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MATHEMATICAL MODEL FOR THE SIMULATION OF DYNAMIC DOCKING TEST SYSTEM (DDTS) ACTIVE TABLE MOTION

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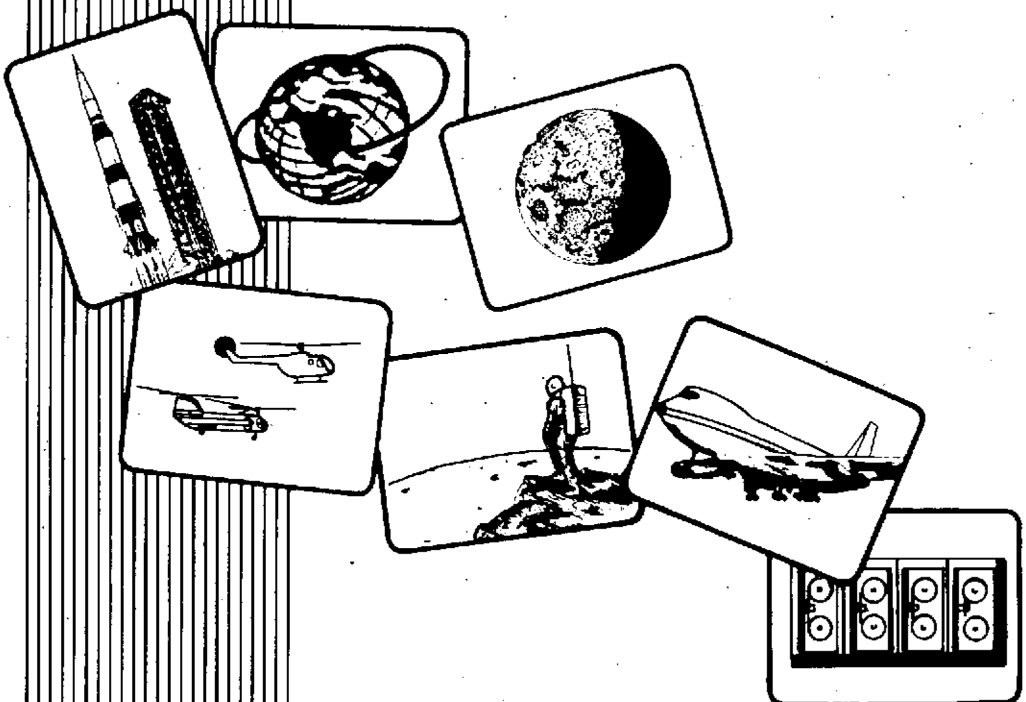

THE BOEING COMPANY

HOUSTON, TEXAS

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MATHEMATICAL MODEL FOR THE SIMULATION OF DYNAMIC DOCKING TEST SYSTEM (DDTS) ACTIVE TABLE MOTION

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ABSTRACT

This document describes the mathematical model developed to describe the three-dimensional motion of the Dynamic Docking Test System (DDTS) active table. The active table is modeled as a rigid body supported by six flexible hydraulic actuators which produce the commanded table motions.

Key Words

Docking Simulator
Dynamic Docking Test System (DDTS)
Equations of Motion
Hydraulic Actuator
Mathematical Model
Motion Simulator

/

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ILLUSTRATIONS

REFERENCES

1. D2-118544-2, "Dynamic Docking Test System (DDTS) Active Table Computer Program NASA Advanced Docking System (NADS)," August 30, 1974.

2. Merritt, H. E., Hydraulic Control Systems, John Wiley & Sons Inc., 1967.

1.0 INTRODUCTION

The development of the three-dimensional mathematical model and computer program which simulates the dynamic motion of the DDTS active table in response to table motion commands is documented in two volumes. Volume 1 presents the derivation of the mathematical model and Volume 2 (Reference 1) describes the resulting computer program, "NASA Advanced Docking System (NADS)."

The active table shown in Figure 1-1 is modeled as a rigid body, and each of the six actuators is modeled as a flexible rod with pinned ends. The model includes nonlinear hydraulic equations for the hydraulic actuators and a mathematical representation of the electronic control system for each actuator. Actuator position, velocity, and differential pressure across the hydraulic piston are used as feedback signals in the control system. The nomenclature used in the equations is shown in the Appendix.

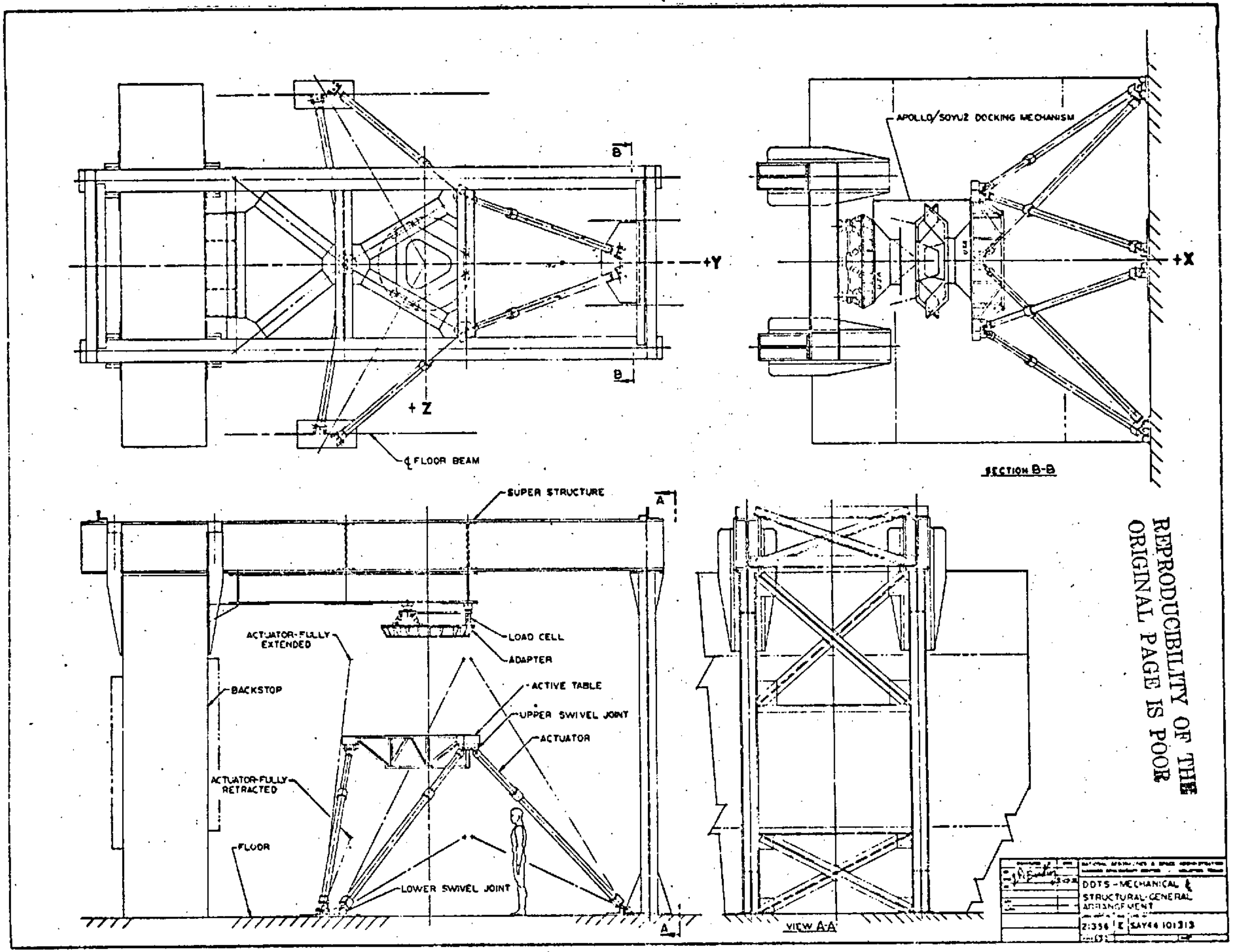


Figure 1-1. DDTS Simulator Facility

2.0 COORDINATE SYSTEMS AND TRANSFORMATIONS

2.1 INERTIAL COORDINATES (x_I, y_I, z_I)

The inertial simulator coordinate system is an orthogonal, right-handed coordinate system whose origin is on the simulator centerline in the plane of the floor swivel joints. The y_I and z_I axes form a horizontal plane, and the x_I axis is positive down (see Figure 2-1).

2.2 TABLE COORDINATES (x_T, y_T, z_T)

The table coordinate system is an orthogonal, right-handed coordinate system whose origin is at the center of gravity of the simulator table. The y_T and z_T axes lie in the plane of the table, and the x_T axis is positive "down" (see Figure 2-1).

2.3 ACTUATOR COORDINATES (x_{s_i}, y_{s_i}, z_{s_i})

Each actuator has its own coordinate system. The x_{s_i} axis is colinear with the actuator centerline. The y_{s_i} axis is perpendicular to the x_{s_i} axis and the inertial gravity vector. The z_{s_i} axis is perpendicular to both x_{s_i} and y_{s_i} and is positive "up" as shown in Figure 2-1.

2.4 TRANSFORMATION FROM INERTIAL TO TABLE COORDINATES

Euler angles shown in Figure 2-2 are used to transform from inertial coordinates to table coordinates.

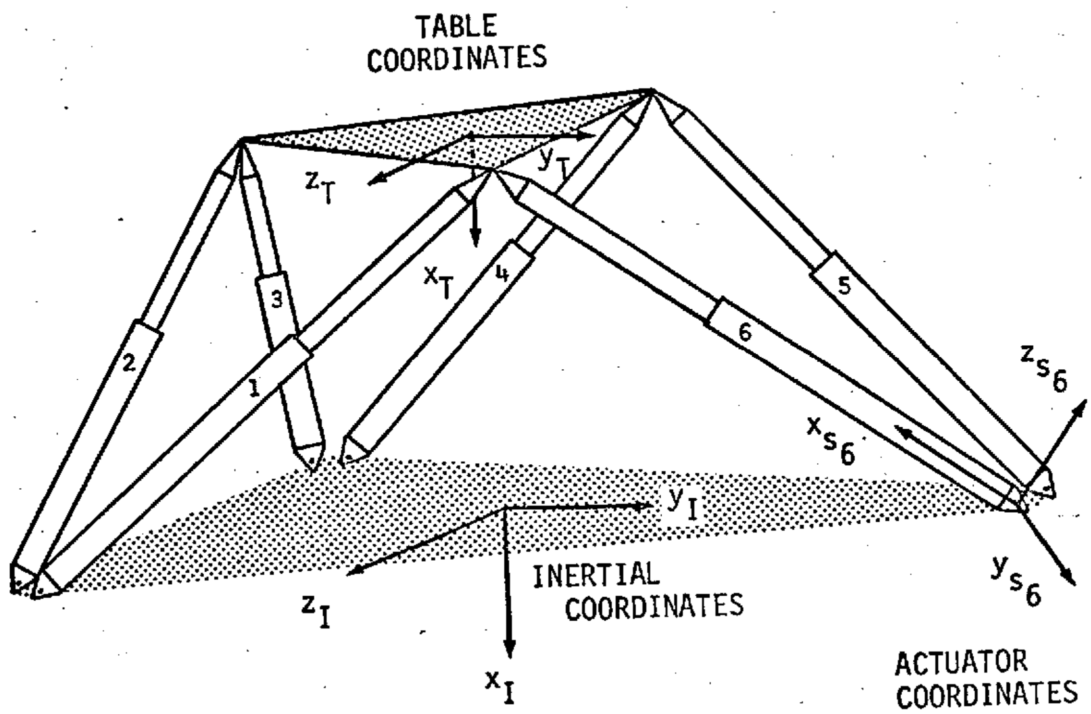


Figure 2-1. Active Table Coordinate Systems

2.4 (Continued)

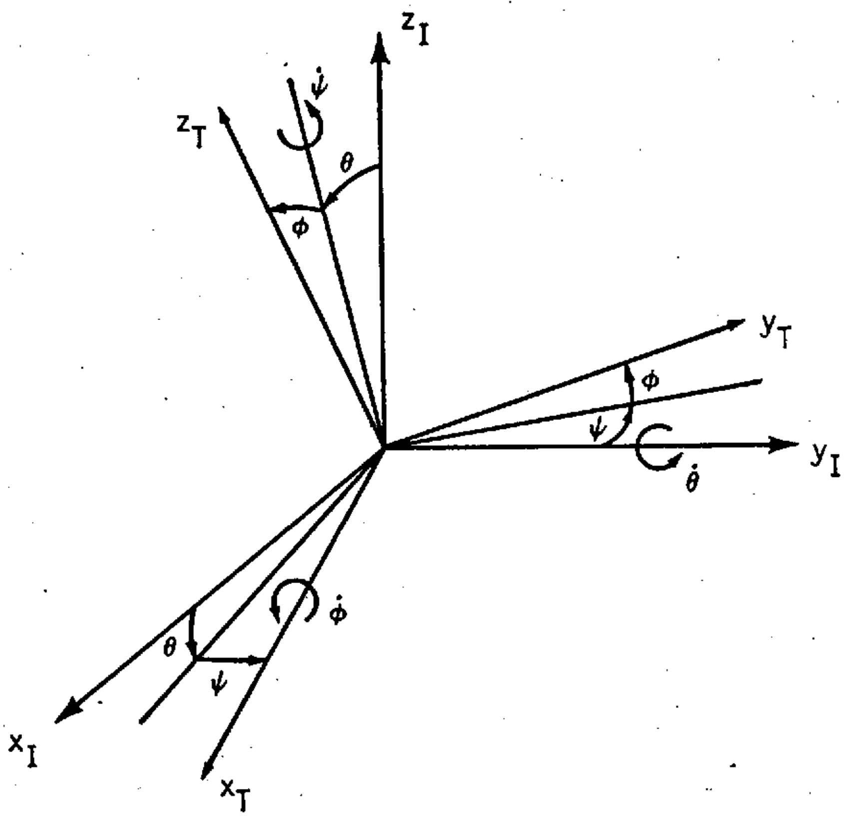


Figure 2-2. Euler Angles

The order of rotation is θ, ψ, ϕ which corresponds to rotation about the y, z, x axes, respectively. The transformation from table to inertial coordinates is:

$$\begin{Bmatrix} x_I \\ y_I \\ z_I \end{Bmatrix} = [A] \begin{Bmatrix} x_T \\ y_T \\ z_T \end{Bmatrix} \tag{1}$$

Therefore:

$$\begin{Bmatrix} x_T \\ y_T \\ z_T \end{Bmatrix} = [A]^T \begin{Bmatrix} x_I \\ y_I \\ z_I \end{Bmatrix} \tag{2}$$

2.4 (Continued)

$$\begin{Bmatrix} \dot{\theta} \\ \dot{\psi} \\ \dot{\phi} \end{Bmatrix} = \begin{bmatrix} 0 & \frac{\cos\phi}{\cos\psi} & -\frac{\sin\phi}{\cos\psi} \\ 0 & \sin\phi & \cos\phi \\ 1 & -\cos\phi\tan\psi & \sin\phi\tan\psi \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} \tag{3}$$

Where:

$$[A] = \begin{bmatrix} C\theta \cdot C\psi & -C\phi \cdot C\theta \cdot S\psi + S\theta \cdot S\phi & S\phi \cdot C\theta \cdot S\psi + C\phi \cdot S\theta \\ S\psi & C\phi \cdot C\psi & -S\phi \cdot C\psi \\ -S\theta \cdot C\psi & C\phi \cdot S\theta \cdot S\psi + S\phi \cdot C\theta & -S\phi \cdot S\theta \cdot S\psi + C\phi \cdot C\theta \end{bmatrix} \tag{4}$$

C = cosine

S = sine

2.5 TRANSFORMATION FROM ACTUATOR TO INERTIAL COORDINATES

The transformation from actuator coordinates to inertial coordinates uses the following angles:

θ_A - the angle between the horizontal plane through the floor joint and the actuator x_s axis (Figure 2-3)

β_A - the angle between the inertial z_I axis and the projection of the actuator x_s axis in the $y_I - z_I$ plane (Figure 2-3)

$$\sin \theta_{A_i} = -\frac{r_{s_{x_i}}}{l_{p_i}} \tag{5}$$

$$\cos \theta_{A_i} = \frac{\sqrt{r_{s_{y_i}}^2 + r_{s_{z_i}}^2}}{l_{p_i}} \tag{6}$$

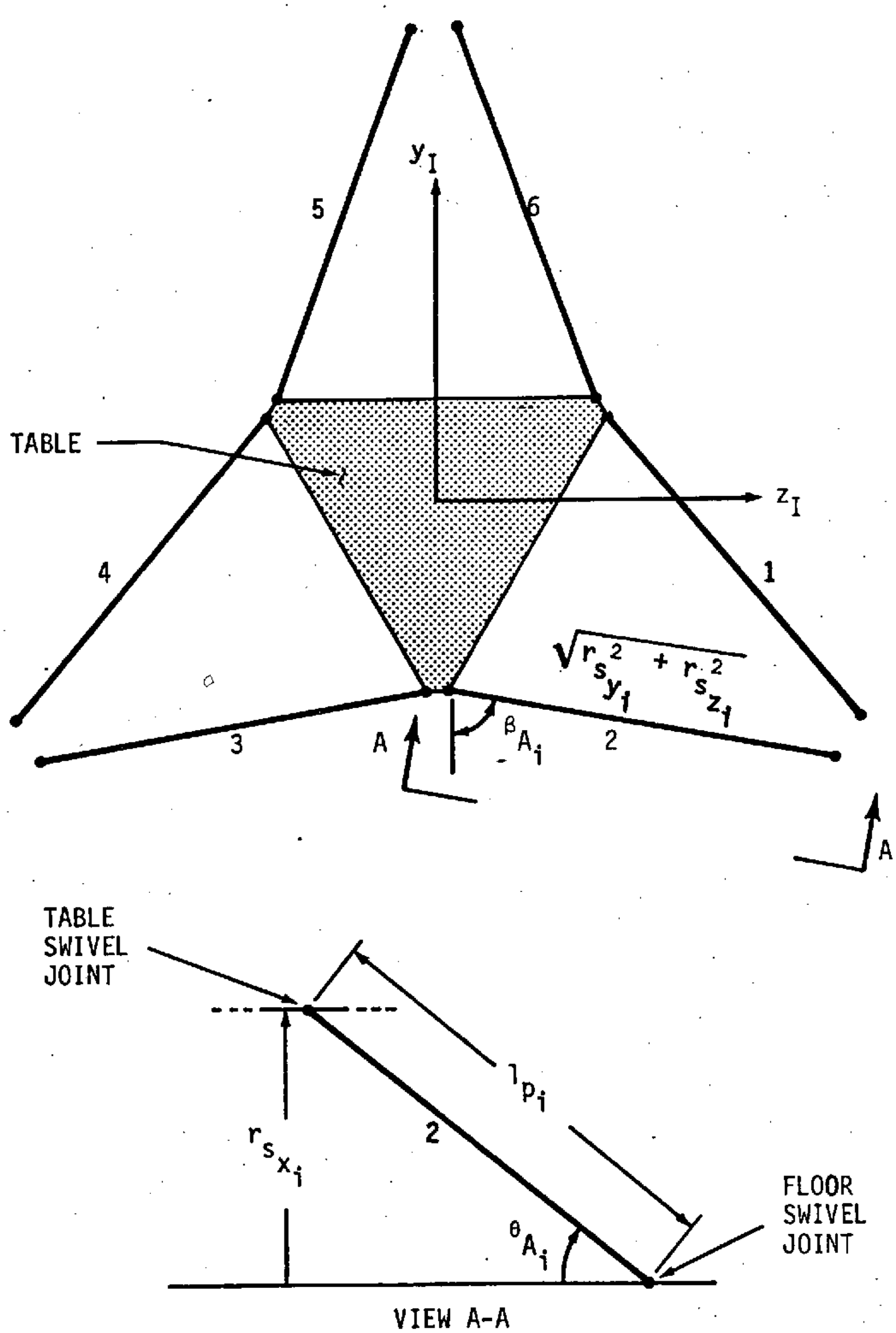


Figure 2-3. Actuator Transformation Angles

2.5 (Continued)

$$\sin \beta_{A_i} = \frac{r_{s_{y_i}}}{\sqrt{r_{s_{y_i}}^2 + r_{s_{z_i}}^2}} \tag{7}$$

$$\cos \beta_{A_i} = \frac{r_{s_{z_i}}}{\sqrt{r_{s_{y_i}}^2 + r_{s_{z_i}}^2}} \tag{8}$$

where $r_{s_{z_i}}$, $r_{s_{y_i}}$, and $r_{s_{z_i}}$ are vector components of actuator length, l_{p_i}, in the inertial coordinate system.

The transformation from actuator to inertial coordinates is then:

$$\begin{Bmatrix} x_I \\ y_I \\ z_I \end{Bmatrix} = \left[TI_i\right] \begin{Bmatrix} x_{s_i} \\ y_{s_i} \\ z_{s_i} \end{Bmatrix} \tag{9}$$

where:

$$\left[TI_i\right] = \begin{bmatrix} -S\theta_{A_i} & 0 & -C\theta_{A_i} \\ C\theta_{A_i} \cdot S\beta_{A_i} & C\beta_{A_i} & -S\theta_{A_i} \cdot S\beta_{A_i} \\ C\beta_{A_i} \cdot C\theta_{A_i} & -S\beta_{A_i} & -C\beta_{A_i} \cdot S\theta_{A_i} \end{bmatrix} \tag{10}$$

2.5 (Continued)

The equivalent "Euler angle" rotations for this transformation (from inertial to actuator coordinates) would:

a. Rotate -90^{0} about y_I

b. Rotate β_{A_i} about the intermediate z axis

c. Rotate $-\theta_{A_i}$ about the intermediate y axis

2.6 TRANSFORMATION FROM INDIVIDUAL ACTUATOR COORDINATES TO TABLE COORDINATES

Using the previous transformations, the transformation from individual actuator coordinate systems to the table coordinate system becomes:

$$\begin{Bmatrix} x_T \\ y_T \\ z_T \end{Bmatrix} = \left[T_i\right]^T \begin{Bmatrix} x_{s_i} \\ y_{s_i} \\ z_{s_i} \end{Bmatrix} \qquad (11)$$

where

$$\left[T_i\right]^T = \left[A\right]^T \left[TI_i\right] \qquad (12)$$

Likewise:

$$\begin{Bmatrix} x_{s_i} \\ y_{s_i} \\ z_{s_i} \end{Bmatrix} = \left[T_i\right] \begin{Bmatrix} x_T \\ y_T \\ z_T \end{Bmatrix} \qquad (13)$$

where:

$$\left[T_i\right] = \left[TI_i\right]^T \left[A\right] \qquad (14)$$

3.0 TABLE MOTION COMMANDS

Table commands are specified in the inertial coordinate system. Actuator commands for two types of commands will be discussed: sinusoidal position commands and constant velocity commands.

3.1 SINUSOIDAL POSITION COMMANDS

Let $\begin{Bmatrix} \Delta x_I \\ \Delta y_I \\ \Delta z_I \end{Bmatrix}$ and $\begin{Bmatrix} \Delta\theta \\ \Delta\psi \\ \Delta\phi \end{Bmatrix}$ be the amplitude of commanded sinusoidal table motion in the inertial coordinate system. The total inertial commands are then obtained by adding the commanded sinusoidal motion to the initial inertial position of the table.

$$
\begin{aligned}
x_{I_c} &= x_{I_o} + \Delta x_I \sin \omega_c t \\
y_{I_c} &= y_{I_o} + \Delta y_I \sin \omega_c t \\
z_{I_c} &= z_{I_o} + \Delta z_I \sin \omega_c t \\
\theta_c &= \theta_o + \Delta\theta \sin \omega_c t \\
\psi_c &= \psi_o + \Delta\psi \sin \omega_c t \\
\phi_c &= \phi_o + \Delta\phi \sin \omega_c t \\
\dot{x}_{I_c} &= \Delta x_I\, \omega_c \cos \omega_c t \\
\dot{y}_{I_c} &= \Delta y_I\, \omega_c \cos \omega_c t \\
\dot{z}_{I_c} &= \Delta z_I\, \omega_c \cos \omega_c t \\
\dot{\theta} &= \Delta\theta\omega_c \cos \omega_c t \\
\dot{\psi} &= \Delta\psi\omega_c \cos \omega_c t \\
\dot{\phi} &= \Delta\phi\omega_c \cos \omega_c t
\end{aligned}
\qquad (15)
$$

3.2 CONSTANT VELOCITY COMMANDS

For constant velocity commands $(\Delta\dot{x}_I,\ \Delta\dot{y}_I,\ \Delta\dot{z}_I,\ \Delta\dot{\theta},\ \Delta\dot{\psi},\ \Delta\dot{\phi})$, the total inertial commands are:

$$
\begin{aligned}
x_{I_c} &= x_{I_o} + \Delta\dot{x}_I t \\
y_{I_c} &= y_{I_o} + \Delta\dot{y}_I t \\
z_{I_c} &= z_{I_o} + \Delta\dot{z}_I t \\
\theta_c &= \theta_o + \Delta\dot{\theta} t \\
\psi_c &= \psi_o + \Delta\dot{\psi} t \\
\phi_c &= \phi_o + \Delta\dot{\phi} t \\
\dot{x}_{I_c} &= \Delta\dot{x}_I \\
\dot{y}_{I_c} &= \Delta\dot{y}_I \\
\dot{z}_{I_c} &= \Delta\dot{z}_I \\
\dot{\theta}_c &= \Delta\dot{\theta} \\
\dot{\psi}_c &= \Delta\dot{\psi} \\
\dot{\phi}_c &= \Delta\dot{\phi}
\end{aligned}
\qquad (16)
$$

3.3 ACTUATOR COMMANDS

The total inertial position and velocity commands are transformed to individual actuator commands.

Let $\left[A_c\right]$ be the transformation from the table coordinate system to the inertial coordinate system (equation 4) with the Euler angles replaced by the commanded Euler angles. Then the commanded inertial velocities of the actuator/table attachment points are:

3.3 (Continued)

$$\begin{Bmatrix} \dot{r}_{s_{x_i}} \\ \dot{r}_{s_{y_i}} \\ \dot{r}_{s_{z_i}} \end{Bmatrix} = \begin{Bmatrix} \dot{x}_{I_c} \\ \dot{y}_{I_c} \\ \dot{z}_{I_c} \end{Bmatrix} + \left[A_c\right] \begin{bmatrix} 0 & -\omega_{z_c} & \omega_{y_c} \\ \omega_{z_c} & 0 & -\omega_{x_c} \\ -\omega_{y_c} & \omega_{x_c} & 0 \end{bmatrix} \begin{Bmatrix} r_{xa_i} \\ r_{ya_i} \\ r_{za_i} \end{Bmatrix} \tag{17}$$

where

$$\begin{Bmatrix} \omega_{x_c} \\ \omega_{y_c} \\ \omega_{z_c} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & S\psi_c \\ 0 & S\phi_c & C\psi_c \cdot C\phi_c \\ 0 & C\phi_c & -C\psi_c \cdot S\phi_c \end{bmatrix} \begin{Bmatrix} \dot{\phi}_c \\ \dot{\psi}_c \\ \dot{\theta}_c \end{Bmatrix} \tag{18}$$

The commanded inertial components of actuator length are:

$$\begin{Bmatrix} r_{s_{x_i}} \\ r_{s_{y_i}} \\ r_{s_{z_i}} \end{Bmatrix} = \begin{Bmatrix} x_{I_c} \\ y_{I_c} \\ z_{I_c} \end{Bmatrix} + \left[A_c\right] \begin{Bmatrix} r_{xa_i} \\ r_{ya_i} \\ r_{za_i} \end{Bmatrix} - \begin{Bmatrix} 0 \\ y_{f_i} \\ z_{f_i} \end{Bmatrix} \tag{19}$$

Commanded actuator lengths are then:

$$l_{c_i} = \sqrt{r_{s_{x_i}}^2 + r_{s_{y_i}}^2 + r_{s_{z_i}}^2} \tag{20}$$

3.3 (Continued)

and commanded actuator velocities are:

$$\dot{l}_{c_i} = \frac{1}{l_{c_i}} \left[r_{s_{x_i}} \cdot \dot{r}_{s_{x_i}} + r_{s_{y_i}} \cdot \dot{r}_{s_{y_i}} + r_{s_{z_i}} \cdot \dot{r}_{s_{z_i}} \right] \qquad (21)$$

4.0 SERVO ELECTRONICS

The servo electronics consist of actuator position and rate command signals; forward loop compensation network; servo valve; and position, rate, and differential pressure feedback loops as shown in Figure 4-1. The forward loop compensation network, the valve dynamics representation, and the position feedback filter are designed to be optional in the computer program. For example, if $\beta<1$, then the forward loop compensation network is not included in the simulation. Similarly, the valve dynamics and position feedback filter are neglected if $\omega_v<1$ and $\omega_s<1$, respectively.

4.1 ACTUATOR COMMANDS

Define

$$\delta_i = K_f \, l_{c_i} + K_{rc} \, \dot{l}_{c_i} \qquad (22)$$
$$\dot{\delta}_i = K_f \, \dot{l}_{c_i} + K_{rc} \, \ddot{l}_{c_i}$$

where l_{c_i}, $\dot{l}_{c_i}$, and $\ddot{l}_{c_i}$ are the commanded actuator length, velocity, and acceleration, respectively.

4.2 POSITION FEEDBACK FILTER

The differential equation for the actuator position feedback filter is:

If $\omega_s \geq 1$

$$\ddot{x}_{s_i} = \omega_s^2 \left(l_{p_i} - \frac{2\zeta_s}{\omega_s} \dot{x}_{s_i} - x_{s_i} \right) \qquad (23)$$

If $\omega_s<1$

$$\ddot{x}_{s_i} = \ddot{l}_{p_i}$$
$$\dot{x}_{s_i} = \dot{l}_{p_i} \qquad (24)$$
$$x_{s_i} = l_{p_i}$$

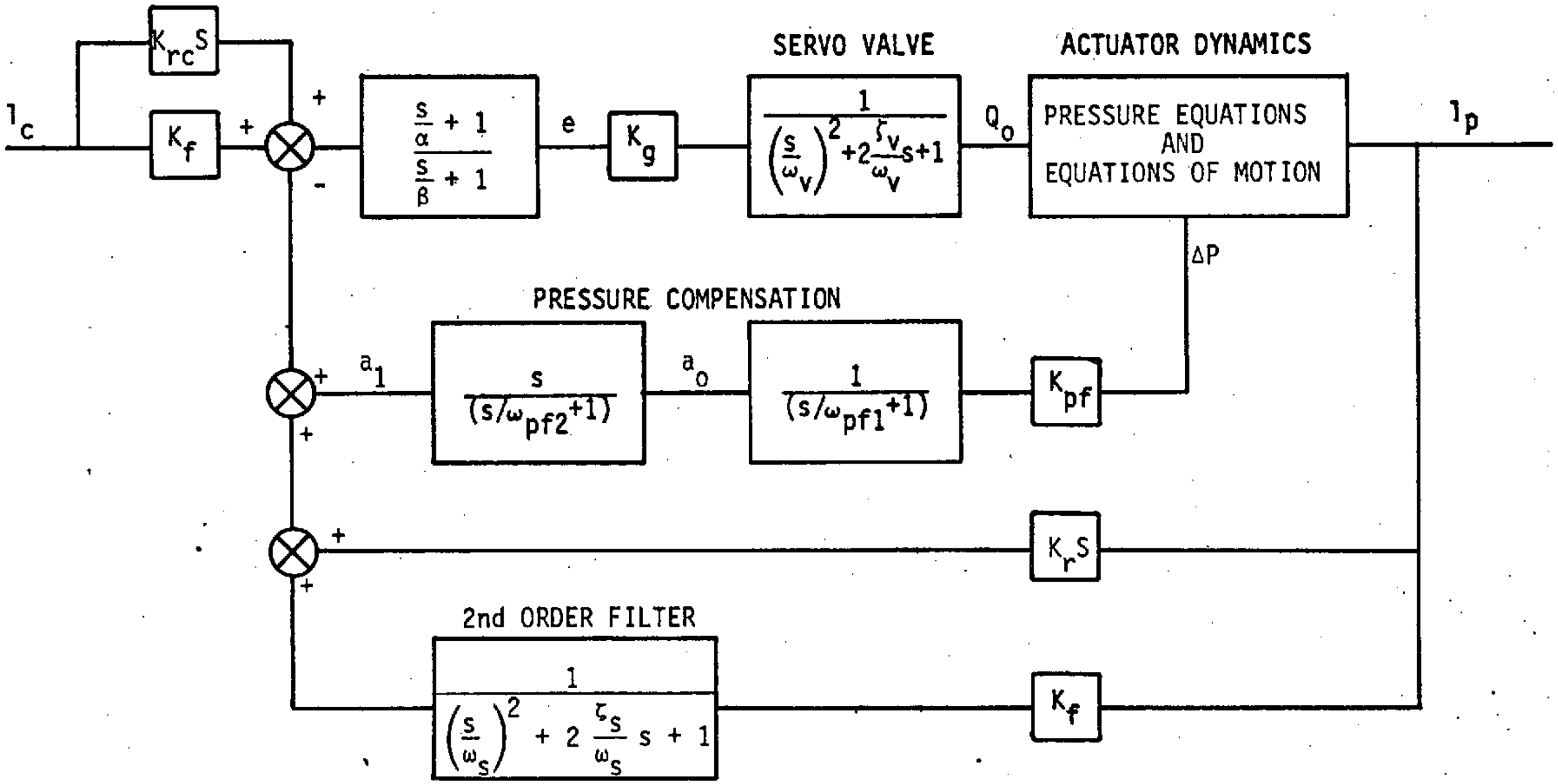


Figure 4-1. Servo Electronics Block Diagram

4.3 DIFFERENTIAL PRESSURE FEEDBACK COMPENSATION

Pressure feedback compensation consists of two parts--a first-order lag filter and a high-pass filter. The first-order lag attenuates the higher frequency pressure fluctuations, while the high-pass filter eliminates the static differential pressure caused by unequal piston areas. The differential equations for these filters are:

$$\dot{a}_{o_i} = \omega_{pf1} \left[K_{pf} \left(p_{1_i} - p_{2_i} \right) - a_{o_i} \right]$$

$$\dot{a}_{1_i} = \omega_{pf2} \left(\dot{a}_{o_i} - a_{1_i} \right) \quad (25)$$

where K_{pf} is the pressure feedback gain.

4.4 FORWARD LOOP COMPENSATION NETWORK

The forward loop compensation network consists of a lead-lag filter with corner frequencies α and β.

For $\beta \geq 1$:

$$\dot{e}_i = \beta \left[\frac{1}{\alpha} (\dot{\delta}_i - \dot{a}_{1_i} - K_r \ddot{x}_{s_i} - K_f \dot{x}_{s_i}) + (\delta_i - a_{1_i} - K_r \dot{x}_{s_i} - K_f x_{s_i}) - e_i \right] \quad (26)$$

where $\dot{a}_{1_i}$, a_{1_i}, $\ddot{x}_s$, $\dot{x}_s$, and x_s are signals from the feedback loops.

For $\beta < 1$:

$$\dot{e}_i = 0$$

$$e_i = \delta_i - a_{1_i} - K_r \dot{x}_{s_i} - K_f x_{s_i} \quad (27)$$

where K_f and K_r are the displacement and rate feedback gains, respectively.

4.5 SERVO VALVE DYNAMICS

The dynamics of the servo valve are represented by a single-degree-of-freedom system with a natural frequency ω_v and damping ratio ζ_v.

If $\omega_v \geq 1$:

$$\ddot{Q}_{o_i} = \omega_v^2 \left(K_g e_i - \frac{2\zeta_v}{\omega_v} \dot{Q}_{o_i} - Q_{o_i} \right) \tag{28}$$

where K_g is the forward loop amplifier gain and Q_o is the no-load flow through the valve.

If $\omega_v < 1$:

$$\ddot{Q}_{o_i} = \dot{Q}_{o_i} = 0$$
$$Q_{o_i} = K_g e_i \tag{29}$$

5.0 ACTUATOR MODEL

Each actuator is modeled as a flexible rod with pinned ends. Hydraulic forces are calculated using nonlinear hydraulic flow equations and unequal push and pull piston areas. Actuator control system electronics are modeled and include differential pressure, velocity, and position feedback. The actuator geometry and nomenclature are shown in Figure 5-1.

5.1 ACTUATOR MASS AND INERTIA CHARACTERISTICS

The mass moment of inertia of the piston rod is:

$$I_p = \frac{m_p \, l_r^2}{12} \qquad (30)$$

where m_p is the driven mass (piston and piston rod). The mass moment of inertia of the entire actuator assembly about the floor pivot is:

$$I_{A_i} = I_{A_c} + I_p + m_p \left(l_{p_i} - \frac{l_r}{2} \right)^2 \qquad (31)$$

where:

I_{A_c} = Mass moment of inertia of the cylinder structure about the floor pivot (excludes driven mass, m_p).

The effective rigid lateral mass of each actuator assembly for use in the equations of motion is then:

$$m_{L_i} = I_{A_i} \, / \, l_{p_i}^{\,2} \qquad (32)$$

5.2 ACTUATOR FLEXIBILITY

The dynamic bending characteristics of each actuator are calculated assuming that the cylinder is rigid compared to the piston rod and that the effective dynamic mass is lumped at the rod end seal of the cylinder. The bending characteristics are also assumed to be identical for each of the two bending planes of the actuator.

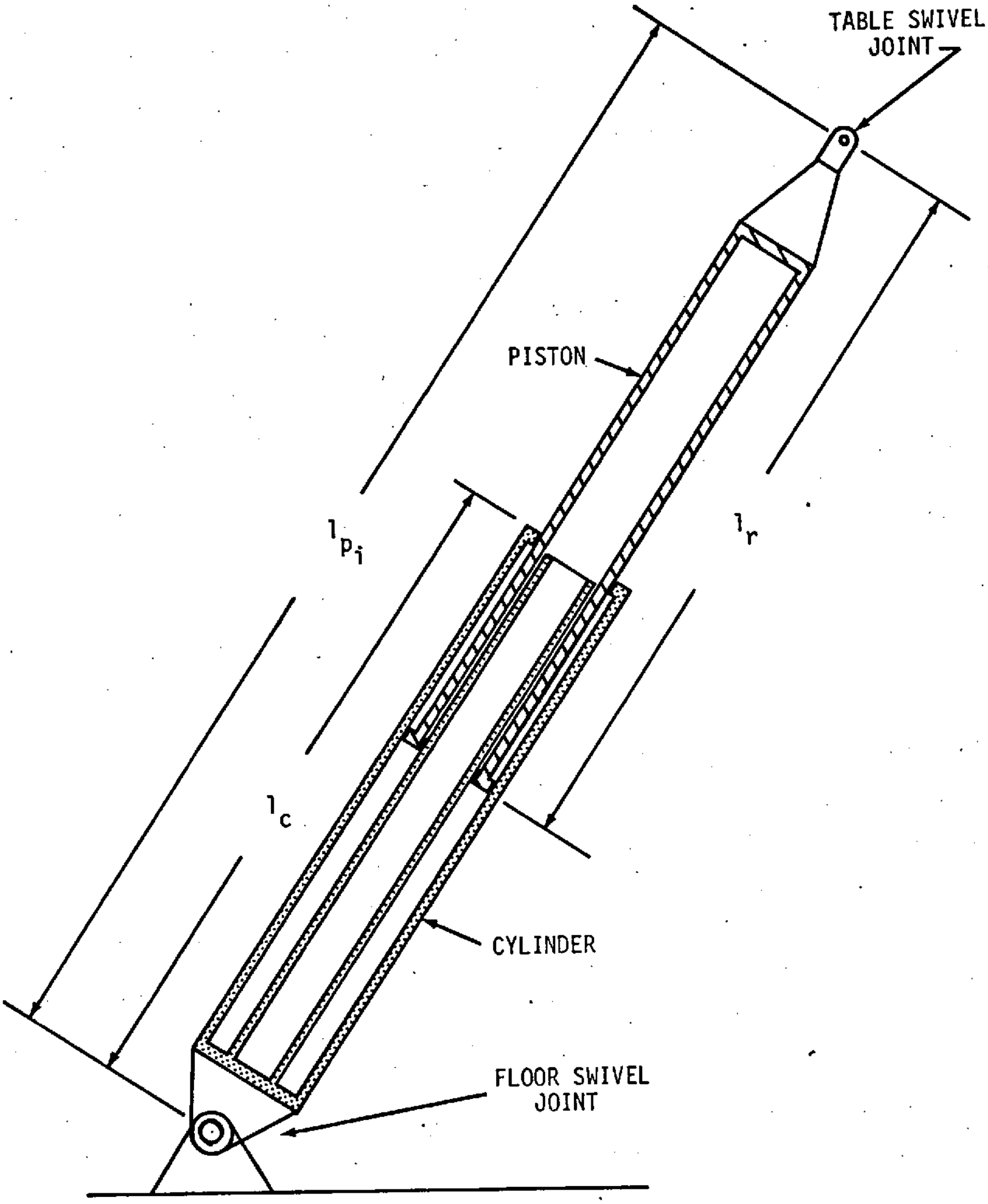


Figure 5-1. Hydraulic Actuator

5.2 (Continued)

The effective dynamic mass lumped at the cylinder end is approximated as:

$$m_{q_i} = \left[I_{A_c} / l_c^2\right] + \left[m_p/2\right] \quad (33)$$

Assuming pinned joints between the cylinder and piston rod, the piston rod stiffness is:

$$k_{r_i} = \frac{3\,(EI)_r\, l_r}{l_{r1_i}^2\, l_{r2_i}^2} \quad (34)$$

where l_{r1_i} and l_{r2_i} are defined as follows:

$$l_{r1_i} = l_{p_i} - l_c$$

$$l_{r2_i} = l_r - l_{r1_i}$$

The effective lateral stiffness of the actuator with a rigid cylinder is:

$$k_{e_i} = \frac{3\,(EI)_r \left(l_r l_c - l_{r1_i} l_{p_i}\right)\left(l_r + l_{p_i}\right)\left(l_c - l_{p_i}\right)}{l_{r1_i}^2\, l_{r2_i}^2\, l_c^2\, l_r} \quad (35)$$

The actuator bending frequency is then:

$$\omega_{e_i}^2 = k_{e_i} / m_{q_i} \quad (36)$$

5.3 HYDRAULIC FLOW EQUATIONS

The nonlinear hydraulic flow equations are based on the derivations presented in Reference 2 for a double-acting hydraulic piston. A schematic of the hydraulic servo valve and actuator is shown in Figure 5-2.

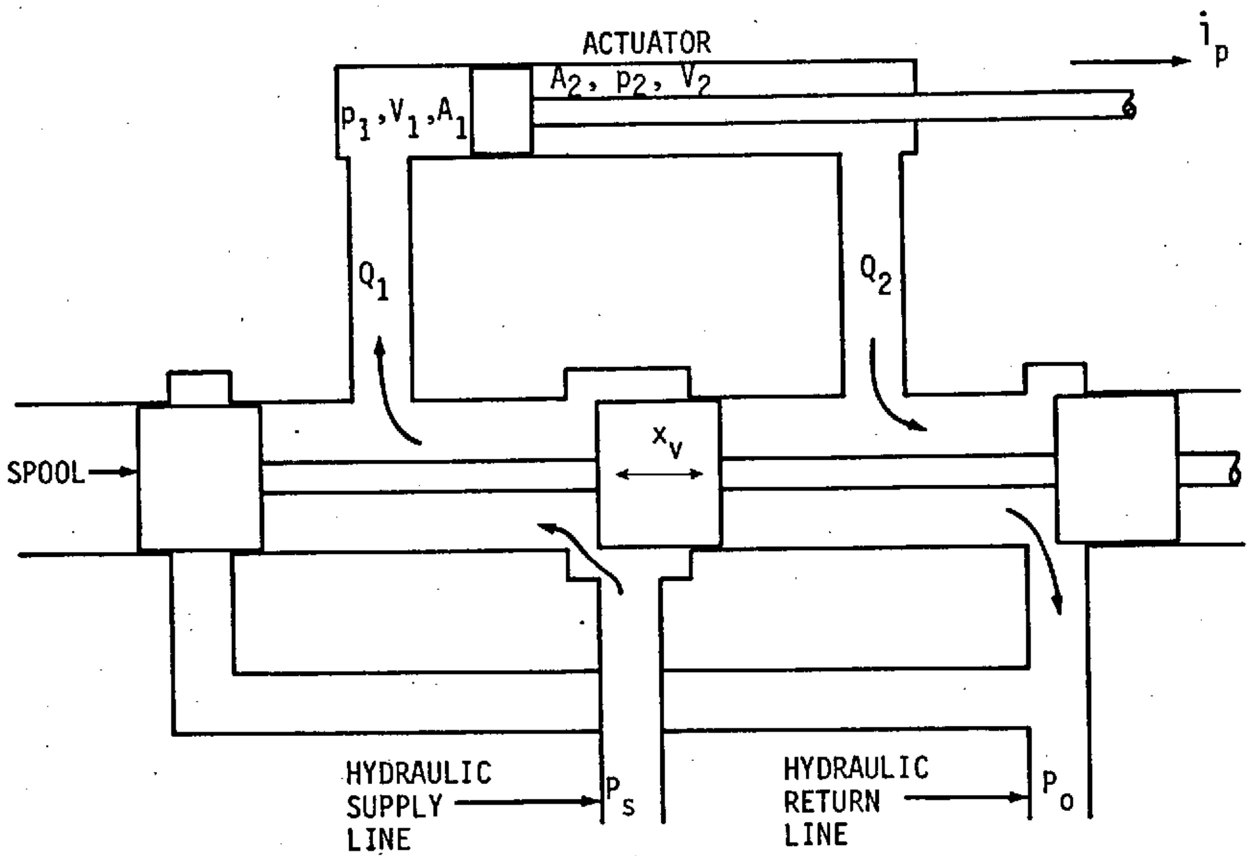


Figure 5-2. Hydraulic Servo Valve Schematic

5.3 (Continued)

The flow continuity equations are:

$$Q_1 = C_p\,(p_1 - p_2) + \dot{V}_1 + \frac{V_1}{\beta_e}\,\dot{p}_1$$

$$Q_2 = C_p\,(p_1 - p_2) - C_{e_p}\,p_2 - \dot{V}_2 - \frac{V_2}{\beta_e}\,\dot{p}_2 \qquad (37)$$

where:

$$Q_1 = Q_o - 2\,K_c\,p_1$$

$$Q_2 = Q_o + 2\,K_c\,p_2 \qquad (38)$$

and

Q_o = The no-load flow of the valve

K_c = Valve pressure flow coefficient

C_p = Leakage coefficient across the piston

C_{e_p} = Leakage coefficient past the piston rod seal

The volume-stroke relationships are:

$$V_1 = V_{o_1} + A_1(l_p - l_o)$$

$$V_2 = V_{o_2} - A_2(l_p - l_o) \qquad (39)$$

$$\dot{V}_1 = A_1\,\dot{l}_p$$

$$\dot{V}_2 = -A_2\,\dot{l}_p$$

where V_{o_1} and V_{o_2} are the hydraulic volumes at zero stroke.

5.3 (Continued)

Therefore, neglecting piston rod seal leakage, the hydraulic flow equations for each actuator are:

$$\dot{p}_1 = \frac{\beta_e}{V_1}\left[Q_o - 2K_c\,p_1 - C_p\,(p_1 - p_2) - A_1\,\dot{i}_p\right]$$

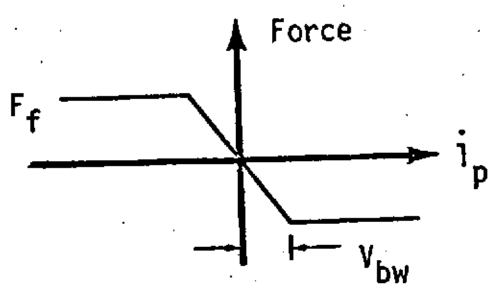
$$\dot{p}_2 = \frac{\beta_e}{V_2}\left[-Q_o - 2K_c\,p_2 + C_p\,(p_1 - p_2) + A_2\,\dot{i}_p\right] \qquad (40)$$

5.4 ACTUATOR FORCES

Actuator forces $\left(F_{p_i}\right)$ are calculated from the differential pressure across the piston. In addition to the viscous damping forces associated with the actuators, coulomb friction is also included.

$$F_p = A_1p_1 - A_2p_2 - B_p\dot{i}_p - C_FF_f \qquad (41)$$

A velocity "bandwidth" for coulomb friction is used to prevent a discontinuity at zero velocity.



The coulomb friction force is:

$$F_{CF} = -C_F\,F_f \qquad (42)$$

5.3 (Continued)

where C_F is a coefficient which is a function of actuator velocity:

$$\text{If } |\dot{i}_p| \geq V_{bw}\text{, then } C_F = \frac{\dot{i}_p}{|\dot{i}_p|}$$

$$\text{If } |\dot{i}_p| < V_{bw}\text{, then } C_F = \frac{\dot{i}_p}{V_{bw}} \qquad (43)$$

6.0 EQUATIONS OF MOTION

Table and actuator equations of motion are written in the body fixed table coordinates in the following form:

$$\{\ddot{X}\} = [M]^{-1} \{C\} \quad (44)$$

where: $\{\ddot{X}\}$ is a column of accelerations for each degree of freedom (six degrees of freedom for the table and two elastic degrees of freedom for each actuator)

$[M]$ is the 18 x 18 coupled mass matrix

$\{C\}$ is a column of generalized forces for each degree of freedom

The mass coupling effects of the actuators due to table motions are derived by Lagrange's method. The three-dimensional rigid motions of the actuators are completely constrained (i.e., they are dependent upon the motions of the table). These constraints are expressed by the velocity substitutions in the energy expressions.

The final equations are much simplified when compared with the equations which would result from a rigorous derivation. Due to the nonorthogonality between actuator and table motions, a large number of nonlinear velocity coupling terms results. All of these terms were assumed negligible since, for expected table velocities, they are quite small and their omission prevents the equations from becoming unwieldy.

6.1 MASS MATRIX

The kinetic energy of the rigid table and actuators is:

$$T = \tfrac{1}{2} m_T \left(\dot{\vec{r}}_T \cdot \dot{\vec{r}}_T\right) + \tfrac{1}{2} \vec{\omega}_T \cdot \left(\tilde{I}_T \vec{\omega}_T\right) + \tfrac{1}{2} m_p \sum_{i=1}^{6} \dot{l}_{p_i}^2 + \tfrac{1}{2} \sum_{i=1}^{6} \int_0^{l_{p_i}} \left(\dot{y}_{a_i}^2 + \dot{z}_{a_i}^2\right) dm_i \quad (45)$$

6.1 (Continued)

The first two terms in this expression are the energy of the rigid table; the third term is the energy of the piston rod due to stroking; and the fourth term is the energy of the actuator assembly due to rotation about the floor pivot.

Using the transformation from table coordinates to actuator coordinates, $[T_i]$, shown in equation (14):

$$\dot{l}_{p_i} = T_{i_{11}} \left(\dot{x}_T - \omega_z r_{ya_i} + \omega_y r_{za_i} \right) + T_{i_{12}} \left(\dot{y}_T + \omega_z r_{xa_i} - \omega_x r_{za_i} \right) + T_{i_{13}} \left(\dot{z}_T - \omega_y r_{xa_i} + \omega_x r_{ya_i} \right) \tag{46}$$

$$= T_{i_{11}} \dot{x}_{a_i} + T_{i_{12}} \dot{y}_{a_i} + T_{i_{13}} \dot{z}_{a_i}$$

$$\dot{y}_{a_i} = \left[T_{i_{21}} \left(\dot{x}_T - \omega_z r_{ya_i} + \omega_y r_{za_i} \right) + T_{i_{22}} \left(\dot{y}_T + \omega_z r_{xa_i} - \omega_x r_{za_i} \right) + T_{i_{23}} \left(\dot{z}_T - \omega_y r_{xa_i} + \omega_x r_{ya_i} \right) \right] \frac{x_s}{l_{p_i}} + \dot{y}_{e_i} \phi_i (x_s) \tag{47}$$

$$= \dot{y}_{r_i} \frac{x_s}{l_{p_i}} + \dot{y}_{e_i} \phi_i (x_s)$$

6.1 (Continued)

$$\dot{z}_{a_i} = \left[T_{i_{31}} \left(\dot{x}_T - \omega_z \, r_{ya_i} + \omega_y \, r_{za_i} \right) \right.$$

$$+ T_{i_{32}} \left(\dot{y}_T + \omega_z \, r_{xa_i} - \omega_x \, r_{za_i} \right)$$

$$\left. + T_{i_{33}} \left(\dot{z}_T - \omega_y \, r_{xa_i} + \omega_x \, r_{ya_i} \right) \right] \frac{x_s}{l_{p_i}} \qquad (48)$$

$$+ \dot{z}_{e_i} \, \phi_i \, (x_s)$$

$$= \dot{z}_{r_i} \frac{x_s}{l_{p_i}} + \dot{z}_{e_i} \, \phi_i \, (x_s)$$

where:

$\phi_i \, (x_s)$ is the actuator bending mode shape as a function of x_s, and y_{e_i} and z_{e_i} are bending velocities of the actuators.

The elastic bending modes of each actuator are assumed to be a simple mode shape normalized to unity at the upper end of the cylinder ($x_s = l_c$). There are two identical modes for each actuator. The generalized mass for each mode is assumed to be lumped at the upper end of the cylinder; thus, the mass distribution terms can be integrated.

e.g. $\int_0^{l_{p_i}} x_s^2 \, dm = I_{A_i}$ = mass moment of total actuator assembly about the floor pivot.

6.1 (Continued)

and:

$$\int_0^{l_{p_i}} x_s \, \phi(x_s) \, dm_i = l_c \, m_{q_i}$$

where m_{q_i} is the generalized mass of $i^{\underline{th}}$ actuator for each bending mode.

Lagrange's equation requires the determination of $\frac{d}{dt}\frac{\partial T}{\partial \dot{Q}_j}$ where Q_j is the $j^{\underline{th}}$ generalized coordinate in the equation of motion. In this simulation:

$$\begin{aligned}
\dot{Q}_1 &= \dot{x}_T & \dot{Q}_7 &= \dot{y}_{e_1} \\
\dot{Q}_2 &= \dot{y}_t & \dot{Q}_8 &= \dot{z}_{e_1} \\
\dot{Q}_3 &= \dot{z}_T & & \vdots \\
\dot{Q}_4 &= \omega_{T_x} & & \\
\dot{Q}_5 &= \omega_{T_y} & \dot{Q}_{17} &= \dot{y}_{e_6} \\
\dot{Q}_6 &= \omega_{T_z} & \dot{Q}_{18} &= \dot{z}_{e_6}
\end{aligned} \tag{49}$$

Then:

$$\begin{aligned}
\frac{\partial T}{\partial \dot{Q}_j} &= \tfrac{1}{2} m_T \frac{\partial}{\partial \dot{Q}_j} (\dot{\vec{r}}_T \cdot \dot{\vec{r}}_T) + \tfrac{1}{2} \frac{\partial}{\partial \dot{Q}_j} \left[\vec{\omega}_T \cdot (\tilde{I} \cdot \omega_T) \right] \\
&+ m_p \sum_{i=1}^{6} \frac{\partial \dot{l}_{p_i}}{\partial \dot{Q}_j} \dot{l}_{p_i} + \sum_{i=1}^{6} \int_0^{l_{p_i}} \left(\frac{\partial \dot{y}_{a_i}}{\partial \dot{Q}_j} \dot{y}_{a_i} + \frac{\partial \dot{z}_{a_i}}{\partial \dot{Q}_j} \dot{z}_{a_i} \right) dm_i
\end{aligned} \tag{50}$$

6.1 (Continued)

For $j \le 6$:

$$\frac{\partial \dot{y}_{a_i}}{\partial \dot{Q}_j} = \frac{x_s}{l_{p_i}} C_{y_{ij}} \qquad (51)$$

where $C_{y_{ij}}$ are coefficients from equation (47). e.g.

$$C_{y_{i1}} = T_{i_{21}}$$

$$C_{y_{i4}} = -T_{i_{22}} r_{za_i} + T_{i_{23}} r_{ya_i}$$

$$\vdots$$

etc.

For j = 7, 9, 11, 13, 15, 17:

$$\frac{\partial \dot{y}_{a_i}}{\partial \dot{Q}_j} = \phi_i = 1. \qquad (52)$$

and, for j = 8, 10, 12, 14, 16, 18

$$\frac{\partial \dot{y}_{a_i}}{\partial \dot{Q}_j} = 0 \qquad (53)$$

Likewise:

$$\frac{\partial \dot{z}_{a_i}}{\partial \dot{Q}_j} = \frac{x_s}{l_{p_i}} C_{z_{ij}} \quad \text{for } j \le 6 \qquad (54)$$

$$= 0 \quad \text{for } j = 7, 9, 11, 13, 15, 17 \qquad (55)$$

$$= 1 \quad \text{for } j = 8, 10, 12, 14, 16, 18 \qquad (56)$$

6.1 (Continued)

Therefore, for $j \leq 6$, the last term in equation (50) becomes:

$$\frac{\partial T}{\partial \dot{Q}_j} = \sum_{i=1}^{6} \left[\left(\frac{x_s}{l_{p_i}}\right)^2 \dot{y}_{r_i} C_{y_{ij}} + \frac{x_s}{l_{p_i}} C_{y_{ij}} \dot{y}_{e_i} \phi_i (x_s) + \left(\frac{x_s}{l_{p_i}}\right)^2 \dot{z}_{r_i} C_{z_{ij}} + \frac{x_s}{l_{p_i}} C_{z_{ij}} \dot{z}_{e_i} \phi_i (x_s) \right] dm_i \tag{57}$$

$$= \sum_{i=1}^{6} \left[\frac{I_{A_i}}{l_{p_i}^2} \left(\dot{y}_{r_i} C_{y_{ij}} + \dot{z}_{r_i} C_{z_{ij}} \right) + m_{q_i} \frac{l_c}{l_{p_i}} \left(C_{y_{ij}} \dot{y}_{e_i} + C_{z_{ij}} \dot{z}_{e_i} \right) \right] \tag{58}$$

where, from equations (47) and (48):

$$\dot{y}_{r_i} = T_{i_{21}} \dot{x}_{a_i} + T_{i_{22}} \dot{y}_{a_i} + T_{i_{23}} \dot{z}_{a_i} \tag{59}$$

$$\dot{z}_{r_i} = T_{i_{31}} \dot{x}_{a_i} + T_{i_{32}} \dot{y}_{a_i} + T_{i_{33}} \dot{z}_{a_i} \tag{60}$$

and, from equation (46):

$$\dot{x}_{a_i} = \dot{x}_T - \omega_z r_{ya_i} + \omega_y r_{za_i} \tag{61}$$

$$\dot{y}_{a_i} = \dot{y}_T + \omega_z r_{xa_i} - \omega_x r_{za} \tag{62}$$

$$\dot{z}_{a_i} = \dot{z}_T - \omega_y r_{xa_i} + \omega_x r_{ya_i} \tag{63}$$

6.1 (Continued)

Also, for j = 7, 9, 11, 13, 15, 17, the last term in equation (50) becomes:

$$\frac{\partial T}{\partial \dot{Q}_j} = \left(\dot{y}_{r_i} \frac{x_s}{l_{p_i}} \phi_i \ (x_s) + \dot{y}_{e_i} \ \phi_i^2 \ (x_s) \right) dm_i \tag{64}$$

$$= m_{q_i} \left(\dot{y}_{r_i} \frac{l_c}{l_{p_i}} + \dot{y}_{e_i} \right) \tag{65}$$

Likewise, for j = 8, 10, 12, 14, 16, 18:

$$\frac{\partial T}{\partial \dot{Q}_j} = m_{q_i} \left(\dot{z}_{r_i} \frac{l_c}{l_{p_i}} + \dot{z}_{e_i} \right) \tag{66}$$

Differentiating equations (50), (58), (65), and (66) to obtain $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right)$:

for $j \leq 6$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) = \frac{d}{dt} \left\{ \tfrac{1}{2} \, m_T \frac{\partial}{\partial \dot{Q}_j} (\dot{\vec{r}}_T \cdot \dot{\vec{r}}_T) + \tfrac{1}{2} \frac{\partial}{\partial \dot{Q}_j} \left[\vec{\omega}_T \cdot (\tilde{I}_T \cdot \vec{\omega}_T) \right] \right\}$$

$$+ m_p \sum_{i=1}^{6} \left[\frac{\partial \dot{l}_{p_i}}{\partial \dot{Q}_j} \ddot{l}_{p_i} + \dot{l}_{p_i} \frac{d}{dt} \frac{\partial \dot{l}_{p_i}}{\partial \dot{Q}_j} \right]$$

$$+ \sum_{i=1}^{6} \left[\frac{1}{l_{p_i}^4} \left(l_{p_i}^2 \ \dot{I}_{A_i} - 2 \ I_A \ l_{p_i} \ \dot{l}_{p_i} \right) \left(\dot{y}_{r_i} \ C_{y_{ij}} + \dot{z}_{r_i} \ C_{z_{ij}} \right) \right.$$

$$+ \frac{I_{A_i}}{l_{p_i}^2} \left(\ddot{y}_{r_i} \ C_{y_{ij}} + \ddot{z}_{r_i} \ C_{z_{ij}} + \dot{y}_{r_i} \ \dot{C}_{y_{ij}} + \dot{z}_{r_i} \ \dot{C}_{z_{ij}} \right)$$

6.1 (Continued)

$$+ \frac{1}{l_{p_i}^2}\left(l_{p_i}\, l_c\, \dot{m}_{q_i} - m_{q_i}\, l_c\, \dot{l}_{p_i}\right)\left(C_{y_{ij}}\, \dot{y}_{e_i} + C_{z_{ij}}\, \dot{z}_{e_i}\right)$$

$$+ m_{q_i}\, \frac{l_c}{l_{p_i}}\left(C_{y_{ij}}\, \ddot{y}_{e_i} + C_{z_{ij}}\, \ddot{z}_{e_i} + \dot{C}_{y_{ij}}\, \dot{y}_{e_i} + \dot{C}_{z_{ij}}\, \dot{z}_{e_i}\right)\Bigg] \tag{67}$$

for j = 7, 9, 11, 13, 15, 17:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) = \dot{m}_{q_i}\left(\dot{y}_{r_i}\, \frac{l_c}{l_{p_i}} + \dot{y}_{e_i}\right) + m_{q_i}\left(\ddot{y}_{r_i}\, \frac{l_c}{l_{p_i}} + \ddot{y}_{e_i} - \dot{y}_{r_i}\, \frac{l_c \dot{l}_{p_i}}{l_{p_i}^2}\right) \tag{68}$$

and, for j = 8, 10, 12, 14, 16, 18:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_i}\right) = \dot{m}_{q_i}\left(\dot{z}_{r_i}\, \frac{l_c}{l_{p_i}} + \dot{z}_{e_i}\right) + m_{q_i}\left(\ddot{z}_{r_i}\, \frac{l_c}{l_{p_i}} + \ddot{z}_{e_i} - \dot{z}_{r_i}\, \frac{l_c \dot{l}_{p_i}}{l_{p_i}^2}\right) \tag{69}$$

At this point, it is convenient to redefine several quantities to simplify the notation.

Let:

$$\begin{Bmatrix} \dot{x}_{a_i} \\ \dot{y}_{a_i} \\ \dot{z}_{a_i} \end{Bmatrix} \equiv \begin{Bmatrix} \dot{r}_{a_{i1}} \\ \dot{r}_{a_{i2}} \\ \dot{r}_{a_{i3}} \end{Bmatrix} \equiv \dot{\vec{r}}_{a_i} \tag{70}$$

6.1 (Continued)

also:

$$C_{y_{ij}} \equiv T_{i_{2j}}$$
$$C_{z_{ij}} \equiv T_{i_{3j}} \qquad (71)$$

Then equations (46), (59), and (60) become:

$$\dot{i}_{p_i} = \sum_{k=1}^{3} T_{i_{1k}} \dot{r}_{a_{ki}} \qquad (72)$$

$$\dot{y}_{r_i} = \sum_{k=1}^{3} T_{i_{2k}} \dot{r}_{a_{ki}} \qquad (73)$$

$$\dot{z}_{r_i} = \sum_{k=1}^{3} T_{i_{3k}} \dot{r}_{a_{ki}} \qquad (74)$$

Then, for j = 1, 2, 3:

$$\frac{\partial \dot{i}_{p_i}}{\partial \dot{Q}_j} = T_{i_{1j}} \qquad (75)$$

$$\ddot{i}_{p_i} = \sum_{k=1}^{3} \dot{T}_{i_{1k}} \dot{r}_{a_{ik}} + T_{i_{1k}} \ddot{r}_{a_{ik}} \qquad (76)$$

$$\ddot{y}_{r_i} = \sum_{k=1}^{3} \dot{T}_{i_{2k}} \dot{r}_{a_{ik}} + T_{i_{2k}} \ddot{r}_{a_{ik}} \qquad (77)$$

$$\ddot{z}_{r_i} = \sum_{k=1}^{3} \dot{T}_{i_{3k}} \dot{r}_{a_{ik}} + T_{i_{3k}} \ddot{r}_{a_{ik}} \qquad (78)$$

6.1 (Continued)

Therefore, for j = 1, 2, 3:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) = \left[j^{\underline{th}} \text{ row of } m_T\ (\ddot{\vec{r}}_T + \vec{\omega}_T \times \dot{\vec{r}}_T)\right]$$

$$+ m_p \sum_{i=1}^{6} \left[T_{i_{1j}} \sum_{k=1}^{3} \left(\dot{T}_{i_{1k}}\ \dot{r}_{a_{ik}} + T_{i_{1k}}\ \ddot{r}_{a_{ik}}\right)\right.$$

$$\left. + \dot{T}_{i_{1j}} \sum_{k=1}^{3} T_{i_{1k}}\ \dot{r}_{a_{ik}}\right] + \sum_{i=1}^{6} \left\{\frac{1}{l_{p_i}^3}\left(l_{p_i}\ \dot{I}_{A_i}\right.\right.$$

$$\left. - 2\, I_{A_i}\ \dot{l}_{p_i}\right)\left(T_{i_{2j}} \sum_{k=1}^{3} T_{i_{2k}}\ \dot{r}_{a_{ik}} + T_{i_{3j}} \sum_{k=1}^{3} T_{i_{3k}}\ \dot{r}_{a_{ik}}\right)$$

$$+ \frac{I_{A_i}}{l_{p_i}^2}\left[T_{i_{2j}} \sum_{k=1}^{3} \left(\dot{T}_{i_{2k}}\ \dot{r}_{a_{ik}} + T_{i_{2k}}\ \ddot{r}_{a_{ik}}\right)\right.$$

$$+ T_{i_{3j}} \sum_{k=1}^{3} \left(\dot{T}_{i_{3k}}\ \dot{r}_{a_{ik}} + T_{i_{3k}}\ \ddot{r}_{a_{ik}}\right)$$

$$\left. + \dot{T}_{i_{2j}} \sum_{k=1}^{3} T_{i_{2k}}\ \dot{r}_{a_{ik}} + \dot{T}_{i_{3j}} \sum_{k=1}^{3} T_{i_{3k}}\ \dot{r}_{a_{ik}}\right]$$

$$+ \frac{l_c}{l_{p_i}^2}\left(l_{p_i}\ \dot{m}_{q_i} - m_{q_i}\ \dot{l}_{p_i}\right)\left(T_{i_{2j}}\ \dot{y}_{e_i} + T_{i_{3j}}\ \dot{z}_{e_i}\right)$$

$$\left. + m_{q_i}\ \frac{l_c}{l_{p_i}}\left(T_{i_{2j}}\ \ddot{y}_{e_i} + T_{i_{3j}}\ \ddot{z}_{e_i} + \dot{T}_{i_{2j}}\ \dot{y}_{e_i} + \dot{T}_{i_{3j}}\ \dot{z}_{e_i}\right)\right\} \quad (79)$$

The general equations are extremely complex, particularly because of all the centrifugal and coriolis acceleration terms. These terms can be shown

6.1 (Continued)

to be small (less than 0.01 g) for the expected table velocities. Neglecting these terms, equation (79) becomes (for j = 1, 2, 3):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) = \left[j^{\underline{th}} \text{ row of } m_T \left(\ddot{\vec{r}}_T + \vec{\omega}_T \times \dot{\vec{r}}_T \right) \right]$$

$$+ m_p \sum_{i=1}^{6} T_{i_{1j}} \sum_{k=1}^{3} T_{i_{1k}} \ddot{r}_{a_{ik}}$$

$$+ \sum_{i=1}^{6} \frac{I_{A_i}}{l_{p_i}^2} \left(T_{i_{2j}} \sum_{k=1}^{3} T_{i_{2k}} \ddot{r}_{a_{ik}} + T_{i_{3j}} \sum_{k=1}^{3} T_{i_{3k}} \ddot{r}_{a_{ik}} \right) \tag{80}$$

$$+ \sum_{i=1}^{6} m_{q_i} \frac{l_c}{l_{p_i}} \left(T_{i_{2j}} \ddot{y}_{e_i} + T_{i_{3j}} \ddot{z}_{e_i} \right)$$

Likewise for j = 4, 5, 6:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) = \left[\text{row (j-3) of } [\tilde{I}_T] \{\dot{\vec{\omega}}_T\} + \vec{\omega}_T \times (\tilde{I}_T \cdot \vec{\omega}_T) \right]$$

$$+ m_p \sum_{i=1}^{6} \frac{\partial \dot{l}_{p_i}}{\partial \dot{Q}_i} \ddot{l}_{p_i} + \sum_{i=1}^{6} \frac{I_{A_i}}{l_{p_i}^2} \left(\ddot{y}_{r_i} C_{y_{ij}} + \ddot{z}_{r_i} C_{z_{ij}} \right) \tag{81}$$

$$+ \sum_{i=1}^{6} m_{q_i} \frac{l_c}{l_{p_i}} \left(C_{y_{ij}} \ddot{y}_{e_i} + C_{z_{ij}} \ddot{z}_{e_i} \right)$$

where:

$$\frac{\partial \dot{l}_{p_i}}{\partial \dot{Q}_4} = -T_{i_{12}} r_{za_i} + T_{i_{13}} r_{ya_i} \equiv C_{xx_i} \tag{82}$$

$$\frac{\partial \dot{l}_{p_i}}{\partial \dot{Q}_5} = T_{i_{11}} r_{za_i} - T_{i_{13}} r_{xa_i} \equiv C_{xy_i} \tag{83}$$

6.1 (Continued)

$$\frac{\partial \dot{1}_{p_i}}{\partial \dot{Q}_6} = -T_{i_{11}} r_{ya_i} - T_{i_{12}} r_{xa_i} \equiv C_{xz_i} \tag{84}$$

Define:

$$C_{y_{i4}} = -T_{i_{22}} r_{za_i} + T_{i_{23}} r_{ya_i} \equiv C_{yx_i} \tag{85}$$

$$C_{z_{i4}} = -T_{i_{32}} r_{za_i} + T_{i_{33}} r_{ya_i} \equiv C_{zx_i} \tag{86}$$

$$C_{y_{i5}} = T_{i_{21}} r_{za_i} - T_{i_{23}} r_{xa_i} \equiv C_{yy_i} \tag{87}$$

$$C_{y_{i6}} = -T_{i_{21}} r_{ya_i} + T_{i_{22}} r_{xa_i} \equiv C_{yz_i} \tag{88}$$

Then, using equations (72), (73), and (74), equation (81) becomes:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) = & \left[\text{row } (j-3) \text{ of } [\tilde{I}]\ \{\dot{\vec{\omega}}_T\} + \vec{\omega}_T \times (\tilde{I} \cdot \vec{\omega}_T)\right] \\ & + m_p \sum_{i=1}^{6} \left(\frac{\partial \dot{1}_{p_i}}{\partial \dot{Q}_j}\right) \sum_{k=1}^{3} T_{i_{1k}} \ddot{r}_{a_{ik}} \\ & + \sum_{i=1}^{6} \frac{I_{A_i}}{1_{p_i}^2} \left(C_{y_{ij}} \sum_{k=1}^{3} T_{i_{2k}} \ddot{r}_{a_{ik}} + C_{z_{ij}} \sum_{k=1}^{3} T_{i_{3k}} \ddot{r}_{a_{ik}}\right) \\ & + \sum_{i=1}^{6} m_{q_i} \frac{1_c}{1_{p_i}} \left(C_{y_{ij}} \ddot{y}_{e_i} + C_{z_{ij}} \ddot{z}_{e_i}\right) \end{aligned} \tag{89}$$

Simplifying equation (68), for j = 7, 9, 11, 13, 15, 17:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_i}\right) \approx m_{q_i} \left(\frac{1_c}{1_{p_i}} \ddot{y}_{r_i} + \ddot{y}_{e_i}\right)$$

6.1 (Continued)

$$\approx m_{q_i} \ddot{y}_{e_i} + m_{q_i} \frac{l_c}{l_{p_i}} \sum_{k=1}^{3} T_{i_{2k}} \ddot{r}_{a_{ik}} \tag{90}$$

and, simplifying equation (69), for j = 8, 10, 12, 14, 16, 18:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{Q}_j}\right) \approx m_{q_i} \ddot{z}_{e_i} + m_{q_i} \frac{l_c}{l_{p_i}} \sum_{k=1}^{3} T_{i_{3k}} \ddot{r}_{a_{ik}} \tag{91}$$

Expanding some of the summations in equations (89), (90), and (91):

$$\begin{aligned} \sum_{k=1}^{3} T_{i_{1k}} \ddot{r}_{a_{ik}} = T_{i_{11}} & \left(\ddot{x}_T - \dot{\omega}_z r_{ya_i} + \dot{\omega}_y r_{za_i} \right) \\ & + T_{i_{12}} \left(\ddot{y}_T + \dot{\omega}_z r_{xa_i} - \dot{\omega}_x r_{za_i} \right) \\ & + T_{i_{13}} \left(\ddot{z}_T - \dot{\omega}_y r_{xa_i} + \dot{\omega}_x r_{ya_i} \right) \end{aligned} \tag{92}$$

$$\begin{aligned} \sum_{k=1}^{3} T_{i_{2k}} \ddot{r}_{a_{ik}} = T_{i_{21}} & \left(\ddot{x}_T - \dot{\omega}_z r_{ya_i} + \dot{\omega}_y r_{za_i} \right) \\ & + T_{i_{22}} \left(\ddot{y}_T + \dot{\omega}_z r_{xa_i} - \dot{\omega}_x r_{za_i} \right) \\ & + T_{i_{23}} \left(\ddot{z}_T - \dot{\omega}_y r_{xa_i} + \dot{\omega}_x r_{ya_i} \right) \end{aligned} \tag{93}$$

$$\sum_{k=1}^{3} T_{i_{3k}} \ddot{r}_{a_{ik}} = T_{i_{31}} \left(\ddot{x}_T - \dot{\omega}_z r_{ya_i} + \dot{\omega}_y r_{za_i} \right)$$

6.1 (Continued)

$$+ T_{i_{32}} \left(\ddot{y}_T + \dot{\omega}_z r_{xa_i} - \dot{\omega}_x r_{za_i} \right)$$

$$+ T_{i_{33}} \left(\ddot{z}_T - \dot{\omega}_y r_{xa_i} + \dot{\omega}_x r_{ya_i} \right) \tag{94}$$

Using the definitions in equations (82) through (88), these equations reduce to:

$$\sum_{k=1}^{3} T_{i_{1k}} \ddot{r}_{a_{ik}} = T_{i_{11}} \ddot{x}_T + T_{i_{12}} \ddot{y}_T + T_{i_{13}} \ddot{z}_T + C_{xx_i} \dot{\omega}_x + C_{xy_i} \dot{\omega}_y + C_{xz_i} \dot{\omega}_z \tag{95}$$

$$\sum_{k=1}^{3} T_{i_{2k}} \ddot{r}_{a_{ik}} = T_{i_{21}} \ddot{x}_T + T_{i_{22}} \ddot{y}_T + T_{i_{23}} \ddot{z}_T + C_{yx_i} \dot{\omega}_x + C_{yy_i} \dot{\omega}_y + C_{yz_i} \dot{\omega}_z \tag{96}$$

$$\sum_{k=1}^{3} T_{i_{3k}} \ddot{r}_{a_{ik}} = T_{i_{31}} \ddot{x}_T + T_{i_{32}} \ddot{y}_T + T_{i_{33}} \ddot{z}_T + C_{zx_i} \dot{\omega}_x + C_{zy_i} \dot{\omega}_y + C_{zz_i} \dot{\omega}_z \tag{97}$$

The mass matrix shown in upper triangular form shown in Figure 6-1 is obtained by combining equations (95) through (97) and (82) through (88) with equations (80), (89), (90), and (91).

6.2 GENERALIZED FORCES

The generalized forces are obtained by considering the work required to produce a unit displacement for each degree of freedom.

	$\ddot{x}_T$	$\ddot{y}_T$	$\ddot{z}_T$	$\dot{\omega}_x$	$\dot{\omega}_y$	$\dot{\omega}_z$	$\ddot{y}_{e_i},\ i = 1\text{-}6$	$\ddot{z}_{e_i},\ i = 1\text{-}6$
$\ddot{x}_T$	$m_T + m_p \sum T_{i_{11}}^2 + \sum m_{L_i}\left(T_{i_{21}}^2 + T_{i_{31}}^2\right)$	$m_p \sum T_{i_{11}} T_{i_{12}} + \sum m_{L_i}\left(T_{i_{21}} T_{i_{22}} + T_{i_{31}} T_{i_{32}}\right)$	$m_p \sum T_{i_{11}} T_{i_{13}} + \sum m_{L_i}\left(T_{i_{21}} T_{i_{23}} + T_{i_{31}} T_{i_{33}}\right)$	$m_p \sum T_{i_{11}} C_{xx_i} + \sum m_{L_i}\left(T_{i_{21}} C_{yx_i} + T_{i_{31}} C_{zx_i}\right)$	$m_p \sum T_{i_{11}} C_{xy_i} + \sum m_{L_i}\left(T_{i_{21}} C_{yy_i} + T_{i_{31}} C_{zy_i}\right)$	$m_p \sum T_{i_{11}} C_{xz_i} + \sum m_{L_i}\left(T_{i_{21}} C_{yz_i} + T_{i_{31}} C_{zz_i}\right)$	$\cdots T_{i_{21}} m_{q_i} l_c / l_{p_i} \cdots$	$\cdots T_{i_{31}} m_{q_i} l_c / l_{p_i} \cdots$
$\ddot{y}_T$		$m_T + m_p \sum T_{i_{12}}^2 + \sum m_{L_i}\left(T_{i_{22}}^2 + T_{i_{32}}^2\right)$	$m_p \sum T_{i_{12}} T_{i_{13}} + \sum m_{L_i}\left(T_{i_{22}} T_{i_{23}} + T_{i_{32}} T_{i_{33}}\right)$	$m_p \sum T_{i_{12}} C_{xx_i} + \sum m_{L_i}\left(T_{i_{22}} C_{yx_i} + T_{i_{32}} C_{zx_i}\right)$	$m_p \sum T_{i_{12}} C_{xy_i} + \sum m_{L_i}\left(T_{i_{22}} C_{yy_i} + T_{i_{32}} C_{zy_i}\right)$	$m_p \sum T_{i_{12}} C_{xz_i} + \sum m_{L_i}\left(T_{i_{22}} C_{yz_i} + T_{i_{32}} C_{zz_i}\right)$	$\cdots T_{i_{22}} m_{q_i} l_c / l_{p_i} \cdots$	$\cdots T_{i_{32}} m_{q_i} l_c / l_{p_i} \cdots$
$\ddot{z}_T$			$m_T + m_p \sum T_{i_{13}}^2 + \sum m_{L_i}\left(T_{i_{23}}^2 + T_{i_{33}}^2\right)$	$m_p \sum T_{i_{13}} C_{xx_i} + \sum m_{L_i}\left(T_{i_{23}} C_{yx_i} + T_{i_{33}} C_{zx_i}\right)$	$m_p \sum T_{i_{13}} C_{xy_i} + \sum m_{L_i}\left(T_{i_{23}} C_{yy_i} + T_{i_{33}} C_{zy_i}\right)$	$m_p \sum T_{i_{13}} C_{xz_i} + \sum m_{L_i}\left(T_{i_{23}} C_{yz_i} + T_{i_{33}} C_{zz_i}\right)$	$\cdots T_{i_{23}} m_{q_i} l_c / l_{p_i} \cdots$	$\cdots T_{i_{33}} m_{q_i} l_c / l_{p_i} \cdots$
$\dot{\omega}_x$				$I_{xx} + m_p \sum C_{xx_i}^2 + \sum m_{L_i}\left(C_{yx_i}^2 + C_{zx_i}^2\right)$	$-I_{xy} + m_p \sum C_{xx_i} C_{xy_i} + \sum m_{L_i}\left(C_{yx_i} C_{yy_i} + C_{zx_i} C_{zy_i}\right)$	$-I_{xz} + m_p \sum C_{xx_i} C_{xz_i} + \sum m_{L_i}\left(C_{yx_i} C_{yz_i} + C_{zx_i} C_{zz_i}\right)$	$\cdots C_{yx_i} m_{q_i} l_c / l_{p_i} \cdots$	$\cdots C_{zx_i} m_{q_i} l_c / l_{p_i} \cdots$
$\dot{\omega}_y$				SYMMETRICAL	$I_{yy} + m_p \sum C_{xy_i}^2 + \sum m_{L_i}\left(C_{yy_i}^2 + C_{zy_i}^2\right)$	$-I_{yz} + m_p \sum C_{xy_i} C_{xz_i} + \sum m_{L_i}\left(C_{yy_i} C_{yz_i} + C_{zy_i} C_{zz_i}\right)$	$\cdots C_{yy_i} m_{q_i} l_c / l_{p_i} \cdots$	$\cdots C_{zy_i} m_{q_i} l_c / l_{p_i} \cdots$
$\dot{\omega}_z$						$I_{zz} + m_p \sum C_{xz_i}^2 + \sum m_{L_i}\left(C_{yz_i}^2 + C_{zz_i}^2\right)$	$\cdots C_{yz_i} m_{q_i} l_c / l_{p_i} \cdots$	$\cdots C_{zz_i} m_{q_i} l_c / l_{p_i} \cdots$
$\ddot{y}_{e_i},\ i = 1\text{-}6$							$\ddots\ m_{q_i}\ \ddots$	0
$\ddot{z}_{e_i},\ i = 1\text{-}6$								$\ddots\ m_{q_i}\ \ddots$

Note: $\sum \equiv \sum_{i=1}^{6}$

$m_{L_i} = I_{A_i} / l_i^2$

$C_{xx_i} = -T_{i_{12}} r_{za_i} + T_{i_{13}} r_{ya_i}$

$C_{xy_i} = T_{i_{11}} r_{za_i} - T_{i_{13}} r_{xa_i}$

$C_{xz_i} = -T_{i_{11}} r_{ya_i} + T_{i_{12}} r_{xa_i}$

$C_{yx_i} = -T_{i_{22}} r_{za_i} + T_{i_{23}} r_{ya_i}$

$C_{yy_i} = T_{i_{21}} r_{za_i} - T_{i_{23}} r_{xa_i}$

$C_{yz_i} = -T_{i_{21}} r_{ya_i} + T_{i_{22}} r_{xa_i}$

$C_{zx_i} = -T_{i_{32}} r_{za_i} + T_{i_{33}} r_{ya_i}$

$C_{zy_i} = T_{i_{31}} r_{za_i} - T_{i_{33}} r_{xa_i}$

$C_{zz_i} = T_{i_{31}} r_{ya_i} + T_{i_{32}} r_{xa_i}$

Figure 6-1. Mass Matrix

6.2 (Continued)

$$\delta W_j = Q_j\ (\delta q_j) \qquad (98)$$

where:

Q_j = Generalized force

δq_j = Unit displacement

Let F_p be the net piston force along the local x_s actuator axis. The work done is then:

$$\delta W_i = F_{p_i}\ x_{s_i} \qquad (99)$$

But, since:

$$\begin{Bmatrix} x_{s_i} \\ y_{s_i} \\ z_{s_i} \end{Bmatrix} = \begin{bmatrix} T_i \end{bmatrix} \begin{Bmatrix} x_T \\ y_T \\ z_T \end{Bmatrix} \qquad (100)$$

then:

$$\delta W_i = F_{p_i} \lfloor T_{i_{11}}\ T_{i_{12}}\ T_{i_{13}} \rfloor \begin{Bmatrix} x_T \\ y_T \\ z_T \end{Bmatrix} \qquad (101)$$

The generalized force for the table translational degrees of freedom are obtained by letting $x_T = 1$, $y_T = z_T = 0$ and then $y_T = 1$, $x_T = z_T = 0$, etc.

6.2 (Continued)

$$\begin{Bmatrix} F_{H_x} \\ F_{H_y} \\ F_{H_z} \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{6} F_{p_i} \; T_{i_{11}} \\ \sum_{i=1}^{6} F_{p_i} \; T_{i_{12}} \\ \sum_{i=1}^{6} F_{p_i} \; T_{i_{13}} \end{Bmatrix} \tag{102}$$

The displacements at the table swivel joints due to rotations of the table are:

$$\begin{Bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{Bmatrix}_T = \begin{bmatrix} 0 & -\Delta\theta_z & \Delta\theta_y \\ \Delta\theta_z & 0 & -\Delta\theta_x \\ -\Delta\theta_y & \Delta\theta_x & 0 \end{bmatrix} \begin{Bmatrix} r_{xa_i} \\ r_{ya_i} \\ r_{za_i} \end{Bmatrix} \tag{103}$$

For $\Delta\theta_x = 1, \;\; \Delta\theta_y = \Delta\theta_z = 0$:

$$\begin{Bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{Bmatrix}_T = \begin{Bmatrix} 0 \\ -r_{za_i} \\ r_{ya_i} \end{Bmatrix} \tag{104}$$

For $\Delta\theta_y = 1, \;\; \Delta\theta_x = \Delta\theta_z = 0$:

$$\begin{Bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{Bmatrix}_T = \begin{Bmatrix} r_{za_i} \\ 0 \\ -r_{xa_i} \end{Bmatrix} \tag{105}$$

6.2 (Continued)

For $\Delta\theta_z = 1$, $\Delta\theta_x = \Delta\theta_y = 0$:

$$\begin{Bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{Bmatrix}_T = \begin{Bmatrix} -r_{ya_i} \\ r_{xa_i} \\ 0 \end{Bmatrix} \tag{106}$$

Transforming these displacements to the servo actuator coordinate system:

$$\begin{aligned} \Delta x_{s_i} &= -T_{i_{12}} r_{za_i} + T_{i_{13}} r_{ya_i} && \text{for } \Delta\theta_x = 1 \\ \Delta x_{s_i} &= T_{i_{11}} r_{za_i} - T_{i_{13}} r_{xa_i} && \text{for } \Delta\theta_y = 1 \\ \Delta x_{s_i} &= -T_{i_{11}} r_{ya_i} + T_{i_{12}} r_{xa_i} && \text{for } \Delta\theta_z = 1 \end{aligned} \tag{107}$$

Then:

$$\delta W = \sum_{i=1}^{6} F_{p_i} \Delta x_{s_i} \tag{108}$$

or:

$$\begin{Bmatrix} M_{H_x} \\ M_{H_y} \\ M_{H_z} \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{6} F_{p_i} \left(-T_{i_{12}} r_{za_i} + T_{i_{13}} r_{ya_i} \right) \\ \sum_{i=1}^{6} F_{p_i} \left(T_{i_{11}} r_{za_i} - T_{i_{13}} r_{xa_i} \right) \\ \sum_{i=1}^{6} F_{p_i} \left(-T_{i_{11}} r_{ya_i} + T_{i_{12}} r_{xa_i} \right) \end{Bmatrix} \tag{109}$$

These generalized forces are combined with the $m_T (\vec{\omega}_T \times \dot{\vec{r}}_T)$ terms from equation (80), the $\vec{\omega}_T \times (\tilde{I}_T \cdot \vec{\omega}_T)$ terms from equation (81), the actuator

6.2 (Continued)

damping and stiffness terms and the external forces and moments to obtain the $\{C\}$ matrix of equation (44):

$$\begin{Bmatrix} \ddot{x}_T \\ \ddot{y}_T \\ \ddot{z}_T \\ \hline \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \hline \vdots \\ \ddot{y}_{e_i} \\ \vdots \\ \hline \vdots \\ \ddot{z}_{e_i} \\ \vdots \end{Bmatrix} = [M]^{-1} \begin{Bmatrix} -m_T \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_T \\ \dot{y}_T \\ \dot{z}_T \end{Bmatrix} + \begin{Bmatrix} F_{H_x} \\ F_{H_y} \\ F_{H_z} \end{Bmatrix} + \begin{Bmatrix} F_{E_x} \\ F_{E_y} \\ F_{E_z} \end{Bmatrix} \\ \hline - \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} \tilde{I} \end{bmatrix} \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} + \begin{Bmatrix} M_{H_x} \\ M_{H_y} \\ M_{H_z} \end{Bmatrix} + \begin{Bmatrix} M_{E_x} \\ M_{E_y} \\ M_{E_z} \end{Bmatrix} \\ \hline \vdots \\ -2\zeta_e \omega_{e_i} m_{q_i} \dot{y}_{e_i} - \omega_{e_i}^2 m_{q_i} y_{e_i} \\ \vdots \\ \hline \vdots \\ -2\zeta_e \omega_{e_i} m_{q_i} \dot{z}_{e_i} - \omega_{e_i}^2 m_{q_i} z_{e_i} \\ \vdots \end{Bmatrix} \qquad (110)$$

7.0 CALCULATION OF ACTUATOR VELOCITIES AND POSITIONS

Actuator lengths and velocities are calculated from the equations of motion variables for use in the servo loop feedbacks and for the determination of actuator friction forces.

Actuator velocities are calculated by first determining the velocities of the actuator/table attachment points in table coordinates:

$$\begin{Bmatrix} \dot{r}_{a_{i_1}} \\ \dot{r}_{a_{i_2}} \\ \dot{r}_{a_{i_3}} \end{Bmatrix} = \begin{Bmatrix} \dot{x}_T \\ \dot{y}_T \\ \dot{z}_T \end{Bmatrix} + \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{Bmatrix} r_{xa_i} \\ r_{ya_i} \\ r_{za_i} \end{Bmatrix} \qquad (111)$$

These velocities are then transformed to actuator coordinates to obtain:

$$\dot{l}_{p_i} = \sum_{k=1}^{3} T_{i_{1k}} \dot{r}_{a_{i_k}} \qquad (112)$$

Actuator lengths are calculated by first obtaining the components of actuator length in the inertial coordinate system.

$$\begin{Bmatrix} r_{s_{x_i}} \\ r_{s_{y_i}} \\ r_{s_{z_i}} \end{Bmatrix} = \begin{Bmatrix} x_I \\ y_I \\ z_I \end{Bmatrix} + \begin{bmatrix} A \end{bmatrix} \begin{Bmatrix} r_{xa_i} \\ r_{ya_i} \\ r_{za_i} \end{Bmatrix} - \begin{Bmatrix} 0 \\ y_{f_i} \\ z_{f_i} \end{Bmatrix} \qquad (113)$$

where y_{f_i} and z_{f_i} are the inertial coordinates of the floor swivel joints of each actuator.

7.0 (Continued)

Then the actuator lengths are calculated as follows:

$$l_{p_i} = \sqrt{r_{s_{x_i}}^2 + r_{s_{y_i}}^2 + r_{s_{z_i}}^2} \tag{114}$$

These actuator lengths and velocities are used in the feedback loops in the servo electronics shown in Figure 4-1.

APPENDIX

NOMENCLATURE

Symbol	Description
$[A]$	Transformation matrix from table to inertial coordinates
A_1, A_2	"Push" and "pull" stroke working areas of actuators
$\dot{a}_0$	Output of the pressure feedback first-order lag filter
$\dot{a}_1$	Output of the pressure feedback high-pass filter
B_p	Viscous damping coefficient of actuator
$\{C\}$	Column of generalized forces for equations of motion solution
C_p	Leakage coefficient across piston seals
e	Output of the forward loop compensation network
EI_r	Bending modulus of piston rod
F_{EXT}	External force
F_f	Coulomb friction force of actuator
F_H	Total hydraulic and friction forces acting on pistons
F_P	Net forces on actuator piston
$\tilde{I}$	Inertia tensor of the active table
I_A	Mass moment of inertia of entire actuator assembly about floor pivot
I_{AC}	Mass moment of inertia of cylinder (excluding the mass of the piston) about floor swivel joint
I_p	Mass moment of inertia of the piston rod

Symbol	Description
I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}	Moment and products of inertia
k_e	Effective lateral stiffness of the actuator
k_r	Piston rod stiffness
K_c	Valve pressure flow coefficient
K_f	Displacement feedback and command gain
K_g	Electronics and valve forward loop gain
K_{pf}	Pressure feedback loop gain
K_r	Velocity feedback loop gain
K_{rc}	Velocity command gain
l_c	Distance from floor swivel to center line of piston rod seal at end of cylinder
l_{c_i}	Commanded actuator length
$\dot{l}_{c_i}$	Commanded actuator velocity
$\ddot{l}_{c_i}$	Commanded actuator acceleration
l_o	Retracted length (between swivel joints) of actuators
l_{pm}	Maximum stroke of actuators
l_r	Length of piston rod
l_p	Actuator length
$\dot{l}_p$	Actuator velocity

Symbol	Description
$\ddot{l}_p$	Actuator acceleration
m_l	Effective rigid lateral mass of actuator assembly
m_p	Mass of piston rod and piston
m_q	Effective bending mass lumped at rod seal of cylinder
m_t	Table mass
M, M^{-1}	Mass matrix and mass matrix inverse
M_{EXT}	External moments
M_H	Moment acting about table c.g. from hydraulic and friction forces
P_s	Supply pressure
P_1, P_2	"Push" and "pull" actuator hydraulic pressure
Q_j	j^{th} generalized coordinate
Q_o	No-load valve flow
Q_1, Q_2	Hydraulic flow into and out of the actuator
r_s	Inertial vector components of actuator length
r_{xa}	X axis table station of actuator swivel joints with respect to the table c.g.
r_{ya}, r_{za}	Y, Z table coordinates of swivel joints with respect to the table c.g.
[T]	Transformation matrix transforming vectors from table coordinates to local actuator coordinates
[TI]	Transformation from actuator to inertial coordinates
t	Time
T	Kinetic energy of the system

Symbol	Description
V_{bw}	Velocity bandwidth for coulomb friction
V_o	Initial hydraulic volumes of push and pull strokes of fully retracted actuator
V_1, V_2	"Push" and "pull" hydraulic volumes
x_I, y_I, z_I	Inertial coordinates
x_s, y_s, z_s	Actuator coordinates
x_T, y_T, z_T	Table coordinates
X_{I_o}, Y_{I_o}, Z_{I_o}	Initial inertial coordinates of table c.g.
y_e, z_e	Bending displacements of the actuators
y_f, z_f	$\bar{Y}$ and Z inertial coordinates of floor swivel joints
α	Break frequency of first order filter
β	Break frequency of first order filter
β_A	The angle between the inertial z_I axis and the projection of the actuator x_s axis in the y_I-z_I plane
β_e	Equivalent hydraulic system bulk modulus
δ	Total actuator command signal
Δx_I, Δy_I, Δz_I, $\Delta\theta$, $\Delta\psi$, $\Delta\phi$	Sinusoidal amplitudes of translational commands for table c.g. and of table Euler angles
θ, ψ, ϕ	Euler angles
θ_A	The angle between the y_I-z_I plane and the actuator x_s axis

Symbol	Description
θ_o, ψ_o, ϕ_o	Initial Euler angles of the table coordinate system with respect to the inertial system
ϕ_i	Actuator bending mode shape
ζ_e	Damping constant for actuator bending
ζ_s	Damping constant of second order filter on displacement feedback
ζ_v	Damping constant of valve dynamics
ω_1, ω_2	Break frequencies of first order filters
ω_c	Displacement command signal frequency
ω_e	Actuator bending frequency
ω_f	Frequency of sinusoidal external forces and moments
ω_{pf1}, ω_{pf2}	Break frequencies of pressure feedback filters
ω_s	Frequency of second order filter on displacement feedback
ω_v	Frequency of valve dynamics
ω_x, ω_y, ω_z	Table rotational rates