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GREENBELT, MARYLAND

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OF EMISSION MEASURE IN SOLAR X-RAY PLASMAS

I - Non-Flaring Active Regions

By

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ABSTRACT

X-ray and ultra-violet line emission from hot, optically thin material forming coronal active regions on the sun may be described in terms of an emission measure distribution function, $\emptyset(T) = n_e^2\ dV/dT$. We develop the relationship between line flux and $\emptyset(T)$, a theory which assumes that the electron density is a single-valued function of temperature. The sources of error involved in deriving $\emptyset(T)$ from a set of line fluxes are examined in some detail. These include errors in atomic data (collisional excitation rates, assessment of other mechanisms for populating excited states of transitions, element abundances, ion concentrations, oscillator strengths) and errors in observed line fluxes arising from, e.g., poorly - known instrumental responses. We discuss two previous analyses in which $\emptyset(T)$ for a non-flaring active region is derived. A least-squares method of Batstone et al. uses x-ray data of low statistical significance, a fact which appears to influence their results considerably. The work of Chambe, in which a functional form for $\emptyset(T)$ is deduced, uses only approximate atomic parameters. Replacing these with improved data, we calculate line fluxes using Chambe's $\emptyset(T)$ and compare these with observed values. The comparison indicates that Chambe's $\emptyset(T)$ describes the observed line emission very well.

Two methods for finding $\emptyset(T)$ ab initio are developed. In the first, $\emptyset(T)$ is expanded in the form $\emptyset(T) = \Sigma\ a_n\ T^n$, the a_n s being evaluated by least-squares analysis of fluxes of a number of lines. In the second, the integral in the expression for line flux in terms of $\emptyset(T)$ is replaced by a Gauss-Laguerre quadrature sum. The coefficients are again evaluated by least squares. These two methods should have application not only to active-region plasmas, but also to hot, flare-produced plasmas.

1. INTRODUCTION

It is now becoming clear that the great variety of x-ray and ultraviolet line emission observed from coronal active regions cannot be explained in terms of the emitting region's having a single temperature. This was the assumption in earlier work using data from rocket-borne instrumentation (e.g. Evans and Pounds, 1968). Often, the justification for this was that the observed lines were emitted by a set of ions which were known to be produced over a fairly narrow region of temperature, so that temperatures and emission measures could be deduced simply from the intensity ratio of any two lines. With the great improvement in data quality being achieved at the present time, this simple model is not adequate. It is necessary to consider models for coronal active regions in which there is a continuous distribution of emitting material with temperature.

A number of recent works (Batstone *et al.*, 1970; Chambe, 1971; Acton *et al.*, 1972; Walker *et al.*, 1974) have investigated such models using soft x-ray observations of active regions. The x-ray line fluxes are analyzed to give what Batstone *et al.* have called "differential emission measure", but as we shall see this is in fact the function $\emptyset(T) = n_e^2\, dV/dT$ (n_e = electron density, dV an element of volume, T the electron temperature). In this paper, we shall consider two of these works in some detail, particularly from the point of view of possible sources of error, to see what exactly has been established about the form of the distribution function, $\emptyset(T)$. We shall then develop two *ab initio* methods of finding $\emptyset(T)$ which have not hitherto been used.

2. X-RAY LINE FORMATION IN CORONAL ACTIVE REGIONS

We shall first develop the relation between the flux of x-ray line emission and the function $\emptyset(T) = n_e^2\ dV/dT$, taking such emission to arise from optically thin material.

Considering the strong resonance lines that have been used in the works mentioned earlier, excitation of the upper level occurs predominantly by electron collisions from the ground state. For particular transitions, cascading from higher levels or recombination of the next higher ion may contribute significantly to the upper-level populations. When the effects of cascades and recombinations can be neglected, the flux (F) observed at the earth may be written as

$$F = \frac{1}{4\pi r^2} \int_V n(\text{ion})\ n_e\ C_{12}\ dV \text{ photons cm}^{-2}\ \text{s}^{-1} \qquad (1)$$

where r is the sun-earth distance, C_{12} is the rate coefficient of collisional excitation from level 1 to 2, n(ion) the ion number density, and the integral extends over the emitting volume, V. For a maxwellian distribution of electron velocities, C_{12} is related to the cross-section for collisional excitation, Q_{12}, by

$$C_{12} = \sqrt{\frac{8}{\pi m}}\ \frac{1}{(kT)^{3/2}} \int_W^\infty Q_{12}(E)\ E_e^{-E/kT}\ dE \text{ cm}^3\ \text{s}^{-1}$$

where m is the electron mass, E the energy of the incident electron, and W the threshold energy for excitation. Cross-sections are often expressed in the Bethe approximation, for which Q_{12} is proportional to the absorption oscillation strength, f_{12}, and to the Gaunt factor, g. This approximation assumes that excitations occur mostly as a result of distant electron encounters, and is valid for electron energies much larger than the threshold

energy, W. Following van Regemorter (1962), who uses the Bethe approximation for Q_{12}, we have

$$C_{12} = 1.70 \times 10^{-3} \frac{e^{-W/kT} f_{12}}{W\sqrt{T}} P(W/kT) \quad cm^3\ s^{-1} \qquad (2)$$

where the function P is related to the Gaunt factor, g, by

$$P(W/kT) = \int_{W/kT}^{\infty} g e^{-E/kT}\, d(E/kT).$$

Van Regemorter has constructed an interpolation formula which approximates various measurements and calculations of g for s → p transitions, and tabulates the resulting values of P as a function of W/kT. These indicate that, for positive ions, P tends to an asymptotic value of 0.20 for $W \geq kT$.

From (1) and (2), we may now express F as

$$F = 6.05 \times 10^{31} \frac{f_{12}}{W} \frac{n(\text{element})}{n(H)} \int_V \frac{n(\text{ion})}{n(\text{element})}\, n(H)\, n_e \frac{e^{-W/kT} P(W/kT)}{\sqrt{T}}\, dV$$

$$\text{photons } cm^{-2}\ s^{-1} \qquad (3)$$

where n(element) is the number density of all ion stages of the element in question, and n(H) the number density of H atoms. For weaker lines arising from higher levels, the right side of (3) must be multiplied by an appropriate branching ratio. Taking the abundance of helium relative to hydrogen to be n(He)/n(H) = 0.04 (e.g. Robbins et al., 1974), we have

$$n(H) = \frac{n_e}{1+2n(He)/n(H)} \simeq 0.93\ n_e.$$

We can now write

$$F = K \int_V G(T)\ n_e^2\ dV \qquad (4)$$

where

$$K = 5.60 \times 10^{31}\ \frac{f_{12}}{W}\ \frac{n(\text{element})}{n(H)}\ , \qquad (5)$$

and, following a notation of Pottasch (1964), G(T) is given by

$$G(T) = \frac{n(\text{ion})}{n(\text{element})} \cdot \frac{e^{-W/kT}\ P(W/kT)}{\sqrt{T}}\ . \qquad (6)$$

Jefferies, Orrall, and Zirker (1972) have discussed equations describing line emission in optically thin material which have forms similar to (4). Their procedure is to express $n_e dV = dN_e$ in terms of a bivariate distribution function $\mu(n_e,T)$ given by

$$n_e dV = dN_e = N_e\ \mu(n_e,T)\ dn_e\ dT,$$

with N_e the total number of electrons in the emitting volume, V. Applying this to (4), we have

$$F = K\ N_e \int_0^\infty \int_0^\infty G(T)\ n_e\ \mu(n_e,T)\ dn_e\ dT. \qquad (7)$$

The data will not in general permit us to determine the complete functional form of $\mu(n_e,T)$. However, we can find the integral of $\mu(n_e,T)$ over all densities providing that $n_e = n_e(T)$ is a single-valued function of T, i.e. to each T, there is a single value of n_e. Evidence that this is so is

provided by the few estimates we have of coronal active-region densities from white-light measurements (Newkirk 1967). Thus (7) may be expressed in the form

$$F = KN_e \int_0^\infty G(T)\ n_e(T)\ \Phi(T)\ dT$$

where

$$\Phi(T) = \int_0^\infty \mu(n_e,T)\ dn_e .$$

For the purposes of our problem, we put

$$\emptyset(T) = N_e\ n_e(T)\ \Phi(T)$$

so that

$$F = K \int_0^\infty G(T)\ \emptyset(T)\ dT \qquad (8)$$

Comparing this with (4), we see that (8) is equivalent to putting

$$\emptyset(T) = n_e^2(T)\ \frac{dV(T)}{dT} .$$

$\emptyset(T)$ may be considered to be the function describing the manner in which the active region's emitting plasma is distributed with temperature.

3. SOURCES OF ERROR

A great deal of atomic data is required in any derivation of $\emptyset(T)$, as can be seen from the various terms occurring in Equation (3). For the weaker lines in the soft x-ray region, many of the parameters may be very poorly known, while even for the strong resonance lines used in

previous $\emptyset(T)$ analyses significant uncertainties may also exist. We shall discuss some of the more important sources of error.

(i) Rate Coefficients for Direct Collisional Excitation.

Expression (2) with van Regemorter's (1962) values for P(W/kT) has been extensively used in the literature for the rate of direct collisional excitation. The resulting rate coefficients agree well with calculations using more recent cross-sections from Coulomb-Born or similar approximations for many of the strong resonance lines which involve $s \rightarrow p$ transitions in the soft x-ray region. In particular, the asymptotic value of $P = 0.20$ for $W \geq kT$ agrees to within $\pm$ 50% with values for $1s^2\ ^1S - 1s\ 2p\ ^1P$ transitions in He-like Si, Ca, and Fe ions using Coulomb-Born cross-sections of Blaha (1973), and for 1s - 2p transitions in hydrogenic ions using the Coulomb-Born-Oppenheimer cross-sections of Burgess, Hummer, and Tully (1970). The work of Burgess et al, which also has application to certain complex ions, includes the effects of exchange of the incident electron and the bound electron. Application of van Regmorter's procedure to more complex ions and transitions other than $s \rightarrow p$ may not have high accuracy, however, because P(W/kT) is expected to depend on both ion charge and transition type (Bely and van Regemorter, 1970). Also, for many x-ray transitions, we are interested in values of the collisional excitation cross-section close to the threshold energy, W, since, when $W > kT$ (as is usually the case), only electrons in the high-energy tail of the maxwellian distribution are capable of exciting the transition concerned. Unfortunately, the Coulomb-Born approximation is unsuited for such a condition and may thus lead to incorrect results.

For certain ions and transitions, the occurrence of resonances in the collisional excitation cross-sections, arising from the formation of intermediate auto-ionizing states whose lifetimes determine the width of the resonances, may be of some importance astrophysically (e.g., Gerjuoy, 1966). As an illustration, Gerjuoy mentions the case of a resonance structure at 22.4 eV in the excitation cross-section of the 1s 2s $^3S_1$ state in neutral helium. This appears to arise from the production of He^- in a 1s $3s^2$ $^2S_{\frac{1}{2}}$ state, which auto-ionizes to form neutral He in a 2 $^3S_1$ state. Hershkowitz and Seaton (1973) have pointed to the importance of resonances near the threshold energy for a C III transition visible in the optical spectra of quasi-stellar objects. We cannot assess the significance of such effects for x-ray lines from the sun since there are not yet any data referring to resonances in the cross-sections for the transitions involved.

(ii) Population of Excited States By Cascading and Recombination.

The contribution to excited-state populations by cascading has been considered only approximately, but values given by Mewe (1972) indicate that cascading will be unimportant (< 10% of the direct collisional rate) for $1s^2$ - 1s 2p ^1P transitions in He-like ions and 1s - 2p transitions in H-like ions. The same may not be true for intercombination and forbidden transitions in He-like ions (although previous analyses have not used these lines) or for more complex ions generally. An example is the Ne-like Fe XVII ion, which emits a number of strong lines between 13Å and 17Å. Statistical equilibrium calculations for this ion by Loulergue and Nussbaumer (1973) suggest that the upper levels in the $2p^6$ - $2p^5$ 3s transition array are populated almost entirely by cascades from $2p^5$ 3p.

The excited states may also be populated by processes of radiative and dielectronic recombination. As discussed by Tucker and Koren (1971), the number of radiative recombinations to any given ion must always be less than the number of excitations from the ground state to the first excited level. Thus, radiative recombination as a mechanism of populating excited states is never more important than direct collisional excitation, and in fact can usually be neglected. With dielectronic recombination, the dielectronic capture of an electron by an ion, X^{+z}, forms a doubly excited state in ion X^{+z-1}:

$$X^{+z}(n\ell) + e \rightarrow X^{+z-1}(n'\ell'; n''\ell''),$$

which is stabilized by one or more radiative transitions to the ground state of the ion, X^{+z-1}. If the energy of the dielectronically-captured electron $(n''\ell'')$ can be neglected, the stabilizing transition,

$$X^{+z-1}(n'\ell'; n''\ell'') \rightarrow X^{+z-1}(n\ell ; n''\ell''),$$

may be indistinguishable from the energy of the resonance-line transition, $X^{+z}(n'\ell') \rightarrow X^{+z}(n\ell)$, in the ion X^{+z}. A discussion by Gabriel (1972) shows that a contribution of $\leq 10\%$ is made to the intensity of the $1s^2\ ^1S$ - $1s\ 2p\ ^1P$ transition in He-like ions by dielectronic-recombination satellite transitions, $1s\ 2\ell\ n''\ell'' \rightarrow 1s^2\ n''\ell''$ $(n'' \geq 3)$, occurring in Li-like ions. An approximate treatment by Tucker and Koren indicates that the contribution made by dielectronic recombination to line intensities in all other ions is less than $\sim 25\%$ of the direct collisional rate for $z \leq 25$.

(iii) Element Abundances

Analyses of intensities of lines emitted by more than one element must assume values for relative abundances. Since x-ray active regions are situated in the corona, it would seem natural to assume coronal abundances. However, there is a possibility that the abundances of heavy elements will be enhanced in regions where the temperature is greater than the general coronal value since, for a highly ionized gas, thermal diffusion tends to drive heavy species to higher temperatures (e.g., Seaton, 1964; Nakada, 1969). This will occur only if mixing by turbulence is small (a condition which seems likely to obtain in view of the presence of magnetic fields in coronal active regions), and if there is a temperature gradient along the magnetic-loop configurations. According to calculations by Nakada (1970), a diffusive equilibrium will be attained for, e.g., O VIII ions in a time comparable with the lifetime of an x-ray active region.

(iv) Ion Concentrations

Ionization equilibrium in a non-flaring active region has been assumed in all previous analyzes to deduce ion concentrations. This should be justified in view of the fact that, with electron densities of $\geq 10^9$ cm^{-3} (e.g. Newkirk, 1967), such an equilibrium will be set up very quickly. (Gabriel's (1972) evidence that an active region is an ionizing plasma must be considered doubtful since it relies upon the observed intensity of an extremely weak spectral feature.) Some existing calculations of ion concentrations may have to be modified slightly to take into account more precisely the effects of electron density on dielectronic

recombination rates (Summers (1973) has done this for C, O, and Ne ions) and adjustments in radiative recombination rates for complex ions (Tarter, 1971), but the effects of each should be small. However, the dielectronic recombination rate may show significant departures from that given by Burgess's (1965) formula which has been applied extensively in ionization-equilibrium calculations. Shore (1969) points out that the formula considerably overestimates the rate whenever the stabilizing transition involves a change in the principal quantum number. This occurs in particular when a H-like ion recombines to form a He-like ion. Both these ions feature prominently active-region x-ray spectra.

(iv) Oscillator Strengths

Many of the weaker x-ray lines may have poorly known oscillator strengths, but the errors in the calculated f-values for the strong lines should now be quite small as increasingly sophisticated formulations are applied.

It would be clearly desirable to assess what effect each of the errors listed above would have on the derived functional form of $\emptyset(T)$. However, we often have only a vague idea of the error to be placed on some of the quantities. To illustrate the point, inter-agreement of several Coulomb-Born calculations of excitation cross-sections (see (i)) would not necessarily imply the cross-sections (and thus the rate of collisional excitation) is known to high accuracy. Each calculation could be appreciably in error if the effect of auto-ionizing resonances is important. If enhancement of heavy-element abundances through thermal diffusion cannot be neglected (see (iii)), the methods of Walker et al.

and Jakimiec et al. (1973), in which abundances are adjustable in the analysis, may have to be tried. Shore's corrected dielectronic-recombination rates (see (iv)) could be included approximately in existing ionization-equilibrium calculations by making empirical adjustments (e.g. Tucker and Koren (1972)) to the H-like and He-like ion concentrations.

In addition to the atomic parameters, the solution of $\emptyset(T)$ from (8) will require the observed fluxes from a set of lines whose G(T) functions completely cover the temperature range of interest ((1-9) x 10^6K, say, for a non-flaring active region). The observed fluxes will depend on the accuracy of the instrumental calibration and may, if small, be uncertain through count-rate statistics. A significant difficulty in the intensity calibration of Bragg crystal spectrometers which have been used in the soft x-ray region is the calculation or measurement of the integrated reflectivity coefficient, $R(\lambda)$. An in-flight calibration would obviate the need for calculating $R(\lambda)$, a procedure which in any case represents an ideal situation unlikely to occur in practice (see, e.g., Guinier, 1963). Much existing data from satellites are uncalibrated, however, and we must rely on theory for the calculation of $R(\lambda)$. Alexandropoulos et al. (1973) have investigated the change of crystal properties ($R(\lambda)$ among them) when the crystal face is momentarily heated by strong visible-light illumination, but it is uncertain how their results, which indicate significant changes in $R(\lambda)$, would refer to illumination by sunlight. The situation seems most critical for rocket-borne detectors without in-flight calibration since, with many of the instruments aboard satellites, the crystal face is either subjected to prolonged sunlight illumination,

during which time $R(\lambda)$ assumes an equilibrium value, or (as with the GSFC OSO-5 spectrometer) is shielded by a filter designed to reduce ultraviolet background.

4. $\emptyset(T)$ FOR AN ACTIVE REGION: THE PRESENT EVIDENCE

Solutions for $\emptyset(T)$ for a non-flaring active region, as we saw in §1, have been claimed by various workers, but since Acton et al. (1972) and Walker et al. (1974) assumed that the functional form for $\emptyset(T)$ was basically that given by Chambe (1971), the methods of analysis tried so far break down to only two: one given by Chambe and the other by Batstone et al. (1970).

Batstone et al. have used the fluxes of five x-ray lines in a least-squares procedure to solve for up to four values of $\overline{\emptyset}_i$ given by

$$\overline{\emptyset}_i = \frac{1}{\Delta T} \int_{T_i}^{T_i+\Delta T} \emptyset(T)\, dT$$

where $\Delta T = 2 \times 10^6$K and $T_i = 2.5 \times 10^6$K, 4.5×10^6K, ... The problem, which can be stated in the form

$$F_j = a_1 \overline{\emptyset}_1 + a_2 \overline{\emptyset}_2 + a_3 \overline{\emptyset}_3 + a_4 \overline{\emptyset}_4 \quad (j = 1,2,\ldots,5),$$

is thus only just over-determined. Three active regions were observed by them with an uncollimated crystal spectrometer. Photon counting rates for individual regions were extracted by de-convolution of the data with the known instrumental response function. Their procedure did not strictly minimize the sum of squared deviations since two constraints were first imposed: the solutions $\overline{\emptyset}_i$ were forced (a) to have non-negative values, and

(b) to decrease monotonically with T. The first of these conditions may appear to be reasonable, but it might have been thought that, with observational data of adequate statistical significance, it would be unnecessary actually to impose this as a constraint on the analysis. A least-squares calculation with no such conditions ought to have only positive solutions unless there is an insufficient number of degrees of freedom or the data have a poor statistical quality.

In view of the low photon counting rates and difficulties associated with the de-convolution process, I have applied an unweighted least-squares analysis on the data exactly as presented by Batstone et al. without the two conditions imposed by them. In addition, I investigated the effects on the solutions, $\overline{\emptyset}_i$, of statistical errors in the observed fluxes, F_j, which I took to be $F_j \sqrt{N_j}/N_j$ (N_j = photon counting rate). Table I lists the values of $\overline{\emptyset}_i$, together with their standard deviations, for the three active regions. These are compared with values given by the calculation of Batstone et al. For many solutions, we have the physically absurd situation of negative $\overline{\emptyset}_i$ s, but it is clear that this arises from the poor photon statistics since the standard deviations are so large. It seems that values of $\overline{\emptyset}_i$ for $T > 3.5 \times 10^6$K are insignificantly different from zero. Data of a much higher quality are evidently needed for this type of procedure.

As already discussed, Chambe (1971) assumes a functional form for $\emptyset(T)$ which, for $T > 2 \times 10^6$K, exponentially falls with T. This is alleged to fit the observed line fluxes very well, but in fact it is not clearly stated that the form of $\emptyset(T)$ derived actually reproduces these fluxes to

TABLE I

Values of $\overline{\phi} = \frac{1}{\Delta T} \int_{T}^{T+\Delta T} \phi(T)\, dT$ for Three Active Regions Seen by Batstone et al.

T - (T+ΔT)(K)		(1.5-3.5) x 10^6	(3.5-5.5) x 10^6	(5.5-7.5) x 10^6	(7.5-9.5) x 10^6
Active Region 1:	B	15	9	7	0
	P	18.5 ± 11.7	4.2 ± 10.8	14.7 ± 13.7	- 4.5 ± 8.4
Active Region 2:	B	8	4	2	1
	P	12.2 ± 7.2	- 1.3 ± 6.4	8.7 ± 8.6	- 2.2 ± 5.4
Active Region 3:	B	16	8	6	0
	P	43.2 ± 9.0	- 13.6 ± 7.3	25.4 ± 10.4	- 9.5 ± 6.7

Units of $\overline{\phi}$: 10^{47} $cm^{-3}(10^6K)^{-1}$. B = values from Batstone et al. (1970)

P = values of this paper (errors are standard deviations).

within the observational errors. This could have been done by comparing observed fluxes, F_{obs}, with the values of $F_{calc} = K \int_0^\infty G(T)\, \emptyset'(T)\, dT$ (see equation (8)), $\emptyset'(T)$ being the derived $\emptyset(T)$. Chambe's analysis is based on twenty-seven x-ray and ultra-vilet lines. The function $\emptyset(T)$ is derived using atomic parameters for the lines which must in many cases be considered very approximate. For example, values of the collisional excitation rates were calculated with the function P(W/kT)(see equation (2)) set equal to 0.3 for all x-ray ($\lambda < 100$Å) lines regardless of the transition type. As we saw, such an approximation may not be good for ions which are not H-like or He-like. For the Ne-like ion, Fe XVII, Chambe multiplies the direct collisional excitation rate, $C(2p^6 \rightarrow 2p\ ^5 3s)$, by three to take account of cascading from the $2p\ ^5 3p$ levels. However, the calculations of Loulergue and Nussbaumer (1973) indicate that the cascading contribution will be much more significant, and a correcting factor of at least twenty is required. Loulergue and Nussbaumer calculate that the cascading contribution is also significant for the $2p\ ^5 3d\ ^3P_1 - 2p^6\ ^1S_o$ transition at 15.45Å. This was totally neglected by Chambe. Finally, Chambe uses Bely's (1967) calculations of the ionization equilibrium of iron which do not include radiative recombination and deal only approximately with the dielectronic recombination rate.

In view of the possibility that the uncertainties in the atomic parameters have led to inaccuracies in the derived functional form of $\emptyset(T)$, I have investigated how well the observed fluxes of the spectral lines Chambe considers are reproduced by his model. A comparison was made between the observed line fluxes, F_{obs}, and the values calculated from F_{calc}=

$K \int G(T)\ \emptyset'(T)\ dT$, $\emptyset'(T)$ being the function $\emptyset(T)$ given by Chambe. The values of KG(T) were taken from Tucker and Koren's paper where available, except for lines due to Fe XVII for which Loulergue and Nussbaumer's treatment was used. Thus, element abundances as given by Tucker and Koren were assumed, as were the values given by them for the other atomic parameters of equations (5) and (6). There are three lines in Chambe's list at ultraviolet ($\lambda > 100$Å) wavelengths due to the ions Fe XV and Fe XVI. These were considered in the same way as for the x-ray lines, although the ultraviolet lines are marginally optically thick in their Doppler cores. A rough calculation, in which the extent of a typical coronal active region is taken to be 2×10^5 km, indicates that for all three transitions the optical depth in the Doppler core is about two. This should not affect these results significantly, however, since if we assume that at most half of the resonantly-scattered photons are directed towards the chromosphere or photosphere (where they would be absorbed), then the total emission would be reduced by no more than $\sim 40\%$. For these three lines, the values of KG(T) were calculated using van Regemorter's values of P(W/kT), Crossley and Dalgarno's (1965) f-values, and Jordan's (1969) tabulations of n(ion)/n(element). n(Fe)/n(H) was taken to be 5×10^{-5}, the value used in Tucker and Koren's work.

There is a good correspondence between F_{obs} and F_{calc}, as can be seen from the plot of Figure 1. The differences between F_{obs} and F_{calc} are mostly within a factor of two or so. This is acceptable in view of the uncertainties in the atomic parameters. Discrepancies much larger than this (e.g. the lines of O VII at 21.60Å and O VIII at 14.82Å, fluxes

of which were taken from Evans and Pounds (1968)), seem to be explicable in terms of possible errors in the observational data. The O VII line, for example, occurs in a region where the efficiency of Evans and Pounds' detector was very low, while the O VIII line had a very low count rate. The three ultraviolet lines due to Fe XV and Fe XVI are rather more intense than is calculated from Chambe's model, not less as would be expected if some of the resonantly-scattered photons were directed downwards and absorbed by the photosphere. Since the observation of these lines were taken on an occasion different from those of the x-ray lines, there is a possibility that the enhanced ultraviolet emission arose from a different level of solar activity. In conclusion, then, the form of $\emptyset(T)$ given by Chambe would seem to describe non-flaring active regions rather closely.

Some further evidence that the form of $\emptyset(T)$ which Chambe discusses is close to the correct form is presented by Walker et al. Their functional form has a similar shape to Chambe's, and with it they are able to reproduce the observed fluxes of eight lines between 4 and 8Å very well indeed. However, this has been done through adjustment of element abundances which appears to affect the analysis to a greater extent than the precise form of $\emptyset(T)$. Moreover, although the continuum flux at $\lambda = 8$Å is correctly predicted by their model, that at the shorter wavelengths is not, suggesting a greater quantity of emitting material at $T \gtrsim 7 \times 10^6$K than is suggested in their model.

5. SOME OTHER METHODS FOR THE SOLUTION OF $\emptyset(T)$

In this section, we discuss two methods for the derivation of the functional form of $\emptyset(T)$ which hitherto have not been tried but which ought

to have an advantage over Chambe's method in that a determination of $\phi(T)$ is made ab initio. Both procedures enable a large number of spectral lines to be used with a least-squares analysis, so that there is some hope that the effect of poorly known atomic parameters may be reduced to some extent.

(a) $\phi(T)$ Taken to be a Power Series in T.

At first, we shall take $\phi(T)$ to be given by

$$\phi(T) = \sum_{n=-N_1}^{+N_2} a_n T^n \quad (N_1, N_2 \geq 0). \tag{9}$$

With a set of m line fluxes (m greater than or equal to the number of terms in the summation), $F_i (i = 1, \ldots, m)$, we can solve for $\phi(T)$ from (9) and (8) using

$$F_i = K_i \sum_{n=-N_1}^{N_2} a_n \int_0^\infty G_i(T)\, T^n\, dT, \quad i = 1, \ldots, m.$$

A least-squares procedure could be used to solve for the unknown coefficients a_n if m exceeds the number of terms in the sum.

There are two difficulties with using the form (9). The first is that terms of the summation with $n \geq 3$ tend to produce spurious oscillations which may not be distinguishable from possible real structure in $\phi(T)$. The second is that any representation of $\phi(T)$ involving a sum of terms may give negative values for $\phi(T)$ in certain temperature ranges. Using x-ray data for a flare plasma (the results to appear in Paper II of this series), it has been found that both effects should be avoidable by having a set of lines in which the temperature ranges of the G(T) functions (defined, say, by the range within which G(T) is at least 1/e x its maximum

value) overlap over the entire temperature interval (T_1, T_2) of interest. The results of Walker et al. suggest that, for a non-flaring active region, temperatures between $T_1 = 1 \times 10^6$K and $T_2 \geq 9 \times 10^6$K should be considered, while OSO-5 results to be reported in Paper II indicate that, for a flare, $T_1 = 1 \times 10^6$K, $T_2 \simeq 50 \times 10^6$K. Spurious oscillations may still occur just above the temperature interval (T_1, T_2), but these may be considerably damped by including in the data sample lines which have zero or near-zero fluxes and whose G(T) functions extend well beyond the temperature T_2.

A series of solutions for $\emptyset(T)$ in the form of (9) can be obtained for various values of N_1 and N_2. The goodness of fit in each case may be tested by comparing the observed line fluxes, F_{obs}, with the value of $K \int_{T_1}^{T_2} G(T)\, \emptyset'(T)\, dT$, $\emptyset'(T)$ being one of the solutions. The integration over (T_1, T_2) should not include any region where $\emptyset'(T) < 0$; any solutions having this property should be rejected. If the solutions giving best agreement between observed and calculated line fluxes show any marked similarity with a well-known functional form (e.g. Chambe's expression resembles the form $\emptyset(T) = A\, T^N \exp(-\beta T)$), such an expression could be tested by varying its parameters to see if there is any further improvement in reproducing the observed line fluxes.

The analytic expression for $\emptyset(T)$ which is finally derived may be used to find the flux of continuum emission which may then be compared with that observed. The x-ray continuum is made up of principally free-free and free-bound emission for temperatures typical of non-flaring active regions. At wavelengths of about 20-60Å, two-photon emission from H-like and He-like ions may also be an important contributor (e.g. Tucker and Koren, 1971). Writing the continuum radiation per unit $n_e^2\, dV/dT$ at

energy E (keV) and temperature T as $\eta(E,T)$ photons cm^{-2} s^{-1} $keV^{-1}/(cm^{-3}$ $K^{-1})$, the flux of continuous emission predicted by the $\phi(T)$ derived as above is

$$\mathcal{F}(E) = \int_0^\infty \phi(T)\ \eta(E,T)\ dT \quad \text{photons } cm^{-2}\ s^{-1}\ keV^{-1}.$$

This may then be compared with observed values of $\mathcal{F}(E)$ at spectral locations relatively free of line emission.

(b) Gauss-Laguerre Quadrature Method

In this method, we replace the integral of (8) with a quadrature sum. We substitute $x = cT$ (c = constant) for the integrating variable, T, so that (8) becomes

$$F = \frac{K}{c} \int_0^\infty G(x)\ \phi(x)\ dx. \tag{10}$$

Writing

$$f(x) = G(x)\ \phi(x)\ e^x, \tag{11}$$

and replacing the infinite integral of (10) by a Gauss-Laguerre quadrature sum, we have

$$F = \frac{K}{c} \sum_{k=1}^{n} A_k\ f(x_k) \tag{12}$$

where the x_ks are zeros of the nth Laguerre polynomial and the A_ks weight factors. Replacing the integral of (10) with the quadrature sum of (12) is exact if f(x) is a polynomial in x of degree $\leq 2n - 1$. Values for x_k and A_k are tabulated for $n = 1,2,3,\ldots,16$ by Krylov (1962). Writing (12) in terms of temperature, we have for the flux of the ith line of a sample of m lines,

$$F_i = \frac{K_i}{c} \sum_{k=1}^{n} A_k\ G_i(T_k)\ \phi(T_k)\ e^{cT_k}.$$

If $m > n$, we may then solve for the n unknowns, $\emptyset(T_k)(k=1,2,\ldots,n)$, using a least-squares procedure having chosen some suitable value for the constant, c.

The method thus finds discrete values for $\emptyset(T)$. It may not be suitable if the function f(x) in (11) cannot be adequately described by a polynomial of degree $\leq 2n - 1$. Values of $\emptyset(T_k)$ may only be established if all the T_k are within the temperature range of the various G(T) functions for the m lines.

6. SUMMARY

The functional form of $\emptyset(T) = n_e^2\, dV/dT$ can in principle be derived for a coronal active region using intensities of x-ray or ultraviolet lines from optically thin plasma. The procedure assumes that the electron density, $n_e = n_e(T)$, is a single-valued function of temperature. A large number of lines will give a better determination of $\emptyset(T)$ if all the atomic parameters and the observed fluxes are reasonably well-known. At the present time, errors may still be significant for the rates of electron collisional excitation and cascade contributions, particularly for ions more complex than the He-like stage. Errors in values for element abundances (arising from the possibility of heavy-element enhancement by diffusion) and ion concentrations may also have to be considered.

Previous attempts to determine $\emptyset(T)$ are examined critically. The least-squares method of Batstone et al. (1970) is based on very low photon count rates, a fact which appears to explain why some values of $\int_T^{T+\Delta T} \emptyset(T)\, dT$ are negative when no constraints are imposed on the analysis. The results of the re-analysis are summarized in Table I. Chambe's (1971)

method was to take some arbitrary functional form for $\emptyset(T)$ and to adjust its shape until a good fit with observations is achieved. It is not shown, however, that the derived $\emptyset(T)$ actually reproduces the observed line fluxes. The atomic parameters used in the analysis, moreover, were in many cases only rough approximations. A re-analysis is made in which the x-ray line fluxes are calculated from (8) using the $\emptyset(T)$ as given by Chambe and more accurate atomic data. The agreement between the observed fluxes and those calculated as a result is mostly to within a factor of two, so that we must conclude that Chambe's form for $\emptyset(T)$ is able to describe the x-ray line emission rather well.

Two ab initio methods are developed in which $\emptyset(T)$ may be derived from x-ray line fluxes. In the first, $\emptyset(T)$ is expressed as a power series in T, with coefficients to be determined by least-squares methods if the number of lines is sufficiently great. That solution ($\emptyset'(T)$) which is able best to reproduce the observed line fluxes should be selected. If this $\emptyset'(T)$ resembles some familiar analytic form, then this form should be tested to see whether any improvement in reproducing observed fluxes can be obtained.

The second method approximates the integral of (8) with a Gauss-Laguerre quadrature sum with n terms. This procedure is exact if the right side of (11) can be expressed as a polynomial of degree $\leq 2n - 1$. With a set of m ($> n$) x-ray lines, solutions for the n unknown values of $\emptyset(T_k)$, $k = 1, \ldots, n$, may be obtained by a least squares procedure.

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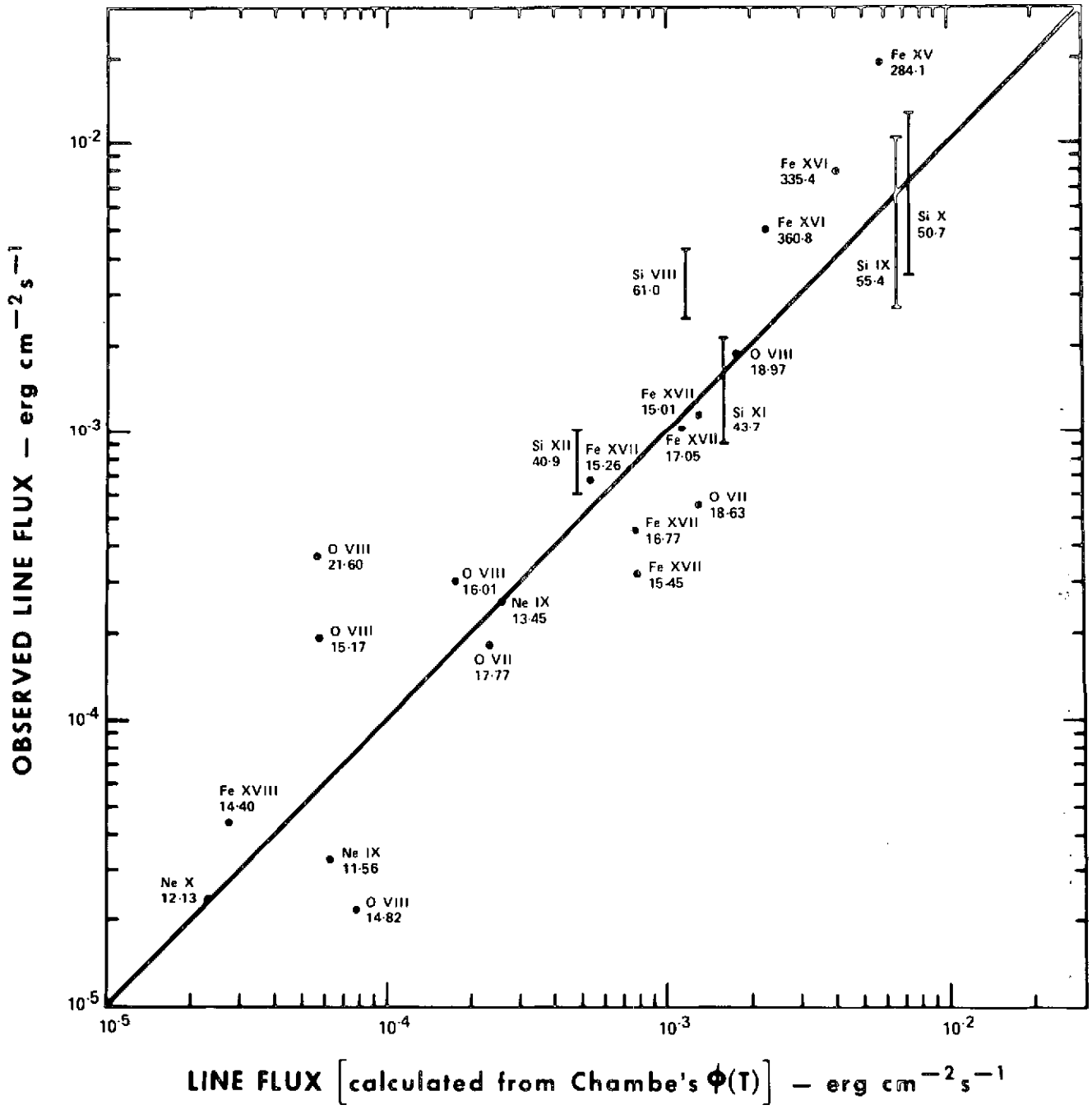
FIGURE CAPTION

Plot of observed flux (erg cm^{-2} s^{-1}) vs. flux calculated from Chambe's Ø(T) function. Points are annotated with ion and line wavelength in Å.


OBSERVED LINE FLUX — erg cm^{-2}s^{-1}
LINE FLUX [calculated from Chambe's Φ(T)] — erg cm^{-2}s^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Fe XV 284·1
Fe XVI 335·4
Fe XVI 360·8
Si X 50·7
Si IX 55·4
Si VIII 61·0
O VIII 18·97
Fe XVII 15·01
Si XI 43·7
Fe XVII 17·05
Si XII 40·9
Fe XVII 15·26
O VII 18·63
Fe XVII 16·77
Fe XVII 15·45
O VIII 21·60
O VIII 16·01
Ne IX 13·45
O VIII 15·17
O VII 17·77
Fe XVIII 14·40
Ne IX 11·56
Ne X 12·13
O VIII 14·82