LMSC-D403169

(NASA-CR-120505) SPAR DEMONSTRATION PROBLEMS (Lockheed Missiles and Space Co.) 128 p HC $5.75 CSCL 20K
N75-10496
Unclas
G3/39 53611

SPAR DEMONSTRATION PROBLEMS

by

C. L. Yen, R. A. Moore, W. D. Whetstone

June 1974

Corrections attached.

FOREWORD

This report was prepared by Lockheed Missiles and Space Company for the National Aeronautics and Space Administration under Contract NAS8-26352. The contract was funded jointly by NASA's Marshall and Langley Centers, and the U. S. Army Missile Command. The Contracting Officer's representatives were L. A. Kiefling of the MSFC Aero-Astrodynamics Laboratory and J. C. Robinson of the Structures Division at Langley.

The demonstration problems presented in this report were prepared and executed by C. L. Yen and R. A. Moore under the direction of W. D. Whetstone.

INTRODUCTION

A series of examples are presented to indicate some of the principal functions of the SPAR system (Reference 1), and to illustrate SPAR's control card-data card structure. For each of the eleven examples, information in the following categories is given:

- A description of the problem and, in most cases, comparisons with analytical solutions.
- A list of the input cards.
- A printout of the Table of Contents of the direct access library into which all SPAR output was directed.
- A few representative plots.

Many comments are embedded in the input cards. All text to the right of a $ symbol is ignored by SPAR's input decoder. In Problem 1 the input to processors TAB and ELD is extensively annotated. Problem 1 also illustrates the most general form of applied loading. In Problem 10 the INC and MOD commands are extensively used in ELD input. Problem 11 illustrates the use of a spectral shift in solving an eigenproblem.

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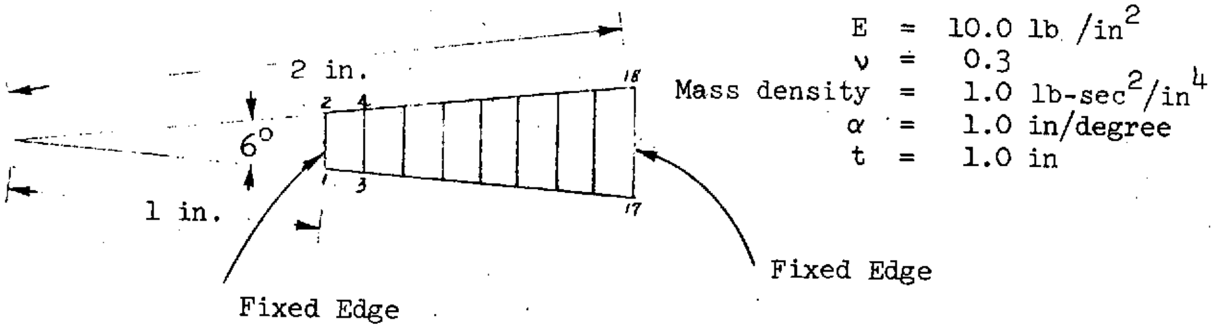
1. RING SECTOR

Solutions were obtained for a segment of a ring in order to explain in detail various data for processors TAB and ELD, and to demonstrate the use of combined loadings in one load case definition in the Q processor. A 6-degree sector of a ring was divided into an 8 x 1 quadrilateral finite element mesh. The geometrical and material properties of the ring are shown on the figure below:



$E = 10.0$ lb /in^2

$\nu = 0.3$

Mass density $= 1.0$ lb-sec^2/in^4

$\alpha = 1.0$ in/degree

$t = 1.0$ in

Linear and quadratic thermal loads were applied to the ring, as well as a combination loading of nodal forces, thermal loads, specified nodal motion, surface pressure, and an inertia load.

```
@XQT TAB                                        . GENERATE BASIC TABLES DEFINING STRUCTURE
 $                                                STRUCTURE HAS 18 NODES .
 $                                                DEGREES OF FREEDOM 1,2,3,4,5 ARE ACTIVE
 START 18   6$
 $                                                TITLE DESIGNATION TO BE ASSOCIATED WITH
 $                                                DIRECT ACCESS LIBRARY (DAL)
 TITLE'   RING WITH COMBINATION LOADINGS
 $                                                EXPLANATORY TEXT MATERIAL TO BE DISPLAYED
 TEXT$
 'A RING UNDER LINEAR/QUADRATIC TEMPERATURE LOADING AND COMBINED LOADINGS
 MATERIAL CONSTANTS$
 $                                                ENTRY 1 OF MATERIAL PROPERTIES IN WHICH
 $                                                MODULUS OF ELASTICITY = 10.0
 $                                                POISSON'S RATIO = 0.3
 $                                                MASS DENSITY = 1.0
 $                                                THERMAL EXPANSION COEFFICIENT = 1.0
      1 10. .3 1. 1.$
 JOINT LOCATIONS$
 $                                                NODAL LOCATIONS DEFINED IN CYLINDRICAL
 $                                                COORDINATES
      FORMAT=2$
 $                                                NODE 1 HAS COORDINATES R=1.0, THETA=0.0,
 $                                                AND Z=0.0
 $                                                NODE 17 HAS COORDINATES R=2.0, THETA=0.0,
 $                                                AND Z=0.0
 $                                                LINEAR INTERPOLATION OF COORDINATES
 $                                                BETWEEN NODES 1 AND 17 FOR NODES
 $                                                3,5,7,9,11,13 AND 15
      1 1. 0. 0. 2. 0. 0. 9 2 2$
 $                                                NODE 2 HAS COORDINATES R=1.0, THETA=6.0
 $                                                AND Z=0.0
 $                                                NODE 18 HAS COORDINATES R=2.0, THETA=6.0.
 $                                                AND Z=0.0
 $                                                LINEAR INTERPOLATION OF COORDINATES
 $                                                BETWEEN NODES 2 AND 18 FOR NODES
 $                                                4,6,8,10,12,14 AND 16
      1 1. 6. 0. 2. 6. 0.$
 JOINT REFERENCE FRAMES$
 $                                                ALL JOINT REFERENCE FRAMES WILL BE
 $                                                TANGENT TO GLOBAL CYLINDRICAL COORDINATE
 $                                                LINES
      NREF=-1$
      1 18 $
 SHELL SECTION PROPERTIES$
 $                                                ENTRY 1 IS ISOTROPIC SECTION WITH T=1.0
      1 1.$
 CONSTRAINT CASE 1$
 $                                                CONSTRAIN ALL DEGREES OF FREEDOM AT ONE
 $                                                END.  CONSTRAINTS AND SPECIFIED MOTIONS
 $                                                AT THE OTHER END.
      ZERO 1 2 3 4 5: 1 2$
      ZERO 1 2 4 5  : 17 18$
      NONZERO 3     : 17 18$
@XQT ELD                                        . READ ELEMENT DEFINITIONS
 E43 $
 $                                                DEFINE A QUADRILATERAL MEMBRANE/BENDING
 $                                                NETWORK. ELEMENT NODE NUMBERS: 1 3 4 2,
 $                                                3 5 6 4, .... (A TOTAL OF 8 ELEMENTS,
 $                                                THE LAST OF WHICH IS 15 17 18 16)
      1 3 4 2   1 8 1$
@XQT TOPO                                       . ANALYZE ELEMENT INTERCONNECTIVITY
```

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

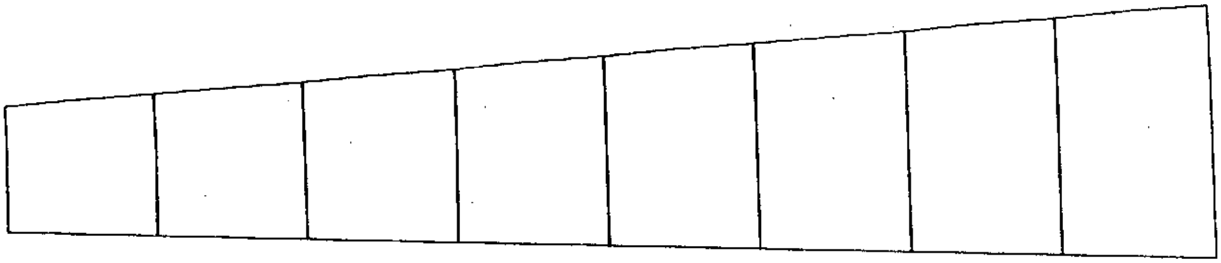
```
@XQT E
@XQT EKS                                  . FORM ELEMENT DATA PACKETS
@XQT K                                    . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT INV                                  . FORM SYSTEM K
@XQT M                                    . FACTOR MATRIX IN K SPAR FORMAT
@XQT Q                                    . FORM SYSTEM M
     RESET MLIB=1$                        . DEFINE APPLIED LOADS
     CASE 1:LINEAR RADIAL TEMPERATURE LOAD
      NODAL TEMPERATURES
 $
 $                                          GENERATE A THERMAL LOADING WITH
 $                                          THE THERMAL EXPANSION COEFFICIENT = 1.
 $                                          FOR THE MATERIAL DEFINED
 $                                          NODES 3,5,7,9,11,13,15   LOAD 0.5
         3 .5 7 2 .625-1 2 1$               NODES 4,6,8,10,12,14,16  LOAD 0.0625
     CASE 2:QUADRATIC RADIAL TEMPERATURE LOAD
      NODAL TEMPERATURES
 $
 $                                          GENERATE A SERIES OF THERMAL LOADS
 $                                          NODES  3  4  LOAD 0.3164
 $                                          NODES  5  6  LOAD 0.3906
 $                                            .
 $                                            .
 $                                            .
 $                                          NODES 15 16  LOAD 0.8787
         3 .3164  2 1$
         5 .3906  2 1$
         7 .4727  2 1$
         9 .5625  2 1$
        11 .6602  2 1$
        13 .7656  2 1$
        15 .8787  2 1$
     CASE 3:COMBINED LOADING
      NODAL FORCES, MOMENTS
 $                                          LOAD ON NODE  9 IN NORMAL DIRECTION,
 $                                          MAGNITUDE=-1.0
 $                                          LOAD ON NODE 10 IN NORMAL DIRECTION,
 $                                          MAGNITUDE= 1.0
         9 3 -1.$
        10 3  1.$
      NODAL TEMPERATURES
 $                                          UNIFORM TEMPERATURE LOAD ALONG ONE EDGE
 $                                          NODES 3,5,7,9,11,13,15  MAGNITUDE=1.
         3 1. 7 2$
      NODAL MOTIONS
 $                                          SPECIFIED MOTION AT ONE END
 $                                          NODES 17 18,  MAGNITUDE=0.1
 $                                          DIRECTION 3 (SPECIFIED AS NONZERO
 $                                          MOTION ON THE CONSTRAINT CASE)
        17 3 .1 2 1$
      NODAL PRESSURES
 $                                          UNIFORM PRESSURE ACROSS THE SURFACE
 $                                          NODES 3-15,  MAGNITUDE=1.0
         3 1. 2 1.0. 7 2$
 $                                          GRAVITY LOAD NORMAL TO THE SURFACE
 $                                          MAGNITUDE=-32.2 OR -1 G
      INERTIA FORCE 3 -32.2$
@XQT DSOL                                 . COMPUTE STATIC SOLUTIONS
 $                                          PRINT NODAL DISPLACEMENTS AND REACTION
 $                                          FORCES FOR ALL SOLUTION CASES
     RESET UP=1,FP=1$
@XQT GSF                                  . GENERATE STRESS DATA
@XQT PSF                                  . PRINT STRESS DATA
@XQT DCU                                  . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
TABLE OF CONTENTS, DAL  1

   RING WITH COMBINATION LOADINGS

                                    F              ROWS   BLK   T    DATA SET NAME
SEQ       RR   DATE    TIME         R     WORDS    /BLK   SIZE  Y    N1    N2    N3     N4
  1       11  060574  114309        0        18       1     18   0  JDF1  BTAB    1      8
  2       12  060574  114309        0        18      18     18   0  JREF  BTAB    2      6
  3       13  060574  114309        0        12       1     12   1  ALTR  BTAB    2      4
  4       14  060574  114309        0        19       1     19   4  NDAL          0      0
  5       15  060574  114310        0        20       1     20   3  TEXT  BTAB    2      1
  6       16  060574  114310        0        10       1     10   1  MATC  BTAB    2      2
  7       17  060574  114311        0        54      18     54   1  JLOC  BTAB    2      5
  8       19  060574  114311        0        18      18     18   0  JREF  BTAB    2      6
  9       20  060574  114312        0        19       1     19   1  SA    BTAB    2     13
 10       21  060574  114312        0        18      18     18   0  CON           1      0
 11       22  060574  114312        0       162      18    162   1  QJJT  BTAB    2     19
 12       28  060574  114315        0       144       9    896   0  DEF   E43    11      4
 13       60  060574  114315        0         2       1      2   0  GD    E43    11      4
 14       61  060574  114315        0        15       1     15   0  GTIT  E43    11      4
 15       62  060574  114315        0        12      12     12   0  NELZ  BTAB    1     11
 16       63  060574  114315        0         5       1      5   0  KE            0      0
 17       64  060574  114315        0         6       1      6   0  NS            0      0
 18       65  060574  114315        0         1       1      1   3  ELTS  NAME    0      0
 19       66  060574  114315        0         1       1      1   0  ELTS  LTYP    0      0
 20       67  060574  114315        0         1       1      1   0  ELTS  NNOD    0      0
 21       77  060574  114315        0         1       1      1   0  ELTS  ISCT    0      0
 22       78  060574  114315        0         1       1      1   0  ELTS  NELS    0      0
 23       79  060574  114315        0         1       1      1   0  ELTS  LES     0      0
 24       80  060574  114318        0      1344      18   1344   0  KMAP         59     10
 25      128  060574  114319        0      1344      18   1344   0  AMAP  0000   59     10
 26      176  060574  114324        0      2772       9    308   4  E43          11      4
 27      275  060574  114321        0        20      20     20   0  DIR   E43    11      4
 28      276  060574  114326        0      2240      18   2240   1  K     SPAR   25     59
 29      356  060574  114330        0      3136      18   3136   1  INV   K       1     59
 30      468  060574  114334        0      2240      18   2240   1  M     SPAR   25     59
 31      548  060574  114339        0       108      18    108  -1  RBMX  VEC     1      0
 32      552  060574  114339        0       108      18    108  -1  RBMX  VEC     2      0
 33      556  060574  114339        0       108      18    108  -1  RBMX  VEC     3      0
 34      560  060574  114339        0        18      18     18  -1  NTMP          0      1
 35      561  060574  114340        0       108      18    108  -1  NFM           0      1
 36      565  060574  114340        0        15       1     15   4  CASE  0000    0      1
 37      566  060574  114340        0        18      18     18  -1  NTMP          0      2
 38      567  060574  114341        0       108      18    108  -1  NFM           0      2
 39      571  060574  114341        0        15       1     15   4  CASE  0000    0      2
 40      572  060574  114342        0        18      18     18  -1  NTMP          0      3
 41      582  060574  114342        0       108      18    108  -1  NMOT          0      3
 42      586  060574  114343        0       108      18    108  -1  NFM           0      3
 43      590  060574  114343        0        15       1     15   4  CASE  0000    0      3
 44      591  060574  114343        0         0       0      0   0  LDIR          1      3
 45      591  060574  114350        0       108      18    108   1  SSOL  U       1      1
 46      595  060574  114351        0       108      18    108   1  SSOL  U       1      2
 47      599  060574  114352        0       108      18    108   1  SSOL  U       1      3
 48      603  060574  114358        0       441       9   5600  -1  STRS  E43     0      1
 49      803  060574  114400        0       441       9   5600  -1  STRS  E43     0      2
 50     1003  060574  114400        0       441       9   5600  -1  STRS  E43     0      3
```

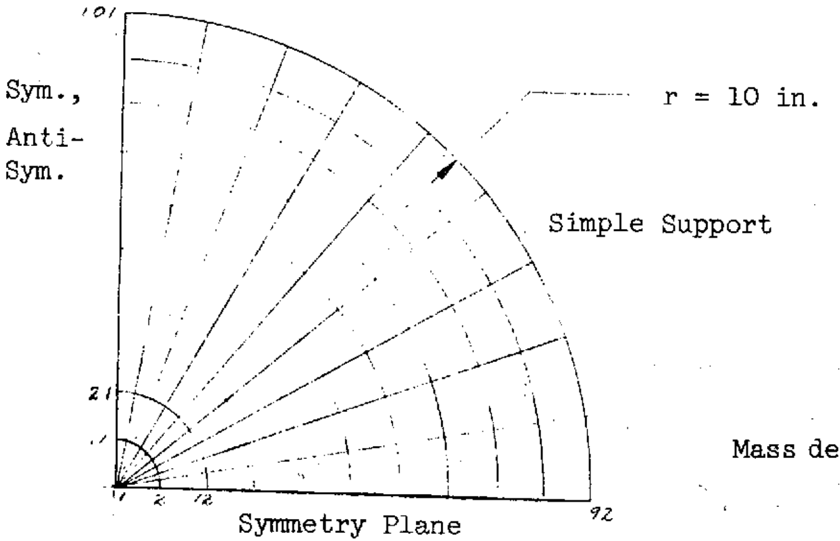
UNDEFORMED STRUCTURE



RING SECTOR

2. VIBRATION OF A CIRCULAR MEMBRANE

Natural frequencies were obtained for lateral vibration of a simply supported circular membrane in uniform tension. A quarter of the membrane was divided into 9 x 10 finite elements to obtain the numerical solution. The geometrical and material properties of the membrane are shown on the figure below:



$E = .3 \times 10^8 \text{ lb/in}^2$
$\nu = .3 \text{ lb/in}^2$
Mass density $= .1 \text{ lb-sec}^2/\text{in}^4$
$t = .1 \text{ in}$

Numerical results are listed below:

	SPAR	Analytical
ω_{01}^2	$.58622 \times 10$	$.57792 \times 10$
ω_{11}^2	$.15003 \times 10^2$	$.14684 \times 10^2$
ω_{21}^2	$.26999 \times 10^2$	$.26337 \times 10^2$
ω_{02}^2	$.31389 \times 10^2$	$.30471 \times 10^2$
ω_{31}^2	$.41982 \times 10^2$	$.40692 \times 10^2$
ω_{12}^2	$.51580 \times 10^2$	$.49224 \times 10^2$

```
@XQT TAB                                        . GENERATE BASIC TABLES DEFINING STRUCTURE
START 101,4 5 6$
TITLE' VIBRATION OF A CIRCULAR MEMBRANE
MATERIAL CONSTANTS$
    1 3.+7 .3 .1 1.-5 1.+4$
JOINT REFERENCE FRAME ASSIGNMENTS$
    NREF=-1: 1 101 1$
JOINT LOCATIONS$
    FORMAT=2: 1 0. 0. 0.: 2 1. 0. 0. 1. 90. 0. 10 1 10: 10 10. 0. 0. 10. 90. 0.$
SHELL SECTION PROPERTIES$
    1 .1$
CONSTRAINT 1$
$
$                                                 CONSTRAINED AT EDGES AND SYMMETRY AT
                                                  THETA=0 AND 90 DEGREE PLANES
    SYMMETRY PLANE=2$
    SYMMETRY PLANE=1$
    ZERO 1 2 3: 92 101 1$
CONSTRAINT 2$
$
$                                                 CONSTRAINED AT EDGES, SYMMETRY AT THETA=0
$                                                 DEGREE PLANE, ANTI-SYMMETRY AT THETA=90
                                                  DEGREE PLANE
    ANTISYMMETRY PLANE=1$
    SYMMETRY PLANE =2$
    ZERO 1: 1$
    ZERO 1 2 3: 92 101 1$
CONSTRAINT 3$
$
$                                                 SAME AS CASE 1 EXCEPT RADIAL DIRECTION IS
                                                  FREE AT EDGES.
    SYMMETRY PLANE=1$
    SYMMETRY PLANE=2$
    ZERO 2 3: 1 101 1$
@XQT ELD                                        . READ ELEMENT DEFINITIONS
E41 $
    2 12 13 3      1 9 9$
E31 $
    1 2 3      2 9 1$
@XQT TOPO
$                                               . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E
$                                               . FORM ELEMENT DATA PACKETS
@XQT EKS
$                                               . INSERT K'S INTO ELEMENT DATA PACKETS
@XQT K                                          . FORM SYSTEM K
@XQT INV                                        . FACTOR MATRIX IN K SPAR FORMAT
    RESET CON=3$
@XQT M
$                                               . FORM SYSTEM M
@XQT Q
    CASE 1: RADIAL TENSION AT EDGES             . DEFINE APPLIED LOADS
    NODAL FORCES, MOMENTS$
    92 1 .872665 2 9: 93 1 1.74533 8 1$
@XQT DSOL
    RESET CON=3$                                . COMPUTE STATIC SOLUTIONS
@XQT KG
@XQT LCM                                        . FORM SYSTEM KG, UNIFORM TENSION
$                                               . FORM LINEAR COMBINATION OF MATRICES
$                                                 FORM STIFFNESS MATRIX FOR RADIALLY
@XQT INV                                          STRESSED CIRCULAR MEMBRANE
    RESET K=K+KG$
@XQT EIG
$                                               . SOLVE SYSTEM EIGENPROBLEM
                                                  FIND VIBRATIONAL MODES OF THE STRESSED
```

```
$                                      CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 1
    RESET K=K+KG,INIT=8$
PRINT 0 0 0 0 1$
@XQT INV
    RESET K=K+KG,CON=2$
@XQT EIG
$                                      FIND VIBRATIONAL MODES OF THE STRESSED
$                                      CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 2
    RESET K=K+KG,INIT=8,CON=2$
    PRINT 0 0 0 0 -1$
@XQT DCU                               EXECUTE DATA COMPLEX UTILITY PROGRAM
    TOC 1$
```

```
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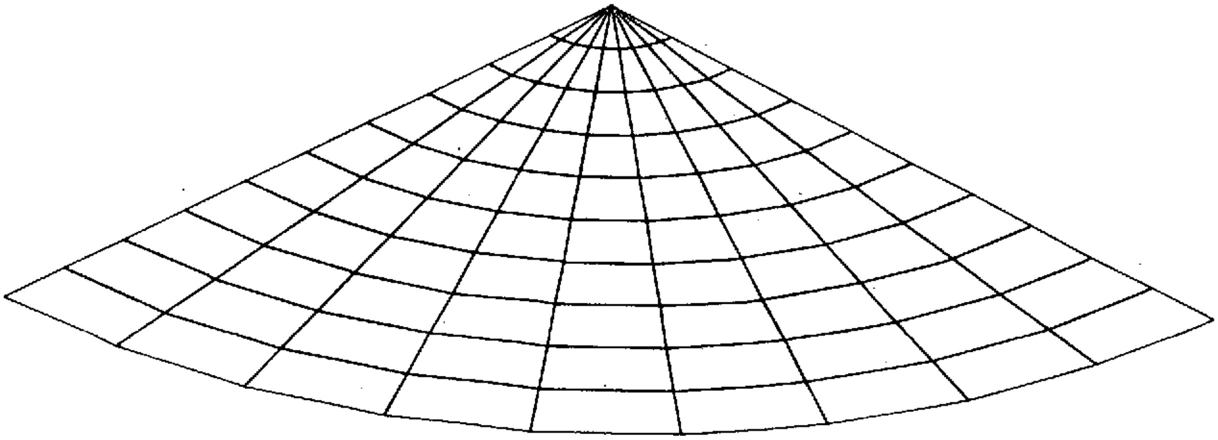
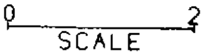
 VIBRATION OF A CIRCULAR MEMBRANE

                                   E              ROWS     BLK   T    DATA SET NAME
SEQ         RR   DATE     TIME    R    WORDS    /BLK    SIZE   Y    N1    N2     N3     N4
  1         11 051474  042706    0       18       1       18   0 JDF1  BTAB     1      8
  2        -12 051474  042706    0      101     101      101   0 JREF  BTAB     2      6
  3         16 051474  042706    0       12       1       12   1 ALTR  BTAB     2      4
  4         17 051474  042706    0       19       1       19   4 NDAL            0      0
  5         18 051474  042706    0       10       1       10   1 MATC  BTAB     2      2
  6         19 051474  042706    0      101     101      101   0 JREF  BTAB     2      6
  7         23 051474  042707    0      303     101      303   1 JLOC  BTAB     2      5
  8         34 051474  042707    0       19       1       19   1 SA    BTAB     2     13
  9         35 051474  042707    0      101     101      101   0 CON.            1      0
 10         39 051474  042708    0      101     101      101   0 CON             2      0
 11         43 051474  042709    0      101     101      101   0 CON             3      0
 12         47 051474  042709    0      909     101      909   1 QJJT  BTAB     2     19
 13         80 051474  042715    0     1296      81      896   0 DEF   E41      9      4
 14        144 051474  042715    0        2       1        2   0 GD    E41      9      4
 15        145 051474  042715    0       15       1       15   0 GTIT  E41      9      4
 16        146 051474  042715    0      144       9      896   0 DEF   E31      6      3
 17        178 051474  042715    0        2       1        2   0 GD    E31      6      3
 18        179 051474  042715    0       15       1       15   0 GTIT  E31      6      3
 19        180 051474  042715    0       12      12       12   0 NELZ  BTAB     1     11
 20        181 051474  042715    0       10       2       10   0 KE              0      0
 21        191 051474  042715    0       12       2       12   0 NS              0      0
 22        192 051474  042716    0        2       2        2   3 ELTS  NAME     0      0
 23        193 051474  042716    0        2       2        2   0 ELTS  LTYP     0      0
 24        194 051474  042717    0        2       2        2   0 ELTS  NNOD     0      0
 25        195 051474  042717    0        2       2        2   0 ELTS  ISCT     0      0
 26        196 051474  042718    0        2       2        2   0 ELTS  NELS     0      0
 27        197 051474  042718    0        2       2        2   0 ELTS  LE3      0      0
 28        198 051474  042721    0     2688     101     1344   0 KMAP          453     10
 29        294 051474  042726    0     9408     101     1344   0 AMAP  @@@(  1137     78
 30        630 051474  042734    0     1260       9      140   4 E31             6      3
 31        675 051474  042730    0       20      20       20   0 DIR   E31      6      3
 32        676 051474  042736    0    13608      81      168   4 E41             9      4
 33       1162 051474  042732    0       20      20       20   0 DIR   E41      9      4
 34       1163 051474  042743    0     6720     101     2240   1 M     SPAR     9    453
 35       1403 051474  042745    0      606     101      606  -1 NFM             0      1
 36       1425 051474  042745    0       15       1       15   4 CASE  @@@@     0      1
 37       1426 051474  042745    0        0       0        0   0 LDIR            1      1
 38       1426 051474  042751    0     6720     101     2240   1 K     SPAR     9    453
 39       1666 051474  042755    0     9408     101     3136   1 INV   K        3   1137
 40       2002 051474  042758    0      606     101      606   1 SSOL  U        0      1
 41       2035 051474  042806    0     6720     101     2240   1 KG    SPAR     9    453
 42       2275 051474  042807    0     6720     101     2240   1 K*KG  SPAR     9    453
 43       2513 051474  042815    0    12544     101     3136   1 INV   K+KG     1   1137
 44       2961 051474  042816    0        8       8        8  -1 VIBR  EVAL     1      0
 45       2962 051474  042817    0     4848     101      606  -1 VIBR  U        1      0
 46       3138 051474  042919    0    12544     101     3136   1 INV   K+KG     2   1137
 47       3586 051474  042921    0        8       8        8  -1 VIBR  EVAL     2      0
 48       3587 051474  042922    0     4848     101      606  -1 VIBR  U        2      0
```

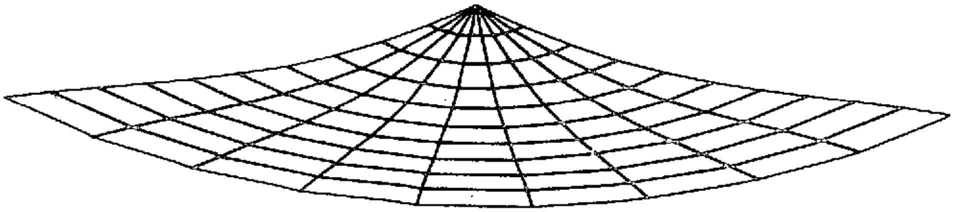
UNDEFORMED STRUCTURE



CIRCULAR MEMBRANE



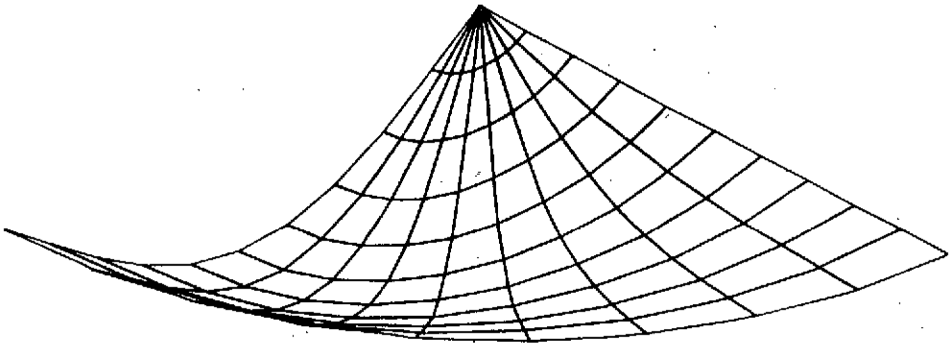
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.385344\text{X}10^{+00}$



CIRCULAR MEMBRANE

0 3
SCALE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.616456\text{X}10^{+00}$

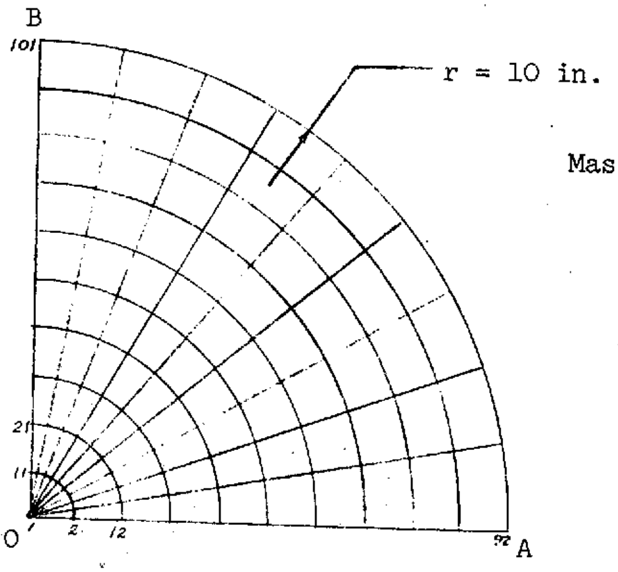


CIRCULAR MEMBRANE

0 3
SCALE

3. CIRCULAR PLATE

The quarter-circle finite element mesh shown on Figure 3-1 was used to solve the problems described in Sections 3.1, 3.2, and 3.3.



$E = .3 \times 10^8$ lb /in^2

$\nu = .3$

Mass density = .1 lb-sec^2/in^4

$t = .1$ in

Fig. 3-1 Circular Plate

3.1 Vibrational Characteristics. With edge AB fixed, and edge OA a symmetry plane, solutions were obtained for (1) OB a symmetry plane, and (2) OB an anti-symmetry plane. SPAR results are compared with Reference 2, pp 449, 450.

	OB = Symmetry Plane			OB = Anti-Symmetry Plane
	ω_{10}^2	ω_{12}^2	ω_{20}^2	ω_{11}^2
SPAR	$.28991 \times 10^4$	$.33770 \times 10^5$	$.43923 \times 10^5$	$.12547 \times 10^5$
Ref. 2	$.28638 \times 10^4$	$.33470 \quad 10^5$	$.43474 \times 10^5$	$.12371 \times 10^5$

3.2 Buckling due to Compressive Edge Load. Both OA and OB are symmetry planes. Unit uniform compression is developed by applying $N_r = 1$ along free edge AB. With edge AB then clamped, N_r (critical) is computed. The result indicated below is compared with Reference 3, pp 389-392.

	N_r (Critical)
SPAR	$.40298 \times 10^3$
Ref. 3	$.40326 \times 10^3$

3.3 <u>Lateral Deflection, including Prestress Effects</u>. OA and OB are symmetry planes. Uniform tension is developed by applying $N_R = 100$ along free edge AB. AB is then clamped, and unit uniform lateral pressure is applied. The result indicated below is compared with Reference 4, pp 391-393.

	Lateral Deflection at Center
SPAR	$.75739 \times 10^{-1}$
Ref. 4	$.75960 \times 10^{-1}$

```
@XQT TAB                                 . GENERATE BASIC TABLES DEFINING STRUCTURE
START 101,6$
TITLE!  CIRCULAR PLATE PROBLEMS
TEXT$
!      THE FOLLOWING CIRCULAR PLATE PROBLEMS ARE SOLVED IN THIS RUN!
!        1  VIBRATION OF A CIRCULAR PLATE
!        2  BUCKLING OF A CIRCULAR PLATE
!        3  LATERAL DEFLECTION OF A PRESTRESSED CIRCULAR PLATE
!
MATEIRAL CONSTANTS$
     1 .3+8  .3  .1  .1-4$
JOINT LOCATIONS$
     FORMAT=2$
         1  .0  .0  .0
         2  1.0  .0  .0   1.0 90.0  .0  10 1 10
        10 10.0  .0  .0  10.0 90.0  .0
JOINT REFERENCE FRAME ASSIGNMENTS
     NREF=-1:  1,101$
SHELL SECTION PROPERTIES
         1 0.1$
CONSTRAINT CASE 1$
$                                         CLAMPED AT EDGES, SYMMETRY AT BOTH THETA=
$                                         0 AND 90 DEGREE PLANES
     SYMMETRY PLANE=2$
     SYMMETRY PLANE=1$
     ZERO 2,3,4,5: 92,101$
CONSTRAINT CASE 2$
$                                         CLAMPED AT EDGES, SYMMETRY AT THETA=0
$                                         PLANE AND ANTI-SYMMETRY AT THETA=90
$                                         DEGREE PLANE.
     ANTISYMMETRY PLANE=1$
     SYMMETRY PLANE=2$
     ZERO 1,2,3,4,5: 92,101$
     ZERO 1: 1$
@XQT ELD                                 . READ ELEMENT DEFINITIONS
E33 $
     1 2 3  2 9$
E43 $
     2 12 13 3  1 9 9$
@XQT TOPO                                . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E                                   . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                 . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                   . FORM SYSTEM M
$
@XQT K                                   . FORM SYSTEM K
$
@XQT INV                                 . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT EIG                                 . SOLVE SYSTEM EIGENPROBLEM
$                                          FIND VIBRATIONAL MODES OF THE CIRCULAR
$                                          PLATE UNDER CONSTRAINT CONDITION 1
       RESET INIT=8$
@XQT INV
       RESET CON=2
@XQT EIG
$                                          FIND VIBRATIONAL MODES OF THE CIRCULAR
$                                          PLATE UNDER CONSTRAINT CONDITION 2
       RESET INIT=8,CON=2$
@XQT Q                                   . DEFINE APPLIED LOADS
```

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

```
    CASE 1' UNIFORM RADIAL COMPRESSIVE FORCE AT EDGES
      NODAL FORCES, MOMENTS
        92 1 -0.87266463  2 9$
        93 1 -1.7453293   8 .1$
    CASE 2' UNIFORM LATERAL PRESSURE LOAD
      NODAL PRESURE
        1  1.0  101 1$
@XQT DSOL                                 . COMPUTE STATIC SOLUTIONS
    RESET L1=1,L2=1$
@XQT KG                                   . FORM SYSTEM KG
@XQT EIG
$                                           FIND CRITICAL RADIAL COMPRESSIVE LOAD.
     RESET PROB=STAB,INIT=4,UPPER=1,NDYN=10$
@XQT LCM                                  . FORM LINEAR COMBINATION OF MATRICES
$                                           FORM STIFFNESS MATRIX FOR PRE-COMPRESSED
$                                           CIRCULAR PLATE, COMPRESSIVE FORCE=100.
     RESET R=KMKG,Q1=K,Q2=KG,C2=100.0$
@XQT INV
     RESET K=KMKG$
@XQT DSOL
$                                           FIND LATERAL DEFLECTION OF PRE-COMPRESSED
$                                           CIRCULAR PLATE UNDER UNIFORM PRESSURE
     RESET K=KMKG,L1=2,L2=2,UP=1$
@XQT DCU                                  . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

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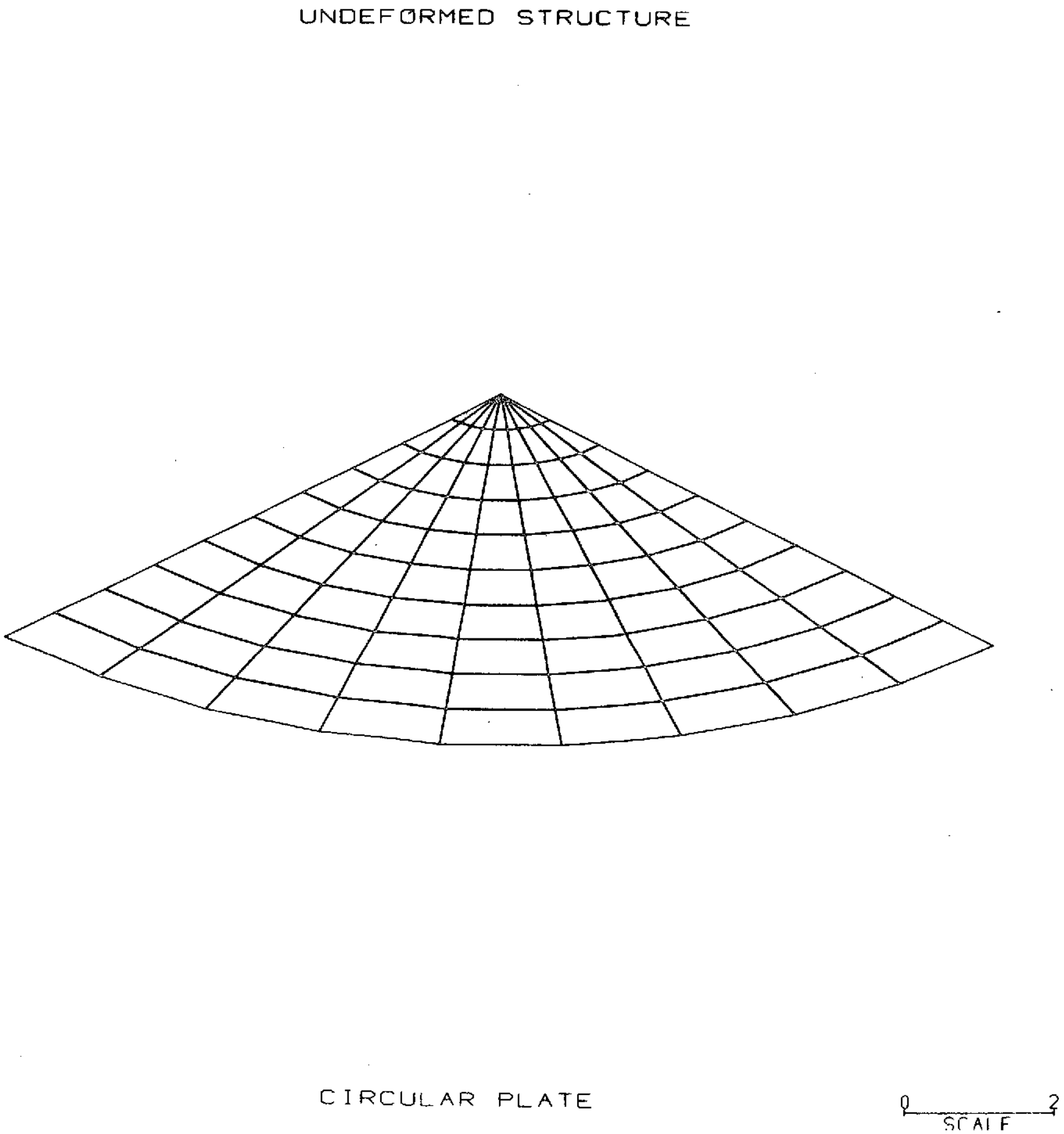
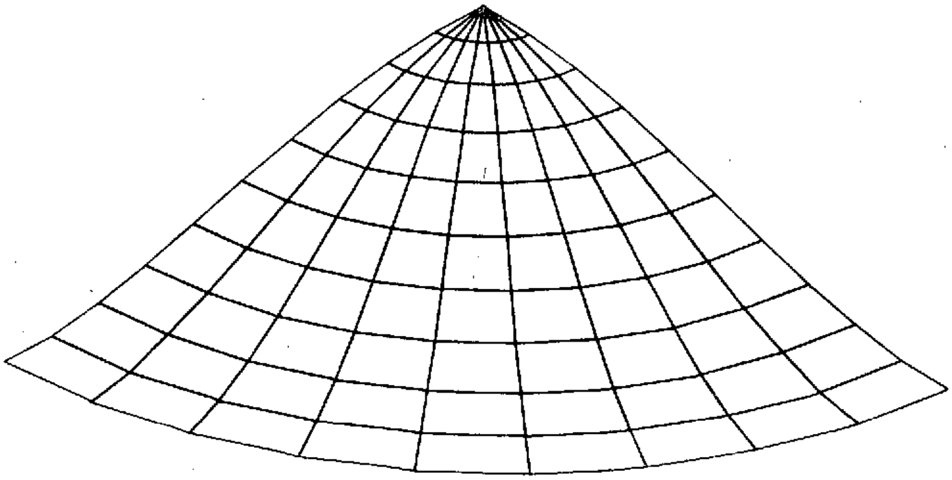
CIRCULAR PLATE PROBLEMS

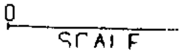
```
                                E         ROWS   BLK  T    DATA SET NAME
SEQ     RR   DATE    TIME     R  WORDS  /BLK   SIZE  Y   N1   N2     N3   N4
  1     11 051374 213904     0     18      1     18  0  JDF1 BTAB     1    8
  2     12 051374 213904     0    101    101    101  0  JREF BTAB     2    6
  3     16 051374 213904     0     12      1     12  1  ALTR BTAB     2    4
  4     17 051374 213904     0     19      1     19  4  NDAL          0    0
  5     18 051374 213904     0     80      4     80  3  TEXT BTAB     2    1
  6     21 051374 213905     0     10      1     10  1  MATC BTAB     2    2
  7     22 051374 213905     0    303    101    303  1  JLOC BTAB     2    5
  8     33 051374 213905     0    101    101    101  0  JREF BTAB     2    6
  9     37 051374 213906     0     19      1     19  1  SA   BTAB     2   13
 10     38 051374 213906     0    101    101    101  0  CON           1    0
 11     42 051374 213907     0    101    101    101  0  CON           2    0
 12     46 051374 213907     0    909    101    909  1  QJJT BTAB     2   19
 13     79 051374 213911     0    144      9    896  0  DEF  E33      8    3
 14    111 051374 213911     0      2      1      2  0  GD   E33      8    3
 15    112 051374 213911     0     15      1     15  0  GTIT E33      8    3
 16    113 051374 213912     0   1296     81    896  0  DEF  E43     11    4
 17    177 051374 213912     0      2      1      2  0  GD   E43     11    4
 18    178 051374 213912     0     15      1     15  0  GTIT E43     11    4
 19    179 051374 213912     0     12     12     12  0  NELZ BTAB     1   11
 20    180 051374 213912     0     10      2     10  0  KE            0    0
 21    190 051374 213913     0     12      2     12  0  NS            0    0
 22    191 051374 213913     0      2      2      2  3  ELTS NAME     0    0
 23    192 051374 213913     0      2      2      2  0  ELTS LTYP     0    0
 24    193 051374 213913     0      2      2      2  0  ELTS NNOD     0    0
 25    194 051374 213914     0      2      2      2  0  ELTS ISCT     0    0
 26    195 051374 213914     0      2      2      2  0  ELTS NELS     0    0
 27    196 051374 213915     0      2      2      2  0  ELTS LE3      0    0
 28    197 051374 213918     0   2688    101   1344  0  KMAP        453   30
 29    293 051374 213921     0   9408    101   1344  0  AMAP @@@(  1137   78
 30    629 051374 213930     0   2016      9    224  4  E33           8    3
 31    701 051374 213924     0     20     20     20  0  DIR  E33      8    3
 32    702 051374 213937     0  24948     81    308  4  E43          11    4
 33   1593 051374 213927     0     20     20     20  0  DIR  E43     11    4
 34   1594 051374 213954     0  13440    101   2240  1  M    SPAR    25  453
 35   2074 051374 213956     0    606    101    606 -1  NFM           0    1
 36   2096 051374 213957     0     15      1     15  4  CASE @@@@     0    1
 37   2097 051374 213958     0    606    101    606 -1  NFM           0    2
 38   2119 051374 213959     0     15      1     15  4  CASE @@@@     0    2
 39   2120 051374 213959     0      0      0      0  0  LDIR          1    2
 40   2120 051374 214008     0  13440    101   2240  1  K    SPAR    25  453
 41   2609 051374 214023     0  31360    101   3136  1  INV  K        1 1137
 42   3729 051374 214026     0      8      8      8 -1  VIBR EVAL     1    0
 43   3730 051374 214027     0   4848    101    606 -1  VIBR U        1    0
 44   3906 051374 214215     0  28224    101   3136  1  INV  K        2 1137
 45   4914 051374 214218     0      8      8      8 -1  VIBR EVAL     2    0
 46   4915 051374 214219     0   4848    101    606 -1  VIBR U        2    0
 47   5091 051374 214359     0    606    101    606  1  SSOL U        0    1
 48   5113 051374 214416     0  13440    101   2240  1  KG   SPAR    25  453
 49   5593 051374 214518     0      6      6      6 -1  STAB EVAL     1    0
 50   5594 051374 214519     0   3636    101    606 -1  STAB U        1    0
 51   5726 051374 214649     0  13440    101   2240  1  KMKG SPAR    25  453
 52   6206 051374 214713     0  31360    101   3136  1  INV  KMKG     1 1137
 53   7326 051374 214723     1    606    101    606  1  SSOL U        0    2
```


UNDEFORMED STRUCTURE
CIRCULAR PLATE
0
2
SCALE

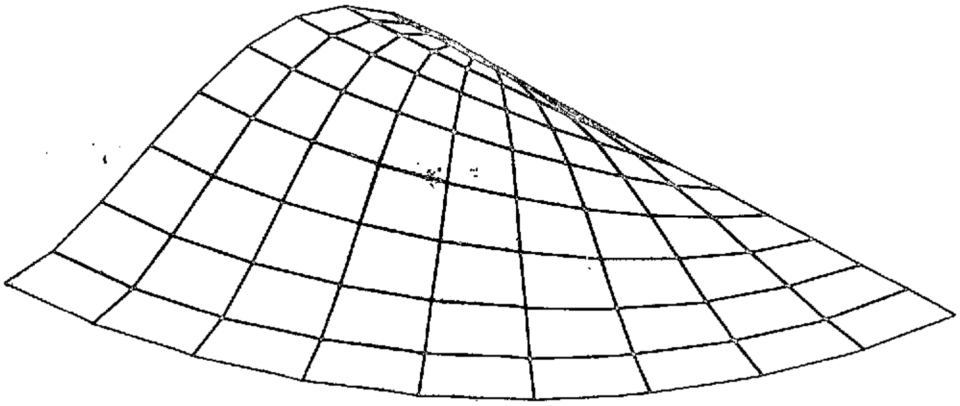
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.856945 \times 10^{+01}$



CIRCULAR PLATE



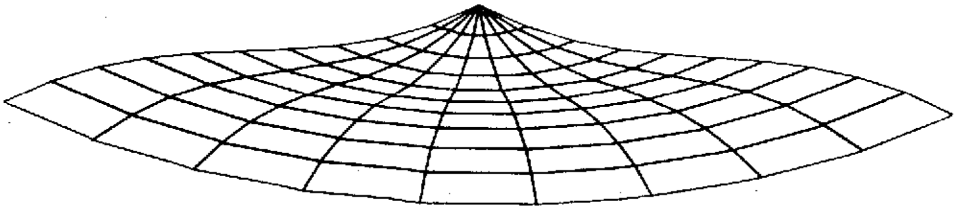
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.178274\text{X}10^{+02}$



CIRCULAR PLATE

0 3
SCALE

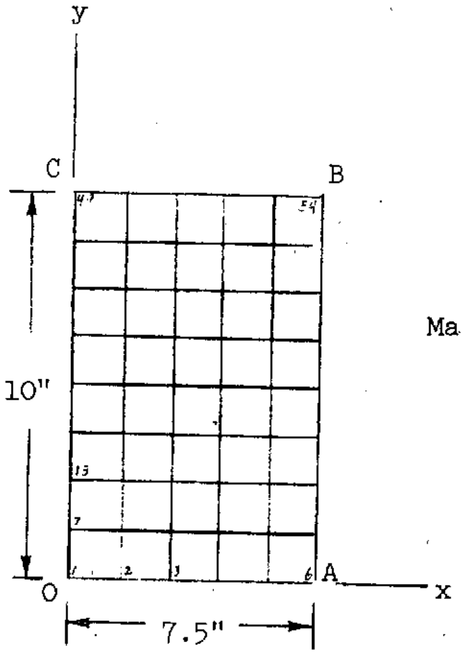
SEQ 1 BUCKLING MODE. CRITICAL LOAD = $.402798 \times 10^{+03}$



CIRCULAR PLATE

4. RECTANGULAR PLATE

The 5 by 8 rectangular mesh shown on Figure 4-1 was used to solve the problems described in Sections 4.1-4.9.



$E = .3 \times 10^8$ lb/in^2

$\nu = .3$

$\alpha = .1 \times 10^{-4}$ in degrees

Mass density = .1 lb-sec^2/in^4

Thickness = .1 in

Fig. 4-1 Rectangular Plate

4.1 Vibrational Characteristics. Edges AB and BC are simply supported; and OA and OC are in symmetry planes. Results are compared with Reference 2, p 443, Eq. (188)

	ω_{11}^2	ω_{13}^2	ω_{31}^2
SPAR	$.12906 \times 10^4$	$.19435 \times 10^5$	$.48406 \times 10^5$
Ref. 2	$.12906 \times 10^4$	$.19428 \times 10^5$	$.48336 \times 10^5$

4.2 Buckling due to Unidirectional Compression. OA and OC are in symmetry planes. With AB free, a unit compressive edge load, $N_y = -1$, is applied along BC. AB and BC are then simply supported, and the critical value of N_y computed. Results are compared with Reference 3, pp 351-353

	N_y (Critical)
SPAR	$.52306 \times 10^3$
Ref. 3	$.52304 \times 10^3$

4.3 Effect of Prestress on Buckling Load. OA and OC are in symmetry planes. With edge BC free, a tensile load, $N_x = 2000$, is applied uniformly along AB. Effects on lateral stiffness of this constant load are included in the subsequent analysis. A unit edge load, $N_y = -1$, is applied along BC. AB and BC are then simply supported and N_y (critical) is computed. Results are compared with Reference 3, pp 356-358.

	N_y (Critical)
SPAR	$.12704 \times 10^4$
Ref. 3	$.12700 \times 10^4$

4.4 Buckling due to Uniform Compression. OA and OC are in symmetry planes. Uniform compression, $N = N_x = N_y = -1$, is applied along AB and BC. AB and BC are then simply supported and N (critical) is computed. Results are compared with Reference 3, pp 356-358.

	N (Critical)
SPAR	$.18830 \times 10^3$
Ref. 3	$.18829 \times 10^3$

4.5 Effects of Prestress on Vibrational Characteristics. OA and OC are in symmetry planes. The plate is uniformly pre-tensioned by applying $N_x = N_y = 1000$, along AB and BC. With AB and BC simply supported, vibrational modes were then computed. Results are compared with Reference 2, p 435, 443.

	ω_{11}^2	ω_{13}^2	ω_{31}^2
SPAR	$.81445 \times 10^4$	$.46029 \times 10^5$	$.90354 \times 10^5$
Ref. 2	$.81444 \times 10^4$	$.46022 \times 10^5$	$.90282 \times 10^5$

4.6 Buckling due to Linearly Varying Edge Load. OC is in a symmetry plane. With horizontal edge OA and BC free, edge AB is loaded with N_x varying linearly from N = 1 at A to N = -1 at B. Edges OA, AB, and BC are then simply supported and N (critical) computed. Results are compared with Reference 3, pp 373-379.

	N (Critical)
SPAR	$.69434 \times 10^4$
Ref. 3	$.65346 \times 10^4$

4.7 Buckling due to Edge Shear. A constant unit shear load, N_{xy}, is applied to the edges. All edges are then simply supported, and the critical shear load computed. Results are compared with Reference 3, pp 379-393.

	N_{xy} (Critical)
SPAR	$.36190 \times 10^4$
Ref. 3	$.36152 \times 10^4$

4.8 Buckling due to Thermal Load. OA and OC are in symmetry planes. Edges AB and BC are simply supported. The critical temperature, T (critical), producing lateral buckling is computed. Results are compared with Reference 3, pp 356-360, and Reference 10, Chapter 14.

	T (Critical)
SPAR	.43937 x 10
Ref. 3	.43935 x 10

4.9 Effect of Thermal Prestress on Vibrational Characteristics. OA and OC are in symmetry planes. AB and BC are simply supported. Effects of a thermal load, T = 2, are included in computing vibrational modes. Results are compared with Reference 2, p 435, p 443.

	ω_{11}^2	ω_{13}^2	ω_{31}^2
SPAR	$.70315 \times 10^3$	$.17156 \times 10^5$	$.44810 \times 10^5$
Ref. 2	$.70307 \times 10^3$	$.17149 \times 10^5$	$.44741 \times 10^5$

```
@XQT TAB                                    . GENERATE BASIC TABLES DEFINING STRUCTURE
START 54,6$
TITLE'RECTANGULAR PLATE PROBLEMS$
TEXT$
'      THE FOLLOWING PLATE PROBLEMS ARE SOLVED IN THIS RUN:
'        1 FREE VIBRATION OF A RECTANGULAR PLATE.
'        2 BUCKLING OF A RECTANGULAR PLATE, COMPRESSED IN ONE DIRECTION.
'        3 BUCKLING OF A PRE-STRESSED RECTANGULAR PLATE.
'        4 BUCKLING OF A RECTANGULAR PLATE, UNIFORM COMPRESSION.
'        5 FREE VIBRATION OF A PRE-STRESSED RECTANGULAR PLATE.
'        6 BUCKLING OF A RECTANGULAR PLATE, BENDING LOAD.
'        7 BUCKLING OF A RECTANGULAR PLATE, SHEAR LOAD.
'        8 BUCKLING OF A RECTANGULAR PLATE, TEMPRATURE LOAD.
'        9 FREE VIBRATION OF A HEATED RECTANGULAR PLATE.
'
MATERIAL CONSTANTS
    1  .3+8  .3  .1  .1-4$
JOINT LOCATIONS
    1 0.0 0.0 0.0  7.5 0.0 0.0  6 1 9
    6 0.0 10. 0.0  7.5 10. 0.0
SHELL SECTION PROPERTIES
    1 0.1$
CONSTRAINT CASE 1$
$                                             SYMMETRY AT BOUNDARY X=0.,Y=0. AND SIMPLY
$                                             SUPPORTED AT BOUNDARY X=7.5, Y=10.
    SYMMETRY PLANE=1
    SYMMETRY PLANE=2
    ZERO  3,4:      6,48,6$
    ZERO  3,5:      49,53$
    ZERO  3:        54$
CONSTRAINT CASE 2$
$                                             SYMMETRY AT X=0., SIMPLY SUPPORTED AT
$                                             Y=0., X=7.5 AND Y=10.
    SYMMETRY PLANE=1
    ZERO 2:         25$
    ZERO 3:          1,6:  12,54,6:  49,53$
    ZERO 4:         12,48,6$
    ZERO 5:          1,5:  49 53$
CONSTRAINT CASE 3$
$                                             SIMPLY SUPPORTED AT ALL BOUNDARIES.
    ZERO 1:          1,6$
    ZERO 2:          1,49,6$
    ZERO 3:          1,6:  12,54,6:  7,49,6:  50,53$
    ZERO 4:         12,48,6:  7,43,6$
    ZERO 5:          2,5:  50 53$
CONSTRAINT CASE 4$
$                                             SAME AS CASE 1, ADDITIONALLY, MOTIONS
$                                             IN X- AND Y-DIRECTION ARE CONSTRAINED AT
$                                             THE BOUNDARY X=7.5 AND Y=10.
    SYMMETRY PLANE=1
    SYMMETRY PLANE=2
    ZERO 1,2,3,4:  6,48,6$
    ZERO 1,2,3,5:  49,53$
    ZERO 1,2,3:    54$
@XQT ELD                                    . READ ELEMENT DEFINITIONS
E43 $
    1 2 8 7  1 5 8
@XQT TOPO                                   . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                      . FORM ELEMENT DATA PACKETS
$
```

```
@XQT EKS                                        . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                          . FORM SYSTEM M
$
@XQT K                                          . FORM SYSTEM K
$
@XQT INV                                        . FACTOR K
$
@XQT EIG                                        . SOLVE SYSTEM EIGENPROBLEM
$                                                 FIND FREQUENCIES FOR FREE VIBRATION OF
$                                                 THE SIMPLY SUPPORTED RECTANGULAR PLATE.
     RESET INIT=6,MAXIT=1$
@XQT Q                                          . DEFINE APPLIED LOADS
     CASE 1: COMPRESSIVE FORCES IN Y-DIRECTION. APPLY AT Y=10.
       NODAL FORCES, MOMENTS
          49 2 -0.75     2 5$
          50 2 -1.5      4 1$
     CASE 2: COMPRESSIVE FORCES IN X- AND Y-DIRECTION. APPLY AT Y=10. AND X=7.5
       NODAL FORCES, MOMENTS
          49 2 -0.75     2 5$
          50 2 -1.5      4 1$
           6 1 -0.625    2 48$
          12 1 -1.25     7 6$
     CASE 3: BENDING FORCES. APPLY AT X=7.5
       NODAL FORCES, MOMENTS
           6 1  0.57292$
          12 1  0.9375  3 6  -0.3125$
          36 1 -0.3125   3 6  -0.3125$
          54 1 -0.57292$
     CASE 4: SHEAR FORCES. APPLY AT X=7.5 AND Y=10.
       NODAL FORCES, MOMENTS
           6 2  0.625    2 48$
          12 2  1.25     7 6$
          49 1  0.75     2 5$
          50 1  1.5      4 1$
     CASE 5: UNIFORM TEMPERATURE LOAD. APPLY AT ALL NODES
       NODAL TEMPERATURES
           1  1.0  54 1$
     CASE 6: TENSILE FORCES IN X-DIRECTION. APPLY AT X=7.5
       NODAL FORCES, MOMENTS
           6 1  1250.0  2 48$
          12 1  2500.0  7 6$
@XQT DSOL                                       . SOLVE LOAD CASE 1
     RESET L1=1,L2=1$
@XQT KG                                         . FORM SYSTEM KG, LOAD CASE 1
$
@XQT EIG
$                                                 FIND CRITICAL AMPLITUDE, LOAD CASE 1
     RESET PROB=STAB,INIT=6,UPPER=1$
@XQT DSOL                                       . SOLVE LOAD CASE 6
     RESET L1=6,L2=6$
@XQT KG                                         . FORM SYSTEM KG, LOAD CASE 6
     RESET QSEQ=6$
@XQT LCM
$                                                 ADD KG AND K TO FORM STIFFNESS MATRIX FOR
$                                                 THE PRESTRESSED PLATE.
@XQT INV                                        . FACTOR K+KG
     RESET K=K+KG$
@XQT DSOL
$                                                 SOLVE LOAD CASE 1 FOR THE PRESTRESSED
$                                                 PLATE.
```

```
      RESET K=K+KG,L1=1,L2=1$
 @XQT KG
 $                                          FORM GEOMETRICAL STIFFNESS MATRIX FROM
 $                                          LOAD CASE 1 FOR THE PRESTRESSED PLATE.
 @XQT EIG
 $                                          FIND BUCKLING VALUE OF LOAD CASE 1 FOR
 $                                          THE PRESTRESSED PLATE BY LOAD CASE 6
      RESET K=K+KG,PROB=STAB,INIT=4,UPPER=1$
 @XQT DSOL
      RESET L1=2,L2=2$
 @XQT KG                                  . FORM SYSTEM KG, LOAD CASE 2
      RESET QSEQ=2$
 @XQT EIG
 $                                          FIND BUCKLING VALUE OF LOAD CASE 2.
      RESET PROB=STAB,INIT=4,UPPER=1$
 @XQT LCM
 $
 $                                          FORM STIFFNESS MATRIX FOR A UNIFORMLY
                                            PRESTRESSED PLATE.(NX=NY=1000)
      RESET R=KMKG,Q1=K,Q2=KG,C2=-1000.0$
 @XQT INV
      RESET K=KMKG$
 @XQT EIG
 $
 $                                          FIND FREQUENCY FOR FREE VIBRATION OF THE
                                            UNIFORMLY PRESTRESSED PLATE.
      RESET K=KMKG,INIT=6$
 @XQT INV
      RESET CON=2$
 @XQT DSOL                                . SOLVE LOAD CASE 3
      RESET CON=2,L1=3,L2=3$
 @XQT KG
      RESET QSEQ=3$
 @XQT EIG
 $                                          FIND BUCKLING VALUE OF LOAD CASE 3.
      RESET CON=2,PROB=STAB,INIT=8,UPPER=1$
 @XQT INV
      RESET CON=3$
 @XQT DSOL                                . SOLVE LOAD CASE 4
      RESET CON=3,L1=4,L2=4$
 @XQT KG                                  . FORM SYSTEM KG, LOAD CASE 4
      RESET QSEQ=4$
 @XQT EIG
 $                                          FIND BUCKLING VALUE OF LOAD CASE 4.
      RESET CON=3,PROB=STAB,INIT=6,UPPER=1$
 @XQT INV
      RESET CON=4$
 @XQT DSOL                                . SOLVE LOAD CASE 5
      RESET CON=4,L1=5,L2=5$
 @XQT KG                                  . FORM SYSTEM KG, LOAD CASE 5
      RESET QSEQ=5$
 @XQT EIG
 $                                          FIND BUCKLING TEMPERATURE OF THE PLATE
      RESET PROB=STAB,CON=4,INIT=4,UPPER=1$
 @XQT LCM
 $
 $                                          FORM STIFFNESS MATRIX OF THE PLATE AT
                                            TEMPERATURE=2.
      RESET R=KTKG,Q1=K,Q2=KG,C2=2.0$
 @XQT INV
      RESET K=KTKG,CON=4$
 @XQT EIG
 $
 $                                          FIND FREQUENCIES FOR FREE VIBRATION OF
                                            THE HEATED PLATE.
      RESET CON=4,K=KTKG,INIT=6$
 @XQT DCU                                 . EXECUTE DATA COMPLEX UTILITY PROGRAM
       TOC 1$
```

```
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RECTANGULAR PLATE PROBLEMSS

                                       E           ROWS    BLK   T     DATA SET NAME
SEQ        RR   DATE    TIME     R    WORDS    /BLK    SIZE  Y    N1    N2     N3     N4
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  2        12  051374  172618    0       54      54      54  0  JREF  BTAB     2      6
  3        14  051374  172618    0       12       1      12  1  ALTR  BTAB     2      4
  4        15  051374  172619    0       19       1      19  4  NDAL           0      0
  5        16  051374  172619    0      200      10     200  3  TEXT  BTAB     2      1
  6        24  051374  172619    0       10       1      10  1  MATC  BTAB     2      2
  7        25  051374  172619    0      162      54     162  1  JLOC  BTAB     2      5
  8        31  051374  172620    0       19       1      19  1  SA    BTAB     2     13
  9        32  051374  172621    0       54      54      54  0  CON            1      0
 10        34  051374  172621    0       54      54      54  0  CON            2      0
 11        36  051374  172621    0       54      54      54  0  CON            3      0
 12        38  051374  172622    0       54      54      54  0  CON            4      0
 13        40  051374  172622    0      486      54     486  1  QJJT  BTAB     2     19
 14        58  051374  172626    0      640      40     640  0  DEF   E43     11      4
 15        90  051374  172626    0        2       1       2  0  GD    E43     11      4
 16        91  051374  172626    0       15       1      15  0  GTIT  E43     11      4
 17        92  051374  172626    0       12      12      12  0  NELZ  BTAB     1     11
 18        93  051374  172627    0        5       1       5  0  KE             0      0
 19        94  051374  172627    0        6       1       6  0  NS             0      0
 20        95  051374  172627    0        1       1       1  3  ELTS  NAME     0      0
 21       105  051374  172627    0        1       1       1  0  ELTS  LTYP     0      0
 22       106  051374  172627    0        1       1       1  0  ELTS  NNOD     0      0
 23       107  051374  172627    0        1       1       1  0  ELTS  ISCT     0      0
 24       108  051374  172627    0        1       1       1  0  ELTS  NELS     0      0
 25       109  051374  172627    0        1       1       1  0  ELTS  LE3      0      0
 26       110  051374  172630    0     1344      54    1344  0  KMAP         227     19
 27       158  051374  172631    0     2688      54    1344  0  AMAP  0000   387     36
 28       254  051374  172640    0    12320      40     308  4  E43           11      4
 29       694  051374  172634    0       20      20      20  0  DIR   E43     11      4
 30       695  051374  172648    0     6720      54    2240  1  M     SPAR    25    227
 31       935  051374  172649    0      324      54     324 -1  NFM            0      1
 32       947  051374  172650    0       15       1      15  4  CASE  0000     0      1
 33       948  051374  172650    0      324      54     324 -1  NFM            0      2
 34       960  051374  172651    0       15       1      15  4  CASE  0000     0      2
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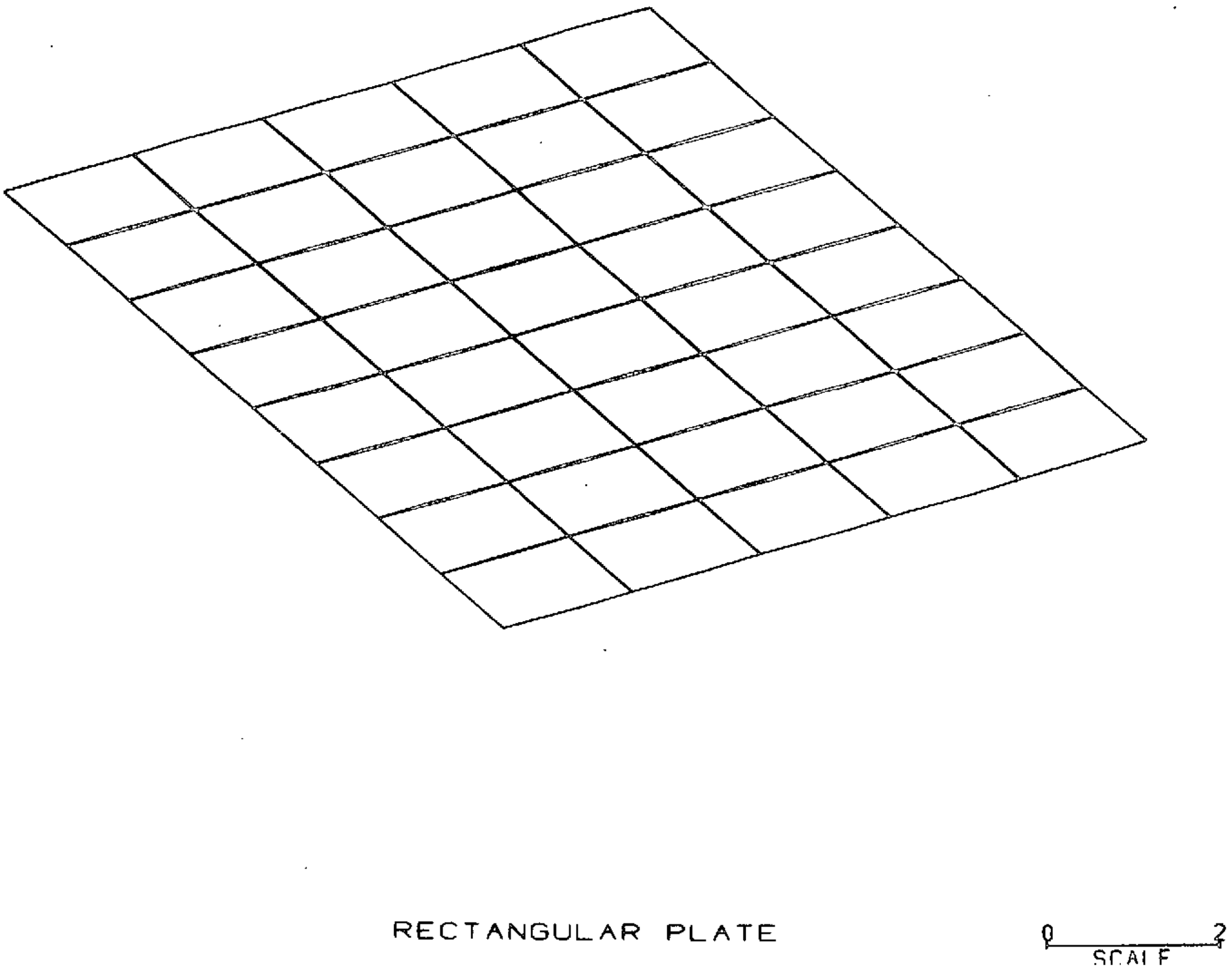
 45      1024  051374  172701    0     6720      54    2240  1  K     SPAR    25    227
 46      1264  051374  172705    0     9408      54    3136  1  INV   K        1    387
 47     -1600  051374  172707    0        6       6       6 -1  VIBR  EVAL     1      0
 48     -1601  051374  172708    0     1944      54     324 -1  VIBR  U        1      0
 49     -1673  051374  172754    0      324      54     324  1  SSOL  U        0      1
 50     -1685  051374  172802    0     6720      54    2240  1  KG    SPAR    25    227
 51     -1925  051374  172804    0        6       6       6 -1  STAB  EVAL     1      0
 52     -1926  051374  172804    0     1944      54     324 -1  STAB  U        1      0
```

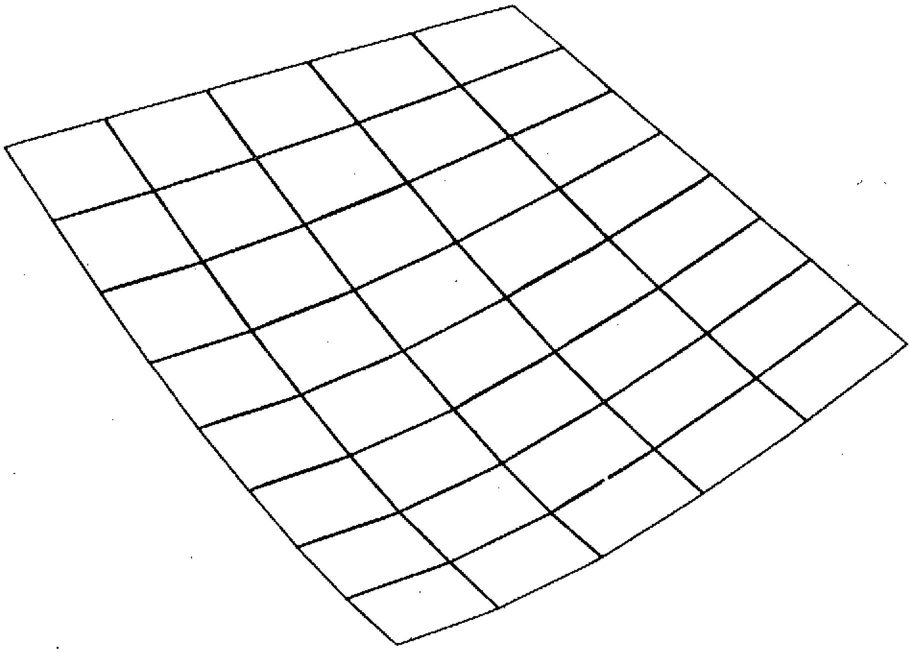
```
53     1998 051374 172842  0    324    54   324  1 SSOL U       0    6
54    -2010 051374 172850  0   6720    54  2240  1 KG   SPAR   25  227
55     2250 051374 172851  0   6720    54  2240  1 K+KG SPAR   25  227
56     2490 051374 172858  0   9408    54  3136  1 INV  K+KG    1  387
57     2826 051374 172902  0    324    54   324  1 SSOL U       0    1
58    -2838 051374 172910  0   6720    54  2240  1 KG   SPAR   25  227
59    -3078 051374 172912  0      4     4     4 -1 STAB EVAL    1    0
60    -3079 051374 172912  0   1296    54   324 -1 STAB U       1    0
61     3136 051374 172950  0    324    54   324  1 SSOL U       0    2
62    -3148 051374 173000  0   6720    54  2240  1 KG   SPAR   25  227
63     3388 051374 173003  0      4     4     4 -1 STAB EVAL    1    0
64     3389 051374 173003  0   1296    54   324 -1 STAB U       1    0
65     3437 051374 173037  0   6720    54  2240  1 KMKG SPAR   25  227
66     3677 051374 173048  0   9408    54  3136  1 INV  KMKG    1  387
67     4013 051374 173053  0      6     6     6 -1 VIBR EVAL    1    0
68     4014 051374 173054  0   1944    54   324 -1 VIBR U       1    0
69     4086 051374 173231  0   9408    54  3136  1 INV  K       2  387
70     4422 051374 173245  0    324    54   324  1 SSOL U       0    3
71    -4434 051374 173254  0   6720    54  2240  1 KG   SPAR   25  227
72     4674 051374 173257  0      8     8     8 -1 STAB EVAL    2    0
73     4675 051374 173258  0   2592    54   324 -1 STAB U       2    0
74     4771 051374 173420  0   9408    54  3136  1 INV  K       3  387
75     5107 051374 173425  0    324    54   324  1 SSOL U       0    4
76    -5119 051374 173434  0   6720    54  2240  1 KG   SPAR   25  227
77     5359 051374 173501  0      6     6     6 -1 STAB EVAL    3    0
78     5360 051374 173502  0   1944    54   324 -1 STAB U       3    0
79     5432 051374 173722  0   9408    54  3136  1 INV  K       4  387
80     5768 051374 173753  0    324    54   324  1 SSOL U       0    5
81     5789 051374 173815  0   6720    54  2240  1 KG   SPAR   25  227
82     6029 051374 173819  0      4     4     4 -1 STAB EVAL    4    0
83     6030 051374 173821  0   1296    54   324 -1 STAB U       4    0
84     6078 051374 173934  0   6720    54  2240  1 KTKG SPAR   25  227
85     6318 051374 174003  0   9408    54  3136  1 INV  KTKG    4  387
86     6654 051374 174020  0      6     6     6 -1 VIBR EVAL    4    0
87     6655 051374 174021  0   1944    54   324 -1 VIBR U       4    0
```

UNDEFORMED STRUCTURE



RECTANGULAR PLATE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.571766X10^{+01}$



RECTANGULAR PLATE

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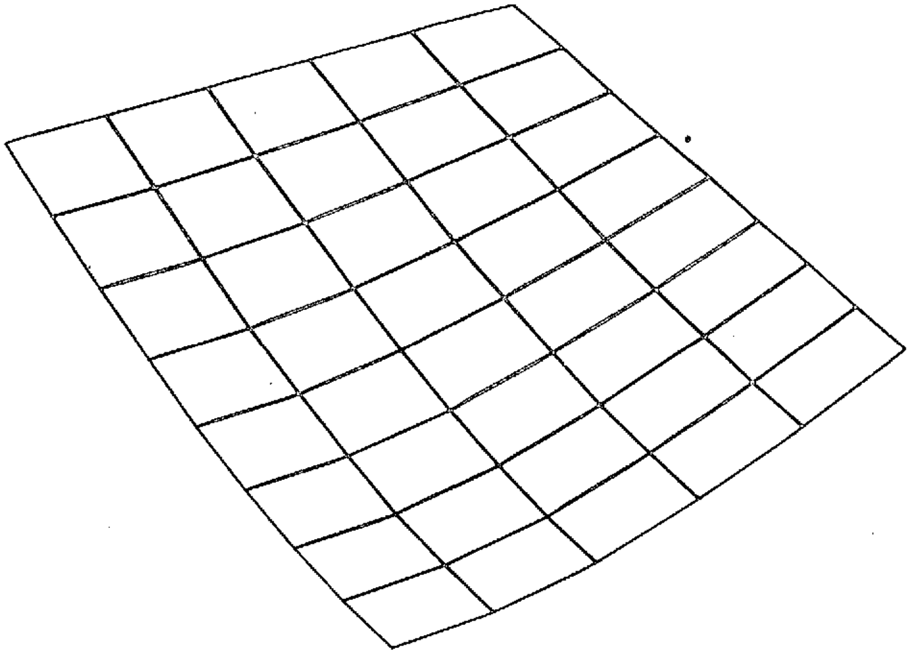
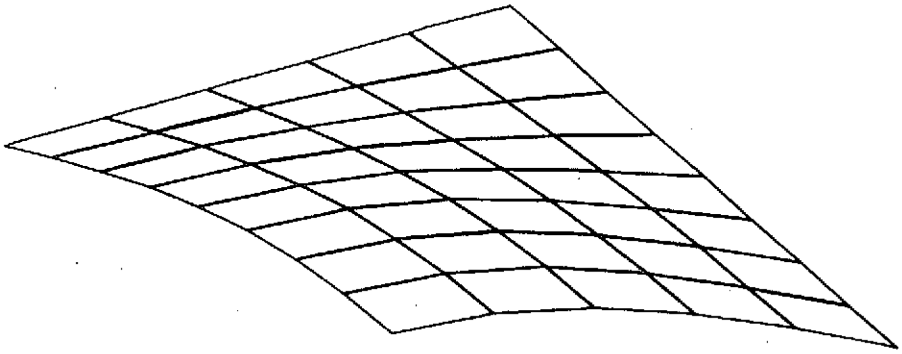


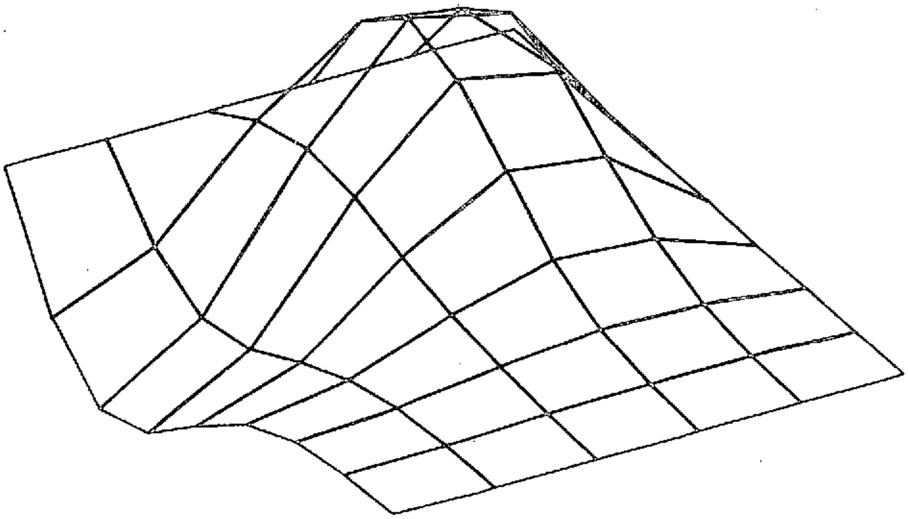
RECTANGULAR PLATE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = $.188302\times10^{+03}$



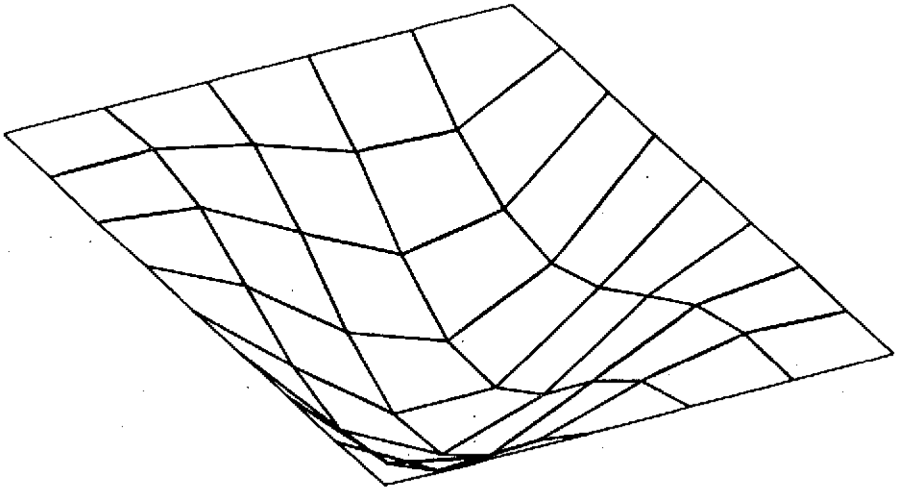
RECTANGULAR PLATE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = .694336X10^{+04}



RECTANGULAR PLATE

SEQ 1 BUCKLING MODE. CRITICAL LOAD = $.361898 \times 10^{+04}$



RECTANGULAR PLATE

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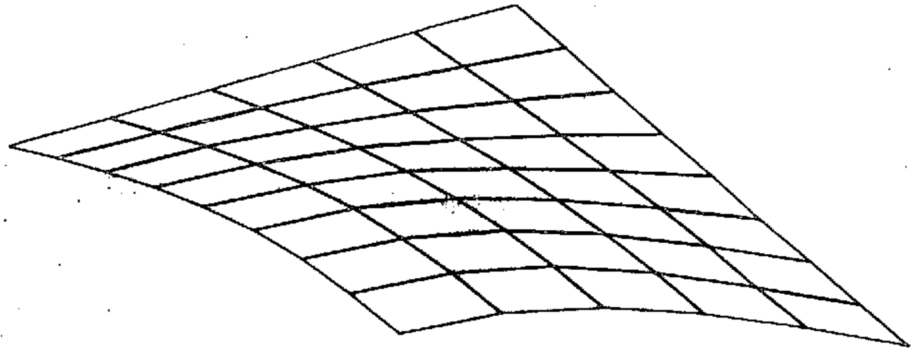
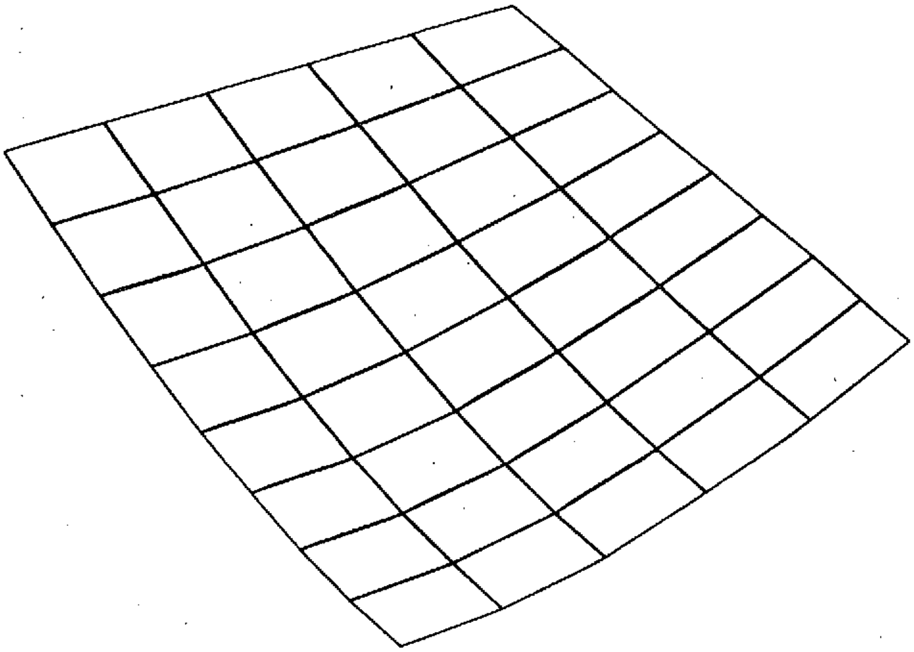


RECTANGULAR PLATE

0 2
SCALE

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.422032 \times 10^{+01}$

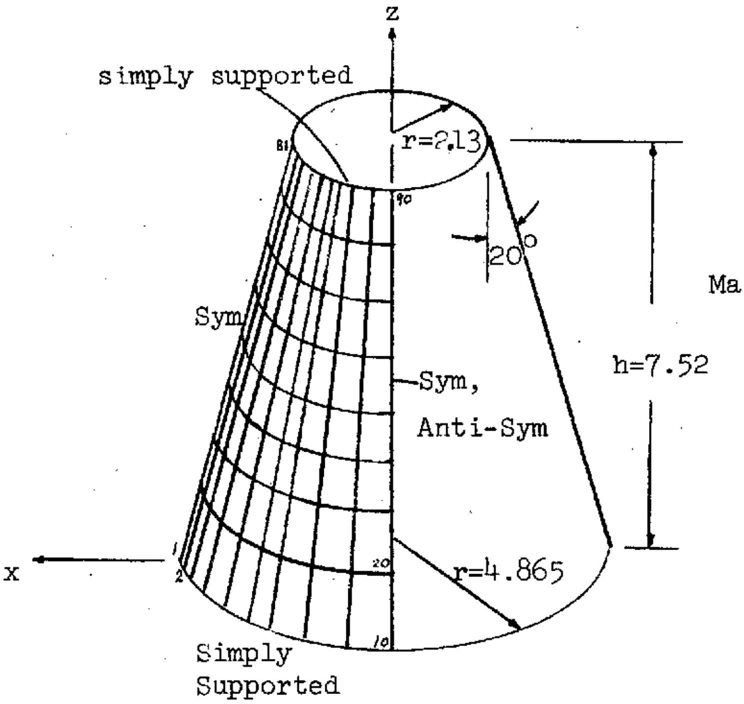


RECTANGULAR PLATE

0 SCALE

5. VIBRATION OF A CONICAL SHELL

Vibrational modes of a simply supported truncated conical shell were obtained. One quarter of the shell was used for the finite element model. The quarter shell was divided into a 9 x 8 finite element network. The seven lowest frequencies were obtained. These seven frequencies correspond to half waves in the meridional direction with wave numbers three to nine in the circumferential direction. The geometrical and material properties of the shell are indicated in the figure below:



$E = .3 \times 10^{8}$ lb /in^{2}

$\nu = .3$

Mass density = .1 lb-sec^{2}/in^{4}

t = .02 in

Results are compared with Reference 11:

Circumferential Wave No. n	SPAR	ω_{on}^{2} Seide
3	$.19244 \times 10^{6}$	$.22078 \times 10^{6}$
4	$.80675 \times 10^{5}$	$.89014 \times 10^{5}$
5	$.52175 \times 10^{5}$	$.56966 \times 10^{5}$
6	$.55235 \times 10^{5}$	$.59480 \times 10^{5}$
7	$.73549 \times 10^{5}$	$.77386 \times 10^{5}$
8	$.99957 \times 10^{5}$	$.10340 \times 10^{6}$
9	$.13408 \times 10^{6}$	$.13724 \times 10^{6}$

Since Reference 11 neglected longitudinal inertia terms, the resulting frequency values are higher than the exact solution. For low values of circumferential wave number, most of the energy of the shell is associated with membrane action. Therefore, neglecting the longitudinal inertia terms results in significant error.

```
@XQT TAB                                         . GENERATE BASIC TABLES DEFINING STRUCTURE
START 90$
TITLE'VIBRATION OF A TRUNCATED CONICAL SHELL
MATERIAL CONSTANTS$
      1 .3+8 .3 .3 1.-5 1.+4$
ALTERNATE REFERENCE FRAMES$
$                                                  DEFINE REFERENCE FRAMES PARALLEL TO
$                                                  MERIDIONAL, RADIAL, CIRCUMFERENTIAL
$                                                  COORDINATE LINES OF THE CONE,
       2 3  0. 2 -20.$
       3 3 10. 2 -20.$
       4 3 20. 2 -20.$
       5 3 30. 2 -20.$
       6 3 40. 2 -20.$
       7 3 50. 2 -20.$
       8 3 60. 2 -20.$
       9 3 70. 2 -20.$
      10 3 80. 2 -20.$
      11 3 90. 2 -20.$
JOINT REFERENCE FRAMES$
      NREF= 2: 1,81,10$
      NREF= 3: 2,82,10$
      NREF= 4: 3,83,10$
      NREF= 5: 4,84,10$
      NREF= 6: 5,85,10$
      NREF= 7: 6,86,10$
      NREF= 8: 7,87,10$
      NREF= 9: 8,88,10$
      NREF=10: 9,89,10$
      NREF=11:10,90,10$
JOINT LOCATIONS$
      FORMAT=2$
       1 2.130 0. 7.52 2.130 90. 7.52 10 1 9$
      10 4.860 0. 0.00 4.865 90. 0.00$
SHELL SECTION PROPERTIES$
      1 .02$
CONSTRAINT CASE 1$
$                                                  SIMPLY SUPPORTED AT BOTH ENDS OF THE CONE
$                                                  SYMMETRY AT THETA=0 PLANE, ANTI-SYMMETRY
$                                                  AT THETA=90 DEGREE PLANE.
      ANTISYMMTRY PLANE=1$
      SYMMETRY PLANE=2$
      ZERD 1,2: 1,10: 81,90$
CONSTRAINT CASE 2$
$                                                  SIMPLY SUPPORTED AT BOTH ENDS OF THE CONE
$                                                  SYMMETRY AT BOTH THETA=0 AND 90 DEGREE
$                                                  PLANES.
      SYMMETRY PLANF=1$
      SYMMETRY PLANE=2$
      ZERO 1,2: 1,10: 81,90$
@XQT ELD                                         . READ ELEMENT DEFINITIONS
E43 $
      1 2 12 11  1 9 8$
@XQT E                                           . FORM FLEMENT DATA PACKETS
@XQT EKS                                         . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                           . FORM SYSTEM K
@XQT INV                                         . FACTOR MATRIX IN K SPAR FORMAT
@XQT M                                           . FORM SYSTEM M
@XQT EIG                                         . SOLVE SYSTEM FIGENPROBLEM
$                                                  FIND FREQUENCIS AND CORRESPONDING MODE
$                                                  SHAPE FOR FREE VIBRATION OF THE CONE
```

```
$                                      UNDER CONSTRAINT CONDITION 1.
     RESET INIT=12$
     PRINT 0 0 0 0 1$
@XQT INV
     RESET CON=2$
@XQT EIG
$                                      FIND FREQUENCIS AND CORRESPONDING MODE
$                                      SHAPE FOR FREE VIBRATION OF THE CONE
$                                      UNDER CONSTRAINT CONDITION 2.
     RESET INIT=12,CON=2$
     PRINT 0 0 0 0 1$
@XQT DCU                               EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
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VIBRATION OF A TRUNCATED CONICAL SHELL

                                    E            SUB-    BLK   T     DATA SET NAME
SEQ      RR   DATE     TIME      R    WORDS    SETS   SIZE   Y    N1    N2     N3    N4
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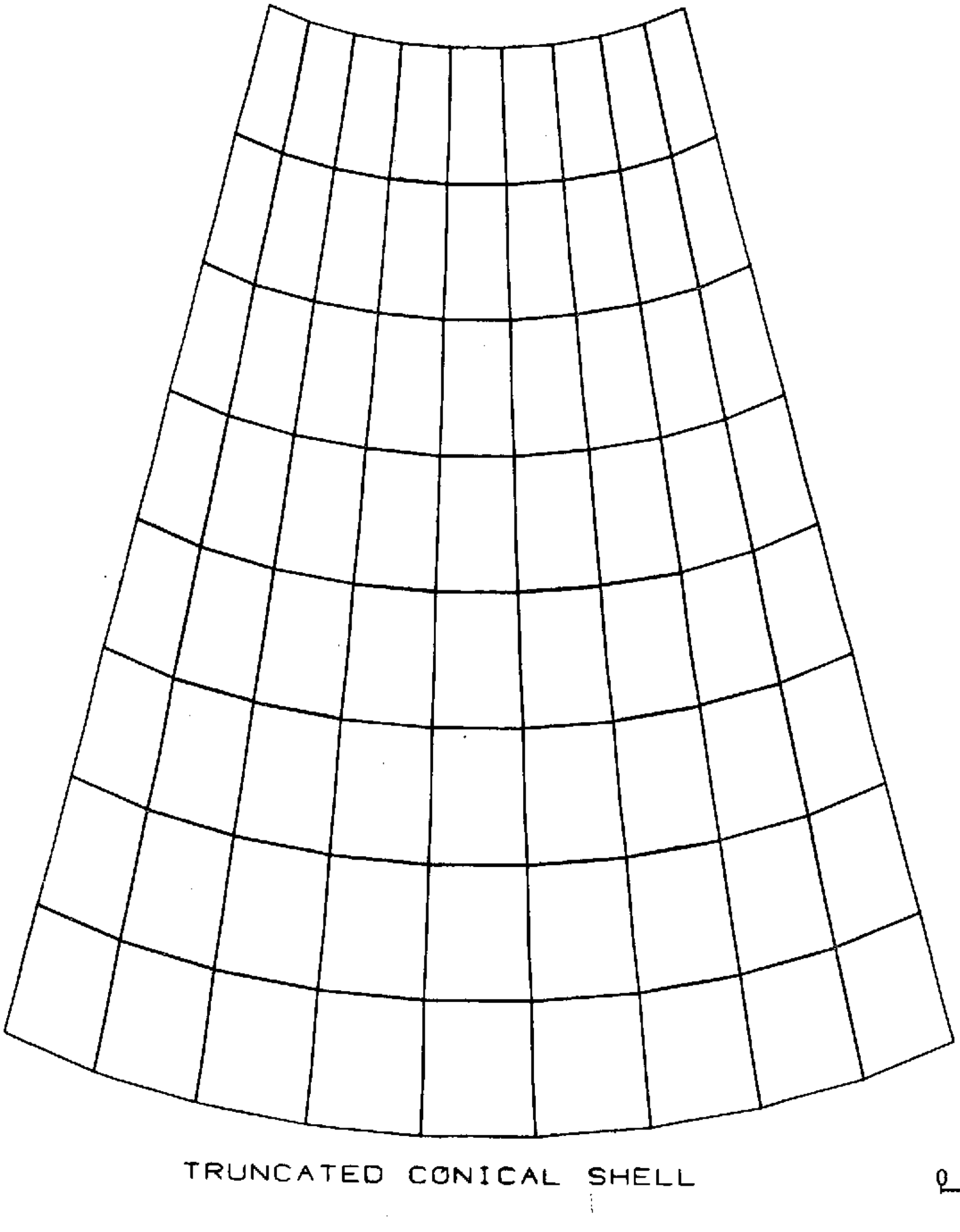
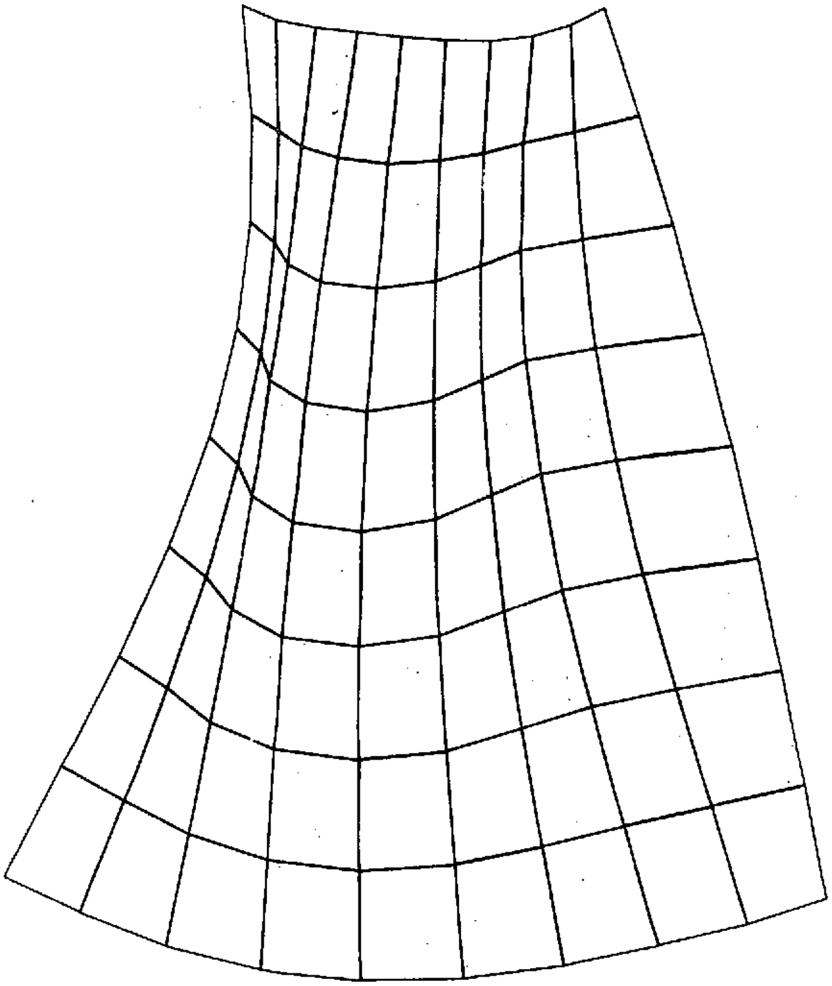
  2     -12  041874  131339      0       90      90      90   0  JREF  BTAB     2     6
  3     -16  041874  131339      0       12       1      12   1  ALTR  BTAB     2     4
  4      17  041874  131339      0       19       1      19   4  NDAL           0     0
  5      18  041874  131339      0       10       1      10   1  MATC  BTAB     2     2
  6      19  041874  131340      0      132      11     132   1  ALTR  BTAB     2     4
  7      24  041874  131340      0       90      90      90   0  JREF  BTAB     2     6
  8      28  041874  131340      0      270      90     270   1  JLOC  BTAB     2     5
  9      38  041874  131340      0       19       1      19   1  SA    BTAB     2    13
 10      39  041874  131342      0       90      90      90   0  CON            1     0
 11      43  041874  131342      0       90      90      90   0  CON            2     0
 12      47  041874  131342      0      810      90     810   1  QJJT  BTAB     2    19
 13      76  041874  131418      0     1152      72     896   0  DEF   E43     11     4
 14     140  041874  131418      0        2       1       2   0  GD    E43     11     4
 15     141  041874  131418      0       15       1      15   0  GTIT  E43     11     4
 16     142  041874  131418      0       12      12      12   0  NELZ  BTAB     1    11
 17     143  041874  131418      0        5       1       5   0  KE             0     0
 18     144  041874  131418      0        6       1       6   0  NS             0     0
 19     145  041874  131418      0        1       1       1   3  ELTS  NAME     0     0
 20     146  041874  131418      0        1       1       1   0  ELTS  LTYP     0     0
 21     156  041874  131418      0        1       1       1   0  ELTS  NNOD     0     0
 22     157  041874  131419      0        1       1       1   0  ELTS  ISCT     0     0
 23     158  041874  131419      0        1       1       1   0  ELTS  NELS     0     0
 24     159  041874  131419      0        1       1       1   0  ELTS  LE3      0     0
 25     160  041874  131421      0     2688      90    1344   0  KMAP         395    27
 26     256  041874  131424      0     8064      90    1344   0  AMAP  ***(  971    78
 27     544  041874  131433      0    22176      72     308   4  E43           11     4
 28    1336  041874  131427      0       20      20      20   0  DIR   E43     11     4
 29    1337  041874  131440      0    15680      90    2240   1  K     SPAR    36   395
 30    1897  041874  131501      0    37632      90    3136   1  INV   K        1   971
 31    3241  041874  131518      0    15680      90    2240   1  M     SPAR    36   395
 32    3801  041874  131521      0       12      12      12  -1  VIBR  EVAL     1     0
 33    3802  041874  131521      0     6480      90     540  -1  VIBR  U        1     0
 34    4042  041874  131855      0    37632      90    3136   1  INV   K        2   971
 35    5386  041874  131858      0       12      12      12  -1  VIBR  EVAL     2     0
 36    5387  041874  131858      0     6480      90     540  -1  VIBR  U        2     0
```

UNDEFORMED STRUCTURE



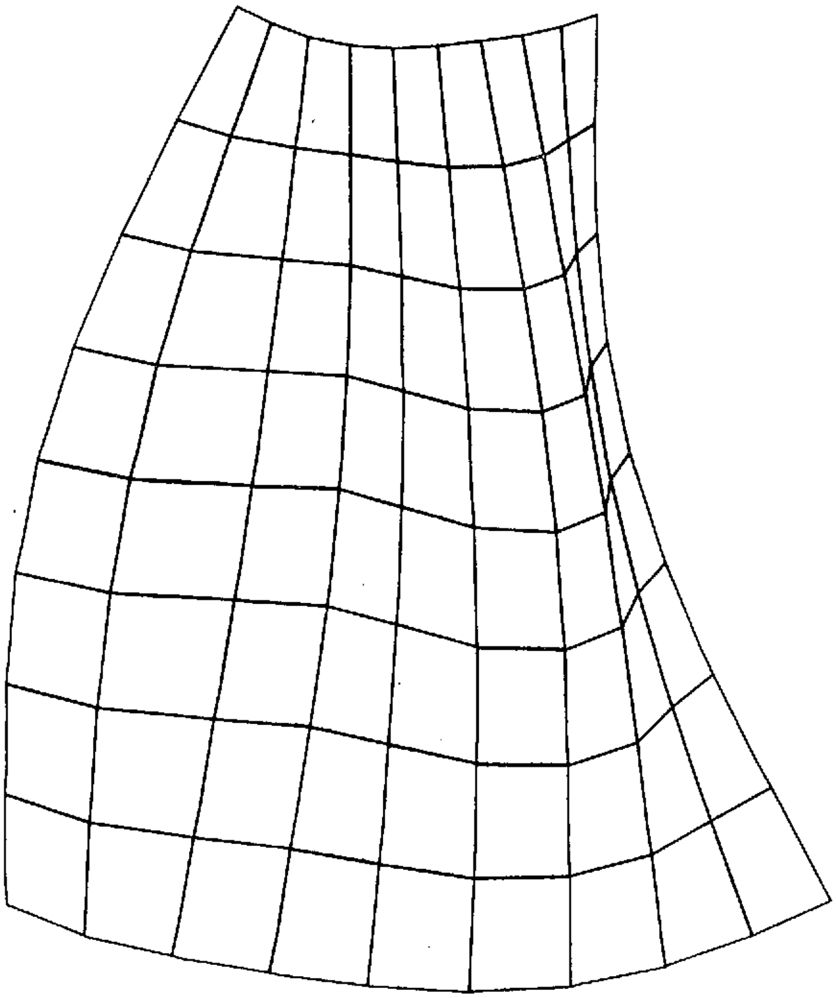
TRUNCATED CONICAL SHELL

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.363540 \times 10^{+02}$



TRUNCATED CONICAL SHELL

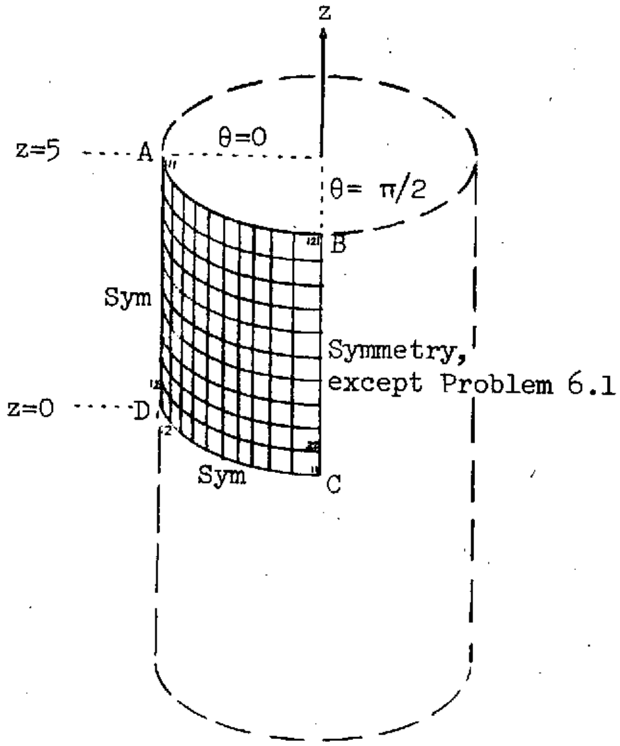
SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.374047\times10^{+02}$



TRUNCATED CONICAL SHELL

6. CYLINDRICAL SHELL

The 10 by 10 cylindrical mesh shown on Figure 6-1 was used to solve the problems discussed in Sections 6.1 through 6.5.



$E = .1 \times 10^8$ lb/in^2

$\nu = .3$

Mass density = .1 lb-sec^2/in^4

Thickness = .1 in

Radius = 10. in

Fig. 6-1 Cylindrical Shell

6.1 Vibrational Characteristics. With edge AB simply supported, vibrational modes were computed with (1) BC in a symmetry plane, and (2) BC in an anti-symmetry plane. Results are compared with Reference 6, p 2154.

Circumferential Wave No., n	ω_{on}^2 SPAR	Reference 6
4	$.1457 \times 10^6$	$.1425 \times 10^6$
5	$.8870 \times 10^5$	$.8863 \times 10^5$
6	$.6393 \times 10^5$	$.6330 \times 10^5$
7	$.5828 \times 10^5$	$.5813 \times 10^5$
8	$.6608 \times 10^5$	$.6596 \times 10^5$
9	$.8548 \times 10^5$	$.8493 \times 10^5$
10	$.1168 \times 10^6$	$.1125 \times 10^6$
11	$.1609 \times 10^6$	$.1587 \times 10^6$

6.2 Buckling due to Axial Compression. A unit compressive edge load, N_z, is applied to AB. AB is then simply supported and critical value of N_z computed. Results are compared with Reference 3, pp 457-458.

	N_z (Critical)
SPAR	$.6068 \times 10^4$
Ref. 3	$.6075 \times 10^4$

6.3 Effects of Pretensioning on Buckling due to External Pressure. The shell is pretensioned by a line load, $N_z = 2200$, along AB. AB is then simply supported, and the critical external pressure, P_c, is computed. Results are compared with Reference 5, p 425, Figure 9.

	P_c
SPAR	$.1415 \times 10^3$
Ref. 5	$.1406 \times 10^3$

6.4 Buckling due to External Pressure. Edge AB is simply supported. The critical pressure, P_c, is computed. Results are compared with Reference 5, p 433, Figure 15.

	P_c
SPAR	$.1084 \times 10^3$
Ref. 5	$.1045 \times 10^3$

6.5 Effects of Pressurization on Buckling due to Axial Compression. With edge AB free, an internal pressure, P = 220, is applied. The stiffening effect of this constant preload is included in the subsequent analysis. A unit compressive load, N_z, is applied along AB. AB is then simply supported, and N_z (critical) computed. Results are compared with Reference 4, p 425, Figure 9.

	N_z (Critical)
SPAR	$.6082 \times 10^4$
Ref. 4	$.6150 \times 10^4$

```
@XQT TAB                                     . GENERATE BASIC TABLES DEFINING STRUCTURE
START 121$
TITLE' CYLINDRICAL SHELL PROBLEMS
TEXT$
'     THE FOLLOWING PROBLEMS IN CYLINDRICAL SHELLS ARE SOLVED IN THIS RUN:
'        1  VIBRATION OF A CYLINDRICAL SHELL
'        2  BUCKLING OF A CYLINDRICAL SHELL DUE TO UNIFORM AXIAL COMPRESSION
'        3  BUCKLING OF A AXIALLY PRE-STRESSED CYLINDER DUE TO PRESSURE LOAD.
'        4  BUCKLING OF A CYLINDRICAL SHELL DUE TO PRESSURE LOAD
'        5  BUCKLING OF A PRESSURIZED CYLINDER DUE TO AXIAL COMPRESSION
'
MATERIAL CONSTANTS
    1  .1+8  .3  .1  .1-4$
JOINT LOCATIONS$
    FORMAT=2$
      1  10.0 0.0  0.  10.0 90.0  0.  11 1 11
     11  10.0 0.0  5.  10.0 90.0  5.
JOINT REFERENCE FRAME ASSIGNMENTS$
    NREF=-1: 1,121$
SHELL SECTION PROPERTIES$
    1 0.1$
CONSTRAINT CASE 1$
$                                             SIMPLY SUPPORTED AT ONE END, SYMMETRY AT
$                                             THE OTHER END AND AT PLANES THETA=0 AND
$                                             90 DEGREES.
    SYMMETRY PLANE=3
    SYMMETRY PLANE=2
    SYMMETRY PLANE=1
     ZERO 1,2: 111,121$
CONSTRAINT CASE 2$
$                                             SIMPLY SUPPORTED AT ONE END, SYMMETRY AT
$                                             THE OTHER END AND AT PLANE THATA=0, ANTI-
$                                             SYMMETRY AT THETA=90 DEGREES,
    SYMMETRY PLANE=3
    SYMMETRY PLANE=2
    ANTISYMMETRY PLANE=1
     ZERO 1,2: 111,121$
CONSTRAINT CASE 3$
$                                             SYMMETRY AT PLANES AT THETA=0 AND 90
$                                             DEGREES, AND AT ONE END OF THE CYLINDER.
    SYMMETRY PLANE=3
    SYMMETRY PLANE=2
    SYMMETRY PLANE=1
@XQT ELD                                      . READ ELEMENT DEFINITIONS
E43 $
    1 2 13 12  1 10 10$
@XQT TOPO                                     . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                        . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                      . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT K                                        . FORM SYSTEM K
$
@XQT INV                                      . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT M                                        . FORM SYSTEM M
$
@XQT EIG                                      . SOLVE SYSTEM EIGENPROBLEM
$                                               FIND FREQUENCIES AND CORRESPONDING MODE
$                                               SHAPES FOR FREE VIBRATION OF THE CYLINDER
```

```
$                                       UNDER CONSTRAINT CASE 1
     RESET INIT=10$
     PRINT 0 0 0 0 1$
@XQT INV
     RESET CON=2$
@XQT EIG
$                                       FIND FREQUENCIES AND CORRESPONDING MODE
$                                       SHAPES FOR FREE VIBRATION OF THE CYLINDE
$                                       UNDER CONSTRAINT CASE 2
     RESET INIT=10,CON=2$
     PRINT 0 0 0 0 1$
@XQT Q                                . DEFINE APPLIED LOADS
     CASE 1! UNIFORM AXIAL COMPRESSIVE FORCE APPLY AT ONE END
       NODAL FORCES, MOMENTS
         111 3  -0.7854  2 10$
         112 3  -1.5708  9 1$
     CASE 2! UNIFORM RADIAL PRESSURE
       NODAL PRESSURE
         1  -1.0   121 1$
@XQT INV
     RESET CON=3$
@XQT DSOL                             . SOLVE LOAD CASE 1
     RESET L1=1,L2=1,CON=3$
@XQT KG                               . FORM SYSTEM KG, AXIAL COMPRESSION
@XQT EIG
$                                       FIND BUCKLING AXIAL LOAD OF THE CYLINDER
     RESET PROBLEM=STABILITY,INIT=4,UPPER=1,NDYN=10$
@XQT LCM                              . FORM LINEAR COMBINATION OF MATRICES
$                                       FORM STIFFNESS MATRIX OF AN AXIALLY PRE-
$                                       STRESSED CYLINDRICAL SHELL, AXIAL TENSIL
$                                       FORCE=2200.
     RESET R=KXKG,Q1=K,Q2=KG,C2=-2200.$
@XQT INV
     RESET K=KXKG,CON=3$
@XQT DSOL
     RESET L1=2,L2=2,CON=3,K=KXKG$
@XQT KG
     RESET QSEQ=2$
@XQT INV
     RESET K=KXKG$
@XQT EIG
$                                       FIND BUCKLING PRESSURE OF PRE-STRESSED
$                                       CYLINDER
     RESET INIT=4,UPPER=1,PROB=STAB,NDYN=10,K=KXKG$
@XQT DSOL
     RESET L1=2,L2=2,CON=3$
@XQT KG                               . FORM SYSTEM KG, UNIFORM PRESSURE
     RESET QSEQ=2$
@XQT EIG
$                                       FIND BUCKLING PRESSURE OF THE CYLINDER
     RESET INIT=4,UPPER=1,PROB=STAB,NDYN=10$
@XQT LCM
$                                       FORM STIFFNESS MATRIX OF A PRESSURIZED
$                                       CYLINDER, INTERNAL PRESSURE=220
     RESET R=KYKG,Q1=K,Q2=KG,C2=-220.$
@XQT INV
     RESET K=KYKG,CON=3$
@XQT DSOL
     RESET K=KYKG,CON=3,L1=1,L2=1$
@XQT KG
@XQT INV
```

```
      RESET K=KYKG$
 @XQT EIG
 $                                        FIND BUCKLING AXIAL LOAD OF THE PRE-
 $                                        PRESSURIZED CYLINDER
      RESET INIT=4,UPPER=1,PROB=STAB,NDYN=10,K=KYKG$
 @XQT DCU                               . EXECUTE DATA COMPLEX UTILITY PROGRAM
      TOC 1$
```

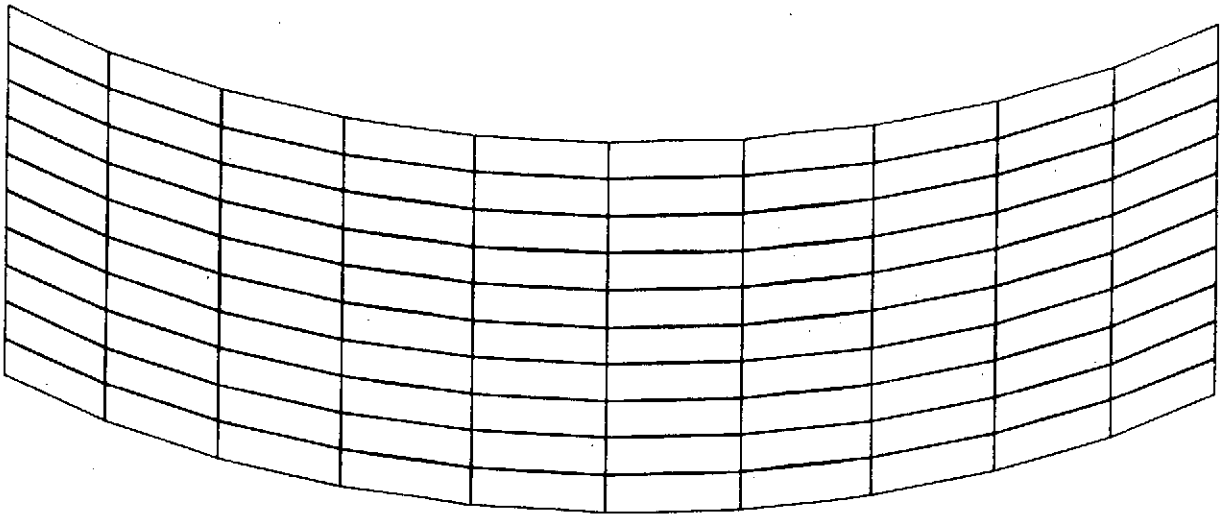
TABLE OF CONTENTS, DAL 1

CYLINDRICAL SHELL PROBLEMS

```
                                   E             ROWS   BLK  T     DATA SET NAME
SEQ      RR   DATE    TIME         R     WORDS   /BLK  SIZE  Y   N1   N2      N3    N4
  1      11 061174  121857         0        18      1    18  0 JDF1 BTAB       1     8
  2     -12 061174  121857         0       121    121   121  0 JREF BTAB       2     6
  3      17 061174  121857         0        12      1    12  1 ALTR BTAB       2     4
  4      18 061174  121857         0        19      1    19  4 NDAL            0     0
  5      19 061174  121858         0       140      7   140  3 TEXT BTAB       2     1
  6      24 061174  121858         0        10      1    10  1 MATC BTAB       2     2
  7      25 061174  121901         0       363    121   363  1 JLOC BTAB       2     5
  8      38 061174  121901         0       121    121   121  0 JREF BTAB       2     6
  9      43 061174  121901         0        19      1    19  1 SA   BTAB       2    13
 10      44 061174  121902         0       121    121   121  0 CON             1     0
 11      49 061174  121903         0       121    121   121  0 CON             2     0
 12      54 061174  121905         0       121    121   121  0 CON             3     0
 13      59 061174  121906         0      1089    121  1089  1 QJJT BTAB       2    19
 14      98 061174  121909         0      1600    100   896  0 DEF  E43       11     4
 15     162 061174  121909         0         2      1     2  0 GD   E43       11     4
 16     163 061174  121910         0        15      1    15  0 GTIT E43       11     4
 17     164 061174  121910         0        12     12    12  0 NELZ BTAB       1    11
 18     165 061174  121910         0         5      1     5  0 KE              0     0
 19     166 061174  121910         0         6      1     6  0 NS              0     0
 20     167 061174  121910         0         1      1     1  3 ELTS NAME       0     0
 21     177 061174  121910         0         1      1     1  0 ELTS LTYP       0     0
 22     178 061174  121910         0         1      1     1  0 ELTS NNOD       0     0
 23     179 061174  121910         0         1      1     1  0 ELTS ISCT       0     0
 24     180 061174  121910         0         1      1     1  0 ELTS NELS       0     0
 25     181 061174  121911         0         1      1     1  0 ELTS LE3        0     0
 26     182 061174  121922         0      4032    121  1344  0 KMAP          541    29
 27     326 061174  121925         0     12096    121  1344  0 AMAP ●●●1    1441    91
 28     758 061174  121954         0     30800    100   308  4 E43            11     4
 29    1858 061174  121936         0        20     20    20  0 DIR  E43       11     4
 30    1859 061174  122004         0     22400    121  2240  1 K    SPAR      36   541
 31    2659 061174  122108         0     53312    121  3136  1 INV  K         1  1441
 32    4563 061174  122128         0     22400    121  2240  1 M    SPAR      36   541
 33    5363 061174  122133         0        10     10    10 -1 VIBR EVAL       1     0
 34    5384 061174  122133         0      7260    121   726 -1 VIBR U          1     0
 35    5624 061174  122536         0     53312    121  3136  1 INV  K          2  1441
 36    7528 061174  122543         0        10     10    10 -1 VIBR EVAL       2     0
 37    7529 061174  122543         0      7260    121   726 -1 VIBR U          2     0
 38    7789 061174  123040         0       726    121   726 -1 MEM             0     1
 39    7815 061174  123040         0        15      1    15  4 CASE ●●●●       0     1
 40    7818 061174  123041         0       726    121   726 -1 MEM             0     2
 41    7851 061174  123042         0        15      1    15  4 CASE ●●●●       0     2
 42    7852 061174  123042         0         0      0     0  0 LDIR            1     2
 43    7852 061174  123209         0     53312    121  3136  1 INV  K          3  1441
 44   -9756 061174  123250         0       726    121   726  1 SSOL U          1     1
 45   -9782 061174  123312         0     22400    121  2240  1 KG   SPAR      36   541
 46  -10582 061174  123325         0         4      4     4 -1 STAB EVAL       1     0
 47  -10583 061174  123325         0      2904    121   726 -1 STAB U          1     0
 48   10687 061174  123712         0     22400    121  2240  1 KXKG SPAR      36   541
 49   11487 061174  123848         0     53312    121  3136  1 INV  KXKG       3  1441
 50  -13391 061174  123900         0       726    121   726  1 SSOL U          1     2
 51  -13417 061174  123918         0     22400    121  2240  1 KG   SPAR      36   541
 52   14217 061174  124009         0     53312    121  3136  1 INV  KXKG       1  1441
```

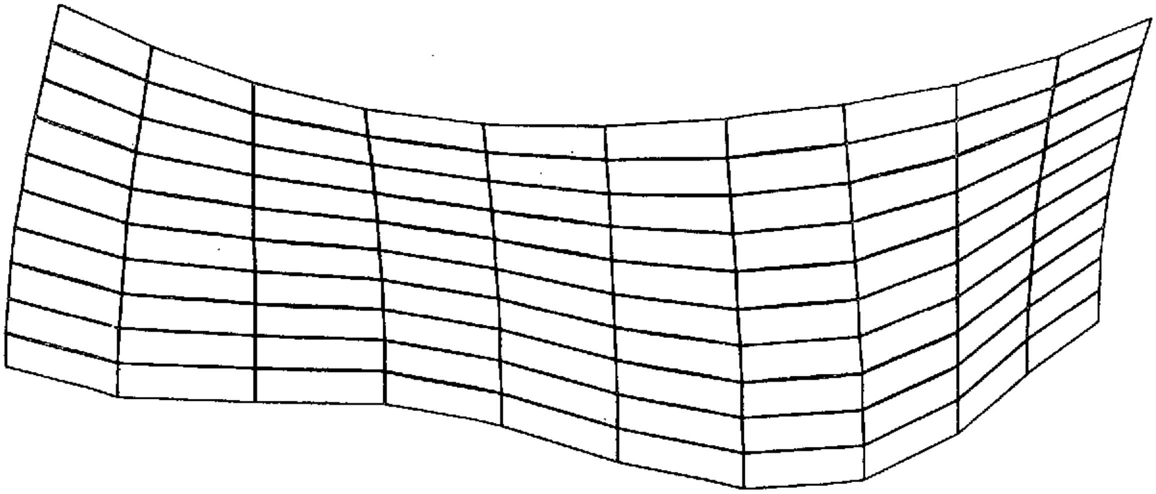
```
53  -16121 061174 124021  0      4    4     4 -1 STAB ĒVAL    1    0
54  -16122 061174 124021  0   2904  121   726 -1 STAB Ū       1    0
55   16226 061174 124308  0    726  121   726  1 SSOL Ū       1    2
56  -16252 061174 124323  0  22400  121  2240  1 KG   S̄PAR   36  541
57  -17052 061174 124327  0      4    4     4 -1 STAB ĒVAL    1    0
58  -17053 061174 124327  0   2904  121   726 -1 STAB Ū       1    0
59   17157 061174 124527  0  22400  121  2240  1 KYKG S̄PAR   36  541
60   17957 061174 124602  0  53312  121  3136  1 INV  K̄YKG    3 1441
61   19870 061174 124612  0    726  121   726  1 SSOL Ū       1    1
62   19896 061174 124639  0  22400  121  2240  1 KG   S̄PAR   36  541
63   20696 061174 124717  0  53312  121  3136  1 INV  K̄YKG    1 1441
64   22600 061174 124731  0      4    4     4 -1 STAB ĒVAL    1    0
65   22601 061174 124731  0   2904  121   726 -1 STAB Ū       1    0
```

UNDEFORMED STRUCTURE



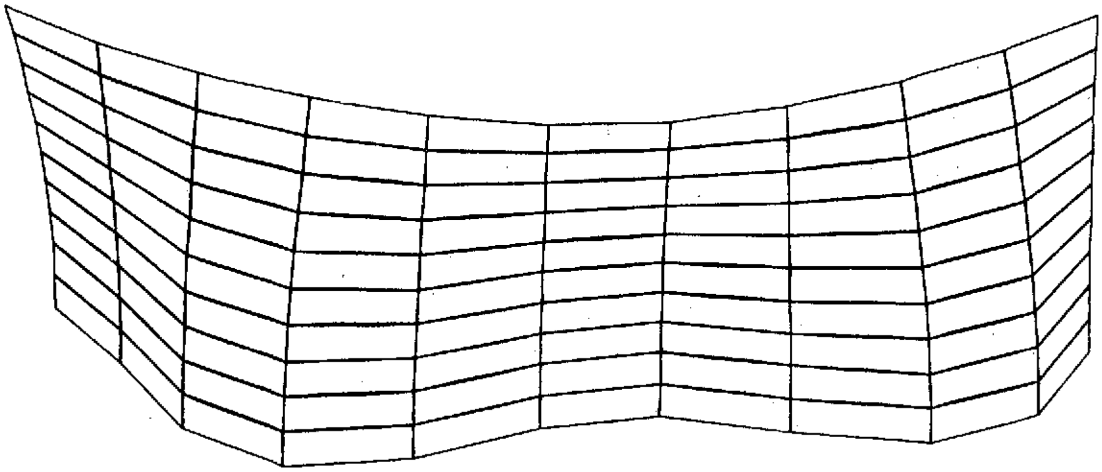
CYLINDRICAL SHELL PROBLEM

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.402415X10^{+02}$



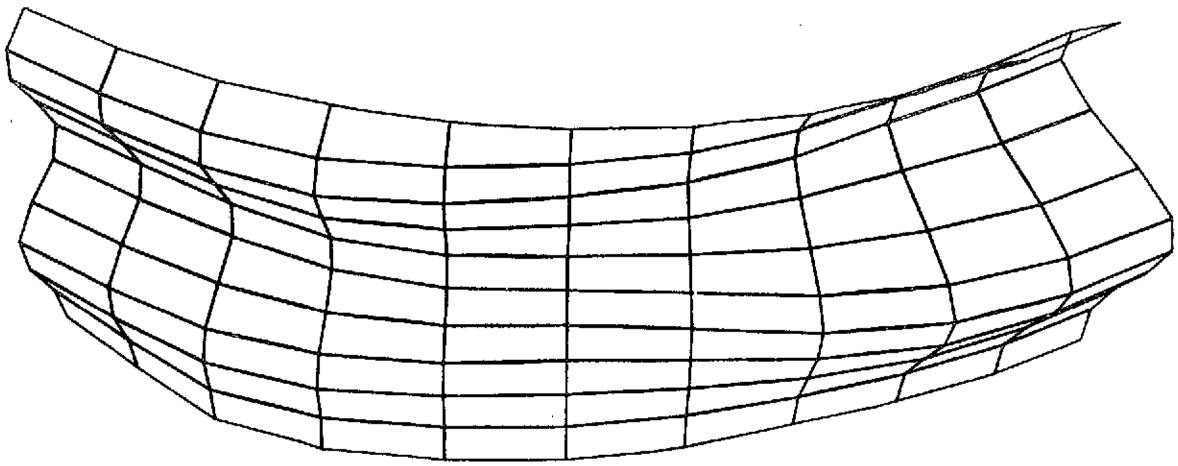
CYLINDRICAL SHELL PROBLEM

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.384223\text{X}10^{+02}$



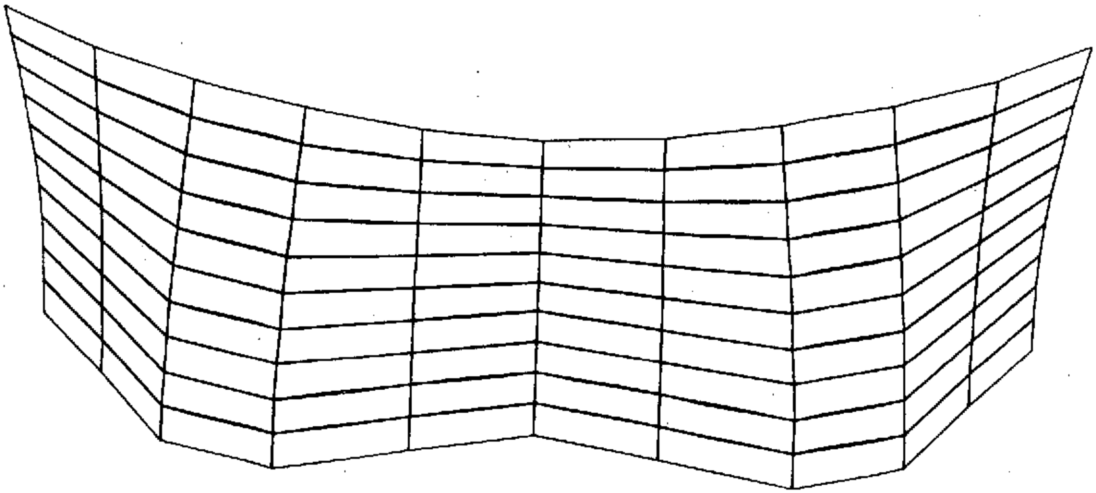
CYLINDRICAL SHELL PROBLEM

SEQ 1 BUCKLING MODE. CRITICAL LOAD = $.606797 \times 10^{+04}$



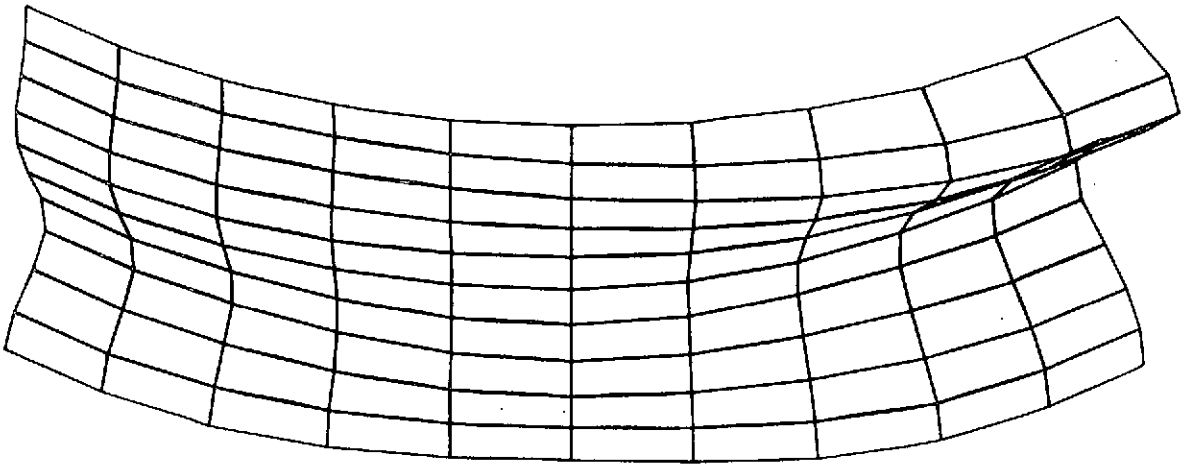
CYLINDRICAL SHELL PROBLEM

SEQ 1 BUCKLING MODE. CRITICAL LOAD = $.108406\text{X}10^{+03}$



CYLINDRICAL SHELL PROBLEM

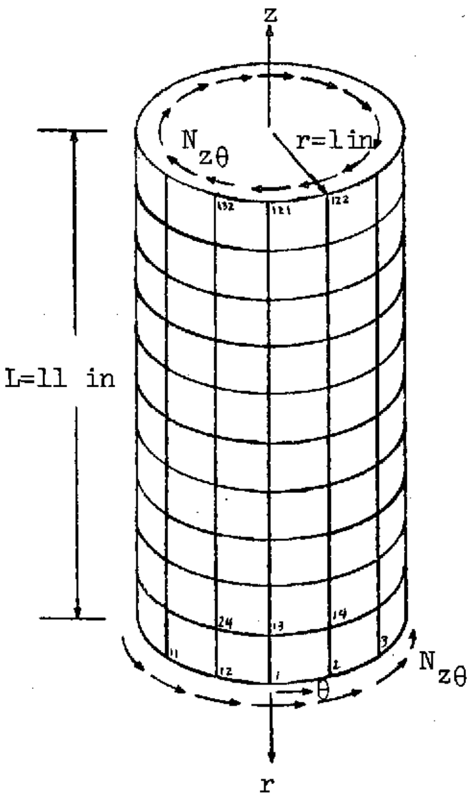
SEQ 1 BUCKLING MODE, CRITICAL LOAD = $.608179 \times 10^{+04}$



CYLINDRICAL SHELL PROBLEM

7. BUCKLING OF A CYLINDRICAL SHELL DUE TO TORSIONAL LOADING

A simply supported cylindrical shell is subjected to shear load at the ends. The shell is divided into ten elements along the axial direction and twelve elements along the circumferential direction. The geometrical and material properties of the shell are shown on the figure below:



$E = .3 \times 10^8$ lbs/in^2

$\nu = .3$

$t = .1$ in

Results are compared with Ref. 5, p 448, Figure 21.

	$N_{z\theta}$ (Critical)
SPAR	$.3437 \times 10^5$
Ref. 5	$.3423 \times 10^5$

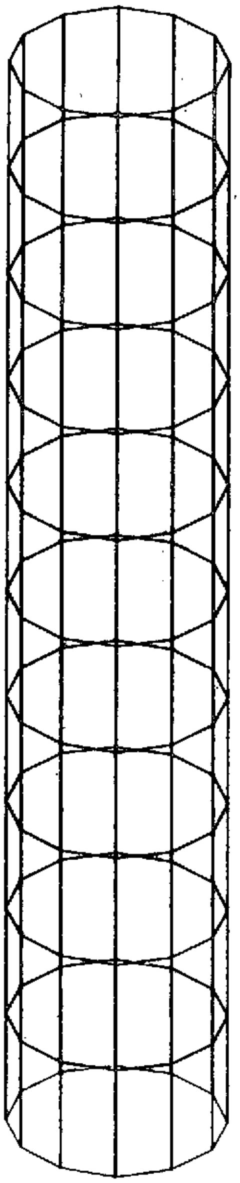
```
@XQT TAB                                   . GENERATE BASIC TABLES DEFINING STRUCTURE
START 132$
TITLE!BUCKLING OF A CYLINDRICAL SHELL DUE TO TORSIONAL LOAD.
MATERIAL CONSTANTS$
     1 3.+7 .3 .1 1.-5 1.+4$
JOINT REFERENCE FRAMES$
     NREF=-1$
     1 132 1$
JOINT LOCATIONS$
     FORMAT=2$
      1 1. 0.  0. 1. 330.  0. 12 1 11$
     12 1. 0. 11. 1. 330. 11.$
SHELL SECTION PROPERTIES$
     1 .1$
CONSTRAINT CASE 1$
$                                          CONSTRAIN 1,2 AND 3 DIRECTION MOTION AT
$                                          ONE END OF THE CYLINDER.
     ZERO 1 2 3:121 132 1$
CONSTRAINT CASE 2$
$                                          SIMPLY SUPPORTED AT BOTH ENDS OF CYLINDER
$                                          AND SUPPORT AT TWO POINTS AT MIDDLE OF
$                                          THE CYLINDER TO AVOID RIGID BODY MOTION.
     ZERO 1 2:  1   12 1: 121 132 1$
     ZERO 3  : 61   67 6$
@XQT ELD                                   . READ ELEMENT DEFINITIONS
E43 $
     1 13 14 2  2 12 10$
@XQT TOPO                                  . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                     . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                   . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT K                                     . FORM SYSTEM K
$
@XQT INV                                   . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT Q                                     . DEFINE APPLIED LOADS
$                                          APPLY UNIFORM SHEAR FORCES AT FREE END TO
$                                          OBTAIN UNIFORM SHEAR STRESS.
     CASE 1$
      NODAL FORCES, MOMENTS$
       1 2 0.523 12 1$
@XQT DSOL                                  . COMPUTE STATIC SOLUTIONS
$
@XQT KG                                    . FORM SYSTEM KG
$
@XQT INV
     RESET CON=2$
@XQT EIG                                   . SOLVE SYSTEM EIGENPROBLEM
$                                          FIND BUCKLING VALUE OF THE SHEAR STRESS.
     RESET PROB=STAB,INIT=8,CON=2,NDYN=10,UPPER=1$
@XQT DCU                                   . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
TABLE OF CONTENTS, DAL  1

BUCKLING OF A CYLINDRICAL SHELL DUE TO TORSIONAL LOAD.

                                  E          ROWS    BLK   T    DATA SET NAME
SEQ      RR   DATE    TIME        R   WORDS  /BLK   SIZE   Y   N1    N2    N3
  1      11  061174  073619       0      18     1     18   0 JDF1  BTAB     1
  2      12  061174  073619       0     132   132    132   0 JREF  BTAB     2
  3      17  061174  073619       0      12     1     12   1 ALTR  BTAB     2
  4      18  061174  073619       0      19     1     19   4 NDAL           0
  5      19  061174  073619       0      10     1     10   1 MATC  BTAB     2
  6      20  061174  073620       0     132   132    132   0 JREF  BTAB     2
  7      25  061174  073621       0     396   132    396   1 JLOC  BTAB     2
  8      40  061174  073622       0      19     1     19   1 SA    BTAB     2    1
  9      41  061174  073622       0     132   132    132   0 CON            1
 10      46  061174  073623       0     132   132    132   0 CON            2
 11      51  061174  073623       0    1188   132   1188   1 QJJT  BTAB     2    1
 12      94  061174  073631       0    1920   120    896   0 DEF   E43     11
 13     190  061174  073631       0       2     1      2   0 GD    E43     11
 14     191  061174  073631       0      15     1     15   0 GTIT  E43     11
 15     192  061174  073631       0      12    12     12   0 NELZ  BTAB     1    1
 16     193  061174  073631       0       5     1      5   0 KE             0
 17     194  061174  073631       0       6     1      6   0 NS             0
 18     195  061174  073631       0       1     1      1   3 ELTS  NAME     0
 19     196  061174  073631       0       1     1      1   0 ELTS  LTYP     0
 20     197  061174  073631       0       1     1      1   0 ELTS  NNOD     0
 21     207  061174  073631       0       1     1      1   0 ELTS  ISCT     0
 22     208  061174  073631       0       1     1      1   0 ELTS  NELS     0
 23     209  061174  073631       0       1     1      1   0 ELTS  LE3      0
 24     210  061174  073640       0    4032   132   1344   0 KMAP         624    3
 25     354  061174  073643       0   17472   132   1344   0 AMAP  ●●●#  1803   12
 26     978  061174  073727       0   36960   120    308   4 E43           11
 27    2298  061174  073648       0      20    20     20   0 DIR   E43     11
 28    2299  061174  073738       0   24640   132   2240   1 K     SPAR    36   62
 29    3179  061174  073827       0   75264   132   3136   1 INV   K        1  180
 30    5867  061174  074049       0     792   132    792  -1 NFM            0
 31    5896  061174  074049       0      15     1     15   4 CASE  ●●●●     0
 32    5897  061174  074049       0       0     0      0   0 LDIR           1
 33    5897  061174  074145       0     792   132    792   1 SSOL  U        1
 34    5926  061174  074249       0   24640   132   2240   1 KG    SPAR    36   62
 35    6806  061174  074343       0   75264   132   3136   1 INV   K        2  180
 36    9494  061174  074348       0       8     8      8  -1 STAB  EVAL     2
 37    9495  061174  074348       0    6336   132    792  -1 STAB  U        2
```

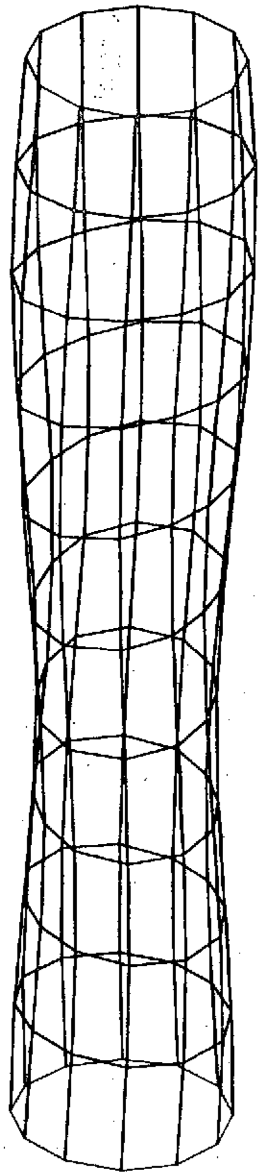
UNDEFORMED STRUCTURE



SHEAR BUCKLING OF A CYLINDRICAL SHELL

0 2
SCALE

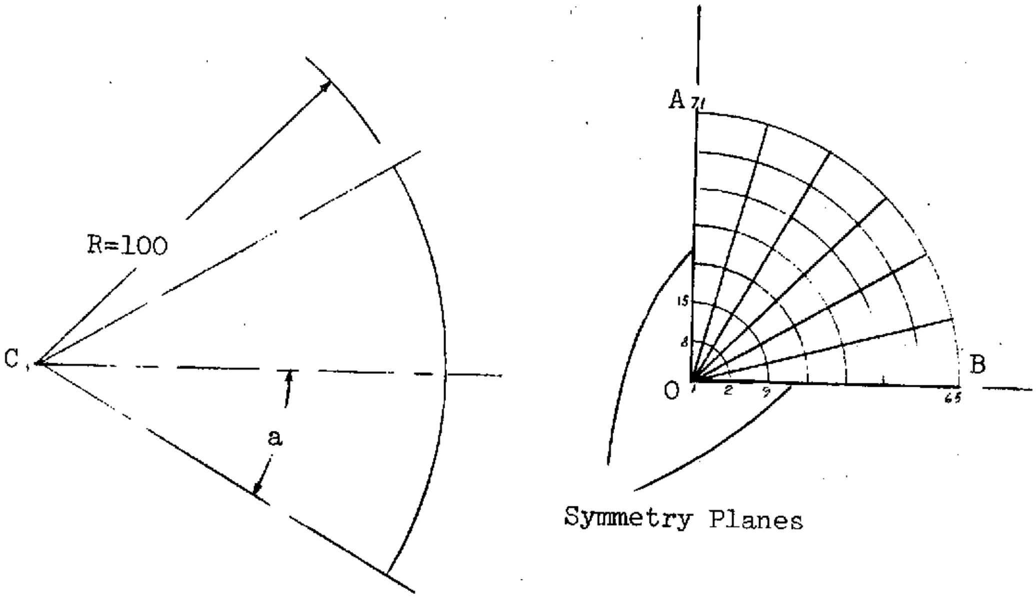
SEQ 1 BUCKLING MODE. CRITICAL LOAD = .343730X10^{+05}



SHEAR BUCKLING OF A CYLINDRICAL SHELL

8. SPHERICAL SHELL

The 6 by 10 mesh shown on Figure 8-1 was used to solve the problems discussed in Section 8.1 through 8.5



Section	a degrees	t in.	E lb/in^2	ν	Mass Density lb-sec^2/in^4
8.1	60°	5.	10^7	.3	.1
8.2	30°	5.	10^7	.3	.1
8.3	35°	3.	10^7	1/6	
8.4	10°	.396	10^7	1/3	

Fig. 8-1 Spherical Shell

8.1 Vibrational Characteristics of a 60° Sector. With edge AB simply supported, vibrational modes are computed. Results are compared with Reference 6, p 1363, Table 2.

	ω_{01}^2	ω_{02}^2
SPAR	$.9209 \times 10^4$	$.1736 \times 10^5$
Ref. 6	$.9197 \times 10^4$	$.1764 \times 10^5$

8.2 Vibrational Characteristics of a 30° Sector. With edge AB fixed, vibrational modes are computed. Results are compared with Reference 6, p 1363.

	ω_{12}^2
SPAR	$.4743 \times 10^5$
Ref. 6	$.4623 \times 10^5$

8.3 Stresses due to Pressure Loading. With edge AB fixed, unit internal pressure was applied. Results are compared with Reference 4, pp 553-554, Figure 273-274.

	Hoop Stress at Apex	Meridional Moment at AB
SPAR	48.0	35.8
Ref. 4	47.5	36.6

8.4 Buckling of 10° Sector due to Pressure. To obtain a state of uniform compression, unit pressure, P, was applied with displacements normal to conical surface ABC constrained along AB. Edge AB was then fixed, and P (critical) computed. Results are compared with Reference 8, p 450, Figure 6.

	P (Critical)
SPAR	$.236 \times 10^3$
Ref. 7	$.234 \times 10^3$

```
 @XQT TAB                                          . GENERATE BASIC TABLES DEFINING STRUCTURE
 START 71$
 TITLE' SPHERICAL SHELL PROBELMS
 TEXT$
 '     THE FOLLOWING PROBLEMS IN SPHERICAL SHELL ARE SOLVED IN THIS RUN;
 '      1  VIBRATION OF SIMPLY SUPPORTED 60 DEGREE SPHERICAL DOME
 '      2  VIBRATION OF FIXED EDGED 30 DEGREE SPHERICAL DOME
 '      3  STRESSES IN FIXED EDGED 35 DEGREE SPHERICAL DOME UNDER UNIFORM PRESSURE
 '      4  BUCKLING OF FIXED EDGED 10 DEGREE SPHERICAL CAP UNDER UNIFORM PRESSURE
 '
 MATERIAL CONSTANTS
      1  .1+8  .3      .1  .1-4$
      2  .1+8  .1667   .1  .1-4$
      3  .1+8  .3333   .1  .1-4$
JOINT LOCATIONS$
 $                                                  JOINT LOCATIONS, 60 DEGREE DOME
      FORMAT=2$
        1   0.0     0.0  0.0
        2  10.453   0.0  0.5478 10.453 90.0  0.5478  7 1
        9  20.791   0.0  2.1852 20.791 90.0  2.1852  7 1
       16  30.902   0.0  4.8944 30.902 90.0  4.8944  7 1
       23  40.674   0.0  8.6455 40.674 90.0  8.6455  7 1
       30  50.0     0.0 13.3975 50.0   90.0 13.3975  7 1
       37  58.779   0.0 19.0983 58.779 90.0 19.0983  7 1
       44  66.913   0.0 25.6855 66.913 90.0 25.6855  7 1
       51  74.315   0.0 33.0869 74.315 90.0 33.0869  7 1
       58  80.902   0.0 41.2215 80.902 90.0 41.2215  7 1
       65  86.603   0.0 50.0   86.603 90.0 50.0      7 1
JOINT REFERENCE FRAME ASSIGNMENT$
      NREF=-1; 1,71$
SHELL SECTION PROPERTIES
      1 5.0$
    NMAT=2
      2 3.0$
    NMAT=3
      3 0.396$
CONSTRAINT CASE 1
$                                                 SIMPLY SUPPORTED AT EDGES, SYMMETRY AT
$                                                 THETA=0 AND 90 DEGREE PLANES
     SYMMETRY PLANE=1$
     SYMMETRY PLANE=2$
     ZERO 1,2,3,4,6; 65,71$
CONSTRAINT CASE 2
$                                                 CONSTRAINED AT EDGES, SYMMETRY AT THETA=0
$                                                 AND 90 DEGREE PLANES
     SYMMETRY PLANE=1$
     SYMMETRY PLANE=2$
     ZERO 1,2,3,4,5,6; 65,71$
@XQT ELD                                        . READ ELEMENT DEFINITIONS
E43 $
     2 9 10 3  1 9 6$
E33 $
     1 2 3     2 6$
@XQT TOPO                                       . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                          . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                        . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                          . FORM SYSTEM M
$
```

```
@XQT K                                            . FORM SYSTEM K
$
@XQT INV                                          . FACTOR MATRIX IN K SPAR FORMAT
@XQT EIG                                          . SOLVE SYSTEM EIGENPROBLEM
$                                                   FIND FREQUENCIES AND CORRESPONDING MODE
$                                                   SHAPES FOR FREE VIBRATION OF THE SIMPLY
$                                                   SUPPORTED 60 DEGREE SPHERICAL DOME
     RESET INIT=8$
     PRINT 0 0 0 0 1$
@XQT TAB
JOINT LOCATION$
$                                                   JOINT LOCATIONS. 30 DEGREE DOME
     FORMAT=2$
     1  0.0     0.0  0.0$
     2  5.2336 0.0  0.1371  5.2336 90.0  0.1371  7 1$
     9 10.4529 0.0  0.5478 10.4529 90.0  0.5478  7 1$
    16 15.6435 0.0  1.2312 15.6435 90.0  1.2312  7 1$
    23 20.7912 0.0  2.1852 20.7912 90.0  2.1852  7 1$
    30 25.8819 0.0  3.4074 25.8819 90.0  3.4074  7 1$
    37 30.9017 0.0  4.8944 30.9017 90.0  4.8944  7 1$
    44 35.8368 0.0  6.6420 35.8368 90.0  6.6420  7 1$
    51 40.6737 0.0  8.6455 40.6737 90.0  8.6455  7 1$
    58 45.3991 0.0 10.8994 45.3991 90.0 10.8994  7 1$
    65 50.0    0.0 13.3975 50.0    90.0 13.3975  7 1$
@XQT E
@XQT EKS
@XQT M
@XQT K
@XQT INV
     RESET CON=2$
@XQT EIG
$                                                   FIND VIBRATIONAL MODES OF THE FIXED
$                                                   EDGED 30 DEGREE SPHERICAL DOME
     RESET INIT=8,CON=2$
     PRINT 0 0 0 0 1$
@XQT TAB
JOINT LOCATION$
$                                                   JOINT LOCATIONS. 35 DEGREE DOME
     FORMAT=2$
     1  0.0     0.0  0.0$
     2  5.4944 0.0  0.1679  5.4944 90.0  0.1679  7 1$
     9 10.9682 0.0  0.6709 10.9682 90.0  0.6709  7 1$
    16 16.4012 0.0  1.5071 16.4012 90.0  1.5071  7 1$
    23 21.7730 0.0  2.6734 21.7730 90.0  2.6734  7 1$
    30 27.0635 0.0  4.1655 27.0635 90.0  4.1655  7 1$
    37 32.2531 0.0  5.9778 32.2531 90.0  5.9778  7 1$
    44 37.3224 0.0  8.1035 37.3224 90.0  8.1035  7 1$
    51 42.2525 0.0 10.5347 42.2525 90.0 10.5347  7 1$
    58 47.0249 0.0 13.2624 47.0249 90.0 13.2624  7 1$
    65 51.6218 0.0 16.2763 51.6218 90.0 16.2763  7 1$
@XQT ELD
E43 $
     NSEC=2$
     NMAT=2$
     2 9 10 3  1 9 6$
E33 $
     NSEC=2$
     NMAT=2$
     1 2 3     2 6$
@XQT E
@XQT EKS
```

```
@XQT K
@XQT INV
     RESET CON=2$
@XQT Q                                              . DEFINE APPLIED LOADS
     CASE 1: UNIFORM PRESSURE LOAD
       NODAL PRESSURE
         1  1.0  71 1$
@XQT DSOL                                           . COMPUTE STATIC SOLUTIONS
$                                                     FIND DISPLACEMENT OF THE 35 DEGREE DOME
$                                                     UNDER UNIFORM PRESSURE
     RESET UP=1,CON=2$
@XQT GSF                                            . GENERATE STRESS DATA
$                                                     FIND STRESSES OF THE DOME
@XQT PSF                                            . PRINT STRESS DATA
$                                                     PRINT STRESSES OF THE DOME
@XQT TAB
JOINT LOCATIONS$
$                                                     JOINT LOCATIONS, 10 DEGREE DOME
     FORMAT=2$
     1  0.0     0.0  0.0$
     2  1.7452 0.0  0.0152  1.7452 90.0  0.0152  7 1$
     9  3.4900 0.0  0.0609  3.4900 90.0  0.0609  7 1
    16  5.2336 0.0  0.1371  5.2336 90.0  0.1371  7 1$
    23  6.9757 0.0  0.2436  6.9757 90.0  0.2436  7 1$
    30  8.7156 0.0  0.3805  8.7156 90.0  0.3805  7 1$
    37 10.4529 0.0  0.5478 10.4529 90.0  0.5478  7 1$
    44 12.1869 0.0  0.7454 12.1869 90.0  0.7454  7 1$
    51 13.9173 0.0  0.9732 13.9173 90.0  0.9732  7 1$
    58 15.6434 0.0  1.2311 15.6434 90.0  1.2311  7 1$
    65 17.3648 0.0  1.5192 17.3648 90.0  1.5192  7 1$
ALTERNATE REFERENCE FRAME
$                                                     DEFINE JOINT REFERENCE FRAMES PARALLEL TO
$                                                     SPHERICAL COORDINATE LINES AT EDGES
     2  3   0.0  2 -10.0$
     3  3  15.0  2 -10.0$
     4  3  30.0  2 -10.0$
     5  3  45.0  2 -10.0$
     6  3  60.0  2 -10.0$
     7  3  75.0  2 -10.0$
     8  3  90.0  2 -10.0$
JOINT REFERENCE FRAME ASSIGNMENTS
     NREF=-1; 1,64$
     NREF=2; 65$
     NREF=3; 66$
     NREF=4; 67$
     NREF=5; 68$
     NREF=6; 69$
     NREF=7; 70$
     NREF=8; 71$
CONSTRAINT CASE 3
$                                                     EDGES ARE SIMPLY SUPPORTED BUT FREE TO
$                                                     MOVE IN MERIDIONAL DIRECTION, SYMMETRY AT
$                                                     THETA=0 AND 90 DEGREE PLANES
     SYMMETRY PLANE=1$
     SYMMETRY PLANE=2$
     ZERO 1,2,4,6; 65,71$
CONSTRAINT CASE 4
$                                                     SAME AS CONSTRAINT CASE 2
     SYMMETRY PLANE=1$
     SYMMETRY PLANE=2$
     ZERO 1,2,3,4,5,6; 65,71$
```

```
@XQT ELD
E43 $
      NSEC=3$
      NMAT=3$
      2 9 10 3  1 9 6$
E33 $
      NSEC=3$
      NMAT=3$
      1 2 3     2 6$
@XQT E
@XQT EKS
@XQT K
@XQT INV
      RESET CON=3$
@XQT Q
      CASE 1: UNIFORM PRESSURE LOAD
        NODAL PRESSURE
          1  1.0  71 1$
@XQT DSOL
      RESET CON=3$
@XQT KG                                   . FORM SYSTEM KG
$
@XQT INV
      RESET CON=4$
@XQT EIG
$                                           FIND BUCKLING VALUE OF THE PRESSURE FOR
$                                           THE 10 DEGREE FIXED EDGED SPHERICAL CAP
      RESET PROB=STAB,CON=4,INIT=4,NDYN=10,UPPER=1$
      PRINT 0 0 0 0 1$
@XQT DCU
      TOC 1$
```

```
TABLE OF CONTENTS, DAL  1

 SPHERICAL SHELL PROBELMS

                                   E                ROWS    BLK    T     DATA SET NAME
SEQ       RR   DATE     TIME       R        WORDS   /BLK   SIZE    Y     N1    N2     N3      N4
  1       11 051574   171123       0           18      1      18   0  JDF1  BTAB      1       8
  2      -12 051574   171123       0           71     71      71   0  JREF  BTAB      2       6
  3      -15 051574   171123       0           12      1      12   1  ALTR  BTAB      2       4
  4       16 051574   171123       0           19      1      19   4  NDAL            0       0
  5       17 051574   171124       0           80      4      80   3  TEXT  BTAB      2       1
  6       20 051574   171124       0           30      3      30   1  MATC  BTAB      2       2
  7      -22 051574   171124       0          213     71     213   1  JLOC  BTAB      2       5
  8      -30 051574   171124       0           71     71      71   0  JREF  BTAB      2       6
  9       33 051574   171125       0           57      3      57   1  SA    BTAB      2      13
 10       36 051574   171126       0           71     71      71   0  CON             1       0
 11       39 051574   171126       0           71     71      71   0  CON             2       0
 12      -42 051574   171127       0          639     71     639   1  QJJT  BTAB      2      19
 13      -65 051574   171204       0          864     54     896   0  DEF   E43      11       4
 14      -97 051574   171204       0            2      1       2   0  GD    E43      11       4
 15      -98 051574   171204       0           15      1      15   0  GTIT  E43      11       4
 16      -99 051574   171204       0           96      6     896   0  DEF   E33       8       3
 17     -131 051574   171204       0            2      1       2   0  GD    E33       8       3
 18     -132 051574   171204       0           15      1      15   0  GTIT  E33       8       3
 19     -133 051574   171204       0           12     12      12   0  NELZ  BTAB      1      11
 20     -134 051574   171204       0           10      2      10   0  KE              0       0
 21     -144 051574   171205       0           12      2      12   0  NS              0       0
 22     -145 051574   171205       0            2      2       2   3  ELTS  NAME      0       0
 23     -146 051574   171205       0            2      2       2   0  ELTS  LTYP      0       0
 24     -147 051574   171206       0            2      2       2   0  ELTS  NNOD      0       0
 25     -148 051574   171206       0            2      2       2   0  ELTS  ISCT      0       0
 26     -149 051574   171206       0            2      2       2   0  ELTS  NELS      0       0
 27     -150 051574   171207       0            2      2       2   0  ELTS  LE3       0       0
 28      151 051574   171225       0         2688     71    1344   0  KMAP          309      21
 29      247 051574   171227       0         4032     71    1344   0  AMAP  ****    594      45
 30     -391 051574   171307       0         1344      6     224   4  E33             8       3
 31     -439 051574   171251       0           20     20      20   0  DIR   E33       8       3
 32     -440 051574   171311       0        16632     54     308   4  E43            11       4
 33    -1034 051574   171252       0           20     20      20   0  DIR   E43      11       4
 34    -1035 051574   171325       0        13440     71    2240   1  M     SPAR     36     309
 35    -1515 051574   171335       0        13440     71    2240   1  K     SPAR     36     309
 36     1995 051574   171358       0        21952     71    3136   1  INV   K         1     594
 37     2779 051574   171404       0            8      8       8  -1  VIBR  EVAL      1       0
 38     2780 051574   171404       0         3408     71     426  -1  VIBR  U         1       0
 39    -2908 051574   171700       0          213     71     213   1  JLOC  BTAB      2       5
 40    -2916 051574   171700       0          639     71     639   1  QJJT  BTAB      2      19
 41    -2948 051574   171722       0         1344      6     224   4  E33             8       3
 42    -2996 051574   171705       0           20     20      20   0  DIR   E33       8       3
 43    -2997 051574   171727       0        16632     54     308   4  E43            11       4
 44    -3591 051574   171707       0           20     20      20   0  DIR   E43      11       4
 45     3592 051574   171746       0        13440     71    2240   1  M     SPAR     36     309
 46    -4072 051574   171753       0        13440     71    2240   1  K     SPAR     36     309
 47    -4552 051574   171858       0        21952     71    3136   1  INV   K         2     594
 48     5336 051574   171902       0            8      8       8  -1  VIBR  EVAL      2       0
 49     5337 051574   171903       0         3408     71     426  -1  VIBR  U         2       0
 50    -5465 051574   172107       0          213     71     213   1  JLOC  BTAB      2       5
 51    -5473 051574   172109       0          639     71     639   1  QJJT  BTAB      2      19
 52    -5496 051574   172127       0          864     54     896   0  DEF   E43      11       4
```

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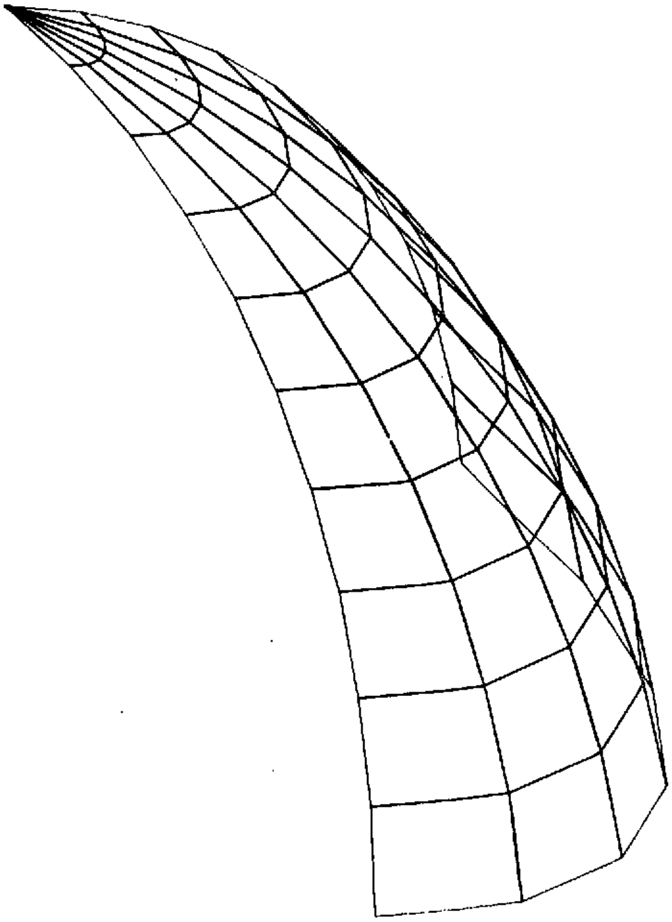
 53    -5528 051574 172128   0        2     1     2  0 GD   E43    11    4
 54    -5529 051574 172128   0       15     1    15  0 GTIT E43    11    4
 55    -5530 051574 172129   0       96     6   896  0 DEF  E33     8    3
 56    -5562 051574 172130   0        2     1     2  0 GD   E33     8    3
 57    -5563 051574 172130   0       15     1    15  0 GTIT E33     8    3
 58    -5564 051574 172130   0       12    12    12  0 NELZ BTAB    1   11
 59    -5565 051574 172131   0       10     2    10  0 KE           0    0
 60    -5566 051574 172131   0       12     2    12  0 NS           0    0
 61    -5576 051574 172132   0        2     2     2  3 ELTS NAME    0    0
 62    -5577 051574 172133   0        2     2     2  0 ELTS LTYP    0    0
 63    -5578 051574 172133   0        2     2     2  0 ELTS NNOD    0    0
 64    -5579 051574 172133   0        2     2     2  0 ELTS ISCT    0    0
 65    -5580 051574 172134   0        2     2     2  0 ELTS NELS    0    0
 66    -5581 051574 172134   0        2     2     2  0 ELTS LE3     0    0
 67    -5582 051574 172158   0     1344     6   224  4 E33          8    3
 68    -5630 051574 172148   0       20    20    20  0 DIR  E33     8    3
 69    -5631 051574 172204   0    16632    54   308  4 E43         11    4
 70    -6225 051574 172151   0       20    20    20  0 DIR  E43    11    4
 71    -6226 051574 172212   0      426    71   426 -1 NFM          0    1
 72    -6242 051574 172212   0       15     1    15  4 CASE @@@@    0    1
 73    -6243 051574 172212   0        0     0     0  0 LDIR         1    1
 74    -6243 051574 172220   0    13440    71  2240  1 K    SPAR   36  309
 75     6723 051574 172231   0    21952    71  3136  1 INV  K       2  594
 76    -7507 051574 172248   0      426    71   426  1 SSOL U       0    1
 77    -7523 051574 172309   0      186     6  5600 -1 STRS E33     0    1
 78    -7723 051574 172310   0     2646    54  5600 -1 STRS E43     0    1
 79     7923 051574 172325   0      213    71   213  1 JLOC BTAB    2    5
 80     7931 051574 172326   0       96     8    96  1 ALTR BTAB    2    4
 81     7944 051574 172328   0       71    71    71  0 JREF BTAB    2    6
 82     7947 051574 172330   0       71    71    71  0 CON          3    0
 83     7950 051574 172331   0       71    71    71  0 CON          4    0
 84     7953 051574 172333   0      639    71   639  1 QJJT BTAB    2   19
 85     7976 051574 172340   0      864    54   896  0 DEF  E43    11    4
 86     8008 051574 172340   0        2     1     2  0 GD   E43    11    4
 87     8009 051574 172341   0       15     1    15  0 GTIT E43    11    4
 88     8010 051574 172342   0       96     6   896  0 DEF  E33     8    3
 89     8042 051574 172342   0        2     1     2  0 GD   E33     8    3
 90     8043 051574 172343   0       15     1    15  0 GTIT E33     8    3
 91     8044 051574 172344   0       12    12    12  0 NELZ BTAB    1   11
 92     8045 051574 172345   0       10     2    10  0 KE           0    0
 93     8046 051574 172345   0       12     2    12  0 NS           0    0
 94     8047 051574 172346   0        2     2     2  3 ELTS NAME    0    0
 95     8048 051574 172347   0        2     2     2  0 ELTS LTYP    0    0
 96     8049 051574 172348   0        2     2     2  0 ELTS NNOD    0    0
 97     8050 051574 172348   0        2     2     2  0 ELTS ISCT    0    0
 98     8051 051574 172349   0        2     2     2  0 ELTS NELS    0    0
 99     8052 051574 172350   0        2     2     2  0 ELTS LE3     0    0
100     8053 051574 172403   0     1344     6   224  4 E33          8    3
101     8110 051574 172357   0       20    20    20  0 DIR  E33     8    3
102     8111 051574 172409   0    16632    54   308  4 E43         11    4
103     8705 051574 172400   0       20    20    20  0 DIR  E43    11    4
104     8706 051574 172433   0    13440    71  2240  1 K    SPAR   36  309
105     9186 051574 172449   0    21952    71  3136  1 INV  K       3  594
106     9970 051574 172510   0      426    71   426 -1 NFM          0    1
107     9986 051574 172510   0       15     1    15  4 CASE @@@@    0    1
108     9987 051574 172511   0        0     0     0  0 LDIR         1    1
109     9987 051574 172529   0      426    71   426  1 SSOL U       0    1
110    10003 051574 172550   0      186     6  5600 -1 STRS E33     0    1
```

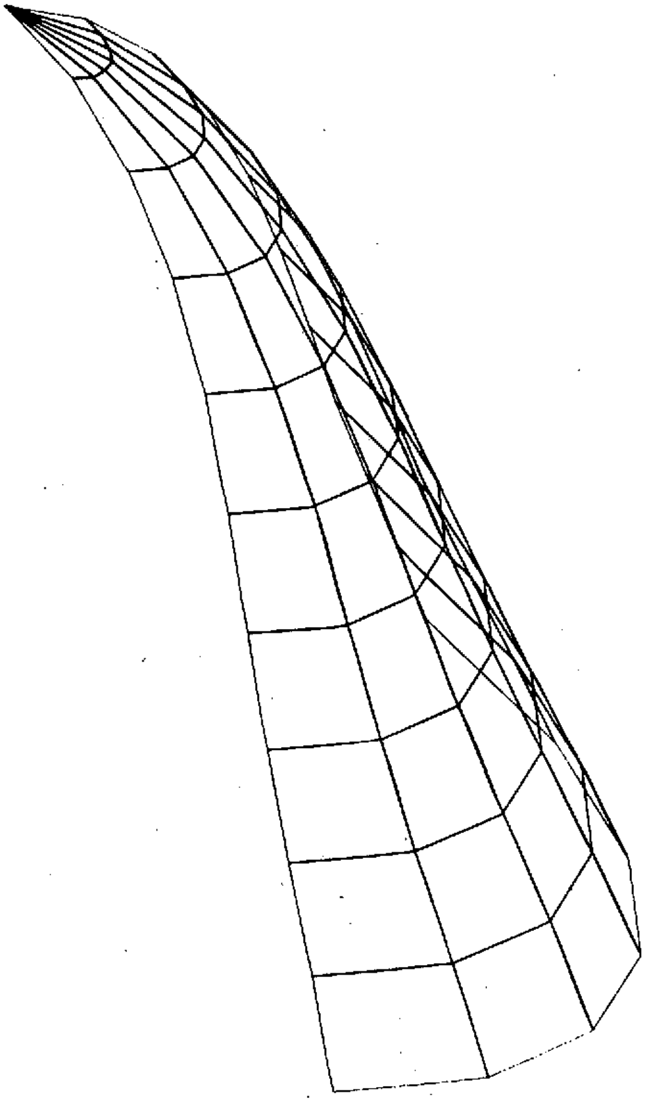
```
111   10203 051574 172551  0     2646     54  5600 -1 STRS E43      0    1
112   10403 051574 172615  0    13440     71  2240  1 KG   SPAR    35  309
113   10883 051574 172628  0    21952     71  3136  1 INV  K        4  594
114   11667 051574 172640  0        4      4     4 -1 STAB EVAL     4    0
115   11668 051574 172641  0     1704     71   426 -1 STAB U        4    0
```

UNDEFORMED STRUCTURE



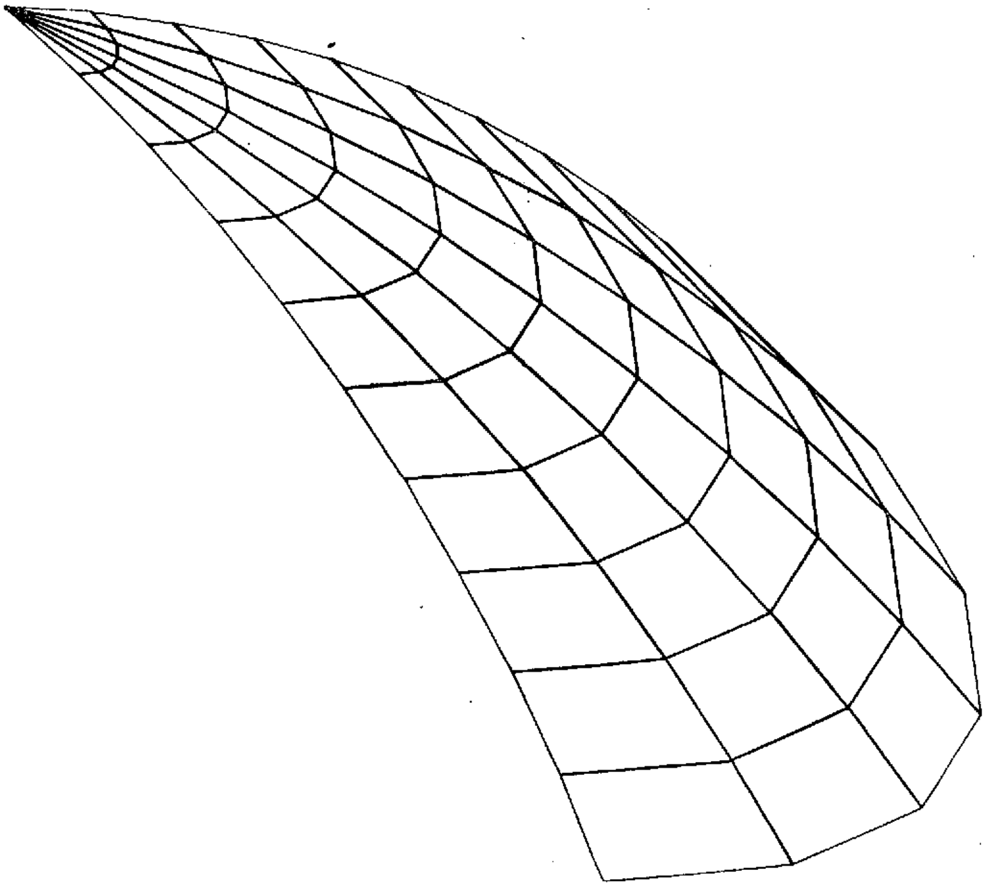
60-DEGREE
SPHERICAL SHELL

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.152732 \times 10^{+02}$



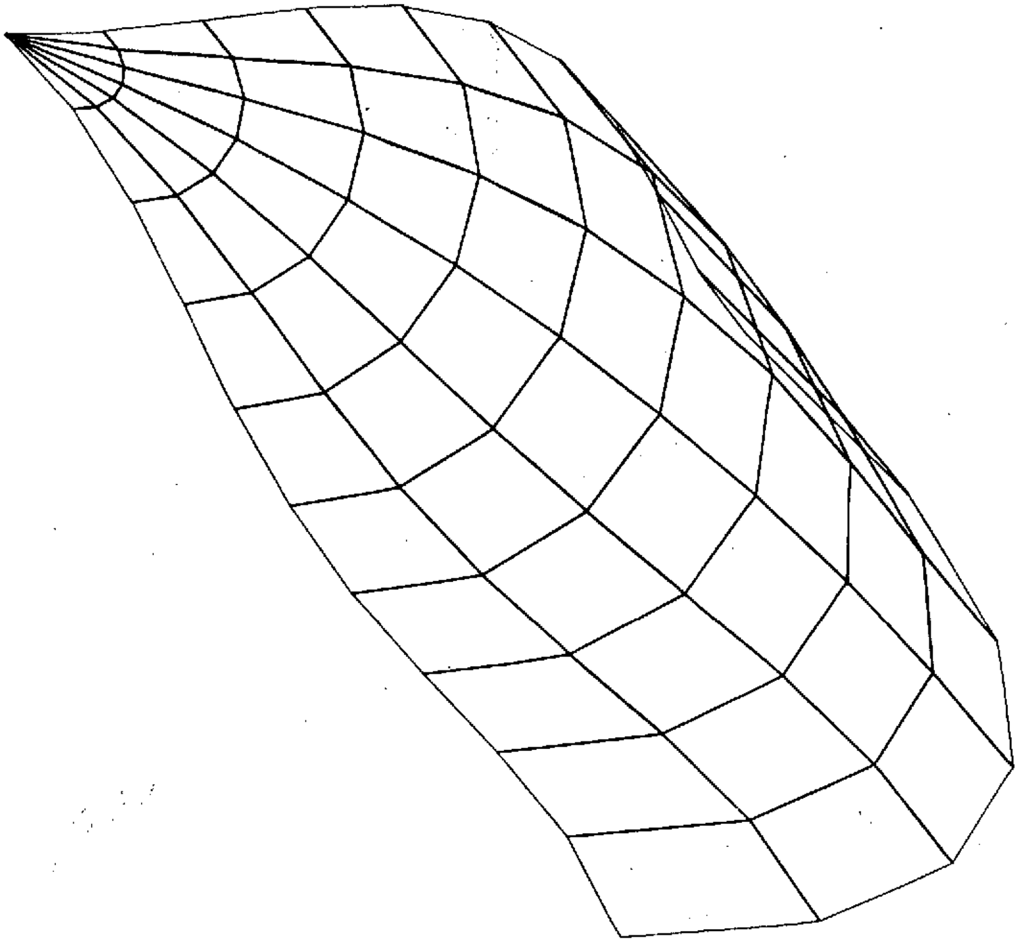
60-DEGREE
SPHERICAL SHELL

UNDEFORMED STRUCTURE



30-DEGREE
SPHERICAL SHELL

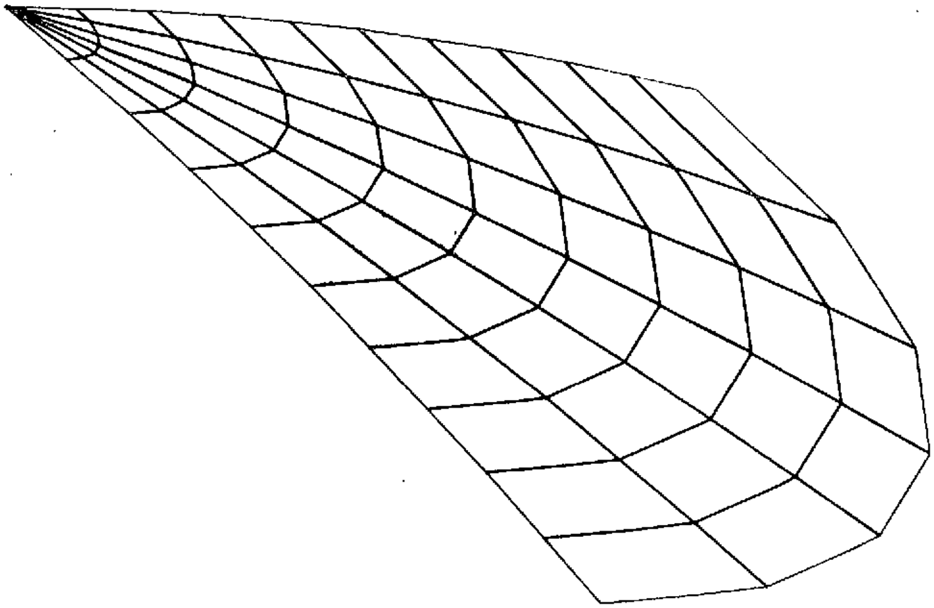
SEQ 2 VIBRATIONAL MODE. FREQ (HZ) = $.346612 \times 10^{+02}$



30-DEGREE

SPHERICAL SHELL

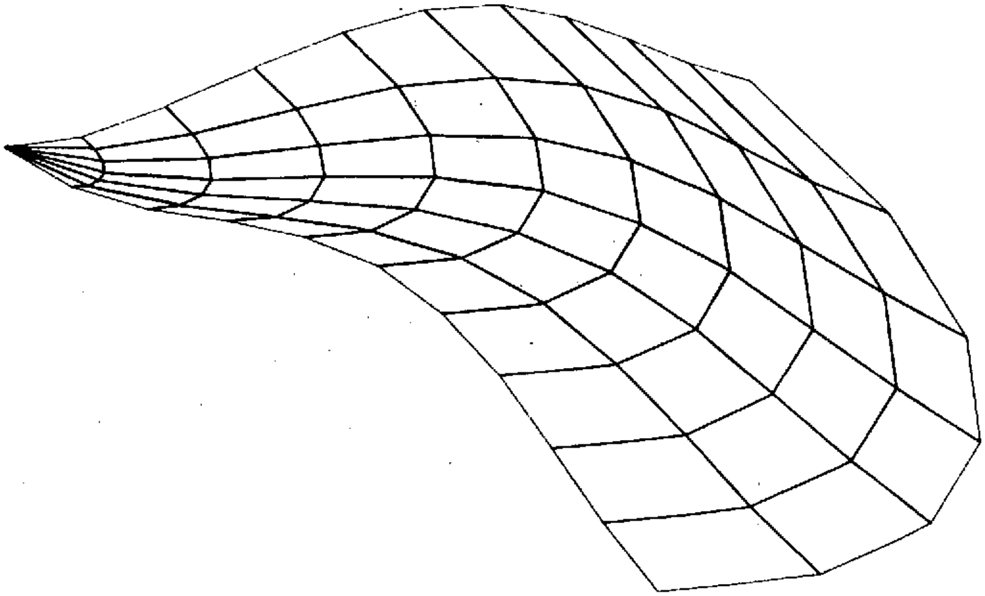
UNDEFORMED STRUCTURE



10-DEGREE

SPHERICAL SHELL

SEQ 1 BUCKLING MODE. CRITICAL LOAD = $.236468 \times 10^{+03}$



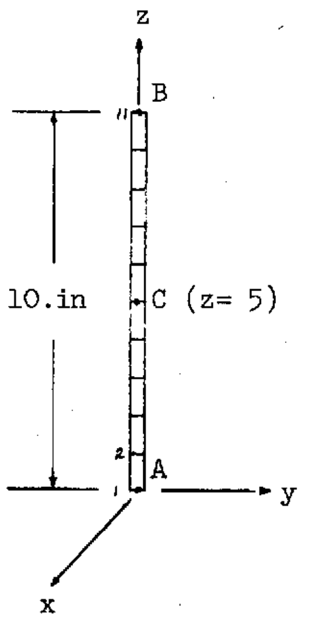
10-DEGREE

SPHERICAL SHELL

0 SCALE

9. BEAM

The 10 element mesh shown on Figure 9-1 was used to solve the problems discussed in Section 9.1 through 9.8.



$E = 10^7$ lb/in^2

$G = .5 \times 10^7$ lb/in^2

Mass density = .1 lb-sec^2/in^4

$\alpha = .1 \times 10^{-4}$ in/degree

$I_x = .36 \times 10^{-2}$ in^4

$I_y = .4 \times 10^{-3}$ in^4

$I_z = .4 \times 10^{-2}$ in^4

$J = 0.16 \times 10^{-2}$ in^4

$A = .12$ in^2

Fig. 9-1 Beam

9.1 Vibration of a Cantilever. With point A fixed and the beam constrained to move in the yz plane, vibrational modes were computed. Results are compared with Reference 1, p 339.

	SPAR	Ref. 2
ω_1^2	$.41213 \times 10^3$	$.41199 \times 10^3$
ω_2^2	$.16185 \times 10^5$	$.16182 \times 10^5$
ω_3^2	$.12695 \times 10^6$	$.12690 \times 10^6$
ω_4^2	$.48817 \times 10^6$	$.48732 \times 10^6$
ω_5^2	$.13382 \times 10^7$	$.13314 \times 10^7$

9.2 Column Buckling. With point A fixed, the lowest critical value of compressive load at B was computed.

SPAR	$.98695 \times 10^2$
$\frac{EI_y\pi^2}{4\ell^2}$	$.98695 \times 10^2$

9.3 Beam-Column Effects. A compressive pre-load of 1000 lbs was applied. With simple support at A and B, lateral displacement under a 10 lb lateral load at C was computed.

	Lateral Deflection at C
SPAR	$.80231 \times 10^{-2}$
Ref. 8	$.80233 \times 10^{-2}$

9.4 Effects of Pre-Tension on Vibration. A tensile pre-load of 1000 lbs was applied. With simple support at A and B, vibrational modes were computed. Results are compared with Reference 2, pp 374-376.

	SPAR	Ref. 2
ω_1^2	$.11472 \times 10^5$	$.11472 \times 10^5$
ω_2^2	$.84861 \times 10^6$	$.84850 \times 10^6$
ω_3^2	$.33731 \times 10^6$	$.33703 \times 10^6$
ω_4^2	$.96559 \times 10^6$	$.96282 \times 10^6$
ω_5^2	$.22511 \times 10^7$	$.22350 \times 10^7$

9.5 Buckling of a Cantilever due to Lateral Load. With A fixed, the critical value of an x-direction point force at B was computed. Results are compared with Reference 3, pp 259-260.

	Critical Force
SPAR	$.22760 \times 10^3$
Ref. 3	$.22674 \times 10^3$

9.6 Buckling due to Pure Bending. Critical values of equilibrating point moments(about the y axis) applied at A and B are computed. Results are compared to Reference 2, pp 253-257.

	Critical Moment
SPAR	$.17845 \times 10^4$
Ref. 3	$.17750 \times 10^4$

9.7 Buckling due to Thermal Load. With A and B simply supported, the critical temperature was computed.

	T (Critical)
SPAR	$.32899 \times 10^2$
$\frac{EI\pi^2}{L\ AE\alpha}$	$.32899 \times 10^2$

9.8 Effects of Thermal Pre-Stress on Vibration. A 10° temperature load was applied to the beam, with A and B simply supported. Vibrational modes were computed. Results are compared with Reference 2, pp 374-375

	SPAR	Ref. 2
ω_1^2	$.22600 \times 10^4$	$.22600 \times 10^4$
ω_2^2	$.48015 \times 10^5$	$.48004 \times 10^5$
ω_3^2	$.25440 \times 10^6$	$.25412 \times 10^6$
ω_4^2	$.81819 \times 10^6$	$.81543 \times 10^6$
ω_5^2	$.20208 \times 10^7$	$.20047 \times 10^6$

```
@XQT TAB                             . GENERATE BASIC TABLES DEFINING STRUCTUR
START 11$
TITLE' BEAM PROBLEMS
TEXT$
'     THE FOLLOWING BEAM PROBLEMS ARE SOLVED IN THIS RUN:
'       1  VIBRATION OF A CANTILEVERED BEAM
'       2  BUCKLING OF A COLUMN
'       3  LATERAL DEFLECTION OF A COMPRESSED BEAM
'       4  VIBRATION OF A PRESTRESSED BEAM
'       5  LATERAL BUCKLING OF A CANTILEVERED BEAM
'       6  LATERAL BUCKLING OF A BEAM IN PURE BENDING
'       7  BUCKLING OF A BEAM DUE TO TEMPERATURE
'       8  VIBRATION OF A HEATED BEAM
'
MATERIAL CONSTANTS
    1  .1+8  .0   .1   .1-4$
JOINT LOCATIONS
    1  .0  .0  .0   .0  .0  10.0  11  1$
BEAM ORIENTATION SPECIFICATIONS$
    1  1  2  1  0.$
E21 SECTION PROPERTIES$
    GIVN  1 .0004   .0  .0036   .0 .12  .0016$
CONSTRAINT CASE 1$
$                                      THE BEAM IS CONSTRAINED AT ONE END
    FIXED PLANE = 3$
CONSTRAINT CASE 2$
$                                      SIMPLY SUPPORTED AT BOTH ENDS
    ZERO 1,2,3,6:   1,11,10$
@XQT ELD                             . READ ELEMENT DEFINITIONS
E21 $
    1 2  1 10$
@XQT TOPO                            . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                               . FORM ELEMENT DATA PACKETS
$
@XQT EKS                             . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                               . FORM SYSTEM M
$                                      EXCLUDE ROTATORY INERTIA EFFECTS
     RESET IBEAM=1$
@XQT K                               . FORM SYSTEM K
$
@XQT INV                             . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT EIG                             . SOLVE SYSTEM EIGENPROBLEM
$                                      FIND VIBRATIONAL MODES OF THE CANTILEVE
$                                      BEAM.
     RESET INIT=10,REPR=1$
@XQT Q                               . DEFINE APPLIED LOADS
     CASE 1' AXIAL COMPRESSIVE FORCE APPLY AT FREE END.
       NODAL FORCES, MOMENTS
         11 3 -1.0$
     CASE 2' SHEAR FORCE IN 1-DIRECTION APPLY AT FREE END.
       NODAL FORCES, MOMENTS
         11 1 1.0$
     CASE 3' BENDING MOMENT IN 2-DIRECTION APPLY AT BOTH ENDS.
       NODAL FORCES, MOMENTS
          1 5 1.0:  11 5 -1.0$
     CASE 4' CONCENTRATED FORCE AT MID-SPAN
       NODAL FORCES, MOMENTS
          6 1 10.0$
```

REPRODUCIBILITY OF THE
ORIGINAL PAGE IS POOR

```
      CASE 5: TEMPERATURE LOAD AT ALL JOINTS.
        NODAL TEMPERATURES
           1  1.0  11 1$
  @XQT DSOL                                    . SOLVE LOAD CASE 1
      RESET L1=1,L2=1$
  @XQT KG                                      . FORM SYSTEM KG, AXIAL COMPRESSION
  $
  @XQT EIG
  $                                              FIND BUCKLING LOAD FOR CANTILEVERED
  $                                              COLUMN
      RESET INIT=10,PROB=STAB$
  @XQT LCM                                     . FORM LINEAR COMBINATION OF MATRICES
  $                                              FORM STIFFNESS MATRIX FOR AXIALY PRE-
  $                                              COMPRESSED BEAM. COMPRESSIVE FORCE=1000.
      RESET R=KXKG,Q1=K,Q2=KG,C2=1000.0$
  @XQT INV
      RESET K=KXKG,CON=2$
  @XQT DSOL                                    . SOLVE LOAD CASE 4
  $                                              FIND DEFLECTION OF THE PRESTRESSED BEAM
  $                                              UNDER LOAD CASE 4.
      RESET K=KXKG,CON=2,L1=4,L2=4$
  @XQT LCM
  $                                              FORM STIFFNESS MATRIX FOR AXIALY PRE-
  $                                              STRESSED BEAM. TENSILE FORCE=1000.
      RESET R=KYKG,Q1=K,Q2=KG,C2=-1000.0$
  @XQT INV
      RESET K=KYKG,CON=2$
  @XQT EIG
  $                                              FIND FREQUENCY FOR FREE VIBRATION OF THE
  $                                              PRE-STRESSED BEAM.
      RESET K=KYKG,CON=2,INIT=10$
  @XQT DSOL                                    . SOLVE LOAD CASE 2
      RESET L1=2,L2=2$
  @XQT KG                                      . FORM SYSTEM KG, LOAD CASE 2
      RESET QSEQ=2$
  @XQT EIG
  $                                              FIND LATERAL BUCKLING VALUE OF LOAD CASE
  $                                              2 FOR THE CANTILEVER BEAM.
      RESET INIT=6,PROB=STAB$
  @XQT INV
      RESET CON=2$
  @XQT DSOL
      RESET CON=2,L1=3,L2=3$
  @XQT KG                                      . FORM SYSTEM KG, PURE BENDING
      RESET QSEQ=3$
  @XQT EIG
  $                                              FIND BUCKLING LOAD OF A SIMPLY SUPPORTED
  $                                              BEAM IN PURE BENDING.
      RESET INIT=6,PROB=STAB,CON=2$
  @XQT DSOL
      RESET L1=5,L2=5,CON=2$
  @XQT KG                                      . FORM SYSTEM KG, TEMPERATURE LOAD
      RESET QSEQ=5$
  @XQT EIG
  $                                              FIND BUCKLING TEMPERATURE LOAD FOR A
  $                                              SIMPLY SUPPORTED BEAM.
      RESET CON=2,INIT=6,PROB=STAB$
  @XQT LCM
      RESET R=K5KG,Q1=K,Q2=KG,C2=10.0$
  @XQT INV
      RESET K=K5KG,CON=2$
```

```
@XQT EIG
$                                        FIND FREQUENCIES FOR FREE VIBRATION OF
$                                        THE HEATED BEAM
     RESET K=K5KG,CON=2,INIT=10$
     PRINT 0 0 0 0 1$
@XQT DCU                               . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
TABLE OF CONTENTS, DAL  1

  BEAM PROBLEMS

                                 E              ROWS    BLK   T     DATA SET NAME
SEQ        RR   DATE    TIME     R     WORDS   /BLK    SIZE   Y    N1   N2    N3     N4
  1        11 061274 160919      0        18      1      18   0 JDF1 BTAB      1      8
  2        12 061274 160919      0        11     11      11   0 JREF BTAB      2      6
  3        13 061274 160919      0        12      1      12   1 ALTR BTAB      2      4
  4        14 061274 160919      0        19      1      19   4 NDAL           0      0
  5        15 061274 160919      0       200     10     200   3 TEXT BTAB      2      1
  6        23 061274 160920      0        10      1      10   1 MATE BTAB      2      2
  7        24 061274 160921      0        33     11      33   1 JLOC BTAB      2      5
  8        26 061274 160921      0         5      1       5   1 MREF BTAB      2      7
  9        27 061274 160921      0        31      1      31   1 BA   BTAB      2      9
 10        29 061274 160922      0        11     11      11   0 CON            1      0
 11        30 061274 160922      0        11     11      11   0 CON            2      0
 12        31 061274 160922      0        99     11      99   1 QJJT BTAB      2     19
 13        35 061274 160930      0       160     10     896   0 DEF  E21       1      2
 14        67 061274 160930      0         2      1       2   0 GD   E21       1      2
 15        68 061274 160930      0        15      1      15   0 GTIT E21       1      2
 16        69 061274 160930      0        12     12      12   0 NELZ BTAB      1     11
 17        70 061274 160930      0         5      1       5   0 KE             0      0
 18        71 061274 160930      0         6      1       6   0 NS             0      0
 19        72 061274 160930      0         1      1       1   3 ELTS NAME      0      0
 20        73 061274 160930      0         1      1       1   0 ELTS LTYP      0      0
 21        83 061274 160930      0         1      1       1   0 ELTS NNOD      0      0
 22        84 061274 160930      0         1      1       1   0 ELTS ISCT      0      0
 23        85 061274 160930      0         1      1       1   0 ELTS NELS      0      0
 24        86 061274 160931      0         1      1       1   0 FLTS LE3       0      0
 25        87 061274 160937      0      1344     11    1344   0 KMAP          21      3
 26       135 061274 160939      0      1344     11    1344   0 AMAP @@@@     21      3
 27       183 061274 160949      0      1400     10     140   4 E21            1      2
 28       233 061274 160946      0        20     20      20   0 DIR  E21       1      2
 29       234 061274 160953      0      2240     11    2240   1 M    SPAR     36     21
 30       314 061274 160956      0      2240     11    2240   1 K    SPAR     36     21
 31       394 061274 161010      0      3136     11    3136   1 INV  K         1     21
 32       506 061274 161024      0        10     10      10  -1 VIBR EVAL      1      0
 33       507 061274 161024      0       660     11      66  -1 VIBR U         1      0
 34       537 061274 161101      0        66     11      66  -1 NFM            0      1
 35       540 061274 161101      0        15      1      15   4 CASE @@@@      0      1
 36       541 061274 161101      0        66     11      66  -1 NFM            0      2
 37       544 061274 161101      0        15      1      15   4 CASE @@@@      0      2
 38       545 061274 161101      0        66     11      66  -1 NFM            0      3
 39       548 061274 161101      0        15      1      15   4 CASE @@@@      0      3
 40       549 061274 161101      0        66     11      66  -1 NFM            0      4
 41       561 061274 161102      0        15      1      15   4 CASE @@@@      0      4
 42       562 061274 161102      0        11     11      11  -1 NTMP           0      5
 43       563 061274 161102      0        66     11      66  -1 NFM            0      5
 44       566 061274 161103      0        15      1      15   4 CASE @@@@      0      5
 45       567 061274 161103      0         0      0       0   0 LDIR           1      5
 46       567 061274 161112      0        66     11      66   1 SSOL U         1      1
 47      -570 061274 161116      0      2240     11    2240   1 KG   SPAR     36     21
 48      -650 061274 161121      0        10     10      10  -1 STAB EVAL      1      0
 49      -651 061274 161121      0       660     11      66  -1 STAB U         1      0
 50       681 061274 161150      0      2240     11    2240   1 KXKG SPAR     36     21
 51       761 061274 161157      0      3136     11    3136   1 INV  KXKG      2     21
 52       873 061274 161203      0        66     11      66   1 SSOL U         1      4
```

```
53     876 061274 161208  0      2240     11    2240   1 KYKG SPAR   36    21
54     956 061274 161212  0      3136     11    3136   1 INV  KYKG    2    21
55   -1068 061274 161227  0        10     10      10  -1 VIBR EVAL    2     0
56   -1069 061274 161227  0       660     11      66  -1 VIBR U       2     0
57    1099 061274 161302  0        66     11      66   1 SSOL U       1     2
58   -1102 061274 161307  0      2240     11    2240   1 KG   SPAR   36    21
59    1182 061274 161314  0         6      6       6  -1 STAB EVAL    1     0
60    1183 061274 161314  0       396     11      66  -1 STAB U       1     0
61    1210 061274 161336  0      3136     11    3136   1 INV  K       2    21
62    1322 061274 161342  0        66     11      66   1 SSOL U       1     3
63   -1325 061274 161348  0      2240     11    2240   1 KG   SPAR   36    21
64   -1405 061274 161441  0         6      6       6  -1 STAB EVAL    2     0
65   -1406 061274 161441  0       396     11      66  -1 STAB U       2     0
66    1424 061274 161519  0        66     11      66   1 SSOL U       1     5
67    1427 061274 161522  0      2240     11    2240   1 KG   SPAR   36    21
68    1507 061274 161528  0         6      6       6  -1 STAB EVAL    2     0
69    1508 061274 161528  0       396     11      66  -1 STAB U       2     0
70    1526 061274 161554  0      2240     11    2240   1 K5KG SPAR   36    21
71    1606 061274 161558  0      3136     11    3136   1 INV  K5KG    2    21
72    1718 061274 161605  0        10     10      10  -1 VIBR EVAL    2     0
73    1719 061274 161606  0       660     11      66  -1 VIBR U       2     0
```

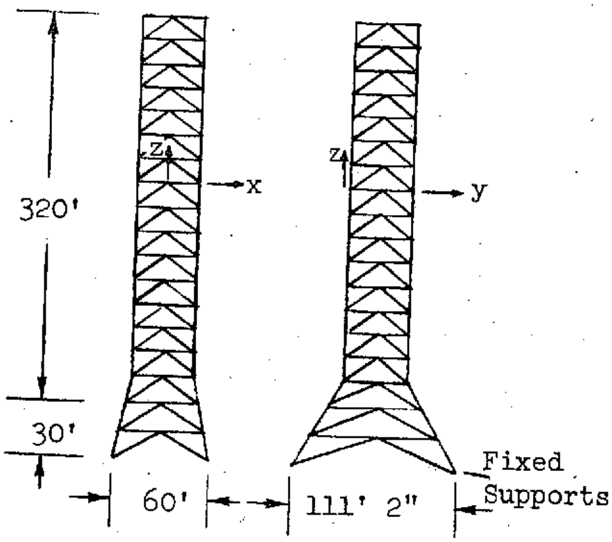
10. LUT (SATURN 5 LAUNCHER UMBILICAL TOWER)

Inertia prestress vibrational modes were calculated for the LUT. This problem illustrates use of the MOD and INC commands in processor ELD. The LUT is a multi-story frame structure, the floors of which are supported by columns and diagonals. Non-structural mass was attached at the four corners of each floor as rigid lumped mass. Varying cross-sectional properties for the general beam sections were defined throughout the structure. The accompanying figures and table show the section property data set distribution. The geometrical and material properties for the LUT are listed below:

$E = 3.0 \times 10^7 \ \text{lb/in}^2$

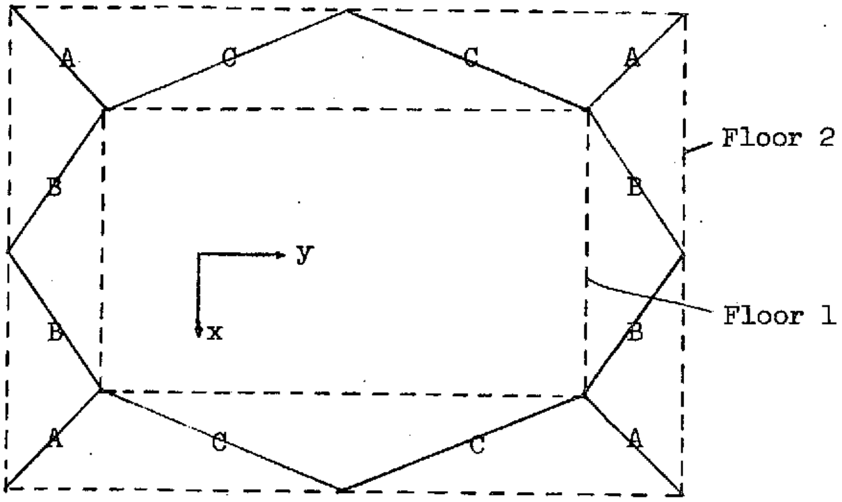
$\nu = 0.2998$

$\text{Specific Weight} = 0.28 \ \text{lb/in}^3$

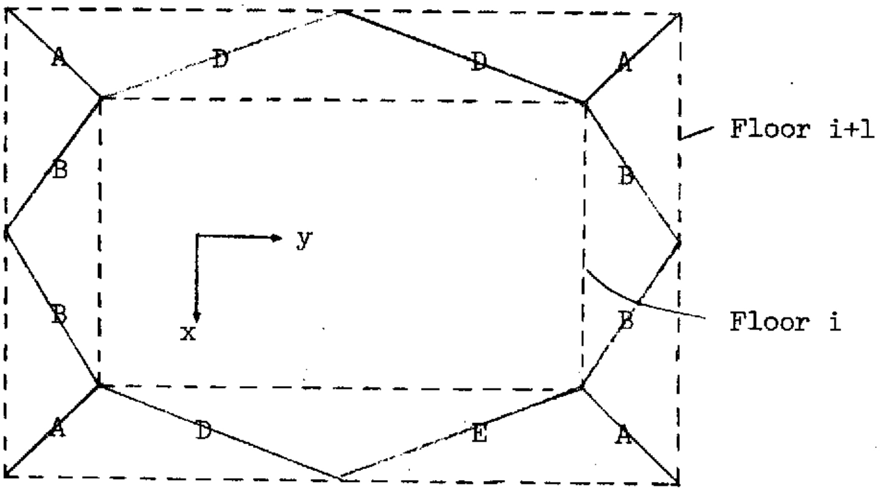


A downward inertia load of 386.088 (equivalent to a one-g loading) was used as a prestress load, and cantilevered vibrational modes were calculated for the prestressed structure.

Floor 1 Bay Layout



Floors 2-3 Bay Layout



Floors 4-19 Layout

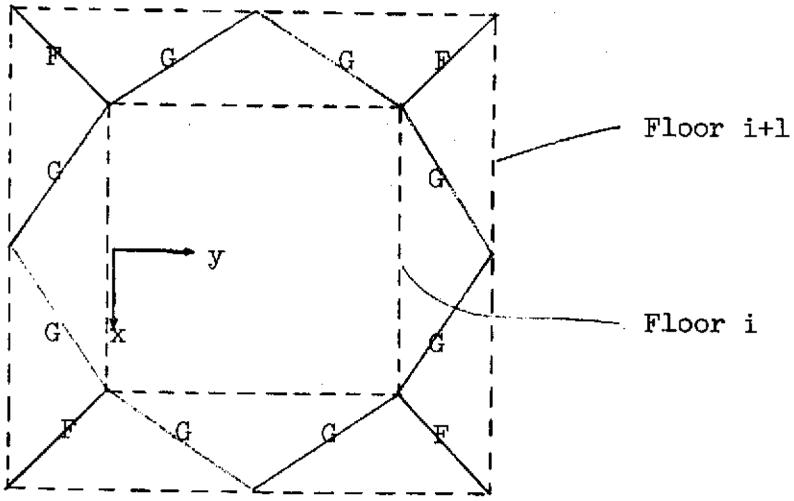
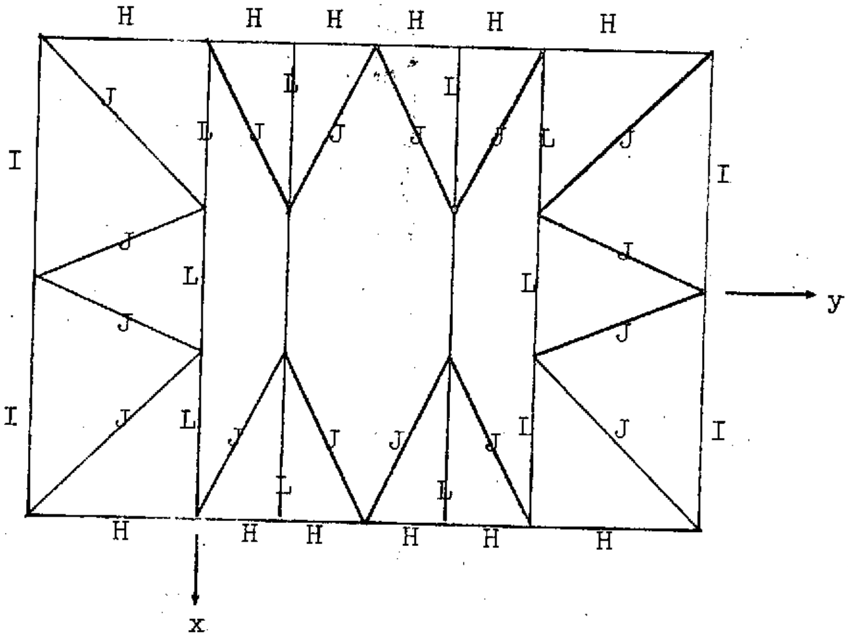


Fig. 10.1

Floors 2-3 Layout



Floors 4-19 Layout

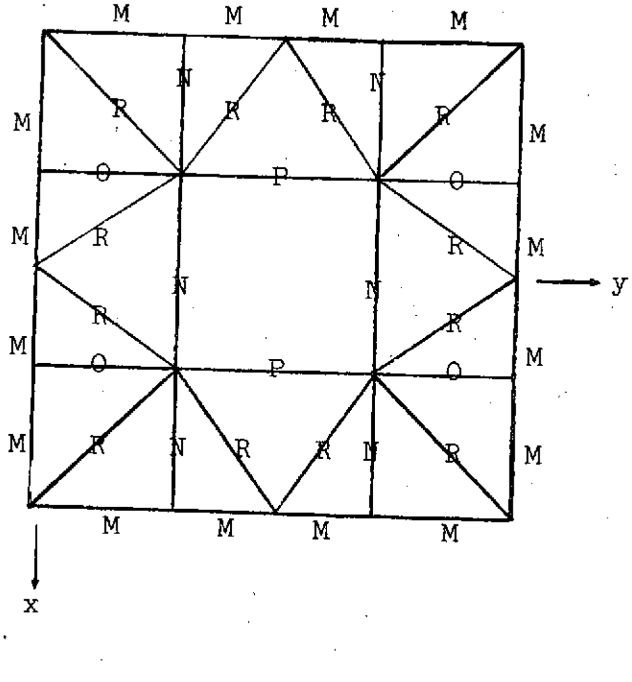


Fig. 10.1 (Continued)

Floor Bay Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	72	72	76																
B	73	73	77																
C	74																		
D		75	78																
E		72	78																
F				16	19	22	25	28	31	34	37	40	43	46	49	52	55	58	
G				17	20	23	26	29	32	35	38	41	44	47	50	53	56	59	
H		1	6																
I		2	7																
J		3	8																
K		4	9																
L		5	10																
M				18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	66
N				11	11	11	11	11	11	11	11	11	11	11	11	11	11	61	67
O				12	12	12	12	12	12	12	12	12	12	12	12	12	12	62	68
P				13	13	13	13	13	13	13	13	13	13	13	13	13	13	63	69
Q				14	14	14	14	14	14	14	14	14	14	14	14	14	14	64	70
R				15	15	15	15	15	15	15	15	15	15	15	15	15	15	65	71

Table 10.1 Section Data Sets for Floors and Bays

The elastic modes are summarized below:

Mode		ω^2
1	Bending, Direction -2	7.7389
2	Bending, Direction -1	8.9712
3	Torsional	34.3664

```
@XQT TAB                                       . GENERATE BASIC TABLES DEFINING STRUCTURE
START 372$
TITLE'SATURN V LAUNCHER UMBILICAL TOWER (LUT)
TEXT$
'SATURN V LAUNCHER UMBILICAL TOWER (LUT)
'
'DATA DESIGNED TO EXEMPLIFY THE USE OF 'INC'
'AND 'MOD' COMMANDS IN THE 'ELD' PROCESSOR.
MATERIAL CONSTANTS$
$                                               MATERIAL PROPERTY IS DEFINED WITH A
$                                               WEIGHT DENSITY
      1 3.+7 .29982668 .28$
CONSTRAINT CASE 1$
$                                               JOINTS 1-4 ARE COMPLETELY CONSTRAINED
      ZERO 1 2 3 4 5 6: 1 4 1$
BEAM ORIENTATION SPECIFICATIONS$
      1 1 3 1 0.$
      2 1 1 1 0.$
E21 SECTION PROPERTIES$
      GIVN  1  9012.1  0.   250.4  0.  44.16     9.47$
      GIVN  2  2096.4  0.    76.5  0.  22.4      8.4 $
      GIVN  3    63.35 0.    63.35 0.   7.3    126.7 $
      GIVN  4  4461.   0.   135.1  0.  31.77     4.66$
      GIVN  5  1814.5  0.    63.8  0.  20.       1.7 $
      GIVN  6  9012.1  0.   250.4  0.  44.16     9.47$
      GIVN  7  2096.4  0.    76.5  0.  22.4      8.4 $
      GIVN  8    28.14 0.    28.14 0.   5.58    56.28$
      GIVN  9  2024.8  0.    95.7  0.  24.71     2.57$
      GIVN 10  1140.7  0.    44.   0.  16.18     1.14$
      GIVN 11  1140.7  0.    44.   0.  16.18     1.14$
      GIVN 12    21.7  0.     2.89 0.   3.53      .08$
      GIVN 13   105.3  0.     2.79 0.   4.86      .1 $
      GIVN 14   446.3  0.    22.1  0.  10.59      .5 $
      GIVN 15    15.16 0.    15.16 0.   4.3     30.32$
      GIVN 16 21200.   0. 21200.   0. 276.   31100.  $
      GIVN 17   475.7  0.   475.7  0.  26.04   951.4 $
      GIVN 18  3988.6  0.   116.9  0.  29.11     3.47$
      GIVN 19 16667.   0. 16667.   0. 228.   24554.  $
      GIVN 20   475.7  0.   475.7  0.  26.04   951.4 $
      GIVN 21  3988.6  0.   116.9  0.  29.11     3.47$
      GIVN 22 14284.   0. 14284.   0. 205.   21538.  $
      GIVN 23   475.7  0.   475.7  0.  26.04   951.4 $
      GIVN 24  3266.7  0.   115.1  0.  27.65     3.8 $
      GIVN 25 12609.   0. 12609.   0. 182.   18683.  $
      GIVN 26   361.5  0.   361.5  0.  19.24   723.  $
      GIVN 27  3266.7  0.   115.1  0.  27.65     3.8 $
      GIVN 28 10773.   0. 10773.   0. 160.   16000.  $
      GIVN 29   361.5  0.   361.5  0.  19.24   723.  $
      GIVN 30  3266.7  0.   115.1  0.  27.65     3.8 $
      GIVN 31  9068.   0.  9068.   0. 138.   13476.  $
      GIVN 32   361.5  0.   361.5  0.  19.24   723.  $
      GIVN 33  3266.7  0.   115.1  0.  27.65     3.8 $
      GIVN 34  7459.   0.  7459.   0. 117.   11233.  $
      GIVN 35   361.5  0.   361.5  0.  19.24   723.  $
      GIVN 36  2364.3  0.    88.3  0.  24.7      3.57$
      GIVN 37  5969.   0.  5969.   0.  96.    8917.  $
      GIVN 38   361.5  0.   361.5  0.  19.24   723.  $
      GIVN 39  2364.3  0.    88.3  0.  24.7      3.57$
      GIVN 40  4585.   0.  4585.   0.  76.    6859.  $
      GIVN 41   248.5  0.   248.5  0.  12.88   497.  $
      GIVN 42  2096.4  0.    76.5  0.  22.4      8.4 $
```

```
          GIVN 43   2402.4   0.    930.1   0.   56.73     34.45$
          GIVN 44    248.5   0.    248.5   0.   12.88    497.  $
          GIVN 45   2096.4   0.     76.5   0.   22.4        8.4 $
          GIVN 46   2402.4   0.    930.1   0.   56.73     34.45$
          GIVN 47    248.5   0.    248.5   0.   12.88    497.  $
          GIVN 48   1814.5   0.     63.8   0.   20.         1.7 $
          GIVN 49   1266.5   0.    454.9   0.   32.65       7.63$
          GIVN 50    192.3   0.    192.3   0.    9.84    384.6 $
          GIVN 51   1814.5   0.     63.8   0.   20.         1.7 $
          GIVN 52   1266.5   0.    454.9   0.   32.65       7.63$
          GIVN 53    192.3   0.    192.3   0.    9.84    384.6 $
          GIVN 54   1814.5   0.     63.8   0.   20.         1.7 $
          GIVN 55    641.5   0.    107.3   0.   17.94       2.01$
          GIVN 56    192.3   0.    192.3   0.    9.84    384.6 $
          GIVN 57   1814.5   0.     63.8   0.   20.         1.7 $
          GIVN 58    641.5   0.    107.3   0.   17.94       2.01$
          GIVN 59    192.3   0.    192.3   0.    9.84    384.6 $
          GIVN 60   1814.5   0.     63.8   0.   20.         1.7 $
          GIVN 61   2987.3   0.    203.5   0.   29.43       4.52$
          GIVN 62   1140.7   0.     44.    0.   16.18       1.14$
          GIVN 63   1326.8   0.     53.1   0.   18.23       1.71$
          GIVN 64    446.3   0.     22.1   0.   10.59        .5 $
          GIVN 65     15.16  0.     15.16  0.    4.3       30.32$
          GIVN 66   5886.9   0.    170.3   0.   34.71       4.9 $
          GIVN 67   2987.3   0.    203.5   0.   29.43       4.52$
          GIVN 68   2987.3   0.    203.5   0.   29.43       4.52$
          GIVN 69   2987.3   0.    203.5   0.   29.43       4.52$
          GIVN 70   2987.3   0.    203.5   0.   29.43       4.52$
          GIVN 71     15.16  0.     15.16  0.    4.3       30.32$
          GIVN 72 26478.     0. 26478.     0. 326.     38584.   $
          GIVN 73    562.    0.    562.    0.   18.41   1124.   $
          GIVN 74    732.    0.    732.    0.   24.35   1464.   $
          GIVN 75   1157.    0.   1157.    0.   40.19   2314.   $
          GIVN 76 26300.     0. 26300.     0. 294.     38700.   $
          GIVN 77    562.    0.    562.    0.   18.41   1124.   $
          GIVN 78   1556.    0.   1556.    0.   56.6    3112.   $
   JOINT LOCATIONS$
              1  360.  907. -2880.  270.  589. -2160. 2 48$
              2 -360.  907. -2880. -270.  589. -2160. 2 48$
              3 -360. -427. -2880. -270. -109. -2160. 2 48$
              4  360. -427. -2880.  270. -109. -2160. 2 48$
              5  316.  348. -2520.  270.  348. -2160. 2 24$
              6 -316.  348. -2520. -270.  348. -2160. 2 24$
              7 -316.  132. -2520. -270.  132. -2160. 2 24$
              8  316.  132. -2520.  270.  132. -2160. 2 24$
              9  316.  480. -2520.  270.  480. -2160. 2 24$
             10 -316.  480. -2520. -270.  480. -2160. 2 24$
             11 -316.    0. -2520. -270.    0. -2160. 2 24$
             12  316.    0. -2520.  270.    0. -2160. 2 24$
             13  108.  480. -2520.  108.  480. -2160. 2 24$
             14 -108.  480. -2520. -108.  480. -2160. 2 24$
             15  108.  348. -2520.  108.  348. -2160. 2 24$
             16 -108.  348. -2520. -108.  348. -2160. 2 24$
             17  108.  132. -2520.  108.  132. -2160. 2 24$
             18 -108.  132. -2520. -108.  132. -2160. 2 24$
             19  108.    0. -2520.  108.    0. -2160. 2 24$
             20 -108.    0. -2520. -108.    0. -2160. 2 24$
             21  316.  240. -2520.  270.  240. -2160. 2 24$
             22    0.  751. -2520.    0.  589. -2160. 2 24$
             23 -316.  240. -2520. -270.  240. -2160. 2 24$
             24    0. -271. -2520.    0. -109. -2160. 2 24$
```

```
          25  316.  751. -2520.$
          26 -316.  751. -2520.$
          27 -316. -271. -2520.$
          28  316. -271. -2520.$
          53  240.  348. -1920.  240.  348.  1440. 15 20$
          54  108.  480. -1920.  108.  480.  1440. 15 20$
          55 -108.  480. -1920. -108.  480.  1440. 15 20$
          56 -240.  348. -1920. -240.  348.  1440. 15 20$
          57 -240.  132. -1920. -240.  132.  1440. 15 20$
          58 -108.    0. -1920. -108.    0.  1440. 15 20$
          59  108.    0. -1920.  108.    0.  1440. 15 20$
          60  240.  132. -1920.  240.  132.  1440. 15 20$
          61  108.  348. -1920.  108.  348.  1440. 15 20$
          62 -108.  132. -1920. -108.  132.  1440. 15 20$
          63 -108.  348. -1920. -108.  348.  1440. 15 20$
          64  108.  132. -1920.  108.  132.  1440. 15 20$
          65  240.  240. -1920.  240.  240.  1680. 16 20$
          66    0.  480. -1920.    0.  480.  1680. 16 20$
          67 -240.  240. -1920. -240.  240.  1680. 16 20$
          68    0.    0. -1920.    0.    0.  1680. 16 20$
          69  240.  480. -1920.  240.  480.  1680. 16 20$
          70 -240.  480. -1920. -240.  480.  1680. 16 20$
          71 -240.    0. -1920. -240.    0.  1680. 16 20$
          72  240.    0. -1920.  240.    0.  1680. 16 20$
         353  240.  290.  1680.$
         354   50.  480.  1680.$
         355  -50.  480.  1680.$
         356 -240.  290.  1680.$
         357 -240.  190.  1680.$
         358  -50.    0.  1680.$
         359   50.    0.  1680.$
         360  240.  190.  1680.$
         361   50.  290.  1680.$
         362  -50.  190.  1680.$
         363  -50.  290.  1680.$
         364   50.  190.  1680.$
RMASS$
$                                              RIGID LUMPED MASSES
         REPEAT  4 1$
          49 100.025$
         REPEAT  4 1$
          89  43.880$
         REPEAT  4 1$
         109  24.788$
         REPEAT  4 1$
         129  47.480$
         REPEAT  4 1$
         149  30.632$
         REPEAT  4 1$
         169  64.710$
         REPEAT  4 1$
         189  56.938$
         REPEAT  4 1$
         209  84.460$
         REPEAT  4 1$
         229  72.472$
         REPEAT  4 1$
         249 110.585$
         REPEAT  4 1$
         269 116.048$
         REPEAT  4 1$
```

```
        289 121.750$
        REPEAT  4 1$
        309 106.598$
        REPEAT  4 1$
        329 151.376$
        REPEAT  4 1$
        349 231.478$
        REPEAT  4 1$
        369 308.282$
  @XQT ELD                                         . READ ELEMENT DEFINITIONS
  E21 $
   GROUP 1        'FLOOR 2
       NSECT=1  !  9 25   1   1 4 1    $
                   5  9   1   1 4 1    $
                   5 21   1   1 2 2    $
                   8 21                $
                   6 23                $
       NSECT=2  ! 22 26   1   1 2 2    $
                  22 25   1   1 2 2    $
       NSECT=3  ! 13 25   1   1 2 1    $
                  13 22   1   1 2 6    $
                  14 22                $
                  20 27                $
                   9 15   1   1 2 1    $
                  11 18   1   1 2 5    $
                  12 17                $
                  17 21   1   1 2 3    $
                  15 21                $
                  18 23   1   1 2 1    $
       NSECT=4  !  5 15   1   1 2 1    $
                   7 18                $
                   8 17                $
                  15 16   1   1 2 2    $
       NSECT=5  !  9 13   1   1 2 1    $
                  11 20                $
                  12 19                $
                  13 14   1   1 2 6    $
   GROUP 2        'FLOOR 3
     MOD JOINT=24 ! MOD NSECT=5 $
       NSECT=1  !  9 25   1   1 4 1    $
                   5  9   1   1 4 1    $
                   5 21   1   1 2 2    $
                   8 21                $
                   6 23                $
       NSECT=2  ! 22 26   1   1 2 2    $
                  22 25   1   1 2 2    $
       NSECT=3  ! 13 25   1   1 2 1    $
                  13 22   1   1 2 6    $
                  14 22                $
                  20 27                $
                   9 15   1   1 2 1    $
                  11 18   1   1 2 5    $
                  12 17                $
                  17 21   1   1 2 3    $
                  15 21                $
                  18 23   1   1 2 1    $
       NSECT=4  !  5 15   1   1 2 1    $
                   7 18                $
                   8 17                $
                  15 16   1   1 2 2    $
       NSECT=5  !  9 13   1   1 2 1    $
```

```
                 11 20                 $
                 12 19                 $
                 13 14    1  1  2  6   $
  GROUP 3       !FLOORS 4-19
   INC NSECT=3 $
    MOD JOINT=0 ! MOD NSECT=0  $
      NSECT=18 ! 54 66    1  1  15 20  $
      NSECT=18 ! 55 66    1  1  15 20  $
      NSECT=18 ! 55 70    1  1  15 20  $
      NSECT=18 ! 54 69    1  1  15 20  $
      NSECT=18 ! 56 70    1  1  15 20  $
      NSECT=18 ! 56 67    1  1  15 20  $
      NSECT=18 ! 57 67    1  1  15 20  $
      NSECT=18 ! 57 71    1  1  15 20  $
      NSECT=18 ! 58 71    1  1  15 20  $
      NSECT=18 ! 58 68    1  1  15 20  $
      NSECT=18 ! 59 72    1  1  15 20  $
      NSECT=18 ! 59 68    1  1  15 20  $
      NSECT=18 ! 60 72    1  1  15 20  $
      NSECT=18 ! 60 65    1  1  15 20  $
      NSECT=18 ! 53 69    1  1  15 20  $
      NSECT=18 ! 53 65    1  1  15 20  $
   INC NSECT=0 $
    MOD JOINT=300 $
      NSECT=66 ! 54 66    1  1   2  6  $
                 55 66    1  1   2  1  $
                 54 69    1  1   2  1  $
                 56 70    1  1   2  1  $
                 57 67    1  1   2  1  $
                 58 71    1  1   2  1  $
                 59 68                 $
                 60 65                 $
                 53 69                 $
                 53 65                 $
    MOD JOINT=0$
      NSECT=11 ! 53 61    1  1  14 20  $
                 61 63    1  1  14 20  $
                 56 63    1  1  14 20  $
                 57 62    1  1  14 20  $
                 62 64    1  1  14 20  $
                 60 64    1  1  14 20  $
      NSECT=12 ! 55 63    1  1  14 20  $
                 58 62    1  1  14 20  $
                 59 64    1  1  14 20  $
      NSECT=13 ! 62 63    1  1  14 20  $
                 61 64    1  1  14 20  $
      NSECT=14 ! 54 61    1  1  14 20  $
      NSECT=15 ! 61 66    1  1  14 20  $
                 63 66    1  1  14 20  $
                 63 67    1  1  14 20  $
                 62 67    1  1  14 20  $
                 63 70    1  1  14 20  $
                 62 71    1  1  14 20  $
                 62 68    1  1  14 20  $
                 64 68    1  1  14 20  $
                 64 72    1  1  14 20  $
                 64 65    1  1  14 20  $
                 61 65    1  1  14 20  $
                 61 69    1  1  14 20  $
   INC NSECT=6 $
    MOD JOINT=280$
```

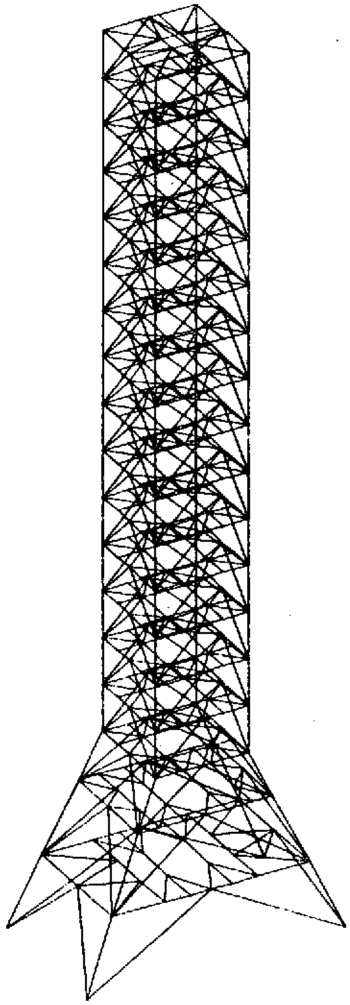
```
        NSECT=61 : 53 61     1 1  2 20 $
        NSECT=61 : 61 63     1 1  2 20 $
        NSECT=61 : 56 63     1 1  2 20 $
        NSECT=61 : 57 62     1 1  2 20 $
        NSECT=61 : 62 64     1 1  2 20 $
        NSECT=61 : 60 64     1 1  2 20 $
        NSECT=62 : 55 63     1 1  2 20 $
        NSECT=62 : 56 62     1 1  2 20 $
        NSECT=62 : 59 64     1 1  2 20 $
        NSECT=63 : 62 63     1 1  2 20 $
        NSECT=63 : 61 64     1 1  2 20 $
        NSECT=64 : 54 61     1 1  2 20 $
        NSECT=65 : 61 66     1 1  2 20 $
        NSECT=65 : 63 66     1 1  2 20 $
        NSECT=65 : 63 67     1 1  2 20 $
        NSECT=65 : 62 67     1 1  2 20 $
        NSECT=65 : 63 70     1 1  2 20 $
        NSECT=65 : 62 71     1 1  2 20 $
        NSECT=65 : 62 68     1 1  2 20 $
        NSECT=65 : 64 68     1 1  2 20 $
        NSECT=65 : 64 72     1 1  2 20 $
        NSECT=65 : 64 65     1 1  2 20 $
        NSECT=65 : 61 65     1 1  2 20 $
        NSECT=65 : 61 69     1 1  2 20 $
   GROUP 4      'INTER-FLOOR COLUMNS AND DIAGONALS
    INC NSECT=0 $
     MOD JOINT=0$
      NREF=2     $
        NSECT=72 :  1 25     1 1  4  1 $
                   25 49     1 1  4  1 $
                   25 45               $
        NSECT=73 :  2 22     1 1  2 24 $
                    1 22     1 1  2 24 $
                    3 24     1 1  2 24 $
                    4 24     1 1  2 24 $
        NSECT=74 :  2 23               $
                    4 21               $
                    1 21     1 1  2  2 $
        NSECT=75 : 26 47               $
                   27 47               $
                   28 45               $
        NSECT=76: 49 69      1 1  4  1 $
        NSECT=77 : 49 66     1 1  2  2 $
                   50 66     1 1  2  2 $
        NSECT=78 : 50 67               $
                   49 65     1 1  2  2 $
                   52 65               $
    INC NSECT=3 $
        NSECT=16 : 69 89     1 1 15 20   $
        NSECT=16 : 70 90     1 1 15 20   $
        NSECT=16 : 71 91     1 1 15 20   $
        NSECT=16 : 72 92     1 1 15 20   $
        NSECT=17 : 69 86     1 1 15 20   $
        NSECT=17 : 70 86     1 1 15 20   $
        NSECT=17 : 70 87     1 1 15 20   $
        NSECT=17 : 71 87     1 1 15 20   $
        NSECT=17 : 71 88     1 1 15 20   $
        NSECT=17 : 72 88     1 1 15 20   $
        NSECT=17 : 72 85     1 1 15 20   $
        NSECT=17 : 69 85     1 1 15 20   $
  @XQT TOPO                              ' ANALYZE ELEMENT INTERCONNECTIVITY
```

```
$
@XQT E                                   . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                 . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT K                                   . FORM SYSTEM K
$
@XQT INV                                 . FACTOR K
$
@XQT M                                   . FORM SYSTEM M
$                                          FORM SYSTEM M WITH MASS ENTRIES BY
$                                          CONVERTING MATERIAL PROPERTY WEIGHT
$                                          DENSITY INTO MASS DENSITY
     RESET G=386.088$
@XQT MRM                                 . ADD RIGID MASS DATA TO M
$
@XQT Q                                   . DEFINE APPLIED INERTIA PRESTRESS LOADS
     RESET MLIB=1$
     RESET M=M+RM,MTYPE=SPAR$
$                                          INERTIA PRESTRESS LOAD
     CASE 1:       INERTIA LOAD FOR PRESTRESS
        INERTIA FORCE 3 -386.088$
@XQT DSOL                                . COMPUTE SOLUTION FOR PRESTRESS LOAD
     RESET MAXIT=2,UP=1,FP=1$
@XQT KG                                  . FORM SYSTEM KG, PRESTRESS SOLUTION
$
@XQT LCM                                 . FORM LINEAR COMBINATION OF MATRICES
$
@XQT INV                                 . FACTOR K+KG
     RESET K=K+KG$
@XQT EIG                                 . SOLVE SYSTEM EIGENPROBLEM
$                                          PRESTRESS VIBRATION ANALYSIS
     RESET INIT=5,NDYN=5,MAXIT=2,REPRT=1$
     RESET K=K+KG,M=M+RM$
@XQT DCU                                 . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
SATURN V LAUNCHER UMBILICAL TOWER (LUT)

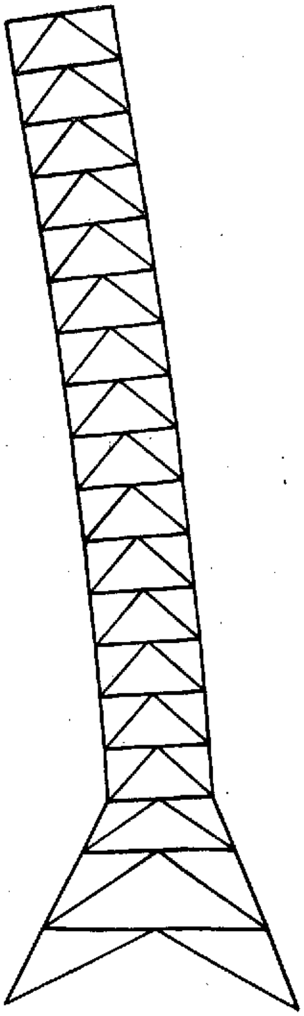
                                     E              ROWS    BLK    T    DATA SET NAME
SEQ      RR   DATE    TIME           R      WORDS   /BLK    SIZE   Y    N1    N2      N3    N4
  1      11 061074  112901           0         18      1      18   0  JDF1  BTAB       1     8
  2      12 061074  112901           0        372    372     372   0  JREF  BTAB       2     6
  3      26 061074  112901           0         12      1      12   1  ALTR  BTAB       2     4
  4      27 061074  112901           0         19      1      19   4  NDAL             0     0
  5      28 061074  112902           0         80      4      80   3  TEXT  BTAB       2     1
  6      31 061074  112902           0         10      1      10   1  MATC  BTAB       2     2
  7      32 061074  112903           0        372    372     372   0  CON              1     0
  8      46 061074  112903           0         10      2      10   1  MREF  BTAB       2     7
  9      47 061074  112906           0       2418     78    2418   1  BA    BTAB       2     9
 10     134 061074  112909           0       1116    372    1116   1  JLOC  BTAB       2     5
 11     174 061074  112909           0       2232    372    2232   1  RMAS  BTAB       2    18
 12     254 061074  112909           0       3348    372    3348   1  QJJT  BTAB       2    19
 13     374 061074  112921           0      15104    944     896   0  DEF   E21        1     2
 14     918 061074  112921           0          8      4       8   0  GD    E21        1     2
 15     919 061074  112921           0         60      4      60   0  GTIT  E21        1     2
 16     922 061074  112921           0         12     12      12   0  NELZ  BTAB       1    11
 17     923 061074  112921           0          5      1       5   0  KE               0     0
 18     924 061074  112921           0          6      1       6   0  NS               0     0
 19     925 061074  112921           0          1      1       1   3  ELTS  NAME       0     0
 20     926 061074  112921           0          1      1       1   0  ELTS  LTYP       0    -0
 21     936 061074  112921           0          1      1       1   0  ELTS  NNOD       0     0
 22     937 061074  112921           0          1      1       1   0  ELTS  ISCT       0     0
 23     938 061074  112921           0          1      1       1   0  ELTS  NELS       0     0
 24     939 061074  112921           0          1      1       1   0  ELTS  LE3        0     0
 25     940 061074  112929           0      10752    372    1344   0  KMAP          1316    20
 26    1324 061074  112932           0      12096    372    1344   0  AMAP  ♦♦♦]   2223    71
 27    1756 061074  112957           0     132160    944     140   4  E21              1     2
 28    6476 061074  112943           0         20     20      20   0  DIR   E21        1     2
 29    6477 061074  113057           0      51520    372    2240   1  K     SPAR      36  1316
 30    8317 061074  113129           0      90944    372    3136   1  INV   K          1  2223
 31   11565 061074  113242           0      51520    372    2240   1  M     SPAR      36  1316
 32   13405 061074  113249           0      51520    372    2240   1  M+RM  SPAR      36  1316
 33   15245 061074  113258           0       2232    372    2232  -1  M+RM  DIAG       0     0
 34   15325 061074  113303           0       2232    372    2232  -1  RBMX  VEC        1     0
 35   15405 061074  113306           0       2232    372    2232  -1  RBMX  VEC        2     0
 36   15485 061074  113309           0       2232    372    2232  -1  RBMX  VEC        3     0
 37   15565 061074  113310           0       2232    372    2232  -1  NFM              0     1
 38   15645 061074  113310           0         15      1      15   4  CASE  ♦♦♦♦       0     1
 39   15646 061074  113310           0          0      0       0   0  LDIR             1     1
 40   15646 061074  113331           0       2232    372    2232   1  SSOL  U          1     1
 41   15735 061074  213213           0      51520    372    2240   1  KG    SPAR      36  1316
 42   17575 061074  213215           0      51520    372    2240   1  K+KG  SPAR      36  1316
 43   19415 061074  213303           0      90944    372    3136   1  INV   K+KG       1  2223
 44   22663 061074  213311           0          5      5       5  -1  VIBR  EVAL       1     0
 45   22664 061074  213312           0      11160    372    2232  -1  VIBR  U          1     0
```

UNDEFORMED STRUCTURE



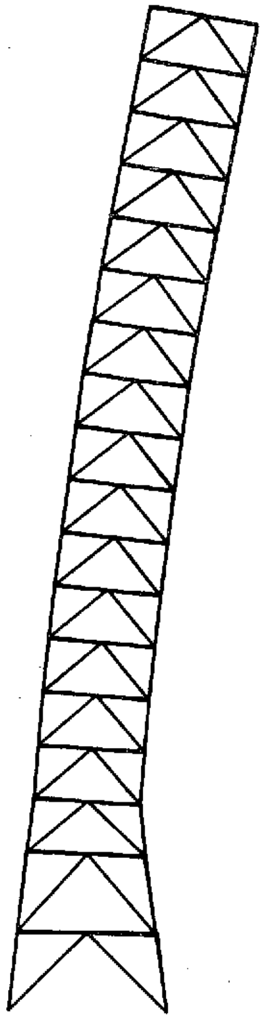
LAUNCHER UMBILICAL TOWER

SEQ 1 VIBRATIONAL MODE. FREQ (HZ) = $.442753 \times 10^{+00}$



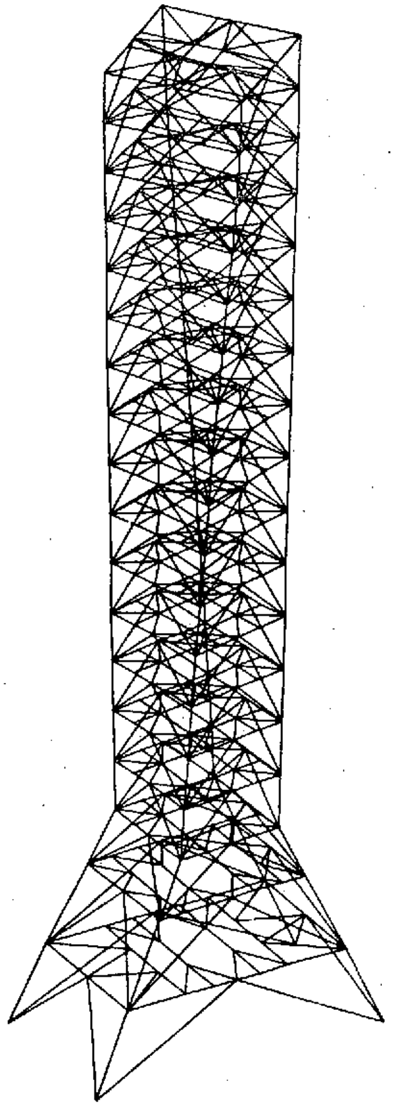
LAUNCHER UMBILICAL TOWER

SEQ 2 VIBRATIONAL MODE. FREQ (HZ) = $.476699 \times 10^{+00}$



LAUNCHER UMBILICAL TOWER

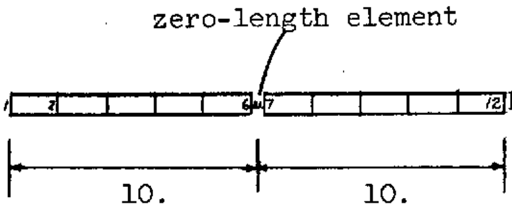
SEQ 3 VIBRATIONAL MODE. FREQ (HZ) = $.933012 \times 10^{+00}$



LAUNCHER UMBILICAL TOWER

11. MECHANISM

Free-free vibrational modes were obtained for a linkage in order to demonstrate the application of a spectral shift in the eigensolution process. The linkage was defined as a system of two beam segments connected through a zero-length 6 x 6 directly specified stiffness matrix. Planar motion was specified for the otherwise unconstrained configuration. The geometrical and material properties of the linkage are shown on the figure below:



$E = .1 \times 10^{8}\ \text{in}^{2}$

$\nu = .3$

Mass density $= .1 \times 10^{-4}\ \text{lb-sec}^{2}/\text{in}^{4}$

$I = 1.0\ \text{in}^{4}$

$A = 1.0\ \text{in}^{2}$

Eigenvalues were desired in the neighborhood of the value $c = 0.1 \times 10^{11}$, which was selected as the shift point for the eigenvalue solution of K-cM (see Reference 1, Section 3.17). Four "zero" eigenvalues were obtained, representing the three rigid body motions and a mechanism mode.

The elastic modes are summarized below:

	SPAR
ω_1^2	$.24584 \times 10^{11}$
ω_2^2	$.34629 \times 10^{11}$
ω_3^2	$.36038 \times 10^{11}$

```
@XQT TAB                                       . GENERATE BASIC TABLES DEFINING STRUCTURE
START 12   3 4 5$
TITLE'TWO-BEAM LINKAGE
TEXT$
'SYSTEM OF TWO BEAM SEGMENTS CONNECTED
'BY A ZERO-LENGTH ELEMENT.
'
'PROBLEM DESIGNED TO ILLUSTRATE THE USE OF A
'SHIFT POINT IN CALCULATION OF FREE-FREE
'VIBRATIONAL MODES.
MATC$                                            MATERIAL CONSTANTS
     1 1.+7 .3 .1-4$
JLOC$                                            JOINT LOCATIONS
     1  0. 0. 0. 10. 0. 0. 6 1 2$
     6 10. 0. 0. 20. 0. 0. 6 1$
CON 1$                                           CONSTRAINT CASE
MREF$                                            BEAM ORIENTATION SPECIFICATIONS
     1 1 2 1 1.$
E21 SECTION PROPERTIES$
     GIVN 1 1. 1. 1. 1. 1. 1.$
BB$                                              BEAM S6X6 STIFFNESS COEFFICIENTS
$                                                DIRECTLY SPECIFIED STIFFNESS MATRIX,
$                                                THE LAST FOUR ROWS OF WHICH ARE ZERO.
     1 1.+8: 0. 1.+8: : : : $
@XQT ELD                                       . READ ELEMENT DEFINITIONS
E21 $
$                                                BEAM SEGMENT DEFINITIONS
     1 2  1 5 2 6$
E25 $
$                                                ZERO-LENGTH ELEMENT DEFINTION
     6 7$
@XQT TOPO                                      . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E                                         . FORM ELEMENT DATA PACKETS
@XQT EKS                                       . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                         . FORM SYSTEM K
@XQT M                                         . FORM SYSTEM M
@XQT LCM                                       . FORM LINEAR COMBINATION OF MATRICES
$                                                SHIFT POINT SELECTED = .1+11
$                                                FOR THE EQUATION R=C1*Q1+C2*Q2,
$                                                R=K+LM
$                                                C1=1.0 (DEFAULT)
$                                                Q1=K   (DEFAULT)
$                                                C2=-.1+11
$                                                Q2=M
     RESET R=K+LM,Q2=M,C2=-.1+11$
@XQT INV                                       . FACTOR K+LM
     RESET K=K+LM$
@XQT EIG                                       . SOLVE SYSTEM EIGENPROBLEM
$                                                SOLVE THE EIGENPROBLEM FOR THE 'SHIFTED'
$                                                EIGENSYSTEM.
$                                                USE K=K+LM AS THE 'SHIFTED MATRIX'.
$                                                ADD  SHIFT=.1+11  TO THE RESULTING
$                                                EIGENVALUES.
     RESET INIT=10,NDYN=10,REPRT=1$
     RESET K=K+LM$
     RESET SHIFT=.1+11$
     PRINT 0 0 0 1 1$
@XQT DCU                                       . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
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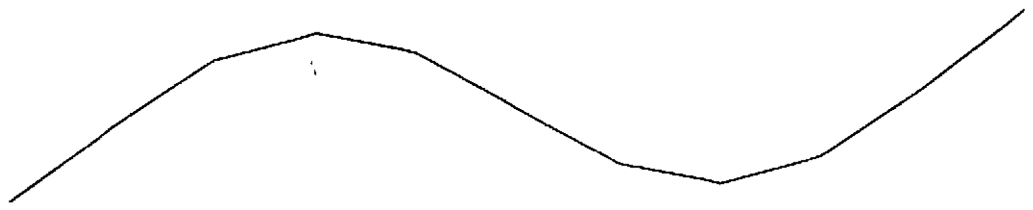
TWO-BEAM LINKAGE

                            E              ROWS   BLK   T    DATA SET NAME
SEQ      RR  DATE   TIME    R     WORDS    /BLK  SIZE   Y   N1    N2     N3     N4
  1      11 062074 001823   0        18       1    18   0 JDF1  BTAB     1      8
  2      12 062074 001823   0        12      12    12   0 JREF  BTAB     2      6
  3      13 062074 001823   0        12       1    12   1 ALTR  BTAB     2      4
  4      14 062074 001823   0        19       1    19   4 NDAL           0      0
  5      15 062074 001823   0       120       6   120   3 TEXT  BTAB     2      1
  6      20 062074 001824   0        10       1    10   1 MATC  BTAB     2      2
  7      21 062074 001824   0        36      12    36   1 JLOC  BTAB     2      5
  8      23 062074 001824   0        12      12    12   0 CON            1      0
  9      24 062074 001824   0         5       1     5   1 MREF  BTAB     2      7
 10      25 062074 001824   0        31       1    31   1 BA    BTAB     2      9
 11      27 062074 001824   0        21       1    21   1 BB    BTAB     2     10
 12      28 062074 001824   0       108      12   108   1 QJJT  BTAB     2     19
 13      32 062074 001826   0       160      10   896   0 DEF   E21      1      2
 14      64 062074 001826   0         2       1     2   0 GO    E21      1      2
 15      65 062074 001826   0        15       1    15   0 GTIT  E21      1      2
 16      66 062074 001826   0        16       1   896   0 DEF   E25      5      2
 17      98 062074 001826   0         2       1     2   0 GO    E25      5      2
 18      99 062074 001826   0        15       1    15   0 GTIT  E25      5      2
 19     100 062074 001826   0        12      12    12   0 NELZ  BTAB     1     11
 20     101 062074 001826   0        10       2    10   0 KE             0      0

 21     111 062074 001826   0        12       2    12   0 NB             0      0
 22     112 062074 001826   0         2       2     2   3 ELTS  NAME     0      0
 23     113 062074 001826   0         2       2     2   0 ELTS  LTYP     0      0
 24     114 062074 001826   0         2       2     2   0 ELTS  NNOD     0      0
 25     115 062074 001826   0         2       2     2   0 ELTS  ISCT     0      0
 26     116 062074 001826   0         2       2     2   0 ELTS  NELS     0      0
 27     117 062074 001827   0         2       2     2   0 ELTS  LE3      0      0
 28     118 062074 001828   0      1344      12  1344   0 KMAP          23      3
 29     166 062074 001828   0      1344      12  1344   0 AMAP  @@@@    23      3
 30     214 062074 001831   0      1400      10   140   4 E21            1      2
 31     264 062074 001830   0        20      20    20   0 DIR   E21      1      2
 32     265 062074 001831   0       112       1   112   4 E25            5      2
 33     269 062074 001830   0        20      20    20   0 DIR   E25      5      2
 34     270 062074 001833   0      2240      12  2240   1 K     SPAR     9     23
 35     350 062074 001834   0      2240      12  2240   1 M     SPAR     9     23
 36     430 062074 001835   0      2240      12  2240   1 K*LM  SPAR     9     23
 37     510 062074 001837   0      3136      12  3136   1 INV   K+LM     1     23
 38     622 062074 001839   0        10      10    10  -1 VIBR  EVAL     1      0
 39     623 062074 001839   0       720      12    72  -1 VIBR  U        1      0
```

SEQ 5 VIBRATIONAL MODE. EIGENVALUE = $.458397\times10^{+10}$

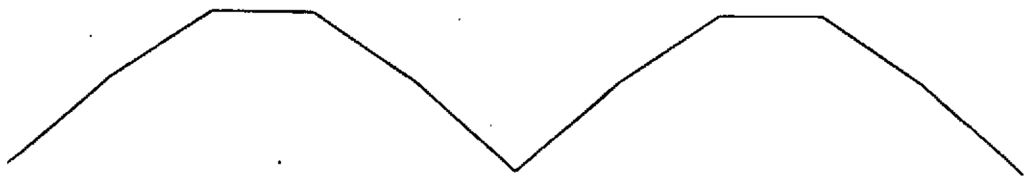


BEAM MECHANISM

SEQ 6 VIBRATIONAL MODE. EIGENVALUE = $.146287 \times 10^{+11}$

BEAM MECHANISM

SEQ 7 VIBRATIONAL MODE. EIGENVALUE = $.160383 \times 10^{+11}$



BEAM MECHANISM

REFERENCES

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2. Timoshenko, S. and D. H. Young, "Vibration Problems in Engineering", Third Edition, Van Nostrand, 1955
3. Timoshenko, S. and J. M. Gere, "Theory of Elastic Stability", 2nd Edition, McGraw-Hill, 1961
4. Timoshenko, S. and S. Woinowsky-Krieger, "Theory of Plates and Shells", 2nd Edition, McGraw-Hill, 1959
5. Flügge, W. "Stresses in Shells", 2nd printing, Springer-Verlag, 1962
6. Forsberg, K. "Influence of Boundary Conditions on the Modal Characteristics of Thin Cylindrical Shells", AIAA Journal, Vol. 2, No. 12, December 1964
7. Kalnins, A., "Free Vibration of Rotationally Symmetric Shells", The Journal of the Acoustic Society of America, Vol. 36, No. 7, July 1964
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9. Timoshenko, S., "Strength of Materials", Part 2, Third Edition, Van Nostrand, 1956
10. Timoshenko, S. and J. N. Goodier, "Theory of Eleasticity", McGraw-Hill, New York, 1951
11. Seide, "On the Free Vibration of Simply Supported Truncated Conical Shells", SSD-TDR-64-15, 27 February 1964

CORRECTIONS FOR REPORT

LMSC-D403169

CORRECTIONS FOR REPORT

LMSC-D403169

```
$                                          CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 1
    RESET K=K+KG,INIT=8$
PRINT 0 0 0 0 1$
@XQT INV
    RESET K=K+KG,CON=2$
@XQT EIG
$                                          FIND VIBRATIONAL MODES OF THE STRESSED
$                                          CIRCULAR MEMBRANE UNDER CONSTRAINT CASE 2
    RESET K=K+KG,INIT=8,CON=2$
    PRINT 0 0 0 0 -1$
@XQT DCU                                 . EXECUTE DATA COMPLEX UTILITY PROGRAM
    TOC 1$
```

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ORIGINAL PAGE IS POOR

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 VIBRATION OF A CIRCULAR MEMBRANE

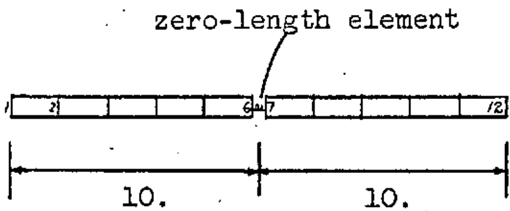
                                 E            ROWS   BLK   T   DATA SET NAME
SEQ     RR   DATE     TIME       R    WORDS   /BLK   SIZE  Y   N1   N2    N3    N4
  1     11 051474  042706        0       18      1     18  0  JDF1 BTAB   1     8
  2    -12 051474  042706        0      101    101    101  0  JREF BTAB   2     6
  3     16 051474  042706        0       12      1     12  1  ALTR BTAB   2     4
  4     17 051474  042706        0       19      1     19  4  NDAL        0     0
  5     18 051474  042706        0       10      1     10  1  MATC BTAB   2     2
  6     19 051474  042706        0      101    101    101  0  JREF BTAB   2     6
  7     23 051474  042707        0      303    101    303  1  JLOC BTAB   2     5
  8     34 051474  042707        0       19      1     19  1  SA   BTAB   2    13
  9     35 051474  042707        0      101    101    101  0  CON.        1     0
 10     39 051474  042708        0      101    101    101  0  CON         2     0
 11     43 051474  042709        0      101    101    101  0  CON         3     0
 12     47 051474  042709        0      909    101    909  1  QJJT BTAB   2    19
 13     80 051474  042715        0     1296     81    896  0  DEF  E41    9     4
 14    144 051474  042715        0        2      1      2  0  GD   E41    9     4
 15    145 051474  042715        0       15      1     15  0  GTIT E41    9     4
 16    146 051474  042715        0      144      9    896  0  DEF  E31    6     3
 17    178 051474  042715        0        2      1      2  0  GD   E31    6     3
 18    179 051474  042715        0       15      1     15  0  GTIT E31    6     3
 19    180 051474  042715        0       12     12     12  0  NELZ BTAB   1    11
 20    181 051474  042715        0       10      2     10  0  KE          0     0
 21    191 051474  042715        0       12      2     12  0  NS          0     0
 22    192 051474  042716        0        2      2      2  3  ELTS NAME   0     0
 23    193 051474  042716        0        2      2      2  0  ELTS LTYP   0     0
 24    194 051474  042717        0        2      2      2  0  ELTS NNOD   0     0
 25    195 051474  042717        0        2      2      2  0  ELTS ISCT   0     0
 26    196 051474  042718        0        2      2      2  0  ELTS NELS   0     0
 27    197 051474  042718        0        2      2      2  0  ELTS LE3    0     0
 28    198 051474  042721        0     2688    101   1344  0  KMAP      453    30
 29    294 051474  042726        0     9408    101   1344  0  AMAP @@@( 1137    78
 30    630 051474  042734        0     1260      9    140  4  E31         6     3
 31    675 051474  042730        0       20     20     20  0  DIR  E31    6     3
 32    676 051474  042736        0    13608     81    168  4  E41         9     4
 33   1162 051474  042732        0       20     20     20  0  DIR  E41    9     4
 34   1163 051474  042743        0     6720    101   2240  1  M    SPAR   9   453
 35   1403 051474  042745        0      606    101    606 -1  NFM         0     1
 36   1425 051474  042745        0       15      1     15  4  CASE @@@@   0     1
 37   1426 051474  042745        0        0      0      0  0  LDIR        1     1
 38   1426 051474  042751        0     6720    101   2240  1  K    SPAR   9   453
 39   1666 051474  042755        0     9408    101   3136  1  INV  K      3  1137
 40   2002 051474  042758        0      606    101    606  1  SSOL U      0     1
 41   2033 051474  042806        0     6720    101   2240  1  KG   SPAR   9   453
 42   2273 051474  042807        0     6720    101   2240  1  K*KG SPAR   9   453
 43   2513 051474  042815        0    12544    101   3136  1  INV  K+KG   1  1137
 44   2961 051474  042816        0        8      8      8 -1  VIBR EVAL   1     0
 45   2962 051474  042817        0     4848    101    606 -1  VIBR U      1     0
 46   3138 051474  042919        0    12544    101   3136  1  INV  K+KG   2  1137
 47   3586 051474  042921        0        8      8      8 -1  VIBR EVAL   2     0
 48   3587 051474  042922        0     4848    101    606 -1  VIBR U      2     0
```

```
@XQT TAB                                   . GENERATE BASIC TABLES DEFINING STRUCTURE
START 11$
TITLE'  BEAM PROBLEMS
TEXT$
'     THE FOLLOWING BEAM PROBLEMS ARE SOLVED IN THIS RUN:
'       1  VIBRATION OF A CANTILEVERED BEAM
'       2  BUCKLING OF A COLUMN
'       3  LATERAL DEFLECTION OF A COMPRESSED BEAM
'       4  VIBRATION OF A PRESTRESSED BEAM
'       5  LATERAL BUCKLING OF A CANTILEVERED BEAM
'       6  LATERAL BUCKLING OF A BEAM IN PURE BENDING
'       7  BUCKLING OF A BEAM DUE TO TEMPERATURE
'       8  VIBRATION OF A HEATED BEAM
'
MATERIAL CONSTANTS
   1  .1+8  .0  .1  .1-4$
JOINT LOCATIONS
   1  .0  .0  .0   .0  .0  10.0  11 1$
BEAM ORIENTATION SPECIFICATIONS
   1  1  2  1  0.$
E21 SECTION PROPERTIES$
   GIVN  1  .0004  .0  .0036  .0  .12  .0016$
CONSTRAINT CASE 1$
$                                           THE BEAM IS CONSTRAINED AT ONE END
   FIXED PLANE = 3$
CONSTRAINT CASE 2$
$                                           SIMPLY SUPPORTED AT BOTH ENDS
   ZERO 1,2,3,6:   1,11,10$
@XQT ELD                                  . READ ELEMENT DEFINITIONS
E21 $
   1 2  1 10$
@XQT TOPO                                 . ANALYZE ELEMENT INTERCONNECTIVITY
$
@XQT E                                    . FORM ELEMENT DATA PACKETS
$
@XQT EKS                                  . INSERT K, S INTO ELEMENT DATA PACKETS
$
@XQT M                                    . FORM SYSTEM M
$                                           EXCLUDE ROTATORY INERTIA EFFECTS
    RESET IBEAM=1$
@XQT K                                    . FORM SYSTEM K
$
@XQT INV                                  . FACTOR MATRIX IN K SPAR FORMAT
$
@XQT EIG                                  . SOLVE SYSTEM EIGENPROBLEM
$                                           FIND VIBRATIONAL MODES OF THE CANTILEVER
$                                           BEAM.
    RESET INIT=10,REPR=1$
@XQT Q                                    . DEFINE APPLIED LOADS
    CASE 1' AXIAL COMPRESSIVE FORCE APPLY AT FREE END.
      NODAL FORCES, MOMENTS
        11 3 -1.0$
    CASE 2' SHEAR FORCE IN 1-DIRECTION APPLY AT FREE END.
      NODAL FORCES, MOMENTS
        11 1 1.0$
    CASE 3' BENDING MOMENT IN 2-DIRECTION APPLY AT BOTH ENDS.
      NODAL FORCES, MOMENTS
         1 5 1.0:  11 5 -1.0$
    CASE 4' CONCENTRATED FORCE AT MID-SPAN
      NODAL FORCES, MOMENTS
         6 1 10.0$
```

```
        CASE 5: TEMPERATURE LOAD AT ALL JOINTS.
            NODAL TEMPERATURES
              1   1.0   11  1$
   @XQT DSOL                                        . SOLVE LOAD CASE 1
        RESET L1=1,L2=1$
   @XQT KG                                          . FORM SYSTEM KG, AXIAL COMPRESSION
   $
   @XQT EIG
   $                                                  FIND BUCKLING LOAD FOR CANTILEVERED
   $                                                  COLUMN
        RESET INIT=10,PROB=STAB$
   @XQT LCM                                         . FORM LINEAR COMBINATION OF MATRICES
   $                                                  FORM STIFFNESS MATRIX FOR AXIALY PRE-
   $                                                  COMPRESSED BEAM, COMPRESSIVE FORCE=1000.
        RESET R=KXKG,Q1=K,Q2=KG,C2=1000.0$
   @XQT INV
        RESET K=KXKG,CON=2$
   @XQT DSOL                                        . SOLVE LOAD CASE 4
   $                                                  FIND DEFLECTION OF THE PRESTRESSED BEAM
   $                                                  UNDER LOAD CASE 4.
        RESET K=KXKG,CON=2,L1=4,L2=4$
   @XQT LCM
   $                                                  FORM STIFFNESS MATRIX FOR AXIALY PRE-
   $                                                  STRESSED BEAM, TENSILE FORCE=1000.
        RESET R=KYKG,Q1=K,Q2=KG,C2=-1000.0$
   @XQT INV
        RESET K=KYKG,CON=2$
   @XQT EIG
   $                                                  FIND FREQUENCY FOR FREE VIBRATION OF THE
   $                                                  PRE-STRESSED BEAM.
        RESET K=KYKG,CON=2,INIT=10$
   @XQT DSOL                                        . SOLVE LOAD CASE 2
        RESET L1=2,L2=2$
   @XQT KG                                          . FORM SYSTEM KG, LOAD CASE 2
        RESET QSEQ=2$
   @XQT EIG
   $                                                  FIND LATERAL BUCKLING VALUE OF LOAD CASE
   $                                                  2 FOR THE CANTILEVER BEAM.
        RESET INIT=6,PROB=STAB$
   @XQT INV
        RESET CON=2$
   @XQT DSOL
        RESET CON=2,L1=3,L2=3$
   @XQT KG                                          . FORM SYSTEM KG, PURE BENDING
        RESET QSEQ=3$
   @XQT EIG
   $                                                  FIND BUCKLING LOAD OF A SIMPLY SUPPORTED
   $                                                  BEAM IN PURE BENDING.
        RESET INIT=6,PROB=STAB,CON=2$
   @XQT DSOL
        RESET L1=5,L2=5,CON=2$
   @XQT KG                                          . FORM SYSTEM KG, TEMPERATURE LOAD
        RESET QSEQ=5$
   @XQT EIG
   $                                                  FIND BUCKLING TEMPERATURE LOAD FOR A
   $                                                  SIMPLY SUPPORTED BEAM.
        RESET CON=2,INIT=6,PROB=STAB$
   @XQT LCM
        RESET R=K5KG,Q1=K,Q2=KG,C2=10.0$
   @XQT INV
        RESET K=K5KG,CON=2$
```

11. MECHANISM

Free-free vibrational modes were obtained for a linkage in order to demonstrate the application of a spectral shift in the eigensolution process. The linkage was defined as a system of two beam segments connected through a zero-length 6 x 6 directly specified stiffness matrix. Planar motion was specified for the otherwise unconstrained configuration. The geometrical and material properties of the linkage are shown on the figure below:



$E = .1 \times 10^{8}$ in^{2}

$\nu = .3$

Mass density = $.1 \times 10^{-4}$ lb-sec^{2}/in^{4}

$I = 1.0$ in^{4}

$A = 1.0$ in^{2}

Eigenvalues were desired in the neighborhood of the value $c = 0.1 \times 10^{11}$, which was selected as the shift point for the eigenvalue solution of K-cM (see Reference 1, Section 3.17). Four "zero" eigenvalues were obtained, representing the three rigid body motions and a mechanism mode.

The elastic modes are summarized below:

	SPAR
ω_1^2	$.24584 \times 10^{11}$
ω_2^2	$.34629 \times 10^{11}$
ω_3^2	$.36038 \times 10^{11}$

```
@XQT TAB                                     . GENERATE BASIC TABLES DEFINING STRUCTURE
START 12   3 4 5$
TITLE'TWO-BEAM LINKAGE
TEXT$
'SYSTEM OF TWO BEAM SEGMENTS CONNECTED
'BY A ZERO-LENGTH ELEMENT.
'
'PROBLEM DESIGNED TO ILLUSTRATE THE USE OF A
'SHIFT POINT IN CALCULATION OF FREE-FREE
'VIBRATIONAL MODES.
MATC$                                          MATERIAL CONSTANTS
     1 1.+7 .3 .1-4$
JLOC$                                          JOINT LOCATIONS
     1  0. 0. 0. 10. 0. 0. 6 1 2$
     6 10. 0. 0. 20. 0. 0. 6 1$
CON 1$                                         CONSTRAINT CASE
MREF$                                          BEAM ORIENTATION SPECIFICATIONS
     1 1 2 1 1.$
E21 SECTION PROPERTIES$
     GIVN 1 1. 1. 1. 1. 1. 1.$
BB$                                            BEAM S6X6 STIFFNESS COEFFICIENTS
$                                              DIRECTLY SPECIFIED STIFFNESS MATRIX,
$                                              THE LAST FOUR ROWS OF WHICH ARE ZERO.
     1 1.+8: 0. 1.+8: : : : $
@XQT ELD                                     . READ ELEMENT DEFINITIONS
E21 $
$                                              BEAM SEGMENT DEFINITIONS
     1 2  1 5 2 6$
E25 $
$                                              ZERO-LENGTH ELEMENT DEFINTION
     6 7$
@XQT TOPO                                    . ANALYZE ELEMENT INTERCONNECTIVITY
@XQT E                                       . FORM ELEMENT DATA PACKETS
@XQT EKS                                     . INSERT K, S INTO ELEMENT DATA PACKETS
@XQT K                                       . FORM SYSTEM K
@XQT M                                       . FORM SYSTEM M
@XQT LCM                                     . FORM LINEAR COMBINATION OF MATRICES
$                                              SHIFT POINT SELECTED = .1+11
$                                              FOR THE EQUATION R=C1*Q1+C2*Q2,
$                                              R=K+LM
$                                              C1=1.0 (DEFAULT)
$                                              Q1=K   (DEFAULT)
$                                              C2=¬.1+11
$                                              Q2=M
     RESET R=K+LM,Q2=M,C2=-.1+11$
@XQT INV                                     . FACTOR K+LM
     RESET K=K+LM$
@XQT EIG                                     . SOLVE SYSTEM EIGENPROBLEM
$                                              SOLVE THE EIGENPROBLEM FOR THE 'SHIFTED'
$                                              EIGENSYSTEM.
$                                              USE K=K+LM AS THE 'SHIFTED MATRIX'.
$                                              ADD  SHIFT=.1+11  TO THE RESULTING
$                                              EIGENVALUES.
     RESET INIT=10,NDYN=10,REPRT=1$
     RESET K=K+LM$
     RESET SHIFT=.1+11$
     PRINT 0 0 0 1 1$
@XQT DCU                                     . EXECUTE DATA COMPLEX UTILITY PROGRAM
     TOC 1$
```

```
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TWO-BEAM LINKAGE

                                         E             ROWS    BLK   T    DATA SET NAME
SEQ       RR   DATE     TIME     R    WORDS    /BLK    SIZE  Y   N1    N2     N3    N4
  1       11 062074  001823    0       18        1      18   0 JOF1  BTAB      1     8
  2       12 062074  001823    0       12       12      12   0 JREF  BTAB      2     6
  3       13 062074  001823    0       12        1      12   1 ALTR  BTAB      2     4
  4       14 062074  001823    0       19        1      19   4 NDAL            0     0
  5       15 062074  001823    0      120        6     120   3 TEXT  BTAB      2     1
  6       20 062074  001824    0       10        1      10   1 MATC  BTAB      2     2
  7       21 062074  001824    0       36       12      36   1 JLOC  BTAB      2     5
  8       23 062074  001824    0       12       12      12   0 CON             1     0
  9       24 062074  001824    0        5        1       5   1 MREF  BTAB      2     7
 10       25 062074  001824    0       31        1      31   1 BA    BTAB      2     9
 11       27 062074  001824    0       21        1      21   1 BB    BTAB      2    10
 12       28 062074  001824    0      108       12     108   1 QJJT  BTAB      2    19
 13       32 062074  001826    0      160       10     896   0 DEF   E21       1     2
 14       64 062074  001826    0        2        1       2   0 GD    E21       1     2
 15       65 062074  001826    0       15        1      15   0 GTIT  E21       1     2
 16       66 062074  001826    0       16        1     896   0 DEF   E25       5     2
 17       98 062074  001826    0        2        1       2   0 GD    E25       5     2
 18       99 062074  001826    0       15        1      15   0 GTIT  E25       5     2
 19      100 062074  001826    0       12       12      12   0 NELZ  BTAB      1    11
 20      101 062074  001826    0       10        2      10   0 KE              0     0

 21      111 062074  001826    0       12        2      12   0 NS              0     0
 22      112 062074  001826    0        2        2       2   3 ELTS  NAME      0     0
 23      113 062074  001826    0        2        2       2   0 ELTS  LTYP      0     0
 24      114 062074  001826    0        2        2       2   0 ELTS  NNOD      0     0
 25      115 062074  001826    0        2        2       2   0 ELTS  ISCT      0     0
 26      116 062074  001826    0        2        2       2   0 ELTS  NELS      0     0
 27      117 062074  001827    0        2        2       2   0 ELTS  LES       0     0
 28      118 062074  001828    0     1344       12    1344   0 KMAP           23     3
 29      166 062074  001828    0     1344       12    1344   0 AMAP  @@@@     23     3
 30      214 062074  001831    0     1400       10     140   4 E21             1     2
 31      264 062074  001830    0       20       20      20   0 DIR   E21       1     2
 32      265 062074  001831    0      112        1     112   4 E25             5     2
 33      269 062074  001830    0       20       20      20   0 DIR   E25       5     2
 34      270 062074  001833    0     2240       12    2240   1 K     SPAR      9    23
 35      350 062074  001834    0     2240       12    2240   1 M     SPAR      9    23
 36      430 062074  001835    0     2240       12    2240   1 K*LM  SPAR      9    23
 37      510 062074  001837    0     3136       12    3136   1 INV   K*LM      1    23
 38      622 062074  001839    0       10       10      10  -1 VIBR  EVAL      1     0
 39      623 062074  001839    0      720       12      72  -1 VIBR  U         1     0
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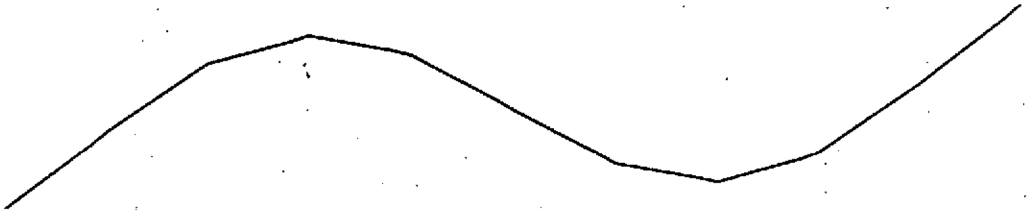
SEQ 5 VIBRATIONAL MODE. EIGENVALUE = $.458397 \times 10^{+10}$



BEAM MECHANISM

0 SCALE