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IMPROVED SOLUTION FOR POTENTIAL FLOW ABOUT ARBITRARY AXISYMMETRIC BODIES BY THE USE OF A HIGHER-ORDER SURFACE SOURCE METHOD

PART II. USER'S MANUAL FOR COMPUTER PROGRAM

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SUMMARY

This portion of the report explains the use of the computer program that is described in Part 1 of the report. Certain detailed calculation procedures are documented. However, the formulas of Part 1 are not duplicated here. The main purpose is to explain the input to the program so that the reader is prepared to use it for his application.

I. INTRODUCTION

An exact general method of solving the Neumann (or Second Boundary Value) problem has been applied to the calculation of low speed flows about or within bodies of axially symmetric shape. Solid body, inlet, and purely internal flow problems can be solved. The method is capable of dealing with several bodies at once in the presence of one another, and consequently, interference problems can be treated with ease. Boundaries need not be solid, that is, flows involving area suction can be calculated. Velocities can be computed not only for points on the body but for the entire flowfield.

A surface source distribution is used as the basis for solution. This leads to a Fredholm Integral Equation of the second kind, which is solved as a set of linear algebraic equations. The source distribution can be taken as constant, linearly varying, or quadratically varying across each of the elements which form the body. The elements themselves can be flat or curved (parabolas).

At the present time, it is possible to obtain the solution for the flow about any body that has the previously mentioned characteristics and whose profile can be defined by no more than 500 coordinate points.

This method is a generalization of a previous program [1-6]. In particular the analysis of part 1 of this report is incorporated into the program of [5].

II. EQUATIONS AND METHOD OF SOLUTION

A. Formation of the Elements

Each body is described to the present program by the input of an ordered sequence of points distributed along that body's surface. By convention, the flowfield of interest is assumed to be "overhead" to an observer who is "walking" along the body surface in the order of the input point sequence. This convention is necessary in order to establish the direction of the "outward" normal vector.

Surface elements are constructed by connecting consecutive points in the sequence by either straight lines or parabolas. (Recall that since the body is assumed to be axisymmetric, and if we take the x-axis to be the axis of symmetry, these lines or arcs should be thought of as "conoids" formed by their rotation about the x-axis). In the case of straight (hereafter "flat") elements, each element is simply determined by straight lines constructed between consecutive points. However, for the case of parabolic arc (hereafter "curved") elements, the curvatures may either be input by the user or determined automatically by the program in the following way: the curvature of each element (except for the first and the last elements of each body) is the geometric mean of the curvatures of the two circles defined by the two end-points of each interval and the next sequential point on either side of that interval. For the first and last elements, the curvatures are simply those of the circles defined by the first three and last three points of the body, respectively. The curvature, of each of these circles, is calculated from the formulas of Appendix A of Part 1 of this report.

$$K \equiv d/r$$

where,

$$d \equiv 4[(x_2-x_1)(y_2-y_3) - (x_2-x_3)(y_2-y_1)]$$

and

$$r \equiv \sqrt{(x_2 d-d_h)^2 + (y_2 d-d_k)^2}$$

where,

$$d_h \equiv 2[r_1{}^2(y_3-y_2) + r_2{}^2(y_1-y_3) + r_3{}^2(y_2-y_1)]$$

and

$$d_k \equiv 2[r_1{}^2(x_2-x_3) + r_2{}^2(x_3-x_1) + r_3{}^2(x_1-x_2)]$$

where,

$$r_1^2 \equiv x_1^2 + y_1^2$$

$$r_2^2 \equiv x_2^2 + y_2^2$$

$$r_3^2 \equiv x_3^2 + y_3^2$$

Once the correct geometric means of these curvatures are calculated to determine the local element curvatures, the control point locations of each element are obtained by approximating the circular arc with a parabola having that curvature at its vertex (this is correct through third order in Δs).

B. Calculation of the Influence Coefficient Matrices

The velocity induced by an element at a control point is computed by integrating the basic ring source (or vortex) formulas over the element as described in Part 1. For $j \neq 1$ the integrals of equation (9) of Part 1 are evaluated by Simpson's rule in the following manner.

The integrand is evaluated at an odd number NI of points equally spaced in s over the element, where

$$NI = \text{Integer Part of} \left[2\left(8 \frac{\Delta s}{d_{min}} + 0.9\right) + 1\right] \qquad (1)$$

where Δs is the total arc length of the element and d_{min} is the distance between the point where velocity is being evaluated and the centroid or one of the endpoints of the elements, whichever is closest. (If NI turns out to be less than 3, it is set equal to 3; and if NI turns out to be greater than 129, it is set equal to 129.) Once these NI values of $\vec{V}$ have been determined, the Simpson rule integration formula is applied:

$$\int_{\Delta s_i} \vec{V}\, ds \simeq \frac{\Delta s}{3} \left[\vec{V}_1 + 4\vec{V}_2 + 2\vec{V}_3 + 4\vec{V}_4 + 2\vec{V}_5 + \ldots + 2\vec{V}_{NI-2} + 4\vec{V}_{NI-1} + \vec{V}_{NI}\right] \qquad (2)$$

where

$$\vec{V}_1 \equiv \vec{V}\left[\bar{x}_i,\ \bar{y}_i,\ b(-s_j'),\ a(-s_j'),\right]$$

$$\vec{V}_2 \equiv \vec{V}\left[\bar{x}_i,\ \bar{y}_i,\ b(-s_j'+\delta s),\ a(-s_j'+\delta s)\right]$$

etc.

and where $\delta s \equiv s_j/(NI-1)$.

The second integral in equation (9) of Part 1 is now easily determined by multiplying each of the $(\vec{V}_m,\ m = 1,\ NI)$ values by the local value of s, and then apply the Simpson rule formula to the result:

$$\int_{\Delta s_j} \vec{V}s\ ds = \frac{\Delta s}{3}\left[(\vec{V}_1)(-s_j') + 4\ (\vec{V}_2)(-s_j'+\delta s) + \ldots\right]$$

This procedure is repeated to evaluate the third integral in equation (9).

It should be pointed out that equation (9) of Part 1 encounteres difficulty when the parameter:

$$\frac{\bar{y}_i}{\sqrt{(\bar{x}_i-b_j)^2 + a_j^2}} \leq \varepsilon, \qquad (4)$$

where $\varepsilon \equiv 0.01$. When this happens (termed "small y"), expansions of these equations (in this parameter) are used. Their expansions are in reference 3.

For $j = i$, use is made of the singular subelement expansions of Appendix B of Part 1.

C. Matrix Assembly

The assumption is made that the surface source strength, σ, varies parabolically between three successive values of σ. The derivatives, $\sigma^{(1)}$ and $\sigma^{(2)}$, may then be expressed in terms of the local source strength values at the control points of the elements on both sides and then may be replaced in equation (10) of Part 1 as follows:

$$\begin{aligned}\vec{V}_{ij}{}^* = {} & \vec{V}_{ij}{}^{(0)}\sigma_j{}^{(0)} \\ & +\vec{V}_{ij}{}^{(1)}\left[A_{11j}\sigma_{M-1}{}^{(0)} + A_{12j}\sigma_M{}^{(0)} + A_{13j}\sigma_{M+1}{}^{(0)}\right] \\ & +\vec{V}_{ij}{}^{(2)}\left[A_{21j}\sigma_{M-1}{}^{(0)} + A_{22j}\ {}_M{}^{(0)} + A_{23}\ \sigma_{M+1}{}^{(0)}\right]\end{aligned} \qquad (5)$$

where,

$$M = \begin{cases} j+1 & \text{for the first element of each "body"} \\ j-1 & \text{for the last element of each body} \\ j & \text{for the other element of each body} \end{cases}$$

$$A_{11j} = \begin{cases} A_j & \text{for the first element of each body} \\ J_j & \text{for the last element of each body} \\ D_j & \text{for the other elements of each body} \end{cases}$$

$$A_{12j} = \begin{cases} B_j & \text{for the first element of each body} \\ K_j & \text{for the last element of each body} \\ E_j & \text{for the other elements of each body} \end{cases}$$

$$A_{13j} = \begin{cases} C_j & \text{for the first element of each body} \\ L_j & \text{for the last element of each body} \\ F_j & \text{for the other elements of each body} \end{cases} \qquad (6)$$

$$A_{21j} = \begin{cases} G_2 & \text{for the first element of each body} \\ G_{N-2} & \text{for the last element of each body} \\ G_j & \text{for the other elements of each body} \end{cases}$$

$$A_{22j} = \begin{cases} H_2 & \text{for the first element of each body} \\ H_{N-2} & \text{for the last element of each body} \\ H_j & \text{for the other elements of each body} \end{cases}$$

$$A_{23j} = \begin{cases} I_2 & \text{for the first element of each body} \\ I_{N-1} & \text{for the last element of each body} \\ I_j & \text{for the other elements of each body} \end{cases}$$

where,

$$A_j = -\frac{s_{j+1} + s_j + 1/2\left(s_{j+1}+s_{j+2}\right)}{\left(s_{j+1}+s_j\right)\left[s_{j+1}+1/2\left(s_j+s_{j+2}\right)\right]}$$

$$B_j = 2\frac{s_{j+1} + 1/2\left(s_j+s_{j+2}\right)}{\left(s_j+s_{j+1}\right)\left(s_{j+1}+s_{j+2}\right)}$$

$$C_j = -\frac{1}{2}\;\frac{s_j + s_{j+1}}{\left(s_{j+1}+s_{j+2}\right)\left[s_{j+1}+1/2\left(s_j+s_{j+2}\right)\right]}$$

$$D_j = \left(\frac{-1}{2\left[s_j+1/2\left(s_{j-1}+s_{j+1}\right)\right]}\right)\left(\frac{s_j+s_{j+1}}{s_j+s_{j-1}}\right) \qquad (7)$$

$$E_j = \left(\frac{1}{2\left[s_j+1/2\left(s_{j-1}+s_{j+1}\right)\right]}\right)\left(\frac{s_j+s_{j+1}}{s_j+s_{j-1}} - \frac{s_j+s_{j-1}}{s_j+s_{j+1}}\right)$$

$$F_j = \left(\frac{1}{2\left[s_j+1/2\left(s_{j-1}+s_{j+1}\right)\right]}\right)\left(\frac{s_j+s_{j-1}}{s_j+s_{j+1}}\right)$$

$$G_j = \frac{1}{2\left[s_j+1/2\left(s_{j-1}+s_{j+1}\right)\left(s_j+s_{j-1}\right)\right]}$$

$$H_j = \frac{-1}{\left(s_j+s_{j-1}\right)\left(s_j+s_{j+1}\right)}$$

$$I_j = \frac{1}{2\left[s_j+1/2\left(s_{j-1}+s_{j+1}\right)\right]\left(s_j+s_{j+1}\right)}$$

$$J_j = \frac{1}{2}\,\frac{s_{j-1}+s_j}{\left(s_{j-2}+s_{j-1}\right)\left[s_{j-1}+1/2\left(s_{j-2}+s_j\right)\right]}$$

$$K_j = -2\,\frac{s_{j-1}+1/2\left(s_{j-2}+s_j\right)}{\left(s_{j-2}+s_{j-1}\right)\left(s_{j-1}+s_j\right)}$$

$$L_j = \frac{s_{j-1}+s_j+1/2\left(s_{j-1}+s_{j-2}\right)}{\left(s_{j-1}+s_j\right)\left[s_{j-1}+1/2\left(s_{j-2}+s_j\right)\right]} \qquad (7)$$

The above $A_j, \ldots, L_j$ are functions only of the geometry, and are calculated prior to initiating the actual V_{ij} matrix formulation. The procedure for handling the V_{ij} matrix to produce the normal- and tangential-flow matrices is identical to that employed in the original program (see reference 5). The vector velocity is dotted into the local element normal $(\hat{u}_N)$ and tangential $(\hat{u}_T)$ directions

$$A_{ij} = \vec{V}_{ij} \cdot \hat{u}_{N_j} = \vec{V}_{ij} \cdot (-\sin\alpha_j\,\hat{u}_x + \cos\alpha_j\,\hat{u}_y)$$

$$T_{ij} = \vec{V}_{ij} \cdot \hat{u}_{T_j} = \vec{V}_{ij} \cdot (\cos\alpha_j\,\hat{u}_x + \sin\alpha_j\,\hat{u}_y) \qquad (8)$$

The normal velocity matrix is then used to calculate the source strength values for each onset flow as:

$$\sum_{j=1}^{N} A_{ij}\sigma_j = -V_{\infty_i} \qquad (9)$$

where V_{∞_i} is the onset normal flow at the i-th control point. For the axisymmetric onset flow:

$$V_{\infty_i} = \sin \alpha_i \tag{10}$$

and $\vec{V}_{ij}$ is determined as described in Section B.1. For the crossflow case as explained in reference 2, the normal onset flow truly varies as the cosine of the circumferential angle; but the source density, which is truly cosinusoidal may be obtained by applying the zero normal flow requirement at $\theta = 0$. For this case, the right-hand side of equation (9) is found from:

$$V_{\infty_i} = \cos \alpha_i . \tag{11}$$

Note that the A_{ij} matrix for the crossflow case is different from that of the axisymmetric case.

D. Solution of the Matrix Equations

The solution of equation (9) for the axisymmetric onset flow and for the crossflow cases is obtained by inversion of the A_{ij} matrix, as explained in detail in reference 6. This procedure allows multiple "right-hand sides" to be computed with very little increase in program run time (beyond that of a single right-hand side).

It should be realized that the constant surface vorticity option automatically generates a number of non-uniform axisymmetric onset flows equal to the number of "bodies" that have been input.

E. Calculation of the Surface Velocities

Once the source strengths have been determined, the net surface velocities are calculated as follows:

$$\vec{V}_i = \sum_{j=1}^{N} \vec{V}_{ij}\sigma_j + \vec{V}_{\infty_i} \tag{12}$$

For axisymmetric flows, the velocity is presented in tangential component form. For the crossflow case, however, as discussed in reference 2, the velocity is presented in terms of the "fundamental velocity components, T_2 and T_3," where

$$T_2 = \cos \alpha_i \sum_{j=1}^{N}\left[\left(\frac{\partial\phi}{\partial x}\right)_{ij} \sigma_j\right] + \sin \alpha_i \sum_{j=1}^{N}\left[\left(\frac{\partial\phi}{\partial y}\right)_{ij} \sigma_j\right] + \sin \alpha_i \tag{13}$$

$$T_3 = \sum_{j=1}^{n} \left[\left(\frac{1}{y} \frac{\partial \phi}{\partial \theta} \right)_{ij} \sigma_j \right] + 1. \qquad (14)$$

Thus, the velocity tangent to a meridian curve (at any θ value) is $T_2 \cos \theta$. The "crossflow" velocity (i.e., the velocity component tangent to the circular cross-section) is $-T_3 \sin \theta$.

III. INPUT INSTRUCTIONS

A. Card Column Parameter Locations

Card 1 - Header Card.

cc1-60	Header	Any alphanumeric run description.
cc63-68	Case	Case number.

Card 2 - Control Flag Card

cc1	NB	Number of bodies ($1 \leq NB \leq 9$).
cc2	NNU	Number of non-uniform flows ($0 \leq NNU \leq 5$).
cc3	IAXI	Axisymmetric flow flag.
cc4	ICRØSS	Crossflow flag.
cc5	IØFF	Off-body point input flag.
cc6	IØNLY	Basic-data-only flag.
cc7	IELPSE	Ellipse generator flag (see also Card 5).
cc8-10	(blank)	
cc11	IPRTRB	Perturbation velocities only.
cc12	IPØTNL*	Solve potential matrix.
cc14	IPTANV	Prescribed tangential velocity (for the last IPTANV bodies).
cc15	IVØRT	Strip-ring vorticity flag.
cc16	IØMITA	Omit axisymmetric uniform flow solution.
cc17	IOMITC	Omit crossflow uniform flow solution.
cc18	ISURFV	Surface vorticity (instead of sources) for the final ISURFV bodies.
cc19	IPRSCV	Prescribed values of the surface vortex strengths for the final ISURFV bodies will be input.
cc20	IALLV	All bodies are surface vorticity bodies.
cc21	IEXCRS*	Extra crossflow.
cc22	IGENBC	Generated boundary conditions.
cc23	IRNGW	Ring wing option.
cc28	IPNCH	Punched output.
cc29,30	IUNIT	Unit number for input coordinates (default = 05).
cc31	IVIJ	Matrix print flag.
cc32	ICØEF	Matrix-assembly coefficient print flag.
cc33	IPRINT	Very detailed matrix construction print flag.
cc34	IRAKF	Automatic rake generation flag (see also Cards 8 & 9)

Card 3 - Chord/Mach number card.

cc1-10	CHØRD	Reference chord length (default = 1.0).
cc11-20	XMACH	Mach number for Goethert correction (0.0 implies incompressible).
cc21-80	(blank)	

* Available if and only if NØNEWF = 1, ISIGF = 1 and IGEØMF = 1.

Card 4 - Body Control Card 1 of 2.

cc1	IGEØMF	0 = curved elements; 1 = flat elements.
cc2	ISIGF	0 = parabolic σ; 1 = linear σ; 2 = constant σ (on each element).
cc3	ICURVN	0 = internally calculated element curvatures; 1 = input curvature (see card 7).
cc4	NØNEWF	0 = use the newest formulae; 1 = use the old formulae (implies flat elements and constant σ).
cc5	IFØRMT	Input format flag (see Card Set 6)
cc6-10	NN	Number of defining endpoints for this body.
cc11-20	XMULT	x-multiplier value (default = 1.0).
cc21-30	YMULT	y-multiplier value (default = 1.0).
cc31-40	THETA	Coordinate rotation value (degrees, measured about - Z-axis).
cc41-50	ADDX	x-increment (to be applied to all the input coordinate for this body).
cc51-60	ADDY	y-increment (to be applied to all the input coordinates for this body).

Card 5 - Body Control Card 2 of 2

cc1-10	IBDN	"Body" number (sequential for bodies, zero for off-body points).
cc11-20	ISUBKS	Subcase flag.
cc21-30	NLF	Non-lifting flag (for combination cases, only)
cc31-40	A	Semi-major axis for ellipse cases } If IELPSE≠0.
cc41-50	B	Semi-minor axis for ellipse cases } If IELPSE≠0.

Card (Set) 6 - Body Definition Cards

IF	IFØRMT=0:	X-coordinates (6F10.5), then Y-coordinates (6F10.5)
IF	IFØRMT=1:	X, Y coordinates (2F10.5) (i.e., one "point-set" per card)
IF	IFØRMT=2:	X, Y coordinates (F10.5,10X,F10.5) (i.e., one "point-set" per card).

Card (Set) 7 - Input curvature values (needed only if ICURVN ≠ 0).

(6F10.5) CURV(I),I=1,NN-1 The curvature values for the NN-1 elements which constitute this body.

Repeat Cards 4-7 a total of (NB+IØFF) times

Card 8 - Rake Number Card (needed only if IRAKF ≠ 0)

cc1-10	NRAKES	The number of "automatically" generated mass-flow rakes.

Card 9 - Rake Definition Card (needed only if IRAKF $\neq$ 0)

cc1-10	X1	Coordinates of "start" of the rake.
cc11-20	Y1	
cc21-30	X2	Coordinates of the "end" of the rake.
cc31-40	Y2	
cc41-45	N	Number of intervals to be used in the rake (note, $4 \leq N \leq 200$, and N must be an even integer).

Repeat Card 9 a total of NRAKES times.

Card 10 - Non-Uniform Flow Control Card (needed only if NNU $\neq$ 0)

cc1-10	NUN	Flow identification number.
cc11-20	MSF	0 = axisymmetric onset flow; 1 = crossflow onset flow, 2 = both 0 and 1.
cc21-30	TYPE	+1.0 = velocity will be input in x,y component form; 0 = velocities will be input in normal, tangential form; -1 = automatic generation of flow due to rotation about the Z-axis (for crossflow, only).
cc31-40	FG	Flow generator constant.

Card (Set) 11 - Non-Uniform Flow Velocities (needed only if NNU $\neq$ 0)

(6F10.5) VX(I) or VN(I), I = 1, total number of control points.
(6F10.5) VY(I) or VT(I), I = 1, total number of control points.

Repeat Cards 10 and 11 a total of NNU times.

Card (Set) 12 - Specified Tangential Flow Velocities (needed only if IPTANV $\neq$ 0

(6F10.5) TG(I), I = 1, total number of control points on the last IPTANV bodies

B. Discussion of Parameters

In all cases, the integer values (indicated by the standard FØRTRAN IV naming convention) must be right adjusted within the specified input fields. For floating-point type input, the input decimal point will override the quoted FØRMAT specifications.

Card 1. Self-explanatory.

Card 2. Although most of these flags are unchanged from the original program, a brief description of each one will be given here for completeness.

NB The total number of separate "bodies" ($1 \le NB \le 8$).

NNU The total number of (user specified) input non-uniform flows ($0 \le NNU \le 5$).

IAXI Axisymmetric flow flag (0: no axisymmetric flow, 1: axisymmetric flow)

ICROSS Crossflow flag (0: no crossflow, 1: crossflow)

IOFF Off-body point input flag (0: no off-body points will be input, 1: off body points will be input)

IONLY Basic-data-only flag (0: full execution, 1: basic data only)

IELPSE Ellipse generator flag (see also card 5)(0: standard body defining coordinates will be input, 1: coordinates for the ellipse whose properties are specified on card 5 are to be created automatically by the program).

IPRTRB Perturbation velocities only (0: standard total net velocities including the onset flow will be printed, 1: perturbation velocities only will be printed).

IPØTNL Solve the potential matrix (0: standard solution for the surface velocities, 1: solution for the surface velocity potential). Since the velocity potential problem for the higher order elements has not been coded in the present version, the solution for the potential is available only for the case of flat elements and, in particular, through the usage of the "old" velocity formulae. These "old" formulae have been retained as a subsection of this new, higher order program and may be "reached" by setting the following quantities (on card 4): IGEØMF = 1, ISIGF = 2, and NØNEWF = 1; in such a case, the solution for the potential is still available.

IPTANV Prescribed tangential velocity flag (0: standard zero normal velocity solution is to be obtained, 1: a user defined set of prescribed tangential velocities will be input on the last IPTANV bodies; obviously, IPTANV ≤ NB.) See the Appendix for further details on the use of this flag.

IVØRT Strip ring vorticity onset flows for each of the bodies (0: no strip ring vorticity, 1: automatic generation of the strip ring vorticity onset flows).

IØMITA Omit the uniform axisymmetric flow solution (0: calculate the standard uniform axisymmetric flow solution if IAXI ≠ 0, 1: omit this uniform axisymmetric flow solution even if IAXI ≠ 0).

IØMITC Omit the uniform crossflow solution (0: calculate the uniform crossflow solution if ICRØSS ≠ 0, 1: omit the uniform crossflow solution even if ICRØSS ≠ 0)

ISURFV Surface vorticity for axisymmetric flow (in place of surface sources) flag (0: use the standard source distribution on all elements, ≠ 0:

use vorticity instead of a source distribution as the singularity on the elements of the last ISURFV bodies). Note that if ISURFV ≠ 0, then both IVØRT and IPTANV must be ≠ 0, and then in particular, IPTANV must = ISURFV). If we assume that IPTANV = ISURFV = k, and that M is the total number of elements on the last k bodies, then the surface vorticity option causes the source induced velocity formulae to be used on the first N-M elements of each matrix row, and the vortex formulae to be used on the remaining M elements of each matrix row. The solution for the unknown strengths then proceeds as it would have if only IPTANV was nonzero.

IPRSCV Prescribed vorticity flag for axisymmetric flow (0: prescribed vorticity values will not be input, 1: prescribed vorticity values (W_i) for the last IPRSCV bodies will be input; note that if IPRSCV is ≠ 0, then both IVØRT and IPTANV must ≠ 0, and then, in particular, IPTANV must = IPRSCV). If IPRSCV is ≠ 0, then the vortex strengths of the affected elements are taken to be $W_i/4\pi$.

IALLV Total vorticity flag for axisymmetric flow (0: all elements are assumed to be source type elements [unless ISURFV ≠ 0], 1: all elements are assumed to be vortex type elements). Note that if IALLV is ≠ 0, the use of IPTANV is optional, but not mandatory.

IEXCRS Extra crossflow flag (0: no "extra" crossflow, 1: generate an "extra" crossflow [i.e., having a potential which varies as the cosine of twice the circumferential angle]). Note that if IEXCRS ≠ 0, then NØNEWF = 1, ISIGF = 2, and IGEOMF must = 1, since the higher order formulae for this kind of velocity potential have not been included in this program.

IGENBC Generated boundary condition flag (0: do not generate any additional boundary conditions for the crossflow case; 1: generate the onset crossflow due to rotation about an axis normal to the axis of symmetry [see also the notes for Card 10]).

IRNGW Ring wing option (0: no ring wing option, 1: use the ring wing option [see MDC Report J0741/01, April 1970 for further details]).

IPNCH Punched card output flag (0: no punched card output, 1: punched card output).

Card 3. This card is usually left blank, which results in a default chord length of unity, and no Mach number corrections (i.e., incompressible results). If a non-zero value of the Mach number is input, the program uses the Goethert correction to account for compressibility.

Card 4. Usage of the IGEOMF and ISIGF flags permit the user to "turn-off" any or all of the higher order element curvature and/or varying source density terms. The default values are curved elements with parabolically varying source density. If IGEØMF = 0, the program will automatically calculate the local element curvature values by the procedure described in Section II.A, unless ICURVN is non-zero, in which case the user must supply these curvature values (see Card 7). The value for the NØNEWF flag is ordinarily left blank (or zero), even if a flat element, constant source density solution is required. However, since certain of the original Douglas-Neumann Axisymmetric Potential Flow Program capabilities have not been made available in the higher order program (e.g., calculation of the potential [hence the added mass, etc.) the original formulae have been preserved within this version (see description of the IPØTNL and IEXCRS flags). The original capabilities can be obtained by setting IGEØMF = 1, ISIGF = 2, and NØNEWF = 1 for each of the input bodies, in which case the input instructions of reference 5 apply.

The other parameters on Card 4 are self-explanatory, with the understanding that the order of coordinate transformations are as listed on the input.

Card 5. The "bodies" are normally loaded prior to any off-body points (although the latter require the usage of body control Cards 1 and 2, also). For this reason, the value of IBDN should be sequentially increasing beginning with unity. A non-zero value for ISUBKS means that the body definition cards for this body (Card Set 6), (say this is the ith body input under this header card) will not be included, but that the program is to use the ith set of points that were input under the previous header card. Obviously, this capability is useful only with "stacked" input cases.

Card 6. The three input formats that are available are as shown. The value of IFØRMT determines which format is used. The default format is the "old" format: x-coordinates, followed by the y-coordinates.

Card 7. These values are to be input only if ICURVN $\neq$ 0.

Card 8. This card is needed only if IRAKF $\neq$ 0. Note that $1 \leq$ NRAKES ≤ 20.

Card 9. These are the rake definition cards, which define the "start" and "end" of each mass flow rake. The sign convention that is employed is as follows: a positive mass flux means that the flow is from left to right to an observer traversing the rake from point (X1, Y1) to point (X2, Y2). For example, for an inlet without a bullet, point 1 is typically at y = 0.0, and point 2 would have a y-coordinate located on the inlet wall. Note that the x-coordinates need not be the same, i.e., a tilted rake may be used, if desired. Note also that one rake may "overlap" another if so desired, since they are each treated independently. It should be pointed out that a Simpson Rule integration is performed over the "N" intervals (and therefore N must be an even integer), with the program automatically generating N-1 "intermediate" points as "off-body points" for each input rake (program limit is 500 total off-body points). The velocity values at the first and last rake points are obtained by linear extrapolation of the two nearest values. In this way, the difficulty associated with calculating induced velocities at off-body points which lie very near to, or on, the surface is avoided. Typically, values of N between 10 and 20 appear to be satisfactory for most cases.

Card 10. Self-explanatory, except that FG, if entered is used in the following way to generate the rotation onset flow:

$$V_{x_i} = y_i$$

$$V_{y_i} = FG - x_i$$

$$V_N = V_{x_i} \sin \alpha_i - V_{y_i} \cos \alpha_i$$

$$V_T = V_{x_i} \cos \alpha_i + V_{y_i} \sin \alpha_i .$$

Card 11. Self-explanatory.

IV. OUTPUT FORMATS

The main output of the program, which consists of calculated surface velocities and/or pressures has not been changed. Its format is that of [5]. The surface coordinates with which the calculated velocities are associated are the transformed coordinates, i.e., the original input coordinates altered by any specified translations, multiplications, or rotations. Similarly, the format of the calculated velocities at off-body points is similar to that of [5]. The only change is that output at rake points (q.v.) is included.

The initial output, which details the surface geometry, has been changed considerably. The header identifies what kind of solution has been computed: (1) curved or flat elements; (2) constant, linear, or parabolic source density element curvatures. The header also specifies whether "new" or "old" velocity formulae have been used, the latter of which apply only to the flat-element constant-source case. In the body of the output, the first two columns are the untransformed (input) coordinates, and the third and fourth columns are the transformed coordinates, which are the endpoints of the surface elements. The fifth and sixth columns are the coordinates of the control points of the elements. Element arc lengths are given in column 7 and a running total of arc length is displayed in column 8. Column 9 lists the differences between average slopes of successive elements, i.e., the differences in the slopes of the two straight lines, through the respective element endpoints. The actual slope discontinuity at an endpoint between two parabolic elements is normally much smaller than the difference between the average slopes of the two elements. Moreover, this discontinuity approaches zero if the body contour approaches a parabola. The final (tenth) column lists elements curvatures.

A new output is entitled Automatic Rake Calculation. For each rake the two input points that bound the rake and the input identification are output together with three calculated quantities. The first is the surface area of the cone frustum defined by the two input rake points. The third is the total flux of fluid that crosses this area per unit time. The second quantity, average velocity, is the ratio of the third and the first quantities. There is a rake output for all axisymmetric flows, both uniform and non-uniform, but there is no rake output for cross-flow, because its circumferential variation guarantees zero flux.

With regard to surface vorticity solutions, there are only two possibilities. Either there are no vorticity solutions or there is one solution for each body, which corresponds to a unit vorticity strength on that body and zero vorticity strength on all other bodies. The order of these solutions is the same as the order in which the bodies are input. Thus, in particular, an inlet with centerbody has two vorticity solutions - one with vorticity on the inlet and one with vorticity on the centerbody. The second of these is not meaningful and should be discarded. (It may, of course, occur first on the output).

V. TEST CASE

To help the user verify that the program is performing correctly with his equipment, a test case is included. The geometry is a sphere defined by 13 points. Axisymmetric, vortex, and crossflow solutions are computed using the curved element, parabolic source density, and internally-computed curvature options. The input data and the computer output for this case follows.

TEST CASE INPUT

```
 12 ELEMENT SPHERE                                                             1
1 11          1                                                                2
                                                                               3
00000   13                                                                     4
         1                                                                     5
-1.0      -0.96593  -0.86602  -0.70711  -0.5      -0.25882                     6
 0.0       0.25882   0.5       0.70711   0.86602   0.96593                     6
 1.0                                                                           6
 0.0       0.25882   0.5       0.70711   0.86602   0.96593                     7
 1.0       0.96593   0.86602   0.70711   0.5       0.25882                     7
 0.0                                                                           7
```

DOUGLAS AIRCRAFT COMPANY
LONG BEACH DIVISION

PROGRAM EODF -- PARABOLIC AXISYMMETRIC AND CROSSFLOW

***** CASE CONTROL DATA *****

12 ELEMENT SPHERE. CASE NO.

BODIES = 1
NNU = 0
CHORD = 1.0000000
MACH NO.= 0.0
TCNST = 0.0
PSF NO. =

SURFACE OF REVOLUTION
CROSSFLOW
MATRIX SOLUTION BY TRIANGULARIZATION (SOLVIT)
STRIP VORTEX
INPUT TAPE NO. FOR COORDINATES AND NON-UNIFORM FLOW ONLY = 5

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LONG BEACH DIVISION

12 ELEMENT SPHERE.

NN = 13 MX = 0.0 MY = 0.0
THETA = 0.0 ADDX = 0.0 ADDY = 0.0
XE = 0.0 YE = 0.0

CURVED ELEMENTS(IGECMF=0) PIECEWISE-PARABOLIC SOURCE DENSITIES(ISIGF=0) INTERNALLY-COMPUTED ELEMENT CURVATURES(ICURVN=0)
NEW VELOCITY FORMULAE ARE USED.

O N - B O D Y C O O R D I N A T E S

	UNTRANSFORMED		*** TRANSFORMED ***							
	X	Y	X	Y	X C.P.	Y C.P.	DELTA S	SUMDS	D ALPHA	CURVATURE
1	-1.00000	0.00000	-1.00000	0.00000	-0.99141	0.13052	0.26179	0.26179	0.0	-1.00001
2	-0.96593	0.25882	-0.96593	0.25882	-0.92385	0.38267	0.26179	0.52359	-15.00024	-1.00000
3	-0.86602	0.50000	-0.86602	0.50000	-0.79332	0.60874	0.26179	0.78538	-14.99982	-1.00000
4	-0.70711	0.70711	-0.70711	0.70711	-0.60874	0.79332	0.26179	1.04717	-14.99993	-1.00000
5	-0.50000	0.86602	-0.50000	0.86602	-0.38267	0.92385	0.26179	1.30896	-14.99989	-1.00000
6	-0.25882	0.96593	-0.25882	0.96593	-0.13052	0.99141	0.26179	1.57076	-15.00015	-1.00000
7	0.0	1.00000	0.0	1.00000	0.13052	0.99141	0.26179	1.83255	-14.99993	-1.00000
8	0.25882	0.96593	0.25882	0.96593	0.38267	0.92385	0.26179	2.09434	-15.00003	-1.00000
9	0.50000	0.86603	0.50000	0.86603	0.60874	0.79332	0.26179	2.35614	-14.99991	-1.00000
10	0.70711	0.70711	0.70711	0.70711	0.79332	0.60874	0.26179	2.61793	-15.00006	-1.00000
11	0.86602	0.50000	0.86602	0.50000	0.92385	0.38267	0.26179	2.87972	-14.99994	-1.00000
12	0.96593	0.25882	0.96593	0.25882	0.99141	0.13052	0.26179	3.14152	-15.00008	-1.00000
13	1.00000	0.0	1.00000	0.0						

DOUGLAS AIRCRAFT COMPANY
LONG BEACH DIVISION

12 ELEMENT SPHERE. CASE NO. PSF =

ON-BODY UNIFORM AXISYMMETRIC FLOW
TRANSFORMED COORDINATES

	X	Y	T1	CP	SIN A	COS A	SIGMA	N	PHI
1	-1.0000000	0.0000010							
	-0.9914083	0.1305223	0.1956868	0.9617067	0.99145	0.13053	-0.1183583	-0.0000031	0.0
2	-0.9659260	0.2588230							
	-0.9238451	0.3826699	0.5737243	0.6708405	0.92388	0.38269	-0.1102906	-0.0000015	0.0
3	-0.8660250	0.5000000							
	-0.7933235	0.6087390	0.9128096	0.1667787	0.79335	0.60876	-0.0947085	-0.0000004	0.0
4	-0.7071060	0.7071069							
	-0.6087385	0.7933239	1.1896343	-0.4152298	0.60876	0.79335	-0.0726728	-0.0000004	0.0
5	-0.5000000	0.8660250							
	-0.3826694	0.9238452	1.3853798	-0.9192762	0.38269	0.92388	-0.0456845	-0.0000004	0.0
6	-0.2588190	0.9659260							
	-0.1305214	0.9914082	1.4867077	-1.2102995	0.13053	0.99145	-0.0155819	-0.0000005	0.0
7	0.0	1.0000000							
	0.1305214	0.9914082	1.4867067	-1.2102966	-0.13053	0.99145	0.0155820	0.0	0.0
8	0.2588190	0.9659260							
	0.3826688	0.9238456	1.3853798	-0.9192762	-0.38268	0.92388	0.0456843	-0.0000002	0.0
9	0.4999990	0.8660260							
	0.6087386	0.7933245	1.1896353	-0.4152317	-0.60876	0.79335	0.0726725	0.0000002	0.0
10	0.7071069	0.7071067							
	0.7933239	0.6087395	0.9128108	0.1667765	-0.79335	0.60876	0.0947084	-0.0000001	0.0
11	0.8660250	0.5000010							
	0.9238452	0.3826699	0.5737217	0.6708434	-0.92388	0.38268	0.1102907	0.0000015	0.0
12	0.9659260	0.2588190							
	0.9914082	0.1305214	0.1956850	0.9617074	-0.99145	0.13053	0.1183582	0.0000015	0.0
13	1.0000000	0.0							

ADDED MASS = 0.0 VOLUME = 4.1762981 SUM (T)(DELTA S) = 2.9989319

DOUGLAS AIRCRAFT COMPANY
LONG BEACH DIVISION

12 ELEMENT SPHERE. CASE NO. PSF =

ON-BODY STRIP VORTEX FLOW
TRANSFORMED COORDINATES

	X	Y	T1	CP	SIN A	COS A	SIGMA	N	PHI
1	-1.0000000	0.0000010							
	-0.9914083	0.1305223	1.3354311	-0.7833757	0.99145	0.13053	2.2962484	0.0000916	0.0
2	-0.9659260	0.2588200							
	-0.9238451	0.3826699	-0.2542801	0.9353417	0.92388	0.38269	1.2829266	0.0000048	0.0
3	-0.8660250	0.5000000							
	-0.7933235	0.6087390	-0.0182657	0.9996664	0.79335	0.60876	0.8337094	0.0000057	0.0
4	-0.7071060	0.7071069							
	-0.6087385	0.7933239	-0.0054169	0.9999707	0.60876	0.79335	0.5359750	0.0000038	0.0
5	-0.5000000	0.8660250							
	-0.3826694	0.9238452	-0.0023832	0.9999943	0.38269	0.92388	0.3025500	0.0000048	0.0
6	-0.2588190	0.9659260							
	-0.1305214	0.9914082	-0.0013103	0.9999983	0.13053	0.99145	0.0980312	0.0000039	0.0
7	0.0	1.0000000							
	0.1305214	0.9914082	-0.0013151	0.9999983	-0.13053	0.99145	-0.0980305	0.0000006	0.0
8	0.2588190	0.9659260							
	0.3826688	0.9238456	-0.0023851	0.9999943	-0.38268	0.92388	-0.3025491	-0.0000010	0.0
9	0.4999990	0.8660260							
	0.6087386	0.7933245	-0.0054121	0.9999707	-0.60876	0.79335	-0.5359737	-0.0000010	0.0
10	0.7071069	0.7071069							
	0.7933239	0.6087395	-0.0182714	0.9996662	-0.79335	0.60876	-0.8337085	-0.0000029	0.0
11	0.8660250	0.5000010							
	0.9238452	0.3826699	-0.2542858	0.9353387	-0.92388	0.38268	-1.2829294	-0.0000010	0.0
12	0.9659260	0.2588190							
	0.9914082	0.1305214	1.3353949	-0.7832794	-0.99145	0.13053	-2.2962656	-0.0000763	0.0
13	1.0000000	0.0							

ADDED MASS = 0.0 VOLUME = 4.1762981 SUM (T)(DELTA S) = 0.5501668

DOUGLAS AIRCRAFT COMPANY
LONG BEACH DIVISION

12 ELEMENT SPHERE. CASE NO. PSF =

CN-BODY UNIFORM CROSS FLOW
TRANSFORMED COORDINATES

	X	Y	T2	T3	SIN A	COS A	SIGMA	N	PHI
1	-1.0000000	0.0000010							
	-0.9914083	0.1305223	1.4870138	1.4993658	0.99145	0.13053	0.0155821	0.0000004	0.0651784
2	-0.9659260	0.2588200							
	-0.9238451	0.3826699	1.3826275	1.4991264	0.92388	0.38269	0.0456054	0.0000003	0.1910008
3	-0.8660250	0.5000000							
	-0.7933235	0.6087390	1.1867304	1.4989996	0.79335	0.60876	0.0724524	0.0000006	0.3037606
4	-0.7071060	0.7071069							
	-0.6087365	0.7933239	0.9115025	1.4991226	0.60876	0.79335	0.0944496	0.0000003	0.3959663
5	-0.5000000	0.8660250							
	-0.3826694	0.9238452	0.5741063	1.4993992	0.38269	0.92388	0.1101502	-0.0000001	0.4613678
6	-0.2588190	0.9659260							
	-0.1305214	0.9914082	0.1961133	1.4996233	0.13053	0.99145	0.1183575	0.0	0.4953313
7	0.0	1.0000000							
	0.1305214	0.9914082	-0.1961144	1.4996233	-0.13053	0.99145	0.1183576	0.0000001	0.4953312
8	0.2588190	0.9659260							
	0.3826688	0.9238456	-0.5741080	1.4993992	-0.38268	0.92388	0.1101502	0.0000001	0.4613683
9	0.4999990	0.8660260							
	0.6087386	0.7933245	-0.9115077	1.4991217	-0.60876	0.79335	0.0944493	0.0000002	0.3959657
10	0.7071069	0.7071069							
	0.7933239	0.6087395	-1.1867332	1.4989996	-0.79335	0.60876	0.0724517	0.0000003	0.3037609
11	0.8660250	0.5000010							
	0.9238452	0.3826699	-1.3826275	1.4991226	-0.92388	0.38268	0.0456047	0.0	0.1909994
12	0.9659260	0.2588190							
	0.9914082	0.1305214	-1.4870119	1.4993696	-0.99145	0.13053	0.0155822	0.0000002	0.0651785
13	1.0000000	0.0							

ADDED MASS = 2.0856047 VOLUME = 4.1762981

VI. REFERENCES

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2. Hess, J.L.: Calculation of Potential Flow about Bodies of Revolution Having Axes Perpendicular to the Freestream Direction, DAC Report ES 29812, October 1960.

3. Hess, J.L.: Improved Ring Source Formulae for Small Values of Distance from the Symmetry Axis. A Modification of the Douglas-Neumann Program for Axisymmetric Bodies, DAC Report 70002, August 1969.

4. Hess, J. L.: Extension of the Douglas Axisymmetric Potential Flow Program to Include the Effects of Ring Vorticity with Application to the Problem of Specified Tangential Velocity, DAC Report 33195, June 1966.

5. Hess, J. L. And Clissold, C.T.: Instructions for Use of the Douglas-Neumann Potential Flow Program for Calculation of Low-Speed Flows about Bodies of Revolution.

6. Hess, J. L. and Riddell, T. M.: Direct Solution of a Square Matrix Whose Size Greatly Exceeds High-Speed Storage, DAC Report 70000, July 1969.

APPENDIX:

USAGE OF THE IPTANV FLAG

Usage of the IPTANV flag permits tangential velocity to be specified over part of the surface of the body or bodies about which flow is being computed. On the remainder of the surface the normal velocity is made zero in the usual way. At locations where tangential velocity is specified the normal velocity will not in general be zero. This feature can be used only for axisymmetric flows with a uniform free stream.

If the flow about several bodies is to be calculated, these bodies are considered in sequence as determined by the order in which they are input. This feature uses a flag to legislate that normal velocity conditions will be applied on the first several bodies of this sequence and tangential velocity conditions will be applied on the remaining bodies. All bodies on which normal velocities are specified must be input before any of the bodies on which tangential velocities are specified. For example, suppose it is desired to specify tangential velocity over the region near the middle of a body, while the usual normal velocity conditions is applied over the forward and aft regions. This body must be input as three bodies, the first two of which are the forward and aft regions and the third of which is the middle region.

The desired tangential velocities are input as a set of numbers. The first number in the set is the tangential velocity on the first element of the first body on which tangential velocity is specified. The subsequent numbers of the set give tangential velocitites on the remaining elements in the order that they were input. The order of the input of the tangential velocities is used to associate them with elements, and it is the responsibility of the user to insure that this order is correct and that the number of input tangential velocities equals the number of elements on the bodies on which tangential velocities are specified. The tangential velocities should be input as total tangential velocities, including the contribution of the uniform free stream.

If the desired tangential velocity is a constant over the body or bodies on which tangential velocity is specified, only this single constant need be input. The constant should represent a total tangential velocity. It is not possible to input a separate constant for each body on which tangential velocity is specified.

It is possible to specify tangential velocity over the entire surface of the body or bodies that are input, so that no normal velocity condition is applied.

The final output is similar to that of the usual case. On the bodies where tangential velocities were specified the printed values of T should be the same as those input. The normal velocities will not in general be zero, but their magnitude is a measure of how near the input shape is to an impervious body having the same tangential velocity distribution. The printed values of N are outward normal velocities. A plus sign denotes flow out of the body, and a minus sign denotes flow into the body.

The option of specifying tangential velocity is excercised by punching some non-zero integer into C.C. 14 of the control flags card (IPTANV). This value indicates the number of bodies on which tangential velocity is being specified and therefore must be less than or equal to the total number of bodies being input. In particular, if a three body case is being run and tangential velocity is to be specified on two of the bodies, then those two bodies must be input last in the input sequence, and IPTANV must have a value of 2. The use of IPTANV affects the matrix solution portion of the program as follows:

Whereas the usual equations to be solved for a for uniform axi-symmetric flow are

$$\sum_{j+1}^{N} A_{ij}a_j = \sin a_i \qquad \text{(for } i = 1, 2, \ldots, N\text{)}$$

Where N is the total number of elements on all the bodies, use of IPTANV directs the program to solve for a in the following ways:

I. If IPTANV is equal to the total number of bodies input, the equations to be solved for a become

$$\sum_{j+1}^{N} B_{ij}a_j = TG_i = \cos a_i \qquad \text{(for } i = 1, 2, \ldots, N\text{)}$$

where B_{ij} is the usual axi-symmetric B_{ij} matrix and TG_i is an array of N numbers consisting of either (i) the same constant number (this number to be specified by the user in the TCNST field of input card no. 3), or (ii) N values input from cards (see card set 12).

II. If IPTANV is less than the total number of bodies (say, IPTANV = k), then tangential velocities are to be specified on the last k bodies input. That is, if M is the total number of elements on the last k bodies, then N-M is the number of elements on the first NB-k bodies (where NB is the total number of bodies input), and the equations to be solved for a become

$$\sum_{j=1}^{N} A_{ij} a_j = \sin a_i \qquad \text{(for } i = 1, 2, \ldots, N\text{-}M)$$

$$\sum B_{ij} a_j = TG_i - \cos a_i \qquad \text{(for } i = N\text{-}M\text{+}1, \ldots, N)$$

If all values of the TG array are input by cards, the user must take care to input precisely M values.

Note that his special matrix formation finally affects only the values of a for axi-symmetric cases (crossflow matrix formation and solution remains unchanged); all remaining calculations are performed as usual.

Restrictions exist in the use of IPTANV. In particular, IPTANV cannot be used in conjuction with any non-uniform flow solution, whether axi-symmetric or crossflow. Neither can IPTANV be used if the total number of body elements exceeds 275. Nor can the strip vortex option (IVORT) be used.

END

DATE

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