

(NASA-CR-120499) ENGINEERING ANALYSIS AND
TEST RESULTS OF THE PRE-STAGE PLANETARY
GEAR TRAINS FOR WRIST ROTATION AND PITCH
ASSEMBLY AND AZIMUTH AND (USM Corp.,
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FINAL REPORT

ENGINEERING ANALYSIS AND TEST
RESULTS OF THE PRE-STAGE PLANETARY
GEAR TRAINS FOR WRIST ROTATION AND
PITCH ASSEMBLY AND AZIMUTH AND
ELEVATION ASSEMBLY OF THE EXTENDABLE
STIFF ARM MANIPULATOR KIT ASSEMBLY.

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PREPARED FOR

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1.0 INTRODUCTION

In order to improve the performance capability of the Extendable Stiff Arm Manipulator (ESAM) it was deemed necessary to increase the overall gear ratio by a factor of approximately four. Previous studies at USM resulted in a recommendation that this could be accomplished with minimum effect to existing hardware by the interposition of a planetary gear transmission between the respective drive motors and the Harmonic Drive transmissions.

The Engineering analysis in support of this design approach and the subsequent no-load test results are the subject of this report.

Assembly drawings showing the detail design of each of the four planetary assemblies are included in Appendix I of this report. They are as follows:

Azimuth Drive	USM 7320997
Elevation Drive	USM 7320998
Wrist Roll Drive	USM 7320999
Wrist Pitch Drive	USM 7319000

2.0 DISCUSSION

2.1 Design Parameters

The following is a list of the input power parameters to the planetary assemblies as shown in the NASA Scope of Work:

	<u>Wrist Rotation</u>	<u>Wrist Pitch</u>	<u>Azimuth</u>	<u>Elevation</u>
Motor Stall Torque (oz. in.)	15	15	120	120
Motor No Load Speed (RPM)	5500	5500	550	550

Since a duty cycle was not defined, the planetary transmissions were designed on the basis of delivering the maximum useable horsepower. This condition exists where the D.C. motor is delivering one-half of its stall torque.

2.2 Design Details

For a fixed ring, input to the sun gear planetary transmission the ratio is:

$$\frac{T_{\text{ring}}}{T_{\text{sun}}} + 1$$

For these planetary transmissions:

$$\text{Teeth, Ring} = 108$$

$$\text{Teeth, Planet} = 36$$

$$\text{Teeth, Sun} = 36$$

Therefore, the transmission ratio for all the transmissions is:

$$\frac{108}{36} + 1 = 4$$

A detailed stress and bearing load analysis is included in this report as Appendix II. This analysis is based upon a determination of the bending stresses and Hertzian contact stresses for each of the gear meshes. Also included is a calculation of the B-10 life for the planet bearings.

The selection of materials to be used for these drives proved to be a difficult problem. Since the wrist roll and pitch assemblies are such compact devices, it was necessary to make the motor stator housing and the internal ring gear a single unit. This type of construction requires that this housing, ring gear be made of a material with low magnetic permeability, yet have sufficient strength to support the gear loads. SAE 660 Continuous Cast bronze was selected for this application. All other gears in all of the transmissions are fabricated from 4340 Alloy Steel thru hardened to 300 -320 BHN.

Hertzian stress capacity data for various materials is difficult to obtain. The values used for this analysis are extrapolated from data shown in "Load/Life Curves for Gear and Cam Materials" by Morrison; Machine Design, August 1, 1968, a copy of which is included as Appendix III.

Appendix IV lists the selected materials, the stress levels and resultant factors of safety for each of the meshes.

Prior to the disassembly of the manipulator, no-load speed measurements were attempted for each of the four degrees of freedom. Operation of the manipulator showed

that the Azimuth and Elevation Drive Assemblies were not operational when 28 volts D.C. was applied to the control console.

Disassembly of the manipulator showed that the control electrical cabling to the Azimuth Drive had been damaged to an apparent coast thru the electrical stop. The Elevation Drive when not attached to the remainder of the manipulator arm appeared to function normally but would not handle the resultant load from the remainder of the arm. There appeared to be no apparent reason for this anomaly.

The Manipulator Kit was re-assembled with the modifications as shown on the enclosed USM drawings and the required repairs were made to the Azimuth circuitry. No-load testing was accomplished for the manipulator in the "as modified" condition. The results of the testing are shown in Appendix V.

3.0 CONCLUSIONS

The stress and bearing load analysis (Appendix II) and the Factors of Safety calculation (Appendix IV) show that the design is adequate for its intended purpose. The lowest factor of safety is 1.96 and is based upon the conservative assumption that each one of the two planet gears is carrying 1.5 times its normal tooth load. The capacity of the planetary transmissions to withstand the stall torque of the motors is assured since this is a static torque which results in only tooth bending stresses. These static bending stresses are twice the values calculated, but still well below the capacity of the materials.

The B-10 life of the planet bearings is also calculated. This life is shown to be as a minimum 16,000 hours and is based upon the same assumption as the gear tooth loads. The static capacity of the planet bearings is above the load that would be applied at motor stall torque conditions.

The test results show that the required no-load speeds were not met and that the actual reduction in speed was not four to one as would be expected from the mathematics associated with the gear design.

The reason for this anomaly is clear. The interposition of the planetary gear train between the motor and Harmonic Drive allowed a lower reflected torque to the motor, thus allowing the operating point for the DC motor to move up the motor torque speed curve to a

higher operating speed. This higher operating speed results in a speed change of less than the theoretical four to one. Thus, the actual speed change is a function of the slope of the motor torque speed curve as well as the no-load efficiency of the planetary gear assembly and Harmonic Drive unit.

In order to accomplish an actual change in the no-load output speed of four to one, additional testing would be required to determine the effect of speed on the no-load input torque of both the planetary gear train and Harmonic Drive. This information could then be used to calculate the necessary gear ratio to obtain the desired speed ratio change.

4.0 APPENDIX I
ASSEMBLY DRAWINGS

5.0 APPENDIX II
STRESS AND BEARING LOAD ANALYSIS



DESIGN ANALYSIS
NASA, HUNTSVILLE
USM ORDER NO 21-15815

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WRIST ROTATION & PITCH ASSEMBLIES:

MOTOR PERFORMANCE DATA:

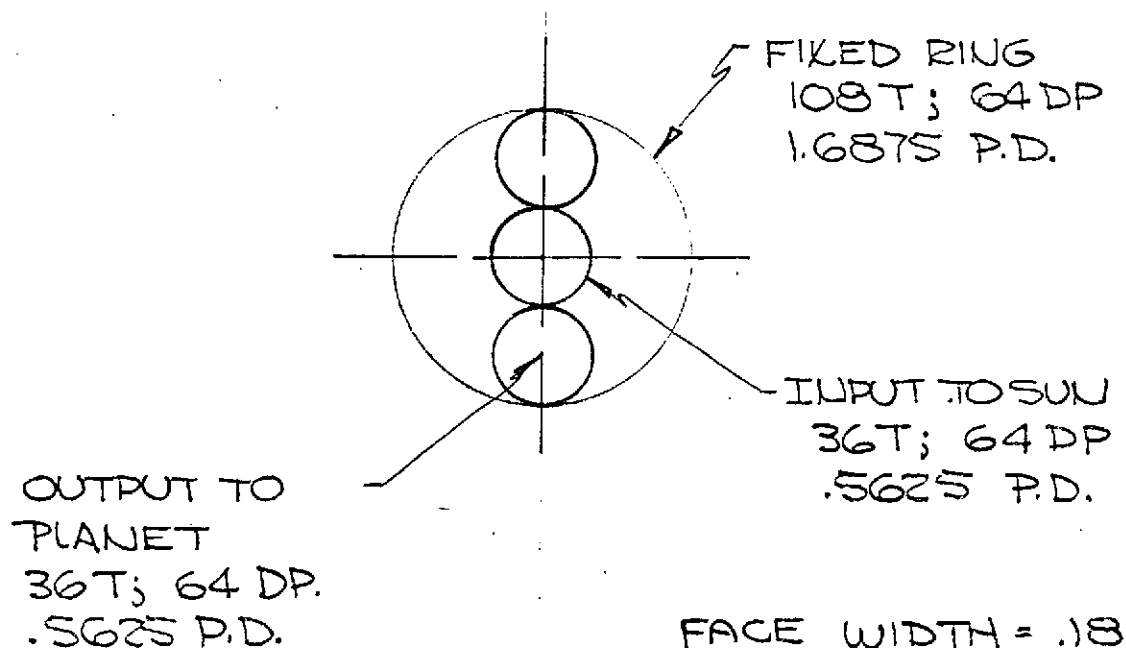
$W_{NOLOAD} = 5500 \text{ RPM}$
 $T_{STALL} = .94 \text{ LB-IN}$

MAX. POWER IS ATTAINED AT $\frac{W_{NL}}{2}$

OR $\frac{5500}{2} = 2750 \text{ RPM.}$

$$\text{TORQUE} = .94 - \frac{.94}{5500} (2750) = .47 \text{ LB-IN}$$

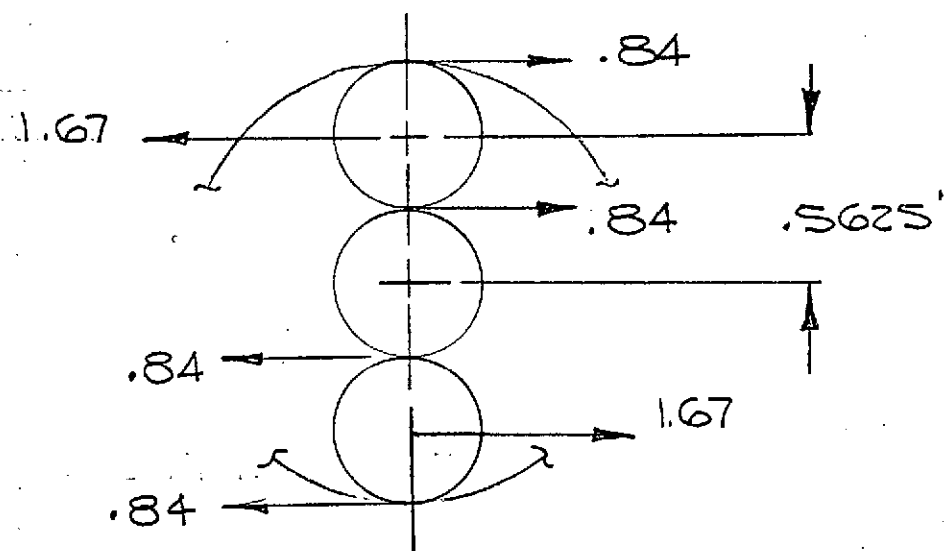
BASIC ASSEMBLY CONFIGURATION:



TOOTH LOAD @ MAX. POWER (W_t)

$$= \frac{(.47)(2)}{.5625} = 1.67 \text{ LBS}$$

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SINCE IT IS NOT PRACTICAL TO ASSUME EQUAL
LOAD SHARING FOR THE TWO PLANETS, WE SHALL
ASSUME THE TOOTH LOAD PER GEAR IS:

$$(.84)(1.5) = 1.26 \text{ LBS}$$

DYNAMIC LOADS:

PITCH LINE VELOCITY, SUN: (V)

$$\frac{2750(\pi)(.5625)}{12} = 405 \text{ FT/MIN}$$

DYNAMIC LOAD (W_d) =

$$\frac{W_t}{W_d} = \frac{50}{50 + \sqrt{\quad}} = \frac{50}{70}$$

∴

$$W_d = 1.4 W_t$$

THEREFORE, STRESS CALCULATIONS SHALL
BE BASED UPON A TOOTH LOAD OF:

$$1.4(126) = 1.77 \text{ LBS}$$

RING, PLANET MESH:

BENDING STRESS:

$$\sigma_b = \frac{1.77}{\frac{\pi}{64}(.18)(.12)} = 1669 \text{ PSI} \quad (\text{NO NOTCH FACTOR})$$

HERTZ STRESS:

$$\sigma_H = \left[\frac{(.7)(1.77) \left(\frac{1}{.5625} - \frac{1}{1.6875} \right)}{.18 \sin 20^\circ \left(\frac{1}{14.5(10^6)} + \frac{1}{30(10^6)} \right)} \right]^{\frac{1}{2}}$$
$$= 15,269 \text{ PSI}$$

PLANET, SUN MESH:

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BENDING STRESS:

$$\sigma_B = 1669 \text{ PSI}$$

(NO NOTCH FACTOR)

HERTZ STRESS:

$$\sigma_H = \left[\frac{(.7)(1.77) \left(\frac{1}{.5625} + \frac{1}{.5625} \right)}{.18 \sin 20^\circ \left(\frac{1}{30(10^6)} + \frac{1}{30(10^6)} \right)} \right]^{\frac{1}{2}}$$
$$= 32,762 \text{ PSI}$$

PLANET BEARING:

THE DYNAMIC LOADS ASSOCIATED WITH THE GEAR TEETH ARE NOT IMPOSED UPON THE BEARING. THE RADIAL LOAD ON THE BEARING IS TWICE THE TRANSMITTED TOOTH LOAD:

$$= 2(1.77) = 3.54 \text{ LBS}$$

THE BEARING:

MPB CORP.

R2:

RADIAL CAPACITY: 68 LBS

500 HRS 310 LIFE

AT 33.3 RPM

STATIC CAPACITY: 21 LBS

ROTATIONAL VELOCITY OF PLANET
ABOUT ITS OWN AXIS:

$$= 2750 \left(\frac{1.6875}{.5625} \right) \left(\frac{.5625}{1.6875 + .5625} \right)$$

$$= 2062 \text{ RPM}$$

PLANET BRG B-10 LIFE:

$$\text{LIFE} = \frac{16667}{2062} \left(\frac{68}{3.54} \right)^3$$

$$= 57,290 \text{ HRS}$$

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AZIMUTH & ELEVATION ASSEMBLIES:

MOTOR PERFORMANCE DATA:

$$\begin{aligned} W_{\text{NOLOAD}} &= 550 \text{ RPM} \\ T_{\text{STALL}} &= 7.50 \text{ LB-IN} \end{aligned}$$

TORQUE @ MAX POWER:

$$= 7.50 - \frac{7.50}{550} (275)$$

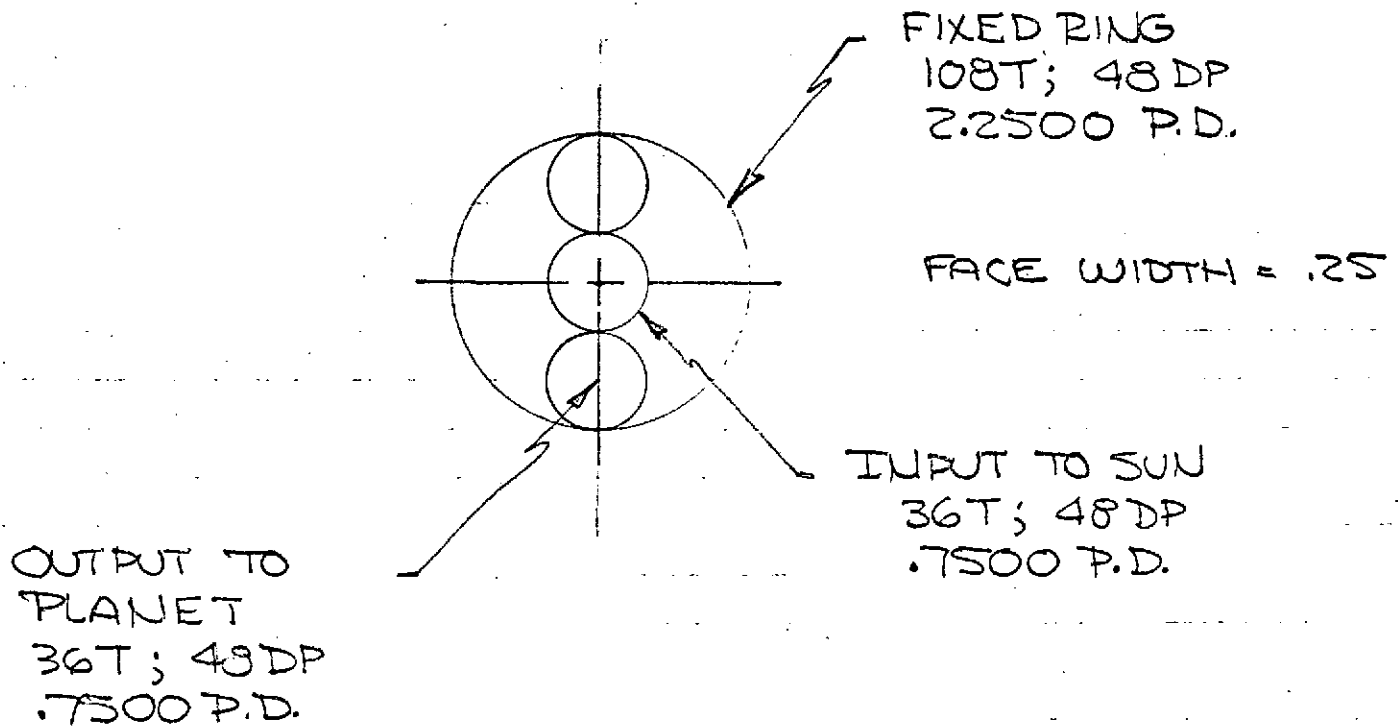
$$= 3.75 \text{ LB-IN}$$

SPEED @ MAX POWER:

$$= 275 \text{ RPM}$$

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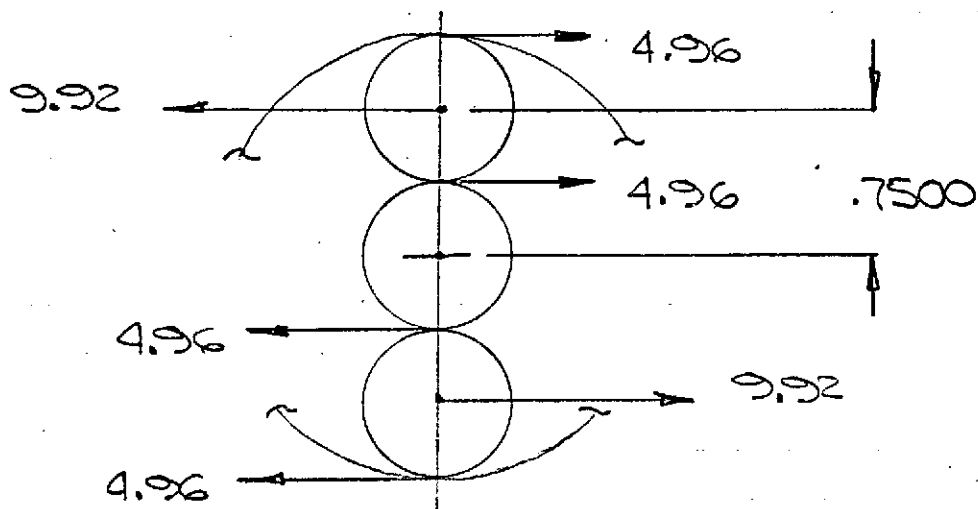
BASIC ASSEMBLY CONFIGURATION:



TOOTH LOAD @ MAX POWER (W_t):

$$= \frac{3.75(2)}{.7500}$$

$$= 9.92 \text{ LBS}$$



TOOTH LOAD PER GEAR:

$$4.96(1.5) = 7.44 \text{ LBS}$$

DYNAMIC LOADS:

PITCH LINE VELOCITY; $SUN(V)$:

$$\frac{550(\pi)(.7500)}{12} = 108 \text{ FT/MIN.}$$

DYNAMIC LOAD (W_d):

$$\frac{W_t}{W_d} = \frac{50}{50 + \sqrt{V}} = \frac{50}{60.4}$$

\therefore

$$W_d = 1.21 W_t$$

THEREFORE, STRESS CALCULATIONS SHALL BE BASED UPON A TOOTH LOAD OF:

$$1.21(7.44) = 9.00 \text{ LBS}$$

RING, PLANET MESH:

BENDING STRESS:

$$\sigma_B = \frac{9.00}{\frac{\pi}{48}(.25)(.12)} = 4583 \text{ PSI} \quad (\text{NO NOTCH FACTOR})$$

HERTZ STRESS:

$$\sigma_H = \left[\frac{(.7)(9.00) \left(\frac{1}{.7500} - \frac{1}{2.2500} \right)}{.25 \sin 20^\circ \left(\frac{1}{30(10^6)} + \frac{1}{30(10^6)} \right)} \right]^{\frac{1}{2}}$$

$$= 31,343 \text{ PSI}$$

PLANET, SUN MESH:

BENDING STRESS:

$$\sigma_B = 4583 \text{ PSI}$$

(NO NOTCH FACTOR)

HERTZ STRESS:

$$\sigma_H = \left[\frac{(.7)(9.00) \left(\frac{1}{.7500} + \frac{1}{.7500} \right)}{.25 \sin 20^\circ \left(\frac{1}{30(10^6)} + \frac{1}{30(10^6)} \right)} \right]^{\frac{1}{2}}$$

$$= 54,288 \text{ PSI}$$

PLANET BEARING:

$$\text{RADIAL LOAD} = 2(9.00) = 18.00 \text{ LBS}$$

THE BEARING:

MPB CORP: R3

RADIAL CAPACITY: 140 LBS

500 HRS B10 LIFE

AT 33.3 RPM.

STATIC CAPACITY: 48 LBS

ROTATIONAL VELOCITY OF PLANET
ABOUT ITS OWN AXIS:

$$= 550 \left(\frac{2.2500}{.7500} \right) \left(\frac{.7500}{2.2500 + .7500} \right)$$
$$= 413 \text{ RPM}$$

PLANET BRG B-10 LIFE:

$$\text{LIFE} = \frac{16667}{413} \left(\frac{140}{18} \right)^3$$
$$= 18,987 \text{ HRS}$$

6.0 APPENDIX III

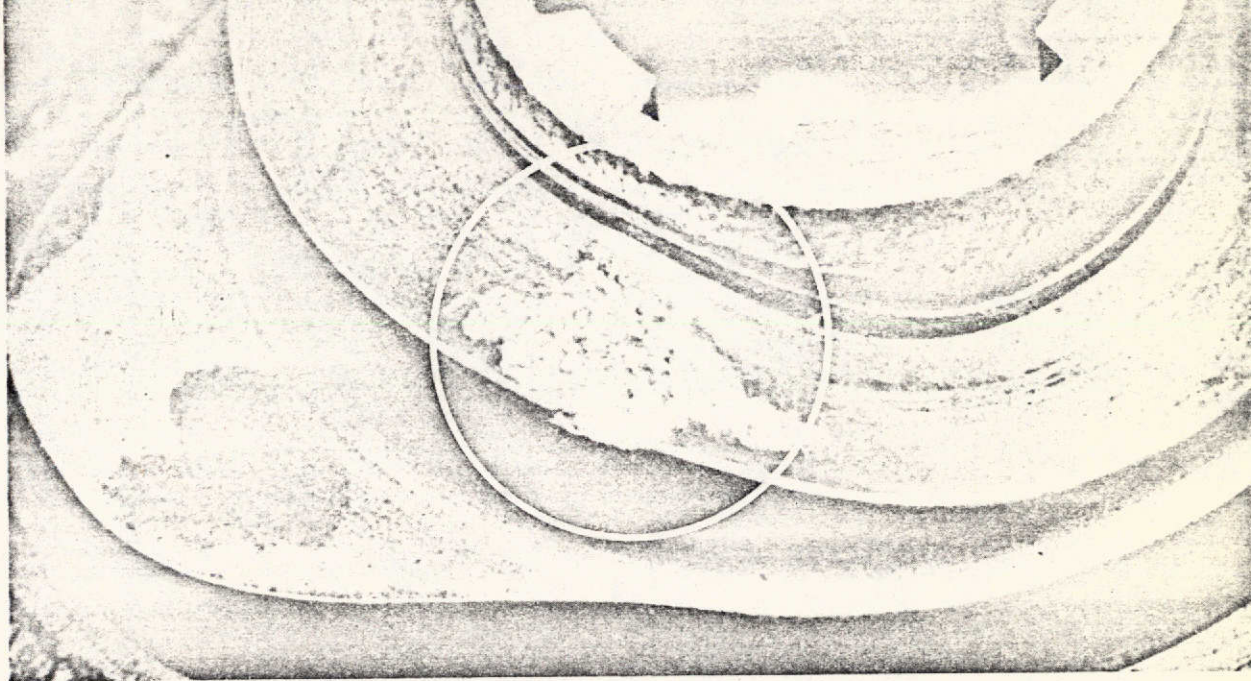
"LOAD/LIFE CURVES FOR GEARS AND CAM MATERIALS"

Test data let you develop your own

**Load / life curves
for gear and
cam materials**

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Cam material: Class 30 gray iron. Applied load: 800 lb per in. of face. Surface failure: 3,000,000 cycles.

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Test data let you develop your own Load / life curves for

SURFACE fatigue is the most frequent cause of failure in gears and cams. Unfortunately, sufficient data has never been available to determine the surface endurance of gear and cam materials.

For some years, tests have been carried out to

determine the surface endurance limits of many of the most widely used materials. This has been done by conducting rolling and sliding tests on radially loaded contacting cylinders, and correlating the resulting data with the action of gears and cams.

BASIC CONSIDERATIONS

Tests run under rolling only simulate the action of gear teeth at their active pitch diameters and the normal rolling action of a cam and cam follower in continuous intimate contact. In these cases, the only sliding action is the minute differential motion possible under the area of surface deformation.

Tests run under combined rolling and sliding simulate the action of gear teeth in any zone of contact other than at the active pitch circles. Here, sliding varies both in direction and magnitude across the profile of the gear; the sliding commonly ranges from zero to several hundred percent, depending on geometry of the gear pair, Fig. 1. Early tests by Buckingham established that surface fatigue begins near the active pitch circles and that something on the order of 9% sliding on the test rolls can be used to evaluate spur and helical gears.

Percent sliding is the ratio of relative sliding

velocity to the surface velocity of the test rolls or gear teeth. As such, it is dimensionless and independent of roll or gear speed. Percent sliding should not be confused with gear pitch-line velocity or rate of sliding between gear teeth. Because of the nature of this parameter, the experimental load-stress factors given in this article for similar percent-sliding conditions can be used even though test-roll and gear or cam speeds differ.

The zone of contact usually surrounding the active gear pitch diameter—where surface fatigue by pitting may be expected to occur—is shown in Fig. 1. This zone carries the entire transmitted load without the assistance of adjacent teeth. It is in this zone that case crushing of carburized or otherwise case-hardened teeth may be initiated¹.

The remainder of the tooth profile, unless in gross geometric error, not only shares the load but is also subjected to higher percent of sliding,

Manufacturers have long supplied load/life relationships for bearings. Gear and cam designers haven't been so lucky. Result: many gears and cams are over-designed or under-designed. In this article, the author presents the results of 30 years of testing to determine the surface endurance of many common—and uncommon—gear and cam materials.

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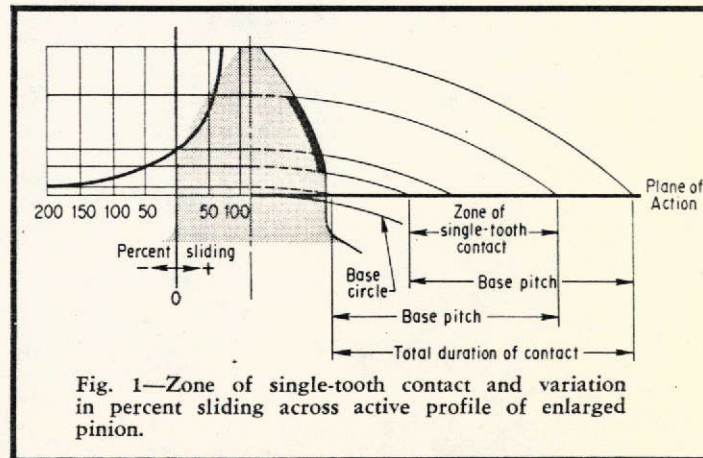
gear and cam materials

as shown in Fig. 1. Here, if lubrication is not adequate, abrasive wear, welding, and scoring will occur. Adequate lubrication will avoid these problems (unless scoring occurs because of high gear speeds and lubricant flashing). If the basic endurance limit of the material combination is exceeded, the capability of the lubricant is also likely to be exceeded and failure may occur.

High percent sliding in the region of pitch planes is common on cross-axis gearing such as spiral and worm-gear drives. Here, percent sliding refers to the critical combination of tangential and normal sliding conditions introduced by the screwing action of the mesh and the normal involute action in the plane of gear rotation. There is mounting evidence that tests designed around 100% sliding action may be acceptable for many of these designs.

The value of the simplified roll test over the far more expensive procedure of running gears together becomes obvious when one considers the

complications of dynamic loading, alignment, and overlapping of failure modes inherent in gearing systems.



HOW TO DETERMINE SAFE SURFACE LOADS

Because Hertzian stresses vary with distance below the surface², Fig. 2, analysis of cam or gear materials having surface or subsurface treatments requires comparison of stress and material fatigue strength at each level of stress penetration. Ob-

viously, parts with superficial surface hardening or case hardening of insufficient depth and low core hardness are susceptible to subsurface failure under some loading conditions. The emphasis here is on materials of a more homogeneous structure,

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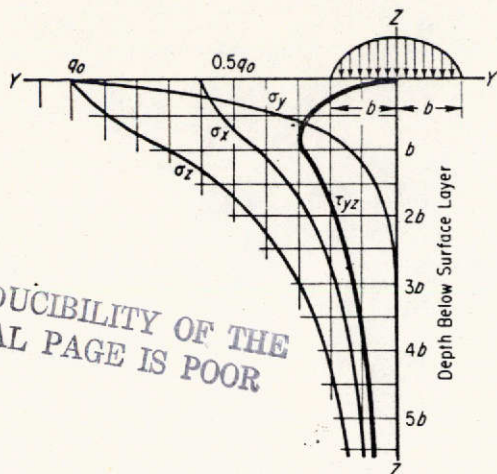
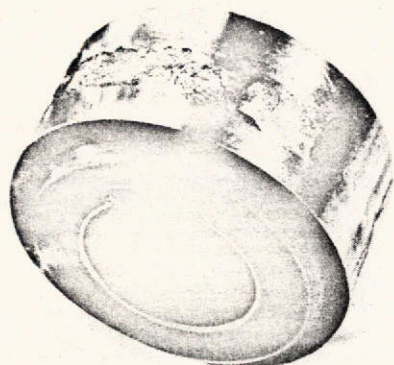
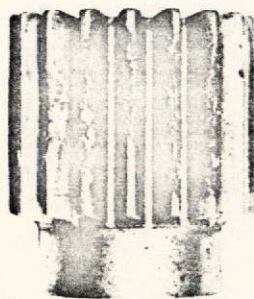


Fig. 2—Variation of Hertz stresses with depth below surface.



Field failure of oil-quenched nickel cast-iron roll under rolling action. Only a few hours before failure, the surface showed no visible sign of surface breakdown.



Failure of nitrided gear, initiated by cracks in the tooth section, running generally along the tooth face. Crushing of tooth tips resulted from case particles jamming into the root of the mating rack. By increasing case depth and drawing case hardness to approximately 58 Rc, these failures were eliminated. Experience and test data have shown that case depth should always be twice, and preferably three times, the depth to the calculated point of maximum shear stress.

which may be evaluated through the following procedure.

Basic Equations: Maximum compressive contact stress between cylinders of materials with Poisson's ratio of 0.3 is

$$s_c^2 = \frac{0.35 W \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}{L \left(\frac{1}{E_1} + \frac{1}{E_2} \right)}$$

where W = normal load between surface, L = length of contact, and R_1 , R_2 , E_1 , and E_2 = radii of curvature and Young's Moduli. Transposing gives

$$W = \frac{s_c^2 L \left(\frac{1}{E_1} + \frac{1}{E_2} \right)}{0.35 \left(\frac{1}{R_1} + \frac{1}{R_2} \right)}$$

The terms which are a function of material only can be combined. Then, by definition, they can represent the experimental load-stress factor K_1 . Thus,

$$K_1 = \frac{s_c^2 \left(\frac{1}{E_1} + \frac{1}{E_2} \right)}{0.35}$$

and the general equation for safe endurance load, W_e , becomes

$$W_e = \frac{K_1 L}{\frac{1}{R_1} + \frac{1}{R_2}} \quad (1)$$

where the sign is positive for convex curvatures and negative for concave curvatures. If test rolls are found to fail at N repetitions of stress at some load W_e , then K_1 for that combination of materials at that number of stress cycles is calculated from Equation 1. By repeating the test at other loads, K_1 can be determined as a function of life, Fig. 3. Factors K_1 for several gear and cam materials are given in Table 1.

Although the contact stresses determined by the Hertzian equation are commonly used as a measure of loading, they are not theoretically correct for other than static conditions. Distortion of the stress pattern by the tangential friction forces generated by sliding results in an increase in compressive and shear stresses as well as in a substantial relocation of maximum shear point below the surface³. For this reason, the Hertzian stresses given in Table 1 are only indicative values. However, this does not affect the correlation between test rolls and gears and cams, since similar distortion occurs on both the test roll and the gear or cam as long as conditions of combined rolling and sliding are the same.

Equation 1 is applicable to cams, gears, rollers or any other kinematic pair with line contact under rolling or rolling and sliding conditions. All that is required are the radii of curvature of cam and roller at the point of maximum contact stress or the radii of curvature of gear teeth at or near the active pitch circles. Then, with an experimen-

Table 1—Experimental Load/Stress Factors for Gear and Cam Materials

Material	Rolling Only				Rolling & 9% Sliding			
	K_t	Hertz Stress	A	B	K_t	Hertz Stress	A	B
Material Running Against Tool-Steel Roll Hardened to 60-62 Rc								
1020 steel, carburized, 0.0-0.5 in. min depth, 50-60 Rc	12,700	256,000	7.39	38.33	10,400	99,000	13.20	61.06
1020 steel, 130-150 Bhn	1,720	89,000	4.21	21.41	1,720	94,000	4.78	23.45
1117 steel, 130-150 Bhn	1,500	89,000	4.21	21.41	1,150	77,000	3.63	19.12
X1340 steel, induction-hardened, 0.045 in. min depth, 45-58 Rc	10,000	227,000	6.56	34.24	8,200	206,000	8.51	41.31
Blue tempered spring-steel stampings, 40-50 Rc	2,470	113,000	4.00	21.57
4140/4150 steel, 350-370 Bhn (etd 180)	11,300	242,000	17.76	80.90
4150 steel, heat-treated, 270-300 Bhn, phosphate-coated	12,000	249,000	11.40	54.52	8,650	211,000	15.47	68.92
4150 steel, heat-treated, 270-300 Bhn, flash chromium-plated	6,060	177,000	11.18	50.29
4150 steel, heat-treated, 270-300 Bhn phosphate-coated	9,000	216,000	8.80	42.81	6,260	180,000	11.56	51.92
4150 ceramic cast steel, heat-treated, 270-300 Bhn	2,850	121,000	17.86	69.72
4340 steel, induction-hardened 0.045 in. min depth, 50-58 Rc	13,000	259,000	14.15	66.22	9,000	216,000	14.02	63.44
4340 steel, heat-treated, 270-300 Bhn	5,500	189,000	18.05	75.55
6150 steel, 300-320 Bhn	1,170	78,000	3.10	17.51
6150 steel, 270-300 Bhn	1,820	97,000	8.30	35.06
18% Ni maraging tool, air-hardened, 48-50 Rc	4,300	146,000	3.90	22.18
Gray-iron, Class 20, 160-190 Bhn, phosphate-coated	940	53,000	3.90	19.60
Gray-iron, Class 20, 140-160 Bhn	790	49,000	3.83	19.09	740	47,000	4.09	19.72
Gray-iron, Class 30, 200-220 Bhn	1,120	63,000	4.24	20.92
Gray-iron, Class 30, heat-treated (austempered), 255-300 Bhn, phosphate-coated	2,920	102,000	5.52	27.11	2,510	94,000	6.01	28.44
Gray-iron, Class 30, oil-quenched, 270-415 Bhn	1,850	81,000	5.45	25.79
Gray-iron, Class 35, 225-255 Bhn	2,000	88,000	11.62	46.35	1,900	84,000	8.39	35.51
Gray-iron, Class 45, 220-240 Bhn	1,070	85,000	3.77	19.41
Nodular-iron, Grade 80-60-03, 207-241 Bhn	2,100	96,000	10.09	41.53	1,960	93,000	5.56	26.31
Nodular-iron, Grade 100-70-03, heat-treated, 240-260 Bhn	3,570	122,000	13.04	54.33
High-strength yellow brass, drawn, 157-162 Bhn	1,280	67,000	3.69	19.45
Nickel bronze, 80-90 Bhn	1,390	73,000	6.01	26.89
SAE 65 phosphor-bronze sand casting, 65-75 Bhn	730	52,000	2.84	16.13	350	36,000	2.39	14.08
SAE 680 continuous-cast bronze, 75-80 Bhn	320	33,000	1.94	12.87
Aluminum bronze	2,500	98,000	5.87	27.97
Zinc die casting, 70 Bhn	250	28,000	3.07	15.35	220	26,000	3.11	15.29
Random-fiber cotton-base phenolic	1,000	6.03	26.11	900	5.95	25.60
Graphitized laminated phenolic	900	6.58	27.43
Nema Grade I, laminated phenolic	880	9.39	35.64	830	5.53	24.13
Linen-base laminated phenolic	830	8.54	32.90	670	6.46	26.25
Acetal resin	620	580
Polyurethane rubber	240
Polycarbonate resin	60
High-molecular-weight polyethylene	370	8.03	28.61
Material Running Against Same								
1020 steel, 130-170 Bhn, and same but phosphate-coated	2,900	122,000	7.84	35.17	1,450	87,000	6.38	28.23
1144 CD steel, 260-290 Bhn, (stress-proof)	2,290	109,000	4.10	21.79
4150 steel, heat-treated, 270-300 Bhn, and same but phosphate-coated	6,770	187,000	10.46	48.09	2,320	110,000	9.58	40.24
4150 leaded steel, phosphate-coated, heat-treated, 270-300 Bhn	3,050	125,000	6.63	31.10
4340 steel, heat-treated, 320-340 Bhn, and same but phosphate-coated	10,300	230,000	18.13	80.74	5,200	164,000	26.19	105.31
Gray-iron, Class 20, 130-180 Bhn	960	45,000	3.05	17.10	920	43,900	3.55	18.52
Gray-iron, Class 30, heat-treated (austempered), 270-290 Bhn	3,800	102,000	7.25	33.97	3,500	97,000	7.87	35.90
Nodular-iron, Grade 80-60-03, 207-241 Bhn	3,500	117,000	4.69	24.65	1,750	82,000	4.18	21.56
Meehanite, 190-240 Bhn	1,600	80,000	4.766	23.27	1,450	76,500	4.94	23.64
6061-T6 aluminum, hard anodic coat	350	10.27	34.15	260	5.02	20.12
HK31XA-T6 Magnesium, HAE coat	175	6.46	22.53	275	11.07	35.02

tally determined K_t for the materials, sliding conditions, and required life, a comparison of safe endurance load with the expected dynamic tooth or cam loading can be made. Buckingham⁴ introduced a useful endurance load equation for gears:

$$W_e = DFK_gQ \quad (2)$$

where D = active pitch diameter of smaller gear, F = active length of face, $K_g = K_t$ factor converted for gears, and Q = gear-ratio factor = $2N/(N \pm n)$. Also, N = number of teeth in large gear, and n = number of teeth in small gear. For a gear and rack, $Q = 2$. For external gears it is

positive, and for internal gears negative. Also,

$$K_g = \frac{K_t \sin \phi}{4} \quad (3)$$

where ϕ = active pressure angle of the gear mesh.

Application of Test Data: Table 1 lists experimental load-stress factors, K_t , for 100 million repetitions of stress for materials combinations subjected to rolling only and rolling combined with 9% sliding action. The values for rolling only are applicable to cam-type applications where sliding is not anticipated. The values listed under 9%

sliding are applicable to spur and helical gears, or to cams where a small amount of sliding is anticipated.

Many tests have been performed at other combinations of rolling and sliding for specific applications and to explore the behavior of materials under a variety of conditions. Having limited utility in practical design situations, they are not included in Table 1. The load-life curves in Fig. 3 show typical data and the effect of various parameters on surface fatigue for several groups of materials.

Plotting Load/Life Curves: In plotting K_1 vs life, a statistically determined regression line is drawn through the normal scatter of significant points obtained from tests. The equation for the K_1 factor as a function of life is derived from the slope of these regression lines:

$$\log_{10} K_1 = \frac{B - \log_{10} N}{A} \quad (4)$$

where constants A and B are taken from Table 1, and N = number of stress cycles for which the K_1 factor is required. From Equation 4, useful reference curves of K_1 versus any desired range of stress cycles can easily be plotted.

Table 1 is divided into a group of materials run against hardened tool steel and a group of similar materials run together. The first group is most often useful in cam applications because the cam roller follower is usually of hardened steel. The data derived from combinations of similar materials is most useful in gear applications.

Because up to six months are required to establish minimum data for a given material combination, not enough tests can be carried out to satisfy a strict statistician. However, these factors have been successfully used for over 30 years. But regardless of the degree of data refinement available, engineering judgment must still be applied in allowing for mounting errors, lubrication deficiencies, thermal effects, and other operating or geometric variables not accounted for in the laboratory tests. The regression lines obtained are only considered lines of probability; $\frac{3}{4}$ of the resulting load stress factors in Table 1 is considered a reasonable working value.

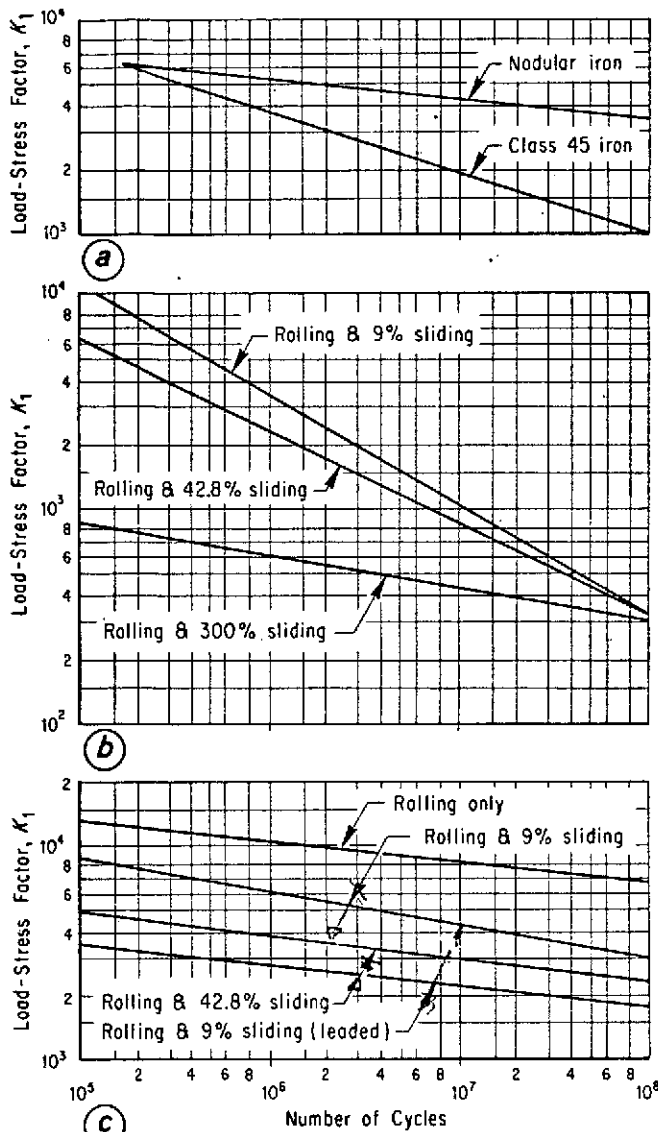


Fig. 3—Typical curves showing load/life relationships for common gear and cam materials. Curves in *a* are for 100-70-03 nodular iron (240-260 Bhn) and Class 45 gray iron (220-240 Bhn), both materials running on carbon tool steels (60-62 Rc). Curves in *b* are for continuous-cast bronze running on hardened steel. Curves in *c* are for heat-treated 4150 steel running against the same material, but phosphate-coated. In all charts, 9% sliding velocity is 54 fpm; 42.8% sliding velocity is 221 fpm.

WHAT TESTS REVEALED ABOUT SURFACE FATIGUE

The tests confirmed several facts important in selecting the most suitable material.

Sliding: An orderly transition occurs from pitting fatigue to abrasive wear as percent sliding is increased. Where the transition is completed depends on material combination, percent sliding, and, to a large extent, the lubricant. Pitting failure has been observed on some irons under as high as 300% sliding with only straight mineral oil as the lubricant. Abrasive wear has been observed under as low as 9% sliding on highly

stressed and less compatible hardened steels with similar lubrication.

As percent sliding is increased, the coefficient of friction follows a regulated upward progression. Also, the point of maximum shear stress approaches the surface until some critical value is reached where surface fatigue is preempted by surface abrasion³. This conclusion has been supported by observation and measurement of progressively shallower fatigue pits as percent sliding is increased.

Observations have also been made on the ef-

As early as 1931, Professor Earle Buckingham of MIT observed the similarity of contact conditions between radially loaded parallel cylinders and gears and cams. He designed a roll-test machine to determine surface endurance limits for rolling or combined rolling and sliding contact conditions.^{5,6,7}

The use of this and other similar machines has been documented in the literature. Radially loaded test rolls are varied in diameter and gear ratio to provide pure rolling or some desired combination of rolling and sliding at the loaded interface. Then, load-life relationships are established by plotting load-stress factors, obtained from the calculated Hertzian contact stress for each test, against stress cycles to failure.

The test lubricant is a straight mineral oil of 280-320 SSU at 100 F. The more expensive fortified or activated oils and surface treatments are reserved for tests involving very high loads or high sliding conditions not usually associated with spur or helical gearing and cams.

Effectiveness of oxide coatings, fortified lubricants, and the addition of lead as a material constituent to reduce the tangential-stress component developed by sliding friction. These modifications raise the allowable percent sliding to a point short of abrasion, and thereby increase surface endurance limits. There is also considerable evidence that pitting can and probably does start at the surface under higher percent sliding, but can originate from subsurface failures under rolling or low percent sliding, Fig. 4.

In general, surface endurance limits go down as percent sliding goes up—but not proportionally. For example, tests in nodular iron under 9% sliding give a *K* factor only 55% of that for rolling only. Usually, for cast irons the *K* values for 9% sliding are from 80% to 90% of those for rolling. Also, the slope of load/life curves vary sharply as a function of percent sliding for some ductile materials and very little for heat-treated or hardened steels.

In many early tests on highly loaded steel and iron rolls, material sparking and flashing of lubricant occurred under relatively low values of percent sliding. To relieve this problem, phosphate conversion coatings were tried on the premise that contaminating the surface in this way would inhibit welding action during the periods of material work hardening and surface-finish refinement. Not

efficient was found to decrease; also, a marked increase in surface endurance was observed for many material combinations.

Although the test program has not up to now included a regulated effort to compare and evaluate lubricants or their role in the surface-fatigue process, future plans include the testing of several material coatings, lubricants, and their combinations on common gear and cam materials. One important question to be answered is how applicable lubricant test data from a given combination of steel-roll tests might be on other materials.

It has been established that certain lubricants designed to operate under gross sliding conditions can have a very adverse effect on surface fatigue.

Speed: The rate of stress cycling has no significant effect on surface endurance of metals. This is not the case, however, for nonmetallic materials with low heat-conduction properties. Under these conditions, high speed was found to cause blistering or excessive material flow in the test specimens.

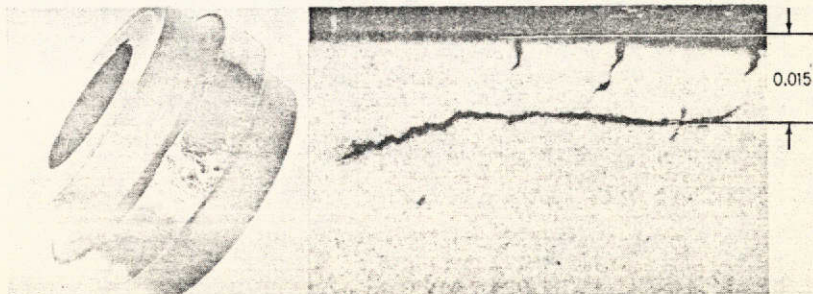
Modulus: Tests have proven that, as shown by the Hertzian equations, contact stress is reduced if materials of lower modulus are used. For example, cast iron run against cast iron shows a higher endurance limit than cast iron on hardened steel. The same is true of cast iron on bronze.

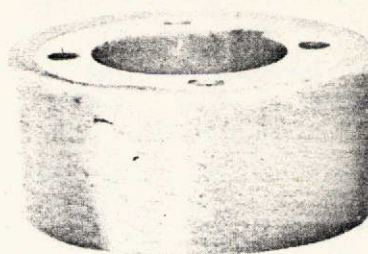
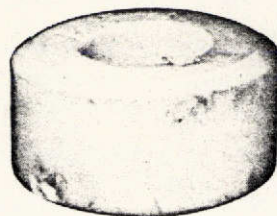
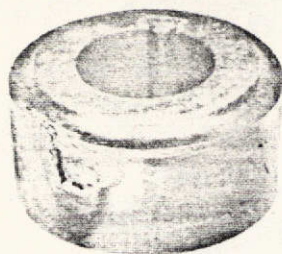
Using a material of lower modulus can provide increased surface endurance and a reduction in dynamic tooth loading resulting from decreased stiffness of the teeth. As an added bonus, gears operating under sporadic lubrication can often survive by virtue of the free graphite inherent in the structure of the iron.

Hardness: Although surface endurance usually increases with hardness in a given material, hardness alone is a dangerous criterion for comparing different materials. For example, SAE 6150 steel at 300 Bhn shows a surface endurance only slightly greater than the much less expensive and easier to machine SAE 1020 at 130 Bhn.

Initially, the mechanical properties of 18% nickel air-hardening tool steel aroused considerable interest as a gear material. Because of its low distortion under heat treatment, good welding properties, very high tensile strength, and toughness, the material seemed ideal. However,

Fig. 4—Nitrided SAE 4150 leaded steel roll (25 Rc core) showing radial and subsurface cracks in zone exhibiting no visible signs of failure. Surface failure occurred at 51 million cycles under rolling only. Calculated maximum Hertzian stress was 197,000 psi.





Typical failure modes for plastics. The laminated phenolic roll has failed by blistering caused by heat buildup resulting from excessive speed. The acetal roll has failed from pitting fatigue similar to that observed on steel specimens. The third roll, of high molecular-weight polyethylene, shows gross deformation caused by material flow. No cracking, pitting, or blistering could be induced in this material.

tests showed that at 50 Rc, surface endurance was almost the same as SAE 4340 at 37 Rc. Tests are now in process on nitrided samples; early results indicate at least a doubling of the surface endurance limit.

Hardened materials have not been extensively tested. It is most economical to avoid the complications of distortion and need for finishing operations by using materials heat-treated before machining. Through normal work hardening and surface-finish refinement, together with the reduction of stress concentration factors, these materials are adequate in many cases.

Early tests on hardened materials have shown the harmful effects of stress concentrations at the edge of rolls, similar to what may occur in gears. To eliminate this problem sharp edges and nicks should be avoided, and generous fillet or corner radii should be used. Although all tests were carefully checked for alignment, edge failure has been common because of the stress concentrations inevitable with hardened materials. As much as 30% increase in surface endurance limit has been shown for test rolls designed to avoid sharp discontinuities at edges. This is a strong argument for careful gear alignment and even slight crowning of gear teeth. Where case hardening is used, premature edge failure can be expected if these factors are overlooked.

Although the Class 20 gray irons have a low endurance limit, Class 30, 45, and others have shown considerable advantages over free-machining steel. Austempering of Meehanite or Class 30 iron approximately doubles surface endurance over untreated iron of similar hardness; this material has proved well suited for gears and cams, probably because of the fine grain structure, excellent dispersion of graphite, and conversion of retained austenite.

Cold-working of the surface lamina of materials with some degree of plasticity builds up a mechanical case which allows the material to carry a greater load. This condition is usually accompanied by an improvement in surface finish. On some iron tests, a hardness increase of 50 to 60 points (Bhn) as well as an improvement of from 100 to 20 rms in surface finish was common.

General Conclusions: No material has been found to have an infinite life-endurance limit. For some time, it was felt that for ductile materials a load stress factor for 40 million cycles represented the point where infinite life could be expected. Similarly, it was expected that something on the order of 500 million cycles might be the turning point on hardened materials. However, enough test history is now available to suggest that no significant change in slope of stress-life curves can be expected in these regions.

Recent tests on leaded 4150 steel have shown higher surface-endurance limits than for the more difficult to machine unleaded types—this in spite of the fact that there is indication of considerable life scatter at load-stress factors above 5000.

Ceramic cast steel of 4150 composition at 270-300 Bhn showed only 35% of the endurance limit found in wrought material at the same hardness. Nodular (ductile) iron of the 100-70-03 grade surpasses the cast steel in this respect and has the added advantage of less sensitivity to lubrication variations.

Tests on nodular irons show a strong dependence on hardness. On the 100-70-03 series at 207-240 Bhn, for 9% sliding $K = 1750$. For the same material in the 240-260 range, $K = 3480$.

Failure of some of the phenolics and other polymers was by material flow rather than by fatigue. An exception is polyurethane which failed by surface cracking; high-density polyethylene exhibited no surface fatigue before severe material flow occurred.

ACKNOWLEDGMENT

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7.0 APPENDIX IV
MATERIALS AND FACTORS OF SAFETY

MATERIALS:

1. SAE 660 Continuous Cast Bronze:

Tensile Strength:	45,000 PSI
Yield Strength:	27,000 PSI
Endurance Limit:	20,000 PSI
Hertzian Limit:	30,000 PSI

2. 4340 Alloy Steel thru hardened to 300 -320 BHN:

Tensile Strength:	150,000 PSI
Yield Strength:	132,000 PSI
Endurance Limit:	50,000 PSI
Hertzian Limit:	169,000 PSI

FACTORS OF SAFETY

WRIST ROTATION & PITCH ASSEMBLIES

<u>Mesh</u>	<u>Material</u>	<u>Allowable Stress (PSI)</u>	<u>Actual Stress (PSI)</u>	<u>Factor of Safety</u>
Ring, Planet	1	20,000 (endurance)	1,669	11.98
		30,000 (Hertz)	15,269	1.96
	2	50,000 (endurance)	1,669	29.95
		169,000 (Hertz)	18,586	9.09
Planet, Sun	2	50,000 (endurance)	1,669	29.95
		169,000 (Hertz)	32,762	5.16

AZIMUTH & ELEVATION DRIVE ASSEMBLIES:

<u>Mesh</u>	<u>Material</u>	<u>Allowable Stress (PSI)</u>	<u>Actual Stress (PSI)</u>	<u>Factor of Safety</u>
Ring, Planet	2	50,000 (endurance)	4583	10.91
		169,000 (Hertz)	31,343	5.39
Planet, Sun	2	50,000 (endurance)	4583	10.91
		169,000 (Hertz)	54,288	3.11

8.0 APPENDIX V
TEST RESULTS

TEST RESULTS

<u>MOTION</u>	<u>SPECIFIED NO LOAD SPEED (DEGREES/SEC)</u>	<u>PRE-MODIFICATION NO LOAD SPEED (DEGREES/SEC)</u>	<u>POST MODIFICATION NO LOAD SPEED (DEGREES/SEC)</u>	<u>ACTUAL SPEED RATIO CHANGE</u>
Azimuth	6.5	No Data	8.3	--
Elevation				
Up	3.7	No Data	4.8	--
Down	3.7	No Data	5.3	--
Wrist Rotation	60	110.8	45.8	2.42
Wrist Pitch				
Up	60	70.0	28.6	2.45
Down	60	140.0	42.4	3.30

9.0 APPENDIX VI

PRICE PROPOSAL

9.0. PRICE PROPOSAL

Our firm fixed price (valid for ninety (90) days) to upgrade the five (5) remaining ESAM Motor Drive Assemblies in the same manner as has been accomplished under this contract is] each, for a total amount of ~~12,400~~ .

Twenty (20) sets of components will be provided for retrofitting the Manipulator Assemblies. Other work to be performed will include disassembly, inspection and cleaning of the five (5) GFE Manipulator Kits and assembly and test of the new transmission assemblies. Testing will consist of no-load and stall load functional testing of each assembly.

The efforts described above could be completed four (4) months after receipt of authorization to proceed.