NASA TECHNICAL NOTE

NASA TN D-7663

NASA TN D-7663

(NASA-TN-D-7663) COMPUTER SIMULATION OF THE THRESHOLD SENSITIVITY DETERMINATIONS (NASA) 33 p HC $3.75 CSCL 12A

N75-12690

Unclas
H1/65 04202

COMPUTER SIMULATION OF THRESHOLD SENSITIVITY DETERMINATIONS

J. B. Gayle

John F. Kennedy Space Center
Kennedy Space Center, Fla. 32899

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1974

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-7663		

4. Title and Subtitle	5. Report Date
Computer Simulation of Threshold Sensitivity Determinations	December 1974
	6. Performing Organization Code
7. Author(s) J. B. Gayle	8. Performing Organization Report No.
9. Performing Organization Name and Address Support Operations - Laboratories Division Kennedy Space Center, Florida 32899	10. Work Unit No. QP00
	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Note
12. Sponsoring Agency Name and Address	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A computer simulation study was carried out to evaluate various methods for determining threshold stimulus levels for impact sensitivity tests. In addition, the influence of a number of variables (e.g., initial stimulus level, particular stimulus response curve, and increment size) on the apparent threshold values and on the corresponding population response levels was determined. Finally, a critical review of previous assumptions regarding the stimulus response curve for impact testing is presented in the light of the simulation results.

17. Key Words	18. Distribution Statement
Sensitivity testing, impact testing, oxygen compatibility, stimulus response curves, threshold determinations, computer simulations	Unlimited CAT. 18

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	31	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22151

ABSTRACT

A computer simulation study was carried out to evaluate various methods for determining threshold stimulus levels for impact sensitivity tests. In addition, the influence of a number of variables (e.g., initial stimulus level, particular stimulus response curve, and increment size) on the apparent threshold values and on the corresponding population response levels, was determined. Finally, a critical review of previous assumptions regarding the stimulus response curve for impact testing is presented in the light of the simulation results.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

COMPUTER SIMULATION OF THRESHOLD SENSITIVITY DETERMINATIONS

by J.B. Gayle

Kennedy Space Center

INTRODUCTION

It is characteristic of sensitivity tests that relatively large numbers of trials are required to obtain results of acceptable precision. This is of little concern so long as the cost per trial (measured in dollars or time) remains small. However, when the cost per trial becomes appreciable, it tends to limit both the quantity and quality of data collected. Costs have always been a problem in the impact testing of materials at ambient pressure for compatibility with liquid oxygen for which the cost per trial is of the order of 2 dollars. However, recent modifications of the apparatus to permit testing of materials in both liquid and gaseous oxygen at pressures up to 6.89×10^7 N/m^2 (10,000 psi) have greatly increased the cost per trial. Also, damage to the test fixture, which frequently occurs when a reaction is obtained at high pressures, has further increased the cost of those particular trials. Moreover, it appears that due to the extreme reactivity of most materials with oxygen at high pressures, testing at a single stimulus level (as has usually been the case in the past) may not be sufficient and it may be necessary to modify the test procedure to include determination of some form of threshold stimulus level, i.e., the maximum level at which no reactions occur under the test conditions.

To assist in selecting a procedure which would maximize the information return per unit cost, a computer simulation study was carried out to compare the number of trials, the number of reactions, and the probability of reaction corresponding to the apparent threshold stimulus level determined by each of several procedures using a wide range of stimulus response curves. The influence of a number of other variables was also studied.

RESULTS OF PREVIOUS INVESTIGATORS

Sensitivity tests are common to many technical disciplines (e.g., bacteriology, metallurgy, chemistry, etc.) and have been studied extensively by various workers. A comprehensive survey of the literature was conducted by Rothman, Alexander, and Zimmerman under contract to NASA in 1965 (ref. 1). In general, the more extensive studies have been carried out to develop procedures for estimating the stimulus level corresponding to some particular percentage response (usually 50, 5, or 1) either with or without some specific assumption as to the form of the stimulus response curve. For example, much attention has been devoted to the up and down or Bruceton method for determining the stimulus level corresponding to 50 percent responses under the assumption that the response curve is the cumulative normal distribution function. This method should be ideal when the mid-portion of the curve is of direct interest for some particular application. However, most sensitivity tests are carried out for reliability or safety purposes and the portions of the curve which are of direct interest for these applications generally fall between 95 and 100 percent or between 0 and 5 percent, thus requiring extrapolation beyond the range of most of the data. Although the Bruceton method has been used extensively in conjunction with impact tests, the response curve is not always cumulative normal; in which case, extrapolation can lead to gross errors.

A number of procedures for direct determination of response levels in the 95 to 100 or 0 to 5 percent range have been developed and evaluated by both simulation and analytical methods, and some consideration has been given to the determination of threshold values. Although some of these procedures appear well suited to impact testing, little attention has been given to the significance of the apparent threshold values or to the manner in which these values are influenced by the particular stimulus response curve and other variables.

DETAILS OF SIMULATION

In developing the simulation program, an attempt was made to obtain information with regard to sensitivity testing in general, yet having specific application to the impact testing of materials for compatibility with oxygen.

The simulation of sensitivity test data consists essentially of the simulation of results for individual trials together with the various decision steps necessary to determine if the testing sequence should be terminated, at what response level the next sequence is to be initiated, and the necessary data collection and analysis steps to produce summary reports. The simulation of the result for an individual trial for a given population response level (e.g., probability of reaction of 0.20) is accomplished by generating a uniformly distributed pseudo random number, (e.g., 0.37192) and comparing this number with the reaction probability. If the random number is less than the reaction probability, the trial is said to have resulted in a reaction. Otherwise (as in the example), the trial is recorded as a nonresponse.

To be consistent with previous testing, the starting stimulus level was taken as 43 units for most test series. Since it appeared unlikely that materials giving very high percentages of reactions at this level would be considered candidates for high pressure oxygen service, all stimulus response curves were scaled to give a probability of reaction of 0.3300 at this level. Also since a major objective of the simulation was to determine the characteristics of threshold stimulus levels obtained by various methods, each response curve was similarly scaled or truncated to give an absolute threshold value of 10 units.

The first response curve studied was as follows:

$$\text{Prob. Rx.} = 1. - e^{A\,(\text{Stimulus Level} - \text{Threshold})^{B}} \qquad (1)$$

Where:

Prob. Rx. equals zero when Stimulus Level is equal to or less than Threshold.

B = 3.3000

A = -.0000039037

This equation is of the form of the two-parameter cumulative Weibull distribution function and closely approximates the cumulative normal distribution function for the particular value of B used.

The second response curve was as follows:

$$\text{Prob. Rx.} = A\ (\text{Stimulus Level} - \text{Threshold})^{B} \quad (2)$$

Where:

Prob. Rx. equals zero when Stimulus Level is equal to or less than Threshold.

$B = 2.0$

$A = 0.00030303$

The third and fourth response curves were similar to the second except for the values of the constants A and B which were as follows:

$$B = 1.0, \quad A = 1.0 \quad (3)$$

$$B = 0.5, \quad A = 0.057446 \quad (4)$$

These particular curves were selected as being representative of those used by previous investigators and also because they are consistent with much of the available experimental data. Figure 1 depicts the four stimulus response curves graphically. Inspection of the plots shows that, for the first curve, the probability of a reaction fell to a relatively low value well above the threshold stimulus level and decreased slowly thereafter. For the fourth curve, the probability of reaction did not fall to a low value until the stimulus level had decreased to a value just above the population threshold. Beyond this point, the probability of reaction decreased rapidly to zero. The second and third curves were between the first and fourth.

The first method (method 1) used to determine an apparent threshold value was that described by Schwinghamer and Key (ref. 2). This method consisted of running 20 trials at the starting stimulus level. The material was then accepted if the number of reactions was zero, rejected if the number was two or more, or subjected to an additional 40 trials if the number was exactly one. The material was subsequently accepted if no reactions were noted in the additional trials; otherwise it was rejected. In the event the material was rejected at the initial stimulus level, this level was decreased by some pre-established increment and the process repeated. The stimulus level at which the material was finally accepted was taken as the threshold value.

The other methods used represented successive modifications of the first, intended to reduce both the total number of trials and the total number of reactions sustained in any test series. The second method (method 2) was similar to the first except for terminating the sequence at any level as soon as two reactions had been noted, i.e., as soon as it became evident that the material would be rejected at that stimulus level.

The third method (method 3) deleted the requirement for additional trials when exactly one reaction was noted in the first 20. In this modification, the material was accepted if there were no reactions in 20 trials. Otherwise, the sequence was terminated as soon as a reaction occurred and the stimulus level was decreased by one increment prior to the next sequence.

Two other methods were used in which the stimulus level was decreased by two increments when testing at any given level was terminated early in the sequence. Thus, the fourth method (method 4) was identical to the second except that when two reactions were noted within the first 10 trials, the stimulus level was decreased by two increments. Similarly, the fifth method (method 5) was identical to the third except that the stimulus level was decreased by two increments when a reaction was noted within the first five trials.

Increments of 4 to 6 units have frequently been used in connection with impact testing. However, since this variable has a major influence on all the results of interest, increments of 1, 1.5, 2, 2.5, 4, 6, 8, 10, and 12 were included. Also, the initial stimulus level was varied for some test series.

The random number generator was of the multiplicative congruential type and was subjected to the usual Chi square and similar tests. The adequacy of the cumulative Weibull distribution function with B equal 3.3 for approximating the cumulative normal distribution function was confirmed by plotting values calculated from this function on probability paper. The validity of some portions of the simulation program was confirmed by generating hypothetical data for tests on a wide range of materials using the basic test procedure at a fixed stimulus level and comparing the results with those expected for a binomial process. The results of this exercise are included in the Appendix.

RESULTS OF SIMULATIONS

Results for roughly 10 percent of the various simulation runs are summarized in Tables 1 through 5 and Figures 2 through 6. The initial stimulus level was 43 units for all simulations. Since 100 simulation runs were made in each case, the standard errors of the mean threshold values are just 10 percent of the tabulated values for the standard deviations.

Table 1 gives data obtained by method 1 which consisted of carrying out the full MSFC SPEC 106B procedure at each stimulus level. Included in the table are results using increments or step sizes ranging from 1 to 12 units for each of the four response curves. Both the large number of total tests and the large number of reactions (not given) make use of this procedure extremely expensive for routine testing.

Table 2 was obtained by use of method 2 which was identical to method 1 except that testing at any given stimulus level was terminated after two reactions had been noted. Differences in threshold values for Tables 1 and 2 are small and are due to sampling variations resulting from use of different sequences of random numbers. The total number of trials for method 2 ranged from 40 to 70 percent of the corresponding values for method 1.

A further marked decrease in the number of trials was noted for method 3 which was similar to method 2 except that testing at any given stimulus level was terminated after the first reaction. The data, given in Table 3, indicate that the total number of trials ranged from 22 to 43 percent of the values for method 1. The total number of reactions (not given) was generally approximately half that for method 2. Corresponding changes in threshold and standard deviation values were small and in most instances were not significant at the 5 percent level.

Tables 4 and 5 present results obtained by methods 4 and 5 which were similar to methods 2 and 3 except that the stimulus level was decreased by two increments when testing at any level was terminated early in the sequence, i.e., within 10 trials for method 4 or within five trials for method 5. Comparison of these results with those for Tables 2 and 3 indicate some further decrease in the total number of trials. Although results for method 4 do not differ appreciably from those for method 2, over half the threshold values for method 5 differed significantly at the 5 percent level from those for method 3. Further inspection of the data suggests that in the latter case the modification is roughly equivalent to increasing the size of increment used. Thus, the changes in both the total number of trials and the threshold values are roughly similar to those that could be obtained by increasing the size of the predetermined increment for any given test series.

Taken together, the data indicate that method 3 can be used in place of method 1 without significant changes in the resulting threshold values or standard deviations. This is not unexpected when it is recognized that little information is gained by completing the planned testing sequence at any level once it is evident that testing must be continued at the next level. Also, the number of instances in which the threshold value is the result of exactly one reaction in the first 20 trials followed by zero reactions in the next 40 trials is small. Eliminating this possibility, therefore, has little influence on the test results but permits a considerable reduction in both the total number of trials and the total number of reactions. Decreasing the stimulus level by two increments in certain instances caused significant changes in many of the threshold values and was not pursued further.

Still other methods could be devised for determining threshold values and, to the extent that they modify the size of increment or other steps in the procedure to take account of information obtained during testing, some further reduction in the total number of tests and/or reactions may be possible. However, method 3 permits a major reduction both in total number of trials and reactions by comparison with method 1 without impairing the existing data base which has been accumulated over a period of some 15 years.

Although the influence of size of increment, starting stimulus level, and stimulus response curve was studied for each of the five methods, only the results for method 3 will be discussed since those for the other methods were similar.

The effects of varying the size of increment from 1 to 12 units are shown graphically in Figure 2 for each of the stimulus response curves. In general, any increase in increment size caused a decrease in the threshold value. The relation appears to be roughly linear with a slope of -0.62. However, decreasing the initial stimulus level caused a corresponding decrease in the value of this slope. Thus, Figure 3 shows that for an initial level of 22 units, the relation appears to be linear with a slope of -0.18.

The influence of the initial stimulus level is further illustrated by Figures 4 and 5 in which these values are plotted against the threshold values for selected sizes of increments for the first and fourth response curves. Inspection of the data for Figure 4 indicates that for the cumulative normal distribution function, decreasing the initial stimulus level has little effect until a value of approximately 30 units is reached. This level corresponds to a probability of reaction of approximately 8 percent for a single trial. Decreasing the initial level below this value resulted in a rapid decrease in threshold values for initial levels down to 22 units. Extrapolation of the decreasing portion of the curve suggests a threshold value of approximately 10 for an initial stimulus level of 10 which is consistent with the population value.

Results given in Figure 5 for the fourth response curve differed somewhat in that there was no tendency for lower threshold values to be determined when the initial stimulus level was reduced below some critical value. The pronounced saw tooth pattern noted for the results using an increment size of 6 units, however, suggests that for this type of response curve, the ratio of the difference between the initial stimulus level and the population threshold value to the size of increment is important. Thus, initial stimulus levels for which this ratio was an integer value yielded apparent threshold values almost one-half increment higher than those initial stimulus levels for which this ratio was equal to some integer value plus one-half.

The nature of the various methods for threshold determination makes it unlikely that the apparent threshold stimulus level resulting from any test series will coincide exactly with the population value. Inspection of the results of the various simulation runs confirms this expectation. Some insight as to the practical significance of the differences between the apparent threshold stimulus levels and the population values is afforded by calculating the probabilities of reactions corresponding to the apparent threshold values by use of the stimulus response functions for the various populations. Data given in the last column of Table 3 indicate that the population responses corresponding to the apparent threshold values range from 10.5 to 0.0 percent. If the size of increment is limited to either 4 or 6 units, as has frequently been done in practice, the range of responses is from 4.6 to 0.0 percent. It is therefore apparent that, on the average, the apparent threshold values correspond to population probabilities of reaction which are well within the 10 percent limit cited by Schwinghamer and Key for the impact test for compatibility of materials in liquid oxygen at ambient pressure. However, the standard deviations of the various threshold values ranged from 2.8 to 5.9 units which indicates that results for individual determinations will exceed this limit in some instances.

Inspection of data given in Table 3 indicates the effects of the various stimulus response curves on the apparent threshold values. For the cumulative normal curve, all threshold values were well above the population value. By way of contrast for the fourth response curve, average values considerably smaller than the population value were frequently determined, particularly for the larger increments.

In view of the marked dependence of the test results on the stimulus response curve, surprisingly little experimental data have been reported which could be used to establish these curves. Instead, most investigators have assumed one or more curves without extensive supporting data, the most common assumption being that the curve is cumulative normal. Inspection of such experimental data as is available does little to resolve the matter in that most determinations have been based on results of 20 trials at each of three to six arbitrarily spaced stimulus levels. The precision of such data is not generally sufficient to permit discrimination between alternate forms of interest. With regard to the cumulative normal curve, the lack of support for this particular assumption is evident when it is recognized that even a much weaker assumption does not always hold, i.e., the probability of a reaction does not always increase monotonically with increasing stimulus level (ref. 3).

Due to the large number of trials needed (several thousand), a rigorous experimental study of this problem would be quite expensive. However, Figure 6 gives liquid oxygen impact results at ambient pressure for two materials determined as a part of the present study. These are a titanium alloy and a styrene butadiene rubber and therefore represent a wide range of mechanical properties.

Inspection of the data for the titanium alloy illustrates the difficulty in the experimental determination of stimulus response curves. Thus, even though each plotted point is based on 100 trials, the scatter of the data is such that considerable judgement must be used in inferring the shape and location of the population curve. For example, the responses between drop heights of 33 and 98 centimeters (13 and 38 inches) could be considered to represent either a smoothly increasing function with considerable scatter as indicated by the solid line, or another instance of non-monotonic behavior as indicated by the dashed line. In any event, the shape of the lower portion of the curve appears to be roughly linear and suggests a threshold drop height of approximately 2.5 centimeters (1 inch). Results for the styrene butadiene rubber are similar to those for the titanium alloy and indicate a threshold drop height of roughly 3.2 centimeters (1.3 inches) with the lower portion of the curve again appearing to be roughly linear.

Curves based on smaller numbers of trials (usually 20 at each stimulus level) are generally more or less similar to those shown above. Thus, threshold determinations for a variety of materials have been reported in connection with a study of the effect of nitrogen dilution on liquid oxygen impact sensitivity (ref. 4). Plots of these data suggest a variety of response functions similar to those selected for the simulation portion of this study. Taken together, the data suggest that stimulus response curves for some materials may be approximated by the cumulative normal distribution. However, much of the existing data is not consistent with this type of relation and assumption of this particular stimulus response curve for materials in general does not appear to be warranted.

SUMMARY AND CONCLUSIONS

The computer simulation study was carried out to evaluate various methods for determining threshold stimulus levels for impact sensitivity tests. The influence of a number of variables on the test results was also considered. The principle conclusions are as follows:

1. Minor modifications to the method of Schwinghamer and Key permit major reductions in the total number of trials and total number of reactions without significantly affecting the results.

2. The apparent threshold values generally correspond to population reaction probabilities of less than 5 percent.

3. The assumption of the cumulative normal distribution function as the stimulus response curve applicable to impact testing in general is not consistent with much of the available experimental data and is not warranted.

Table 1. Simulation Results of Method 1

Response Curve	Increment Size	Apparent Threshold	Standard Deviation	Number of Levels	Total No. of Trials	Reaction Probability
I	1.0	30.4	3.0	13.5	334	.0797
I	1.5	28.5	3.0	10.6	281	.0581
I	2.0	28.6	3.2	8.1	216	.0590
I	2.5	28.1	3.5	6.9	181	.0537
I	4.0	26.0	4.0	5.2	145	.0367
I	6.0	24.2	4.0	4.1	107	.0246
I	8.0	23.5	4.9	3.4	87	.0211
I	10.0	22.9	5.5	3.0	76	.0179
I	12.0	21.2	5.3	2.8	72	.0115
II	1.0	26.8	2.9	17.1	428	.0861
II	1.5	25.3	3.9	12.7	324	.0711
II	2.0	24.1	3.3	10.4	270	.0608
II	2.5	23.9	4.5	8.6	221	.0588
II	4.0	21.8	4.9	6.3	167	.0422
II	6.0	20.8	5.2	4.6	121	.0357
II	8.0	18.1	4.6	4.1	111	.0200
II	10.0	18.4	5.5	3.4	88	.0214
II	12.0	17.3	6.1	3.1	80	.0162
III	1.0	19.6	4.1	24.3	582	.0964
III	1.5	18.4	4.1	17.3	422	.0844
III	2.0	17.2	4.7	13.8	343	.0726
III	2.5	16.2	4.3	11.7	285	.0622
III	4.0	15.1	5.9	7.9	197	.0512
III	6.0	13.3	5.6	5.9	145	.0336
III	8.0	12.8	5.0	4.7	118	.0284
III	10.0	9.7	5.3	4.3	111	.0000
III	12.0	9.0	4.8	3.8	90	.0000
IV	1.0	14.4	5.8	29.5	658	.1206
IV	1.5	12.4	4.5	21.5	480	.0901
IV	2.0	11.3	3.7	16.8	389	.0665
IV	2.5	10.8	4.0	13.8	320	.0514
IV	4.0	9.4	3.7	9.3	214	.0000
IV	6.0	8.0	2.8	6.8	155	.0000
IV	8.0	6.0	4.5	5.6	130	.0000
IV	10.0	4.7	4.2	4.8	109	.0000
IV	12.0	7.4	2.3	3.9	86	.0000

Table 2. Simulation Results of Method 2

Response Curve	Increment Size	Apparent Threshold	Standard Deviation	Number of Levels	Total No. of Trials	Reaction Probability
I	1.0	30.6	3.0	13.3	163	.0813
I	1.5	29.7	3.4	9.8	131	.0710
I	2.0	28.4	3.1	8.2	116	.0568
I	2.5	27.9	3.4	7.0	96	.0518
I	4.0	26.0	3.6	5.2	79	.0361
I	6.0	24.4	4.3	4.0	64	.0260
I	8.0	23.8	4.8	3.3	51	.0227
I	10.0	22.1	5.5	3.0	49	.0145
I	12.0	20.8	4.9	2.8	43	.0100
II	1.0	27.6	4.4	16.3	194	.0941
II	1.5	25.5	4.2	12.6	163	.0735
II	2.0	24.3	3.8	10.3	135	.0625
II	2.5	24.3	4.4	8.4	117	.0622
II	4.0	21.6	4.5	6.3	91	.0413
II	6.0	20.7	4.5	4.7	72	.0350
II	8.0	18.7	5.1	4.0	63	.0233
II	10.0	17.5	5.1	3.5	54	.0170
II	12.0	15.7	5.8	3.2	55	.0101
III	1.0	20.2	4.9	23.8	254	.1020
III	1.5	18.6	4.6	17.2	194	.0864
III	2.0	18.1	5.7	13.4	157	.0810
III	2.5	16.2	4.6	11.7	145	.0622
III	4.0	14.0	4.2	8.2	108	.0400
III	6.0	12.8	4.6	6.0	81	.0288
III	8.0	12.3	4.6	4.8	67	.0236
III	10.0	10.5	5.9	4.2	58	.0050
III	12.0	8.3	3.7	3.8	52	.0000
IV	1.0	13.0	3.7	31.0	297	.0995
IV	1.5	12.5	4.6	21.3	210	.0915
IV	2.0	11.2	3.6	16.8	174	.0635
IV	2.5	10.6	3.2	13.9	148	.0472
IV	4.0	9.0	3.4	9.5	104	.0000
IV	6.0	8.1	2.6	6.8	75	.0000
IV	8.0	6.6	4.7	5.5	66	.0000
IV	10.0	3.9	2.8	4.9	56	.0000
IV	12.0	7.1	1.2	3.9	45	.0000

Table 3. Simulation Results of Method 3

Response Curve	Increment Size	Apparent Threshold	Standard Deviation	Number of Levels	Total No. of Trials	Reaction Probability
I	1.0	29.8	3.3	14.1	90	.0726
I	1.5	28.8	3.7	10.4	73	.0612
I	2.0	27.9	3.7	8.5	61	.0526
I	2.5	27.3	4.0	7.2	54	.0469
I	4.0	25.4	4.7	5.3	44	.0324
I	6.0	24.2	4.5	4.1	38	.0246
I	8.0	22.0	5.7	3.5	34	.0166
I	10.0	21.3	5.8	3.1	32	.0116
I	12.0	19.8	5.7	2.9	30	.0074
II	1.0	26.7	3.9	17.2	105	.0850
II	1.5	26.0	4.2	12.2	79	.0782
II	2.0	23.9	4.2	10.5	72	.0585
II	2.5	23.4	4.4	8.8	64	.0550
II	4.0	21.0	5.1	6.4	51	.0372
II	6.0	19.2	5.9	4.9	42	.0259
II	8.0	18.1	5.7	4.1	36	.0200
II	10.0	16.2	5.2	3.6	34	.0116
II	12.0	15.4	7.1	3.3	32	.0088
III	1.0	19.6	4.8	24.3	135	.0963
III	1.5	18.3	4.2	17.4	100	.0834
III	2.0	16.9	4.9	14.0	85	.0696
III	2.5	16.2	4.7	11.7	74	.0625
III	4.0	14.5	4.6	8.1	55	.0456
III	6.0	12.1	5.4	6.1	47	.0216
III	8.0	10.6	5.4	5.0	41	.0068
III	10.0	9.6	6.0	4.3	36	.0000
III	12.0	8.9	4.4	3.8	34	.0000
IV	1.0	13.3	4.3	30.6	149	.1050
IV	1.5	11.6	3.3	21.9	114	.0735
IV	2.0	11.2	3.3	16.8	91	.0645
IV	2.5	10.8	3.9	13.8	78	.0522
IV	4.0	9.2	3.3	9.4	56	.0000
IV	6.0	8.0	2.8	6.8	45	.0000
IV	8.0	5.8	4.3	5.6	40	.0000
IV	10.0	3.9	3.2	4.9	38	.0000
IV	12.0	7.2	1.6	3.9	33	.0000

Table 4. Simulation Results of Method 4

Response Curve	Increment Size	Apparent Threshold	Standard Deviation	Number of Levels	Total No. of Trials	Reaction Probability
I	1.0	30.4	2.7	12.2	155	.0795
I	1.5	29.2	3.0	8.9	123	.0648
I	2.0	28.4	3.3	7.1	104	.0568
I	2.5	27.8	3.4	5.9	88	.0509
I	4.0	25.3	3.7	4.2	72	.0313
I	6.0	23.8	4.1	3.1	53	.0223
I	8.0	22.6	4.3	2.6	49	.0169
I	10.0	21.1	4.1	2.2	35	.0109
I	12.0	18.6	3.1	2.1	33	.0048
II	1.0	27.1	4.2	15.5	194	.0895
II	1.5	25.6	4.3	11.3	150	.0746
II	2.0	25.1	4.5	8.6	119	.0693
II	2.5	23.3	3.7	7.7	115	.0536
II	4.0	22.3	4.6	5.0	75	.0463
II	6.0	19.4	4.9	3.8	67	.0269
II	8.0	18.0	5.4	3.2	52	.0196
II	10.0	16.8	5.8	2.6	46	.0140
II	12.0	15.9	5.7	2.3	36	.0108
III	1.0	20.1	5.1	22.5	249	.1019
III	1.5	18.5	4.8	15.9	188	.0853
III	2.0	17.1	4.8	12.7	160	.0712
III	2.5	17.0	4.7	10.0	129	.0707
III	4.0	12.9	3.8	7.3	104	.0296
III	6.0	12.7	4.5	4.9	73	.0270
III	8.0	11.8	4.4	3.8	56	.0180
III	10.0	8.4	5.9	3.3	55	.0000
III	12.0	8.0	4.3	2.9	44	.0000
IV	1.0	12.9	3.7	29.7	289	.0980
IV	1.5	13.2	5.4	19.5	196	.1029
IV	2.0	11.4	3.4	15.4	160	.0689
IV	2.5	10.3	2.6	12.7	140	.0315
IV	4.0	9.2	3.1	8.1	96	.0000
IV	6.0	8.2	2.7	5.4	63	.0000
IV	8.0	5.9	4.0	4.4	61	.0000
IV	10.0	3.4	1.9	3.8	49	.0000
IV	12.0	6.1	3.4	3.0	37	.0000

Table 5. Simulation Results of Method 5

Response Curve	Increment Size	Apparent Threshold	Standard Deviation	Number of Levels	Total No. of Trials	Reaction Probability
I	1.0	29.4	3.5	9.3	66	.0674
I	1.5	28.0	3.3	7.1	55	.0529
I	2.0	27.2	3.6	5.8	46	.0463
I	2.5	26.2	3.8	5.2	45	.0375
I	4.0	24.0	4.7	3.9	37	.0234
I	6.0	23.0	5.5	3.0	31	.0184
I	8.0	20.0	6.5	2.7	29	.0079
I	10.0	18.9	7.1	2.4	26	.0053
I	12.0	17.0	5.8	2.2	25	.0024
II	1.0	25.7	3.8	11.5	77	.0752
II	1.5	23.9	4.2	8.9	65	.0586
II	2.0	22.5	4.3	7.3	57	.0478
II	2.5	21.8	4.5	6.1	48	.0426
II	4.0	19.0	5.2	4.7	40	.0248
II	6.0	17.0	5.7	3.6	34	.0149
II	8.0	14.6	6.3	3.1	32	.0066
II	10.0	13.5	7.5	2.7	29	.0037
II	12.0	13.5	7.5	2.4	26	.0038
III	1.0	17.3	4.1	16.4	100	.0732
III	1.5	16.5	4.6	11.7	77	.0652
III	2.0	15.3	4.7	9.3	63	.0534
III	2.5	14.7	4.4	7.8	56	.0477
III	4.0	12.2	5.5	5.6	45	.0228
III	6.0	10.2	5.5	4.2	36	.0066
III	8.0	9.5	4.9	3.4	32	.0000
III	10.0	6.8	6.2	3.2	31	.0000
III	12.0	6.5	5.9	2.8	30	.0000
IV	1.0	12.1	3.6	18.8	100	.0836
IV	1.5	11.4	3.5	13.1	74	.0689
IV	2.0	9.9	3.3	10.6	64	.0000
IV	2.5	9.7	3.1	8.7	54	.0000
IV	4.0	7.8	3.2	6.1	43	.0000
IV	6.0	6.5	3.4	4.5	35	.0000
IV	8.0	4.5	4.1	3.8	33	.0000
IV	10.0	3.0	2.8	3.3	31	.0000
IV	12.0	4.1	4.5	2.9	28	.0000

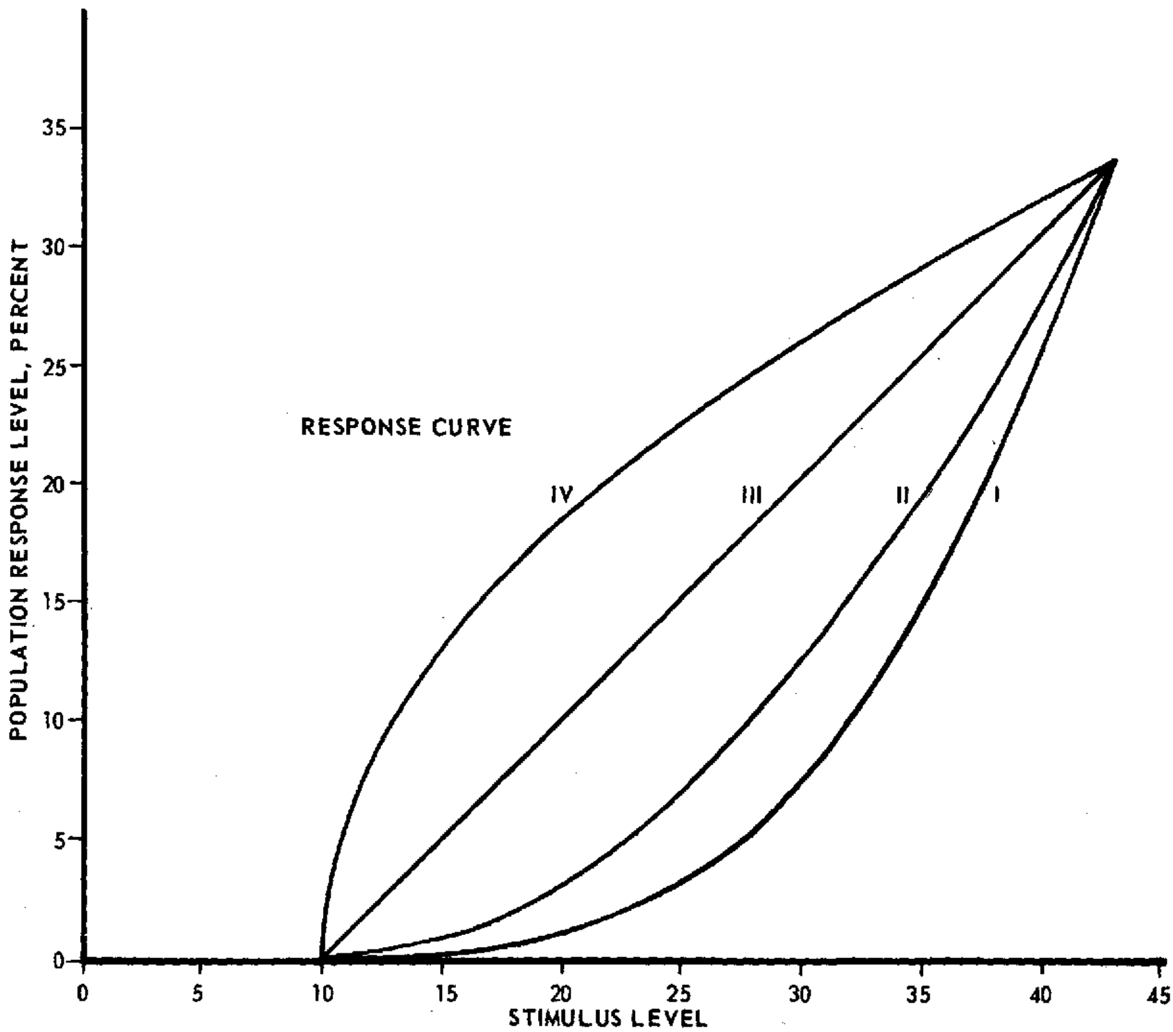

Figure 1 Stimulus Response Curves used for Simulation

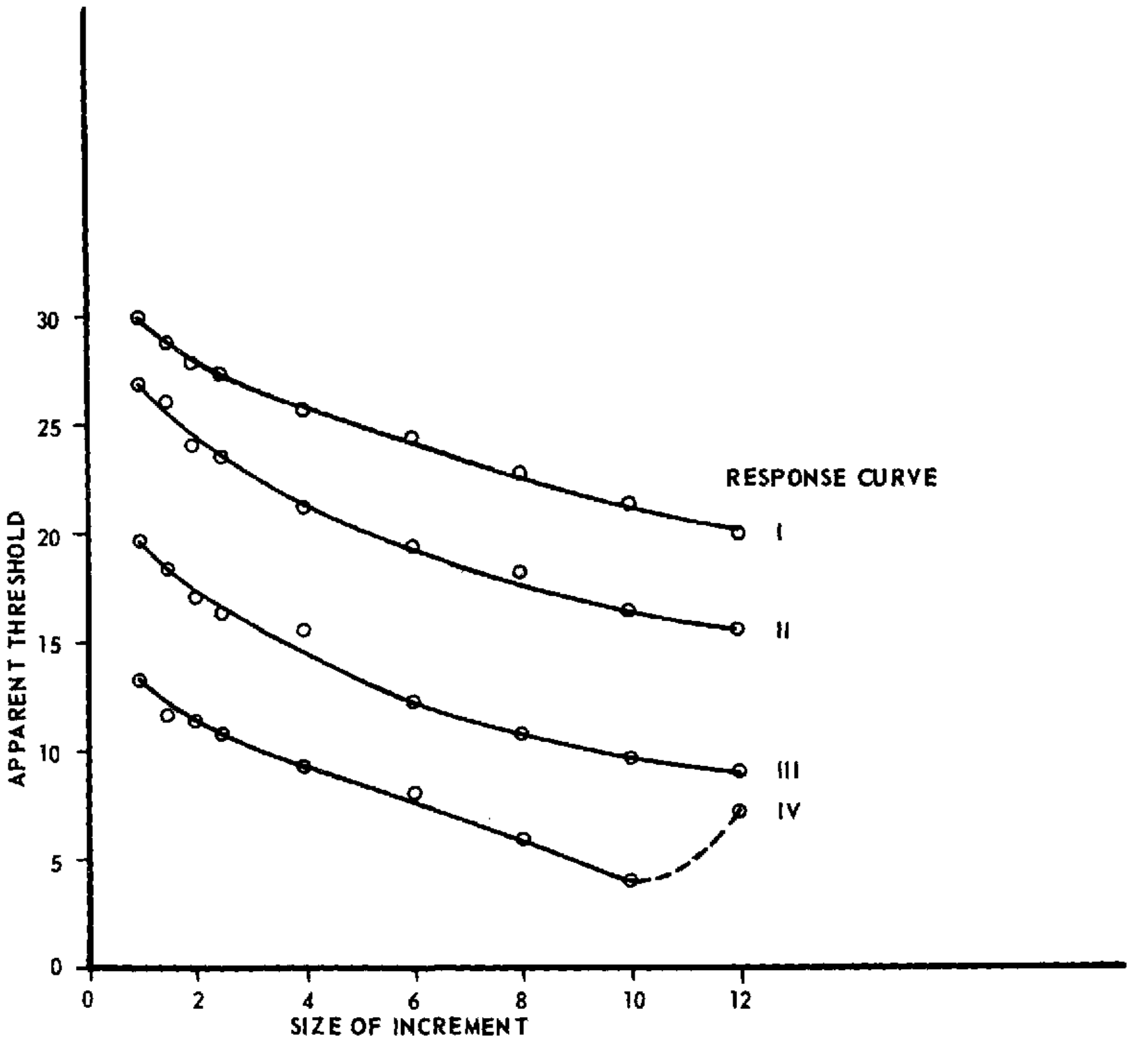

Figure 2. Effect of Size of Increment on Apparent Threshold Values for Different Response Curves

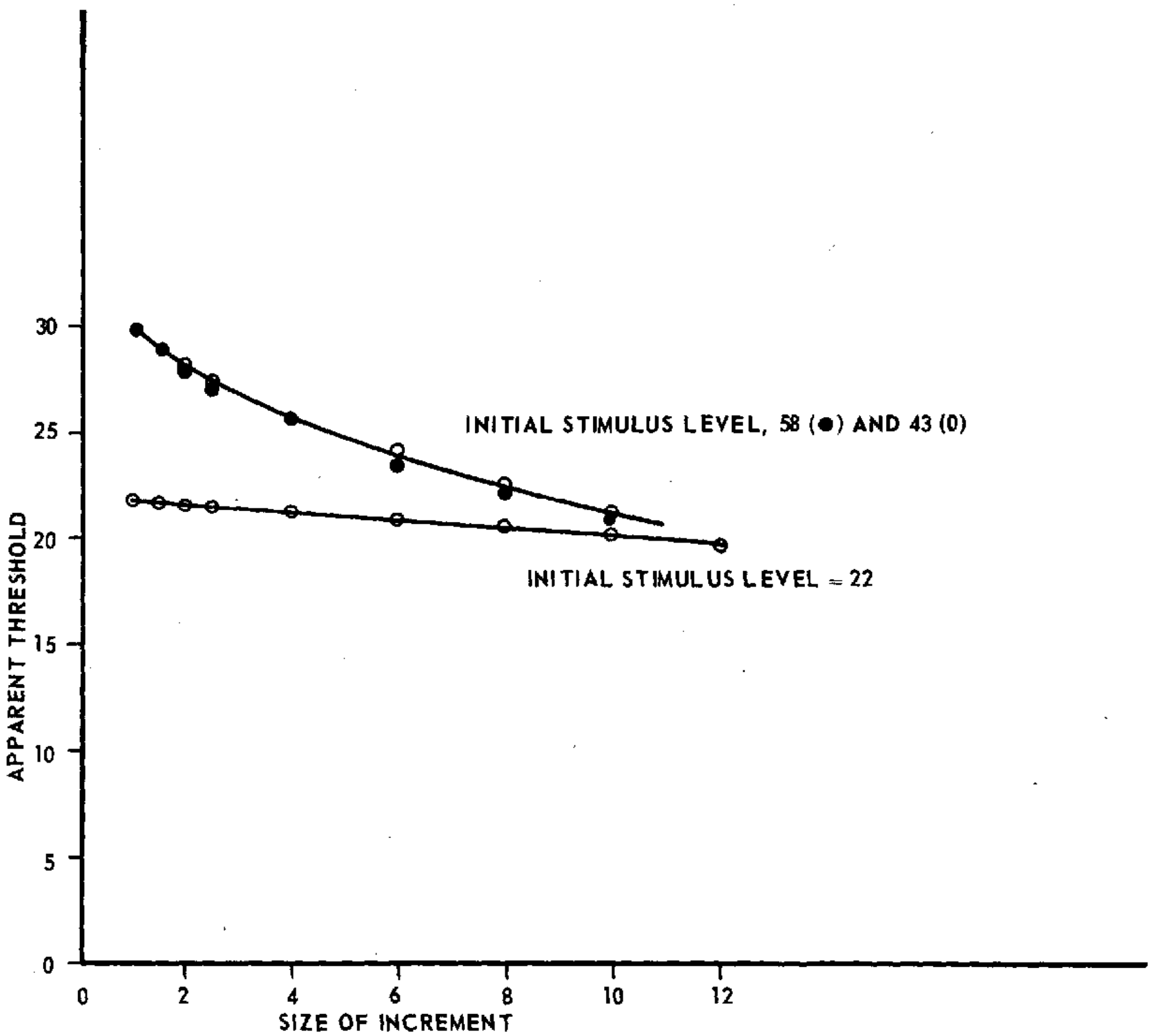

Figure 3. Effect of Initial Stimulus Level on Size of Increment Versus Apparent Threshold Value Relation for First Response Curve

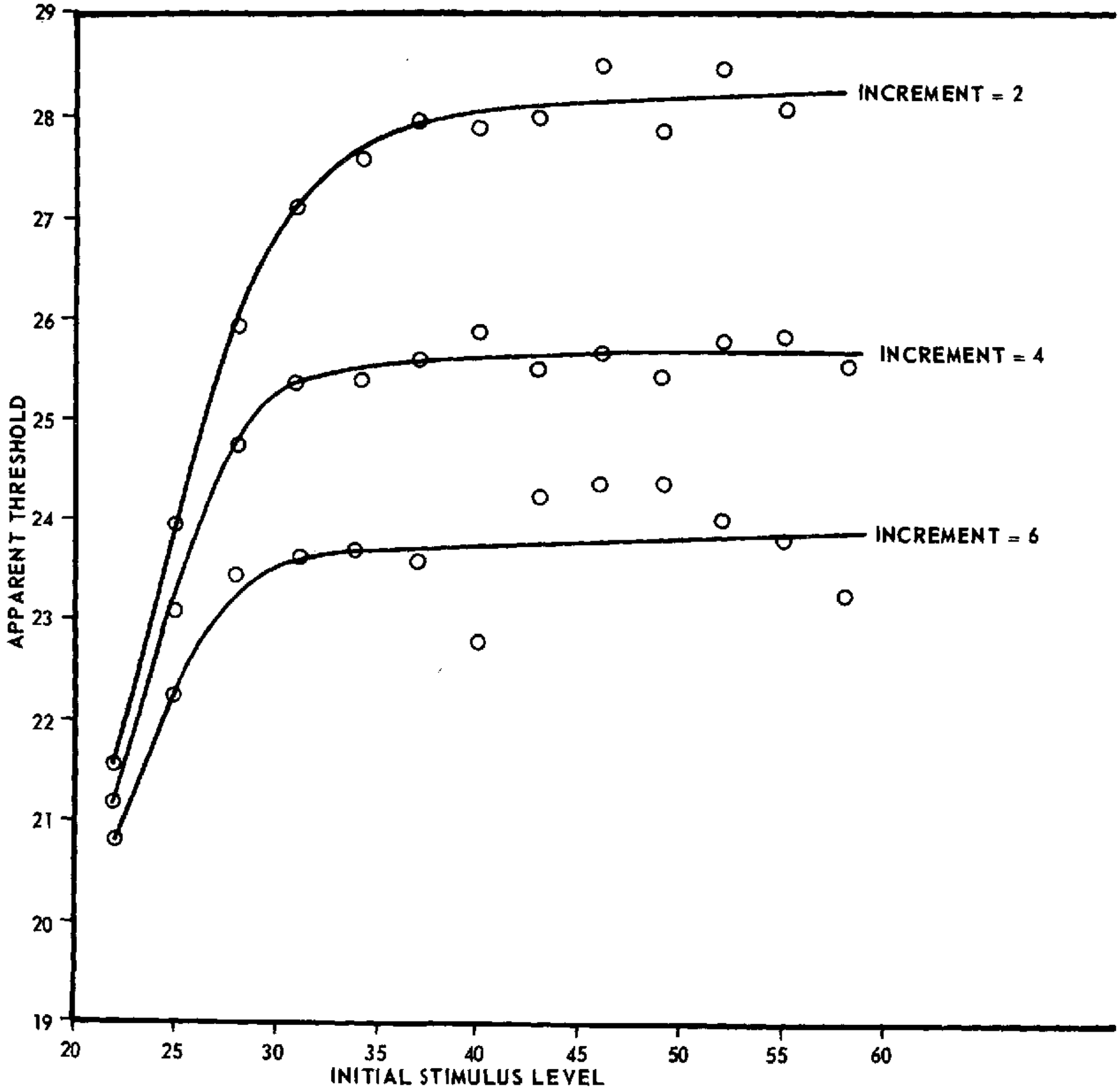

Figure 4. Effect of Initial Stimulus Level on Apparent Threshold Values for First Response Curve

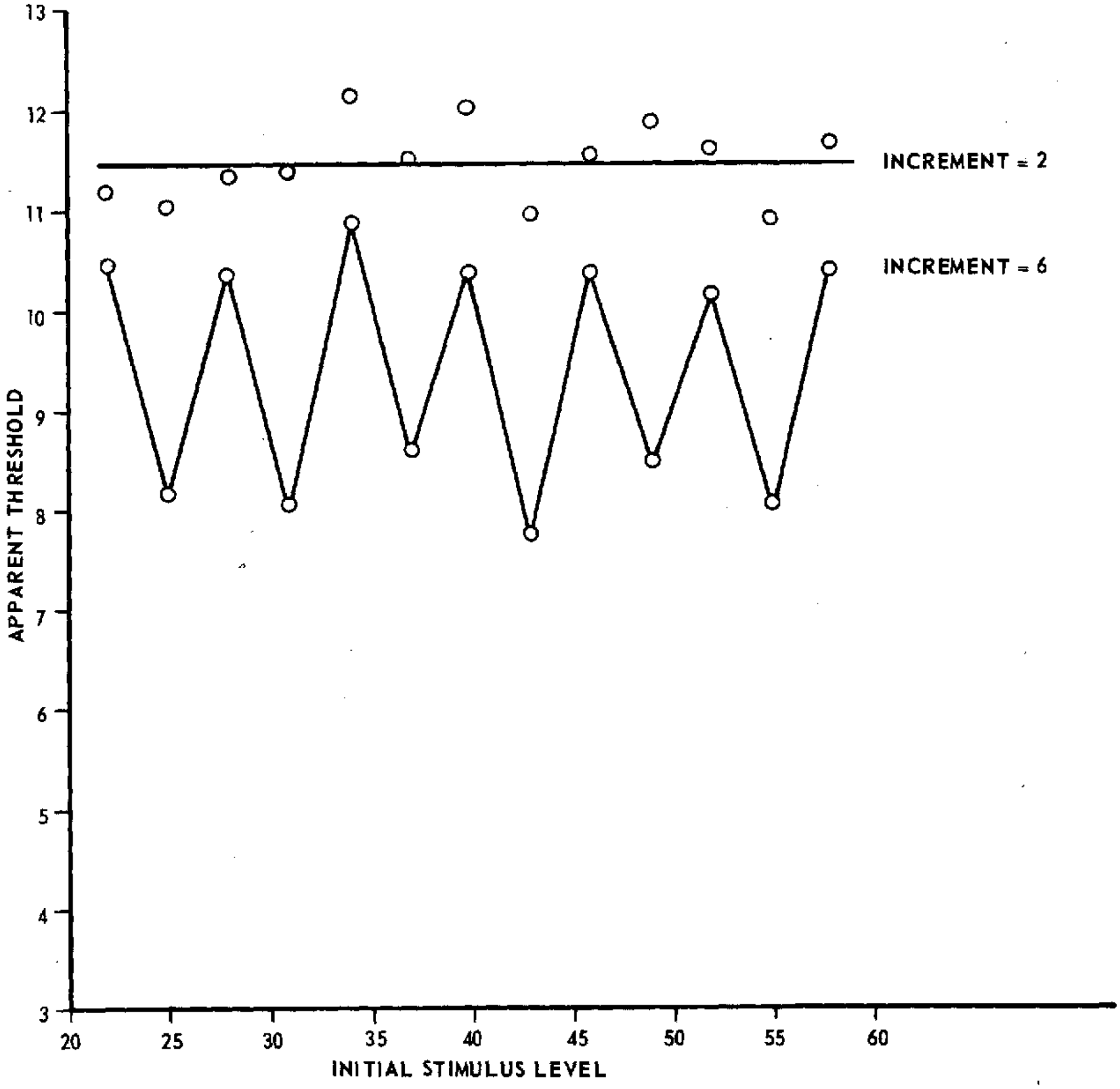

Figure 5. Effect of Initial Stimulus Level on Apparent Threshold Values for Fourth Response Curve

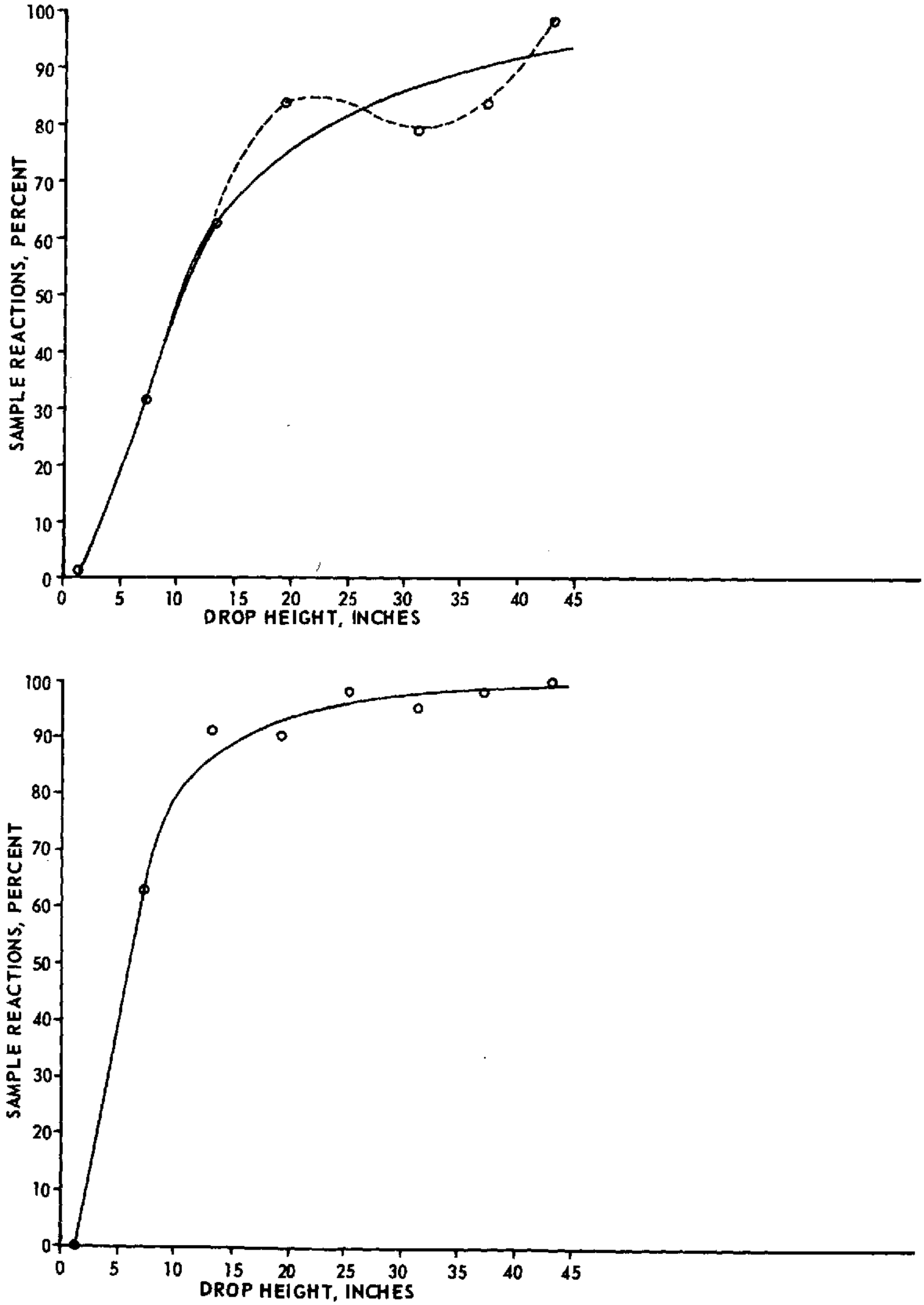

Figure 6. Experimental Determination of Stimulus Response Curves

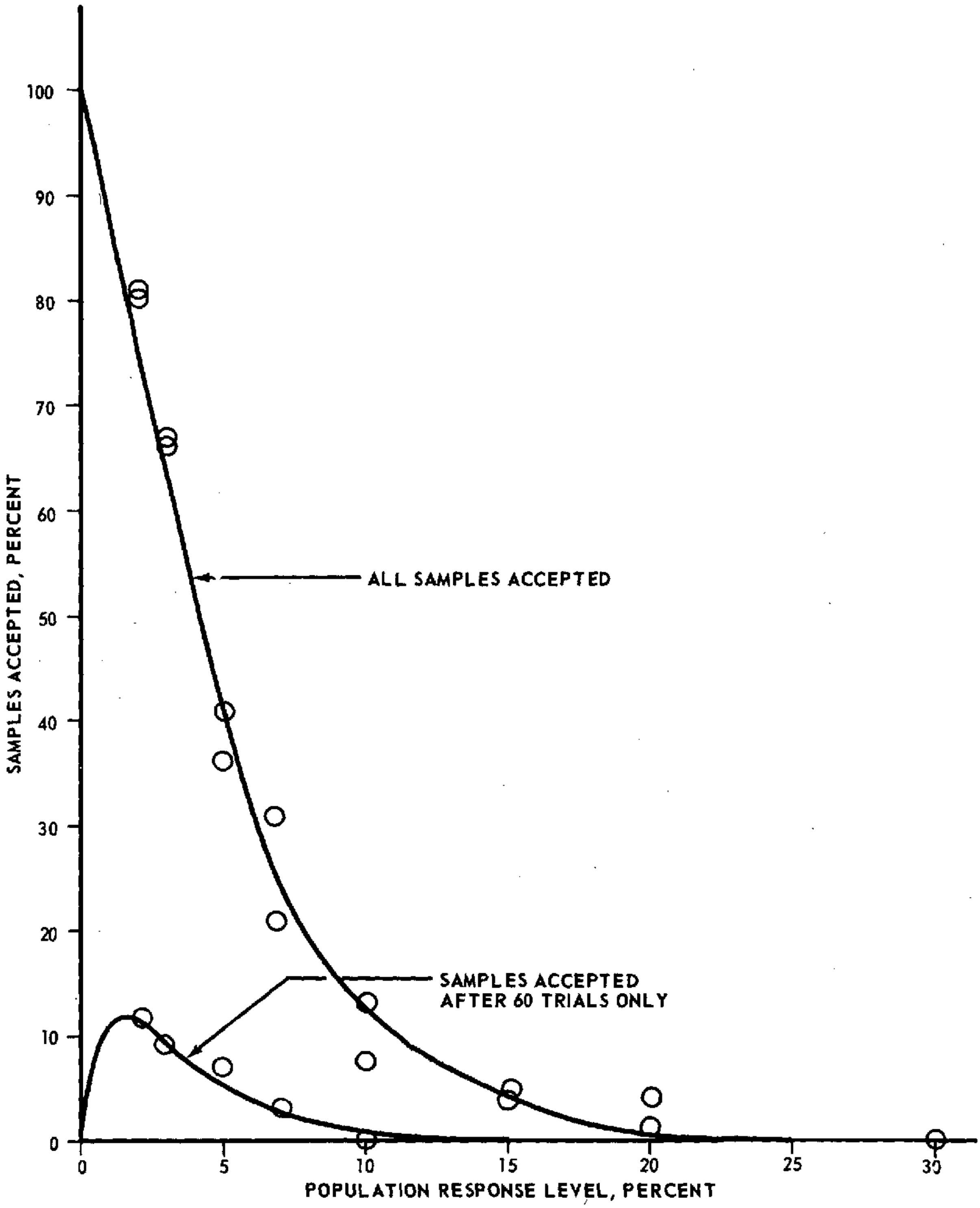

Figure 7. Simulation of MSFC SPEC 106B Results for Materials with Wide Range of Population Response Levels

APPENDIX

One method used to validate the random number generator together with some parts of the program listing was to simulate results for standard liquid oxygen impact tests by MSFC SPEC 106B for materials with a range of population response values. Figure 7 gives the percent acceptance based on groups of 100 samples each for populations with responses ranging from 2 to 30 percent. The results are in close agreement with the expected values based on the binomial distribution which are shown as a smooth curve. Also shown in the figure are samples for which final acceptance was based on 60 tests, i.e., samples giving exactly one reaction in the first 20 trials followed by zero in the next 40. As mentioned previously, such samples represent a small fraction of all samples accepted.

REFERENCES

1. Rothman, D.; Alexander, M.J.; and Zimmerman, J.M.: The Design and Analysis of Sensitivity Experiments, Vol. 1 and 2, Final Report for NASA Contract NAS 8-11061 DCN 1-3-84-80257-01 (1F) CPB 02-1200-63, Rocketdyne, June 28, 1965.

2. Schwinghamer, R.J.; and Key, C.F.: High Pressure Oxygen Test Evaluation; presented to AIChE Oxygen Utilization Under Pressure Symposium; AIChE 74th National Meeting, March 15, 1973.

3. Gayle, J.B.: Nomonotonicity in Sensitivity Test Data. Materials Research and Standards, March 1966, pp. 147-148.

4. Key, C.F.; and Gayle, J.B.: Effect of Liquid Nitrogen Dilution on LOX Impact Sensitivity. Journal of Spacecraft and Rockets, February, 1966.

NASA-Langley, 1974