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SOME CABLE SUSPENSION SYSTEMS AND THEIR EFFECTS ON THE FLEXURAL FREQUENCIES OF SLENDER AEROSPACE STRUCTURES

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16. Abstract

The effects of several cable suspension configurations on the first free-free flexural frequency of uniform beams have been determined by experiment and analysis. The results of this study confirm that in general the larger the test vehicle the larger is the flexural frequency measurement error attributable to a given cable suspension configuration. For horizontally oriented beams representing modern aerospace vehicles of average size and flexibility, the restraining effects of all but the shortest support cables were minor. The restraining effects of support cables of moderate length attached near the base of vertically oriented vehicles were overshadowed by the effects of beam compression due to gravity.

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SOME CABLE SUSPENSION SYSTEMS AND THEIR EFFECTS ON THE FLEXURAL FREQUENCIES OF SLENDER AEROSPACE STRUCTURES

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SUMMARY

The effects of several cable suspension configurations on the first free-free flexural frequency of uniform beams have been determined by experiment and analysis. The following trends established by this study should also be applicable to slender aerospace structures.

For replica vehicles, the natural flexural frequency measurement error induced by the constraint of cable suspensions of similar geometry is generally proportional to the dimensionless scaling parameter $g/\omega_r^2 L$, where g is the gravity acceleration, ω_r a reference structural frequency, and L the vehicle length. The larger the test vehicle, therefore, the larger is the flexural frequency measurement error attributable to suspension constraint.

For horizontally oriented beams, the suspension constraint effects (natural frequency measurement error) were inversely proportional to support cable length. For test vehicles of average size and flexibility, however, the restraint effects were minor.

The restraining effects of support cables of moderate length attached near the base of vertically oriented vehicles were overshadowed by the effects of beam compression due to gravity; i.e., the natural frequencies of the restrained beams were appreciably less than the frequencies of the free-free beams.

Restraint effects were less when the minimum static stability of vertically oriented beams was provided by lateral springs near each end of the beam than when all the restraint is concentrated at the base.

INTRODUCTION

The accurate determination of the free-free lateral response characteristics of an aerospace structure from ground vibration tests is made difficult by the necessity to restrain the structure against the forces of gravity. A massless spring restraint applied anywhere except at a nodal point will manifest itself as an apparent increase in the natural frequencies of the structure. The magnitude of the apparent frequency increase for a given mode of vibration is dependent on both the location and the magnitude of the restraint. For minimal effect on natural bending frequencies, the lateral restraint should be the minimum required to provide static stability of the test vehicle.

Several ingenious soft suspension systems for ground vibration tests of structures have been devised and used successfully. For example, reference 1 describes a combination suspension-excitation system with feedback control for maintaining vehicle attitude with a minimum of lateral restraint. The foregoing system effectively simulates the launch phase of liquid-fueled vehicles. The hydraulic support used in vibration tests of the full-scale Apollo-Saturn V is described in reference 2. This support allows six degrees of freedom and consists mainly of four frictionless hydraulic jacks incorporating hydrostatic bearings, capillary seals, and gas springs. Although these suspension systems fulfilled their intended purpose, it is probable that the requirements of most dynamic test programs can be met at a considerable saving in cost by utilizing less complex passive suspension systems.

Reference 3 is chiefly an experimental paper dealing with several cable suspension systems for model vibration studies. The apparent increase in the natural frequency induced by the constraints of the cable suspension system is investigated by comparing the unrestrained free body flexural frequencies of a simplified model with the corresponding frequencies of the constrained model. In the present paper many of the experiments of reference 3 have been corroborated by analysis which includes the effects of beam compression due to gravity (or longitudinal acceleration) where applicable. Some variations of vertical suspension systems and their merits are also discussed.

It was not the purpose of the study reported herein to determine the precise effect of a given type of suspension system on the dynamics of a specific launch vehicle or aerospace structure but rather to show trends and provide some insight into the magnitude of the effects of various suspension systems on the flexural frequencies of slender bodies in general. The experimental and analytical models selected for these investigations therefore were uniform beams of varying length and stiffness.

SYMBOLS

a	support cable separation distance, m (fig. 15)
a_o	minimum value of a required to maintain model vertical, m (fig. 16)
b	length of upper horizontal restraining cables, m
c	support cable separation distance at vehicle base, m

d	vertical distance from vehicle support point to center of gravity, m
f	vertical height from support cable anchor to juncture of restraining cables with support cables, m (figs. 15, 16, and 18)
g	acceleration, m/s^2
h	vertical distance between lateral restraints, m
K	total lateral restraint, N/m
K_L	lower lateral restraint, N/m
K_u	upper lateral restraint, N/m
K_v	total vertical restraint, N/m
k	lateral spring restraint per unit length of beam, N/m^2
L	length of beam, m
l	a characteristic length, m
M	beam mass, kg
m	beam mass per unit length, kg/m
r	radial distance from vehicle center line to point of application of vertical force, m
S	support cable length, m
T_r	tension in restraining cables, N
x	distance from end of horizontal beam to cable support, m
Δ	lateral deflection of the support cables, m
ω	first flexural natural frequency of restrained beam, rad/s

ω_o lowest rigid-body frequency of restrained beam, rad/s

ω_r reference frequency: first free-free flexural natural frequency, rad/s

RESULTS AND DISCUSSION

General

Results showing the effect on free body bending frequencies of the five basic suspension systems depicted in figure 1 are presented in figures 2 to 18. Modifications to allow multidegree of freedom testing are illustrated in figures 19 and 20. The effects of the two-cable horizontal suspension system (fig. 1(a)) are given in figures 2 and 3 while those of the multicable horizontal suspension (fig. 1(b)) are presented in figures 4 and 5. The data included in figures 6, 8, and 9 show the restraining effects of the one-cable vertical suspension (fig. 1(c)) and compare them with the effects of beam tension due to gravity. The calculated effect of pitch restraint applied at the base of a vertically oriented uniform beam (fig. 1(d)) is shown in figure 10. The data presented in figures 12 to 14 show the effects of two lateral restraints on a vertically oriented beam (fig. 1(e)) as well as the effects of beam compression due to gravity.

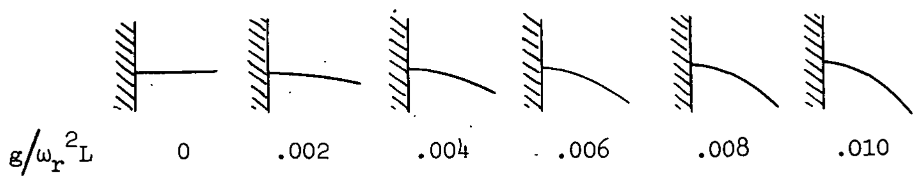
All of the suspension systems investigated experimentally were simple cable suspensions. Only the restraining effects of the suspension systems were investigated. Because of the small scale of the test beams, the cable mass was insignificant, thus allowing the cable dynamics to be ignored. Also, inasmuch as the ratio of the minimum support cable mass to test structure mass increases with test structure mass, cable resonances are more apt to be a problem with very large test structures such as full-scale launch vehicles than with small-scale models. In the event a cable resonance does coincide with a structural resonance, it may be feasible to detune the cables by the judicious application of small concentrated masses at cable antinodes.

In most of the figures the restraining effects of the suspension systems shown in figure 1 on the flexural frequencies of uniform beams are indicated by plotting the ratio of a restrained structural frequency, ω, to the corresponding free-free reference frequency, ω_r, as a function of the dimensionless ratio $g/\omega_r^2 l$, where g is the acceleration due to gravity and l is a characteristic length. Because a given lateral spring restraint generally has the most pronounced effect on the lowest flexural mode, the first free-free bending frequency was chosen as the reference frequency ω_r, and thus, ω denotes the restrained first bending frequency.

The first flexural frequency of vertically oriented beams is affected not only by the restraint of the suspension system but also by the beam compression or tension due to gravity. The gravity effects are a function of the dimensionless parameter $g/\omega_r^2 L$, where L is the beam length.

The restraining effects of all the suspension systems are shown as a function of this parameter in order that the relative magnitudes of the restraint and gravity effects may be compared directly.

The significance of the parameter $g/\omega_r^2 L$ is illustrated by the fact that for a uniform cantilever the static tip deflection relative to the beam length is equal to $62.8g/\omega_r^2 L$. Thus, the static deflection of horizontal uniform cantilevers would appear as depicted below



for various values of $g/\omega_r^2 L$. For similar beams or structures (all dimensions proportional) in a constant gravitational field, $g/\omega_r^2 L$ is proportional to structure size (length). The approximate values of $g/\omega_r^2 L$ for some launch vehicles are: Vanguard, 0.0008; Redstone, 0.0011; Saturn SA-1, 0.0011; Titan IIIC, 0.0017; and Saturn V, 0.0020. The value of $g/\omega_r^2 L$ tends to be larger for the larger vehicles.

Two-Cable Horizontal Suspension Systems

The two-cable horizontal suspension illustrated in figure 1(a) is attractive because of its simplicity. The suspension has no effect on free-free bending modes and frequencies when the support cables are attached at nodal points. It is not always feasible, however, to attach support cables at nodal points, and the magnitude of the increase in the resonant frequency of the test configuration due to the restraining effects of the support cables then depends on the location of the support points, the relative length of the cables, and the value of the parameter $g/\omega_r^2 L$.

The effect of the support cables was determined experimentally utilizing three aluminum rods of 9.52-mm diameter and lengths of 2.49, 3.12, and 3.58 meters resulting in values of the parameter $g/\omega_r^2 L$ of 0.002, 0.004, and 0.006, respectively. The results are shown in figure 2 where the ratio ω/ω_r is plotted as a function of cable attachment location x/L for relative cable lengths S/L of 1/8, 1/4, and 1/2. The free-free reference frequency ω_r, used in this and other figures pertaining to uniform beams, is the experimental frequency obtained with the cables located at the nodal points.

As indicated in figure 2, a short cable support $(S/L = 1/8)$ on a flexible beam $\left(g/\omega_r^2 L = 0.006\right)$ and on a somewhat stiffer beam $\left(g/\omega_r^2 L = 0.002\right)$ can be attached within $\pm 0.08L$ and $\pm 0.14L$ of the node, respectively, without exceeding 1-percent error in the first free-free bending frequency. The stiffer beam still represents an extremely large or flexible vehicle which is not likely to be suspended in this manner. For vehicles of the size and flexibility which would normally be suspended on two cables, the cables could probably be attached at any point along the vehicle length with negligible effect on the first bending frequency. Although not shown, the effects of the suspension system on the higher bending modes would in general be even less than those shown here for the first mode.

In addition to the experimental results a computer program utilizing exact linear beam theory was used to determine the bending frequencies of uniform beams with lateral end restraints. For small oscillations, the lateral restraint imposed by each cable is $Mg/2S$. The calculated frequency ratios ω/ω_r for the three beams are denoted by the solid symbols in figure 2 at $x/L = 0$ and are seen to be in good agreement with the experiment.

For horizontal suspension systems these data can be presented in a simpler form by plotting as a function of the dimensionless parameter $g/\omega_r^2 S$ which is equal to the square of the ratio of the lowest rigid-body frequency ω_o to the first free-free bending frequency ω_r. The data of figure 2 for $x/L = 0$ are presented in this fashion in figure 3. The data for other values of x/L were omitted from this figure because the effects of cable location are best shown in the form of figure 2.

Multicable Horizontal Suspension Systems

In cases where it is not feasible to support a vehicle horizontally at two points due to excessive bending moments or localized stresses, it may be possible to suspend it from many vertical cables distributed along the length of the beam as depicted in figure 1(b).

As noted in reference 4, where an infinite number of lateral springs constrain a uniform beam, the frequency equation is

$$\omega^2 = \frac{k}{m} + \omega_r^2 \tag{1}$$

In this equation k is the lateral spring stiffness of the test vehicle per unit length and m the mass per unit length. If k is constant along the length, then

$$\frac{k}{m} = \frac{K}{M} \tag{2}$$

where K is the total lateral restraint and M the total beam mass. For cables of length S,

$$K = \frac{Mg}{S} \tag{3}$$

Substituting equations (2) and (3) into equation (1) and rearranging,

$$\frac{\omega}{\omega_r} = \sqrt{\left(\frac{g}{\omega_r^2 L}\right)\left(\frac{L}{S}\right) + 1} \tag{4}$$

The increase in frequency ω/ω_r due to cable restraint was determined by equation (4) and is plotted in figure 4 as a function of $g/\omega_r^2 L$ for relative cable lengths S/L of 1/16, 1/8, 1/4, and 1/2.

As with the two-cable suspension, the effect on the first free-free frequency is approximately proportional to $g/\omega_r^2 L$ and inversely proportional to the cable length. For cable lengths greater than 1/8 of the beam length and for the common range of $g/\omega_r^2 L < 0.002$, the measurement error is less than 1 percent. For corresponding cable lengths and values of $g/\omega_r^2 L$, the error is approximately one-quarter of the error obtained when the beam is supported by a cable at each end ($x/L = 0$ in fig. 2).

To obtain equal tension in the many cables, it is advisable to use either soft springs in series with cables or elastic shock cords. One experimental data point for which 12 elastic shock cords were utilized is shown in figure 4. The experimental and calculated results are in good agreement.

In figure 5, the data of figure 4 are plotted as a function of the ratio of pendulum frequency to reference frequency, $g/\omega_r^2 S$. The effects of the multicable horizontal suspension system are indicated by a single curve described by

$$\frac{\omega}{\omega_r} = \sqrt{\frac{g}{\omega_r^2 S} + 1} \tag{5}$$

One-Cable Vertical Suspension

The results of vibration tests on vertically oriented beams suspended by a single cable attached to the upper end, as in figure 1(c), are given in figure 6. The ratio of the measured first bending frequency to the beam free-free frequency ω/ω_r is plotted as a function of the beam length relative to the cable length L/S for four beams having values of $g/\omega_r^2 L$ of -0.0029, -0.0063, -0.0114, and -0.0255.

The data are plotted with L/S as the abscissa to emphasize that when the curves are extrapolated to a value of $L/S = 0$ (no lateral restraint) there is still a substantial increase in the natural frequency over the free-free frequency. This increase can be attributed to tension in the beam due to its own weight. Thus the results shown in figure 6 include the combined effects of cable restraint and beam tension due to gravity. The separate effects of cable restraint and beam tension are illustrated in figures 7 and 8.

The effect of constant longitudinal acceleration on the transverse vibration of uniform beams has been computed by Seide (ref. 5) and is shown in figure 7 as a function of the dimensionless parameter $g/\omega_r^2 L$. The independent effect of the spring restraint Mg/S applied to one end of a uniform beam has been computed and is illustrated in figure 8, also as a function of $g/\omega_r^2 L$. From these two figures it may be observed that at comparable values of $g/\omega_r^2 L$ and for cable lengths greater than one-quarter the length of the beam, the tension in the beam has a greater effect on the natural frequency than does the cable restraint.

It is of interest to note that for like cable lengths and values of $g/\omega_r^2 L$, the effect of the restraint Mg/S applied to one end of a uniform beam (fig. 8) is essentially the same as the effect of half this restraint applied to each end of a beam ($x/L = 0$ in fig. 2).

The approximate combined effect of the spring restraint Mg/S and beam tension acting on a uniform beam suspended at its upper end was obtained by multiplying the ordinates of figures 7 and 8 at corresponding values of $g/\omega_r^2 L$. This calculated combined effect is compared in figure 9 with experimental results cross-plotted from figure 6. The correlation is very good.

Vertical Orientation With Base Restraint Only

Illustrated in figure 1(d) is a type of restraint sometimes used for vibration tests of liquid-fueled launch vehicles. In this system, the pitch and lateral restraint is concentrated at the base of the vehicle. Any of several types of restraining springs may be used, such as a series of coil springs spaced around the periphery at the base, pneumatic bags, or a combination of vertical cables to provide lateral restraint and coil springs to provide longitudinal and pitch restraint.

Regardless of the type of restraint used, a minimum spring constant of Mgd must be provided in pitch for static stability. The quantity d is the distance from the vehicle support point to its center of gravity. If a variation in the value of $g/\omega_r^2 L$ is assumed to represent a change in the gravitational field acting on a given vehicle, it is apparent that the minimum pitch restraint required for static stability (Mgd) is proportional to $g/\omega_r^2 L$. As the assumed acceleration due to gravity acting on a given vehicle is increased, the minimum spring restraint must also be increased thus increasing the effect on the natural frequencies. Similarly, the natural frequencies of a full-scale vehicle tested in a 1g field would be affected more by the minimum pitch restraint than would the natural frequencies of a replica model tested in the same gravitational field.

The effect that such a minimum pitch restraint has on the first free-free frequency has been computed and is shown by the lowest solid curve of figure 10. In this analysis the lateral restraint was assumed to be zero. Although these restraints are impractical from an experimental standpoint, the results indicate the minimum increase in the natural frequencies that can be obtained with this type of restraint.

Figure 10 indicates that for a uniform beam having a value of g/ω_r^2L corresponding to that of Saturn V (0.002), the minimum pitch restraint $\left(\text{denoted by } \omega_o/\omega_r = 0\right)$ increases the first natural frequency 4 percent, a relatively large increase when compared to the results obtained utilizing horizontal suspensions with reasonable cable lengths.

When the pitch restraint at the base is increased to provide a finite rigid-body pitch frequency $\left(\omega_o\right)$, the effect on the first flexural frequency can be quite pronounced. Although a ratio of rigid-body frequency to first free-free frequency of 1/5 may intuitively indicate a soft suspension, it is noted that for a value of $g/\omega_r^2L = 0.002$ the increase in frequency over that of a free-free beam is 12 percent.

An alleviating factor for this case is the compression in the beam due to gravity. The approximate combined effects of the suspension restraint and compression are indicated by the dashed lines in figure 10, which were obtained by multiplying the ordinates of the solid curves by the ordinates of figure 7 at corresponding values of g/ω_r^2L. Comparison of solid and dashed curves shows that at the higher values of g/ω_r^2L, the increase in natural frequency due to the restraint is largely offset by the decrease in frequency due to compression in the beam. The alleviation is not so great, however, in the more practical range of $g/\omega_r^2L < 0.002$.

Vertical Orientation With Two Lateral Restraints

In the preceding section, the destabilizing effect of gravity, Mgd, for a vertically oriented beam supported a distance d below its center of gravity was balanced with a pitch restraint concentrated at the base of the beam. In this section, the required pitch restraint is provided by two lateral springs (fig. 1(e)): one, K_u, near the upper end of the beam, and the other, K_L, near the lower end. It can be shown that a beam will be marginally stable if

$$\frac{1}{K_u} + \frac{1}{K_L} = \frac{h^2}{Mgd} \qquad (6)$$

where h is the vertical distance between lateral springs. From this relationship it may be seen that for minimum lateral restraint, h should be as large as feasible.

A method used at the Langley Research Center to provide the lateral restraint K_L is to support the test vehicle on two nearly vertical cables of length S attached to opposite sides near the base of the vehicle. For small deflections, the effective spring restraint at the cable attachment point is given by

$$K_L = \frac{Mg}{S} \tag{7}$$

Vibration excitation is in the direction normal to the plane of the cables since pitch restraint is very high in the plane of the cables.

The value of the upper lateral spring restraint required to stabilize a vehicle may be found by substituting equation (7) into equation (6). Thus,

$$K_u = \frac{Mg}{\frac{h^2}{d} - S} \tag{8}$$

Intuitively it may seem desirable to make the vertical support cables as long as feasible to minimize the restraint at the base. Equation (8) indicates, however, that as the cable length S is increased, the restraint required at the upper end to hold the vehicle erect becomes larger. For a uniform beam restrained at its ends, equations (7) and (8) reduce to

$$\frac{K_L L}{Mg} = \frac{1}{S/L} \tag{9}$$

and

$$\frac{K_u L}{Mg} = \frac{1}{2 - \frac{S}{L}} \tag{10}$$

These nondimensional spring constants are plotted in figure 11 as a function of the support cable length relative to the length of the beam. The spring restraint curves are seen to be symmetric about a value of $S/L = 1$. It is obvious that as the support cable length approaches zero, the lower restraint approaches infinity. It is not so apparent that as the support cable length approaches twice the beam length, the upper restraint required to hold the beam erect also approaches infinity.

The effect of these restraints on the first free-free bending frequencies of a uniform beam has been computed and is indicated in figure 12 in which the restrained bending frequency relative to the unrestrained frequency is plotted as a function of the relative cable length S/L for different values of the dimensionless parameter $g/\omega_r^2 L$. The restraints are seen to have a minimum effect on the first bending frequency when the support cables are equal in length to the beam. For a realistic value of

$g/\omega_r^2L < 0.002$, the increase in frequency due to the restraint is 0.8 percent or less. The frequency curves are seen to be quite flat over the middle range of cable lengths with a rather abrupt increase in frequency for values of S/L less than 0.2 and greater than 1.8.

To facilitate comparison with other suspension systems, these same data are replotted (solid curve) in figure 13 as a function of the dimensionless scaling parameter g/ω_r^2L for several values of S/L. The dashed curve in this figure is for the case where the upper restraint is applied at the uppermost node point of the first free-free bending mode. The only restraint affecting the bending mode is, therefore, that at the base, Mg/S. Thus the dashed curves of figure 13 are identical to those of figure 8. The shaded area represents a range of frequencies corresponding to upper restraint locations between the tip of the beam and the upper node. For the shorter support cable lengths, there is little difference in the frequency whether the upper restraint is located at the node or at the tip of the beam. When $S/L = 1$, however, the effect of the restraints on the first bending frequency is twice as great when the upper restraint is located at the tip as when it is applied at the nodal point. The effect, however, is small in either case. When these restraining effects are compared with those of figure 10 for support cables of reasonable length, the two-cable vertical suspension is seen to have appreciably less effect on the first free-free bending frequencies than does a suspension system in which the restraint is concentrated at the base of a vehicle.

The approximate combined effects of the restraint of the two-cable vertical suspension and the compression due to gravity were obtained by multiplying the ordinates of figures 7 and 13. The results are indicated in figure 14. In computing the flexural modes of aerospace vehicles, the effects of compression due to gravity are usually considered of secondary importance and ignored, whereas in cases of discrepancy between analytical and experimental results, the effects of the suspension system are usually suspect. The results of figure 14, however, indicate that for support cable lengths greater than approximately one-sixth the length of the beam, the ratio of the restrained to unrestrained frequency is less than unity, indicating that in this case the restraining effects of the suspension are overshadowed by the gravity effects.

Variations of Two Lateral Restraints

Long support cables.- A method of supporting a launch vehicle in a vertical attitude with a minimum of restraint while maintaining static stability has been used at the Langley Research Center for several years and is shown in figure 15. The weight of the vehicle is carried by two nearly vertical support cables running between overhead supports and the base of the vehicle. Stability is provided by two restraining cables tied horizontally between the support cables and the periphery of the vehicle at some point above the vehicle's center of gravity.

The minimum upper restraint K_u required to hold the vehicle erect is obtained by varying the support cable separation distance, a, which in turn varies the tension in the horizontal restraining cables until the vehicle is marginally stable. Based on dimensions defined in figure 15 an analysis of this restraint system shows the minimum cable separation required to cause the vehicle to stand erect to be

$$a_o = \left(\frac{bf}{h - d} - \frac{cf}{h}\right) + b \tag{11}$$

When $a < a_o$, the vehicle will tilt a few degrees to a stable position, and when $a > a_o$, the vehicle will have a finite rigid-body frequency in pitch.

Experimental verification of the critical cable separation distance a_o is shown in figure 16. For a given vertical location, h, of the restraining cables, the support cable separation distance, a, was progressively increased until the vehicle was marginally stable in a vertical attitude. Correlation of the experimental results with calculated values shows excellent agreement.

The effective spring restraint of this suspension system is decidedly nonlinear with pitch angle. The nonlinearity increases as the location of the restraining cables is raised further above the center of gravity. The nonlinear spring is advantageous because it can apply the minimum pitch restraint necessary to hold the vehicle erect but quickly stiffens with increasing angle of tilt. As the weight and center-of-gravity conditions change for a given test vehicle, the only adjustment needed is the cable separation distance, a, as the longitudinal (vertical) location of the center of gravity changes.

The effects of this type of suspension on the first free-free bending mode frequency of three very flexible uniform beams $\left(g/\omega_r^2 L = 0.002, 0.003, \text{and } 0.006\right)$ are illustrated in figure 17. The support cables for the experiments were 1.25 times the beam length. Frequencies were obtained for the upper minimum spring restraint K_u located at the tip of the beam and at the upper nodal point. Approximate calculated frequencies were determined as in figure 14 and show very good agreement with the experimental results. Again, the decrease in beam frequency due to compression completely overshadows the small increase in frequency attributable to the restraints. Further experiments with these models restrained at the tip indicated that the cable separation distance, a, is not critical in that the natural frequency increases rather slowly after the cable separation distance exceeds the minimum required for stability.

Short support cables.- Although the suspension system described in the previous section has proved very useful in vibration tests of numerous dynamic models of launch vehicles, two factors limit its usefulness when applied to full-scale launch vehicles. One factor is the requirement that the restraining cables and, hence, the support cable tiedown points, be well above the center of gravity of the vehicle. This condition necessitates a tall and massive service tower capable of supporting the entire mass of the vehicle. The other factor is the need to provide a mechanism (such as a horizontal track and dollies) at the top of the tower which will permit adjustment of the support cable separation while supporting the mass of the vehicle.

The foregoing shortcomings have been largely overcome in the suspension system which utilizes short support cables as depicted in figure 18. The principle is the same as for the long cable suspension (fig. 15) in that the horizontal component of the support cable tension is used to apply the necessary tension to the restraining cables to keep the vehicle statically stable and in a vertical attitude. Rather than attaching the restraining cables directly between the support cables and the vehicle, the restraining cables are routed from the support cables upward, through pulleys attached to the service tower, to a point on the vehicle well above its center of gravity. The tension in the restraining cables necessary to hold the vehicle erect is obtained by adjusting the length of the restraining cables through the use of turnbuckles. The cable length adjustment in turn varies the horizontal component of the support cable tension. The use of turnbuckles to adjust the restraining cable tension is much simpler and less costly than the overhead tracks and dollies described previously.

The main improvement achieved by the foregoing anchored upper pulleys stems from the fact that the upper ends of the support cables may now be located lower than the vehicle center of gravity, thereby reducing the height of that portion of the service tower which must support the entire mass of the vehicle. The total height of the service tower must still approach the length of the vehicle in order to support the upper restraining cable pulleys. That portion of the tower above the upper ends of the support cables, however, may be of relatively light weight since the tension in the restraining cables is only a small percentage of that in the supporting cables.

For small lateral displacements, the effective upper lateral restraint offered by the restraining cables at their point of attachment to the test vehicle is equal to $4T_r/b$, where T_r is the tension in the restraining cable below the turnbuckle and b is the distance between pulleys (fig. 18). The required tension can be determined by equating $4T_r/b$ to equation (8) which results in the expression

$$T_r = \frac{Mg\frac{b}{h}}{4\left(\frac{h}{d} - \frac{S}{h}\right)} \tag{12}$$

For a typical test vehicle with restraint at the tip, b/h might equal approximately 1/4, $h/d \approx 3$, and $S/h \approx 1/3$. The tension in the restraining cable below the turnbuckle would then be approximately one-fortieth the weight of the vehicle.

With the assumption of small angles and near vertical support cables, it can be shown that the lateral deflection Δ of the support cables due to the lateral force T_r is

$$\Delta = \frac{2ST_r}{Mg}\left(\frac{f}{S}\right)\left(1 - \frac{f}{S}\right) \tag{13}$$

Substituting equation (12) into equation (13) yields the following expression for the ratio of lateral displacement to cable length:

$$\frac{\Delta}{S} = \frac{\frac{f}{S}\frac{b}{h}\left(1 - \frac{f}{S}\right)}{2\left(\frac{h}{d} - \frac{S}{h}\right)} \tag{14}$$

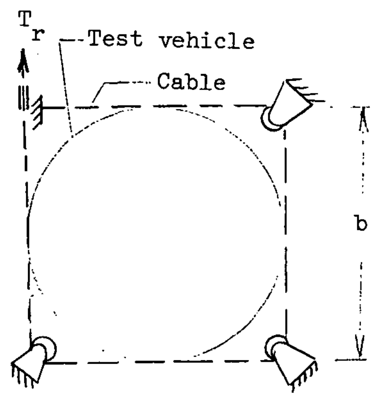
If for the example cited above, f/S is assumed to be equal to 1/3, the required lateral deflection of the support cable is found from equation (14) to be approximately 1 percent of the cable length S.

As shown in figure 12 for a uniform beam restrained at its extremities, the minimum effect of restraint is obtained when the vehicle and support cables are of equal length $\left(S = L \text{ and } K_u = K_L\right)$. It might appear therefore that shorter support cables would introduce an unnecessary penalty. As noted previously, however, the restraining effects are rather insensitive to cable length as evidenced by the flatness of the middle portion of the curves of figure 12. More importantly, since many launch vehicles are not of uniform construction but are relatively stiff and massive at the base, the minimum effect on frequency would be achieved if $K_L > K_u$, i.e., $S/L < 1$. As an example, if the restraint K_u is applied at the tip $(h/L = 1)$ and the support cables are equal in length to the vehicle $(S/L = 1)$ but the c.g. is moved from midlength to a distance of one-third vehicle length from the base $(d/L = 1/3)$, equation (8) shows that K_u must be infinite to hold the vehicle upright. If, on the other hand, K_u is set equal to K_L, equations (7) and (8) are satisfied when $S/L = 1/2$. When the reduced flexibility at the tip is considered, it is apparent that for minimum effect on frequency in this hypothetical case, K_u should be less than K_L and hence, $S/L < 1/2$. In some instances where the tip of the test vehicle is extremely flexible relative to the rest of the structure (such as the escape tower of the Apollo-Saturn V), the restraining cables would be more appropriately located at a hard point further aft from the nose of the vehicle.

Additional degrees of freedom.- In the two-cable suspension described above, excitation for the lateral bending modes was normal to the plane of the cables since both the pitch restraint at the base and the upper lateral restraint are relatively stiff in the plane of the cables. A method used at the Langley Research Center to eliminate the pitch restraint in the plane of the support cables is illustrated in figure 19(a) and consists of passing a continuous support cable around a pair of pulleys affixed to the base.

Assuming frictionless pulleys, the lateral restraint K_L is then Mg/S in all directions. Where static stresses are high, such as for very large vehicles or uncommonly fragile smaller ones, it may be expedient to distribute the loads to several pulleys around the periphery of the base as in figure 19(b).

A convenient method of providing the minimum upper restraint in all directions is illustrated in the sketch at right. As indicated, a flexible cable is passed over three pulleys located so that the cable is tangent to the periphery of a cross section of the vehicle at four points. The minimum restraining cable tension T_r to hold the test vehicle erect is given by equation (12). This tension may be supplied by hanging weights on the end of the restraining cable, making it a simple matter to adjust the required lateral restraint as the fuel load is varied. It should be noted that the value of tension given by equation (12) will result in a neutrally stable vehicle in pitch. Thus, if the vehicle is deflected to one side, the restraining cable will not restore it to a vertical attitude. In practice, therefore, a slightly larger tension will be required to provide a restoring moment.



A more satisfactory method of applying tension to the restraining cable is through a linear spring attached to a winch or jack. In this arrangement, if the vehicle is deflected to one side, the spring will increase the tension in the restraining cable and restore the vehicle to a vertical attitude. The net effect in pitch is that of a nonlinear spring with a minimum of restraint when the test vehicle is vertical. The degree of nonlinearity may be controlled by the stiffness of the linear spring in series with the restraining cable.

Suspension Systems for Longitudinal Testing

The horizontal suspensions discussed earlier are satisfactory for the excitation and measurement of longitudinal responses since they provide extremely small restraint in this direction. The vertical suspensions of figures 9, 14, 15, 18, and 19(a) may be altered for this purpose, without affecting the required lateral restraints, by the addition of suitable springs in series with the support cables.

Any of several types of springs may be distributed about the base of a test vehicle (fig. 20) to provide longitudinal resilience and some pitch restraint while the remainder of the required pitch restraint is provided as before, by two lateral restraints K_u and K_L. That portion of the pitch restraint provided by the longitudinal springs is equal to $\frac{1}{2} K_v r^2$ where K_v is the total vertical restraint and r is the radial distance to these springs from the vehicle center line. Thus an extension of equation (6) to include the pitch component of the vertical restraint becomes

$$\frac{h^2}{\frac{1}{K_u} + \frac{1}{K_L}} + \frac{1}{2} K_v r^2 \geqq Mgd \tag{15}$$

For very small values of K_v or r^2, equation (15) reverts to equation (6) while for sufficiently large combinations of K_v and r, no lateral restraints are needed for static stability. The latter arrangement may, as noted previously, result in a larger effect on the natural frequencies.

CONCLUDING REMARKS

The effects of several cable suspension configurations on the first free-free flexural frequency of uniform beams have been determined experimentally and by analysis. The trends established by this study should be applicable to slender aerospace structures. The results given herein include the stiffening effect but not the mass effects of the suspensions. Because the ratio of the minimum support cable mass to vehicle mass increases with vehicle size, cable resonances are more apt to be a problem with very large test vehicles than with smaller models. Based on the results of this study the following general comments are warranted.

The natural flexural frequency measurement error induced by the constraint of cable suspensions of similar geometry is generally proportional to the dimensionless scaling parameter $g/\omega_r^2 L$, where g is the gravity acceleration, ω_r a reference structural frequency, and L the vehicle length. Hence, the larger the test vehicle (under 1g gravity conditions), the larger is the flexural frequency measurement error attributable to suspension constraint.

For horizontally oriented beams the suspension constraint effects (natural frequency measurement error) were inversely proportional to support cable length. For test vehicles of conventional size and flexibility, however, these restraint effects were minor.

For vertically oriented vehicles supported by vertical cables of moderate length attached near the base, the restraining effects were overshadowed by the effects of beam compression due to gravity; i.e., the natural frequencies of the restrained beams were appreciably lower than the frequencies of the free-free beams.

Restraint effects were less when the minimum static stability of vertically oriented beams was provided by a lateral spring near each end of the beam than when all the restraint was concentrated at the base.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., June 3, 1974.

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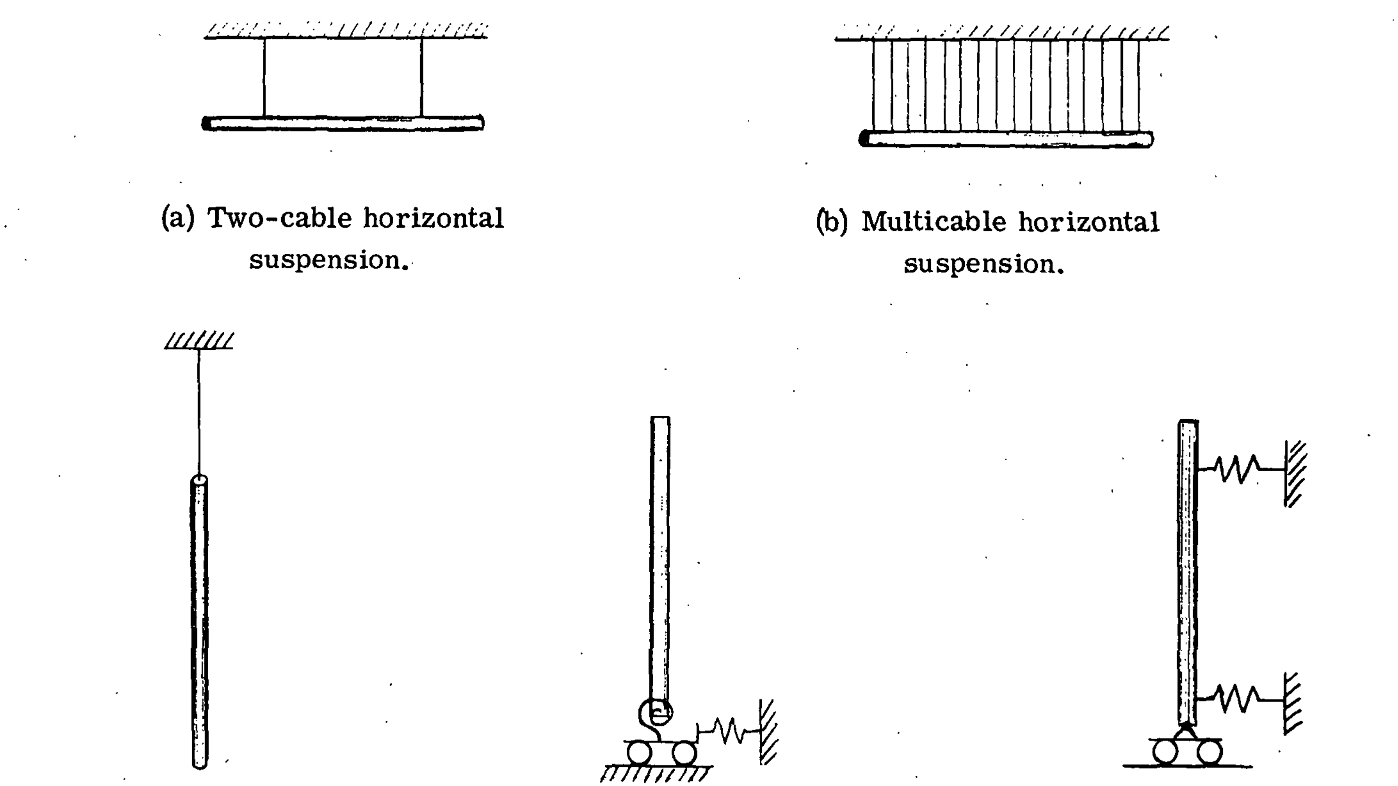


(a) Two-cable horizontal suspension.

(b) Multicable horizontal suspension.

(c) One-cable vertical suspension.

(d) Vertical orientation with restraint at base.

(e) Vertical orientation with two lateral restraints.

Figure 1.- Types of suspension systems investigated.

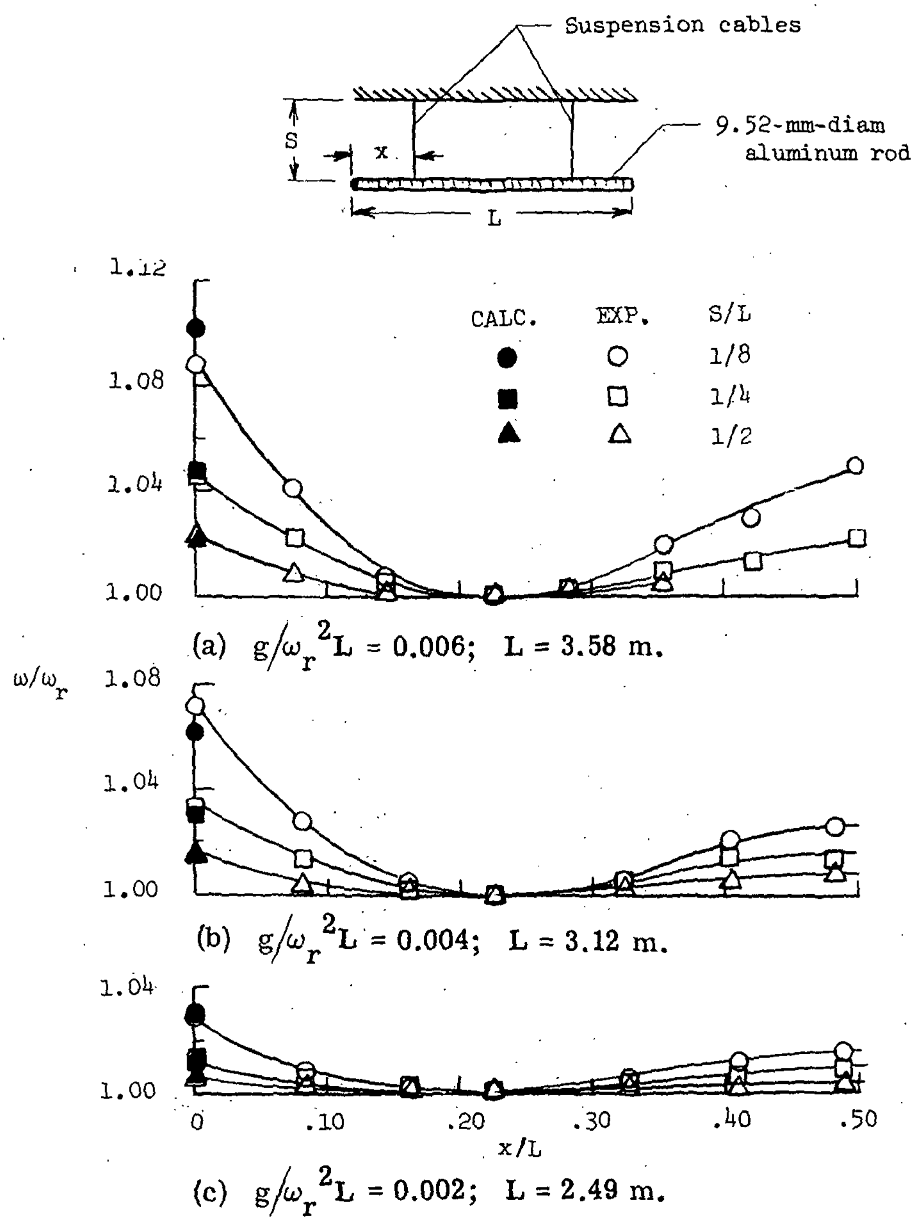


Figure 2.- Effects of two-cable horizontal suspension system on experimental and calculated first free-free bending frequency.

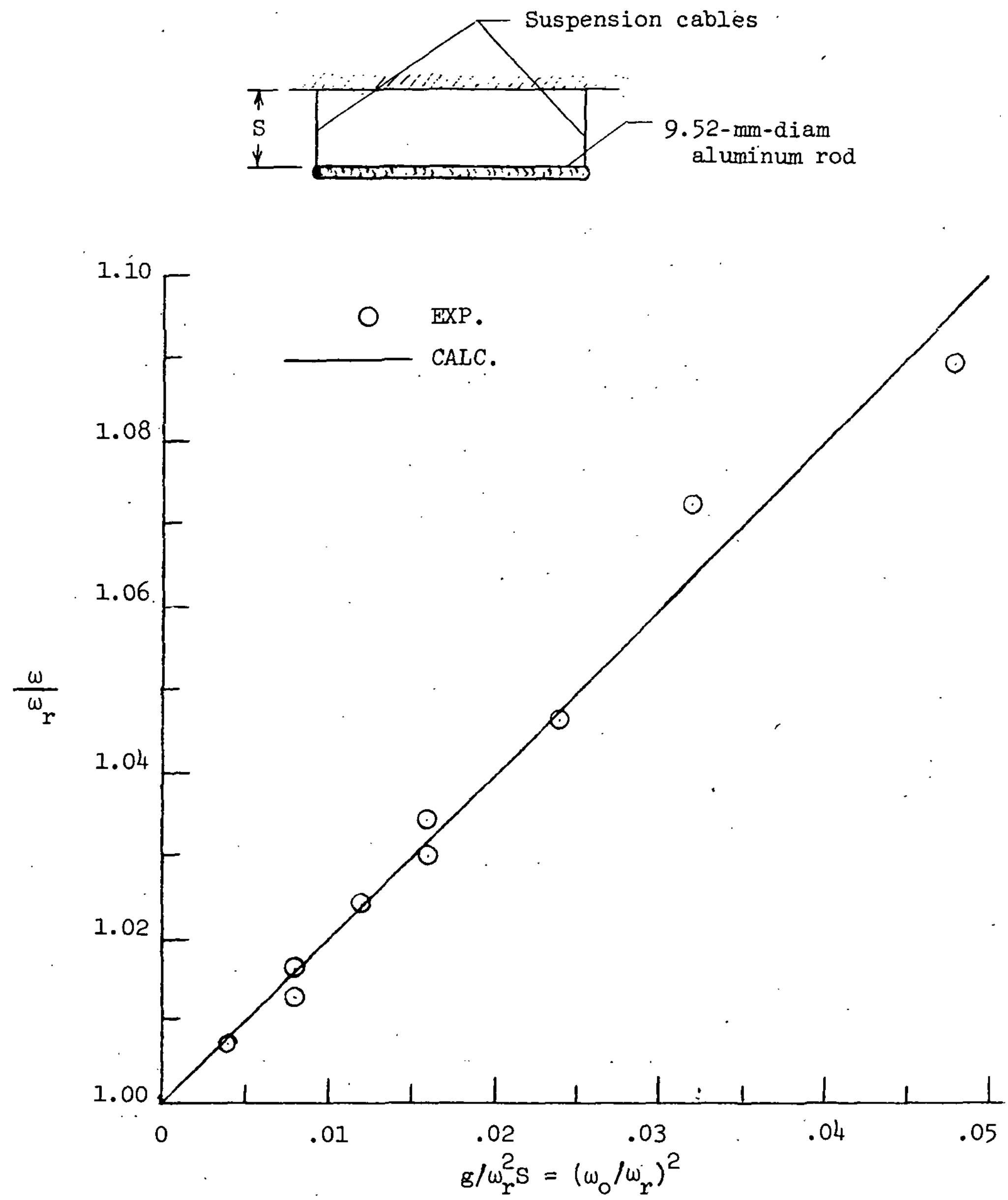


Figure 3.- Effects of two-cable horizontal suspension system as a function of pendulum frequency. $x/L = 0$.

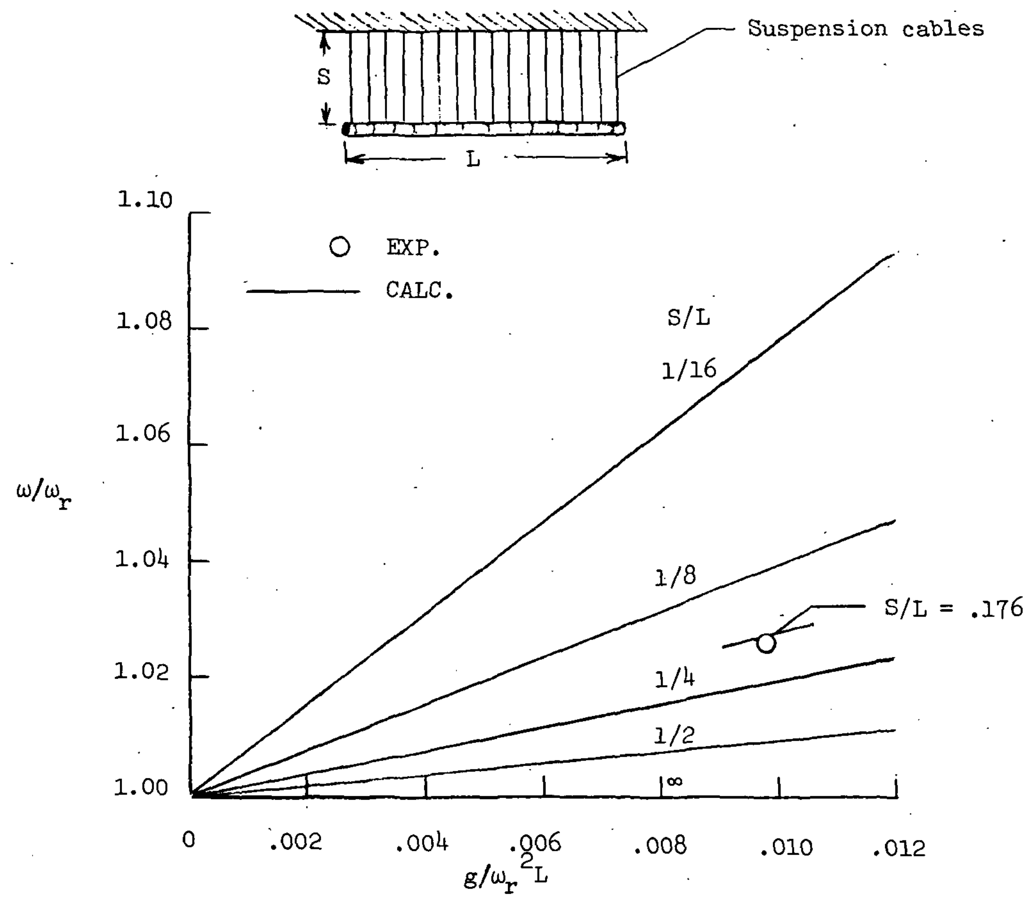


Figure 4.- Effects of multicable horizontal suspension system on calculated and experimental first free-free bending frequency.

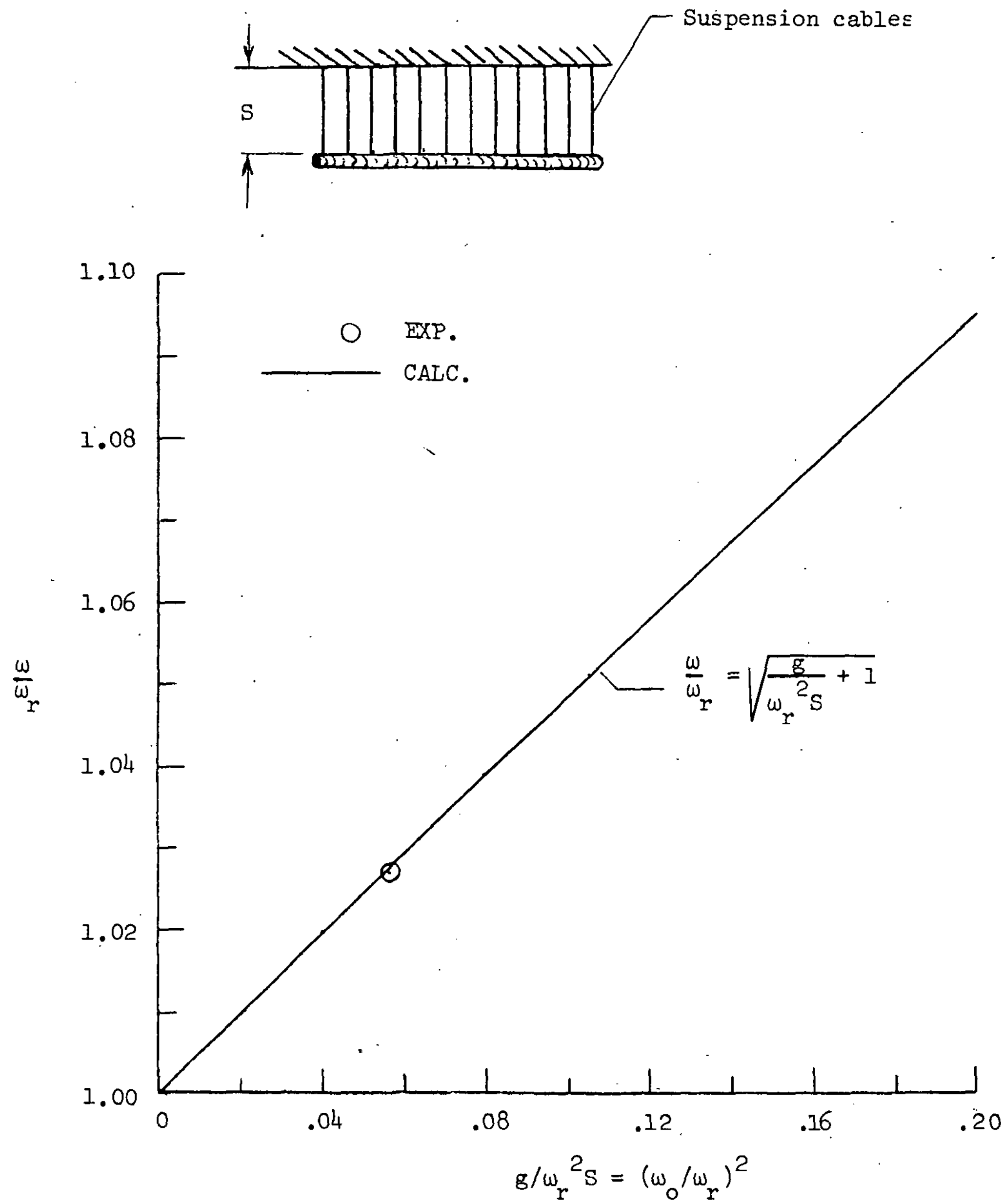


Figure 5.- Effects of multicable horizontal suspension system as a function of pendulum frequency.

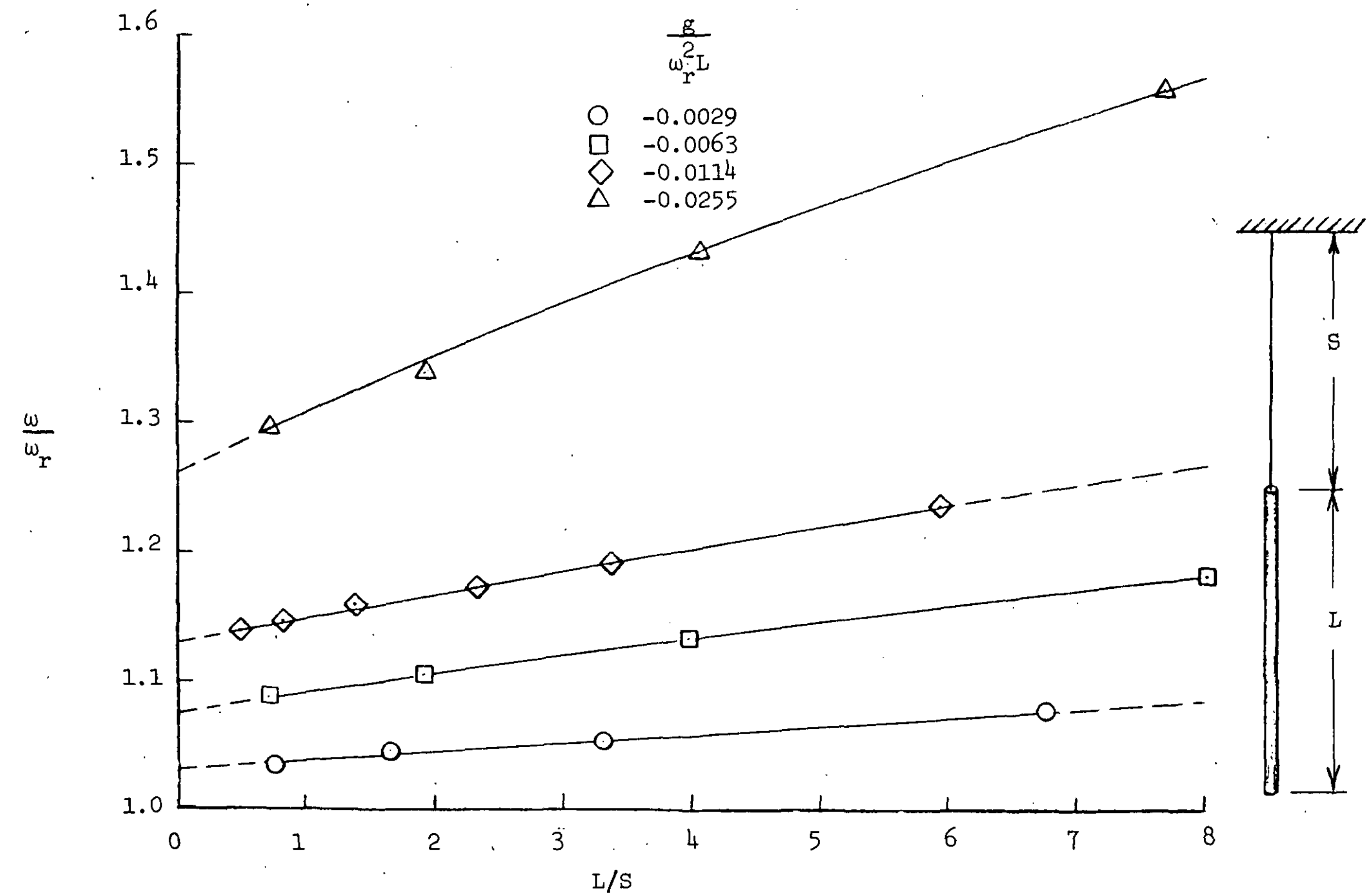


Figure 6.- Effects of one-cable vertical suspension on experimental first free-free bending frequency.

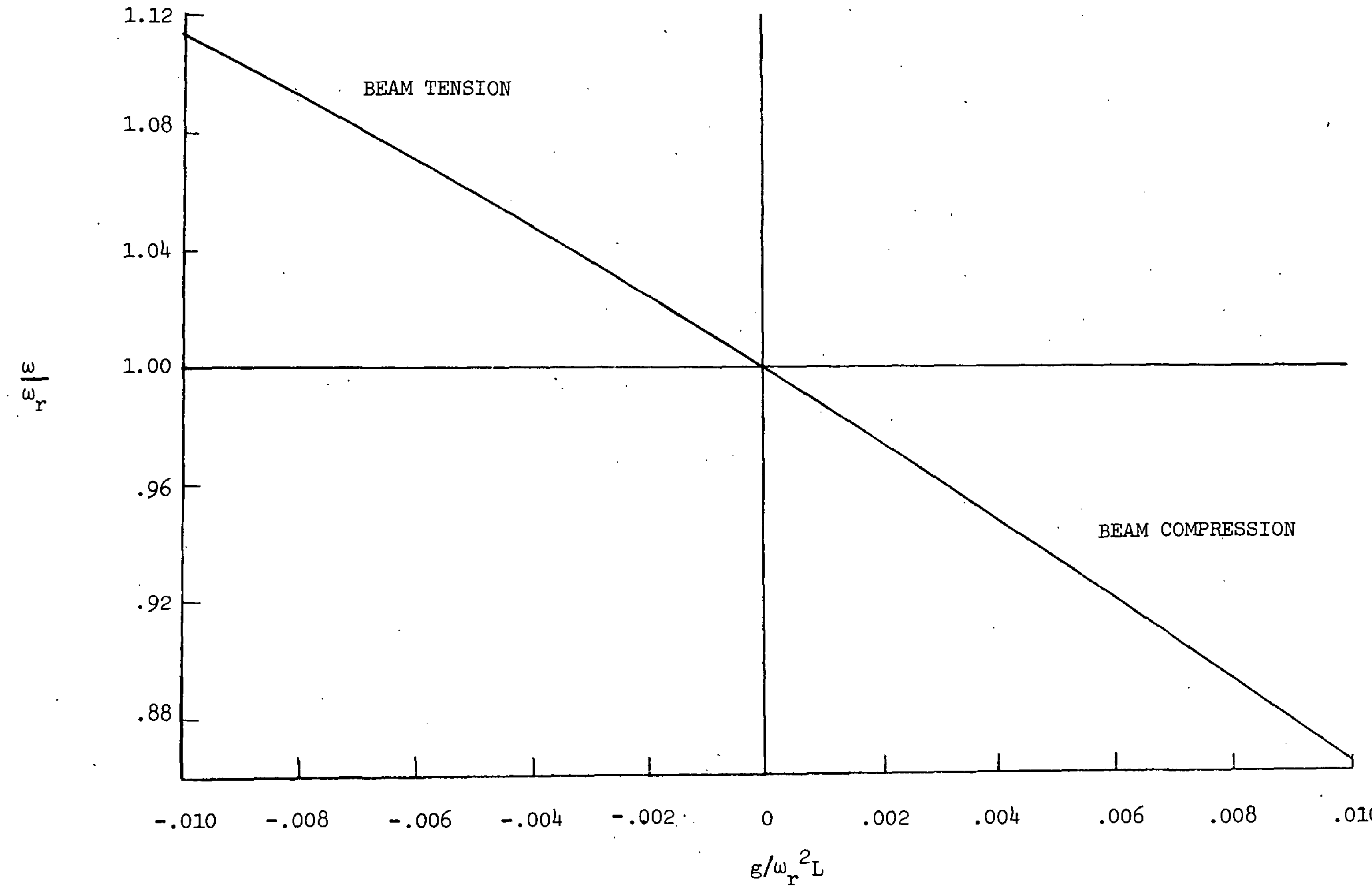


Figure 7.- Effect on longitudinal acceleration on calculated first free-free bending frequency of a uniform beam (ref. 5).

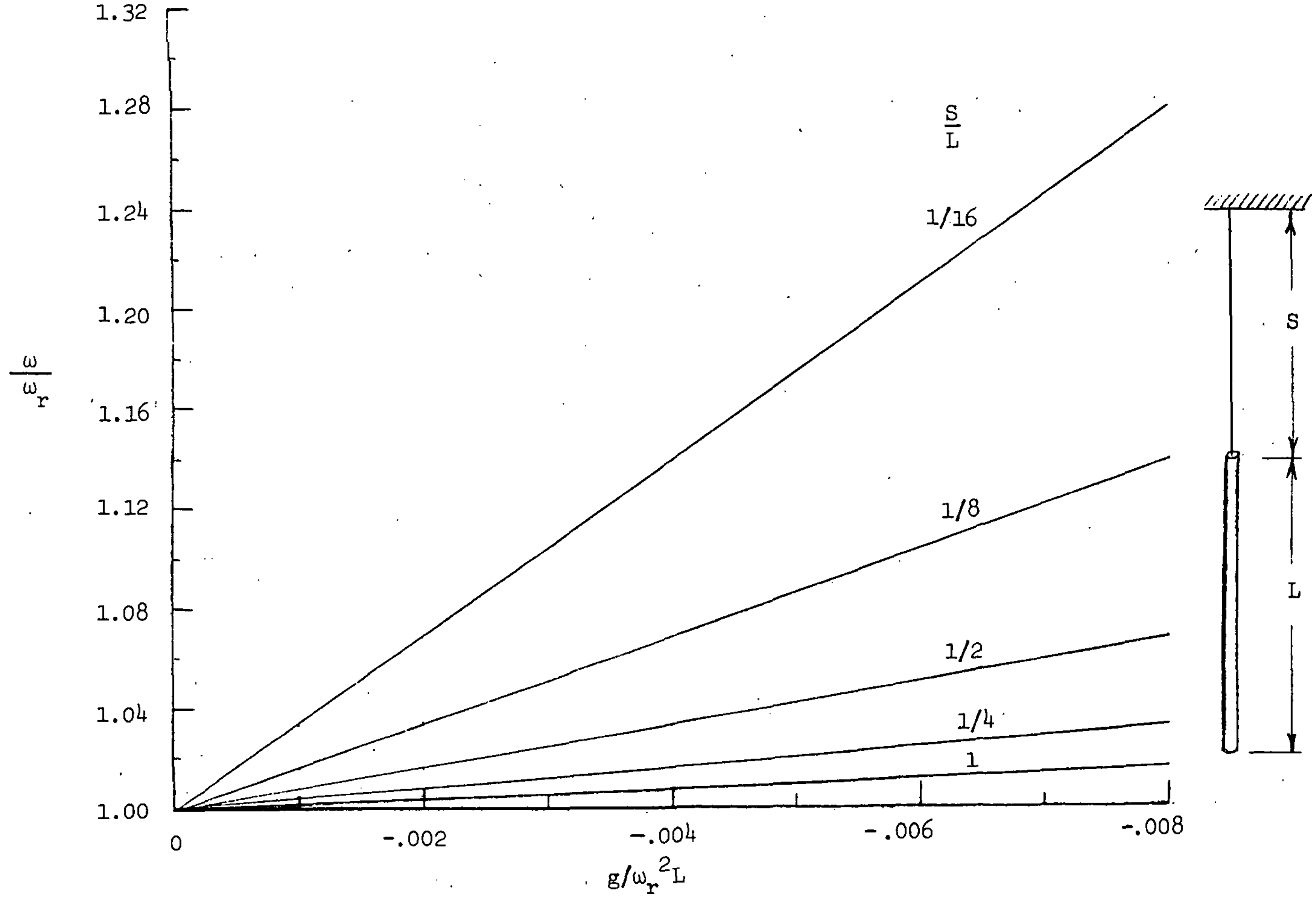


Figure 8.- Calculated effect of spring restraint Mg/S applied at one end of uniform beam.

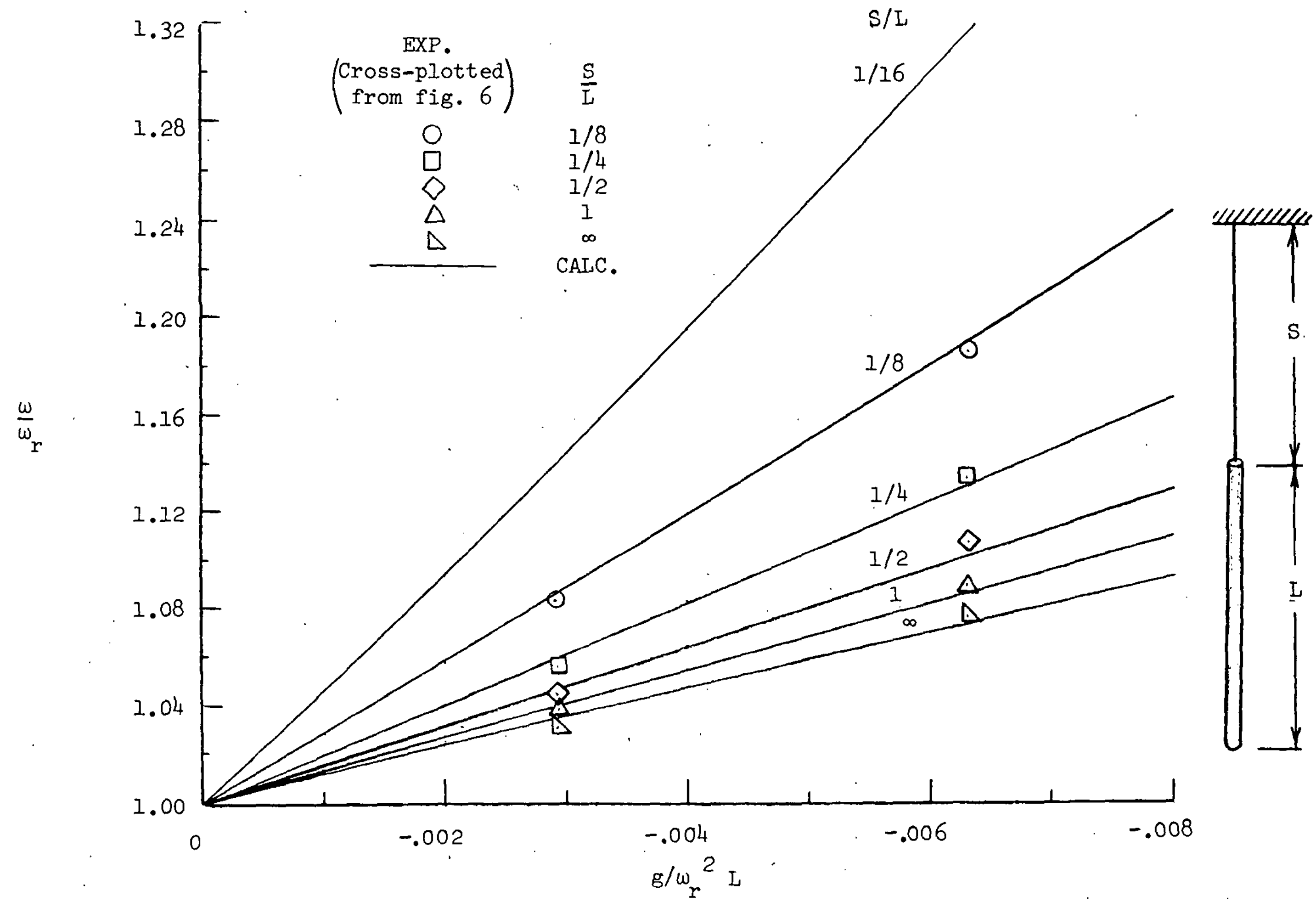


Figure 9.- Approximate combined effect of spring restraint Mg/S and beam tension compared with experimental results.

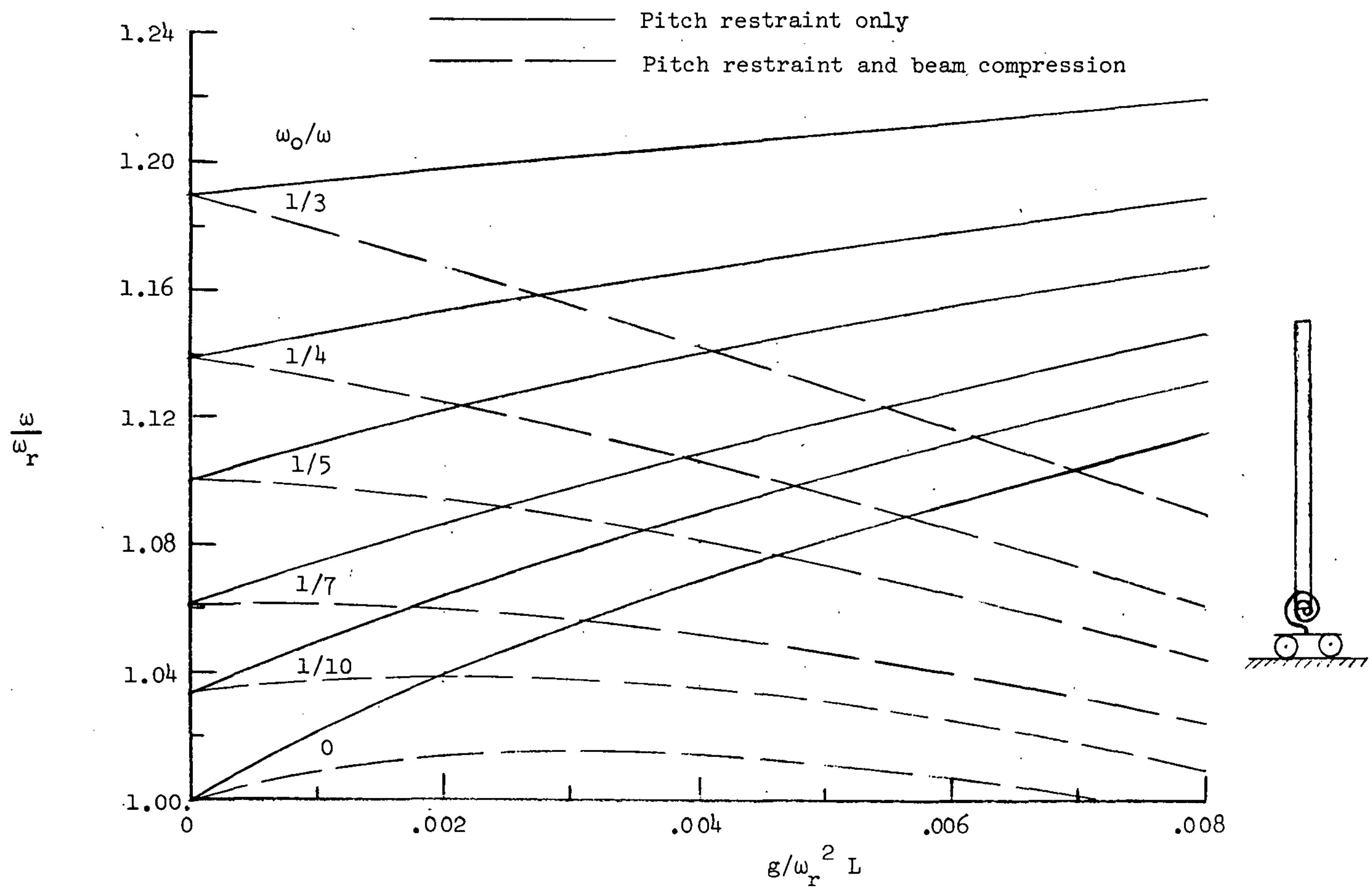


Figure 10.- Calculated effect of pitch restraint applied to base of uniform beam and beam compression.

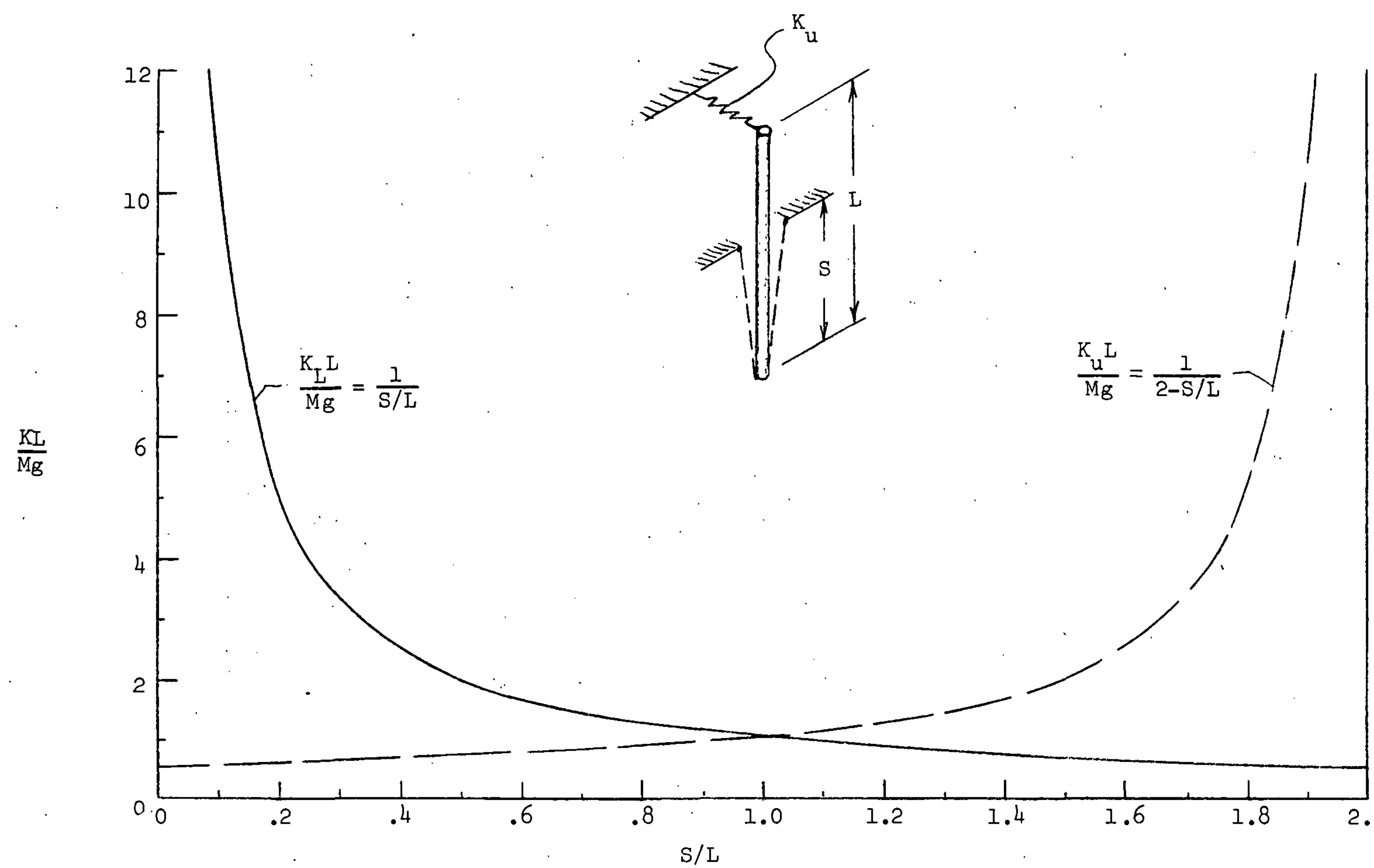


Figure 11.- Calculated lateral spring restraints for static stability of uniform beam supported by vertical cables attached to its base.

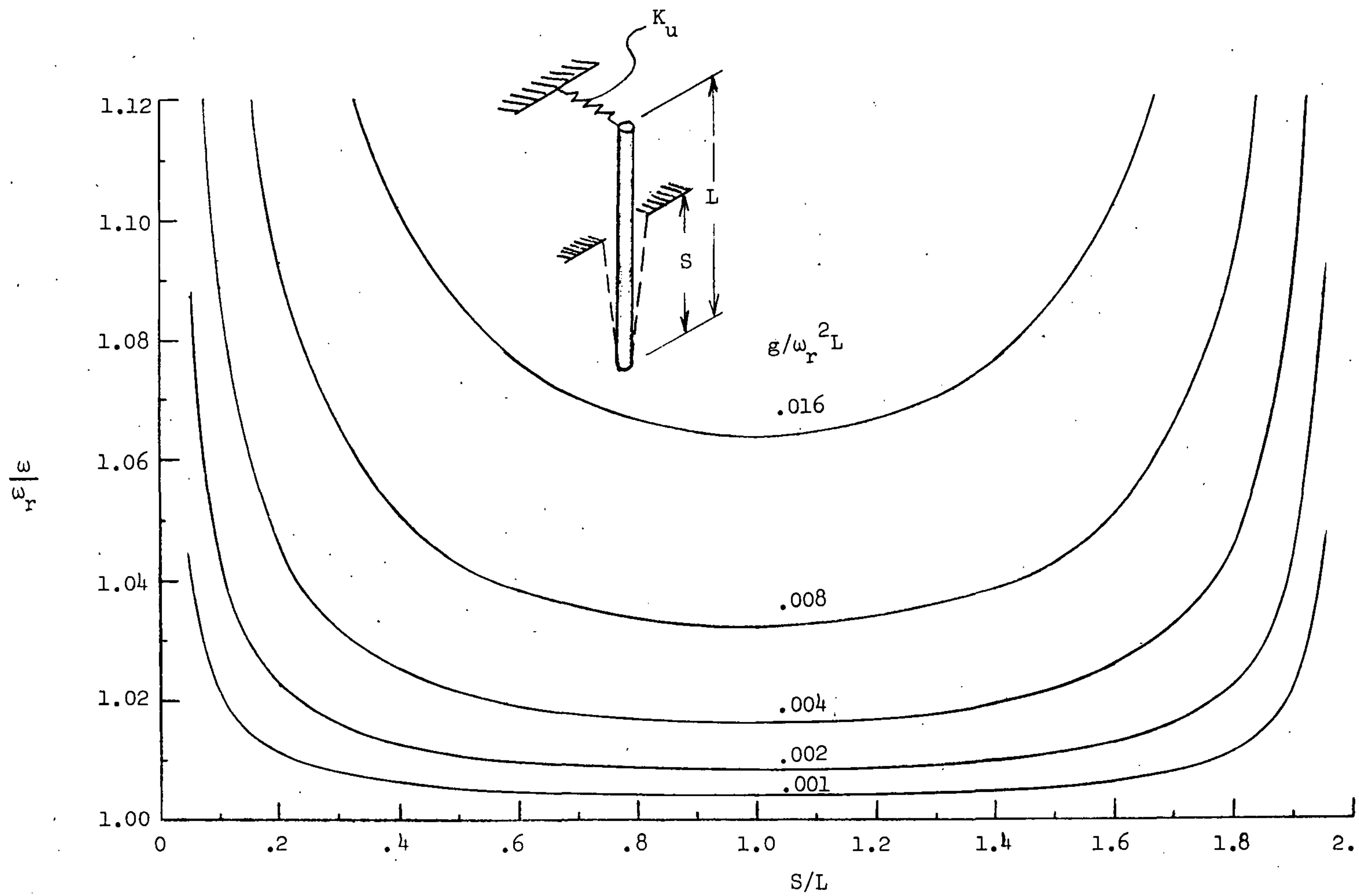


Figure 12.- Calculated effects of two-cable suspension system with minimum upper restraint K_u applied at tip of uniform beam.

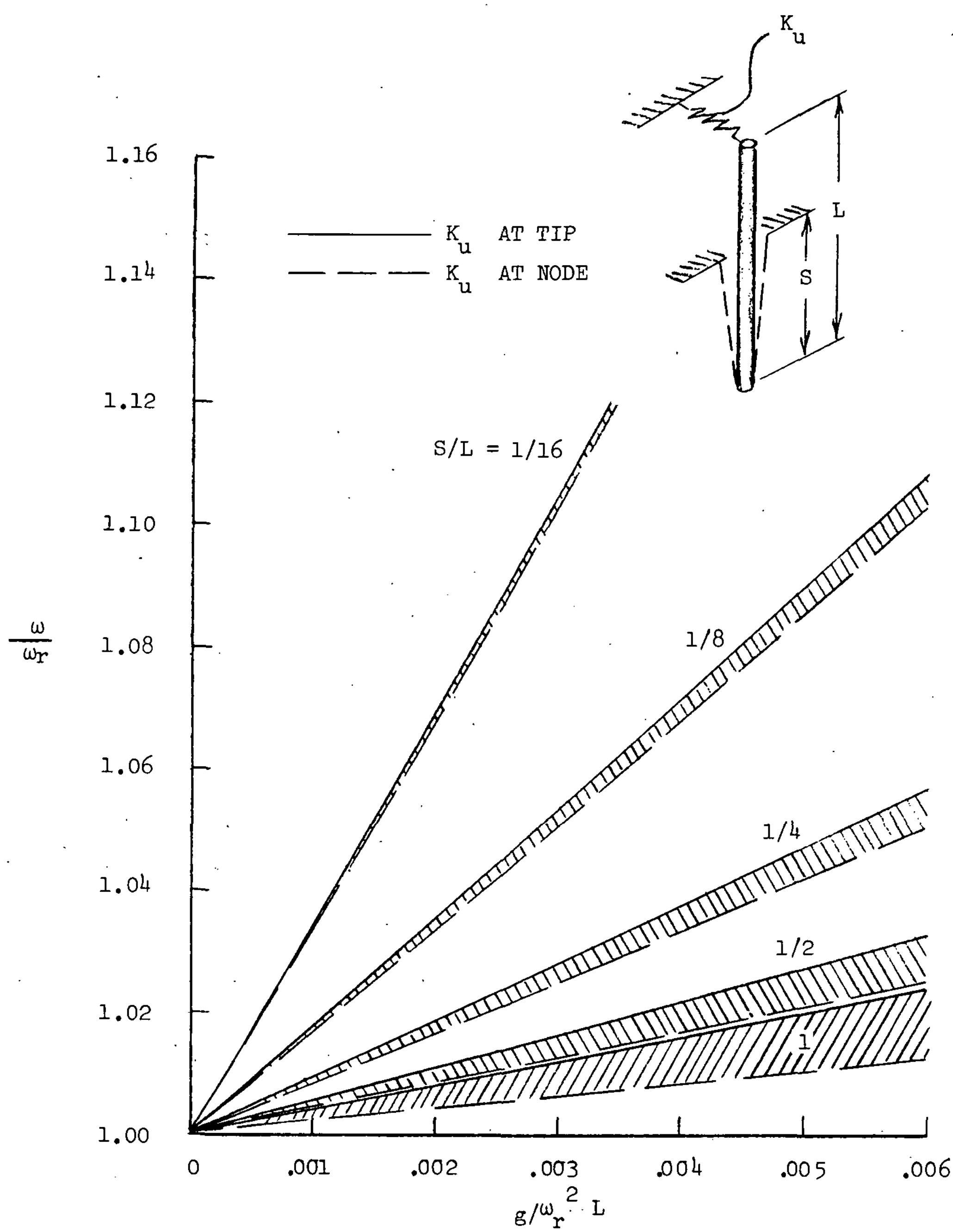


Figure 13.- Calculated effects of two-cable vertical suspension system with minimum upper restraint K_u applied at tip or at node point.

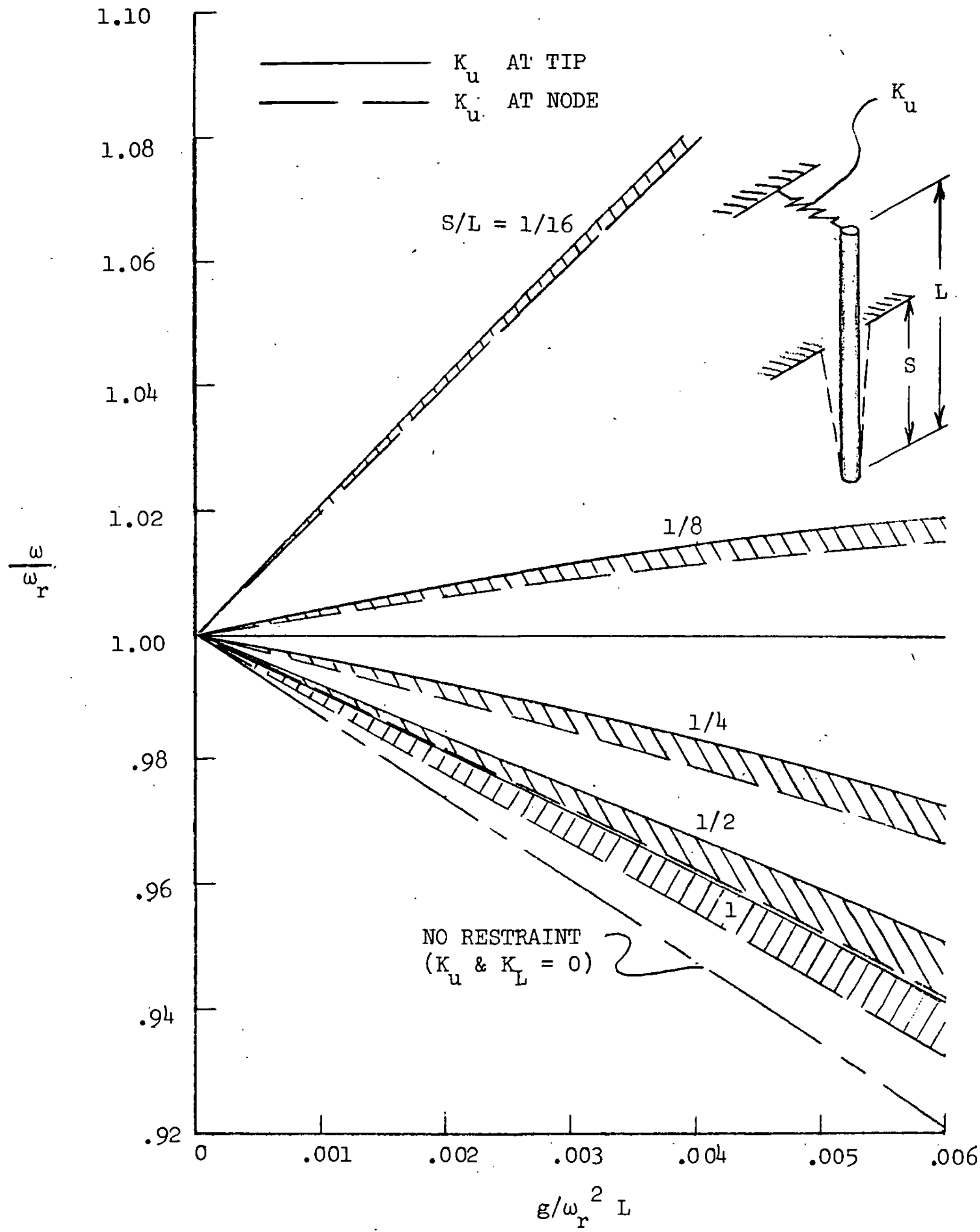


Figure 14.- Calculated approximate combined effect of two-cable vertical suspension system and beam compression due to gravity.

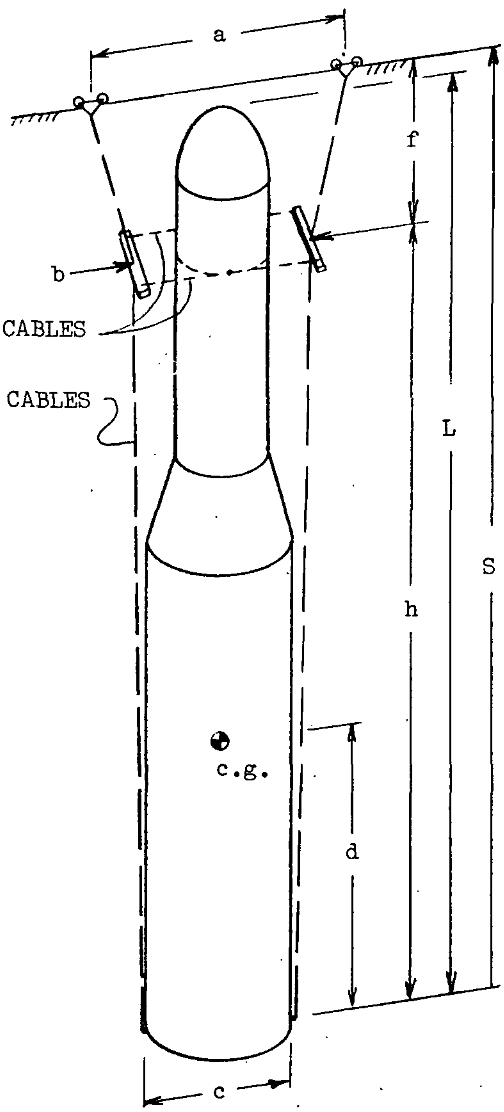


Figure 15.- Vertical orientation with long support cables.

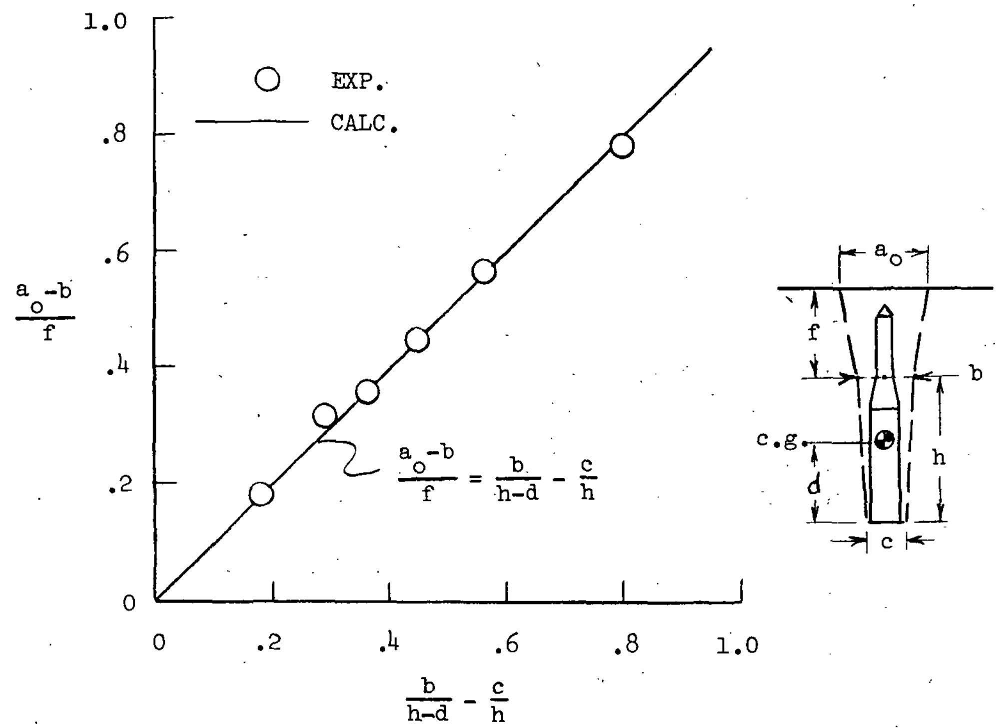


Figure 16.- Comparison of experimental and calculated values of minimum support-cable separation a_o required to maintain vertical attitude.

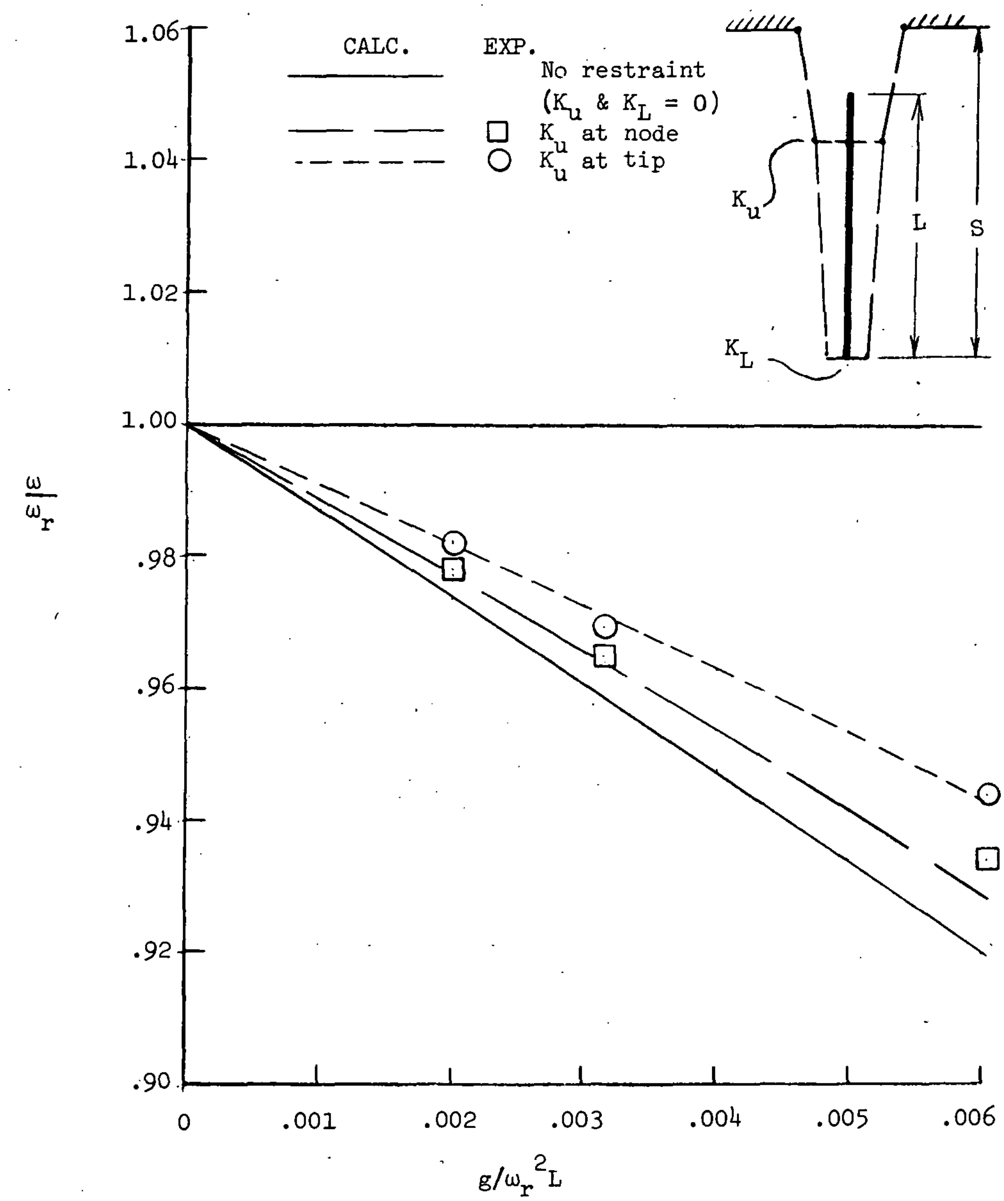


Figure 17.- Comparison of experiment with calculated approximate combined effect of two-cable vertical suspension system and beam compression. S/L = 1.25.

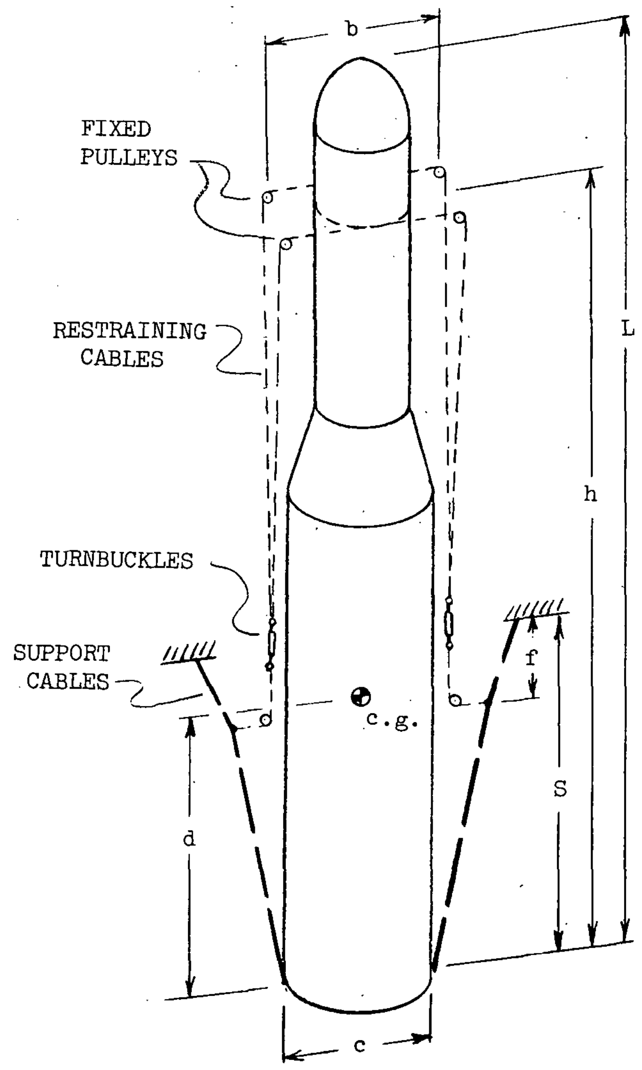
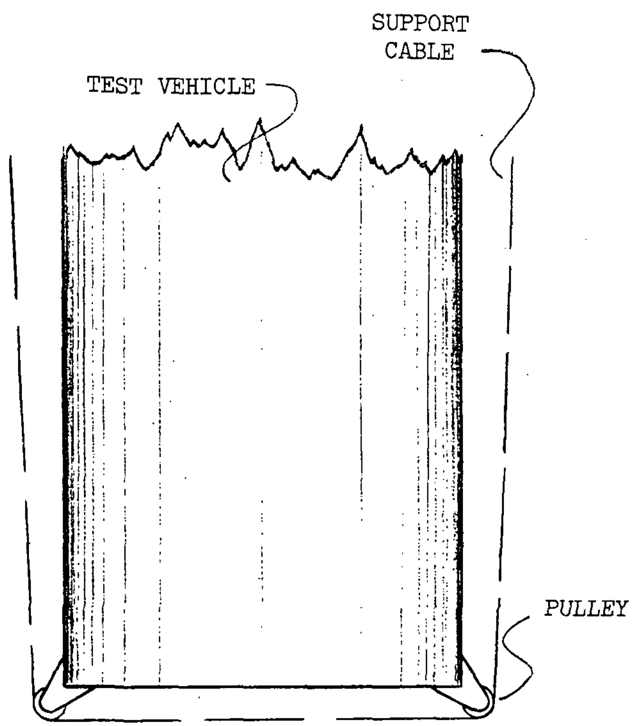
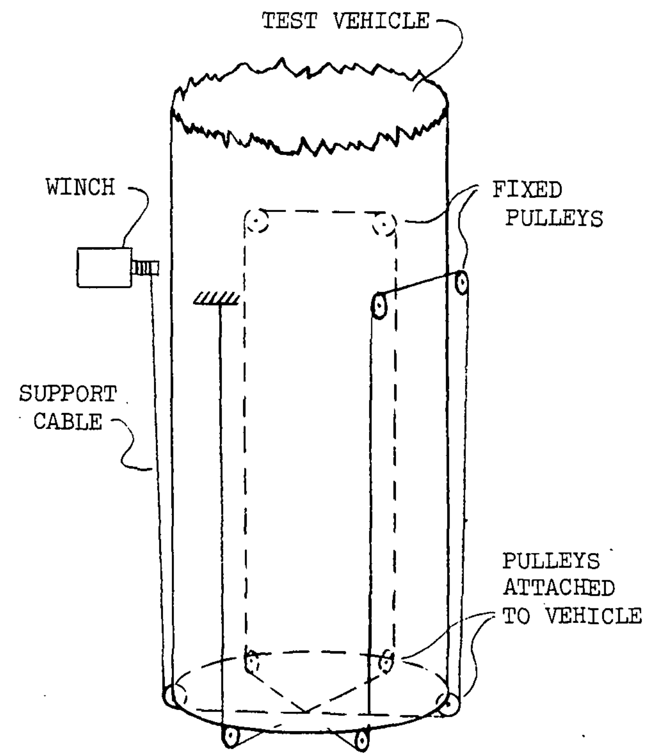


Figure 18.- Vertical orientation with short support cables.



(a) Two-cable suspension.



(b) Multicable suspension.

Figure 19.- Methods of obtaining omnidirectional freedom in pitch at base of test vehicle.

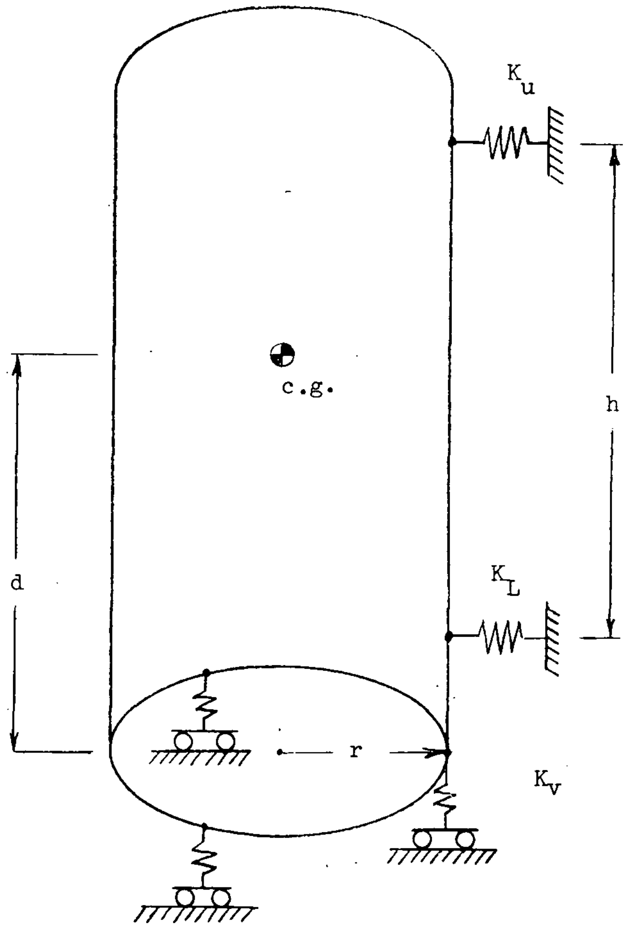


Figure 20.- Combined longitudinal and lateral restraint.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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