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INTERIM REPORT

on

Contract NAS8-25562

THRESHOLD DETECTION IN AN ON-OFF BINARY COMMUNICATIONS CHANNEL WITH ATMOSPHERIC SCINTILLATION

by

William E. Webb
Principal Investigator

and

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Research Assistant

Prepared for

National Aeronautics and Space Administration
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ABSTRACT

The optimum detection threshold in an on-off binary optical communications system operating in the presence of atmospheric turbulence has been investigated assuming a poisson detection process and log normal scintillation. The dependence of the probability of bit error on log amplitude variance and received signal strength has been analyzed and semi-emperical relationships to predict the optimum detection threshold derived. On the basis of this analysis a piecewise linear model for an adaptive threshold detection system is presented. The bit error probabilities for non-optimum threshold detection systems have also been investigated.

INTRODUCTION

It is well known that atmospheric scintillation not only increases the bit error probability in a pulse code modulated optical communications channel but also influences the decision level for optimum threshold detection. The later effect is of considerable practical importance to the design of efficient pulse code modulated optical communications systems regardless of whether fixed or adaptive threshold detection is used. Fried and Schmelzer[1] have considered the bit error rates in an optical communications channel by assuming gaussian detection statistics, an approximation which is valid only for large numbers of signal photons. Titterton and Speck[2] have treated the problem using Poisson statistics so that their results are valid for small numbers of signal photons also. Although both previous investigators have recognized that the optimum detection threshold changes in the presence of scintillation, neither has studied the effect in depth.

In this paper we investigate the optimum detection threshold in an on-off binary optical communications channel as a function of the number of received photoelectrons and the strength of scintillation. Poisson detection statistics and log normal scintillation are assumed and, on the basis of this model, expressions to predict the optimum detection threshold are derived. In addition, we have investigated the bit error probabilities for sub-optimum choices of detection threshold. This is of importance to the design and analysis of optical communications systems since exact optimization of the threshold level is never possible, especially in the presence of atmospheric effects. Application of these results to the design of fixed and adaptive threshold optical receivers is discussed.

ERROR PROBABILITY IN THE OPTICAL CHANNEL.

We shall consider an on-off binary optical communications channel in which the laser transmitter may send either a '1', which corresponds to a pulse being transmitted, or a '0' which corresponds to no pulse transmitted. Let S be the number of signal photoelectrons per pulse received at the detector when a '1' is transmitted and εS be the number of received signal photoelectrons when a '0' is transmitted. Here ε is the reciprocal of the modulator extinction ratio. We will assume that in addition to the signal photoelectrons the detector receives N noise photoelectrons per pulse, mostly due to background. In a threshold detection system the receiver interprets the received signal as either a '1' or a '0' depending on whether the total number of received photoelectrons is greater or less than some threshold level T.

In the presence of scintillation the probability of a detection error is

$$P_E = P(0)P_{FA} + P(1)(1-P_D) \quad (1)$$

Where P(0) and P(1) are the apriori probabilities of sending '0' and a '1' respectively, P_{FA} is the false alarm probability, ie. the probability of a '1' being received given that a '0' was sent, and P_D is the detection probability, ie. the probability of a '1' being received given that a '1' was sent. P_D and P_{FA} are given by

$$P_{FA} = \int_0^\infty \sum_{j=t}^{\infty} \exp\left[-(N+\varepsilon S)\right] \frac{(N+\varepsilon S)^j}{j!} f(S)dS \tag{2}$$

$$P_D = \int_0^\infty \sum_{j=t}^{\infty} \exp\left[-(N+S)\right] \frac{(N+S)^j}{j!} f(S)dS \quad . \tag{3}$$

Assuming log normal scintillation and a symmetrical pulse code we obtain

$$P_E = \frac{1}{8C_\ell} + \left(\frac{e^{C_\ell 2T(T-1)}}{4\sqrt{2\pi C_\ell}}\right)\left(-(S_0)^T\int_{-\infty}^{\infty} e^{-8C_\ell Z_1^2}\,\gamma^*(T, S_0 e^{(Z_1+4C_\ell(T-1/2))})dZ_1\right.$$

$$\left. + ((S_0-N)\varepsilon+N)^T\int_{-\infty}^{\infty} e^{-8C_\ell Z_0^2}\,\gamma^*(T,((S_0-N)\varepsilon+N)e^{(Z_0+4C_\ell(T-1/2))})dZ_0\right) \tag{4}$$

where γ^* is the incomplete gamma function

$$\gamma^*(T,n) = \frac{1}{n^T}\sum_{X=T}^{\infty} \frac{e^{-n}n^X}{X!} \tag{5}$$

S_0 is the average number of signal photoelectrons per pulse and C_ℓ is the variance of ln(S).

The bit error probability has been evaluated by numerical integration of equation 4 on a UNIVAC 1108 computer. The threshold T was initially taken to be the no scintillation optimum threshold given by 3

$$T_0 = \frac{S(1-\varepsilon) + \ln(P(0)/P(1))}{\ln((S+N)/(N+\varepsilon S))} \quad . \tag{6}$$

An iterative procedure was then used to minimize the error probability as a function of T, while holding S_0, N, ε and C_ℓ constant. In this way we were able to simultaneously determine the optimum threshold in the presence of scintillation and study the effect of sub-optimum threshold on the error rates. Calculations were repeated for values of the parameters S_0, ε, N and C_ℓ over the ranges likely to be encountered in practice.

Figure 1 shows the probability of a bit error at optimum threshold as a function of the number of received signal photoelectrons and the log amplitude variance of scintillation with one background photoelectron and an extinction ratio of 15 db. The computed values of bit error probability agree with those previously published by Titterton and Speck[2].

One feature of figure 1 which deserves note is the difference in the slope of the curves for large and small values of the log amplitude variance. When C_ℓ is less than about 0.02 the error probability decreases rapidly with increasing number of signal photoelectrons, whereas for large values of C_ℓ the error probability tends to become relatively independent of the number of signal photoelectrons provided that S_0 is not too small. Thus there appears to be two regions, one in which the error probability is determined primarily by scintillation and the other in which it is mainly determined by signal strength. This dicotomy, which we might call the signal limited and scintillation limited cases, will be seen more clearly when we consider the optimum detection threshold in the presence of scintillation.

An alternate method of displaying these results is in terms of the transmitted power required to achieve a given bit error rate in the presence of scintillation as compared to the power required when there is no scintillation. In figure 2 the required power margin is shown as a function of bit error rate and log amplitude variance. These results are essentially independent of the choice of N and ϵ as long as both are relatively small. Fried and Schmeltzer[1] have developed a similar set of curves based on the assumption of gaussian detection statistics. Comparison of our results with those of Fried and Schmeltzer shows that the two models agree reasonably well in predicting the required power margin for values of C_ℓ less than about 0.02, but that the gaussian model may underestimate the necessary power margin by as much as 8 db for C_ℓ equal to 0.03 and 30 db for C_ℓ equal to .05 at error rates of 10^{-6}. For a log amplitude variance of 0.1 the difference in the power margins predicted by the gaussian and Poisson models may exceed 80 db. Thus the gaussian model is reasonably accurate in the case of weak scintillation but gives an extremely over-optimistic estimate of a communications system performance in the presence of strong scintillation.

Since the Poisson distribution approaches a gaussian distribution as the mean increases, it might be expected that our results should reduce to those of Fried and Schmeltzer in the case of a large average number of received signal photoelectrons. However, for the system under consideration a large value of S_0 is not sufficient to insure that the Poisson counting statistics can be adequately approximated by a gaussian distribution. The means of the Poisson distributions in question are not S_0 but $(N + S)$ and $(N + \varepsilon S)$, where the variable S is averaged over all values from zero to infinity. Thus if N is small, the mean of the Poisson distribution will be small over part of the range of S, even though S_0 is large. Clearly, when C_ℓ is large the contribution to the error probability from that part of the range of S in which the gaussian approximation is invalid will be greater. Physically this means that during a deep fade there are only a few photoelectrons per pulse; therefore gaussian statistics can not be used. A second, and perhaps more significant, problem with Fried and Schmeltzer's model is the assumption of additive noise independent of the signal strength. This assumption is equivalent to approximating the Poisson distribution by a gaussian distribution with constant variance. Since the variance of a Poisson distribution is equal to the mean, this assumption is valid only if

$$N + S \approx N + \varepsilon S \approx \text{constant} \qquad (7)$$

for all values of S with non negligible probability of occurance. In particular, unless N is very large compared to S_0, the noise associated with a zero being received is different from the noise associated with a one. Therefore Fried and Schmeltzer's analysis can only be valid if the number of noise photoelectrons per pulse is much greater than the number of signal photoelectrons. In this case the two analyses should yield the same results.

As a check on our results we have calculated the error probabilities assuming fourty noise photoelectrons per pulse, log amplitude variances of 0.0 and 0.05 and error probabilities from 10^{-1} to 3×10^{-3}. For these cases the computed power margin required to compensate for scintillation agreed with that reported by Fried and Schmeltzer to within one db. The residual error is due to the fact that even for fourth noise photoelectrons the condition of equation 7 is only approximately satisfied.

On the basis of our analysis we conclude that Fried and Schmeltzer's results contain a fundamental inconsistancy; namely, the number of signal photoelectrons required to obtain error probabilities less than about 10^{-3} is so large that equation 7 can not be satisfied unless the log amplitude variance is very small. Thus the assumption of background limited operation is invalid except for the case of weak scintillation or very high bit error rates.

OPTIMUM THRESHOLD

The optimum threshold in the presence of scintillation is shown in figure 3 as a function of the average number of received signal photoelectrons per pulse. As is to be expected the optimum threshold decreases with increasing scintillation. For S_0 greater than about 10 the curves are very nearly straight lines whose slopes are dependent on the strength of the scintillation. This linearity can be understood in the no scintillation case by noting that when εS is large compared to N equation 6 reduces to

$$T = \frac{1-\varepsilon}{\ln(1/\varepsilon)} S. \tag{8}$$

With scintillation we may replace equation 8 by a linear relation of the form

$$T = \alpha(C_\ell)S_0 + \beta \tag{9}$$

where the parameters α and β can be determined by linear least mean square fits to the data of figure 3. To a reasonable approximation we may take β to be 2.5, independent of C_ℓ. The slope α, on the other hand, is strongly dependent on C_ℓ as shown in figure 4. The dependence of the slope on C_ℓ can be represented by the relation

$$\alpha(C_\ell) \approx .145(e^{7.5X10^3 C_\ell{}^2} + 1) \tag{10}$$

Perhaps the most striking feature of figure 4 is the distinct knee that occurs for values of C_ℓ near 0.02, and corresponds to the transition from signal strength limited to scintillation limited conditions. For C_ℓ much larger than 0.02 the slope of the threshold curve varies only slowly with increasing C_ℓ whereas for weak scintillation the slope is a very strong function of C_ℓ. In fact we can approximate α fairly well in these two regions by straight lines as shown by the dashed curves in figures 4. Hence

$$T_0 = a_i C_\ell S_0 + b_i S_0 + C \tag{11}$$

where the coefficients a_i and b_i take on either one of two values depending upon whether C_ℓ is greater or less than the break point. A piecewise linear approximation of this sort is appealing from an engineering point of view since it provides a convenient model for implementing an adaptive threshold detection system. One could envision, for example, a detection system in which the mean and variance of the received signal power were continuously monitored and the threshold set in accordance with equation 11. Such a system would require only linear operations and a single discrete discontinuity to control the threshold level.

As Titterton and Speck have pointed out (footnote 4 of reference 2) this analysis is applicable to systems in which the threshold is varied on the basis of a long term estimate (ie. on the order of seconds) of scintillation and is limited to systems with bit rates of 10^7 or less. Tycz, Fitzmaurice and Premo [5] have considered a perfectly adaptive threshold system for both the log normal and beta channels.

SUB-OPTIMUM THRESHOLDS

In practice the detection threshold of an optical communications system operating through the atmosphere will never be perfectly optimized. If the threshold is fixed then changing atmospheric conditions will deoptimize the system and even if an adaptive threshold is used the system will be incapable of precisely tracking changes in signal strength and turbulence. In order to properly predict a communication system's performance it is necessary to know the expected bit error probabilities for non-optimum detection thresholds. Investigation of the performance of non-optimum threshold detection is also necessary to the design of adaptive threshold systems.

Figure 5 shows the bit error probability as a function of detection threshold with log amplitude variance as a parameter. The data plotted is for the case of 40 signal photoelectrons and no background photoelectrons per pulse and an extinction ratio of 15 db. Other choices of parameters yielded curves which were essentially similar.

Inspection of figure 5 shows that with a fixed detection threshold a decrease in scintillation will always result in an improvement in system performance even though decreasing the scintillation deoptimizes the system. That is to say that the bit error probability decreases more rapidly with decreasing log amplitude variance than it is increased by the corresponding deoptimization of the threshold level. Thus if one selects the detection threshold that is optimum for the strongest expected scintillation and predicts the error rate on this basis, one is assured that the system performance will not be worse under conditions of weaker scintillation. This strategy might be appropriate if one wishes to insure that a given error rate will be obtained under all expected conditions.

ACKNOWLEDGMENT

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REFERENCES

1. D. L. Fried and R. A. Schmeltzer, Appl. Opt. 6, p 1729-1737, 1967.

2. P. J. Titterton and J. P. Speck, Appl. Opt. 12, p 425-426, 1973.

3. This corresponds to Titterton and Speck's (op. cit.) equation 5. However these authors' expression contains a typographical error in that two pairs of parenthesis in the denominator were omitted.

4. M. Tycz, M. W. Fitzmaurice and D. A. Premo, IEEE Trans. on Communications, p 1069-1071, 1973.

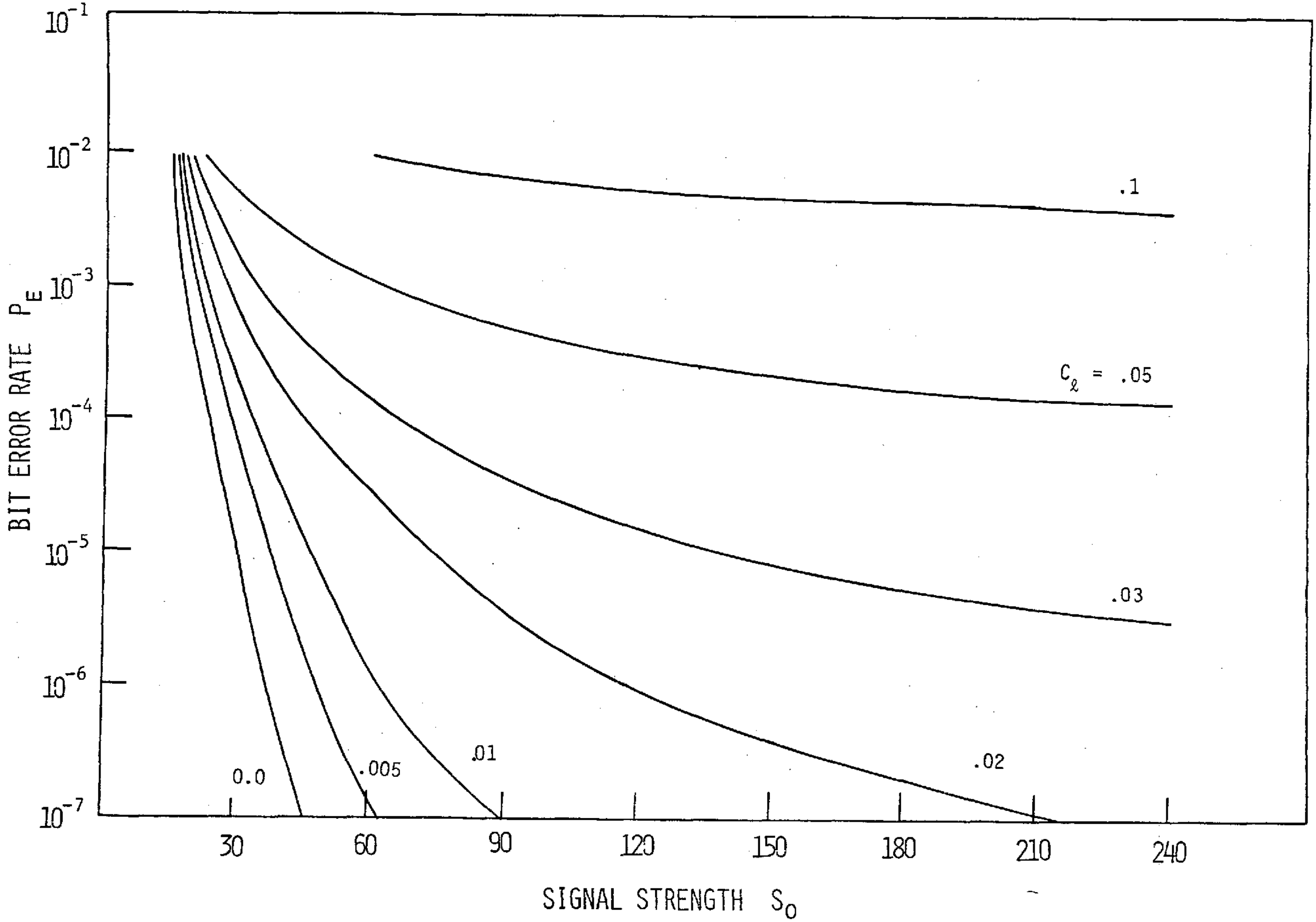
Figure 1. Probability of bit error for an on-off binary PCM optical communications system as a function of the number of signal photoelectrons for log amplitude variances between 0,0 and 0.1. One background photoelectron per pulse and a modulation extinction ratio of 15 db. was assumed.

Figure 2. The Loss Factor, defined as the additional transmitted power required to compensate for atmospheric scintillation, is given as a function of the number of received photoelectrons per pulse and the log-amplitude variance of scintillation. Other parameters are the same as the preceeding figure.

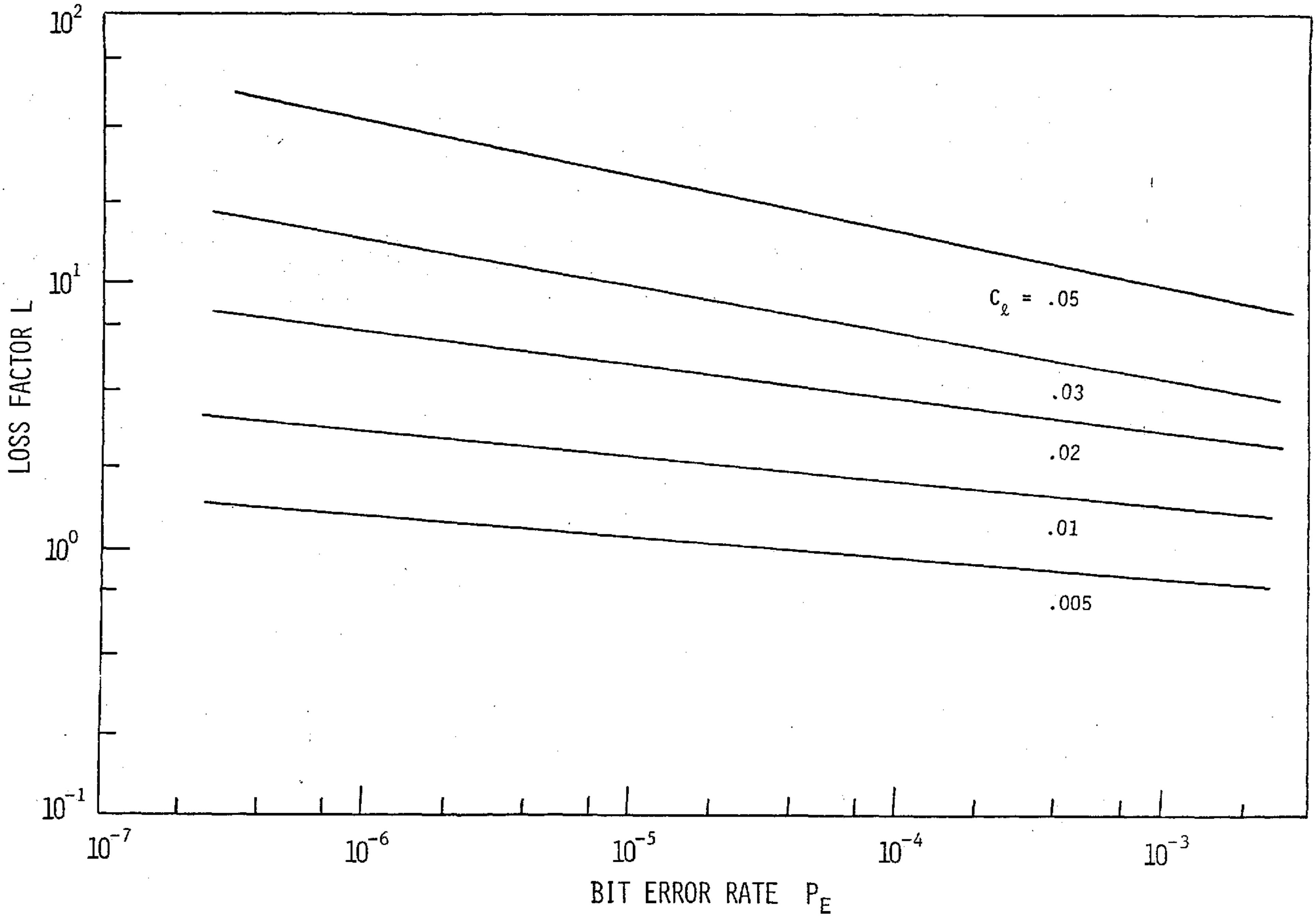
Figure 3. The optimum detection threshold as a function of number of signal photoelectrons per pulse and log amplitude variance of scintillation. Other parameters are the same as in the preceeding figures.

Figure 4. The slope of the linear portion of the detection threshold curves (figure 3) as a function of the log amplitude variance of scintillation. The dashed lines indicate a piecewise linear approximation discussed in the text.

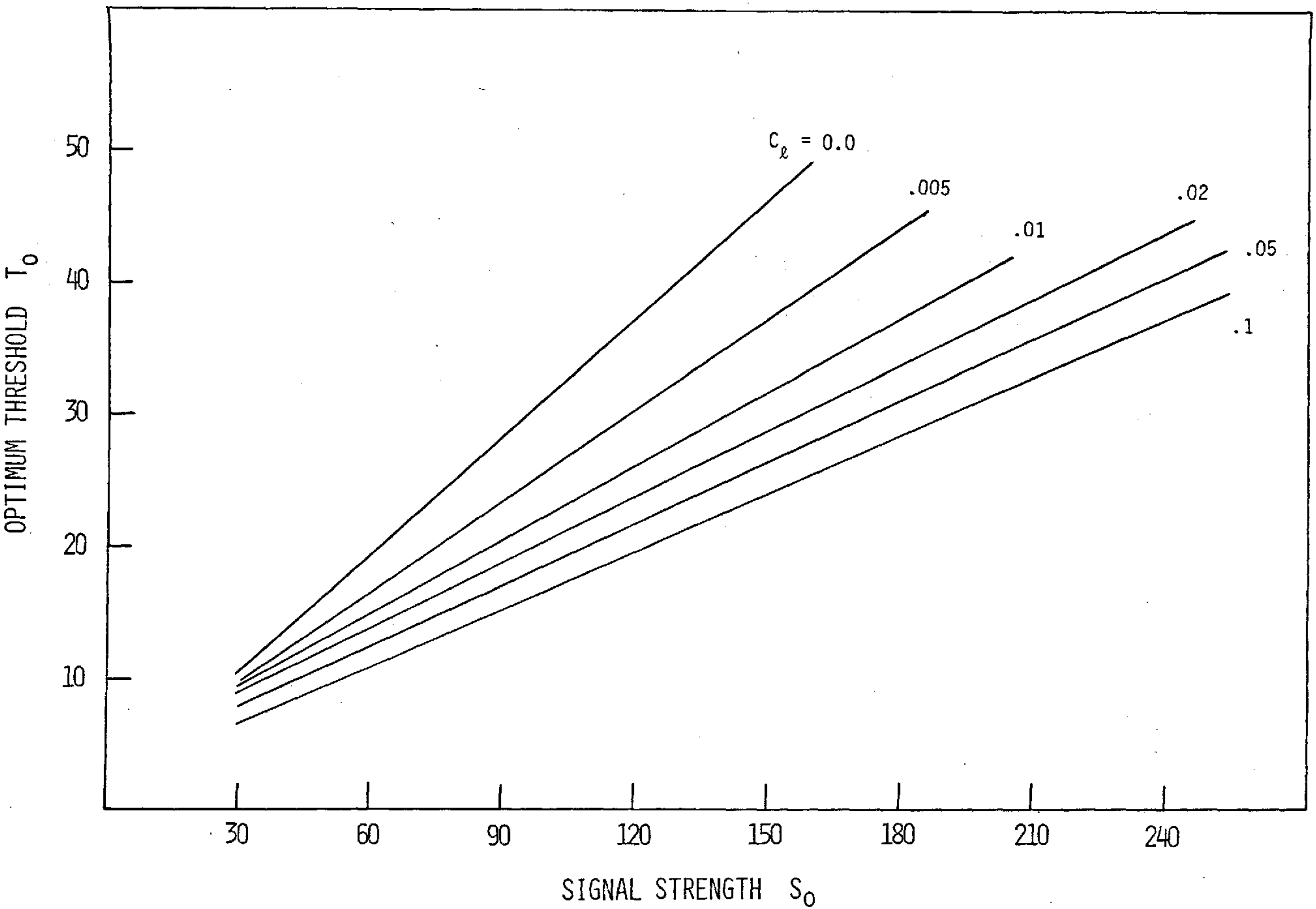
Figure 5. Dependence of the bit error probability on detection threshold for non-optimum detection. Error probability is plotted as a function of the normalized threshold, T/S_0, for $S_0 = 40$.



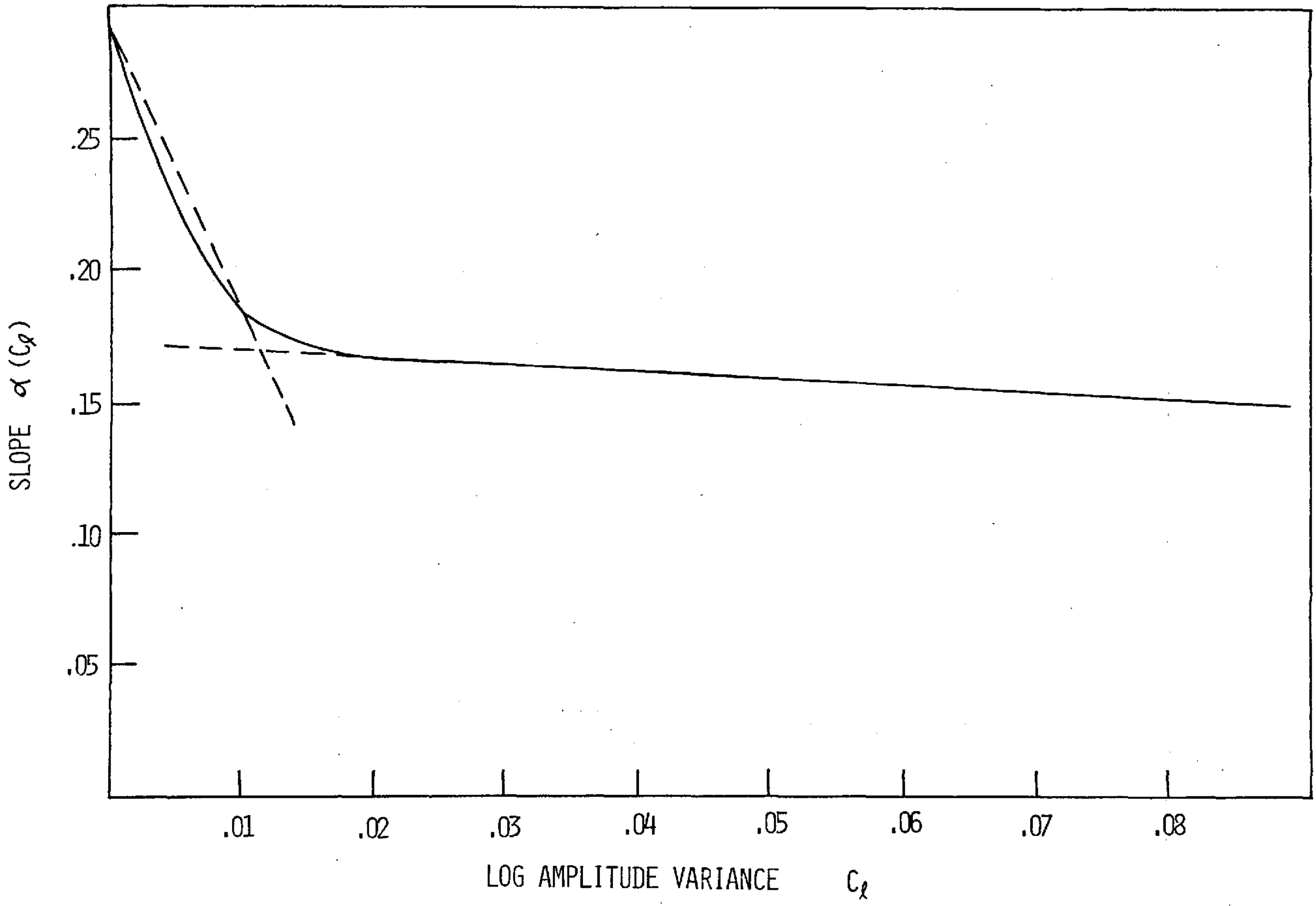
BIT ERROR RATE P_E
SIGNAL STRENGTH S_0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
30
60
90
120
150
180
210
240
.1
C_ℓ = .05
.03
.02
.01
.005
0.0

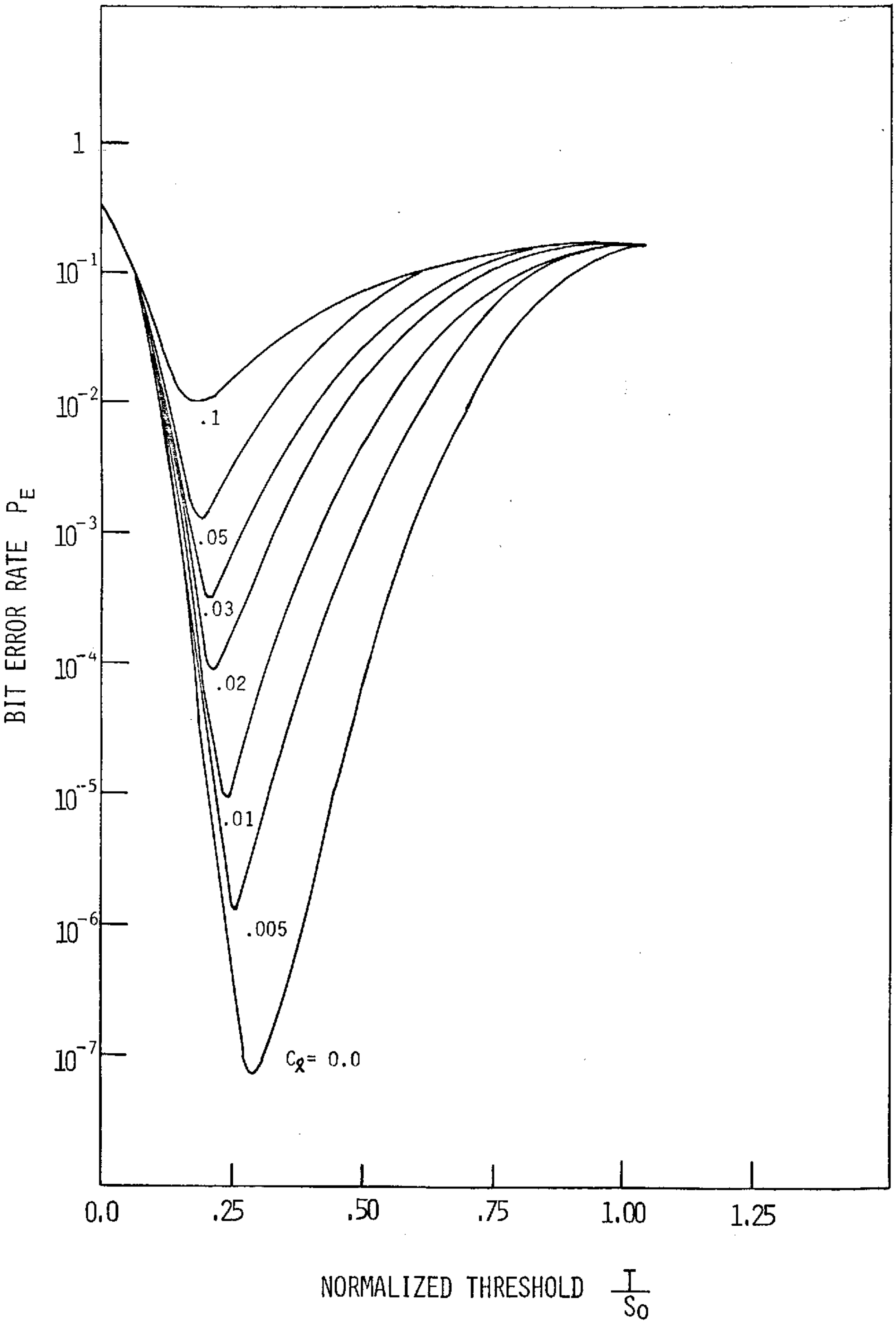


10^2
10^1
10^0
10^{-1}
LOSS FACTOR L
C_ℓ = .05
.03
.02
.01
.005
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
BIT ERROR RATE P_E



OPTIMUM THRESHOLD T_O
SIGNAL STRENGTH S_O
C_ℓ = 0.0
.005
.01
.02
.05
.1
50
40
30
20
10
30
60
90
120
150
180
210
240


.25
.20
.15
.10
.05
SLOPE $\alpha(C_\ell)$
.01
.02
.03
.04
.05
.06
.07
.08
LOG AMPLITUDE VARIANCE C_ℓ


BIT ERROR RATE P_E
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
.1
.05
.03
.02
.01
.005
C_ℓ = 0.0
0.0
.25
.50
.75
1.00
1.25
NORMALIZED THRESHOLD $\frac{T}{S_0}$