NASA CONTRACTOR REPORT

NASA CR-2463

NASA CR-2463

UNSTEADY SUBSONIC AND TRANSONIC POTENTIAL FLOW OVER HELICOPTER ROTOR BLADES

by Morris P. Isom

Prepared by
POLYTECHNIC INSTITUTE OF NEW YORK
for Ames Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1974

ABSTRACT

Differential equations and boundary conditions for a rotor blade in forward flight, with subsonic or transonic tip Mach number, are derived. A variety of limiting flow regimes determined by different limits involving blade thickness ratio, aspect ratio, advance ratio and maximum tip Mach number is discussed. The transonic problem is discussed in some detail, and in particular the conditions that make this problem quasi-steady or essentially unsteady are determined. Asymptotic forms of equations and boundary conditions that are valid in an appropriately scaled region of the tip and an azimuthal sector on the advancing side are derived. These equations are then put in a form that is valid from the blade tip inboard through the strip theory region.

ACKNOWLEDGMENT

This research was supported by the U. S. Army Air Mobility Research and Development Laboratory, Ames Directorate, under Contract NAS2-6813. I am greatly indebted to the contract monitor, Mr. Francis X. Caradonna, for helpful discussion throughout the course of the contract.

SYMBOLS

$\bar{a} \equiv -(\bar{\Omega} \times \bar{r} + \bar{V})$	free stream velocity relative to blade fixed coordinates
c	sound speed
$f = f(x, y)$	non-dimensional blade geometry in blade attached co-ordinate system
$F = F(x, y, z, t) = 0$	blade geometry relative to an inertial coordinate system
$g = g(x, y)$	dimensional blade geometry in blade attached coordinate system
i, j	unit vectors along x and y axes
ℓ	blade chord
$M = \Omega R/c$	reference Mach number
$\mathcal{M} = M(1 + \mu)$	tip Mach number on advancing side
p	static pressure
$\bar{r}$	position vector
R	blade radius
s	distance in chords measured toward center of rotation from blade tip
$S = \tau^{1/3} s$	span coordinate for transonic problem
t	time
T	scaled time variable for transonic problem
V	forward velocity of rotor
x, y, z	coordinates along the blade chord, span and normal to the plane of rotation
γ	ratio of specific heats
δ	scale parameter for time in transonic problem
ϵ	inverse of blade aspect ratio
μ	advance ratio
τ	thickness ratio

φ	velocity potential
ω	reduced frequency
Ω	rotational velocity

A prime refers to a fixed (inertial) coordinate system.

Subscript ∞ refers to conditions in undisturbed air.

A bar denotes a vector.

UNSTEADY SUBSONIC AND TRANSONIC POTENTIAL FLOW OVER HELICOPTER ROTOR BLADES

I. INTRODUCTION

Helicopter rotor blades that could support a nearly shock free transonic flow in the advancing blade tip region would improve helicopter aerodynamic performance and would reduce noise produced by the blade. The design of better high speed rotor blades has so far been guided mostly by qualitative consideration of how blade geometry influences the flow in the complex aerodynamic environment of the tip region. While such considerations are undoubtedly useful, they necessarily involve some guessing of how blade geometry influences flow behavior, particularly at high speeds. Verification of a particular design must rely on expensive (and time consuming) wind tunnel and flight testing. It is therefore clear that blade design (as in any other design problem) would benefit from analytical and numerical methods that would predict the details of a transonic flow field in the tip region for a given blade geometry and specified flight parameters. Calculation of the nonlinear, three-dimensional and possibly unsteady flow near a rotor tip is, of course, only a first step toward better design. The problem here is that while there have been recent advances in the analytical and numerical methods that will generate shock-free two-dimensional airfoil sections, there are no such methods available for three-dimensional flows. There is also no indication yet that a shock-free two-dimensional section would show better performance in a three-dimensional environment than any other standard airfoil section, although it may be that the two-dimensional designs do have advantages over standard sections in a three-dimensional application.

Preliminary calculations of transonic pressure distributions on rotor blades were made by Caradonna and Isom (1). They considered what is probably the simplest case, a non-lifting, hovering rectangular blade with 6% thick biconvex sections operating in the lower transonic regime. The calculations show how direct numerical methods suitable for two-dimensional problems can be extended in a fairly straightforward way to the three-dimensional problem of steady transonic flow near a rotor tip.

The analysis of reference (1) is restricted to a rotor with zero advance ratio, so that the flow is steady in a coordinate system attached to the blade. In this report we shall consider the more complicated problem of a rotor with non-zero advance ratio. The primary objective is to obtain suitably scaled differential equations that describe transonic flow over the advancing rotor tip, and to show how possible flow regimes depend on various limiting cases of scale parameters. In particular, it is important to isolate what might be called the "worst" case, which is a three-dimensional, nonlinear and unsteady (as opposed to quasi-steady) flow regime on the advancing side of the blade.

The hover problem of reference (1) is characterized by three parameters: blade thickness ratio τ, aspect ratio $AR \equiv \epsilon^{-1}$ and tip Mach number $M = \Omega R/c$ (Ω is angular velocity, R is blade radius and c the speed of sound in undisturbed air). It was shown in reference (1) that the hover problem is in some ways similar to the problem of flow over a fixed wing, and the analysis was guided by this (sometimes rough) analogy. The advancing rotor problem involves one additional parameter, the advance ratio μ, as well as the time variable t. The reference Mach number is then the maximum free-stream Mach number on the advancing side,

$\mathcal{M} = M(1 + \mu)$. Our objective is to determine how τ, ϵ, $\mathcal{M}$ and μ combine as coefficients in the governing differential equations and boundary conditions, and then to consider asymptotic forms of these equations, with τ and ϵ small, $\mathcal{M}$ either subsonic or transonic, and μ either order one or small. The asymptotic results always simplify unwieldly equations, and, more importantly, they serve to put equations into a form that is subject to useful physical interpretation. It will be noted, however, that pursuit of a variety of limiting cases may produce distinctions that are rather fine, and in some instances too fine. For example, we consider three limiting values of advance ratio: a high advance ratio problem where μ is of the order of 0.4 and fixed; a case for which $\mu = O(\tau^{2/3})$; and one for which $\mu = O(\tau)$, with $\tau \rightarrow 0$ in each case. Each of these limiting cases results in a distinct set of governing equations and a distinguished flow physics. However, when the blade thickness ratio τ is about 0.1, then $\mu = 0.4$, or $\mu = O(\tau^{2/3})$ or $\mu = O(\tau)$ are not widely different values for advance ratio. Nevertheless, the usual scaling procedure followed here is always based on taking appropriate limits ($\tau \rightarrow 0$, $\epsilon \rightarrow 0$, etc.), and whether the resulting equations describe sharply distinguished physical regimes encountered in practice must depend on detailed numerical calculations and experiments.

II. FORMULATION OF PROBLEMS

Consider a two-blade rotor advancing at a constant velocity $\overline{V}$ and rotating at angular velocity $\overline{\Omega}$ perpendicular to the direction of advance. For our purposes, the number of blades is not important. Attention is restricted to the outer few chords of the advancing blade. In the near flow field, close to the advancing tip region, the flow is not influenced (in a first approximation) by other portions of the blade system. Boundary conditions on the blade are formulated for arbitrary planform, although special care may be required if the planform has some unusual shape, as for one with a kink. We shall not consider the lifting problem because too little is known about rotor wake properties, this being particularly true for high speed rotors. It will also be assumed that the flow is inviscid and isentropic. A velocity potential therefore exists for the flow described in an inertial coordinate system. We take this system to be one at rest relative to the undisturbed air.

The potential equation for φ is an inertial reference system is

$$\varphi_{tt} + \frac{\partial}{\partial t} (\nabla\varphi)^2 + \nabla\varphi \cdot \nabla[\frac{1}{2} (\nabla\varphi)^2] = c^2 \nabla^2 \varphi \tag{1}$$

where c is the local adiabatic sound speed. Bernoulli's equation, relating c and φ is,

$$\varphi_t + \frac{1}{2} (\nabla\varphi)^2 + \frac{c^2}{\gamma-1} = \frac{c_\infty^2}{\gamma-1} \tag{2}$$

where c_∞ is the sound speed in the undisturbed gas.

In a fixed Cartesian (x, y, z) coordinate system, let the blade geometry and location be described by F(x, y, z, t) = 0. The condition of flow tangency at the blade surface is

$$F_t + \nabla\varphi \cdot \nabla F = 0, \tag{3}$$

and we require that φ_t and $\nabla\varphi$ go to zero at infinity.

In all cases of interest here (including nonlinear problems), the third term on the left hand side of equation (1), the cubic term, is negligible. That term will be neglected in all further discussion.

It is convenient to reformulate the problem in a coordinate system attached to the blade, with the origin at the center of rotation. The velocity of advance $\overline{V}$ will be taken in the direction of the negative x-axis of the fixed reference frame. Coordinate z for both fixed and moving frames is normal to the plane of rotation, and the angular velocity of rotation $\overline{\Omega}$ points in the positive z-direction. In the rotating frame, the blade-attached x-axis is in the chordwise direction and the y-axis is along the blade span.

We now transform equations (1)-(3) and rewrite them in a coordinate system attached to the blade. For a moment it will be necessary to distinguish between time and space coordinates of the inertial frame and time and space coordinates of the moving frame. Let primed variables refer to the inertial system, and unprimed variables refer to the blade-attached system. Let $\overline{r}' = (x', y', z')$ and $\overline{r} = (x, y, z)$ be position vectors in these two systems. The two reference systems are related by

$$\overline{r}' = \overline{V}\, t + R(t)\, \overline{r},$$

$$t' = t,$$

where $R(t)$ is the rotation matrix

$$R(t) = \begin{bmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The inverse transformation is

$$\overline{r} = R^{-1}(t')\, \overline{r}' - R^{-1}(t')\, \overline{V}\, t'$$

$$t = t'.$$

where R^{-1} is the inverse of R. For the purposes of equations (1)-(3), the gradient operation in the inertial system may be replaced by the gradient operation in the moving system. Time differentiation in the inertial system transforms as

$$\frac{\partial \varphi}{\partial t'} = \frac{\partial \varphi}{\partial t} + \frac{\partial x_i}{\partial t'} \frac{\partial \varphi}{\partial x_i} \tag{4}$$

with a sum over the i-subscript. It is easily shown that

$$\frac{\partial x_i}{\partial t'} = - [\dot{R}(0)\, \overline{r}]_i - [R^{-1}(t)\, \overline{V}]_i = - (\overline{\Omega} \times \overline{r} + \overline{V})_i$$

where $\dot{R}(0)$ is dR/dt evaluated at $t = 0$. In a more convenient notation, equation (4) becomes

$$\frac{\partial \varphi}{\partial t'} = \frac{\partial \varphi}{\partial t} - (\overline{\Omega} \times \overline{r} + \overline{V}) \cdot \nabla\varphi \tag{5}$$

The velocity $\overline{V}$ in equation (5) must be expressed as it appears to an observer in the moving frame. In the fixed frame, $\overline{V} = - i' V$. In the moving frame,

$$\overline{V} = - i\, V \cos \Omega\, t + j\, V \sin \Omega\, t.$$

An observer in the moving frame sees a free-stream velocity $-(\overline{\Omega} \times \overline{r} + \overline{V})$. Let

$$\overline{a} = - (\overline{\Omega} \times \overline{r} + \overline{V}) .$$

Then equation (5) is

$$\frac{\partial \varphi}{\partial t'} = \frac{\partial \varphi}{\partial t} + \overline{a} \cdot \nabla \varphi .$$

To transform equations (1)-(3) to the moving frame, we first note that a second application of equation (5) gives

$$\varphi_{t' t'} = \varphi_{tt} + \bar{a} \cdot \nabla \nabla \varphi \cdot \bar{a} + 2 \bar{a} \cdot \nabla \varphi_t - \bar{\Omega} \times \bar{a} \cdot \nabla \varphi - \dot{\bar{V}} \cdot \nabla \varphi .$$

The second term in equation (1) is

$$\frac{\partial}{\partial t'} (\nabla \varphi)^2 = (\frac{\partial}{\partial t} + \bar{a} \cdot \nabla) (\nabla \varphi)^2$$

$$= 2 \nabla \varphi \cdot \nabla \varphi_t + 2 \bar{a} \cdot \nabla \nabla \varphi \cdot \nabla \varphi$$

The right hand side of equation (1), $c^2 \nabla^2 \varphi$, requires c^2 from Bernoulli's equation (2), and c^2 must be expressed in the moving frame. Bernoulli's equation transforms to

$$c^2 = c_\infty^2 - (\gamma - 1) [\varphi_t + \bar{a} \cdot \nabla \varphi + \frac{1}{2} (\nabla \varphi)^2] .$$

The complete equation for φ written in the moving frame is now

$$\varphi_{tt} + \bar{a} \cdot \nabla \nabla \varphi \cdot \bar{a} + 2 \bar{a} \cdot \nabla \varphi_t - \bar{\Omega} \times \bar{a} \cdot \nabla \varphi - \dot{\bar{V}} \cdot \nabla \varphi$$

$$+ 2 \nabla \varphi \cdot \nabla \varphi_t + 2 \bar{a} \cdot \nabla \nabla \varphi \cdot \nabla \varphi$$

$$= \{ c_\infty^2 - (\gamma - 1) [\varphi_t + \bar{a} \cdot \nabla \varphi + \frac{1}{2} (\nabla \varphi)^2] \} \nabla^2 \varphi \qquad (6)$$

Equation (6) is the exact potential equation written in a coordinate system that translates at constant velocity $\bar{V}$ and that rotates with constant angular velocity $\bar{\Omega}$. As far as equation (6) is concerned, $\bar{V}$ and $\bar{\Omega}$ are arbitrary vectors as they appear in the definition $\bar{a} = - \bar{\Omega} \times \bar{r} - \bar{V}$. Later, $\bar{V}$ and $\bar{\Omega}$ will be specified as vectors appropriate for a rotating blade problem.

It is very useful to simplify equation (6) by non-dimensionalizing and scaling it in a way that is appropriate for the particular problems at hand. This simplification requires a choice of suitable length and time scales, and a further specification of the relative magnitudes of all non-dimensional parameters that enter the problem. The choice of length and time scales will in turn depend on what part of the flow field one is interested in. For the rotor problem, there are several distinguished flow regions. Some of their features are listed here.

<u>Region 1.</u> This is the region near the blade and sufficiently far inboard from the tip that tip effects are not important. It is the strip theory or blade element region. If the blade tip Mach number is subsonic or transonic, equation (6) in Region 1 will become linear and quasi-steady; that is, all nonlinear terms and time derivatives may be neglected. In addition, the flow in this region becomes approximately two-dimensional. Span coordinate and time appear only as parameters in the problem. The length scale is blade chord and time scale is Ω^{-1}.

<u>Region 2.</u> This is the region near the blade surface and near the blade tip. If the blade tip Mach number is subsonic (not transonic), equation (6) will be linear, quasi-steady and three-dimensional. The length scale is distance in chords measured from the tip and time scale is Ω^{-1}. In a first approximation, the blade will appear to be semi-infinite as viewed from the tip. When a quantity (e.g., φ) is calculated in the tip region, then, as the calculation marches inboard from the tip, the results must match those obtained in Region 1.

If the tip Mach number is transonic, analysis of equation (6) in the tip region is more complicated. The equation will be nonlinear in spatial derivatives, and may be quasi-steady or essentially unsteady, depending on certain limiting cases to be discussed later.

Region 3. This region will be called the first far field region. The characteristic length scale is blade span. For subsonic flow, an observer in this region would view the blade as a line rotating about its mid-point. Disturbances measured in this region would be considered as caused by point singularities continuously distributed along the blade line. Equation (6) becomes linear, three-dimensional and unsteady for both subsonic and transonic Mach numbers. The inner boundary of this region is the combined flow of Regions 1 and 2; the outer boundary is the field at infinity.

Region 4. This is the distant far field, defined by letting $x^2 + y^2 + z^2 \to \infty$. It is the region appropriate for far field acoustic calculations. At any point in this far field, the local flow is described by retaining all linear terms in equation (1). Time scale is Ω^{-1} and distance is a characteristic wavelength $2\pi c/\Omega$.

We are primarily interested in Region 2, the tip region. A preliminary simplification of equation (6) will now be made, and it can be verified later that terms dropped here are negligible. All first order derivatives on the left-hand side of equation (6) may be neglected. The term $2\,\nabla\varphi \cdot \nabla\varphi_t$ may be neglected compared to the term $2\,\bar{a} \cdot \nabla\varphi_t$ because $\bar{a}$ is essentially a first order term and $\nabla\varphi$ is the perturbation velocity. On the right hand side, the terms φ_t and $\frac{1}{2}(\nabla\varphi)^2$ in the square bracket that multiplies $\nabla^2\varphi$ may be neglected. With these observations, the basic equation (6) becomes

$$\varphi_{tt} + \bar{a} \cdot \nabla\nabla\varphi \cdot \bar{a} + 2\,\bar{a} \cdot \nabla\varphi_t + 2\,\bar{a} \cdot \nabla\nabla\varphi \cdot \nabla\varphi$$

$$= [c_\infty^2 - (\gamma - 1)\,\bar{a} \cdot \nabla\varphi]\,\nabla^2\varphi. \qquad (7)$$

The condition that the flow be tangent to the blade surface is easily expressed in the moving coordinate system, where the blade surface appears stationary. Let blade geometry be defined by $z = g(x, y)$. Then the blade boundary condition is

$$\varphi_z = (-\overline{\Omega} \times \overline{r} - \overline{V} + \nabla\varphi) \cdot \nabla g,$$

evaluated on $z = g(x, y)$. The term $\nabla\varphi$ is negligible, and we use the mean surface approximation to get

$$\varphi_z (x, y, 0, t) = -(\overline{\Omega} \times \overline{r} + \overline{V}) \cdot \nabla g. \tag{8}$$

Scaling of equations (7) and (8) requires that they be written out in scalar form. Using the definition of $\overline{a}$, and rearranging some terms, equation (7) expands into the following lengthy form:

$$\begin{aligned}
&\varphi_{tt} + 2(\Omega y + V \cos \Omega t)\, \varphi_{xt} - 2(\Omega x + V \sin \Omega t)\, \varphi_{yt} \\
&\quad = [c_\infty^2 - (\Omega y + V \cos \Omega t)^2 - (\gamma+1)(\Omega y + V \cos \Omega t)\, \varphi_x \\
&\qquad\quad - (\gamma - 1)(\Omega x + V \sin \Omega t)\, \varphi_y]\, \varphi_{xx} \\
&\quad + [2(\Omega x + V \sin \Omega t)(\Omega y + V \cos \Omega t) + 2(\Omega x + V \sin \Omega t)\, \varphi_x \\
&\qquad\quad - 2(\Omega y + V \cos \Omega t)\varphi_y]\, \varphi_{xy} \\
&\quad + [c_\infty^2 - (\Omega x + V \sin \Omega t)^2 - (\gamma - 1)(\Omega x + V \cos \Omega t)\, \varphi_x \\
&\qquad\quad + (\gamma + 1)(\Omega x + V \sin \Omega t)\, \varphi_y]\, \varphi_{yy} \\
&\quad - 2(\Omega_y + V \sin \Omega t)\, \varphi_z\, \varphi_{xz} + 2(\Omega_x + V \sin \Omega t)\, \varphi_z\, \varphi_{xz} \\
&\quad + [c_\infty^2 - (\gamma - 1)(\Omega_y + V \cos \Omega t)\varphi_x + (\gamma - 1)(\Omega_x + V \sin \Omega t)]\varphi_{zz}
\end{aligned} \tag{9}$$

The expanded form of the boundary condition, equation (8), is

$$\varphi_z(x, y, 0, t) = (\Omega\, y + V \cos \Omega\, t)\, g_x - (\Omega\, x + V \sin \Omega\, t)\, g_y \tag{10}$$

Equation (9) and (10), with a suitable boundary condition at infinity, describe the aerodynamics of the nonlifting rotor blade. These equations will be further simplified by scaling them. This scaling will be done in the next section where the two most important problems are considered, the blade with subsonic advancing tip Mach number, and then the blade with transonic Mach number on the advancing tip.

III. SCALED EQUATIONS

1. The subsonic problem

We first consider the problem of a blade with advancing tip Mach number that is subsonic.

The introduction of new symbols for nondimensional scaled variables can be avoided by a convenient notation. The combination Ωt in equation (9) is simply replaced by t, with the understanding that from now on t stands for Ωt. Similarly, let ℓ be the blade chord, and use the substitution $x/\ell \rightarrow x$, where the x on the left is dimensional and on the right nondimensional. The complete set of time and spatial substitutions is then

$$\begin{aligned} \Omega t &\rightarrow t \\ x/\ell &\rightarrow x \\ y/R &\rightarrow y \\ z/\ell &\rightarrow z. \end{aligned} \tag{11}$$

A reference Mach number will be taken as

$$M = \Omega R / c \tag{12}$$

and advance ratio

$$\mu = V/\Omega R . \tag{13}$$

The blade has reference thickness ratio τ. Using the substitution scheme (11), the scaled potential for the subsonic problem is

$$\frac{\varphi}{\tau \Omega R \ell} \rightarrow \varphi . \tag{14}$$

The blade has aspect ratio $AR = R/\ell$. Define the parameter ϵ

$$\epsilon = AR^{-1} = \ell / R , \tag{15}$$

which is generally a small number for a rotor blade. We now use the substitutions (11) - (15) in the differential equation (9). All nonlinear terms may be neglected in the subsonic tip problem. The scaled differential equation is therefore

$$M^2 \epsilon^2 \varphi_{tt} + 2M^2 \epsilon (y + \mu \cos t) \varphi_{xt} - 2M^2 \epsilon^2 (\epsilon x + \mu \sin t) \varphi_{yt}$$

$$= [1 - M^2 (y + \mu \cos t)^2] \varphi_{xx} + 2M^2 \epsilon (\epsilon x + \mu \sin t)(y + \mu \cos t) \varphi_{xy}$$

$$+ \epsilon^2 [1 - M^2 (\epsilon x + \mu \sin t)^2] \varphi_{yy} + \varphi_{zz}. \tag{16}$$

A properly posed problem for the tip region requires some y-derivatives in the differential equation. The scale length for this region is distance s in chords measured from the tip toward the center of rotation:

$$s = \frac{1-y}{\epsilon} , \qquad y = 1 - \epsilon s \tag{17}$$

Equation (16) becomes

$$M^2\epsilon^2\varphi_{tt} + 2M^2\epsilon(1-\epsilon s + \mu\cos t)\varphi_{xt} + 2M^2\epsilon(\epsilon x + \mu\sin t)\varphi_{st}$$

$$= [1-M^2(1-\epsilon s + \mu\cos t)^2]\varphi_{xx} - 2M^2(\epsilon x + \mu\sin t)(1-\epsilon s + \mu\cos t)\varphi_{xs}$$

$$+ [1-M^2(\epsilon x + \mu\sin t)^2]\varphi_{ss} + \varphi_{zz}. \qquad (18)$$

Keep M, μ, x, s, z and t fixed. Let $\epsilon \to 0$. Equation (18) becomes

$$[1-M^2(1+\mu\cos t)^2]\varphi_{xx} - 2M^2\mu\sin t\,(1+\mu\cos t)\varphi_{xs}$$

$$+ (1-M^2\mu^2\sin^2 t)\varphi_{ss} + \varphi_{zz} = 0 \qquad (19)$$

In the boundary condition, equation (10), let blade geometry be further specified (in dimensional form) by

$$z = g(x, y) = \ell\,\tau f(\frac{x}{\ell}, \frac{y}{R})$$

where τ is a reference thickness ratio. Nondimensional scaled geometry is

$$z = \tau f(x, y).$$

Equation (10) transforms to

$$\varphi_z(x, y, 0, t) = (y + \mu\cos t)f_x - \epsilon(\epsilon x + \mu\sin t)f_y. \qquad (20)$$

Replacing y by $1-\epsilon s$ and letting $\epsilon \to 0$ gives

$$\varphi_z(x, 1, 0, t) = (1 + \mu\cos t)\, f_x(x, 1) + \mu\sin t\, f_s(x, 1) \qquad (21)$$

The differential equation (19) and boundary condition (21) describe the tip flow for all values of time, that is, for all azimuthal angles around the rotor disk. The scaled problem is also quasi-steady, with time appearing only as a parameter. Note also that the problem posed by equations (19) and (21) is exactly the same as the problem of the flow in the tip region of a fixed yawed wing in subsonic motion, moving with forward velocity $\overline{V} = -i'(\Omega R + V\cos\Omega t) + j' V\sin\Omega t$ relative to the undisturbed air.

It has been assumed so far that the advance ratio μ is not small. If μ were small, say of the order of thickness ratio τ, then it would be inconsistent to retain terms of order μ and smaller in equations (19) and (21). If $\mu = O(\tau)$, then all terms involving μ must be neglected, and time no longer enters the problem at all: for small μ, the flow in the tip region of an advancing rotor with subsonic tip Mach number becomes equivalent to the flow in tip region of an unyawed fixed wing moving at constant velocity $\overline{V} = -i' V$ relative to undisturbed air.

While equations (19) and (21) describe the tip flow for the subsonic rotor, they are not convenient for use in numerical calculations. A single set of equations that describes the flow in the tip region, and which remains valid inboard from the tip through the strip theory region, is found by inspection of equations (19) and (21) to be

$$[1-M^2(y+\mu\cos t)^2]\varphi_{xx} + 2M^2\epsilon\,\mu\sin t(1+\mu\cos t)\varphi_{xy}$$

$$+\epsilon^2[1-M^2\mu^2\sin^2 t]\varphi_{yy} + \varphi_{zz} = 0 \tag{22}$$

$$\varphi_z(x, y, 0, t) = (y+\mu\cos t)f_x - \epsilon\,\mu\sin t\, f_y\,.$$

2. The subsonic far field

In numerical calculations, some condition must be imposed on a spatial boundary that is, in some sense, far from the blade. There are two possibilities. The first is to set φ and its derivatives equal to zero on a boundary that is some number of blade chords distant from the blade itself.

The second alternative is to derive an asymptotic far field representation of the solution, and impose this representation on a boundary that is again suitably far from the blade. This second approach has been used by Murman and Cole (2) for a two-dimensional airfoil in transonic potential flow, and extended by Klunker (3) to the case of a three-dimensional fixed wing in transonic flow. For the subsonic rotor problem, the asymptotic far field is most easily derived in an inertial coordinate system, where the potential satisfies $\varphi_{tt} = c^2 \nabla^2 \varphi$. The far field is then represented by the radiative part of a potential field that is caused by a system of retarded doublets continuously distributed along the span of the wing. The strength of the doublets will vary along the span, and also with time for an advancing rotor. When this representation has been found in the inertial system, it can be transformed to the moving frame and used as an outer boundary condition for equations (19) and (21). It is not clear, however, whether the additional labor involved in finding and using an accurate far field representation is worth the effort. Numerical experiments with two- and three-dimensional fixed wings indicate that simply setting $\varphi = 0$ on the outer numerical boundary leads to sufficiently accurate results.

3. The transonic problem

The starting point for the nonlinear transonic problem is equation (7), and its expanded form (9). We are again guided by the close analogy between the flow near the tip of the rotor and the corresponding flow near the tip of a fixed wing. The preliminary scaling of spatial coordinates is exactly the same as for the fixed wing problem. Thus, in equation (9) we make substitutions

$$\Omega t \rightarrow t$$
$$x/\ell \rightarrow x$$
$$y/R \rightarrow y$$
$$\tau^{1/3} z/\ell \rightarrow z$$
$$\varphi/\tau^{2/3}\Omega R \ell \rightarrow \varphi$$

After deleting some terms by inspection from equation (9), we get

$$\frac{M^2\epsilon^2}{\tau^{2/3}}\varphi_{tt} + 2M^2\frac{\epsilon}{\tau^{2/3}}(y+\mu\cos t)\varphi_{xt} - \frac{2M^2\epsilon^2}{\tau^{2/3}}(\epsilon x+\mu\sin t)\varphi_{yt}$$

$$= \left[\frac{1-M^2(y+\mu\cos t)^2}{\tau^{2/3}} - (\gamma+1)M^2(y+\mu\cos t)\varphi_x\right]\varphi_{xx}$$

$$+ \frac{2M^2\epsilon}{\tau^{2/3}}(\epsilon x+\mu\sin t)(y+\mu\cos t)\varphi_{xy} + \frac{\epsilon^2}{\tau^{2/3}}\varphi_{yy} + \varphi_{zz} \qquad (23)$$

The blade boundary condition is

$$\varphi_z(x, y, 0, t) = (y+\mu\cos t)f_x - \epsilon(\epsilon x+\mu\sin t)f_y. \qquad (24)$$

In contrast to the subsonic problem, some assumption must now be made about the relative orders of magnitude of $\epsilon = AR^{-1}$ and τ. A typical blade has thickness ratio of the order .06 to 0.1 near the tip, and aspect ratio of order 20 to (in an extreme case) 10. Therefore we assume the limits $\tau \rightarrow 0$, $\epsilon \rightarrow 0$ and $\tau/\epsilon = O(1)$. We first assume that the advance ratio is of order 0.3 to 0.4 and is held fixed when any limits are taken.

When the flow is transonic only on the advancing side of the rotor disk, there is some finite azimuthal angle measured about t (or Ωt) = 0 within which the flow is transonic and outside of which the flow should become subsonic. It is therefore necessary to scale time in a suitable way.

Also, when the flow is transonic there is some characteristic region, measured in chords, near the tip when transonic conditions exist and inboard of which the flow becomes subsonic and follows strip theory. It is therefore also necessary to appropriately scale the span coordinate y to obtain a properly posed boundary value problem in the tip region. Following Reference (1), we introduce a new span variable:

$$S = \tau^{1/3}\ \frac{(1-y)}{\epsilon}, \qquad y = 1 - \frac{\epsilon}{\tau^{1/3}}\ S \tag{25}$$

Introduce a time scale parameter δ by

$$t = \delta\ T, \tag{26}$$

with δ to be determined. Use of equations (25) and (26) in (23) gives

$$\begin{aligned}
&\frac{M^2\,\epsilon^2}{\tau^{2/3}\delta^2}\,\varphi_{TT} + \frac{2M^2\epsilon}{\tau^{2/3}\delta}\left(1 - \frac{\epsilon}{\tau^{1/3}}\,S + \mu\cos\delta T\right)\varphi_{xT} \\
&\qquad + 2\,\frac{M^2}{\tau^{1/3}}\,(\epsilon x + \mu\sin\delta T)\varphi_{ST} \\
&= \left[\frac{1 - M^2\left(1-\frac{\epsilon}{t^{1/3}}\,S + \mu\cos\delta T\right)^2}{\tau^{2/3}} - (\gamma+1)M^2\left(1-\frac{\epsilon}{\tau^{1/3}}\,S+\mu\cos\delta T\right)\varphi_x\right]\varphi_{xx} \\
&\qquad - \frac{2M^2}{\tau^{1/3}}\,(\epsilon x + \mu\sin\delta T)\left(1 - \frac{\epsilon}{\tau^{1/3}}\,S + \mu\cos\delta T\right)\varphi_{xS} \\
&\qquad + \varphi_{SS} + \varphi_{zz}.
\end{aligned} \tag{27}$$

The boundary condition is

$$\varphi_z\left(x,\ 1 - \frac{\epsilon}{\tau^{1/3}}\,S,\ 0,\ \delta T\right) = \left(1 - \frac{\epsilon}{\tau^{1/3}}\,S + \mu\cos\delta T\right)f_x + \tau^{1/3}(\epsilon x+\mu\sin\delta t)f_S.$$

The maximum Mach number of the free stream relative to the blade occurs when T = 0, and is $\mathfrak{M} = M(1 + \mu)$. As for a fixed wing, we assume $(1 - \mathfrak{M}^2)\,\tau^{-2/3} = O(1)$. Consider first the coefficient of φ_{xx} in equation (27), and in particular consider the part

$$\frac{1 - M^2\left(1 - \frac{\epsilon}{\tau^{1/3}}\ S + \mu \cos \delta T\right)^2}{\tau^{2/3}} = \frac{1 - M^2(1+\mu)^2}{\tau^{2/3}} + 2M^2(1+\mu)\frac{\epsilon}{\tau}\ S$$

$$+ \frac{2M^2}{\tau^{2/3}}\left(1 + \mu - \frac{\epsilon}{\tau^{1/3}} S\right)\mu\,(1 - \cos \delta T) - \frac{\mu^2 M^2}{\tau^{2/3}}(1 - \cos \delta T)^2$$

$$+ \frac{\epsilon^2}{\tau^{4/3}}\ S^2 \tag{28}$$

Let S and T be O(1), let $\epsilon \to 0$, $\tau \to 0$, $\epsilon/\tau = 0(1)$, $(1 - \mathfrak{M}^2)/\tau^{2/3} = O(1)$, and for the moment assume also $\delta \to 0$ as $\tau \to 0$. Equation (28) becomes

$$\frac{1 - M^2\left(1 - \frac{\epsilon}{\tau^{1/3}}\ S + \mu \cos \delta T\right)}{\tau^{2/3}} \sim \frac{1 - \mathfrak{M}^2}{\tau^{2/3}} + 2M^2(1+\mu)\frac{\epsilon}{\tau}\ S$$

$$+ M^2\ \mu\ (1 + \mu)\frac{\delta^2}{\tau^{2/3}}\ \ T^2 \tag{29}$$

The second part of the square bracket that multiplies φ_{xx} becomes, in the above limit and with $x = 0(1)$, $(\gamma + 1)\ M^2(1 + \mu)\varphi_x$, which provides the usual nonlinearity. We assume this term is O(1).

The magnitude of the last term in equation (29) is determined by the limiting value of $\delta\ \tau^{-1/3}$, with both δ, $\tau \to 0$. Examine the coefficient of φ_{xS} in equation (27) in the above limits. The coefficient of φ_{xS} becomes

$$- 2M^2\ \mu\ \delta\ \tau^{-1/3}\ (1 + \mu)\ T.$$

If $\delta\ \tau^{-1/3}$ becomes unbounded as $\tau \to 0$, the φ_{xS} and φ_{xx} terms would dominate all other terms in equation (27), and the scaling procedure followed so far would be incorrect. Alternatively, if $\delta\ \tau^{-1/3} \rightarrow 0$ as $\tau \rightarrow 0$, inspection of equation (27) shows that the φ_{xT} term alone dominates, leading to another useless result. The conclusion is that δ should be chosen so that $\delta\ \tau^{-1/3} = O(1)$ as $\tau \rightarrow 0$, and we therefore take

$$\delta = \tau^{1/3} .$$

Combining these results, the asymptotic form of equation (27), valid in the tip region on the advancing side, is

$$2M^2(1 + \mu)\frac{\epsilon}{\tau}\ \varphi_{xT} = [\frac{1-\mathcal{M}^2}{\tau^{2/3}} + 2M^2(1 + \mu)\frac{\epsilon}{\tau} S + M^2(1 + \mu)\ \mu\ T^2$$

$$- (\gamma + 1)\ M^2(1 + \mu)\ \varphi_x]\ \varphi_{xx}$$

$$- 2M^2\ \mu\ (1 + \mu)\ T\ \varphi_{xS} + \varphi_{SS} + \varphi_{zz}, \qquad (30)$$

while the blade boundary condition is

$$\varphi_z(x,\ 1,\ 0,\ 0) = (1 + \mu)\ f_x. \qquad (31)$$

The original choice of the relative order of magnitude of ϵ and τ was based on the observation that $\epsilon = O(\tau)$ for a typical rotor blade. However, it now emerges from equation (27) that, analytically, a necessary condition for the flow to be transonic and nonlinear is that precisely this relation between ϵ and τ should hold.

The transonic problem defined by equations (30) and (31) differs from the subsonic problem in two essential respects: the transonic problem is nonlinear (in the usual way, and as is to be expected), and it is essentially an unsteady problem with the coefficient of $\varphi_{xT} = O(1)$. Thus the simplifying feature of quasi-steady flow does not appear. This latter result naturally complicates numerical calculations because solutions must be obtained in a four-dimensional (x, y, z, t) space. The term in φ_{TT}, however, again disappears. The hypersurface T = constant (or t = constant) is therefore characteristic (the necessary and sufficient condition for T = constant to be characteristic is that the term in φ_{TT} not appear in the differential equation).

Notice that the tip flow is transonic, unsteady and nonlinear over a sector of the rotor disk of order t (or Ωt) $\sim \tau^{1/3}$, with t measured about t = 0. If $\tau = 0.1$, the azimuthal extent of the transonic region is about 54°, or 27° either sice of t = 0.

While equations (30) and (31) are a first approximation to the flow in the tip region, they are not convenient for numerical calculations. Following the same procedure that was used for the subsonic problem, we retain only the appropriate terms in equations (23) and (24) and find

$$2M^2 \frac{\epsilon}{\tau^{2/3}} (1 + \mu \cos t)\varphi_{xt} = \left[\frac{1-M^2(y+\mu \cos t)^2}{\tau^{2/3}} - (\gamma+1)M^2(1+\mu \cos t)\varphi_x\right]\varphi_{xx}$$

$$+ \frac{2M^2\epsilon}{\tau^{2/3}} \mu \sin t\,(1 + \mu \cos t)\varphi_{xy} + \frac{\epsilon^2}{\tau^{2/3}} \varphi_{yy} + \varphi_{zz},$$

$$\varphi_z = (y + \mu \cos t)f_x \qquad \text{on} \qquad z = 0.$$

These two equations describe the flow in the transonic tip region and remain valid inboard through the subsonic strip theory region.

4. Reduced frequency

A fixed wing with chord ℓ moving at constant velocity V and oscillating with angular frequency Ω has associated with it a nondimensional reduced frequency $\omega = \Omega \ell / V$. This parameter is a measure of the number of oscillations the wing goes through as a fluid particle goes from leading to trailing edge in its mean motion. Unsteady flow is generally characterized by large ω and quasi-steady flow by small ω. Usually ω is multiplied by some other parameter or parameters in the governing differential equation and boundary condition, and appropriate limits must be taken to specify what is meant by large or small ω. In the rotor problem, a reduced frequency in precisely the form $\Omega \ell / V$ does not appear, but a similar parameter can be interpreted as a reduced frequency. The reference velocity near the tip on the advancing side is $\Omega R + V$. The reference transit time of a particle in its motion from leading edge to trailing edge is $\ell(\Omega R + V)^{-1}$. The number of radians traversed by the tip in this time is

$$\omega \equiv \frac{\ell}{\Omega R + V} \cdot \Omega$$

$$= \frac{\ell / R}{1 + \mu} ,$$

or

$$\omega = \frac{\epsilon}{1 + \mu} .$$

This parameter may be defined as the reduced frequency for an advancing rotor. Since μ is never large, the reduced frequency is effectively the inverse of the blade aspect ratio ϵ. Reference to the basic differential equation (23) indicates that ϵ does indeed play the role of reduced frequency wherever it multiplies time derivatives.

5. The transonic far field

The specification of a far field boundary condition in the transonic problem is considerably more complicated than for the subsonic problem. This is particularly true if an asymptotic representation more accurate than

$\varphi = 0$ is to be prescribed on a boundary some chords distant from the blade. If a distant far field solution based on the limit $x^2 + y^2 + z^2 \to \infty$ is required, then finding such a solution is equivalent to finding the radiative part of the solution at infinity, and getting this solution means solving the acoustic problem for the rotor. One method for analyzing the acoustic far field for a body in transonic motion has been described by Ffowcs Williams and Hawkings (4), but it has not yet been applied to rotor problems. The method is rather complicated and involves the near field transonic solution in an integral representation of the far field. The essential difficulty here is that an element of blade surface in transonic motion causes a disturbance that is not compact, and the far field of the disturbance cannot be represented as a simple point multipole, as it could be for subsonic motion. These remarks indicate that without a much deeper analysis, there is no alternative, at the present time, to using the condition $\varphi = 0$ at a far field boundary in numerical calculations.

6. The transonic problem with small advance ratio

It has already been noted that for a subsonic blade at moderate advance ratio the flow is quasi-steady and linear; and when advance ratio is small, time does not enter the problem at all: if, say, $\mu = O(\tau)$, then the effect of forward motion becomes a second order effect, and the first approximation to the flow is simply the flow over a hovering blade.

We now consider the transonic problem for small advance ratio and determine the appropriate equations in the limit $\mu \to 0$.

Refer again to the basic differential equation (23) and consider the first term in the square bracket that multiplies φ_{xx}:

$$\frac{1 - M^2(y+\mu \cos t)^2}{\tau^{2/3}} = \frac{1-M^2(1 - \frac{\epsilon}{\tau^{1/3}} S + \mu \cos \delta T)^2}{\tau^{2/3}}$$

$$\equiv \frac{1-M^2(1+\mu)^2}{\tau^{2/3}} + 2M^2(1+\mu)\frac{\epsilon}{\tau} S$$

$$+ \frac{2M^2}{\tau^{2/3}}\left(1+\mu - \frac{\epsilon}{\tau^{1/3}} S\right)\mu(1-\cos \delta T) - \frac{\mu^2 M^2}{\tau^{2/3}}(1-\cos \delta T)^2$$

$$+ \frac{\epsilon^2}{\tau^{4/3}} S. \tag{32}$$

We first consider the case for which $\mu \to 0$, $\mu = 0(\tau^{2/3})$. Set the time scale factor $\delta = 1$ in equation (32), and revert to $\delta T = t$. Let $S = 0(1)$, $t = O(1)$, $\epsilon/\tau = O(1)$, $\mu = O(\tau^{2/3})$, $\epsilon \to 0$, $\tau \to \theta$, $\mu \to 0$. The asymptotic form of equation (32) is

$$\frac{1-M^2(t+\mu \cos t)^2}{\tau^{2/3}} \sim \frac{1-M^2}{\tau^{2/3}} - 2M^2 \frac{\mu}{\tau^{2/3}} + 2M^2 \frac{\epsilon}{\tau} S$$

$$+ 2M^2 \frac{\mu}{\tau^{2/3}}(1-\cos t)$$

Time derivatives become negligible, and the asymptotic forms of the differential equation (23) and boundary condition (24) are

$$\left[\frac{1-M^2}{\tau^{2/3}} - 2M^2 \frac{\mu}{\tau^{2/3}} + 2M^2 \frac{\epsilon}{\tau} S + 2M^2 \frac{\mu}{\tau^{2/3}}(1-\cos t) - (\gamma+1)M^2\varphi_x\right]\varphi_{xx},$$

$$+ \varphi_{SS} + \varphi_{zz} = 0,$$

$$\varphi_z(x, 1, 0, t) = f_x.$$

The cross derivative φ_{xy} is also negligible in this limit, its coefficient being $O(\tau^{1/3})$. The flow is therefore quasi-steady and sweep-back effects are of second order. The flow is, however, transonic and nonlinear around the entire rotor disk.

The differential equation and boundary condition in forms suitable for computations from the tip inboard through the strip theory region are

$$\left[\frac{1-M^2(y+\mu\cos t)^2}{\tau^{2/3}} - (\gamma+1)M^2\varphi_x\right]\varphi_{xx} + \frac{\epsilon^2}{\tau^{2/3}}\varphi_{yy} + \varphi_{zz} = 0, \tag{33}$$

$$\varphi_z = y\, f_x, \qquad \text{on} \qquad z = 0. \tag{34}$$

There is also a limiting case based on the limit $\mu/\tau^{2/3} \to 0$. It is apparent from equations (33) and (34) that in this case time derivatives, sweep back effects and time itself do not enter the problem in both the tip region and the strip theory region. In this limit, the flow over the rotor from the tip inboard is therefore equivalent, in the first approximation, to the flow over a hovering rotor with tip Mach number M.

7. The very large aspect-ratio blade

A final limiting case emerges naturally from equations derived in previous sections. It is based on the limit $\tau \to 0$, $\epsilon \to 0$, $\epsilon/\tau \to 0$ and corresponds to a very large (and perhaps impracticably large) aspect ratio blade. In this limit, time derivatives and sweep-back effects disappear, and also the effect of a mean Mach number gradient along the blade span becomes negligible. For a hovering blade, this limit implies that the rotor problem as described in a blade fixed coordinate system is entirely equivalent to the problem of a fixed wing with free-stream Mach number $\Omega R/c$. For the advancing blade, there are several flow regimes associated with $\epsilon/\tau \to 0$, depending on the order of magnitude of the advance ratio μ. In all cases, the flow over the very large aspect ratio blade is analogous to some fixed wing problem. Further details are unnecessary here, since each limiting case can be read directly from the differential equation and boundary condition corresponding to a particular choice of μ.

IV. PRESSURE COEFFICIENT

The pressure coefficient C_p is based on the local chordwise free stream velocity at each span station. In nondimensional form, this velocity is $y + \mu \cos t$. The leading terms in the expression for C_p are

$$\frac{C_p}{\tau} \equiv \frac{p - p_\infty}{\frac{1}{2}\rho_\infty \Omega^2 R^2 (y+\mu \cos t)^2 \tau}$$

$$= -\frac{2\varphi_x}{y + \mu \cos t} + \frac{2\epsilon \mu \sin t \, \varphi_y}{(y+\mu \cos t)^2} . \tag{35}$$

For subsonic flow in the tip region this becomes

$$\frac{C_p}{\tau} \sim \frac{2\varphi_x}{1 + \mu \cos t} - \frac{2\mu \sin t \, \varphi_s}{(1 + \mu \cos t)^2} . \tag{36}$$

Strictly speaking, if $\mu = O(1)$, the second term involving the spanwise derivative φ_s must be retained. While this term may be neglected in a sufficiently small sector about $t = 0$, it should be noted that there is no characteristic azimuthal angle on the advancing side for a subsonic tip (in contrast to the advancing transonic tip, for which a characteristic azimuthal angle about $t = 0$ does play an essential role). The terms in φ_y in equation (35) and φ_s in equation (36) should therefore be retained in a subsonic calculation when $\mu = O(1)$.

The second term in equation (35) is never important in transonic flow. Therefore, for this case

$$\frac{C_p}{\tau^{2/3}} = - \frac{2\,\varphi_x}{y + \mu \cos t} \qquad (37)$$

Equations (35) and (37) represent C_p from the tip inboard through the strip theory region for subsonic and transonic tip Mach numbers.

V. CONCLUSIONS

The most difficult problem for numerical calculations occurs when the advancing tip in transonic and advance ratio is about 0.4. Under these conditions, a shock wave begins to form as the blade enters the advancing side of the rotor disk. It grows to some maximum strength in the advancing region and then decays as the blade retreats. Both the shock wave and blade thickness cause weak waves to be propagated very slowly, relative to the blade, into the upstream region. The slow propagation speed of these waves causes the essential unsteadiness in the flow and complicates the governing equations. If the tip is transonic and advance ratio is 0.4, a blade of radius 20 feet would have a forward velocity of about 190 knots and 375 rpm at sea level conditions. These conditions could be encountered in cruise or maneuver for a high performance helicopter.

The computational problem is much simpler when advance ratio is small. The flow becomes quasi-steady, sweep-back effects are negligible and the flow is nonlinear and transonic around the entire rotor disk. Computer programs for a hovering transonic rotor can be used here because time enters only as a parameter.

If advance ratio is very small (of the order of thickness ratio or smaller), sweep-back effects, time itself and therefore time derivatives do not enter the problem in both the tips and strip theory regions. In a first approximation, this problem is equivalent to a hovering rotor. It differs from a fixed wing problem only by the mean Mach number gradient along the span.

Other limiting flow regimes for subsonic and transonic tip Mach numbers are discussed in the main text. These regimes have interesting physical features, but they are of secondary importance in their numerical, aerodynamic and acoustic properties relative to the high speed, high advance ratio blade.

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NASA-Langley, 1974

1. Report No. NASA CR-2463	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle "Unsteady Subsonic and Transonic Potential Flow Over Helicopter Rotor Blades"		5. Report Date OCTOBER 1974
		6. Performing Organization Code
7. Author(s) Morris P. Isom		8. Performing Organization Report No. 74-2
		10. Work Unit No.
9. Performing Organization Name and Address National Aeronautics & Space Administration Washington, D.C. 20546		11. Contract or Grant No. NAS 2-6813
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address Polytechnic Institute of New York		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

Differential equations and boundary conditions for a rotor blade in forward flight, with subsonic or transonic tip Mach number, are derived. A variety of limiting flow regimes determined by different limits involving blade thickness ratio, aspect ratio, advance ratio and maximum tip Mach number is discussed. The transonic problem is discussed in some detail, and in particular the conditions that make this problem quasi-steady or essentially unsteady are determined. Asymptotic forms of equations and boundary conditions that are valid in an appropriately scaled region of the tip and an azimuthal sector on the advancing side are derived. These equations are then put in a form that is valid from the blade tip inboard through the strip theory region.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Rotor Blades, Transonic Potential Flow, Unsteady	Unclassified - unlimited CAT. 12

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	27	$3.25

*For sale by the National Technical Information Service, Springfield, Virginia 22151