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Turbomachinery Design Described by Similarity Considerations

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The dimensionless representation of the *operating conditions* of turbomachinery by the specific speed and the suction specific speed is well known, although specific speed and suction specific speed are often not used in truly dimensionless form. It is also known that these expressions of operating conditions are related to the design of the machine. This paper presents first an extension of this approach, particularly in the direction of mechanical design characteristics. Secondly, the paper attempts to establish a reasonably comprehensive picture of the dimensionless field of turbomachinery design. This attempt results in the concept of a "space of dimensionless operating conditions." Every point in this space can be associated with a set of dimensionless *design* parameters such as diameter ratios, flow coefficients, head coefficients, and the like, provided certain "design choices" regarding the fluid and the machine have been made. The core of the design process then relates such sets of design parameters to the dimensionless design form of the machine. This part of the design process is not described in this paper but only "located" relative to other parts of the overall design process.

For over half a century, the design of turbomachines has been related to a principally dimensionless expression of operating conditions, the "specific speed," which is defined by the statement that any fixed value of the specific speed describes that combination of operating conditions which permits similar flow conditions in geometrically similar turbomachines. Figure 1 depicts the well-known relation between the runner design of hydrodynamic pumps and the specific speed.

The specific speed, called in this paper the "basic specific speed," is here used in strictly dimensionless form:

$$n_s = \frac{n\sqrt{Q}}{(g_oH)^{3/4}} \tag{1}$$

A list of symbols is provided at the end of this paper. Metric or any other units may also be used, as long as consistent units are employed for length and time. This is often not the case, as for example with the pump specific speed usually used in the United States. There the speed of rotation, n,

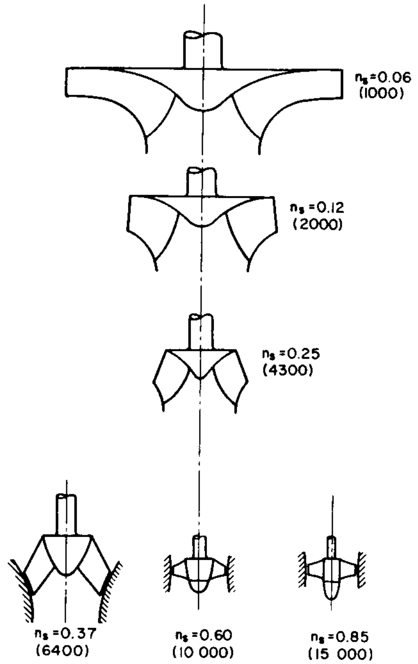

FIGURE 1.—*Pump runner profiles as a function of basic specific speed.*

is measured in revolutions per minute (rpm), Q in gallons per minute (GPM), and H in feet (ft), omitting the standard acceleration, g_o. This dimensional specific speed is related to the dimensionless specific speed n_s given by equation (1) as follows:

$$\frac{n(\text{rpm})\sqrt{Q(\text{GPM})}}{H(\text{ft})^{3/4}}=17{,}200\ n_s$$

The dimensional values of the pump specific speed are given in figure 1 (and in fig. 18) in parentheses.

The relation between the design form of the machine and the dimensionless specific speed is easily obtained by putting

$$n=\frac{U_o}{D_o\,\pi}\quad\text{and}\quad Q=V_{m_i}\cdot\frac{D_i^2\pi}{4}\left(1-\frac{D_h^2}{D_i^2}\right)$$

The notations used are defined in figure 2 and are given in the list of symbols. Substituting these expressions into equation (1) leads to the following equation:

$$n_s=\frac{n\sqrt{Q}}{(g_oH)^{3/4}}=\frac{1}{2^{1/4}\pi^{1/2}}\left(\frac{U_o^2}{2g_oH}\right)^{3/4}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(\frac{D_i}{D_o}\right)^{3/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{2}$$

By assuming fixed values for the flow coefficient V_{m_i}/U_i and for the maximum head coefficient $2g_oH/U^2_{o\min}$, it is not difficult to derive the runner forms shown in figure 1. However, one can, by using the condition of continuity for incompressible fluids, relate the basic specific speed also to other form characteristics of the runner or of stationary flow passages adjacent to the runner.

About 35 years ago the concept of specific speed was extended to include cavitation conditions, leading to the now well-known concept of "suction specific speed." Suction specific speed can be expressed in exactly the same form as the basic specific speed (eq. (2)) simply by replacing the total head of the machine (H) by the total suction head of the machine above the vapor pressure (H_{sv}). A more useful expression of the suction specific speed (in dimensionless form) is

$$S=\frac{nQ^{1/2}}{(g_oH_{sv})^{3/4}}=\frac{1}{2^{1/4}\pi^{1/2}}\left(\frac{V_{m_i}^2}{2g_oH_{sv}}\right)^{3/4}\frac{U_i}{V_{m_i}}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{3}$$

because it is concerned only with conditions at the low-pressure side of the runner, and $2g_oH_{sv}/V_{m_i}^2$ is approximately constant for a wide range of S values.

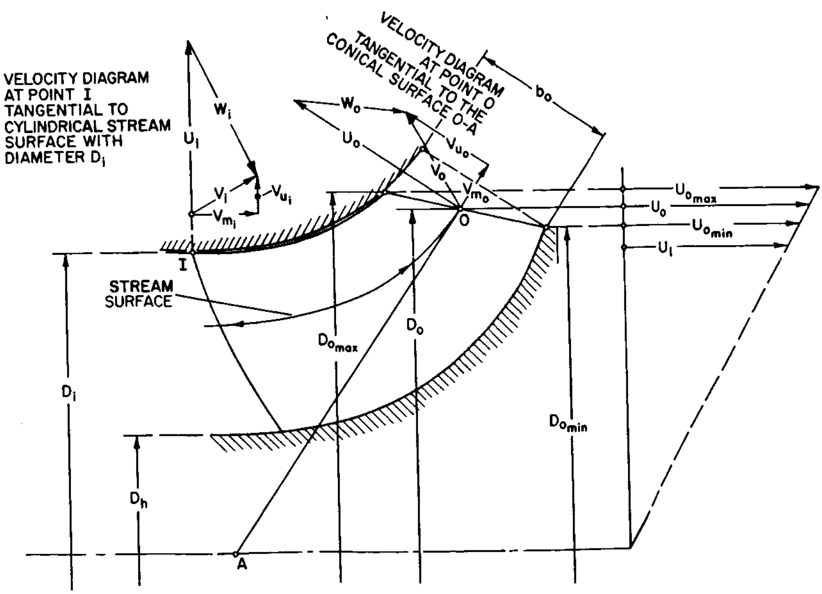

FIGURE 2.—*Profile of a turbomachine runner, defining notations.*

During the last few years it has become apparent that the basic specific speed and the suction specific speed are only examples of a much wider range of dimensionless expressions of operating conditions. References 1 and 2 demonstrate the application of this type of consideration to a variety of engineering design problems. The "power factor" $B_P = n\sqrt{P}/V^{5/2}$ is a "specific speed" that has been used in marine engineering for many years.

This paper represents an attempt to present the relation between the dimensionless operating conditions of turbomachines, expressed here in the form of various specific speeds, to the design form of turbomachines. A *design form* shall be defined as the three-dimensional, geometric form of an engineering device (i.e., a device conceived and judged by functional, not artistic, criteria). The term "design" is broader, including properties of the structural materials. Furthermore, the word "design" denotes a plan (like drawings, specifications, or a computer program) for an object as well as the action of creating such a plan.

It shall be understood that all geometrically similar machines represent only *one* design form so that the entire field of all possible design forms of turbomachines is dimensionless. Since the corresponding field of all possible, continuously variable operating conditions can also be presented in dimensionless form (the specific speeds), it is seen that the relation between operating conditions and design forms is a relation between two fields of dimensionless technical informations.

It is hoped that a presentation of the physical facts involved in this relation will stimulate a mathematically competent engineer or a mathematician to formulate this subject in a theoretically more satisfactory fashion. In view of this possibility, it should be mentioned that the terminology used in this paper is that of an engineer, not that of a mathematician. For example, the word "field," when used here, does not have its mathematical meaning. In some cases the mathematicially inclined reader might substitute "class" for "field."

OPERATING CONDITIONS EXPRESSED IN THE FORM OF SPECIFIC SPEEDS

There are, of course, many other operating conditions besides n, Q, H, and H_{sv} that determine the design of a turbomachine. These operating conditions may be expressed in dimensionless form in many different ways. It is merely for reasons of tradition that this writer has chosen the form of the specific speed for this purpose.

Table I presents the operating conditions and corresponding specific speeds that have been considered by this writer. No doubt other operating conditions could be considered. The right sides of the equations presented can be expressed in as many ways as the rate of volume or mass flow may be related to the fluid velocities in various parts of the machine (see table II).

It is evident that all dimensionless operating conditions (i.e., all specific speeds) follow the same general scheme. In all cases, the kinematically determined velocity $(n\sqrt{Q})^{2/3}$, raised to the $\frac{3}{4}$ power, is divided by the $\frac{3}{2}$ power of a velocity expressing some forces that determine the action of the machine; for example, $(\sqrt{2g_oH})^{3/2}$, $(\sqrt{2g_oH_{sv}})^{3/2}$, the acoustic velocity $a^{3/2}$, $(\sqrt{\sigma/\rho})^{3/2}$, and so on. Designating this general, force-describing velocity by v, tables II and III present the form of all specific speeds considered here and its relations to various parts of the machine.

TABLE I.—*Partial List of Dimensionless Operating Conditions (Specific Speeds)*

Basic specific speed:

$$n_s=\frac{nQ^{1/2}}{(g_oH)^{3/4}}=\frac{1}{2^{1/4}\pi^{1/2}}\left(\frac{U_o^2}{2g_oH}\right)^{3/4}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(\frac{D_i}{D_o}\right)^{3/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{a}$$

Suction specific speed:

$$S=\frac{nQ^{1/2}}{(g_oH_{sv})^{3/4}}=\frac{1}{2^{1/4}\pi^{1/2}}\left(\frac{V_{mi}^2}{2g_oH_{sv}}\right)^{3/4}\frac{U_i}{V_{mi}}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{b}$$

Compressibility specific speed:

$$n_a=\frac{nQ^{1/2}}{a^{3/2}}=\frac{1}{2\pi^{1/2}}\left(\frac{w_i}{a}\right)^{3/2}\frac{(V_{mi}/U_i)^{1/2}}{\left(1-2\frac{V_{u_i}}{U_i}+\frac{V_{u_i}^2}{U_i^2}+\frac{V_{mi}^2}{U_i^2}\right)^{3/4}}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{c}$$

Viscosity specific speed:

$$n_\nu=\frac{nQ^{1/2}}{(n\nu)^{3/4}}=\frac{\pi^{1/4}}{2}\left(\frac{U_oD_o}{\nu}\right)^{3/4}\left(\frac{D_i}{D_o}\right)^{3/2}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{d}$$

Stress specific speed:

$$n_\sigma=\frac{nQ^{1/2}}{(\sigma/\rho)^{3/4}}=\frac{1}{2^{1/4}\pi^{1/2}}\left(\frac{\rho U_o^2}{2\sigma}\right)^{3/4}\left(\frac{D_i}{D_o}\right)^{3/2}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{e}$$

Gravity specific speed:

$$n_g=\frac{nQ^{1/2}}{(g/n)^{3/2}}=\frac{1}{2\pi^2}\left(\frac{U_o^2}{gD_o}\right)^{3/2}\left(\frac{D_i}{D_o}\right)^{3/2}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{f}$$

Vibration specific speed:

$$n_v=\frac{nQ^{1/2}}{(E/\rho_s)^{3/4}}=\frac{1}{2^{1/4}\pi^{1/2}}\left(\frac{\rho_sU_o^2}{2E}\right)^{3/4}\left(\frac{D_i}{D_o}\right)^{3/2}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2} \tag{g}$$

TABLE II.—*Specific Speeds Based on the Representative Velocity (v) Defined in Table III*

$$\frac{nQ^{1/2}}{(v)^{3/2}}=\frac{1}{2\pi^{1/2}}\left(\frac{U_o}{(v)}\right)^{3/2}\left(\frac{D_i}{D_o}\right)^{3/2}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2}$$

$$\frac{nQ^{1/2}}{(v)^{3/2}}=\frac{1}{2\pi^{1/2}}\left(\frac{U_i}{(v)}\right)^{3/2}\left(\frac{V_{mi}}{U_i}\right)^{1/2}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2}$$

$$\frac{nQ^{1/2}}{(v)^{3/2}}=\frac{1}{2\pi^{1/2}}\left(\frac{V_{mi}}{(v)}\right)^{3/2}\left(\frac{U_i}{V_{mi}}\right)\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2}$$

$$\frac{nQ^{1/2}}{(v)^{3/2}}=\frac{1}{2\pi^{1/2}}\left(\frac{w_i}{(v)}\right)^{3/2}\frac{(V_{mi}/U_i)^{1/2}}{\left(1-2\frac{V_{u_i}}{U_i}+\frac{V_{u_i}^2}{U_i^2}+\frac{V_{mi}^2}{U_i^2}\right)^{3/4}}\left(1-\frac{D_h^2}{D_i^2}\right)^{1/2}$$

$$\frac{nQ^{1/2}}{(v)^{3/2}}=\frac{1}{\pi^{1/2}}\left(\frac{U_o}{(v)}\right)^{3/2}\left(\frac{V_{mo}}{U_o}\right)^{1/2}\left(\frac{b_o}{D_o}\right)^{1/2}$$

$$\frac{nQ^{1/2}}{(v)^{3/2}}=\frac{1}{\pi}\left(\frac{U_o}{(v)}\right)^{3/2}\left(\frac{V_{u_o}}{U_o}\right)^{1/2}\left(\frac{D_o}{D_{th}}\right)^{1/2}\frac{A_{th}^{1/2}}{D_o}N^{1/2}$$

One may also compare the general velocity (v) with the head of the machine in the form $(v)^2/g_oH$.

TABLE III.—*Definitions of the Representative Velocity (v)*

$v=(g_oH)^{1/2}$ for the basic specific speed
$v=(g_oH_{sv})^{1/2}$ with respect to cavitation
$v=a=$ the velocity of sound, with respect to compressibility
$v=(n\nu)^{1/2}=(U\cdot\nu/\pi D)^{1/2}$ with respect to viscosity
$v=(\sigma_c/\rho_s)^{1/2}$ with respect to centrifugal stresses
$v=(\sigma_f/\rho_f)^{1/2}$ with respect to fluid-induced stresses
$v=g/n=g\pi D/U$ with respect to any general acceleration g of the system as a whole
$v=(E/\rho)^{1/2}=D\cdot f$ with respect to vibrations of the machine at a frequency f

The foregoing comments raise the question whether the $\frac{2}{3}$ power of the specific speeds would not be a better dimensionless expression of operating conditions than the specific speeds used here, as the $\frac{2}{3}$ power would be a velocity ratio. Since any power of such a ratio serves essentially the same purpose, this writer prefers not to depart from the conventional specific speed more than to use it in its dimensionless form, recognizing that raising the specific speeds to *any* power does not make any essential difference.

PHYSICAL SIGNIFICANCE OF THE DESIGN PARAMETERS

The various dimensionless ratios appearing on the right sides of the specific speed equations (tables I and II) are here denoted as "design parameters." This does not require any explanation regarding the simple ratios of linear dimensions D_i/D_o, b_o/D_o, D_h/D_i, etc., as these ratios have clear meanings regarding the *form* of the "profile" of the machine to be designed (see fig. 2). *The flow coefficient* V_{m_i}/U_i *and the head coefficient* $2g_oH/U_o^2=2\eta_h V_{u_o}/U_o$ (for zero rotation of the fluid at the suction side of the runner) *determine the velocity vector diagrams at the inlet and discharge edges of the impeller vanes,* using the condition of continuity. By some extensions, this is true also for the nearby stationary vane systems, provided the general form of the machine has been selected, as will be discussed later. Figure 3 depicts the information that can be derived from equations (2) and (3) regarding the design of a radial- or mixed-flow runner, considering that the direction of the vane ends is closely related to the direction of the relative velocity of flow (w). (It illustrates that this relation exists also if the flow on the suction side of the runner has a circumferential component (V_{u_i}, V_u, V_{u_h}).) With the information given in figure 3 the true design process starts, connecting these pieces of information to a geometrically, hydrodynamically, and mechanically consistent structure.

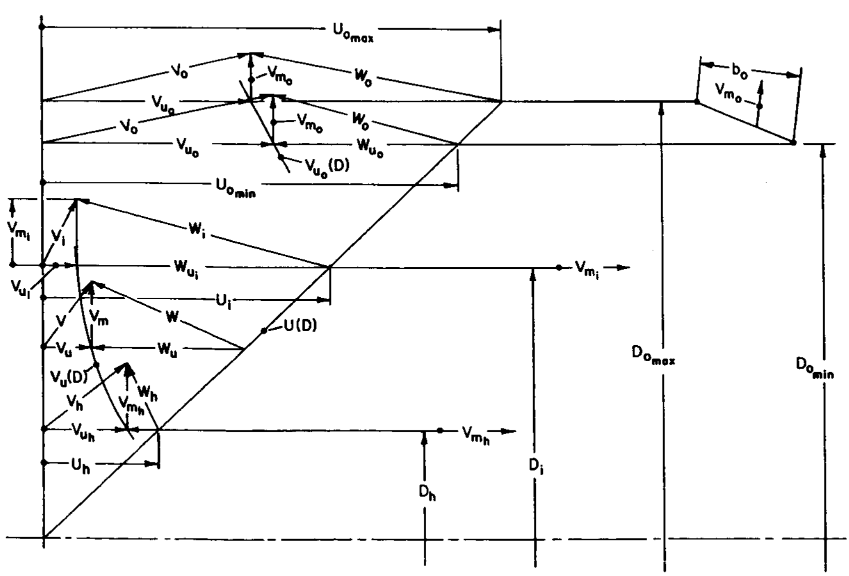

FIGURE 3.—*Information on runner design derived from basic and suction-specific speeds.*

With respect to mechanical design characteristics, the "stress specific speed"

$$n_\sigma = \frac{nQ^{1/2}}{(\sigma/\rho)^{3/4}} = \frac{1}{2^{1/4}\pi^{1/2}} \left(\frac{\rho U_o^2}{2\sigma}\right)^{3/4} \left(\frac{D_i}{D_o}\right)^{3/2} \left(\frac{V_{m_i}}{U_i}\right)^{1/2} \left(1 - \frac{D_h^2}{D_i^2}\right)^{1/2} \quad (4)$$

is of particular interest. The stress (σ) is here an *allowable* stress and is therefore known from the properties and the working conditions of the structural material. The stress coefficient $\rho U_o^2/2\sigma$ is of the most direct significance regarding the centrifugal stress σ_c of a structural rotating element having the mass per unit volume ρ_s. For example, a thin, freely rotating hoop has the centrifugal stress coefficient $\rho_s U_o^2/2\sigma_c = 1/2$.[1]

For a radial strut of constant cross section, the maximum stress coefficient at the center of rotation is $\rho_s U_o^2/2\sigma_c = 1$.[1] Straight radial structural members are of importance for runner blades (as shown in fig. 4), having radial blade elements to achieve maximum resistance against centrifugal forces. This condition is fulfilled almost automatically for axial-flow runners. In such cases the radial blade elements usually do not have constant cross sections, but radially diminishing cross sections. Figure 5 shows the cross-section distribution of straight radial struts having

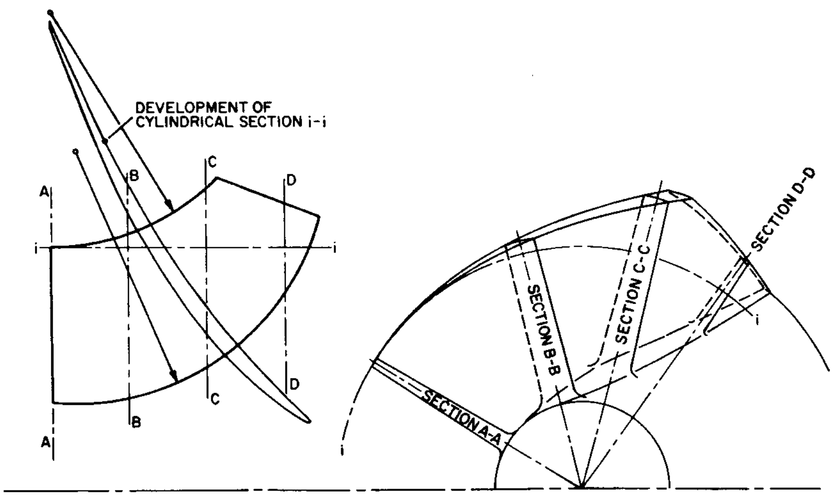

FIGURE 4.—*Turbomachinery runner with radial blade elements.*

[1] If the fluid rotates at the same angular velocity as the solid, rotating parts, then the difference between the mass-density of the structural material and the fluid should be used in these expressions.

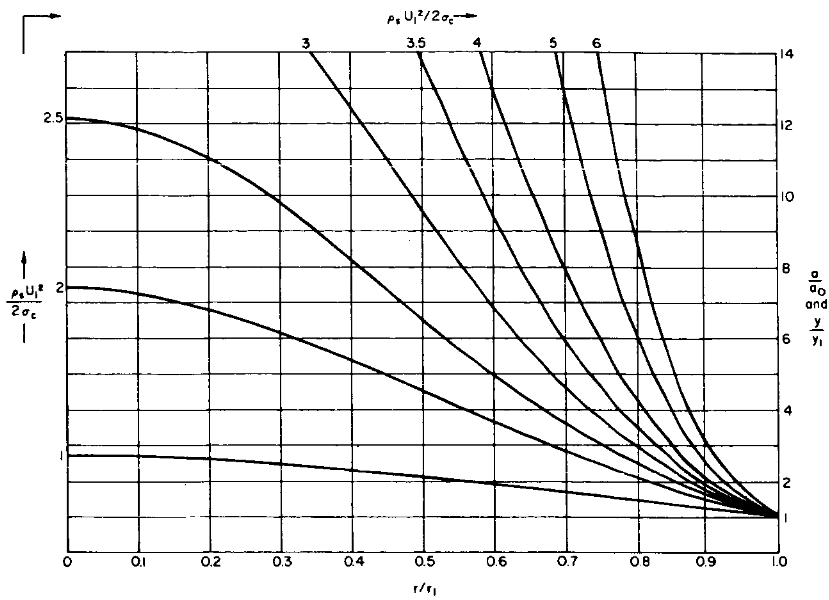

FIGURE 5.—*Area distribution of a rotating radial strut (vane element) of constant stress, and thickness distribution of a rotating disc of constant stress.*

radially constant centrifugal stress including their outer cross section at radius r_1, where such a stress might be imposed by an outer shroud or by an extension of the radial element with nonuniform, radially diminishing stress. The curves shown in figure 5 describe also the thickness distribution y/y_1 of a rotating disc of constant stress. In this case the radial stress at the outer periphery is generated by the vanes and vane-holding rim, considering a standard axial-flow turbine or compressor runner as shown in figure 6. If the stress coefficient is calculated with the maximum peripheral velocity U_o at radius $r_o > r_1$, its value will be higher than shown in figure 5. For radial blade elements, the stress at radius r_1 permits a constant area radial extension of the blade beyond r_1, say to $r_o > r_1$. The resulting cross-section distribution and stress coefficients $\rho_s U_o^2/2\sigma_c$ are given in figure 7.

Axially extending, nonradial vanes (as used, for example, with standard centrifugal pump or compressor runners with backward-bent vanes) obviously lead to bending stresses in the vanes. The resulting centrifugal stress coefficient referred to the peripheral velocity at the vane section considered is

$$\frac{\rho_s U^2}{2\sigma_c} = \frac{q}{12} \frac{h}{b} \frac{r}{b} \frac{1}{\sin \beta} \tag{5}$$

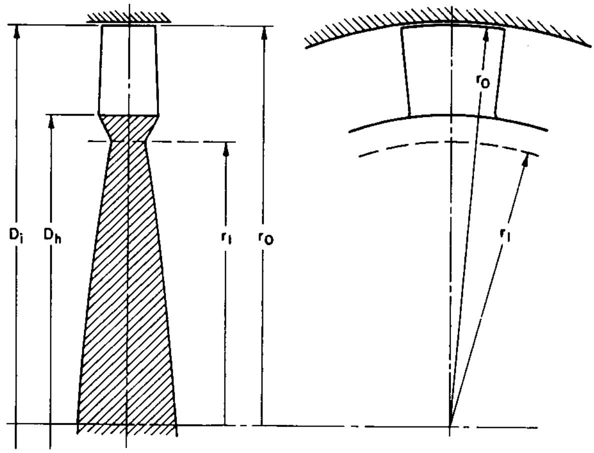

FIGURE 6.—*Axial-flow runner of high stress-specific speed, defining notations.*

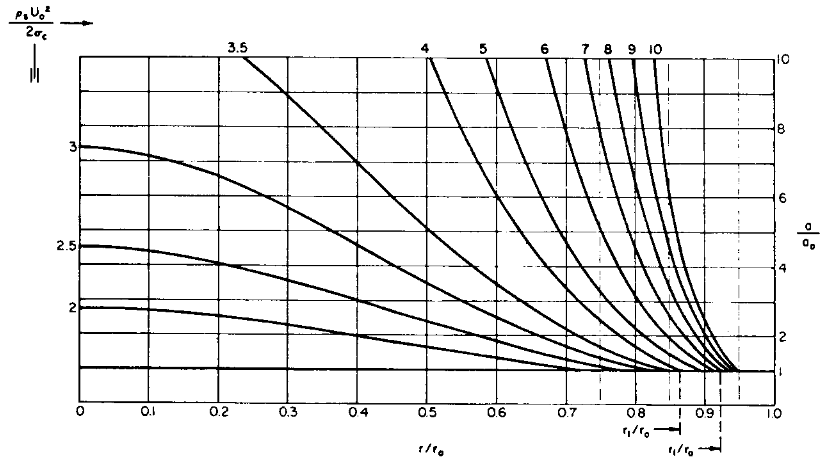

FIGURE 7.—*Area distribution of a rotating radial element with zero stress at* r_0 *and constant stress from* $r=r_1$ *to* $r=0$.

The notations used are defined by figure 8, except q, which varies from 8 for no bending stiffness at the axial end supports of the vanes (i.e., very thin shrouds) to 32 for completely rigid end supports; q is 2 for vanes cantilevered axially from a single shroud.

From the above, one can readily draw some conclusions regarding the centrifugal stress coefficients of runners of various forms. For example, an axial-flow runner as shown in figure 6 will have a centrifugal stress coefficient of $\rho_s U_o^2/2\sigma_c=4$ *if* $r_o/r_1=\sqrt{2}$, if the disc thickness ratio $y/y_1=7$ at

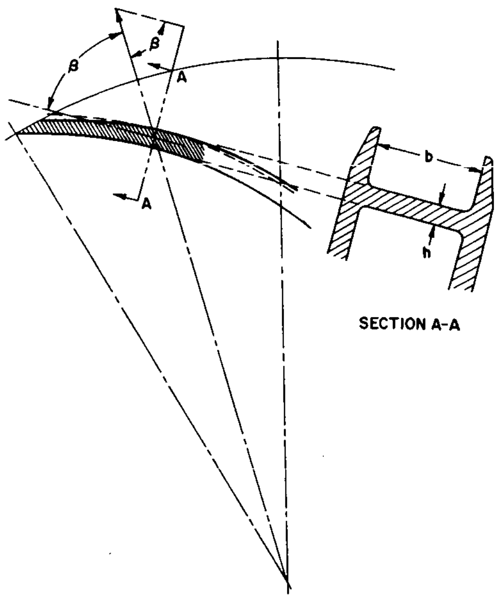

FIGURE 8.—*Bending stress configuration of nonradial vanes, defining notations.*

$r=0$, and if the blade cross-section ratio $a/a_o=2.5$ at a blade root-to-tip radius ratio of $r/r_o=0.75$. This may well be close to a maximum for turbomachinery runners. The corresponding stress-specific speed would be, according to the equation given for n_{σ_c} in table I:

$$\frac{nQ^{1/2}}{\sigma_c/\rho_s}=n_{\sigma_c}=0.487 \tag{6}$$

where it was assumed that $D_h/D_i=0.75$, $V_{m_i}/U_i=0.3$, and, of course, $D_o=D_i$.

For a runner with axially extending nonradial vanes, one finds from equation (5) that $\rho_s U_o^2/2\sigma_c=1.52$, where it was assumed that $h/b=\frac{1}{4}$, $r/b=4.2$, and $\beta=60°$. The coefficient q was assumed to be 15. The corresponding centrifugal stress specific speed was found to be in the neighborhood of $n_{\sigma_c}=0.17$, where the basic specific speed was assumed to be about 0.12 (2060). The fact that this stress specific speed is only about one-third that previously calculated for an axial flow runner of very favorable proportions (in terms of stress) reflects correctly the less favorable design form of a centrifugal runner with backward-bent vanes. Nevertheless, even this runner form does not come close to representing a minimum of centrifugal stress specific speed. Comparable runners of higher specific speeds would have lower stress specific speeds than 0.17.

The most important fluid-dynamically generated stresses σ_f are probably the bending stresses in the vanes, and are therefore proportional to the average vane-pressure difference $C_L\rho_f w^2/2$. A corresponding bending-stress coefficient would be

$$\frac{\rho_f U_o^2}{2\sigma_f}=q\frac{U_o^2}{C_L w^2}\frac{m_s}{b^2 l} \tag{7}$$

where m_s is the root section modulus of the vane, b the vane span, l the vane length, and $q=2$ for cantilevered vanes; or $q=8$ to 32 for vanes supported on both ends.

It is of interest to compare the fluid-dynamically induced stress (σ_f) with the centrifugal stress (σ_c). This comparison is most direct for centrifugal bending stresses in axially extending, nonradial vanes. One finds

$$\frac{\sigma_f}{\sigma_c}=\frac{\rho_f}{\rho_s}\cdot\frac{r}{2h}\frac{C_L w^2}{U_o^2 \sin\beta} \tag{8}$$

with the additional notations defined by figure 8.

Evidently $C_L w^2/U_o^2 \sin\beta$ is of the order *one*. $r/2h$ is for geometric reasons much larger than one, usually in the neighborhood of ten. For gases, ρ_s/ρ_f is about 1000 so that $\sigma_c > \sigma_f$. For liquids with densities in the neighborhood of that of water, ρ_s/ρ_f lies between 2 and 10 so that $\sigma_f \geqq \sigma_c$. For liquid hydrogen, $\rho_s/\rho_f > 10$ so that $\sigma_c > \sigma_f$.

It should be evident that next to the basic specific speed, the suction specific speed, and (perhaps) the compressibility specific speed, the stress specific speeds have the best defined influence on the design of turbomachines. It is for this reason and because this specific speed and coefficient are not yet used extensively that this type of stress consideration has been given a prominent place in this paper.

RELATION BETWEEN VELOCITY DIAGRAMS AND VANE SHAPE

Figure 3 presented an example of the information on design form represented by the design parameters derived from the flow-determined specific speeds. It was pointed out that the complete design form of the vane system has to be derived from this information by what may well be considered as the core of the design process. This derivation requires the entire knowledge and experience available in the field and is a problem far too extensive to be explored here. Two considerations will be presented because they describe the nature of the design problem: A real-flow limitation of the velocity diagrams and the relation between the velocity diagrams and the shape of cylindrical, coaxial vane sections through axial-flow runners.

The limitation of the velocity diagrams results from the "separation" or "stall" limits of the flow of a real fluid in vane systems. It has been found by approximate theoretical considerations, supported by test results, that the mean relative velocity of flow cannot be reduced in a single vane system to less than about 60 percent of its initial mean value (i.e., $w_2/w_1 \geq 0.6$ for rotating vane systems and $V_2/V_1 \geq 0.6$ for stationary vane systems). The reason for this limitation is outlined in chapters 12 and 25 of reference 3 and does not lie within the scope of this paper. However, the *existence* of such a limitation is important for the present considerations. It means that there *is* a limitation on the velocity vector diagrams (i.e., on the conditions of design) before the design process is started. This limitation is different from other limitations such as cavitation limitations insofar as it is not a function of a continuously varying operating condition, because the dependence of this limit on the Reynolds number (and thereby on the viscosity specific speed) is not known.

With this limitation of the velocity vector diagrams in mind, these diagrams will now be related to the vane shapes as appearing in coaxial, cylindrical sections through axial-flow vane systems. Figure 9 shows the

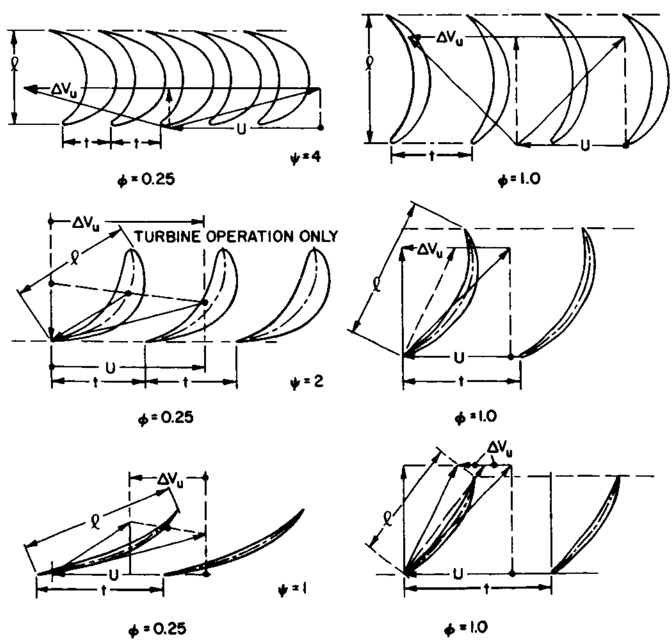

FIGURE 9.—*Family of axial-flow vane systems.*

vane shape as a function of the flow coefficient $\phi = V_{m_i}/U_i$ and of the head coefficient $\psi = 2g_oH/U^2$. The velocity vector diagrams are shown for pump (or compressor) flow (vertically up) except for the case $\phi = 0.25$, $\psi = 2$, where this condition leads to $w_2/w_1 < 0.6$ for pump operation. Thus this system is shown for turbine operation (flow from top to bottom) only.

The family of axial-flow vane systems indicated diagrammatically in figure 9 is not uniquely related to the flow and head coefficients ϕ and ψ as the *rate* of changing the angular momentum along the blade; i.e., the *distribution* of blade pressure difference affects the blade shape for given inlet and discharge conditions. However, for some given (say, optimum) pressure distribution, the (optimum) blade shape may well become a unique function of the inlet and discharge conditions, or of ϕ and ψ.

Before closing this discussion it is necessary to say something about the vane *spacing*. Its determination is theoretically included in criteria concerning a satisfactory vane pressure *distribution*. However, a more practical approach is one by way of the blade lift coefficient, which should for this purpose be expressed in the following form:

$$C_L = \frac{2\Gamma}{w_\infty l}$$

where Γ is the circulation per blade. For axial-flow machines, this reduces to the simple form

$$C_L = 2\,\frac{\Delta V_u}{w_\infty}\,\frac{t}{l} \tag{10}$$

and, for radial-flow machines,

$$C_L = 2\,\frac{V_{u_o}}{w_\infty}\left(1 - V_{u_i}\,\frac{D_i}{D_o}\right)\frac{t_o}{l} \tag{11}$$

One can only estimate that, for vane systems with very little change in mean static pressure, C_L has about the same maximum value as for a single airfoil in an infinitely extended stream (say, $C_{L_{max}} = 1.5$). For systems with retarded relative flow ($w_2 < w_1$), $C_{L_{max}}$ is lower (say, between 1 and 1.5), whereas, for accelerated relative flow, $C_{L_{max}}$ may be chosen higher than 1.5 (say, 2, and perhaps still higher, although a true maximum value of C_L has never been established).

For any estimated value of C_L, equations (10) or (11) determine the vane spacing t or t_o for a given vane length l.

"DESIGN CHOICES" AND THE RELATION BETWEEN OPERATING CONDITIONS AND DESIGN FORMS

Assuming the relation between the velocity diagrams and the vane shape is extended into the field of radial-flow runners, including the three-dimensional relation between vane shape and profile shape of the runner, one can imagine a complete line of connections between the operating conditions (the specific speeds), the design parameters (which determine certain elementary design form characteristics and the velocity vector diagrams), and, finally, the complete design form of a turbomachine. This connection is as complete as the given operating conditions and the knowledge available regarding the design of such machines.

However, even under the most ideal circumstances this relationship is not unique. Certainly there is a vast difference in design between machinery handling liquids and machinery handling gases. There is another fundamental difference between pumps (and compressors) on one hand and turbines on the other. Distinctions of this type will be called "design choices," although many of them are obviously not made by the design engineer but by the customer. Obviously design choices of such fundamental nature have to be made before the design process can start. However, even regarding what many may consider as design details, certain choices must be made before one can start with the most elementary analytical procedures. For example, it is well known that the runner forms given in figure 1 are not the only forms by which the given specific speed values can be satisfied.

Figure 10 shows a series of *axial-flow* runners covering essentially the same range of basic specific speeds as figure 1, but agreeing in design form

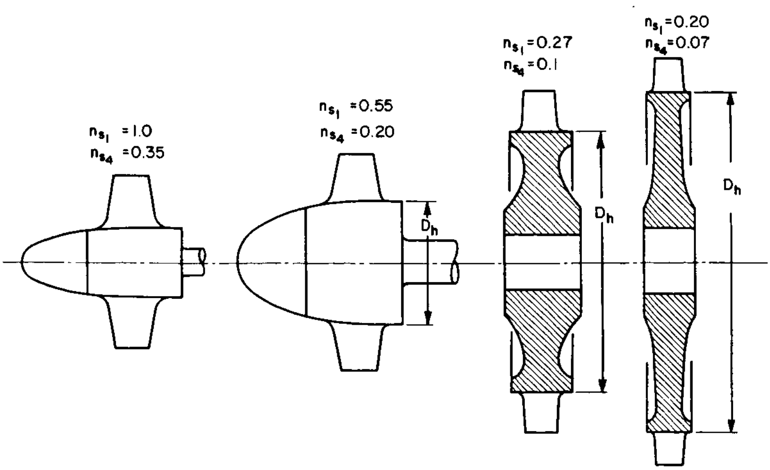

FIGURE 10.—*Axial-flow runner profiles as a function of basic specific speed.*

with figure 1 only in the region of high specific speeds. (n_{s_1} denotes the basic specific speed achieved with a root head coefficient $\psi=1$, and n_{s_4} with a blade root head coefficient $\psi=4$. For the axial vane sections, see fig. 9.) On the other hand, figure 11 shows the same range covered entirely by radial- and mixed-flow runners. It is thus apparent that a *choice* between radial- and axial-flow design must be made before the design form can be determined from the specific speeds. Figure 11 also shows diagrammatically the stationary parts encasing the runner, indicating that for the highest specific speeds only a diffuser with axial discharge is a practically useful solution, thus demonstrating why in this range of n_s axial-flow machines are usually preferred.

Figure 12 describes the design choice between a single-stage, radial-flow pump and a multistage, axial-flow pump. Here the choice is often debatable. Both types of machines have about the same volume and weight, and, for medium basic specific speeds (about 0.12), about the same efficiency. The radial-flow pump has a wider stable operating range (at constant speed). The axial-flow pump has a mechanically much better casing with respect to high internal pressures, and, by virtue of its large number of vanes, probably lower amplitudes of pressure fluctuations at the discharge. In the aircraft gas turbine field, the lower "frontal area" was decisive for the choice of the axial-flow design.

Another design choice is that between "single-suction" and "double-suction" machines shown in figure 13. The double-suction machine has the advantage of a higher suction specific speed or (for gases) a lower Mach

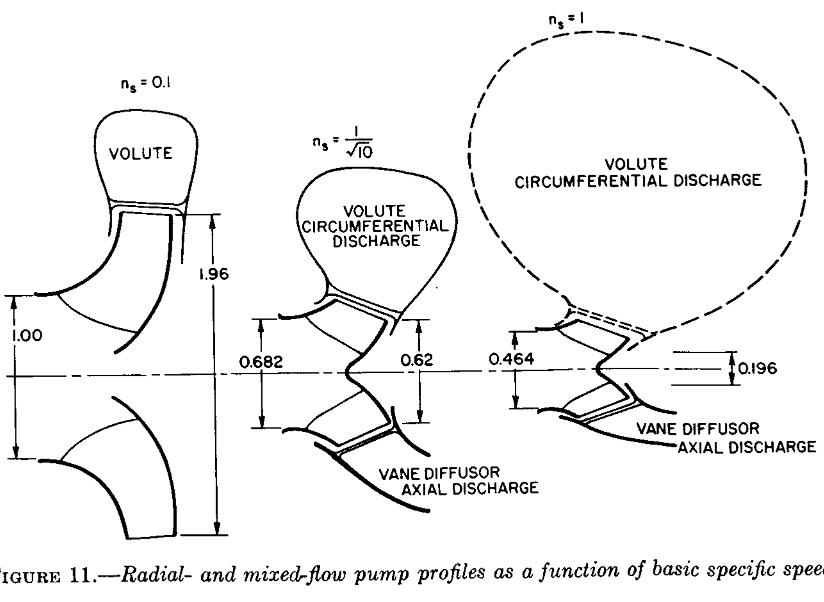

FIGURE 11.—*Radial- and mixed-flow pump profiles as a function of basic specific speed.*

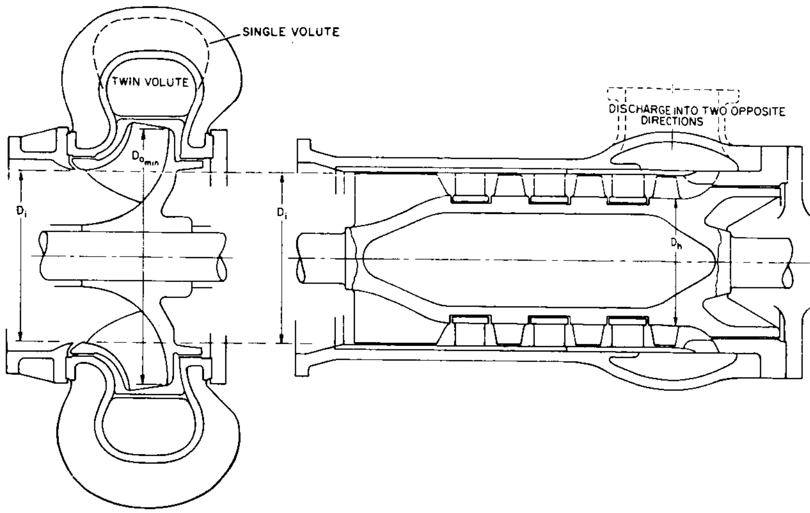

FIGURE 12.—*Design choice between a single-stage radial-flow and a multistage axial-flow turbomachine.*

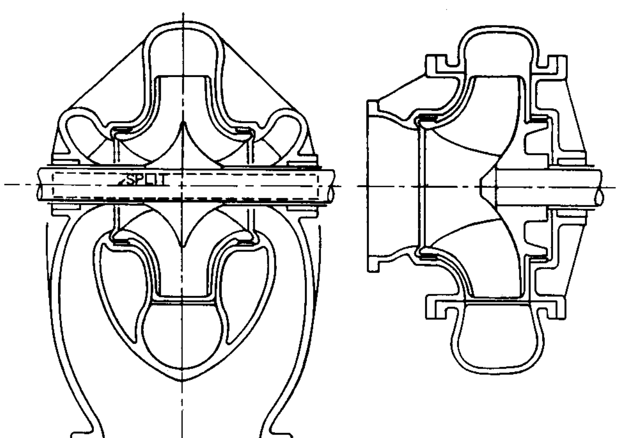

FIGURE 13.—*Design choice between double-suction and single-suction turbomachines.*

number of the relative flow entering the runner, or higher compressibility specific speed. The single-suction machine has the advantage of greater mechanical simplicity.

Figure 13 also demonstrates still another design choice; namely, that between the so-called horizontally split and vertically split casing construction. "Horizontally split" actually means that the casing is divided along a plane containing the axis of rotation; "vertically split" means the casing is divided along planes normal to the axis of rotation. Figures 14 and 15 show the principle of this distinction with respect to multistage machines.

Horizontally split machines are much easier to assemble or disassemble than vertically split machines, particularly multistage machines. This construction is mostly used for commercial multistage pumps and compressors and large steam turbines. The vertically split construction has the advantage of greater reliability and is therefore preferred in rocket pumps; pumps for highly corrosive, toxic, or otherwise dangerous fluids; aircraft turbine engines; etc.

There are other design choices; examples include choices regarding number of stages and "volute" versus "diffuser" pumps. From the examples given, it will be clear that the term "design choice" is used here whenever one is concerned with a very limited number of *alternates*—often only two. In contrast, operating conditions are understood to be given in terms of *continuous variables* (speed, rate of flow, head, allowable stress, density, etc.).

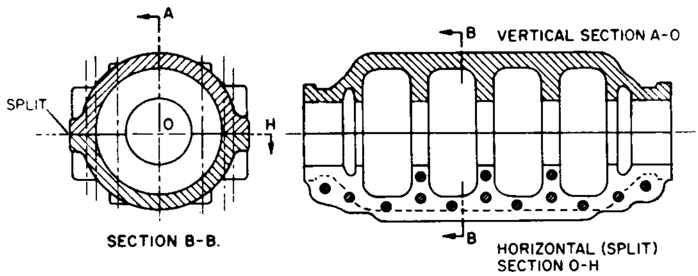

FIGURE 14.—*"Horizontally split" multistage pump casing.*

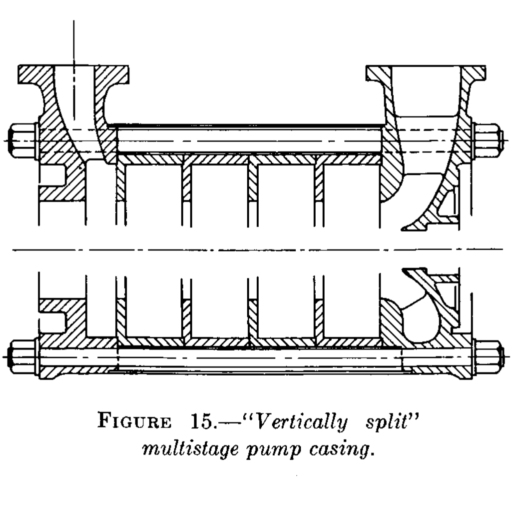

FIGURE 15.—*"Vertically split" multistage pump casing.*

It has been mentioned before that under idealized conditions of knowledge one can imagine a definite relation between the continuously varying operating conditions and the corresponding continuously varying design forms. Furthermore, it was stated that this relationship is not unique even under the idealized assumption that for completely given conditions there is only one optimum design form. *It can now be stated that the multivalued character of the relation between operating conditions and design forms is dictated by the discrete design choices described in this section.*

REPRESENTATION OF THE FIELD OF TURBOMACHINERY DESIGN

The last step in this presentation of turbomachinery design is the construction of a mental picture or scheme representing what was said before

The field of all possible dimensionless operating conditions—here presented as a number of specific speeds (the *left* sides of the equations in tables I and II)—is imagined as a multidimensional space, each coordinate being one of the dimensionless, continuously variable operating conditions (one specific speed). A point in this space represents one complete set of dimensionless operating conditions.

According to what was said before, every specific speed can be related to a number of design parameters as expressed by the *right* sides of the equations in tables I and II. This relation is multivalued, every particular solution depending on a number of design choices. However, after all pertinent design choices have been made, one can imagine that every point in the space of dimensionless operating conditions can be associated with a set of numerical values of the design parameters appearing on the right sides of the specific-speed equations. Accordingly one can draw in the multidimensional space of operating conditions the loci (lines, surfaces, etc.) of constant values of the design parameters concerned.

It is somewhat difficult to demonstrate this situation, not only because of the multidimensional nature of this space, but also because the design information available for most of the specific speeds is as yet far too incomplete to permit such a demonstration in definite terms. Therefore a highly simplified case will be considered for this demonstration.

Only two specific speeds will be considered as being variable. The best design information available today falls in the hydrodynamic field, represented by the basic specific speed and the suction specific speed. These two specific speeds shall therefore be the variable operating conditions considered in this demonstration, with all other specific speeds having fixed values in ranges where sufficient design information is available. One may consider this example as a plane section through the multidimensional space of operating conditions, this section being parallel to the n_s and S axes, and normal to all other coordinate axes of this space.

The next step is that of making the necessary design choices. These choices shall be the following:

(a) The machine is a pump (not a turbine).

(b) The fluid is a liquid of low viscosity; this determines a sufficiently low value of the compressibility specific speed and a sufficiently high value of the viscosity specific speed to make the effects of changes in compressibility and changes in viscosity (and size) negligible.

(c) The peripheral velocities are sufficiently low and the strength-to-weight ratio of the structural material sufficiently high to practically eliminate stress considerations; specifically, the centrifugal stress specific speed shall be well below $1/10$.

(d) The gravity specific speed is sufficiently high and the vibration specific speed sufficiently low to practically eliminate gravity and vibration effects from the design consideration.

(e) The runner design form will be single-suction and will vary continuously from radial (outward) flow for low basic specific speeds to axial flow for high basic specific speeds.

These design choices are not yet sufficient to solve the equations for the basic specific speed and the suction specific speed for the design parameters. However, certain design parameters can be chosen on theoretical and empirical grounds, as follows.

On theoretical grounds, one can select the inlet velocity head ratio $2g_oH_{sv}/V_{m_i}{}^2 = 3.5$, and, on empirical grounds, the hub diameter ratio $D_h/D_i = 0.25$. With these assumptions, the equation for the suction specific speed is reduced to

$$S = \frac{1}{5.585} \frac{U_i}{V_{m_i}} \tag{12}$$

With the flow coefficient V_{m_i}/U_i so determined by the suction specific speed alone, the basic specific speed equation can be solved for D_i/D_o if one can make a rational assumption about the head coefficient. It will be assumed empirically that the maximum value of this head coefficient, which exists at the minimum discharge diameter ($D_{o_{min}}$), is $2g_oH/U^2_{o_{min}} = 1$. With this assumption and equation (12), one finds:

$$n_s = 0.4585 \left(\frac{V_{m_i}}{U_i}\right)^{1/2} \left(\frac{D_i}{D_{o_{min}}}\right)^{3/2} = \frac{0.4585}{(5.585S)^{1/2}} \left(\frac{D_i}{D_{o_{min}}}\right)^{3/2} \tag{13}$$

Figure 16 shows a graphical evaluation of equations (12) and (13); this evaluation indeed represents the beforementioned "section" through the multidimensional space of operating conditions. In this section appear

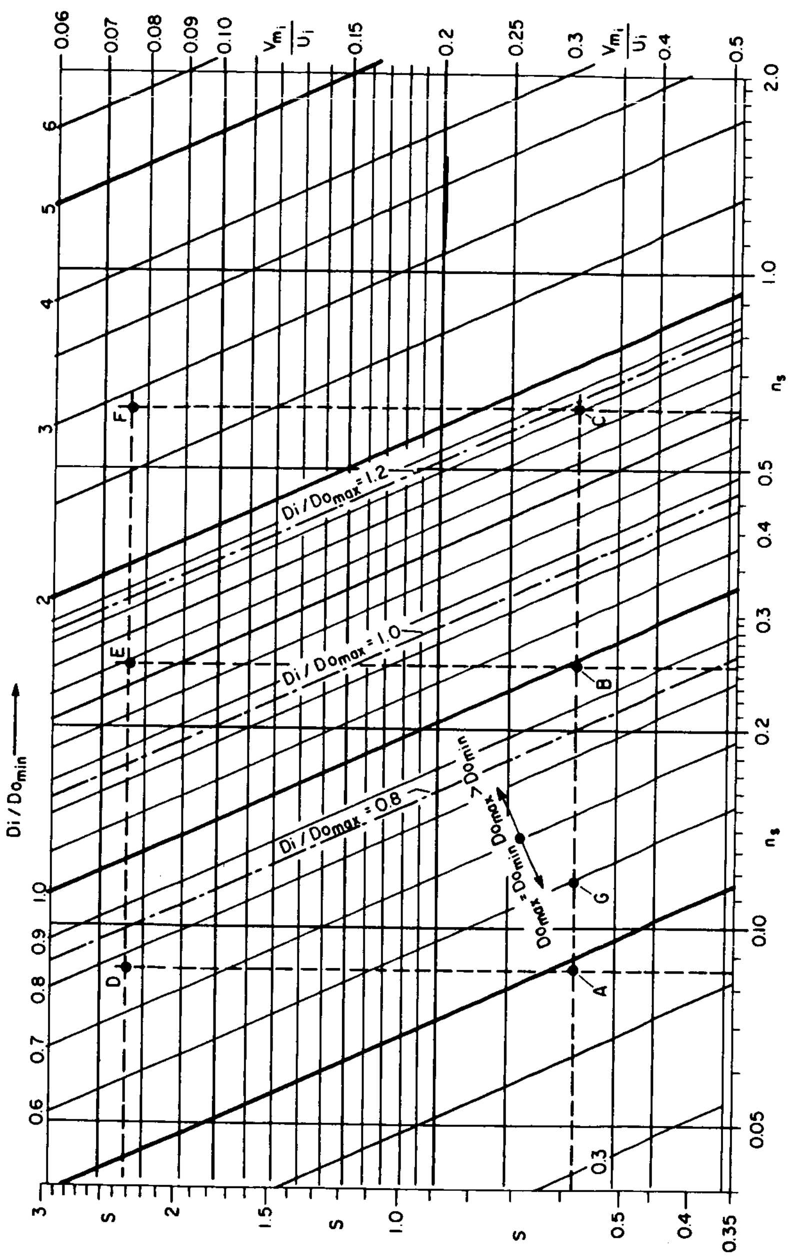

FIGURE 16.—*Design parameters as functions of* n_s *and* S *for* $2g_0H/U_{0_{min}}^2 = 1$.

two systems of lines—lines of constant values of the inlet flow coefficient V_{m_i}/U_i and lines of constant values of the diameter ratio $D_i/D_{o_{\min}}$. It is quite proper to use, for dimensionless coordinates, logarithmic scales, which give this section the qualities of a computation chart. Thus this initially somewhat abstract concept of a section through the space of dimensionless operating conditions appears in a practically useful form, giving hope that the concept of a space of dimensionless operating conditions may have eventually some practical usefulness.

The diameter ratio $D_i/D_{o_{\min}}$ and the flow coefficient V_{m_i}/U_i are of course not the only design parameters that are of interest and can be related to the specific speeds. Of particular significance is the maximum outside diameter, $D_{o_{\max}}$. Often this diameter must be larger than $D_{o_{\min}}$ because of the previously mentioned limit of the retardation of the relative flow. To arrive at a simple solution, it was here assumed that the circumferential component of the relative flow should not be retarded more than according to $w_{u_o}/w_{u_i} \geqq 0.65$.

Using Euler's turbomachinery momentum equation as well as $w_{u_i} = -U_i$ (for zero rotation of the absolute flow at the impeller inlet), and with $(2g_oH/U_{o_{\min}})^2 = 1$, one arrives at

$$\frac{D_{o_{\max}}}{D_i} \geqq \left[\frac{1}{2\eta_h}\left(\frac{D_{o_{\min}}}{D_i}\right)^2 + 0.325^2\right]^{1/2} + 0.325 \qquad (14)$$

where η_h is the so-called hydraulic efficiency of the machine, accounting only for head losses, not for leakage or parasite torque increases. Figure 17 shows the evaluation of equation (14) under the assumption that $\eta_h = 0.90$, and using only the equality sign, so that $D_{o_{\max}}/D_i$ has its minimum value. Evidently it would be possible to enter this information into the section through the space of operating conditions represented by figure 16. This has been done only for $D_i/D_{o_{\max}} = 0.8$, 1.0, and 1.2. These values are represented in figure 16 as dash-and-dot lines to indicate this family of lines diagrammatically. Since, according to equation (14), the lines $D_i/D_{o_{\max}} =$ constant are parallel to the lines $D_i/D_{o_{\min}} =$ constant, it would be difficult to distinguish the $D_i/D_{o_{\max}}$ family of lines clearly from the $D_i/D_{o_{\min}}$ family. However, only such practical considerations of visibility prevent one from showing other parameters such as $D_i/D_{o_{\max}}$, or $2g_oH/U^2_{o_{\max}}$, in the section (fig. 16). The minimum head coefficient $2g_oH/U^2_{o_{\max}}$ is derived easily from the assumed value of the maximum head coefficient $2g_oH/U^2_{o_{\min}} = 1$ by the relation:

$$2g_oH/U^2_{o_{\max}} = (2g_oH/U^2_{o_{\min}}) \frac{D^2_{o_{\min}}}{D^2_{o_{\max}}} \qquad (15)$$

with $D_{o_{\max}}/D_{o_{\min}}$ given in figure 17.

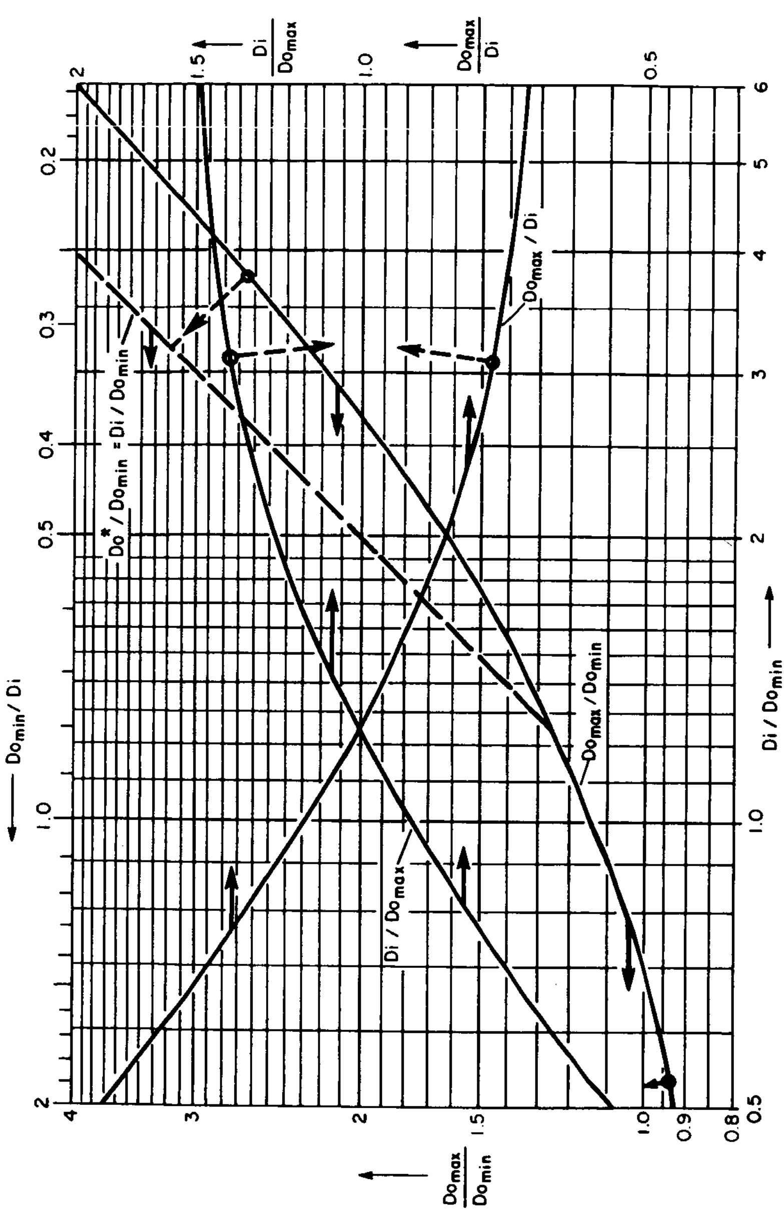

FIGURE 17.—*Maximum pump discharge diameters* $D_{0_{max}}$ *and* D_0^* *as functions of* $D_{0_{min}}/D_i$.

Additional design coefficients can easily be calculated by the elementary laws of turbomachinery design (as for example the discharge width ratio $b_o/D_{o_{min}}$ (see fig. 2) from the condition of continuity, or dimensions of stationary vane passages adjacent to the impeller), primarily on the basis of the law of constant angular momentum. Every point in the section shown in figure 16 therefore represents in principle the whole set of dimensionless design parameters as completely as permitted by the state of knowledge available. Figure 3 depicts the type of information represented by every point in this section (fig. 16), in this case with respect to the design form of the impeller only.

Before returning to multidimensional considerations on the space of operating conditions, it is desirable to illustrate the last step; i.e., *the establishment of a complete design form from the design parameters.* At present this step can be demonstrated (under many simplifying assumptions) only for the relatively well established field of hydrodynamic runners, represented in an only two-dimensional "space" of the operating conditions, n_s vs. S.

Ideally every point in this space, or section, should be associated with a complete design form. This can be demonstrated here only for a very limited number of points A, B, C, D, E, and F in figure 16. Figure 18 illustrates diagrammatically the corresponding design forms by showing the impeller profiles only. The various profiles are correlated with the six points in figure 16 by the same letters, as well as by the values of the basic specific speed (n_s) and the suction specific speed (S). (Values of the specific speeds in conventional pump units are given in parentheses.)

Since the impeller design forms actually include the vane shapes as derived from the velocity diagrams (shown in fig. 3), it is evident that this last step is a very major step, demanding all the knowledge, experience, and skill available in the pump design field. It is the core of the design process. Hopefully, what has been said shows this process in its proper position within the overall design procedure.

Since $D_{o_{max}}/D_i$ given by figure 17 is a minimum value of this ratio, it is permissible, even desirable, to show in figure 18 a larger ratio $D_o^*/D_i=1$ whenever $D_{o_{max}} \leqq D_i$.

It is now time to return to the original, multidimensional picture of the space of dimensionless operating conditions. To aid our imagination, we will consider at one time the interaction of the n_s vs. S section (shown in fig. 16) with only *one* of the other coordinates (specific speeds) of this space. This particular coordinate axis is of course normal to the n_s vs. S plane, giving a three-dimensional picture of this "interaction" with a third specific speed.

The lines in figure 16 represent in this picture surfaces intersecting the n_s vs. S plane. Under the design choices (or assumptions) made at the

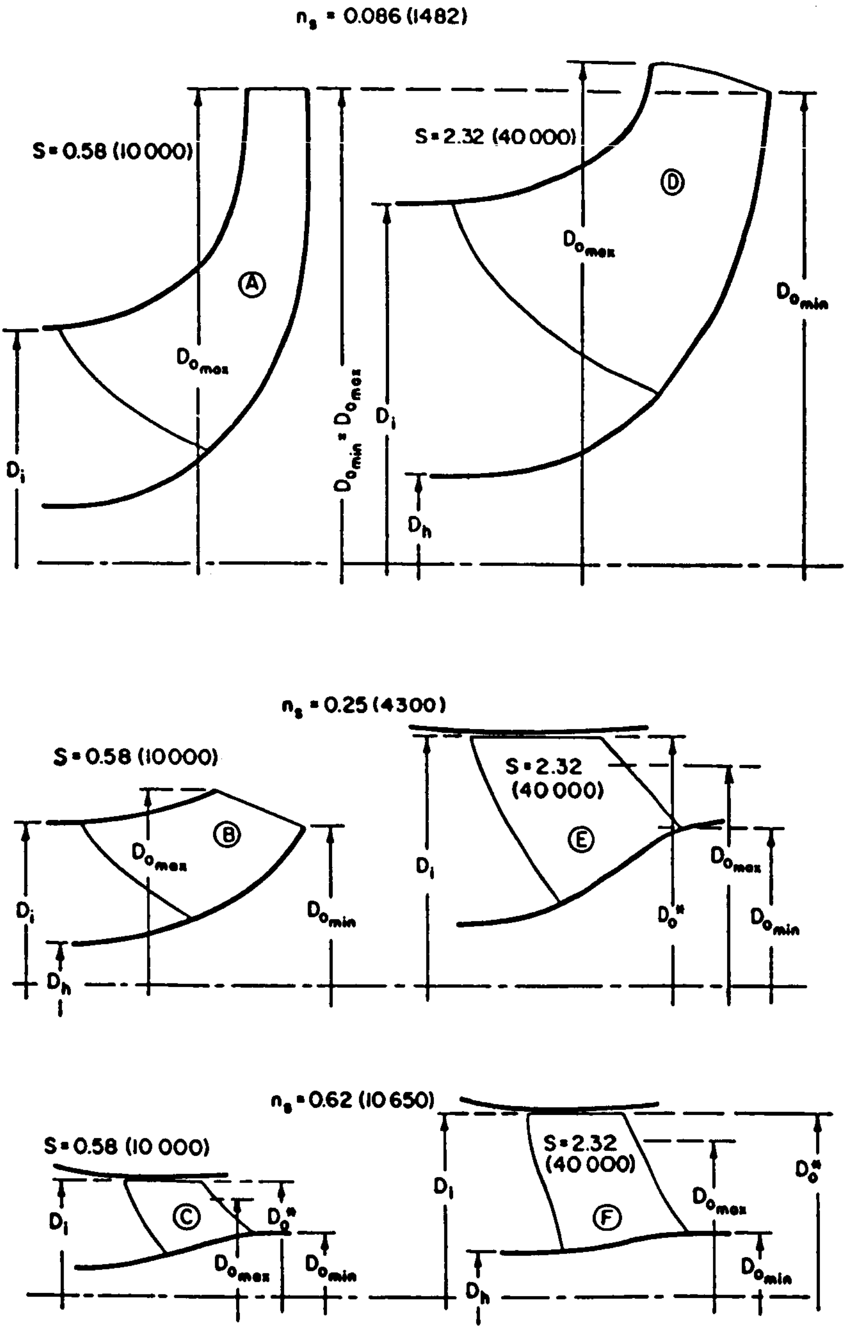

FIGURE 18.—*Runner profiles as functions of* n_s *and* S.

beginning of this example, these surfaces will intersect the particular n_s vs. S plane at right angles if the third specific speed is either the compressibility specific speed or the viscosity specific speed, the stress, the gravity, or the vibration specific speed, because it was assumed that the effects of small changes in the compressibility, viscosity, stress, gravity, and vibrations characteristics should be negligible. This set of assumptions or design choices therefore eliminates (as intended) the multidimensional character of this space, leaving n_s and S as the only significant variables.

As soon as one extends considerations to large changes in compressibility, viscosity, stress characteristics, etc., the picture becomes quite different. The surfaces, represented in the n_s vs. S plane by lines $D_i/D_{o_{min}}$=constant and V_{m_i}/U_i=constant, *curve* in planes *normal* to the n_s vs. S plane. For example, at low viscosity specific speeds (low Reynolds numbers) the lines in the n_s vs. S plane have different positions, and the surfaces they represent intersect the n_s vs. S plane at angles substantially different from 90°. This is just a geometric way of saying that the viscosity of the fluid (the viscosity specific speed), and changes thereof, have substantial effects on the design parameters of the machine.

The very same type of statement can be made for other specific speeds as the third coordinate, for example, the stress specific speed. It is to be expected that at high stress specific speeds the surfaces of constant design parameters, such as $D_i/D_{o_{min}}$ or V_{m_i}/U_i, will intersect an n_s vs. S plane not at right angles and at a substantially different place than at the low stress specific speed assumed before. To describe this three-dimensional space of the three operating conditions n_s, S, and n_σ, one could investigate relations in planes normal to the n_s vs. S plane, for example, in several n_σ vs. n_s planes at different constant values of S. A series of diagrams, analogous to figure 16, representing n_σ vs. n_s, n_σ vs. S, and n_s vs. S planes at different, constant values of S, n_s, and n_σ, respectively, would describe the field of single-suction centrifugal and axial-flow pump design forms rather completely and would be of great practical value, particularly for preliminary design. Unfortunately, presently available information on the design of such pumps is not nearly sufficient for arriving at an answer for such a representation that is even approximately unique.

To avoid the impression that the foregoing mental pictures are merely abstract speculations, figure 19 presents a somewhat qualitative picture of the final results that might be obtained from a step in the direction of the n_{σ_c} axis at constant values of n_s and S. The step is taken from the point G in figure 16, from a centrifugal stress specific speed $n_{\sigma_c}<0.1$ to a value between $n_{\sigma_c}=0.2$ and $n_{\sigma_c}=0.3$. (It would require a fairly detailed stress analysis to arrive at more definite figures.) As mentioned previously with respect to hydrodynamic design, it will take the entire available knowledge, experience, and skill in hydrodynamic and mechanical

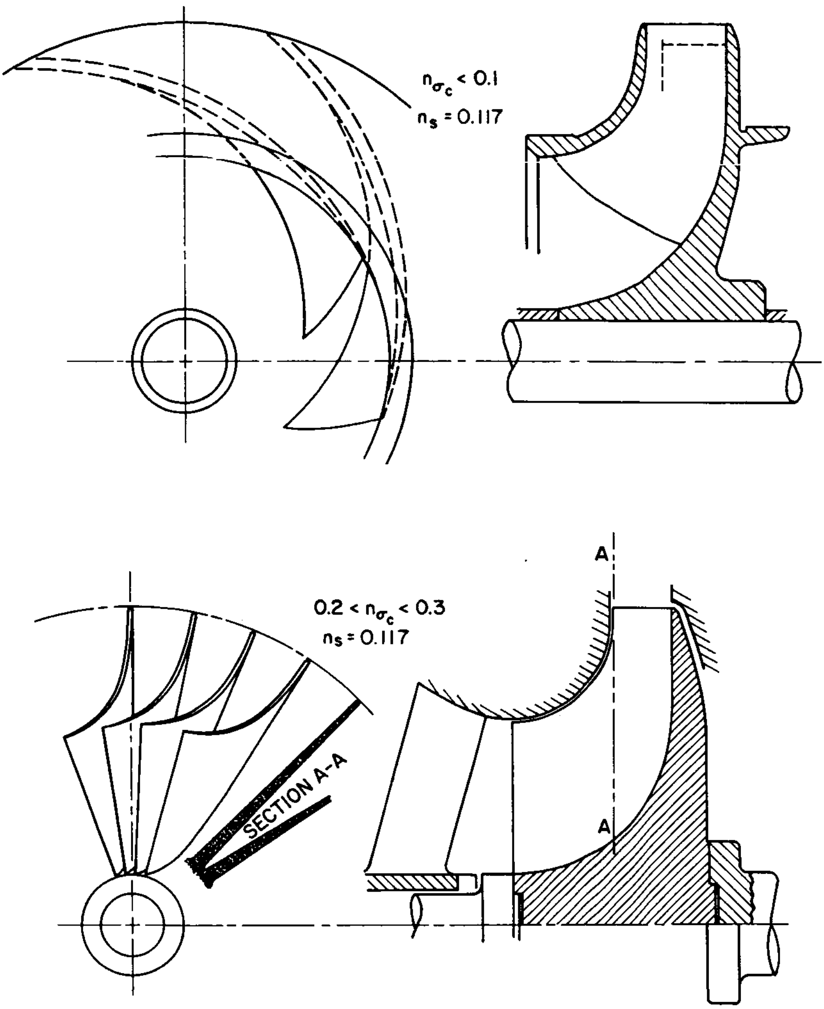

FIGURE 19.—*Radial-flow runner design as a function of stress-specific speed.*

design, and more, to make a reasonably useful attack on the design problems of the n_s, S, and n_{σ_c} space. This situation can hardly be better in the n_s, n_a, and n_{σ_c} space of gas-dynamic machines, and it appears to be much farther from a practically useful solution when the viscosity, gravity, or vibration specific speeds are involved.

SUMMARY

The field of turbomachinery design is first represented by a number of analytical relations among various dimensionless operating conditions, the "specific speeds," and dimensionless design parameters such as ratios of important linear dimensions and flow and head coefficients. Every set of operating conditions is associated with a corresponding set of design parameters that can be reasonably unique only after certain design choices have been made regarding the nature of the fluid, purpose and type of the machine, and so on.

This situation can be represented as a space of dimensionless operating conditions, every coordinate representing one of these operating conditions (i.e., one "specific speed"). Every point in this space represents a complete set of dimensionless operating conditions (as complete as possible within the present state of knowledge).

After all pertinent design choices have been made, every point in this space can be associated with a corresponding set of design parameters, so that one can locate in this space the loci of constant values of all design parameters appropriate for the design choices made. A two-dimensional section through this space shows these loci as lines of constant values of a design parameter (fig. 16). Such sections may thus be charts from which one can read values of the design parameters. *Modern means of computation may not restrict this possibility to the use of two-dimensional, graphical charts.*

The core of the form design process consists in associating with points in this space—i.e., with complete sets of design parameters (as complete as permitted by the state of knowledge)—corresponding *design forms* (see figs. 18 and 19). This process requires all the knowledge, experience, and skill available in the field of design and is not described, only located, in this paper relative to other aspects of the *overall* design process.

Persons qualified in mathematical or computational matters should take over at this juncture.

ACKNOWLEDGMENT

Some of the thoughts expressed in this paper were expressed originally in 1967 in a report by this writer to the National Science Foundation. The present development of these thoughts is part of an effort currently sponsored by the National Aeronautics and Space Administration. The permission by NASA to use this material, and the encouragement received from Messrs. Melvin J. Hartman and Cavour H. Hauser of the Lewis Research Center, Cleveland, are very much appreciated. Furthermore, the writer would like to thank Dr. Thomas G. Lang and Dr. James W. Bond of the Naval Undersea Research and Development Center, San Diego, for their interesting comments and suggestions.

The writer feels greatly indebted to the Committee chaired by Dr. B. Lakshminarayana which initiated and organized the Symposium on Turbomachinery at The Pennsylvania State University and the publication of its proceedings.

LIST OF SYMBOLS

A, a	area	ft^2
a	velocity of sound	ft/sec
b	width of impeller, vane span	ft
C_L	lift coefficient	
D	diameter	ft
E	Modulus of Elasticity	lbs/ft^2
g	acceleration of the whole system	ft/sec^2
g_o	standard gravitational acceleration	ft/sec^2
H	head ft lbs/lb =	ft
H_{sv}	total suction head above vapor pressure	ft
h	vane thickness	ft
l	vane length	ft
m_s	section modulus	ft^3
n	speed of rotation	rps
$n_a, n_\nu, n_\sigma, n_g, n_v$	See table I	
n_s	basic specific speed	
N	number of vanes	
p	pressure in a fluid	lbs/ft^2
Q	rate of volume flow	ft^3/sec
r	radius	ft
S	suction specific speed	
t	circumferential vane spacing	ft
U	circumferential velocity of solid rotating parts	ft/sec
V	absolute fluid velocity	ft/sec
w	relative fluid velocity	ft/sec
w_∞	vectorial mean relative fluid velocity between inlet and discharge of a vane system	ft/sec
y	axial thickness of a rotating disc	ft
y/y_1	ratio of axial thickness of a rotating disc to its thickness at outer radius r_1	
β	angle from the meridional direction	
Γ	circulation, vane circulation	ft^2/sec
η	efficiency: η_h "hydraulic efficiency"	

ν	kinematic viscosity	ft²/sec
ρ	mass density	lbs-sec²/ft⁴
σ	stress	lbs/ft²
ϕ	flow coefficient V_m/U	
ψ	head coefficient $2g_oH/U^2$	

Subscripts Not Listed in Connection With Symbol

f	fluid, fluid-induced
h	hub, measured at hub diameter
i	inside, inlet of pumps or compressors
m	meridional
o	outside, measured at outside diameter
s	structural, solid material
th	"throat," discharge cross section of the volute
U	circumferential, in direction of velocity U

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DISCUSSION

T. G. LANG (Naval Undersea Research and Development Center). This paper is a refreshing and enlightening new approach to the design of turbomachines. By utilizing a nondimensional approach it collapses a large number of design problems into a basic set of design problems, leading to a better understanding of the field of turbomachinery design. The fundamental operating conditions which determine most design problems are listed and the steps leading to the design solutions are described.

It is pointed out that certain design choices must be made which depend largely upon the specific design situation. The design solution is then that form which "best" satisfies the problem in view of both theoretical relationships and empirical design results which have lead to highest efficiency, ease of manufacture, etc.

New operating conditions are introduced, such as the stress, gravity, compressibility, and vibration specific speeds. The latter might be more generally called the elasticity specific speed. The use of this approach should be helpful to the designer who is designing a complete system, since this paper indicates how the turbomachine design might vary as the inlet pressure, shaft speed, flow rate, head, strength of the material, etc., vary. Consequently, this approach could help in selecting the power source, gearing, and other aspects of a complete design problem by quickly showing how the resulting turbomachinery form would vary.

The design example of relating the design form with variations in n_s and S clearly demonstrates the usefulness of this procedure.

J. H. HORLOCK (Cambridge University): Professor Wislicenus' paper gives me much food for thought. My own use of the concept of specific speed has been limited to the basic specific speed, which means the ratio (flow coefficient)/(head coefficient)$^{3/2}$ to me. A low basic specific speed means (relatively) a machine with a small hole at the front for the flow to go through, but a machine with a strong capacity for changing tangential momentum and doing work. Hence the geometry of the first diagram of figure 1, and hence my preference for the term "shape parameter" instead of "basic specific speed." But Professor Wislicenus' introduction of other specific speeds greatly widens visual concepts for the designer.

My main contribution to discussion of his paper must be to add a complication. In the multidimensional space of specific speeds, room should be found for one or more (dependent, not independent) coordinates that give the performance of the machine. For example, in figure 16, the probable efficiency of each of the eight machines should be plotted perpendicular to the paper so that the designer can be assisted in his design choice. (Fig. 1.15 of ref. D-1 was a crude attempt to go in this direction.)

May I also add a philosophical point, or rather a question? Why is it that pump designers and consulting engineers are the widest users of the concept of specific speeds in turbomachinery design? Is it in fact that the gas turbine or steam turbine designer is wallowing in Professor Wislicenus' multidimensional space and cannot cope with the complexities of the problem? Or is it that he has been forced by long experience very close to one point in that multidimensional space, and he cannot get away from it? I suspect the consulting engineer faced with continuously new pump designs can get good estimates of where to design from a graph such as Prof. Wislicenus' figure 16. The situation is more quickly understood because of the effective elimination of many variables (or extra specific speeds)—hence the ready use of the concept.

One final reaction. My first thought on reading this paper was that the designer could turn to the computer to obtain the answer within the multidimensional space of specific speeds. But I think he will get little advantage in doing so. Any "solution" of a design point in that space must be associated with an empirical knowledge of performance for such a design. He may have obtained the geometry (design parameters) that meets the requirement of a dozen specified specific speeds, but it may be a poor machine. He will have to compromise to meet his design point. The computer cannot eliminate the designer's judgment based on experimental observation.

WISLICENUS (author): The writer should like to express his appreciation to all who took the time to discuss this paper. Not every comment can be explicitly recognized here.

Dr. Lang may be interested to hear that in a report to the National Science Foundation entitled "Form Design in Engineering" this writer used in 1967 the term "elastic specific speed" for what he has called now "vibration specific speed." The reason for this change in name is that the writer is as yet not certain that this specific speed is given here in the best form with respect to *steady* elastic deformations. The parameter $U/\sqrt{E/\rho_s}$ on the right side of this specific speed equation is, for simple elastic systems, proportional to n/f, where n is the frequency of excitation (e.g., the speed of rotation or a blade-passing frequency) and f a natural or "critical" frequency of the system. Thus $U/\sqrt{E/\rho_s}$ has a clear *dynamic* significance.

The writer is not sure that he agrees with Dr. Horlock's term "shape factor" for the basic specific speed. Any specific speed is primarily a dimensionless expression of operating conditions. Its relation to the form of the machine is derived, and it is multivalued, depending on several "design decisions."

The space of dimensionless operating conditions has, indeed, "room" for performance characteristics such as efficiency, stall margins, and others. However, most performance characteristics of this type are not rigorously determined by the specific speeds even after the necessary design decisions have been made. The lines, surfaces, etc., of constant efficiency, etc., that can be drawn into the space of dimensionless operating conditions are empirically, not rigorously located, compared with the loci of constant design parameters such as diameter ratios considered in this paper. Of course, this distinction may be only temporary if one assumes that efficiencies, stall margins, etc., will be predictable under some future state of knowledge and design of turbomachinery. With a more complete knowledge than available at present it should certainly be possible to draw (for example) lines of constant efficiencies (at design flow conditions) into figure 16 of the paper.

The basic specific speed is not generally used with machines for compressible fluids because the rate of volume flow (Q) does not have one single value. However, if Q is measured always at the same place in the machines compared, say at their low-pressure ends, then n_s can be defined and employed advantageously. In principle one can use the basic specific speed, the compressibility specific speed, and the stress specific speed for compressible fluids just as effectively and in the same manner as the basic, the suction, and the stress specific speeds for machines handling liquids.

Computers can do, of course, much more accurately what is done by computation charts such as that shown in figure 16, and the computer is not restricted to two (or three) dimensions, as are graphical charts. However, whether graphical or computerized, the results can only represent the knowledge, experience, and skill that originally went into the construction of the charts or into the programming of the computer.

REFERENCES

D-1.—*Axial Flow Turbines*. Butterworth Scientific Publications, 1968.