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Introduction

In this report, the progress we have made on the following projects during the period of this grant, with particular emphasis on the work carried out since submitting the semi-annual report in June 1974:

A. Calculation of the energy-density relation for pure neutron matter in the density range relevant for neutron stars using four different hard-core potentials.

B. Calculation of the properties of the superfluid state of the neutron component and the superconducting state of the proton component and the effects of polarization in neutron star matter.

A. Neutron Matter - Normal State Calculations

Since the preparation of the semi-annual status report, we have been able to obtain a better method of constrained minimization of energy. This new method gives energies considerably lower than those reported in the semi annual report. Therefore we have adopted this method and carried out the full range of calculations for Ohmura-Morita - Yamada (OMY-4), standard hard core (SHC), Reid Hard Core (RHC) and Hamada-Johnston (HJ) potentials as outlined in the proposal.

The correlation factor used in our calculation has the following form:

$$f(r) = \begin{cases} \left[1 - e^{-\mu_1 (r - r_c)}\right]\left[1 + \gamma\, e^{-\mu_2 (r - r_c)}\right], & r > r_c \\ 0, & r < r_c \end{cases} \qquad (C3)$$

The choice of this three-parameter correlation factor is based on considerable numerical experience involving the use of two simpler correlation factors; A two-parameter form (C2) which can be obtained from (C3) by choosing $\mu_1 = \mu_2$ and a one-parameter form (C1) obtained from (C3) when $\gamma = 0$. We have found that when we determine μ_1 in (C1) by energy minimization, this is associated with the violation of several of the conditions discussed in the semi annual report, except in the low-density region. The additional flexibility of (C2) enables us to extend the region, where the conditions are satisfied, beyond the low densities. But (C2) turns out to be inadequate in the moderate to high density region. For example, it has been found in our earlier nuclear matter calculation using the SHC potential that in order to extend the region of validity of the method up to and beyond the region of equilibrium density, we have to use the three-parameter form (C3). It should also be noted that the magnitude of the cluster correction $\mathcal{E}_3$ relative to $\mathcal{E}_2$ progressively diminishes as we moved from (C1) to (C2) to (C3) in our calculations. These remarks summarize some important general features of our numerical experience in neutron matter and symmetrical nuclear matter calculations.

The necessary conditions on the radial distribution function as well as other physically motivated conditions on the trial wave function discussed in the semi annual report are summarized below for convenience

(IA) $\Delta N = 0$ (Normalization condition or structure-factor sum rule to lowest order)

(IB) $\overline{\Delta N} = 0$

(II) $S(0) = 0$ (Structure-factor sum rule)

(III) $S(k) \geq 0$ (Structure-factor inequality)

(IV) $I_C \leq 1.243$ (Coulomb inequality)

(VA) $I_B = 0$ (Pauli condition averaged over the fermi sea)

(VB) $\overline{I_B} = 0$ (Pauli condition for the "average pair")

We recall that some of these are mutually exclusive, if we attempt to impose all of them exactly on a state-independent correlation factor. However, we may try to choose a subset of these to be imposed exactly and hope to have the results thus obtained satisfy the remaining ones approximately. Our objective is to seek the best one among the various procedures that are possible within this general framework - best, in the sense that it gives the lowest energy.

In order to obtain a calculational method which is reliable in the high-density region, we chose to try different procedures first at the density corresponding to $k_F = 3.5\ fm^{-1}$ using the OMY-4 potential. The difference in the methods is due to different choices of constraints on the energy minimization as described below.

Method A1. At each k_F, the parameter γ in the correlation factor $f(\mu_1, \mu_2, \gamma; r)$ is determined by the condition (IA), for every chosen set of values (μ_1, μ_2). With $\gamma = \bar{\gamma}$ thus determined, we find that for a given μ_2, energy $\mathcal{E} \equiv \mathcal{E}_F + \mathcal{E}_2 + \mathcal{E}_3$ exhibits a minimum as a function of μ_1 at some value $\bar{\mu}_1$. The two-body approximation for energy, $(\mathcal{E}_F + \mathcal{E}_2)$, also has a minimum with respect to μ_1 at approximately the same value $\bar{\mu}_1$. Therefore, we determine $\mu_1 = \bar{\mu}_1$ by minimizing $(\mathcal{E}_F + \mathcal{E}_2)$. The values of

$\mathcal{E}(k_F, \bar{\mu}_1, \mu_2, \bar{\gamma})$, S(o) and some other associated quantities defined in the semi-annual report, calculated for various values of μ_2 with $k_F = 3.5\,fm^{-1}$ are given in Table A-1. We then plot S(o) as a function of μ_2 to determine $\mu_2 = \mu_2^o$ where S(o)=0. But, it is seen from Table A-1 that S(o) never reaches zero. At somewhat lower densities, $k_F \lesssim 3.0\,fm^{-1}$, S(o) crosses over from a negative value to a positive value at some μ_2. At $k_F = 3.5\,fm^{-1}$, we have to be satisfied with choosing μ_2^o as the value where S(o) is closest to zero, this value is $\mu_2^o = 11.90\,fm^{-1}$. At this μ_2^o, the other parameters $\gamma = \gamma^o$ and $\mu_1 = \mu_1^o$ are redetermined from the condition (IA) and the minimization of $\mathcal{E}$ respectively. In this manner, we arrive at the optimum set of parameters $(\mu_1^o, \mu_2^o, \gamma^o)$ for the correlation factor at each density. For $k_F = 3.5\,fm^{-1}$ and the OMY-4 potential, the optimum set is $(13.23\,fm^{-1}, 11.90\,fm^{-1}, 1.897)$. The corresponding values for energy and other related quantities are entered under method (A1) in Table A-2. We note that $I_c = 0.811$, which satisfies condition (IV), namely, the Coulomb inequality. By calculating $S(k)$ for a wide range of k values, we have also verified that condition (III), namely, the structure-factor inequality, is satisfied by the optimum correlation factor $f(\mu_1^o, \mu_2^o, \gamma^o, r)$. It also satisfies the Pauli condition approximately since $|I_B| = 0.011$. Finally, the correlation parameter $|\xi| = 0.112$, is sufficiently small indicating good convergence, especially since the ratios R_c and $R_s(0)$ are also small. Method A2. In this method, condition (VA) is used instead of (IA) to determine $\gamma = \gamma^o$ otherwise the procedure in method A1 is followed. For the OMY-4 potential, the results obtained at $k_F = 3.5\,fm^{-1}$ using this method are given in Table A-2. We note that the energy obtained by this method, namely, $\mathcal{E} = 1219$ MeV, is higher than the value $\mathcal{E} = 1022$ MeV obtained by method A1. Furthermore, this energy is associated with S(o) = −0.091 compared to S(o) = −0.007 for method A1. Therefore method A1 is better than method A2.

Method A3. The condition $\xi = 0$ is used to determine $\gamma = \gamma^o$ in this method. (In this case, $2\Delta N$ may be assumed to play the role of the "smallness parameter."). Otherwise the procedure in the two previous methods is followed. The results given in Table A-2 for $k_F = 3.5\,\mathrm{fm}^{-1}$ and OMY-4 potential show that with $\mathcal{E} = 1156$ MeV and $S(o) = -0.042$, , this method is better than A2, but still not as good as A1.

We have carried out calculations using two other methods in which the condition determining γ is changed to (IB) and (VB) respectively. These also result in values of $\mathcal{E}$ higher than that obtained in method A1 and also associated with a greater degree of violation of the condition S(o)=0.

The method we adopted in calculating the results reported in this paper is method A, which is an improvement over method A1, as we shall see below. In A1, at a given k_F, the value of μ_1 which minimizes $(\mathcal{E}_F + \mathcal{E}_2)$ or $\mathcal{E}$ for every chosen value of μ_2 is associated with S(o) < 0, except in the very low density region. However, at that stage of the procedure, we have ignored this violation of the structure-factor sum rule, anticipating that μ_2 will be adjusted to make S(o)=0. In studying S(o) as a function of μ_1 for given μ_2, we have observed the behavior schematically indicated in figure A-1. The five sketches are for five different values of μ_2 namely β_i, such that $\beta_1 < \beta_2 < \beta_3 < \beta_4 < \beta_5$. Our numerical results show that the region near the first (deeper) minimum in S(o) is associated with extremely large values of $\mathcal{E}$ and severe violations of all the conditions, except the one explicitly imposed to determine γ. We are seeking the lowest value of $\mathcal{E}$ consistent with S(o)=o. But α_1 is not an acceptable value of μ_1 even though S(o) is zero there, because of the reasons mentioned above. As the curves (a) through (e) in figure A-1 indicate, the value $\bar{\mu}_1$ of μ_1 where $\mathcal{E}$ is minimum is roughly the same location where S(o) also has a minimum, which is negative unless μ_2 is sufficiently large. In method A1, we take a high enough value of μ_2 to make S(o)=0, so that the situation in curve (d) is obtained. In general, the energy $\mathcal{E}$ increases with μ_2. In order to see whether we can get a lower value of $\mathcal{E}$ than obtained in method A1, we attempt to determine $\bar{\mu}_1$ by not merely minimizing but by seeking its minimum consistent with S(o)=o. For the value $\mu_2=\beta_1$, we see that this is not possible because (1) we have already seen that α_1 is not acceptable and (2) the only other zero of S(o) occurs at a very high μ_1 where, once again, the value of ξ is found to be too large and some of the conditions are violated. (It may be appropriate here to point out that we have found the following general result in our numerical calculations using various methods: the coulomb inequality and the structure-factor inequality along with the conditions that ξ, I_B and $\Delta\mathcal{N}$ be small would restrict the parameter space to a region where the difference between the values of μ_1 and μ_2 is not very large. Thus we find, referring to curves (a) through (e), that the zero of S(o) occurring at a very high μ_1

and the zero of S(o) at a very small $\mu_1=\alpha_1$ are both unacceptable.) Thus we are forced to go to higher values of μ_2. For $\mu_2=\beta_2$ (curve (b), S(o) is zero at $\mu_1=\alpha_0$. Note that since $\mathcal{E}(\bar{\mu}_1)$ is the minimum of $\mathcal{E}(\mu_1)$, $\mathcal{E}(\alpha_0)>\mathcal{E}(\bar{\mu}_1)$. Now, if we go to curve (C) with $\mu_2=\beta_3$, S(o) is zero at two values of μ_1, namely α_1 and α_2. The energies are related as $\mathcal{E}(\alpha_2)>\mathcal{E}(\alpha_3)>\mathcal{E}(\bar{\mu}_1)$. In curve (d), for $\mu_2=\beta_4$, S(o) = o when μ_1 is α_2 and α_4, with the relation between the energies, $\mathcal{E}(\alpha_2)>\mathcal{E}(\alpha_4)$. For values of μ_2 greater than β_4, S(o) has only the zero at $\mu_1=\alpha_2$, Since the second minimum in S(o) is already on the positive side and it becomes more positive with increasing μ_2. This situation is illustrated in curve (e).

Thus, in method A, for each μ_2, we are determining μ_1 by minimizing $\mathcal{E}$ subject to the condition, S(o) =o. The remaining task is to find the minimum of $\mathcal{E}$ in the range of values spanned by $\mathcal{E}(\alpha_0)$, $\mathcal{E}(\alpha_3)$ and $\mathcal{E}(\alpha_4)$ which are all consistent with the structure-factor sum rule. At some densities we find that the minimum of $\mathcal{E}(\mu_2)$ occurs when $\mu_2=\alpha_0$. At others, $\mathcal{E}(\alpha_4)$ is the minimum of $\mathcal{E}(\mu_2)$. At still other densities the value of μ_2 at which $\mathcal{E}(\mu_2)$ is lowest lies between α_0 and α_4. Denoting this minimizing value by μ_2^o, we have $(\mu_1^o, \mu_2^o, \gamma^o)$ for the optimum set of parameters, where μ_1^o is the value of μ_1 at which $\mathcal{E}(\mu_1, \mu_2^o)$ has the lowest value consistent with S(o)=o, as explained above. The results for $\mathcal{E}(\mu_1^o, \mu_2^o, \gamma^o)$ and the associated quantities at $k_F = 3.5\ \mathrm{fm}^{-1}$ obtained using the OMY-4 potential are given in table A-2. We note the significant improvement in the results over those from method A1, especially in the value of energy, namely, $\mathcal{E} = 722.5$ Mev compared to $\mathcal{E} = 1022$ Mev from method A1.

The results from the calculation for the OMY-4 potential using method A for the entire density range $0.25\ \mathrm{fm}^{-1} \leq k_F \leq 5.00\ \mathrm{fm}^{-1}$ are given in table A-3. The values of $\mathcal{E}$ determined by this method are plotted against the density parameter k_F in figure A-2. The most notable feature of the $\mathcal{E}(k_F)$ curve is the occurrence of a local minimum near $k_F = 1.75\ \mathrm{fm}^{-1}$
and the discontinuity in the slope of $\mathcal{E}(k_F)$ near $k_F = 1.30\ \mathrm{fm}^{-1}$. The energies at all densities, except in the very low density region, determined by our constrained minimization procedure have come out much higher than the results obtained by other authors. The convergence of the cluster series for energy and other associated quantities appears to be excellent judging from the data in table A-3. One of the most important aspects of our method that emerge from these results is the crucial role played by the structure factor sum rule in achieving this convergence. It is also responsible for raising the energies to such high values. For example, if the condition s(o) = o is ignored, at $k_F = 1.5\ \mathrm{fm}^{-1}$, we will get $\mathcal{E} = 13$ Mev compared to $\mathcal{E} = 23$ Mev we have in table A-3, for the OMY-4 potential. This effect, of course, becomes more pronounced at higher densities. Thus the answer to the question whether the energies given by method A are reliable depends crucially on the justification for the central importance we have assigned to the structure-factor sum rule. (We may also point out here that once S(o) =o is imposed in the manner we have done in method A, $S(k) \geq o$ is always automatically satisfied. Choosing one of the other zeroes of S(o) leads to a violation of $S(k) \geq 0$ for some range of k values and, associated with this, to large values of ξ, except in the very low density region, where a straightforward minimization of $\mathcal{E}(\mu_1, \mu_2)$ with respect to μ_1 and μ_2 yields a minimum that is consistent with all the conditions.) Since our method is variational, it may be contended that

our high values of energy may constitute only a poor upper bound to the true energy. In reply to this criticism, we can only say that our numerical experience as summarized in this report suggests that this is the lowest energy we can obtain for our choice of constraints with the three-parameter correlation factor (C3). We also note that the constraints chosen in method A are not arbitrary, but have been justified by the physical arguments referred to in the original proposal and by the numerical evidence for their central role in obtaining apparently rapid convergence of the energy series and the associated cluster expansions.

The results from the various methods we have attempted indicate that any one of the conditions (IA), (IB), (VA) and (VB) tend to make the correlation parameter ξ small. But we also find that smallness of ξ by itself does not assure the convergence of the cluster expansions. Only if the smallness of ξ is accompanied by the satisfaction of $S(k) \geq o$, it seems possible to obtain over-all convergnece. In spite of this statement, ξ still plays a useful role as an ordering parameter for cluster expansions in the CBF formalism. We have repeated our calculation at $k_F = 3.5$ fm^{-1} for OMY-4 potential using method A1, omitting the term $\mathcal{E}_{IV}$ from the expression for $\mathcal{E}_3$. The energy obtained in this way is higher than the value $\mathcal{E} = 1022$ Mev given in table A-2. We recall that $\mathcal{E}_{IV}$ is a (reducible) four-body correlation term whereas all other terms in $\mathcal{E}_3$ contain the effects of (irreducible) three-body correlations. If we are to use the "number of bodies" to classify various orders in the cluster series, we must exclude $\mathcal{E}_{IV}$ from $\mathcal{E}_3$. But, according to the ξ - classification scheme, $\mathcal{E}_{IV}$ is of the same order as the other terms constituting $\mathcal{E}_3$ and hence must be included in $\mathcal{E}_3$. Hence the numerical results mentioned above seem to justify the use of ξ as an ordering parameter for cluster expansions.

The energy per particle of neutron matter calculated using the SHC, RHC and HJ potentials are given in figures A-3, A-4, and A-5 respectively. These also show local minima and discontinuity in slope as we have seen for the OMY-4 potential. We cannot yet attach any physical significance to these nonmonotonic regions of the curve for the following reason. The parameter values in the correlation factor vary significantly with density in these regions as seen from table A-3 for the case of the OMY-4 potential. It is possible that when perturbation corrections $\Delta\mathcal{E}$ are calculated using these optimum correlatinn factors, the final result, namely, $(\mathcal{E}_F + \mathcal{E}_2 + \mathcal{E}_3) + \Delta\mathcal{E}$ when plotted against k_F, may not have such local minima or discontinuities in its slope. When $\Delta\mathcal{E}$ is calculated on the basis of a convergent perturbation expansion, if it is found that the final energy-density curve still displays these features, then only we can consider these as possible indications of phase transitions.

In the meantime, we also need to look critically at our choice of the structure factor-sum rule, namely S(o) = o, as a vital constraint in the energy minimization. In particular, we need to assure ourselves that it is realistic to impose that condition on a short range correlation factor and that the raising of energies caused by it is not an artificial effect. We want to emphasize, however, that we have given some physical arguments that justify the use of this condition and also the fact that it definitely helps to obtain better convergence of the cluster expansions in our formalism.

$\bar{\mu}_1$ (fm^{-1})	μ_2 (fm^{-1})	$\bar{\gamma}$	$\lvert\xi\rvert$	$S(0)$	$R_S(0)$	I_c	R_c	$\lvert I_B\rvert$	$\Delta^2 N$	$\mathcal{E}_F+\mathcal{E}_2$ (MeV)	$\mathcal{E}_3$ (MeV)	$\mathcal{E}$ (MeV)	$R_\mathcal{E}$
3.8	3.0	0.744	0.385	−0.843	0.840	1.159	0.096	0.018	0.070	111.2	76.35	187.5	1.86
4.4	3.5	0.795	0.342	−0.574	0.572	1.113	0.052	0.016	0.055	129.9	75.40	205.3	3.37
5.0	4.0	0.846	0.307	−0.420	0.418	1.070	0.028	0.015	0.044	152.9	74.53	227.4	125.
5.7	4.5	0.865	0.277	−0.322	0.321	1.032	0.014	0.014	0.037	179.6	73.88	253.4	2.70
6.2	5.0	0.948	0.252	−0.256	0.256	1.000	0.007	0.013	0.031	209.5	73.49	283.0	1.28
6.8	5.5	0.999	0.231	−0.208	0.207	0.972	0.004	0.013	0.027	242.6	73.49	316.1	0.81
7.4	6.0	1.051	0.213	−0.170	0.170	0.948	0.003	0.012	0.023	278.8	73.92	352.7	0.17
7.9	6.5	1.129	0.197	−0.140	0.139	0.927	0.004	0.012	0.019	317.8	74.77	392.6	0.45
8.4	7.0	1.204	0.184	−0.114	0.114	0.908	0.006	0.012	0.016	359.9	76.07	436.0	0.37
9.0	7.5	1.255	0.172	−0.092	0.092	0.891	0.008			404.9	77.82	482.7	0.31
9.5	8.0	1.328	0.162	−0.073	0.073	0.876	0.012			452.7	79.89	532.6	0.27
10.0	8.5	1.400	0.153	−0.057	0.056	0.863	0.015			503.5	82.21	525.8	0.23
10.5	9.0	1.471	0.144	−0.043	0.042	0.851	0.018			557.3	84.70	642.0	0.21
11.0	9.5	1.541	0.137	−0.031	0.031	0.841	0.020			613.9	87.23	701.1	0.19
11.5	10.0	1.610	0.131	−0.022	0.022	0.832	0.022			673.4	89.71	763.1	0.17
11.9	10.5	1.699	0.125	−0.015	0.015	0.825	0.023			735.8	91.97	827.7	0.16
12.4	11.0	1.765	0.120	−0.010	0.010	0.819	0.024	0.011	0.002	801.0	93.97	894.9	0.15
12.9	11.5	1.831	0.115	−0.008	0.008	0.814	0.023	0.011	0.001	869.0	95.55	964.5	0.13
13.3	12.0	1.916	0.111	−0.007	0.007	0.810	0.022	0.011	0.002	939.7	96.62	1036.	0.12
13.8	12.5	1.980	0.107	−0.009	0.009	0.808	0.020	0.011	0.002	1013.	97.10	1110.	0.11
14.2	13.0	2.062	0.103	−0.012	0.012	0.806	0.017	0.011	0.003	1089.	96.88	1186.	0.10
14.7	13.5	2.124	0.100	−0.017	0.017	0.806	0.013	0.011	0.004	1167.	95.93	1263.	0.10

Table A.2 : $\mathcal{E}(\bar{\mu}_1, \mu_2, \gamma)$ and associated results calculated for the OMY-4 potential at $k_F = 3.5\ fm^{-1}$ using method A1.

Method	$\mathcal{E}_2$ (MeV)	$\mathcal{E}_F+\mathcal{E}_2$ (MeV)	$\mathcal{E}_3$ (MeV)	$\mathcal{E}$ (MeV)	$R_{\mathcal{E}}$	κ_1^0 (fm^{-1})	κ_2^0 (fm^{-1})	γ^0	$\lvert\xi\rvert$	$\lvert I_B\rvert$	$\lvert\Delta N\rvert$	$\langle\kappa_J\rangle$	$\lvert\bar{I}_B\rvert$	$\lvert\overline{\Delta N}\rvert$	$\bar{\kappa}_J$	$\lvert\Delta^2 N\rvert$	I_c	R_c	$S(0)$	$R_S(0)$
A	519.9	672.2	50.3	722.5	0.10	5.705	8.10	3.612	0.094	0.012	0	0.024	0.153	0.027	0.253	0.002	0.810	0.01	0.000	0.00
A1	773.0	925.3	96.5	1022.	0.12	13.23	11.90	1.897	0.112	0.011	0	0.022	0.265	0.032	0.233	0.001	0.811	0.02	-0.007	0.01
A2	1048.	1201.	17.7	1219.	0.02	11.50	11.00	2.665	0.259	0	0.148	0.296	0.030	0.097	0.254	0.077	0.777	0.22	-.091	0.55
A3	928.8	1082.	73.5	1156.	0.08	12.80	12.00	2.256	0	0.008	0.043	0.102	0.102	0.017	0.238	0.026	0.802	0.04	-.042	0.14

Table A.2 Comparison of the results obtained from various methods for the OMY-4 potential at $k_F = 3.5\ fm^{-1}$.

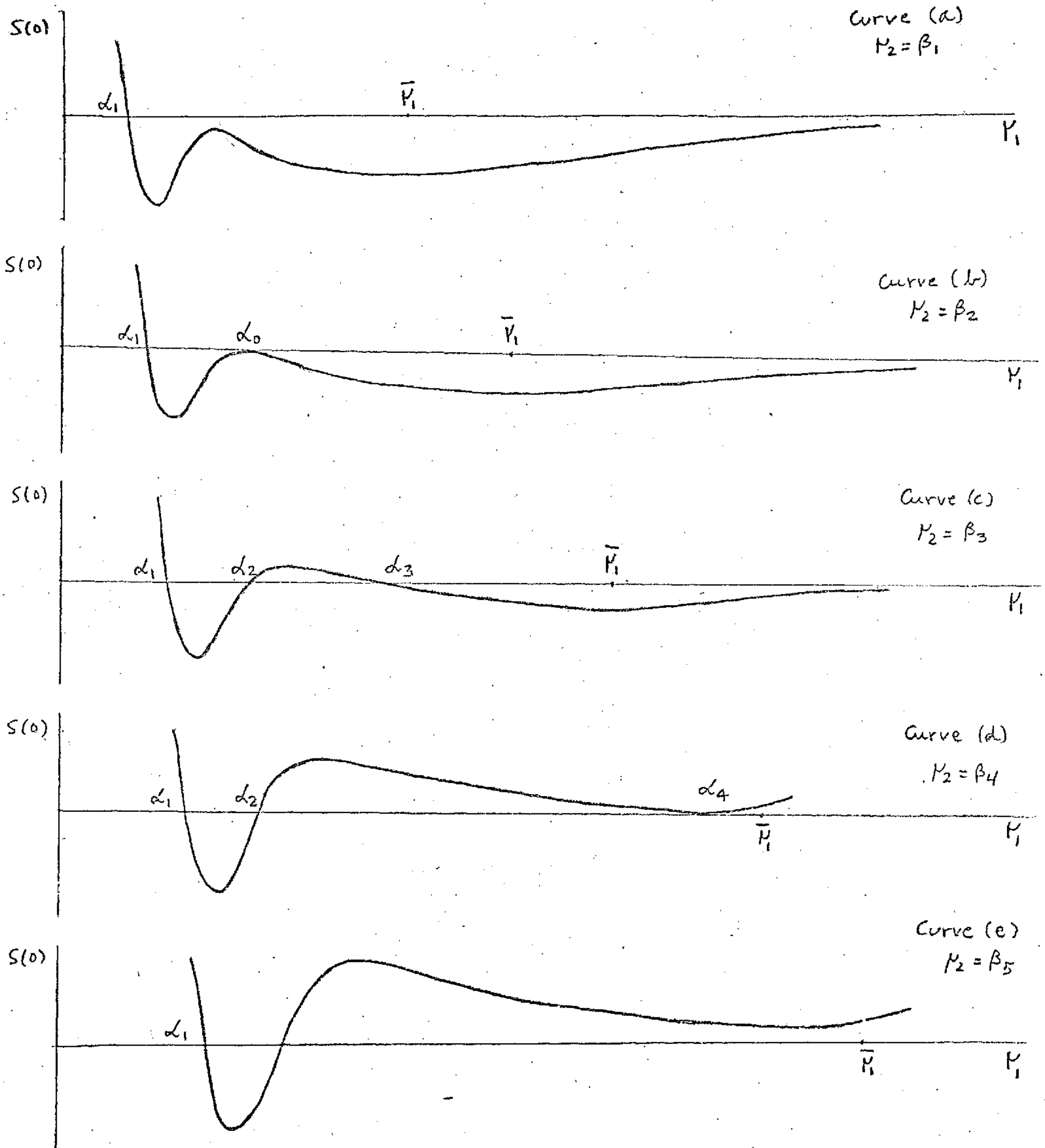


Figure A.1 The behavior of S(0) as a function of μ_1 for various values of μ_2 (schematic); $\beta_1 < \beta_2 < \beta_3 < \beta_4 < \beta_5$

k_F (fm^{-1})	ε_2 (MeV)	$\varepsilon_F+\varepsilon_2$ (MeV)	ε_3 (MeV)	ε (MeV)	R_ε	κ_1^0 (fm^{-1})	κ_2^0 (fm^{-1})	γ^0	$-\xi$	$-I_B$	ΔN	$\langle K_J \rangle$	$-\bar{I}_B$	$-\overline{\Delta N}$	$\bar{K}_J$	$-\Delta^2 N$	I_C	R_C	$S(0)$	$R_S(0)$
0.25	-0.129	0.648	0.000	0.648	0.00	1.800	2.00	1.709	0.000	0.000	0	0.000	0.000	0.000	0.000	0.000	0.605	0.00	0.000	0.00
0.50	-0.952	2.156	0.000	2.156	"	1.755	2.00	1.834	0.000	0.000	0	0.000	0.001	0.000	0.001	"	0.635	0.00	"	"
1.00	-5.911	6.522	-0.001	6.521	[illegible]	1.620	1.90	1.971	0.002	0.000	0	0.000	0.008	0.003	0.010	"	0.669	0.01	"	"
1.30	-9.964	11.05	+0.014	11.06	"	2.270	2.74	2.183	0.003	0.001	0	0.001	0.011	0.003	0.017	"	0.677	0.00	"	"
1.50	-4.657	23.32	0.123	23.44	0.03	7.070	5.80	1.101	0.012	0.001	0	0.002	0.012	0.002	0.020	"	0.680	0.01	"	"
1.73	-13.49	23.72	0.220	23.94	0.02	2.676	3.35	2.435	0.009	0.002	0	0.003	0.026	0.007	0.037	"	0.698	0.01	"	"
2.00	-8.694	41.04	0.643	41.68	0.07	2.980	3.86	2.688	0.011	0.002	0	0.005	0.038	0.011	0.054	0.001	0.710	0.02	"	"
2.50	55.32	133.0	3.929	136.9	0.07	4.334	5.80	3.032	0.029	0.004	0	0.008	0.058	0.011	0.095	0.000	0.732	0.01	"	"
3.00	203.1	315.0	16.94	331.9	0.08	5.102	7.00	3.284	0.058	0.007	0	0.014	0.097	0.018	0.160	0.002	0.768	[illegible]	"	"
3.50	519.9	672.2	50.31	722.5	0.10	5.705	8.10	3.612	0.094	0.012	0	0.024	0.153	0.027	0.253	"	0.810	"	"	"
4.00	1144.	1343.	104.5	1447.	0.09	6.020	9.06	4.147	0.131	0.018	0	0.037	0.236	0.043	0.385	0.005	0.860	0.04	"	"
4.50	2138.	2389.	180.1	2569.	0.08	6.226	9.80	4.590	0.184	0.028	0	0.055	0.347	0.066	0.563	0.015	0.939	0.09	"	"
5.00	3894.	4205.	206.5	4411.	0.05	6.335	10.62	5.263	0.237	0.040	0	0.081	0.504	0.102	0.805	0.031	1.036	0.18	"	"

Table A.3 : Results (obtained using method A) for the OMY-4 potential in the density range $0.25\ fm^{-1} \le k_F \le 5.00\ fm^{-1}$

FIGURE A.2 : $\mathcal{E}(k_F)$ FOR NEUTRON MATTER (OMY-4 POTENTIAL)

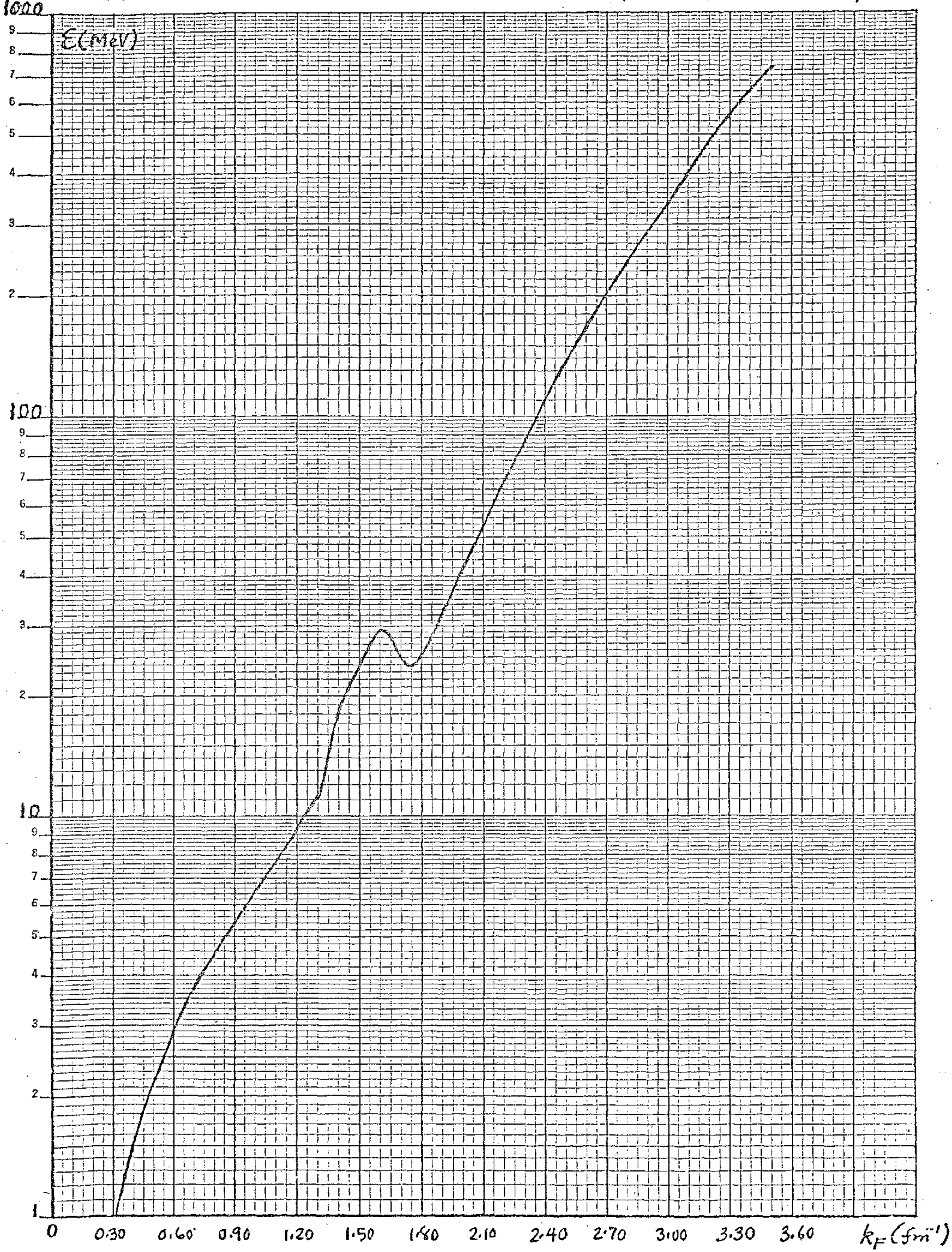


FIGURE A.3: $\mathcal{E}(k_F)$ FOR NEUTRON MATTER (SHG POTENTIAL) 14

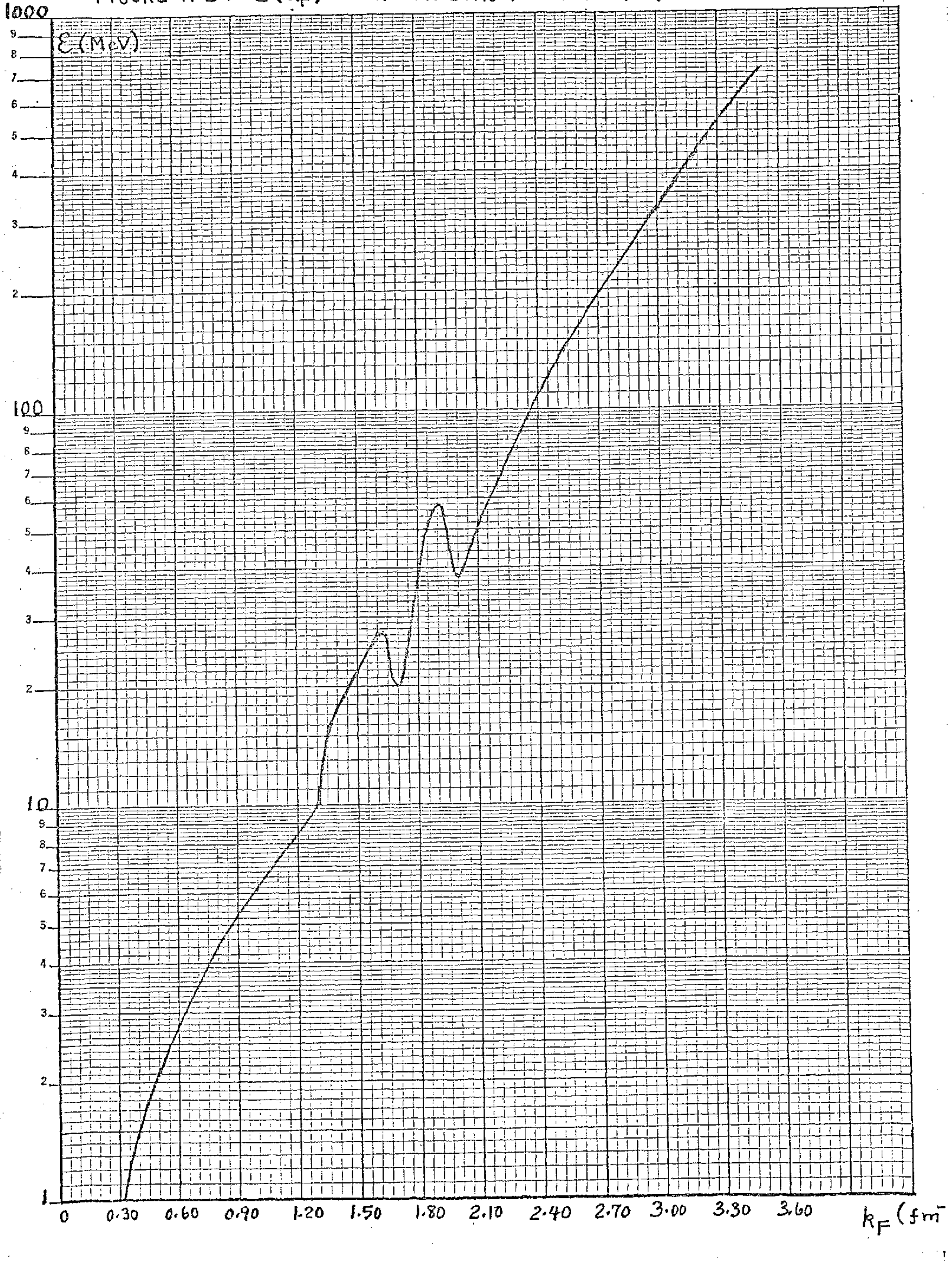


FIGURE A.4: $\mathcal{E}(k_F)$ FOR NEUTRON MATTER (RHC POTENTIAL)

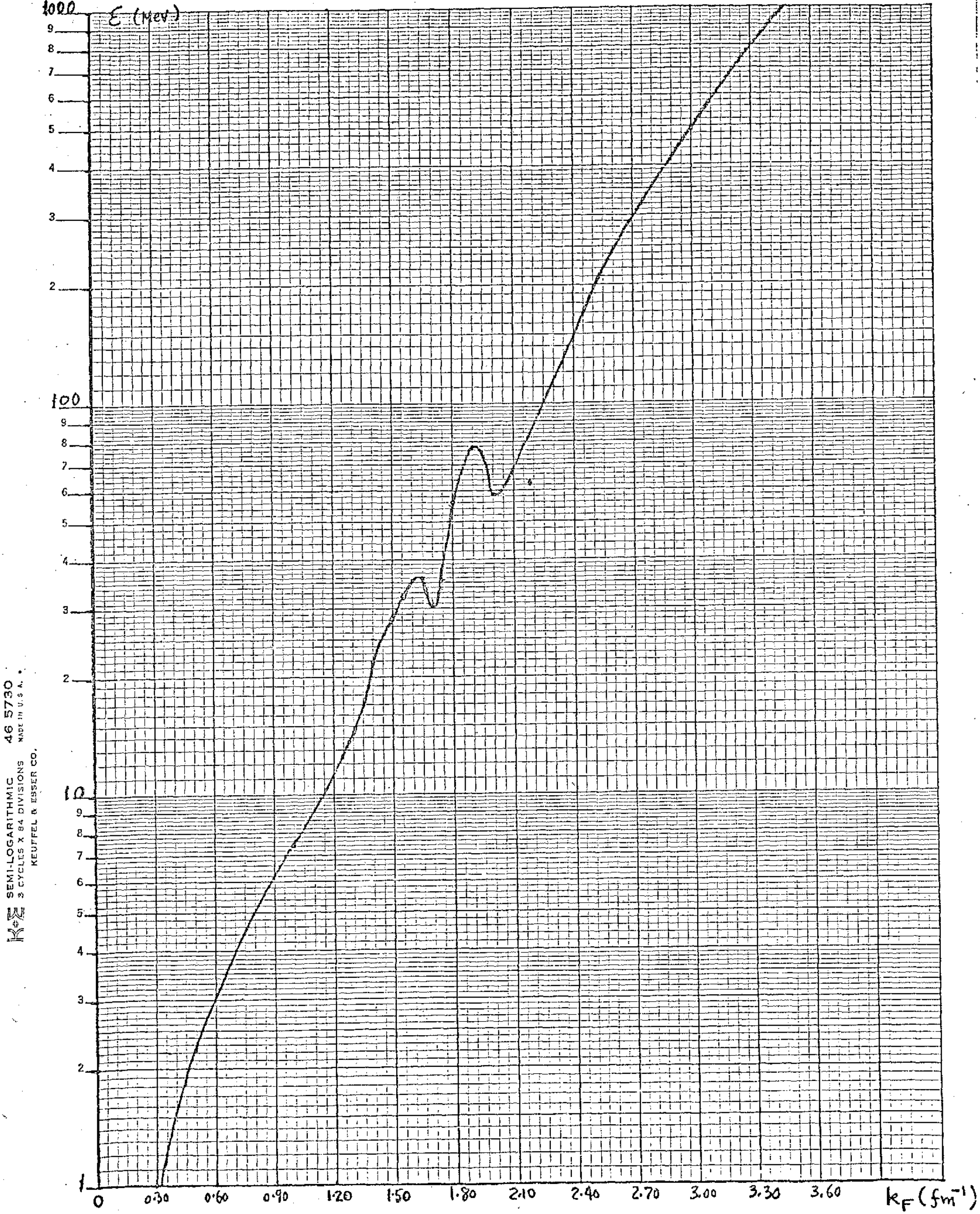
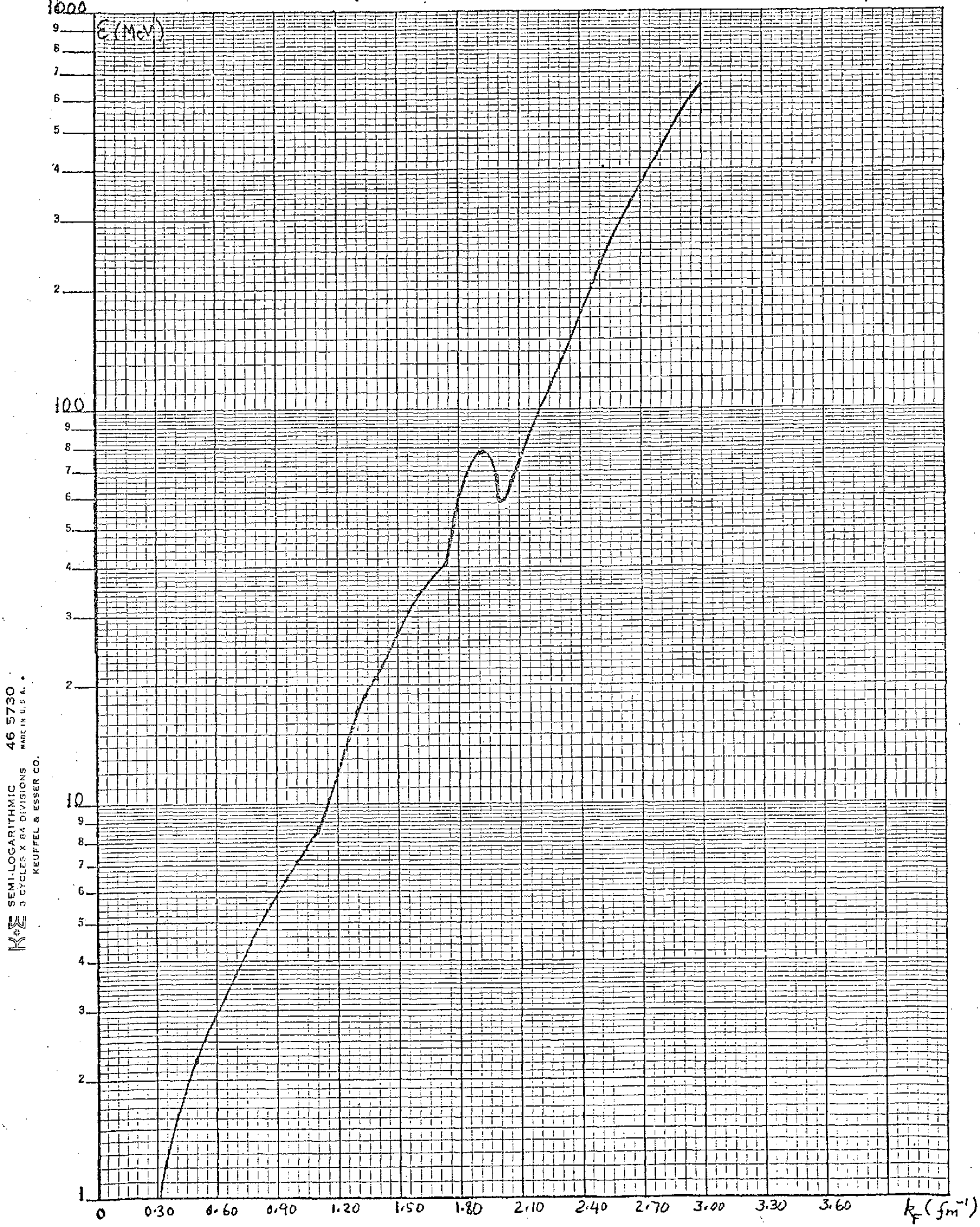


FIGURE A.5 : $\mathcal{E}(k_F)$ FOR NEUTRON MATTER (HJ POTENTIAL)



B. Effects of Polarization on Neutron Star Matter

The final numerical work on this preliminary study of this subject has been completed. We plan to present a paper on this subject at the APS meeting to be held in Washington, D. C. next April.

We have completed the following additional calculations since our last semi-annual report:

1. The three-particle contributions to the normal state energy (per particle) in the density region interested to us has been carefully checked out. They are as following:

At density (expressed in term of k_F (fm^{-1}))	the three particle contributions, $\mathcal{E}_3$(MeV)
0.25	0.0000
0.50	-0.0012
1.00	-0.0858

These numerical results for normal state indicate that the three-particle contributions in the density region interested to us are indeed sufficient small as compared to their corresponding single and two particle contributions to the energy expectation value. That leads us to believe that the three-particle contributions to the energy expectation value is ngligible as compared with the single-and two-particle contributions in the superfluid state because of the same cluster expansion employed in both cases.

b. The numerical accuracy of the condensation energy calculations were tested by changing our computer programs for those calculation from the 12-point Gaussian integral to the 20-point Gaussian integral for the enhanced factor β = 1.50 for the calculations mentioned in our last annual report (see Table B-2).

The results are as following:

at density, k_F (fm^{-1})	$\mathcal{E}_c$ (MeV) 12-PT. G. I.	20-pt. G. I.	Difference
0.60	2.0695	2.0642	0.0053
0.96	1.1047	1.1095	0.0048

Thus the numerical evaluations for condensation energies are reliable up to the third place after decimal point (the difference is less than one-half of one percent).

3. In addition to the numerical results given in the last two reports, we have made the following two sets of calculations to complete our preliminary study of this subject:

   (a) The condensation energy is calculated in such a way that it has the effect of incorporating polarization prior to the short range correlations and superfluidity. Numerically, the condensation energy, $\mathcal{E}_c$, is calculated by using the optimal energy gap, Δ_k, for each given enhanced factor β in both $\mathcal{E}_{c_1}$ and $\mathcal{E}_{c_2}$, but only the A_0 in $P_{k\ell}$ is multiplied by β. The results are given in Table (B-1) for enhanced factor β = 1.20, 1.30 and 1.40.

   (b) The condensation energy is calculated in such a way that it has the effect of incorporating the short range correlations and superfluidity prior to polarization. Numerically, the condensation energy, $\mathcal{E}_c$, is calculated by $\mathcal{E}_c = \mathcal{E}_{c_1} + \mathcal{E}_{c_2}$ (i.e. the whole ω_2 in $P_{k\ell}$ is enhanced by the factor β). The results are given in Table (B-2) for enhanced factor β = 1.15, 1.20, 1.30 and 1.40.

The results of Table (B-1) of this report and Table (B-2) of the last annual report all seem to indicate that the polarization effect indeed enhances the condensation energy and the gap function and there is a tendency of the neutron star matter to undergo a first-order phase transition at a relatively lower density region (10^{12}-10^{13} gm-cm^{-3}). This preliminary study reaffirms the necessity and interest of a first principle theory study of the problem.

Table (B-1)

In this set of calculations, the condensation energy, $\mathcal{E}_c$, is calculated by using the $(\Delta_{k_F})_{\text{optimal}}$ for each corresponding β in both $\mathcal{E}_{c_1}$ and $\mathcal{E}_{c_2}$, but only the A_o in P_{kk} is multiplied by β

β =			1.20				1.30				1.40			
k_F (fm^{-1})	n ($\times 10^{-2} fm^{-3}$)	$\mathcal{E}_n$ (MeV)	$\mathcal{E}_c$ (MeV)	$\mathcal{E}_s$ (MeV)	$n\mathcal{E}_s$ ($\times 10^{-2}$ MeV-fm^{-3})	$(\Delta_{k_F})_{opt.}$ (MeV)	$\mathcal{E}_c$ (MeV)	$\mathcal{E}_s$ (MeV)	$n\mathcal{E}_s$ (10^{-2} MeV-fm^{-3})	$(\Delta_{k_F})_{opt.}$ MeV	$\mathcal{E}_c$ (MeV)	$\mathcal{E}_s$ (MeV)	$n\mathcal{E}_s$ ($\times 10^{-2}$ MeV-fm^{-3})	$(\Delta_{k_F})_{opt.}$ (MeV)
0.24	0.0477	0.606	1.157	-0.551	-0.026	1.693	2.516	-1.910	-0.091	2.517	4.199	-3.593	-0.171	3.539
0.36	0.1576	1.253	1.355	-0.102	-0.016	2.634	2.706	-1.453	-0.229	3.576	4.458	-3.205	-0.505	4.667
0.48	0.3735	2.050	1.427	0.623	0.233	3.508	2.489	-0.439	-0.164	4.548	4.016	-1.966	-0.694	5.726
0.60	0.7295	2.960	1.362	1.598	1.166	4.153	2.316	0.644	0.470	5.304	3.586	-0.626	-0.457	6.608
0.72	1.2506	3.990	1.171	2.819	3.525	4.489	1.992	1.998	2.499	5.750	3.070	0.920	1.151	7.116
0.84	2.0018	5.150	0.938	4.212	8.432	4.440	1.643	3.507	7.020	5.803	2.566	2.584	5.173	7.265
0.96	2.9881	6.450	0.715	5.735	17.137	3.937	1.304	5.146	15.377	5.375	2.085	4.365	13.043	6.919
1.08	4.2545	7.935	0.420	7.515	31.973	3.005	0.881	7.054	30.011	4.457	1.507	6.428	27.348	6.023
1.20	5.8361	9.655	0.176	9.479	55.320	1.790	0.467	9.188	53.622	3.098	0.940	8.715	50.862	4.643
1.32	7.7680	11.650	0.000	11.650	90.497	0.070	0.144	11.506	89.379	1.588	0.402	11.248	87.374	2.874

FOLDOUT FRAME 1

FOLDOUT FRAME 2

Table (B-2)

In this set of calculations, the condensation energy, $\mathcal{E}_c$, is calculated by $\mathcal{E}_c = \mathcal{E}_{c_1} + \beta\mathcal{E}_2$ (i.e. the whole ω_2 in Pkq is enhanced by the factor β)

$\beta =$			1.15			1.20			1.30			1.40		
k_F	n	$\mathcal{E}_n$	$\mathcal{E}_c$	$\mathcal{E}_s$	$n\mathcal{E}_s$	$\mathcal{E}_c$	$\mathcal{E}_s$	$n\mathcal{E}_s$	$\mathcal{E}_c$	$\mathcal{E}_s$	$n\mathcal{E}_s$	$\mathcal{E}_c$	$\mathcal{E}_s$	$n\mathcal{E}_s$
(fm^{-1})	($\times 10^{-2} fm^{-3}$)	(MeV)	(MeV)	(MeV)	(MeV-fm^{-3})	(MeV)	(MeV)	(MeV-fm^{-3})	(MeV)	(MeV)	($\times 10^{-2}$ MeV-fm^{-3})	(MeV)	(MeV)	($\times 10^{-2}$ MeV-fm^{-3})
0.24	0.0477	0.606	0.271	0.335	0.016	0.323	0.283	0.014	0.428	0.178	0.009	0.533	0.073	0.004
0.36	0.1576	1.253	0.418	0.835	0.132	0.479	0.774	0.122	0.599	0.654	0.103	0.720	0.533	0.084
0.48	0.3735	2.050	0.496	1.554	0.580	0.562	1.488	0.556	0.695	1.355	0.506	0.831	1.219	0.455
0.60	0.7295	2.960	0.486	2.474	1.805	0.546	2.414	1.761	0.667	2.293	1.673	0.788	2.172	1.584
0.72	1.2506	3.990	0.362	3.628	4.537	0.410	3.580	4.477	0.504	3.486	4.358	0.598	3.392	4.242
0.84	2.0018	5.150	0.219	4.931	9.871	0.249	4.901	9.811	0.308	4.842	9.693	0.368	4.782	9.573
0.96	2.9881	6.450	0.132	6.318	18.879	0.146	6.304	18.837	0.175	6.275	18.750	0.204	6.246	18.664
1.08	4.2545	7.935	0.034	7.901	33.615	0.038	7.897	33.598	0.048	7.887	33.555	0.057	7.878	33.517
1.20	5.8361	9.655	0.001	9.654	56.342	0.002	9.653	56.336	0.002	9.653	56.336	0.002	9.653	56.336
1.32	7.7680	11.650	0.000	11.650	90.497	0.000	11.650	90.497	0.000	11.650	90.497	0.000	11.650	90.497