## TOPICAL REPORT

## ACOUSTIC ATTENUATION ANALYSIS PROGRAM FOR DUCTS WITH MEAN FLOW



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NASA-LEWIS RESEARCH CENTER CLEVELAND, OHIO
H. BLOOMER, PROJECT MANAGER
 V/STOL AND NOISE DIVISION
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## FOREWORD

The work described herein was done by The Boeing Company, Wichita Division, under NASA contract NAS 3-14321 with Mr. H. Bloomer, V/STOL and Noise Division, NASA-Lewis Pesearch Center, as Project Manager.


#### Abstract

A computerized acoustic attenuation prediction procedure has been developed to evaluate acoustically lined ducts for various geometric and environmental parameters. The analysis procedure is based on solutions to the acoustic wave equation, assuming uniform airflow on a duct cross section, combined with appropriate mathematical lining impedance models. The impedance models included in the analysis procedure are representative of either perforated sheet or porous polyimide impregnated fiberglass facing sheet coupled with a cellular backing space. Advantages and limitations of the analysis procedure are reviewed.


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## NOMENCLATURE

c
d
$f$
$h_{x}, h_{y}$
k
$k_{x}, k_{y}, k_{z}$
$k_{r}, m, k_{z}$
n
p
t
r, $\theta, z$
$x, y, z$
$p, q, r, s$
$J_{\nu}, Y_{\nu}$
$\mathrm{H}_{\nu}$ (1), $\mathrm{H}_{\nu}$
L
L

M

R
U
V
Z

Velocity of sound, $\mathrm{cm} / \mathrm{sec}$
Acoustic lining backing depth, cm
Frequency, Hz
Rectangular duct cross section dimension, cm
$k=\omega / \mathrm{c} \quad 1 / \mathrm{cm}$
Rectangular duct wave numbers
Annular duct wave numbers
Hard wall mode number
Acoustic pressure, dynes $/ \mathrm{cm}^{2}$
Time, sec
Cylindrical coordinates
Rectangular coordinates
Cross products of Bessel functions
Bessel functions, first and second kind, respectively, of order $\nu$
Hankel functions of order $\nu$, Type 1 and Type 2
Lining treatment length, cm
$\bar{L}=1 / 2$
Mach number
Attenuation rate, $\mathrm{db} / \mathrm{cm}, \mathrm{db} /$ inch
Duct mean airflow velocity, $\mathrm{cm} / \mathrm{sec}$
Acoustic particle velocity, $\mathrm{cm} / \mathrm{sec}$
Specific acoustic impedance c.g.s. rayls
Particle displacement, cm
Bessel and Hankel function order
Density, $\mathrm{gm} / \mathrm{cm}^{3}$
Angular frequency, radians/sec
Viscosity (poise)

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5 Input Format

A systematic acoustic lining design procedure was developed for the NASA Fan Noise Suppression Program. This design procedure consists of several computerized modules, including the Acoustic Attenuation Analysis Program. The analysis procedure is based on solutions to the acoustic wave equation, combined with appropriate mathematical lining impedance models, when all the modes that can propagate are considered at the duct entrance with equal amplitude.
The analytical model is that of a semi-infinite duct with airflow. The airflow is assumed inviscid and nonturbulent, with a uniform velocity on a duct cross section. For the rectangular duct case, one wall or two opposing walls may be acoustically treated. The trw walisined case is restricted to the walls having the same impedance. The environmental parameters considered are temperature, pressure, Mach number, and the acoustic spectrum within the duct.
The impedance models included in the analysis procedure are representative of either perforated sheet or porous polyimide impregnated fiberglass facing sheet coupled with a cellular backing space. These models consist of a mathematical description of the porous facing sheet acoustic impedance dependence on material porosity characteristics, temperature, pressure, and particle velocity effects.

The basic analysis determines the modal propagation constants within a given duct increment as a function of duct geometry, environment, and lining impedance. The overall analysis procedure involves applying the basic analysis to sequential lining increments to establish the acoustic performance of the total duct. Variations in Mach number, lining impedance, and geometry are permissible on an incremental basis. However, the analytical procedure does neglect the effects of wave reflection resulting from geometric changes and/or lining impedance discontinuities.

A detailed description of the derivation and corresponding application of this technology are presented in this report.

INTRODUCTION

### 1.1 Background

A systems acoustic lining design procedure was developed for the NASA Fan Noise Suppression Program. This design procedure consists of several computerized modules, including the Acoustic Attenuation Analysis Program. The program combines the basics of two technologies, acoustic propagated wave analysis and material acoustic impedance modeling, into an acoustic lined duct performance simulation.

The zero airflow work of Cremer (Reference 1) and the mean flow results of Eversman (Reference 2) were incorporated into an early version of this program, using a rudimentary impedance model for perforated sheet impedance (Reference 3). This version was utilized in the NASA Langley Treated Tailpipe Program (Reference 4). More recent studies at Boeing-Wichita (Reference 5) have led to improvements in the impedance model for perforated sheet and development of an impedance model for polyimide liners. A portion of the experimental impedance data for these models was included in work reported by Rice (Reference 6). These models have been incorporated along with expanded wave analysis applications, into the current version of the program. A detailed description of the analytical techniques is presented in the following sections.

### 1.2 Technical Approach

The technical approach used in the development of this analytical program is the solution of the mean flow acoustic duct wave equation. The duct is assumed to be infinite in length with one or two walls lined. A rectangular duct is assumed for computational purposes, and annular geometry applications are treated by approximating a sector of the annulus by a rectangle. Normally reacting duct walls are assumed for lined surfaces. Variations in Mach number, wall impedance, and duct dimensions in the direction of airflow are treated approximately, and acoustic wave reflections due to these variations are neglected. Wall impedances are provided by semiempirical impedance models for perforated plate and polyimide facing sheets with cellular air backing cavities.

The computer application of the acoustic wave equation solution to the acoustic lined duct performance simulation is shown in the block diagram of Figure (1). The major components of this program will be discussed in detail in the following sections.

## ANALYTICAL MODEL

The analytical model for this study is that of a semi-infinite duct with airflow. The airflow is assumed inviscid and nonturbulent, with a uniform axial velocity $U$ on a duct cross section. The acoustic pressure is assumed to be small in magnitude in comparison with the uniform pressure.

For a duct with the flow axis in the $\mathbf{Z}$ coordinate direction, the acoustic wave equation is

$$
\begin{equation*}
c^{2} \nabla^{2} P=\left(\frac{\partial}{\partial t}+U \frac{\partial}{\partial z}\right)^{2} P \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathbf{c}=\text { velocity of sound } \\
& \mathrm{p}=\text { acoustic pressure } \\
& \mathbf{U}=\text { uniform airflow velocity }
\end{aligned}
$$

The solution and boundary condition analysis for a rectangular duct, as depicted in Figure 2, follows.

An assumed solution to (1) is

$$
\begin{equation*}
P=e^{i \omega \dagger} e^{-i k_{z} z}\left[A e^{i k_{x} x}+B e^{-i k_{x} x}\right]\left[C e^{i k_{y} y}+D e^{-i k y y}\right] \tag{2}
\end{equation*}
$$

Substitution of this solution into (1) results in the following relationship between the wave numbers $k, k_{x}, k_{y}$ and $k_{z}$;

$$
\begin{equation*}
k_{z}=\left(\frac{1}{1-M^{2}}\right)\left(-k M \pm \sqrt{k^{2}-\left(1-M^{2}\right)\left(k_{x}^{2}+k_{y}^{2}\right)}\right) \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
& \omega=\text { angular frequency }- \text { radians } / \mathrm{sec} \\
& k=\omega / \mathrm{c} \\
& M=U / \mathrm{c}
\end{aligned}
$$

The expressions for $k_{x}$ and $k_{y}$ are determined from boundary conditions at the duct walls. These conditions are obtained from the assumption of continuity of particle displacement at the walls. Let $\xi_{1}$ be the particle displacement normal to the wall at $X=$ $h_{x}$. The equation of motion for fluid flow at $X=h_{x}$ in this direction is

$$
\begin{equation*}
\rho_{1}\left(\frac{D}{D t}\left(\xi_{1}\right)\right)^{2}=-\left(\frac{\partial P}{\partial x}\right)_{x=h_{x}} \tag{4}
\end{equation*}
$$

where
$\rho=$ air density $\mathrm{gm} / \mathrm{cm}^{3}$ and ( $\mathrm{D} / \mathrm{Dt}$ ) is the total derivative with respect to time ( t )

$$
\frac{D}{D t}=\frac{\partial}{\partial t}+\frac{\partial x}{\partial t} \frac{\partial}{\partial x}+\frac{\partial y}{\partial t} \frac{\partial}{\partial y}+\frac{\partial z}{\partial t} \frac{\partial}{\partial z}
$$

For the wall at $x=0,(4)$ becomes

$$
\begin{equation*}
\rho\left(\frac{D}{D t}\left(\xi_{2}\right)\right)^{2}=\left(\frac{\partial P}{\partial x}\right)_{x=0} \tag{5}
\end{equation*}
$$

where $\xi_{2}$ is the particle displacement normal to the wall at $x=0$. The difference in sign between (4) and (5) is due to the change in direstion of the normal to the wall with respect to the $\times$ coordinate axis.

The mean velocity components in the $x$ and $y$ direction are zero, thus (4) and (5) become, respectively

$$
\begin{align*}
& {\left[\rho\left(\frac{\partial}{\partial t}+U \frac{\partial}{\partial z}\right)^{2} \xi 1=-\frac{\partial p}{\partial x}\right]_{X=h_{x}}}  \tag{6}\\
& {\left[\rho\left(\frac{\partial}{\partial t}+U \frac{\partial}{\partial z}\right)^{2} \xi 2=\frac{\partial p}{\partial x}\right]_{X=0}} \tag{7}
\end{align*}
$$

The acoustic impedance ( $Z / \rho \mathrm{c}$ ) and admittance ( $\overline{\mathrm{L}}$ ) may be defined by

$$
\begin{align*}
& {\left[\frac{1}{\rho C} \frac{P}{\frac{\partial \xi}{\partial t}}=\frac{z}{\rho C}=\frac{1}{\bar{L}}\right]_{x=x_{n}}}  \tag{8}\\
& {\left[\frac{1}{\rho C} \frac{P}{\frac{\partial \xi}{\partial t}}=-\frac{z}{\rho C}=-\frac{1}{\bar{L}}\right]_{x=0}} \tag{9}
\end{align*}
$$

For harmonic motion

$$
\frac{\partial}{\partial t}(\xi)=i \omega \xi
$$

and from (2)

$$
\frac{\partial}{\partial z} P=-i k_{z} P
$$

applying these relations to (6), (7), (8) and (9) we obtain the boundary value equations

$$
\begin{align*}
& {\left[i \omega \bar{L}\left(1-M k_{z} / k\right)^{2} p=\frac{-\partial P}{\partial X}\right]_{X=h_{X}}}  \tag{10}\\
& {\left[i \omega \bar{L}\left(1-M k_{z} / k\right)^{2} P=\frac{\partial P}{\partial X}\right]_{X=0}} \tag{11}
\end{align*}
$$

application of the boundary condition (10) and (11) to a duct with one lined wall at $X=$
$h_{x}$ leads to

$$
\text { at } \begin{aligned}
y & =0: \bar{L}=0 \text { and } \frac{\partial P}{\partial y}=0 \\
y & =h_{y}: \bar{L}=0 \text { and } \frac{\partial P}{\partial y}=0 \\
x & =0: \bar{L}=0 \text { and } \frac{\partial p}{\partial x}=0 \\
\text { at } x & =h_{x}: \bar{L}=\overline{L_{1}}
\end{aligned}
$$

and
$i L_{1} k\left(1-\frac{M k_{z}}{k}\right)^{2} P=-\frac{\partial P}{\partial X}$
for the $y$ direction we obtain

$$
\operatorname{SiN}\left(k_{y} h_{y}\right)=0
$$

or

$$
\begin{equation*}
k_{y}=\frac{n \pi}{h_{y}} ; n=0,1,2, \ldots \tag{12}
\end{equation*}
$$

for the $x$ direction

$$
\begin{aligned}
& i L_{1} k\left(1-\frac{M k_{z}}{k}\right)^{2} \cos \left(k_{x} h_{x}\right)=-k_{x} \operatorname{SiN}\left(k_{x} h_{x}\right) \\
& i L_{-} k h_{x}=h_{x} k_{x} \operatorname{TAN}\left(h_{x} k_{x}\right) /\left(1-\frac{M k_{z}}{k}\right)^{2}
\end{aligned}
$$

In the case of a duct with equal admittance walls at both $x=0$ and $x=h_{x}$, we obtail $x$ boundary equations in the following manner from (2), (10), and (11),

$$
\begin{aligned}
& i L_{1} k\left(1-\frac{M k_{z}}{k}\right)^{2}\left[A e^{i k_{x} h_{x}}+B e^{-i k_{x} h_{x}}\right] \\
&=-i k_{x}\left[A e^{i k_{x} h_{x}}-B e^{-i k_{x} h_{x}}\right]
\end{aligned}
$$

$$
\begin{array}{r}
i L_{1} k\left(1-\frac{M k_{z}}{k}\right)^{2}[A+B] \\
=i k_{x}[A-B]
\end{array}
$$

where $L_{1}$ is the admittance at $x=0$ and $x=h_{x}$
These two relations may be combined to eliminate $A$ and $B$, resulting in a qua ( $\mathrm{i} \mathrm{L}_{\mathrm{j}} \mathrm{kh}$ ) :

$$
\begin{aligned}
& h_{x}^{2}\left(i L_{\mid} k\right)^{2}\left(1-\frac{M k_{z}}{k}\right)^{4} \sin \left(h_{x} k_{x}\right) \\
& +2 i L_{1} h_{x} k\left(1-\frac{M k_{z}}{k}\right)^{2} h_{x} k_{x} \cos k_{x} h_{x}-h_{x}{ }^{2} k_{x}{ }^{2} \operatorname{Sin} k_{x} h_{x} \\
& =0
\end{aligned}
$$

or

$$
i L_{1} h_{x} k=\frac{-h_{x} k_{x}\left(\cos \left(h_{x} k_{x}\right) \pm 1\right) / \sin \left(k_{x} h_{x}\right)}{\left(1-M k_{z} / k\right)^{2}}
$$

using half angle trigonometric identities (17) becomes

$$
\text { for the " }+ \text { " sign }
$$

$$
i L_{1} k h_{x} / 2=\frac{-\left(h_{x} k_{x} / 2\right) \operatorname{CoT}\left(h_{x} k_{x} / 2\right)}{\left(1-M k_{z} / k\right)^{2}}
$$

for the "-" sign

$$
\begin{equation*}
i L_{1} k h_{x} / 2=\frac{\left(h_{x} k_{x} / 2\right) \operatorname{TAN}\left(h_{x} k_{x} / 2\right)}{\left(1-M k_{z} / k\right)^{2}} \tag{19}
\end{equation*}
$$

### 3.0 METHOD OF APPLICATION

The acoustic attenuation prediction program provides an analytical evaluation of the attenuation characteristics of an acoustically treated rectangular duct. Lining configurations may be either a single treated wall or two opposite walls with identical treatment. In addition, propagation of the acoustic pressure wave may be in opposition to the flow or in the same direction as the flow.

### 3.1 General Description

The analytical acoustic attenuation prediction program evaluates the attenuation characteristics of rectangular ducts with mean airflow and acoustically lined walls, having either one or two walls treated.

The basic analysis consists of determining the acoustic modal propagation constants for a given duct geometry, environment, and lined wall impedance. The environmental parameters considered are temperature, pressure, Mach number, and duct acoustic source spectrum. The lined wall impedance is obtained from acoustic impedance mathematical models, which is discussed in Section 3.2

Initially, the input acoustic source spectrum is subdivided at each frequency into modal pressures. The wall impedance at each frequency is then determined for the first lining increment, and the boundary value problem solutions are obtained. The resulting modal attenuations are now applied to the modal pressures, and the corresponding attenuated acoustic spectrum is calculated.

This process is repeated for each lining increment until the entire lined length has been analyzed. However, the source spectrum is subdivided into modal pressures for the first increment only, with each succeeding incremental treatment utilizing the attenuated modal pressure spectrum resulting from the preceding increment. Also, the resultant acoustic spectrum at each increment is used for impedance determination in the next increment.

### 3.2 Impedance Models

The impedance models in the present program include mathematical representations for both perforated plate and porous polyimide facing sheets with cellular air backings terminated by an impervious sheet (Figure 3).

These models consist of a mathematical description of the impedance of the porous face sheet dependence on material porosity characteristics, temperature, pressure, and total particle velocity. The total particle velocity $V_{T}$ is a combination of particle velocities resulting from both grazing airflow, $\mathrm{V}_{\mathrm{gf}}$, at the face sheet, and acoustic pressure excitation $\mathrm{V}_{\mathrm{a}}$ :

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The explicit expression for the complex acoustic face sheet impedance of polyimide and perforated sheet, including expressions for the respective grazing flow particle velocities, are as follows:

$$
\begin{align*}
& \text { Polyimide } \\
& Z=Q+.305 \mu N^{2} f+2.86 \rho N^{2} V p+i\left\{.549 \rho N^{3 / 2} f \cdot 5.12 \cdot 10^{-5} \rho N^{3} f \vee p\right\}(21) \\
& V_{g f}=\frac{.05 \mathrm{cM}^{2}}{\left[0 / \rho \mathrm{c}+\sqrt{(\mathrm{Q} / \rho \mathrm{c})^{2}+.71} \mathrm{~N}^{2} \mathrm{M}^{2}\right]}  \tag{22}\\
& \text { where } Q=4640 . \mu N^{4 / 3} \\
& t=\text { viscosity (Poise) } \\
& i=\sqrt{-1} \\
& c=\text { velocity of sound in air ( } \mathrm{cm} / \mathrm{SEC} \text { ) } \\
& N=\text { number of plies of polyimide } \\
& \rho=\text { density of air ( } \mathrm{gm} / \mathrm{cm}^{3} \text { ) } \\
& V_{p}=\text { particle velocity } \\
& f=\text { frequency }
\end{align*}
$$

Perforated Sheet

$$
\begin{gathered}
\mathrm{Z} / \rho \mathrm{c}=0+\frac{3.83 \cdot 10^{-5} \theta \cdot 75 \sqrt{\mathrm{f}}}{P_{\mathrm{OA}} \sqrt{\delta(\theta+.416)}}\left[\mathrm{T} / \mathrm{D}+1 \cdot \mathrm{P}_{\mathrm{OA}}\right]+\pi / 2 \sqrt{2} \mathrm{Vp/c} \mathrm{Ee}^{-1.8 \mathrm{~S} \eta} \\
\\
+i\left\{\frac{4.69 \cdot 10^{-4} \mathrm{f}}{\mathrm{P}_{\mathrm{OA}} \sqrt{\theta}}(\mathrm{~T}+\Delta)\right\} \\
\mathrm{V}_{\mathrm{gf}}=\frac{.25 \mathrm{KcM}^{2}}{\mathrm{Q} / 2+\sqrt{(\mathrm{O} / 2)^{2}+\mathrm{K} / 2 \mathrm{EM}^{2}}}
\end{gathered}
$$

$$
\begin{aligned}
& \text { where } \Delta=\left\{\begin{array}{c}
\Delta_{1} ; \Delta_{1}>0 \\
0 ; \Delta_{1} \leq 0
\end{array}\right\} \\
& \Delta_{1}=.850\left(1-.7 \sqrt{\left.P_{O A}\right)}-\frac{1.2 \cdot 10^{-5}}{P_{O A}} V_{P}\right. \\
& f=\text { frequency }\left(\mathrm{H}_{\mathrm{z}}\right) \\
& S_{\eta}=\frac{2.54 D+P_{\mathrm{OA}}}{V p} \\
& T=\text { sheet thickness (inches) (cm.) } \\
& D=\text { perforate hole diameter (inches) (cm.) } \\
& \left.\theta=T_{T} \text { (total temperature }{ }^{\circ}{ }_{\mathrm{R}}\right) / 519\left(=\mathrm{T}_{\mathrm{T}}\left({ }^{\circ} \mathrm{K}\right) / 288.33\right) \\
& \delta=\frac{\text { static pressure (PSIA) }}{14.7}=\left(\frac{\text { static pressure }\left(\text { dynes } / \mathrm{cm}^{2}\right)}{\left(1.013 \times 10^{6}\right)}\right) \\
& 0=\frac{.077 T}{\delta P_{O A}}\left(\frac{\theta 2}{\theta+.416}\right) \\
& E=1.0251\left(1 / P_{O A}{ }^{2-1}\right)\left(1 / P_{O A}\right) \cdot{ }^{1} \mathrm{e}-.5072 \mathrm{~T} / \mathrm{D} \\
& K=.05+.11 \mathrm{D} / \theta \eta \\
& \theta \eta=\text { boundary layer momentum thickness (inches) (cm.) } \\
& V_{P}=\text { particle velocity }
\end{aligned}
$$

These face sheet impedance models are described in greater detail in Reference (5).

The face sheet impedance $Z_{F}$ is modeled as a function of total particle velocity, and the wall impedance, $Z_{w}$, is given by

$$
Z_{w}=Z_{F}-i \cot (k d)
$$

where $d$ is the depth of the air backing cavity.
The acoustic pressure particle velocity due to the acoustic pressure at a single frequency $P_{i}$ is given by

$$
\begin{equation*}
v_{a i}=P_{i} / Z_{w} \tag{25}
\end{equation*}
$$

Particle velocities and impedances are calculated assuming that the acoustic pressure particle velocity impedance dependence is based on either the acoustic pressure at single frequencies or the combined effects of the acoustic pressure at all frequencies.

In the first case, the expression for total particle velocity calculation is

$$
\begin{equation*}
v_{T}(f)=\left(V_{g f}^{2}+\left(\frac{P(f)}{\mid Z_{w}\left(f, V_{T}(f) \mid\right.}\right)\right)^{1 / 2} \tag{26}
\end{equation*}
$$

where

$$
f=\text { frequency }
$$

$V_{T}(f)=$ total particle velocity at frequency $f$
$P(f)=$ acoustic pressure at frequency $f$
$Z_{W}\left(f, V_{T} f\right)=$ wall impedance at frequency $f$ and particle velocity $V_{T}(f)$
In the second case, the total particle velocity is denoted $V_{T_{r m s}}$ to indicate the dependence on acoustic pressure at all frequencies, and the expression becomes

$$
\begin{align*}
& v_{T_{r m s}}=\left(v_{g f}{ }^{2}+\sum_{f} v_{a_{f}}^{2}\right)  \tag{27}\\
& v_{a_{f}}=\frac{P_{f}}{\left|Z_{w_{f}}\left(V_{T_{r m s}}\right)\right|}
\end{align*}
$$

where

These expressions are solved by iteration techniques based on the Newton Raphson method to obtain $V_{T}(f)$ or $V_{T_{r m s}}$ and the wall impedances.

### 3.3 Modal Pressure Subdivision and Recombination

The acoustic pressure at a given frequency is subdivided according to the number of modes which can propagate at that frequency. The cutoff frequency for any given mode number $n$ is given by $p=\frac{n c \sqrt{1-M \bar{M}}}{2 h y}$ for unlined ducts. The cutoff frequency is decreased for lined ducts, and a factor of 1.1 has been arbitrarily introduced in the denominator of the above equation to account for this effect.

The subdivision of acoustic pressure into modal pressure amplitudes is accomplished assuming the acoustic modes are uncorrelated and initially have equal modal pressure amplitudes. The total sound pressure level at any frequency ( SPL $_{\text {total }}$ ) is given in terms of the total acoustic pressure ( $\mathrm{P}_{\text {total }}$ ) by

$$
\begin{equation*}
S P L_{\text {total }}=10 \log _{10}\left(P_{\text {total }} / .00022\right)^{2} \tag{28}
\end{equation*}
$$

where $P_{\text {total }}$ is in dynes $/ \mathrm{cm}^{2}$
Defining $S P L_{i_{\text {modal }}}$ as the sound pressure level of the ith modal pressure amplitude, for n propagating modes the expression for modal amplitude subdivision is

$$
\begin{equation*}
S P L_{i_{\text {modal }}}=S P L_{\text {total }}-10 \log _{10} n \tag{29}
\end{equation*}
$$

The recombination of attenuated modal sound pressure levels to obtain a total attenuated SPL is given by

$$
\begin{equation*}
S P L_{\text {total }}=10 \log 10\left[\sum_{i=1}^{n} 10\left(S P L_{i_{\text {modal } / 10}}\right)\right] \tag{30}
\end{equation*}
$$

where all SPL's in this expression have been attenuated. The attenuation for any frequency is then the difference between the initial and attenuated values of $S P L_{\text {total }}$.

### 3.4 Boundary Value Solution Application

The boundary value problem solutions are obtained for each frequency of interest and for each increment of wall lining analyzed. These solutions are obtained by iterative procedures based on the Newton Raphson method. Rewriting the boundary value equation for the symmetric modes of a duct with two walls lined, equation (19) becomes

$$
\begin{equation*}
i L_{1} k h_{x} / 2=\left(h_{x} / 2\right) k_{x} \tan \left(h_{x} / 2\right) k_{x} /\left(1-M k_{z} / k\right)^{2} \tag{31}
\end{equation*}
$$

It is noted that the required solutions for physical wall impedances lie in the first quadrant of the complex plane $\left(h_{x} / 2\right) k_{x}$. (The third quadrant is a mirror image of the first, while the second and fourth quadrant corresponds to wall impedances whose real part is negative.)

The solution to (31) for zero Mach number and unlined walls are easily obtained real quantities.

The starting points for the iterative solutions for no-flow, lined ducts are chosen near the unlined wall solutions. Similarly, the lined wall duct with flow is solved using the lined wall no-flow solutions as starting points. However, the interative solution for the flow case must be accomplished in incremental Mach number steps to obtain all solutions of interest. The incrementation of Mach number by .05 steps appears to be sufficient for most applications.

In some cases, the iterative procedure crosses into the second or fourth quadrant of the complex plane. This effect is checked in the solution procedure, and countered by returning the iteration to a new starting point. This point is specified by selecting the point on a line segment connecting the non-first quadrant point to the last iteration point in the first quadrant.

### 3.5 Modal Ordering

The eigenvalue solutions are ordered according to increasing modal attenuation rate. This ordering scheme assures that the least attenuated mode is always considered. Under the above stated assumption of initially equal modal pressure amplitude, it can be seen that the least attenuated mode is the most significant contributor to the resultant sound pressure level after some length $L$ of treatment. Consider the simplified case of two modes with attenuation rates $R_{1}$ and $R_{2}$. If the final total sound pressure level is

$$
\begin{equation*}
S P L_{T}=10 \log _{10}\left(10 \mathrm{SPL}_{1} / 10+10 \mathrm{SPL} 2 / 10\right) \tag{32}
\end{equation*}
$$

where $S P L_{1}$ and $S P L_{2}$ are the sound pressure levels of modes 1 and 2, respectively, after length $L$. The difference, $\Delta$, between $S P L_{T}$ and $S L_{1}$ can be expressed as:

$$
\begin{equation*}
\Delta=10 \log _{10}\left(1+10^{\left.-\left(R_{2}-R_{1}\right) L / 10\right)}\right. \tag{33}
\end{equation*}
$$

From this representation, it can be seen that the maximum difference is 3 dB , which results if $R_{1}=R_{2}$ and that for $R_{2}=R_{1}+.58$, the difference becomes 1 dB .

### 3.6 Lining Length Incrementation

The analysis of a duct by incremental lengths is incorporated to account for effects of
changes in Mach number, duct, height, and lining characteristics with progressive position in the direction of acoustic propagation. These effects are treated by specification of the varying parameters for each increment analyzed. Acoustic wave reflection is not considered, and thus the analysis is applicable for small parameter variations.

### 4.0 PROGRAM APPLICATION AND LIMITATION

The assumptions of acoustic modal content, mean airflow, and neglection of acoustic wave reflections are the major limiting factors in the application of this program. Some of the pertinent characteristics of these assumptions as related to program application are as follows:
4.1 Modal Content

The equal modal pressure amplitude assumption is incorporated since experimental modal pressure amplitude definition is unavailable, and for ease of application. As definitive model content information becomes available, it can be incorporated in this program.

### 4.2 Duct Length Incrementation

The sensitivity of the impedance models to sound pressure level spectrum variation requires the analysis of an acoustic liner in several incremental lengths. A check on the validity of any increment length utilized may be accomplished by comparison of the attenuation rates of each lining segment; gross differences indicate the need for further subdivision of the increment length used.

A further consideration for increment selection is the degree of variation of the other environmental parameters with length, e.g., Mach number, geometry, etc. Variations in these parameters may require a further reduction in the increment length than for fixed parameters.

### 4.3 Multiple Lining Analysis

The treatment of cases having sequential segments of differing acoustic liners should be accomplished with one program execution in preference to an execution for each lining type. This approach maintains the continuity of modal pressure contributions for the entire lining treatment, conversely, an execution for each lining type would redistribute the modal pressures, and the final duct attenuations would be overestimated.

### 4.4 Application to Annular Geometry Ducts

The subject program can be utilized in the performance prediction of annular geometry ducts by approximating a section of the duct annulus with a rectangle. Details of this type of application are treated in Appendix I.
4.5 Application to Ducts with a Sheared Flow

The applicability of the subject program to ducts with a sheared flow (nonuniform
velocity profile on a duct cross section) depends upon the relative direction of the acoustic wave propagation and the airflow, and upon the magnitude of the boundary layer thickness and Mach number. Using the nomenclature "inlet mode" and "exhaust mode" to denote cases in which the acoustic flow and airflow are in the opposing and identical direction, respectively, the work of Eversman (Reference 7) and Munger and Plumblee (Reference 8) indicate that the shear flow effects are generally small for the exhaust mode, but can be very significant in the inlet mode. Analytical methods have been developed to correct for the sheared flow effect, but have not been included in this document.

### 5.0 CONCLUDING REMARKS

The acoustic attenuation analysis program provides an analytical capability to evaluate lined duct configurations having uniform flow on a duct cross section. Variations in duct geometry, Mach number, and lining impedance are permissible within the duct. However, variations in these parameters are treated approximately in that acoustic wave reflection effects are neglected. Initial modal pressure amplitudes are assumed equal, modal interaction effects are neglected and modes are assumed to be in phase. Acoustic wall impedances are obtained from semiempirical impedance models.

The acoustic attenuation analysis program was developed primarily for application to aircraft fan jet engines. The major limitation is considered to be the assumption of mean airflow within the inlets of these engines. For this case, the program results should be corrected to account for the sheared flow effects to obtain a more accurate estimate of the acoustic attenuation.

Further analysis and subsequent program improvements are indicated in the following areas:

- Sheared flow effects
- Modal pressure amplitude definition
- Modal phasing and interaction
- Acoustic wave reflection effects

The first item requires a reformulation of the analytical procedure so that the sheared flow effects can be evaluated as an integral part of the analysis program. This modification would provide a more detailed and rigorous analysis than correcting the mean flow results.

The second and third items are contingent on having adequate space-time data from which modal amplitude and phase information can be extracted. These data are necessary to compute the initial modal energies and to track these energies down the treated duct.

The fourth item deals with the reflection of acoustic modes in the treated duct. These reflections can result from changes in the wall impedance and/or geometry. This problem can be approached using either assumed modal amplitudes and phase or when available the results of items two and three.



## RECTANGULAR DUCT GEOMETRY

FIGURE 2


ACOUSTIC LINING PANEL
FIGURE 3


## ANNULAR DUCT GEOMETRY FIGURE 4

FORMAT
72 Column Alphanumeric Field 8 Fields, 8 Columns each
72 Column Alphanumeric Field 9 Fields, 8 Columns each

VARIABLE NAMES
Title
A, SWM, FWM, H, PSPL, RMS, REORD, OVRMOD SPL NAM

SPL (1), SPL (2), . . . . . . . . . . . SPL (9)
SPL (10),

SPL (99)
TL, DL, DHH, G, TT, PT, A, B
ANPLY, AM, DHS, OPA NP, D, DIA, R, MTHK
AAM (1), AAM (2)

ADHS (1), ADHS (2) $\qquad$
$\square$
POAC (1), POAC (2) $\qquad$

DCON (1), DCON (2) $\qquad$

DIAC (1), DIAC (2) $\qquad$
.
-

THCK (1), THCK (2) $\qquad$

MTHK (1), MTHK (2)

## INPUT FORMAT

FIGURE 5

## APPENDIX I

## ANNULAR GEOMETRY ANALYTICAL MODEL, APPROXIMATION BY RECTANGULAR GEOMETRY

The annular geometry analysis follows that of the rectangular duct solution, using cylindrical coordinates in the wave equation solution. The duct geometry and coordinate system is shown in Figure 4. The acoustic wave equation is again

$$
\begin{equation*}
c^{2} \nabla^{2} p=\left(\frac{\partial}{\partial t}+U \frac{\partial}{\partial z}\right)^{2} p \tag{A-1}
\end{equation*}
$$

Separation of variables leads to the solution:

$$
\begin{equation*}
P=\left(A J_{m}\left(r k_{r}\right)+B Y_{m}\left(r k_{r}\right)\right) \cos (m \theta) e^{j}\left(\omega t-k_{z} Z\right) \tag{A-2}
\end{equation*}
$$

and

$$
\begin{equation*}
k_{z}=\left(\frac{1}{1-M^{2}}\right)\left(-M k \pm \sqrt{k^{2}-\left(1-M^{2}\right) k_{r}^{2}}\right) \tag{A-3}
\end{equation*}
$$

where $r, \theta, z$ are the system cylindrical coordinates

$$
\begin{aligned}
& k=\omega / c \\
& M=U / c \\
& m=\text { angular wave number }=0,1,2, \ldots \ldots \\
& J_{m}\left(r k_{r}\right)=\text { Bessel function, first kind, order } m \\
& Y_{m}\left(r k_{r}\right)=\text { Bessell function, second kind, order } m
\end{aligned}
$$

Both walls are assumed lined with identical wall admittance $\bar{\tau}$. The boundary condition at $r_{0}$, the outer annular wall, and $r_{i}$, the inner annular wall for this assumption become

$$
\begin{align*}
& \text { at } r_{i}: i k \bar{L}\left(1-M \frac{k_{z}}{k}\right)^{2} P=\frac{\partial P}{\partial r}  \tag{A-4}\\
& \text { at } r_{0}: i k-\left(1-M \frac{k_{z}}{k}\right)^{2} P=-\frac{\partial P}{\partial r} \tag{A-5}
\end{align*}
$$

Substitution of A-2 into A-4 and A-5 and solving, the following eigenvalue equation is obtained:

$$
\begin{equation*}
\left[(i k L)\left(1-M \frac{k_{2}}{k}\right)^{2}\right]^{2} P_{m}-(i k L)\left(1-M \frac{k_{z}}{k}\right)^{2}\left(a_{m}-r_{m}\right)-S_{m}=0 \tag{A-6}
\end{equation*}
$$

where

$$
\begin{aligned}
& a=r_{o} k_{r} \\
& b=r_{i} k_{r}
\end{aligned}
$$

and

$$
\begin{align*}
& R_{y}=J_{y}(a) Y_{y}(b)-J_{y}(b) Y_{y}(a)  \tag{A-7}\\
& a_{y}=\left(J_{y}(a) Y_{y}(b)-J_{y}^{\prime}(b) Y_{y}(a)\right) k_{r}  \tag{A-8}\\
& r_{y}=\left(J_{y}(a) Y_{y}(b)-J_{y}(b) Y_{y}(a)\right) k_{r}  \tag{A-9}\\
& S_{y}=\left(J y^{\prime}(a) Y_{y}^{\prime}(b)-J_{y}^{\prime}(b) Y_{y}{ }^{\prime}(a)\right) k_{r}^{2} \tag{A-10}
\end{align*}
$$

(Primes denote differentiation of the Bessel function with respect to the particular function argument.)

As in the rectangular case, A- 6 may be written as the quadratic solution

$$
\begin{equation*}
i k L=\frac{1}{2 P_{m}}\left\{\left(a_{m}-r_{m}\right) \pm \sqrt{\left(a_{m}-r_{m}\right)^{2}+4 P_{m} s_{m}}\right\} /\left(1-M \frac{k_{z}}{k}\right)^{2} \tag{A-11}
\end{equation*}
$$

The approximation of an annulus by a rectangle might be presumed, by inspection, to be valid whenever the radius ratio, $r_{i} / r_{0}$, is close to unity. This presumption may be mathematically verified in part by applying large argument Bessel function approximations to A-6. Taking the definition for Hankel functions in terms of Bessel functions:

$$
\begin{aligned}
& H_{y}{ }^{(1)}(Z)=J y(Z)+i Y y(Z) \\
& H_{y}^{(2)}(Z)=J_{y}(Z)-i Y y(Z)
\end{aligned}
$$

where $\mathrm{H}_{\gamma}{ }^{(1)}(Z)$ and $\mathrm{H}_{\gamma}{ }^{(2)}(Z)$ are Hankel functions, type 1 and type 2, respectively. The large where $\mathrm{H} \gamma \quad(\mathrm{Z})$ and $\mathrm{H} \gamma$
argument approximations for $\mathrm{H} \gamma$
$(\mathrm{Z})$${ }_{(\mathrm{Z}) \text { and } \mathrm{H} \gamma}{ }^{(2)}(\mathrm{Z})$ are (Reference 9 ):

$$
\begin{equation*}
H y^{(1)}(z) \sim \sqrt{2 / \pi z} e^{i\left(z-\frac{y_{\pi}}{2}-\frac{\pi}{4}\right)} \tag{A-12}
\end{equation*}
$$

$$
\begin{equation*}
H_{y}{ }^{(2)}(Z) \sim \sqrt{2 / \pi Z} e^{-i\left(z-\frac{\gamma_{\pi}}{2}-\frac{\pi}{4}\right)} \tag{A-13}
\end{equation*}
$$

The Bessel cross products $\mathrm{P} \gamma, \mathrm{a} \gamma, \mathrm{r} \gamma, \mathrm{S} \gamma, \mathrm{A}-7, \mathrm{~A}-8, \mathrm{~A}-9$ and $\mathrm{A}-10$ may be written in terms of Hankel function cross product

$$
\begin{align*}
& P_{y}=-\frac{1}{2 i}\left(H_{y}{ }^{(1)}(a) H_{y}{ }^{(2)}(b)-H_{y}{ }^{(1)}(b) H_{y}{ }^{(2)}(a)\right)  \tag{A-14}\\
& a_{y}=-\frac{k_{r}}{2 i}\left(H_{y}{ }^{(1)}(a) H_{y}{ }^{(2)^{\prime}}(b)-H_{y}{ }^{(1)^{\prime}(b) H_{y}(2)}(a)\right)  \tag{A-15}\\
& r_{y}=-\frac{k_{r}}{2 i}\left(H_{y}{ }^{(1)^{\prime}}(a) H_{y}{ }^{(2)}(b)-H_{y}{ }^{(1)}(b) H_{\gamma}{ }^{(2)^{\prime}}(a)\right)  \tag{A-16}\\
& S_{y}=-\frac{k_{r}{ }^{2}}{2 i}\left(H_{y}{ }^{(1)^{\prime}}(a) H_{y}{ }^{(2)^{\prime}}(b)-H_{y}{ }^{(1)^{\prime}}(b) H_{y}(2)^{\prime}(a)\right) \tag{A-17}
\end{align*}
$$

Applying A-12 and A-13 to the above yields:

$$
\begin{align*}
& P_{\gamma} \simeq \frac{-2}{\pi \sqrt{a b}} \sin (a-b)  \tag{A-18}\\
& a_{y} \simeq k_{r}\left(\frac{2}{\pi \sqrt{a b}} \cos (a-b)+\frac{y}{b} P_{y}\right)  \tag{A-19}\\
& r_{y} \simeq k_{r}\left(\frac{-2}{\pi a b} \cos (a-b)+\frac{\gamma}{b} P_{\gamma}\right)  \tag{A-20}\\
& s_{\gamma} \simeq-k_{r} 2\left(\left(\frac{\gamma^{2}}{a b}+1\right)\left(\frac{2}{\pi \sqrt{a b}}\right) \sin (a-b)+\left(\frac{y}{b}-\frac{y}{a}\right) \frac{2}{\pi \sqrt{a b}} \cos (a-b)\right) \tag{A-21}
\end{align*}
$$

Using the above relations in A-11 results in

$$
\begin{align*}
i L_{1} k \simeq & \frac{k_{r}}{2 \sin (a-b)}\left\{-2 \cos (a-b)-\left(\frac{\gamma}{a}-\frac{\gamma}{b}\right) \sin (a-b) \pm\right.  \tag{A-22}\\
& {\left.\left[4+\left(\frac{\gamma}{a}+\frac{\gamma}{b}\right) \sin ^{2}(a-b)+8\left(\frac{\gamma}{a}-\frac{\gamma}{b}\right) \cos (a-b) \sin (a-b)\right] 1 / 2\right\} }
\end{align*}
$$

noting ( $a-b)=h k_{r}$. where $h$ is the distance between lined walls for the annular duct. For the case
where $r=o$ (radial modes only), A-22 becomes

$$
\begin{equation*}
i L_{i} k h=n k r \frac{(\operatorname{Cos}(h k r) \pm 1)}{\operatorname{SiN}(h k r)} /\left(1-M \frac{k_{z}}{k}\right)^{2} \tag{A-23}
\end{equation*}
$$

which is identically the rectangular solution (Equation 17).
The use of the large argument approximation has two implications: first, for large radii and/or $\mathrm{k}_{\mathrm{i}}$, the rectangular approximation appears valid, and second, a further limitation on the relative magnitudes of $r_{0}$ and $r_{i}$ is implied. This limitation is that if $r_{0} k_{r}$ satisfies the large argument approximation, then $r_{i} / r_{o}$ should not be nearly zero, since the approximation is applied to both $r_{0} k_{r}$ and $r_{i} k_{r}$.

## APPENDIX II

## PROGRAM DESCRIPTION

This program consists of a mainline control program and subroutines. The mainline program calls subroutine "input" to obtain all input data and obtain preliminary processed data. The incrementation loop is then set up and the subroutine "ATNSPL" is called for each lining increment. "ATNSPL" serves as overall control for all calculations for each increment except for the calculation of an attenuation spectra at the current increment. A maximum of 50 increments may be treated.

## Subroutine "INPUT"

This subroutine reads all input data, calculates speed of sound, density, and the numbers of propagating modes for all frequencies, and model pressure spectra based on these data. The input format is shown in Figure 5. The variable name definitions and the input options are as follows:

TITLE Identification only
A Number of 100 Hz bandwidth SPL values to be input (beginning with frequency of 200 Hz , maximum value 99)

SWM Maximum number of soft wall modes treated
HWM Maximum number of hard wall modes treated
H Number of walls lined (1 of 2)
PSPL Flat SPL spectrum value; if zero, will read " $A$ " values of SPL, if nonzero, sets up flat spectrum array of " $A$ " values of SPL equal to "PSPL"

RMS 0. Impedance programs calculate particle velocity and impedance as function of SPL at one frequency

1. Impedance programs calculate an "RMS" particle velocity based on entire SPL spectrum

REORD 0. Modal ordering by least attenuated mode

1. Modal ordering by largest real part of downstream wave numbers

OVRMOD 0. Treats number of modes propagating to limits of "SWM", " HWM "

1. Treats "SWM", "HWM" modes, cutoff conditions not applied

SPL NAM SPL spectrum identification title

SPL Array of up to " $A$ " sound pressure levels
TL Total duct lined length (inches)
DL Increment length (inches)
DHH Hard wall duct height (inches)
G Ratio of specific heats for duct environment
TT Total temperature $\left({ }^{\circ} \mathrm{R}\right)$
PT Total pressure (PSIA)
" $A$ ", " $B$ " Frequency limits for calculation of attenuation $\left(H_{Z}\right)$
ANPLY Impedance model selection
0. Perforated sheet

1. Polyimide

AM Mach number
DHS Lined wall duct height (inches)
OPA NP Perforated sheet fractional open area or polyimide number of plies
D Backing depth (inches)
DIA Hole diameter (inches)
R Material thickness (inches)
MTHK Boundary layer momentum thickness (inches)
("DIA", "R", and "MTHK" are required for perforated sheet impedance model only.)

The following data arrays are required when the corresponding variables above are zero. The number of values per array is equal to the number of increments as determined from the total length and incremental length:

AAM(I) Mach number
ADHS(I) Lined wall duct height (inches)

DCON(I) Liner backing depth (inches)
DIAS(I) Perforated sheet hole diameter (inches)
THKC(I) Perforated sheet thickness (inches)
MTHKC(I) Boundary layer momentum thickness (inches)

## Subroutine "ATNSPL"

Subroutine "ATNSPL" is the control routine for the calculations pertaining to one duct increment. Initially, if RMS impedance treatment has been selected, the RMS particle velocity for the desired lining type is obtained by a call sequence to the "ENTRY" section of the appropriate impedance subroutine. If the discrete frequency impedance treatment option has been selected, the above call sequence is skipped over, and the frequency increment "do loop" is entered.

The frequency increment "do loop" controls calculation of impedance, eigenvalue equation solutions, modal attenuation ordering, and attenuated SPL spectra for each frequency in the range specified.

The impedance is obtained by standard calls to one of the impedance routines "IMPQD" or "IMPPD". A parameter "TSP" in the impedance routine call sequence specifies whether the routine is to calculate discrete or RMS particle velocity impedance. The impedance and other required data are then entered to the subroutine " $U X$ " which determines eigenvalue solutions for the lined walls.

This subroutine returns the solutions ordered in terms of increasing attenuation. The subroutine "RORDER" is then called if ordering is desired according to decreasing real part of the down stream propagation constant.

The ( $n, o$ ) modal attenuations are next applied to the appropriate modal spectra. If more than one hard wall mode is specified, the remaining eigenvalue solutions are obtained from subroutine "HWDBS" and the resultant attenuations applied to the modal spectra. Finally, the attenuated SPL spectra are synthesized from the modal spectra, and control returned to the "Mainline" program.

Impedance Subroutines "IMPQD" and "IMPPD"
These subroutines are the impedance models of the program. The models are semiempirical models for perforated sheet and polyimide, respectively, with cellular air backing.

Each routine is set up in two parts - the first accessed by a standard subroutine call, and the second accessed by an "entry" call. The second part is used to determine an "RMS" particle velocity for the lining, while the first part calculates either an impedance from an imput "RMS" particle velocity or an impedance based on a discrete frequency particle velocity.

Subroutine "UX"
This subroutine controls the calculation of lined wall eigenvalue solutions for one frequency. Th: zero Mach number solutions are obtained from subroutine "RTZ" and are filtered for duplicate ans invalid solutions. The filtered set of solutions are then input to subroutine "MRT" from whicl solutions for the required Mach number are obtained. Control is then returned to subroutint "ATNSPL".

## Subroutines "RTZ" and "NRAPH"

"RTZ" controls the calculation of zero Mach number solutions accomplished in "NRAPH". The maximum number of modes possible is limited only by the size of the array "ALZRT" in "RTZ' and "UX", and array " $A$ " in "ANTSPL". Presently, these arrays are dimensioned at 42, which give: a maximum of twenty modes. If $N$ is the required number of modes, $2(N+1)$ is the array siz: required. "NRAPH" calculates eigenvalue solutions by the Newton Raphson method. The forms of equations solved are:

$$
\begin{aligned}
& 0=F-Z \tan (Z) \\
& 0=F+Z \cot (Z)
\end{aligned}
$$

where $F$ is some complex constant and $Z$ is the required solution.

## Subroutine "MRT"

This routine accepts a filtered set of zero Mach number eigenvalue solutions and obtains solutions for the required Mach number. These solutions are then ordered according to least attenuated mode. For Mach number specified zero, the Mach $\neq 0$ calculation section is skipped, and the only function of this routine is the ordering process.

The Mach $=0$ solutions are accomplished in incremental Mach number steps. The number of increments is based on a Mach number increment nearest .05 which subdivides the specified Mach number an integer number of times. The eigenvalue equations appropriate to the duct geometry and number of lined walls are solved for each incremental Mach number. The Newton Raphson method is employed with modifications necessary to restrain the solutions to the first quadrant of the complex plane.

## Subroutine "HWDBS"

This routine calculates the eigenvalue solutions for lined wall-hard wall combination modes having hard wall mode numbers greater than zero. The soft wall solutions for zero hard wall mode number are used as starting points. The Newton Raphson method is employed for the solution.

Subroutine "RORDER"
This routine accepts lined wall eigenvalue solutions in any order and orders them according to the largest real part of the downstream propagation constants. These constants are calculated in "RORDER".

Naming Conventions for Transfer Variables
Variables are transferred between routines through "common" statements and through the "call" statements. The naming conventions for these variables are given here for program reference.

An unnamed common statement is used for the "mainline" routines and subroutines "INPUT", "ATNSPL", "IMPQD", and "IMPPD". In the "mainline" and "input" routines, all names are identical and are as follows:

AAM, ADHS,
POAC, DCON,
DIAC, THKC,
MTHKC Arrays of input variables of which each array member corresponds to a particular duct increment; the array names correspond to Mach number, lined wall duct height, perforated sheet fractional open area or polyimide number of plies, lining backing depth, hole diameter, material thickness, and boundary layer momentum thickness, respectively. The last three arrays pertain to perforated sheet linings only.

OSPL Input sound pressure level spectrum. This array remains unchanged throughout the program.

CFRQ Standard, 100 Hz bandwidth center frequency array ( 200 Hz to 10 KHz ).

SPL Attenuated sound pressure level spectrum.
TSPL Three-dimensional array of modal sound pressure level spectra; the first index refers to the hard wall mode number, the second to the lined wall mode number, and the third to the frequency at which these modes are calculated.

NSPL Number of input sound pressure levels.
H Number of lined walls.
RMS Impedance option; RMS or discrete particle velocity treatment.
REORD Modal ordering option.

OVERMOD Option to treat fixed number of modes irrespective of cuts. considerations.

Hard wall duct height (inches)
Thermodynamic ratio of specific heats.

Total pressure, psia
ANPLY Number of polyimide plies or perforated sheet fractional op: area if these parameters are constants, otherwise zero.

Number of lined increments.
LOW, IUP Index constants for the array "CFRQ" indicating the towest ar highest frequencies for which eigenvalue solutions will t obtained.

Static pressure, psia
Velocity of sound, $\mathrm{cm} / \mathrm{sec}$

Maximum number of lined wall acoustic modes to be treated.

X
DHSI
TSR
The only differences in naming the common statement for routine "ATNSPL" are for variz names "AK" and "J" which are renamed "AKDUM" and "JDUM", respectively. In "ATNSPL" variable names " $A K$ " and " J " are used for other purposes and have no relationship to the va used in the "MAINLINE" and "INPUT" routines.

The "commons" occurring in "IMPQD" and "IMPPD" differ from the above representation in only the following variables are required, and all other variables are unused:

```
"CFRQ" (renamed "FQ")
"SPL"
"NSPL"
```

Variable transfer through the "call" statements occur between the following routines:

| From "ATNSPL" | to | "GETV", "GETVPP", "IMPPD", "IMPQD", "UX", <br> "'RORDER","HWDBS". |
| :---: | :---: | :---: |
| From "UX" | to | "RTZ" and "MRT". |
| From "RTZ" | to | "NRAPH" |

The variable names and meanings for these transfers are as follows (names in parentheses indicate name conventions in the called routine where differences exist).

Calls from "ATNSPL"
"GETV" and "IMPPD"
POA (PLY) Number of polyimide plies
V(VP) Particle velocity ( $\mathrm{cm} / \mathrm{sec}$ )
AM Mach number
RO Density ( $\mathrm{gm} / \mathrm{cm} / \mathrm{cm}$ )
TSR(TSDR) Static temperature (degrees R )
D(BSP) Lined wall backing depth
C Speed of sound
$\mathrm{PC}(\mathrm{ROC}) \quad$ Environmental characteristic impedance (cgs Rayls) ${ }^{\prime}$
FTPI Logical variable indicating first frequency pass through IMPPD
$\operatorname{FRQ}(F) \quad$ Frequency
TSP(TSPL) Real variable indicating method of particle velocity treatment

ZZ Calculated impedance

The variable names and meanings are the same as for "GETV" and "IMPPD" wi: the following exceptions:

| POA | Perforated sheet fractional open area |
| :--- | :--- |
| TT | Total temperature (degrees Rankine) |
| PS | Static pressure (psia) |
| THK, DIA | Perforated sheet thickness and hole diameter, respectively |
| MTHK | Boundary layer momentum thickness |
| 'UX" |  |
| FRQ(F) | Frequency |
| AM | Mach number |
| H | Number of lined walls |
| C | Velocity of sound |
| DBN | Modal attenuation rates |
| MCUX | Number of lined wall eigenvalue solutions required |
| Z(ZS) | Modal lined wall eigenvalue solutions |
| NN(NS) | Variable indicating sequence of symmetric and antisymmetric mos: |
| DHS(S) | Lined wall duct height |
| FSMAL | Combined admittance, frequency and geemetry parameter |
| ZZ(Z) | Lined wall impedance |
| "RORDER" |  |

The variables are identical with those of "UX" immediately above except for 1 . following:
AK
$\omega / \mathrm{c}$

G Duct height parameter
Z Modal lined wall eigenvalue solutions
"'HWDBS"
$Z(R) \quad$ Lined wall eigenvalue solution
DB Model attenuation rate
$N \quad$ Variable indicating symmetric or antisymmetric mode
AM . Mach number
FRQ Frequency
C Velocity of sound
$\mathrm{K}(\mathrm{M}) \quad$ Hard wall mode number
DHS Lined wall duct height
DHH Hard wall duct height
FSMAL Combined admittance frequency and geometry parameter
H Number of walls lined

Calls from "UX"
"RTZ"
ALZRT Zero Mach number eigenvalue solutions
MCUX(NALZ) Input is number of modes requested, output is number of eigenvalue solutions supplied
(Variables "FSMAL" and " H " are identical with those of "HWDBS" above.)
"MRT"
ALZRT(ZRT) Zero Mach number eigenvalue solutions input ás starting points for nonzero Mach numbef́ eigenvalue problem

RK $\omega / \mathrm{c}$
All other parameters are as defined for inputs to " $U X$ " and retain the same names except "MCUX" which is named "NROOT" in "MRT".
, Calls from "RTZ" to "NRAPH"
FSMAL As defined for "HWDBS"

## $\mathbf{Z}(Z S) \quad$ Starting point for eigenvalue solution <br> ALZRT(ZF) Eigenvalue solution

The variable " N " in routine "NRAPH" indicates that the solution is required for a symmetric or antisymmetric mode.

APPENDIX III
PROGRAM LISTING AND SAMPLE PROGRAM OUTPUT

REAL MTHK, MTHKC
IVTFGER FIRST F
COMMCN $\triangle A M(57), ~ A D+S(50)$.
THKत1501. MTHKC(50)
DIMFNSION DLY（En）
FOIIVALENCF IMDA ND，DTA，POAC（1），PLY（1）），（AM，AAMII）， （OHS，AOHS（ll），（D，DCON（I）），（CIA，DIAC（1））． （THK，२，THKこ（1）1，（MTHK，MTHKC（1））
C
FOUIVALFNCE（VMACH，AAM（2））．（VDHS，ADHSI？））．（VOPA ND，DT $? \quad$ IVN MOTVI？II：
C
CALI INPUT
$x=0 L$
$I N L=1$
DHS $=\mathrm{DHS}$
Gก Tח 40
$38 x=x+n$ ．
$\mathrm{IDL}=\mathrm{IDL}+1$
IF（VMACH ANF．M．O！AM＝AAM（IDL）
IF IVNHS •NF．n．n）THS $=A D H S$ IIDL）
IF IVDPA ND．．NF．R．CI POA $=$ PDAC（IDI）
IF（VD ．NF．R．OI）＝DCON IINL）
IF IVDIA ．NF．N．nl JIA＝NIAC IINLI
IF IVTHK AME．「．n THK $=$ THKK ITML
IF（VMTHK ．NF．N．n）MTHK $=$ MTHKC（INL）
IE（VMarH．EQ．n．ni ，GO Tn $4 n$
$P S=D T \quad /(1 . n+(r,-1 . n) \& A M * A M / ? . n) * *(G /(G-1 . C) 1)$
TSR $=T T / 11 . n+1 G-1.01 * A M * A M / 2.01$

$C=33145.0 * S O R T(T S / 273.161$
$R Q=27^{2} .1 A / T S * D S / 1$. R13）5E6＊1．293F－3＊6RO47．
DC＝R $\cap * C$
$4 \cap$ CALL $\triangle T A S P L$
กา $5 n \mathrm{t}=\mathrm{LOW}+\mathrm{IUD}^{\circ}$
59 D3D（1）$=$ CSPLII）－SPLII
WVITF $(A, 5) x$
WOTTE（E，7）（CFDO（I），OSD（I），I＝LO甘，IUD）
WRITE（1，5）X
WPITE 11．7）（TFROITI．D30（II．I＝LTW．TUP）

```
    IF (ARSIX-TLI .GT. N.I) GO TO 38
    WRTTF (t,13)
    WR|TE (1.1?)
    FIRST F = 10n*LnW + 10n
    IFIDST = LTW
RO LASTF=MINOIFIRST F*2NO, ION*IUP * 1:01
    ILAST = MINO(IFIOST + N.IUD)
    WPITE (G,RIIFIOST F, LAST F, (SPLII),I = IFIRST, ILASTI
    W\ITF (1,QI) FIRSTF, LASTF. (SPLIII,I = IFIRST, (LASTI
    IF (ILAST.EQ. IUP) GT TD 9?
    FIRCTF= LASTF + 10?
    IFIRST = ILACT + I
    G) T0 9C
0) [M| = I||P - 1
    07 70 I = LOW,IM!
    OROII| = (DRO{I) + NRO(I+1)|/?.
70CFRQ\I)=(TFRO(II + CFROII+II)/Z.
    WQ\TF (1. 15) X
    WQITF I, IT) (DRDIII,I=LOW,IMI)
    STOD
    5 FTRMAT ("! DR DIFFFRFNEF AFTER',F6.2.' INCHES OF IINING'.//I
    7 FORMAT (: ,FQ.C.FIN.2)
13 FORMAT ('1 FRFOUENCIFS FINAL SPERTRUMO)
15 FDRMAT IO ER OIFFERFV=ES (SMOOTHENI AFTFR',FG.2.' INCHFS OF'.
    l LINING'//I
17 FORMAT (: ',5FIC.21
81 FORMAT {'C', 14, ,TJ', 16.4F11.21
    END
```



```
    SIIRROIJTINF INDUT
    REAT. MTHK, MTHKE.
    INTEGER FIRST F
    COMMNV AAM(59), AOHS(5,), POAC{E:I, DCON(5O). DIACIG
        THK`(5N), MTHKC(50)
                    . OSPL(99), CFRO(991. SOL190), TSPL16.29.091,
                                NSPI., H, 2MS, REORD, OVRMOD, TI, DL, DHH, G,
                                TT, DT, \triangleNDLY, J , LOW, IIIP, PS, r., RO, PC.,
                                NSWM,NHWM, X, DHSI, TSR
    OIMENSION PLY(SN), SPL NAM(I&)
        FJUTVAL. ENCE (7DA ND, DJAL(I), PLY(1)), (AM, AAM(1)),
                        (OHS. AOHS(I)), (D,DCON(I)I,(CIA, OIAC(1)).
                        (THK, ?, THKE(1)), (MTHK, MTHKC,11)।
    COMMON /NJN/ NJNI?,OQ)
        FOUIVALFNCE (VMACH, AAM(2)|, IVDHS, AOHS(2)), (VOPA NO, POACI
        l IVO ,OCTVI?I), IVDIA, DIACITII, IVTHK NO, POACI
                (VMTHK,MTHKC(2)।
            SFT CONSTANT VALIIF FLAGSIVARIATION AIONC, DUCT LENGTH IF NOM.
        VYACH=n.n
        VOHS =n.!
        VJDA NP = n.n
        Vn =n.n
        VIIA = C.n
        VTHK = n.?
        VMTHK =0.0
    C
        DIMENSION TITLE 119:
        RFAN (5, 4) TITIF
        RFAN (5,23)A,SWM, HWM, H, PSPI, RMS, REORO. NVRMOD
        NSPL = A
        IF (PSPI .FQ. \.n) Gח Tח 44
        O\cap 43 1= 1. NSPI
        43 SDI(I)= OSPL
            G7 T\ 45
        44 PFAN {5, 41 SOL NAS
        READ (5. 2.) (SOL(I), I = 1. NSPL)
        45 IF (HWM .FO. O.n) HWM = 1.C
            HNM = AMINI (A.N. HWM)
            NHWM = HWM
            IF (SNM .FQ. O.O) SWM = 1.N
            SWM = AMINI(?O.r. SWM)
            NSWM = SWM
    K = 1
    00 4 क J = 1. 2?
    07 46 1=1.NSP1.
46 TSPI{K,J,I|= = .n
    CFROIII = 2nn.
    OOS I = 2.NSOL
56 CFRDIII = CFOOII-1) + 19N.
    RFAN (5,22) TL, OL, OHH,G, TT, PT, A, &
```

```
    J=1TL}+0.11/n
    RFAD (5,2?) ANPIY, AM, DHS, MPA NP, O, OIA, P, MTHK
    WOITF (t, l) TITLF
    WRITE (1, 6) TITLF
    IF (ANPLY .FO. O.n! WRITE (G, ?)
    IF (ANPLY .NE, R.OI WPITE (G, 3)
    WRITE (t,5)
    LOW = A/ICr. -.g
    IF (LOW .FO. C) LOW = 1
    IUP = 8/100. -. - 
    IF I||P .FO. CI IUD = VSPL
    WOITE (E, G: NSPI, NSWM, NHWM, H, PSPL, RMS, REORD, DVRMNO
    HRITE (1, व) NSPL, NSWM. NHWM, H, PSPI, PMS, REORI, OVRMON
    WRITE (G, R) TL, NL, )HH, G, TT, PT, CFRO(LOW), CFPOIIUPI
    WRITF (I, R) TL, IL, )HH,G. TT, PT, CFROILNW), CFRO(IUPI
    WRITE (E,ICI ANPLY, AM, JHS
    WRITF (1,1n) ANDIY, AM, NHS
    IF IAvPIY .NF. O.CI
                                    GO TH GR
    WRITE (E, ll) OPA ND
    WPITE (1, ll| nPa ND
    HRITE (A, 14) D, IIA, R, MTHK
    WRITE (I, 14) D, DIA, P, MTHK
    GO TO 69
    K3 WRITF (A. 121 חPA ND, D
    W2ITE (1, 1?) חPA NP, D
    69 FJFCTT = AMINI (ABSIAM), JHS, חPA ND, O, DIA, R, MTHK)
    jF (AM .NF. O.C) (п) TOTO
    PEAN (5, 23) (AAM ([), 1 = 1. J)
    WYITE {E, 3\cap) (AAN IT), I = l. JI
    WरITE !1, 3n) \AAM (II,I=1. J
    70 IF (DHS.NF. O.N) GOTO 71
    RFAO (5, 23) (ADHS (|), I = 1. J)
    W`ITE (f, ?l) (ADHS (l), I = 1. J)
    WRITE (1, 3l) (AOHS (T), I = 1, J)
    71 [F (חDA ND .NF. O.N)
    READ (5, 23) (POAR (I), I = 1. J)
    IF {AVPIY NF. r.ri GO TO 77
        PERFORATFO SHFFT PPER EFNT' DPEN ARFAS
            INTFRNALIY, FRASTIONAL
            FXTERNALLY, DER CFNT
    WRITE (t, 2?) (DOAT. (I), 1 = 1, J)
    WRITE (1, 32) (POA: (ll, I = 1. Jl
    GO TO 7?
        DOLYIMID - NUMRER OF PLYS
    77 HRITE (t, 3T) IPLY (II, I=1, J
    WRITF (1, 37) (PLY (I), I = 1. J
    7) IF ID .NE.O.\capI GOTO 72
    READ (5, 2`) (OCON (II, I = 1, J)
    WQITE (t, 2?) (DCON (I). I = 1, J)
    WRITE (1, 33) {DRON (I), 1 = 1, J)
73 IF IOIA .NE. O.N.TR. ANDLY.NE. O.CI ROTO T4
```

```
    QFAO (5,.23) (DIAC, (I), I= !, J!
    W\ITF (t, 34) (DIAC (I|, l=1, J)
    W\ITF (I, 24! (NIMC. (I), I= = J)
7 4
    RFAN (5. 2?) (THKC (I), I = 1. J)
    WQ\TE (6, 25) (THKR (I|, l=1, J)
    WRITF \1, 35| {THKP \II, I= =, J!
75 IF IMTHK .VF.N.^.JR. ANDIY.NF. N.CI GOTO 76
    RFAN (5, 23) {MTHKC:\1, I = l, J)
    W2ITE (E, 2G) (MTHKC(I), I = 1. Jl
    WOITF (1, 36) {MTMKCIII, I = 1. Jl
76 [F PPSPL. NE. O.O\ Oח TO 120
    IF {EJETT NF.N.N|G\ TM 70
    WRITF (6, 6)
    WRITF (1.6)
79 WRITF (6.13) SPI NAM
    WQITE (1, 131 SP1. NAM
    FIPCT F=209
    IFIRST=1
    A\cap {AST F = MINOIFIRSTF + 3nO. INO*NSPL& 1rOI
    ILAST = 4INO(IFTRST + 3. NSPI)
    W2ITE (G,PI) FIRST F, LAST F, (SPLII),I = IFIRST. ILASTI
    IF (LOW -LF. IFIRCT .AN`. IFIRST.LF. IUP .IN.
    I LOW .LF. ILAST .AND. ILAST .LF. IHP)
    \ WRITF (1.QI) FIRST F,IASTF.ISPLIII,I = IFIRST, ILAST:
    IF ITLAST .FO. NSPLI NO TN LPN
    FIRST F = LAST F + 10n
    IFIRST = ILAST +1
    G% Tח $0
120 W2ITE (6, EI
    WRITF (1,6)
130 CONTINUF
```




```
    TSR \(=T T /(1.9+(\Pi \quad-1.0) * A M * A M / ?\). 01
```

    TSR \(=T T /(1.9+(\Pi \quad-1.0) * A M * A M / ?\). 01
    \(T S=272.16+5.719 .0 *(T S R-491.69)\)
    \(T S=272.16+5.719 .0 *(T S R-491.69)\)
    \(C=33145.0 *\) SORTITS/273.161
    ```
    \(C=33145.0 *\) SORTITS/273.161
```






```
    WRITF \((4,19)\)
```

    WRITF \((4,19)\)
    D) \(501=10 W, 1 \cup D\)
    D) \(501=10 W, 1 \cup D\)
    \(S W=(2.7 *(F R \cap(T) * \Gamma H S * 2.54 / C+1.0) / S O R T(1.0-\Delta M \not \subset A M)\)
    ```
    \(S W=(2.7 *(F R \cap(T) * \Gamma H S * 2.54 / C+1.0) / S O R T(1.0-\Delta M \not \subset A M)\)
```




```
    IF ISW.GT. SWM. חR. OVRMOD GT. O.NI SW = SWM
```

    IF ISW.GT. SWM. חR. OVRMOD GT. O.NI SW = SWM
    IF IHW . TT. HWM OQ. OVRMON . GT. O. Ol HW \(=\mathrm{HWM}\)
    IF IHW . TT. HWM OQ. OVRMON . GT. O. Ol HW \(=\mathrm{HWM}\)
    \(\mathrm{IH}=\mathrm{HW}\)
    \(\mathrm{IH}=\mathrm{HW}\)
    \(I S=S W\)
    \(I S=S W\)
    IF IIH -IT. II \(14=1\)
    IF IIH -IT. II \(14=1\)
    IF IIS LT. 11 IS = 1
    IF IIS LT. 11 IS = 1
    \(A=I S\)
    \(A=I S\)
    \(A=S O L I I-10 * * A L O G 1)(A * I H)\)
    ```
    \(A=S O L I I-10 * * A L O G 1)(A * I H)\)
```

C

```
NJN(1,I: = IS
NJN(2,I) = IH
O\cap 2? L = 1.IS
0) 2, K = 1.1H
```

2) TSPL(K,L,I)=1
WRITE \{E.? II CFROIII. NJVII.II. NJNI?.II, TSPLII.I,II
$5 \cap$ OSPLII $=$ SPLIII
RFTIJRN
1 FORNAT (1*S*\&*F'.18A4)
2 FIRMAT ('C RFCTANCIILAZ ANALYSIS - POLYIMIDF MODEL')
3 FनRMAT IO QE
4 FORMAT (19A4)
5 FORMAT IC INDUT DARAMETERS'I
g FJRMAT ("!",19A4)
\& FJRMAT 'C'. FI3.h. $6 \times$. TOTAL DUCT LFNGTH'/
6F14.6, 6X, "INCRFMFNT LFNGTH*//
FFI4.6, $6 X$, HARN WALL DUCT HFIGHTV//
9F14.6, 6X. 'GAS CONSTANT'/
QFI4.6. $6 \times$, THTAL TEMDERATURE $!$
7F1.4.6. $6 X_{\text {, 'TITAL DRFSSURE'// }}$ /
C.F14.6, $E X$, 'IOWEST FRFDUENCY CONSIOFREN• $/$
DFI4. $G$. $A X$, HICHFST FRFOUFNEY CONSINFRFR'/I

$\rightarrow 11496 X$, MAXIMIJM NUM3ER OF SOFT WALI MODES CONSIDERED'I
HII4, $\quad$ GX, MAXIMUM NUM 3 EQ OF HARD WALL MODES CONSITERFD'II
3 Fi4.6. EX, 'NUM3EQ OF WALLS LINFO II
JF14.5. $6 X$, UNIFORM SPL I/I
KFI4.6, $6 X$, IRMS PAOTITLF VFLOCITY $/$
LFI4.6, $G X$, QFOR DFR חPTITN: $/$
MFI4.6. EX. OVFRRITE MODF OPTION:/I
10 FORMAT ION•, FI3.6. $5 x$, MODEL SELFCTMR•//
1F14.6. 6 E . MAC.H NUMRFR ///
FFI4. G, $6 X$. SOFT WALI DUCT HETGHT•/I
11 FORMAT $12 P F 14.6$. $6 X$, PFRCENT OPEN $A R E A \cdot / 1$
12 FIRMAT IFI4.G, GX, NUMRFR OF POLYIMIN PIYS'//
1F!4.6, GX, BACKIN' SOACET/I'I
13 FIOMAT ( $1 \times$, 1 RA4) O FREDUFNCIES SOUND PRESSIJRF LEVFLS')
14 FIRMAT (F14.5, $6 X$, •RAEKINT SPACE'//
YF14.6. $6 X$. HILE $T$ TAMFTER:/
AFI4.6, 6X, MATERIAL THICKNESS:/
ZFI4.6. $6 X$, MOMENTIM THICKNFSS*IM:I
19 FJRMAT I'N EOUAL ORFSSURE MOCAL SPI./
1
1
21 FJOMAT $(1 X, F B . C, I 6,111, F 13.1)$

30 FIRMAT ('O MACH NUMAEQS


33 FJRMAT IO RACKINC.SOA.FS URGOFI2.K)I
24 FJRMAT ('O DIAMFTFRS
```
    SIJPPDIITINE ATNSPL
    REAI MTHK, YTHKC., PI/3.14159/
    INTFGER FIRST F
    COMMON
    l
    ? -
    3
    4
    5
    DIMFNSITNN PLY(SC)
    FOUIVALENCF {OPA ND, POA, POAC(1), PLY(1)|, (AM, AAM\I|),
                        (OHS, AOHS(1)|, (C,DRON(I)),(OIA, OIAC(1)),
                        (THK, R, THKE(1)), {MYHK, MTHKC(1))
    EOUIVALFNCE (VMACH, AAMI?I), (VOHS, ATHS(?)I, (VOPA NP, POAC(2)I,
    COMMON /NJN/ NJN(2.OO!
    COMPIEX 2.14\), FSMAL,FS,TZ, SAVE 7.7
    DIMENSICN DRN(4?),MN(42)
C
```

```
    V=40.
```

    V=40.
    FTPI = .TRUE.
    FTPI = .TRUE.
    IF IRMS LLE N.I GO T\ 5C
    IF IRMS LLE N.I GO T\ 5C
    WRITE (f, 19)
    WRITE (f, 19)
    19
19
IF {ANPLY.NF.n, n)CALL GFTVIONA,V,AM,RO,TSP,O,C,OC, +5こ)
IF {ANPLY.NF.n, n)CALL GFTVIONA,V,AM,RO,TSP,O,C,OC, +5こ)
CALL GFTVPD IOOA, V, AM, TT, PS, THK, DIA, MTHK, O, R, OCI
CALL GFTVPD IOOA, V, AM, TT, PS, THK, DIA, MTHK, O, R, OCI
5n DO 7n I = LOW, JID
5n DO 7n I = LOW, JID
TSP= SPL{1)
TSP= SPL{1)
IF (RMS IF.N.) TSP = -TSP
IF (RMS IF.N.) TSP = -TSP
FRO = CFRO(I)
FRO = CFRO(I)
|F IANPIY ,NF. N. NI CALL IMPPN IFTPI, FRO, PNA, V, AM, RON.
|F IANPIY ,NF. N. NI CALL IMPPN IFTPI, FRO, PNA, V, AM, RON.
! , TSP, O. R,P(, TSD, 77. +55).
! , TSP, O. R,P(, TSD, 77. +55).
CALL TMPOD IFTPI, FRG, POA, V, AM.
CALL TMPOD IFTPI, FRG, POA, V, AM.
55 WQITE(G,O) FRO, T7,V, PS, THK, NIA, MTHK, D, C, PC, TSP, IZI
55 WQITE(G,O) FRO, T7,V, PS, THK, NIA, MTHK, D, C, PC, TSP, IZI
A = 0.0
A = 0.0
NS ={2.2*FRO*NHST*2.54/E + 1.n) / SOOT(1.n - AM*AM)
NS ={2.2*FRO*NHST*2.54/E + 1.n) / SOOT(1.n - AM*AM)
NH={?.*FDO*\capHH*?.54/こ +1.NI/ SORTII.O-AM*AM|
NH={?.*FDO*\capHH*?.54/こ +1.NI/ SORTII.O-AM*AM|
IF INS.GT.NSWM .OR. OVZMON.GT.C.OI NS = NSWM
IF INS.GT.NSWM .OR. OVZMON.GT.C.OI NS = NSWM
IF (NH.GT. VHWM .ПR. ПVZMON .GT. O.OI NH = NHWM
IF (NH.GT. VHWM .ПR. ПVZMON .GT. O.OI NH = NHWM
IF (NS .LT. I) NS = !
IF (NS .LT. I) NS = !
IF INH \&IT. I| NH=1
IF INH \&IT. I| NH=1
MCIIX = NS
MCIIX = NS
CALI UX IFRO, AM, H, E. TRN, MCUX, Z,NN, DHS,FSMAL, Z7)
CALI UX IFRO, AM, H, E. TRN, MCUX, Z,NN, DHS,FSMAL, Z7)
WRITE (E,5) (7(J), DBN(J),NN{J),J=1,MCUX)
WRITE (E,5) (7(J), DBN(J),NN{J),J=1,MCUX)
AK = 2.*P\*CFRO(1)/C

```
    AK = 2.*P\*CFRO(1)/C
```

```
G=OHS*2.54/H
    IF (REORC .CGT. N.I CALL ROROFR (AM,MK,G,MCUX,7,NN,DBN)
    M= MINC(MCUX,NS, NJN(1,I)\
    NH=MINC(NH, NJNI?,I\I
    WDITF (A, 2)
```



```
    IF IRFALITII
```

    WRITE (E, 22) CFRD(II
    
21 जरTTF (t.6)
DO.2CJ $=1,4$
TSPL(I.J.I) $=$ TSOL(1.J.I)- ORNIJi*DL
IF (NH .FO. 1) rin TO 20
$N=$ NN\{J\}
IF (DRN(J) .FO. n.I GO TO 20
DO $10 \mathrm{~K}=2, \mathrm{NH}$
CALL HWDRS $(Z(J), D R, V, A M, F R Q, C, K, O H S, O H H, F S M A L, H, F S)$
WRITF (E.7) J.K.Z(J).DS.N
TSOLIK.J.II = TSDLIK.J.I)-OR*DL
20 CONTINUE
NS $=$ M\{MC(NS, NJN(1,I))
D) $x C J=1, N S$
D) $x \cap K=1, \mathrm{NH}$
$2 n A=1+1 \cap$.**(TSOL\{K,J,1)/IO.1
SPLIII = ! C.*ALOCI I (A)
IF (MOD(I-LTW, 2) .FD. 1) WRITE (6. 21
70 CONTINUF
WPITE (t,1) $x$
FIRCT F = >CN
IFIRST $=1$ MINOIFIRST F +3 OO. 100 *NSPI +1001
RO LASTF $\quad$ = MINAIFIDSTFF 3NO:
IIAST = MINN(IFIQCT + LASTF, (SPLITI,I = IFIRST, ILAST)
IF (ILAST.FO. NSDI.) RETIIRN
FIRST $F=$ LAST $F+10 n$
IFIOST $=$ ILAST +1
GO Tn 80

ORN NN'II


6 FJRMAT ! SWM HWM ZREAL ITMAG.
7 FORMAT IT5. I4, 1DF14.5. F13.5
 1 V V'Fl4.51
END

```
        SIIRRIUTTINF IMPOMIFTPI,F,DLY,VP,AM,RN,TSDP,RSP,C,ROC,TSPL,ZL, *
    IMOFNANCF - OOLYIMITE
        C\capMPLEX RMOLX,7]
        LOGICAL*& FTOT
        DATA P1/{.141501
        CTMMON RUNCH(449), FO(99), SP((99).MORF(11,RRO), NSPL, THE,RFSTI
        OIMFNSICN PSPL{C\etaI
        COT(x)= (COS (x)/S[N{x]
        0=RSO
        P= 2. 2F-4*1n.**(ARSTTCDL;/2N.)
        TKO=?.*D|*F/C*O*).54
        ITN=n
        IN=0
        IF(.vרT.FTDI\ G, TO 50
        U=3.17E-P/2.MQQ#TSTRR**1.5*734.7/(TSOR+216.)
        RV\capC=464C.*U*P{Y**1.333
        VOGF=.C5*T*AM*AM/(QVOC/ROC+SORTIPRVOC/ROCI**?+.71*PLY*PLY*AM*AV
        IF (TSPL &LT. N.OI WRITF (E,I) VPGF
    I FORMAT IN GRALIVI, FITW PARTICLE VELOCITY=*.F11.4."FRIJM SURD.
    l OUUTINE IMOOD'I
    FTDI = FALSF.
    5) CONTINUF
    RIAC=2. F&*R\cap#PL Y*PLY*VP
    RVAC=|*. 2n5*D|Y*OLY*F
    RT=R|AC+RVAC+QVOR
    RX=R!*F*!.c49*DLY**1.5-5.12E-5*口{Y**3*VP)
    IFITSPL.CF.N.1 Gつ rn Rn
    RZPV=RIAC/VD
    RX口V=-5.12F-5*RO*OLY** 2*F
    IT=(MPLX(RZ,PX-ROC*COT(TKD))
    OAR = CARS{TT)
    SORTV = SORT(VDr,F**) + (D/QAR)**2)
    FVR = VO - SORTV
    DDDZ = - -./\AQ** 4*(RZ*R P.PV + RX*RXPV)*P*P
    FNRD = 1.C - DPOT*. 5/SORTV
    VDN=VP-FNR /FNPD
    IF{VPN.LT.O.} GO TN 4]
    IF((AGS(VON-VO)-.NOCI*VO).I T.n.) GO Tח 6n
    ITN=ITN+I
    IF(ITN.GT.5n| GM TM TN
    VP=VPN
    GO TO 5n
    4! VD=1.0
    IN=IN+1
    {F(IN.GT.3} r.n Tח 7n
    Gn TO 5n
70 RONTINUE
    7l = I7/ROR.
    RETURN I
GO CONTINUF
    ZL = IL/R\capC
```

```
    VP=VPN
    RETIJRN I
8\cap T< D=?.*PI*F/R*R <n*?.54
    ZL=CMDLX(RT/OOC.RXIRON-COT(TKO))
    RETIIRN I
    ENTRY GFTV (PIY, VP, AM, OO, TSOR, RSP, C. ROC, *)
    0) 5 I = 1,NSPL
    5PSPLII|=2.PE-4*1^.**(SPLII)/20.1
    U=3.17E-2/7. AgG#TSNR**1.5*734.7/(TSNR+21%.)
    DCM = 8SP* 2.54
    RVDr =464f.*U*PLVA*1.333
    VDGF=.O5*C*AM*AM/1QVORT/2OC+SORT((RVDC/ROC)**2+.71*DLY*P&Y*AM*AM))
    GRITE (6. 11 VOC,F
    TK = 2.C*DI/C
    VO}=10\cap.
17 RIAC=2.RG*R\cap*PLY*DIV*VP
    POOL=9.
    PO7=0.
    Oก 15 I =1, NSDL
    TKD=TK*FQ(I)*OCM
    RVAC=U*. .n⿻上丨口LY*OI.Y*FO(I)
    RZ=R\AC+RVAC+RVDR.
    RX=RO*FO(I)*(.540*OI,Y**1.5-5.12F-5*P{Y**\**VP)
    ZL=(MPLX{RT,QX-ROR*COTITKD)
    OAB=CARS(T) 
    P] PF=(口SPI(I|/OAR)**?
    POZ =PO7 +PO7F
    PZI=-5.12F-5*R ח*DLY**2*FO(I)
    PIR=2.8G*R\cap*DLY*口LY
15 POחZ=PPOZ-2./OAR**4*(२Z*PZR*RX*PZI)*PSPLII)*PSPL(I)
    SORTV=SORT (VPGF*VPr,F+P\capI)
    FAI=VO -SQRTV
    FA1P=1.- OPOZ*.5/SORTV
    VO1 =VP -'FAI/FAIP
    IFIARSIVPI -VP I.LT..NOOI*VP I RETURNI
    VP = VPI
    Gח TO 17
    FND
```



SURROUT TNE IMPQIIFTPI,F, DחA, VP, AM, TT, DS, THK, DIA, MTHK,RSP, C, 1RIC,TSDL,771

7DM9!7 3
C IMDFRAVCE - PFOFODATFD SHEFT
COMMON RUNCH(449)
OIMFNSION DSDI:CTI
COMPLFX CMPLX, 27
LOGICAL*4 FTOI
RFAL MTHK
OATA OI/?.14159/
$\operatorname{cor}(x)=\cos (x) / \operatorname{Sin}(x)$
$P=2.2 F-4 * 1^{n}$.**(ARS(TSDL)/2C.)

I $\mathrm{r} N=0$
$\mathrm{N}=\mathrm{C}$
IFI.NIT.FTPTI GR Tn $5^{n}$
$\mathrm{rH}=\mathrm{T} T / 51 \mathrm{~S}$.
$\mathrm{pp} ?=\mathrm{DI} / 7.1$ SORT1?.1
OFL $=P S / 14.7$
$F=1 . \cap 251 *(1.10 \cap A * * 2-1) / P \cap A * *$.1/EXO(.5n72*THK/DIA)
RVOC= = ก77*THK/DFL/OOA*TH*TH/(TH+.416)
RV?=RVクC/2.

1MTHK) \# F \# \& M*AM 1
2 * O.F ITSPL.IT. C. N1 WRITE (6.1) VPCFF
 1

COUTINF (MPO):1
FTDI $=. F A L S F$.
59 RONTINUF
$S V=2.54 *$ חIA * F * D ПA/VO R|AC=DP2*VP/C*E*FXD(-1. $9 * S N * S N$ )
RVAC=3. P3F-5*TH**. 7 5*SORT(F)*(THK/DIA+1.-DIA)/(POA*SORT(DEL*ITH
$1+.416111$
$R T=(R I A C+P \vee A C * R$ VNC $) * R O S$
חI. $T=.85 * C 1 A *(1 .-7 * S O R T(O \cap A))-1.2 F-5 / P \cap A * V D$
IFIOLT.LT.O.I DIT=O.
DX $=4.59 \mathrm{~F}-4 / 0 \cap A * F / S O R T(T H I *(T H K+D L T) * R \cap C$
PFITSPI.CF.C. $\mathcal{C O}$ TT R R
RTDV $=$ RIAC/VD* $1 .+3.5 * S N * S N) * R \cap C$
R $\times P V=-R \times /(T H K+\cap 1 T) * 1.2 E-5 / P O A$
IF (DLT - FO. n.r) RXDV $=0.0$
$77=C M D L X(R Z, R X-R O T * C \cap T(T K O) 1$
QAB $=$ CABSIZZ
SORTV $=$ SQRT(VPGF**2 $+(P / Q A B) * * 21$
$F V R=V P-S Q P T V$
$P P \cap Z=-2.1 D A R * * 4 *(R Z * 2 Z P V+R \times * R \times P V) * P * P$
FVR $=1.0-$ PPNZ*. $5 / 5$ SRTV
VPN = VO-FNR/FNRD
IF (VPN.IT.O.) GO T? 41
IF ((ARS (VPN-VO)-. $\cap \cap \cap l * V O) .1 T .0 .1$ GO Tn 60
ITN = ITN+1

REPRODUCIBILITY OF THE
$\qquad$

```
    IFIITN.GT.5^I GO TO 7O
    VO=VON
    GO TO 5n
41VO=1.0
    IN =IN+I
    IF (AM .NE. O.O) VO = VOGF & 1.O
    IFIIN.OT.ZI CO In 70
    GO TH 5n
7n CONTINUE
    WQITF (6, 71I IN, ITN
```



```
    ZZ = ZZIPNC
    RETURN
60 CONTINUF
    7% = 22/ROC
    VD=VON
    RFTURN
8の TKn=2.*P!*F/C*SSD*2.54
    ZI = CMPLXIRZ/OOC,RX/ROR-COT(TKOII
    RFTIIRN
    EVTRY GETVPD IOחA, VP, AM, TT, PS, THK, DIA, MTHK, BSP, C, ROC,I
    DO & I = 1.NSOL
5 PSPL(I)=2.2F-4*10.**{SPL(I)/2?.}
    REALK = .C5 + .11# חIA/MTHK
    TH=TT/519.
    PD2=PI/SORT(Q.O)
    DEL=PS/14.7
    E=1.n\51*(1./DOA**2-1)/DJA**.1/FXP(.5^72*THK/DIA)
    RVDC = RVRC/IOPOC)
    RVDC=.077*THK/DFL*TH*TH/(TH+.416)/POA/2.O
    VDCF=0.5*RFAL K*C*AM*AM/(RVOC+SORT(RVOC**2+.5*REAL K*F*AM*AM))
    2*0.5
    WRITF \E. I| VPGF
    VD=1CO.C
    IN = n
17 PDO2=9.
    POZ=0.
    IN =IN +I
    D) 15 1=1, NSPL
    TKn=2.*P\*FO(I)/C*RSD*2.54
    RVAC=.CCCC3R3*THA*.75*SORTIFOIII)/POA/SORT(OEL*(TH+.416)I
1 * THK/\cap{\Delta+!.\cap-O@A}
    SV = 2.5ム*\GammaTA*FO[!|*POA/VD
    RIArェロP ?*VD/C*F*EXD(-1.S*SN*SN)
    RI=R I Ar, +RVAC +RV\capC+RVDC
```



```
    RX=.(CC469*FD(T)*(THK+DLT)
    RX=.CCC&GO*FQ{T)*{THK+N{T)/OחA/SQRT(TH)
    l=CMPLY(RL,RX-C\capTITKDI|*ROC
    OAR=CARS\771
    P\7F=(PSPL(F)/QAR)**2
```

```
    POZ =POZ*POZF
    RYPV=-RX/(THK + OLT)*.O\capOO12/P\capA
    RTPV=Q\AC/VP*{1.n+?.6*SN*SN)
    RX = QX-rnT(TKn)
```



```
    STRRTV=SORT(VDRF*VORFF+D\capZI
    FAl=VD - SQRTV
    FAlP=1.- DOOl*.5/502TV
    VP1 =VP }->\mathrm{ FA1/FA1O
    IF (VDI .IT. 0.CI GO rO 20
    IF(ABSTVPI -VP I.LT..ONOI*VP I RETIIRN
    VD = VOI
    GO T0 17
20 IF IIN .EQ. 3 GN TO >1
    VD = 1. n
    IF (AM .NF. O.O) VP = VDGF + 1.O
    Gก TO 17
2! WRITF (6.22)
?2 FOOMAT I: RETIIRNEN VIA 21 FROM GETVPD IN = . %3)
    RETIIRN
    FNO
```

```
            SURR\capUTINF UYPF, A&, H, 二, ORN, MCUX, TS,NS, S, FSMAL, 2)
            RFAL*4 CRN(42),M,FSMAL.ALZRT(4), 2S(42)
            O{MENSION NS(42)
            **** RK IS K=W/C
            PI=3.14150
            AI = CMDLX(\cap.r.,l.r)
            RK=2.*口|*F/C
            OHS =S
            AD=1./7.
            IF (REAQ(AO).LT.O.O) rO rn 5n
            FSMAL = AI#A\Gamma*RK*OHS*?.54/H
            CALL QTI(FSMAL, Al /RT,MCUX,H)
            WRITE (E,I) FSMAL, (ALTOT(J), J=1, MCUY)
            JF (REALIFSMAL) ORE.N.N) GO TO }6
            N = MCUX - 1
            \cap\cap ? J=1,N
            l=J+1
            DO 4K}K=1,MCU
            ARSRF= ARS(RFAL(ALZRT(J))-RFAL(ALZRT(K)))
            ARSIM= =ARS(AIMACIALJRT(J))-AIMAG(AITRT(K)))
            IF(ABSRE.GT. 1.F-5) B.C TO 4
            IFIARSIM .TT. 1.F-5) ती Tत 4
            ALIRT(J)= CMPLXII.F2C,n.n)
            GO TO ?
            4 CONTINUF
            3 CONTINUF
6 4 ~ C A L L ~ M R ~
            RFTURN
    5n WRITE (E, lN!) AO
ICI FOQMAT IIX,?S(: *
            I IIMRENANCF ':O SIVCF THE REAL DART OF THF ABOVE ',
                        IMPENANCF (', 2F11.4, ') IS'/, NAFTOATIVF ABOVE ',
                            ULATIONS ARE RYDASSFD FOR THIS FRFOIIENOY, SMME CAIC..
1C) FTRMAT (1, 1)?) AN
    l IMX,95\"* 'I/MN SINRF THE REAL, PART NF THE % %
```



```
    MCUX = C RESULTS ARF INVALIO. //2FI' * 'II
    RFTURN
    1 FTRMAT
```



```
        FNO DRELIMINAOY MACH=O SNLUTIONS',/13(E13.4,E12.41)
```

```
            SHRROUTINF RTY(FSMAI,ALTRT,NALZ,HI
            COMPLEX FSMAL,ALJOT(L2),?
            P=3.14150
            IF (REAL(FSMAL).FFF.O.N) GO TO 21
            l = CMOLX(ATMAGIFSMAL), -REALIFSMAL))
            CALL NRAPH (FSMAL.?,ALZRT(1),1)
            N = NAL7
            IF (H.FQ. 2.0) N=(N+1)/2
            IF 1H.FO. 1.N1 gत, ro In
                    l = CMPLX(ATMAG(FSMALI,-QFAL(FSMAL))
                    CALL NPAPH {FSMAL.T,AL7RT(2*N+2),-1)
10 DO x J=1,N
            Z = C.MO{X(O*(J-.5) +.05,.C5)
            CALI NRAPH (FSMAL,Z.ALZRT(J+1).1)
            l = CMPLX(P*J -.O5..N5)
            CALL NRAPH (FSMAL,7,AL7RT(J+N+1).1)
            IF (H.FQ. 1.N) rn Tत 3
                    7=CMPLX(O*(J-.5)-.05.0.0.0)
                    CALI NRAPH (FSMAL, I,ALTRT(J+2*N+2),-1)
                    7 = CMPIX\P*J+.N5..NS)
                    CALI NRAPH (FSMAI,7,AL'ZRTIJ+3*N+21,-1)
3 CONTINUF
NALT = 2*N + !
    IF (H.FO. ?.MI VALY= 4*N+?
    RFTURN
21 IF (H.FO. 1.N1 NAIZ = NALZ*2
    D7 7 J = 1.NAL?
    1F (H EO. 1.NI ती ron 20
    Z = CMDLX{D*J-.C5..n?)
    CALI VRAPH {FSMAI,7.ALTRYIJ+NALZI,-1)
?? Z = CMPLX(O*(J-.5)-.r5,.\cap2)
    7CALL NRAPH (FSMAL,7,ALZRT(J),1)
        IF (H.FQ. 2.C) NALY = NALZ*2
        RFTURN
    FN:
```

```
SIIRROUTINF MRT (FSMAL,TRT,RK,H,NROIT, ORN,AM,NHS,TS,NSI
COMPLEX FSMAI,.7PT(4?),\SI47)
C\capMPLEX*1G 7,F,P,rTANT,CSFCT,RR,DFS
OFAI.*4 RRN(4?), nQ(4)?
DIMENSION NNI42),N\(4?)
REAL*Q 7R,ZI,S(2),OMAR,T,DAM,A,RO,TMM,ML,R
FOUIVALFNCF 17.S(1)|
ORS = 1^n.
A = nHS&2.54/H
RO = QK
OFS = FSMAL
LH=H
NYAC.H= APS(AN)/.N5+1.n
NAM = AM/NMAF.H
NR = ?
D7 lQ\ 1 = NOMNT
OB (L)=r.n
Z = FRTILI
STC=SI!1
IF (SIC .rFF. L.FSN) FON TM IRC
TMM = 1.CO
7RT(L)= MMOLXI1.FPr,`.O)
OMARH=0.OO
N=1
IF (L*|H .TTT. NRO\capT) N= =1
IF IAM EO. O.rI rin rn IOn
On OO J=I,NMACH
OMACH= CMACH + NAM
I= ?
TMM = 1.DN - OMAT.H#DMACH
n\cap a1. k=1,1\capn
7Q = S(1)
7%=S%21
IF IOABSIZRI GT. 3.471?.CR. DARSIZII .GT. 174.0nIGOTO 18?
IF {N.RT. n) CTANZ = COSINITI/CNCOSIZI
IF (N.LT. O) RTANZ =rOROSIT)/COSINITI
IF {N .GT. OI CSFRT = 1./CDCOSIT)
IF (N .LT. C) CSECT = 1.ITOSIN(Z)
RR= COSDPT(I.nn_TMM* L*Z/(PD*RD*A*A)।
F = OFS/TMM/TMM*(1.-DMA(H*RR)**? + 7*CTANI
O = RD*RC*A*A*TMN
\rho=2.Пr*ПFS*(1.\capn-ПMACH*RR)*ПMATH/RQ*1/D
p=p+rTANT - l#CSFC,Z*&SFC.T
CARP = CCAASPDI
IF (CARP .LT. L.F-4n) GO TO lan
l = l - F/P*.70n
```



```
I= I + 1
IF||.GT. {| GN TM IRM
Z=Z +F/O*.500
```



```
    15 IF IDARSITR-SIIII .LT. 1.D-6 .AND. DARSIZI-SI2II .LT. 1.D-GIGN.
    al continuf
        go ro 1%n
    0n gINTINUF
ION 7R = S(1)/A
    lI = S(z)/A
    AL = RO*RN - TMM*(IR*IR - ZI*II)
    R = TMM*12.*7R*ZII
    NR = NR + 1
    MR(NR)=R.GRNC* OSORTI.5DC*DARS(OSORT(AL*AL+R*R)-ALII/TMM*2.S*
    NN(NR)=N
180 CONTINISE
    O\cap 50 J = 1.NP
    O7 40 I = 1,NR
    IF (DR(I) &LT. NAS) IS = I
40 IF (DR(I) .LT. DRS) ORS = DR(I)
    ORNIJI = DRS
    DRS = 1.EIC
    ZS(J)= ZRT(IS)
    NS(J)= AN(IS)
50 DR(IS)=1.F?n
    NQOMT = NR
    RFTURN
    FND
```

```
SIIRROUTINE NRAPH (FSMAL,TS,IF,NI
```

CЭMDLEX*P FSMAL.7S.7F
CNMOLEX*1世 7,F,D,CTANT,CSECZ
REAL*R RI, II, H( $)$ )
FOUIVALFNCE $(7, H\{1)$
$1=7 \mathrm{~S}$
$J=$ ?
กก 2n $1=1.109$
$Q I=H(1)$
ZI = Hİ

IF (N GT. OI CTAN7 = - OLOS(T)/COSIN(7)
IF IN.GT. nI CCFCT $=1 . / \operatorname{COCOS} Z 1$
IF (N.IT. A) CSEC7 = 1. ICDSIN!T)
$F=F S M A L+Z *$ CTANT
$\mathrm{p} \quad=\mathrm{CTANL}-7 * C S F C Z \angle S F C Z$
IF (COARS(P).LT. 1.n-4C) GOTO 22

$J=J+1$
IF IJ.CT. SI GC TN 27
$L=7+F / 0 * .5 \cap ก$

15 IE(NABS(P7-H(1)).1T.1.D-6.AND.OABS(7)-H(2)).1. .1.0-6) GOTO 21
30 RONTINUF
$227 F=(M P L \times(1, F \rightarrow n, \cap . n)$
RETIJRN

RFTIIRN
FND

```
            SURROIITINE HWIAS (D,OR,N,AM,FRO,C,M,OHS,DHH,FSMAL,H,FS)
            COMPLEX FSMAL,R,FS
            COMPLEX*IA 7,F,D,CTANZ,CSFCZ,RR,DFS
            REAI.*Q QZ,TI,SI2I,A,RK,PI,YK,TMM,DAM,AL, R
            FOUIVALFNCE {7,S(1)|
            A = DHS*2.54/H
            PI = 3.1415026536
            RK = 2.DC*D{#FRQ/C.
            DAM = AM
            YK = OI*(M-1)/0HH/?.54
            OFS = FSMAL
            OQ = O.0
            Z = R
            IF (S(1) GF. 1.O2O) RFTURN
            R = CMP\X(1.F3n.N.N)
            TMM = 1.DC - DAM*DAM
            IF (AM.FD. O.O) Gר TT 1CO
            J=0
            On 91 k = 1.1nn
            RZ=S\11
            7I = S(?)
                            IF (CABSIR7),GT. 3.E371O15 . חR.DABS\7II.GT. 174.67ON) DETUR.
                            IF IN .GT. O\ C.rANZ = -COSIN(7)/CNCOSITI
                            IF (N -LT. १) CTANT =COCOS(T)/CDSIN(7)
                            IF {N ©TT. n) CSFCT=1./CDCOS(II
                            IF IN -LT. n\ CSFCT = 1./CDSINITI
                            RQ= CISQRT(1.NO-TMM/RK/RK*{7*7/A/A & YK*VKII
                            F= OFS/TMM/TMM*(1.DN - OAM*RR)**?
    F=F* Z*CTAN ?
    D = RK*RK*A*A*TMM
    P=2.OC*\capFS*(1.\capח-ПAM*RR)*ПAM/RR*//P
    O = D CTANZ - T*CSECZ*CSECT
    IF (CDARSIO).LT. 1.O-4OI RFTURN
    Z=7-F/O
    IF (SII\.GF.N.ON.AND.SI2).GE.O.DRI GOTO 16
    J=J +1
    IF IJ GT. EI QFTIIRA
    7=l + F/O*.50r
    IF (STI\.LY.C.DN .OR.SITI LLT. O.ONI RETURN
    15 IF IDABSIRT-SC1H.LT.1.n-G AND NARSITRETURN
    CONTINIJE
    RFTIIRN
1OO 2O = S(11/A
    ZI= S121/A
    R=2
    AL = RK#RK - TMM* (7D*7? - IT* \II + YK*YK)
    R=-TMM*(?.*7O*II)
    NQ=Q.GODO*ПSORT(.5חO*OABS(DSORT(AL*AL+R*R)-AL)//TMM*2.54
    FNO
```

```
    SIIRROIITINF ROROFQ (AM,AK,G,MCUX,Z,NN,OP)
    COMOLFX*? 2121).,Y{211.7NR(?11.ZKNR(211.71
    OIMFNSITN NN(14),NNR(14),OR(14)
    WQITE (t,?)
    ON In ! = l.Mr.UX
    ZI=\DeltaK*AK - (1.-\DeltaM*AM|*Z(I)*Z|I:/G/G
1^7K1!1=-\DeltaM*AK rSOQTII11
    \cap7 3n J = 1,MCUX
    AIT = - 1.F?n
    On }20\textrm{K}=1.MCJ
    IF {PFAL(IK(K)},LT. RIGI GO TO 2?
    RIG= RFAI (ZK(K))
    KS =K
OO CONTINUF
    7NQ(J)=7(KS)
    7KNR(J)=ZK(KS)
    DS(J)= - 2.54*R.60*AIMAGIZK(KS))
    NNR(J)=NN{KS}
20 LK(KS)= CNPLX(-1.525.0.9)
    OO 4?L = 1,MCUX
    Z(L)= 2AP(L)
    7K(L)=7KNR{L}
    NV(L)= NNQ(L)
4O WRITE (A,?) 7(1), ZK(L),O9(L)
    DFTIIRN
l FIRMAT (*I 7,KI,OR FROM SUBROUTINE RORNFP:I
3 FOOMAT 1'OA,1PKF14.51
    FNO
```

```
    SUBROUTINE RORDER (AM,AK,G,MCUX,Z,NN,OB)
    COMPLEX*8 Z(21).2K(21).lNR(21),2KNR(21),71
    DIMENSION NN(14),NNR(14),DE(14)
    WRITE (G.1)
    DO 10 I = 1,MCUX
    Z1 = AK*AK - (1.-AM*AM)*Z|I)* L(I)/G/G
10 LK(I) = (-AM*AK + CSORT(II|)/(1.0 - AN*AM)
    DO 2 J = 1,MCUX
    AIG=-1.E20
    OO 20 K = 1,MCUX
    IF (REAL(ZK(K)).LT. BIG) GO TO 20
    AIG= REAL(IK(K))
    KS = K
20 CONTINUE
    LNR(J) = Z(KS)
    ZKNR(J)= 2K{KS)
    OR{J)=-2.54*B.68*AIMAG{ZK(KS\)
    NNR(J)= NN(KS)
30 ZK(KS)=CMPLX(-1.E25.0.0)
    OD 4OL=1.MCUX
    Z(L)= ZNR(L)
    ZK(L)= ZKNR(L)
    NN(L)=NNR(L)
40 WRITE (t,3) L(LI,ZK(t),DB(L)
    RFTURN
l FORMAT ('1 2,KZ, OB FROM SUBROUTINE PORDER')
3 FORMAT ('C',1PSF14.5)
END
```




DB OIFFERENCE AFTER 4.00 INCHES OF LINING

| 2000. | 2.60 |
| :--- | :--- |
| 2100. | 4.09 |
| 2200. | 4.30 |
| 2300. | 4.60 |
| 2400. | 4.98 |
| 2530. | 5.43 |
| 2600. | 5.92 |
| 2700. | 5.40 |
| 2800. | 7.04 |
| 2900. | 7.63 |
| 3003. | 8.15 |
| 3100. | 8.91 |
| 3200. | 8.53 |
| 3300. | 8.21 |
| 3400. | 7.93 |
| 3500. | 7.65 |
| 36.30. | 7.33 |
| $37 C 0$. | 6.98 |
| 3800. | 6.62 |
| 3903. | 6.28 |
| 4000. | 5.97 |

DB DIFFERENCE AFTER 8.00 INCHES OF LINING

| 2000. | 4.93 |
| :--- | ---: |
| 2100. | 7.10 |
| 2200. | 7.78 |
| 2300. | 8.55 |
| 2400. | 9.44 |
| 2500. | 10.41 |
| 2630. | $11.4 t$ |
| 2700. | 12.54 |
| 2800. | 13.64 |
| 290. | 14.72 |
| 30.00. | 14.88 |
| 3100. | 14.02 |
| 3200. | 12.60 |
| 3300. | 11.59 |
| 340. | 10.82 |
| 35.00. | 10.21 |
| 3600. | 9.68 |
| 3700. | 9.20 |
| 3830. | 8.76 |
| 3900. | 8.36 |
| 4000. | 7.99 |


| 2000. | 7.03 |
| :--- | ---: |
| 2100. | 9.68 |
| 22100. | 10.80 |
| 2390. | 12.05 |
| 2400. | 13.42 |
| 2530. | 14.94 |
| 2600. | 16.52 |
| 2730. | 18.04 |
| 2800. | 19.53 |
| 290. | 20.79 |
| 3000. | 19.89 |
| 3100. | 17.62 |
| 3200. | 15.48 |
| 330. | 14.01 |
| 3400. | 12.91 |
| 3500. | 12.05 |
| 3600. | 11.35 |
| 3700. | 10.76 |
| 3800. | 10.24 |
| 3930. | 9.78 |
| 4000. | 9.36 |

FREQUENCIES FINAL SPECTRUA

| 2000 | TO | 2300 | 127.97 | 125.32 | 124.20 | 122.95 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2400 TO 2700 | 121.57 | 120.06 | 118.48 | 116.96 |  |  |
| 2800 rO 3100 | 115.47 | 114.21 | 115.11 | 117.38 |  |  |
| 3200 TO 3500 | 119.52 | 120.99 | 122.09 | 122.95 |  |  |
| 3600 TO 3900 | 123.65 | 124.24 | 124.76 | 125.22 |  |  |
| 4000 TO 4000 | 125.64 |  |  |  |  |  |
| UB CIFFERENCES (SHOOTHEDI AFTER 12.00 INCHES OF LINING |  |  |  |  |  |  |


| 8.35 | 10.24 | 11.42 | 12.74 | 14.18 |
| ---: | ---: | ---: | ---: | ---: |
| 15.73 | 17.28 | 18.79 | 20.16 | 20.34 |
| 18.75 | 16.55 | 14.75 | 13.46 | 12.48 |
| 11.70 | 11.05 | 10.50 | 10.01 | 9.57 |

APPENDIX IV
PROGRAM FLOW CHART










## REPRODUCIBILTTY OF THE <br> ORIGINAL PAGE IS POOR















$v$

## REFERENCES

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