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EFFECT OF NONSYMMETRICAL FLOW RESISTANCE UPON ORIFICE IMPEDANCE

by

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16. Abstract

Previous laboratory work has indicated that an orifice in a thin sheet behaves in a quasi-steady manner under acoustical excitation. Also, it has been found that the steady flow resistance of an orifice may be dependent upon the direction of flow, especially in the presence of a crossflow on one side of the hole. Here, an analytical study is presented which assumed a non-reactive orifice in an infinite baffle. The pressure difference Δp across the orifice varies sinusoidally with amplitude 1.0 and average value $-P$. The orifice resistance, $\Delta p/u$, is discontinuous at zero velocity and exhibits the constant values R_+ and R_- for $u > 0$ and $u < 0$, respectively. The resultant velocity has power in all harmonics of the excitation frequency, providing an explanation of the even harmonic excitation observed by other investigators, but not predicted by symmetric nonlinearity. A quasi-linear resistance is defined and found to be relatively insensitive to the presence or absence of a resonant backing cavity; however, it does vary from $1.33\ R_+$ to $0.67\ R_+$ for a resistance ratio R_+/R_- between 0.5 and 2.0.

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EFFECT OF NONSYMMETRICAL FLOW RESISTANCE

UPON ORIFICE IMPEDANCE

by

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INTRODUCTION

The acoustical behavior of orifices has been under study for more than a century (ref. 1), primarily because of their usefulness in resonators. During the past four decades, work has concentrated upon the nonlinearity of the response of an orifice to an acoustical excitation (refs. 2 - 5). For low frequencies, oscillating flow through an orifice may be assumed to be quasi-steady, so than an understanding of the dc flow situation can be straight forwardly applied to the transient case. For steady flow, the orifice behavior may be completely specified by the function $R(u)$, defined as the ratio of the pressure difference, Δp, across the hole to the resulting velocity u through the hole.

$$R(u) = \Delta p/u \qquad (1)$$

Budoff and Zorumski (ref. 6) measured the dc flow resistance of perforated plates mounted in the side wall of a flow duct, and they reported an apparent discontinuity at zero through flow ($u=0$). The graph in figure 1 shows with a solid line, the trend of the observed resistance $R(u)$. Similar measurements recently reported by Feder (ref. 7) also indicate a strong asymmetry in $R(u)$ for some circumstances. If there is a cavity behind the hole as is the normal situation, then there is less resistance to the cavity inflow than to its outflow. Even in the absence of a tangential flow, an orifice of asymmetrical construction (either by accident or by design) will exhibit a nonsymmetrical resistance. The asymmetry may be a rounded entry on one side and a sharp entry on the other, as is shown in figure 1, or it may be the presence of knicks or burrs on the material.

In the current paper, the non-symmetrical resistance to be studied is of the form indicated by the dashed lines in figure 1, i.e.,

$$R(u) = \begin{cases} R_+ , & u > 0 \\ R_- , & u < 0 \end{cases} \qquad (2)$$

LIST OF SYMBOLS

M_∞	Mach number of grazing flow
N	harmonic index
p_1	pressure on side 1 of orifice
p_2	pressure on side 2 of orifice
P	average pressure rise $(p_2 - p_1)$ over time T
R	steady flow orifice resistance, $\Delta p/u$
R_{eq}	quasi-linear orifice resistance, $1/u_1$
R_+	constant value of R for $u > 0$
R_-	constant value of R for $u < 0$
t	time
T	period of Δp, $2\pi/\omega$
u	orifice through flow velocity from side 1 to side 2
u_N	amplitude of $u(t)$ component at frequency $N\omega$, $\lvert U_N \rvert$
U_N	complex Fourier coefficient of $u(t)$ at frequency $N\omega$
Δp	pressure difference, $p_1 - p_2$
ϕ_N	phase of $u(t)$ sine component at frequency $N\omega$, $\frac{\pi}{2}$ - phase U_N
ω	angular frequency of Δp

Mathematical Development

For the sake of simplicity, the problems of wave excitation and wave transmission and reflection are ignored. Thus, the absolute values of $p_1(t)$ and $p_2(t)$, the pressures on the two sides of the orifice, are not specified, but the pressure drop, $\Delta p = p_1 - p_2$, is assumed to be sinusoidal with angular frequency ω, unit amplitude and average value $-P$.

$$\Delta p(t) = \sin \omega t - P \tag{3}$$

This pressure history and the resulting velocity history are plotted in figure 2. Notice that one period T of the velocity is made up of segments from two different sine waves.

$$u(t) = \begin{cases} \dfrac{\sin \omega t - P}{R_+} & , \sin \omega t > P \\ \dfrac{\sin \omega t - P}{R_-} & , \sin \omega t < P \end{cases} \tag{4}$$

As with any nonlinear deterministic system, energy in a sinusoidal excitation Δp is distributed into harmonics of the excitation frequency ω. This is easily quantized by expressing $u(t)$ as the sum of its average value u_0 and contributions from harmonics 1 and above.

$$u(t) = u_0 + \sum_{N=1}^{\infty} u_N \sin(N\omega t + \phi_N) \tag{5}$$

Here, $u_N = |U_N|$ and $\phi_N = \frac{\pi}{2} - \text{phase } U_N$

where, $$U_N = \frac{1}{\pi}\int_0^T u(t)\, e^{i\omega t}\, dt$$

The appropriate expressions for u_N and ϕ_N when $u(t)$ is as given in equation 4 are shown in figure 3. Notice that u_N is a function of the two resistances, R_+ and R_-, and the average pressure rise P. Also, u_N may have a non-zero value at any harmonic number N. Ingard (ref. 8) in a relevant experiment observed energy transfer into both even and odd harmonics (as did Thurston, Hargrove and Cook, ref. 3), even though his nonlinear theory predicted excitation of only the odd harmonics. Ingard postulated that the energy in the even harmonics might be the result of his using an asymmetrical orifice in the test, and the present study substantiates that conjecture.

In some situations, one might be interested only in the velocity response at the fundamental frequency of pressure excitation ω. Thus, it is appropriate to examine the equivalent, quasi-linear resistance R_{eq} defined as the amplitude of the exciting sinusoidal pressure differential divided by the amplitude of the first harmonic component of the resulting velocity. Since Δp has unit amplitude,

$$R_{eq} = \frac{1}{u_1} \tag{6}$$

The full expression for $R_{eq}(R_+, R_-, P)$ is given in figure 4.

Special Cases

In order to obtain a better idea of the physical significance of having a directional resistance, two special cases representing physical extremes (though not mathematical extremes) are examined in this section. If each of the two sides of the orifice is in communication with a common pressure reservoir (such as the atmosphere) then the average pressure rise P across the orifice over a period T must be zero. On the other hand, if the orifice is backed by a closed, resonant cavity then in the steady state the average flow velocity u_0 over a period T must be zero. These two cases are examined in turn.

In the case P = 0, the expression for u_0 reduces to

$$u_o = \frac{1}{\pi R_+} \left(1 - \frac{R_+}{R_-} \right) , \quad P = 0 \tag{7}$$

as plotted in figure 5. The net flow vanishes only in the trivial case $R_+/R_- = 1$ and varies linearly with the resistance ratio. In the same figure, the rms power levels of harmonics 0 through 6 are shown in dB relative to the rms power of u(t) for a resistance ratio of 0.5. Since there is no bias pressure differential, all of the flow is due to the pressure oscillation, $\Delta p = \sin \omega t$, yet a steady flow is induced which is only 11dB below the total power. This zeroth harmonic together with the second harmonic (-17 dB) accounts for most of the energy not in the fundamental (-0.5dB). Odd harmonics above the fundamental are not excited at all, and the even harmonics fall off rapidly.

Consider next the more common case of a resonant cavity behind the hole which results in zero net flow. When u_0 is set equal to zero, P becomes a function of the resistance ratio. In particular, $P(R_+/R_-)$ must satisfy the following transendental equation.

$$\frac{\pi}{2}\left(1+\frac{R_+}{R_-}\right) P - \left(1-\frac{R_+}{R_-}\right)\left[P \sin^{-1} P + (1-P^2)^{1/2}\right] = 0 , \quad u_o = 0 \tag{8}$$

The solution is plotted in figure 6. For R_+/R_- less than unity, the cavity is pumped up; i.e., the average gage pressure P in the cavity is greater than zero. When the resistance ratio exceeds unity, there is more resistance to cavity inflow than to outflow, so that it becomes pumped down. In any event the induced value of P cannot be greater than 1 or less than -1. A resistance ratio of 0.5 implies $P = 0.217$, and the subsequent harmonic power distribution in u(t) is as shown in figure 6. The fundamental is only 0.1 dB below the total, and the second harmonic is 17 dB down. All higher harmonics are excited, but each is at least 30 dB below the fundamental.

In each of the two special cases discussed here, the normalized equivalent resistance R_{eq}/R_+ is a function only of the resistance ratio R_+/R_-. As is indicated by the graph in figure 7, R_{eq}/R_+ varies from 1.33 to 0.67 over what might be considered the practical range of $0.5 < R_+/R_- < 2.0$; however, in this range the two curves corresponding to $u_0 = 0$ and $P = 0$ are virtually indistinguishable. Thus, while it is important to be aware of the effect of a resistance asymmetry upon the effective resistance, the cavity backing condition seems to have little effect upon R_{eq}.

SUMMARY

This study shows that the even harmonic excitation noted in earlier experimental work on orifice impedence may be due to asymmetrical flow resistance rather than finite amplitude effects. Under excitation by a sinusoidal pressure difference, the velocity through the orifices is nonsinusoidal. It contains contributions at all harmonics of the excitation frequency, and the second harmonic component may be less than 20 dB below the component at the fundamental frequency. Even in situations where the presence of higher harmonics may not be important, the existance of a nonsymmetrical resistance may result in an equivalent quasi-linear resistance which varies by as much as 30% or more from the steady flow resistance measured for a single flow direction.

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STEADY FLOW ORIFICE RESISTANCE

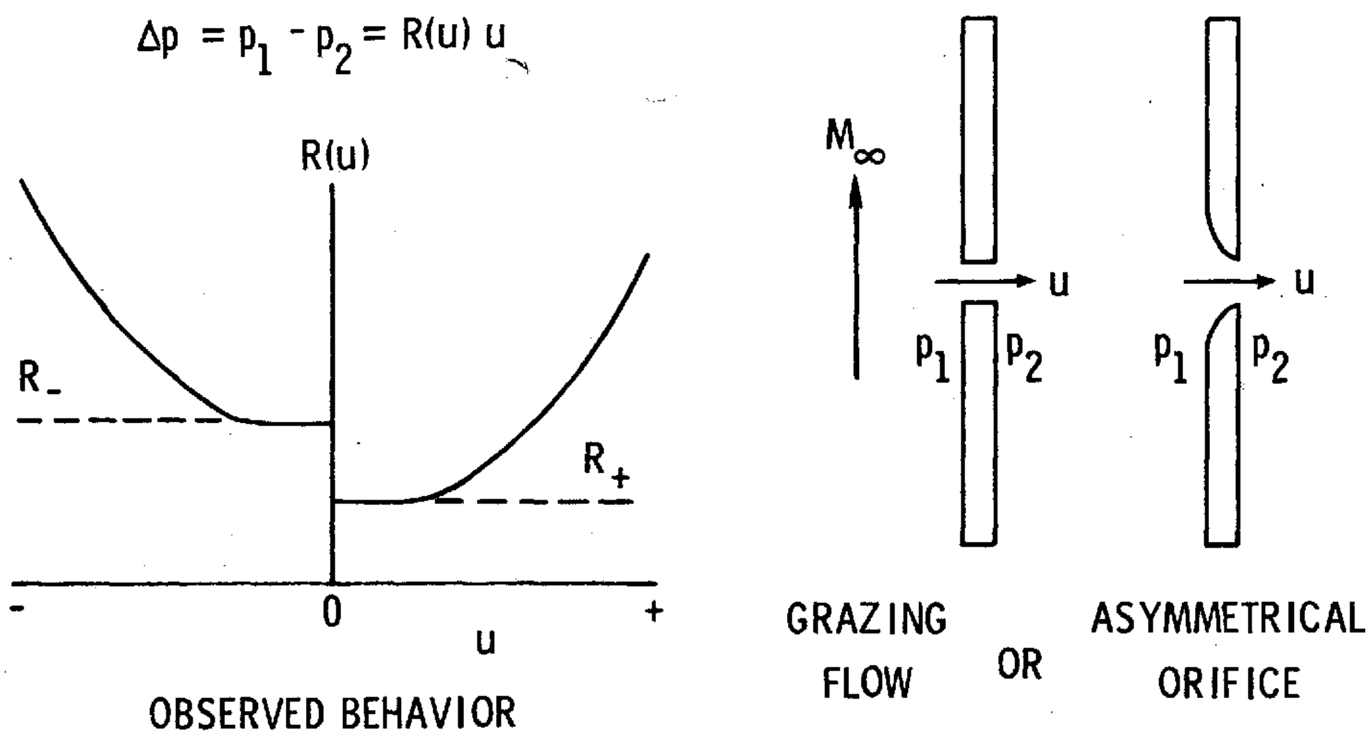


FIGURE 1

ASSUMED PRESSURE AND VELOCITY HISTORIES

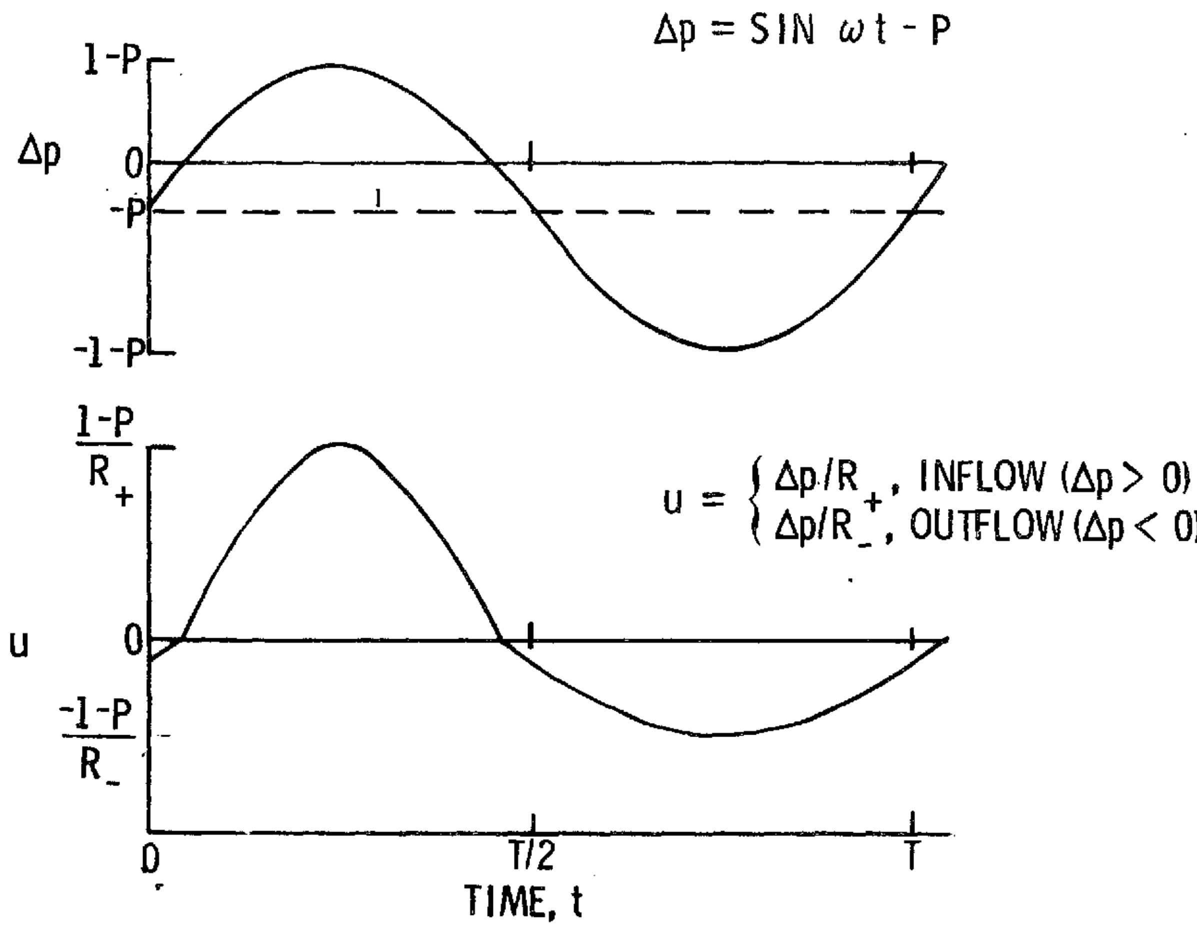


FIGURE 2

HARMONIC ANALYSIS OF u(t)

$$u_0 + \sum_{N=1}^{\infty} u_N \,\mathrm{SIN}\,(N\omega t + \Phi_N) \equiv \underline{u(t)} = \begin{cases} \dfrac{1}{R_+}\ \mathrm{SIN}\,\omega t - \dfrac{P}{R_+}\ , \text{ INFLOW} \\ \dfrac{1}{R_-}\ \mathrm{SIN}\,\omega t - \dfrac{P}{R_-}\ , \text{ OUTFLOW} \end{cases}$$

$$u_0 = \frac{1}{R_+}\left\{\frac{P}{2}\left(1 + \frac{R_+}{R_-}\right) + \frac{1}{\pi}\left(1 - \frac{R_+}{R_-}\right)\left[P\,\mathrm{SIN}^{-1}P + (1 - P^2)^{1/2}\right]\right\}$$

$$u_1 = \frac{1}{R_+}\left\{\frac{1}{2}\left(1 + \frac{R_+}{R_-}\right) - \frac{1}{\pi}\left(1 - \frac{R_+}{R_-}\right)\left[\mathrm{SIN}^{-1}P + P\,(1 - P^2)^{1/2}\right]\right\}$$

$$u_N = \frac{1}{\pi R_+}\left(\frac{R_+}{R_-} - 1\right) V_N\,, \quad N > 1$$

$$V_N = \begin{cases} \dfrac{\mathrm{SIN}\,[(N-1)A]}{N-1} - \dfrac{\mathrm{SIN}\,[(N+1)A]}{N+1} + \dfrac{2P}{N}\ \mathrm{COS}(NA)\ , \text{ N ODD} \\ \dfrac{\mathrm{COS}\,[(N-1)A]}{N-1} - \dfrac{\mathrm{COS}\,[(N+1)A]}{N+1} - \dfrac{2P}{N}\ \mathrm{SIN}(NA)\ , \text{ N EVEN} \end{cases}$$

WHERE $A = \mathrm{SIN}^{-1}P$

$$\Phi_N = \begin{cases} 0, \text{ N ODD} \\ \pi/2, \text{ N EVEN} \end{cases}$$

FIGURE 3

QUASI-LINEAR ACOUSTICAL RESISTANCE, R_{eq}

$$\text{EQUIVALENT LINEAR RESISTANCE} = \frac{\text{SINUSOIDAL PRESSURE AMPLITUDE}}{\text{1ST HARMONIC AMPLITUDE OF } u}$$

$$R_{eq} = \frac{1}{u_1}$$

$$R_{eq} = \frac{2R_+R_-}{R_++R_-}\left\{1 - \frac{1 - R_+/R_-}{\pi\left(1 + R_+/R_-\right)}\left[P\left(1 - P^2\right)^{1/2} + \mathrm{SIN}^{-1}\, P\right]\right\}^{-1}$$

FIGURE 4

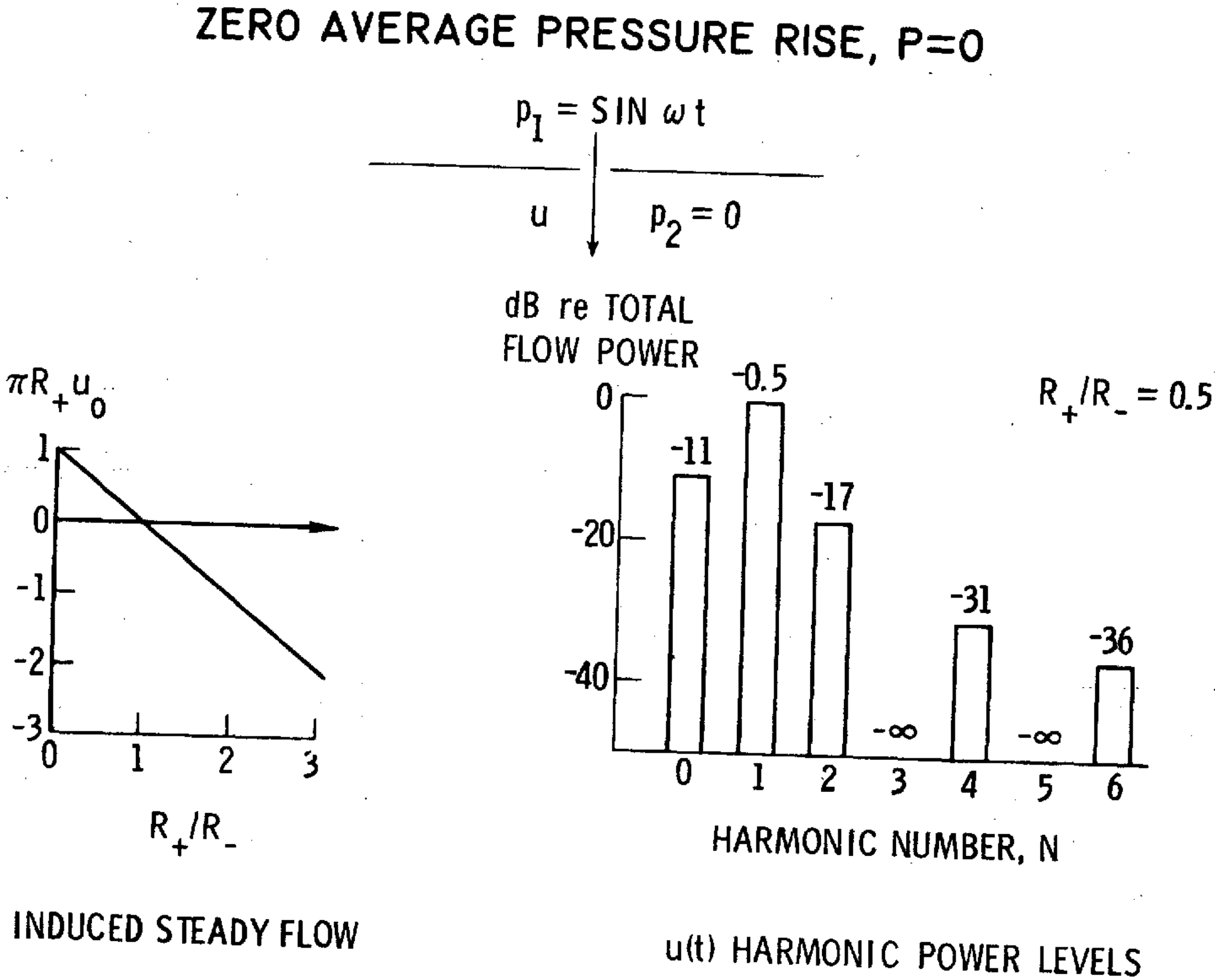
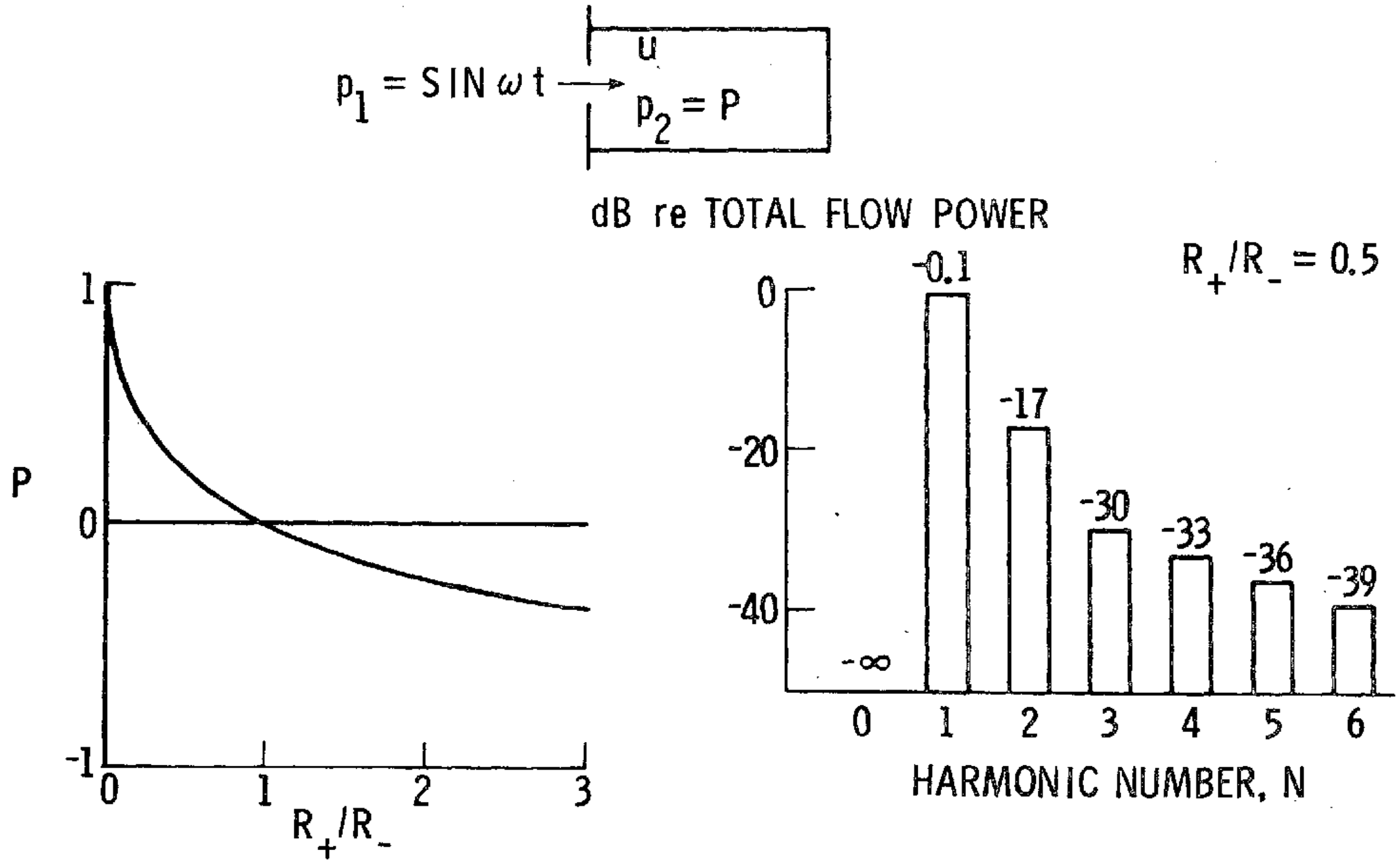

ZERO AVERAGE PRESSURE RISE, P=0
p_1 = SIN ωt
u
p_2 = 0
πR+u0
1
0
-1
-2
-3
0
1
2
3
R+/R-
INDUCED STEADY FLOW
dB re TOTAL
FLOW POWER
0
-20
-40
-11
-0.5
-17
-∞
-31
-∞
-36
R+/R- = 0.5
0
1
2
3
4
5
6
HARMONIC NUMBER, N
u(t) HARMONIC POWER LEVELS

FIGURE 5

ZERO AVERAGE VELOCITY, $u_o = 0$

$$u_o = \frac{1}{T} \int_T u(t)dt = 0 \Rightarrow P = P(R_+/R_-)$$



INDUCED PRESSURE RISE

u(t) HARMONIC POWER LEVELS

FIGURE 6

QUASI–LINEAR RESISTANCE VERSUS RESISTANCE RATIO

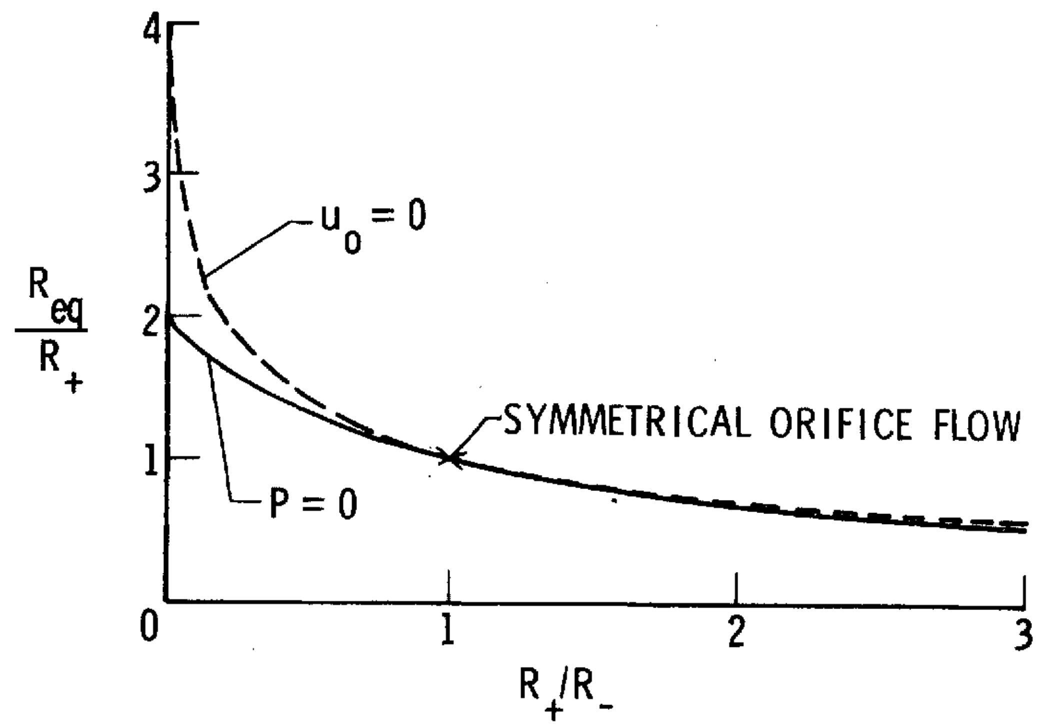


FIGURE 7