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**CONTROL SYSTEM DESIGN  
USING FREQUENCY DOMAIN MODELS  
AND PARAMETER OPTIMIZATION,  
WITH APPLICATION TO  
SUPERSONIC INLET CONTROLS**

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16. Abstract A technique is described for designing feedback control systems using frequency domain models, a quadratic cost function, and a parameter optimization computer program. FORTRAN listings for the computer program are included in the report. The approach is applied to the design of shock position controllers for a supersonic inlet. Considered are a deterministic or random system disturbance and the presence of random measurement noise. The cost function minimized is formulated in the time domain, but the problem solution is obtained using a frequency domain system description. A scaled and constrained conjugate gradient algorithm is used for the minimization. In applying the approach to a typical supersonic inlet, both optimal proportional-plus-integral (PI) and proportional-plus-integral-plus-derivative (PID) controllers were calculated. For the inlet considered, a single-loop PI controller was judged to be the most desirable of the various designs considered.					
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SUMMARY

This report described a technique for designing feedback control systems using frequency domain models, a quadratic performance index, and a parameter optimization computer program. The approach is applied to the design of a terminal shock position controller for a mixed-compression supersonic inlet. The computer program described can be used to design controllers for any two-loop linear feedback system having a single control input and a cascade structure. The problem formulation assumes the system is acted upon by a single deterministic or random disturbance plus random measurement noise introduced in each loop. The quadratic performance index chosen to be minimized is a weighted sum of averaged square system output, output rate, control, and control rate. The performance index is expressed in the frequency domain and is minimized, given a frequency domain system description, using a scaled and constrained conjugate gradient search algorithm.

For the supersonic inlet problem, the disturbance is a deterministic airflow perturbation at the diffuser exit, and measurement noises are assumed to contaminate inlet duct pressure measurements. The cost function chosen to be minimized is a weighted sum of averaged square values of the output (throat exit static pressure) and its derivative plus the control (inlet bypass door area) and its derivative. Optimal parameters were calculated for both proportional-plus-integral (PI) and proportional-plus-integral-plus-derivative (PID) single-loop controllers for a NASA designed mixed compression inlet. Feedback signals used were either throat exit static or diffuser exit static pressures. Designs were evaluated on the basis of averaged square values of output and control signals. PI control on throat exit static pressure proved to be the most effective compromise between complexity and ability to attenuate disturbances. Appendixes are included to describe the computer program as well as outline the solution to a sample design problem.

A supersonic aircraft propulsion system consists of a supersonic inlet and either a turbofan or turbojet engine. The function of the inlet is to convert high velocity, low pressure air ahead of the inlet to low velocity, high pressure air more suitable for the engine's compressor. An efficient inlet for flight at Mach numbers above about 2 is the mixed compression type, having a convergent supersonic region followed by a divergent subsonic region. An increase in static pressure occurs in both regions as the flow is decelerated. A terminal shock separates the supersonic and subsonic flows. To maximize efficiency, the terminal shock should be located near the throat in the divergent duct. If the terminal shock moves upstream into the convergent region, it jumps forward to form a strong shock wave ahead of the inlet. This occurrence is known as an inlet unstart, and results in increased drag and a rapid loss in pressure recovery. This in turn may lead to a compressor stall and/or combustor flameout. Conversely, if the shock moves too far downstream, the pressure recovery is reduced and distortion is increased at the compressor face, which may also cause compressor stall. Thus, controls are needed to maintain the terminal shock close to but yet downstream of the throat for good pressure recovery without inlet unstarts.

Figure 1 is a schematic diagram of a mixed compression inlet with a typical terminal shock control loop shown. The terminal shock is positioned using the bypass door loop. The door opens and closes to maintain a match between the inlet and engine airflow as engine airflow demand changes. This tends to prevent the terminal shock from moving too far rearward, causing increased distortion, or too far forward, causing an unstart. Throat exit and diffuser exit static pressure signals shown give indications of shock position. They are fed back through the controller to drive the bypass door servo.

The controller transfer functions are significant factors in the dynamic regulation of shock position. The design of such transfer functions has been studied in references 1 to 4. A root locus design technique was reported in reference 1. A stochastic optimal control theory approach was reported in references 2 and 3, where the expected frequency of unstarts was minimized. A parameter optimization approach, where the parameters in a fixed-form controller were selected so that the response approximated a desired closed-loop transfer function, was reported in reference 4.

The approach taken in this report is also based on optimizing the parameters in a fixed-form controller. However, the cost function chosen to be minimized is the weighted sum of average integral square errors, due to a deterministic disturbance, and the mean square errors due to random noise on the measurements. This approach was motivated by the inlet control problem (ref. 5), where the compressor face disturbance is most conveniently described as a deterministic signal but where the noise on duct pressure measurements is definitely random in nature. The variables included in the

spective first derivatives.

While the problem is formulated in the time domain, the optimization is done in the frequency domain. This is because system, disturbance, and noise models are most often obtained in frequency domain form. The conjugate gradient search (ref. 6) is used for the optimization. However, the standard procedure is modified to reduce search convergence time and insure that the resulting controller is stable. This is done by scaling and constraining the search parameters.

In the next section, the general control problem investigated is defined and the cost function and gradient calculations are described. Then the parameter optimization program used is described, followed by a description of the inlet model to which the controller design method was applied. Finally, the results are presented, followed by appendixes, which include one on the use of the computer program and one in which a sample problem is presented.

## GENERAL CONTROL PROBLEM DESCRIPTION

### Description of Plant, Noise, and Disturbance

Motivated by the supersonic inlet control problem, a control system structure was selected as a framework for the controller design problem. This structure is shown in figure 2. The linear plant (blocks  $G_1$  and  $G_2$ ) is assumed to have two measurements  $x_1$  and  $x_2$  available. (Symbols are defined in appendix A.) Variable  $x_1$ , the outer-loop measurement, is considered the output. The control is to consist of two blocks,  $H_1$  and  $H_2$ , whose forms are specified at the outset, but whose parameters are to be optimized. The two inputs to the controller are assumed contaminated with independent Gaussian noises  $v_1$  and  $v_2$ , each having the same power spectral density  $\Phi_v(\omega)$ . The controller output drives actuator  $G_a$  to produce control  $u$ . The plant is acted upon by deterministic disturbance  $d$  which occurs at time zero.

### Cost Function Evaluation

It is assumed that the purpose of the controller is to keep  $x_1$  and  $\dot{x}_1$  as close to zero as possible but at the same time limiting the excursions in control  $u$  and control rate  $\dot{u}$ . Thus, a cost function must be defined which adequately reflects the average deviations in output and control caused by random measurement noises  $v_1$  and  $v_2$  and deterministic disturbance  $d$ . We now proceed to develop such a cost function.

For convenience, define components of  $x_1$  and  $u$  such that

$$\left. \begin{aligned} x_1 &= x_{1v} + x_{1d} \\ \dot{x}_1 &= \dot{x}_{1v} + \dot{x}_{1d} \\ u &= u_v + u_d \\ \dot{u} &= \dot{u}_v + \dot{u}_d \end{aligned} \right\} \quad (1)$$

where the first quantities on the right sides of equation (1) are the components due to measurement noise  $v_1$  and  $v_2$  and the second quantities on the right sides are components due to deterministic disturbance  $d$ . First, separate cost functions will be derived for the measurement noise and deterministic inputs. Then, these separate cost functions will be combined into a single cost function. Consider first the case where only measurement noise is present. Define the cost function

$$C_v = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \left[ q_1 x_{1v}^2(t) + q_2 \dot{x}_{1v}^2(t) + r_1 u_v^2(t) + r_2 \dot{u}_v^2(t) \right] dt \quad (2)$$

The  $q$ 's and  $r$ 's are arbitrarily selected scalar penalties. Thus,  $C_v$  is simply a weighted sum of mean square values. Next, consider the case where only a deterministic disturbance  $d$  is present. Disturbance  $d$  is assumed to be zero for  $t < 0$ . Define the cost function

$$C_d = \frac{1}{T} \int_0^T \left[ q_1 x_{1d}^2(t) + q_2 \dot{x}_{1d}^2(t) + r_1 u_d^2(t) + r_2 \dot{u}_d^2(t) \right] dt \quad (3)$$

Time  $T$  is defined as the (arbitrarily selected) period during which control is to be effective in minimizing the effects of  $d$ . Factor  $1/T$  is included to make  $C_d$  an average integral squared quantity, and thus comparable to mean square quantity  $C_v$ . Now, the total cost function can be defined as

$$C = C_v + C_d \quad (4)$$

Cost function  $C$  is to be minimized by proper selection of controller transfer function parameters. The problem may be simplified if we assume that  $T$  is large enough so

that  $x_{1d}$ ,  $\dot{x}_{1d}$ ,  $u_d$ , and  $\dot{u}_d$  all go to zero before  $t = T$ . Then the total cost can be written as

$$C = C_v + C_{d\infty} \quad (5)$$

$$C = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} \left[ q_1 x_{1v}^2(t) + q_2 \dot{x}_{1v}^2(t) + r_1 u_v^2(t) + r_2 \dot{u}_v^2(t) \right] dt \\ + \frac{1}{T} \int_0^{\infty} \left[ q_1 x_{1d}^2(t) + q_2 \dot{x}_{1d}^2(t) + r_1 u_d^2(t) + r_2 \dot{u}_d^2(t) \right] dt \quad (6)$$

where the upper limit on the second integral  $C_{d\infty}$  has now been made equal to  $\infty$ .

Our aim is now to express  $C$  in the frequency domain, since, as was indicated previously, we desire to perform the parameter optimization using frequency domain system models and data. In the following development, an expression for  $C$  will be obtained which is an integral in the frequency domain, over a range of 0 to  $\infty$ . The integrand will be obtained in terms of the following known quantities: noise power spectral density (PSD), plant, and controller transfer functions, and the Fourier transform of the disturbance.

As previously noted, cost function  $C_v$  (eq. (2)) is a weighted sum of mean square values of stationary random variables  $x_{1v}$ ,  $\dot{x}_{1v}$ ,  $u_v$ , and  $\dot{u}_v$ . It can be shown (ref. 7) that the mean square value of any stationary random variable  $y$  can be expressed in the frequency domain as

$$\overline{y^2} = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{\tau} y^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_y(\omega) d\omega \quad (7)$$

where  $\overline{y^2}$  is defined as the mean square value of  $y$  and  $\Phi_y(\omega)$  is defined as the PSD of  $y$ . It can also be shown (ref. 7) that the PSD of the output of a single-input - single-output linear system whose transfer function is  $G(s)$  is given by

$$\Phi_y(\omega) = \Phi_z(\omega) |G(j\omega)|^2 \quad (8)$$

where  $\Phi_z(\omega)$  is the PSD of the input to the system. If the system of concern is a differentiator, that is,  $G(s) = s$ , or  $y(s) = s z(s)$ , then  $\Phi_y(\omega) = \omega^2 \Phi_z(\omega)$ . We can use this fact plus equation (7) to express  $C_v$  in terms of PSD's, obtaining

$$C_v = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ (q_1 + \omega^2 q_2) \Phi_{x_1}(\omega) + (r_1 + \omega^2 r_2) \Phi_u(\omega) \right] d\omega \quad (9)$$

where  $\Phi_{x_1}(\omega)$  is the PSD of  $x_{1v}$  and  $\Phi_u(\omega)$  is the PSD of  $u_v$ .

Next we must express  $C_{d\infty}$  in terms of frequency dependent variables. Given a time function  $y(\tau)$ , having a Fourier transform  $y(j\omega)$ , Parseval's theorem (ref. 7) states that

$$\int_{-\infty}^{\infty} y(t)^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |y(j\omega)|^2 d\omega \quad (10)$$

Then  $C_{d\infty}$  of equation (6) can be put in the form of the left side of equation (10) by allowing the lower limit in equation (6) be  $-\infty$ . This can be done because the integrand of  $C_{d\infty}$  is zero for time  $t < 0$ . If this fact is used in addition to  $\dot{y}(j\omega) = j\omega y(j\omega)$ , a frequency domain expression can be obtained for  $C_{d\infty}$  as

$$C_{d\infty} = \frac{1}{2\pi T} \int_{-\infty}^{\infty} \left[ (q_1 + \omega^2 q_2) |x_{1d}(j\omega)|^2 + (r_1 + \omega^2 r_2) |u_d(j\omega)|^2 \right] d\omega \quad (11)$$

Since the integrands of equations (9) and (11) are even functions of frequency, the lower limits can be set to zero, and an expression for cost function  $C$  obtained as

$$C = \frac{1}{\pi} \int_0^{\infty} \left\{ \left[ (q_1 + \omega^2 q_2) \Phi_{x_1}(\omega) + (r_1 + \omega^2 r_2) \Phi_u(\omega) \right] \right. \\ \left. + \frac{1}{T} \left\{ \left[ (q_1 + \omega^2 q_2) |x_{1d}(j\omega)|^2 + (r_1 + \omega^2 r_2) |u_d(j\omega)|^2 \right] \right\} \right\} d\omega \quad (12)$$

To be able to numerically evaluate  $C$ , the PSD's and the absolute value squares must be expressed in terms of known system transfer functions, noise PSD  $\Phi_v(\omega)$ , and disturbance Fourier transform absolute value squared,  $|d(j\omega)|^2$ . Refer now to figure 2. First, variable  $x_{1v}$ , the component of  $u_1$  due to measurement noise, can be written as



$$x_{1v}(s) = G_a(s)G_{cl}(s) \left[ H_1(s)v_1(s) + H_2(s)v_2(s) \right] \quad (13)$$

where

$$G_{cl}(s) = \frac{G_1(s)G_2(s)}{1 + G_s(s) \left[ G_1(s)H_2(s) + G_1(s)G_2(s)H_1(s) \right]} \quad (14)$$

Variable  $x_{1d}$ , the component of  $x_1$  due to the disturbance, is

$$x_{1d}(s) = G_{cl}(s) d(s) \quad (15)$$

Similarly, the component of  $u$  due to measurement noise is

$$u_v(s) = \frac{G_{cl}(s)G_a(s)}{G_1(s)G_2(s)} \left[ H_1(s)v_1(s) + H_2(s)v_2(s) \right] \quad (16)$$

and the component of  $u$  due to the disturbance is

$$u_d(s) = G_{cl}(s)G_a(s) \left[ \frac{H_2(s)}{G_2(s)} + H_1(s) \right] d(s) \quad (17)$$

Using equations (13) and (16), the fact that  $v_1$  and  $v_2$  are uncorrelated, and letting  $s = j\omega$ , the PDS's of  $x_{1v}$  and  $u_v$  become

$$\Phi_{x_{1v}}(\omega) = \Phi_v(\omega) |G_a(j\omega)G_{cl}(j\omega)|^2 \left[ |H_1(j\omega)|^2 + |H_2(j\omega)|^2 \right] \quad (18)$$

and

$$\Phi_{u_v}(\omega) = \Phi_v(\omega) \left| \frac{G_{cl}(j\omega)G_a(j\omega)}{G_1(j\omega)G_2(j\omega)} \right|^2 \left[ |H_1(j\omega)|^2 + |H_2(j\omega)|^2 \right] \quad (19)$$

Also, using equation (15) results in

$$|x_{1d}(j\omega)|^2 = |G_{cl}(j\omega)|^2 |d(j\omega)|^2 \quad (20)$$

and using equation (17) gives

$$u_d(j\omega)^2 = |G_{cl}(j\omega)G_a(j\omega)|^2 \left| \frac{H_2(j\omega)}{G_2(j\omega)} + H_1(j\omega) \right|^2 |d(j\omega)|^2 \quad (21)$$

Substituting equations (18) to (21) into the cost function equation (12), cost  $C$  becomes

$$C = C(\underline{b}) = \frac{1}{\pi} \int_0^\infty F(\omega, \underline{b}) d\omega \quad (22)$$

where  $F(\omega, \underline{b})$  is defined as

$$F(\omega, \underline{b}) \triangleq |G_{cl}(j\omega, \underline{b})|^2 \left\{ [V_q(\omega) + V_r(\omega)] \left[ |H_1(j\omega, \underline{b})|^2 + |H_2(j\omega, \underline{b})|^2 \right] + D_q(\omega) + D_r(\omega) \left| \frac{H_2(j\omega, \underline{b})}{G_2(j\omega)} + H_1(j\omega, \underline{b}) \right|^2 \right\} \quad (23)$$

and

$$\left. \begin{aligned} V_q(\omega) &\triangleq (q_1 + \omega^2 q_2) \Phi_v(\omega) |G_a(j\omega)|^2 \\ V_r(\omega) &\triangleq (r_1 + \omega^2 r_2) \Phi_v(\omega) \left| \frac{G_a(j\omega)}{G_1(j\omega)G_2(j\omega)} \right|^2 \\ D_q(\omega) &\triangleq \frac{1}{T} (q_1 + \omega^2 q_2) |d(j\omega)|^2 \\ D_r(\omega) &\triangleq \frac{1}{T} (r_1 + \omega^2 r_2) |d(j\omega)|^2 |G_a(j\omega)|^2 \end{aligned} \right\} \quad (24)$$

The symbolism  $H_1(j\omega, \underline{b})$  and so forth has been introduced to indicate which terms are functions of controller parameter vector  $\underline{b}$ . Note that  $V_q(\omega)$ ,  $V_r(\omega)$ ,  $D_q(\omega)$ , and  $D_r(\omega)$  are all independent of the controller parameters.

Having now obtained an equation from which to evaluate  $C$  as a function of known system parameters and variable controller parameters, the parameter optimization problem to be solved is: minimize  $C(\underline{b})$  by proper selection of parameter vector  $\underline{b}$ .

The problem has thus far been formulated for the case where the system is acted upon by a single deterministic disturbance plus two random measurement noises. However, the problem can be considered to be a completely stochastic one if the deterministic disturbance  $d(t)$  is replaced by a Gaussian random variable with a PSD of  $\Phi_d(\omega)$ . Then,  $\Phi_d(\omega)$  would be used in place of  $(1/T)|d(j\omega)|^2$  in equations (24). Also, the problem becomes entirely deterministic if measurement noise  $v$  is considered to be zero. In that case the noise PSD,  $\Phi_v(\omega)$  is set to zero in equations (24).

### Cost Function Gradient Evaluation

An expression for the cost function (eq. (22)) is not usually sufficient to allow an effective solution to the parameter optimization problem. Most parameter optimization methods (conjugate gradients, Fletcher-Powell, steepest descents, etc.) require also that the gradient of  $C$  be calculated. One way of computing an approximate gradient is to make finite perturbations in  $\underline{b}$ , compute  $C$  each time, and find the gradient as the change in  $C$  divided by the change in the respective  $\underline{b}$  vector components. However, a more efficient method is to derive an explicit expression for the gradient  $\nabla C$  in terms of  $\underline{b}$ . Such an approach was taken in this report. Equation (22) can be used to write the gradient as

$$\nabla C(\underline{b}) = \frac{1}{\pi} \int_0^{\infty} \nabla F(\omega, \underline{b}) d\omega \quad (25)$$

Using equations (23) and (14) makes it possible to express  $\nabla F(\omega, \underline{b})$  as

$$\begin{aligned} \nabla F(\omega, \underline{b}) = & 2 \operatorname{Real} \left[ F(\omega, \underline{b}) \left\{ -G_{cl}(j\omega, \underline{b}) G_a(j\omega) \cdot \nabla \left[ \frac{H_2(j\omega, \underline{b})}{G_2(j\omega)} + H_1(j\omega, \underline{b}) \right] \right\}^* \right. \\ & + |G_{cl}(j\omega, \underline{b})|^2 \left( D_r(\omega) \left[ \frac{H_2(j\omega, \underline{b})}{G_2(j\omega)} + H_1(j\omega, \underline{b}) \right] \nabla \left[ \frac{H_2(j\omega, \underline{b})}{G_2(j\omega)} + H_1(j\omega, \underline{b}) \right] \right. \\ & \left. \left. + [V_q(\omega) + V_r(\omega)] \cdot \left\{ H_2(j\omega, \underline{b}) \nabla [H_2(j\omega, \underline{b})]^* + H_1(j\omega, \underline{b}) \nabla [H_1(j\omega, \underline{b})]^* \right\} \right) \right] \quad (26) \end{aligned}$$

where the fact that  $\nabla |y(j\omega)|^2 = 2 \text{ Real } [y(j\omega)\nabla y(-j\omega)]$  was used. Once the forms of  $H_1(j\omega, \underline{b})$  and  $H_2(j\omega, \underline{b})$  have been chosen, the required gradients can also be calculated. In the computer program described in appendix B,  $H_1(s, \underline{b})$  and  $H_2(s, \underline{b})$  are assumed to be of the following form:

$$H_i(s, \underline{b}) = Kb_1 s^{m_1} \prod_{j=1}^{m_2} \left( \frac{s}{b_{2j}} + 1 \right)^{p_{2j}} \prod_{j=1}^{m_3} \left( \frac{s^2}{b_{4j}^2} + \frac{2sb_{3j}}{b_{4j}} + 1 \right)^{p_{3j}} \quad i = 1, 2 \quad (27)$$

Here,  $m_1$ ,  $m_2$ , and  $m_3$  are given. Exponents  $p_{2j}$  and  $p_{3j}$  indicate whether the factors appear in the numerator or denominator, are  $\pm 1$ , and are given for all  $j$ . The parameter  $K$  could represent a transducer gain (for instance, conversion from a pressure measurement to a controller input voltage) and is given. With these restrictions, a closed form calculation can be made for  $\nabla H_1(j\omega, \underline{b})$ . For example, if  $H_1(s, \underline{b})$  has four parameters and is given as

$$H_1(s, \underline{b}) = \frac{Kb_1}{s} \left( \frac{s}{b_{21}} + 1 \right)^{p_{21}} \left( \frac{s^2}{b_{41}^2} + \frac{2sb_{31}}{b_{41}} + 1 \right)^{p_{31}} \quad (28)$$

where  $\underline{b}^T = (b_1, b_{21}, b_{31}, b_{41})$ , then,

$$\nabla H_1(s, \underline{b}) = H_1(s, \underline{b}) \cdot \begin{bmatrix} \frac{1}{b_1} \\ \frac{-p_{21}s}{(s+b_{21})b_{21}} \\ \frac{2p_{31}s}{\left( \frac{s^2}{b_{41}^2} + \frac{2sb_{31}}{b_{41}} + 1 \right) b_{41}} \\ -2p_{31} \left( \frac{b_{31}}{b_{41}} + \frac{s^2}{b_{41}^2} \right) s \\ \frac{\left( \frac{s^2}{b_{41}^2} + \frac{2sb_{31}}{b_{41}} + 1 \right) b_{41}} \end{bmatrix} \quad (29)$$

### Programming Considerations

A computer program was written (see appendix B) which calculates the optimum parameters  $\underline{b}$  using the conjugate gradient method (ref. 6). In the program, the cost function (eq. (22)) and cost function gradient (eq. (25)) are computed using numerical integration. In particular, using the trapezoidal rule, the cost function can be written as

$$C = \frac{1}{2\pi} \sum_{i=1}^{N_d} F(\omega_i)(\omega_{i+1} - \omega_{i-1}) \quad (30)$$

where  $N_d$  is the number of frequency points over which the integral is to be computed ( $\geq 3$ ),  $\omega_i$  is the frequency at the  $i^{\text{th}}$  data point,  $\omega_{N_d+1} = \omega_{N_d}$ , and  $\omega_0 = \omega_1$ . A similar expression can be written for the gradient. Judgment is required in selecting the spacing of the frequency points, such that integration errors are minimized. In addition, a variable step size feature is introduced to further reduce errors. This is done by using the fact that errors caused by a fixed step size are greatest near a system resonance. Resonance is defined herein as the portion where the closed-loop transfer function magnitude  $|G_{cl}(s)|$  exceeds the open-loop transfer function magnitude. The program detects this condition and when it occurs,  $|G_{cl}(j\omega)|^2$  is evaluated at additional points within each frequency interval using interpolation between adjacent prespecified frequency points. This feature increases accuracy and also tends to prevent occurrence of another computational problem, namely, that of the closed-loop system transfer function becoming unstable during the search. Since the magnitude squared of an unstable  $G_{cl}(j\omega)$  is the same as a stable one having the same pole magnitudes, the cost function calculation won't differentiate between the desired stable and an unwanted unstable solution. However in becoming unstable during the search procedure,  $G_{cl}(j\omega)$  will have an increasingly large resonant peak, which the program will tend to detect. Then the addition of the extra function values will ensure adequate accuracy in computing  $C$  and  $\nabla C$ . That is, the decrease in system stability will be properly reflected as an increase in the cost function.

The search procedure used is similar to the conjugate gradient method of reference 6. Two modifications were made to the conjugate gradient search to improve the speed of convergence in this application. The first was to constrain the search vector  $\underline{b}$  components to not change sign. With the transfer functions defined by equation (27), this just means that only stable controllers are allowed as candidates for the optimal one. The second modification concerns scaling of the  $\underline{b}$  vector. It is known that "spherical" cost functions (where the cost is more or less equally sensitive to each element of the  $\underline{b}$  vector) tend to lend themselves to rapidly converging searches. Thus, a scaling was

incorporated into the search procedure such that each parameter has approximately equal influence. This was done by defining a new scaled parameter vector  $\underline{p}$  using the diagonal matrix  $\Lambda$  as follows:

$$\underline{p} \triangleq \begin{bmatrix} 1/\lambda_1 & & & \\ & 1/\lambda_2 & & \\ & & \ddots & \\ & & & 1/\lambda_{m_p} \end{bmatrix} \underline{b} \triangleq \Lambda^{-1} \underline{b} \quad (31)$$

where  $\lambda_i$  is the magnitude of the  $i^{\text{th}}$  element of  $\underline{b}$  at the end of the previous iteration of the conjugate gradient algorithm. The cost function and gradient in terms of  $\underline{p}$  are thus:

$$C(\underline{b}) = C(\Lambda \underline{p}) \quad (32)$$

and

$$\nabla F(\underline{p}) = \Lambda \nabla F(\underline{b}) \quad (33)$$

Theory states that the unmodified conjugate gradient search will always converge for quadratic cost functions. In the modified conjugate gradient search, the coordinate system is changed at the start of each iteration. Thus in theory, the modified conjugate gradient algorithm may not always converge for quadratic cost functions. However, experience to date on nonquadratic cost functions confirms that the aforementioned scaling algorithm gave convergence times less than or equal to those for nonscaled cases. Appendix C shows the results for one test case where scaling was particularly useful.

## APPLICATION OF PARAMETER OPTIMIZATION TO INLET CONTROL DESIGN

The plant, to which the parameter optimization method is applied in this study, is a NASA designed two-dimensional, mixed-compression inlet. A description of the inlet is given in references 8 and 5. Reference 8 gives experimental open-loop frequency responses of the inlet's terminal shock and subsonic duct static pressures to overboard bypass door area. Experimental frequency responses of the inlet with control are given in

reference 5. In these tests, one set of overboard bypass doors was used to generate the disturbance and a second set was used for control.

### Inlet and Noise Dynamics

The structure studied in this report is shown in figure 3. The output  $x_1$  to be controlled is  $P_{57}$ , a throat exit static pressure 57 centimeters from the cowl lip and downstream of the terminal shock, which is used as an indication of shock position. Measured signal  $x_2$  is pressure  $P_{87}$ , which is closer to the compressor face station, where the airflow rate disturbance  $d$  originates. Controllers  $H_1$  and  $H_2$  drive bypass doors  $G_a$  which pass more or less flow to counteract the effects of disturbance  $d$ . Control  $u$  is  $u_{bp}$ , bypass door area (or airflow rate).

The inlet dynamics and bypass door frequency responses were found in reference 8 and are tabulated in table I. Magnitude data are shown normalized to the values at 1 hertz. Experimental frequency response data were available over the range of 1 to 150 hertz; points at 0.001, 300, and 600 hertz were extrapolated using transfer function models similar to those given in reference 8.

From a limited amount of experimental data, measurement noises  $v_1$  and  $v_2$  were found to be uncorrelated and to have the same PSD  $\Phi_v(\omega)$ . The PSD is tabulated in table I, with the data for 300 and 600 hertz extrapolated to be equal to the value at 150 hertz.

### Cost Function

To simplify the discussion of results, the cost function of equation (12), by substituting and collecting terms, can be written in terms of averaged square values as

$$C = q_1 \overline{x_1^2} + q_2 \overline{\dot{x}_1^2} + r_1 \overline{u^2} + u_2 \overline{\dot{u}_2^2} \quad (34)$$

where

$$\overline{x_1^2} \triangleq \frac{1}{\pi} \int_0^\infty \left[ \Phi_{x_{1v}}(\omega) + \frac{1}{T} |x_{1d}(j\omega)|^2 \right] d\omega \triangleq \text{averaged square output} \quad (35)$$

$$\overline{\dot{x}_1^2} \triangleq \frac{1}{\pi} \int_0^\infty \omega^2 \left[ \Phi_{x_{1v}}(\omega) + \frac{1}{T} |x_{1d}(j\omega)|^2 \right] d\omega \triangleq \text{averaged square output rate} \quad (36)$$

$$\overline{u^2} \triangleq \frac{1}{\pi} \int_0^{\infty} \left[ \Phi_u(\omega) + \frac{1}{T} |u_d(j\omega)|^2 \right] d\omega \triangleq \text{averaged square control} \quad (37)$$

$$\overline{\dot{u}^2} \triangleq \frac{1}{\pi} \int_0^{\infty} \omega^2 \left[ \Phi_u(\omega) + \frac{1}{T} |u_d(j\omega)|^2 \right] d\omega \triangleq \text{averaged square control rate} \quad (38)$$

### Disturbance Model

One of the critical assumptions in this study is the selection of the form of the disturbance  $d(s)$ . The disturbance is assumed to have the form

$$d(t) = Ae^{-at} \quad (39)$$

The disturbance represents a corrected flow rate change at the diffuser exit. For convenience,  $d(t)$  is taken as an equivalent diffuser exit area change ( $\text{cm}^2$ ) instead of corrected flow. A range of parameter  $a$  values is considered, from  $a = 4$  to 400 radians per second, so as to account for our uncertainty in establishing the exact nature of the disturbance.

### RESULTS

The majority of the results were obtained for a case designated as the reference case, defined as follows:

(1) A single-loop control is used with  $H_2 = 0$ ;  $H_1$  is assumed to be of the following form:

$$H_1(s) = K \left( \frac{b_1}{s} + \frac{b_1}{b_{21}} \right) = K \frac{b_1}{s} \left( \frac{s}{b_{21}} + 1 \right)$$

which is a proportional-plus-integral (PI) controller. PI control was chosen in view of the results obtained using PI control in previous inlet control programs.

(2) An averaging time  $T$  of 1.0 second is assumed.

(3) The disturbance pole  $a$  is assumed to be 40 radians per second. Disturbance



amplitude  $A$  is assumed to equal 84 square centimeters. This is about the same amplitude that was used in unstart tests reported in reference 5.

### Averaged Square Value Comparisons for Reference Case

Reference case designs are compared on the basis of their averaged square values. Three different cases of  $q_2$ ,  $r_1$ , and  $r_2$  cost function penalties are considered. Since the values of  $q_1$ ,  $q_2$ ,  $r_1$ , and  $r_2$  cannot all be varied independently,  $q_1$  is set to one for all cases. For each case, two of the penalties are set equal to zero and the third is varied from zero to infinity. Thus, although not all combination of penalties are examined, the ones used will give a representative sample of all possible results.

In figure 4(a), the normalized averaged square output rate  $(\dot{x}_1^2)_N$  is plotted as a function of normalized averaged square output  $(x_1^2)_N$ . Quantities  $\overline{x_1^2}$  and  $\overline{\dot{x}_1^2}$  are normalized with respect to their open-loop values. For each curve, the penalty indicated goes from zero to infinity in the direction of the arrow. The  $r_1$  and  $r_2$  curves go to the open-loop condition for  $r_1$  or  $r_2$  equal  $\infty$ . The  $q_2 = \infty$  case, which is equivalent to  $C = \overline{\dot{x}_1^2}$ , appears to also have an open-loop solution, but in fact approaches it quite closely but doesn't reach it. Each of the three curves has a "knee" type characteristic, thus a design trade-off exists between  $(\dot{x}_1^2)_N$  and  $(x_1^2)_N$ . For the  $q_2$  case, for example, one might choose the design having  $(\dot{x}_1^2)_N$  of 1.2 and  $(x_1^2)_N$  of 0.5 as one which has a fairly low value of  $(x_1^2)_N$  while not having an excessively large value of  $(\dot{x}_1^2)_N$ . The  $q_2$  curve lies below the  $r_1$  and  $r_2$  curves since it is the only case where  $\overline{\dot{x}_1^2}$  is penalized directly.

In selecting a controller design suitable for actual implementation, it is important that the design not require control actuator capabilities beyond those available. Thus, control signal and control rate requirements are examined in figures 4(b) and (c). Here, control and control rate are normalized to their values at the point of minimum  $\overline{x_1^2}$  (with  $r_1 = r_2 = q_2 = 0$ ). Figure 4(b) is a plot of normalized averaged square control as a function of  $(x_1^2)_N$ . For constant  $(x_1^2)_N$ , as would be expected, the case where  $u^2$  is penalized  $r_1$  results in a controller which requires less  $(u^2)_N$  than the other two cases. Similarly in figure 4(c), a plot of  $(u^2)_N$  against  $(x_1^2)_N$ , the  $r_2$  curve falls below the  $r_1$  or  $q_2$  curves. Once the physical limits of control and control rate are known (e.g., for the inlet, bypass door area, and bypass actuator power output) figures

4(b) and (c) can be used in selecting candidate controllers which would not require these physical limits to be exceeded.

Figure 4 can be used in the following manner to assist in coming up with a controller design that minimizes a combination of  $\overline{x_1^2}$  and  $\overline{\dot{x}_1^2}$  while not causing variables  $u^2$  and  $\dot{u}^2$  to exceed limits. First, check to see whether the limits on  $\overline{x_1^2}$ ,  $\overline{u^2}$ , and  $\overline{\dot{u}^2}$  are violated for the case  $q_2 = r_1 = r_2 = 0$ . If the limits are not violated, then this controller is acceptable, being the one of the assumed structure that minimizes  $\overline{x_1^2}$  without regard to variables  $\overline{x_1^2}$ ,  $\overline{u^2}$ , and  $\overline{\dot{u}^2}$ . If one or more of the variables do exceed limits, determine the  $q_2$ ,  $r_1$ , and  $r_2$  values at which the limits are reached ( $(q_2)_l$ ,  $(r_1)_l$ , and  $(r_2)_l$ ), using figures 4(a), (b), and (c), respectively. Finally go back to the computer and conduct a trial-and-error design using penalty combinations in the ranges  $(q_2)_l \leq q_2 \leq \infty$ ,  $(r_1)_l \leq r_1 \leq \infty$ ,  $(r_2)_l \leq r_2 \leq \infty$  until a design is found which minimizes  $\overline{x_1^2}$  and does not exceed the limits.

The results displayed in figure 4 can be examined in a conventional manner by displaying the proportional and integral gains of the reference case controller for various values of  $(\overline{x_1^2})_N$ . This is done in figure 5. It can be noted that for constant  $(\overline{x_1^2})_N$ , proportional gain  $b_1/b_{21}$  is larger for the  $r_1$  case than for the  $r_2$  case; but integral gain  $b_1$  is higher for the  $r_2$  case than for the  $r_1$  case. That is, proportional gain most directly affects  $\overline{u^2}$ .

Three particular designs are compared on a normalized magnitude frequency response basis in figure 6. Each has a different type of penalty ( $q_2$ ,  $r_1$ , or  $r_2$ ) but all are for a value of  $(\overline{x_1^2})_N = 0.4$ . The frequency responses displayed are for  $G_1(j\omega)G_2(j\omega)$  (open-loop response of  $x_1$  to  $d$ ) and  $G_{cl}(j\omega)$  (closed-loop response of  $x_1$  to  $d$ ). It can be seen that the  $r_1$  design behaves more like a proportional controller in that it attenuates the disturbance similarly at low and high frequency, while the  $r_2$  design acts like an integral controller since it attenuates the disturbance more strongly at low frequencies (less than 10 Hz) than at high. The case where  $\overline{\dot{x}_1^2}$  is penalized in a compromise between the other two designs.

### Effect of Disturbance Pole Location on Averaged Square Quantities

The effect of having a disturbance which has a pole value a larger than or less than that of the reference case is shown in figure 7. For simplicity, comparisons are made only for the  $q_2$  cases with averaging time  $T = 1$  second. It can be seen that, since the initial value of  $d(t)$  is the same for all three cases, the open-loop value of  $\overline{x_1^2}$  is largest

for the smallest value of  $a$ . Also the case with the largest value of  $d(t)$  ( $a = 400$ ) has the largest value of  $\overline{\dot{x}_1^2}$ , as would be expected.

### Effect of Adding Derivative Control Action and of Using Inner-Loop

#### Controller on Averaged Square Quantities

In an attempt to improve upon the single-outer-loop PI control, two additional configurations were investigated. The first was the addition of derivative action to the outer-loop PI controller. The transfer function for this controller has the form  $\frac{Kb_1(s, b_{21} + 1)(s, b_{22} + 1)}{s(s/5000 + 1)}$ . The second was an inner-loop PI control, which uses signal  $x_2$ , the diffuser exit static pressure. One reason for considering use of  $x_2$  instead of  $x_1$  is that  $x_2$  is nearer than  $x_1$  to the point at which the disturbance enters. Thus it might be expected that such a controller could better respond to a diffuser exit disturbance. In figure 8 both of these controllers are compared with the outer-loop PI controller for  $q_2$  designs of the reference case. The outer-loop PI and PID controllers exhibit very similar  $(\overline{\dot{x}_1^2})_N$  against  $(\overline{x_1^2})_N$  characteristics. Figure 8(a) shows the PID controller is able to reduce  $(\overline{\dot{x}_1^2})_N$  over that of the PI controller only at large values of  $(\overline{x_1^2})_N$ . The inner-loop PI control is not as effective in reducing  $(\overline{\dot{x}_1^2})_N$  as either of the other controllers, except at low values of  $(\overline{\dot{x}_1^2})_N$ . Although there is less phase lag in the loop between  $x_2$  and  $d$  than between  $x_1$  and  $d$  (see table I); there is also less gain. Consequently, the signal-to-noise ratio at the input to the inner-loop controller ( $H_2(s)$ ) is less than for the outer-loop controller. It is believed that this poorer signal-to-noise ratio accounts for the ineffectiveness of the inner-loop control. It can be noted in figures 8(b) and (c) that  $(\overline{u^2})_N$  and  $(\overline{\dot{u}^2})_N$  are essentially identical for the outer-loop PI and PID controllers. Thus, the added complexity of the PID controller hardly seems warranted. Also, the inner-loop PI controller has poorer performance in terms of  $\overline{x_1^2}$  and  $\overline{\dot{x}_1^2}$ , and also in terms of  $(\overline{u^2})_N$  or  $(\overline{\dot{u}^2})_N$ .

### SUMMARY OF RESULTS

This report has demonstrated the use of parameter optimization techniques in the design of controllers for a supersonic inlet. The basic problem formulation allows the

disturbance to be described as a deterministic signal but includes measurements which are corrupted with random noise. The controller design problem was set up as a parameter optimization problem in the time domain but was solved in the frequency domain. A modified conjugate gradient algorithm was used to compute the optimum controller parameters. Control effectiveness was evaluated in terms of average square values of output, output rate, control, and control rate, and also in terms of frequency responses. In applying the method to the inlet, it was found that, of the controllers investigated, proportional-plus-integral (PI) control using throat exit static pressure feedback was most effective. A proportional-plus-integral plus-derivative (PID) controller showed only marginal improvement over the PI control. A PI controller using diffuser exit static pressure was inferior due to signal-to-noise problems. Also investigated was the effect of disturbance dynamic characteristics on controller performance.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, July 2, 1974,  
501-24.

## APPENDIX A

### SYMBOLS

A	disturbance amplitude, $\text{cm}^2$
a	disturbance pole, rad/sec
$\underline{b}$	controller parameter vector, $m_p \times 1$
$b_1$	controller gain (rad/sec) <sup>-<math>m_1</math></sup>
$b_{2j}$	controller poles or zeroes, rad/sec
$b_{3j}$	controller damping ratio
$b_{4j}$	controller natural frequency, rad/sec
C	total cost function
$C_d$	cost function due to deterministic disturbance
$C_{d-\infty}$	cost function due to deterministic disturbance with upper limit set equal to $\infty$
$C_v$	cost function due to measurement noise
$D_q$	intermediate variable in cost function
$D_r$	intermediate variable in cost function
d	disturbance, $\text{cm}^2$
F	cost function integrand or summand
$G_a$	actuator transfer function, inlet bypass door actuator, $\text{cm}^2/\text{V}$
$G_{cl}$	plant closed-loop transfer function, inlet throat exit static pressure to diffuser exit area disturbance, $(\text{N}/\text{cm}^2)/\text{cm}^2$
$G_1$	plant transfer function, inlet diffuser exit pressure to diffuser exit disturbance, $\text{N}/\text{cm}^2/\text{cm}^2$
$G_2$	plant transfer function, inlet throat exit static pressure to diffuser exit static pressure, $\text{N}/\text{cm}^2/\text{N}/\text{cm}^2$
$H_1$	general controller transfer function
$H_1$	outer-loop controller transfer function, $\text{V}/\text{N}/\text{cm}^2$
$H_2$	inner-loop controller transfer function, $\text{V}/\text{N}/\text{cm}^2$
j	integer, in eq. (27)
$j\omega$	Fourier transform variable, rad/sec

K	measurement gain, pressure transducer gain for inlet, $V/N/cm^2$
$m_p$	integer, number of elements in $\underline{b}$ vector
$m_1$	integer exponent of free $s$ 's in controller transfer functions
$m_2$	integer, number of first-order controller factors
$m_3$	integer, number of second-order controller factors
$N_d$	integer, number of frequency points in numerical integration
$P_{57}$	throat exit static pressure, $N/cm^2$
$P_{87}$	diffuser exit static pressure, $N/cm^2$
$\underline{p}$	transformed parameter vector, $m_p \times 1$
$p_{2j}$	integer, $\pm 1$
$p_{3j}$	integer, $\pm 1$
$q_1$	penalty on output in cost function C
$q_2$	penalty on output rate in cost function C
$r_1$	penalty on control in cost function C
$r_2$	penalty on control rate in cost function C
$s$	Laplace variable, $sec^{-1}$
T	deterministic disturbance averaging time, sec
t	time, sec
u	actuator output, inlet bypass door area, $cm^2$
$u_{bp}$	bypass door area, $cm^2$
$u_d$	component of u due to disturbance
$u_v$	component of u due to measurement noise
$V_q$	intermediate variable in cost function
$V_r$	intermediate variable in cost function
$v_1$	outer-loop measurement noise, noise on throat exit static pressure measurement, $N/cm^2$
$v_2$	inner-loop measurement noise, noise on diffuser exit static pressure measurement, $N/cm^2$
$x_1$	plant output, inlet throat exit static pressure, $N/cm^2$
$x_2$	plant inner-loop variable, inlet diffuser exit static pressure, $N/cm^2$

- $x_{1d}$  component of  $x_1$  due to disturbance
- $x_{1v}$  component of  $x_1$  due to measurement noise
- $y$  dummy variable
- $z$  dummy variable
- $A$  diagonal scaling transformation matrix,  $m_p \times m_p$
- $\lambda$  element of  $A$
- $\tau$  random noise averaging time, sec
- $\Phi_d$  power spectral density of  $d$ ,  $(\text{cm}^2)^2/\text{Hz}$
- $\Phi_u$  power spectral density of  $u$ ,  $(\text{cm}^2)^2/\text{rad}/\text{sec}$
- $\Phi_v$  power spectral density of  $v_1$  and  $v_2$ ,  $(\text{N}/\text{cm}^2)^2/\text{Hz}$
- $\Phi_{x1}$  power spectral density of  $x_{1v}$ ,  $(\text{N}/\text{cm}^2)^2/\text{Hz}$
- $\Phi_y$  power spectral density of  $y$
- $\Phi_z$  power spectral density of  $z$
- $\omega$  frequency, rad/sec
- $\omega_i$  frequency at  $i^{\text{th}}$  data point, rad/sec

Subscripts:

- $( )_N$  averaged square value of, normalized to averaged square value for reference case with  $r_1 = r_2 = q_2 = 0$ ,  $a = 40$ , and  $T = 1$
- $( )_l$  value at which limit is reached

Superscripts:

- $T$  matrix transpose
- $-1$  matrix inverse
- $\text{—}$  averaged value of
- $\cdot$  derivative with respect to time
- $*$  complex conjugate

## APPENDIX B

### COMPUTER PROGRAMS

This appendix describes the FORTRAN IV computer programs which mechanize the controller parameter optimization. The package consists of a main program, two sub-routines, and a block data subprogram, written for an IBM 360/67 TSS time sharing computer. The subroutine CCFM conducts the conjugate gradient search. The subroutine CALFG computes the cost function and gradient.

#### Dimensions

The programs are dimensioned for a maximum number of frequency points  $N_d$  of 25 and a maximum number of controller parameters  $m_p$  of 10. The vectors dimensioned  $N_d$  are AD, A1, A12, DD, DQ, DR, GA, G1G2, HZ, P1, P12, PD, V, VQ, and VR. The vectors dimensioned  $m_p$  are B, G, GS, ID, and Z. Vector H has dimension  $2m_p$  and vector W has dimension  $N_d + 1$ .

#### Main Program

The main program handles data and performs preliminary calculations. The computer variables in the main program are defined in the Main Program Variable List. A flow chart for the main program is presented in figure 9. The following is a FORTRAN listing for the main program:

#### Main Program Listing

```
C MAIN PROGRAM FOR CONTROLLER PARAMETER OPTIMIZATION
  COMPLEX G1(25),G1G2(25),GA(25)
  DIMENSION A1(25),A12(25),AD(25),P1(25),P12(25),PD(25),DD(25)
  DIMENSION HZ(25),W(26),V(25),VQ(25),VR(25),DQ(25),DR(25)
  DIMENSION B(10),G(10),ID(10),H(20)
  COMMON/CALC/FDQ,FVQ,FDR,FVR,G1,G1G2,GA,W,VQ,VR,DQ,DR,IO,-
1 K1,K2,NH1,NH2,INST,NDATA,KNT,GN
  COMMON/FMC/KOUNT,KG
  COMMON/BLOCK/HZ,A1,A12,AD,P1,P12,PD,V,A,AA,T,GA1,GA12,GAD,GK,HD
  LOGICAL KG
  EXTERNAL CALFG
  NAMELIST /NAM3/A,AA,A1,A12,AD,HZ,GA1,GA12,GAD,GK,KPR,-
  1ND,P1,P12,PD,T,V
  NAMELIST /NAM2/K1,K2,NH1,NH2,B,IO,LIMIT,O1,O2,R1,O2
```



```

C PRINT PROGRAM HEADING FOR USER REFERENCE
WRITE(5,961)
961 FORMAT(' NAM3=(A,AA,A1,A12,AD,Hz,GA1,GA12,GAD,GK,-
1KPR,ND,P1,P12,PD,T,V)',/-
2,' NAM2=(K1,K2,NH1,NH2,R,LD,LIMIT,C1,C2,R1,R2)',/-
3,' ID=(1=Z;2=G;3=P;4,5=CZD,CZM;6,7=CPD,CPM)',/-
4,' INST=(1=SEARCH,2=NAM2,3=NAM3,4=PRINT)',/-
5,' IER=(0=CONV,1=NOT CONV,2=ERROR)')
C PROMPT, READ AND PRINT NAM3
907 WRITE(6,930)
931 FORMAT(' NAM3?')
READ(5,NAM3)
IF(KPR.EQ.1) WRITE(8,NAM3)
C CONVERSIONS FROM EPSO RESP TO COMPLEX NOS
NDATA=ND
GN=GA12
DO 110 I=1,NDATA
G1(I)=GA1*A1(I)*CEXP(CMPLX(0.,P1(I)*.0174533))
G1G2(I)=GA12*A12(I)*CEXP(CMPLX(0.,P12(I)*.0174533))
W(I)=HZ(I)*6.2831854
DD(I)=AA**2/(T*(W(I)**2+A**2))
111 GA(I)=GX*GAD*AD(I)*CEXP(CMPLX(0.,PD(I)*.0174533))
W(NDATA+1)=W(NDATA)
C PROMPT, READ AND PRINT NAM2
51 WRITE(6,340)
340 FORMAT(' NAM2?')
READ(5,NAM2)
N=NH1*NH2
WRITE(8,220) K1,K2,NH1,NH2,LIMIT,C1,C2,R1,R2,(ID(J),J=1,N)
220 FORMAT(' K1,K2,NH1,NH2,LIMIT=',5I4,/,
1' C1,C2,R1,R2=',1P4E10.2,' ID=',20I3)
C COMPUTE VARIABLES IN COST WHICH ARE NOT FUNCTIONS OF B
DO 210 I=1,ND
WW=W(I)**2
GASQ=REAL(GA(I)+CONJG(GA(I)))
VQ(I)=V(I)*(Q1+WW*Q2)*GASQ
VR(I)=V(I)*(R1+WW*R2)*GASQ/REAL(G1G2(I)+CONJG(G1G2(I)))
DQ(I)=DD(I)*(Q1+WW*Q2)
210 DR(I)=DD(I)*(R1+WW*R2)*GASQ
GO TO 87
C PROMPT AND READ INST
777 WRITE(6,250)
250 FORMAT(' INST?')
READ(5,42) INST
42 FORMAT(I1)
GO TO (28,51,900,35),INST
C PRINT FREQUENCY RESPONSE HEADING
35 WRITE(8,41)
41 FORMAT(6X,'HZ',12X,'/Y/',7X,'DEG',11X'REAL IMAG')
23 SIZE=.1
EPS=1.E-5
KNT=0
ITER=LIMIT*(1-INST/4)
CALL CGFN(CALFG,N,B,F,G,SIZE,EPS,ITER,IER,H)
C PRINT SEARCH RESULTS
WRITE(8,230) F,IER,KNT,KOUNT,SIZE,FDQ,FDR,FVO,FVR
230 FORMAT(' F=',1P16.2,' IER=',I2,' KNT,KOUNT,SIZE=',
1 2I5,E10.3,/, ' FDO,FDR,FVQ,FVR=',1P4E10.3)

```

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```

87 WRITE(P,151) (R(I),I=1,")
151 FORMAT(' R=',1P20F11.4)
GO TO 777
END

```

### Main Program Variable List

A disturbance time constant  $a$  (input variable)

AA disturbance pulse amplitude (input variable)

AD  $G_d(j\omega)$  normalized, vector (input variable)

A1  $G_1(j\omega)$  normalized, vector (input variable)

A12  $|G_1(j\omega) \cdot G_2(j\omega)|$  normalized, vector (input variable)

B controller parameter  $b$ , vector (input variable). When inputting, those in outer-loop controller transfer function  $H_1$  must precede those in inner-loop transfer function  $H_2$ . Damping ratios must precede their natural frequency terms.

CALFG subroutine (declared external in main program) which computes the cost function and gradient

DD  $|d(j\omega)|^2$ , vector

DQ  $D_q(\omega)$ , vector

DR  $D_r(\omega)$ , vector

EPS parameter change defining search convergence, for example,  $10^{-5}$

F  $F = C = FDQ + FDR + FVQ + FVR$ , cost function

FDQ costs of averaged square output and output rate due to disturbance

FDR costs of averaged square control and control rate due to disturbance

FVQ costs of averaged square output and output rate due to measurement noise

FVR costs of averaged square control and control rate due to measurement noise

G cost function (scaled) gradient, vector

GA  $G_a(j\omega)$ , vector

GAD gain normalizing AD (input variable)

GA1 gain normalizing A1 (input variable)

GA12 gain normalizing A12 (input variable)

GK transducer gain  $K$  (input variable)

GN        GN = GA12  
 G1         $G_1(j\omega)$ , vector  
 G1G2       $G_1(j\omega) \cdot G_2(j\omega)$ , vector  
 H        storage, vector  
 HZ        frequency in hertz, vector (input variable)  
 I        index of element in vector, integer  
 ID        integer vector which identifies corresponding parameter in B as to type  
           (input variable). 1 = zero, 2 = gain, 3 = pole, 4 = complex zero damping,  
           5 = complex zero natural frequency, 6 = complex pole damping, 7 = com-  
           plex pole natural frequency.  
 IER       search convergence parameter. 0 = convergence to EPS in LIMIT; 1 = con-  
           vergence to EPS in LIMIT not obtained; 2 = probable error occurred.  
 INST      branching instruction parameter (input variable). 1 = search for optimum;  
           2 = return to NAM2 namelist; 3 = return to NAM3 namelist; 4 = print fre-  
           quency, system closed-loop frequency response (normalized by GN), and  
           system Nyquist plot.  
 ITER      set equal to LIMIT except for INST = 4 case when ITER = 0.  
 J        index of element in vector  
 KG        logical variable .TRUE. means compute gradient  
 KNT       count of cost function evaluations  
 KOUNT     count of line search iterations  
 KPR       if equal to 1 causes NAM3 variables to be printed (input variable)  
 K1        exponent of free s in H1 (input variable)  
 K2        exponent of free s in H2 (input variable)  
 LIMIT     maximum number of iterations (input variable)  
 N        NH1 + NH2  
 ND        number of frequency points over which integration is performed  $N_d$   
 NDATA     NDATA = ND  
 NH1       number of parameters of B in H1 (input variable)  
 NH2       number of parameters of B in H2 (input variable)  
 PD         $\angle G_a(j\omega)$  in degrees, vector (input variable)

P1  $-G_1(j\omega)$  in degrees, vector (input variable)  
 P12  $-G_1(j\omega) \cdot G_2(j\omega)$  in degrees, vector (input variable)  
 Q1  $q_1$  (input variable)  
 Q2  $q_2$  (input variable)  
 R1  $r_1$  (input variable)  
 R2  $r_2$  (input variable)  
 SIZE parameter step size; for example, set to 0.1 at start of search  
 T T (input variable)  
 V  $\Phi_v(\omega)$ , vector (input variable)  
 VQ  $V_q(\omega)$ , vector  
 VR  $V_r(\omega)$ , vector  
 W radian frequency  $\omega$ , vector  
 WW frequency squared

### Program Input and Output

The program starts by printing a heading referencing the namelist variables and variable codes. Then the program prompts for NAM3 namelist data. The namelist variables are entered according to the FORTRAN rules for namelist data. The NAM3 variables are A, AA, A1, A12, AD, HZ, GA1, GA12, GAD, GK, KPR, ND, P1, P12, PD, T, and V. Since eight of the variables are vectors, the input could be lengthy. Thus, an alternative to entering NAM3 data at run time is to use the block data subprogram. The following is a listing of the subprogram for the inlet investigated in the report:

```

BLOCK DATA
COMMON/BLOCK/HZ,A1,A12,AD,P1,P12,PD,V,A,AA,T,GA1,GA12,GAD,GK,ND
REAL HZ(25)/.001,1.,3.,7.,10.,15.,20.,25.,30.,40.,50.,60.,70.,-
1 80.,90.,100.,110.,120.,130.,140.,150.,300.,3*600./
REAL A1(25)/1.,1.,1.,.997,.927,.818,.732,.643,.565,.411,.359,.326,-
1 .318,.387,.465,.486,.46,.407,.326,.303,.209,.042,3*.0093/
REAL A12(25)/1.,1.,.95,.774,.607,.671,.628,.602,.53,.437,.347,.313,-
1 .334,.352,.318,.322,.299,.288,.27,.26,.237,.012,3*.0013/
REAL AD(25)/1.,1.,.999,1.,.996,.993,.997,.996,1.,1.012,1.016,1.012,-
1 1.01,1.011,1.005,1.024,.954,.908,.782,.689,.609,.051,3*.0063/
REAL P1(25)/0.,-1.,-8.,-18.,-26.,-39.,-46.,-51.,-58.,-63.,-62.,-55.,-
1 -49.,-55.,-65.,-74.,-84.,-110.,-125.,-115.,-98.,-174.,3*-178./
REAL P12(25)/0.,-2.,-13.,-24.,-31.,-35.,-44.,-57.,-62.,-75.,-87.,-90.,-
1 -92.,-105.,-110.,-128.,-145.,-156.,-166.,-172.,-177.,-251.,3*-266./
  
```

```

REAL PD(25)/0.,-2.,-5.,-10.,-13.,-18.,-23.,-27.,-32.,-42.,-51.,-59.,-
1,-68.,-77.,-85.,-90.,-116.,-128.,-146.,-160.,-173.,-226.,3*-250./
REAL V(25)/.24E-5,4*2.4E-5,4.E-5,2*5.7E-5,7.E-5,6.E-5,4.4E-5,-
1 7.E-5,5.4E-5,4*4.E-5,3*3.3E-5,4.4E-5,7.E-5,3*4.E-5/
DATA T,3,AA/1.,40.,84./
DATA HD,GAI,GAI2,GAD,GK/23.,.01E.,.02502,36.9.,.762/
END

```

The NAM3 variables are printed if KPR is set to one. Then the program prompts for NAM2 namelist data. The NAM2 variables are B, ID, K1, K2, LIMIT, NH1, NH2, Q1, Q2, R1, and R2. Then the NAM2 variables are printed and a prompt for the INST variable is printed.

The INST variable is entered in I1 format. The INST code values are 1, 2, 3, and 4. Making INST = 1 causes a search for the optimum parameters. After the search results are printed, another INST prompt is issued. Making INST = 2 returns the program to request NAM2. Making INST = 3 returns the program to request NAM3. Making INST = 4 causes the system frequency response to be printed. The frequency is printed under HZ. The system closed-loop frequency response normalized magnitude is printed under /Y/ and phase in degrees under DEG. The real part of the system open-loop transfer function is printed under REAL and the imaginary part under IMAG. Then the zero iteration search results are printed, and another INST prompt is issued.

#### Subroutine CALFG(N, E, F, G)

The purpose of this subroutine is to compute the cost function and its gradient. The CALFG program variables are defined in the CALFG Program Variable List. Those variables in the common blocks and subroutine call are labeled the same as those in the main program and are not repeated again.

The Data statement IN and RSNAT values are variables in the variable step size integration. Smaller step sizes are taken near a system resonance; that is, when the absolute value squared of the denominator of the system closed-loop transfer function  $|1 + \text{OPEN}|^2$ , is less than RSNAT. The variable IN is the number of subintervals into which an interval is divided. Upon detecting a resonance, the program fills in extra points starting from the previous frequency by linear interpolation, using data from the present and past frequency points. A flow chart of the integration logic for the subroutine CALFG is given in figure 10.

FORTTRAN Listing of Subroutine CALFG

```

SUBROUTINE CALFG(N, B, F, G)
  COMPLEX Z(10), GA(25), G1(25), G1G2(25)
  COMPLEX Y, CF, CT, H1, H2, S, OPEN, G11, G1G21, GA1, H2G211
  DIMENSION U(26), VR(25), VO(25), DO(25), DR(25)
  DIMENSION GS(10), ID(10), R(1), C(1)
  COMMON/CALC/FDD, FVQ, FDR, FVR, G1, G1G2, GA, U, VO, VR, DO, DR, ID, -
1  K1, K2, NH1, NH2, INST, HDATA, KNT, GN
  COMMON/FNC/KOUNT, KG
  LOGICAL KC, RCNT
  DATA RSNAT, IN/.5, 3/

```

C INITIALIZATIONS

```

      KNT=KNT+1
      RCNT=.TRUE.
      DO 70 J=1, N
70     G(J)=0.
80     H1=CMPLX(0., 0.)
      H2=CMPLX(0., 0.)
      FDD=0.
      FVQ=0.
      FDR=0.
      FVR=0.
      US=U(1)
      IDYN=IN
      R=1./FLOAT(IN)

```

C INDEX I FOR EACH OPEN LOOP DATA POINT; PROVIDE  
C FOR INTERPOLATION BETWEEN POINTS IN RESONANCE

```

      DO 200 I=1, HDATA

```

```

90     IDX=IDXN
100    RI=FLOAT(IDX)*R
      RIM=1.-RI
      HI=U(I-1)*RIM+W(I)*PI
      S=CMPLX(0., HI)

```

C BUILD CONTROLS H1 AND H2

```

      IF(NH1.GT.0) H1=S**K1
      IF(NH2.GT.0) H2=S**K2
      DO 10 J=1, N
      IDJ=ID(J)
      GO TO (1, 3, 1, 2, 10, 2, 10), IDJ
1      ST=(S/B(J)+1.)**(2-IDJ)
      IF(KG) Z(J)=S/(S+R(J))*(FLOAT(IDJ)-2.)
      GO TO 8
2      ST=(S/B(J+1))**2+?.*S*R(J)/B(J+1)+1.
      IF(.NOT.KC) GO TO 9R
      Z(J)=?.*S*R(J)/(B(J+1)*ST)*(5.-FLOAT(IDJ))
      Z(J+1)=-Z(J)*(1.+S/(B(J)+R(J+1)))
88     ST=ST**(5-IDJ)
      GO TO 9
3      ST=B(J)
      Z(J)=CMPLX(1., 0.)
8      IF(J.LE.NH1) H1=H1*ST
      IF(J.GT.NH1) H2=H2*ST
10     CONTINUE

```

C CHECK FOR RESONANCE; COMPUTE SYSTEM COSTS

```

      GA1=GA(I-1)*RIM+GA(I)*RI
      G11=G1(I-1)*RIM+G1(I)*RI
      G1G21=G1G2(I-1)*RIM+G1G2(I)*RI

```

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```

OPEN=GAI*(G11*H2+G1G21*H1)
IF(IDX.LT.IN) GO TO 110
IF(REAL((1.+OPEN)*CONJG(1.+OPEN)).LT.RSNAT) GO TO 120
MPI=W(I+1)
IDYN=IN
RSNT=.FALSE.
GO TO 140
110 WPI=(W(I)*FLOAT(IDX+1)+W(I-1)*FLOAT(IN-IDX-1))*R
GO TO 140
120 IDYN=1
IF(RSNT) GO TO 130
IF(I.GT.2) WSS=W(I-2)
IF(I.EQ.2) WSS=W(1)
CRCT=-1.+((W(I)+FLOAT(IN-I)*W(I-1))*R-WSS)/(W(I)-WSS)
FDO=FDO+CRCT*DOH
FVQ=FVQ+CRCT*VOH*HMAG
FDR=FDR+CRCT*DRH*HM
FVR=FVR+CRCT*VRH*HMAG
DO 125 J=1,N
125 G(J)=G(J)+CRCT*GS(J)
RSNT=.TRUE.
GO TO 90
130 WPI=W(I)*(1.-R)+W(I+1)*R
140 Y=G1G21/(1.+OPEN)
YY=REAL(Y*CONJG(Y))*(MPI-MS)
H2G211=(G11*H2)/G1G21+H1
DOH=(RI*DO(I)+RIM*DO(I-1))*YY
VOH=(RI*VO(I)+RIM*VO(I-1))*YY
VRH=(RI*VR(I)+RIM*VR(I-1))*YY
DRH=(RI*DR(I)+RIM*DR(I-1))*YY
HMAG=REAL(H1*CONJG(H1)+H2*CONJG(H2))
HM=REAL(H2G211*CONJG(H2G211))
FDO=FDO+DOH
FDR=FDR+DRH*HM
FVQ=FVQ+VOH*HMAG
FVR=FVR+VRH*HMAG
IF(.NOT.KG) GO TO 180
C COMPUTE GRADIENT
ST=(DOH*HMAG*(VOH+VRH)+DRH*HM)*CONJG(-Y*GAI)+DRH*H2G211
DO 160 J=1,N
IF(J.EQ.1) GF=(ST+(VOH+VRH)*H1)*CONJG(H1)
IF(J.EQ.NH1+1) GF=(ST*CONJG(G11/G1G21)+(VOH+VRH)*H2)*CONJG(H2)
GS(J)=REAL(ST*CONJG(Z(J)))
160 G(J)=G(J)+GS(J)
180 IF(INST.LT.4) GO TO 190
C PRINT SYSTEM FREQUENCY RESPONSE AND OPEN LOOP
HZ=WI/6.2831854
YMAG=CABS(Y/GN)
DEG=ATAN2(AIMAG(Y),REAL(Y))*57.29578
WRITE(8,300) HZ,YMAG,DEG,OPEN
190 IDX=IDX+1
MS=MI
IF(IDX.LE.IN) GO TO 100
200 CONTINUE
F=FDO+FVQ+FDR+FVR
RETURN
300 FORMAT(1PE10.3,2(5X,2E10.3))
END

```

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If the user desired to use a search routine other than subroutine CGFM, the scaled gradient  $\Delta \nabla C(b)$  may not be desired. In such a case the unscaled gradient may be obtained by dividing each term in the gradient vector by its corresponding parameter in the  $\underline{b}$  vector.

### CALFG Program Variable List

DEG	$\angle G_{cl}(j\omega)$ in degrees
DQW	partial product
DRW	partial product
GF	gradient of F
GS	saved gradient partial sum, vector
G1I	$G_1(j\omega)$ value
G1G2I	$G_1(j\omega) \cdot G_2(j\omega)$ value
HM	temporary value
HMAG	temporary value
H2G2II	temporary value $H_2/G_2 + H_1$
HZ	frequency value
H1	$H_1$
H2	$H_2$
I	frequency index
IDJ	ID(J) value
IDX	counter for data insertions
IDXN	counter value for logic
IN	number of subintervals inserted into an interval near a system resonance; for example, IN = 3 means each interval is divided into thirds
J	parameter index
OPEN	$G_a \cdot G_1(H_2 + G_2H_1)$
R, RI, RIM	temporary values
RSNAT	condition defining existence of system resonance for $ 1 + \text{OPEN} ^2 < \text{RSNAT}$ , that is, for RSNAT = 0.5. The closed-loop frequency response magnitude is $> 1.42 = (1/\sqrt{0.5})$ times its open-loop value.





FORTRAN Listing of Subroutine CGFM

```

SUBROUTINE CGFM(CALEC,N,B,F,G,SIZE,EPS,ITER,IER,H)
DIMENSION B(1),H(1),G(1)
LOGICAL KG
COMMON/EMC/KOUNT,KG
C INITIALIZATIONS
KOUNT=0
STEP=2.
IEP=-5
5 BETA=0.
  NCYC=0
15 K=0
  DO 20 J=1,N
20 H(J)=B(J)
  KG=.TRUE.
  CALL CALEC(N,B,F,G)
C TEST FOR STOPPING SEARCH
  IF(KOUNT.GE.ITER) IER=1
  IF(SIZE.LT.EPS) IER=0
40 IF(IER.GT.-2) RETURN
C COMPUTE PAST GRADIENT WEIGHTING
  TSOR=0.
  DO 50 J=1,N
50 TSOR=TSOR+G(J)**2
  IF(NCYC.EQ.0) GO TO 60
  BETA=TSOR/TSAVE
60 SCALE=0.
  DO 70 J=1,N
  JN=J+N
  H(JN)=-G(J)+BETA*H(JN)
70 SCALE=SCALE+ABS(H(JN))
  IF(SCALE.GT.0.) GO TO 80
  IEP=0
  GO TO 40
80 SCALE=SIZE/SCALE
  TSAVE=TSOR
  NCYC=NCYC+1
  FSS=F
  L=1
C UPDATE B'S
100 DO 110 J=1,N
  JN=J+N
110 B(J)=H(J)*ABS(1.+SCALE*H(JN))
  FS=F
  KG=.FALSE.
  CALL CALEC(N,B,F,G)
C LOGIC CHANGES LINE SEARCH STEP SIZE OR CONCLUDES SEARCH
  IF(K.GT.0) GO TO 120
  IF(F.LT.FS) GO TO 130
  IF(L.GT.1) GO TO 140
  GO TO 150
120 IF(F.LT.FSS) GO TO 140
  GO TO 160
130 SCASV = SCALE
  L = L+1
  SCALE = SCALE*STEP

```

```

      FSS=FS
      IF(L.LT.15) GO TO 100
      IER=2
      GO TO 40
C   FIT QUADRATIC CURVE TO 3 PTS. BRACKETING LINE SEARCH MIN.
140  DO 148 J=1,N
      JN=J+N
      R1=H(J)
      R2=H(J)*ABS(1.+SCASV*H(JN))
      R3=B(J)
      IF(L.GT.3) R1=R1+(R2-R1)/STEP
      X1=(FSS-FS)*(R1-R3)
      X2=(FSS-F)*(R2-R1)
      IF(ABS(R2-R3).GT.EPS/4.) GO TO 147
      IF(L.GT.1) B(J)=R2
      GO TO 148
147  B(J)=(X1*(R1+R3)+X2*(R1+R2))/((X1+X2)*2.)
148  IF(B(J)*H(J).LE.0.) B(J)=-.1*B(J)+EPS*R3
C   UPDATE SEARCH VARIABLES
      SIZE=SIZE*(FLOAT(L)+2.)/4.
      KOUNT=KOUNT+1
      IF(NCYC.GT.N) GO TO 5
      GO TO 15
150  SCASV= SCALE
      SCALE= SCALE/(1.+STEP)
      K= K+1
      SIZE= SIZE/(1.+STEP)
      GO TO 100
160  SIZE= SIZE/(1.+STEP)
      DO 180 J=1,N
180  B(J)=H(J)
      GO TO 5
      END

```

### CGFM Program Variable List

BETA	conjugate direction weighting
FS	saved F
FSS	saved FS
J	parameter index
JN	J + N
K	indicator for step size reductions
L	number step size increases within iteration
NCYC	number of iterations before restarting conjugate search
R1,R2,R3	terms in quadratic curve fit
SCALE	step size scale factor

SCASV	saved SCALE
STEP	step size
TSAVE	saved TSQR
TSQR	squared gradient terms sum_____
X1,X2	partial product

APPENDIX C

SAMPLE PROBLEM

A sample problem, using the inlet data from table I, is presented. The following is a computer terminal printout of the problem solution:

```

1  NAM3=(A,AA,A1,A12,AD,HZ,GA1,GA12,GAD,GK,KPR,ND,P1,P12,PD,T,V)
2  NAM2=(K1,K2,NH1,NH2,B,LD,LIMIT,O1,O2,P1,R2)
3  ID=(1=Z;2=G;3=P;4,5=CZD,CZW;6,7=CPD,CPW)
4  INST=(1=SEARCH,2=NAM2,3=NAM3,4=PRINT)
5  IFR=(0=CONV,1=NOT CONV,2=ERROR)
6  NAM3?
7*  &nam3 &end
8  NAM2?
9*  &nam2 k1=-1,nh1=2,b=10,10000,ld=2,1,limit=50,
10*  q1=1,q2=.0001,r1=0,r2=0,k2=0,nh2=0 &end
11  K1,K2,NH1,NH2,LIMIT= -1 0 2 0 50
12  O1,O2,R1,R2= 1.00E 00 1.00E-04 0.00 0.00 10= 2 1
13  B= 1.0000E 01 1.0000E 04
14  INST?
15*  1
16  F= 3.11901510E-01 IFR= 0 KNT,KOUNT,SIZE= 102 24 1.301E-06
17  FDO,FDR,FVQ,FVP= 2.884E-01 0.000 2.352E-02 0.000
18  B= 4.7216E 01 1.3056E 02
19  INST?
20*  4
21  HZ /Y/ DEG REAL IMAG
22  1.000E-03 1.931E-04 8.999E 01 2.629E-01-5.462E 03
23  1.000E 00 1.906E-01 7.884E 01 -1.189E-01-5.467E 00
24  3.000E 00 5.111E-01 5.477E 01 -2.067E-01-1.720E 00
25  7.000E 00 7.489E-01 1.248E 01 -1.690E-01-6.145E-01
26  1.000E 01 7.295E-01-6.281E 00 -1.321E-01-3.095E-01
27  1.500E 01 7.023E-01-1.759E 01 -2.836E-02-2.859E-01
28  2.000E 01 6.732E-01-3.047E 01 -9.309E-02-2.183E-01
29  2.500E 01 6.671E-01-4.611E 01 -1.138E-01-1.704E-01
30  3.000E 01 5.864E-01-5.359E 01 -1.060E-01-1.322E-01
31  4.000E 01 4.974E-01-7.013E 01 -1.066E-01-7.616E-02
32  5.000E 01 3.830E-01-9.489E 01 -2.464E-02-3.339E-02
33  6.000E 01 3.425E-01-8.886E 01 -8.622E-02-1.817E-02
34  7.000E 01 3.680E-01-9.165E 01 -2.233E-02-5.503E-03
35  8.000E 01 3.878E-01-1.067E 02 -9.264E-02 2.755E-02
36  9.000E 01 3.410E-01-1.222E 02 -6.885E-02 5.180E-02
37  1.000E 02 3.379E-01-1.324E 02 -4.094E-02 7.316E-02
38  1.100E 02 2.075E-01-1.494E 02 2.260E-03 7.627E-02
39  1.200E 02 2.796E-01-1.599E 02 2.819E-02 6.392E-02
40  1.300E 02 2.585E-01-1.679E 02 4.373E-02 3.531E-02
41  1.400E 02 2.488E-01-1.729E 02 4.486E-02 1.594E-02
42  1.500E 02 2.283E-01-1.771E 02 3.927E-02 1.412E-03
43  3.000E 02 1.200E-02 1.000E 02 -9.206E-05-1.383E-04
44  6.000E 02 1.300E-03 0.000E 01 -1.097E-06-9.075E-07
45  F= 3.11901510E-01 IFR= 1 KNT,KOUNT,SIZE= 1 0 1.000E-01
46  FDO,FDR,FVQ,FVP= 2.884E-01 0.000 2.352E-02 0.000
47  B= 4.7216E 01 1.3056E 02
48  INST?

```

\*User input.

Responses in capital letters are program output (43 lines), and lower case letters are user input (5 starred lines). The line numbers to the left of each line were added for discussion purposes. The listing shows the problem of optimizing a two-parameter controller for a given cost function. The controller was a single-loop proportional plus integral controller with the form  $H_2(s) = 0$  and  $H_1(s)/K = b_1(s/b_{21} + 1)/s = (b_1/s) + (b_1/b_{21})$ .

The cost function was  $C = x_1^2 + 0.0001 \dot{x}_1^2$ . It penalizes system output and output rate but not control or control rate.

Lines 1 to 5 were printed by the computer after the user called the program. These lines list the NAM3 and NAM2 variables and parameters ID, INST, and IER. At line 6, the computer prompted for NAM3 namelist. In line 7 the user signed in and out of the namelist without updating any values from the block data subprogram. Since  $KPR \neq 1$ , there is no print of the NAM3 variables. In line 8 the computer prompted for NAM2. In lines 9 and 10 the user supplied NAM2 input. The initial parameter estimates were  $b_1 = 10$  and  $b_{21} = 1000$ . The transducer gain  $K$  is part of the NAM3 input and not input here. In lines 11 to 13, the computer outputted the updated NAM2 values, and in line 14 it prompted for an INST input.

In line 15 the user entered a 1 to search for the optimum controller parameters. Lines 16 to 18 list the search results. The cost function  $F$  was 0.3119.  $IER = 0$  meant that the (scaled) parameters were changing less than  $EPS$  ( $1.E-5$ ) as can be verified noting that  $SIZE$  was  $1.39 E-6$ . The iteration line search count  $KOUNT$  was 24, and the number of cost function evaluations  $KNT$  was 102. The  $q_1$  and  $q_2$  costs due to the disturbance,  $FDQ$ , was 0.2884 and due to the noise,  $FVQ$ , was 0.0235.  $FDR$  and  $FVR$  costs were zero because  $r_1$  and  $r_2$  were zero. The optimum parameters  $b_1 = 47$  and  $b_{21} = 131$  were returned.

At line 19 the computer prompted for another INST input. The user input  $INST = 4$  in line 20, which directed the program to display the system frequency response. Line 21 contains the column headings for the data given in lines 22 to 44. The first frequency was made 0.001 instead of zero (in NAM3) because  $H_1(s)$  would be infinite at zero. A system stability spot check can be performed using the Nyquist criteria and the open-loop REAL and IMAG data by reading the IMAG column until a sign change occurs in which the REAL part is negative (between  $IMAG = -0.0056$  and  $+0.028$ ). A stability check is that the REAL part is greater than -1.0; -0.09 is. A complete test may not always be so simple but for this problem the response simply spirals into zero.

Lines 45 to 47 repeat the list of costs and search parameters, and at line 48 the computer prompted for another INST input.

This example problem is also used to study convergence and parameter scaling. In figure 12 the cost function contours are plotted as functions of  $b_1$  and  $b_{21}$ . The contours form an elongated valley more sensitive to  $b_1$  than to  $b_{21}$  and thus not well scaled away from the minimum. The search trajectory starting at (10, 10 000) is also

shown. The search run in figure 12 took 3.5 seconds central processor unit time on the IBM 360/67 TSS. The same problem was run using a standard conjugate gradient search for two different but constant scaling rules. One rule scaled the parameters by their initial estimates (10, 10 000). The problem ran about the same time, but it converged to a slightly less accurate  $F = 0.3121$  instead of 0.3119. The other rule was to scale the parameters near the solution (40, 130). In this case the search could not advance significantly from the initial point for  $b_{12}$  which remained at 10 000 while  $b_1$  moved to 46.

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TABLE I. - MIXED COMPRESSION INLET AND BYPASS DOOR FREQUENCY RESPONSE DATA AND MEASUREMENT NOISE POWER SPECTRAL DENSITY (PSD)

Frequency, Hz	Inlet frequency response				Bypass door frequency response bypass door area to command voltage, $G_a(j\omega)$		Measurement noise PSD, $\Phi_v(\omega)$ , $\frac{N \text{ cm}^2}{\text{rad sec}^2}$
	Diffuser exit static pressure to bypass door area $G_1(j\omega)$		Throat exit static pressure to bypass door area $G_1(j\omega)G_2(j\omega)$		Magnitude 36.8 $\text{cm}^2 \text{ V}$	Phase, deg	
	Magnitude 0.016 $\frac{N \text{ cm}^2}{\text{cm}^2}$	Phase, deg	Magnitude 0.026 $\frac{N \text{ cm}^2}{\text{cm}^2}$	Phase, deg			
0.001	1	0	1	0	1	0	$0.024 \cdot 10^{-5}$
1	1	-1	1	-2	1	-2	$.24 \cdot 10^{-4}$
3	1	-8	.95	-13	1	-5	$.24 \cdot 10^{-4}$
7	1	-18	.77	-24	1	-10	$.24 \cdot 10^{-4}$
10	.93	-26	.70	-31	1	-13	$.24 \cdot 10^{-4}$
15	.82	-39	.67	-35	.99	-18	$.40 \cdot 10^{-4}$
20	.73	-46	.63	-44	1	-23	$.57 \cdot 10^{-4}$
25	.64	-51	.60	-57	1	-27	$.57 \cdot 10^{-4}$
30	.56	-58	.53	-62	1	-32	$.70 \cdot 10^{-4}$
40	.41	-63	.44	-75	1.01	-42	$.60 \cdot 10^{-4}$
50	.36	-62	.35	-87	1.02	-51	$.44 \cdot 10^{-4}$
60	.33	-55	.31	-90	1.01	-59	$.70 \cdot 10^{-4}$
70	.32	-49	.33	-92	1.01	-68	$.53 \cdot 10^{-4}$
80	.39	-55	.35	-105	1.01	-77	$.40 \cdot 10^{-4}$
90	.46	-65	.32	-119	1.01	-85	$.40 \cdot 10^{-4}$
100	.49	-74	.32	-128	1.02	-96	$.40 \cdot 10^{-4}$
110	.46	-94	.30	-145	.95	-116	$.40 \cdot 10^{-4}$
120	.41	-110	.29	-156	.91	-128	$.33 \cdot 10^{-4}$
130	.33	-125	.27	-166	.78	-146	$.33 \cdot 10^{-4}$
140	.30	-115	.26	-172	.69	-160	$.33 \cdot 10^{-4}$
150	.21	-98	.24	-177	.61	-173	$.44 \cdot 10^{-4}$
300	.042	-174	.012	-251	.051	-226	$.70 \cdot 10^{-4}$
600	.0093	-178	.0013	-266	.0063	-250	$.40 \cdot 10^{-4}$

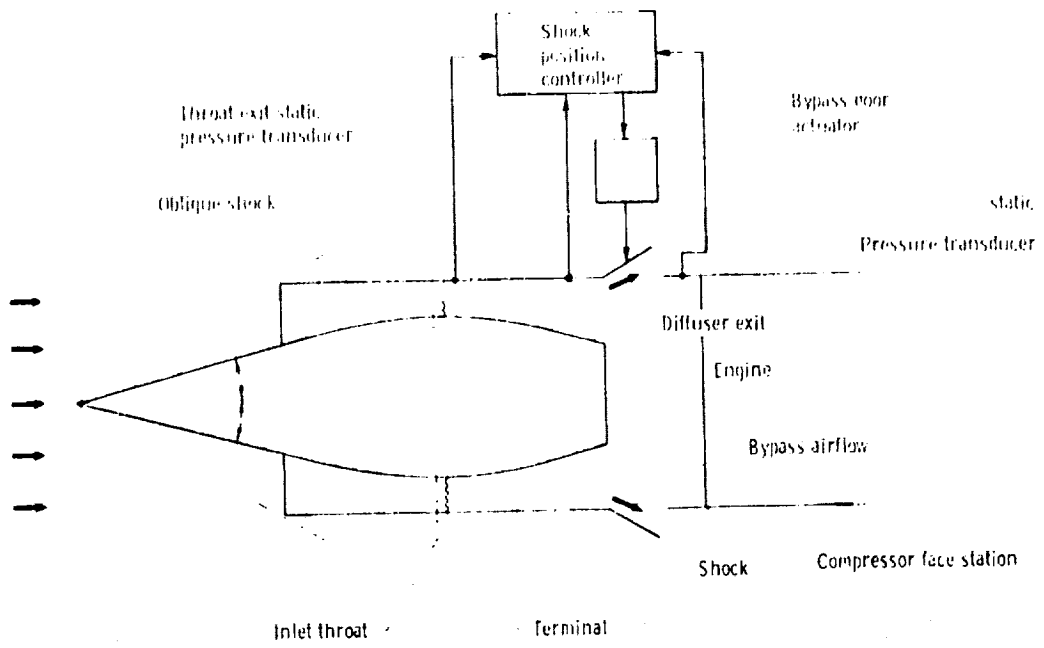


Figure 1. - Schematic of mixed compression inlet with terminal shock position control.

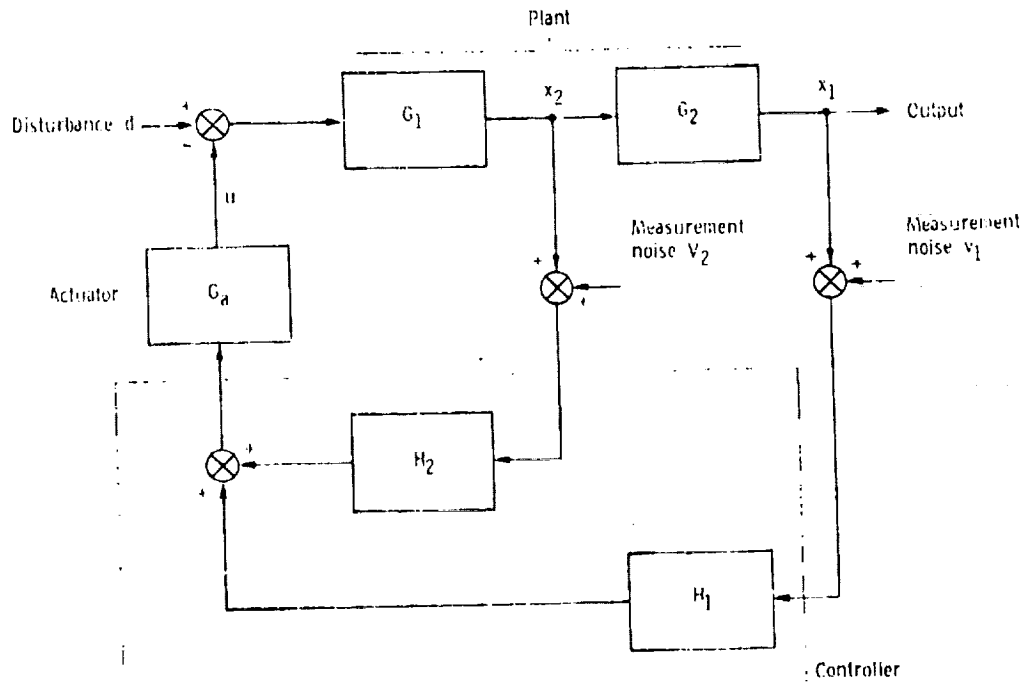


Figure 2. - General control system block diagram.

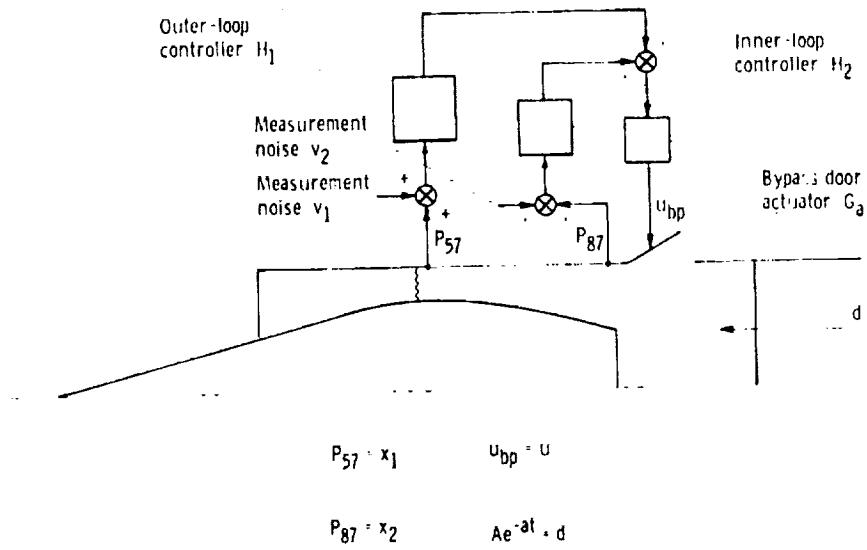
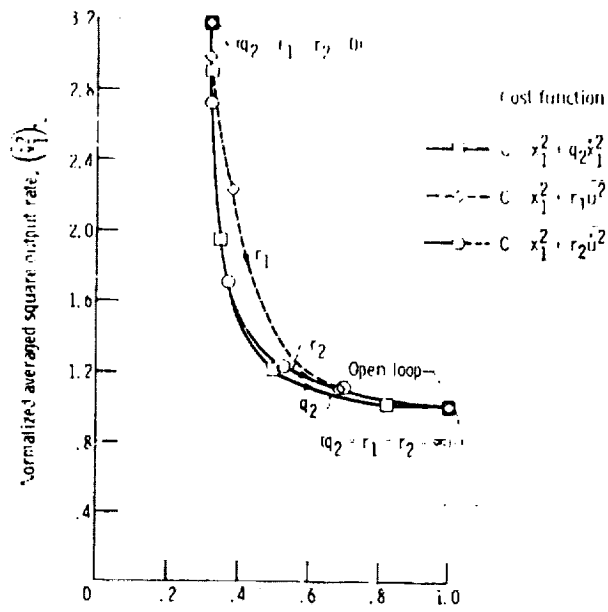
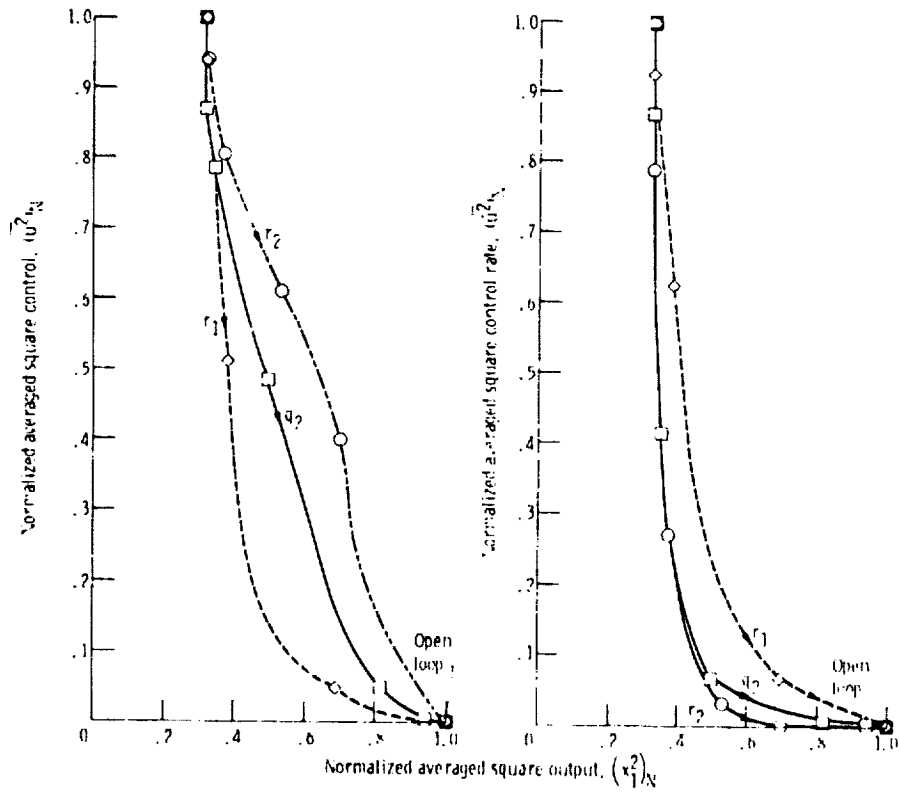


Figure 3. - Inlet control system block diagram.



(a) Output rate as function of output.



(b) Control as function of output.

(c) Control rate as function of output.

Figure 4. Normalized averaged square output rate, control and control rates as functions of normalized averaged square output as  $q_2$ ,  $r_1$ , and  $r_2$  penalties vary from 0 to  $r_2$  reference case.

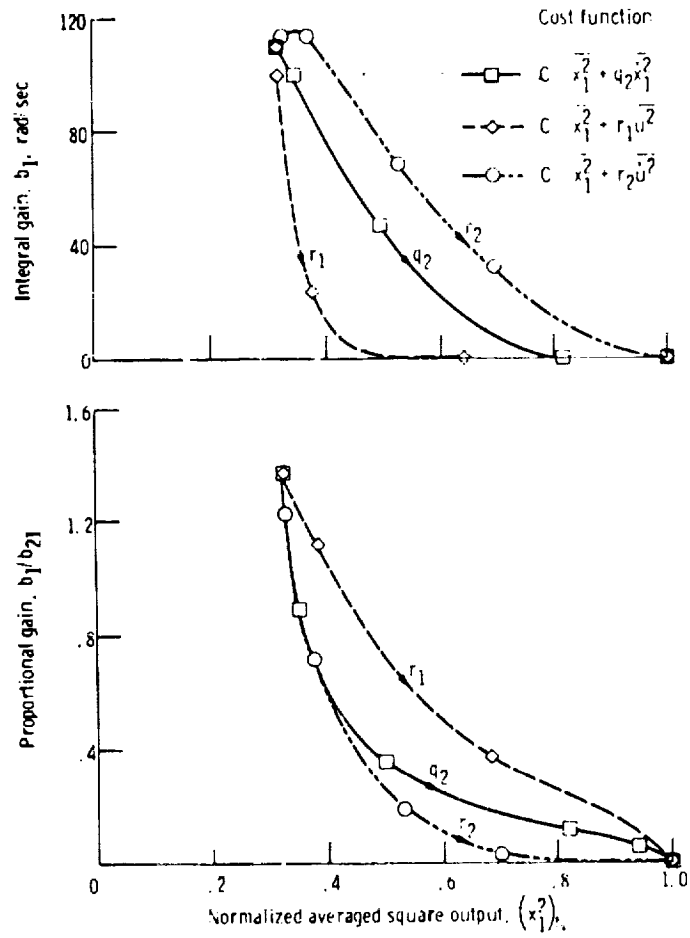


Figure 5. - Proportional and integral gains as function of normalized averaged square output as  $q_2$ ,  $r_1$ , and  $r_2$  penalties vary from 0 to  $\infty$ ; reference case.

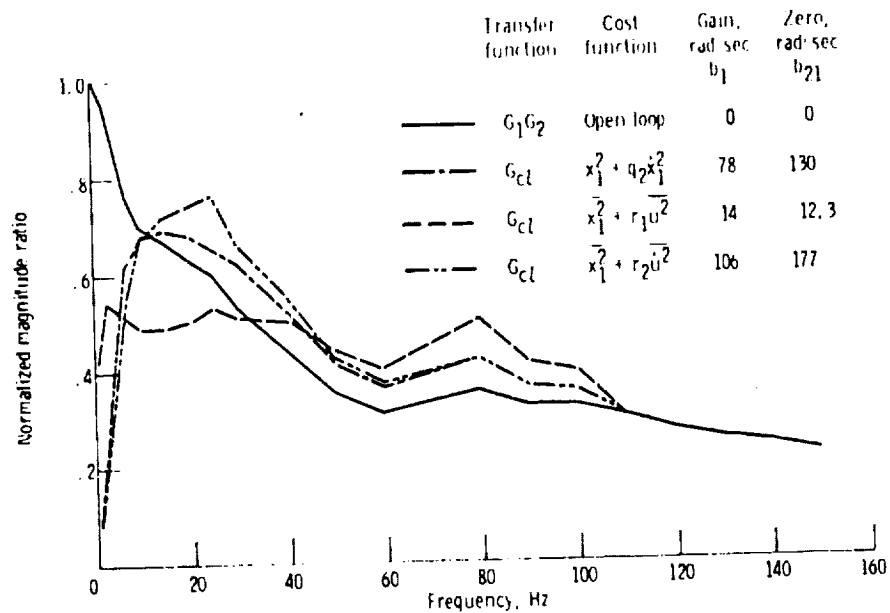


Figure 6. - Frequency response of output  $x_1(j\omega)$  in response to disturbance  $d(j\omega)$ ; normalized to magnitude of open loop response at 1 hertz; reference case;  $(x_1^2)_N = 0.4$  for all closed-loop cases.

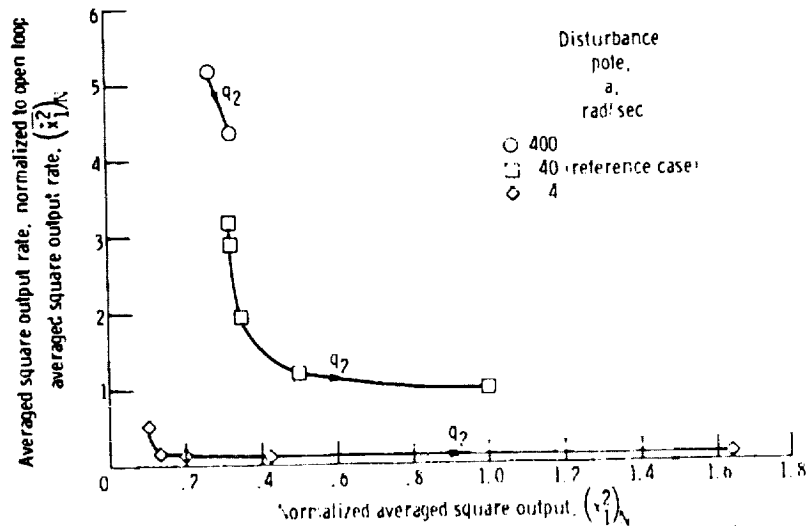


Figure 7. - Comparison of normalized averaged square output and output rate for controllers designed for different disturbance pole locations;  $q_2$  designs only. Averaging time  $T = 1$  second;  $H_{2ps} = 0$ ;  $H_{1ps}$  - proportional plus integral control

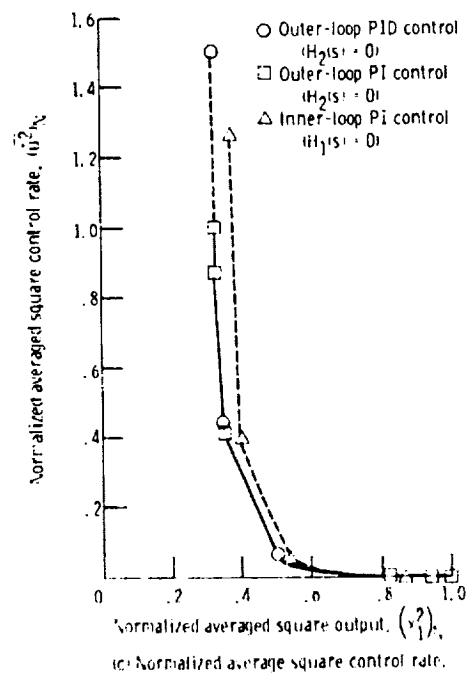
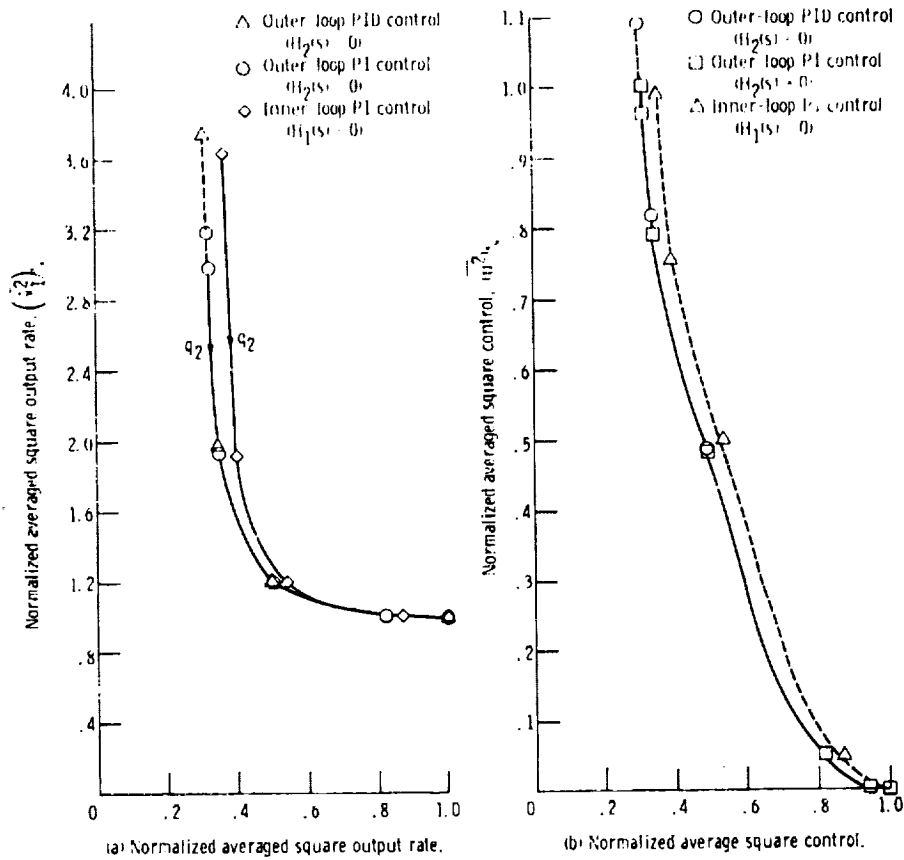


Figure 8. Comparison of normalized averaged square quantities for inner- and outer-loop PI controllers and outer-loop PID controller;  $q_2$  designs only. Averaging time  $T = 1$  second; disturbance pole  $\mu = 40$  radians per second.

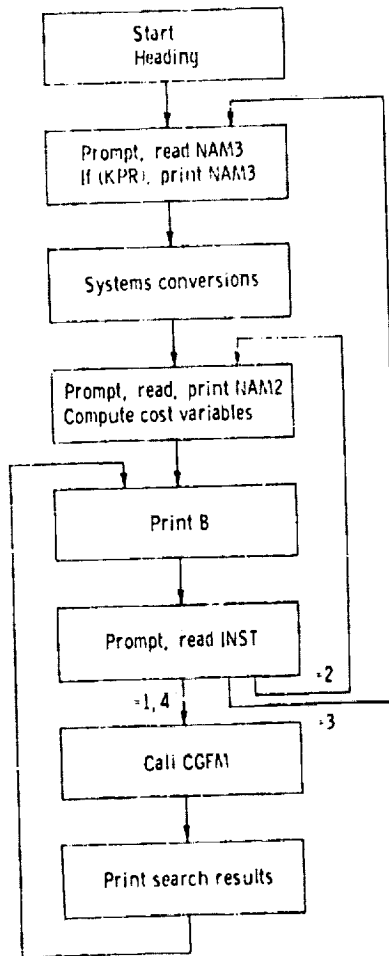


Figure 9. - Flow chart for main program.



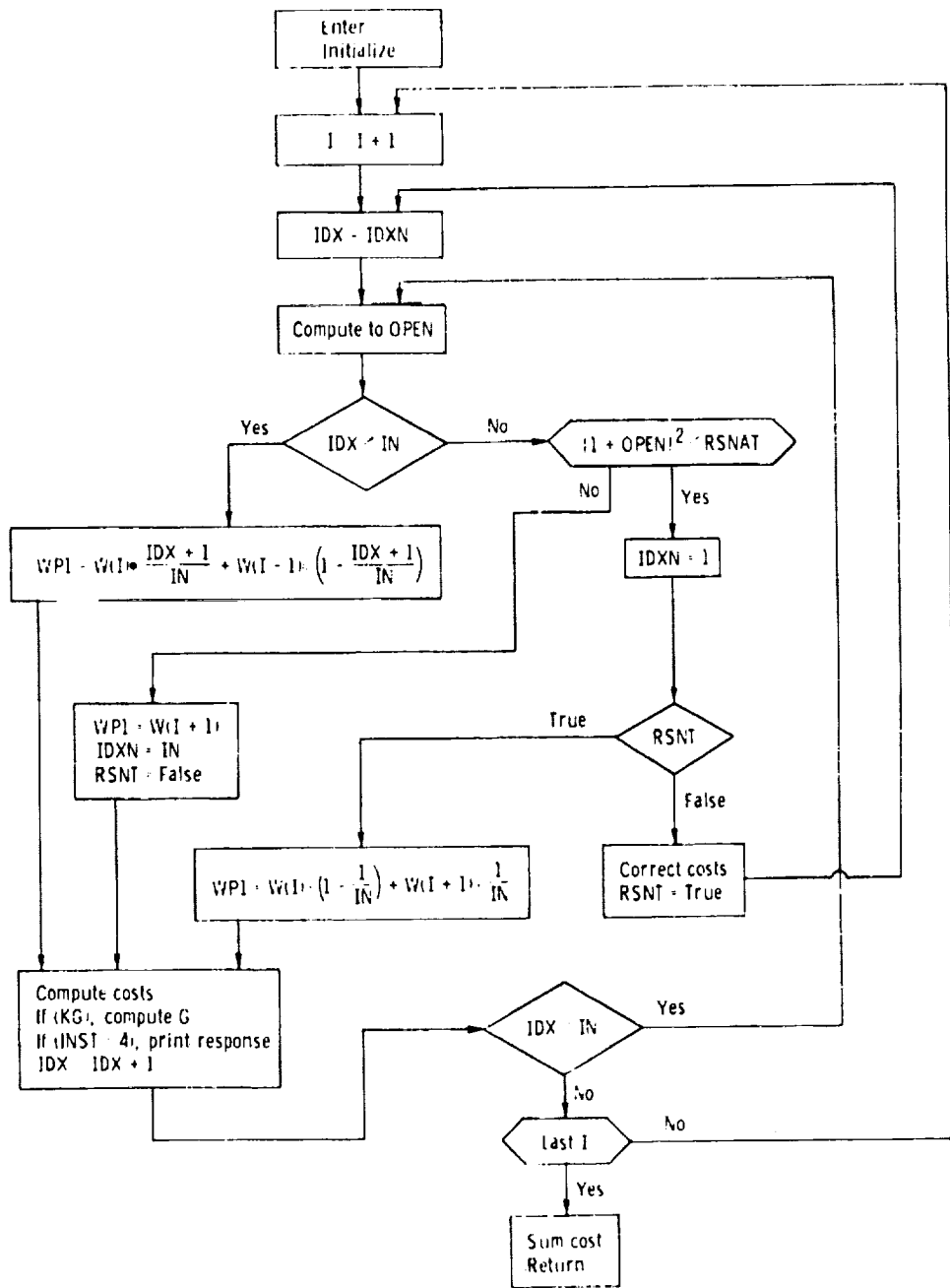


Figure 10. CALFG flow chart.

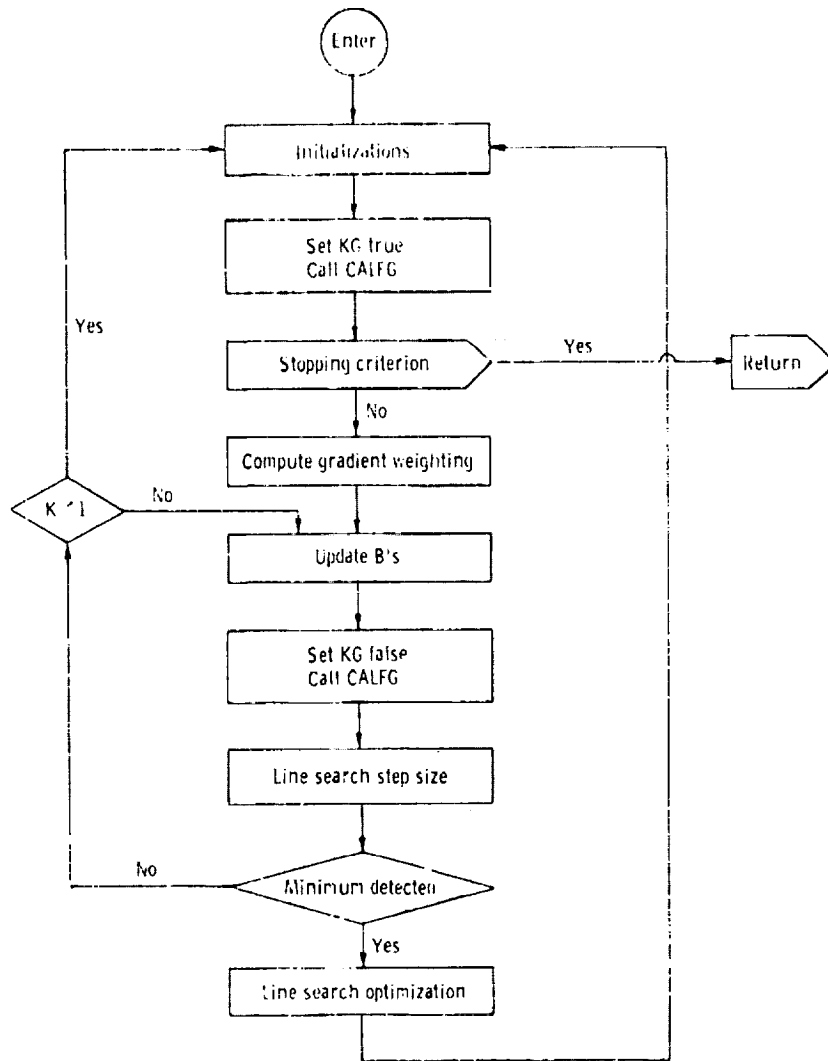


Figure 11. - Flow chart for subroutine CGFAI.

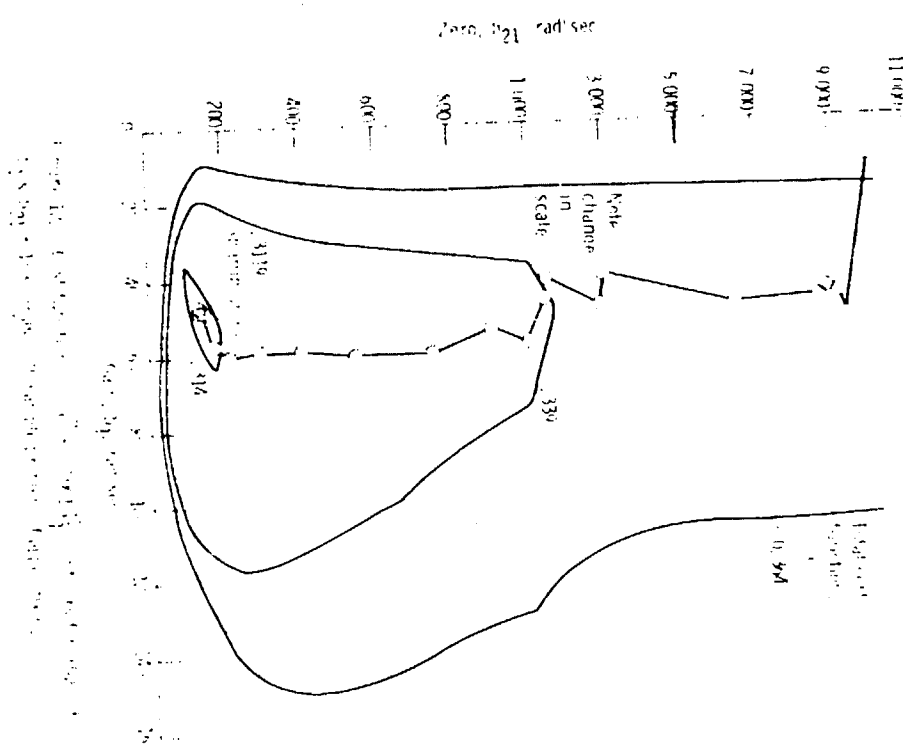


Figure 12. Plot of  $\frac{d\theta}{dt}$  vs.  $\theta$  for the data of Figure 11. The dashed line is a straight line fit to the data points at  $\theta = 3$  and  $\theta = 7$ .

