Impeller Blade Design Method for Centrifugal Compressors

WILLEM JANSEN AND ANDREAS M. KIRSCHNER

Northern Research and Engineering Corporation

The design of a centrifugal impeller must yield blades that are aerodynamically efficient, easy to manufacture, and mechanically sound. The blade design method described here satisfies the first two criteria and with a judicious choice of certain variables will also satisfy stress considerations. The blade shape is generated by specifying surface velocity distributions and consists of straight-line elements that connect points at hub and shroud. The method may be used to design radially elemented and backward-swept blades. The background, a brief account of the theory, and a sample design are described in this paper.

The design of impeller blades is a difficult task due to the fact that, ideally, three separate and equally important requirements must be satisfied. The first requirement is that the impeller provide acceptable distributions of relative velocity on both driving and trailing surfaces of the blades in order to minimize the possibility of flow separation and the accompanying loss in performance. In addition, the selected blade shape must be such that it can be manufactured accurately and economically by means of automated fabrication procedures. Finally, the blades should be designed so as to keep the stresses at a safe level, eliminating the possibility of excessive distortion or fracture occurring during operation.

For many applications, these three requirements are somewhat incompatible. For example, it is possible to define a blade shape which theoretically would give ideal velocity distributions throughout. However, it is often the case that such a shape is virtually impossible to fabricate in any reasonable manner. Therefore, any practical design method must involve some reasonable compromises with regard to the conflicting requirements. The method described in this paper, the prescribed loading method, is intended specifically to produce impeller designs which satisfy the aerodynamic and manufacturing requirements mentioned above.

Although the method does not include stress considerations explicitly, an intelligent choice of inputs to the method and a careful examination of the results obtained will enable safe designs to be produced with a moderate amount of effort.

AERODYNAMIC CONSIDERATIONS

Losses in a centrifugal compressor are generally caused by boundary-layer and separated flow developments in the impeller and their subsequent effects in the diffuser. The development of the boundary layers and separated flows is directly a result of the velocity distributions along the blade, hub, and shroud surfaces. There are four important parameters associated with the velocity distribution that affect the boundary-layer behavior. They are

(1) The streamwise velocity reduction. This velocity gradient is also recognized as being the main variable that affects boundary-layer behavior in two-dimensional flows.

(2) The reduction and increase in turbulence intensity along the suction and pressure blade surfaces, respectively. This enhances flow separation along the suction surface due to partial laminarization. The change in turbulence intensity is directly related to the differences in suction and pressure surface velocities or loading (ref. 1).

(3) The influx of flow along the blade surfaces from one streamtube (a thin annular channel containing the primary flow) to the next caused by secondary flow. This flow usually consists of low-momentum fluid; the boundary layers are more susceptible to separation when they contain a large amount of low-momentum flow. The differential loading of one streamtube compared to the next is directly related to the development of secondary flows (ref. 2).

(4) The higher flow losses of the shroud streamtube caused by clearance effects. These losses cause a pressure drop which leads to flow separation. Increased work input will offset the rise in pressure losses (ref. 3).

Extensive experimental research is currently being carried on to measure these effects and refine relevant theories.

The method should therefore be capable of generating a blade surface for which the following items are specified:

(1) The streamwise velocity distribution. This quantity is controlled by both the hub and shroud contours and the blade loading.

(2) The difference in suction and pressure surface velocity (or loading) to minimize the turbulence attenuation at the suction surface.

(3) The difference between the work addition to a streamtube near the hub and that near the shroud to minimize the transport of low-momentum flow by the secondary flow phenomenon.

(4) The higher work input into the flow in the shroud streamtube to offset the clearance losses.

From an aerodynamic point of view, it is thus desirable to specify a velocity distribution along the pressure and suction surfaces of the blade such that the generation of separated flows is kept to a minimum. This general principle would lead to a blade defined by a set of streamlines whose individual loading schedule is controlled by the designer.

MANUFACTURING AND GEOMETRICAL CONSIDERATIONS

The ideal blade which would be composed of an arbitrary number of individually loaded streamlines represents a very general three-dimensional blade surface. It is questionable whether such a blade could be economically produced with the required accuracy. Impeller casting patterns are very difficult to manufacture, particularly when the surface of the blade has an arbitrary shape, when close tolerances have to be maintained, or when the impeller size is small. A more desirable blade shape is one defined by straight-line elements; such a shape facilitates setting up the pattern and allows close tolerances to be maintained.

The machining of an arbitrary blade surface would generally be impossible because of the extreme local curvature of the blade surface. At best, the tool would have point contact only and too much time would be required to machine away all the filler material. A more appropriate machining method is one where the cutter has line contact and hence large cutting rate. However, the blade shape must still be restricted to moderate inclinations against normals to the hub surface to avoid tool interferences.

In summary, from manufacturing considerations, a blade shape consisting of straight-line elements on both surfaces is preferred. The thickness distribution of the blades is then necessarily restricted to a constant taper between hub and shroud, and the camber line surface of the blades will also consist of straight-line elements.

Since the camber-line surface is defined by a set of streamlines with controlled individual loading, the number of streamlines for which loading can be controlled and still yield a straight-line element camber surface is limited. In general, the individual streamlines form curves in space and a straight line can only connect a maximum of three curves in space. Consequently, a straight-line element blade can be constructed when the loading of three streamlines is specified. These streamlines are assumed to be those adjacent to the shroud and the hub and an intermediate one.

This specification is considered adequate aerodynamically, and the ruled blade surface is accepted as a balanced compromise.

There are three general categories of blades which can be defined by straight-line elements. These are termed radial, axial, and arbitrary blades. The first two blade types are characterized by the fact that the angular orientations of the line elements defining their camber surfaces are fixed by the nature of the blades; that is, in the radial direction and axial direction, respectively. As a result, the angular orientation (θ) can be prescribed along only one streamline; the obvious choice is the shroud streamline where the flow conditions are critical.

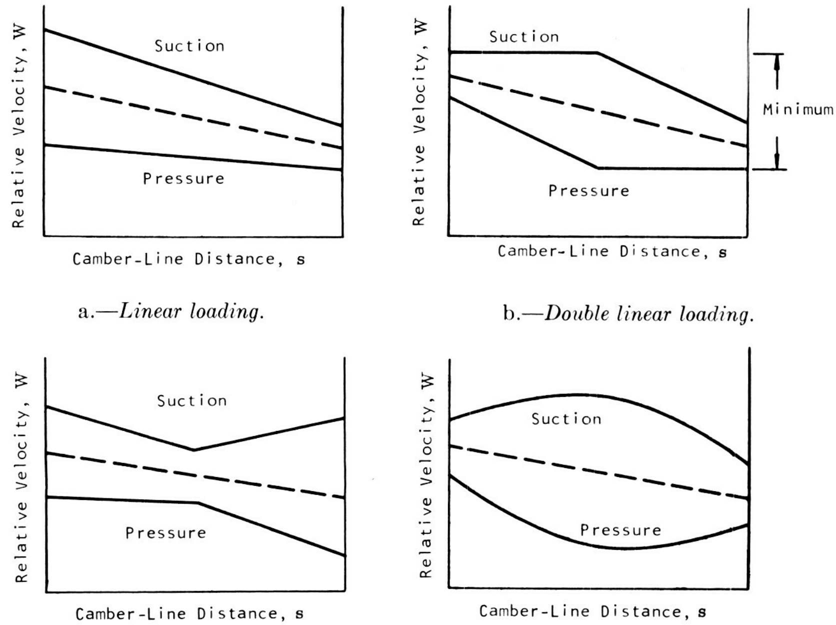
The arbitrary type of blade is characterized by the general and usually unspecified orientations of the line elements (except those at the leading and trailing edges which are usually radial elements) defining the camber surface. The procedure followed in designing an arbitrary blade involves, in general, the loading of the three streamlines in an appropriate manner. The orientation of each blade element is then determined by requiring that the straight line pass through each of the three space curves defined by the loaded streamlines.

In some cases when for an arbitrary blade the loading is specified at three streamlines, the resulting blades may still exhibit a large variation in circumferential angle between the hub and shroud streamlines which would be unsatisfactory from manufacturing or stress considerations; that is, the blade would have excessive lean throughout its length. This difficulty was recognized during the development of the procedure and special provisions were introduced in the procedure to circumvent this by specifying at the outset the desired distribution of line element orientation and by prescribing the loading along the shroud streamline only.

REQUIRED AND GENERATED INFORMATION

The prescribed loading method can be utilized to design an impeller when its rotative speed, flow, average efficiency, overall geometry (that is, hub and shroud coordinates, distribution of blade thickness along hub and shroud, location of leading and trailing edge, number of blades, and exit blade angles), desired swirl distribution at inlet, and angular momentum distribution at discharge are given. The designer must also specify any one of four possible types of loading distribution:

(1) Linear loading from inlet to discharge (fig. 1*a*).
(2) Double linear loading from inlet to discharge (fig. 1*b*).
(3) Double linear loading from inlet to discharge, maximum loading near leading or trailing edge (fig. 1*c*).
(4) General loading distribution (fig. 1*d*).



a.—*Linear loading.*

b.—*Double linear loading.*

c.—*Inverse double linear loading.*

d.—*General loading.*

FIGURE 1.—*Available options of loading distributions.*

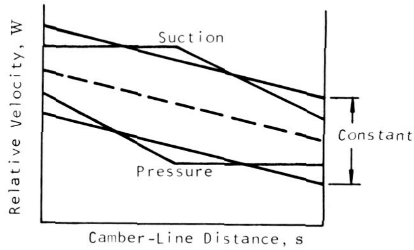
The choice of a preferable loading is difficult to make, since practically no systematic experiments have been performed to determine the relative merits of possible loadings. It is suggested that in most applications either a constant linear loading distribution, thereby minimizing secondary flows (ref. 4), or a double linear loading, minimizing the streamwise velocity reduction, will be quite satisfactory. Figure 2 shows these two loadings superposed.

The results of the calculation are a full description of the blade surface in cylindrical coordinates and the associated thickness distribution.

LIMITATION OF THE BLADE DESIGN METHOD

The prescribed loading method can be used to design radial, axial, and "arbitrary" blade shapes whose surfaces are defined by straight-line elements. However, there are a number of limitations incorporated in this approach. These limitations are discussed briefly below. In some cases

FIGURE 2.—*Linear and double linear loading superposed.*



these limitations are not serious; in others some additional effort is necessary before the generated blading can be produced.

Thickness Distribution

The generated blade shapes are restricted to having trapezoidal thickness distributions; i.e., constant taper along any line element between hub and shroud. An exponential thickness distribution may be preferred when blade stresses are an important design consideration. This alteration may be made by adding or subtracting thickness from the generated surface; no appreciable change in aerodynamic variables is expected when thickness changes are kept moderate.

Blade Shape Restrictions

One fairly important limitation of the prescribed loading method as applied to arbitrary blades is that it is possible to obtain unusual shapes which may be difficult to manufacture or which would not be expected to be capable of withstanding the operating stresses. Such a blade shape may result from different loading requirements at the hub and shroud streamtube. This blade can be redesigned by specifying a new and more desirable straight-line orientation. For this case the blade loading at the shroud is still specified as before, but now the blade loading at the hub and the intermediate streamline can no longer be specified. The sample case is an illustration of this procedure.

Leading Edge Geometry

The method generates the blade shape near the leading edge such that the flow enters at zero incidence. However, when the relative inlet Mach number is high, this may not result in an optimum blade shape. Therefore, separate procedures are necessary to design a blade leading edge capable of efficiently handling supersonic flows.

Hub and Shroud Contours

A complete aerodynamic impeller passage design method would include a provision for automatically adjusting the hub and shroud contours to give a linear or any other prescribed distribution of average relative velocity. Due to the phenomenon of slip, the average relative velocity near the trailing edge decreases and then rises. In order to straighten out this distribution, the blade height of the impeller near the tip becomes larger at higher radii, which is usually unacceptable. Considerable effort has been expended to incorporate this design feature, but it was found that hub and shroud contours which yield a linear relative velocity distribution are in general not acceptable, and this hub-shroud contour generation procedure was abandoned.

Trailing Edge Flow

The specified blade loading does not vanish near the trailing edge as is required from the Kutta or "slip" conditions. However, appropriate steps are taken in the analysis to satisfy the Kutta condition, thereby overriding the specified blade loading over the last 10 percent of the blade chord. In addition, the procedure adjusts the loading such that the specified amount of work will still be imparted to the flow.

GENERAL THEROY OF PRESCRIBED LOADING METHOD

The overall procedure used in designing an impeller by the method of prescribed loading involves the following four basic steps:

(1) An initial estimate is made of the meridional velocities in the impeller passage as described below.

(2) The meridional velocities are then used in loading the required number of streamlines in the prescribed manner.

(3) A blade surface the shape of which conforms to loaded streamlines is developed.

(4) A detailed analysis of the flow field in the impeller with the developed blade surface is performed.

The procedure, starting with the loading of the streamlines, is repeated with the new estimates of the flow field obtained in the detailed analysis. Experience has shown that two repetitions of the procedure are sufficient to ensure convergence. This is defined as occurring when the estimated flow field used in loading the streamlines and defining the blade shape is approximately the same as that obtained in the subsequent detailed analysis. The various steps involved in the design procedure are discussed below in more detail.

INITIAL VELOCITY ESTIMATE

The distributions of meridional velocity along the shroud, hub, and intermediate streamlines are required for use in the loading procedure.

Initial estimates of these distributions are obtained by means of the one-dimensional analysis. First, a number of normals are constructed which are normal to both the hub and shroud at the points of intersection. These normals to the hub and shroud are circular arcs.

Neglecting curvature effects, the mean meridional velocity at a normal is given by the continuity equation

$$C_{m,m} = \frac{\dot{m}}{2\pi r_m n \rho_m} \tag{1}$$

The density ρ_m is a function of the mean meridional velocity, $C_{m,m}$, and equation (1) is solved by an iterative procedure using compressible flow equations to relate static density to total temperatures and pressures. The values of $C_{m,m}$ calculated in this manner are taken as the initial estimates of the meridional velocities along the intermediate streamline.

If it is assumed that zero circulation exists in an area bounded by two adjacent normals and the hub and shroud streamlines, the following expression relating $C_{m,s}$ and $C_{m,h}$ can be derived.

$$C_{m,h}\, dm_h = C_{m,s}\, dm_s \tag{2}$$

If the additional assumption is made that the quantity $\rho_m C_m r$ varies linearly with normal length between hub and shroud, then for constant density

$$C_{m,m} r_m = \tfrac{1}{2}(C_{m,h} r_h + C_{m,s} r_s) \tag{3}$$

A rearrangement and simultaneous solution of equations (2) and (3) results in the following explicit relations for $C_{m,s}$ and $C_{m,h}$.

$$C_{m,s} = \frac{2(r_m/r_h)C_{m,m}}{(r_s/r_h) + (dm_s/dm_h)} \tag{4}$$

$$C_{m,h} = \frac{dm_s}{dm_h} C_{m,s} \tag{5}$$

The derivatives dm_s/dm_h are obtained from the ratio of meridional length between two normals at hub and shroud. The initial estimates of the meridional velocities at the calculation stations along the hub and shroud are then obtained from the latter two equations along with the intermediate values, $C_{m,m}$.

STREAMLINE LOADING PROCEDURE

At the outset of the procedure for loading a streamline, the meridional coordinates (r,z), figure 3, of a number of points or calculation stations along the streamline and the values of meridional velocity (C_m) at these

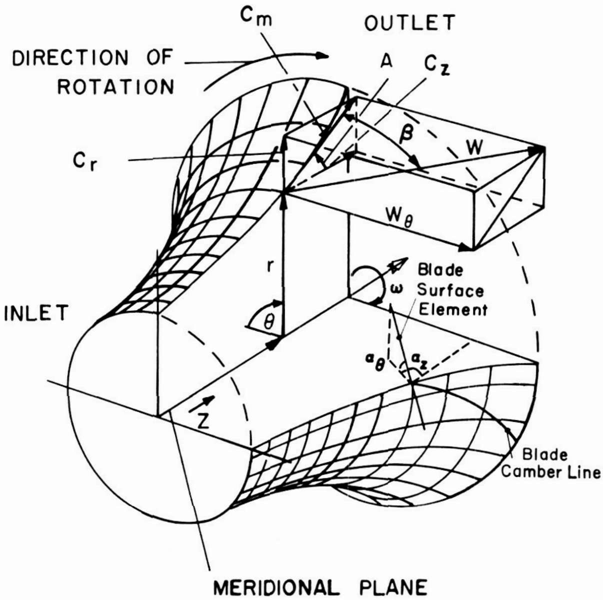


FIGURE 3.—*Nomenclature of relative flow and blade-element geometry in a centrifugal compressor.*

points are known from the results of the previous steps in the design method. First, the distribution of angular momentum is calculated from the given loading distribution $W_t - W_d$ from

$$\frac{d(rC_\theta)}{ds} = \frac{Z}{2\pi}(W_t - W_d) \tag{6}$$

where s is the camber-line length and Z the number of blades.

Assuming for the moment that the blade camber-line distance from the leading edge is also known at each point, then the swirl can be distributed in the specified manner and a value of rC_θ can be computed anywhere along the streamline. Combined with the meridional velocity distribution, sufficient information is then available to construct the velocity triangle and evaluate the flow angle (β) at each calculation station from

$$\beta = \tan^{-1}\left(\frac{C_\theta - r\omega}{C_m}\right) \tag{7}$$

In the above description of the loading procedure, it was assumed that the blade camber-line distance from the leading edge was known at each calculation station. In actual fact, this is not the case at the outset of the procedure. Therefore, this quantity must be determined iteratively. The initial estimate of the blade camber-line distance is taken as the meridional arc length from the leading edge. The streamline is then loaded in the manner described to obtain values of β along the streamline. The angular coordinate (θ) of the blade at each station is obtained by integrating the calculated β-distribution as follows:

$$\theta = \int_{m_1}^{m} \frac{\tan \beta}{r} \, dm \tag{8}$$

In this integration, θ is arbitrarily initialized to be zero at the trailing edge point on the shroud. Finally, the three-dimensional coordinates (r,z,θ) of the blade camber line are used to compute the camber-line distance at each point. The procedure is repeated using the calculated values of camber-line distance as the new estimates. This is done until the estimated and calculated values of the total blade camber-line distance from leading edge to trailing edge are the same. An additional iteration on the distribution of angular momentum is necessary when geometrical constraints require the relative streamline to pass through a given angular interval. Such conditions arise whenever the blade surface is to be determined from the loading specification on more than one streamline. The requirement follows from the stipulation that all streamlines intersect with an initial and final straight-line blade element.

The relative streamline coordinates that result from a specified loading distribution and the meridional velocity distribution define the blade camber line for an infinite number of blades. A modification of this camber line is introduced to account for flow deviation associated with a finite number of impeller blades (i.e., slip). For the major part of the blade passage, the average flow direction is assumed to coincide with that of the blade camber line. In the latter third of the blade passage, a progressive degree of deviation is assumed to occur. At the impeller discharge the amount of deviation is equivalent to the slip.

BLADE CONSTRUCTION

The third major step in the procedure is the synthesis of a mean blade surface from the three camber lines that have been generated in the previous step and the two line elements at the leading and trailing edge.

The general procedure for constructing a line element is illustrated in figure 4 which shows three general space curves and an initial element intersecting each of them. The incremental arc length, Δs_i, along the first curve from the initial line element is chosen arbitrarily to provide the origin of the adjacent element which is to be constructed. The incremental arc lengths, Δs_j and Δs_k, represent the distances from the initial element to the points of intersection of the adjacent element with the remaining two curves. It can be shown that the distance ratios defining the adjacent

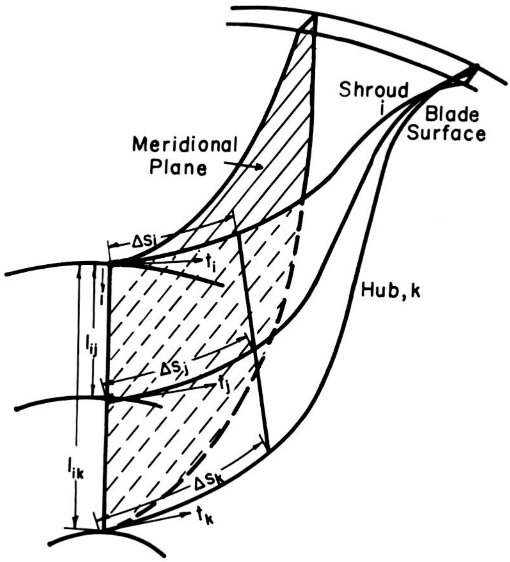


FIGURE 4.—*Vector representation of the straight-line element surface.*

line element are given by the following expressions:

$$\frac{\Delta s_j}{\Delta s_i}=\left(\frac{l_{ik}-l_{ij}}{l_{ik}}\right)\cdot\left(\frac{\mathbf{t}_k\cdot(\mathbf{t}_i\times\mathbf{l})}{\mathbf{t}_k\cdot(\mathbf{t}_j\times\mathbf{l})}\right) \tag{9a}$$

$$\frac{\Delta s_k}{\Delta s_i}=\left(\frac{l_{ik}-l_{ij}}{l_{ij}}\right)\cdot\left(\frac{\mathbf{t}_j\cdot(\mathbf{t}_i\times\mathbf{l})}{\mathbf{t}_j\cdot(\mathbf{t}_k\times\mathbf{l})}\right) \tag{9b}$$

These relations are based on the property of ruled surfaces by which the infinitesimal displacement of a straight-line element is related to vectors which are tangent to the surface at different points on the line element.

In the above equations, the unit vector, $\mathbf{l}$, represents the direction of the initial line element from the first curve, and $\mathbf{t}$ is the unit tangential vector of each curve at the point of intersection as shown in figure 4. The value of $\mathbf{t}$ is found from differentiation of the previously defined camber line with respect to the distance s measured along it.

The procedure for defining the camber surface of the blade is begun by generating on the loaded shroud streamline between the leading and trailing edge a number of equally spaced points representing the origins of the line elements to be constructed. Starting from the leading edge element, the points of intersection of the next line with the intermediate and hub streamlines are determined by computing the incremental arc lengths along these curves with equations (9a) and (9b). The construction procedure is performed in a stepwise fashion with the generation of each line element being the basis for the generation of the next one until the trailing edge is reached. This process is necessarily an iterative one, since initially the exact values of **t** and **l** are not known.

As has been pointed out before, the method for constructing a blade surface from three streamlines in space may lead to undesirable geometries. If this occurs, the case may be redesigned by specifying the orientation of the straight-line elements in advance in terms of two angles, α_θ and α_z, in addition to the loading distribution along the shroud line.

The straight line and its normal projection on the meridional plane include an angle of lean or dihedral (α_θ) as shown in figure 3. The angle between the projection and the impeller axis defines the slope (angle α_z) of the straight-line element. The described definition of blade element orientation leads naturally to the following often used special blade surfaces:

(1) The radial element blade surface is determined by a camber line through which radial line elements are drawn to define the surface. The elements are sloped vertically without lean ($\alpha_\theta=0$, $\alpha_z=90°$).

(2) The axial element blade surface is determined by axial line elements originating from a camber line. These elements are sloped horizontally without lean ($\alpha_\theta=0$, $\alpha_z=0$).

(3) The arbitrary blade surface is defined by straight-line elements emanating with arbitrary orientation from a camber line. The elements may have slope and lean ($\alpha_\theta \neq 0$, $\alpha_z \neq 0$).

The camber surface and the given distribution of normal blade thickness along the hub and shroud contour determine the suction and pressure surface of the blade. At each point of intersection of camber line and straight-line element, the normal thickness is interpolated from the given data and one-half of this value is applied in each direction of the local normal to the mean blade surface to define points on the outer blade surfaces. The remainder of these outer surfaces is again described by straight-line elements joining corresponding hub and shroud points. For purposes of the following flow analysis, the blade geometry is defined by a matrix of discrete point coordinates and tangential blade thicknesses on the straight-line elements of the mean blade surface. The interpolation of blade thicknesses and subsequent conversion to tangential thickness requires the determination of the local normal to the mean blade surface. On straight-line elements this normal vector is a linear combination of the known normal vectors at the hub and shroud points of the element. During the flow analysis, planar interpolation of the blade geometry is used for any blade point not explicitly located on a specified straight-line element.

DETAILED FLOW ANALYSIS

The final step in the blade design procedure is an analysis of the flow field that corresponds to the designed impeller blade passages. The analysis serves to reevaluate the meridional flow field from which the blade shape has evolved, by means of prescribed loading distributions on one or more streamlines. Three cycle iterations of the blade design are usually sufficient to achieve convergence on the meridional velocity distribution.

The most convenient way of expressing the governing equations is in so-called streamline coordinates. These coordinates lie in a meridional plane (see fig. 3). A set of fictitious streamlines is defined as the intersections of annular stream surfaces and the meridional plane. The stream surface itself is obtained by rotating the actual streamline for the infinite-number-of-blades solution around the axis.

Using the distance along streamlines (the meridional distance), m, and the distance normal to the streamlines, n, as coordinates, the governing equations reduce to the following:

(1) The normal momentum equation, a differential equation which primarily determines the distribution of meridional velocity along normals.

(2) The continuity equation, an integral equation which primarily determines the position of the streamlines along normals for a given meridional velocity distribution.

(3) The conservation of angular momentum equation (upstream and downstream of the wheel) or the Euler work equation (within the wheel), which is an algebraic equation applied along streamlines between normals.

(4) The energy equation, an algebraic relation between the change in stagnation enthalpy and the energy input along streamlines.

(5) The equivalent of the streamwise momentum equation, an algebraic relation which primarily determines total pressure loss along streamlines.

These equations are generally expressed in two slightly different forms depending upon whether the flow region considered is inside or outside the blade passage. In addition, for gases, the equation of state provides the necessary relationship between fluid properties. The remaining equations, including the many useful relations derivable from the equation of state and the definitions of stagnation properties, are well known and not presented here. Detailed discussions of this flow analysis method may be found in references 3, 5, and 6.

Briefly, the solution procedure is an iterative one which involves first estimating the positions of the streamlines throughout the flow field, solving the normal momentum equation and the continuity equation simultaneously along each normal (using other equations to relate flow properties on each streamline to those of the upstream normal), revising the estimate of streamline positions, and repeating the process until convergence on the streamline position is obtained. The solution so obtained is based on the assumption that the flow is everywhere parallel to the blade surfaces and, in effect, accomplishes the determination of appropriate streamtube widths throughout the flow field from which the meridional velocities are determined by the use of quasi-one-dimensional flow relationships.

SAMPLE IMPELLER DESIGN

The blade design method is illustrated by a sample case with the following design specifications:

Rotational speed	27 900 rpm
Design mass flow	4.61 lbm/sec
Inlet total pressure	14.7 psia
Inlet total temperature	528° R
Discharge total temperature	708° R
Impeller isentropic efficiency	0.945
Impeller discharge blade angle	−25°
Impeller inlet hub radius	1.5 in.
Impeller inlet shroud radius	3.33 in.
Impeller discharge radius	5.08 in.
Impeller discharge width	0.539 in.
Impeller axial length	2.75 in.
Leading edge element orientation	$\alpha_z = 90°$
	$\alpha_\theta = 0$
Trailing edge element orientation	$\alpha_z = 0$
	$\alpha_\theta = -30°$
Blade number	16
Blade thickness along the shroud	0.05 in.
Average blade thickness along the hub	0.15 in.

The impeller hub and shroud contours are shown in figure 5. An arbitrary mean blade surface of unspecified blade element orientation was selected for the initial design. The loading distribution along the shroud streamline was chosen for minimum diffusion on the blade suction and pressure surfaces. This loading schedule is as shown in figure 1*b*. Along the hub and an intermediate streamline, a linear loading schedule was maintained as shown in figure 1*a*. At the blade inlet this loading was adjusted to meet the geometrical constraint that all three streamlines should pass through the initial straight-line element at the leading edge and the final line element at the trailing edge.

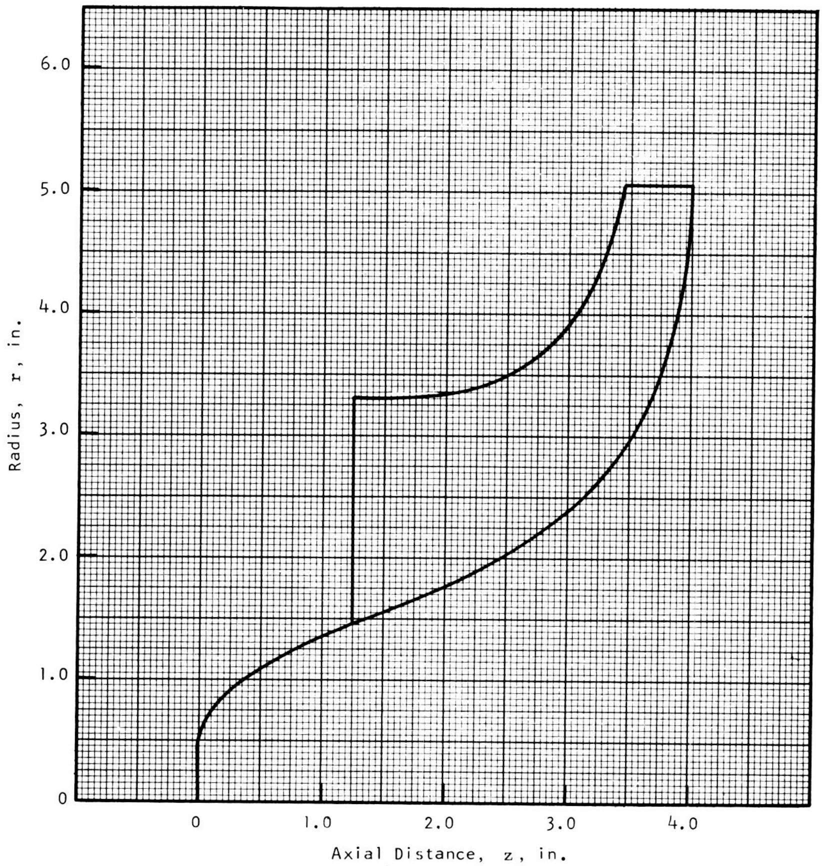


FIGURE 5.—*Impeller contours for the datum stage (25-degree backslope impeller).*

Figure 6 shows the blade element orientation resulting from the initial design. The leading edge blade element is radial while a negative or forward lean, α_θ, is indicated in the inducer region. The blade lean reverses its sign toward the middle of the blade and has a negative or forward lean of 30° at the trailing edge as prescribed. Blade element slope, α_z, varies uniformly from vertical at the leading edge to horizontal at the trailing edge.

Several design considerations led to a revision of the blade element orientation. A first concern arose from blade root bending stresses at the inducer inlet. The subject impeller forms a basic design standard in a series of research compressors that represent systematic variations of design parameters. For this reason, a simpler variation of blade element orientation was selected for the impeller by specifying zero blade lean throughout and a variation of slope as shown in figure 6. This design

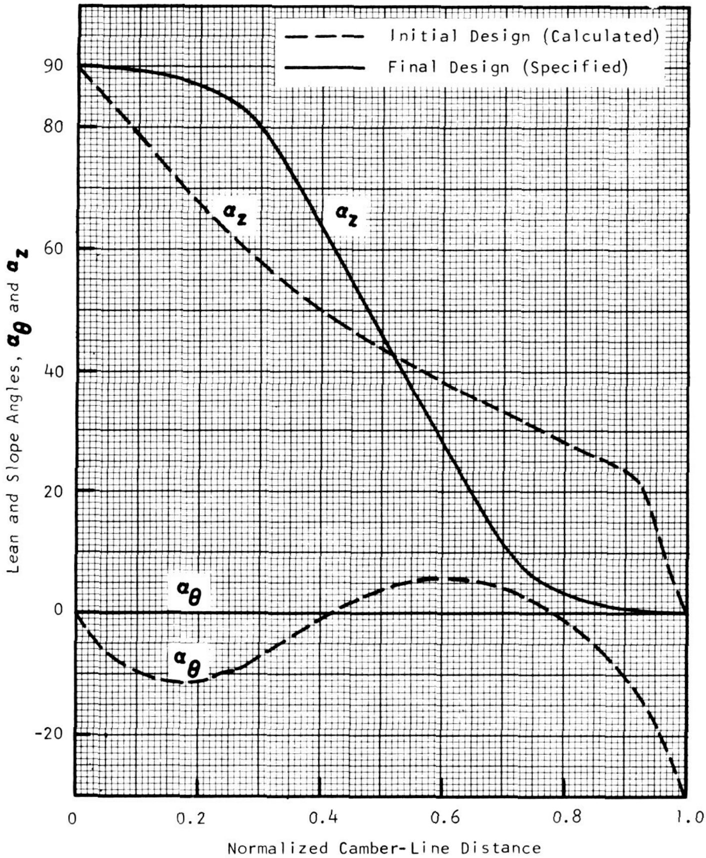


FIGURE 6.—*Blade-element orientation for sample impeller design.*

modification eliminated blade root bending stresses in the inducer and the originally calculated negative blade lean, α_θ, at the impeller exit.

The final velocity distributions on the blade surfaces are shown for the shroud streamtube in figure 7. The velocity variation along the shroud was not affected by the revised distribution of blade element orientation, since the loading schedule on the shroud streamline was not altered. Within the numerical accuracy of this application, the loading along the shroud streamtube varies in the prescribed manner. A suitable modification of the hub and shroud contour would have produced a more linear variation of the average relative velocity and a resemblance to the idealized velocity distributions of figure 1, but the present distribution was considered satisfactory.

The final velocity variation along the hub streamtube as shown in figure 8 was not significantly affected by the revised orientation of blade

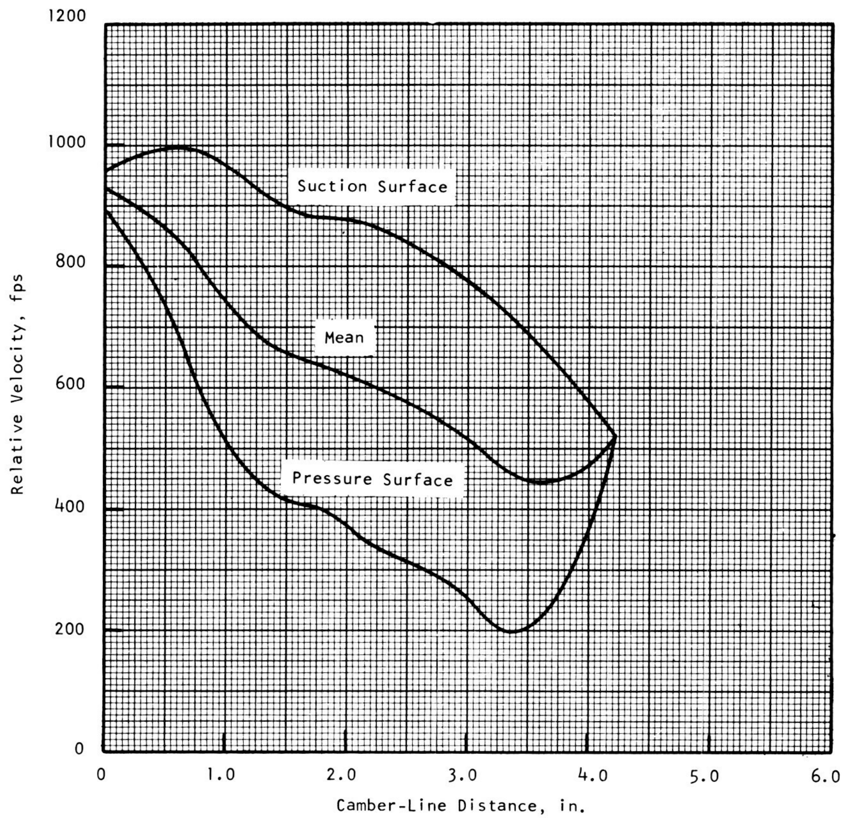


FIGURE 7.—*Design-point blade-surface-velocity distributions for sample impeller (shroud streamtube).*

elements, although blade loading was increased along the forward portion of the hub camber line. The large variation in average relative velocity along the hub could be reduced by modification of the hub contour but was considered acceptable in its present form.

CONCLUSIONS AND RECOMMENDATIONS

The impeller blade design method has been utilized to generate impellers with 0°, 25°, 45°, and 55° backslope at the trailing edge. Some of these designs were cast and others machined, but due to the straight-line element principle the manufacturing of these designs was relatively straightforward. The test results of these designs showed that their aerodynamic performance was very satisfactory. Dynamic and static pressure measurements have shown that the actual surface velocity distributions agree

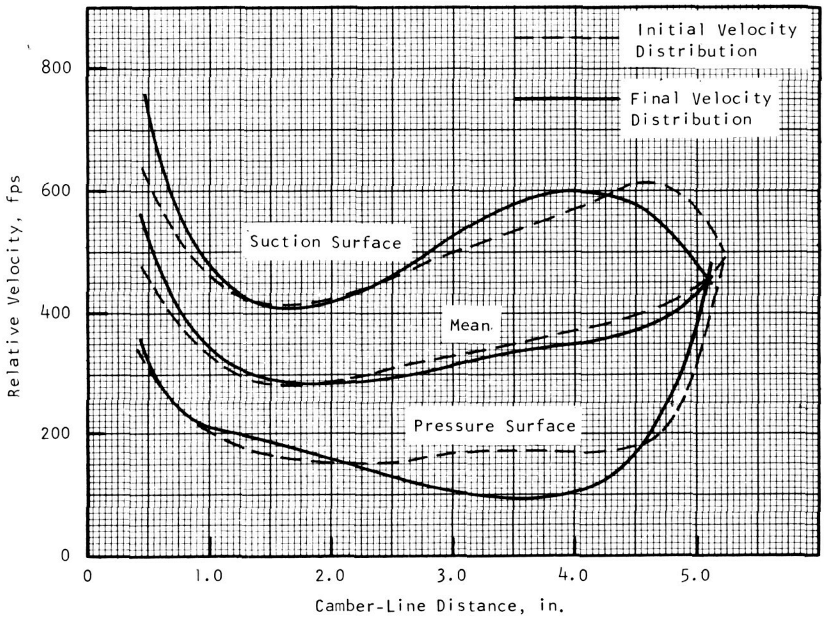


FIGURE 8.—*Design-point blade-surface-velocity distributions for impeller (hub streamtube).*

closely with those calculated with the prescribed loading procedure as described above.

The method involves a large and complicated set of calculation steps which are not presented here; these steps are computerized so that a solution is readily obtainable.

There are two areas where future activities in impeller blade design are essential. The first is linking the velocity distribution along the blade surfaces at different streamlines to the impeller and impeller-induced diffuser losses. When this link is established, there is more confidence that a specified velocity distribution will achieve low impeller losses. The second area is the attenuation of impeller losses by various boundary-layer treatments such as suction, blowing, split blades, tandem blades, and wall treatment. Both areas are currently under investigation in various research establishments.

A logical extension of the present method is to generate directly from the computer program a paper tape to be used in a numerically controlled five-axis milling machine for manufacturing impellers which cannot be easily cast at present (i.e., titanium).

LIST OF SYMBOLS

A	Angle between meridional velocity and impeller axis, deg
C	Absolute velocity, ft/sec
$\mathbf{l}$	Vector tangent to line element, ft
$\dot{m}$	Mass flow rate, lbm/sec
m	Streamline meridional distance, ft
n	Distance along normal to meridional streamline (positive in direction of increasing radius), ft
r	Radius, ft
s	Camber-line distance, ft
$\mathbf{t}$	Vector tangent to streamline, ft
t	Blade thickness, ft
W	Relative velocity, ft/sec
Z	Number of blades
z	Axial distance, ft
α	Angle of blade surface orientation, deg
β	Relative angle from meridional plane (positive in direction of wheel rotation); when overscored ($\bar{\beta}$), signifies flow angle, deg
θ	Cylindrical coordinate angle around z-axis (positive in direction of rotor rotation), deg
ρ	Density, lbm/ft^3
ω	Angular velocity, rad/sec

Subscripts

1	Impeller inlet
2	Impeller exit
d	Pressure surface
h	Hub
m	Meridional value
m,m	Mid-passage meridional value
r	Radial direction
s	Shroud
t	Suction surface
z	Axial direction
θ	Tangential direction

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DISCUSSION

F. DALLENBACH (Consultant, U.S. Army): The authors present a method whereby the blade shape may be determined by specifying the blade loading distribution along the flow path. Starting with a prescribed meridional shape, the meridional velocities are computed along the shroud and hub by assuming that zero vorticity exists along the streamline normals and that the quantity $\rho C_m r$ varies linearly with normal length between hub and shroud. These assumptions are incorrect, since the vorticity along the normals in general is not zero but varies with the normal distance and the normal gradient of the quantity $\rho C_m r$ is not constant. The resulting meridional velocities must therefore be considered quite approximate and, in fact, may be in considerable error from the correct values.

The relative flow angle β is computed by prescribing the blade loading along a particular streamline as a function of the camber-line length s. The angular momentum rC_θ is then readily calculated. However, until the relationship between the camber-line length s and the meridional length m is established, the angle β cannot be determined.

The solution of the camber-line length as a function of the meridional length leads to the solution of a nonlinear differential equation in which the overall camber-line length is a variable. The final solution which may be obtained by numerical methods must satisfy the given overall meridional length. This procedure is very time consuming.

If, instead of specifying the blade loading distribution along the camber-line length, the angular momentum rC_θ is specified along the meridional length, the angle β may be directly computed and the blade-to-blade velocities readily determined along the meridional and camber-line lengths. This procedure requires little time and affords a ready means of adjusting the blade loading in the event the first trial does not give the desired result.

By specifying blade loading distributions along the shroud and hub contours, the total angular displacement between inlet and exit is determined from the distribution of the blade angle (not the flow angle). As pointed out by the authors, the total angular displacement for the shroud and hub should be essentially the same in order that the leading edge should be a straight radial line. The problem of specifying blade loading distributions for both the shroud and hub contours for backward curved impellers is that the angular displacements of the shroud and hub are, in general, not equal. The authors recognize this and point out that adjustments of the shroud and hub blade loadings are required to satisfy equal angular displacement but they do not treat in any detail what compromises are necessary.

The authors' proposal to generate the impeller blade surfaces by straight lines connecting the hub and shroud is quite practical and, indeed, is being done today where such impellers are machined on a five-axis milling machine.

The authors' discussion of the impeller design philosophy is very well presented and they are to be commended for presenting this paper covering a new design approach for centrifugal impellers.

D. G. WILSON (Massachusetts Institute of Technology): This is a request for further information about the actual boundary-layer behavior on the tested impellers. It is stated that measurements of pressure distributions have shown good agreement with predictions, and this agreement is very heartening for proponents of analytical design methods. It would be very helpful to be shown the measurements.

In particular, such a demonstration of success would help to quiet a strong criticism of theoretical methods of design of centrifugal compressors. It is easy to show that, unless the radial velocity at outlet is made unacceptably high, the mean velocity through a centrifugal compressor with near-radial blades must undergo a deceleration which in two-dimensional flow would result in separation. Conditions must therefore be worse for the pressure surface than for the mean, and worse at the hub than at the shroud. Figures 7 and 8 confirm these rather obvious statements. But flow analysis by these methods is useless once separation has taken place. Do the authors agree? Does separation take place near enough to the impeller outlet that neither the work output (slip factor) nor the impeller efficiency is greatly affected? Even so, the pressure distributions would be strongly, if locally, affected. Or do the secondary flows, instead of causing earlier separation as one might imagine, thin down the important parts of the boundary layer and concentrate all the low-momentum fluid in one noncritical location?

I believe that the significance of these questions is that, although many previous tests on centrifugal compressors using flow visualization of various types have shown serious flow breakdown which has often been ascribed to secondary flows, none of these impellers, to my knowledge, was designed in the first place to have the kind of controlled velocity distributions which it has been the aim of the authors to produce. They are to be congratulated on a stimulating paper.

H. D. LINHARDT (Airco Cryogenics): The authors are to be congratulated for reporting on their continued effort in the development of aerodynamic impeller design methods for centrifugal compressors. Since the complex flow field of centrifugal and, in particular, high pressure ratio centrifugal compressors is not completely understood yet and does not lend itself to rigorous analytical treatment, approximate solutions and/or analysis of assumed flow fields are being utilized for most compressor designs. The authors' suggested loading technique, which lacks experimental foundation and physical understanding of the impeller flow, falls into the latter category of approximate solutions. The application to practice of the authors' method requires a large and complicated set of calculation steps (the authors neglected to present and discuss the equations) which can only be handled by a computer with a large storage capacity. I am sure continued experimental and analytical effort will eventually refine the proposed technique, thus providing a reasonable design of centrifugal compressors of moderate or medium pressure ratio.

Considering industrial applications, one should not forget or underestimate the classic hydraulic approach as outlined in reference D–1. The advantage of the "Conformal Transformation Technique" is that the designer can prescribe the blade loading according to his personal understanding of the anticipated flow and his experience with manufacturing of similar machines. In addition, a clear picture of the impeller passage can be readily obtained, which allows application of basic diffusion and boundary layer stability analysis. The conformal transformation or geometric technique has been applied successfully in the past to industrial pumps and rocket type turbopumps and presents a good and rapid design tool for custom-engineered industrial process compressors. As a matter of fact, the conformal transformation technique has recently been computerized at Airco and successfully applied to medium and high pressure ratio industrial compressors. For this purpose the computer-generated blade coordinates were punched on a paper tape which has been utilized for automatic machining of impeller passages with a tape controlled machine. For example, figure D–1 shows a high specific speed, mixed flow, 2:1 pressure ratio, 5000-horsepower process compressor that achieved 86-percent efficiency at design point and exhibited a wide flow range due to the 35° back-swept design.

JANSEN AND KIRSCHNER (authors): Mr. Dallenbach raised three points, and they are discussed below.

The meridional velocities calculated by assuming zero vorticity are those used as the first values in an iterative scheme. As is pointed out in the text, subsequent velocities in the iteration are determined by a detailed flow analysis of the type described in references 3, 4, 5, or 6. We agree that the initial estimates are quite approximate, but the final velocities are accurate.

We agree that the procedure for loading the blade along the camber line is very time-consuming, since again an iterative scheme is required.

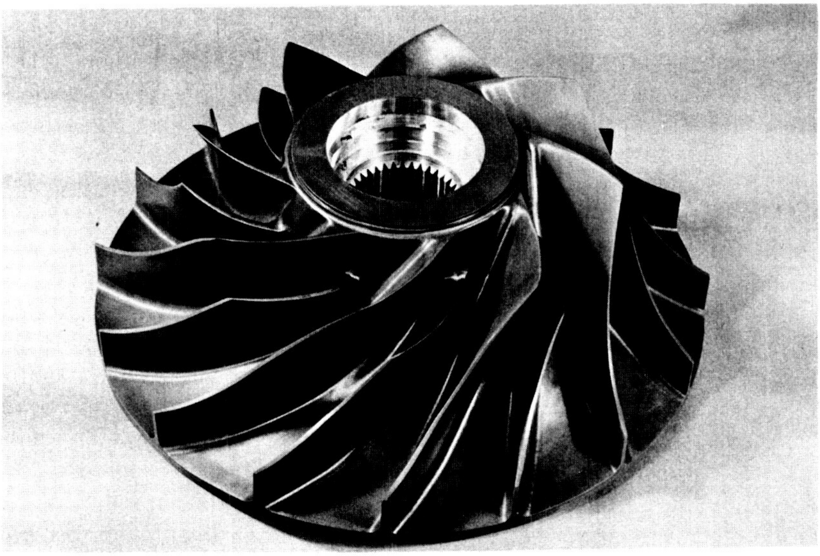

FIGURE D-1.—*5000-horsepower compressor wheel.* (*Airco Cryogenics*)

However, the velocity distribution along the blade length (and not that along the meridional length) is the item to be controlled, since it governs the boundary-layer characteristics and secondary flows. Thus, a technically superior procedure was preferred over a method which yielded a shorter computer running time.

The compromise which has to be accepted when the blade orientation (α_θ and α_z) is specified is that the velocity distribution along the hub and an intermediate streamtube cannot be specified. However, the shroud streamtube velocity distribution should still be specified.

Professor Wilson asks valid questions about the applicability of theoretical methods to centrifugal impeller design. What the present paper attempts is to bridge the gap between research and design. From research we know that boundary-layer separation more than anything else will generate large losses. Also from research we have identified the significant flow mechanisms that govern separation in impellers. Each of these flow mechanisms may be controlled by manipulation of the flow field and this paper provides a procedure by which this may be accomplished and which yields a producible impeller. It is true that more research is needed to associate one flow field with a superior impeller performance, but current work at various establishments, including the authors' own firm, has uncovered some definite trends in velocity distributions (in addition to those associated with limiting the inlet to discharge relative velocity ratio) which improve impeller performance.

The usefulness of the procedure is further exemplified by considering the correlation between calculated and measured results. Figures D–2 and D–3 show the calculated and superposed measured velocities along the shroud streamtube for an impeller with 50° backslope generating a pressure ratio of 2.4 at an rpm of 28 000. The impellers for figures D–2 and D–3 are similar in all aspects, except that the exit width for the case of figure D–3 is 13 percent less than that for figure D–2. The measured data shown are reduced from measurements of average static pressures and dynamic pressures taken along the shroud, together with total pressure traverses at the impeller discharge. The agreement between calculated and measured data is reasonable in the initial 60 percent of the blade passage. After this point, the measured velocities are higher, which indicates that the flow is separated. The actual passage flow is compared with the calculated flow in figure D–4. By prescribing a more reasonable

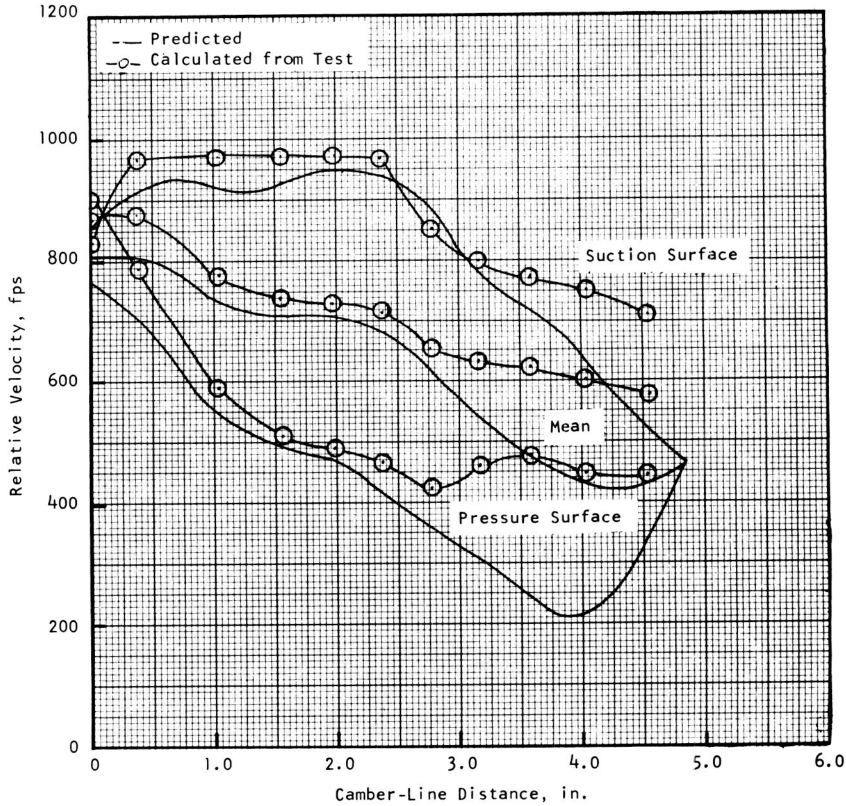

FIGURE D–2.—*Comparison of calculated and measured blade-surface-velocity distribution along the shroud for 50-degree backslope impeller (separation starts at a camber-line distance of 2.9 inches).*

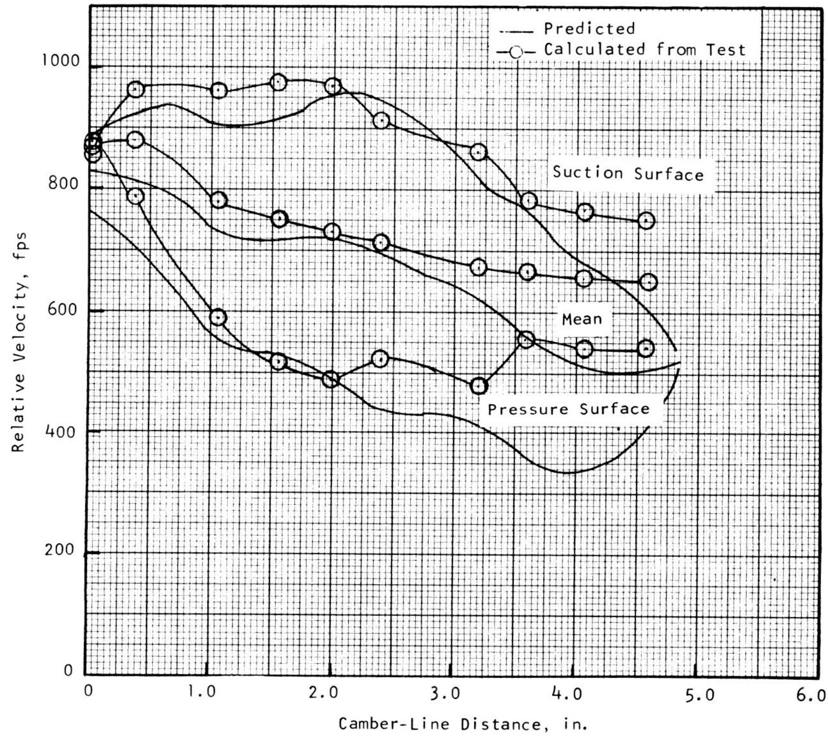

FIGURE D–3.—*Comparison of calculated and measured blade-surface-velocity distribution along the shroud for 50-degree backslope impeller (separation starts at camber-line distance of 3.2 inches; impeller width is 0.87 times that of fig. D–1).*

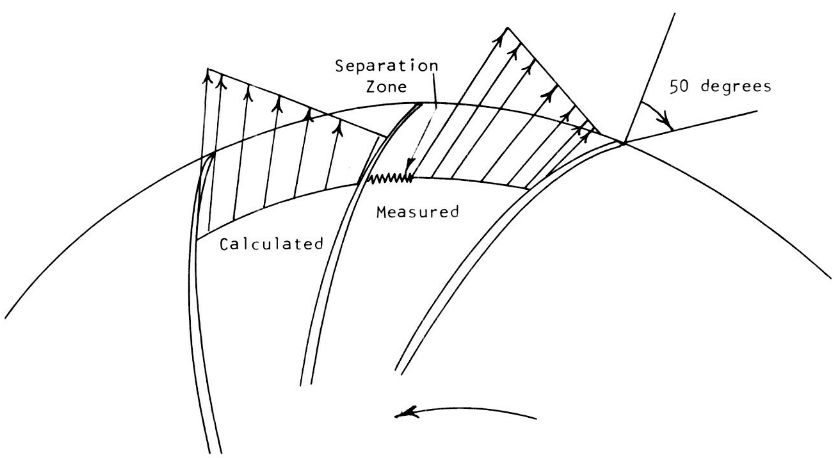

FIGURE D–4.—*Comparison of calculated and measured impeller-passage velocities (flow is separated in actual case; this figure complements fig. D–1).*

velocity distribution, a new impeller can now be designed with the procedure. The occurrence of separation can thus be delayed or eliminated in the new impeller, ensuring higher efficiencies.

The writers appreciate Dr. Linhardt's comments and interpret his discussion as a brief outline of an alternative blade design method.

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