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NINETEENTH PROGRESS REPORT

on

CALIBRATION AND EVALUATION OF SKYLAB ALTIMETRY FOR GEODETIC DETERMINATION OF THE GEOID (Contract NAS9-13276, EPN 440) September 1 to November 30, 1974

to

NASA Johnson Space Center
Principal Investigation Management Office
Houston, Texas 77058

from

BATTELLE

Columbus Laboratories

December 12, 1974

Prepared by: S. Gopalapillai and M. Kuhner

and

A. G. Mourad (Principal Investigator)

Z. H. Byrns, Code TF6 - NASA/JSC Technical Monitor

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PROGRESS

During this period the tapes containing the modified SKYBET data have been received. The main effort during this period was aimed at

(1) matching the altimetry data from ECT S071-1 tapes (received in August) with satellite ephemeris data from SKYBET tapes with respect to the time of observation,

(2) modifying the computer program used in the simulation studies to make it more general, efficient and economical with respect to computer time to suit the data on tapes and to enable the solution of a larger system of equations simultaneously, and

(3) testing this program with the real data.

The details of matching the altimetry data with the ephemeris data are presented in Appendix A. The reduced algorithm for modifying the existing computer programs are presented in Appendix B. The data and documents received, during this period, are reported in Appendix C.

DATA PROCESSING RESULTS

In the earlier analysis, only one bias term was considered for all the data from various submodes. The obvious discrepencies in the heights between different submodes had been manually accounted for. The modified algorithm assumes a different bias term for each submode so that these terms can absorb the height discrepencies in the submodes. This assumption is well supported by the results obtained from the real data analysis. These results include:

(1) the recovered bias terms for each submode are distinctly different. The magnitude of the differences agrees with the height discrepencies in the observed data,

(2) the insensitivity of the bias terms to the a priori weights of the observations indicate the stability and efficiency of the mathematical model used for evaluating them,and

(3) the accuracy of estimates obtained from the analysis are highly correlated with the number of available ground truth data points.

PROBLEMS AND RECOMMENDATIONS

There are no significant problems or recommendations to be made for this period.

NEXT PERIOD

The major efforts during the next reporting period include:

(1) Continuation of the processing and analysis of data from the remainder of EREP Pass No. 9,

(2) Evaluation of the geoidal profile obtained for Pass No. 9 with the ground truth profile, and

(3) Initiation of the processing and analysis of data from Passes No. 4, 6, and 7.

TRAVEL

No plans for travel are anticipated at this time.

APPENDIX A

MATCHING OF ALTIMETRY DATA WITH SKYBET DATA

For each altimeter range to be corrected, the precise location in space of the satellite is required. Therefore, altimetry data points must be combined with ephemeris orbit data points from the SKYBET tapes. Unfortunately the times for which these data are recorded are different for the two types of tapes. On the SKYBET tapes the time interval between data points is exactly 1/8 second. On the altimetry tapes the time interval is 1.04/8 seconds. The result is that the two sets of data wander slowly in and out of phase. The data needed from the SKYBET tapes are the satellite latitude, longitude, height above reference ellipsoid and the earth-fixed geocentric X, Y, and Z coordinates. The first three of these parameters do not change significantly in 1/8 second. However, the X, Y, and Z coordinates which must be known to better than 1 meter do change by several meters in 1/8 second. Therefore, a simple interpolation scheme was used to compute the X, Y, and Z coordinates to better than one meter for the times at which the altimeter ranges are given.

For each time, t, on the SKYBET tape not only X, Y, and Z coordinates are given but also the velocity components $\dot{X}$, $\dot{Y}$, and $\dot{Z}$. Therefore, at a later time $t+\Delta t$ the approximation,

$$X(t+\Delta t) = X(t) + \Delta t\, \dot{X}(t)$$

$$Y(t+\Delta t) = Y(t) + \Delta t\, \dot{Y}(t)$$

$$Z(t+\Delta t) = Z(t) + \Delta t\, \dot{Z}(t)$$

can be used. It can be shown that if no forces other than gravity affect the satellite, $\dot{X}$, $\dot{Y}$, and $\dot{Z}$ are very nearly constant over 1/8 second so that this approximation is valid (the errors should be of the order of 0.06 meters or so). To test the interpolation scheme, the X, Y, and Z coordinates were computed for $\Delta t = 1/8$ second and the results were compared with those given on the SKYBET tapes. The maximum error was 0.095 meter, which is of the same order of magnitude as estimated from theory. Therefore, this interpolation scheme was incorporated into a computer code which combines the necessary SKYBET data with the necessary altimetry data and writes them onto a single tape for later processing.

APPENDIX B

MODIFIED ALGORITHM FOR COMPUTER PROCESSING OF ALTIMETRY DATA

1.0 CONDITION EQUATION OF INTRINSIC PARAMETERS

The adjusted value R_i^a, corresponding to the measured altimeter range, R_i^o, is intrinsically related to (a) the geocentric coordinates, X_{si}, Y_{si}, Z_{si}, of the satellite at the instant of measurement, (b) the geoidal undulation, N_i^a, at the satellite subpoint referred to a given reference ellipsoid, and (c) the biases in all the measurement systems involved.

There are two types of condition equations considered in this investigation, depending on how the system biases are modeled. In the case where the system biases are considered proportional to the measured ranges, the condition equation is given by

$$R_i^a\ (1+\Delta f) + N_i^a - D_i = 0 \tag{1}$$

where

$$D_i = F\ (X_{si}, Y_{si}, Z_{si}, a, e) \quad , \tag{1a}$$

being the height of the satellite above the reference ellipsoid given as a function of the geocentric coordinates of the satellite at the instant of observation, and the parameters (a, e) defining the size and shape, respectively, of the assumed reference ellipsoid. $R_i \Delta f$ is the system bias for this observation. The second type of equation is

$$R_i^a + \Delta f + N_i^a - D_i = 0 \tag{2}$$

where the bias Δf is assumed to be constant independent of the range.

The first form was used in the studies reported in the previous progress reports. However, the variation in R_i is very small compared to R_i itself so that $R_i \Delta f$ remains constant for all practical purposes. Preliminary results of the numerical comparisons of these two models support this contention. Therefore, the model given in equation (2) will be used throughout the rest of this investigation.

2.0 THE OBSERVATION EQUATION

In order to determine the bias, Δf, the other quantities in equation (2) are needed: R_i is the measured altimetry range; N_i is the geoid undulation available as part of the ground truth data, and D_i is assumed to be known for each of the measured ranges. Both R_i and N_i are associated with random errors which have to be determined in a least squares adjustment.

The observation equation for the model described in equation (2) is given by

$$V_i + \Delta f + \Delta N_i + W_i = 0 \tag{3}$$

where V_i and ΔN_i are residuals of R_i and N_i respectively and

$$W_i = R_i^o + N_i^o - D_i \tag{4}$$

where the superscript o refer to the observed values of the quantities R_i and N_i. Equation (3) refers to only one altimetry range. There will be as many equations of this type as there are altimetry observations for which the ground truth data is available. The system of all these equations can be written in matrix notation as follows.

$$V + AX + W = 0 \tag{5}$$

where

$$V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \tag{6}$$

$$A = \begin{bmatrix} 1 & 1 & & & & & & 0 \\ 1 & & 1 & & & & & \\ 1 & & & 1 & & & & \\ 1 & & & & 1 & & & \\ \cdot & & & & & \cdot & & \\ \cdot & & & & & & \cdot & \\ \cdot & & 0 & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ 1 & & & & & & & & 1 \end{bmatrix} \tag{7}$$

$$X = \begin{bmatrix} \Delta f \\ \Delta N_1 \\ \Delta N_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta N_n \end{bmatrix} \tag{8}$$

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ W_n \end{bmatrix} \tag{9}$$

Preliminary examinations of the altimetry range ovservations for different submodes indicate different biases for the observations in each submode. In this case equation (3) for the i^{th} observation which corresponds to the k^{th} submode will be of the form

$$V_i + \Delta f_k + \Delta N_i + W_i = 0 \qquad (10)$$

which will result in the A matrix of the form

$$\begin{array}{cc} & \begin{array}{ccccccccc} \Delta f_1 & \Delta f_2 & \Delta f_k & \Delta N_1 & \Delta N_2 & & & & \Delta N_n \end{array} \\ A = & \left[\begin{array}{ccccccccc} 1 & 0 & 0 & 1 & & & & & \\ 0 & 1 & 0 & & 1 & & & 0 & \\ 1 & 0 & 0 & & & 1 & & & \\ 0 & 0 & 1 & & & & 1 & & \\ \cdot & \cdot & \cdot & & & & & \cdot & \\ \cdot & \cdot & \cdot & & & & & \cdot & \\ \cdot & \cdot & \cdot & & 0 & & & \cdot & \\ 0 & 0 & 1 & & & & & & 1 \end{array}\right] \end{array} \qquad (11)$$

while the structure of other matrices will remain unchanged.

3.0 MATRIX SOLUTION

If the weight matrices for the observations R and N are P and P_x respectively, their least squares solution in matrix form is given by

$$X = -(A^1PA + P_x)^{-1} A^1PW \qquad (12)$$

$$V = -AX-W \qquad (13)$$

where the superscript 1 referes to the transpose of the matrix.

The variance covariance matrix, Σ_x, for the vector X is given by

$$\Sigma_x = \sigma_o^2 (A^1PA + P_x)^{-1} \qquad (14)$$

with σ_o^2 being the variance of unit weight given by

$$\sigma_o^{\ 2} = \frac{(X^1 P_x X + V^1 PV)}{df} \tag{15}$$

where, df is the degree of freedom.

4.0 SPECIAL CHARACTERISTICS OF THE MATRICES INVOLVED

4.1 Weight Matrices

The observations R and the a priori estimates for N are assumed to be independent. The biases corresponding to each of the submodes are also assumed to be independent of each other. Under these assumptions, the matrices P and P_x are diagonal. If the P_x matrix is partitioned along a line separating the biases from the undulations then, P_x can be written in the form

$$P_x = \left[\begin{array}{c|c} P_{x1} & 0 \\ \hline 0 & P_{x2} \end{array}\right] \tag{16}$$

Further, if the accuracy of all the observations in each of the groups R, N, and Δf is assumed to be equal then,

$$\begin{aligned} P &= p_1 \cdot I \\ P_{x1} &= px_1 \cdot I \\ \text{and } P_{x2} &= px_2 \cdot I \end{aligned} \tag{17}$$

where I is the identity matrix.

Partitioning (the matrix A) along the same line as in matrix P_x in equation (16), A can be rewritten in the form

$$A = \left[\begin{array}{c|c} A_1 & I \end{array}\right] \tag{18}$$

where A_1 is the submatrix of A containing the columns corresponding to the bias terms Δf_k. Similarly X is partitioned into components X_1 and X_2 such that

$$X = \begin{bmatrix} X_1 \\ \hline X_2 \end{bmatrix} \tag{19}$$

The special structures of the matrices, as described above, are taken advantage of in the numerical evaluation of equation (12) - (15).

5.0 REDUCED ALGORITHM FOR DIGITAL COMPUTER EVALUATION

The nature of the weight matrices assumed and the structure of the design matrix on either side of the partition make it possible to simplify the equations given in equations (12) - (15) for computer coding so that this program could handle the data more efficiently and economically.

The partition of matrices enables the solution of the normal equation (12) to be sequential, i.e., to solve for x_1, and then for x_2. Equation (12) can be rewritten as

$$X = - N^{-1} U \tag{20}$$

with

$$\begin{aligned} N &= (A^1 PA + P_x) \\ \text{and } U &= A^1 PW \end{aligned} \tag{21}$$

Using the partition approach, the submatrices of N and U will be

$$N = \left[\begin{array}{c|c} N_{11} & N_{12} \\ \hline N_{21} & N_{22} \end{array}\right] = \left[\begin{array}{c|c} pA_1^1 A_1 + px_1 . I & pA_1^1 \\ \hline pA_1 & (p+px_2) . I \end{array}\right] \tag{22}$$

$$U = \begin{bmatrix} U_1 \\ \hdashline U_2 \end{bmatrix} = \begin{bmatrix} pA_1^1 W \\ \hdashline pW \end{bmatrix} \tag{23}$$

Now, let

$$\left[\begin{array}{c:c} Q_{11} & Q_{12} \\ \hdashline Q_{21} & Q_{22} \end{array}\right] = \left[\begin{array}{c:c} N_{11} & N_{12} \\ \hdashline N_{21} & N_{22} \end{array}\right]^{-1}$$

Then (Faddeev and Faddeeva, 1963)

$$Q_{11} = (N_{11} - N_{12} N_{22}^{-1} N_{12}^{1})^{-1} \tag{24}$$

$$Q_{12} = - Q_{11} N_{12} N_{22}^{-1} \tag{25}$$

$$Q_{22} = N_{22}^{-1} (I - N_{12}^{1} Q_{12}) \tag{26}$$

and the solutions X_1 and X_2 are given by

$$X_1 = -Q_{11} \left[U_1 - N_{12} N_{22}^{-1} U_2 \right] \tag{27}$$

$$X_2 = - N_{22}^{-1} N_{12}^{1} X_1 - N_{22}^{-1} U_2 \tag{28}$$

With the submatrices of N defined as in equation (22) in terms of the weight matrices and the design matrix A, it can be shown that

$$Q_{11} = \frac{p \cdot px_2}{p + px_2} \cdot A_1^1 A_1 + px_{1}.I \tag{29}$$

It should be noted that since $A_1^1 A_1$ is diagonal, Q_{11} is also diagonal.

and $$\overline{U} = U_1 - N_{12} N_{22}^{-1} U_2 = \frac{p \cdot px_2}{p + px_2} A_1^1 W \tag{30}$$

Then, from equations (27) and (28)

$$X_1 = -Q_{11}\bar{U} \tag{31}$$

$$X_2 = -\frac{p}{p+px_2}\left[A_1X_1+W\right] \tag{32}$$

The weight coefficients matrix for the geoid undulations N (Ground Truth) is given by Q_{22} which is given by

$$Q_{22} = \frac{1}{p+px_2}\left[I + \frac{p^2}{p+px_2} A_1Q_{11}A_1^1\right] \tag{33}$$

One of the advantageous features in this system is that both the matrices (Q_{11} and N_{22}) whose inverses are required are diagonal. This enables one to solve a system (of normal equations) of any size with relatively small computer storage requirement.

Once the bias for each submode is determined, the geoid undulation for any other point along the pass is determined from equation (2) as

$$N = D - R - \Delta f \tag{34}$$

6.0 TEST RESULTS AND DISCUSSIONS

Using the algorithm presented in the last section, the data corresponding to the first 3 submodes in mode 5 of EREP pass 9. (Start Time 163:13:1:37.181, Stop Time 163:13:4:26.701) was evaluated.

The input data are:

(a) The altimetry observations at 5 seconds interval.

(b) The SKYBET ephemeris data corresponding to these altimetry data.

(c) Geoid undulations from Marsh-Vincent geoid map.

(d) The weight matrices corresponding to the altimetry observation (p), bias (px_1), and to the geoid undulations (px_2).

The 5 seconds interval for altimetry observations is selected simply because this is the least interval for which the satellite sub-points can be plotted on the available Marsh-Vincent geoid map.

The reference ellipsoid used is the one used by Marsh and Vincent (1974) where it is defined by

semi-major axis a = 6378142 m

flattening f = 1/298.255

Five test runs were made varying the relative weights p, px_1, and px_2. The results are presented in Table 1, where σ_o^2 is a posteriori variance of unit weight.

TABLE 1

	Test 1	Test 2	Test 3	Test 4	Test 5
p	1/1.0	1/9.0	1/9.0	1/4.0	1/1.0
px_1	1/100.0	1/100.0	$1/\infty$	$1/\infty$	$1/\infty$
px_2	1/0.01	1/4.0	1/4.0	1/4.0	1/4.0
Δf_1	-20.75	-20.15	-20.80	-20.80	-20.80
Δf_2	-7.26	-7.22	-7.27	-7.27	-7.27
Δf_3	2.81	2.77	2.81	2.81	2.81
σ_o	9.15	0.92	0.85	1.05	1.35

The a priori estimate assumed for the variance of unit weight is unity. Comparing this value to the a posteriori variance, it is felt that the relative weights between the altimetry observations and the ground truth are equal (tests 2, 3, and 4). However, the estimates obtained for the bias terms are significantly insensitive to the relative weights used.

The a posteriori standard deviations obtained for the bias terms are

Bias	Standard deviation (in meters)	No. of points of Ground Truth
Δf_1	1.50	4
Δf_2	0.66	21
Δf_3	0.95	10

These values indicate that the standard deviation is inversely proportional to the square root of the number of ground truth points available for the observations in each submode.

APPENDIX C

REPORTS AND DATA RECEIVED

	Title	Identification Number	No. of Copies
(1)	SKYLAB PROGRAM EARTH RESOURCES EXPERIMENT PACKAGE July 10, 1974 Contract NAS8-24000 Amendment JSC-14S Sensor Performance Report, Vol IV (S193 R/S) (Engineering Baseline, SL2 and SL3 evaluation)	MSC-05529 (SL3)	1
(2)	NASA TECHNICAL MEMORANDUM June, 1974 NASA TM X-58122	JSC-08461	1
(3)	SKYLAB EREP S191 INFRARED SPECTROMETER DATA ACQUISITION CAMERA SCENE LIST FOR SL-2, SL-3, and SL-4	440	1
(4)	SKYLAB 4 S190A 461636 461536 PI 4X Transparencies - 1 each pos. Mag: 52 FRAMES: 083/090 Mag: 70 FRAMES: 194/207 Mag: A4 FRAMES: 366/462		1
(5)	EREP — Tape No. 4 — V14612 6 — V03546 7 — V05607 9 — V03248 54 — V03284 85 — V03208 97 — V09770		7
(6)	SKYLAB 4 October 21 2 x Prints - 1 each Mag 90 057/60 61 thru 62 blank	W06079	4
(7)	SKYLAB PROGRAM EARTH RESOURCES EXPERIMENT PACKAGE September 6, 1974 Contract NAS8-24000 Amendment JSC-14S Sensor Performance Report, Volume 1 (S190A) (Engineering Baseline, SL2, SL3 and SL4 Evaluation)	MSC-05528	1

	Title	Identification Number	No. of Copies
(8)	SKYLAB PROGRAM EARTH RESOURCES EXPERIMENT PACKAGE September 6, 1974 Contract NAS8-24000 Amendment JSC-14S Sensor Performance Report, Volume II (S191) (Engineering Baseline, SL2, SL3 and SL4 Evaluation)	MSC-05528	1
(9)	REQUIREMENTS FOR EREP ELECTRONICS SENSORS October 18, 1974	PHO-TR524 Rev. A, Ch. 2	3
(10)	SKYLAB PROGRAM EARTH RESOURCES EXPERIMENT PACKAGE September 6, 1974 Contract NAS8-24000 Amendment JSC-14S Sensor Performance Report, Volume V (S193 ALT) (Engineering Baseline, SL2, SL3 and SL4 Evaluation)	MSC-05528	1
(11)	SKYLAB PROGRAM EREP September 6, 1974 Contract NAS8-24000 Amendment JSC-14S Sensor Performance Report, Volume V (S192) (Engineering Baseline, SL2, SL3 and SL4 Evaluation)	MSC-05528	1
(12)	EARTH RESOURCES EXPERIMENT PACKAGE (EREP) EXPERIMENT CALIBRATION DATA	MSC-07744	1