.001$ in. Some points in the near neighborhood of the wall had to be eliminated in certain cases because the calibration was found to be unreliable for velocities below $2.5 \mathrm{ft} / \mathrm{sec}$. This is discussed in Section 3.11 .

This probe was also used for measuring the rms value of the axial velocity component and the temperature fluctuations. This matter is discussed in Section 3.9

The horizontality of the wire with respect to the wall was measured by means of an optical comparator, and the difference between the ordinates of the two prong tips was found to be 0.001 in .

### 3.8 Hot Wire Instrumentation - The Measurement of Turbulent Quantities

The details of turbulence in isothermal flows are usually measured by means of two wires following an $X$ configuration, usually in a plane perpendicular to the wall and parallel to the mean flow field. Two anemometer units, for adding and subtracting signals analogically, and a multiplier are necessary to measure $\overline{u^{\prime} v^{\prime}}, \overline{v^{\prime 2}}, \overline{u^{\prime 2}}$. If each wire in the $X$ configuration is at $45^{\circ}$ (absolute value) with respect to the mean flow direction, one of them will respond to the sum $u^{\prime}+v^{\prime}$, the other one to the difference $u^{\prime}-v^{\prime}$, to a first order approximation (if small fluctuations are involved in the problem). A simple algebraic manipulation of the signals can generate the desired quantities in a rather quick way. The drawback is, of course, the matching between the two wires. They are not usually alike and manipulations with the analog
signals are necessary. Because of the fact that so many analogic operations are performed, care must be exercised to bring the errors inherent in the electronics to a minimum level. Several other variations have been used for this kind of measurement. Watts [13], for example, used an $X$ wire banked at an angle of $45^{\circ}$ with respect to the plane perpendicular to the wall and parallel to the mean flow field. The latter being twodimensional, both $\overline{u^{\prime} v^{\prime}}$ and $q^{2}\left(=\overline{u^{\prime}}+\overline{v^{\prime 2}}+\overline{w^{\prime 2}}\right)$ could be determined directly even by using wires with unequal sensitivities and with angles that possess departures of greater than $1^{\circ}$ from the nominal $45^{\circ}$ value.

Instead of an X-wire probe, Fujita and Kovasznay [14] used a single, rotatable wire. By using redundant data, a least squares fit gave the best estimate for $\overline{u^{\top} v^{\top}}, \overline{u^{\top} Z}$ and $\overline{v^{\top} 2}$ in two dimensional flows. According to their work the errors in the hot wire response are minimized by calibrating it in place for every measurement, without having to make use of predetermined and accepted hot wire correlations and parameters. The advantage of using only one wire is simplicity; only one anemometer is used and the matching problem between wires is avoided. However, because of the fact that the system may be sensitive to small errors in the rms values, a longer integration time is required and the procedure becomes time consuming. Therefore, the experimenter must be sure that the flow conditions do not change during the data taking procedure. On the other hand, several turbulent quantities can be measured, once a reliable rotating system is designed. This flexibility has tempted some experimenters to use it. Durst and Rodi [15], for example, claim they could use this system in highly turbulent flows (e.g., jet flows), for measuring the turbulent quantities very accurately, provided the mean velocity field is known. Andersen [8] used a rotating wire for measurements of Reynolds stress tensor components and his procedure, with some modifications, has been used in this investigation.

Figure 3.7 shows the probe and its rotating mechanism, used to measure $\overline{u^{\prime} v^{\top}}$ and, in connection with the horizontal wire, $\overline{v^{\top 2}}$ and $\overline{w^{12}}$. nominal $45^{\circ}$ slant wire having a total length of 3 mm with a 1.2 mm sensitive center position. The ends of the wire are gold plated. The
wire is made of tungsten and has a diameter of $5 \mu \mathrm{~m}$. The prongs are parallel to the mean flow direction at any position of its rotation. Both the horizontal and the slant wires were shown to be strong: no great care is needed to handle them. Andersen's work [8] and this investigation confirmed this point. Only one wire for each kind of measurement was used during the whole period of data taking.

The rotatable hot wire probe has a cable drive to permit it to be rotated while in position in the tunnel. The probe spindle incorporates a "lock-drum" which features six radially drilled holes spaced at $60^{\circ}$. A spring loaded "lock pin" which fits into the holes in the lock drum may be lifted by means of a lever located on top of the transversing mechanism. The arrangement permits turning of the probe to any of the six angular positions, $\phi_{n}=(2 n-1) \frac{\pi}{6} ; n= \pm 1, \pm 2, \pm 3$, while operating in the tunnel. The probe is similar to Andersen's [8], which had to be modified to allow measurements to be taken in a plane parallel to the wall. This was dictated by the need of measuring the mean velocity at the point of shear stress measurement, and by the necessity of checking the two-dimensionality of the flow field. The choice of the angular positions is somewhat arbitrary. Watts [13] uses the X wire banked at $45^{\circ}$, which allows direct measurement of the turbulent kinetic energy. A later study in this investigation indicated theoretically that a single slant wire with an angle different from the nominal $45^{\circ}$ could give the same results in a simpler operation. Manufacturing difficulties, however, prevented the testing of this system. Appendix $C$ discusses the theoretical basis for the direct measurement of $\overline{u^{\top} v^{\top}}$ and $\overline{q^{2}}$ with a single rotating wire. The positioning of the slant wire in a plane perpendicular to the wall was concluded in this study to be the correct one when nothing is known about the two dimensionality of the turbulent field ( $\overline{v^{\top} w^{\top}}$ and $\overline{u^{\prime} w^{\top}}$ not zero). This matter is discussed in Appendix $B$, together with the measurement technique for turbulent quantities. Tests of two dimensionality indicated that in the examined cases $\overline{\mathrm{v}^{\top} \mathrm{w}^{\top}}=$ $\overline{u^{\prime} w^{\prime}}=0$. Therefore, it was concluded that the above configuration could be used to measure $\overline{u^{\prime} v^{\top}}$ with the probe placed at two different positions $-\frac{\pi}{6},+\frac{5 \pi}{6}$, the minimum required in such a procedure. The
sensitivity of the wire is higher when normal to the wall. The above angles were chosen for three main reasons: (1) Andersen's probe [8] was available and only small modifications would be required. (The manufacturing of this probe is time consuming and expensive, requiring a high degree of craftsmanship). (2) In isothermal two-dimensional flows errors due to non-zero size of the wire (non-uniform velocity distribution along it) are minimized by placing the probe the closest possible to the horizontal plane, where the two-dimensional assumption about the mean flow field allows the velocity to be uniform along the wire. (3) In nonisothermal flows the departure of the wire prongs from the isothermal plane generates a non-symmetric temperature distribution along the wire and as a consequence the response of the system to temperature fluctuations (e.g., measurement of $\overline{v^{\top} t^{\top}}$ ) becomes more susceptible of having significant errors.

The spacing between the prongs was chosen to provide minimum aerodynamic interference. Dahm and Rasmussen's [11] results show that for a 3 mm spacing the error in measuring mean velocity profiles is reduced to $2 \%$. Strohl and Comte-Bellot [16] show and recommend a prong spacing of 3 mm together with other probe features if the error in measuring $\overline{u^{\top} v^{\top}}$ is to be kept down to $2 \%$.

The angle of the wire and prong system with respect to the wall was measured by means of a toolmaker's microscope. Initially, an optical comparator was chosen for this kind of measurement. However, the uncertainty in the measurement, due to the fact that the shadow of the object is projected onto a screen and amplified, made this system not reliable for measurement of small angles. A direct observation through the use of a toolmaker's microscope gave a maximum departure from the nominal positions of less than $1^{\circ}$. According to Strohl and Comte-Bellot [16], and also some calculations done in this investigation, a maximum error of $2.5 \%$ can be present in the measurement of $\overline{u^{\dagger} v^{\prime}}$ for a $1^{\circ}$ deviation. The wire angle with respect to the mean flow was measured to be $48^{\circ}$ rather than the nominal $45^{\circ}$. This value was therefore used in the data reduction.

The alignment of the hot wire spindle with the mean flow direction was done by placing the wire in a horizontal plane and measuring at two
symmetric positions in this plane. The transversing mechanism fits into the access holes in the top plates and is locked in place before running. A procedure based on the sensitivity of the hot wire to angles was preferred over a mechanical alignment. The spindle was rotated by moving the mechanism body until the difference between the two electrical signals was 2 mv . (out of a 3 volt signal, as measured by a precision integrating digital voltmeter). When measuring velocity, the deviation from its mean value due only to probe misalignment was therefore 1 mv . or $0.1 \mathrm{ft} / \mathrm{sec}$ at the lowest sensitivity. This procedure was naturally carried out in the free stream, so that the fluctuating field could not influence the probe alignment. The probe was then lowered down to the boundary layer and measurements taken. During the Reynolds stress measurement procedure the mean velocity measurement was required because the system sensitivity to velocity is not constant (the linearizer is not used). An integration time of 100 sec . was used and the contribution to the error in the velocity sensitivity is believed to be less than $1 \%$.

The distance from the center of the wire to the wall was measured by means of an optical comparator. The reference point, for determining when the spindle touches the wall, is the interface circumference between the tapered and the straight portion of the spindle. A nominal value of 0.069 in. was assigned to this point and the error was estimated to be $\pm 0.002$ in. To start the measurement procedure the probe was simply lowered down until no light could be seen between the reference line and the wall. The micrometer then was rotated in the opposite direction until a light beam could be seen between the two points. This location is assigned a value of 0.070 in. , indicating the probe has already departed from the wall.

In non-isothermal flows, as in the case of the measurement of $\overline{v^{\top} t^{\top}}$, a careful study of the hot wire response had to be done. Corrsin [17] analyzed in a theoretical way the general problem of the response of the hot wire to temperature, velocity and concentration fluctuations in twodimensional flows. The procedure is basically to assume an expression for the hot wire response, namely

$$
E^{2}=\left(T_{W}-T\right)\left(A+B U_{e f f}^{0.5}\right)
$$

E being the output signal, $T_{W}$ the wire temperature, $T$ the fluid temperature, $U_{\text {eff }}$ an effective velocity (function of $U$ and $V$ ), Differentiation of the above expression gives

$$
\begin{equation*}
d E=\frac{\partial E}{\partial U} d U+\frac{\partial E}{\partial V} d V+\frac{\partial E}{\partial T} d T \tag{3.1}
\end{equation*}
$$

Finally, for small fluctuations, the following is obtained:

$$
\begin{equation*}
e^{\prime}=\frac{\partial E}{\partial U} u^{\prime}+\frac{\partial E}{\partial V} v^{\prime}+\frac{\partial E}{\partial T} t^{\prime} \tag{3.2}
\end{equation*}
$$

The rms value of the anemometer signal is

$$
\begin{align*}
\overline{e^{\prime 2}}=\left(\frac{\partial E}{\partial U}\right)^{2 \overline{u^{\prime}}}+\left(\frac{\partial E}{\partial V}\right)^{2 \overline{v^{\prime 2}}} & +\left(\frac{\partial E}{\partial T}\right)^{2 \overline{t^{\prime}} \overline{2}}+2 \frac{\partial E}{\partial U} \frac{\partial E}{\partial V} \overline{u^{\prime} v^{\prime}}+2 \frac{\partial E}{\partial U} \frac{\partial E}{\partial T} \overline{u^{\prime} t^{\prime}} \\
& +2 \frac{\partial E}{\partial V} \frac{\partial E}{\partial T} \overline{v^{\prime} t^{\prime}} \tag{3.3}
\end{align*}
$$

Measuring at two different symmetrical positions and subtracting one from the other $\overline{e_{1}^{\prime 2}}-\overline{e_{2}^{2}}=a \overline{u^{\prime} v^{\top}}+b \overline{v^{\prime} t^{\prime}}$, where $a$ and $b$ are functions of the sensitivity coefficients, the wire material, the directional properties, and geometry. If two different wire temperatures are also employed, then a system of linear equations can be solved to give values for $\overline{u^{1} v^{\top}}$ and $\overline{v^{\prime} t^{\prime}}$.

Corrsin [17] has estimated the magnitude of each term in the equations and showed the feasibility of the measurement procedure. Arya and Plate [18] applied this technique for measuring $\overline{u^{\top} v^{\top}}$ and $\overline{v^{\top} t^{\top}}$ with a slant wire. The velocity and temperature sensitivity of the anemometer system, as defined by $\partial E / \partial U$ and $\partial E / \partial T$ respectively, were determined experimentally, without having to assume an expression for the variation of the hot wire signal as a function of velocity and temperature. This was done by measuring, for each calibration velocity, the output signal for different $\left(T_{W}-T_{\infty}\right)$ where $T_{W}$ is the wire temperature and $T_{\infty}$ is
the fluid temperature at the measurement point. The set of curves was differentiated to give the sensitivities.

As pointed out by Arya and Plate [18], however, this procedure is subject to some scatter in $\overline{v^{\prime} t^{\top}}$ due to experimental errors in the required rms values of the anemometer signal. A least square fit procedure was therefore used to improve the accuracy of the system.

This procedure has the advantage of simplicity of operation, because only one anemometer and one wire are used. The complexity of calibration is reduced to a minimum level and the reliability of the system is improved because only one wire can be broken and replaced. The spatial resolution is another factor of consideration. The information comes from one wire and, therefore, from one location only. This factor becomes important when steep temperature and velocity gradients are present. However, the procedure is time consuming and due to the fact that the information for the measurement of $\overline{v^{\top} t^{\top}}$ is obtained at different instants of time, the experimenter must be sure that the flow conditions do not change during the data taking procedure. Naturally, as in the case of isothermal flows, a good hot wire rotating system must be supplied for the measurement procedure.

This technique has been used by a few investigators to measure $\overline{u^{\prime} t^{\prime}}$, by means of a horizontal sensor. Among them, one could mention Kudva and Sesonske [19] who used one hot film probe. Only one reference, Fulachier and Dumas [26] has been found, in which measurements of $\overline{v^{\top} t^{\top}}$ are carried out by this procedure.

Another technique requires 3 wires. Two of them, in an $X$ configuration, with the anemometers operated in the constant temperature mode, give directly $v^{\prime}$. The third one, a very small diameter wire, horizontal, with the anemometer operated in the constant current mode, at a very small current, gives directly $t^{\prime}$. The procedure is based on the fact that if two matched wires are placed in symmetrical positions (as in the case of $X$ configuration) the subtraction of the two signals is directly proportional to $\mathrm{v}^{\prime}$, being independent of the temperature fluctuation in low turbulence level flows, as observed by Corrsin [17]. The multiplication of this signal by the one from the temperature wire, gives $\overline{v^{\top} t^{\top}}$
after a time averaging procedure. The main advantage is that instantaneous values of $\overline{v^{\prime} t^{\top}}$ can be obtained, being particularly useful for spectral measurements. This procedure is not time consuming but requires a somewhat complex electronic set up. Three anemometers must be used. Two of them operated in the constant temperature mode, and the other one in the constant current mode, A subtracting and a multiplying circuit must be employed. The wires must be carefully arranged to give a good spatial resolution. The calibration is obviously more complex and in case of breaking one of them extreme care must be exercised to replace it.

Johnston [20] used this procedure to measure $\overline{v^{\prime} t^{\top}}$. He used two 0.00015 in. tungsten wires and one 0.00005 in. platinum wire, Bremhorst and Bullock [21] used this method for the measurement of $\overline{u^{\top} t^{\top}}$, but did not report any $\overline{v^{\top} t^{\top}}$ measurement. Bourke and Pulling [22], and Bradshaw [23] reported having used this technique for obtaining $\overline{v^{\top} t^{\dagger}}$. Blom [24] however slightly modified it by replacing the temperature wire for a very thin platinum film.

Finally, Burchill and Jones [25] present a procedure to obtain directly $u^{\prime}, v^{\prime}, t^{\prime}, \bar{U}, \bar{T}$ by means of a number of hot $f 11 m$ sensors.

### 3.9 Calibration for Mean and RMS Temperature Measurement

The calibration of the $5 \mu \mathrm{~m}$ gold plated tungsten wire was done in a variabie temperature oil bath (Rosemount Engineering Co. Model 910A) the temperature controller being a Thermotrol Model 910-508 from the same company (a Shell development design), using a resistance thermometer as a sensor.

The oil bath temperature was measured by an HP Model DY-2801A quartz thermometer. Its calibration was checked by placing the sensor into three different calibration standards. The first one was a standard stirred ice bath (Rosemount Engineering Co. Model 911); the second one, a Leeds and Northrup Co. steam point apparatus; and the third, a Thermowells Inc. tin freezing point standard. Having set the thermometer at $0.00^{\circ} \mathrm{C}$, in ice, the calibration was checked to within $0.02^{\circ} \mathrm{F}$ for the steam point and $0.04^{\circ} \mathrm{F}$ for the tin point.

The constant current Wheatstone bridge is a DISA 55M20. A gain of about 3500 was used in conjunction with a wire current of 2 ma . The measured rms of the noise was 2 mv . The calibration of the wire showed that a 5 mv variation in the output corresponds to a $0.1^{\circ} \mathrm{F}$ difference. The noise therefore is equivalent to $0.04^{\circ} \mathrm{F}$ which was deducted from the measured rms value of the temperature fluctuation, when reducing the data.

The output from the anemometer was measured by a HP Model 2401C integrating digital voltmeter. About 10 points were used in the calibration covering the range $60^{\circ} \mathrm{F}-100^{\circ} \mathrm{F}$, and the maximum departure from the fitted straight line was $0.1^{\circ} \mathrm{F}$. Also, the resistance of the wire as a function of the temperature was measured by using the same anemometer, which employs a 0.8 ma current for its measurement. A linear expression $R=A T+B$ was fitted and the sensitivity $A$ was found to be $0.0077 \Omega /^{\circ} \mathrm{F}$ for this tungsten wire.

Although the system is very stable, as far as resistance variation is concerned, some contact resistance changes were observed during the operation of the system. Unplugging the cables, plus a small instability In the resistance of the connections combined to give a resistance variation of 0.004 ohms. Due to the small sensitivity $A$ of the wire, this would mean a $0.5^{\circ} \mathrm{F}$ variation. This instability or lack of repeatability was identified by recalibrating the probe after having disconnected the cables. The resistance-temperature curve had the same slope, but shifted up or down at random indicating that the change was not due to drifts in wire properties.

The solution to this difficulty was an in-place temperature calibration using a calibrated iron constantan thermocouple very close to the hot wire probe in an isothermal region of the wind tunnel. This check was done before and after every data taking procedure, and no variation was observed during the run, although changes were observed from one day to the other one.

The relative accuracy of temperature measurement is estimated as $\pm 0.1^{\circ} \mathrm{F}$. The absolute one, about $\pm 0.2^{\circ} \mathrm{F}$.

The in-place calibration and the oil bath calibration were both performed with both the wire and the prongs at the same temperature (no
heating from the very low wire current), hence no conduction errors should be expected, as described in Appendix $A$.

The calibration oil bath is in continuous movement, and the risk of breaking the wire would be large if the probe were placed directly into it. Besides that risk, the oil film deposited on the wire surface would have to be cleaned afterwards, otherwise the frequency response would be degraded (the working fluid is air). Freon is a good solvent and could be used for this purpose, though the risk of breaking the wire is great. To obviate the need for cleaning the wire it was decided, therefore, to put the wire inside a copper test tube, which was placed in the oil bath. The fluid inside the test tube was air, and it was isolated from the ambient air in order to prevent any air currents from convecting heat from or to the probe or generating a temperature gradient in the neighborhood of the wire. The test tube material was copper, so that the time constant of the system was kept small, and the time required for the calibration minimized. The aspect ratio of the tube (length to diameter ratio) was large, so that the region where the wire was placed would be nearly isothermal: in the present case the ratio is about $7 / 1$ with the probe nearly at the bottom of the tube.

An experiment was carried on to check the existence of conduction errors. The probe was covered by an aluminum foil and placed inside the oil bath, in a region where the flow is already isothermal. The foil prevented the oil movement from damaging the wire. No change in calibration was observed. The probe naturally had to be cleaned afterwards.

### 3.10 The Use of a Linearizing Circuit for Processing the Output from the Anemometer

A linearizer circuit can be used to convert the non-linear output of the anemometer to a linear function of velocity. Its use in nonisothermal flows, however, makes the measurement procedure very cumbersome, since the linearizer parameters must be changed to suit each ambient temperature (unless a temperature compensating probe is used). With a compensating probe the system would lack spatial resolution due to the large size of the probes and the small dimensions of a boundary layer
flow. The usual procedure for measuring the rms value of the velocity fluctuation is to assume a linear dependence between the output from the anemometer and the velocity fluctuation about the mean flow field. Sandborn [30] suggests that a linearizer circuit for the hot wire signal will not noticeably improve the accuracy of turbulence measurements unless the turbulent intensity is larger than 30 to $40 \%$ of the mean. This level of intensity is usually found only in the region very close to the wall in a boundary layer flow. In the present investigation, this region can only be reached by the horizontal wire ( $\overline{u^{\prime 2}}$ measurement) and small errors may be present in those data. The measurement of $u^{\prime} v$ ' is done in regions of much lower turbulence levels and negligible errors should be expected. Watts [13] measured the Reynolds stress tensor on a flat plate, with no free stream pressure gradient, and also concluded that the measurement of $\overline{u^{\prime} v^{\prime}}$ could be done without the use of the linearizer. Klebanoff [31] also obtained his flat plate data without using the linearizer. It was decided, therefore, not to use a linearizing circuit for processing the output from the anemometer.

### 3.11 Calibration of the Horizontal Wire for the Measurement of Mean and

 RMS VelocitiesThe gold plated tungsten wire was calibrated in the free stream of the wind tunnel test section. The velocity was measured by means of a pitot probe, and the air stream temperature by means of a calibrated iron-constantan thermocouple. Both were placed nearby the hot wire probe. An experimental investigation of the influence of the proximity of the probes to the hot wire showed that the perturbation of the flow field could be neglected. The dynamic pressure was read by a Statham PM-97 unbonded strain gauge differential pressure transducer, using an integrating digital voltmeter HP Model 2401C, in connection with an external quartz oscillator, to give an integration time of 10 seconds. The same voltmeter was used to read the output from the anemometer and the temperature of the free stream.

The operating wire resistance was set to a fixed value of 6 ohms, giving an overheating value in the neighborhood of 1.50 ohms in the
range of temperature of the examined thermal boundary layers. The calibration was carried out at two different ambient temperatures, $30^{\circ} \mathrm{F}$ apart, approximately the upper and lower limit of the temperature range in question. The cold resistance of the wire was measured before each data point for velocity calibration, and the two calibration curves were correlated by means of the expression

$$
E^{2} /\left(R_{W}-R_{f}\right)=f(U)
$$

as seen in Fig. 3.8. The departure of the data points with respect to the above calibration curve is less than $1 \%$, probably the accuracy of mean velocity measurement. Lower overheating ratio ( $R_{W}=5.40$ ohms) was also satisfactory but higher overheating ratio ( $\mathrm{R}_{\mathrm{W}}=7.50$ ohms ) was less satisfactory.

During the measurement procedure, the cold resistance of the wire, to be used in conjunction with the calibration curve, was calculated from the measurement of mean temperature (Section 3.6) and the calibrated resistance versus temperature curve (Section 3.9).

Several functional relationships $f(U)$ were tried to give the best estimate of the calibration curve in the least square sense.
a) $f(U)=A+B U^{n}$, where $A, B$ and $n$ were determined by means of a non-linear least square fit. The exponent $n$ was found to be in the neighborhood of 0.45 , as originally concluded by Collis and Williams [32].
b) $f(U)=A+B \sqrt{U}+C U$, where $A, B$, and $C$ were determined by means of a linear least square fit. This expression was tried because of the observation of Davies and Patrick [33], according to which, the velocity sensitivity $\partial E / \partial U$ of the hot wire, when determined from this expression, is very close to the value obtained from the dynamic calibration of the wire, important for the rms measurement of the axial velocity $U$.
c) $f(U)=\sum_{i=1}^{n}\left(A_{i}+B_{i} U+C_{i} U^{2}+D_{i} U^{3}\right)$, spline fit. Usually two or
three intervals were tried $(n=2$ or 3 ), with the constraint that the matching of the functions should be with respect to its value, its first and second derivatives at the matching point. This spline fit (in the least square sense) using the concept of contrained optimization (Langrange multipliers approach) gave the best fit of the calibration curve. The procedure is outlined by Klaus and Van Ness [34]. A higher number of cubics (one for every two points) was tried, and it proved to be useful only for the velocity sensitivity ( $\partial \mathrm{E} / \partial \mathrm{U}$ ) determination in the lower velocity range ( 2.5 to $5 \mathrm{ft} / \mathrm{sec}$ ).

The conclusion of this study is that the form of the functional relationship $f(U)$ is not important when the mean velocity measurement is the only required quantity since all forms tried were acceptable. For simplicity, the expression $b$ ) was chosen.

For the determination of the velocity sensitivity $\partial E / \partial U$, in the measurement of $\overline{u^{\prime 2}}$ (see Appendix B), the choice of $f(U)$ turned out to be important. Expression b) and the spline fit with 2 or 3 cubics, and with a cubic for every two points were used. The results are shown in Fig. 3.9. It can be seen that in the range 5 to $32 \mathrm{ft} / \mathrm{sec}$ the departure from the average value of the derivative (obtained from the three procedures) is about $3 \%$ at most. The lower range, 2.5 to $5 \mathrm{ft} / \mathrm{sec}$, however is critical and analysis had to be carried out to determine the best estimate of the derivative. A cubic for every two points was used for two main reasons: (1) it gave the best mean value fit in that region and, (2) the derivative as calculated from this procedure turned out to be nearly constant in the beginning of the region, as expected from the analysis of the heat transfer from wires.

The mean velocity was corrected for proximity from the wall according to Repik [35]. The maximum correction was in the order of 0.2 $\mathrm{ft} / \mathrm{sec}$ for velocities of 2.5 or $3 \mathrm{ft} / \mathrm{sec}$. Measured velocities below 2.5 $f t / s e c$ were considered unreliable due to uncertainties in calibration.

The measurement of mean velocity was done using a Vidar 5206 D-DAS Data Acquisition System, employing a DEC PDP 8/L computer. The anemometer output was read through a Vidar digital voltmeter with a very
high impedance (higher than 10 megohms), and a variable integration time sufficient to give mean velocity to within $0.1 \mathrm{ft} / \mathrm{sec}$. (This does not suggest the overall accuracy of velocity measurement.)

The measurement of the rms value of streamwise velocity was done by feeding the output from the anemometer to a DISA true rms meter, set at a time constant of 30 seconds, to guarantee that the low frequency oscillations were included. The rims meter was calibrated specially by DISA to give a $1 \%$ accuracy on the measured value at the midrange of the meter, and $2 \%$ at the end of the scale. This was done because most of the critical measurements were done in the mid-range of the rms meter. The usual accuracy being $1 \%$ full scale, it would make the measurement rather inaccurate in that range. The calibration was checked using standard sine waves with known rms values. The mean square output was chosen to be read for two reasons. Firstly, it is slightly more accurate than the rms output (the square root can be obtained numerically and very accurately if necessary). Secondly, the output was averaged over a certain period of time, and conceptually there is a difference between averaging the square root and extracting the square root of the averaged value.

The mean square output was read by an integrating digital voltmeter HP 2401C set to an integration time of 100 seconds by means of an external clock. The final figure was obtained by letting the rms meter integrate during 100 seconds, and then by feeding to the DVM and integrating over 100 seconds. During the procedure the rms meter was monitored visually to make sure no sudden flow perturbation occurred. This was particularly important when measuring shear stress and turbulent heat transfer; since many operations are done on the signals, amplifying any errors. Measurements were repeated when perturbations were observed. With this procedure, repeatable values to within $1 \%$ were obtained.

### 3.12 Calibration of the Slant Wire for the Measurement of Turbulence Quantities

The use of the slant wire for turbulence measurements is based on the principle that the wire responds to both temperature and velocity
fluctuation. For small fluctuations, one can write

$$
\begin{equation*}
d E=\frac{\partial E}{\partial U} \mathrm{UU}_{\text {eff }} \quad+\frac{\partial E}{\partial T} d T \tag{3.4}
\end{equation*}
$$

However $d E \approx e^{\prime}, d U_{e f f} \approx u_{\text {eff }}^{\prime}, d T \approx t^{\prime}$. Therefore the following expression can be written

$$
\begin{equation*}
e^{\prime}=\frac{\partial E}{\partial U}{ }_{e f f}^{\prime \prime}{ }_{e f f}+\frac{\partial E}{\partial T} t^{\prime} \tag{3.5}
\end{equation*}
$$

Appendix B gives the following relationships

$$
\begin{gather*}
\frac{\partial E}{\partial U}=\frac{\partial E}{\partial U} \frac{1}{\sqrt{A}}  \tag{3.6}\\
u_{e f f}^{\prime}=\sqrt{A} u^{\prime}+\frac{D}{2 \sqrt{A}} v^{\prime}+\frac{F}{2 \sqrt{A}} w^{\prime} \tag{3.7}
\end{gather*}
$$

Finally

$$
\begin{equation*}
e^{\prime}=\frac{\partial E}{\partial U}\left[u^{\prime}+\frac{D}{2 A} v^{\prime}+\frac{F}{2 A} w^{\prime}\right]+\frac{\partial E}{\partial T} t^{\prime} \tag{3.8}
\end{equation*}
$$

where $U$ is the mean streamwise velocity.
The calibration of the wire consists basically of obtaining experimentally $\frac{\partial E}{\partial U}$ and $\frac{\partial E}{\partial T}$.
a) Calibration for velocity

As indicated by Arya and Plate [18] the response of the wire can be thought of as a function of the velocity and the difference between the wire temperature and the ambient temperature. Therefore, if the cold resistance is measured, and a constant wire to mean ambient temperature is used, only one calibration curve can be applied to the data reduction, simplifying greatly the measurement procedure.

In the present investigation, however, the gold plated $5 \mu \mathrm{~m}$ slant tungsten wire (DISA 55F02) seems to be contaminated by a perceptible
heat conduction to the prongs due to its relatively low L/D ratio ( $\approx 240$, sensitive length) and the adverse effect of the gold plating. As a matter of fact, the plating of the wire was done originally for strengthening purposes; reducing the aerodynamic interference from the prongs requires the use of a long wire. As pointed out in Appendix A, the gold plated region tends to be at the prong temperature level due to its large thermal inertia (as compared to the wire itself).

As a consequence of the heat conduction effect, there is also a slight dependence of the wire response on the ambient temperature. For measurements in isothermal conditions, the wire was calibrated for velocity at a constant wire to ambient temperature difference (corresponding to 1.50 ohms), at the approximate free stream temperature of the measurement. For non-isothermal conditions, the wire was calibrated at approximately the temperature of the so called logarithmic region of the boundary layer, the closest possible to the wall the measurement can be done. This corresponds to a temperature approximately $15^{\circ} \mathrm{F}$ higher than the free stream. A slight difference was then found between the two calibrations. The choice of the overheating ratio ( $\sim 1.50$ ohms) was dictated by reasons as outlined when calibrating the probe for temperature sensitivity (b).

The probe was calibrated in the free stream of the wind tunnel, aligned according to the method described in Section 3.8. The measurement of velocity was made with a pitot probe, placed near the wire. The precautions described in section 3.11 were taken in order to avoid any aerodynamic interference from the pitot probe to the wire. The dynamic pressure was measured by a pressure transducer, and read by a digital voltmeter, as described in Section 3.11. The calculation of the velocity sensitivity was done by following the same procedure as used for the horizontal wire and also described in Section 3.11.
b) Calibration for temperature

Ideally the sensitivity to temperature would have to be measured at a constant velocity, by keeping the wire temperature constant, and varying the ambient temperature. This procedure, however, would be very time consuming and difficult. Arya and Plate [18] suggested that once the output of the anemometer for a constant velocity is only a function
of the difference between the wire and ambient temperatures, the sensitivity of the wire to ambient temperature can be obtained by varying only the wire temperature and by using the following relationship:

$$
\begin{equation*}
-\frac{\partial E}{\partial T_{\infty}}=\frac{\partial E}{\partial\left(T_{W}-R_{\infty}\right)} \frac{\partial R_{W}}{\partial T_{\infty}}=\frac{\partial E}{\partial\left(R_{W}-R_{\infty}\right)} \frac{\partial R_{\infty}}{\partial T_{\infty}}=\frac{\partial E}{\partial\left(R_{W}-R_{\infty}\right)} \frac{\partial R}{\partial T} \tag{3.9}
\end{equation*}
$$

The cold wire resistance at ambient temperature was about 4.40 ohms, and fixed overheating values of 1.50 ohms and 0.50 ohms were used during the measurement. This was achieved by measuring the cold resistance of the wire before each measurement and adding the overheating value to obtain the hot wire resistance.

The procedure can only be effective if two wire temperatures can be chosen so that the signals are distinctly different. The lowest wire temperature gives the highest temperature sensitivity and the lowest velocity' sensitivity. When going from $\left(R_{w}-R_{\infty}\right)=0.50$ to $\left(R_{W}-R_{\infty}\right)=1.50$ the temperature sensitivity is reduced to half its value, and the velocity sensitivity is doubled. This was concluded to be a good pair for the measurement of $\overline{v^{\top} t^{\top}}$. For each velocity (20-26 data points) in the range $2.5 \mathrm{ft} / \mathrm{sec}$ to $30 \mathrm{ft} / \mathrm{sec}$, the ambient temperature was kept constant and about 18 wire resistances ( 0.1 ohm apart) generated a function of $\left(R_{w}-R_{\infty}\right)$. The curve was differentiated by a spline least square fit procedure and the derivatives calculated at $\left(R_{W}-R_{\infty}\right)=0.50$ and $\left(R_{w}-\right.$ $\left.R_{\infty}\right)=1.50$. The curve of resistance as a function of temperature was obtained experimentally the same way as outlined in Section 3.9. Applying Eq. (3.9), the temperature sensitivity was obtained.

A problem appeared when performing a static calibration of the wire. By static calibration it is meant that the oncoming temperature field is not fluctuating, The wire was placed in an ofl bath, as described in Section 3.9 for the horizontal wire, and by varying the ambient temperature, keeping the wire temperature constant, the sensitivity was determined at $\left(R_{w}-R_{\infty}\right)=0.50$ and $\left(R_{w}-R_{\infty}\right)=1.50$. Surprisingly, for the highest overheating value the difference in the two sensitivity measurements was about $2 \%$ only; for the lowest one, $15 \%$. This suggested that the finite length of the wire might be impairing its calibration. A
detailed analysis of the anemometer had to be carried out. A variable temperature calibration device was designed to provide control of flow rate and temperature. Static calibrations of the wire for the two overheating ratios were performed at approximately $31 \mathrm{ft} / \mathrm{sec}$. Again, the difference was approximately the same, suggesting that the output from the anemometer is a function of the velocity, the wire and ambient temperatures individually, and not only on their difference as supposed before.

It was concluded that two higher overheating ratios should be used to measure $\overline{u^{\prime} v^{\top}}$ and $\overline{v^{\top} t^{\top}}$ in non-isothermal flows. However, as the wire temperature increases its sensitivity to temperature decreases, and the measurement of $\overline{v^{\top} t^{\top}}$ becomes more uncertain. The first idea was to use a very high overheating ratio, so that the wire would be only sensitive to velocity, and a direct measurement of $\overline{u^{\top} v^{\top}}$ would be possible. Sandborn [30] however, states that the anemometer output becomes proportional to $\overline{u^{\top} v^{\top}}$, with a $5 \%$ error, at wire temperatures of about $1000^{\circ} \mathrm{C}$. It was concluded therefore that the use of the value for $\overline{u^{\prime} v^{\top}}$, as determined from the correspondent isothermal flow together with the measurement at $\left(R_{W}-R_{\infty}\right)=1.50$ would be the right approach for $\overline{v^{\prime} t^{\prime}}$ measurement.

One question however arose as to the validity of the assumption that $\overline{u^{\prime} v^{\prime}}$ is the same for both isothermal and non-isothermal flows. Johnson [20] measured both isothermal and non-isothermal $\overline{u^{\top} v^{\top}}$ and found a small difference very close to the wall, and practically nothing in the outer region of a flat plate flow. Kudva and Sesonske [19] found a small difference in the outer region of a pipe flow at low Reynolds number. In the present investigation, the measured mean velocity at the point of $\overline{v^{\prime} t^{\top}}$ measurement was the same as in the isothermal profile to within 1 or $2 \%$. The local temperature was at most $15^{\circ} \mathrm{F}$ above the free stream indicating the flow could be considered a constant property flow in the region of $\overline{v^{\prime} t^{\top}}$ measurement (the density variation is about $2 \%$ ). This suggests that the $\overline{u^{1} v^{\top}}$ profile may be the same as the isothermal one in the region of $\overline{v^{\top} t^{\top}}$ measurement and the above hypothesis is justified. The accuracy of measurement can then be checked by comparison with the value obtained from the mean profile in a flat plate case.

Arya and Plate [18] suggested that small experimental errors can give inaccurate results for $\overline{v^{\top} t^{\top}}$, and offered as a solution the measurements of several wire temperatures and not only the minimum required. According to him a $15 \%$ error may be expected when using this procedure. The abundant data is then curve fitted to give the best estimate of $\overline{v^{\top} t^{\top}}$.

In the present investigation, it was preferred to curve fit the anemometer output and the $\overline{u^{\top} v^{\top}}$ profile, because the long integration time reduced the scatter greatly. The profiles presented in Chapter 6 were obtained by smoothing the above quantities. This system was preferred over Arya and Plate's for two reasons. Firstly, only one wire temperature is used; when a number of data points at each station has to be obtalned, the use of several wire temperatures becomes prohibitive as far as time is involved. Secondly, the long integration time used in this investigation greatly reduced the scatter.

The conclusion of this method for the measurement of $\overline{v^{\prime} t^{\prime}}$ is that great care should be exercised to eliminate the prong effect. This can probably be done by using a very small diameter ( $1 \mu \mathrm{~m}$ ) wire (probably platinum). The $L / D$ ratio is large, yet the spatial resolution remains excellent because a relatively short wire is used. By doing so, the static calibration approaches the dynamic calibration and lower wire temperatures can be used. Both $\overline{u^{\prime} v^{\prime}}$ and $\overline{v^{\prime} t^{\prime}}$ can be measured directly and the scatter will be much smaller. This procedure was tried. However, manufacturing difficulties, related to the fragility of the wire, prevented this approach from being used in the present investigation. As a final conclusion to this study, it can be said that a gold plated wire does not seem to be the right choice in non-isothermal flows, and a further investigation should be carried out.

### 3.13 Qualification of the Apparatus

A series of tests was performed to qualify the basic characteristics of the apparatus and they must be referred to Blackwell [9]. No need of repeating them was necessary because the elapsed time between the present investigation and Blackwell's was very short. However, velocity and
temperature measurements were performed under the same conditions as Blackwell's and excellent agreement was found. Stanton number measurements have been carried out, repeating several of Blackwell's cases and excellent agreement (to less than $1 \%$ ) was obtained. They were conducted mainly as a base line for step in wall temperature conditions.

The tests basically consisted of (1) transpiration energy balance, (2) transverse uniformity of the mean velocity and temperature profiles, (3) boundary layer energy balances, and (4) repeatability of earlier studies by Blackwell [9].

Qualification of the Measurement Procedure for Mean Temperature and Velocity Profiles

The velocity and temperature measurement system was checked on a flat plate case reported earlier by Andersen [8] and Blackwell [9]. The data are compared in Figs. 3.10 and 3.11 , and show the accuracy of the measurement procedure. The cold wall data from Andersen [8] are well matched by the hot wall measurements made with the present technique. Similarly, the temperature profiles obtained with the present hot wire anemometer method agree with Blackwell's [9] thermocouple probe data, obtained using a carefully designed boundary layer thermocouple. Also, the mean velocity profile is seen to be invariant with respect to the temperature field for the low wall to free-stream temperature difference $\left(\approx 25^{\circ} \mathrm{F}\right.$ ) used in this investigation. This conclusion has also been important for the measurement procedure employed in obtaining values of $\overline{v^{\top} t^{\top}}$, described in Section 3.12 .

### 3.15 Qualification of the Procedure for Turbulence Measurements

As described in Appendix B , the procedure for obtaining turbulence quantities requires that $\overline{u^{\top} W^{\top}}$ and $\overline{V^{\top} w^{\top}}$ be zero. A test was carried out by measuring at symmetric positions of the probe. The difference between the signals gave a $1.5 \%$ error in the very neighborhood of the wall and $0.5 \%$ in the fully turbulent region. It can be suggested that there is perhaps a very small three dimensionality in the inner region;
however, this might also be attributed to experimental error, since the accuracy of the RMS meter is $1 \%$ of the measured signal. This test was carried out for all examined flows and was conducted during the measurement of $\overline{v^{\prime} t^{\top}}$ with similar conclusions. The basis for this criterion is discussed in Appendix $B$.

The next step was the measurement of $\overline{u^{\dagger} v^{\top}}$ in a known flow field. The calibration flow was a fully developed channel flow where the turbulent shear stress should be linear in the turbulent core. Figure 3.12 presents the experimental results plotted against the theoretical values. The friction velocity $u_{\tau}$ was obtained directly from pressure drop measurements. The experimental set-up used in this test is the same as described by Hussain [38] and a brief analysis is presented in Appendix C.

Finally, measurements of $\overline{-u^{\prime} v^{\top}} / u_{\tau}^{2}$ and $\overline{v^{\prime} t^{\top}} / u_{\tau} T_{\tau}$ were performed in a flat plate flow and they were found to agree with the values obtained from the mean profiles. This is discussed in Chapter 6.


Fig. 3.1 Schematic of the test apparatus.


Fig. 3.2 Photograph of the test section with a traversing mechanism in position.


Fig. 3.3 Spacing of the plate thermocouples.


LONGITUDINAL CROSS-SECTION OF TUNNEL

Fig. 3.4 A longitudinal cross section of the tunnel test section.


1. Porous plate
2. Heater wires
3. Thermocouples
4. Support webs
5. Honeycomb
6. Thermocouple
7. Base casting
8. Pre-plate
9. Balsa insulation
10. Delivery tube

Fig. 3.5 Cross sectional view of a typical compartment.


Fig. 3.6 The horizontal hot-wire probe.


Fig. 3.7 The rotatable hot-wire probe.


Fig. 3.8 The horizontal hot-wire calibration for
different wire temperatures.


Fig. 3.9 The hot-wire sensitivity to velocity.


Fig. 3.10 Hot-wire measurements of velocity on a flat plate: cold wall vs. hot wall.


Fig. 3.11 Temperature measurements on a flat plate: thermocouple vs. resistance thermometer.


Fig. 3.12 Channel flow shear stress measurements: checking the hot-wire system.

## CHAPTER 4

## ANALYSIS OF STANTON NUMBER DATA

In this study, the analysis of Stanton number data consists of two parts: (1) constant wall temperature case, (2) step in wall temperature case.

For a given set of boundary conditions, the Stanton number data were taken twice to check the repeatability of the data. Also, several of Blackwell's [9] runs were repeated under exactly the same boundary conditions, and an excellent agreement, to within $1 \%$, was found. The uncertainty of the data was estimated to be $3 \%$, at most, by following the same procedure used by Blackwell [9]. For the step in wall temperature runs, the first plate downstream of the step does not give reliable data, due to conduction errors in the Stanton number determination procedure. That data point is listed, however, together with the data of all plates.

The Stanton number data are presented in the form of Stanton number vs. enthalpy thickness Reynolds number, $\operatorname{Re}_{\Delta_{T}}$. Values were measured for each of the 24 test plates, but the data from plates \#1, 2 and 24 will not be presented because of entrance and exit effects. Boundary layer measurements of enthalpy thickness Reynolds number are available for the locations at which the temperature and velocity profiles were measured (six stations). Blackwell [9] suggests that $\operatorname{Re}_{\Delta_{T}}$ values obtained by integration of the two-dimensional boundary layer energy integral equation using the measured Stanton numbers may differ from the values obtained by probing the boundary layer for temperature and velocity profiles, due to streamline convergence or divergence. He has suggested that a better estimate for the enthalpy thickness Reynolds number could be obtained by curve-fitting the data from mean profiles and interpolating for the rest of the plates. This was tried during the present tests. The two procedures had an average difference of about $2 \%$. In the worst case (strong suction) a maximum difference of $16 \%$ in $\operatorname{Re}_{\Delta_{T}}$ was found, which gave a variation of $4 \%$ in $S t$. However, only six temperature and
velocity profiles were taken for each run, and, considering that $\operatorname{Re}_{\Delta_{T}}$ as determined from profiles also has some uncertainty, it was decided to use the values from the energy equation method. Stanton number data were taken twice, and the average value chosen as representative of the flow.

### 4.1 Stanton Numbers for Constant Wall Temperature Conditions

Figure 4.1 shows Stanton number plots for a flat plate case and a strong adverse pressure gradient with different suction rates. From these data, it is concluded that an adverse pressure gradient has only a very small effect on Stanton numbers plotted as a function of $\operatorname{Re}_{\Delta_{\mathrm{T}}}$. No difference is found between flat plate and the strong adverse pressure gradient data, to within the uncertainty of measurements. This same conclusion was also reached by Blackwe11 [9] for mild adverse pressure gradient flows.

Figure 4.2 plots Stanton number normalized by the unblown Stanton number, $S t_{o}$, for the same enthalpy thickness Reynolds number as a function of the blowing parameter. The correlation used by Blackwell [9] and Whitten [3] is shown in the same figure by a solid line. The agreement is observed to be excellent, indicating that an adverse pressure gradient does not affect the Stanton number ratio as a function of the blowing parameter, for the same enthalpy thickness Reynolds number. It can be seen that even with strong suction and blowing the data deviate only slightly from this correlation.

The following expression, due to Whitten [3], is therefore recommended for mild and strong adverse pressure gradients, with transpiration:

$$
\begin{equation*}
\left.\frac{S t}{S t_{o}}\right|_{\operatorname{Re}_{\Delta_{T}}}=\left(\frac{\ln \left(1+B_{h}\right)}{B_{h}}\right)^{1.25}\left(1+B_{h}\right)^{.25} \tag{4.1}
\end{equation*}
$$

where the Stanton numbers are to be evaluated at the same enthalpy thickness Reynolds number. $S t_{o}$ is the Stanton number for zero transpiration and for the pressure gradient in question and $B_{h}=F / S t$. Finally, from the evidence in Fig. 4.1, a flat plate Stanton number correlation could be used if data were not available for the pressure gradient in question.

The Reynolds analogy between heat and momentum transfer, as observed by Blackwell [9], is definitely not valid for adverse pressure gradient flows. While the adverse pressure gradient has only a very small effect on Stanton number, the skin friction coefficient varies over a wide range, as discussed in Chapter 5 (see Fig. 5.11).

The correlation expressed in Eqn. (4.1) was originally developed by Whitten [3] for constant $B_{h}$ flows. The present investigation and Blackwell's [9] have verified it for slowly varying $B_{h}$ : the case of constant $F$ flows. The correlation statement can be written by means of the following expression:

$$
\begin{equation*}
\left.\frac{S t}{S t_{o}}\right|_{\operatorname{Re}_{\Delta_{T}}}=f_{1}\left(B_{h}\right) \tag{4.2}
\end{equation*}
$$

Finally, it is interesting to note that the strongest suction run of this investigation approaches an asymptotic suction larger for $U_{\infty} \approx x^{m}$ flows (Fig. 4-1). The constant property, constant wall temperature energy integral equation can be written as

$$
\begin{equation*}
\frac{\mathrm{d} \Delta \mathrm{~T}}{\mathrm{dx}}=\mathrm{St}+\mathrm{F}-\frac{\mathrm{m} \Delta_{\mathrm{T}}}{\mathrm{x}-\mathrm{x}_{\mathrm{o}}}, \quad \mathrm{~m}<0 \tag{4.3}
\end{equation*}
$$

For large values of $x$, the term $\Delta_{T} /\left(x-x_{0}\right)$ must approach zero, because $\Delta_{T}$ is approaching a constant value, which was verified in the present experiment. Thus, an asymptotic suction layer exists such that St $=-\mathrm{F}$.

### 4.2 Stanton Numbers for Variable Wall Temperature Conditions

The purpose of this section is to present a step solution to be used with the superposition principle for calculating the Stanton number distribution resulting from an arbitrary wall temperature profile.

The superposition principle can be applied because the energy equation for constant-property, low-velocity flows is linear.

The step wall temperature boundary condition can be represented as

$$
\begin{array}{ll}
\Delta \mathrm{T}=0, & \mathrm{x}<\ell, \\
\Delta \mathrm{T}=\Delta \mathrm{T}_{\mathrm{o}}, & \mathrm{x}>\ell, \tag{4.4}
\end{array}
$$

where $\Delta T$ is the difference between the wall and free-stream temperatures and $\ell$ is the unheated starting length along the $x$ direction.

The effect on Stanton number of the unheated starting length can be expressed as the ratio of the non-isothermal to the isothermal Stanton number:

$$
\begin{equation*}
\phi(x ; \ell)=\frac{S t}{S t_{T}} \tag{4.5}
\end{equation*}
$$

This ratio is called the "step wall-temperature function" or "kerne1" solution. It depends upon the flow conditions along the surface. Given this function, the total effect of any variation in wall temperature can be computed by the superposition principle.

In the present investigation, the wall temperature was varied in a stepwise manner and the total temperature difference at the i-th plate, $\Delta T(i)$, can be expressed as the sum of the steps in temperature $\Delta T_{j}$, upstream of the plate:

$$
\begin{equation*}
\Delta T(i)(\text { local })=\sum_{j=1}^{i} \Delta T_{j} \text { (upstream steps in } T_{w a 11} \text { ) } \tag{4.6}
\end{equation*}
$$

where

$$
\begin{align*}
& \Delta T(i)=T_{w}(i)-T_{\infty},  \tag{4.7}\\
& \Delta T_{j}=T_{w}(j)-T_{w}(j-1)
\end{align*}
$$

For this case, the Stanton number correction resulting from all steps in $T_{w}$ upstream of plate (i) is given by:

$$
\begin{equation*}
\phi(i)=\frac{S t}{S t_{T}}=\sum_{j=1}^{i} \phi_{i j} \frac{\Delta T_{j}}{\Delta T(i)} \tag{4,8}
\end{equation*}
$$

where $\phi_{i j}$ is the kernel function, defined by Eqn. (4.5). As applied here, $\phi_{i j}$ is the effect on the Stanton number at plate (i), caused by a step in $T_{w a l l}$ at an upstream plate ( $j$ ). The function $\phi_{i j}$ can also be represented formally as $\phi\left(x_{i} ; \ell_{j}\right)$.

The zero pressure gradient flows have already been well studied, and the expressions below for the kernels follow from Reynolds [37] and Whitten [3].

$$
\begin{align*}
& \phi_{0}(x ; \ell ; 0)= {\left[1-\left(\frac{\ell}{x}\right)^{0.9}\right]^{-1 / 9}, }  \tag{4.9}\\
& \phi(x ; \ell ; F)= \exp \left[\frac{F}{S t_{o_{T}}}\right]-1  \tag{4.10}\\
& \exp \left\langle\frac{F}{S t_{o_{T}}}\left[1-\left(\frac{\ell}{x}\right)^{0.9}\right] 1 / 9\right]-1
\end{align*}
$$

where the subscript $o$ refers to the unblown case, and subscript $T$ to the isothermal case.

The insensitivity to adverse pressure gradients of the Stanton number in the isothermal cases, shown in Section 4.1 , suggested that the kernels would be valid in adverse pressure gradients. Analysis of the present step-wall temperature data showed agreement with these kernel functions to within $5 \%$ (even for the first plate downstream of the step) and two test cases with arbitrary wall temperature variations were chosen. These were mild adverse pressure gradient with no transpiration ( $-0.15,0$ ) and strong adverse pressure gradient with strong suction ( $-0.275,-0.004$ ). Fig. 4.3 shows the wall temperature profiles for the two test cases, while Fig. 4.4 shows the measured and calculated Stanton numbers.

### 4.3 Conclusions Regarding the Stanton Number Behavior

Two conclusions came out of this part of the study:
(1) The adverse pressure gradient, in the flows studied (nearly equilibrium flows), does not seem to have any influence on the relationship between the Stanton number and the enthalpy thickness Reynolds number, at least to within the uncertainty of measurements.
(2) The kernel functions as developed for the transpired flat plate cases with no pressure gradient are excellent approximations for adverse pressure gradient flows. They are, therefore, recommended for predicting Stanton number for an arbitrary wall temperature distribution. The notranspiration, no-pressure gradient Stanton number correlation, well established in the literature, can be used as the unblown Stanton number
in the prediction scheme, and it is recommended according to the following expression:

$$
\begin{equation*}
S t_{0}=0.0154\left(\operatorname{Re}_{\Delta_{T}}\right)^{-0.25} \tag{4.11}
\end{equation*}
$$



Fig. 4.1 Stanton number vs. enthalpy thickness Reynolds number -- strong adverse pressure gradient and flat plate values.


Fig. 4.2 The ratio between the actual Stanton number and its value with no transpiration at the same enthalpy thickness Reynolds number.


Fig. 4.3 Wall-temperature distributions for the variable temperature test cases $(-0.15,0)$ and $(-0.275,-0.004)$.


Fig. 4.4 Comparison of measured and predicted Stanton numbers for the variable wall temperature situations.

## CHAPTER 5

## ANALYSIS OF MEAN TEMPERATURE AND VELOCITY PROFILES

The mean velocity and temperature at each point in a boundary layer were measured sequentially with the same probe, as described in Chapter 3 . Thus, no need exists for assuming a velocity profile from a "corresponding" isothermal flow. The uncertainties in the measurements are estimated to be $\pm 1 \%$ for velocity and $\pm 0.2^{\circ} \mathrm{F}$ for temperature.

The $x$-momentum equation, for incompressible flows, can be written as

$$
\begin{equation*}
\bar{u} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}+v \frac{\partial^{2} \bar{u}}{\partial x^{2}}+v \frac{\partial^{2} \bar{u}}{\partial y^{2}}-\frac{\partial}{\partial x}\left(\overline{u^{\prime}}\right)-\frac{\partial}{\partial y}\left(\overline{u^{\top} v^{\top}}\right) \tag{5.1}
\end{equation*}
$$

The energy equation for low-velocity, constant-property flows can be written as

$$
\begin{equation*}
\bar{u} \frac{\partial T}{\partial x}+\bar{v} \frac{\partial T}{\partial y}=\alpha \frac{\partial^{2} T}{\partial x^{2}}+\alpha \frac{\partial^{2} T}{\partial y^{2}}-\frac{\partial}{\partial x}\left(\overline{u^{\prime} t^{\top}}\right)-\frac{\partial}{\partial y}\left(\overline{v^{\top} t^{\prime}}\right) \tag{5.2}
\end{equation*}
$$

The continuity equation for incompressible flows:

$$
\begin{equation*}
\frac{\partial \bar{u}}{\partial \mathrm{x}}+\frac{\partial \bar{v}}{\partial \mathrm{y}}=0 \tag{5.3}
\end{equation*}
$$

5.1 Behavior of $\overline{u^{\prime} v^{\prime}}$ and Its Derivatives in the Region Very Close
to the Wall
Expanding $u^{\prime}$ and $v^{\prime}$ in a Taylor series about $y=0$,

$$
\begin{align*}
& u^{\prime}=\left.u^{\prime}\right|_{0}+\left.\frac{u^{\prime}}{y}\right|_{0} y+\left.\frac{\partial^{2} u^{\prime}}{\partial y^{2}}\right|_{0} \frac{y^{2}}{2}+\ldots,  \tag{5,4}\\
& v^{\prime}=\left.v^{\prime}\right|_{0}+\left.\frac{\partial v^{\prime}}{\partial y}\right|_{0} y+\left.\frac{\partial^{2} v^{\prime}}{\partial y^{2}}\right|_{0} \frac{y^{2}}{2}+\ldots, \tag{5.5}
\end{align*}
$$

but $\left.u^{\prime}\right|_{0}=0$ and $\left.v^{\prime}\right|_{0}=0$, and from continuity

$$
\begin{equation*}
\left.\frac{\partial u^{\prime}}{\partial x}\right|_{0}+\left.\frac{\partial v^{\prime}}{\partial y}\right|_{0}=0 \tag{5.7}
\end{equation*}
$$

so

$$
\begin{equation*}
\left.\frac{\partial v^{\prime}}{\partial y}\right|_{0}=0 \tag{5.8}
\end{equation*}
$$

Therefore,

$$
\begin{align*}
u^{\prime} & =\left.\frac{\partial u^{\prime}}{\partial y}\right|_{0} y  \tag{5.9}\\
v^{\prime} & =\left.\frac{\partial^{2} v^{\prime}}{\partial y^{2}}\right|_{0} \frac{y^{2}}{2} \tag{5.10}
\end{align*}
$$

So

$$
\begin{equation*}
\overline{u^{\prime} v^{\prime}}=\left.\overline{\frac{\partial u^{\prime}}{\partial y} \frac{\partial^{2} v^{\prime}}{\partial y^{2}}}\right|_{0} \frac{y^{3}}{2}+\ldots . \tag{5.11}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
\left.\frac{\partial u^{\prime} v^{\prime}}{\partial y}\right|_{0}=0 \tag{5.12}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\frac{\partial^{2} \overline{u^{\prime} v^{\prime}}}{\partial y^{2}}\right|_{0}=0 \tag{5.13}
\end{equation*}
$$

5.2 Behavior of $\overline{v^{\prime} t^{\prime}}$ and Its Derivatives in the Region Very Close to the Wall
Expanding $t^{\prime}$ in a Taylor series, about $y=0$,

$$
\begin{equation*}
t^{\prime}=\left.t^{\prime}\right|_{0}+\left.\frac{\partial t^{\prime}}{\partial y}\right|_{0} y+\left.\frac{\partial^{2} t^{\prime}}{\partial y}\right|_{0} \frac{y^{2}}{2}+\ldots ; \tag{5.14}
\end{equation*}
$$

but $\left.t^{\prime}\right|_{o}=0$, so

$$
\begin{equation*}
t^{\prime}=\left.\frac{\partial t^{\prime}}{\partial y}\right|_{0} y+\left.\frac{\partial^{2} t^{\prime}}{\partial y^{2}}\right|_{0} \frac{y^{2}}{2} \tag{5.15}
\end{equation*}
$$

Multiplying by (5.10) and time averaging,

Therefore,

$$
\begin{equation*}
\left.\frac{\partial \overline{v^{\prime} t^{\prime}}}{\partial y}\right|_{0}=0 \tag{5.17}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\frac{\partial^{2} \overline{v^{\prime} t^{\prime}}}{\partial y^{2}}\right|_{0}=0 \tag{5.18}
\end{equation*}
$$

5.3 Validity of the Couette Flow Assumption in the Region Very Near the Wall
Expanding $\bar{u}$. in a Taylor series, about $y=0$,

$$
\begin{equation*}
\bar{u}=\left.\bar{u}\right|_{0}+\left.\frac{\partial \bar{u}}{\partial y}\right|_{0} y+\left.\frac{\partial^{2} \bar{u}}{\partial y^{2}}\right|_{0} \frac{y^{2}}{2}+\left.\frac{\partial^{2} \bar{u}}{\partial y^{3}}\right|_{0} \frac{y^{3}}{6}+0\left(y^{4}\right) \tag{5.19}
\end{equation*}
$$

but

$$
\begin{align*}
\left.\bar{u}\right|_{0} & =0  \tag{5.20}\\
\left.\frac{\partial \bar{u}}{\partial y}\right|_{0} & =\frac{u_{\tau}^{2}}{v} . \tag{5.21}
\end{align*}
$$

Evaluating the terms in the $x$-momentum equation, at $y=0$, yields

$$
\begin{equation*}
\left.\frac{\partial^{2} \bar{u}}{\partial y^{2}}\right|_{0}=v_{o} \frac{u_{\tau}^{2}}{v^{2}}+\left.\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}\right|_{0} \frac{1}{v} \tag{5.21a}
\end{equation*}
$$

Differentiating the $x$-momentum equation with respect to $y$, and evaluating the terms at $y=0$,

$$
\begin{align*}
\left.\frac{\partial^{3-u}}{\partial y^{3}}\right|_{0}= & \frac{v}{v}\left[\frac{v_{0} u^{2} \tau}{v^{2}}+\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} \frac{1}{v}\right]+\frac{1}{v} \frac{\partial}{\partial y}\left(\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}\right)- \\
& -\frac{\partial}{\partial y}\left(\frac{\partial^{2} \bar{u}}{\partial x^{2}}\right)_{0}+\frac{\partial}{\partial y}\left(\frac{\partial u^{\prime 2}}{\partial x}\right)_{0} . \tag{5.22}
\end{align*}
$$

Thus,

$$
\begin{align*}
\bar{u}= & \frac{u_{\tau}^{2} y}{v}+\left[v_{0} \frac{u_{\tau}^{2}}{v^{2}}+\left.\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}\right|_{0}\right]^{2} \frac{y^{2}}{2}+\left\{\frac{v_{0}}{v}\left[\frac{v_{0} u_{\tau}^{2}}{v^{2}}+\left.\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}\right|_{0} \frac{1}{v}\right]+\right.  \tag{5.23}\\
& +\frac{1}{v} \frac{\partial}{\partial y}\left(\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}\right)_{0}-\frac{\partial}{\partial y}\left(\frac{\partial^{2} \bar{u}}{\partial x^{2}}\right)_{0}+\frac{\partial}{\partial y}\left(\frac{\partial u^{\prime 2}}{\partial x}\right)_{0} \frac{y^{3}}{6}+\ldots .
\end{align*}
$$

The conclusion is that the Couette flow assumption is valid in general to a second-order approximation very close to the wall and to a third-order approximation within the frame of the boundary layer assumptions. The expression for velocity can be written, therefore, in terms of dimensionless coordinates as

$$
\begin{equation*}
\mathrm{u}^{+}=\mathrm{y}^{+}+\left(\mathrm{p}^{+}+\mathrm{v}_{\mathrm{o}}^{+}\right) \frac{\mathrm{y}^{+^{2}}}{2}+\mathrm{v}_{\mathrm{o}}^{+}\left(\mathrm{v}_{\mathrm{o}}^{+}+\mathrm{p}^{+}\right) \frac{\mathrm{y}^{+^{3}}}{6}+0\left(\mathrm{y}^{+}\right) \tag{5.24}
\end{equation*}
$$

Terms depending on the fluctuating quantities like $\overline{u^{\top} v^{\top}}$ and its derivatives are of higher orders.

The same conclusions can be reached for the temperature field, with respect to $\overline{\mathrm{v}^{\top} \mathrm{t}^{\top}}$, and in dimensionless coordinates the temperature profile can be represented by

$$
\begin{equation*}
\mathrm{T}^{+}=\operatorname{Pr} \mathrm{y}^{+}+\mathrm{v}_{\mathrm{o}}^{+} \operatorname{Pr}^{2} \frac{\mathrm{y}^{+^{2}}}{2}+\mathrm{v}_{\mathrm{o}}^{+^{2}} \operatorname{Pr}^{3} \frac{\mathrm{y}^{+^{3}}}{6}+\mathrm{o}\left(\mathrm{y}^{+^{4}}\right) \tag{5.25}
\end{equation*}
$$

### 5.4 The Location of the First Data Point with Respect to the Wall

As seen in Chapter 3, the first data point position was measured by means of an optical comparator and by assuming that the same conditions of the measurement would hold during the data-taking procedure. Actually, this is not quite true, because small imperfections in the probe alignment can slightly change this distance. The uncertainty is estimated to be 0.001 in., but even this small variation can greatly change the
profile in the region very near the wall. Blackwell [9] assumed that the Couette flow assumption was valid, and determined the position of the first point; he then shifted all others by the same amount. This can be a good procedure if Stanton number, as well as the local temperature, the wall temperature, and the free-stream temperature are each known with good accuracy, and the fluid properties are known, providing that the Couette flow assumption is really valid. This constitutes no problem, since a probing system is usually able to get very close to the wall. However, with so many uncertainties in the flow parameters, it is not really possible to decide whether the difference with respect to the Couette flow solution is due to uncertainties in the $y$-position or the flow parameters. Blackwell [9] had typically to shift upwards the position of his first data point by 0.0015 in. Analysis of the present data seems to indicate that the same trend would have to be followed.

It was decided, however, to keep the nominal value (as measured by the optical comparator) as representative of the position of the first point.

The velocity profile was corrected, in the neighborhood of the wall, for heat conduction losses from the probe according to Repik [35]. Typically, the correction amounted to $0.2 \mathrm{ft} / \mathrm{sec}$ at most.

The present procedure for measuring velocity and temperature sequentially gives a very smooth and accurate functional dependence of temperature upon the velocity, for the same y-position. In applications like the measurement of the turbulent Prandtl number where the ratio $(\partial T / \partial y) /(\partial \bar{u} / \partial y)$ has to be known, the direct differentiation $\partial T / \partial \bar{u}$ can be shown to be much more accurate, as far as numerical errors in the differentiation procedure are concerned.

### 5.5 Mean Velocity Profiles

As seen from Eqn. (5.24), the dimensionless velocity $u^{+}$is a function not only of $\mathrm{y}^{+}$, but also $\mathrm{v}_{\mathrm{o}}^{+}$and $\mathrm{p}^{+}$. If the same set of dimensionless coordinates is used to plot the data in the logarithmic region for all values of $\mathrm{v}_{\mathrm{o}}^{+}$and $\mathrm{P}^{+}$, no similarity between the profiles at different $x$-stations would be expected, since $\mathrm{v}_{\mathrm{o}}^{+}$and $\mathrm{P}^{+}$vary from one station to the other.

Similarity may be recaptured by examining so-called "equilibrium flows." The concept of equilibrium layers as introduced by Clauser is related to an outer region similarity. This usually is referred to in graphs of mean velocity profiles shown in terms of non-dimensional defect velocity $\left(U_{\infty}-\bar{u}\right) / u_{\tau}$ and $y / \Delta_{2}$, where

$$
\Delta_{2}=\delta \int_{0}^{1}\left[\frac{\left(\mathrm{U}_{\infty}-\overline{\mathrm{u}}\right)}{\mathrm{u}_{\tau}}\right]^{2} \mathrm{~d}(\mathrm{y} / \delta)
$$

is the so-called Clauser boundary layer thickness.
Figures $5.1,5.3,5.5,5.7$ show, in these coordinates, the development of the turbulent boundary layer under a strong adverse pressure gradient with different transpiration rates. It can be seen that similarity holds, approximately, for the outer $70 \%$ of the boundary layer and that suction tends to expand this range.

Figure 5.9 presents the Clauser shape factor $G$, defined as

$$
G=\Delta_{2} / \Delta
$$

where

$$
\Delta=\delta \int_{0}^{1}\left(\frac{U_{\infty}-\bar{u}}{u_{\tau}}\right) d(y / \delta)
$$

for different $x$-stations and transpiration rates. As one can see, the desired "nearly equilibrium" condition is not achieved for all of the flows, since $G$ is not constant with each flow. If one refers back to Figs. 5.1, 5.3, 5.5, 5.7, it can be seen that no inner region similarity exists, under the present conditions.

From these observations, plus the variation of $G$, one can deduce that the non-similarity of the inner region affects the value of $G$. The Clauser shape factor, $G$, is the resultant of an integration from the wall throughout the entire boundary layer. The velocity defect coordinates are appropriate only in the outer region. In flat-plate or mild adverse pressure gradients, the region of usefulness of the defect coordinates include the outer $90 \%$ of the boundary layer thickness and the contribution of the inner region to the value of $G$ is thus negligible.

In strong adverse pressure gradients, however, the defect coordinate similarity holds only for the outer $70 \%$ of the boundary layer thickness, and the contribution of the inner region is not negligible. Thus we find relatively good outer similarity but, at the same time, a nonconstant value of $G$.

### 5.6 Mean Temperature Profiles

Equation (5.25) shows that the dimensionless temperature profile $\mathrm{T}^{+}$ is not a direct function of $\mathrm{P}^{+}$. It is expected, therefore, that similarity, in defect coordinates, will hold over a larger portion of the boundary layer for the temperature profile than for the velocity profile. Figs. 5.2, 5.4, 5.6, 5.8 show the temperature defect plotted versus $y / \Delta_{3}$ (defined below). The data confirm the expectations. The $G_{h}$ shape factor, defined by Blackwell [9], can be written in terms of the temperature profile for a low-velocity, constant property flow as

$$
\begin{gather*}
G_{h}=\frac{\Delta_{4}}{\Delta_{3}}  \tag{5.26}\\
\Delta_{4}=\delta_{T} \int_{0}^{1}\left[\frac{\left(T_{\infty}-T\right)}{T_{\tau}}\right]^{2} d\left(y / \delta_{T}\right)  \tag{5.27}\\
\Delta_{3}=\delta_{T} \int_{0}^{1}\left(\frac{T_{\infty}-T}{T_{\tau}}\right) d\left(y / \delta_{T}\right) \tag{5.28}
\end{gather*}
$$

Because of the extended similarity in the temperature case compared to the velocity case, the shape factor $G_{h}$ is expected to have a much smaller scatter than $G$. This can be seen in Fig. 5.10. It can also be observed that this shape factor is not a function of the transpiration rate for the examined flows.

### 5.7 Determination of the Friction Coefficients

Friction coefficients were determined by Andersen's [8] shear stress method. The time-averaged $x$-momentum equation (boundary layer assumption) can be written as

$$
\begin{equation*}
\bar{u} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}+\nu \frac{\partial^{2} \bar{u}}{\partial y^{2}}-\frac{\partial}{\partial y}\left(\overline{u^{\prime} v^{\prime}}\right) \tag{5.29}
\end{equation*}
$$

Integration of the equation from $y=0$ to a point $y$ somewhere in the inner region gives

$$
\begin{equation*}
\int_{0}^{y} \bar{u} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y} d y=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x}+v \frac{\partial \bar{u}}{\partial y}-u_{\tau}^{2}-\overline{u^{\prime} v^{\prime}} . \tag{5.30}
\end{equation*}
$$

The friction coefficient can be taken directly from the definition of $u_{\tau}^{2}=U_{\infty}^{2}\left(C_{f} / 2\right)$, and is given by

$$
\begin{equation*}
C_{f} / 2=\frac{\nu}{U_{\infty}^{2}} \frac{\partial \bar{u}}{\partial y}-\frac{\overline{u^{\top} v^{\top}}}{U_{\infty}^{2}}-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} \frac{y}{U_{\infty}^{2}}-\frac{1}{U_{\infty}^{2}} \int_{0}^{y} \bar{u} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y} d y \tag{5.31}
\end{equation*}
$$

Once $\overline{u^{\prime} v^{\top}}$ is measured at the particular $y$ position, and mean velocity profiles have been taken at different $x$ stations, then $C_{f} / 2$ can be easily determined.

The friction coefficients were also determined by Andersen's law of the wall [8], but these values were found to be lower than the directly measured values. It is difficult to ascertain the cause of this discrepancy, but three candidates were examined. The first was the effect of three-dimensionality, not included in Eqn. (5.31). However, tests for 3-D effects were performed, as described in Chapter 3, Section 3.15, and the conclusion was that 3-D effects could be neglected. The second is related to the difficulty in determining the logarithmic region and its slope, when using Andersen's [8] law of the wall. The uncertainty can be high. Finally, during the course of the qualification procedure for the turbulent measurements, the skin friction was also measured for a zero pressure gradient, no transpiration case. Again, Andersen's law gave lower values, while the measured one was very close to Simpson's [2]. The measured skin friction was then preferred over the calculated one from Andersen's law of the wall.

Mean velocity and temperature profiles were taken at six stations for each run and the corresponding friction factor determined. Care was taken to avoid measurements in regions where the profile was not completely developed for the desired conditions (beginning of the test
section), or in regions where end effects of the test section could influence the profile (end of the test section). Analysis of the data, however, shows that in the $(-0.275,0.000)$ run the profile is not completely developed at the first measured station ( $x=22$ in.); the last station ( $x=82 \mathrm{in}$. ), for each run, may still be disturbed by end effects.

Equation (5.31) for the friction coefficient can be cast in another form. If one uses the continuity Eqn. (5.3), the integral that appears in Eqn. (5.31) can be arranged so one gets (for constant density):

$$
\begin{align*}
\frac{C_{f}}{2}= & K \frac{U_{\infty} y}{v}+\frac{1}{U_{\infty}^{2}}\left(v \frac{\partial \vec{u}}{\partial y}-\overline{u^{1} v^{\top}}\right)-\frac{1}{U_{\infty}^{2}} \frac{\partial}{\partial x} \int_{0}^{y} \bar{u}^{2} d y \\
- & \frac{\bar{u} v_{o}}{U_{\infty}^{2}}+\frac{1}{U_{\infty}^{2}} \frac{\partial}{\partial x} \int_{0}^{y} \bar{u} d y \tag{5.32}
\end{align*}
$$

Table E-1 shows the friction coefficients for the different runs of this investigation and, for comparison, the contribution of each term of Eqn. (5.32).

Figure 5.11 plots the friction coefficients as a function of momentum thickness Reynolds number. If we recall the discussion on Section 4.1, one can see that the friction coefficients are strongly influenced by the adverse pressure gradient and vary over a much wider range than do the Stanton numbers.


Fig. 5.1 Defect velocity profiles at different x-stations ( $-0.275,0$ ).


Fig. 5.2 Defect temperature profiles at different x-stations (-0.275, 0).


Fig. 5.3 Defect velocity profiles at different x-stations (-0.275, -0.001).


Fig. 5.4 Defect temperature profiles at different x-stations (-0.275, -0.001).


Fig. 5.5 Defect velocity profiles at different x-stations
(-0.275, $=0.002$ ).


Fig. 5.6 Defect temperature profiles at different x-stations (-0.275, -0.002).


Fig. 5.7 Defect velocity profiles at different x-stations (-0.275, -0.004).


Fig. 5.8 Defect temperature profiles at different x-stations (-0.275, -0.004).


Fig. 5.9 Clauser shape factors for velocity as a function of $x$ for different suction rates.


Fig. 5.10 Temperature shape factors as a function of $x$ for different suction rates.


Fig. 5.11 Friction factor as a function of momentum thickness Reynolds number for strong adverse pressure gradient with suction.

## CHAPTER 6

## ANALYSIS OF THE TURBULENCE MEASUREMENTS

### 6.1 Hydrodynamics

Measurements of the turbulence quantities have previously been reported for zero pressure gradient boundary layers and completely developed pipe flows. Both spectral and Reynolds stress measurements are available for these cases. This investigation is concerned with Reynolds stress measurements in boundary layer flows subject to an adverse pressure gradient.

The zero pressure gradient flow has been well studied and several sets of measurements are available for reference. Klebanoff [31], for example, is considered a reliable source for a flat plate flow. Comparison is most fruitfully made in terms of similarity variables, as discussed in the following section.

### 6.1.1 Similarity variables for turbulence measurements

The $x$-momentum equation for a boundary layer flow can be written as

$$
\begin{equation*}
\bar{u} \frac{\partial \bar{u}}{\partial x}+\bar{v} \frac{\partial \bar{u}}{\partial y}=-\frac{1}{\bar{p}} \frac{\partial \bar{p}}{\partial x}+v \frac{\partial^{2} \bar{u}}{\partial y^{2}}-\frac{\partial}{\partial y}\left(\overline{u^{1} v^{1}}\right) \tag{6.1}
\end{equation*}
$$

In the inner region, however, the $x$-convective terms can be dropped and many experiments have shown that there is similarity of the mean flow when the following dimensionless coordinates are used:

$$
\begin{equation*}
u^{+}=\frac{\bar{u}}{u_{\tau}} \quad \text { and } \quad y^{+}=\frac{y u_{\tau}}{v} \tag{6.2}
\end{equation*}
$$

where

$$
u_{\tau}=U_{\infty} \sqrt{C_{f} / 2}
$$

Therefore, equation (6.1) can be written as:

$$
\begin{equation*}
v_{o}^{+} \frac{d u^{+}}{d y^{+}}=-\mathrm{P}^{+}+\frac{d^{2} u^{+}}{d y^{+}}-\frac{d}{d y^{+}} \frac{\overline{u^{\prime} v^{+}}}{u_{\tau}^{2}} \tag{6.3}
\end{equation*}
$$

when the Couette flow assumption is used. This means that if there is. to be inner region similarity of the mean flow field in terms of the above variables, the shear stress, $\overline{u^{\prime} v^{\prime}}$, must be normalized on $u_{\tau}^{2}$, and the result should be a function only of $y^{+}$in the inner region. As an extension to this similarity concept, the following dimensionless turbulent quantities, as functions of $\mathrm{y}^{+}$, are offered as representative of flows which are similar in the inner region of a boundary layer:

$$
\begin{equation*}
\frac{\sqrt{u^{\prime 2}}}{u_{\tau}}, \quad \frac{\sqrt{v^{\prime 2}}}{u_{\tau}}, \quad \frac{\sqrt{w^{\prime 2}}}{u_{\tau}}, \quad \frac{-\overline{u^{\prime} v^{\prime}}}{u^{2}} \tag{6.4}
\end{equation*}
$$

In the outer region, when there is a mean flow field similarity, the following variables are used:

$$
\begin{equation*}
\frac{\bar{u}-U_{\infty}}{u_{\tau}} \quad \text { and } \quad y / \delta \tag{6.5}
\end{equation*}
$$

A simflar reasoning would lead to analogous dimensionless turbulence quantities, as a function of $y / \delta$, in the outer region of the boundary layer.

The dimensionless quantities (6.4) are used in this investigation for plotting the experimental data.

### 6.1.2 Comments on the measurement of turbulence quantities

The measurement of $\overline{u^{\prime} v^{\top}}$ is usually done by means of two slant wires placed in a plane perpendicular to the wall and parallel to the mean flow direction. In regions of steep velocity gradients, it is expected that the non-uniform velocity distribution along each wire may cause some errors in the measurements, an effect which has been neglected by most investigators. Watts [13] measured $\overline{u^{\prime} v^{\prime}}$ with two slant wires located in planes parallel to the mean flow direction and banked $45^{\circ}$ with respect to the wall. His measurements of $\overline{u^{\prime} v^{\prime}}$ were found to be higher
$(\approx 6 \%)$ than the ones obtained by the usual procedure, with the wires in a plane perpendicular to the wall. Andersen [8] used just one rotatable slant wire, with the wire in a plane perpendicular to the wall. The present investigation used only a rotatable slant wire, with the measurements made in a plane inclined $60^{\circ}$ with the wall, similar to that of Watts [13]. The present measurements of friction coefficient for a zero pressure gradient flow were found to be higher than Andersen's [8] by $8 \%$, indicating again that the measured $\overline{u^{\prime} v^{\top}}$ may be a function of wire orientation. A test was carried out by Pimenta [40], using the single rotatable wire procedure and the same difference was noticed. A further investigation on the hot-wire response is therefore necessary to resolve this difference. It is believed, however, that the banked position is the best one for this kind of measurement. Actually, the closest possible to the plane of constant velocity is recommended because the nonuniformity of the velocity distribution along the wire would be smaller. The sensitivity of the wire to the normal velocity, however, would be smaller, which limits the accuracy of the hot wire for $\overline{u^{\prime} v^{\top}}$ measurement. The same kind of errors would also be present in the measurement
of $\mathrm{v}^{2}$ and $\mathrm{w}^{2}$.

### 6.1.3 The zero pressure gradient flow

This flow has received a great deal of attention from investigators because of its relative simplicity. A few conclusions can be inferred from several experiments.
(1) The free stream turbulence intensity strongly influences the turbulence structure of the outer region of the boundary layer. Figure 6.1 shows a comparison between Klebanoff's [31] and the higher turbulence intensity data of the present investigation. The same effect was also noted, for accelerated flows, by Kearney [39]. The mean velocity field, however, does not seem to be much influenced by the turbulence. Sharan [41] discusses cases where the mean velocity is the same but the turbulence profiles are different. This suggests that the turbulence profiles should be functions of the free stream turbulence level and not simply functions of the mean velocity profile, as implied by the mixing-length theory.
(2) The fluctuation field seems to extend far beyond the edge of the momentum boundary layer (based on the mean velocity). Analysis of Watts' [13] and Klebanoff's [31] data shows that the free stream turbulence level is reached at $y / \delta \approx 1.4$. The observation is confirmed by the present investigation. This would suggest, therefore, that there is a region of the boundary layer which is characterized by the existence of turbulence in the absence of significant mean velocity deficit.
(3) The stream-wise normal velocity correlation, $-\bar{u}^{\top} v^{\top} / \sqrt{\frac{u^{\prime 2}}{}} \sqrt{\frac{v^{\prime 2}}{}}$ is found to be approximately equal to the Karman constant ( $\mathcal{K}=0.41$ 0.44 ) in the outer region of the boundary layer $(0.2<y / \delta<0.8)$. Klebanoff [31] reports a value in the neighborhood of 0.5. (4) The ratio between the turbulent shear stress $\overline{-u^{\top} v^{\top}}$ and the kinetic energy of turbulence in the outer region of the boundary layer is found to be approximately constant and equal to 0.14 , as already observed by other investigators (i.e., Townsend [42] and Bradshaw [43]).

### 6.1.4 Adverse pressure gradient flows

In the present study, measurements have been taken for all Reynolds stress tensor components in adverse pressure gradient flows with varying amounts of suction.

Comparisons to zero pressure gradient and mild adverse pressure gradient flows show that the production of turbulence increases when the pressure gradient increases. This can be verified by measuring the ratio between the rms value of the longitudinal velocity fluctuation and the local mean velocity, $\sqrt{u^{u^{2}}} / \bar{u}$.

A plot of $\sqrt{\overline{u^{\prime 2}}} / \mathrm{u}_{\tau}$ shows that the turbulence level profile has two peaks. The second one is located in the outer region of the boundary layer $(y / \delta \approx 0.5)$ and can be seen in Fig. 6.2. Suction is observed to suppress the outer peak.

It is observed that when suction increases, the outer peak is reduced in magnitude much more than the inner one. This suggests that a mechanism is present which inhibits the diffusion of turbulent kinetic energy from the inner to the outer region of the boundary layer. This can be also verified by comparison to the zero pressure gradient and
mild pressure gradient data of Andersen [8] and the strong adverse pressure gradient data of Bradshaw [43]. The outer layer peak is observed to be displaced outwards when the pressure gradient increases. This fact, associated with the observation that the production of turbulence is larger for a higher adverse pressure gradient, suggests that the diffusion of turbulent kinetic energy by pressure fluctuation may be important in the formation of the outer layer peak. The $y$-momentum boundary layer equation can be integrated to give the following expression:

$$
\begin{equation*}
\bar{p}_{o}=\bar{p}+\rho \overline{v^{\prime 2}} \tag{6.6}
\end{equation*}
$$

Suction reduces the turbulence level and $\overline{v^{\prime 2}}$ becomes smaller. As a consequence, the local static pressure approaches the free stream static pressure and the local pressure fluctuations become smaller. In flows of relatively low turbulence level, the pressure fluctuation term is not probably large enough to generate a second peak.

Although measurements have been taken by other investigators for adverse pressure gradient flows, like, for example, Andersen [8] and Bradshaw [43], no mention of the presence of the second peak was found in the literature. It seems that a better understanding of this phenomenon would help to improve the predictive capability of turbulent flows. Figs. $6.3,6.4$, and 6.5 show, respectively, profiles for $\sqrt{\frac{v^{2}}{}} / u_{\tau}$, $\sqrt{\bar{w}^{i / 2}} / u_{\tau}$, and the dimensionless shear stress $-\overline{u^{\prime} v^{\top}} / u_{\tau}^{2}$. It can be concluded that a turbulence model used for predicting $\overline{u^{\prime} v^{\prime}}$ must not rely solely on the mean velocity field dependence. The peak of the $\overline{u^{\prime}} v^{\prime}$ profile in the outer region of the boundary layer cannot be explained by a mean field hypothesis, nor can the effects of the free stream turbulence. Some acknowledgment must be made of the turbulence. Many different approaches are possible. The Prandtl-Kolmogorov model, for example, uses both the turbulent kinetic energy and the mean velocity gradient as descriptors of the shear stress $\overline{u^{\top} v^{\prime}}$, as follows:

$$
\begin{equation*}
-\overline{u^{\prime} v^{\prime}}=a \ell \sqrt{q^{2}} \frac{d \bar{u}}{d y} \tag{6.7}
\end{equation*}
$$

It has the potentiality of simulating, at least qualitatively, the turbulent process, when the mean flow pressure gradient is not zero.

For zero pressure gradient boundary layer flows, the use of an equation for the turbulent kinetic energy has been demonstrated to result in only a marginal improvement in mean velocity prediction. It seems likely, however, that its influence will be noticed more clearly when predicting flows under strong adverse pressure gradient and transpiration rate conditions, like some of Andersen's [8] flows.

Figures 6.6 and 6.7 show the correlation coefficients between the longitudinal and normal velocity components. The mild adverse pressure gradient of Andersen [8] and the strong one of this investigation demonstrate that, for equilibrium pressure gradients (defined as flows which have outer-region similarity), the correlation coefficient is approximately the same value as the Karman constant $K(0.41-0.44)$. This indicates, therefore, similarities in the turbulent transport of momentum for these different equilibrium flows.

Figures 6.8 and 6.9 show the ratio between the shear stress and the kinetic energy of turbulence. For equilibrium adverse pressure gradient flows, it is concluded that in the outer region of the boundary layer an approximately constant value of 0.14 is appropriate. This fact has already been observed by Bradshaw [44], who employed a calculation procedure based on the constancy of this ratio for numerically predicting the mean flow field.

### 6.2 Temperature

Measurements of the temperature fluctuations and the turbulent heat transfer do not seem to be common in the literature. The lack of investigations in this area is due mainly to an experimental difficulty which makes the measurements complicated and time-consuming. Among the small number of measurements in the literature, only a few deal with $\overline{v^{\top} t^{\top}}$ measurements, while several report $\overline{u^{\top} t^{\top}}$ data.

### 6.2.1 Similarity variables for temperature and heat flux measurements

The energy equation for a low-velocity, constant property turbulent boundary layer flow can be written as

$$
\begin{equation*}
\bar{u} \frac{\partial T}{\partial x}+\bar{v} \frac{\partial T}{\partial y}=\alpha \frac{\partial^{2} T}{\partial y^{2}}-\frac{\partial}{\partial y}\left(\overline{v^{\prime} t^{\prime}}\right) \tag{6.8}
\end{equation*}
$$

In the inner region, experiments have shown that there is a similarity of the mean temperature field when the following coordinates are used:

$$
\begin{equation*}
y^{+}=\frac{y u_{\tau}}{\nu} \text { and } T^{+}=\frac{T_{w}-T}{T_{\tau}} \text {, } \tag{6.9}
\end{equation*}
$$

where $T_{\tau}=\left(T_{w}-T_{\infty}\right) S t / \sqrt{C_{f} / 2}$.
Equation (6.8) can be written as

$$
\begin{equation*}
\mathrm{v}_{\mathrm{o}}^{+} \frac{\mathrm{dT}^{+}}{\mathrm{dy}^{+}}=\frac{1}{\mathrm{Pr}} \frac{\mathrm{~d}^{2} \mathrm{~T}^{+}}{\mathrm{dy}^{+}{ }^{2}}-\frac{\mathrm{d}}{\mathrm{dy}^{+}} \frac{\overline{\mathrm{v}^{\top} \mathrm{t}^{\top}}}{\mathrm{u}_{\tau^{T} \tau}} \tag{6.10}
\end{equation*}
$$

when the Couette flow assumption is used. If there is similarity of the mean temperature field in terms of the above variables, then the velocitytemperature correlation must be normalized on the product $u_{\tau} T_{\tau}$. Further, from the fact that the mean temperature field is normalized on $T_{\tau}$, the rms value of the temperature fluctuation should also be normalized on $T_{\tau}$, leading to the set of dimensionless variables:

$$
\begin{equation*}
\frac{\sqrt{\frac{t^{\prime 2}}{}}}{T_{\tau}}, \frac{\overline{v^{\prime} t^{\prime}}}{u_{\tau} T_{\tau}}, \quad \frac{T_{w}-T}{T_{\tau}}, \quad \text { and } \quad \frac{y u_{\tau}}{\nu} \text {, } \tag{6.11}
\end{equation*}
$$

A similar reasoning would lead to analogous dimensionless turbulence quantities, as a function of $y / \delta_{T}$, in the outer region of the boundary layer. However, it seems natural to compare the velocity and the temperature field, and this can be done more easily if $y / \delta$ (boundary thickness for the velocity field) is used as the independent variable. Naturally, this can only have a meaning when the momentum layer is approximately as thick as the thermal layer.

### 6.2.2 The zero pressure gradient flow

Only one reference was found in the literature for the measurement of $\sqrt{\frac{t^{\prime 2}}{}}$, when the virtual origin of the momentum and thermal layers nearly coincide -- Fulachier and Dumas [26]. Fig. 6.10 shows a comparison between the flat plate measurements of this investigation and those of Fulachier and Dumas [26]. It can be seen that their values are higher in the outer region. It is not clear whether the comparison was made in inappropriate dimensionless variables or whether the free stream temperature turbulence level was different for the two experiments.

Figure 6.11 shows the dimensionless temperature fluctuation profile compared with the dimensionless longitudinal velocity fluctuation profile. It can be seen that the temperature peaks at a larger distance from the wall than does the velocity. The ratio is approximately 1.4 or the inverse of the molecular Prandtl number, if a generalization can be made upon this one observation. This fact naturally suggests that the thickness of the thermal sublayer is larger than the momentum one by the same amount. This is not totally unexpected; the mean values of the dimensionless temperature and velocity profiles in the sublayer can be written as

$$
\begin{equation*}
\mathrm{u}^{+}=\mathrm{y}^{+}, \quad \mathrm{T}^{+}=\operatorname{Pr} \mathrm{y}^{+} \tag{6.12}
\end{equation*}
$$

Experiments have shown that the thermal sublayer extends farther away from the wall than the momentum sublayer. Eq. 6.12 , therefore, suggests that the ratio between the sublayer thicknesses may be the molecular Prandtl number.

It is suggested, therefore, that any numerical scheme used for predicting the mean temperature profile and employing the sublayer thickness as a parameter (like van Driest's for the hydrodynamics), should have the temperature field related to the velocity field as above.

Cebeci [45] studied such a model for air $(\operatorname{Pr}=0.73)$, and, by adjusting the value of the thermal sublayer thickness $B^{+}$until the mean temperature profile could be predicted reasonably, obtained the following value:

$$
\begin{equation*}
\mathrm{B}^{+} \approx 35 \text {. } \tag{6.13}
\end{equation*}
$$

This compares favorably to the momentum sublayer thickness $A^{+}=26$, as above.

The normalized turbulent heat transfer $\overline{v^{\prime} t^{\top}} / u_{\tau} T_{\tau}$ was measured in this study for a constant wall temperature condition. No reference was found to compare exactly with the experimental data, although Johnston [20] and Blom [24] investigated the case of a stepwise discontinuity in wall temperature. The present data had to be evaluated by comparison with expected values obtained from mean temperature measurements. Equation (6.10) can be integrated from $y^{+}=0$ to a point in the logarithmic region of the boundary layer to give

$$
\begin{equation*}
v_{o}^{+} T^{+}=\frac{I}{P r} \frac{\mathrm{dT}^{+}}{d y^{+}}-1+\frac{\overline{v^{\prime} t^{\prime}}}{u_{\tau}^{T} \tau} \tag{6.14}
\end{equation*}
$$

For no transpiration $\left(v_{0}^{+}=0\right)$ and for a point in the logarithmic region where the $y$-derivative of the temperature is small, we should find

$$
\begin{equation*}
\frac{\overline{v^{\prime} t^{\top}}}{\bar{u}_{\tau}^{T} \tau}=1 \tag{6.15}
\end{equation*}
$$

The experimental data are shown in Fig. 6.12.

### 6.2.3 Adverse pressure gradient flows

Measurements were taken of temperature fluctuations and turbulent heat transfer rates with suction and adverse pressure gradient. The dimensionless temperature fluctuation level (Fig. 6.13), as expected, decreases with suction, but not so much as the velocity fluctuation level. Following the same trend as the velocity field, its peak is displaced outwards for an increasing suction, indicating that the sublayer becomes thicker.

The temperature fluctuation profile is observed to have only one peak. If an analogy is made to the velocity field, a second peak might appear for an adverse free stream temperature gradient condition, as suggested by the following line of reasoning.

The temperature fluctuation equation can be written, from Tennekes and Lumley [46], for example, as

$$
\begin{align*}
\bar{u} \frac{\partial}{\partial x}\left(\overline{\frac{t^{\prime 2}}{2}}\right)+\bar{v} \frac{\partial}{\partial y}\left(\overline{\frac{t^{\prime 2}}{2}}\right)= & -\frac{\partial}{\partial x}\left[\frac{1}{2} \overline{t^{\prime 2} u^{\prime}}-\alpha \frac{\partial}{\partial x} \overline{\left(\frac{t^{\prime 2}}{2}\right)}\right] \\
& -\frac{\partial}{\partial y}\left[\frac{1}{\frac{1}{t^{\prime 2} v^{\prime}}}-\alpha \frac{\partial}{\partial y}\left(\frac{1}{2} \overline{t^{\prime 2}}\right)\right]  \tag{6.16}\\
& -\overline{u^{\prime} t^{\prime}} \frac{\partial T}{\partial x}-\overline{v^{\prime} t^{\prime}} \frac{\partial T}{\partial y} \\
& -\alpha\left[\left(\overline{\frac{\partial t^{\prime}}{\partial x}}\right)^{2}+\left(\frac{\partial t^{\prime}}{\partial y}\right)^{2}\right] .
\end{align*}
$$

It can be seen that a free stream temperature gradient would influence directly the temperature fluctuation production term $\overline{u^{\prime} t^{\top}} \frac{\partial T}{\partial x}$ in the outer region of the boundary layer.

This indicates that for the same hydrodynamics, the turbulent Prandt1 number, defined as the ratio between the eddy diffusivity for momentum and heat, would be a function of the temperature boundary conditions.

The temperature fluctuation profile is observed to peak at a larger distance from the wall than the velocity fluctuation profile. This ratio is approximately 1.4 , or the inverse of the molecular Prandtl number, as for zero pressure gradient flows. This again suggests that the thermal sublayer is larger than the momentum one by the same amount. A numerical prediction scheme for the temperature field would, therefore, make use of this observation, as an extension of zero pressure gradient flows, to include the effects of pressure gradient.

Figure 6.14 plots the normalized $\overline{v^{\top} t^{\top} / u_{\tau} T_{\tau}}$ profile, having $y^{+}$as the independent variable. It can also be seen that it peaks in the outer region of the boundary layer and suction tends to suppress it. A comparison between the temperature fluctuation profile and the normal velocity fluctuation profile shows that the former peaks in the inner layer and the latter in the outer layer. Peaks of $\overline{v^{\prime} t^{\top}}$ are therefore
expected to exist in between. This is actually the case, and Fig. 6-14 shows another feature: the $\overline{v^{\top} t^{\top}}$ profile peaks also in the inner region. Because of the fact that $\sqrt{\frac{\mathrm{u}^{\prime 2}}{}}$ profile varies with suction more than $\sqrt{t^{1^{2}}}$ profile, it is expected that the hydrodynamics plays a more important role in the magnitude of this peak. For steep variations of $\sqrt{\sqrt{v^{2}}}$, this inner peak can even be suppressed, as seems to be the case for no transpiration. The existence of the inner peak may be related to the observed decrease and then rise in the turbulent Prandtl number data of Blackwell [9] and of this investigation (Chapter 7) in the inner region of the boundary layer.

Figure 6.15 shows the correlation coefficient $\overline{v^{\top} t^{\top}} / \sqrt{\overline{v^{\prime 2}}} \sqrt{{t^{\prime 2}}^{2}}$ between the normal velocity and the temperature. It is observed that in the outer region high values of the coefficient are obtained, indicating that the phase shift between the normal velocity and the temperature fluctuations is small.

Finally, from experimental observation, it is apparent that the free stream temperature turbulence level is reached at a much higher distance from the wall than the thermal boundary layer thickness. This suggests that there is a region of the boundary layer which is characterized by the existence of a turbulent fluctuation in temperature without a measurable mean temperature deficit.


Fig. 6.1 Axial velocity fluctuation profiles -- comparison with the data of Klebanoff.


Fig. 6.2 Axial velocity fluctuation profiles for strong adverse pressure gradient flows with different suction rates.


Fig. 6.3 Normal velocity fluctuation profiles for strong adverse pressure gradient flows with different suction rates.


Fig. 6.4 Transverse velocity fluctuation profiles for strong adverse pressure gradients with different suction rates.


Fig. 6.5 Turbulent shear stress profiles for strong adverse pressure gradient with different suction rates.


Fig. 6.6 Correlation coefficients between the longitudinal and normal velocities -- mild adverse pressure gradient.


Fig. 6.7 Correlation coefficients between the longitudinal and normal velocities -- strong adverse pressure gradient.


Fig. 6.8 The ratio between turbulent shear stress and turbulent kinetic energy -- mild adverse pressure gradient flows.


Fig. 6.9 The ratio between turbulent shear stress and turbulent kinetic energy -- strong adverse pressure gradient flows.


Fig. 6.10 Flat plate measurements of temperature fluctuations -comparison with the data of Fulachier and Dumas.


Fig. 6.11 The temperature fluctuation profile compared with the longitudinal velocity fluctuation profile -- flat plate data.


Fig. 6.12 Measured $\overline{v^{\prime} t^{\dagger}}$ profiles normalized on $u_{\tau} T_{T}-$ flat plate.


Fig. 6.13 The temperature fluctuation profiles in a strong adverse pressure gradient with different suction rates.


Fig. 6.14 The temperature-normal velocity correlation for a strong adverse pressure gradient with different suction rates.


Fig. 6.15 The temperature-normal velocity correlation coefficient for a strong adverse pressure gradient with different suction rates.

## CHAPTER 7

## TURBULENT TRANSPORT OF HEAT AND MOMENTUM

There has been lately a considerable interest in predicting the behavior of the momentum and thermal boundary layers by numerical means. The details of the turbulence structure have become more and more important because the present numerical schemes for the solution of the boundary layer equations allow a high degree of sophistication for the specification of the turbulent terms. Elaborate schemes for the prediction of the hydrodynamic behavior of a turbulent boundary layer are pressently available. The temperature counterpart of the problem, however, lacks the information necessary for the accurate prediction of the heat transfer coefficient and the temperature profile.

This chapter outlines the previous studies of the turbulent transport of heat and momentum and presents the results of turbulent heat transfer measurements together with a new experimental procedure for estimating the turbulent Prandtl number at the wall. The behavior in that region is then analyzed both analytically and experimentally, and it is shown that the turbulent Prandtl number must approach a constant value at the wall; i.e., $2 \operatorname{Pr}_{t} / 2 \mathrm{y}=0$ in the vicinity of the wall.

### 7.1 Previous Theoretical and Experimental Studies

The main objective of the study of turbulent transport of heat and momentum is the determination of the functional dependence of $\overline{u^{\top} v^{\top}}$ and $\overline{v^{\top} t^{\top}}$ on the fluid flow parameters. This has been done by defining eddy diffusivities for momentum and heat respectively as

$$
\begin{equation*}
-\overline{u^{\top} v^{\top}}=\varepsilon_{M} \frac{\partial \bar{u}}{\partial y} \quad \text { and } \quad-\overline{v^{\top} t^{\top}}=\varepsilon_{H} \frac{\partial T}{\partial y} \tag{7.1}
\end{equation*}
$$

The next step is the introduction of the so-called turbulent Prandtl number, that is, the ratio $\varepsilon_{M} / \varepsilon_{H}$ between the eddy diffusivities for momentum and heat. Various assumptions have been made about this ratio

In the past and several expressions have been proposed in attempts to predict the mean temperature profile and the heat transfer coefficient in the boundary layer. The simplest one of all is the Reynolds analogy, which implies a turbulent Prandtl number having a value of 1.0. The assumption is that heat and momentum are transferred by similar processes, which leads to the same valuesfor the eddy diffusivities. The arguments which lead to this claim can be summarized as follows. Consider a mean velocity $\bar{u}$ and temperature $T$ profile of a flowing fluid and suppose that a pulse of fluid is carried by a sudden cross-current fluctuation in the turbulent fluid from $y_{1}$ to $y_{2}$, normal to the wall. The enthalpy of this pulse of fluid before its sudden movement is $c_{p} T$, and accordingly the instantaneous excess of energy at the level $y_{2}$ is given by :

$$
\begin{equation*}
\text { excess of energy }=-\left(y_{2}-y_{1}\right) \frac{d}{d y}\left(c_{p} T\right) \tag{7.2}
\end{equation*}
$$

assuming the wall is at a higher temperature than the free stream. The negative sign shows that there is actually an excess of energy which occurs at level $y_{2}$ owing to the sudden translation there of fluid from the higher energy level $y_{1}$.

Assuming that there is no loss of heat or any kind of energy during the movement, there is a sudden excess of temperature $t$ ' and enthalpy $c_{p} t^{\prime}$ at the new level $y_{2}$, 1.e.,

$$
\begin{equation*}
c_{p} t^{\prime}=-\Delta y \frac{d}{d y}\left(c_{p} T\right) \tag{7.3}
\end{equation*}
$$

where $t^{\prime}$ will be positive. Multiplying by $v^{\prime}$ and averaging

$$
\begin{equation*}
-\overline{v^{\top} t^{\top}}=\overline{v^{1} \Delta y} \frac{d T}{d y} \tag{7.4}
\end{equation*}
$$

Representing the rms values of the displacement $\Delta y$ and the velocity fluctuation $v^{\prime}$ as $\ell$ and $\sqrt{v^{\prime 2}}$, respectively,

$$
\begin{equation*}
-\overline{v^{\prime} t^{\prime}}=\ell \sqrt{v^{\prime 2}} \frac{d T}{d y} \tag{7.5}
\end{equation*}
$$

Here $\ell$ is the mean distance of travel of the lumps of fluid before they lose their identities and are mixed into the fluid at the new position, and is referred to as the mixing length.

If an eddy diffusivity for heat is defined as

$$
\begin{equation*}
-\overline{v^{\prime} t^{\prime}}=\varepsilon_{H} \frac{\partial T}{\partial y} \tag{7.6}
\end{equation*}
$$

it can be seen, comparing (7.4) and (7.5), that

$$
\begin{equation*}
\varepsilon_{\mathrm{H}}=\ell \sqrt{\mathrm{v}^{\prime 2}} \tag{7.7}
\end{equation*}
$$

But from the expression for the eddy diffusivity for momentum,

$$
\begin{equation*}
\varepsilon_{H}=\varepsilon_{M}=\ell \sqrt{\frac{v^{\prime 2}}{}} \tag{7.8}
\end{equation*}
$$

If the turbulent Prandtl number $\operatorname{Pr}_{t}$ is defined as

$$
\begin{equation*}
\operatorname{Pr}_{\mathrm{t}}=\frac{\varepsilon_{\mathrm{M}}}{\varepsilon_{\mathrm{H}}} \tag{7.9}
\end{equation*}
$$

it can be seen that the hypothesis of no heat loss during the flight implies unity turbulent Prandtl number.

Momentum mixing length theory, however, was criticized by Taylor [36], who proposed that the eddy, during its flight, would preserve its vorticity rather than momentum. Taylor's predicted temperature profile in the wake region behind a cylinder compared favorably with experiments.

The assertion that the average value of the turbulent Prandtl number is near unity seems to hold well for boundary layer flows when the molecular Prandtl number is unity and there is no streamwise pressure gradient, because of the similarity of the momentum and energy equations.

Experimental data, however, have shown that the turbulent Prandt1 number varies through the layer in a way which depends on both molecular Prandtl number and the flow field. Measurements have shown that the wall region is characterized by values of the turbulent Prandtl number higher than unity, falling to less than 1 in the outer region.

Measurements of $\overline{v^{\prime} t^{\prime}}$ are usually obtained from mean temperature measurements and by means of the time averaged energy equation. However, as pointed out by Simpson [47] the uncertainty in the measurement of $\mathrm{Pr}_{t}$ becomes high in two regions.
(a) Inner region: $\overline{u^{\top} v^{\top}}$ and $\overline{v^{\top} t^{\top}}$ are very small and then determination from subtraction of terms of nearly the same order of magnitude becomes uncertain.
(b) Outer region: the $y$-derivatives of temperature and velocity are very small and their determination becomes uncertain.

In the logarithmic region, though, there seems to be a better chance for accuracy in the determination of the turbulent Prandtl number. A number of experiments have shown that its value for zero pressure gradient flows (air as working fluid) is approximately 0.9 . Among them, one could mention Simpson [47], Blackwell [9], and Chen [48].

The outer region of zero pressure gradient flows seems to be characterized by values of the turbulent Prandtl number smaller than 1. Rotta [49] examined the no transpiration case and presented an expression for $\operatorname{Pr}_{t}$ as a function of $y / \delta$. in the outer region.

There remains only the inner region. Experimental difficulties have been responsible for the lack of a definite conclusion on its behavior. However, numerical experiments show that the temperature field is predicted reasonably only by assuming a value greater than 1 in that region. The same numerical experiments show that the inner region is very important for the description of the heat transfer and that in the range $10<\mathrm{y}^{+}<15$, the turbulent Prandtl number drops considerably from its high value at the wall to its low value in the $\log$ region.

It is obvious therefore, that the inner region has to be investigated in more detail owing to its importance on the description of the heat transfer.

Only a few direct measurements of $\overline{u^{\prime} v^{\top}}$ and $\overline{v^{\prime} t^{\prime}}$ for the study of the turbulent Prandtl number have been reported in the literature. This is probably due to the fact that the measurement of $\overline{v^{\top} t^{\top}}$ is very difficult. Both Johnson [20] and Blom [24] measured $\operatorname{Pr} t$ for a step in wall temperature condition. The purpose of their study was mainly to
generate a kernel function used for the prediction of the heat transfer under a variable wall temperature condition. They found that $\mathrm{Pr}_{\mathrm{t}}$ increased with $y^{+}$close to the wall, reached a maximum of $0.8-1.2$ in the range $\mathrm{y}^{+}=50-80$ and then decreased. The trends are therefore different from the constant wall temperature case. This would then suggest that the turbulent Prandtl number is not only a function of the turbulent Peclet number $\operatorname{Pe}_{t}\left(\operatorname{Pr} \cdot \varepsilon_{M} / \nu\right)$, as usually assumed, but also on other parameters.

The dependence of the turbulent Prandtl number on the molecular Prandtl number, pressure gradient and transpiration rates has been discussed by several investigators, who try to prove that $\operatorname{Pr}_{t}$ has a universal profile as a function of certain dimensionless independent variables.

Mizushina, Ito and Ogino [50] examined the near wall region of a rectangular duct flow by means of a Mach-Zehnder interferometer technique for the range of $\operatorname{Pr} \approx 6-40$ and no dependence of the turbulent Prandtl number on the molecular Prandtl number was observed. Blom [24], on the other hand presents a survey of the experimental data available for $\operatorname{Pr}_{t}$ in which it is concluded that the molecular Prandtl number is important for the description of $\mathrm{Pr}_{t}$. In general, the experimental results show an enormous scatter, leaving the general behavior of $\mathrm{Pr}_{t}$ an unsolved problem. Simpson [47] studied zero pressure gradient flows under different transpiration rates. The results again show a considerable scatter but to within the calculated uncertainty of measurements (which is high) no effect of blowing or suction could be observed. A definite trend was then observed: high values at the wall and low values in the outer region. Thielbahr [6] studied mild favorable pressure gradient flows and by numerical experiments, he found a direct pressure gradient dependence in the inner layer and a direct transpiration rate dependence in the outer layer, besides the dependence on the Reynolds number of turbulence $\varepsilon_{M} / \nu$ and the sublayer thickness $A^{+}$. Kearney [7] studied a strong favorable pressure gradient with different transpiration rates. He observed a large scatter in his data, but examination of the turbulent Prandtl number as a function of the Reynolds number of turbulence, $\varepsilon_{M} / \nu$, does not show a definite trend towards the universality of the
profile. Finally, Blackwell [9] studied mild adverse pressure gradient flows and obtained an expression for the turbulent Prandtl number as a function of local parameters of the velocity field, which depend on the pressure gradient, transpiration rate and the velocity profile itself.

It is not clear, therefore, whether the scatter in the data is a consequence of the uncertainty in the measurements or there is really no trend towards the universality of the turbulent Prandtl number profile. In that case, it would be a function of the pressure gradient, transpiration rate and boundary conditions for both velocity and temperature field, and it would throw some doubts on the usefulness of $\operatorname{Pr}_{t}$. The usefulness of $\mathrm{Pr}_{t}$ is also challenged by Blom's.results [24] showing $\mathrm{Pr}_{t}$ to be a function of the wall temperature distribution.

Different models for the turbulent Prandtl number have been proposed in the literature. All of them use a physical hypothesis as far as the nature of the turbulent heat transfer is concerned.

The simplest of all is the Prandtl theory which suggests that $\operatorname{Pr}_{t}=1$, as outlined in the beginning of this section.

Jenkins [51] was among the first ones to develop a model which allows an eddy during its flight to lose heat to the flow field. A universal dependence on the turbulent Peclet number $\left(\operatorname{Pr} \varepsilon_{M} / \nu\right)$ was then obtained. The eddies were assumed to be spheres having a radius equal to the mixing length. Their surface temperature was assumed to vary linearly during their movement from creation to destruction. The drawback of this model is that it always predicts values for the turbulent Prandtl higher than unity, which has been shown experimentally not to be the case. However, some investigators have been using this model in the inner region of the boundary layer.

Some other models assume a linear variation of the mean temperature field during the eddy flight and an expression for $\mathrm{Pr}_{t}$ is obtained as a function of $\mathrm{Pe}_{t}$. Others like Wassel and Catton [52] are a mixture of this model and a curve fitting to predict the observed experimental data. By doing so $\operatorname{Pr}_{t}$ can be smaller than $l$ in the outer region. Blackwell [9] for example used a similar formulation in the inner region and Rotta's modified expression in the outer region of the boundary layer.

Coefficients were adjusted as a function of local parameters of the velocity boundary layer and the temperature field was predicted quite reasonably for adverse pressure gradient flows and different transpiration rates. The Stanton number prediction however, was not so good. The reason is possibly that his model predicts an infinite turbulent Prandtl number at the wall. The present investigation proves that it must have a finite value at the wall.

Others like Cebeci [45] assume two mixing lengths, similar to the Van Driest formulation. The sublayer thickness $B^{+}$would therefore have to be determined as suggested in Chapter 6. A complete formulation would rely on adjusting the constants to predict flows under adverse pressure gradient conditions and different transpiration rates.

Hinze [29] suggests that the diffusion of heat might be a combination of gradient and large eddy transport. Simpson [47] used his ideas to develop a model for $\operatorname{Pr}_{t}$ under zero pressure gradient conditions and different transpiration rates.

From this discussion it is concluded that there is no consensus as to the behavior of the turbulent Prandtl number very close to the wall. This thesis analyzes it both analytically and experimentally, by means of a new measurement procedure.

The turbulent Prandtl number throughout most of the boundary layer is deduced from sequential measurements of temperature and velocity using a hot-wire probe. Values at the wall are estimated analytically.

### 7.2 Behavior of the Turbulent Prandtl Number Close to the Wall

The turbulent Prandtl number can be written as

$$
\begin{equation*}
\operatorname{Pr}_{t}=\frac{\overline{u^{\top} v^{\top}}}{\overline{v^{\top} t^{\top}}} \frac{\partial T / \partial y}{\partial u / \partial y} \tag{7,10}
\end{equation*}
$$

In the neighborhood of the wall, from (5.11) and (5.16)

$$
\begin{equation*}
\overline{v^{\prime} t^{\prime}}=-\left.\frac{\partial t^{\prime}}{\partial y} \frac{\partial^{2} v^{\prime}}{\partial y^{2}}\right|_{0} \frac{y^{3}}{2}+\cdots=-b \frac{y^{3}}{2} \tag{5.16}
\end{equation*}
$$

and the $\mathrm{Pr}_{\mathrm{t}}$ can be written as

$$
\begin{equation*}
P_{t}=\frac{a}{b} \frac{\partial \mathrm{~T} /\left.\partial \mathrm{y}\right|_{0}}{\partial u /\left.\partial y\right|_{o}} \tag{7.11}
\end{equation*}
$$

It can be concluded therefore that $\operatorname{Pr}_{t}$ approaches a constant value at the wall. This analysis shows then that models requiring an infinite value for the turbulent Prandtl number at the wall cannot be correct

### 7.3 New Measurement Procedure for Estimating $\operatorname{Pr}_{t}$ at the Wall

The purpose of this procedure is to obtain data for analyzing $\operatorname{Pr}_{t}$ at the wall as a function of pressure gradient and transpiration rates.

The procedure is based on the premise that the correlation coefficients between each velocity component and the temperature can be written as

$$
\begin{align*}
& -\overline{u^{\prime} v^{\prime}}=c \sqrt{\overline{u^{\prime 2}}} \sqrt{\overline{v^{\prime 2}}}  \tag{7.12}\\
& -\overline{v^{\prime} t^{\prime}}=d \sqrt{\overline{t^{\prime 2}}} \sqrt{\overline{v^{\prime 2}}} \tag{7.13}
\end{align*}
$$

Experimental data shows that both $c$ and $d$ should tend to small values close to the wall. The following analysis is intended to estimate the ratio $c / d$, as the wall is approached.

The flow visualization data of Runstadler, Reynolds and Kline [54] shows a streaky nature of the flow close to the wall, which appears to have a well defined transverse wavelength. The streaks are oriented so that their direction is in the flow direction. Periodically these streaks break away from the wall and disperse into the main flow. These large and highly coherent components retain their identity for a long time and thus a high correlation should be expected between the longitudinal velocity fluctuation and the temperature fluctuation

$$
\begin{equation*}
\frac{\overline{u^{\prime} t^{\prime}}}{\sqrt{u^{u^{2}}} \sqrt{t^{\prime^{2}}}} \rightarrow 1.0 \tag{7.14}
\end{equation*}
$$

As the wall is approached, the correlation coefficient becomes 1 , indicating that there is no phase shift between $u^{\prime}$ and $t^{\prime}$. As a consequence, the correlation coefficients $c$ and $d$ in (7.12) and (7.13) respectively become identical.

An experimental confirmation of this result was done by Bremhorst and Bullock [21] for fully developed pipe flow, together with the data of Johnson [20] and Morrison [53]. Bremhorst and Walker [55] used a new procedure for measuring $\overline{u^{\prime} v^{\top}}$ very close to the wall, by sensing the wake of a hot wire with a cold one and obtained a model for the transport of momentum which successfully interprets these results.

The turbulent Prandtl number can be written by using Eq. (7.12) and (7.13), and the fact that the correlation coefficients are equal at the wall as:

$$
\begin{equation*}
\operatorname{Pr}_{t_{0}}=\lim _{y \rightarrow 0} \frac{\overline{u^{\prime} v^{\prime}}}{\overline{v^{\prime} t^{\prime}}} \frac{\partial T / \partial y}{\partial u / \partial y}=\lim _{y \rightarrow 0} \frac{\sqrt{u^{\prime 2}}}{\sqrt{t^{\prime 2}}} \frac{\partial T / \partial y}{\partial u / \partial y} \tag{7.15}
\end{equation*}
$$

Writing Eq. (7.15) in terms of dimensionless coordinates

$$
\begin{equation*}
\operatorname{Pr}_{t_{0}}=\lim _{y \rightarrow 0} \frac{\sqrt{u^{\prime 2}} / u_{\tau}}{\sqrt{t^{\prime 2}} / T} \operatorname{Pr} \tag{7.16}
\end{equation*}
$$

The procedure used for estimating the turbulent Prandt1 number at the wall consists of calculating the function

$$
\begin{equation*}
R=\frac{\sqrt{\overline{u^{\prime 2}}} / u_{\tau}}{\sqrt{t^{\prime^{2}}} / T_{\tau}} \operatorname{Pr} \tag{7.17}
\end{equation*}
$$

near the wall and extrapolating it to the wall. The extrapolation does not constitute a problem because both $\sqrt{u^{\prime 2}}$ and $\sqrt{t^{12}}$ are linear with $y$ near the wall. Actually, the function $R$ (Eq. 7.17) does not
change much for points near the wall, making the extrapolation procedure very easy.

There are some uncertainties associated with this procedure. First of all, expression (7.15) shows that the $y$-derivatives of velocity and temperature must be known so that the calculation of the function $R$ can be carried out. As well known, these derivatives are very difficult to calculate, especially near the wall. This problem can be overcome by the knowledge that in the inner layer the similarity variables $T^{+}, u^{+}, y^{+}$ will describe the flow and a Couette flow function will fit the data.

The Couette flow function is sensitive to the value of Stanton number and friction factor and these are known, for several flow conditions, with good accuracy. The data for flat plate flows is the most certain. Figure 7.1 shows the turbulent Prandt 1 numbers deduced from the present measurements, for the flat plate case. The value at the wall is shown as 1.4 based on the present technique of extrapolating Eq. 7.17 to the wall. This value is approximately the inverse of the molecular Prandtl number for air, a relationship which has been noted by other investigators.

The data from the present work were used to generate Figs. 7.2, 7.3, 7.4, and 7.5, which show the turbulent Prandtl number profiles for a strong pressure gradient flow and different suction rates. The determination of the Stanton number, which is used in the extrapolation procedure, is believed to be accurate to within $5 \%$ as in the flat plate case. The friction coefficient was obtained by direct measurement of $\overline{u^{\prime} v^{\top}}$ and the same procedure was used for all flows. The consistency of the measurement procedure provides, thus, a general trend of the influence of the pressure gradient and transpiration rate on the turbulent Prandt1 number.

It is concluded, therefore, that an adverse pressure gradient increases the turbulent Prandtl number at the wall and the suction decreases it.

One could raise doubts as far as the usefulness of the present results are concerned to the prediction of heat transfer. As very well
known, in the laminar sublayer, the turbulent quantities do not contribute much to the description of the mean profile. However, it has been proved in Section 7.2 that the derivative of the turbulent Prandtl number close to the wall is zero. This fact, together with the observation that both $\sqrt{\frac{u^{\prime 2}}{}}$ and $\sqrt{t^{\prime 2}}$ are approximately linear with $y$ in the sublayer, suggests that the turbulent Prandtl number varies little from the wall to the region where there is a sharp drop to its value in the logarithmic region. The present results should be used, therefore, as an upper limit for turbulent Prandtl number in the region which is believed to be the most important one for the turbulent heat transfer (for flat plate $y^{+}<(10-15)$.

### 7.4 Analysis of the Turbulent Prandtl Number Data

Figure 7.1 shows the turbulent Prandtl number profile for zero pressure gradient flow. In the logarithmic region a value of approximately 0.9 is obtained, and approaches 0.5 in the outer region. This measurement constitutes an experimental confirmation of the analysis obtained from mean profile measurement. It can be done by assuming a Couette flow approximation in the inner region.

$$
\begin{align*}
& -\frac{\overline{u^{\prime} v^{\prime}}}{u_{\tau}^{2}}=1-\frac{\mathrm{du}^{+}}{d y^{+}}  \tag{7.18}\\
& +\frac{\overline{v^{\top} t^{\top}}}{u_{\tau^{\top} \tau}^{T}}=1-\frac{d T^{+}}{d y^{+}} \frac{1}{\mathrm{Pr}} \tag{7.19}
\end{align*}
$$

The turbulent Prandtl number can then be written as

$$
\begin{equation*}
\dot{\operatorname{Pr}}{ }_{t}=\frac{\overline{u^{\prime} v^{\prime}}}{\overline{v^{\prime} t^{\prime}}} \frac{\partial T / \partial y}{\partial u / \partial y}=-\frac{\overline{u^{\prime} v^{\prime}} / u_{\tau}^{2}}{\overline{v^{\prime} t^{\prime}} / u_{\tau^{T}}{ }^{T}} \frac{d T^{+}}{d u^{+}} \tag{7.20}
\end{equation*}
$$

Substituting (7.18) and (7.19) into (7.20) and by neg1ecting $\frac{\mathrm{du}^{+}}{\mathrm{dy}^{+}}$and $\frac{\mathrm{dT}^{+}}{\mathrm{dy}}{ }^{+}$in the $\log$ region since they are small:

$$
\begin{equation*}
\operatorname{Pr}_{t}=\frac{\mathrm{dT}^{+}}{\mathrm{du}}{ }^{+} \tag{7.21}
\end{equation*}
$$

This derivative has been calculated from experimental data and found to be approximately 0.9 in the logarithmic region.

Figures $7.2,7.3,7.4$ and 7.5 show the turbulent Prandtl number profiles for adverse pressure gradient and different transpiration rates. It is observed that the adverse pressure gradient seems to decrease $\operatorname{Pr}_{t}$ in both the logarithmic and the outer regions. The same conclusion was reached by Blackwell [9] who obtained the turbulent Prandtl number profile for milder adverse pressure gradient flows from mean temperature and velocity measurements. His data, however, seem to indicate a higher dependence on pressure gradient.

Suction is observed to increase $\mathrm{Pr}_{\mathrm{t}}$ in both logarithmic and outer regions. This is also confirmed by Blackwell [9], and sharply contrasts with the finding that $\mathrm{Pr}_{\mathrm{t}}$ decreases with suction at the wall, as discussed in Section 7.3. This would indicate that the dominant terms in the turbulent transport of heat may be different for the near wall region and the rest of the boundary layer.

It can also be observed in the same figures that there is a region of the boundary layer where the turbulent Prandtl number drops and then rises to its value in the logarithmic region of the boundary layer. This was also observed in Blackwell's [9] data and is a result of a peak in the $\overline{v^{\prime} t^{\top}}$ profile in a different position from the peak in $\overline{u^{\prime} v^{\prime}}$ profile. This was discussed in Chapter 6.

Figure 7.6 shows the variation of $\partial T / \partial \bar{u}$ within an adverse pressure gradient boundary layer. This derivative can be evaluated throughout the boundary layer at any $x$ holding all other parameters of the flow constant, since then both $T$ and $\bar{u}$ are functions of $y$ alone. We can write

$$
\begin{equation*}
\frac{\partial T}{\partial \bar{u}}=\frac{\partial T / \partial y}{\partial \bar{u} / \partial y} \tag{7.22}
\end{equation*}
$$

It can be observed, by comparing the values of $\partial T / \partial \bar{u}$ from Fig. 7.6 with the values of $\mathrm{Pr}_{\mathrm{t}}$ shown in Figs. 7.2 through 7.5, that in any boundary layer where $\partial T / \partial \bar{u}$ changes significantly, then $\operatorname{Pr}_{t}$ changes
significantly. In boundary layers where $\partial T / \partial \bar{u}$ does not change much, then $\mathrm{Pr}_{t}$ is nearly constant. Examination of $\partial \mathrm{T} / \partial \overline{\mathrm{u}}$ throughout a boundary layer may serve to identify the trends of $\mathrm{Pr}_{t}$ which that boundary layer will exhibit. In particular, boundary conditions which affect $\partial T / \partial \bar{u}$ differently in the inner and outer regions will result in variations of turbulent Prandtl number within the layer. Thus, investigators seeking a "universal function" to describe $\mathrm{Pr}_{\mathrm{t}}$ solely in terms of $\mathrm{y}^{+}$or the turbulent Peclet number may be doomed to failure.

Figure 7.7 shows turbulent Prandtl number as a function of $y^{+}$ for four different values of suction. Most nearly uniform are the values for $\mathrm{F}=\mathbf{- 0 . 0 0 4}$ (strong suction). Reference to Fig. 7.6 shows $\partial T / \partial \bar{u}$ for that case to be nearly uniform. Most widely variable are the Prandtl number values for $F=0.00$, and for that case $\partial T / \partial \bar{u}$ is also widely varying.

Figure 7.8 shows the turbulent Prandt1 number data displayed as a function of turbulent Peclet number, $\frac{\varepsilon M}{\nu} \operatorname{Pr}$. Once again, the strong suction data are most nearly uniform, the unblown data least uniform. When pressure gradient effects and suction are present, there does not seem to be a good correlation. Simpson [47] and Kearney [7] found this same variation in data for turbulent Prandtl number in earlier studies. Different investigators have tried different techniques for accommodating the scatter, with more or less limited success. Blackwell [9] proposed a functional relationship between $\operatorname{Pr}_{t}$ and $\operatorname{Pr} \frac{\varepsilon M}{\nu}$ with coefficients which were functions of the local hydrodynamic parameters. Simpson [9] followed Hinze's [29] suggestion that the diffusion of heat might be a combination of gradient and eddy transport.

The possibility exists, in view of the demonstrated variations of turbulent Prandtl number, that no universal correlation exists between $\operatorname{Pr}_{t}$ and $\operatorname{Pr} \frac{\varepsilon M}{\nu}$ and that some more fundamental approach to the energy transport problem must be adopted.

### 7.5 Uncertainty Intervals

Review of the calibration data and consideration of the observed scatter in measurements made during the data taking has led to the fol-
lowing estimates for the stochastic (i.e, random) component of the uncertainty in the following measured quantities (expressed in relative terms):

$$
\begin{aligned}
\sqrt{\overline{u^{2}}} & = \pm 4 \% \\
\sqrt{\overline{t^{\prime 2}}} & = \pm 4 \% \\
\overline{u^{\prime} v^{\prime}} & = \pm 8 \% \\
v^{\prime} t^{\prime} & = \pm 14 \% \\
\sqrt{\frac{v^{\prime 2}}{}} & = \pm 8 \% \\
\sqrt{\frac{w^{\prime 2}}{}} & = \pm 8 \% \\
\frac{d T}{d U} & = \pm 4 \% \\
u_{\tau} & = \pm 5 \% \\
T_{\tau} & = \pm 8 \%
\end{aligned}
$$

Based on these estimates and the defining equations for $\mathrm{Pr}_{t}$ and $\mathrm{Pr}_{\mathrm{t}_{0}}$, the following uncertainties were estimated by a propagation at constant probability (Root Sum Square Combination)

$$
\begin{aligned}
\operatorname{Pr}_{t} & = \pm 17 \% \\
\operatorname{Pr}_{t_{0}} & = \pm 11 \%
\end{aligned}
$$

These intervals are shown in the figures as vertical bars thru the data points.


Fig. 7.1 The turbulent Prandtl number distribution in a flat plate boundary layer ( 0,0 ).


Fig. 7.2 The turbulent randtl number distribution in an adverse pressure gradient $(-0.275,0)$.


Fig. 7.3 The turbulent Prandt 1 number distribution in an adverse pressure gradient with mild suction ( $-0.275,-0.001$ ).


Fig. 7.4 The turbulent Prandt1 number distribution in an adverse pressure gradient with suction ( $-0.275,-0.002$ ).


Fig. 7.5 The turbulent Prandtl number distribution in an adverse pressure gradient with strong suction ( $-0.275,-0.004$ ).


Fig. 7.6 The ratio $\partial T / \partial U$ within bourdary layers subject to strong adverse pressure gradient with different suction rates.


Fig. 7.7 Turbulent Prandtl number as a function of $y^{+}$; strong adverse pressure gradients with suction.


Fig. 7.8 Turbulent Prandtl number as a function of the turbulent Péclét number; strong adverse pressure gradients with suction.

## SUMMARY AND CONCLUSIONS

The behavior of near equilibrium turbulent boundary layers, under strong adverse pressure gradient conditions and with different suction rates has been examined with emphasis on the determination of the Stanton number and on the behavior of the turbulent Prandtl number.

The free stream velocity variation in this investigation may be described by an equation of the form

$$
\begin{equation*}
\mathrm{U}_{\infty} \alpha \mathrm{x}^{\mathrm{m}}, \text { where } \mathrm{m} \leq 0 \tag{8.1}
\end{equation*}
$$

The transpiration boundary condition, when expressed in terms of the blowing fraction $F$, is kept constant along the test section. Both constant and variable wall temperature conditions are examined. Chapter 2 summarizes the boundary conditions of this investigation.

The following are the results and conclusions drawn from the experimental results of this thesis:

1. Mean temperature and velocity profiles were measured sequentially by using only one probe. The friction coefficient was obtained by means of the direct measurement of $\overline{u^{\prime} v^{\top}}$.
2. Direct measurements of the turbulent heat transfer and the temperature fluctuations have been made, and the turbulent Prandtl number calculated from the data for the turbulent transport of heat and momentum.
3. A new procedure has been developed to estimate the turbulent Prandtl number at the wall. It is shown from limited data that an adverse pressure gradient increases and suction decreases it.
4. The Stanton number for uniform wall temperature is shown to be the same function of enthalpy thickness Reynolds number
and blowing parameter in mild and strong adverse pressure gradients as it is in flat plate flows. The relationship between Stanton number for a given transpiration rate and the corresponding Stanton number for the no transpiration case (at the same enthalpy thickness Reynolds number) is, therefore, independent of pressure gradient and only a function of the blowing parameter $B_{h}$.
5. The Stanton number following a step in wall temperature in an adverse pressure gradient region is shown to be related to the isothermal Stanton number by the same function of relative position which describes the flat plate behavior for a step in wall temperature.
6. The analysis of the hydrodynamic turbulence structure of equilibrium flows shows that an adverse pressure gradient tends to increase the turbulence level. The $\overline{u^{\prime 2}}$ profile is shown to develop two peaks. The second one is located in the outer region of the boundary layer ( $y / \delta \sim 0.5$ ) and is displaced outward, when the pressure gradient increases. Suction is shown to suppress this second peak. The correlation coefficient between $u^{\prime}$ and $v^{\prime}$ is shown to have a constant value (approx. equal to the Karman constant) in the outer region of near equilibrium flows. The same constancy is observed for the ratio between the shear stress and the turbulent kinetic energy, which takes a value of approximately 0.14 . It is shown that when simulating $\overline{u^{\prime} v^{\top}}$ the outer layer peak can be predicted, in principle, by models like the Prandtl-Kolmogorov, which employs the turbulent kinetic energy as a descriptor for the eddy diffusivity for momentum.
7. Analysis suggests that the turbulent Prandtl number should tend to a constant value in the region near the wall.
8. In the logarithmic region adverse pressure gradient decreases
and suction increases the turbulent Prandtl number. Once again, it has been confirmed that $\operatorname{Pr}_{T}$ has a high value at the wall and reaches approximately 0.5 in the outer edge of the boundary layer.

## References

1. Moffat, R. J., and Kays, W. M., "The Turbulent Boundary Layer on a Porous Plate: Experimental Heat Transfer with Uniform Blowing and and Juction," Report No. HMT-1, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1967) PhD Thesis.
2. Simpson, R. L., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer on a Porous Plane: An Experimental Study of the Fluid Dynamics with Injection and Juction," Report No. HMT-2, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1967) PhD Thesis.
3. Whitten, D. G., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer on a Porous Plate: Experimental Heat Transfer with Variable Suction, Blowing and Surface Temperature," Report No. HMT-3, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1967) PhD Thesis.
4. Jullen, H. L., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer on a Porous Plate: Experimental Study of the Effects of a Favorable Pressure Gradient, "Report No. HMT-4, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1969) PhD Thesis.
5. Loyd, R. J., Moffat, R. J., and Kays, W. M., "The Turbulent Boundary Layer on a Porous Plate: An Experimental Study of the Fluid Dynamics with Strong Favorable Pressure Gradients and Blowing," Report No. HMT-13, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1970) PhD Thesis.
6. Thielbahr, W. H., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer: Experimental Heat Transfer with Blowing, Suction, and Favorable Pressure Gradient," Report No. HMT-5, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1969) PhD Thesis.
7. Kearney, D. W., Moffat, R. J., and Kays, W. M., 'The Turbulent Boundary Layer: Experimental Heat Transfer with Strong Favorable Pressure Gradients and Blowing," Report No. HMT-12, Thermosciences Division Dept. of Mech. Engrg., Stanford Univ. (1970) PhD Thesis.
8. Andersen, P. S., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer on a Porous Plate: An Experimental Study of the Fluid Mechanics for Adverse Free-Stream Pressure Gradients," Report No. HMT-15, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1972) PhD Thesis.
9. Blackwe11, B. F., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer on a Porous Plate: An Experimental Study of the Heat Transfer Behavior with Adverse Pressure Gradients," Report No. HMT-16, Thermosciences Division, Dept. of Mech. Engrg., Stanford Univ. (1972) PhD Thesis.
10. Maye, F. P., "Error Due to Thermal Conduction Between the Sensing Wire and Its Supports when Measuring Temperatures with a Hot Wire Anemometer Used as a Resistance Thermometer," DISA Information No. G, February 1970, pp. 22-26.
11. Dahm, M. and Rasmussen, C. G., "Effect of Wire Mounting System on Hot-wire Probe Characteristics," DISA Information No. 7, January 1970, pp. 19-24.
12. Thinh, Nguyen Van, "On Some Measurements Made by Means of a Hot Wire in a Turbulent Flow Near a Wall," DISA Information No. 7, January 1970, pp. 13-18.
13. Watts, K. C., "The Development of Asymptotic Turbulent, Transitional, and Laminar Boundary Layers Induced by Suction," Ph.D. Thesis, Department of Mechanical Engineering, University of Waterloo, June 1972.
14. Fujita, H. and Kovasznay, L. S. G., "Measurement of Reynolds Stress by a Jingle Rotated Hot Wire Anemometer," The Review of Scientific Instruments, Volume 39, No. 9, September 1968.
15. Durst, F. and Rodi, W., "Evaluation of Hot Wire Signals in Highly Turbulent Flows," Fluid Dyn. Meas. in Ind. and Med. Envir., Proc., Disc. Conf., New York, Humanities Press, 1972.
16. Strohl, A. and Comte-Bellot, G., "Aerodynamic Effects Due to Configuration of X-Wire Anemometers," ASME paper No. 73-APM-P.
17. Corrsin, S., "Extended Applications of the Hot-wire Anemometer" NACA Technical Note No. 1864, April 1949.
18. Arya, S. P. S. and Plate, E. J., "Hotwire Measurements in NonIsothermal Flow," Instruments \& Control Systems, p. 87, March 1969.
19. Xuova, A. K. and Sesonsk F. A., "Structure of Turbulent Velocity and Temperature Fields in Ethylene Glycol Pipe Flow at Low Reynolds Number," International Journal of Heat and Mass Transfer, Vol. 15, pp. 127-145, 1972.
20. Johnson, D. S., "Velocity and Temperature Fluctuation Measurements in a Turbulent Boundary Layer Downstream of a Stepwise Discontinuity in Wall Temperature," Journal of Applied Mechanics, ASME, September 1959, p. 325.
21. Bremhorst, K. and Bullock, K. J., "Spectral Measurements of Temperature and Longitudinal Velocity Fluctuations in Fully Developed Pipe Flow," International Journal of Heat and Mass Transfer, Vol. 13, pp. 1313-1329, 1970.
22. Bourke, P. J. and Pulling, D. J., "A Turbulent Heat Flux Metex and Some Measurements of Turbulence in Air Flow Through a Heated Pipe," International Journal of Heat and Mass Transfer, Vo1. 13, pp. 1331-1338, 1970.
23. Bradshaw, P. "An Introduction to Turbulence and its Measurement," The Commonwealth and International Library of Science Technology Engineering and Liberal Studies, Pergamon Press, 1971.
24. Blom, J. "Experimental Determination of the Turbulent Prandtl Number in a Developing Temperature Boundary Layer," Fourth International Heat Transfer Conference Paris-Versailles, Volume II, 1970.
25. Burchill, W. E. and Jones, B. G., "Interpretation of Hot-Film Anemometer Response in a Non-Isothermal Field," Proceedings of Symposium on Turbulence on Liquids, October 4-6, 1971. Department of Chemical Engineering, University of Missouri-Rolla.
26. Fulachier, L. and Dumas, R., "Repartitions Spectrales des Fluctuations Thermiques dans une Couche Limite Turbulent," Agard Conference Proceedings No. 93 on Turbulent Shear Flows, Paper 4, January 1972.
27. Clauser, F. H., "Turbulent Boundary Layers in Adverse Pressure Gradients," Journal of Aero Sci., 21, 91 (1954).
28. Bradshaw, P., "The Turbulence Structure of Equilibrium Boundary Layers," Journal of Fluid Mechanics, Vo1. 29, Part 4, pp. 625-645, (1967).
29. Hinze, J. O., "Turbulence, An Introduction to its Mechanism and Theory," McGraw Hill, (1959).
30. Sandborn, V. A., "Hotwire Anemometer Measurements in Large-Scale Boundary Layers," Advances in Hotwire Anemometry, Proceedings of The International Symposium on Hotwire Anemometry, held at University of Maryland, (1967).
31. Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," NACA Report 1247, (1955).
32. Collis, D. C. and Williams, M. J., "Two-Dimensional Convection from Heated Wires at Low Reynolds Numbers," Journal of Fluid Mechanics, Vol. 6, p. 357, (1959).
33. Davies, T. W. and Patrick, M. A., "A Simplified Method of Improving the Accuracy of Hotwire Anemometry," Fluid Dyn. Meas. in Ind. and Med. Envir., Proc., Disc. Conf., New York, Humanities Press, (1972).
34. Dlaus, R. L. and Van Ness, H. C., "An Extension of the Spline Fit Technique and Applications to Thermodynamic Data," AIChE Journal, Vol. 13, Nov. 1967, p. 1132.
35. Repik, Y. E. U. and Ponomareva, V. S., "The Effect of Proximity of Walls on the Readings of a Hotwire Anemometer in Turbulent Boundary Layers," Heat Transfer-Soviet Research, Vol. 2, No. 4, July 1970.
36. Taylor, G. I., "The Transport of Vorticity and Heat Through Fluids in Turbulent Motion," The Scientific Papers of G. I. Taylor, Vo1. II, Cambridge University Press, (1960).
37. Reynolds, W. C., Kays, W. M., and Kline, S. J., "Heat Transfer in the Turbulent Incompressible Boundary Layer with a Step Wall Temperature Distribution," Stanford University Report, Part II, Stanford, California, July 1957.
38. Hussain, K. M. F. and Reynolds, W. C., "The Mechanics of a Perturbation Wave in Turbulent Shear Flow," Report FM-6, Mech. Eng. Dept., Stanford University, May 1970.
39. Kearney, D. W., Kays, W. M., Moffat, R. J., Loyd, R. J., "The Effect of Free-Stream Turbulence on Heat Transfer to a Strongly Accelerated Turbulent Boundary Layer," Report HMT-9, Mech. Eng. Dept., Stanford, Feb. 1970. PhD Thesis.
40. Pimenta, M. M., Private Communication, Heat and Mass Transfer Group, Mechanical Engineering Dept., Stanford University.
41. Sharan, V. Kr., "On the Importance of Turbulence in Boundary Layer Simulation," Int. J. Mech. Sci., Vo1. 15, pp. 643-640, (1973).
42. Townsend, A. A., "The Structure of Turbulent Shear Flow," Cambridge University Press, (1956).
43. Bradshaw, P., Ferris, O. H. and Atwell, N. P., "Calculation of Boundary Layer Development Using the Turbulent Energy Equation," Journal of Fluid Mechanics, Vol. 28, Part 3, pp. 593-616, (1967).
44. Bradshaw, P., "The Turbulence Structure of Equilibrium Boundary Layers," NPL Aero Report 1184, (1966).
45. Cebeci, T., "A Model for Eddy Conductivity and Turbulent Prandt1 Number," ASME Paper No. 72-WA/HT-13.
46. Tennekes, H. and Lumley, J. L., "A First Course in Turbulence," The MIT Press, (1972).
47. Simpson, R. L., Whitten, D. G., and Moffat, R. J., "An Experimental Study of the Turbulent Prandtl Number of Air with Injection and Suction," Int. Journal Heat Mass Transfer, Vol. 13, pp. 125-143, (1970).
48. Chen, Che Pen, "Détermination Expérimentále du Nombre de Prandtl Turbulent près d'une Paroi Lisse," International Journal of Heat and Mass Transfer, Vol. 16, pp. 1849-1862, October 1973.
49. Rotta, J. C., "Turbulent Boundary Layers in Incompressible Flows," Progress in Aeronautical Sciences, Edited by A. Ferri, D. Kuchemann and L. Sterne, Vol. 2, pp. 1-219, Macmillan, New York, (1962).
50. Mizushina, T., Ito, R. and Ogino, F., "Eddy Diffusivity Distribution Near the Wall," 4th International Heat Transfer Conference ParisVersailles, Volume II, (1970).
51. Jenkins, R., "Variation of the Eddy Conductivity with Prandt1 Modulus and its use in Prediction of Turbulent Heat Transfer Coefficients," Heat Transfer and Fluid Mechanics Institute, Stanford University Press, Stanford, California, p. 147, (1952).
52. Wasse1, A. T. and Cotton, I., "Calculation of Turbulent Boundary Layers over Flat Plates with Different Phenomenological Theories of Turbulence and Variable Turbulent Prandtl Number," International Journal of Heat and Mass Transfer, Vol. 16, pp. 1547-1563, (1973).
53. Morrison, R. B., "Two Dimensional Frequency Wave Number Spectra and Narrow Band Shear Stress Correlations in Turbulent Pipe Flow," Ph.D. Thesis, University of Queensland, Australia, (1969).
54. Runstadler, T. W., Kline, S. J. and Reynolds, W. C., "An Experimental Investigation of the Flow Structure of the Turbulent Boundary Layer," Stanford University, Mech. Eng. Dept., Thermosciences Div., Report MD-8, (1963).
55. Bremhorst, K. and Walker, T. B., "Spectral Measurements of Turbulent Momentum Transfer in Fully Developed Pipe Flow," Journal of Fluid Mechanics, Vol. 61, part 1, pp. 173-186, (1973).
56. Jorgensen, F. E., "Directional Sensitivity of Wire and Hot Film Probes," DISA Information No. 11, (1971).
57. McLean, J. D., "The Transpired Turbulent Boundary Layer in an Adverse Pressure Gradient," PhD Thesis, Department of Aerospace and Mechanical Sciences, Princeton University, (1970).
58. Stevenson, T. N., "A Law of the Wall for Turbulent Boundary Layers with Suction or Injection," The College of Aeronautics, Cranfield, Aero Report No. 166 (1963).
59. Friehe, C. A. and Schwartz, W. H., "Deviations from the Cosine Law for Yawed Cy1indrical Anemometer Sensors," Trans. ASME, 35E, 655 (1968).

## APPENDIX A

## ANALYSIS OF A RESISTANCE THERMOMETER RESPONSE TO <br> MEAN AND FLUCTUATING TEMPERATURE

The use of a hot wire probe as a resistance thermometer for temperature measurement is dictated by the necessity of using a transducer with excellent temporal resolution. For boundary layer measurements it seems to be the best transducer available: its small size allows meatsurements of mean and fluctuating temperature even in regions of moderately sharp temperature gradients.

A parasitic effect caused by thermal conduction between the sensing element and its support can give incorrect results, however. The following analysis is intended to estimate the errors, and follows from Mayer [10] and Hinge [29].

A $5 \mu \mathrm{~m}$ gold plated tungsten wire, DISA model 55 F 04 is used in this investigation. The probe was originally designed for reducing aerodynamic interference and has a gold plated portion for strengthening purposes. It turns out, however, that it can contribute towards lowering the heat conduction to the prongs. The analysis can be done by considering an effective resistance to heat transfer of the gold tungsten composite region, as seen in Fig. A. 1 .


Figure A.1. Gold plated tungsten wire $\left(D_{G} / D_{T}=6\right)$.

The axial conduction resistance to heat transfer in this gold plated region can be seen as two resistances in parallel.

The first one, $R_{T}$, is due to tungsten; the second one, $R_{G}$, is due to gold. It follows that

$$
\begin{equation*}
R=\frac{R_{G} R_{T}}{R_{G}+R_{T}} \tag{A-1}
\end{equation*}
$$

but

$$
\begin{equation*}
R_{G}=\frac{L}{K_{G} A_{G}} \quad \text { where } \quad A_{G}=\frac{\pi}{4}\left(D_{G}^{2}-D_{T}^{2}\right) \tag{A-2}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{R}_{\mathrm{T}}=\frac{\mathrm{L}}{\mathrm{~K}_{\mathrm{T}} \mathrm{~A}_{\mathrm{T}}} \quad \text { where } \quad \mathrm{A}_{\mathrm{T}}=\frac{\pi}{4} \mathrm{D}_{\mathrm{T}}^{2} \tag{A-3}
\end{equation*}
$$

Substituting back into the expression for the resistance $R$,

$$
\begin{equation*}
R=\frac{L}{K_{G} A_{G}+K_{T} A_{T}}=\frac{L}{K_{T} A_{T}}\left(1+\frac{K_{G}}{K_{T}}\left(\frac{D_{G}^{2}}{D_{T}^{2}}-1\right)\right)^{-1} \tag{A-4}
\end{equation*}
$$

In order to estimate the magnitude of the resistance, the following values are assumed:

$$
\begin{align*}
& \mathrm{D}_{\mathrm{G}} / \mathrm{D}_{\mathrm{T}}=6 \\
& \mathrm{~K}_{\mathrm{G}}=170 \mathrm{BTU} /(\mathrm{hr}) \mathrm{f} t^{\circ} \mathrm{F} \text { (Gold) } \\
& \mathrm{K}_{\mathrm{T}}=94 \mathrm{BTU} /(\mathrm{hr}) \mathrm{ft}{ }^{\circ} \mathrm{F} \text { (Tungsten) } \\
& \text { Thus, } \quad \mathrm{R}=\frac{\mathrm{L}}{\mathrm{~K}_{\mathrm{T}} A_{\mathrm{T}}} 0.0155 \tag{A-5}
\end{align*}
$$

This means that the resistance to heat transfer of the composite region is much smaller than if only the tungsten wire were present. As a result, the temperature will be nearly uniform in that region. However, in order to take advantage of this feature the prongs must be in the isothermal plane passing through the wire, because the gold plated portion follows nearly the prong temperature. A discussion of this point is made at the end of this appendix.

Maye [10] reports that conduction errors can be important in resistance thermometry. An analysis was carried out assuming that the total wire length is made of tungsten ( 3 mm , therefore).

Consider an element of wire placed in an isothermal plane for temperature measurement. The conservation of energy principle gives the following expression, as indicated in Fig. A. 2.


Figure A.2. Analysis of the wire.

$$
\begin{gather*}
q_{x}-q_{x+d x}-q_{c}+I^{2} R \frac{d x}{\ell}=\rho c_{p} d V \frac{\partial T_{w}}{\partial t}  \tag{A.6}\\
\frac{\pi D^{2}}{4} \frac{\partial}{\partial x}\left(K \frac{\partial T_{w}}{\partial x}\right) d x-h \pi D d x\left(T_{w}-T_{\infty}\right)+I^{2} R \frac{d x}{\ell}=\rho c_{p} \frac{\pi D^{2}}{4} d x \frac{\partial T_{w}}{\partial t} \tag{A.7}
\end{gather*}
$$

Assuming constant properties, which is quite reasonable for the temperature difference between the wire and the free stream, and integrating from $x=-\ell / 2$ to $x=+\ell / 2$

$$
\begin{equation*}
\left.\frac{\pi D^{2}}{4} K \frac{\partial T_{w}}{\partial x}\right|_{x=\frac{\ell}{2}}-h \pi D\left(T_{m}-T_{\infty}\right)+I^{2} \frac{R_{m}}{\ell}=\rho c_{p} \frac{\pi D^{2}}{4} \frac{\partial T_{m}}{\partial t} \tag{A.8}
\end{equation*}
$$

where $T_{m}$ and $R_{m}$ are respectively the average wire temperature and resistance.

The first term to the left represents the thermal conduction along the wire. The non-uniformity of the temperature distribution along the wire is smaller for a low wire current and for small differences between prong and ambient temperature.

The second term represents the heat convection between the wire and the free stream. Under ideal circumstances, i.e., zero wire current and zero thermal inertia, it is zero. The wire temperature is unform and therefore $T_{m}=T_{\infty}$. This is the ultimate objective of the resistance thermometry approach, i.e., to measure the fluid temperature by means of the wire temperature.

The third term represents the heat generation by the wire current. This contributes towards increasing the non-uniformity of the wire temperature, and therefore the heat conduction term.

Finally, the term to the right represents the thermal inertia of the wire.

In practice, ideal conditions are not attainable. The heat transfer coefficient $h$ is velocity dependent, and the temperature of the wire is a function of the fluid velocity.

- This dependence can be minimized by using a very small current, so that an acceptable error in the fluid temperature measurement is obtained.

In order to analyze the thermal inertia term, consider the ideal case of negligible heat conduction, with a cosinusoidal variation of the fluid temperature. The equation becomes

$$
\begin{equation*}
-h \pi D\left(T_{m}-T_{\infty}\right)=\rho c_{p} \frac{\pi D^{2}}{4} \frac{d T_{m}}{d t} \tag{A.9}
\end{equation*}
$$

where $\quad T_{\infty}=T_{0} \cos (\omega t)$

$$
\begin{equation*}
\text { then, } \quad \frac{T_{m o}}{T_{o}}=\frac{1}{\left[1+\left(\omega \cdot \tau_{c}\right)^{2}\right]^{1 / 2}} \quad \text { where } \quad \tau_{c}=\frac{\rho c_{p} D}{4 h} \tag{A.10}
\end{equation*}
$$

Assuming a tungsten wire

$$
\begin{aligned}
\rho & =1208 \mathrm{lb} / \mathrm{ft}^{3} \\
c_{p} & =0.0321 \mathrm{BTU} / \mathrm{Ib}^{\circ} \mathrm{F} \\
\mathrm{D} & =5 \times 10^{-6} \mathrm{~m}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{h}_{\mathrm{h}} & =1182 \mathrm{BTU} / \mathrm{hr} \mathrm{ft}^{\circ} \mathrm{F} \quad\left(30 \mathrm{ft} / \mathrm{sec}, 68^{\circ} \mathrm{F}\right) \\
\tau_{\mathrm{c}} & =0.484 \mathrm{msec} \\
\text { For } \frac{\mathrm{T}_{\mathrm{mo}}}{\mathrm{~T}_{\mathrm{o}}} & =0.90, \quad \omega=159 \text { hertz } .
\end{aligned}
$$

However, the frequencies encountered in the examined flows are higher. The DISA 55M20 temperature bridge employs a compensating network in the feedback loop of the amplifier to account for the low frequency response of the wire; according to the specifications, for a 1 $\mu m$ platinum wire, the system has a flat response up to $3 \mathrm{Kh}_{z}$.

Assuming the thermal inertia can be taken care of by the compensating network (this is discussed in another section)

$$
\begin{equation*}
\frac{d^{2} T_{w}}{d x^{2}}-\frac{4 h}{K D}\left(T_{w}-T_{\infty}\right)+\frac{I^{2} R}{\ell} / \frac{\pi D^{2} K}{\ell}=0 \tag{A,11}
\end{equation*}
$$

but $\quad R=A T_{W}+B$
so

$$
\begin{equation*}
\frac{d^{2} T_{w}}{d x^{2}}-\omega^{2} T_{w}+\lambda=0 \tag{A.12}
\end{equation*}
$$

where

$$
\omega^{2}=\frac{4 h}{K D}-\frac{I^{2}}{\ell} \frac{4}{\pi_{D}^{2} K} A
$$

$$
\lambda=\frac{4 \mathrm{~h}}{\mathrm{KD}} \mathrm{~T}_{\infty}+\frac{\mathrm{I}^{2}}{l} \frac{4}{\pi D^{2} \mathrm{~K}} \mathrm{~B}=\omega^{2} \mathrm{~T}_{\infty}+\frac{\mathrm{I}^{2}}{l} \frac{4}{\pi D^{2} K} \mathrm{R}_{\infty}=\omega^{2} \mathrm{~T}_{\infty}+\alpha
$$

The solution of the above equation, subject to the following boundary conditions:

$$
\begin{array}{lll}
\mathrm{T}_{\mathbf{w}}=\mathrm{T}_{\mathrm{p}} & \text { at } & \mathrm{x}= \pm \frac{\ell}{2} \\
\frac{\mathrm{~d} \ddot{\mathrm{~T}}_{\mathrm{w}}}{\mathrm{dx}}=0 & \text { at } & \mathrm{x}=0
\end{array}
$$

$$
\begin{equation*}
\frac{T_{w}-\lambda / \omega^{2}}{T_{p}-\lambda / \omega^{2}}=\frac{\cosh \omega x}{\cosh \omega \ell / 2} \tag{A.13}
\end{equation*}
$$

Then, the average wire temperature becomes:

$$
T_{m}=\frac{2}{\ell} \int_{0}^{\ell / 2} T_{W} d x
$$

$$
\begin{array}{r}
\frac{T_{m}-\lambda / \omega^{2}}{T_{p}-\lambda / \omega^{2}}=\frac{2}{\omega l} \tanh \frac{\omega \ell}{\alpha}=v  \tag{A.14}\\
\text { or } \quad T_{\infty}=\frac{T_{m}}{1-v}-\frac{v}{1-v} T_{p}-\frac{\alpha}{\omega^{2}}
\end{array}
$$

An estimate of the magnitude of the terms can be done by assuming the following values:

$$
\begin{aligned}
\mathrm{I} & =2 \mathrm{ma} \\
\mathrm{~A} & =0.0124 \Omega /{ }^{\circ} \mathrm{C} \\
\text { Kwire } & =94 \mathrm{BTU} / \mathrm{hr} \mathrm{ft}^{\circ} \mathrm{F} \\
\ell & =3 \mathrm{~mm} \\
\mathrm{R}_{\infty} & =4.5 \Omega
\end{aligned}
$$

At a typical velocity of $10 \mathrm{ft} / \mathrm{sec}$, and $85^{\circ} \mathrm{F}$, the terms are:

$$
\begin{aligned}
& \frac{4 h}{K_{W} D}=2.09 \times 10^{6} \mathrm{sec}^{-1} \\
& \frac{\mathrm{I}^{2} 4 \mathrm{~A}}{\ell \pi D^{2} \mathrm{~K}}=0.48 \times 10^{3} \mathrm{sec}^{-1} \\
& \omega^{2} \mathrm{~T}_{\infty}=1.77 \times 10^{8} \mathrm{o} / \mathrm{sec}
\end{aligned}
$$

$$
\frac{\mathrm{I}^{2} 4 \dot{R}_{\infty}}{\ell \pi \mathrm{D}^{2} \mathrm{~K}}=3.15 \times 10^{5}{ }^{\circ} \mathrm{F} / \mathrm{sec}
$$

Therefore, for this low current of 2 ma, the 5 micron tungsten wire response can be treated as though there were no current through it, and as analyzed by Maye [10]

$$
\begin{align*}
& T_{\infty}=T_{m}+\frac{\nu}{1-v}\left(T_{m}-T_{p}\right)  \tag{A.15}\\
& \nu=\frac{2}{\omega \ell} \tanh \frac{\omega \ell}{2} \\
& \omega^{2}=\frac{4 h}{K D}
\end{align*}
$$

where

The response of the wire to the mean temperature can be analyzed by means of Equation (A.8). The gold plated portion makes the wire temperature nearly uniform in that region. Further, due to its large thermal inertia as compared to the sensitive region, its temperature fluctuates with very low amplitude about the mean temperature of the fluid. According to Maye [10] this can only be true for a low $v$ probe. A small variation of the prong temperature (i.e., when they are not placed in the isothermal plane of the wire) will not be felt appreciably by the sensing portion of the wire.

Table A. 1 shows values of $v$ for different velocities $(\ell / D=600$, $D=5 \mu \mathrm{~m}$ ). This should be viewed as a lower limit for the gold plated probe in question. Naturally the placing of the prongs in a non-isothermal plane will increase the heat conduction and therefore the value of $v$.

| Probe 55F02 | $v$ | 0.24 | 0.17 | 0.14 | 0.13 | 0.13 | 0.12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probe 55F01 | $v$ | 0.21 | 0.17 | 0.16 | 0.15 | 0.15 | 0.15 |

Table A. 1 Values of $V$ as a function of velocity for the DISA $55 F 02$ $5 \mu \mathrm{~m}$ tungsten probe and DISA 55F01 $1 \mu \mathrm{~m}$ platinum probe.

Assuming the prongs in the isothermal plane of the wire, and time averaging Equation (A.8)

$$
\begin{equation*}
\left.\frac{\pi D^{2}}{4} \mathrm{~K} \frac{\partial \mathrm{~T}_{\mathrm{W}}}{\partial \mathrm{x}}\right|_{\ell / 2}-\mathrm{hmD}\left(\mathrm{~T}_{\mathrm{W}}-\mathrm{T}_{\infty}\right)+\frac{\mathrm{I}^{2} \mathrm{R}_{\mathrm{m}}}{\ell}=0 \tag{A.16}
\end{equation*}
$$

where bars mean time average value of the quantities. However

$$
\begin{array}{ll}
\left.\frac{\partial T}{\partial x}\right|_{\ell / 2} \approx 0 & \text { (gold plating) } \\
\frac{I^{2} R_{m}}{\ell} \approx 0 & \text { (very low current) } \tag{A.17}
\end{array}
$$

Thus $\overline{\mathrm{T}}_{\mathrm{m}} \approx \overline{\mathrm{T}}_{\infty}$

An experiment to determine the value of $v$ seems to be a very difficult one. Maye [10] reports that lower values as compared to the calculated ones were obtained by him. This would place the present calculation on the safer side of error prediction. In the present experiments, the probe was placed in the free stream of the wind tunnel and the independence from the fluid velocity was confirmed to a nominal $\pm 0.05^{\circ} \mathrm{F}$. This only means that actually the probe current is very small and its heating effect is negligible. Nothing can be said about the influence on $v$ because the whole system was isothermal. The actual operation would be in a temperature gradient field where the departure from isothermality is evident.

Maye [10] recommends the present probe for mean temperature measurements. The response of the wire to temperature fluctuations requires some further analysis. Equation (A.15) can be used for this purpose. By assuming that $T_{p}$ is nearly time independent due to large inertia of the prongs:

$$
\begin{equation*}
t_{\infty}^{\prime}=\frac{1}{1-v} t_{m}^{\prime} \tag{A.18}
\end{equation*}
$$

$$
\text { where } \quad v=\frac{2}{l \omega} \tanh \frac{\omega \ell}{2}
$$

Referring to Table A.l, it can be seen that the actual rms value of the fluid temperature can differ from the measured one by $31 \%$ at zero velocity and by $13 \%$ at $30 \mathrm{ft} / \mathrm{sec}$. It can also be seen that both probes ( $1 \mu \mathrm{~m}$ and $5 \mu \mathrm{~m}$ ) can have similar performance as far as heat conduction is concerned. However, the $1 \mu \mathrm{~m}$ probe has a much smaller thermal inertia. The 55 M anemometer is compensated for frequencies up to 3 Khz when the $1 \mu \mathrm{~m}$ platinum wire is used. This upper limit must be reduced when the $5 \mu \mathrm{~m}$ wire is used. Figure A. 3 shows the same flat plate case as measured by the two probes. It can be seen that the performances are similar, although the $5 \mu \mathrm{~m}$ probe is in error due to its larger thermal inertia. It is seen that in the outer region where the time scales are larger, there is a much better agreement between the two measurements, indicating that the frequency response of the system may be different for the two probes.

In the above analysis, it has been assumed that the prongs are in the isothermal plane of the wire. This configuration could be ideally achieved by placing them parallel to the wall. However, in regions where the temperature gradient is steeper, i.e., very close to the wall, the prongs can be at a different temperature. In the present set-up, the wall is hotter than the free stream, hence approaching from above the prongs will be colder than the wire. The gold plating follows nearly the prong temperature. Therefore the conduction will be higher and the measured value of the temperature will be below the fluid temperature.

The $1 \mu \mathrm{~m}$ platinum wire has one main advantage over the $5 \mu \mathrm{~m}$ tungsten wire for mean temperature measurement: its frequency response is better. The magnitude of $v$ does not increase in regions of steep temperature gradient (a higher heat conduction tends to increase $v$ ). If a $5 \mu \mathrm{~m}$ wire is to be used, a better compensating network has to be used.

The accuracy of measurement of mean temperature with the $5 \mu \mathrm{~m}$ tungsten wire has been checked by measuring a flat plate case. Excellent agreement to Blackwell's data [9] has been achieved and is discussed in Section 3.14.


Fig. A. 3 Flat plate temperature fluctuations: comparison between the $1 \mu \mathrm{~m}$ and the $5 \mu \mathrm{~m}$ wires.

## APPENDIX B

## THE MEASUREMENT OF TURBULENCE QUANTITIES

The measurement of the turbulence quantities is done by means of the directional properties of the hot wire to velocity and temperature.

Let $E$ be the output from the hot wire anemometer; $U$ eff an effective velocity as felt by the wire and a function of the velocity components $u, v, w ; T$ the ambient temperature.

A small variation of the anemometer signal will be related to the temperature and velocity field by:

$$
\begin{equation*}
d E=\frac{\partial E}{\partial U_{\text {eff }}} \mathrm{dU}_{\text {eff }}+\frac{\partial \mathrm{E}}{\partial T} \mathrm{dT} \tag{B.1}
\end{equation*}
$$

For small fluctuations,

$$
\begin{equation*}
e^{\prime}=\frac{\partial E}{\partial U_{e f f}} u_{e f f}^{\prime}+\frac{\partial E}{\partial T} t^{\prime} \tag{B.2}
\end{equation*}
$$

If the measurement is referred to a system of coordinates in such a way that only the streamwise velocity component has a value different from zero, Eq. (B.2) can be written as:

$$
\begin{equation*}
e^{\prime}=\frac{\partial E}{\partial U} \frac{\partial U}{\partial U} u_{e f f}^{\prime} u_{e f f}+\frac{\partial E}{\partial T} t^{\prime} \tag{B.3}
\end{equation*}
$$

where the sensitivities $\frac{\partial E}{\partial U}$ and $\frac{\partial E}{\partial T}$ are determined directly from a calibration procedure. There remains only the determination of the relationship between the effective velocity and the velocity components.

1. Directional Sensitivity of the Hot Wire

The directional sensitivity of the hot wire to velocity components, as shown by Jorgensen [56] can be approximated by

$$
\begin{equation*}
\mathrm{U}_{\mathrm{eff}}^{2}=\mathrm{U}_{2}^{2}+\mathrm{k}_{1}^{2} \mathrm{~V}_{2}^{2}+\mathrm{k}_{2}^{2} \mathrm{~W}_{2}^{2} \tag{B.4}
\end{equation*}
$$

where $\mathrm{U}_{2}, \mathrm{~V}_{2}$, and $\mathrm{W}_{2}$ are the velocity components in the coordinate system of the wire: $V_{2}$ is the velocity component parallel to the wire, $W_{2}$ is perpendicular to the wire and to the wire supports and $U_{2}$ is perpendicular to the wire and lies in the plane of the wire supports. $k_{1}$ and $k_{2}$ are constants which depend upon the probe design (i.e., wire length and diameter, prong interference etc.) and have been determined by some investigators for the probe used in this experiment to be:

$$
\begin{align*}
& k_{1}=0.2  \tag{B.5}\\
& k_{2}=1.02
\end{align*}
$$

Figure B.I shows the geometry and position of the hot wire probe for the present analysis

B. 1 The geometry and coordinates of the slant wire.

The angle $\phi$ measures the inclination of the wire with respect to a plane perpendicular to the probe axis: for a "slant" wire, $\phi \neq 0$.

The present analysis follows from Andersen [8] and only the final results will be given.

Writing the effective velocity as a function of the velocity components in the frame of the laboratory ( $U_{1}, V_{1}, W_{1}$ )

$$
\begin{equation*}
\mathrm{U}_{\text {eff }}^{2}=A U_{1}^{2}+B v_{1}^{2}+C W_{1}^{2}+D U_{1} V_{1}+E V_{1} W_{1}+F U_{1} W_{1} \tag{B.6}
\end{equation*}
$$

where

$$
\begin{aligned}
& A=\cos ^{2} \phi+k_{1}^{2} \sin ^{2} \phi \\
& B=\left(\sin ^{2} \phi+k_{1}^{2} \cos ^{2} \phi\right) \cos ^{2} \theta+k_{2}^{2} \sin ^{2} \theta \\
& C=\left(\sin ^{2} \phi+k_{1}^{2} \cos ^{2} \phi\right) \sin ^{2} \theta+k_{2}^{2} \cos ^{2} \theta \\
& D=\left(1-k_{1}^{2}\right) \sin ^{2} \phi \cos \theta \\
& E=\left(\sin ^{2} \phi+k_{1}^{2} \cos ^{2} \theta-k_{2}^{2}\right) \sin 2 \theta \\
& F=\left(1-k_{1}^{2}\right) \sin 2 \phi \sin \theta
\end{aligned}
$$

The velocity components in the "mean flow" frame of reference may be expressed as follows, presuming that the mean velocity is aligned with the axis of the probe:

$$
\begin{align*}
& \mathrm{u}_{1}=\bar{u}+\mathrm{u}^{\prime} \\
& \mathrm{v}_{1}=\mathrm{v}^{\prime}  \tag{B.7}\\
& \mathrm{w}_{1}=\mathrm{w}^{\prime}
\end{align*}
$$

Expanding $\mathrm{U}_{\text {eff }}$ in Taylor series about ( $\overline{\mathrm{u}}, 0,0$ )

$$
\begin{gather*}
U_{e f f}=\sqrt{A} u+\sqrt{A} u^{\prime}+\frac{D}{2 \sqrt{A}} v^{\prime}+\frac{F}{2 \sqrt{A}} w^{\prime} \\
+\left(\frac{B}{\sqrt{A}}-\frac{D^{2}}{4 A \sqrt{A}}\right) \frac{v^{\prime 2}}{2 u}+\left(\frac{C}{\sqrt{A}}-\frac{F^{2}}{4 A \sqrt{A}}\right) \frac{w^{\prime}}{2 u} \\
+\left(\frac{E}{\sqrt{A}}-\frac{D F}{2 A \sqrt{A}}\right) \frac{v^{\prime} w^{\prime}}{2 u}+0(3) \tag{B.8}
\end{gather*}
$$

Define $U_{\text {eff }}^{\prime}=U_{\text {eff }}-\sqrt{A} \bar{u}$ from B. 8 .

Note that $\overline{U_{e f f}^{\prime}}$ is zero to $0(1)$ in the fluctuations, satisfying the requirement for a fluctuation term. Squaring $U_{\text {eff }}^{\prime}$, time averaging, and discarding terms of higher order than $O(2)$ yields the working expression for $\overline{\mathrm{U}_{\mathrm{eff}}^{\prime}}$ :

$$
\begin{equation*}
\overline{U_{e f f}^{\prime 2}}=A \overline{u^{\prime 2}}+\frac{D^{2}}{4 A} \overline{v^{\prime 2}}+\frac{F^{2} \overline{4 A} \overline{w^{\prime 2}}+D \overline{u^{\prime} v^{\prime}}+\frac{D F}{2 A} \overline{v^{\prime} w^{\prime}}+F \overline{u^{\prime} w^{\prime}}+0(3), ~(3)}{} \tag{B.10}
\end{equation*}
$$

Therefore, the mean square value of the effective velocity fluctuation can be related to the velocity components in the frame of the mean flow by (B.10) and can be used to measure all the components of the Reynolds stress tensor.
2. Test for Three Dimensional Effects on the Flow Field The probe system design used in this investigation allows measurements to be taken for a $\phi=48^{\circ}$ probe at $\theta=0^{\circ}, \pm 30^{\circ}, \pm 90^{\circ}$, $\pm 150^{\circ}$.

At $\theta=+90$, Eq. $(B-10)$ can be written as

$$
\begin{equation*}
\left.\overline{U_{e f f}^{\prime 2}}\right|_{\theta=90}=A \overline{u^{\prime 2}}+\frac{F^{2} \overline{4 A} \overline{w^{\prime 2}}+F \overline{u^{\prime} w^{\prime}}}{} \tag{B.II}
\end{equation*}
$$

At $\theta=-90$

$$
\begin{equation*}
\left.\overline{U_{e f f}^{\prime^{2}}}\right|_{\theta=-90}=A \overline{u^{\prime 2}}+\frac{F^{2} \overline{w^{\prime 2}}}{4 A}-F \overline{u^{\top} w^{\top}} \tag{B.12}
\end{equation*}
$$

Measurements for all examined strong adverse pressure gradient flows were taken at different $y$-locations of the boundary layer (normal to the wall) and for $\theta=+90^{\circ}$ and $\theta=-90^{\circ}$. Very small differences between the anemometer signals were observed, indicating that $\overline{u^{1} w^{\top}}$ was really very close to zero.

Measurements were also taken at $\theta= \pm 30^{\circ}$ and $\theta= \pm 150^{\circ}$ and it was concluded that $\overline{V^{\prime} w^{\boldsymbol{*}}}$ was also very close to zero.

The $2-D$ hypothesis about the flow field was therefore proved to be true and the analysis that follows will therefore be based on $\overline{u^{\top} w^{\top}}=$ $\overline{v^{\prime} w^{\prime}}=0$.
3. The Measurement of the Axial Velocity Component in Isothermal Flows
The measurement of the axial velocity component in isothermal flows is obtained by using a horizontal wire ( $\phi=0^{\circ}, \theta=90^{\circ}$ ).

Equation (B.10) can then be written as:

$$
\begin{equation*}
\overline{U_{e f f}^{\prime 2}}=\overline{u^{\prime 2}}+0(3) \tag{B.13}
\end{equation*}
$$

and to a second order approximation, the measurement of $\overline{u^{\prime 2}}$ can be done with a horizontal wire. The probe used for obtaining this profile has the advantage, due to its design, of obtaining measurements very close to the wall ( 0.005 in ), which cannot be obtained directly by means of a slant wire.

## 4. The Measurement of the Reynolds Stress Tensor Components in

## Isothermal Flows

Equation (B.10), with the 2-D hypothesis about the flow field $\left.\overline{\left(u^{\prime} w^{\prime}\right.}=\overline{v^{\prime} w^{\prime}}=0\right)$ can be written as:

In Eq. (B.14) it is assumed that the axial velocity fluctuation component $\overline{u^{\prime 2}}$ is known from the horizontal wire measurement. In principle, if data are obtained for 3 different probe angles $\theta\left(\phi=48^{\circ}\right.$ is constant along the path of the probe support rotation), a system of equations can be solved to obtain directly $\frac{v^{\prime 2}}{2}, \frac{w^{\prime 2}}{2}$ and $\overline{u^{\prime} v^{1}}$. This has been done for $\theta=-30^{\circ},-90^{\circ},-150^{\circ}$ using a fixed $\phi=48^{\circ}$.
5. The Measurement of $\overline{v^{\prime} t^{7}}$

The term $\frac{\partial U}{\partial U_{e f f}}$ in Eq. (B.3) can be obtained by differentiating
Eq. (B.9) and neglecting the fluctuating components. Then, it follows that

$$
\begin{equation*}
\frac{\partial U}{\partial U_{e f f}}=\frac{1}{\sqrt{A}} \tag{B.15}
\end{equation*}
$$

Squaring and averaging Eq. (B.3), and using (B.15)

$$
\begin{equation*}
\overline{e^{\prime 2}}=\left(\frac{\partial E}{\partial U} \frac{1}{\sqrt{A}}\right)^{2} \overline{U_{e f f}^{\prime 2}}+\left(\frac{\partial E}{\partial T}\right)^{2} \overline{t^{\prime 2}}+\left(2 \frac{\partial E}{\partial U} \frac{\partial E}{\partial T} \frac{1}{\sqrt{A}}\right)^{u_{e f f}^{\prime} t^{\prime}} \tag{B.16}
\end{equation*}
$$

If measurements are taken at $\theta=-30^{\circ}$ and $\theta=-150^{\circ}$ two equations are obtained. Introducing the definition of $U_{\text {eff }}^{\mathbf{1}}$ (Eq. B.9), subtracting, discarding terms above $0(2)$ :

$$
\begin{equation*}
\left.\overline{e^{\prime 2}}\right|_{\theta=-30}-\left.\overline{e^{\prime 2}}\right|_{\theta=-150}=\left(\frac{\partial E}{\partial U}\right)^{2} \frac{2 D}{A} \overline{u^{\prime} v^{\prime}}+\left(\frac{\partial E}{\partial U} \frac{\partial E}{\partial T}\right) \frac{2 D}{A} \cdot \overline{v^{\top} t^{\top}} \tag{B.17}
\end{equation*}
$$

From knowledge of $\overline{u^{\prime} v^{\top}}$, a value for $\overline{v^{\prime} t^{\top}}$ can be obtained; however, the value of $\overline{u^{\prime} v^{\top}}$ can be also obtained at the same time by using two more measurements at the same $\theta$ positions at different wire temperatures. The solution of a system of two linear equations will give $\overline{u^{1} v^{\top}}$ and $\overline{v^{\prime} t^{\top}}$ both from this pair of values.
6. The Measurement of $\overline{t^{\prime 2}}$

This measurement can be obtained by means of a resistance thermometry approach and was discussed in Appendix A.

## APPENDIX C

## THE MEASUREMENT OF SHEAR STRESS IN COMPLETELY

## DEVELOPED RECTANGULAR CHANNEL FLOW

The present analysis is intended as a baseline for the measurement of the Reynolds stress tensor components in the adverse pressure gradient flows of this investigation.

A completely developed rectangular channel flow was used for checking the new hot wire system because the shear stress is known to follow a theoretically established equation, which is developed in this appendix.

Figure C.1 shows the channel flow and the system of coordinates to be used in this analysis.


Figure C.l Channel flow and the system of coordinates.

The continuity equation for a two dimensional, constant density flow can be written as

$$
\begin{equation*}
\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}=0 \tag{C.1}
\end{equation*}
$$

Because of the fact that the flow is completely developed, $\frac{\partial \bar{u}}{\partial x}=0$. As a consequence, $\vec{v}=0$ because at the wall $v_{0}=0$.

The Navier Stokes equation for $2-D$ completely developed flows, after having dropped the $x$-derivatives of the velocity components and by setting $\overline{\mathrm{v}}=0$, is:

$$
\begin{equation*}
0=-\frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial x}+v \frac{\partial^{2} \bar{u}}{\partial y^{2}}-\frac{\partial}{\partial y}\left(\overline{u^{\prime} v^{\prime}}\right) \tag{C.2}
\end{equation*}
$$

Integration of (C.2) with respect to $y$, and setting $u_{\tau}^{2}=\left.v \frac{\partial \bar{u}}{\partial y}\right|_{o}$ gives

$$
\begin{equation*}
0=-\frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial x} y+v \frac{\partial \bar{u}}{\partial y}-\overline{u^{1} v^{\prime}}-u_{\tau}^{2} \tag{C.3}
\end{equation*}
$$

At the centerline $(y=\delta)$, however, $\frac{\partial \bar{u}}{\partial y}=0, \overline{u^{\prime} v^{\top}}=0$ hence by using (C.3)

$$
\begin{equation*}
u_{\tau}^{2}=-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \mathrm{x}} \delta \tag{C.4}
\end{equation*}
$$

Equation (C.3) can therefore be written as

$$
\begin{equation*}
\tau^{+}=\frac{1}{u_{\tau}^{2}}\left(\nu \frac{\partial \bar{u}}{\partial y}-\overline{u^{\prime} v^{\prime}}\right)=\left(1-\frac{y}{\delta}\right) \tag{C.5}
\end{equation*}
$$

It is therefore concluded that the total stress $\tau^{+}$is a linear function of the distance from the wall. Far away from the wall, the laminar contribution to the total stress is very small and the turbulent stress can be written as

$$
\begin{equation*}
-\frac{\overline{u^{\top} v^{\top}}}{u_{\tau}^{2}}=1-\frac{y}{\delta} \tag{C.6}
\end{equation*}
$$

The determination of the friction velocity $u_{\tau}$ can be made directly by pressure drop measurements along the channel; thus the shear stress $\overline{u^{\prime} v^{\prime}}$ can be known accurately and compared to the value obtained with the new hot wire system.

## APPENDIX D

THE DIRECT MEASUREMENT OF THE TURBULENT KINETIC ENERGY AND THE SHEAR
STRESS BY MEANS OF A SINGLE SLANT, ROTATABLE HOT WIRE PROBE

As shown in Appendix B, the measurement of the Reynolds stress tensor components can be done by using a single rotatable wire, which provides measurements at different positions of the probe. The solution of a system of linear equations will thus give the value of each component separately.

There has been lately, however, a great interest in the turbulent kinetic energy profile in order to provide a closure for the turbulent flow system of equations. Turbulent kinetic energy can be found by adding up the three components of the mean square values of the velocity fluctuation vector to give the turbulent kinetic energy defined as

$$
\begin{equation*}
\overline{q^{2}}=\overline{u^{\prime 2}}+\overline{v^{\prime 2}}+\overline{w^{\prime 2}} \tag{D.I}
\end{equation*}
$$

The uncertainties of the measurement of each component, however, add up to give an even higher uncertainty in $\overline{q^{2}}$. In some applications where only the value of $\overline{q^{2}}$ is required, its calculation based on the sum of each component becomes time consuming and an unnecessary procedure.

A measurement procedure is proposed, to give directly the value of $\overline{q^{2}}$ and $\overline{u^{i} v}$ by means of a single rotating slant wire.

Equation (B.14), in Appendix B, shows that the hot wire response in a two-dimensional flow field can be written in the reference frame of the mean flow as

$$
\begin{equation*}
\overline{u_{i}^{\prime 2}}=A \overline{u^{\prime 2}}+\frac{D^{2}}{4 A} \overline{v^{\prime 2}}+\frac{F^{2}}{4 A} \overline{w^{\prime 2}}+D \overline{u^{\prime} v^{\prime}}+0(3) \tag{B.14}
\end{equation*}
$$

where $A, D, F$ depend on the angles of the wire and the probe design.

If a combination of the parameters is chosen such that

$$
\begin{equation*}
A=\frac{D^{2}}{4 A}=\frac{F^{2}}{4 A} \tag{D.2}
\end{equation*}
$$

Equation (B.14) can be written as

$$
\begin{equation*}
\overline{u_{i}^{\prime 2}}=A \overline{q^{2}}+D \overline{u^{\prime} v^{\prime}} \tag{D.3}
\end{equation*}
$$

Two measurements are therefore sufficient to give $\overline{q^{2}}$ and $\overline{u^{\prime} v^{\prime}}$.
The values of $A, D, F$ can be used from (B.6) to solve a system of equations and give $\theta$ and $\phi$.

The first relationship, $\frac{D^{2}}{4 A}=\frac{F^{2}}{4 A}$ gives

$$
\begin{gather*}
\mathrm{D}= \pm \mathrm{F}  \tag{D.4}\\
\text { or } \quad\left(1-\mathrm{k}_{1}^{2}\right) \sin 2 \phi \cos \theta= \pm\left(1-\mathrm{k}_{1}^{2}\right) \sin 2 \phi \sin \theta \tag{D.5}
\end{gather*}
$$

which gives

$$
\begin{equation*}
\theta=(2 K+1) \frac{\pi}{4}, K=0, \pm 1, \pm 2 \cdots \tag{D.6}
\end{equation*}
$$

This means that the direct measurement can only be done in particular positions given by (D.6).

The second relationship $\quad A=\frac{D^{2}}{4 A}$
gives

$$
\begin{equation*}
D= \pm 2 \mathrm{~A} \text { or } \tag{D.7}
\end{equation*}
$$

$$
\begin{equation*}
\left(1-k_{1}^{2}\right) \sin 2 \phi \cos \theta=\cos ^{2} \phi+k_{1}^{2} \sin ^{2} \phi \tag{D.8}
\end{equation*}
$$

This non-linear equation can be solved for $\phi$, using $\theta= \pm 45^{\circ}$ and the functional dependence of $k_{1}$ on $\phi$ and $\theta$, as indicated by Friehe and Shwartz [59] to give a wire angle of approximately $\phi \approx 57^{\circ}$. This angle is different from the usual $45^{\circ}$ slant wire; the probe would thus have to be built under these specifications and the comparison between its direct measurement of $\overline{q^{2}}$ and the value obtained from adding up the components from a slanted probe would check its accuracy.

Unfortunately, manufacturing difficulties prevented such a probe from being built. Another trial will be made in a future program. The probe would be basically a $57^{\circ}$ gold plated slant wire, diameter of $5 \mu \mathrm{~m}$, with provisions for the rotating mechanism to stop at every $45^{\circ}$. This probe would be the indicated one for measurements in isothermal flows.

Appendix E
TABULATIONS OF EXPERIMENTAL DATA

Table E-1. Friction Factor Data

Table E-1.1 Friction Factor Data ( $-0.275,0.0$ )

|  | $x=22$ | $\mathrm{x}=34$ | $x=46$ | $\mathrm{x}=58$ | $\mathrm{x}=70$ | $\mathrm{x}=82$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k\left(\frac{u_{0} y}{v_{\infty}}\right)$ | -0.00075 | -0.00050 | -0.00038 | -0.00031 | -0.00027 | - |
| $v \frac{\partial u}{\partial y}$ | 0.0687 | 0.0637 | 0.0460 | 0.0545 | 0.0499 | - |
| $-\overline{u^{\prime} v^{\top}}$ | 0.773 | 0.402 | 0.296 | 0.269 | 0.239 | - |
| $\frac{1}{u_{\infty}^{2}}\left(\nu \frac{\partial u}{\partial y}-\overline{u^{\prime} v^{\top}}\right)$ | 0.00178 | 0.00126 | 0.00107 | 0.00108 | 0.00104 | - |
| $-\frac{1}{\rho_{\infty} u_{\infty}^{2}} \frac{\partial}{\partial \mathrm{x}} \int_{0}^{y^{\prime}}{ }_{\text {pu }}{ }^{2} \mathrm{dy}$ | 0.00001 | 0. | 0. | 0. | 0. | - |
| $-\frac{u^{0}}{} \mathbf{u}_{0}^{2}$ | 0. | 0. | 0. | 0. | 0. | - |
| $\frac{1}{\rho_{\infty} u_{\infty}^{2}}$ u $\frac{\partial}{\partial x} \int_{0}^{y^{\prime}}{ }_{\text {dudy }}$ | -0.00009 | -0.00008 | -0.00005 | -0. | -0.00004 | - |
| $\mathrm{c}_{\mathrm{f}} / 2$ | 0.00096 | 0.00069 | 0.00065 | 0.00076 | 0.00081 | - |
| $c_{\text {f }} / 2$ (Andersen) | 0.00079 | 0.00055 | 0.00047 | 0.00053 | 0.00048 | - |
| $\mathrm{Re}_{\mathrm{S}_{2}}$ | 1991 | 3061 | 3844 | 4695 | 5517 | - |

Table E-1.2 Friction Factor Data (-0.275, -0.001)

|  | $\mathrm{x}=22$ | $\mathrm{x}=34$ | $\mathrm{x}=46$ | $\mathrm{x}=58$ | $\mathrm{x}=70$ | $\mathrm{x}=82$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa\left(\frac{u_{0} y}{V_{\infty}}\right)$ | -0.00075 | -0.00050 | -0.00037 | -0.00030 | -0.00027 | -0.00024 |
| $v \frac{\partial u}{\partial y}$ | 0.0788 | 0.0653 | 0.0647 | 0.0587 | 0.0662 | 0.0611 |
| $-\overline{u^{\prime} v^{\top}}$ | 0.833 | 0.457 | 0.3190 | 0.2450 | 0.236 | 0.208 |
| $\frac{1}{u_{\infty}^{2}}\left(v \frac{\partial u}{\partial y}-\overline{u^{\top} v^{\top}}\right)$ | 0.00184 | 0.00136 | 0.00120 | 0.00103 | 0.00105 | 0.00105 |
| $-\frac{1}{\rho_{\infty} u_{\infty}^{2}} \frac{\partial}{\partial x} \int_{0}^{y_{p u}{ }^{2} d y}$ | 0.00001 | 0. | 0. | 0. | 0. | 0. |
| $-\frac{u_{0}}{u_{0}^{2}}$ | 0.00048 | 0.00041 | 0.00038 | 0.00039 | 0.00039 | 0.00035 |
| $\frac{1}{\rho_{004} u_{0}^{2}} \mathrm{u} \frac{\partial}{\partial x} \int_{0}^{y}{ }_{\text {pudy }}$ | -0.00010 | -0.00010 | -0.00008 | -0.00005 | 0. | -0.00004 |
| $c_{f} / 2$ | 0.00146 | 0.00118 | 0.00114 | 0.00108 | 0.00116 | 0.00121 |
| $c_{f} / 2$ (Andersen) | 0.00123 | 0.00097 | 0.00093 | 0.00103 | 0.00114 | 0.00097 |
| $\mathrm{Re}_{\delta_{2}}$ | 1658 | 2392 | 2939 | 3351 | 3846 | 4494 |

Table E-1.3 Friction Factor Data ( -0.275, -0.002)

|  | $\mathrm{x}=22$ | $\mathrm{x}=34$ | $\mathrm{x}=46$ | $\mathrm{x}=58$ | $\mathrm{x}=70$ | $\mathrm{x}=82$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\kappa\left(\frac{u_{\infty} \mathrm{Y}}{\mathrm{v}_{\infty}}\right)$ | -0.00075 | -0.00050 | -0.00037 | -0.00030 | -0.00026 | -0.00021 |
| $v \frac{\partial u}{\partial y}$ | 0.0793 | 0.0707 | 0.0648 | 0.0717 | 0.0754 | 0.0724 |
| $-\overline{u^{\prime} v^{\prime}}$ | 0.878 | 0.482 | 0.347 | O. 299 | 0.261 | 0.217 |
| $\frac{1}{u_{\infty}}\left(v \frac{\partial u}{\partial y}-\overline{u^{\prime} v^{\prime}}\right)$ | 0.00199 | 0.00148 | 0.00130 | 0.00126 | 0.00121 | 0.00114 |
| $-\frac{1}{\rho_{\infty} u_{\infty}^{2}} \frac{\partial}{\partial x} \int_{0}^{y_{p u}{ }^{2} d y}$ | 0.00001 | 0.00001 | 0. | 0. | 0. | 0. |
| $-\frac{u v_{0}}{u_{\infty}^{2}}$ | 0.00110 | 0.00100 | 0.00096 | 0.00099 | 0.00098 | 0.00094 |
| $\frac{1}{\rho_{00} u_{\infty}^{2}} \mathrm{u} \frac{\mathrm{v}_{0}}{u_{0}} \int_{0}{ }^{\mathrm{y}}$ pudy | -0.00014 | -0.00014 | -0.00012 | -0.00008 | -0.00003 | -0.00003 |
| $\mathrm{c}_{\mathrm{f}} / 2$ | 0.00221 | 0.00185 | 0.00178 | 0.00187 | 0.00191 | 0.00190 |
| $c_{\mathrm{f}} / 2$ (Andersen) | 0.00170 | 0.00147 | 0.00146 | 0.00166 | 0.00176 | 0.00158 |
| $\mathrm{Re}_{\delta_{2}}$ | 1312 | 1868 | 2283 | 2430 | 2678 | 3144 |

Table E-1.4 Friction Factor Data ( $-0.275,-0.0004$ )

|  | $\mathrm{x}=22$ | $\mathrm{x}=34$ | $\mathrm{x}=46$ | $x=58$ | $\mathrm{x}=70$ | $x=82$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k\left(\frac{u_{\infty} y}{v_{\infty}}\right)$ | -0.00076 | -0.00050 | -0.00038 | -0.00031 | -0.00027 | -0.00023 |
| $v \frac{\partial u}{\partial y}$ | 0.0649 | 0.0635 | 0.0611 | 0.0605 | 0.0502 | 0.0618 |
| - $\overline{u^{\prime} v^{\prime}}$ | 0.568 | 0.376 | 0.253 | 0.203 | 0.201 | 0.162 |
| $\frac{1}{u_{\infty}}\left(\nu \frac{\partial u}{\partial y}-\overline{u^{\prime} v^{\prime}}\right)$ | 0.00136 | 0.00121 | 0.00103 | 0.00092 | 0.00094 | 0.00093 |
| $-\frac{1}{\rho_{\infty} u_{\infty}^{2}} \frac{\partial}{\partial x} \int_{0}{ }^{y} \mathrm{pu}^{2} d y$ | 0.00043 | 0.00043 | 0.00036 | 0.00023 | 0.00007 | 0.00011 |
| $-\frac{u v_{0}}{u_{\infty}^{2}}$ | 0.00287 | 0.00278 | 0.00273 | 0.00278 | 0.00281 | 0.00263 |
| $\frac{1}{\rho_{00} u_{0 a}^{2}}$ u $\frac{\partial}{\partial x} \int_{0}^{y}{ }^{\text {pudy }}$ | -0.00019 | -0.00020 | -0.00018 | -0.00013 | -0.00007 | 0. |
| $c_{f} / 2$ | 0.00370 | 0.00371 | 0.00357 | 0.00349 | 0.00350 | 0.00323 |
| $c_{f} / 2$ (Andersen | 0.00269 | 0.00256 | 0.00265 | 0.00265 | 0.00296 | 0.00290 |
| $\mathrm{Re}_{\delta_{2}}$ | 834 | 1072 | 1124 | 1211 | 1255 | 1364 |

## E-2 Experimental Stanton Number at a Constant Wall Temperature Condition

The runs are tabulated below and, with the help of the special nomenclature, should be self-explanatory.

| Date | m | F |
| :---: | :---: | :---: |
| 012973 | -0.275 | 0 |
| 021273 | -0.275 | -0.001 |
| 030473 | -0.275 | -0.002 |
| 031273 | -0.275 | -0.004 |

## Special Nomenclature

| Symbol | Explanation | Unit |
| :---: | :---: | :---: |
| TAMB | Ambient temperature | ${ }^{\circ} \mathrm{F}$ |
| TBASE | Casting base temperature | ${ }^{\circ} \mathrm{F}$ |
| TGAS | Free stream static temperature | ${ }^{\circ} \mathrm{F}$ |
| TCOV | Wind tunnel cover (top) temperature | ${ }^{\circ} \mathrm{F}$ |
| PBAR | Barometric pressure | in Hg |
| RHUM | Relative humidity | - |
| PL | Plate number | - |
| X | Stream-wise coordinate | in |
| UINF | Free stream velocity, $\mathrm{U}_{\infty}$ | $\mathrm{ft} / \mathrm{sec}$ |
| T0 | Plate temperature | ${ }^{\circ} \mathrm{F}$ |
| F | $\dot{m}^{\prime \prime} / \rho_{\infty} \mathrm{U}_{\infty}$ (negative for suction) | - |
| ST | Stanton number | - |
| RED2 | Enthalpy thickness Reynolds number | - |
| STO | Stanton number for $\mathrm{F}=0$ (same RED2) | - |
| BH | F/St | - |

DATE $=012973 \quad(-0.275,0.1$
TAMB $=71.9$
TCOV $=70.0 \quad$ TBASE $=76.1 \quad$ PBAR $=30.32 \quad$ TGAS $=59.0$
RHUM $=0.80$

| PL | X | UINF | TO | F | ST | RED2 | ST/STO | BH |
| ---: | ---: | ---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 10. | 25.7 | 91.5 | 0.00000 | 0.00315 | 650. | 1.082 | 0.000 |
| 4 | 14. | 24.1 | 91.3 | 0.00000 | 0.00293 | 814. | 1.065 | 0.000 |
| 5 | 18. | 22.6 | 91.4 | 0.00000 | 0.00278 | 954. | 1.051 | 0.000 |
| 6 | 22. | 21.3 | 91.4 | 0.00000 | 0.00269 | 1081. | 1.049 | 0.000 |
| 7 | 26. | 20.3 | 91.5 | 0.00000 | 0.00253 | 1195. | 1.012 | 0.000 |
| 8 | 30. | 19.4 | 91.3 | 0.00000 | 0.00241 | 1306. | 0.986 | 0.000 |
| 9 | 34. | 18.8 | 91.4 | 0.00000 | 0.00235 | 1397. | 0.977 | 0.000 |
| 10 | 38. | 18.3 | 91.3 | 0.00000 | 0.00229 | 1494. | 0.969 | 0.000 |
| 11 | 42. | 17.9 | 91.5 | 0.00000 | 0.00218 | 1573. | 0.934 | 0.000 |
| 12 | 46. | 17.5 | 91.2 | 0.00000 | 0.00227 | 1670. | 0.987 | 0.000 |
| 13 | 50. | 17.2 | 91.3 | 0.00000 | 0.00226 | 1749. | 0.994 | 0.000 |
| 14 | 54. | 16.9 | 91.4 | 0.00000 | 0.00217 | 1823. | 0.965 | 0.000 |
| 15 | 58. | 16.7 | 91.2 | 0.00000 | 0.00223 | 1915. | 1.004 | 0.000 |
| 16 | 62. | 16.5 | 91.4 | 0.00000 | 0.00217 | 1984. | 0.985 | 0.000 |
| 17 | 66. | 16.3 | 91.3 | 0.00000 | 0.00221 | 2063. | 1.013 | 0.000 |
| 18 | 70. | 16.0 | 91.3 | 0.00000 | 0.00217 | 2138. | 1.004 | 0.000 |
| 19 | 74. | 15.7 | 91.3 | 0.00000 | 0.00214 | 2215. | 0.999 | 0.000 |
| 20 | 78. | 15.4 | 91.2 | 0.00000 | 0.00215 | 2287. | 1.011 | 0.000 |
| 21 | 82. | 15.1 | 91.3 | 0.00000 | 0.00203 | 2353. | 0.962 | 0.000 |
| 22 | 86. | 14.9 | 91.3 | 0.00000 | 0.00210 | 2415. | 1.001 | 0.000 |
| 23 | 90. | 14.7 | 91.4 | 0.00000 | 0.00204 | 2477. | 0.979 | 0.000 |

STANTON NUMBER - CONSTANT WALL TFMPERATURE

$$
\begin{gathered}
\text { DATE }=021273 \quad 1-0.275,-0.0011 \\
\text { TAMB }=72.7 \quad \text { TBASE }=77.9 \quad \text { TGAS }=63.7 \\
\text { TCOV }=70.8 \quad \text { PBAR }=30.20 \quad \text { RHUM }=0.68
\end{gathered}
$$

| PL | $X$ | $U I N F$ | $T 0$ | $F$ | $S T$ | $R E O 2$ | $S T / S T O$ | $B H$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 10. | 26.0 | 88.7 | -0.00099 | 0.00358 | 559. | 1.184 | -0.277 |
| 4 | 14. | 24.4 | 88.7 | -0.00098 | 0.00345 | 695. | 1.205 | -0.284 |
| 5 | 18. | 22.9 | 88.8 | -0.00099 | 0.00331 | 811. | 1.202 | -0.299 |
| 6 | 22. | 21.6 | 88.7 | -0.00100 | 0.00325 | 922. | 1.218 | -0.308 |
| 7 | 26. | 20.6 | 88.7 | -0.00101 | 0.00310 | 1018. | 1.191 | -0.326 |
| 8 | 30. | 19.7 | 88.7 | -0.00102 | 0.00305 | 1107. | 1.197 | -0.334 |
| 9 | 34. | 19.0 | 88.8 | -0.00103 | 0.00296 | 1183. | 1.181 | -0.348 |
| 10 | 38. | 18.4 | 88.8 | -0.00103 | 0.00291 | 1258. | 1.179 | -0.354 |
| 11 | 42. | 18.0 | 88.9 | -0.00103 | 0.00276 | 1324. | 1.133 | -0.373 |
| 12 | 46. | 17.6 | 88.7 | -0.00103 | 0.00286 | 1398. | 1.190 | -0.360 |
| 13 | 50. | 17.3 | 88.8 | -0.00102 | 0.00283 | 1462. | 1.190 | -0.360 |
| 14 | 54. | 17.1 | 88.8 | -0.00100 | 0.00274 | 1527. | 1.165 | -0.365 |
| 15 | 58. | 16.9 | 88.8 | -0.00102 | 0.00280 | 1590. | 1.203 | -0.364 |
| 16 | 62. | 16.6 | 88.7 | -0.00100 | 0.00268 | 1654. | 1.163 | -0.373 |
| 17 | 66. | 16.5 | 88.8 | -0.00100 | 0.00273 | 1709. | 1.194 | -0.366 |
| 18 | 70. | 16.3 | 89.0 | -0.00099 | 0.00269 | 1757. | 1.185 | -0.368 |
| 19 | 74. | 16.1 | 88.8 | -0.00099 | 0.00267 | 1830. | 1.188 | -0.371 |
| 20 | 78. | 15.8 | 88.7 | -0.00100 | 0.00264 | 1887. | 1.184 | -0.379 |
| 21 | 82. | 15.4 | 88.8 | -0.00100 | 0.00258 | 1935. | 1.164 | -0.388 |
| 22 | 86. | 15.1 | 88.7 | -0.00103 | 0.00266 | 1993. | 1.209 | -0.387 |
| 23 | 90. | 14.7 | 88.8 | -0.00103 | 0.00274 | 2041. | 1.253 | -0.376 |

STANTON NUMBER - CONSTANT WALL TEMPERATURE
JATE $=63 C 473 \quad(-2.275,-0.002)$
TAMB $=67.8 \quad$ TBASE $=79.5 \quad$ TGAS $=64.2$
TCDV $=70.4 \quad$ PBAR $=30.30 \quad$ RHUM $=0.63$

| PL | X | UINF | T0 | F | ST | RFO2 | ST/STO | BH |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 10. | 26.0 | 88.5 | -0.00199 | 0.00398 | 409. | 1.218 | -0.500 |
| 4 | 14. | 24.4 | 89.4 | -0.00196 | 0.00386 | 511. | 1.248 | -0.508 |
| 5 | 18. | 22.9 | 88.6 | -0.00198 | 0.00378 | 598. | 1.272 | -0.524 |
| 5 | 22. | 21.6 | 80.6 | -0.00199 | 0.00374 | 681 | 1.370 | -0.532 |
| 7 | 26. | 20.6 | 83.5 | -0.00201 | 0.00361 | 756 | 1.288 | -0.557 |
| 8 | 32. | 19.7 | 88.6 | -0.00204 | 0.00359 | 820. | 1.307 | -0.568 |
| 9 | 34 | $19.1)$ | 88.5 | -0.00204 | 0.00347 | 881. | 1.286 | -0.598 |
| 10 | 38. | 18.4 | 88.7 | -0.00205 | 0.00351 | 932. | 1.319 | -6. 584 |
| 11 | 42. | 17.9 | 38.6 | -0.00206 | 0.00334 | 987. | 1.274 | -0.617 |
| 12 | 46. | 17.5 | 88.5 | - 0.00205 | 0.00341 | 1049. | 1.313 | -0.603 |
| 13 | 53. | 17.3 | 88.5 | -0.0c206 | 0.00342 | 1088. | 1.336 | -5.60? |
| 14 | 54. | 17.0 | 89.5 | -0.00202 | 0.00332 | $1130^{\circ}$ | 1.311 | -1.608 |
| 15 | 58. | 16.8 | 88.5 | -0.00201 | 0.00335 | 1184. | 1.337 | -0.600 |
| 16 | 52. | 16.t | 88.5 | -0.00200 | 0.00324 | 1227. | 1.304 | -0.617 |
| 17 | 66. | 16.5 | 88.5 | -0.00199 | 0.00329 | 1273. | 1.337 | -0.60) 5 |
| 19 | 7. | 16.3 | 83.4 | -0.00199 | 0.00323 | 1329. | 1.324 | -0.616 |
| 19 | 74. | 16.1 | 88.6 | -0.00199 | 0. 00318 | 1352. | 1.312 | -9.626 |
| 20 | 78. | 15.8 | 88.6 | -0.00202 | 0.00326 | 1393. | 1.355 | -9.62C |
| 21 | 82. | 15.5 | 38.6 | -0.00202 | 0.00319 | 1432. | 1.335 | -0.633 |
| 22 | 86. | 15.1 | 88.5 | -0.00204 | 0.00322 | 1475. | 1.357 | -0.6.34 |
| 23 | 90. | 14.8 | 88.6 | -6.00203 | 0.cc321 | 1507. | 1.361 | -r. 0.632 |

STANT TN INUMBER - CINST AMT WALL TEMPFRATURE:
DATE $=031273 \quad(-1) .275,-6) .0041$
TAHB $=77.7 \quad$ TRASE $=81.1 \quad$ TGAS $=63.4$
TCUV $=71.2 \quad$ PBAR $=30.25 \quad$ RHU4 $=0.58$

| PL | $x$ | UINE | $T:$ | F | $\subseteq T$ | \&も?2 | $S T / C T$ | RH: |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 10. | 25.9 | 87.0 | -0.00411 | 0.00520 | 341. | 1.520 | -0. 790 |
| 4 | 14. | 24.3 | 86.9 | -0.00405 | 0.00507 | 398. | 1.54 ? | -12.799 |
| 5 | 18. | 22.9 | 87.0 | -0.00402 | 0.00493 | 4412. | 1. 541 | -0.815 |
| 6 | 22. | 21.7 | 86.8 | -0.00413 | 0.00503 | 492. | 1.612 | -0.821 |
| 7 | 26. | 2). 7 | 86.3 | -3. 56423 | 0.01491 | 527. | 1.630 | $-0.862$ |
| 9 | 30. | 19.8 | 8t. 7 | -0.00434 | 0.00591 | 558. | 1.656 | -9.866 |
| 9 | 34. | 19.1 | 86.9 | -0.00423 | 0.001484 | 580. | 1.616 | -0.874 |
| 1.3 | 38 | 18.4 | 36.9 | -0.00431 | 0.00485 | 604. | 1.636 | $-0.889$ |
| 11 | 42. | 18.9 | 86. 4 | -0.00423 | 0.00467 | 622. | 1.587 | -0.906 |
| 12 | 46. | 17.6 | $8 \mathrm{H}$. | -0.50417 | U.0.9471 | 644. | 1.614 | -r.885 |
| 13 | 50. | 17.3 | 86.9 | -0.00423 | 0.00480 | 6t? | 1.656 | -0.881 |
| 14 | 54. | 17.1 | 86.8 | -0.00414 | 0.00466 | 683. | 1.621 | -0.888 |
| 15 | 58. | 16.9 | 86.8 | -0.004416 | 0.00471 | 703. | 1.659 | -0.843 |
| 15 | 62. | 16.7 | 86.8 | -0.00413 | 0.00457 | 721. | 1.611 | - 0.9004 |
| 17 | 66. | 18.6 | 86.9 | -0.00408 | 0.00454 | 734. | 1.628 | -1. 1.899 |
| 18 | 79. | 15.4 | 83. 7 | -0.00410 | 0.00454 | 756. | 1.619 | -0.903 |
| 19 | 74. | 16.1 | 86.4 | -0.00399 | 0.00440 | 763. | 1.573 | -0.907 |
| 29 | 78. | 15.9 | 86.8 | -0.0c422 | 0.00457 | 773. | 1.642 | -0.923 |
| 21 | 82. | 15.5 | 86.9 | -0.00405 | 0.00447 | 789. | 1.612 | -0.906 |
| 22 | 86. | 15.2 | 86.8 | -0.00406 | 0.00447 | 804. | 1.619 | -0.908 |
| 23 | 90. | 14.8 | 86.8 | -0.0042.0 | 0.00452 | e18. | 1.644 | -8.929 |

## E-3 Experimental Stanton Numbers: Step in Wall Temperature Condition

 The runs are tabulated below and, with the help of the special nomenclature and the one from E-2, the data tabulation should be selfexplanatory. It should be noted that the step is present at plate 10. Before and after that plate the wall temperature is constant.| $\frac{1}{\text { Date }}$ |  | m | F |
| :---: | :---: | :---: | :---: |
|  | 122272 |  | 0 |
|  |  | 0 |  |
| 122872 |  | -0.15 |  |
| 010273 |  | -0.15 |  |
| 0.001 |  |  |  |
| 010473 |  | -0.15 | 0.004 |
| 012973 |  | -0.275 | 0 |
| 021973 |  | -0.275 | -0.001 |
| 030173 |  | -0.275 | -0.002 |
| 031273 |  | -0.275 | -0.004 |

Special Nomenclature

Stanton number at a plate for a step condition
Stanton number at a plate for a constant wall temperature condition (same hydrodynamics)

STC Predicted Stanton number at a plate for a step condition


STANTON NUMBER - STEP IN WALL TEMPERATURE

$$
\begin{gathered}
\text { DATE }=122872(-0.15,0) \\
\text { TAMB }=70.6 \quad \text { TBASE }=7305 \\
\text { TCOV }=69.3 \quad \text { PBAR }=30.37
\end{gathered} \quad \text { RHUS }=6405=0.50 .
$$

| PL | X | UINF | TO | STT | ST | STC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 150 | 26.4 | 65.0 | 0.00312 |  |  |
| 4 | 19\% | 25.5 | 6501 | 0.00289 |  |  |
| 5 | 230 | 24.6 | 6501 | O.C0277 |  |  |
| 6 | 270 | 2401 | 65.2 | 0.00269 |  |  |
| 7 | 310 | 23.6 | 65.4 | 0.00255 |  |  |
| 8 | 350 | 23.1 | 65.5 | 0.00250 |  |  |
| 9 | 390 | 22.8 | 67.3 | 0.00242 |  |  |
| 10 | 430 | 2204 | 90.1 | 0.00239 | 0.00425 | 0.00340 |
| 11 | 470 | 22.0 | 90.4 | 0.00235 | 0.00290 | 0.00299 |
| 12 | 510 | 2108 | 90.4 | 0.00238 | 0000292 | 0.00288 |
| 13 | 550 | 2105 | 90.4 | 0000232 | 0.00278 | 0.00273 |
| 14 | 590 | 21.2 | 90.5 | 0000221 | 0.00262 | 0000255 |
| 15 | 63. | 21.0 | 90.7 | 0.00225 | 0.00263 | 0.00255 |
| 16 | 670 | 20.8 | 90.6 | 0.00217 | 0.00251 | 0.00243 |
| 17 | 710 | 20.6 | 90.6 | 0000222 | 0000248 | 0.00247 |
| 18 | 750 | 20.4 | 90.5 | 0.00216 | Co00239 | 0.00238 |
| 19 | 790 | 20.3 | 90.5 | 0.00213 | 0.00238 | 0.00233 |
| 20 | 83. | 20.1 | 90.5 | 0.00214 | 0.00232 | 0.00233 |
| 21 | 87. | 20.0 | 90.5 | 0.60203 | 0.00224 | 0.00220 |
| 22 | 910 | 1908 | 90.5 | C000209 | 0.00228 | 0.00225 |
| 23 | 950 | 19.7 | 90.4 | 0.00238 | 0.00226 | 0.00223 |

STANTON NUMBER - STEP IN WALL TEMPERATURE
DATE $=010273(-0.15,000011$

| TAMB $=67.6$ | TBASE $=71.8$ | TGAS $=65.0$ |
| :--- | :--- | :--- |
| TCOV $=68.5$ | PBAR $=30.43$ | RHUM $=0.57$ |

$x$ UINF
TO ST

150 $26.4 \quad 66.4 \quad 0.00270$
190 2504 66.5 0.00246
$2302406 \quad 6605 \quad 0.00232$
27. 2400 6606 0.00226
31. $23.6 \quad 66.9 \quad 0.00217$

350 23.1 67.0 0.00200
39. 22.7 68.6 0.00192
$430 \quad 2203 \quad 88.9 \quad 0.0192$
$4702200 \quad 8900$
51\% 22.0
89000.00192
$8900 \quad 0.00193$

| 0.00365 | 0.00304 |
| :--- | :--- |
| 0.00253 | 0.00256 |
| 0.00244 | 0.00243 |
| 0.00227 | 0.00228 |
| 0.00212 | 0.00207 |
| 0.00214 | 0.00211 |
| 0.00206 | 0.00202 |
| 0.00202 | 0.00200 |
| 0.00194 | 0.00192 |
| 0.00192 | 0.00188 |
| 0.00197 | 0.00194 |
| 0.00182 | 0.00179 |
| 0.00181 | 0.00176 |

STANTON NUMBER - STEP IN WALL TEMPERATURE
DATE $=010473 \quad\{-0.15,0.0041$

| TAMB $=68.2$ | TBASE $=69.8$ | TGAS $=63.9$ |
| :--- | :--- | :--- |
| TCOV $=67.1$ | PBAR $=30.51$ | RHUM $=0.62$ |


| PL | $x$ | U INF | 10 | STT | ST | STC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 150 | 26.4 | 67.2 | 0000168 |  |  |
| 4 | 190 | 25.4 | 67.3 | 0.00130 |  |  |
| 5 | 230 | 2406 | 6705 | 0.000113 |  |  |
| 6 | 27。 | 24.0 | 67.6 | 0060114 |  |  |
| 7 | 310 | 23.5 | 67.9 | 0.00107 |  |  |
| 8 | 350 | 23.1 | 68.0 | 0.00090 |  |  |
| 9 | 390 | 22.7 | 69.3 | 0.00087 |  |  |
| 10 | 430 | 22.3 | 86.9 | 0.00089 | 0.00194 | 0.00172 |
| 11 | 470 | 2200 | 87.3 | 0.00061 | 0.00100 | 0.00097 |
| 12 | 51. | 2108 | 87.1 | 0.00076 | 0.00103 | 0.00110 |
| 13 | 55. | 2105 | 87.0 | 0.00070 | 0.00097 | 0.00097 |
| 14 | 59. | 21.3 | 87.1 | 0.00070 | 0.00090 | 0.00094 |
| 15 | 63. | 21.1 | 87.0 | 0.00072 | 0.00094 | 0.00093 |
| 16 | 67. | 20.9 | 87.0 | 0.00072 | 0.00088 | 0.00092 |
| 17 | 710 | 20.7 | 8701 | 0.00068 | 0.00082 | 0.00085 |
| 18 | 75. | 20.6 | 87.0 | 0.00058 | 0.00076 | 0.00071 |
| 19 | 790 | 20.4 | 87.1 | 0.00051 | 0.00065 | 0.00062 |
| 20 | 83. | 20.3 | 87.1 | 0.00068 | 0.00081 | 0.00081 |
| 21 | 87. | 20.1 | 87. 2 | 0000060 | 0.00072 | 0.00072 |
| 22 | 91。 | 20.0 | 87.1 | 0.00060 | 0.00072 | 0.00071 |
| 23 | 950 | 1909 | 87.0 | 0.00051 | 0.00063 | 0.00060 |

DATE $=012973 \quad(-0.275,0$.
TAMB $=72.3 \quad$ TBASE $=7601 \quad$ TGAS $=6608$
TCOV $=72.5 \quad$ PBAR $=30.32 \quad$ RHUM $=0.69$

DATE $=030173 \quad(-0.275,-0.002)$

| TAMB $=7408$ | TBASE $=7603$ | TGAS $=6407$ |
| :--- | :--- | :--- |
| TCOV $=72.6$ | PBAR $=30.52$ | RHUM $=0.57$ |


| PL' | $x$ | UINF | 10 | STT | ST | STC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 130 | 25.9 | 6408 | 0.00396 |  |  |
| 4 | 170 | 24.3 | 64.8 | 0000384 |  |  |
| 5 | 210 | 22.8 | 64.9 | 0.00374 |  |  |
| 6 | 250 | 21.6 | 64.9 | 0.00372 |  |  |
| 7 | 29. | 20.6 | 6500 | O.00359 |  |  |
| 8 | 33. | 19.7 | 65.2 | 0.00356 |  |  |
| 9 | 370 | 18.9 | 66.7 | 0000348 |  |  |
| 10 | 410 | 18.3 | 87.1 | 0.00349 | 0000570 | 0000446 |
| 11 | 450 | 17.9 | 87.5 | 0.00332 | 0.00405 | 0.00390 |
| 12 | 490 | 17.5 | 87.4 | 0.00338 | 0.00402 | 0.00385 |
| 13 | 53. | 17.2 | 870 4 | 0.00341 | 0.00391 | 0000379 |
| 14 | 57. | 1700 | B706 | 0.00331 | 0.00376 | 0.00363 |
| 15 | 610 | 16.8 | 87.5 | 0.00336 | Q000375 | 0,00361 |
| 16 | 65. | 16.6 | 87.6 | 0.00324 | 0.00363 | 9000347 |
| 17 | 690 | 1604 | 8706 | 0.00327 | 0.00358 | 0.00353 |
| 18 | 730 | 16.3 | 8705 | 0.00321 | 0.00350 | 0.00344 |
| 19 | 770 | 16.0 | 87.6 | 0.00318 | 0.00344 | 0.00336 |
| 20 | 81. | 15.7 | 87.5 | 0.00325 | 0.00349 | 0.00343 |
| 21 | 850 | 1504 | 8706 | 0.00319 | 0.00341 | 0.00333 |
| 22 | 890 | 15.1 | 87.5 | 0.00322 | 0.00344 | 0.00336 |
| 23 | 930 | 1407 | 87.6 | 0000322 | 0.00340 | 0.00334 |

STANTGN NUMBER - STEP IN WALL TEMPERATURE

| DATE $=031273 \quad\{-0.275,-0.004\}$ |  |
| :---: | :---: |
| TAMB $=68.8$ | TBASE $=7506 \quad$ TGAS $=63.3$ |
| TCOV $=70.1$ | PBAR $=30.25 \quad$ RHUM $=0.62$ |


| PL | $x$ | UINF | 10 | STT | ST | STC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 130 | 25.9 | 6304 | 0.00521 |  |  |
| 4 | 170 | 24.3 | 63.5 | 0.00508 |  |  |
| 5 | 210 | 2209 | 63.5 | 0.00494 |  |  |
| 6 | 250 | 21.7 | 6305 | 0.00504 |  |  |
| 7 | 29. | 20.7 | 63.6 | Gocon491 |  |  |
| 8 | 330 | 1908 | 6306 | 0000500 |  |  |
| 9 | 370 | 19.1 | 6408 | 0.00485 |  |  |
| 10 | 410 | 1804 | 8502 | 0000486 | 0.00732 | 0.00566 |
| 11 | 450 | 1800 | 8505 | 0000467 | Con0532 | 0.00513 |
| 12 | 490 | 17.6 | 85.7 | 0009471 | 0000524 | 0.00598 |
| 12 | 53. | 17.3 | 8504 | Ooc0480 | 0.00513 | 0.00511 |
| 14 | 57. | 17.1 | 8505 | 0000466 | O000504 | 0.00491 |
| 15 | 610 | 16.9 | 8505 | 0000471 | 0.00506 | 0.00494 |
| 16 | 650 | 1607 | 85.5 | C000456 | 0.00481 | 0.00475 |
| 17 | 69. | 1606 | 8505 | 0000455 | 0.00484 | 0.00473 |
| 18 | 73. | 1604 | 8504 | 0.00454 | 0.00479 | 0.00470 |
| 19 | 770 | 1601 | 85.6 | 0000439 | 0.00453 | 0.00453 |
| 20 | 81. | 15.9 | 8506 | 0.00458 | 0.00473 | 0000472 |
| 21 | 85. | 1505 | 850 6 | 0000446 | 0.00460 | Conn458 |
| 22 | 890 | 1502 | 8506 | 0.00447 | 0000465 | 0000458 |
| 23 | 93. | 1408 | 85.6 | 0.00452 | 0.00467 | 0000462 |

## E-4 Mean Temperature and Velocity Profiles

The runs are tabulated below and, with the help of the special nomenclature, the data tabulation should be self-explanatory.

| Date | m | F | Number of profiles (x-wise) |
| :---: | :---: | :---: | :---: |
| 013073 | -0.275 | 0 | 5 |
| 0.21573 | -0.275 | -0.001 | 6 |
| 030473 | -0.275 | -0.002 | 6 |
| 031173 | -0.275 | -0.004 | 6 |

It should be noted that the above dates correspond to the date of the first profile out of n number indicated to the right of the table. The $x$-wise position of the profile is indicated in the tabulation.

## Special Nomenclature

| Symbol | Explanation | Unit |
| :---: | :---: | :---: |
| RUN | Date and run number | - |
| PLATE | Plate number | - |
| X (IN) | X-wise coordinate in inches | in |
| Z (IN) | Z-wise coordinate (distance from centerline) | in |
| POINTS | Number of points of the profile (different $y$ ) | - |
| UINE | Free stream velocity, $\mathrm{U}_{\infty}$ | $\mathrm{ft} / \mathrm{sec}$ |
| TWALL | Wall temperature | ${ }^{\circ} \mathrm{F}$ |
| DC | Clauser boundary layer thickness (Eq. 2.2) $\Delta$ | In |
| DEL3 | $\Delta_{3}$ (Eq. 2.8) | in |
| TGAS | Free stream temperature | ${ }^{\circ} \mathrm{F}$ |
| K | Acceleration parameter | - |
| BETA | $\beta$ (Eq. 2.5) | - |
| BF | Blowing parameter for velocity ( $F /\left(\mathrm{C}_{\mathrm{f}} / 2\right)$ ) | - |
| F | Blowing fraction ( $\mathrm{v}_{\mathrm{O}} / \mathrm{U}_{\infty}$ ) | - |
| BH | Heat transfer blowing parameter (F/St) | - |
| CF/ 2 | Friction coefficient | - |
| ST | Stanton number | - |
| PPLUS | $\left(\nu / \rho u_{\tau}^{3}\right) d \bar{p} / d x$ | - |
| VOPLUS | $v_{0} / u_{\tau}$ | - |


| REY.NO | Reynolds number | - |
| :---: | :---: | :---: |
| DELM | Momentum boundary layer thickness, $\delta$ | in |
| DEL1M | Displacement thickness, $\delta_{1}$ | in |
| DEL2M | Momentum thickness, $\delta_{2}$ | in |
| REM | Momentum thickness Reynolds number | - |
| H | Shape factor, $\delta_{1} / \delta_{2}$ | - |
| DELH | Temperature boundary layer thickness, $\delta_{\text {T }}$ | in |
| DEL2H | Enthalpy thickness, $\Delta_{T}$ | in |
| REH | Enthalpy thickness Reynolds number | - |
| GH | Heat transfer shape factor, (Eq. $2.8 \Delta_{4} / \Delta_{3}$ ) | - |
| GF | Clauser shape factor (Eq. 2.4) | - |
| I | Profile point number | - |
| Y | Normal to the wall coordinate | in |
| Yplus | yu ${ }_{T} / \nu$ | - |
| U | Local velocity | $\mathrm{ft} / \mathrm{sec}$ |
| UPLUS | $\bar{u} / u_{\tau}$ | - |
| UDE | Defect velocity $\left(\underline{u}-U_{\infty}\right) / u_{\tau}$ | - |
| UBAR | $\overline{\mathrm{u}} / \mathrm{U}_{\infty}$ | - |
| T | Local temperature | F |
| TPLUS | $\left(T_{w}-T\right) / T_{\tau}$ | - |
| TBAR | $\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}\right) /\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\infty}\right)$ | - |
| TDE | Defect temperature ( $\mathrm{T}^{+}-\mathrm{T}_{\infty}^{+}$) | - |



$$
\text { ADVERSE PRESSURE GFAIJIENT }(-0.275 .0 .)
$$

| RUN = | 13073-2 | UINF | $=$ | 19.0 | $k$ |  | 729E-96 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATF | 9 | TWAL.L. | $=$ | 96.3 | BETA | $=$ | 6.581 |
| $x(I N)=$ | 34. | OC | $=$ | 23.894 | EF | = | 0.000 |
| $Z(1 N)=$ | 0. | DEL 3 | $=$ | 3.162 | F | $=$ | 0.00300 |
| POINTS = | 40 | TGAS | $=$ | 66.8 | BH | $=$ | 0.000 |

$\mathrm{CF} / 2=0.00069$
ST $=0.00235$
PPLUS $=0.04044$
VOPLUS $=0.00000$
REY,NJ $=0.386 E 06$

| DELM | = | 1.978 | DELH | 1.793 |
| :---: | :---: | :---: | :---: | :---: |
| DELIM | $=$ | 0.626 | DEL2H | $=0.1332$ |
| DEL2M | $=$ | 0.309 | REH | 1319. |
| REM | = | 3061. | GH | 3.027 |
| H |  | 2.027 | GF | $=19.326$ |

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AOVERSE PRESSUPE GRADIENT 1 -0.275.0.1

| RUN | 20573-1 | UINF | = | 17.6 | K | $=-0.544 E-06$ | CF/2 $=6.00065$ | OELM | $=$ | 2.411 | DELH | = | 2.431 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE | 12 | TWALL | $=$ | 95.9 | BETA | $=7.171$ | ST $=0.00227$ | DELIM | = | 0.861 | DEL2H | = | 0.1670 |
| $X(I N)=$ | 46. | DC | = | 33.663 | EF | 0.000 | PPLUS $=0.03555$ | DEL2M | $=$ | 0.419 | REH | $=$ | 1549. |
| Z(IN) = | 0. | DEL 3 | $=$ | 4.097 | F | 0.00000 | VOPLUS $=0.00000$ | REM | $=$ | 3844. | GH | $=$ | 2.900 |
| POINTS $=$ | 43 | TGAS | \% | 67.3 | SH | $=0.030$ | RFY.NU $=0.46 \mathrm{GE} \mathrm{O6}$ | H | $=$ | 2.053 | GF | $=$ | 20.050 |


| I | Y | YPLES | Y/OFLM | Y/DC | U | UPLUS | UUE | UBAR | Y/CELH | Y/DEL 3 | $T$ | TPLUS | TDE | TBAR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.000 | O. C | 0.0000 | 0.00000 | 0.00 | 0.00 | -39.12 | 0.000 | 0.0000 | 0.0000 | 95.86 | 0.00 | -11.26 | 0.000 |
| 2 | 0.005 | 1.1 | 0.0021 | 0.00015 | 0.83 | 1.84 | -37.28 | 0.047 | 0.0021 | 0.0012 | 92.31 | 1.39 | -9.87 | 0.123 |
| 3 | 0.0 .66 | 1.3 | 0.0025 | 0.00018 | 0.90 | 2.00 | -37.12 | 0.051 | 0.0025 | 0.0015 | 91.97 | 1.59 | -9.74 | 0.135 |
| 4 | 0.067 | 1.5 | 0.0029 | 0.00021 | 0.94 | 2.08 | -37.03 | 0.053 | 0.0029 | 0.0017 | 91.63 | 1.65 | -9.74 | 0.147 |
| 5 | 0.008 | 1.1 | 0.0033 | 0.00024 | 0.92 | 2.04 | -37.08 | 0.052 | 0.0033 | 0.0020 | 91.12 | 1.85 | -9.41 | 0.164 |
| 6 | C.CCS | 1.9 | 0.0037 | 0. 0.0027 | C. 96 | 2.13 | -36.99 | 0.054 | 0.0037 | 0.0022 | 90.82 | 1.97 | -9.27 | 0.175 |
| 7 | 0.011 | 2.3 | 0.0046 | 0.00033 | 1.10 | 2.44 | - 36.68 | 0.062 | 0.0046 | 0.0027 | 89.89 | 2.33 | -8.93 | 0.207 |
| 8 | C.013 | 2.8 | 0.0 .154 | 0. 00039 | 1.19 | 2.64 | - -36.48 | 0.067 | 0.0054 | 0.0032 | 89.46 | 2.50 | -8.75 | 0.222 |
| 9 | 0.015 | 3.2 | 0.0062 | 0.30045 | 1.35 | 2.99 | -36.12 | 0.077 | 0.0062 | 0.0037 | 88.75 | 2.78 | -8.48 | 0.247 |
| 10 | 3.017 | 3.6 | 0.0071 | 0.02051 | 1.46 | 3.24 | $-35.88$ | 0.083 | 0.0071 | 0.0041 | 88.21 | 2.99 | -8.27 | 0.255 |
| 11 | C.020 | 4.3 | 0.0083 | 0.00059 | 1.74 | 3.86 | -35.26 | 0.099 | 0.0083 | 0.0049 | 87.12 | 3.41 | -7.85 | 0.303 |
| 12 | 0.023 | $4 \cdot 9$ | 0.0395 | 0.00 .768 | 1.94 | 4.30 | -34.81 | 0.110 | 0.0096 | 0.0056 | 86.26 | 3.75 | -7.51 | 0.333 |
| 13 | 0.026 | 5.5 | 0.0168 | 0.00677 | 2.22 | 4.92 | -34.19 | 0.126 | 0.01 CB | 0.0063 | 85.26 | 4.14 | -7.12 | 0.358 |
| 14 | 0.629 | 6.2 | 0.0120 | 0.00086 | 2.42 | 5.37 | -33.75 | 0.137 | 0.0121 | 0.0071 | 84.55 | 4.42 | -6.84 | 0.372 |
| 15 16 | C.033 | 7. 5 | 0.0137 | 0.013098 | 2.89 | 6.41 | -32.71 | 0.164 | 0.0137 | 0.0081 | 83.89 | 4.68 | -6.88 | 0.415 |
| 16 17 | 0.037 0.042 | 7.5 9.6 | 0.0153 0.0174 | 0.00110 0.00125 | 2.97 3.22 | 6.59 7.14 | -32.53 | 0.168 0.183 | 0.0154 | 0.0090 | 83.18 82.30 | 4.95 5.30 | -6.31 | 0.440 |
| 18 | C. 047 | 10.6 | 0.0195 | 0.00140 | 3.46 | 7.67 | -31.98 -31.44 | 0.183 0.196 | 0.0175 0.0196 | 0.0103 0.0115 | 82.30 81.91 | 5.30 5.45 | -5.95 | 0.470 |
| 19 | 0.052 | 11.1 | 0.0216 | 0.00154 | 3.63 | 8.05 | -31.07 | 0.206 | 0.0217 | 0.0127 | 81.06 | 5.78 | -5.81 | 0.484 0.513 |
| 20 | 0.057 | 12.2 | 0.0236 | 0.00169 | 3.92 | 8.69 | $-30.42$ | 0.222 | 0.0237 | 0.0139 | 80.44 | 6.02 | -5.23 | 0.535 |
| 21 | 0.062 | 13.2 | 0.0257 | 0.00184 | 4.07 | 9.03 | -30.05 | 0.231 | 0.0258 | 0.0151 | 79.94 | 6.22 | -5.04 | 0.552 |
| 22 | 0.672 | 15.4 | 0.0299 | 0.00214 | 4.24 | 9.40 | -29.71 | 0.240 | 0.0300 | 0.0176 | 79.52 | 6.38 | -4.87 | 0.567 |
| 23 | 0.082 | 17.5 | 0.0340 | 0.00244 | 4.62 | 10.24 | -28.87 | 0.262 | 0.0341 | 0.0200 | 78.66 | 6.72 | -4.54 | 0.597 |
| 24 | 3.657 | 20.1 | 0.0402 | 0.102288 | 4.79 | 10.62 | -28.49 | 0.272 | 0.6404 | 2.0237 | 78.36 | 6.84 | -4.42 | 0.607 |
| 25 | C. 117 | 25.1 | -. 0.9485 | 0.00348 | 5.16 | 11.44 | -27.67 | 0.293 | 0.0487 | 0.0286 | 77.65 | 7.11 | -4.14 | 0.632 |
| 26 27 | 0.142 0.167 | 30.3 35.6 | 0.0589 0.0693 | 0.10422 0.00496 | 5.41 | 12.00 | -27.12 | 0.307 | 0.6591 | 0.0347 | 77.01 | 7.36 | -3.89 | 0.654 |
| 28 | 0.217 | 35.6 46.3 | 0.0693 0.0900 | 0.104496 0.00445 | 5.47 5.95 | 12.13 13.19 | -26.99 -25.92 | 0.310 0.337 | 0.0695 0.0904 | 0.0408 0.0530 | 76.62 | 7.52 | -3.74 | 0.657 |
| 29 | 0.267 | 57. C | 0.1167 | 0.00793 | 6.20 | 13.75 | -25.37 | 0.351 | 0.0904 0.1112 | 0.0530 0.0652 | 75.92 75.34 | 7.79 8.02 | -3.47 -3.24 | 0.692 0.712 |
| 30 | C. 317 | 67.6 | 0.1315 | 0.0J942 | 6.63 | 14.70 | -24.41 | 0.376 | 0.1320 | 0.0774 | 74.98 | 8.16 | $-3.10$ | 0.712 0.724 |
| 31 | C. 417 | 89.6 | 0.1729 | 0.01239 | 7.04 | 15.61 | -23.51 | 0.399 | 0.1737 | 0.1018 | 74.22 | 8.45 | -2.80 | 0.751 |
| 32 | 0.517 | 110.3 | 0.2144 | 0.01536 | 7.72 | 17.12 | -22.00 | 0.438 | 0.2153 | 0.1262 | 73.60 | 8.70 | -2.56 | 0.772 |
| 33 | 0.667 | $142 \cdot \frac{2}{3}$ | 0.2766 | 0.01981 | 8. 26 | 18.32 | $-20.80$ | 0.468 | 0.2778 | 0.1628 | 72.77 | 9.02 | -2.24 | 0.801 |
| 34 35 | 0.817 $1 . C 17$ | 174.3 | 0.3388 0.4218 | 0.02427 0.03021 | 9.04 10.40 | 29.05 23.06 | -19.07 -16.05 | 0.512 | 0.3402 | 0.1994 | 72.11 | 9.28 | -1.98 | 0.824 |
| 36 | 1.217 | 259.7 | 0.5047 | 0.03615 | 11.59 | 23.06 25.70 | -16.05 -13.42 | 0.590 0.657 | 0.4235 0.5068 | 0.2482 0.2970 | 71.25 70.48 | 9.61 | -1.65 -1.34 | 0.853 0.880 |
| 37 | 1.417 | 302.4 | 0.5377 | 0.04209 | 12.94 | 28.69 | -10.42 | 0.734 | 0.5901 | 0.3459 | 69.90 | 10.14 | -1.34 | 0.880 0.900 |
| 38 | 1.667 | 355.7 | 0.6914 | 0.04552 | 14.35 | 31.82 | $-7.30$ | 0.813 | 0.6942 | 0.4069 | 69.00 | 10.49 | -0.77 | 0.931 |
| 39 | 1. 517 | 409.1 | 0.7951 | 0.05695 | 15.70 | 34.81 | $-4.30$ | 0.890 | 0.7983 | 0.4679 | 68.48 | 10.69 | -0.57 | 0.950 |
| 40 | 2. 167 | 462.4 | C. 8.787 | 0.06437 | 16.78 | 37.21 | -1.91 | 0.951 | 0.9025 | 0.5289 | 67.73 | 10.99 | -0.27 | 0.976 |
| 41 | 2.417 | 515.7 | 1.0 .324 | 0.97180 | 17.48 | 38.76 | $-0.35$ | 0.991 | 1.2066 | 0.5899 | 67.29 | 11.16 | -0.10 | 0.991 |
| 42 | 2.667 | 569.1 | 1.1 .361 | 0.07923 | 17.63 | 39.09 | -0.02 | 0.999 | 1.1107 | 0.6509 | 67.01 | 11.27 | 0.01 | 1.001 |
| 43 | 2.517 | 622.4 | 1.2)98 | 0.08665 | 17.64 | 39.12 | 0.00 | 1.000 | 1.2148 | 0.7120 | 67.03 | 11.26 | $-0.00$ | 1.000 |

ADVERSE PRESSURE GFADIENT $(-0.275,0$.

| RUN | $=20$ | 20573-2 | UINF | 17.1 | K | $=-0.5108-06$ |  | CF/2 | 0076 DELM $=3.109$ |  |  |  | $\text { DELH }=2.845$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE | $=$ | 15 | TWALL | 56.8 | BETA | - | 6.224 | ST | C. 002 |  | $L M=$ | $044$ |  |  |
| X(IN) |  | 58. | OC | $=37.868$ | BF | $=$ | 0.000 | PPLUS | 0.024 |  | M | 528 | REH | 1653. |
| Z(IN) |  | 0. | DEL3 | $=4.890$ | $F$ | 0 | 0.00030 | VOPLUS | $=0.000$ | RE | $=$ | 95. | GH | 2.937 |
| POINTS |  | 40 | TGAS | $=67.2$ | BH | $=$ | 0.000 | REY.NO | $=0.5608$ | H | $=1$ | 77 | GF | 17.918 |
| I | $Y$ | YPLUS | Y/DELM | Y/OC | U | UPLUS | UDE | UBAR | Y/DELH | Y/DEL 3 | T | TPLUS | TDE | TBAR |
| 1 | C.CCO | 0 O.C | 0.0000 | 0.00000 | 0.00 | 0.00 | -36.26 | 0.000 | 0.0000 | 0.0000 | 96.82 | 0.00 | -12.37 | 0.000 |
| 2 | 0.005 | 1.1 | 0.0016 | 0.00013 | 0.83 | 1.76 | -34.50 | 0.049 | 0.0018 | 0.0010 | 92.70 | 1.72 | -10.64 | 0.139 |
| 3 | 0.0 C 6 | 1.3 | 0.0019 | $0.000: 6$ | C. 85 | 1. 80 | -34.46 | 0.050 | 0.0021 | 0.0012 | 92.11 | 1.97 | -10.40 | 0.159 |
| 4 | 0.007 | 7 1.t | 0.0023 | 0.01018 | 0.91 | 1.93 | -34.33 | 0.053 | 0.0025 | 0.0014 | 91.98 | 2.02 | -10.34 | 0.164 |
| 5 | C.CC8 | 1.8 | 0.0026 | 0.00021 | C. 93 | 1.97 | -34.29 | 0.054 | 0.0028 | 0.0016 | 91.68 | 2.15 | -10.22 | 0.174 |
| 6 | 0.009 | 2.0 | 0.0029 | 0.00024 | 1.07 | 2.27 | -33.95 | 0.063 | 0.0032 | 0.0018 | 91.21 | 2.34 | -10.02 | 0.190 |
| 7 | 0.010 | 2.2 | 0.0032 | 0.00026 | 1.06 | 2.25 | -34.01 | 0.062 | 0.0035 | 0.0020 | 91.08 | 2.40 | -9.97 | 0.194 |
| 8 | 0.012 | 2.7 | 0.0039 | 0.03032 | 1.16 | 2.46 | -33.80 | 0.068 | 0.0042 | 0.0025 | 90.27 | 2.74 | -9.63 | 0.221 |
| 9 | 0.014 | 4 3.1 | 0.0045 | 0. 00037 | 1.28 | 2. 71 | -33.55 | 0.075 | 0.0049 | 0.0029 | 89.42 | 3.09 | -9.27 | 0.250 |
| 10 | C.C16 | 6 3.t | 0.0051 | 0.00042 | 1.55 | 3.28 | -32.97 | 0.091 | 10. .1056 | 0.0033 | 88.77 | 3.36 | -9.00 | 0.272 |
| 11 | 0.019 | 94.2 | 0.0061 | 0.03050 | 1.63 | 3.45 | -32.81 | 0.095 | 0.0067 | 0.0039 | 87.92 | 3.72 | -8.65 | 0.301 |
| 12 | 0.022 | 24.9 | 0.0071 | 0.03058 | 2.04 | 4.32 | -31.94 | 0.119 | 0.0077 | 0.0045 | 86.89 | 4. 15 | -8.22 | 0.335 |
| 13 | 0.027 | 7 6.C | 0.0087 | 0.0.)071 | 2.26 | 4.79 | -31.47 | 0.132 | $0 . \operatorname{cos5}$ | 0.0055 | 85.70 | 4.65 | -7.72 | 0.376 |
| 14 | 0. 022 | 2 1.1 | 0.0203 | 0. 0.13085 | 2.73 | 5.79 | -30.47 | 0.160 | 0.0112 | 0.0065 | 84.43 | 5.18 | -7.19 | 0.419 |
| 15 | C.C37 | 78.2 | 0.0119 | 0.00098 | 3.07 | 6.51 | -29.75 | 0.179 | 0.0130 | 0.0076 | 83.58 | 5.53 | -6.83 | 0.447 |
| 16 | 0.042 | 29.3 | 0.0135 | 0.00111 | 3.51 | 7.44 | -28.82 | 0.205 | 0.0148 | 0.0086 | 82.67 | 5.91 | -6.45 | 0.478 |
| 17 | 0.052 | 2 11.t | 0.0167 | 0.00137 | 3.73 | 7.90 | -28.35 | 0.218 | 0.0183 | 0.0106 | 81.21 | 6.52 | -5.84 | 0.527 |
| 18 | 0.062 | 213.8 | 0.0199 | 0.00164 | 4.14 | 8.77 | -27.49 | 0.242 | 0.0218 | 0.0127 | 80.48 | 6.83 | -5.54 | 0.552 |
| 19 | 0.077 | 717.1 | 6. 0248 | 0.00203 | 4.56 | 9.66 | -26.60 | 0.267 | 0.0271 | 0.0157 | 79.19 | 7.36 | -5.00 | 0.595 |
| 20 | 0.057 | 721.6 | 0.0312 | 0.00256 | 4.97 | 10.53 | -25.73 | 0.290 | 0.0341 | 0.0198 | 78.48 | 7.66 | -4.70 | 0.619 |
| 21 | C. 122 | 27.2 | 0.0392 | 0.00322 | 5.21 | 11.04 | -25.22 | 0.305 | 0.0429 | 0.0250 | 77.57 | 8.04 | -4.32 | 0.650 |
| 27. | C. 17 ? | ? 38.3 | 0.0553 | C. 00454 | 5.59 | 11.85 | -24.41 | 0.327 | 0.0605 | 0.0352 | 76.38 | 8.54 | -3.82 | 0.690 |
| 23 | 0.222 | 249.4 | 0.0714 | 0.00586 | 5.97 | 12.65 | -23.61 | 0.349 | 0.6780 | 0.0454 | 75.70 | 8.82 | -3.54 | 0.713 |
| 24 | 0.322 | 711.7 | 2. 1036 | 0.00853 | 6.36 | 13.48 | -22.78 | 0.372 | 0.1132 | 0.0659 | 75.13 | 9.06 | -3.30 | 0.733 |
| 25 | 0.422 | 293.8 | 0.1358 | 0.01114 | 6.83 | 14.47 | -21.79 | 0.399 | 0.1483 | 0.0863 | 74.16 | 9.47 | -2.90 | 0.765 |
| 26 | C. 572 | 2127.3 | 0.1840 | 0.01511 | 7.44 | 15.77 | -20.49 | 0.435 | 0.2011 | 0.1170 | 73.50 | 9.74 | -2.62 | 0.788 |
| 27 | C. 722 | 160.7 | 0.2323 | 0.01907 | 7.85 | 16.64 | -19.62 | 0.459 | 0.2538 | 0.1477 | 72.89 | 10.00 | -2.36 | 0.808 |
| 28 | 0.922 | 205.2 | 0.2966 | 0.02435 | 8.74 | 18.52 | -17.74 | 0.511 | 0.3241 | 0.1886 | 72.09 | 10.33 | -2.03 | 0.835 |
| 29 | 1.172 | 2 260.9 | 0.3170 | 0.03095 | 9.86 | 20.90 | -15.36 | 0.576 | 0.4120 | 0.2397 | 71.33 | 10.64 | -1.72 | 0.861 |
| 30 | 1.422 | 2316.5 | 0.4574 | 0.03755 | 11.04 | 23.40 | -12.86 | 0.645 | 0.4998 | 0.2908 | 70.64 | 10.93 | $-1.43$ | 0.884 |
| 31 | 1.672 | 2372.2 | 0.5379 | 0.04415 | 11.82 | 25.05 | -11.2.1 | . 0.691 | 0.5877 | 0.3420 | 69.88 | 11.25 | -1.11 | 0.910 |
| 32 | 1.922 | 2 427.8 | 0.6183 | 0.05076 | 13.37 | 28.33 | -7.93 | 0.781 | 0.6756 | 0.3931 | 69.33 | 11.48 | -0.8B | 0.928 |
| 33 | 2.172 | 4 483.5 | 0.6987 | 0.05736 | 14.36 | 30.43 | -5.83 | 0.839 | 0.7635 | 0.4442 | 68.78 | 11.71 | -0.66 | 0.947 |
| 34 | 2.422 | 2539.1 | 0.7791 | 0.06396 | 15.45 | 32.74 | -3.52 | 0.903 | 0.8513 | 0.4953 | 68.20 | 11.95 | -0.41 | 0.966 |
| 35 | 2.672 | $2594 . \varepsilon$ | 0.8595 | 0.07056 | 16.22 | 34.37 | -1.89 | 0.948 | 0.9392 | 0.5465 | 67.74 | 12. 14 | -0.22 | 0.982 |
| 36 | 2.922 | 2650.4 | 0.9400 | 0.07716 | 16.70 | 35.39 | -0.87 | 0.976 | 1.0271 | 0.5776 | 67.40 | 12.29 | -0.08 | 0.994 |
| 37 | 3.172 | 2706.1 | 1.0204 | 0.08376 | 17.02 | 36.07 | -0.19 | 0.995 | 1.1150 | 0.6487 | 67.25 | 12.35 | -0.02 | 0.999 |
| 38 | 3.422 | 761.7 | 1.10 CB | 0.09037 | 17.11 | 36.26 | 0.00 | 1.000 | 1.2028 | 0.6999 | 67.17 | 12.38 | 0.02 | 1.001 |
| 39 | 3.672 | 8 817.4 | 1.1812 | 0.09657 | 17.08 | 36.20 | -0.06 | 0.998 | 1.2907 | 0.7510 | 67.10 | 12.41 | 0.05 | 1.004 |
| 40 | 3.922 | 8 873.0 | 1.2616 | 0.13357 | 17.11 | 36.26 | 0.00 | 1.000 | 1.3786 | 0.8021 | 67.21 | 12.37 | -0.00 | 1.000 |

AOVEFSF PRESSURE GRADIENT $(-0.275,0$.


AUVERSE PRESSUFE GRADIEATT (-0.275,-0.001)

| RUN | 21573-1 | UINF | = | 22.1 | K | $=-0.946 \mathrm{E}-06$ | CF/2 $=0.00146$ | DELM | $=$ | 0.964 | DELH | $=0.864$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE | 6 | TWALE | = | 89.7 | BETA | $=1.794$ | $\mathrm{ST}=0.00327$ | DELIM | = | 0.239 | DEL2H | $=0.0674$ |
| X(IN) | 22. | OC | = | 6.254 | BF | -0.683 | PPLUS $=0.01688$ | DEL2M | = | 0.143 | REH | 805. |
| ZIIN) = | 0. | DEL3 | = | 1.459 | F | $=-0.00100$ | VOPLUS $=-0.02743$ | REM | = | 1658. | GH | $=3.243$ |
| PUINTS = | 36 | TGAS | = | 63.7 | EH | $=-0.306$ | REY.NS $=0.313 \mathrm{E} 06$ | H | $=$ | 1.675 | GF | $=10.534$ |


|  | 1 | $Y$ | YPLUS | Y/ OE LM | $\mathrm{Y} / 0 \mathrm{C}$ | U | UPLUS | UDE | UBAR | Y/CELH | Y/DEL 3 | T | TPLUS | TDE | TBAR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | C.000 | 0.0 | 0.0000 | 0.3 .1000 | 0.00 | 0.00 | -26.13 | 0.000 | 0.0000 | 0.0000 | 89.71 | 0.00 | -11.70 | 0.000 |
|  | 2 | C.CC5 | 2.c | 0.0052 | 0.00080 | 2.23 | 2.64 | -23.49 | 0.101 | 0.0058 | 0.0034 | 84.54 | 2.33 | -9.37 | 0.199 |
|  | 3 | C. 0.6 | 2.4 | 0.0062 | 0.00056 | 2.49 | 2.95 | -23.18 | 0.113 | 0.0069 | 0.0041 | 84.22 | 2.48 | $-9.22$ | 0.212 |
|  | 4 | O.CC7 | 2.9 | 0.0073 | $0.0) 112$ | 2.83 | 3.35 | -22.78 | 0.128 | 0.0081 | 0.0048 | 83.69 | 2.71 | -8.99 | 0.232 |
|  | 5 | C.CC8 | 3.3 | 0.0083 | 0.03128 | 2.99 | 3.54 | -22.59 | 0.135 | 0.0093 | 0.0055 | 83.11 | 2.98 | -8.72 | 0.254 |
|  | 6 | 0.069 | 3.7 | 0.0093 | 0.00144 | 3.35 | 3.96 | -22.17 | 0.152 | 0.0104 | 0.0062 | 82.63 | 3.19 | -8.51 | 0.273 |
|  | 7 | C. 010 | 4.1 | C. 0104 | 0.00100 | 3.62 | 4.28 | -21.85 | 0.164 | 0.0116 | 0.0069 | 82.17 | 3.40 | -8.30 | 0.290 |
|  | 8 | 0.012 | 4.5 | 0.0124 | 0.150192 | 4.15 | 4.91 | -21.22 | 0.188 | 0.0139 | 0.0082 | 81.04 | 3.91 | -7.79 | 0.334 |
|  | 9 | 0.014 | 5.7 | 0.0145 | 0.UJ224 | 4.89 | 5.78 | -20.35 | 0.221 | 0.0162 | 0.0096 | 80.13 | 4.32 | -7.38 | 0.359 |
|  | 10 | 0.016 | 6.5 | 0.0166 | 0.00256 | 5.24 | 6.20 | -19.93 | 0.237 | 0.0185 | 0.0110 | 79.49 | 4.61 | -7.09 | 0.374 |
|  | 11 | C.C19 | 7.7 | 0.0197 | 0.313304 | 5.83 | 6.90 | $-19.23$ | 0.264 | 0.0220 | 0.0130 | 78.48 | 5.06 | -6.63 | 0.433 |
|  | 12 | C. 022 | 9. 6 | 0.0228 | 0.00352 | 6.67 | 7.89 | -18.24 | 0.302 | 0.0254 | 0.0151 | 77.57 | 5.47 | -6. 22 | 0.458 |
|  | 13 | C. 025 | 10.2 | 0.0259 | 0.00400 | 7.33 | 8.67 | -17.46 | 0.332 | 0.0289 | 0.0171 | 76.76 | 5.84 | -5.86 | 0.499 |
|  | 14 | 0.029 | 11.8 | 0.0301 | 0.00464 | 7.73 | 9.14 | -16.99 | 0.350 | 0.0335 | 0.0199 | 75.94 | 6.21 | -5.49 | 0.530 |
|  | 15 | C.0ミ3 | 13.E | 0.0342 | 0.30528 | B. 31 | 9.83 | -16.30 | 0.376 | 0.0382 | 0.0226 | 75.32 | 6.48 | -5.21 | 0.554 |
|  | 16 | C.C37 | 15.1 | 0.0384 | 0.30592 | 8.69 | 10.28 | -15.85 | C. 393 | 0.6428 | 0.0254 | 74.88 | 6.68 | -5.02 | 0.571 |
|  | 17 | C.041 | 16.7 | 0.0425 | 0.00656 | 9.13 | 10.80 | -15.33 | 0.413 | 0.0474 | 0.0281 | 74.29 | 6.95 | -4.75 | 0.594 |
| $\xrightarrow{-1}$ | 18 | 0.045 | 18.4 | 0.0467 | $0.007 \overline{0}$ | 9.45 | 11.18 | -14.95 | 0.428 | 0.0521 | 0.0309 | 73.71 | 7.21 | -4.49 | 0.616 |
|  | 19 | C. 055 | 22.4 | 0.0570 | 0.09879 | 9.98 | 11.81 | $-14.32$ | 0.452 | 0.0636 | 0.0377 | 73.02 | 7.52 | -4.18 | 0.642 |
|  | 20 | C. 065 | 26. 5 | 0.0674 | 0.01039 | 10.64 | 12.59 | -13.54 | 0.482 | 0.0752 | 0.0446 | 72.39 | 7.80 | -3.89 | 0.667 |
|  | 21 | 0.075 | 20.6 | 0.0778 | 0.01199 | 10.91 | 12.91 | $-13.22$ | 0.494 | 0.0868 | 0.0514 | 71.99 | 7.98 | -3.71 | 0.632 |
|  | 22 | 0.650 | 36.7 | 0.0933 | 0.01439 | 11.21 | 13.26 | -12.87 | 0.507 | 0.1041 | 0.0617 | 71.37 | 8.26 | -3.43 | 0.706 |
|  | 23 | 0.105 | 42.8 | 0.1087 | 0.01679 | 11.61 | 13.73 | -12.40 | 0.526 | 0.1215 | 0.0720 | 70.91 | 8.47 | -3.22 | 0.724 |
|  | 24 | 0.125 | $51 . \mathrm{C}$ | 0.1296 | 0.11999 | 11.55 | 13.66 | -12.47 | C. 523 | 0.1446 | 0.0857 | 70.41 | 8.70 | $-3.00$ | 0.743 |
|  | 25 | 0.150 | 61.2 | 0.1555 | 0.02398 | 12.43 | 14.70 | -11.43 | 0.563 | 0.1735 | 0.1028 | 69.90 | 8.93 | -2.77 | 0.763 |
|  | 26 | 0.175 | 71.4 | 0.1815 | 0.02798 | 12.86 | 15.21 | -10.92 | 0.582 | 0.2024 | 0.1200 | 69.42 | 9.14 | -2.55 | 0.781 |
|  | 27 | 0.225 | S1.8 | 0.2333 | 0. 03598 | 13.71 | 16.21 | -9.92 | 0.620 | 0.2603 | 0.1543 | 68.76 | 9.44 | -2.25 | 0.807 |
|  | 28 | C. 275 | 112.2 | 0.2852 | 0.04397 | 14.34 | 16.96 | -9.17 | 0.649 | 0.3181 | 0.1885 | 68.12 | 9.73 | -1.97 | 0.831 |
|  | 29 | 0.225 | $132 . t$ | 0.3370 | 0.35196 | 15.16 | 17.93 | -8.20 | 0.686 | 0.3760 | 0.2228 | 67.54 | 9.99 | -1.71 | 0.853 |
|  | 30 | 0.425 | 173.3 | $0.44 C 7$ | 0.06795 | 16.56 | 19.59 | -6.54 | 0.750 | 0.4916 | . 0.2914 | 66.52 | 10.45 | -1.25 | 0.893 |
|  | 31 | 0.525 | 214.1 | 0.5444 | D. 08394 | 17.82 | 21.08 | $-5.05$ | 0.807 | 0.6073 | 0.3599 | 65.78 | 10.78 | -0.92 | 0.921 |
|  | 32 | c. 625 | 254.5 | 0.6481 | 0.09993 | 19.15 | 22.65 | -3.48 | 0.867 | 0.7230 | 0.4285 | 65.11 | 11.08 | -0.62 | 0.947 |
|  | 33 | C. 775 | 315.1 | 0.8036 | 0.12392 | 20.78 | 24.58 | -1.55 | 0.941 | 0.8965 | 0.5313 | 64.26 | 11.47 | -0.23 | 0.980 |
|  | 34 | C. 525 | 377.2 | 0.9591 | 0.14750 | 21.81 | 25.80 | -0.33 | 0.987 | 1.0700 | 0.6341 | 63.82 | 11.67 | -0.04 | 0.997 |
|  | 35 | 1.125 | 458.9 | 1. 1665 | 0.17988 | 22.11 | 26.15 | 0.02 | 1.001 | 1.3014 | 0.7713 | 63.74 | 11.70 | -0.00 | 1.000 |
|  | 36 | 1.325 | 540.4 | 1.3739 | 0.21186 | 22.09 | 26.13 | 0.00 | 1.000 | 1.5328 | 0.9084 | 63.73 | 11.70 | -0.00 | 1.000 |

ADVERSE PRESSUFE GRADIENT $(-0.275,-0.0011$

| RUN | $=2$ | 21573-2 | UINF | 19,4 | K | $=-0.705 E-96$ |  | CF/2 $=0.00118$ D |  |  | DELM | $\begin{aligned} & 1.464 \\ & 0.410 \end{aligned}$ | $\text { DELH }=1.355$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE | 析 | 9 | TWALL. | 89.9 | BETA | = | 2.503 | 5 T | 0.002 |  | 1 M |  | DEL2H | $\begin{array}{r} =1.355 \\ =0.1019 \end{array}$ |
| X(IN) |  | 34. | DC | $=11.955$ | BF | $=$ | -0.874 | PPLUS | 0.01 |  | 2M | 235 | REH | 1039. |
| Z(IN) |  | 0. | OEL 3 | 2.193 | F | -0 | 00103 | VOPLU | -0.031 |  | $=2$ | 92. | GH | 2.956 |
| PUINT | S $=$ | 33 | TGAS | 63.9 | BH | $=$ | -0.346 | REY. | =0.398E | H | $=$ | 749 | GF | 12.475 |
| I | $Y$ | YPLLS | Y/OELM | Y/CC | U | UPLUS | UOE | UBAR | Y/RELH | Y/DEL 3 | T | TPLUS | TDE | TBAR |
| 1 | 0.020 | 0 O.C | 0.0503 | 0.30000 | 0.00 | 0.00 | -29.14 | 0.000 | 0.0000 | 0.0000 | 89.85 | 0.00 | -11.52 | 0.000 |
| 2 | D.005 | 1.t | 0.10034 | 0.00042 | 1.71 | 2.57 | -26.57 | 0.088 | 0.0037 | 0.0023 | 84.82 | 2.23 | -9.28 | 0.194 |
| 3 | 0.066 | $61 . c$ | 0.3041 | 0.00050 | 1.95 | 2.93 | -26.21 | 0.100 | 0.0044 | 0.0027 | 84.26 | 2.48 | -9.03 | 0.215 |
| 4 | 0.067 | 72.2 | 0.01348 | 0.0 .3059 | 2.07 | 3.11 | -26.03 | 0.107 | 0.0051 | 0.0032 | 83.82 | 2.68 | -8.84 | 0.232 |
| 5 | C.CCE | \% $2 . \epsilon$ | 0.0055 | 0.00067 | 2.22 | 3.33 | -25.80 | 0.114 | 0.0059 | 0.0036 | 83.46 | 2.84 | -8.68 | 0.246 |
| 6 | 0.010 | シ.2 | 0.0368 | 0.00084 | 2.68 | 4.02 | -25.11 | 0.138 | 0.0073 | 0.0046 | 82.68 | 3.18 | -8.33 | 0.276 |
| 7 | 0.012 | 2.5 | 0.3082 | 0.00100 | 2.89 | 4.34 | -24.80 | 0.149 | 0.0088 | 0.0055 | 81.79 | 3.58 | -7.94 | 0.311 |
| 8 | 0.015 | $5 \quad 4 . \varepsilon$ | 0.01 J 2 | 0.00125 | 3.58 | 5.37 | -23.76 | 0.184 | 0.0110 | 0.0068 | 80.69 | 4.07 | -7.45 | 0.353 |
| 9 | 0.018 | 85.8 | 0.0123 | 0.00151 | 4.04 | 6.06 | -23.07 | 0.208 | 0.0132 | 0.0082 | 79.64 | 4.53 | -6.98 | 0.393 |
| 10 | 0.021 | 16.7 | 0.0143 | 0.00176 | 4.39 | 6.59 | -22.55 | 0.226 | 0.0154 | 0.0096 | 78.82 | 4.90 | -6.62 | 0.425 |
| 11 | 0.024 | $4 \quad 7.7$ | 0.0164 | 0.00201 | 4.77 | 7.16 | -21.96 | 0.246 | 0.0176 | 0.0109 | 78.01 | 5.25 | -6.26 | 0.456 |
| 12 | 0.028 | 8 9.6 | 0.0191 | 0. 30234 | 5.39 | 8.09 | -21.05 | 0.278 | 0.0205 | 0.0128 | 77.25 | 5.59 | -5.92 | 0.436 |
| 13 | 0.033 | 10.t | 0.0225 | 0.00276 | 5.91 | 8.87 | -20.27 | 0.304 | 0.0242 | 0.0150 | 76.43 | 5.96 | -5.56 | 0.517 |
| 14 | 0.038 | 8 12.2 | 0.0260 | 0.00318 | 6.31 | 9.47 | $-19.67$ | 0.325 | 0.0278 | 0.0173 | 75.68 | 6.29 | -5.22 | 0.546 |
| 15 | C.C48 | 815.4 | 0.0328 | 0.00402 | 6.88 | 10.33 | -18.81 | 0.354 | 0.0352 | 0.0219 | 74.46 | 6.83 | -4.68 | 0.573 |
| 16 | 0.058 | 18.t | 0.0396 | 0.30485 | 7.44 | 11.17 | -17.97 | 0.383 | 0.0425 | 0.0264 | 73.95 | 7.06 | -4.45 | 0.613 |
| 17 | 0.073 | 3 23.5 | 0.0499 | 0. 10611 | 7.93 | 11.90 | -17.23 | 0.409 | 0.0535 | 0.0333 | 73.03 | 7.47 | -4.05 | 0.648 |
| 18 | C.CE8 | 28.3 | 0.0601 | 0.00736 | 8.31 | 12.47 | -16.66 | 0.42 .8 | 0.0645 | 0.0401 | 72.44 | 7.72 | $-3.79$ | 0.671 |
| 19 | 0.108 | 34.7 | 0.0738 | 0.00903 | 8.39 | 12.59 | -16.54 | 0.432 | 0.0791 | 0.0492 | 71.95 | 7.94 | -3.57 | 0.689 |
| 20 | 0.133 | 3 42.i | 0.0909 | 0.01113 | 6.99 | 13.50 | -15.64 | 0.463 | 0.0975 | 0.0606 | 71.29 | 8.23 | -3.28 | 0.715 |
| 21 | 0.158 | \% 50.8 | 0.1079 | 0.01322 | 9.31 | 13.98 | -15.16 | 0.480 | 0.1158 | 0.0720 | 70.90 | 8.41 | -3.10 | 0.730 |
| 22 | C.2c8 | 66.8 | 0.1421 | 0. 11740 | 9.81 | 14.73 | -14.41 | 0.505 | 0.1524 | 0.0948 | 70.34 | 8.66 | -2.85 | 0.752 |
| 23 | 0.3 C8 | 99.0 | 0.2104 | 0.02576 | 10.61 | 15.93 | -13.21 | 0.547 | 0.2257 | 0.1404 | 69.17 | 9.17 | -2.34 | 0.797 |
| 24 | C.4C8 | 131.1 | 0.2787 | 0.03413 | 11.48 | 17.23 | -11.90 | 0.591 | 0.2990 | 0.1860 | 68.37 | 9.53 | -1.98 | 0.828 |
| 25 | C. 568 | 163.2 | 0.3470 | 0.04249 | 12.37 | 18.57 | -10.57 | 0.637 | 0.3722 | 0.2316 | 67.72 | 9.82 | -1.69 | 0.853 |
| 26 | 0.658 | 311.4 | 0.4495 | 0.055 .34 | 13.73 | 20.61 | -8.53 | 0.737 | 0.4822 | 0.3000 | 66.82 | 10.22 | -1.29 | 0.887 |
| 27 | C. 868 | 259.t | 0.5529 | 0.06759 | 15.07 | 22.62 | -6.52 | 0.776 | 0.5921 | 0.3684 | 66.06 | 10.55 | -0.96 | 0.916 |
| 28 | 1.CC8 | 323.5 | 0.6386 | 0.08432 | 16.69 | 25.05 | -4.08 | 0.860 | 0.7386 | 0.4596 | 65.24 | 10.92 | -0.59 | 0.948 |
| 29 | 1.208 | 388.1 | 0.8252 | 0.10135 | 18.18 | 27.29 | -1.85 | 0.937 | 0.8852 | 0.5508 | 64.55 | 11.23 | -0.29 | 0.975 |
| 30 | $1.4 C 8$ | 8452.4 | 0.9619 | 0.11778 | 19.14 | 28.73 | -0.41 | 0.986 | 1.0317 | 0.6420 | 64.04 | 11.45 | -0.07 | 0.994 |
| 31 | 1. $\in \leq \varepsilon$ | 532.1 | 1. 1327 | 0.13869 | 19.48 | 29. 24 | 0.11 | 1.004 | 1.2149 | 0.7559 | 63.86 | 11.53 | 0.01 | 1.001 |
| 32 | 1. SC8 | 613.0 | 1.3034 | 0.15960 | 19.41 | 29.14 | 0.00 | 1.000 | 1.3981 | 0.8699 | 63.80 | 11.56 | 0.04 | 1.003 |
| 33 | 2.158 | 693.2 | 1.4742 | 0.18C51 | 19.41 | 29.14 | 0.00 | 1.000 | 1.5813 | 0.9839 | 63.89 | 11.52 | $-0.00$ | 1.000 |

ADVEPSE PRESSUFE GKADIENT (-0.275,-0.001)


ADVERSE PRESSURE GFADIER:T $(-0.275,-0.001)$


ADVERSE PRESSURE GFADIEPIT $(-0.275,-0.001)$



ADVERSE PRESSURF GRADIENT $(-0.275,-0.002)$


ADVERSE PRESSURE GRAOIENT (-0.275,-0.002)


ADVERSE PRESSURE GRADIENT $1-0.275,-0.0021$


ADVERSE PRESSURE GFAOIENT (-0.275,-0.002)

| RIJN $=$ | $30473-4$ |
| :--- | ---: |
| PLATE $=$ | 15 |
| XIINI $=$ | 59. |
| ZIINI $=$ | 0. |
| PUINTS $=$ | 38 |


| UINF | = | 17.0 | $k$ | $=-0.495 \mathrm{E}-06$ |
| :---: | :---: | :---: | :---: | :---: |
| TWALL | = | 88.9 | BLTA | $=0.991$ |
| OC | $=$ | 9.693 | BF | -1.076 |
| DEL3 | = | 2.688 | F | $=-0.30201$ |
| TGAS | $=$ | 64.5 | BH | $=-0.602$ |


| CF/2 $=$ | 0.00187 |
| :--- | ---: |
| $S T$ | $=0.00334$ |
| PPLUS | $=0.00613$ |
| VOPLUS | $=$ |
| RFY.NT | $=0.04862606$ |


| DELA | $=1.875$ |
| :--- | :--- |
| DEL1M | $=0.419$ |
| DEL2M | $=0.272$ |
| REM | $=2430$. |
| $H$ | $=1.539$ |


| DELH | $=1.822$ |
| :--- | :--- |
| DEL2H | $=0.1274$ |
| REH | $=1155$. |
| GH | $=2.922$ |
| GF | $=8.099$ |


| I | Y | YPLUS | Y/DELM | Y/DC | $\cup$ | JPLUS | UUE | UBAR | Y/DELH | Y/DEL 3 | $T$ | TPLUS | TDE | TBAR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | C. C0O | 0.0 | 0.0000 | 0.03000 | 0.00 | 0.00 | -23.14 | 0.000 | 0.0000 | 0.0000 | 88.89 | 0.00 | -12.94 | 0.000 |
| 2 | 0.005 | 1.8 | 0.0027 | 0.03052 | 1.62 | 2.20 | -20.93 | 0.095 | 0.0027 | 0.0019 | 85.13 | 1.99 | -12.94 | 0.154 |
| 3 | 0.006 | 2.1 | 0.0032 | 0.01062 | 1.81 | 2.46 | -20.67 | 0.106 | 0.0033 | 0.0022 | 84.80 | 2.17 | -10.77 | 0.157 |
| 4 | 0.007 | 2.5 | 0.0037 | 0. 30072 | 1.98 | 2.69 | -20.44 | 0.116 | 0.0038 | 0.0026 | 84.34 | 2.41 | -10.53 | 0.186 |
| 5 | 0.0 Cg | 2. 8 | 0.1043 | 0.03083 | 2.18 | 2. 97 | -20.17 | 0.128 | 0.0044 | 0.0030 | 84.09 84.09 | 2.54 | -10.39 | 0.186 0.197 |
| 6 | 0.005 | 3.2 | 0. 0.348 | 0.01093 | 2.32 | 3.16 | -19.98 | 0.136 | 0.0049 | 0.0034 | 83.67 | 2.77 | $-10.17$ | 0.214 |
| 7 | 0.011 | 3.5 | 0.0059 | 0.00113 | 2.72 | 3.70 | $-19.44$ | 0.160 | 0.0060 | 0.0041 | 82.82 | 3.22 | $-9.72$ | 0.249 |
| 8 9 | 0.013 0.015 | 4.6 5.2 | 0.3069 0.0380 | 0.00134 0.00155 | 3.09 3.42 | 4.20 | -18.93 | 0.182 | 0.0071 | 0.0048 | 82.12 | 3.59 | -9.35 | 0.277 |
| 10 | 0.018 | 6.4 | 0.01996 | 0.00186 | 3.42 4.25 | 4.65 5.78 | -18.48 -17.36 | 0.201 0.250 | 0.0082 0.0099 | 0.0056 0.0067 | 81.35 80.31 | 4.00 | -8.94 | 0.309 0.352 |
| 12 | c.021 | 7.5 | 0.0112 | 0.10217 | 4.61 | 6.27 | -16.87 | 0.271 | 0.0115 | 0.0078 | 79.35 | 5.06 | -7.88 |  |
| 12 | 0.025 | 8.5 | 0.0133 | 0.00258 | 5.21 | 7.09 | -16.05 | 0.306 | 0.0137 | 0.0093 | 77.92 | 5.82 | -7.88 | 0.391 0.450 |
| 13 | C. 030 | 10.7 | 0.0160 | 0.1)369 | 5.99 | 8.15 | -14.99 | 0.352 | 0.0165 | 0.0112 | 77.08 | 6.26 | -6.67 | 0.450 0.494 |
| 14 | 0.035 | 12.5 | 0.0187 | 0.30361 | 6.67 | 9.07 | -14.06 | 0.392 | 0.0192 | 0.0130 | 76.01 | 6.83 | -6. 6.10 | 0.528 |
| 15 | 0.040 | 14.2 | 0.0213 | 0. $) 0413$ | 6.98 | 9.49 | -13.64 | 0.410 | 0.0220 | 0.0149 | 75.38 | 7.17 | -5.177 | 0.554 |
| 16 | 0.050 | 17. 8 | 0.0267 | 0.00510 | 7.73 | 10.51 | $-12.62$ | 0.454 | 0.0274 | 0.0186 | 74.04 | 7.88 | -5.06 | 0.699 |
| 17 | 0.060 0.070 | 21.4 | 0.0320 0.0373 | 0.00619 0.00722 | 8.19 | 11.14 | -12.00 | 0.481 | 0.0329 | 0.0224 | 73.20 | 8.32 | -4.61 | 0.643 |
| 19 | 0.085 | 24.4 30.3 | C. 0373 0.0453 | 0.00722 0.0 .977 | 8.68 8.88 | 11.81 12.08 | -11.33 | 0.510 | 0.0384 | 0.0261 | 72.62 | 8.63 | -4.31 | 0.667 |
| 20 | 0.165 | 37.4 | 0.0560 | 0.01083 | 8.88 9.34 | 12.08 12.70 | -11.06 -10.43 | 0.522 0.549 | 0.0467 0.0576 | 0.0317 0.0391 | 71.88 71.22 | 9.02 9.37 | -3.92 -3.57 | 0.697 0.724 |
| 21 | 0.125 | 44.5 | 0.0667 | 0.01290 | 9.57 | 13.02 | -10.12 | 0.563 | 0.0686 | 0.0466 | 70.80 | 9.59 | -3.34 | 0.741 |
| 22 | 0.150 | 53.4 | 0.0800 | 0.01547 | 9.81 | 13.34 | -9.79 | 0.577 | 0.0823 | 0.0559 | 70.31 | 9.85 | -3.08 | 0.751 |
| 23 | C. 175 | 62.3 | 0.0933 | 0.01805 | 9.98 | 13.57 | -9.56 | 0.587 | 0.6960 | 0.0652 | 69.95 | 10.04 | $-2.89$ | 0.776 |
| 24 | 0.225 | 80.1 1 | 0.1200 | 0.02321 | 10.28 | 13.98 | -9.15 | 0.604 | 0.1235 | 0.0838 | 69.54 | 10.26 | $-2.68$ | 0.793 |
| 25 | 0.325 0.425 | 115.7 | 0.1733 | 0.03353 | 10.73 | 14.59 15.33 | -8.54 | 0.631 | 0.1784 | 0.1211 | 68.66 | 10.73 | -2. 21 | 0.829 |
| 7 | 0.425 C. 575 | 151.3 204.7 | 0.2267 0.3067 | 0.04385 0.05932 | 11.27 11.90 | 15.33 16.19 | -7.81 | 0.663 | 0.2333 | 0.1583 | 68.30 | 10.92 | $-2.02$ | 0.844 |
| 28 | 0.725 | 258.1 | 0.3867 | 0.17479 | 12.47 | 16.19 16.96 | -6.95 -6.18 | 0.700 0.733 | 0.3156 0.3979 | 0.2142 0.2701 | 67.65 | 11.26 | -1.67 -1.36 | 0.870 |
| 9 | 0.925 | 329.3 | 0.4934 | 0.09543 | 13.49 | 18.35 | -4.79 | 0.793 | 0.5077 | 0.3446 | 66.53 | 11.86 | -1.08 | 0.894 0.916 |
| 30 | 1.125 | 400.5 | 0.6000 | 0.11606 | 14.24 | 19.37 | -3.77 | 0.837 | 0.6175 | 0.4191 | 65.96 | 12.16 | -0.77 | 0.940 |
| 31 | 1.325 | 471.7 | 0.7067 | 0.13669 | 15.27 | 20.77 | -2.37 | 0.898 | 0.7272 | 0.4936 | 65.55 | 12.38 | -0.56 |  |
| 32 | 1.525 | $543 . c$ | 0.8134 | 1. 15733 | 15.88 | 21.60 | -1.54 | 0.934 | 0.8370 | 0.5681 | 65.18 | 12.57 | -0.37 | 0.972 |
| 33 | 1.725 | 614.2 | 0.9203 | 0.17756 | 16.60 | 22.58 | -0.56 | 0.976 | 0.9468 | 0.6426 | 64.84 | 12.75 | -0.19 | 0.986 |
| 34 | 1.925 | 685.4 | 1.0267 | 0.19859 | 16.92 | 23.01 | -0.12 | 0.995 | 1.0565 | 0.7171 | 64.62 | 12.87 | -0.07 | 0.995 |
| 35 | 2.175 2.475 | 774.4 863.4 | 1.160 .3 | 0.22438 | 17.10 | 23.26 | 0.12 | 1.005 | 1.1938 | 0.8102 | 64.52 | 12.92 | -0.02 | 0.999 |
| 37 | $2.4<5$ 2.675 | 963.4 952.4 | 1.2934 1.4267 | 0.25017 0.27597 | 17.08 17.07 | 23.23 23.22 | 0.10 | 1.004 | 1.3310 | 0.9034 | 64.47 | 12.95 | 0.01 | 1.001 |
| 38 | 2.925 | 1041.4 | 1.5601 | 0.30176 | 17.07 17.01 | 23.22 23.14 | 0.08 0.00 | 1.004 1.000 | 1.4682 1.6054 | 0.9965 1.0846 | 64.53 64.49 | 12.92 12.94 | -0.02 -0.00 | 0.998 |

ADVERSE PRESSURE GRADIENT (-0.275,-0.002)

| RUN | $=30573-5$ |  | UlNF | $=16.6$ | $K \quad=-0.429 \mathrm{E}-06$ |  |  | $C F / 2$ | 0.00191 | DEL.M |  | $2.138$ | $\text { DELH }=2.325$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE | = | 18 | TWALL | 89.5 | BETA | $=$ | 0.998 |  | 0.003 |  | $M=0$ |  | DEL2H = | $0.1534$ |
| X(IN) | $=$ | 70. | DC | $=10.685$ | BF | $=$ | -1.044 | PPLUS | 0.005 |  | $2 M=0$ | 10 | REH | 1301. |
| Z(IN) |  | 0. | DEL 3 | $=3.121$ | F | $=-0$ | . 00199 | VOPL | -0.047 | RE |  |  | GH | 2.812 |
| POINTS |  | 39 | TGAS | $=64.6$ | BH | $=$ | -0.612 | REY.N | 0.649E | H | $=1$ | 07 | GF = | 7.702 |
| I | $Y$ | YPLUS | Y/OELM | Y/DC | U | UPLUS | UDE | UBAR | Y/DELH | Y/DEL 3 | $T$ | TPLUS | TOE | TBAR |
| 1 | 0.000 | 0.0 | 0.0000 | 0.00000 | 0.00 | 0.00 | -22.90 | 0.000 | 0.0000 | 0.0000 | 89.47 | 0.00 | -13.43 | 0.000 |
| 2 | 0.005 | 1.7 | 0.0023 | 0.00047 | 1.58 | 2.19 | -20.72 | 0.095 | 0.0022 | 0.0016 | 85.37 | 2.21 | -11.22 | 0.155 |
| 3 | 0.006 | 2.1 | 0.0028 | 0.00056 | 1.79 | 2.48 | -20.43 | 0.108 | 0.0026 | 0.0019 | 85.12 | 2.35 | -11.08 | 0.175 |
| 4 | 0.007 | 2.4 | 0.0033 | 0.00066 | 2.03 | 2.81 | -20.09 | 0.123 | 0.0030 | 0.0022 | 84.74 | 2.55 | -10.88 | 0.190 |
| 5 | C.CC8 | 2.8 | 0.0037 | 0.00c75 | 2.17 | 3.00 | -19.90 | 0.131 | 0.0034 | 0.0026 | 84.21 | 2.84 | -10.59 | 0.211 |
| 6 | 0.010 | 3.5 | 0.0047 | 0.00094 | 2.62 | 3.63 | $-19.28$ | 0.158 | 0.0043 | 0.0032 | 83.50 | 3.22 | -10.21 | 0.240 |
| 7 | 0.012 | 4.2 | 0.0056 | 0.00112 | 2.87 | 3.97 | -18.93 | 0.173 | 0.0052 | 0.0038 | 82.73 | 3.64 | -9.79 | 0.271 |
| 8 | 0.014 | 4.9 | 0.0065 | 0.00131 | 3.19 | 4.41 | -18.49 | 0.193 | 0.0060 | 0.0045 | 82.02 | 4.02 | -9.41 | 0.299 |
| 4 | 0.017 | 5.9 | 0.0079 | 0.00159 | 3.76 | 5.20 | -17.70 | 0.227 | 0.0073 | 0.0054 | 81.05 | 4.54 | -8.89 | 0.338 |
| 10 | 0.021 | 7.3 | 0.0098 | 0.00197 | 4.46 | 6.17 | -16.73 | 0.269 | 0.0090 | 0.0067 | 79.73 | 5.26 | -8.17 | 0.391 |
| 11 | 0.026 | 9.C | 0.0122 | 0.00243 | 5.46 | 7.56 | -15.35 | 0.330 | 0.0112 | 0.0083 | 78.60 | 5.87 | -7.56 | 0.437 |
| 12 | 0.031 | 10. 8 | 0.0145 | 0.00290 | 5.94 | 8.22 | -14.68 | 0.359 | 0.0133 | 0.0099 | 77.47 | 6.48 | -6.95 | 0.492 |
| 13 | 0.036 | 12.5 | 0.0168 | 0.00337 | 6.38 | 8.83 | -14.07 | 0.385 | 0.0155 | 0.0115 | 76.40 | 7.05 | $-6.38$ | 0.525 |
| 14 | 0.041 | 14.3 | 0.0192 | 0.00384 | 6.92 | 9.58 | -13.33 | 0.418 | 0.0176 | 0.0131 | 75.58 | 7.50 | -5.93 | 0.558 |
| 15 | 0.046 | 16.6 | 0.0215 | 0.00431 | 7.15 | 9.89 | -13.01 | 0.432 | 0.0198 | 0.0147 | 74.90 | 7. 86 | -5.57 | 0.585 |
| 16 | 0.051 | $17 . \varepsilon$ | 0.0238 | 0.00477 | 7.59 | 10.50 | -12.40 | 0.459 | 0.0219 | 0.0163 | 74.32 | 8.18 | -5.25 | 0.609 |
| 17 | 0.061 | 21.2 | 0.0285 | 0.00571 | 8.07 | 11.17 | -11.73 | 0.488 | 0.0262 | 0.0195 | 73.56 | 8.59 | -4.84 | 0.637 |
| 18 | 0.071 | 24.7 | 0.0332 | 0.00664 | 8.33 | 11.53 | -11.38 | 0.503 | 0.0305 | 0.0228 | 72.85 | 8.97 | -4.46 | 0.667 |
| 19 | 0.086 | 29.9 | 0.0402 | 0.00805 | 8.99 | 12.44 | -10.46 | 0.543 | 0.0370 | 0.0276 | 72.19 | 9.32 | -4.11 | 0.674 |
| 20 | 0.101 | 35.2 | 0.0472 | 0.00945 | 9.18 | 12.70 | -10.20 | 0.555 | 0.0434 | 0.0324 | 71.72 | 9.58 | -3.85 | 0.713 |
| 21 | 0.121 | 42.1 | 0.0566 | 0.01132 | 9.28 | 12.84 | -10.06 | 0.561 | 0.0520 | 0.0388 | 70.91 | 10.01 | -3.42 | 0.745 |
| 22 | 0.146 | 50.8 | 0.0683 | 0.01366 | 9.54 | 13.20 | -9.70 | 0.576 | 0.0628 | 0.0468 | 70.65 | 10.15 | -3. 28 | 0.756 |
| 23 | 0.171 | 59.5 | 0.0800 | 0.01600 | 9.79 | 13.55 | -9.35 | 0.592 | 0.0735 | 0.0548 | 70.28 | 10.35 | -3.08 | 0.771 |
| 24 | 0.221 | 76.5 | 0.1033 | 0.02068 | 10.24 | 14.17 | -8.73 | 0.619 | 0.0950 | 0.0708 | 69.64 | 10.70 | -2.73 | 0.796 |
| 25 | 0.321 | 111.7 | 0.1501 | 0.03004 | 10.34 | 14.31 | -8.59 | 0.625 | 0.1381 | 0.1029 | 68.87 | 11.11 | -2.32 | 0.827 |
| 26 | 0.421 | 146.5 | 0.1969 | 0.03940 | 10.98 | 15.19 | -7.71 | 0.663 | 0.1811 | 0.1349 | 68.55 | 11. 29 | -2.14 | 0.840 |
| 27 | 0.571 | 198.7 | 0.2670 | 0.05344 | 11.37 | 15.73 | -7.17 | 0.687 | 0.2456 | 0.1830 | 67.64 | 11.78 | -1.65 | 0.877 |
| 28 | C. 721 | 251.0 | 0.3372 | 0.06748 | 11.96 | 16.55 | -6.35 | 0.723 | 0.3101 | 0.2310 | 67.39 | 11.91 | -1.52 | 0.887 |
| 29 | 0.921 | 320.6 | 0.4307 | 0.08619 | 12.79 | 17.70 | -5.20 | 0.773 | 0.3961 | 0.2951 | 66.93 | 12.16 | -1.27 | 0.905 |
| 30 | 1:121 | 390.2 | 0.5242 | 0.10491 | 13.48 | 18.65 | -4.25 | 0.815 | 0.4821 | 0.3592 | 66.46 | 12.42 | -1.01 | 0.924 |
| 31 | 1.321 | 459.8 | 0.6177 | 0.12363 | 14.06 | 19.46 | -3.45 | 0.850 | 0.5681 | 0.4233 | 65.94 | 12.70 | $-0.73$ | 0.945 |
| 32 | 1.521 | 529.4 | 0.7113 | 0. 14235 | 14.70 | 20.34 | -2.56 | 0.888 | 0.6542 | 0.4874 | 65.72 | 12.81 | -0.62 | 0.954 |
| 33 | 1.721 | 599.0 | 0.8048 | 0.16107 | 15.38 | 21.28 | -1.62 | 0.929 | 0.7402 | 0.5515 | 65.30 | 13.04 | -0.39 | 0.971 |
| 34 | 1.921 | 668.6 | 0.8983 | 0. 17978 | 15.88 | 21.98 | -0.93 | 0.960 | 0.8262 | 0.6156 | 64.99 | 13.21 | $-0.23$ | 0.983 |
| 35 | 2.171 | 755.6 | 1.0152 | 0. 20318 | 16.46 | 22.78 | -0.12 | 0.995 | 0.9337 | 0.6957 | 64.93 | 13.24 | -0.19 | 0.986 |
| 36 | 2.421 | 842.7 | 1.1321 | 0.22658 | 16.61 | 22.99 | 0.08 | 1.004 | 1.0412 | 0.7758 | 64.75 | 13.34 | -0.10 | 0.993 |
| 37 | 2.671 | 929.7 | 1.2490 | 0.24997 | 16.61 | 22.99 | 0.08 | 1.004 | 1.1488 | 0.8559 | 64-64 | 13.40 | -0.04 | 0.997 |
| 38 | 2.921 | 1016.7 | 1.3659 | 0.27337 | 16.64 | 23.03 | 0.12 | 1.005 | 1.2563 | 0.9360 | 64.76 | 13.33 | -0.10 | 0.992 |
| 39 | 3.171 | 1103.7 | 1.4828 | 0.29677 | 16.55 | 22.90 | 0.00 | 1.000 | 1. 3638 | 1.0162 | 64.57 | 13.43 | -0.00 | 1.000 |

ADVERSE PRESSURE GRADIENT (-0.275,-0.002)


ADVERSE PRESSURE GRADIENT $1-0.275,-0.0041$

|  | $\begin{aligned} & \text { RUN }=3 \\ & \text { PLATE }= \\ & X(I N)= \\ & Z(I N)= \\ & \text { POINTS= } \end{aligned}$ |  | $\begin{array}{r} 31173-1 \\ 6 \\ 22 \\ 0 \\ 33 \end{array}$ | UINF <br> TWALL <br> DC <br> OEL 3 <br> TGAS | $\begin{aligned} & =21.5 \\ & =\quad 77.4 \\ & =1.773 \\ & =0.571 \\ & =53.3 \end{aligned}$ | $k$ <br> BETA <br> BF <br> F <br> BH | $\begin{array}{lr} =-0.947 \mathrm{E}-06 \\ = & 0.342 \\ = & -1.244 \\ = & -0.00409 \\ = & -0.812 \end{array}$ |  | $\begin{aligned} & \text { GF/2 }=0.00329 \\ & \text { ST }=0.00504 \\ & \text { PPLUS }=0.00502 \\ & \text { VOPLUS }=-0.07467 \\ & \text { REY.ND }=0.315 E 06 \end{aligned}$ |  | $\begin{aligned} \text { DELM } & =0.646 \\ \text { DELIM } & =0.102 \\ \text { DEL2M } & =0.071 \\ \text { REM } & =834 . \\ H & =1.423 \end{aligned}$ |  |  | $\begin{aligned} & \text { DELH }= \\ & \text { DEL 2H }= \\ & \text { REH }= \\ & \text { GH }= \\ & \text { GF }= \end{aligned}$ | $\begin{array}{r} 0.514 \\ 0.0313 \\ 366 . \\ 3.048 \\ 5.134 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | Y | YPLES | Y/DELM | $Y / D C$ | U | UPLUS | UDE | UBAR | V/CELH | Y/DEL 3 | T | TPLUS | TDE | TBAR |
|  | 1 | 0.000 | 0.0 | 0.0000 | 0.00000 | 0.00 | 0.00 | -17.44 | 0.000 | 0.0000 | 0.0000 | 77.38 | 0.00 | -11.38 | 0.000 |
|  | 2 | 0.065 | 3.1 | 0.0077 | 0.00282 | 4.75 | 3.85 | -13.59 | 0.221 | 0.0097 | 0.0088 | 70.17 | 3.41 | -7.96 | 0.300 |
|  | 3 | 0.006 | 3.7 | 0.0093 | 0.00338 | 5.23 | 4.24 | $-13.20$ | C. 243 | 0.0117 | 0.0105 | 69.54 | 3.71 | -7.66 | 0.326 |
|  | 4 | $\mathrm{C.OC7}$ | 4. 3 | 0.0108 | 0.00395 | 5.95 | 4.83 | -12.61 | 0.277 | 0.0136 | 0.0123 | 68.78 | 4.07 | -7.30 | 0.358 |
|  | 5 | 0.008 | 4.9 | 0.0124 | 0.00451 | 6.21 | 5.04 | -12.40 | 0.289 | 0.0156 | 0.0140 | 68.15 | 4.37 | -7.01 | 0.384 |
|  | 6 | C. 009 | 5.6 | 0.0139 | 0.005013 | 6.61 | 5.36 | -12.08 | 0.307 | 0.0175 | 0.0158 | 67.65 | 4.60 | -6.77 | 0.455 |
|  | 7 | 0.010 | 6.2 | 0.0155 | 0.00564 | 7.22 | 5.86 | -11.58 | 0.336 | 0.0194 | 0.0175 | 66.95 | 4.93 | -6.44 | 0.434 |
|  | 8 | 0.012 | 7.4 | 0.0186 | 0.00677 | 7.97 | 6.47 | -10.98 | 0.371 | 0.0233 | 0.0210 | 66.18 | 5.30 | -6.07 | 0.466 |
|  | 9 | 0.014 | 8.6 | 0.0217 | 0.00790 | 8.84 | 7.17 | -10.27 | 0.411 | 0.0272 | 0.0245 | 65.13 | 5.79 | -5.58 | 0.509 |
|  | 10 | 0.16 | 9.5 | 0.0248 | 0.00903 | 9.58 | 7.77 | $-9.67$ | 0.446 | 0.0311 | 0.0280 | 64.26 | 6.20 | $-5.17$ | 0.545 |
|  | 11 | 0.018 | 11. 1 | 0.0279 | 0.01015 | 10.32 | 8.37 | -9.07 | 0.480 | 0.0350 | 0.0315 | 63.41 | 6.61 | $-4.76$ | 0.581 |
|  | 12 | C. 020 | 12.4 | 0.0310 | 0.01128 | 10.89 | 8.83 | -8.61 | 0.507 | 0.0389 | 0.0351 | 62.83 | 6.88 | -4.49 | 0.635 |
|  | 13 | 0.023 | 14.2 | 0.0356 | 0.01297 | 11.58 | 9.39 | -8.05 | 0.539 | 0.0447 | 0.0403 | 61.94 | 7.30 | -4.07 | 0.642 |
|  | 14 | 0.026 | 16. 1 | 0.0402 | 0.01467 | 12.09 | 9.81 | -7.63 | 0.562 | 0.0505 | 0.0456 | 61.17 | 7.67 | -3.70 | 0.674 |
| $\stackrel{\sim}{\infty}$ | 15 | 0.030 | 18.5 | 0.0464 | 0.01692 | 12.83 | 10.41 | -7.03 | 0.597 | 0.0583 | 0.0526 | 60.13 | 8.16 | -3. 21 | 0.717 |
| $\cdots$ | 16 | 0.035 | 21.6 | 0.0542 | 0.01974 | 13.33 | 10.81 | -6.63 | 0.620 | 0.0680 | 0.0613 | 59.70 | 8.36 | -3.01 | 0.735 |
|  | 17 | 0.040 | 24. 1 | 0.0619 | 0.02256 | 13.80 | 12.19 | -6.25 | 0.642 | 0.0778 | 0.0701 | 59.31 | 8.55 | -2.82 | 0.751 |
|  | 18 | 0.045 | 27.8 | 0.0697 | 0.02538 | 14.14 | 11.47 | -5.97 | 0.658 | 0.0875 | 0.0789 | 58.73 | 8.82 | -2.55 | 0.776 |
|  | 19 | 0.055 | 34.0 | 0.3851 | 0.03102 | 14.55 | 11.80 | -5.64 | $0.677$ | 0.1069 | $0.0964$ | 58.03 | 9.15 | -2.22 | 0.804 |
|  | 20 | 0.070 | 43.2 | 0.1084 | 0.03548 | 15.15 | 12.29 | -5.15 | 0.705 | 0.1361 | 0.1227 | 57.43 | 9.43 | -1.94 | 0.829 |
|  | 21 | 0.085 | 52.5 | 0.1316 | 0.04795 | 15.59 | 12.65 | -4.79 | 0.725 | 0.1652 | 0.1490 | 57.03 | 9.62 | -1.75 | 0.846 |
|  | 22 | 0.100 | 61.8 | 0.1548 | 0.05641 | 15.80 | 12.82 | -4.62 | 0.735 | 0.1944 | 0.1753 | 56.67 | 9.79 | -1.58 | 0.851 |
|  | 23 | 0.120 | 74.1 | 0.1858 | 0.06769 | 16.25 | 13.18 | -4.26 | 0.756 | 0.2333 | 0.2103 | 56.21 | 10.01 | -1.36 | 0.880 |
|  | 24 | C. 170 | 105.6 | 0.2632 | 0.09589 | 16.95 | 13.75 | -3.69 | 0.788 | 0.3304 | 0.2980 | 55.66 | 10.27 | -1.10 | 0.903 |
|  | 25 | 0.220 | 135.5 | 0.3406 | 0.12410 | 17.64 | 14.31 | -3.13 | 0.820 | 0.4276 | 0.3856 | 55.12 | 10.53 | -0.84 | 0.925 |
|  | 26 | 0.320 | 197.6 | 0.4954 | 0.18050 | 18.81 | 15.26 | -2.18 | 0.875 | 0.6220 | 0.5609 | 54.39 | 10.87 | -0.50 | 0.956 |
|  | 27 | 0.420 | 259.4 | 0.6502 | 0.23691 | 19.81 | 16.07 | $-1.37$ | 0.921 | 0.8164 | 0.7362 | 53.87 | 11.12 | -0.26 | 0.977 |
|  | 28 | 0.520 | 321.2 | 0.8050 | 0. 29332 | 20.65 | 16.75 | -0.69 | 0. 960 | 1.0108 | 0.9114 | 53.55 | 11.27 | -0.10 | 0.991 |
|  | 29 | C. 620 | 382.5 | 0.9598 | 0.34572 | 21.22 | 17.21 | -0.23 | 0.987 | 1.2051 | 1.0867 | 53.35 | $11.37$ | -0.01 | 0.999 |
|  | 30 | 0.720 | 444.7 | 1.1146 | 0.40613 | 21.47 | 17.42 | -0.02 | 0.999 | 1.3995 | 1.2620 | 53.27 | 11.40 | 0.03 | 1.002 |
|  | 31 | C. 920 | 568.2 | 1. 4242 | 0.51894 | 21.51 | 17.45 | 0.01 | 1.000 | 1.7883 | $1.6125$ | $53.21$ | $11.43$ | $0.06$ | $1.005$ |
|  | 32 | 1.120 | 691.7 | 1.7338 | 0.63176 | 21.51 | 17.45 | 0.01 | 1.000 | $2.1770$ | $1.9631$ | $53.29$ | $11.40$ | $0.02$ | $1.002$ |
|  | 33 | 1.320 | 815.3 | 2.0434 | 0.74457 | 21.50 | 17.44 | 0.00 | 1.000 | 2.5658 | 2.3136 | 53.33 | 11.38 | $-0.00$ | 1.000 |

ADVERSE PRESSURE GFADIENT (-0.275,-C.004)

|  | $\begin{aligned} & \text { RUN }= \\ & \text { PLATE }= \\ & \text { X(IN) }= \\ & \text { ZIIN) }= \\ & \text { POINTS }= \end{aligned}$ |  | $\begin{array}{r} 31173-2 \\ 9 \\ 34 . \\ 0 . \\ 34 \end{array}$ | UINF <br> TWALL <br> DC <br> DEL 3 <br> TGAS | $\begin{aligned} & =19.0 \\ & =\quad 77.8 \\ & =\quad 2.542 \\ & =\quad 0.735 \\ & =\quad 53.4 \end{aligned}$ | K <br> BETA <br> EF <br> $f$ <br> BH |  | $\begin{array}{lr} =-0.708 E-06 \\ = & 0.323 \\ = & -1.266 \\ = & -0.00417 \\ =-0.880 \end{array}$ |  | $\begin{aligned} & C F / 2=0.00329 \\ & S T=0.00485 \\ & \text { PPLUS }=0.00375 \\ & \text { VOPLUS }=-0 . C 7611 \\ & \text { REY.NO }=0.402 E 06 \end{aligned}$ |  | $\begin{aligned} \text { DELM } & =0.886 \\ \text { DELIM } & =0.146 \\ \text { DEL2M } & =0.104 \\ \text { REM } & =1072 \\ H & =1.404 \end{aligned}$ |  |  | $\begin{aligned} & \text { DELH }= \\ & \text { DEL2H }= \\ & \text { REH }= \\ & \text { GH }= \\ & \text { GF }= \end{aligned}$ | $\begin{array}{r} =0.679 \\ =0.0382 \\ =394 . \\ =\quad 3.095 \\ =5.015 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | $\gamma$ | yplus | Y/DELM | $\mathrm{V} / \mathrm{DC}$ | $u$ |  | uplus | Ude | UBAR | Y/DELH | Y/DEL 3 | $\top$ | tplus | TDE | TBAR |
|  | 1 | 0.000 | 0.0 | 0.0000 | 0.00000 | 0.00 |  | 0.00 | -17.43 | 0.000 | 0.0000 | 0.0000 | 77.76 | 0.00 | -11.83 | 0.000 |
|  | 2 | 0.065 | 2.7 | 0.0056 | 0.00197 | 3.84 |  | 3.52 | -13.90 | 0.202 | 0.0072 | 0.0068 | 70.81 | 3.38 | -8.45 | 0.236 |
|  | 3 | 0.006 | 3.3 | 0.0068 | 0.00236 | 4.24 |  | 3.89 | -13.54 | 0.223 | 0.0086 | 0.0082 | 70.36 | 3.60 | -8.23 | 0.304 |
|  | 4 | $0 . \mathrm{CC7}$ | 3.8 | 0.0079 | 0.00275 | 4.68 |  | 4.29 | -13.13 | 0.246 | 0.0100 | 0.0095 | 69.56 | 3.99 | -7.84 | 0.337 |
|  | 5 | 0.008 | 4.4 | 0.0090 | 0. 00315 | 4.95 |  | 4.54 | -12.89 | 0.261 | 0.0114 | 0.0109 | 69.24 | 4.14 | -7.68 | 0.350 |
|  | 6 | 0.609 | 4.9 | 0.0102 | 0.00354 | 5.27 |  | 4.83 | -12.59 | 0.277 | 0.0129 | 0.0122 | 68.79 | 4.36 | -7.46 | 0.359 |
|  | 7 | 0.011 | 6.1 | 0.0124 | 0.00433 | 5.98 |  | 5.48 | -11.94 | 0.315 | 0.0157 | 0.0150 | 67.67 | 4.91 | -6.92 | 0.415 |
|  | 8 | 0.013 | 7.1 | 0.0147 | 0.00511 | 6.61 |  | 6.06 | -11.36 | 0.348 | 0.0186 | 0.0177 | 66.96 | 5.25 | -6.57 | 0.444 |
|  | 9 | 0.015 | 8.2 | 0.0169 | 0.00590 | 7.14 |  | 6.55 | -10.88 | 0.376 | 0.0215 | 0.0204 | 66.16 | 5.64 | -6.18 | 0.447 |
|  | 10 | 0.017 | 9.3 | 0.0192 | 0.00669 | 7.97 |  | 7.31 | -10.12 | 0.419 | 0.0243 | 0.0231 | 65.21 | 6.11 | -5.72 | 0.516 |
|  | 11 | 0.020 | 10.9 | 0.0226 | 0.00787 | 8.59 |  | 7.88 | -9.55 | 0.452 | 0.0286 | 0.0272 | 64.31 | 6.54 | -5.29 | 0.553 |
|  | 12 | 0.023 | 12.6 | 0.0260 | 0.00905 | 9.33 |  | 8.56 | -8.87 | 0.491 | 0.0329 | 0.0313 | 63.28 | 7.04 | -4.79 | 0.595 |
|  | 13 | 0.027 | 14.7 | 0.0305 | 0.01062 | 9.98 |  | 9.15 | -8.27 | 0.525 | 0.0386 | 0.0367 | 62.26 | 7.54 | -4.29 | 0.637 |
|  | 14 | C.C32 | 17.5 | 0.0361 | 0.01259 | 10.74 |  | 9.85 | -7.58 | 0.565 | 0.0458 | 0.0435 | 61.34 | 7.99 | -3.84 | 0.6375 |
| $\stackrel{\rightharpoonup}{\circ}$ | 15 | C.C37 | 20.2 | 0.0418 | 0.01456 | 11.12 |  | 10.20 | -7.23 | 0.585 | 0.0529 | 0.0503 | 60.56 | 8.37 | -3.46 | 0.707 |
|  | 16 | 0.042 | 22.5 | 0.0474 | 0.01652 | 11.60 |  | 10.64 | -6.79 | 0.611 | 0.0601 | 0.0571 | 60.05 | 8.61 | -3.21 | 0.728 |
|  | 17 | 0.047 | 25.6 | 0.0531 | 0.01849 | 11.98 |  | 10.99 | -6.44 | 0.631 | 0.0672 | 0.0639 | 59.55 | 8.86 | -2.97 | 0.749 |
|  | 18 | C. 057 | 31.1 | 0.0644 | 0.02242 | 12.13 |  | 11.13 | -6.30 | 0.638 | 0.0815 | 0.0775 | 58.61 | 9.31 | -2.51 | 0.787 |
|  | 19 | 0.072 | 39.3 | 0.0813 | 0.02832 | 12.90 |  | 11.83 | -5.59 | 0.679 | 0.1030 | 0.0979 | 57.91 | 9.66 | -2.17 | 0.816 |
|  | 20 | 0.087 | 47.5 | 0.0982 | 0.03422 | 13.35 |  | 12.24 | -5.18 | 0.703 | 0.1245 | 0.1183 | 57.48 | 9.87 | -1.96 | 0.834 |
|  | 21 | 0.107 | 58.4 | 0.1208 | 0. 64209 | 13.56 |  | 12.44 | -4.99 | 0.714 | 0.1531 | 0.1455 | 57.12 | 10.04 | -1.79 | 0.649 |
|  | 22 | 0.132 | 72. C | 0.1491 | 0.05193 | 13.90 |  | 12.75 | -4.68 | 0.732 | 0.1888 | 0.1795 | 56.58 | 10.30 | -1.52 | 0.871 |
|  | 23 | 0.157 | 85.7 | 0.1773 | 0.06176 | 14.07 |  | 12.90 | -4.52 | 0.741 | 0.2246 | 0.2135 | 56.27 | 10.45 | -1.37 | 0.884 |
|  | 24 | C. 267 | $113 . \mathrm{C}$ | 0.2337 | 0.08143 | 14.38 |  | 13.19 | -4.24 | 0.757 | 0.2961 | 0.2815 | 55.84 | 10.66 | -1.17 | 0.901 |
|  | 25 | 0.257 | 140.2 | 0.2962 | 0.10110 | 15.05 |  | 13.80 | -3.62 | 0.792 | 0.3677 | 0.3495 | 55.27 | 10.94 | -0.89 | 0.924 |
|  | 26 | 0.367 | 161.5 | 0.3467 | 0.12071 | 15.40 |  | 14.12 | -3.30 | 0.811 | 0.4392 | 0.4175 | 55.06 | 11.04 | -0.79 | 0.933 |
|  | 27 | C.4C7 | 222.1 | c. 4596 | 0.16011 | 16.09 |  | 14.76 | -2.67 | 0.847 | 0.5823 | 0.5535 | 54.49 | 11.32 | -0.51 | 0.957 |
|  | 28 | 0.567 | $276 . t$ | 0.5725 | 0.19944 | 16.83 |  | 15.44 | -1.95 | 0.886 | 0.7253 | 0.6894 | 54.26 | 11.43 | -0.40 | 0.966 |
|  | 29 | 0.617 | 331.2 | 0.6854 | 0.23878 | 17.56 |  | 16.11 | -1.32 | 0.924 | 0.8684 | 0.8254 | 53.92 | 11.60 | -0.23 | 0.980 |
|  | 30 | 0.757 | 413.1 | 0.8548 | 0.29779 | 18.45 |  | 16.92 | -0.56 | 0.971 | 1.0830 | 1.0294 | 53.52 | 11.79 | -0.04 | 0.996 |
|  | 31 | 0. 967 | 494.5 | 1.0242 | 0.35680 | 18.87 |  | 17.31 | -0.12 | 0.593 | 1.2976 | 1.2334 | 53.35 | 11.87 | 0.04 | 1.003 |
|  | 32 | 1.167 | 604.0 | 1.2500 | 0.43547 | 19.00 |  | 17.43 | 0.00 | 1.000 | 1.5837 | 1.5054 | 53.29 | 11.90 | 0.07 | 1.006 |
|  | 33 | 1.367 | 713.2 | 1.4759 | 0.51415 | 18.93 |  | 17.36 | -0.06 | 0.996 | 1.8698 | 1.7773 | 53.20 | 11.94 | 0.11 | 1.009 |
|  | 34 | 1.567 | 822.3 | 1.7017 | 0.59282 | 19.00 |  | 17.43 | 0.00 | 1.000 | 2.1560 | 2.0493 | 53.43 | 11.83 | -0.00 | 1.000 |

AOVERSE PRESSURE GRADIENT $(-0.275,-0.0041$

| RUN | $=31$ | 73-3 | UINF | 17.4 | K | $=-0.577 \mathrm{E}-06$ |  | CF/2 | 0.00322 | DELM = |  | 1.059 | DELH | 0.915 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE | $=$ | 12 | TWALL | 77.7 | EETA | - | 0.283 | ST | 0.004 |  | $M=0$ | $167$ | DEL $2 \mathrm{H}=$ | 0.0551 |
| X(IN) | $=$ | 46. | DC | 2.951 | BF | $=$ | -1.278 | PPLLS | 0.003 |  | $2 M=$ | 19 | REH | 523. |
| Z(IN) |  | 0. | OEt3 | 1.012 | F | -0 | . 00411 | VOPLU | $=-0.075$ | 7 REM | $=1$ | 4 | GH | 2.906 |
| POINT |  | 36 | TGAS | 53.5 | BH | $=$ | -0.873 | REY.N | 0.481E | 6 H | $=1$ | 405 | GF | 5.033 |
| I | $Y$ | YPLES | Y/DELM | Y/DC | $\cup$ | UPLUS | UDE | UBAR | Y/OELH | Y/DEL 3 | T | TPLUS | TDE | TBAR |
| 1 | C. CGO | 0.0 | 0.0000 | 0.00000 | 0.00 | 0.00 | -17.63 | 0.000 | 0.0000 | 0.0000 | 77.66 | 0.00 | -12.04 | 0.000 |
| 2 | C.0C5 | 2. 5 | 0.0047 | 0.00169 | 2.80 | 2.84 | -14.79 | 0.161 | 0.0055 | 0.0049 | 72.36 | 2.65 | -9.39 | 0.220 |
| 3 | C.0C6 | 3.0 | 0.0057 | 0.00203 | 3.02 | 3.06 | -14.57 | 0.174 | 0.0066 | 0.0055 | 71.91 | 2.87 | -9.17 | 0.238 |
| 4 | $0 . C C 7$ | 3.5 | 0.0066 | 0.00237 | 3.35 | 3.40 | -14.24 | 0.193 | 0.0076 | 0.0069 | 71.36 | 3.14 | -8.89 | 0.261 |
| 5 | O.CC8 | 4. C | 0.0076 | 0.00271 | 3.62 | 3.67 | -13.96 | 0.208 | 0.0087 | 0.0079 | 70.83 | 3.41 | -8.63 | 0.233 |
| 6 | 0.069 | 4.4 | 0.0085 | 0. 00305 | 3.96 | 4.02 | -13.62 | 0.228 | 0.0098 | 0.0089 | 70.48 | 3.58 | -8.45 | 0.298 |
| 7 | 0.011 | 5.4 | 0. $01 \mathrm{C4}$ | 0.00373 | 4.62 | 4.68 | -12.95 | 0.266 | 0.0120 | 0.0109 | 69.39 | 4.13 | -7.91 | 0.343 |
| $\bigcirc$ | 0.013 | 6.4 | 0.0123 | 0.00441 | 5.32 | 5.39 | -12.24 | 0.306 | 0.0142 | 0.0129 | 68.29 | 4.68 | -7.36 | 0.389 |
| 9 | 0.015 | 7.4 | 0.0142 | 0.30508 | 5.87 | 5.95 | -11.68 | 0.338 | 0.0164 | 0.0148 | 67.36 | 5.14 | -6.89 | 0.427 |
| 10 | 0.018 | 8. 5 | 0.0170 | 0.00610 | 6.70 | 6.79 | -10.84 | 0.385 | 0.0197 | 0.0178 | 66.15 | 5.75 | -6. 29 | 0.477 |
| 11 | 0.021 | 10.4 | 0.0198 | 0.00712 | 7.39 | 7.49 | -10.14 | 0.425 | 0.0229 | 0.0208 | 65.06 | 6.29 | -5.74 | 0.523 |
| 12 | 0.024 | 11. 5 | 0.0227 | 0.00813 | 8.20 | 8.31 | -9.32 | 0.472 | 0.0262 | 0.0237 | 64.36 | 6.64 | -5.39 | 0.552 |
| 13 | 0.027 | 13.3 | 0.0255 | 0.00915 | 8.69 | 8.81 | -8.82 | 0.500 | 0.0295 | 0.0267 | 63.64 | 7.00 | -5.04 | 0.581 |
| 14 | 0.031 | 15.3 | 0.0293 | 0.01051 | 9.21 | 9.34 | -8.29 | 0.530 | 0.0339 | 0.0306 | 62.68 | 7.48 | -4.56 | 0.621 |
| 15 | 0.035 | 17.3 | 0.0330 | 0.01186 | 9.68 | 9.81 | -7.82 | 0.557 | 0.0382 | 0.0346 | 61.90 | 7.87 | -4.17 | 0.653 |
| 16 | 0.040 | 19.8 | 0.0378 | 0.01356 | 10.25 | 10.39 | -7.24 | 0.589 | 0.0437 | 0.0395 | 61.05 | 8.29 | -3.74 | 0.639 |
| 17 | C. 045 | 22.2 | 0.0425 | 0.01525 | 10.68 | 10.83 | -6.80 | 0.614 | 0.0492 | 0.0445 | 60.41 | 8.61 | -3.42 | 0.715 |
| 18 | 0.055 | 27.2 | 0.0519 | 0.01864 | 11.25 | 11.41 | -6.23 | 0.647 | 0.0601 | 0.0544 | 59.63 | 9.00 | -3.03 | 0.748 |
| 19 | 0.065 | 32.1 | 0.0614 | 0.02203 | 11.46 | 11.62 | -6.01 | 0.659 | 0.0710 | 0.0643 | 58.80 | 9.42 | -2.62 | 0.782 |
| 20 | 0.080 | 39.5 | 0.0755 | 0.02711 | 11.96 | 12.13 | -5.51 | 0.688 | 0.0874 | 0.0791 | 58.14 | 9.75 | -2.29 | 0.810 |
| 21 | $0 . C 55$ | 46.5 | 0.0897 | 0.03220 | 12.32 | 12.49 | -5.14 | 0.708 | 0.1038 | 0.0939 | 57.73 | 9.95 | -2.08 | 0.827 |
| 22 | 0.115 | 56.8 | 0.1086 | 0.03897 | 12.45 | 12.62 | -5.01 | 0.716 | 0.1256 | 0.1137 | 57.29 | 10.17 | -1.86 | 0.845 |
| 23 | 0.140 | 69.1 | 0.1322 | 0.04745 | 12.72 | 12.90 | -4.74 | 0.731 | 0.1530 | 0.1384 | 56.83 | 10.40 | -1.63 | 0.864 |
| 24 | 0.165 | 81.5 | 0.1558 | 0.05592 | 12.98 | 13.16 | -4.47 | . 0.746 | 0.1803 | 0.1631 | 56.58 | 10.53 | -1.51 | 0.874 |
| 25 | 0.215 | 106.2 | 0.2030 | 0.07286 | 13.29 | 13.48 | -4.16 | 0.764 | 0.2349 | 0.2125 | 56.20 | 10.72 | -1.32 | 0.890 |
| 26 | 0.315 | 155.5 | 0.2974 | 0.10675 | 13.91 | 14.10 | -3.53 | c. 800 | 0.3441 | 0.3114 | 55.57 | 11.03 | $-1.00$ | 0.916 |
| 27 | 0.415 | 204.5 | 0.3919 | 0.14064 | 14.40 | 14.60 | -3.03 | 0.828 | 0.4534 | 0.4103 | 55.11 | 11.26 | -0.78 | 0.935 |
| 28 | 0.515 | 254.3 | 0.4863 | 0.17453 | 14.94 | 15.15 | -2.48 | 0.859 | 0.5626 | 0.5091 | 54.71 | 11.46 | -0.58 | 0.952 |
| 29 | 0.615 | 303.7 | C. 5867 | 0.20842 | 15.52 | 15.74 | -1.90 | 0.892 | 0.6719 | 0.6080 | 54.45 | 11.59 | -0.45 | 0.962 |
| 30 | 0.765 | 377.7 | 0.7224 | 0.25926 | 16.18 | 16.41 | -1.23 | 0.930 | 0.8358 | 0.7563 | 54.09 | 11.77 | -0.27 | 0.977 |
| 31 | C. 515 | 451. $\varepsilon$ | c. 8640 | 0.31010 | 16.82 | 17.05 | -0.58 | 0.967 | 0.9957 | 0.9045 | 53.79 | 11.92 | -0.12 | 0.990 |
| 32 | 1.115 | 550.5 | 1.0528 | 0.37788 | 17.37 | 17.61 | -0.02 | 0.999 | 1.2182 | 1.1023 | 53.57 | 12.03 | -0.01 | 0.999 |
| 33 | 1.315 | 649.3 | 1.2417 | 0.44566 | 17.48 | 17.72 | 0.09 | 1.005 | 1.4367 | 1.3000 | 53.53 | 12.05 | 0.01 | 1.001 |
| 34 | 1.515 | 748.0 | 1.4305 | 0.51344 | 17.45 | 17.69 | 0.06 | 1.003 | 1.6552 | 1.4977 | 53.55 | 12.04 | -0.00 | 1.000 |
| 35 | 1.715 | 846.8 | 1.6194 | 0.58122 | 17.42 | 17.66 | 0.03 | 1.002 | 1.8737 | 1.6954 | 53.51 | 12.06 | 0.02 | 1.002 |
| 36 | 1.915 | 945.5 | 1.8953 | C. 64900 | .17.39 | 17.63 | 0.00 | 1.000 | 2.0922 | 1.8931 | 53.55 | 12.04 | -0.00 | 1.000 |

ADVERSE PRESSURE GRADIENT (-0.275.-0.004)

|  | $\begin{array}{rr} = & 31173-4 \\ = & 15 \end{array}$ |  | UINF <br> TWALL | 16 | K BETA | $=-0.494 \mathrm{E}-06$ |  | $C F / 2=0.00327$ |  | DELM |  | 1.243 | DELH | 0.914 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | $\text { TA }=$ | 0.248 |  |  |  | 1 M | $180$ | DEL2H | 0.0516 |
|  | 1 | 58. |  | DC | 3.1 | BF | $=$ | -1.254 | PPLUS | 0.00 |  | 2 M | 133 | REH | 470. |
| 21 | ) = | 0. | DEL 3 | 0.9 | F | $=-0$ | . 00410 | VOPLU | -0.67 |  | - | 11. | GH | . 0.061 |
|  | TS = | 35 | TGAS | 54 | BH | $=$ | -0.870 | REY. | $=0.574 \mathrm{E}$ | 6 H | $=$ | 56 | GF | 4.592 |
| I | $Y$ | YPLUS | Y/DELM | Y/OC | U | UPLUS | UDE | UBAR | Y/OELH | Y/DEL 3 | T | TPLUS | TDE | TBAR |
| 1 | 0.000 | 0.6 | 0.0000 | 0.00000 | 0.00 | 0.00 | -17.49 | 0.000 | 0.0000 | 0.0000 | 77.62 | 0.00 | -12.14 | 0.000 |
| 2 | 0.005 | 2. 4 | 0.0040 | 0.00159 | 3.01 | 3.13 | -14.36 | 0.179 | 0.0055 | 0.0052 | 72.62 | 0.00 2.76 | -12.14 -9.38 | $0.227$ |
| 3 | 0.066 | 2.9 | 0.0048 | 0.00190 | 3.01 | 3.13 | $-14.36$ | 0.179 | 0.0066 | 0.0062 | 72.09 | 2.84 | -9.30 | 0.234 |
| 4 | 0.007 | 3.4 | 0.0056 | 0.00222 | 3.26 | 3.39 | $-14.10$ | 0.194 | 0.0077 | 0.0072 | 71.65 | 3.07 | -9.07 | 0.253 |
| 5 | C. 008 | 3. 9 | 0.0064 | 0.00254 | 3.57 | 3.71 | -13.78 | 0.212 | 0.0088 | 0.0083 | 71.09 | 3.35 | -8.78 | 0.276 |
| 6 | 0.009 | 4.3 | 0.0072 | 0.00286 | 3.95 | 4.11 | $-13.38$ | 0.235 | 0.0099 | 0.0093 | 70.53 | 3.64 | -8.50 | 0.300 |
| 7 | 0.011 | 5.3 | 0.0088 | 0.00349 | 4.59 | 4.77 | -12.72 | 0.273 | 0.0120 | 0.0114 | 69.71 | 3.04 4.06 | -8.507 | 0.300 0.335 |
| 8 | 0.013 | 6.3 | 0.0105 | 0.00413 | 5.28 | 5.49 | -12.00 | 0.314 | 0.0142 | 0.0134 | 68.67 | 4.60 | -7.54 | 0.379 |
| 9 | 0.016 | 7.7 | 0.0129 | 0. 00568 | 6.06 | 6.30 | -11.19 | 0.360 | 0.0175 | 0.0165 | 67.47 | 5.21 | -6.92 | 0.379 |
| 10 | 0.019 | 9.1 | 0.0153 | 0.00603 | 7.00 | 7.28 | -10.21 | 0.416 | 0.0208 | 0.0196 | 66.19 | 5.87 | -6.27 | 0.484 |
| 11 | 0.022 | 10.6 | 0.0177 | 0.00698 | 7.71 | 8.02 | -9.47 | 0.458 | 0.0241 | 0.0227 | 65.21 | 6.38 | -5.76 | 0.525 |
| 12 | 0.025 | 12.0 | 0.0201 | 0.00794 | 8.13 | 8.45 | -9.03 | 0.483 | 0.0274 | 0.0258 | 64.29 | 6.85 | -5.29 | 0.564 |
| 13 | 0.028 | 13. 5 | 0.0225 | 0.00889 | 8.85 | 9.20 | -8.29 | 0.526 | $0.03 \mathrm{C6}$ | 0.0289 | 63.70 | 7.15 | -4.99 | 0.589 |
| 14 | 0.032 | 15.4 | 0.0257 | 0.01016 | 9.14 | 9.50 | -7.98 | 0.543 | 0.0350 | 0.0331 | 62.64 | 7.69 | -4.45 | 0.633 |
| 15 | C. 037 | 17. 8 | 0.0298 | 0.01175 | 9.90 | 10.29 | -7.19 | 0.589 | 0.0405 | 0.0382 | 62.22 | 7.91 | -4.23 | 0.651 |
| 16 | 0.042 | 20.2 | 0.0338 | 0.01333 | 10.40 | 10.81 | -6.67 | 0.618 | 0.0460 | 0.0434 | 61.12 | 8.47 | -3.66 | 0.698 |
| 17 | 0.052 | 25. C | 0.0418 | 0.01651 | 10.90 | 11.33 | -6.15 | 0.648 | 0.0569 | 0.0537 | 60.12 | 8.99 | -3.15 | 0.740 |
| 18 | 0.062 | 29.c | 0.0499 | 0.01968 | 11.23 | 11.68 | -5.81 | 0.668 | 0.0679 | 0.0641 | 59.30 | 9.41 | -2.73 | 0.775 |
| 19 | 0.072 | 34.7 | 0.0579 | 0.02286 | 11.69 | 12.15 | -5.33 | 0.695 | 0.0788 | 0.0744 | 58.92 | 9.60 | -2.53 | 0.791 |
| 20 | 0.087 | 41.5 | 0.0700 | 0.02762 | 11.92 | 12.39 | -5.09 | 0.709 | 0.0952 | 0.0899 | 58.26 | 9.94 | -2.19 | 0.819 |
| 21 | 0.1 C 2 | 49.1 | 0.0820 | 0.03238 | 12.30 | 12.79 | -4.70 | 0.731 | 0.1117 | 0.1054 | 57.86 |  |  |  |
| 22 | 0.122 | 56.7 | 0.0981 | 0.03873 | 12.54 | 13.04 | -4.45 | 0.746 | 0.1335 | 0.1261 | 57.86 | 10.15 10.36 | -1.99 -1.78 | 0.836 0.853 |
| 23 | 0.147 | 70.8 | 0.1182 | 0.04667 | 12.72 | 13.22 | -4.26 | 0.756 | 0.1609 | 0.1519 | 57.03 | 10.58 | -1.56 | 0.871 |
| 24 | 0.172 | 82.8 | 0.1384 | 0.05460 | 12.80 | 13.31 | -4.18 | 0.761 | 0.1883 | 0.1778 | 56.77 | 10.71 | -1.43 | 0.882 |
| 25 | 0.222 | 106.5 | 0.1786 | 0.07048 | 13.05 | 13.57 | -3.92 | 0.776 | 0.2430 | 0.2294 | 56.27 | 10.97 | -1.17 | 0.903 |
| 26 | 0.322 | 155.1 | 0.2590 | 0.10222 | 13.55 | 14.09 | -3.40 | 0.806 | 0.3525 | 0.3328 | 55.83 | 11.19 | -0.95 | 0.922 |
| 27 28 | 0.472 0.622 | 227.3 299.5 | 0.37¢7 | 0. 14984 | 14.15 | 14.71 | -2.78 | 0.841 | 0.5167 | 0.4878 | 55.19 | 11.52 | -0.62 | 0.949 |
| 29 | 0.772 | 371.8 | 0.6210 | 0.19746 0.24508 | 14.75 15.33 | 15.34 | -2.15 | 0.877 | 0.6809 | 0.6428 | 54.89 | 11.67 | -0.47 | 0.961 |
| 30 | 0.922 | 444.C | 0.7416 | 0.24508 0.29270 | 15.33 15.89 | 15.94 16.52 | -1.55 -0.97 | 0.911 0.945 | 0.8451 1.0093 | 0.7978 0.9528 | 54.45 54.20 | 11.90 12.03 | -0.24 -0.11 | 0.980 0.991 |
| 31 | 1.122 | 540.3 | 0.9025 | 0.35619 | 16.47 | 17.12 | -0.36 | 0.979 | 1.2282 | 1. 1595 | 54.06 | 12.10 | -0.04 |  |
| 32 | 1. 322 | 636.6 | 1.0634 | C. 41968 | 16.77 | 17.44 | -0.05 | 0.997 | 1.4471 | 1.3662 | 53.87 | 12. 20 | -0.04 0.06 | 0.997 1.005 |
| 33 | 1.522 | 732.5 | 1.2243 | 0.48318 | 16.82 | 17.49 | 0.00 | 1.000 | 1.6660 | 1.5729 | 53.81 | 12.23 | 0.09 | 1.007 |
| 34 | 1.722 | 829.2 | 1.3851 | 0.54667 | 16.83 | 17.50 | 0.01 | 1.001 | 1.8850 | 1.7796 | 53.88 | 12.19 | 0.05 | 1.004 |
| 35 | 1.922 | 925.5 | 1.5460 | 0.61016 | 16.82 | 17.49 | 0.00 | 1.000 | 2.1039 | 1.9863 | 53.98 | $12 \cdot 14$ | -0.00 | 1.000 |

ADVERSE PRESSURE GRADIENT (-0.275,-0.004)

| RUN | 31173-5 | UINF | = | 16.2 | K | $=-0.437 E-06$ | $C F / 2=0.00342$ | DELM | $=$ | 1.416 | DELH | 1.145 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PLATE = | 18 | TWALL | $=$ | 77.7 | BETA | $=0.212$ | $S T=0.00454$ | DELIM | $=$ | 0.189 | DEL2H | $=0.0671$ |
| $X(I N)=$ | 70. | DC | $=$ | 3.229 | BF | $=-1.194$ | PPLUS $=0.00218$ | DEL2M | = | 0.143 | REH | 591. |
| Z(IN) = | 0. | DEL3 | = | 1.229 | F | $=-0.00409$ | VOPLUS $=-0.07315$ | REM | $=$ | 1255. | GH | 2.733 |
| POINTS= | 35 | TGAS | = | 53.8 | BH | $=-0.901$ | REY. $\mathrm{NO}=0.660 \mathrm{O} 06$ | H | $=$ | 1.325 | GF | $=4.174$ |


| 1 | Y | YPLUS | Y/DELM | Y/OC | 4 | UPLUS | UDE | UBAR | Y/CELH | Y/DEL 3 | $T$ | TPLUS | TDE | TBAR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | C. 060 | 0.0 | 0.0000 | 0.00000 | 0.00 | 0.00 | -17.09 | 0.000 | 0.0000 | 0.0000 | 77.69 | 0.00 | -12.89 | 0.000 |
| 2 | 0.005 | 2.4 | 0.0035 | 0.30155 | 3.28 | 3.45 | -13.63 | 0.202 | 0.0044 | 0.0041 | 71.45 | 3.36 | -9.53 | 0.251 |
| 3 | 0.006 | 2.5 | 0.0042 | 0.00186 | 3.68 | 3.87 | -13.21 | 0.227 | 0.0052 | 0.0049 | 70.71 | 3.76 | -9.13 | 0.292 |
| 4 | $0.0 \mathrm{C7}$ | 3.3 | 0.0049 | 0.00217 | 4.03 | 4.24 | -12.84 | 0.248 | 0.0061 | 0.0057 | 70.22 | 4.02 | -8.86 | 0.312 |
| 5 | C.0C8 | 3. e | C. 0056 | 0.00248 | 4.32 | 4.55 | -12.54 | 0.266 | 0.0070 | 0.0065 | 69.82 | 4.24 | -8.65 | 0.329 |
| 6 | 0.010 | $4 . \varepsilon$ | 0.0071 | 0.00310 | 4.86 | 5.12 | -11.97 | 0.299 | 0.0087 | 0.0081 | 68.96 | 4.70 | -8.18 | 0.365 |
| 7 | 0.012 | 5.7 | 0.0085 | 0.00372 | 5.51 | 5. 80 | -11.29 | 0.339 | 0.0105 | 0.0098 | 68.07 | 5.18 | $-7.70$ | 0.432 |
| 8 | 0.014 | 6.7 | 0.0099 | 0. 30434 | 5.89 | 6.20 | -10.89 | 0.363 | 0.0122 | 0.0114 | 67.23 | 5.64 | -7.25 | 0.437 |
| 9 | 0.016 | 7.6 | 0.0113 | 0.00496 | 6.46 | 6.80 | -10.29 | 0.398 | 0.0140 | 0.0130 | 66.61 | 5.97 | -6.92 | 0.453 |
| 10 | 0.019 | 9. C | 0.0134 | 0. 00588 | 7.25 | 7.63 | -9.45 | 0.447 | 0.0166 | 0.0155 | 65.62 | 6.50 | -6.38 | 0.505 |
| 11 | 0.022 | 10.5 | 0.0155 | 0.00681 | 7.84 | 8.25 | -8.83 | 0.483 | 0.0192 | 0.0179 | 64.70 | 7.00 | -5.89 | 0.543 |
| 12 | 0.025 | 11.9 | 0.0177 | 0.00774 | 8.30 | 8.74 | -8.35 | 0.511 | 0.0218 | 0.0203 | 64.04 | 7.35 | -5.54 | 0.570 |
| 13 | 0.029 | 13.8 | 0.02 C 5 | 0.00898 | 8.76 | 9.22 | -7.86 | 0.540 | 0.0253 | 0.0236 | 63.12 | 7.85 | -5.04 | 0.609 |
| 14 | 0.033 | 15.7 | 0.0233 | 0.01022 | 9.32 | 9.81 | -7.28 | 0.574 | 0.0288 | 0.0269 | 62.62 | 8.12 | -4.77 | 0.630 |
| 15 | 6.038 | 18.1 | 0.0268 | 0.01177 | 9.81 | 10.33 | -6.76 | 0.604 | 0.0332 | 0.0309 | 61.76 | B. 58 | -4.31 | 0.666 |
| 16 | 0.043 | 20.4 | 0.0304 | 0.01332 | 10.29 | 10.83 | -6.25 | 0.634 | 0.0375 | 0.0350 | 61.10 | 8.94 | -3.95 | 0.693 |
| 17 | 0.048 | 22.E | 0.0339 | 0.01487 | 10.52 | 11.08 | -6.01 | 0.648 | 0.0419 | 0.0391 | 60.44 | 9.29 | -3.60 | 0.721 |
| 18 | 0.058 | 27.6 | 0.0410 | 0.01796 | 11.07 | 11.66 | -5.43 | 0.682 | 0.0506 | 0.0472 | 59.60 | 9.74 | -3.14 | 0.756 |
| 19 | 0.068 | 32.3 | 0.0480 | 0.02106 | 11.28 | 11.88 | -5. 21 | 0.695 | 0.0593 | 0.0553 | 58.92 | 10.11 | -2.78 | 0.784 |
| 20 | 0.683 | 39.5 | C. 0586 | 0.02570 | 11.57 | 12.18 | -4.91 | 0.713 | 0.0724 | 0.0676 | 58.22 | 10.49 | -2.40 | 0.814 |
| 21 | 0.058 | 46.6 | 0.0692 | 0.03035 | 11.79 | 12.41 | -4.67 | 0.726 | 0.0855 | 0.0798 | 57.79 | 10.72 | -2.17 | 0.832 |
| 22 | 0.118 | 56. 1 | 0.0833 | 0.03654 | 12.13 | 12.77 | -4.32 | 0.747 | 0.1030 | 0.0960 | 57.39 | 10.94 | -1.95 | 0.848 |
| 23 | 0.143 | 68.1 | 0.1010 | 0.04429 | 12.42 | 13.08 | -4.01 | 0.765 | 0.1248 | 0.1164 | 57.01 | 11.14 | -1.75 | 0.854 |
| 24 | C. 168 | 79.9 | 0.1186 | 0.05203 | 12.54 | 13.20 | -3.89 | 0.773 | 0.1466 | 0.1367 | 56.79 | 11.26 | -1.63 | 0.873 |
| 25 | C. 218 | 103.6 | 0.1539 | 0.06751 | 12.87 | 13.55 | -3.54 | 0.793 | 0.1902 | 0.1774 | 56.35 | 11.50 | -1.39 | 0.892 |
| 26 | 0.318 | 151.2 | 0.2245 | 0. 19848 | 13.14 | 13.83 | -3.25 | 0.810 | 0.2775 | 0.2588 | 55.85 | 11.77 | -1.12 | 0.913 |
| 27 | 0.418 | 198.7 | 0.2952 | 0.12945 | 13.55 | 14.27 | -2.82 | 0.835 | 0.3647 | 0.3402 | 55.41 | 12.00 | -0.88 | 0.931 |
| 28 | 0.568 | 270.0 | 0.4011 | 0.17551 | 13.97 | 14.71 | -2.38 | 0.861 | 0.4956 | 0.4623 | 55.04 | 12.20 | -0.68 | 0.947 |
| 29 | 0.718 | 341.4 | 0.5070 | 0.22236 | 14.38 | 15.14 | -1.95 | 0.886 | 0.6265 | 0.5844 | 54.81 | 12.32 | -0.57 | 0.956 |
| 30 | 0.918 | 436.4 | 0.6482 | 0. 28430 | 15.03 | 15.82 | -1.26 | 0.926 | 0.8010 | 0.7472 | 54.41 | 12. 54 | -0.35 | 0.973 |
| 31 | 1.118 | 531.5 | 0.7894 | 0.34624 | 15.59 | 16.41 | $-0.67$ | C.961 | 0.9755 | 0.9100 | 54.04 | 12.74 | -0.15 | 0.988 |
| 32 | 1.318 | 626.6 | 0.9306 | 0.40818 | 15.94 | 16.78 | -0.31 | +0.982 | 1.1500 | 1.0728 | 53.74 | 12.90 | 0.01 | 1. 001 |
| 33 | 1.518 | 721.7 | 1.0719 | 0.47012 | 16.20 | 17.06 | -0.03 | 0.998 | 1.3245 | 1.2356 | 53.76 | 12.89 | 0.00 | 1.000 |
| 34 | 1.718 | 816.8 | 1.2131 | 0.53206 | 16.25 | 17.11 | 0.02 | 1.001 | 1.4990 | 1.3983 | 53.76 | 12.89 | -0.00 | 1.000 |
| 35 | 1.918 | 911.9 | 1. 3543 | 0. 59400 | 16.23 | 17.09 | 0.00 | 1.000 | 1.6735 | 1.5611 | 53.76 | 12.89 | -0.00 | 1.000 |

ADVERSE PRESSURE GRADIENT (-0.275,-0.004)


## E-5. Reynolds stress tensor components (isothermal)

The runs are tabulated below and they were all taken at plate 18, x 70 in. See also 9.3 for symbol explanation

| Date | m | F |
| :---: | :---: | :---: |
| 062473 | 0 | 0 |
| 050973 | -0.275 | 0 |
| 051673 | -0.275 | -0.001 |
| 052373 | -0.275 | -0.002 |
| 052973 | -0.275 | -0.004 |

## Special Nomenclature

| Symbol | Explanation | Unit |
| :---: | :---: | :---: |
| UTAU | Friction velocity, $U_{\infty} \sqrt{C_{f} / 2}$ | ft/sec |
| DEL | Momentum boundary layer thickness, $\delta$ | in |
| $U^{\prime}$ | RMS value of longitudinal velocity fluctuation, $\sqrt{\mathrm{u}^{\prime 2}}$ | $\mathrm{ft} / \mathrm{sec}$ |
| $V^{\prime}$ | RMS value of normal velocity fluctuation, $\mathrm{v}^{\prime 2}$ | $\mathrm{ft} / \mathrm{sec}$ |
| W' | RMS value of tangential velocity fluctuation, $\sqrt{\mathrm{w}^{2}}$ | $\mathrm{ft} / \mathrm{sec}$ |
| Q | Turbulent kinetic energy, $u^{\prime 2}+v^{\prime 2}+w^{\prime 2}$ | $\mathrm{ft}^{2} / \mathrm{sec}^{2}$ |
| UV | Correlation between $u^{\prime}$ and $v^{\prime}$ (shear stress), $\overline{u^{\prime} v^{\prime}}$ | $\mathrm{ft}^{2} / \mathrm{sec}^{2}$ |
| RUV | Correlation coefficient, $-u^{\prime} v^{\prime} / \sqrt{u^{\prime 2}} \sqrt{v^{\prime 2}}$ | - |

REYNOL DS STRESS TENSOR COMPONENTS (ISOTHERMAL) FLAT PLATE (O.,O.)
$D A T E=62473$
UI NF $=31.60 \mathrm{FT} / \mathrm{SEC}$
CF/2=0.00181
UTAU $=1.344$

| Y | Y/DEL | YPLUS | U | UPLUS | U'/UTAU | v*/utau | W*/UTAU | Q/UTAU**2 | -UV/UTAU**2 | RUV | UV/Q |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.072 | 0.056 | 49.5 | 18.88 | 14.04 | 2.113 | 1.069 | 1.557 | 8.030 | 0.969 | 0.429 | 0.121 |
| 0.082 | 0.064 | 56.4 | 19.35 | 14.39 | 2.031 | 1.078 | 1.492 | 7.513 | 0.977 | 0.446 | 0.130 |
| 0.102 | 0.080 | 70. 1 | 19.82 | 14.74 | 1.971 | 1.116 | 1.495 | 7.367 | 0.969 | 0.441 | 0.132 |
| 0.132 | 0.103 | 90.7 | 20.69 | 15.39 | 1.897 | 1.133 | 1.469 | 7.041 | 0.977 | 0.455 | 0.139 |
| 0.182 | 0.143 | 125.1 | 21.41 | 15.93 | 1.860 | 1.048 | 1.382 | 6.467 | 0.915 | 0.469 | 0.142 |
| 0.282 | 0.221 | 193.8 | 23.13 | 17.20 | 1.726 | 1.181 | 1.407 | 6.352 | 0.907 | 0.445 | 0.143 |
| 0.382 | C. 299 | 262.6 | 23.79 | 17.70 | 1.651 | 1.086 | 1.317 | 5.641 | 0.820 | 0.457 | 0.145 |
| 0.582 | 0.456 | 400.0 | 26.74 | 19.89 | 1.517 | 1. 109 | 1. 298 | 5.218 | 0.752 | 0.447 | 0.144 |
| 0.782 | 0.613 | 537.5 | 27.78 | 20.66 | 1.272 | 0.992 | 1.143 | 3.908 | 0.572 | 0.453 | 0.146 |
| 1. 082 | 0.848 | 743.7 | 30.30 | 22.54 | 0.907 | 0.664 | 0.734 | 1.804 | 0.251 | 0.416 | 0.139 |
| 1. 282 | 1.005 | 881.1 | 31.23 | 23. 23 | 0.566 | 0.438 | 0.398 | 0.671 | 0.085 | 0.344 | 0.127 |

REYNDL DS STRESS TENSOR EOMPONENTS (I SUTHERMAL) ADVERSE PRESSURE GFADIENT (-1).275.0.)

```
DATE= 50973
UINF= 16.13 FT/SEC
CF/2=0.00081
UTAU= 0.0.459
```

| $\gamma$ | Y/DEL | YPLUS | U | UPLUS | U*/UTAU | vo/utau | W*/UTAU | Q/UTAU\#\#2 | -UV/UTAU**2 | RUV | UV/Q |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.069 | 0.019 | 16.2 | 3.72 | 8.10 | 3.652 | 1. 080 | 2.272 | 19.664 | 1.134 | 0.287 | 0.058 |
| 0.089 | 0.024 | 20.9 | 4.13 | 9.00 | 3.368 | 1.534 | 2.493 | 19.910 | 1.357 | 0.263 | 0.068 |
| 0.129 | 0.035 | 30.3 | 4.49 | 9.78 | 3.216 | 1.601 | 2.561 | 19.464 | 1.348 | 0.262 | 0.069 |
| 0.199 | 0.054 | 46.1 | 5.27 | 11.48 | 2.995 | 1.912 | 2.545 | 19.123 | 1.599 | 0.278 | 0.084 |
| 0.299 | 0.080 | 71.2 | 5.64 | 12.29 | 2.931 | 2. 034 | 2.652 | 19.759 | 1.727 | 0.290 | 0.087 |
| 0.499 | C. 134 | 117.1 | 6.12 | 13.33 | 3.057 | 2.330 | 2.716 | 22.155 | 2.510 | 0.352 | 0.113 |
| 0. 799 | 0.215 | 187.5 | 6.74 | 14.68 | 3.246 | 2.489 | 2.827 | 24.722 | 2.980 | 0.369 | 0.121 |
| 1. 209 | C. 325 | 283.7 | 7.95 | 17.32 | 3.451 | 2. 863 | 3.186 | 30.260 | 3.981 | 0.403 | 0.132 |
| 1.905 | 0.513 | 447.1 | 10.63 | 23.16 | 3.684 | 3.199 | 3.695 | 37.458 | 5.196 | 0.441 | 0.139 |
| 2.909 | 0.783 | 682.7 | 14.07 | 30.65 | 2.840 | 2.804 | 3.024 | 25.073 | 3.459 | 0.434 | 0.138 |

FEYNULES STRESS TEMSOR CIMPUNENTS (ISOTHERMALI AOVERSE YRESSUKE GRADIENT (-9.275.-0.001)

```
JATE= 51673
UNF= 1E.28 FT/SEC
CF/2= =1.0C116
UTAUN=0.554
```

| $\gamma$ | Y/OEL | yplus | U | UPLUS | U*/UTAU | V'/UTAU | W'/UTAU | Q/UTAU** 2 | -UV/UTAU**2 | RUV | UV/Q |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.065 | C. 025 | 19.7 | 0.35 | 11.45 | 2.788 | 0.655 | 1.740 | 11.231 | 0.755 | 0.413 | 0.067 |
| 0.089 | 0.033 | 25.4 | 6.74 | 12.16 | 2.693 | 3. 731 | 1.757 | 10.331 | 0.816 | 0.433 | 0.075 |
| 0.119 | C.044 | 33.9 | 7.30 | 13.17 | 2.614 | 0.946 | 1.738 | 50.747 | 1.005 | 0.407 | 0.094 |
| 0.159 | C. 059 | 45.4 | 7.67 | 13.83 | 2.479 | 1.166 | 1.829 | 10.851 | 1.383 | 0.375 | 0.100 |
| 0.239 | 0.077 | 59.6 | 7.99 | 14.41 | 2.372 | 1.328 | 1.891 | 10.965 | 1.197 | 3. 380 | 3. 109 |
| 0.309 | 0.114 | 88.1 | 8.56 | 15.44 | 2.365 | 1.454 | 1.983 | 11.641 | 1.311 | 0.381 | 0.113 |
| 0.409 | 0.151 | 116.7 | 8.90 | 10.05 | 2.420 | 1.535 | 1.972 | 12.100 | 1.454 | 0.392 | 0.120 |
| 0.600 | 0.225 | 173.7 | 7.44 | 17.03 | 2.531 | 1.613 | 2.070 | 13.293 | 1.734 | 0.425 | 0.130 |
| 0.909 | 0.335 | 259.3 | 10.56 | 19.34 | 2.663 | 1.900 | 2. 264 | 15.827 | 2.153 | 0.426 | 0.136 |
| 1.409 | -. 520 | 4 J .9 | 12.52 | 22.58 | 2.741 | 1.902 | 2.470 | 17.229 | 2.488 | 0.477 | 0.144 |
| 1.900 | 0.701 | 541.9 | 14.11 | 25.45 | 2.433 | 1.728 | 2.694 | :3. 313 | 1.935 | 0.460 | 0.145 |
| 2.409 | c. 888 | 687.1 | 15.51 | 27.97 | 1.653 | 1.318 | 1. 348 | 6.287 | 0.914 | 1). 420 | 0.145 |

REYNOL OS STRESS TENSOR COMPCINENTS (ISOTHERMAL) ACVERSE PRESSURE GRAMIENT (-O.275.-0.002)

## OATE $=52373$ UNF $=16.37 \mathrm{FT} / S E C$ <br> UFNF $=16.37$ <br> UTAU $=0.715$

| $Y$ | $\mathrm{Y} / \mathrm{O}=\mathrm{L}$ | YPLUS | U | UPLUS | W* /UTAU | VO/UTAU | W' / UTAJ | 6/JTAU**2 | -UV/UTAU**2 | PUV | UV/0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.669 | C. 033 | 25.4 | 8.07 | 11.28 | 2.383 | 0.673 | 1.398 | 6.746 | 1. 533 | 0.380 | 0.079 |
| 0.088 | 0.041 | 32.4 | 8.52 | 12.35 | 1.987 | 0.819 | 1.390 | 6.549 | 0.529 | U. 326 | 0.081 |
| 0.128 | C.060 | 47.1 | 9.18 | 12.83 | 1.817 | 0.851 | 1.419 | 6.041 | 0.569 | 9.368 | 0.094 |
| 0.198 | 0.093 | 72.9 | 9.60 | 13.42 | 1.671 | 1.030 | 1.304 | 5.795 | 0.608 | 0.353 | 0.105 |
| 0.298 | C. 139 | 199.7 | 10.17 | 14.22 | 1.642 | 1.115 | 1.437 | 6.004 | 0.717 | 1. 392 | 0.119 |
| 0.398 | C. 186 | 146.5 | 10.63 | 14.86 | 1.660 | 1.211 | 1.472 | 6.389 | 0.835 | 0.413 | 0.130 |
| 3. 598 | C. 280 | 220.1 | 11.42 | 15.96 | 1.718 | 1.342 | 1.568 | 7.211 | 0.961 | 0.417 | 0.133 |
| 0.798 | 0.373 | 293.7 | 12.21 | 17.07 | 1.751 | 1. 341 | 1.641. | 7.561 | 1.324 | 0.436 | 0.135 |
| 9. 978 | 0.467 | 367.3 | 12.30 | 17.87 | 1.768 | 1.337 | 1.623 | 7.549 | 1.055 | 0.446 | 0.140 |
| 1. 248 | C. 584 | 459.3 | 13.52 | 18.913 | 1.718 | 1. 248 | 1.543 | 6.887 | 1.002 | 0.468 | 0.146 |
| 1.498 | 0.701 | 551.3 | 14.49 | 20.25 | 1.569 | 1.131 | 1. 436 | 5.920 | 0.879 | 1).474 | 0.149 |
| 1.912 | C. 890 | 700.0) | 15.60 | 21.81 | 1.153 | 0.924 | 0.975 | 3.134 | 0.422 | 0.396 | 0.135 |

REYNOLES STRESS TEASOR C JAPOMENTS (ISDTHEPMAL) ADVERSE PRESSURE GRANIEAT (-0.275,-0.004)

## DATF $=52973$ <br> UINF $=16.54 \mathrm{FT} / \mathrm{SEC}$ <br> C.F/2=0.0C342

JTAU $=0.967$

| Y | Y/DEL | YPLUS | J | UPLUS | j*/utau | vorutau | W'/ UTAU | Q/UTAU**2 | -UV/UTAU**2 | PuV | $U \vee / Q^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.065 | 0.049 | 34.1 | 11.38 | 11.77 | 1.414 | 0.498 | 0.913 | 3.080 | 0.215 | 0.305 | 0.070 |
| 0.087 | 0.063 | 44.0 | 11.79 | 12.19 | 1.279 | 0.537 | 9.885 | 2.7 Cl | 0.200 | 0.291 | 0.074 |
| 0.119 | C.084 | 53.8 | 12.33 | 12.75 | 1.128 | 0.646 | 9.896 | 2.492 | 0.238 | 0.327 | 0.096 |
| 0.149 | C. 105 | 73.7 | 12.40 | 12.82 | 1.049 | 0.550 | 0. 846 | 2.119 | 0.215 | 0.372 | 0.101 |
| 0.199 | 0.141 | 98.4 | 12.64 | 13.07 | 0.953 | 0.671 | 0. 855 | 2.090 | 0.249 | 0.389 | 0.119 |
| 0.299 | O. 211 | 147.9 | 13.04 | 13.48 | 0.907 | 0.6 .31 | 0.807 | 1.873 | 0.228 | 0.398 | 0.122 |
| 0.399 | 0.282 | 197.3 | 13.40 | 13.85 | 0.877 | 0.6889 | 0.826 | 1.927 | 0.262 | 0.433 | 0.136 |
| $0.59 \%$ | C. 423 | 296.2 | 14.15 | 14.6 .3 | 0.871 | 0.668 | 7. 793 | 1.842 | 0.261 | 0.448 | 0.142 |
| 0.799 | C. 564 | 395.1 | 14.71 | 15.21 | 2. 834 | -0.623 | 0. 738 | 1.627 | 0.243 | 0.467 | 0.149 |
| J. 999 | O. 706 | 494.0 | 15.37 | 15.89 | 0.746 | 0.566 | 0.655 | 1.305 | 0.193 | 0.458 | 0.148 |
| 1.299 | C.917 | 642.4 | 16.19 | $16.7 / 4$ | 0.496 | 0.440 | 7.434 | 0.627 | 0.087 | 0.397 | 0.138 |

## E-6. Velocity and temperature fluctuation profiles

The runs are tabulated below and they were all taken at plate 18, $x=70$ in. See also 9.3 for symbol explanation.

| Date | m | E |
| :---: | :---: | :---: |
| 041473 | 0 | 0 |
| 050273 | -0.275 | 0 |
| 051773 | -0.275 | -0.001 |
| 052173 | -0.275 | -0.002 |
| 060373 | -0.275 | -0.004 |

It should be noted that when the calibration curves are not reliable (low velocities), no tabulation is provided. This justifies the blanks in the tables.

Special Nomenclature

| Symbol | Explanation | Unit |
| :--- | :--- | :---: |
| TW-TINF | Wall to free stream temperature differences | F |
| STANTON | Stanton number | - |
| UTAU | Friction velocity, $U_{\infty} \sqrt{\mathrm{C}_{f^{\prime}} / 2}$ | $\mathrm{ft} / \mathrm{sec}$ |
| TTAU | $\left(\mathrm{T}_{\mathrm{o}}-\mathrm{T}_{\infty}\right) \sqrt{\mathrm{C}_{\mathrm{f}} / 2} / \mathrm{ST}$ | ${ }^{\circ} \mathrm{F}$ |
| $\mathrm{U}^{\prime}$ | RMS value of longitudinal velocity fluctuation, |  |
|  | $\sqrt{\overline{\mathrm{u}^{\prime 2}}}$ | $\mathrm{ft} / \mathrm{sec}$ |
| $T^{\prime}$ | RMS value of temperature fluctuation, $\sqrt{\mathrm{t}^{\prime 2}}$ | ${ }^{\circ} \mathrm{F}$ |
| $T^{\prime} \mathrm{RAW}$ | Raw value of $T^{\prime}$ | ${ }^{\circ} \mathrm{F}$ |
| R-FUNC | Eq 7.17 | - |

velocity and temperature fluctuation profiles

## flat plate (0.,0.) plate 18



VELOCITY AND TEMPEPATURE FLUCTUATION PROFILES ADVERSE PRESSURE GRADIENT (-0.275,0.) PLATE 18

| DATE | $=50273$ |
| ---: | :--- |
| UINF | $=16.13 \mathrm{FT} / \mathrm{SEC}$ |
| TWTTINF | $=31.0 \mathrm{~F}$ |
| CF/2 | $=0.00081$ |
| STANTON | $=0.00217$ |
| UTAU | $=0.459$ |
| TTAU | $=2.364$ |


| $Y$ | Y/DELM | Y/DELH | yplus | $u$ ' | U./UTAU | TPRAW | T' | T'/TTAU | R-FUNC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.006 | 0.002 | 0.002 | 1.3 |  |  | 0.63 | 0.78 | 0.33 |  |
| 0.011 | 0.203 | 0.003 | 2.4 |  |  | 0.99 | 1.22 | 0.52 |  |
| 0.016 | 0.304 | 0.004 | 3.4 |  |  | 1.52 | 1.87 | 0.79 |  |
| 0.021 | 0.006 | 0.006 | 4.5 |  |  | 2.10 | 2.57 | 1.09 |  |
| 0.026 | 0.007 | 0.007 | 5.6 |  |  | 2.39 | 2.91 | 1.23 |  |
| 0.031 | 0.008 | 0.009 | 6.6 |  |  | 2.63 | 3.19 | 1.35 |  |
| 0.036 | 0.010 | 0.010 | 7.7 |  |  | 2.80 | 3.38 | 1.43 |  |
| 0.045 | 0.012 | 0.013 | 9.6 |  |  | 2.99 | 3.59 | 1.52 |  |
| 0.055 | 0.015 | 0.015 | 11.8 | 1.74 | 3.79 | 3.06 | 3.69 | 1.56 | 1.77 |
| 0.065 | 0. 217 | 0.018 | 13.9 | 1.69 | 3.68 | 3.05 | 3.67 | 1.55 | 1.73 |
| 0.085 | 0.023 | 0.024 | 18.2 | 1.55 | 3.38 | 2.95 | 3.54 | 1.50 | 1.65 |
| 0.118 | 0.032 | 0.033 | 25.3 | 1.50 | 3.27 | 2.72 | 3.25 | 1.38 | 1.73 |
| 0.148 | C. 1340 | 0.041 | 31.7 | 1.42 | 3.09 | 2.56 | 3.06 | 1.29 | 1.75 |
| 0.198 | 0.053 | 0.055 | 42.5 | 1.38 | 3.01 | 2.31 | 2.75 | 1.16 | 1.88 |
| 0.248 | 0.067 | 0.069 | 53.2 | 1.35 | 2.94 | 2.14 | 2.55 | 1.08 | 1.99 |
| 0.298 | 0.080 | 0.083 | 83.9 | 1.35 | 2.94 | 1.95 | 2.32 | 0.98 | 2.19 |
| 0.398 | 0.107 | 0.111 | 85.3 | 1.38 | 3.01 | 1.77 | 2.10 | 0.89 | 2.47 |
| 0.498 | 0.134 | 0.139 | 106.8 | 1.40 | 3.05 | 1.62 | 1.92 | 0.81 | 2.74 |
| 0.698 | 0.188 | 0.195 | 149.7 | 1.47 | 3.20 | 1.44 | 1.70 | 0.72 | 3.25 |
| 0.998 | 0.268 | 0.278 | 214.0 | 1.54 | 3.35 | 1.27 | 1.50 | 0.64 | 3.86 |
| 1.498 | 0.403 | 0.418 | 321.2 | 1.65 | 3.59 | 1.10 | 1.29 | 0.55 | 4.81 |
| 1.920 | 0.517 | 0.535 | 411.7 | 1.69 | 3.68 | 0.99 | 1.16 | 0.49 | 5.50 |
| 2.420 | 0.651 | 0.675 | 518.9 | 1.54 | 3.35 | 0.92 | 1.07 | 0.45 | 5.41 |
| 2.920 | 0.786 | 0.814 | 626.2 | 1.29 | 2.81 | 0.82 | 0.95 | 0.40 | 5.10 |
| 3.420 | 0.920 | 0.954 | 733.4 | 0.80 | 1.74 | 0.66 | 0.76 | 0.32 | 3.94 |
| 3.920 | 1.055 | 1.093 | 840.6 | 0.36 | 0.78 | 0.37 | 0.43 | 0.18 | 3.17 |

VELOCITY AND TEMPERATURE FLUCTUATION PROFILES
ADYERSE PRESSURE GRADIENT $(-0.275,-0.001)$ PLATE 18

## OATE $=51773$

UINF $=16.28 \mathrm{FT} / \mathrm{SEC}$
TW-TINF $=25.6 \mathrm{~F}$
$\mathrm{CF} / 2=0.00116$
STANTON $=0.00272$
UTAU $=0.554$
TTAU $=2.044$

| $Y$ | Y/OELM | Y/DELH | YPLUS | U. | U./UTAU | T.RAW | T. | T./TTAU R-FIUNC |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.006 | 0.002 | 0.002 | 1.6 |  |  |  | 0.52 | 0.64 | 0.31 |
| 0.008 | 0.003 | 0.003 | 2.1 |  |  | 0.64 | 0.78 | 0.38 |  |
| 0.011 | 0.004 | 0.004 | 2.9 |  |  | 0.92 | 1.11 | 0.54 |  |
| 0.014 | 0.005 | 0.005 | 3.7 |  |  | 1.15 | 1.38 | 0.68 |  |
| 0.018 | 0.007 | 0.007 | 4.8 |  |  | 1.42 | 1.70 | 0.83 |  |
| 0.023 | 0.008 | 0.009 | 6.1 | 1.56 | 2.81 | 1.64 | 1.98 | 0.97 | 2.12 |
| 0.028 | 0.010 | 0.011 | 7.4 | 1.64 | 2.96 | 1.90 | 2.28 | 1.12 | 1.94 |
| 0.038 | 0.014 | 0.015 | 10.0 | 1.65 | 2.98 | 2.35 | 2.80 | 1.37 | 1.59 |
| 0.048 | 0.018 | 0.019 | 12.7 | 1.64 | 2.96 | 2.34 | 2.77 | 1.36 | 1.59 |
| 0.058 | 0.021 | 0.022 | 15.3 | 1.61 | 2.90 | 2.36 | 2.79 | 1.37 | 1.55 |
| 0.069 | 0.025 | 0.027 | 18.2 | 1.55 | 2.80 | 2.30 | 2.72 | 1.33 | 1.54 |
| 0.089 | 0.033 | 0.034 | 23.5 | 1.49 | 2.69 | 2.16 | 2.55 | 1.25 | 1.58 |
| 0.119 | 0.044 | 0.046 | 31.4 | 1.45 | 2.62 | 1.99 | 2.34 | 1.14 | 1.87 |
| 0.159 | 0.059 | 0.062 | 42.0 | 1.37 | 2.47 | 1.76 | 2.07 | 1.01 | 1.78 |
| 0.209 | 0.077 | 0.081 | 55.2 | 1.32 | 2.38 | 1.55 | 1.82 | 0.89 | 1.95 |
| 0.309 | 0.114 | 0.120 | 81.6 | 1.31 | 2.36 | 1.32 | 1.55 | 0.76 | 2.28 |
| 0.409 | 0.151 | 0.158 | 108.0 | 1.34 | 2.42 | 1.18 | 1.38 | 0.68 | 2.61 |
| 0.609 | 0.225 | 0.236 | 160.8 | 1.41 | 2.54 | 1.01 | 1.18 | 0.58 | 3.22 |
| 0.909 | 0.335 | 0.352 | 240.0 | 1.48 | 2.67 | 0.87 | 1.01 | 0.50 | 3.93 |
| 1.159 | 0.427 | 0.449 | 306.0 | 1.55 | 2.80 | 0.78 | 0.91 | 0.44 | 4.61 |
| 1.409 | 0.520 | 0.545 | 372.0 | 1.52 | 2.74 | 0.71 | 0.82 | 0.40 | 4.97 |
| 1.909 | 0.704 | 0.739 | 504.0 | 1.36 | 2.45 | 0.59 | 0.68 | 0.33 | 5.37 |
| 2.409 | 0.888 | 0.933 | 636.1 | 0.92 | 1.66 | 0.47 | 0.54 | 0.27 | 4.57 |
| 2.909 | 1.073 | 1.126 | 768.1 | 0.37 | 0.67 | 0.26 | 0.30 | 0.15 | 3.33 |

VELOCITY AND TEMPERATURE FLUCTUATION PRIFFILES ADVERSE ORESSURE GRADIENT ( $\mathbf{- 0 . 2 7 5 ; - 0 . 0 0 2 1}$ PLATE 18
DATE $=52173$
UINF $=16.37 \mathrm{FT} / \mathrm{SEC}$
TW-TINF $=23.8 \mathrm{~F}$
CF/2 $=0.00191$
STANTON $=0.00325$
UTAU $=0.715$
TTAU $=1.779$

| $Y$ | Y/DELM | Y/DELH | YPLUS | U' | U'/UTAU | T'RAW | T* | T'/TTAU | R-FUNC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.006 | 0.003 | 0.003 | 2.0 |  |  | 0.89 | 1.08 | 0.61 |  |
| 0.010 | 6. 0.005 | 0.004 | 3.4 |  |  | 0.92 | 1.11 | 0.63 |  |
| 0.014 | 0.007 | 0.006 | 4.8 | 1. 35 | 1.89 | 1.20 | 1.45 | 0.82 | 1.68 |
| 0.018 | 0.008 | 0.008 | 6.1 | 1.42 | 1.98 | 1.47 | 1.76 | 1.00 | 1.45 |
| 0.023 | 0.011 | 0.010 | 7.8 | 1.51 | 2.11 | 1.73 | 2.06 | 1.17 | 1.32 |
| 0.028 | 0.013 | 4.012 | 9.5 | 1. 53 | 2.14 | 1.85 | 2. 20 | 1.24 | 1.26 |
| 0.038 | 0.018 | 0.016 | 12.9 | 1.60 | 2.24 | 2.06 | 2.43 | 1.38 | 1.19 |
| 0.048 | 0.022 | 0.021 | 16.4 | 1.61 | 2.25 | 2.08 | 2.45 | 1.38 | 1.19 |
| 4.069 | 0.032 | 0.030 | 23.5 | 1.47 | 2.08 | 1.97 | 2.31 | 1.31 | 1.16 |
| 0.088 | 0.041 | 0.038 | 30.0 | 1.42 | 1.98 | 1.83 | 2.14 | 1.21 | 1.20 |
| 0.128 | 0.060 | 3.055 | 43.6 | 1.30 | 1.82 | 1.61 | 1.88 | 1.06 | 1.25 |
| 0.198 | 0.093 | 0.085 | 67.5 | 1. 20 | 1.68 | 1.31 | 1.53 | 0.86 | 1.42 |
| 0.298 | 0.139 | 0.128 | 101.5 | 1.17 | 1.64 | 1.09 | 1.27 | 0.72 | 1.66 |
| 0.398 | 0.186 | 0.171 | 135.6 | 1. 19 | 1.66 | 0.95 | 1.11 | 0.63 | 1.94 |
| 0.598 | 0.280 | 0.257 | 203.7 | 1. 23 | 1.72 | 0.81 | 0.94 | 0.53 | 2.36 |
| 0.798 | 0.373 | 0.343 | 271.9 | 1.25 | 1.75 | 0.72 | 0.84 | 0.47 | 2.70 |
| 0.999 | 0.467 | 0.429 | 340.0 | 1.27 | 1.78 | 0.65 | 0.75 | 0.43 | 3.04 |
| 1. 248 | 0.584 | 0.537 | 425.2 | 1.23 | 1.72 | 0.57 | 0.66 | 0.37 | 3.37 |
| 1.498 | 0.701 | 0.644 | 510.3 | 1.12 | 1.57 | 0.51 | 0.59 | 0.33 | 3.43 |
| 1.920 | 0.898 | 0.826 | 654.1 | 0.81 | 1.13 | 0.41 | 0.47 | 0.27 | 3.09 |
| 2.428 | 1.136 | 1.044 | 827.2 | 0. 30 | 0.42 | 0.21 | 0.24 | 0.14 | 2.24 |

VELOCITY AND TEMPERATURE FLUCTUATION PROFILES

```
ADVERSE PRESSURE GRADIENT (-D. \(275,-0.004\) ) PLATE 18
DATE = 60373
UINF = 16.54 FT/SEC
TW-TINF = 23.0 F
CF/2 =0.00342
STANTON=0.00454
UTAU = 0.967
TTAU = 1.786
```

| Y | Y/DELM | Y/DELH | YPLUS | $U$. | $U$ U/UTAU | T.RAW | T. | T./TTAU R-FUNC |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.006 | 0.004 | 0.005 | 2.7 | 0.92 | 0.95 | 0.65 | 0.78 | 0.44 | 1.59 |
| 0.010 | 0.007 | 0.009 | 4.6 | 1.09 | 1.13 | 0.91 | 1.09 | 0.61 | 1.35 |
| 0.014 | 0.010 | 0.012 | 6.4 | 1.25 | 1.29 | 1.16 | 1.38 | 0.77 | 1.22 |
| 0.018 | 0.013 | 0.016 | 8.2 | 1.39 | 1.44 | 1.38 | 1.63 | 0.91 | 1.15 |
| 0.023 | 0.016 | 0.020 | 10.5 | 1.51 | 1.56 | 1.61 | 1.89 | 1.05 | 1.07 |
| 0.028 | 0.020 | 0.024 | 12.8 | 1.57 | 1.62 | 1.70 | 1.99 | 1.12 | 1.06 |
| 0.038 | 0.027 | 0.033 | 17.4 | 1.56 | 1.61 | 1.81 | 2.12 | 1.18 | 0.99 |
| 0.048 | 0.034 | 0.042 | 22.0 | 1.53 | 1.58 | 1.78 | 2.08 | 1.16 | 0.99 |
| 0.069 | 0.049 | 0.060 | 31.6 | 1.37 | 1.42 | 1.63 | 1.90 | 1.05 | 0.97 |
| 0.089 | 0.063 | 0.078 | 40.8 | 1.24 | 1.28 | 1.44 | 1.67 | 0.94 | 1.00 |
| 0.119 | 0.084 | 0.104 | 54.5 | 1.09 | 1.13 | 1.24 | 1.44 | 0.81 | 1.02 |
| 0.149 | 0.105 | 0.130 | 68.2 | 1.01 | 1.04 | 1.08 | 1.25 | 0.70 | 1.69 |
| 0.199 | 0.141 | 0.174 | 91.1 | 0.92 | 0.95 | 0.89 | 1.03 | 0.58 | 1.20 |
| 0.299 | 0.211 | 0.261 | 136.9 | 0.88 | 0.91 | 0.70 | 0.81 | 0.45 | 1.46 |
| 0.399 | 0.282 | 0.348 | 182.7 | 0.85 | 0.88 | 0.59 | 0.68 | 0.38 | 1.68 |
| 0.599 | 0.423 | 0.523 | 274.3 | 0.84 | 0.87 | 0.47 | 0.54 | 0.30 | 2.08 |
| 0.799 | 0.564 | 0.697 | 365.9 | 0.81 | 0.84 | 0.39 | 0.45 | 0.25 | 2.42 |
| 0.999 | 0.706 | 0.872 | 457.5 | 0.72 | 0.74 | 0.33 | 0.38 | 0.21 | 2.55 |
| 1.299 | 0.917 | 1.134 | 594.9 | 0.48 | 0.50 | 0.24 | 0.28 | 0.15 | 2.34 |
| 1.499 | 1.359 | 1.308 | 686.5 | 0.30 | 0.31 | 0.17 | 0.20 | 0.11 | 2.06 |

## E-7. Turbulent Prandt1 numbers

The runs are tabulated below and they were all taken at plate 18, $\times 70 \mathrm{in}$. See also 9.5 for symbol explanation.

| Date | $\frac{m}{0}$ | $\frac{\mathrm{~F}}{0}$ |
| :---: | :---: | :---: |
| 060473 | 0 | 0 |
| 050873 | -0.275 | -0.001 |
| 051573 | -0.275 | -0.002 |
| 052273 | -0.275 | -0.004 |

## Special Nomenclature

| Symbol | Explanation | Unit |
| :---: | :---: | :---: |
| VT | Normal velocity-temperature correlation, $\overline{v^{\top} t^{\top}}$ | ft. F/sec |
| VTPLUS | $\overline{v^{\prime} t^{\top} / u_{\tau} \tau_{t}}$ |  |
| PRT | Turbulent Prandtl number | - |
| VT/V'T' |  | - |
|  | Square root of the turbulent kinetic energy, $Q$ | $\mathrm{ft} / \mathrm{sec}$ |

TURBUL ENT PRANDTL NUMBER
FLAT PLATE (O.,0.) PLATE 18
DATE $=60473$
UINF $=31.60 \mathrm{FT} / \mathrm{SEC}$
TW-TINF $=24.8 \mathrm{~F}$
$C F / 2=0.00181$
STANTON $=0.30200$
UTAU $=1.344$
TTAU $=2.166$

| $Y$ | Y/OELA | Y/DELH | YPLUS | VTPLUS | PRT | VT/V'T' | $V T / G^{\prime \prime}$ | UV | V T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.072 | 0.056 | 0.051 | 45.7 | 0.931 | C. 95 | 0.612 | 0.231 | $-1.750$ | 1.459 |
| 0.082 | 0.064 | 0.058 | 52.0 | 0.963 | 0.93 | 0.643 | 0.253 | $-1.750$ | 1. 510 |
| 0.102 | 0.380 | 0.072 | 64.7 | 0.886 | 1.01 | 0.605 | 0.249 | -1.750 | 1.388 |
| 0.132 | 0.103 | 0.093 | 83.7 | 0.904 | 0.99 | 0.646 | 0.276 | -1.750 | 1.417 |
| 0.182 | 0.143 | 0.128 | 115.5 | 0.928 | 0.88 | 0.765 | 0.315 | -1.736 | 2.455 |
| 0.282 | 0.221 | 0.159 | 178.9 | 0.797 | 0.97 | 0.635 | 0.297 | -1.660 | 1.249 |
| 0.382 | 0.299 | 0.270 | 242.3 | 0.803 | 0.85 | 0. 731 | 0.334 | -1.570 | 1.259 |
| 0.582 | 0.456 | 0.411 | 369.2 | 0.634 | 0.95 | 0.623 | 0.302 | $-1.360$ | 0.993 |
| 0.782 | 0.613 | 0.552 | 496.1 | 0.511 | 6.88 | 0.626 | 0.314 | -1.030 | 0.801 |
| 1.082 | 0.848 | 0.764 | 686.4 | 0.327 | C. 61 | 0.843 | 0.417 | -0.450 | 0.512 |
| 1.282 | 1.005 | 0.905 | 813.3 | 0.125 | c. 54 | 0.555 | 0.297 | $-0.150$ | 0.196 |

## TURBULENT PRANDTL NUMBER

ADVERSF PRESSURE GRADIENT(-0.275.0.) PLATE 18
DATE = 50873
UINF $=16.13 \mathrm{FT} / \mathrm{SEC}$
TWTINF $=31.0 \mathrm{~F}$
CF/2 $=0.01081$
STANTON $=0.00217$
UTAU $=0.459$
TTAU $=2.364$

| Y | Y/HEL 1 | $Y / \mathrm{O}=\mathrm{LH}$ | YPLUS | VTPLUS | PRT | VT/V'T* | VT/O'T' | UV | $V T$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.089 | 0.1524 | 0.025 | 19.1 | 0.315 | 2.28 | 0.139 | 0.048 | -0. 255 | 0.342 |
| 0.129 | 0.035 | 0.036 | 27.7 | 0.417 | 1.84 | 0.193 | 0.070 | -0.290 | 0.452 |
| 0.199 | 0.054 | 0.055 | 42.7 | 0.594 | 1.32 | 0.266 | 0.117 | -0.340 | 0.645 |
| 0.299 | 5.083 | 0. 083 | 6.4 .1 | 0.785 | 1.09 | 0.393 | 0.18 u | -0.390 | 0.852 |
| 0.499 | 0.134 | 0.139 | 107.0 | 1.047 | 0.81 | 0. 553 | 0.274 | -0.485 | 1.136 |
| 0.797 | 0.215 | 0.223 | 171.3 | 0.968 | 6. 92 | 0.564 | 0.282 | -0.630 | 1.050 |
| 1.209 | 0.325 | 0.337 | 259.3 | 1.104 | 0.86 | 0.646 | C. 335 | -0.84C | 1.198 |
| 1.905 | 0.51 .3 | 0.531 | 408.5 | 1.030 | 0.66 | 0.656 | U. 343 | -1.095 | 1.118 |
| 2.909 | 0.783 | 0.811 | 623.8 | 0.974 | 0.33 | 0.864 | 0.484 | -0.730 | 1.057 |
| TURBULENT PRANDTL NJMBER. |  |  |  |  |  |  |  |  |  |
| OATE $=51573$ (EA |  |  |  |  |  |  |  |  |  |
| UIAF $=16.28 \mathrm{FT} / \mathrm{SEC}$ |  | FT/SEC |  |  |  |  |  |  |  |
| TW-TTNF $=25.6$ - |  |  |  |  |  |  |  |  |  |
| CFI2 = 2.00116 |  |  |  |  |  |  |  |  |  |
| STANTON=0.0027? |  |  |  |  |  |  |  |  |  |
| UTAU $=0.554$ |  |  |  |  |  |  |  |  |  |
| TTAU | 2.044 |  |  |  |  |  |  |  |  |


| $Y$ | Y/DEL 1 | Y/DELH | YPLUS | VTPLUS | PRT | VT/V'T* | VT/Q'T' | UV | VT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.069 | 0.1325 | 0.027 | 18.2 | 0.315 | 1.32 | 0. 361 | 0.071 | -0.260 | 0.357 |
| 0.089 | 0.033 | 0.034 | 23.5 | 0.403 | 1.10 | 0.461 | 0.098 | -0.280 | 0.457 |
| 0.119 | 3.744 | 0.046 | 31.4 | 0.514 | 0.88 | 0.475 | 0.137 | -0.300 | 0.583 |
| 0.159 | 0.059 | 0.062 | 42.0 | 0.428 | 1.11 | 0.362 | 0.128 | -0.330 | 0.485 |
| 0.209 | 0.077 | 0.081 | 55.2 | 0.410 | 1.23 | 0.347 | 0.139 | -0.360 | 0.465 |
| 0.369 | 9.114 | 0.120 | 31.6 | 0.485 | 1.11 | 0.440 | 0.188 | -0.400 | 0.550 |
| 0.409 | 0.151 | 0.158 | 108.0 | C. 532 | 1.10 | 0. 514 | 0.227 | -0. 0.450 | 0.603 |
| 0.609 | 0.225 | C. 236 | 160.3 | 0.581 | 1.08 | 0.624 | 0.276 | -0.530 | 0.659 |
| 0.907 | 0.335 | 13.352 | 240.0 | 0.593 | 1.18 | 0.632 | 0.302 | -0.660 | 0.672 |
| 1.409 | 0.520 | 0.545 | 372.0 | 0.602 | 1.06 | 0.789 | 0.361 | -0.760 | 0.682 |
| 1.903 | 0.791 | 0.736 | 501.7 | 0.525 | C. 81 | 0.913 | 0.433 | -0.600 | 0.595 |
| 2.409 | 0.888 | 0.933 | 636.1 | 0.326 | 0.54 | 0.935 | 0.491 | -0.280 | 0.36 |

TURBULLENT PRANOTL NUMBER
ACVERSE PRESSURE GRADIENTI-0.275.-0.002) PLATE 18
DATE $=52273$
UTNF $=16.37 \mathrm{FT} / \mathrm{SEC}$
TW-TINF $=23.8 \mathrm{~F}$
CF/2 $=0.00191$
STANTDN=0.00325
UTAU $=0.715$
TTAU $=1.770$

| $Y$ | Y/DEL 1 | Y/OELH | YPLUS | VTPLUS | PRT | VT/V'T' | $V T / 0^{\prime} T^{\prime}$ | UV | VT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.069 | 0.032 | 0.030 | 23.5 | 0.274 | 1.26 | 0.312 | 0.081 | -0.262 | 0.347 |
| 0.088 | 0.041 | 0.038 | 30.0 | 0.374 | 0.93 | 0.377 | 0.121 | -0.271 | 0.473 |
| 0.12 H | 0.360 | 0.055 | 43.6 | 0.490 | 0.71 | 0.541 | 0.188 | -0.290 | 0.620 |
| 0.198 | 0.093 | 0.085 | 67.5 | 0.466 | 0.82 | 0.523 | 0.224 | -0.322 | 0.590 |
| 0.298 | 0.139 | 0.128 | 101.5 | 0.405 | 1.04 | 0.507 | C. 230 | -0.367 | 0.513 |
| 0.398 | 0.186 | 0.171 | 135.6 | 0.340 | 1.28 | 0.448 | 0.215 | -0.410 | 0.431 |
| 0.598 | 0.280 | 0.257 | 203.7 | 0.355 | 1.35 | 0.499 | 0.249 | -0.492 | 0.450 |
| 0.798 | 0.373 | 0.343 | 271.9 | 0.359 | 1.28 | 0.504 | 0.275 | -0.524 | 0.455 |
| 0.998 | 0.467 | 0.429 | 340.0 | 0.389 | 1.20 | 0.686 | C. 334 | -0.540 | 0.492 |
| 1.248 | 0.584 | 0.537 | 425.2 | 0.385 | L. 98 | 0.828 | 0.394 | -0.513 | 0.488 |
| 1.498 | 0.701 | 0.644 | 510.3 | 0.318 | C. 97 | 0.808 | 0.392 | -0.450 | 0.403 |
| 1.902 | 0.890 | ) . 818 | 648.0 | 0.213 | 0.65 | 0.869 | 0.454 | -0.216 | 0.270 |

TURBULENT PRANDTL MUMBER.
ADVERSE PPESSURE GFADIENT(-0.275.-0.304) PLATE 18
DATE $=53173$
UINF $=16.54 \mathrm{FT} / \mathrm{SEC}$
TW-TINF $=23.0 \mathrm{~F}$
C.F/2 $=0.00342$

STAN:TJM $=0.00454$
UTAU $=0.967$
$T T A U=1.786$

| Y | Y/OEL 1 | Y/DES LH | YPLUS | VTPLUS | PRT | VT/V'T' | VT/Q'T' | UV | VT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.069 | 0.049 | 0.060 | 31.6 | 0.140 | 1. 21 | 0.264 | 0.075 | -0.201 | 0.242 |
| 0.783 | 1. 063 | 0.078 | 40.8 | 0.150 | 1.17 | 0.258 | 0.097 | -0.211 | 0.259 |
| 0.119 | 0.084 | 0.104 | 54.5 | 0.166 | 1.07 | 0.319 | 0.131 | -0.220 | 0.287 |
| 0.149 | 0.105 | 0.130 | 68.2 | 0.151 | 1.21 | 0.391 | 0.148 | -0.227 | 0.260 |
| 0.199 | 0.141 | 0.174 | 91.1 | 0.151 | 1.24 | 0.389 | 0.181 | -0.235 | 0.260 |
| 0.299 | 0.211 | 0.261 | 136.9 | 0.150 | 1.25 | 0.524 | 0.242 | -0.242 | 0.259 |
| ). 399 | 0.282 | 0.343 | 182.7 | 0.157 | 1.18 | 0.600 | C. 298 | -0.245 | 0.272 |
| 0.599 | 0.423 | 0.523 | 274.3 | 0.175 | 1.02 | 0.869 | 0.427 | -0.244 | 0.303 |
| 0.769 | 0.504 | 0.657 | 365.9 | 0.138 | 1.18 | 0.878 | C. 429 | -0.22.6 | 0.238 |
| 0.999 | 0.706 | 1.872 | 457.5 | 0.116 | 1.13 | 0.961 | 0.476 | -0.186 | 0.200 |
| 1.299 | 0.917 | 1.134 | 594.9 | 0.069 | 0.82 | 0.999 | 0.555 | -0.081 | 0.119 |

