CALCULATIONS OF THE MOON'S HEAT HISTORY AT DIFFERENT CONCENTRATIONS OF RADIOACTIVE ELEMENTS TAKING ACCOUNT OF THE MATERIAL DIFFERENTIATION WITH MELTING

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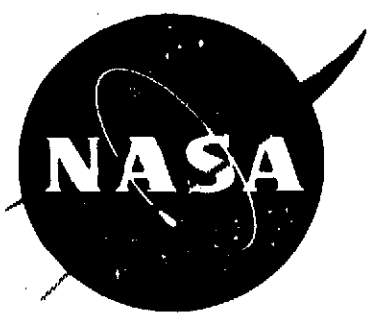

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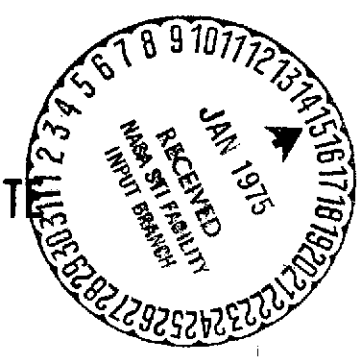

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CALCULATIONS OF THE MOON'S HEAT HISTORY AT DIFFERENT CONCENTRATIONS OF RADIOACTIVE ELEMENTS TAKING ACCOUNT OF THE MATERIAL DIFFERENTIATION WITH MELTING

(3)

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I. Introduction

Papers by G.Urey, G.MacDonald, E.A.Lubimova, B.YuLevin, S.V.Maeva, P.Friker, R.Reynolds and A.Summers, J.Iriyama and J.Schimazu, D.Anderson and R.Phinerey, T.Hanks and D.Anderson, R.MacConnel, H.Toksoz and S.Solomon, et al [2,4,5,6,8-17,26,29-31] are devoted to the investigation of the Moon's heat history based on the solution of the heat conductivity equation with the given values of the Moon's material parameters, the initial and boundary conditions and with the heat sources- radioactive elements with the given concentrations. Our papers [20,21] are devoted to the same problem. Up to 1966-67 the homogeneous and laminated Moon models were considered separately. Then in 1966-67 we and irrespective of us P.Friker, R.Reynolds and A.Sommers took into account in the calculation the differentiation of the Moon's material and convection of the radioactive elements to the surface with melting. The consideration of continuous differentiation in the calculation permits to investigate the Moon's heat history in more detail. In 1971 the differentiation was also taken into account in the paper by S.V.Maeva [15] and in 1973 in the paper by H.Toksoz and S.Solomon [26]

Thus, presently, the calculation scheme of the Moon's heat history is sufficiently full developed by the computer and much new results are obtained. Yet this problem is of interest so far and this interest still increases. This associated with the large uncertainty

in the Moon's parameters values entering in the equation of the heat conductivity. This refers also to the uncertainty in the given values of the density, the heat capacity and the heat conductivity and their dependence on the Moon's material state (for example, when melting) and to the uncertainty in the given curve of melting and to the initial conditions (in particular, to the initial temperature) and to the Moon's age related to its origin. However, the determinative in the Moon's heat history is the accepted concentration of radioactive elements in the Moon's material and their redistribution in the Moon interior in time.

In the above paper except fo [20,21] the concentration of radioactive elements is taken according to their contents in chondrite meteorites (for example, Urey and MacDonald), or in the Earth (Levin, Maeva), or in the "Earth mixture". The values of the heat flux through the Moon's surface were obtained to be of the order of (0.3-0.6) 10^{-6} $cal.cm^{-2}sec^{-1}$.

At the same time the results of radio astronomical investigations by V.S.Troitsky and V.D.Krotikov [27,28] give the flux value equal to 0.85-0.95 10^{-6} cal $cm^{-2}sec^{-1}$ which was considered for a long time to be rather doubtful but was confirmed by the resent investigations of the lunar material from Appolo-15 [7]. As a result it is necessary to calculate the Moon's heat history at different variants of the radioactive elements contents which may account for so large heat flux.

The first attempt of this calculation was made in our papers in 1966-1967 [20,21] . We use four variants of radioactive elements contents two of which corresponds to concentrations accepted by V.Yu. Levin, S.V.Maeva [9] (variant "C" mean contents in the Earth) and G.Urey and G.Mac Donald, and the rest two give relatively large concentrations with the flux value close to the radio astronomical data.

The assumption was used on the exponential differentiation (when melting) of radioactive elements. Since radioactive elements with large concentration were close to the surface of the Moon (in IO-20km) they weakly influence on the temperature of the main lunar mass which quickly cooled. As a result the thickness of the hard cover in this variants was equal to 600-700km, i.e. larger than at the mean (by B. Yu.Levin) and at the small (by Urey,MacDonald) concentrations where the thickness of the cover was 250km and 400 km.

We use in [20,2I]the exponential model of radioactive elements covection to the surface at differentiation corresponds to a certain degree to the so-colled "sudden differentiation". This model permits to find out the main physical peculiarities of the heat history related to differentiation yet it limits the number of variated parameters. In this case laminated differentiation proposed by Friker,et al [5] is proved to be evidently close to the real processes. That is why we used in the present paper the laminated differentiation of radioactive elements when melting with different portions of elements (from I00% up to 20%) convected from layer to layer. The variants of concentration of radioactive elements were taken the same as in [20, 2I.]

It should be noted that we are not specialists in geophysik and we have no rights to discuss questions of the Moon origin and its chemical composition. For this reason for the calculation we take parameters either from the expected values of the heat fluxes (at the given concentration of radioactive elements)or from the fundamental papers by other authors. Thus the initial temperatures ($273^{\circ}K$ and $900^{\circ}K$) were taken on the basis of papers [5,6] , the heat capacity - on the basis of the paper by S.V.Maeva [I3,I4] where a sharp change of the heat capacity at the time of melting is considered.

Remarks by Lubimova [I2] are taken into account on the possibility of the flux increasing at the reasonable concentrations of radioactive elemnts considering the contribution from exciton component of the heat conductivity and from the amplification of the efficiency of the radiational component.

Consideration carried out involves significant idealization reffered first of all to the process of lamination. That is why the results obtaines are rightful only in the limits of our mathematical model.

2. The statement of the problem

As a rule the Moon is assumed to be a sphere with the radius $r_0 = 1735$ km, with homogeneous density $\rho = 3.34 g/cm^3$, with the heat capacity C and the heat conductivity K, defined according to Lubimova [I2] (heat conductivity) and Maeva [I3, I4] (heat capacity) by the following relations:

The heat capacity

$$C[T, r(T)] = \begin{cases} C_0 = 0.25 \frac{cal}{gm \cdot ^\circ K}, & T \le T_{in.m.}; \; T \ge T_{in.m.} + \Delta T, \\ C_0 + \frac{4L}{(\Delta T)^2}(T - T_{in.m.}) \frac{cal}{gm \cdot ^\circ K}, & T_{in.m.} < T < T_{in.m.} + \Delta T/2, \\ C_0 - \frac{4L}{(\Delta T)^2}(T - T_{in.m.}) \frac{cal}{gm \cdot ^\circ K} + \frac{4L}{\Delta T} \frac{cal}{gm \cdot ^\circ K}, & (T_{in.m.} + \frac{\Delta T}{2}) < T < T_{in.m.} + \Delta T \end{cases} \quad (I)$$

$L = 100 cal/gm$ is the heat of melting; $\Delta T = 200$ K;

r = in cm, T = in K.

$T_{init.melt.} = 1373 + 500 (1 - r^2/r_0^2)$ is the beginning of the material melting, of the silica (Urey [29]).

We consider in (I) gradual absorption of melting heat in the interval of melting temperature ΔT (the effective heat capacity in the interval ΔT first increases linear and then decreases).

The heat conductivity

$$K = f(T) + \frac{16 n^2 \sigma T^3}{3\varepsilon} + 1{,}76\, T^2 \left[\left(\frac{E}{\kappa T} + 2 \right)^2 + 2 \right] e^{-\frac{E}{\kappa T}} \qquad (2)$$

In (2) the grid(phonon), radiational (photon) and exciton components of the heat conductivity [I2] are taken into account.

Phonon heat conductivity outside the interval of melting changes inversly proportional to the temperature and during melting it shaply increases[+)] and remains constant:

$$f(T) = A/T \; ; \; A = 10{,}3 \cdot 10^7 \frac{cal.}{cm.\,year} \; , \quad T \le T_{in.m.} \; ; \; T \ge T_{in.m.} + \Delta T \; ;$$

$$f(T) = 0{,}04 \cdot 10^7 \frac{cal}{cm.\,year} = const \; , \quad T_{in.m.} < T < (T_{in.m.} + \Delta T).$$

Photon and exciton components of the heat conductivity are defined by the well known relations where $n^2 = 3$ is the refractive index, $\sigma = 4.27\ 10^{-5}$ cal/cm^2 grad^4 year is Stephan-Boltzmann constant, $\varepsilon = 25$ cm^{-1} is the absorption coefficient of the material, $\bar{E} = 5{,}17 \times 10^{-20}$ cal is the energy of exciton activation[++)].

In the assumption of isotropic distribution of the heat sources and when the beginning of coordinate is in the centre of the sphere the heat conductivity equation is written in the form:

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(K r^2 \frac{\partial T}{\partial r} \right) + H \; , \qquad (3)$$

where H is the heat sources - long lived radioactive elements - uranium, thorium, kalium-40.

[+)] See the paper by Tikhonova, Lubimova, Vlasova 34

[++)] The values are taken for the pyrolit (the mixture of basalt and peridotit) Moon's composition similar to the Earth mantle - see Ringwood [25].

$$H = \sum_{j=1}^{3} H_j^{\circ} e^{-\lambda_j t} F(r,t) \rho \alpha_j , \qquad (4)$$

where the function $F(r,t)$ defines the distribution of radioactive elements depending on the radius at different time. Indexes j =I,2,3 refer to uranium, thorium and kalium-40, respectively, λ_1=I.54 10^{-10} year^{-1}, λ_2 =5 10^{-11} year^{-1}, λ_3 =5.7 10^{-10} year^{-1}, the heat release of one gramm of the radioactive element is α_1 - 0.805 cal g^{-1}year^{-1}, α_2=0.193, α_3=0.224 and values of H_j° - the concentration at the beginning of consideration - are defined based on the expected presently concentration of $H_j^{t_0}$ over the formula $H_j^{\circ} = H_j^{t_0} e^{\lambda_j t_0}$ where t_0=4.7 10^9 years is the Moon age. We consider four variants of values of radioactive elements concentration given in table I. Radioactive elements concentrations are chosen based on the stationary heat fluxes equal to I.35 and 0.9I cal cm^{-2}sec^{-1} for variants I and II^{++} and corresponding to radio astronomical data by Troitsky and Krotikov [27] and to the Appolo-I5 [?] , and 0.6I and 0.236 cal.cm^{-2} sec^{-1} for variants II and III coinsided with one of the variants by Levin and Maeva [9] (II) and close to concentrations accepted by Urey and MacDonald (III). The relation of thorium concentration to uranium one is taken to be equal to 4 (see [9]), uranium concentration is taken in variants I,I^{++},II based on the pyrolit Moon composition and chondrite composition in variant III. This assumption on the pyrolit Moon composition is founded on the similarity of the Moon composition and the Earth mantle noted by Anderson [I] (for example, the mean atomic weight of the Earth mantle $\bar{M}$ =22.4 which refers to ejected rocks,coaly,ordinary and enstative chondrites. The same atomic weight has the Moon's material 22.0) and also on the data by Sorveyer - V and on the paper by MacCrea [I9],Reynolds,Summers [24] et al. From all these data it follows that the Moon consists of silicate (for example,basalt) similar to those at the Earth

mantle[+]). The table gives uranium concentrations and the relation of K-40 and uranium concentrations for different rocks.

Rocks:	Granite:	Basalt:	Aklogite:	Peridotit:	Dunit:	Chondrite:
$U \cdot 10^{-8}$	400	80	4.3	0.6	0.1	1.1.
K40/U	1.05	1.13	1.44	1.2	1.2	9

From this table and the given stationary fluxes we choose the above variants I, I[++] and II with the content of basalt equal to 14%, 10% and 5%, peridotit -73%, 77% and 82% and iron - 13% (Reynolds, Summers [24]) with impoverished content of radioactive elements (iron, has $U=0.1 \cdot 10^{-8}$ g/g, $Th=0.4 \cdot 10^{-8}$, $K^{40}=0$) and the relation $K^{40}/U=1.13$ and 1.6 (see Levin [10] and Lubimova [12]). We take also variant III - chondrite model (Urey and MacDonald).

Note that variant II corresponds to variant "C" of the mean content of radioactive elements in the Earth (see Starkova [33] and Lubimova [12] [++]).

As the initial condition we take two variants of the "Cold" and the "hot" Moon's model at the moment of its formation according to initial temperatures 273°K and 900°K. These are the same initial condition taken by Urey, MacDonald, Irijama, Schimazu, Friker[+++] based

[+] From this it is not necessary followed that the hypothesis is true on the Moon formation by the way of tearing off the Earth since even interstellar material contains grains of high melting silicates (Mac Crea [19]).

[++] Running ahead we note that just this variant gives the flux corresponding to Appolo-15 data and the radio astronomical ones, i.e. evidently, the terrestrial composition of the Moon is rather probable.

[+++] Maeva [15] took the parabolic tenperature distribution with the value in the Moon's centre equal to 500°K.

on the different hypothesis of the Moon origin and the time of its accumulation and the initial heating we do not give consideration.

The boundary condition ($r=r_0$) is defined by relation (5)

$$K\left(\frac{\partial T}{\partial r}\right)_{r=r_0} = 6T^4 - B \qquad (5)$$

where $B = 0.9\ 10^{-2}$ cal cm^{-2}sec^{-1} consideres the Moon heating by the solar radiation. This condition leads practically at the calculation to the same results that the more simple condition $T = 273^{\circ}K$ =const does.

The mechanism of differentiation

The mechanism of differentiation is defined by the function of distribution of radioactive elements $F(r,t)$ which is given by the following manner. At first, we consider a homogeneous model of distribution of radioactive elements, i.e. $F(r,t) = 1$ (analogous to [20,21]) is the first stage of calculation. Then, after reaching at the moment t_1, at the point r_1 , the temperature corresponding to the beginning of melting $T_{init.\ melt}$ the distribution of temperature obtained to the time of $t_2 = t_1 + \Delta t$ (where Δt is the time of melting - see below) is taken for the new initial distribution, and the distribution of radioactive elements is assumed to be laminated, namely, radioactive elements from the layer $r_1 \pm \Delta r$ transfer to the adjacent upper layer ($r_2 \pm \Delta r$) and the calculation is carried out with the pooled layer ($r_2 \pm \Delta r$). However, for the time Δt defined by the necessity of the layer heatingby by $\Delta T = 200^{\circ}K$ more deep layers have reached the temperature of melting (since $T_{init.melt}$ increases with the depth, i.e. with the decrease of r). Radioactive elements from these layers go to the layer $r_1 \pm \Delta r$, i.e. their content in this layer is equalized and so on, until the lower layers

stop melting. Such laminated differentiation (similar to those accepted by Fricker, Reynolds, Summers) is defined by five variants differed by the value n which is the portion of radioactive elements convection from the layer. We accept $n = 1$ (the complete convection), 0.8 (80% of radioactive elements convected from the layer), 0.6, 0.4, 0.2. In this way after differentiation in each of the melted layer there remains from 0 up to 80% from the content of radioactive elements in nondifferential material (note that in the latest paper by Maeva [15] the residue was from 2% up to 20%, i.e. $n \geq 0.8$).

The present paper does not consider the possibility of convection of the melted material to the external hard layer as it was done in the paper by Maeva [15] since the concentration of radioactive elements in the thin surface-attached layer leading, in reality, to the more rapid cooling of the Moon and to the larger thickness of the hard cover is obtained in the model of exponential differentiation which is considered in detail in [20,21].

3. The method of calculation

For the numerical solution of the heat conductivity equation we use the method of line coordinates where the section $[0, r_0]$, (where r_0 is the radius of the Moon) is divided into 100 layers and only arbitraries over the spacial coordinate are replayed by the difference ratio. Thus the solution of the boundary problem is reduced to the solution of Cauchy problem for the system of ordinary differential equations. It should be noted that the approximation error the larger the closer is the layer to the surface. That is why nonuniform grid is used in the calculation: the layer thickness close to the

surface (≈ 5km) is in several times smaller in comparison with the depth layer (≈ I8 km). The system obtained is solved by the approximation method providing the accuracy of temperature calculation (at each step over the time) of the order of 10^{-3}. All calculations are made by the computer BESM -6 .

4. Results of calculation

Results of calculation are given in Figs I-I4 and in Tables 2 and 3.

The process of melting and differentiation

Figs I-4 shows the time temperature dependence in different Moon layers (along the radius beginning from the centre O) for all variants at η =I . Here also temperatures are plotted of the beginning of melting corresponding to each layer and the time dependence of the beginning of melting in different layers. From Figs I-4 it is claer that melting in all variants begins in the layer 80[+)] near I500 km from the centre (i.e. at the depth ≃ 250 km under the surface) at the time period from 0.4 up to I.5 milliard years from the beginning of the Moon history) and rapidly (for 0.I -0.3 milliard of years) the region of melting propagates to the centre and to the Moon's surface. At the same layer where melting begins for the first time, in 0.2 -0.5 milliard of years the differentiation of

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[+)] Altogether there are I00 layers.

the material begins for the first time. The region of melting at the moment of the beginning of differentiation $t_{init.dif.}$ reaches I600km and at the same time a sharp temperature increase occurs in the layers close to the surface. As a result the temperature in these layers is close to or exceeds $T_{init.\ melt.}$. It is clear since melted radioactive elements convected to these layers from the more deep layers. The deep layers (the depth more 500-600 km, i.e. r =II00-I200 km) to the moment of differentiation turn to be heated higher than $T_{init.melt}$ and slowly cooling due to the convection of radioactive elements to the upper layers+). From Figs 5-7 it is seen that in this way almost simultaneous heating of the Moon material occurs up to the depth of 200-250 km under the surface. Layers which are closer to the surface heating slower and the depth of these layers increases as time goes by. Numerical data are given in Table 2 and 3.

The beginning and the duration of the maximum melting of the interior.

Figs 8-II give time dependence of the radius of melting. From these Figures it is clear that in all variants the period of the maximum melting is observed continued I-2 milliard of years (depending on the variant - see table 2 and 3) and beginning in the interval 0.7-2.3 milliard of years from the beginning of the Moon history. It is naturally that for large concentrations and "hot" models the maximum melting begins earlier than for the concentrations of small and "cold" models.So for the variants I and I++ the maximum melting

+)Differentiation occurs in all variants except for III (cold model) where the energy of radioactive elements is not sufficient for the beginning of differentiation.

begins approximately 3.5 - 4 milliard of years ago and continues during I.5 - I milliard of years and in the case of small (variant III) and mean concentrations and the large convection ($n \geq 0.8$) of radioactive elements (variantII) the maximum melting begins 2.5 - 3 milliard of years ago and continues I.5 - 2 milliard of years, i.e. it begins later and continues longer as should be expected[+]. The peculiar case is variant II ("cold" model) at the small convection of radioactive elements ($n \leq 0.6$) where the period of maximum melting beginning 2.5 milliard of years ago and continues at the present time. Here the specific combination of conditions is realized. The concentration of radioactive elements is not so large and the process of differentiation is not so effective that a fast convection of radioactive elements to the surface occurs when their influence on the further history turns to be small but not sufficiently small for the natural cooling of the Moon material.

Thus, in all variants except for the mentioned II ($n \leq 0.6$) the cooling of the Moon takes place presently.

The thickness of the hard cover

As it is seen from Figs 8-II and Tables 2 and 3 the thickness of the hard cover at the period of maximum melting ranges in variants I, I^{++}, II, III I5-20 km, 25-30 km, 35-45 km and I00-200 km , respectively, i.e. up to I - 2.5 milliard of years ago the Moon was almost completely melted (in variants II with $n \leq 0.6$ the Moon even now should be melted up to the depth $\simeq$ 45 km under the surface), then cooling began and presently the thickness of the hard cover in all variants is I50-250 km (except for the mentioned "peculiar case"). The same is

[+] The smaller concebtration is the more quite are the precess passed.

followed from Figs I3-I2 where the distribution of the temperature in the Moon at the present time is given.

The heat flux through the Moon surface

Fig I3 gives time dependence of the heat flux through the lunar surface. From these curves it is clear that at the period of the maximum melting of the Moon flux as it could be expected is also maximum one exceeding 2-3 times the stationary value at the given concentration of radioactive elements. Now at all variants (except for II with $\Lambda \leq 0.6$) the flux decrease is observed and its approximation to the stationary value $q_{stat.}$, yet even now the flux exceeds nearly I.5 times $q_{stat.}$. The flux value in variants I, I^{++}, II ($\Lambda \geq 0.8$) and III is equal to $I.8 \cdot I0^{-6}$ cal $cm^{-2}sec^{-I}$, $I.27 \cdot I0^{-6}$, and $0.9 - 0.95 \cdot I0^{-6}$ $cal.cm^{-2}sec^{-I}$, respectively, in variant III ("cold" and "hot" models) - 0.28 and 0.55 $I0^{-6}$cal $cm^{-2}sec^{-I}$ and in variant II ($\Lambda \leq 0.6$) is a rather large flux equal to I.6 $I0^{-6}cal.cm^{-2}$ sec^{-I}, that associated with the melting in this variant extended up to the depth of 45 km.

Thus fluxes close to the measured ones by the radio astronomical methods and obtained by Appolo-I5 are realized in variants I^{++} and II ($\Lambda \geq 0.8$), i.e. at the terrestrial or "mantle" - pyrolit content of the Moon material.

5. Conclusion

Calculations are given where different concentrations of radioactive elements are accepted which refered both to the chondrite composition (III) and to the terrestrial one (II) and to the Earth mantle -pyrolite composition (I and I^{++}). The material differentiation and the convection of radioactive elements to the surface due to

melting are taken into account at the process of calculation over the scheme close to the scheme by Fricker,Reynolds and Summers. Calculationd show that the Moon's interior has been heated up to the melting during the first 0.7-2.3 milliard of years after its formation. The maximum melting of the Moon covering practically the entire Moon up to the depth I5-45 km from its surface began 3.5-4 milliard of years ago in the assumption on the pyrolite composition of the Moon or 2.5-3 milliard of years ago in the assumption on the terrestrial or chondrite composition of the Moon. This maximum melting continues during I-2 milliard of years and at present the Moon is cooling (there is a scarcely probable variant of weak differentiation at the "terrestrial" composition of the Moon when melting is continuing even now). The present depth of the hard cover is I50-250 km and the value of the heat flux approximately I.5 times exceeds the stationary one, and is equal for variants I, I^{++}, II and III to I.8, I.27,0.9-0.95,and 0.28-0.55 $I0^{-6}$ cal $cm^{-2}sec^{-I}$,respectively.

From Tables 2 and 3 it follows that in the most probable variants with pyrolite-"mantle" (I^{++}) and terrestrial (II) composition of the initially "hot" Moon the maximum melting of the Moon began 3.3-3.8 milliards of years ago thet corresponds to the age of the basalt and maskones (3.I6-3.7I milliard of years). This testify to the vulcanic activity and was completed approximately 2.7-3 milliard of years ago that agrees with the absence of the material samples at the Moon yunger than 3 milliard of years (Papanastassiou,Wasserburg [22,23,32]) when the activity stops. So, limitations stated resently by Toksoz and Solomon [26] are satisfied, yet the thickness of the hard cover obtained by us is essentially small (in [26] the thickness of the cover is 600 km).

It should be noted that the most close to reality is the variant of the "terrestrial" composition of the Moon (composition "C"

according to Starkova) giving inspite of moderate concentration a sufficiently large heat flux[+)] corresponding to the radio astronomical measurements and Appolo-I5 data, i.e. to obtain the large flux observed it is not necessary large concentrations of radio active elements to be present.

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[+)] This possibility was noted by Lubimova in [I2]

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T A B L E I

Variants		Concentration of radioactive elements [g/g] Uranium	Thorium	Kalium-40	q_{stat} cal cm^{-2} sec^{-1}
Basalt 14% Peridotit 73% Iron 13% (K 40/U) =1.6	I	$11,55 \cdot 10^{-8}$	$46,7\ 10^{-8}$	$18,5 \cdot 10^{-8}$	$1,35 \cdot 10^{-6}$
Basalt 10% (K 40/U)=1.13	I++	$8,5 \cdot 10^{-8}$	$34 \cdot 10^{-8}$	$9,6 \cdot 10^{-8}$	$0,91 \cdot 10^{-6}$
Basalt 5% Variant by Levin k40/U =1.6	II	$5,2 \cdot 10^{-8}$	$21 \cdot 10^{-8}$	$8,4 \cdot 10^{-8}$	$0,61 \cdot 10^{-6}$
Chondrite model Variants by Urey and MacDonald	III	$1,16 \cdot 10^{-8}$	$4,67 \cdot 10^{-8}$	$9,35 \cdot 10^{-8}$	$0,236 \cdot 10^{-6}$

TABLE 2

Variant	N	Flux q 10^{-6} cal $cm^{-2}sec^{-1}$			Time t milliard years			Distance r km				
		q_{stat}	q_{max}	q_o	$t_{init\ melt}$ / $t_{init.\ dif}$	$t_{max.\ melt}$	$t_{max\ q}$	$r_{ini.mel\ d}$ / $r_{mel\ ini\ d}$	$r_{max\ mel}$	r^{o}_{hard}	$r_{h.min}$	r hard (cover)
I Cold.mod.	I		5		(0,4)	0,7-2,2	0,8-2			I540		I95
	0,6	I,35	5,5	I,8	0,6	0,7-I,8	0,7-I,8	I500	I720	I460	I5-20	275
	0,2					0,7-2,2	0,8-2	I660	I7I5	I540		I95
I^{++} Cold model	I									I500		235
	0,8				(0,85)			I500		I520		2I5
	0,6	0,9I	2,85	I,27	I,I5	I,4- 2	I,4- 2	I6I0	I7I0	I540	25-30	I95
	0,4								I705	I540		I95
	0,2									I540		I95
I^{++} Hot model	I				(0,35)			I500	I7I0	I520		2I5
	0,6	0,9I	2,85	I,27	0,8	0,9- 2	0,9-I,5	I640	I705	I560	25-30	I75
II Cold model	I			0,95	1,2	2,3-3,5	2,3-2,6			I600		I35
	0,8					2,3-3,5	2,3-2,6	1500		I600		I35
	0,6	0,6I	I,85	1,6	I,8	2,3-4,7	2,3-4,7	1580	I695	I690	40-45	45
	0,4					2,3-4,7	2,3-4,7		I690	I690		45
	0,2					2,3-4,7	2,3-4,7			I690		45
II Hot model	I				0,55					I560		I75
	0,6	0,6I	2,I	0,88	I,2	I,4-2,5	I,5-2	I500 I620	I700	I540	35-40	I95
	0,2								I695	I520		2I5
III Axl.model		0,23б	0,4I	0,28	I,55	2,I-3,6	-	I300	I500	I480	235	255
III Hot model	I	0,236	0,73	0,55	(0,7) I,8	2,3-3,7	2,5-3,7	1500 I580	I635	I580 I600	95-I00	I55 I35

T A B L E 3

Variant	$[cal.cm^{-2}sec^{-1}]\ 10^{-6}$ $q_{stat.}$	q_{max}	q_0	[milliard years] $t_{init.\ dif.}$	[milliard years] t_{max} mel.flux	r melt.init.dif. [km]	Δr hard min [km]	$\Delta r°$ hard [km]
I	I.35	5.5	I.8	0.6 (0.4)	0.7 - 2	I660	I5-20	200-250
I++	0.9I	2.85	I.27	0.8 (C.5)	I-2	I630	25-30	I80-220
II	0.6I	2	Cold $n_1 \geq 0.8$ Hot 0.9-0.95 Cold $n_1 < 0.8$ I.6	I.5(0.9)	Cold $n_1 \geq 0.8$ Hot I.5-2+ 2.5-3.5 Cold $n_1 < 0.8$ 2.3 + 4.7	I600	35-45	Cold $n_1 \geq 0.8$ I35 Hot 200 Cold $n_1 < 0.8$ 45
III	cold 0.24	hot 0.4 - 0.7	cold 0.28-0.55	hot I.8(I.I)	2.3-3.7	hot I600	cold 235	hot IOC

Variant	r init mel. and init dif. = I500 km	$\Delta r°$ hard [km]
III	hot 250	cold I50

Fig 1

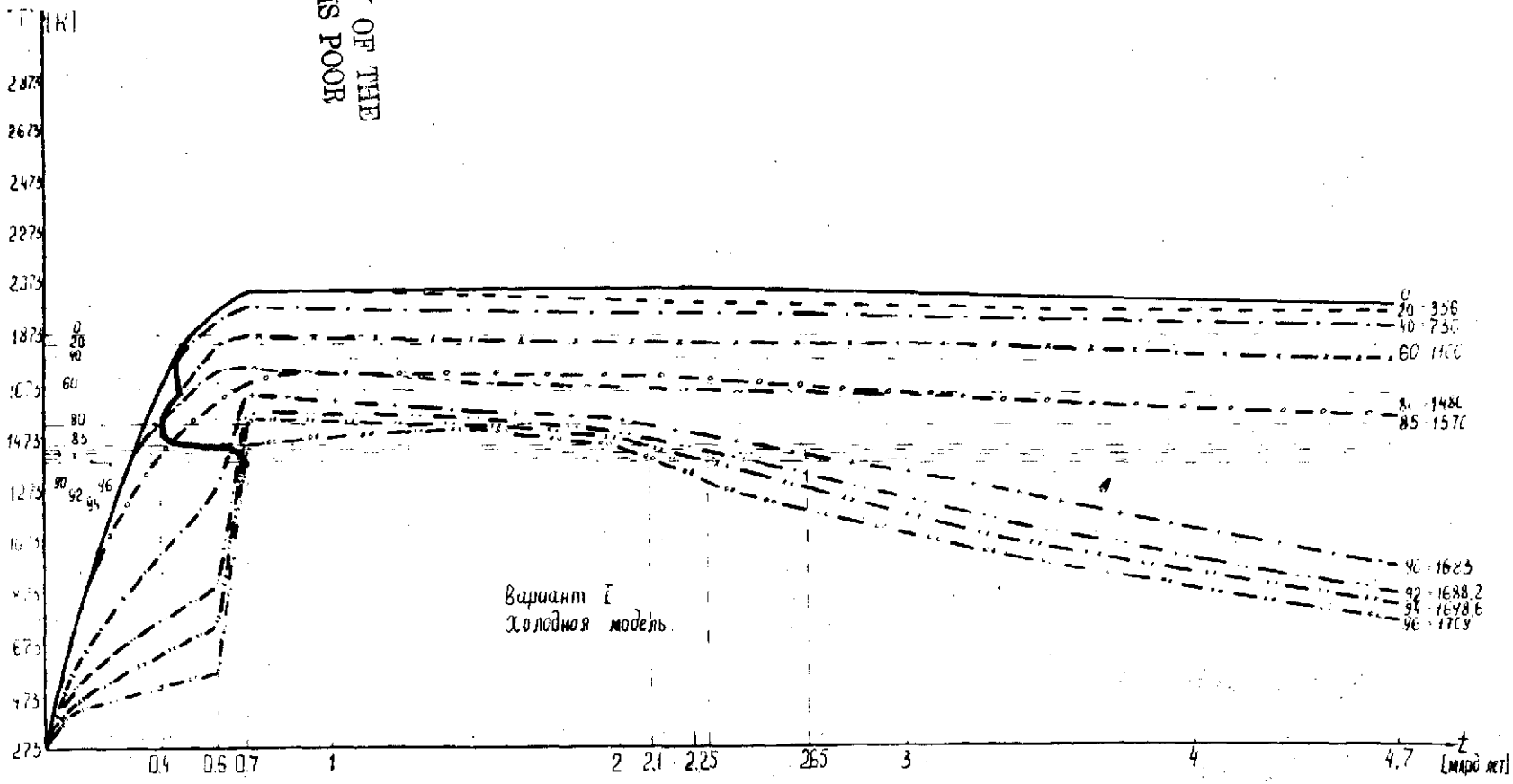
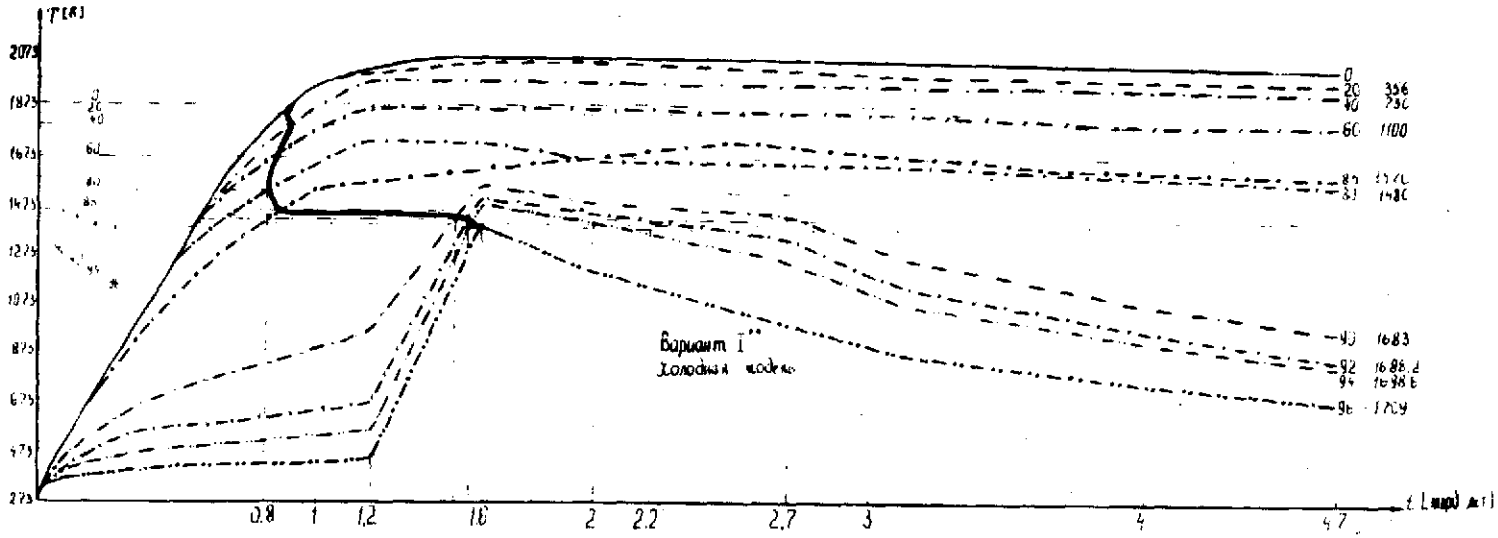


Рис 1


T[K]
2075
1875
1675
1475
1275
1075
875
675
475
275
0
20
40
60
80
92
94
96
356
750
1100
1480
1683
1686.2
1698.6
1709
Вариант I''
Холодная модель
0.8
1
1.2
1.6
2
2.2
2.7
3
4
4.7

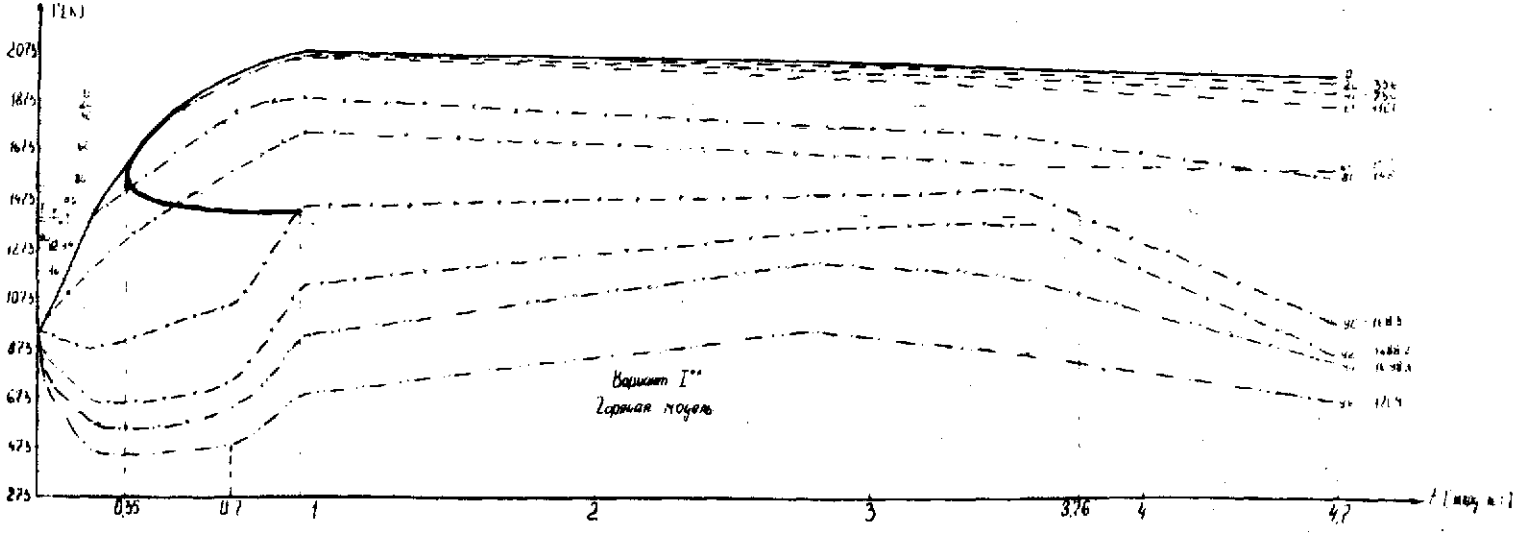

T[K]
2075
1875
1675
1475
1275
1075
875
675
475
275
Вариант I''
Горячая модель
0.55
0.7
1
2
3
3.76
4
4.7

Fig. 2.

Рис. 2

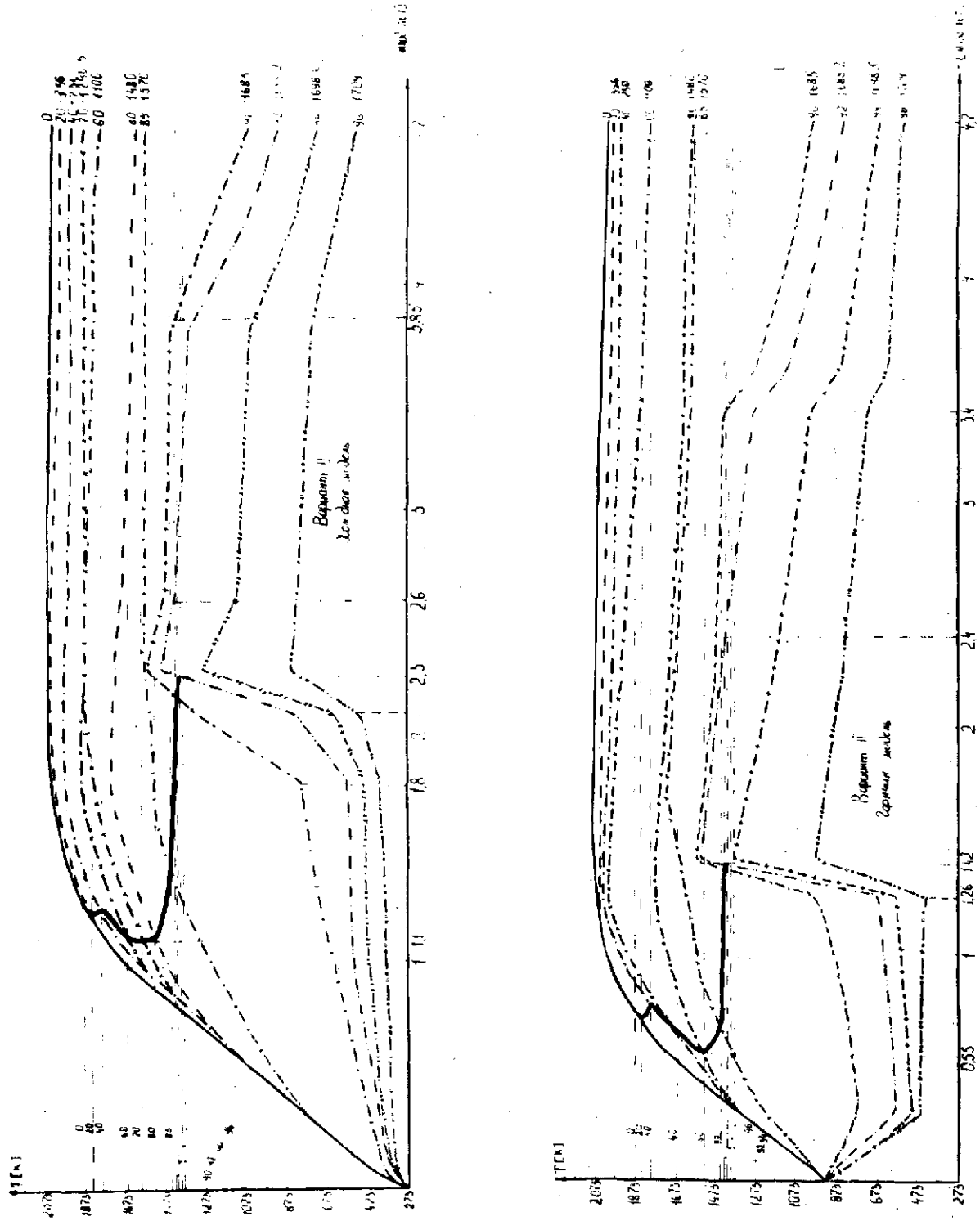
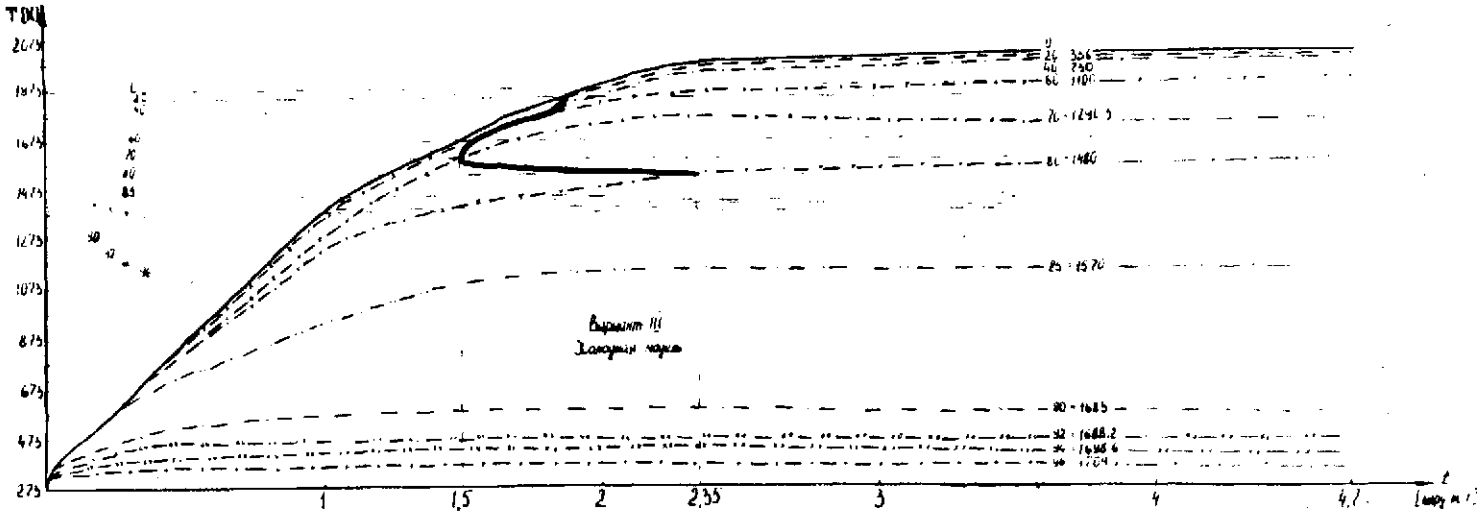
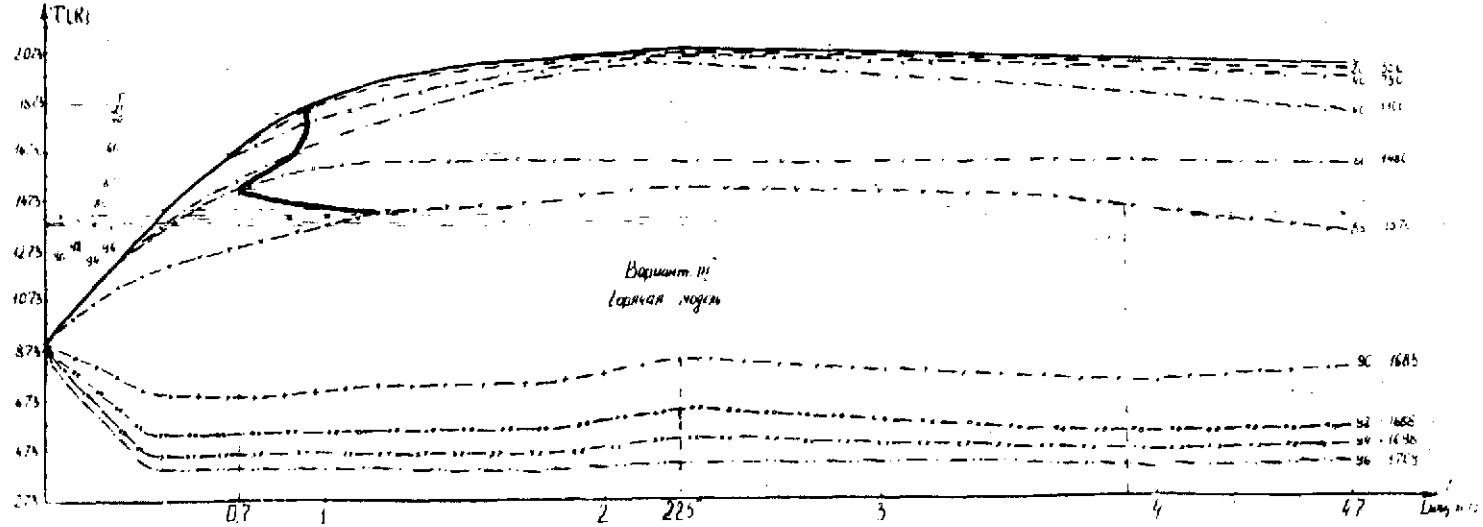


Fig. 3


T(K)
Вариант III
Холодная модель
t
1
1,5
2
2,35
3
4
4,7


T(K)
Вариант III
Горячая модель
0,7
1
2
2,25
3
4
4,7

Фиг. 4.

Fig. 5

Фиг. 9

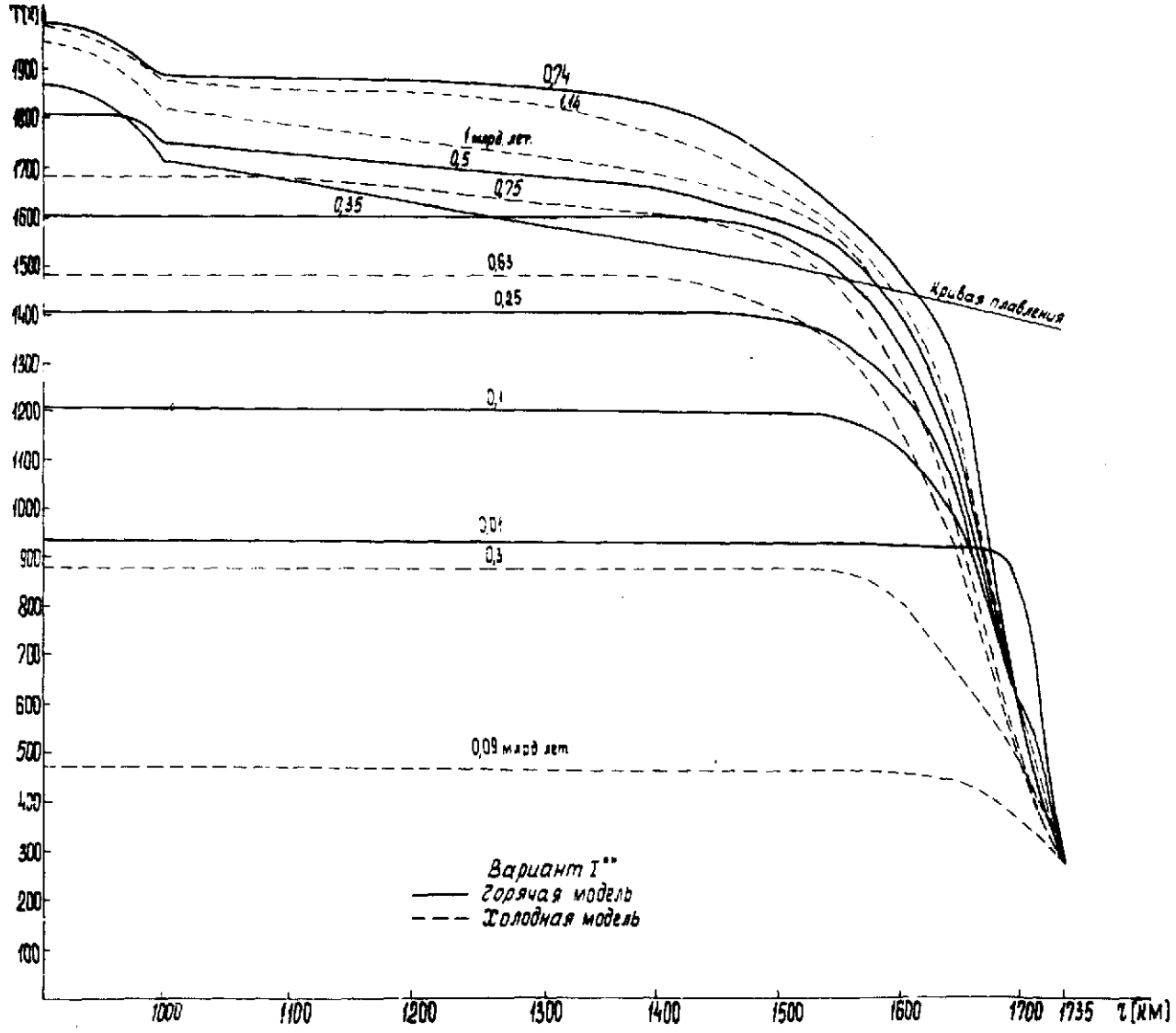

T[K]
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0,4
0,44
1 млрд лет
0,5
0,75
0,35
0,65
0,25
0,1
0,01
0,3
0,09 млрд лет
Кривая плавления
Вариант I**
Горячая модель
Холодная модель
1000
1100
1200
1300
1400
1500
1600
1700
1735
r [км]

Рис 6

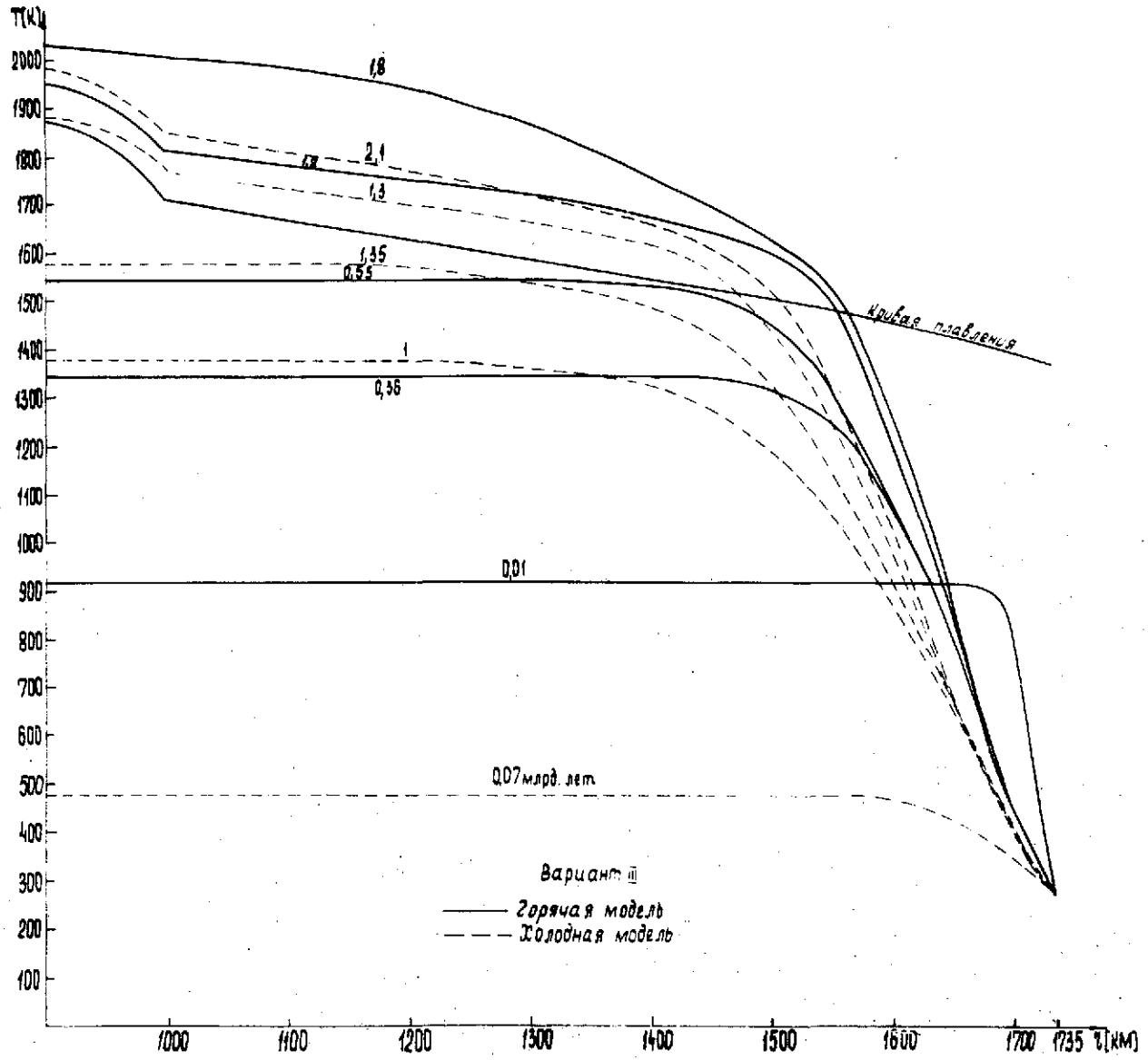
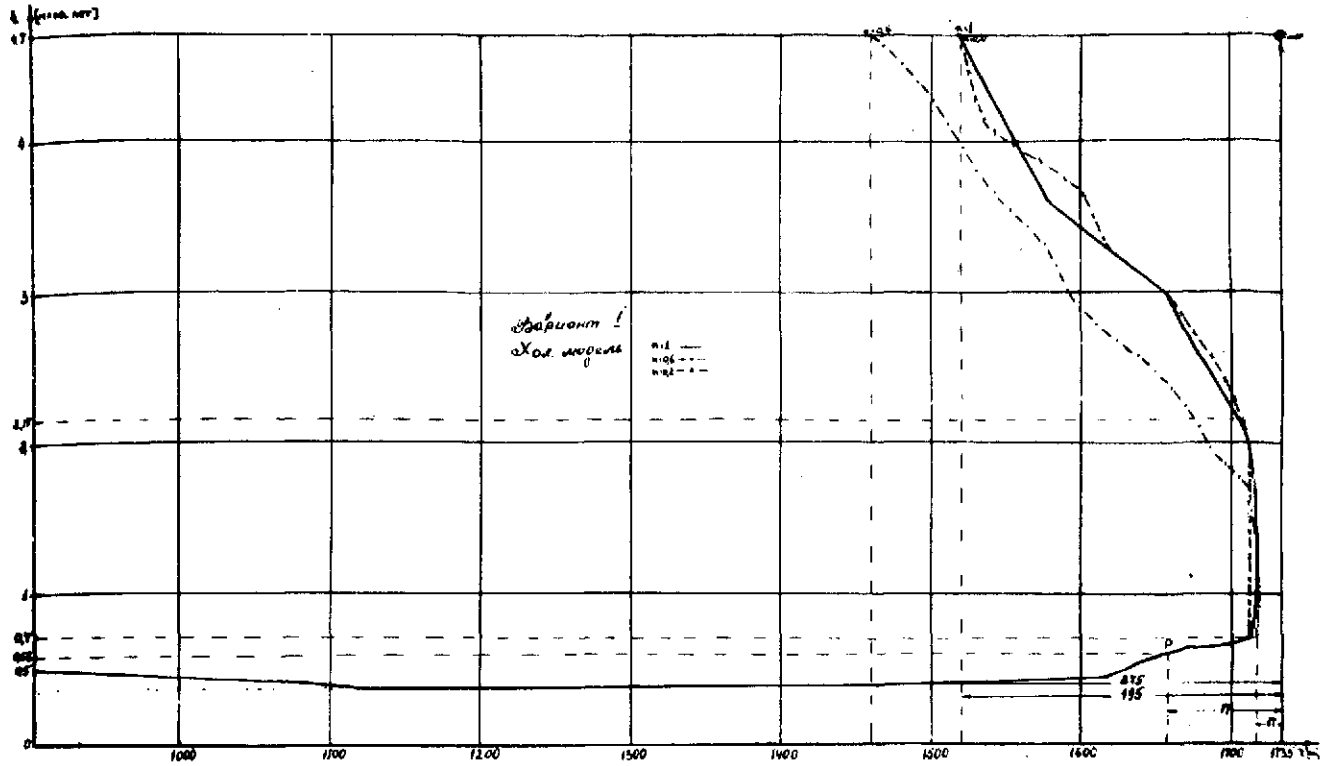

T(K)
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
1,8
2,1
1,3
1,35
0,55
1
0,36
0,01
0,07млрд. лет
кривая плавления
Вариант II
Горячая модель
Холодная модель
1000
1100
1200
1300
1400
1500
1600
1700
1735 r(км)

Рис 7

Fig. 7



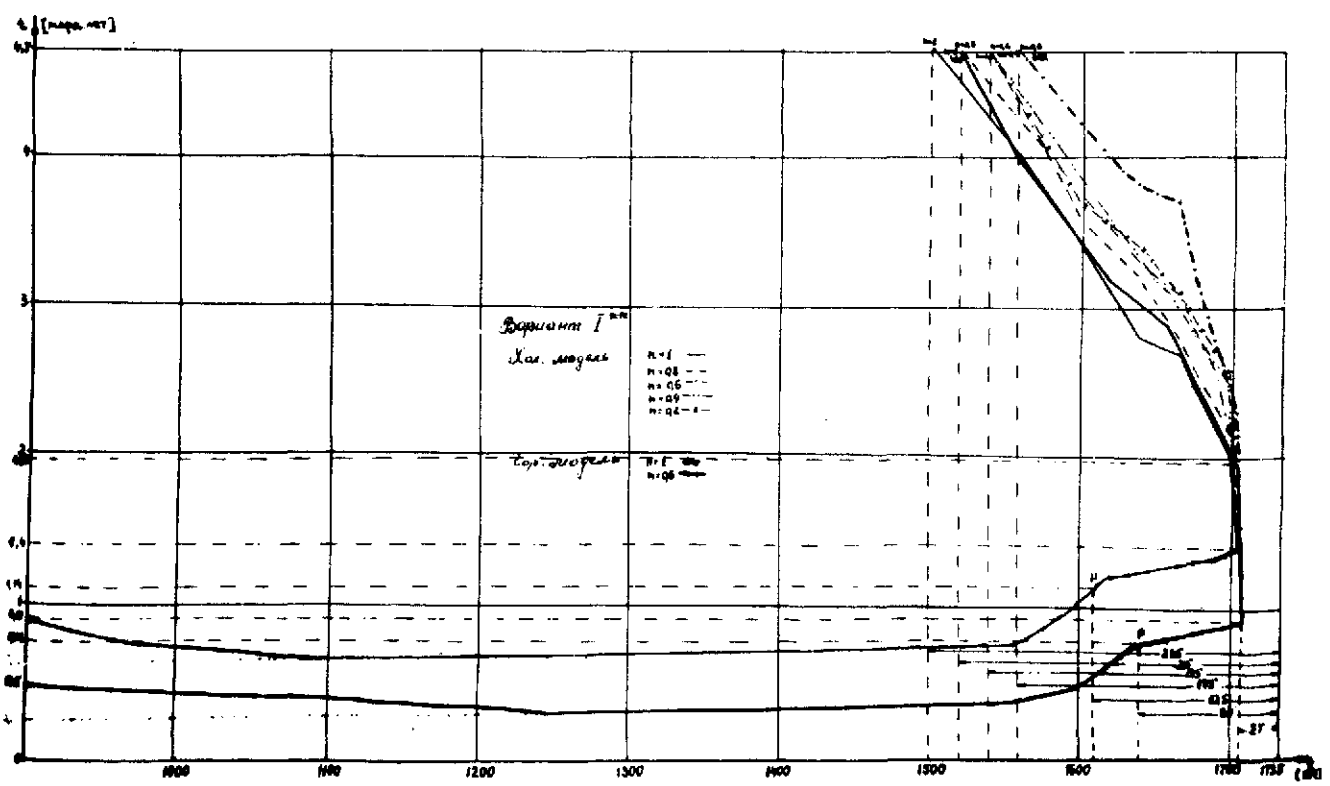


Рис 8

Fig. 8

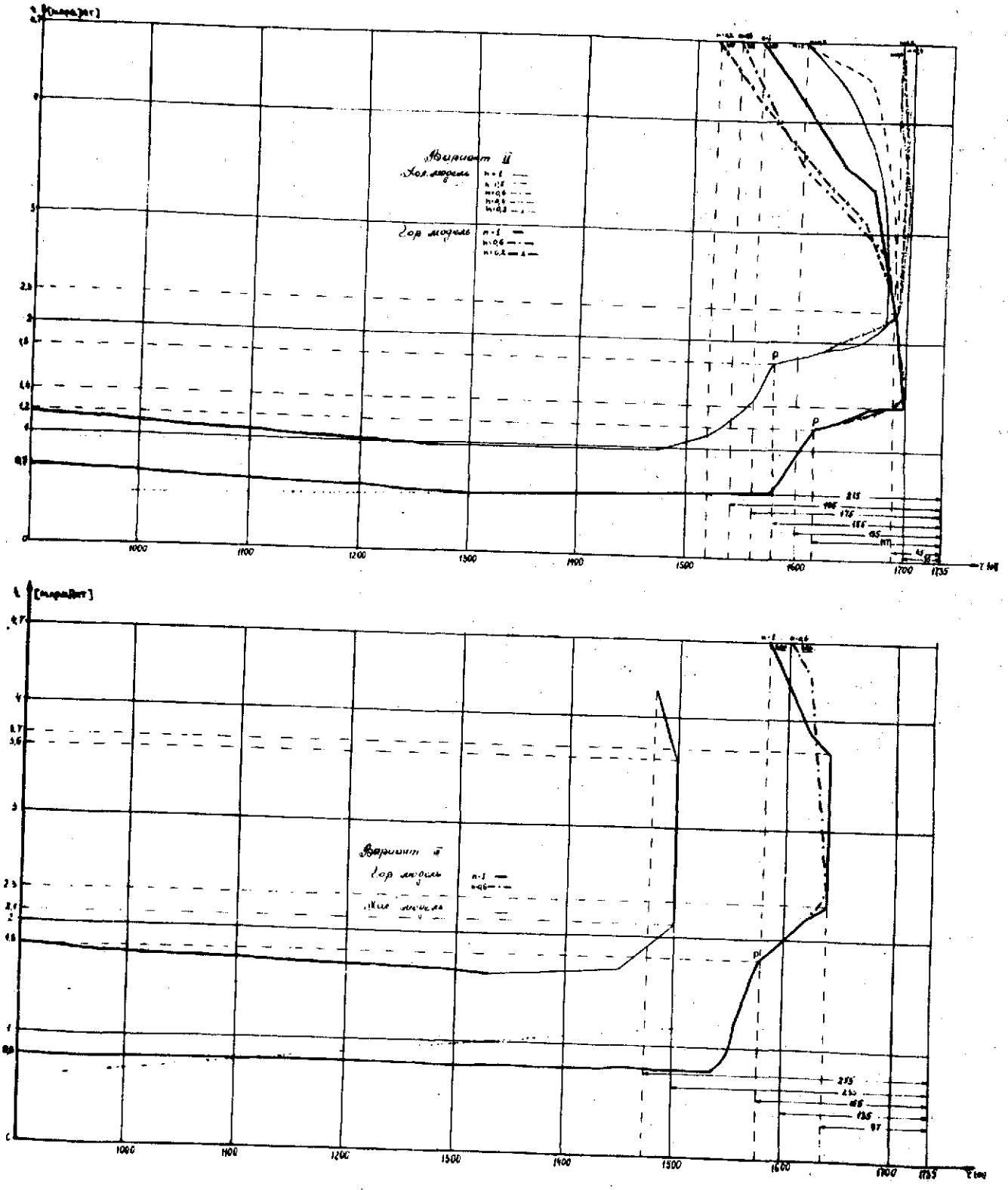


Рис. 9

Fig 9.

Fig. 10

Рис. 10

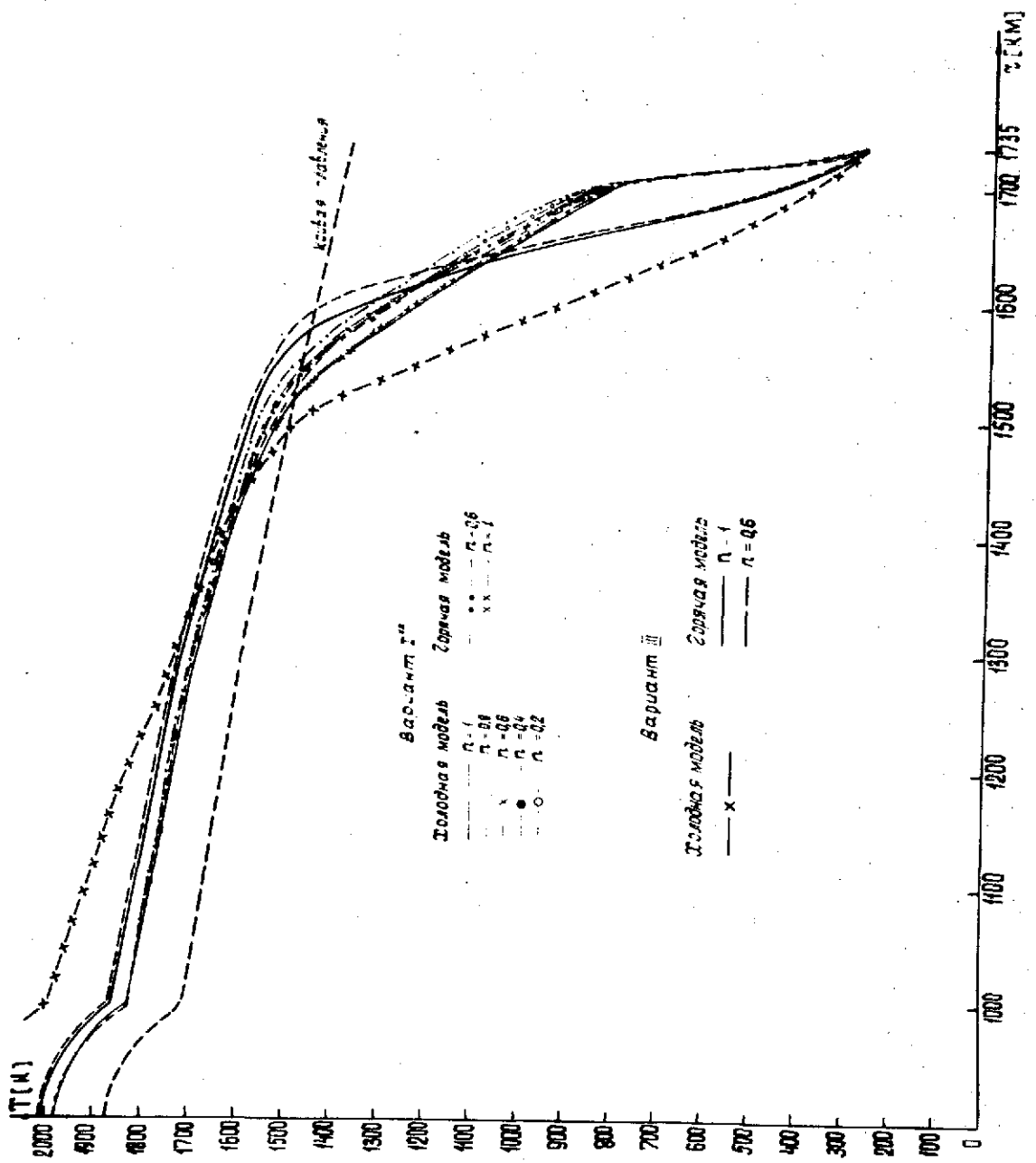
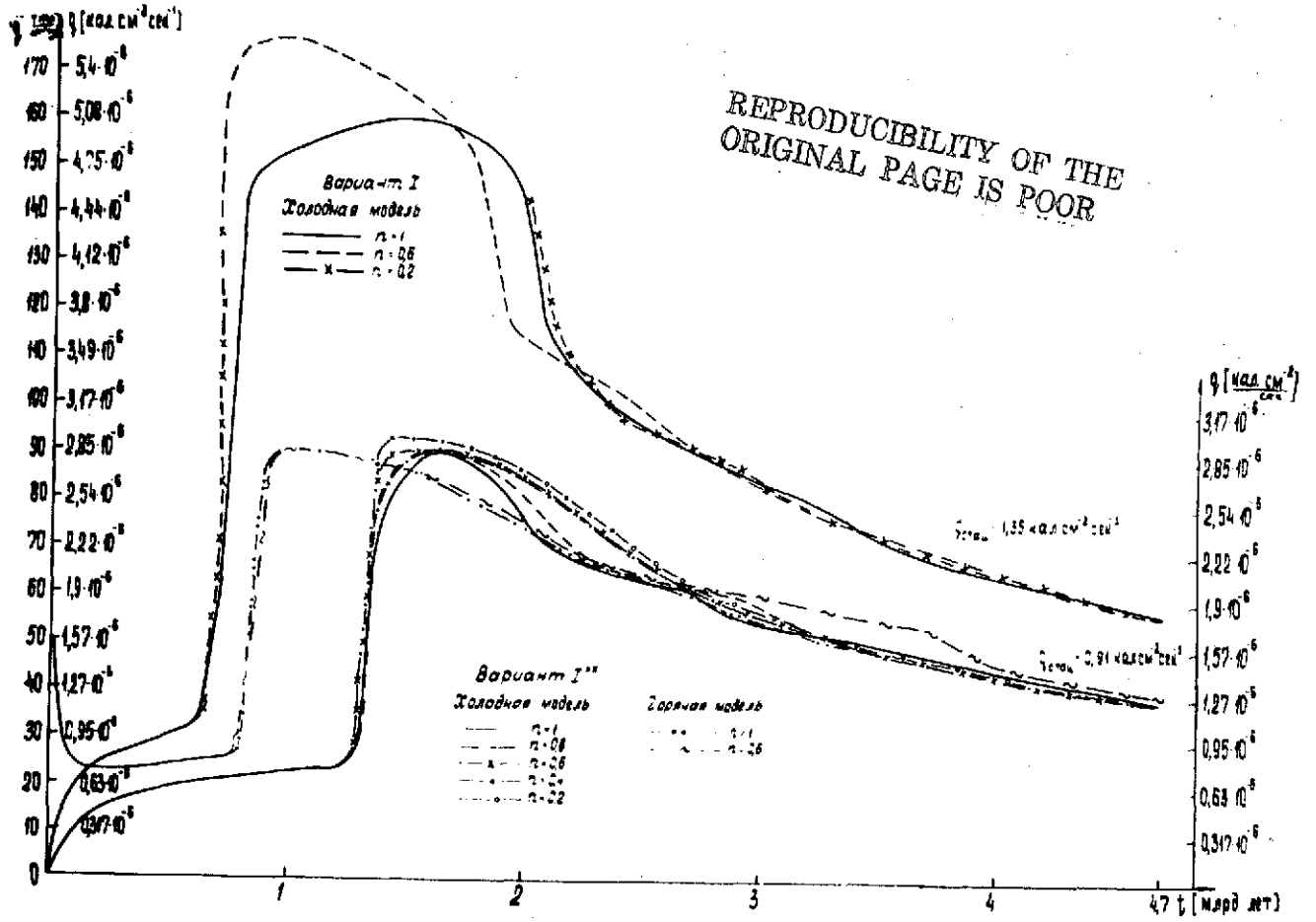


Fig. 11



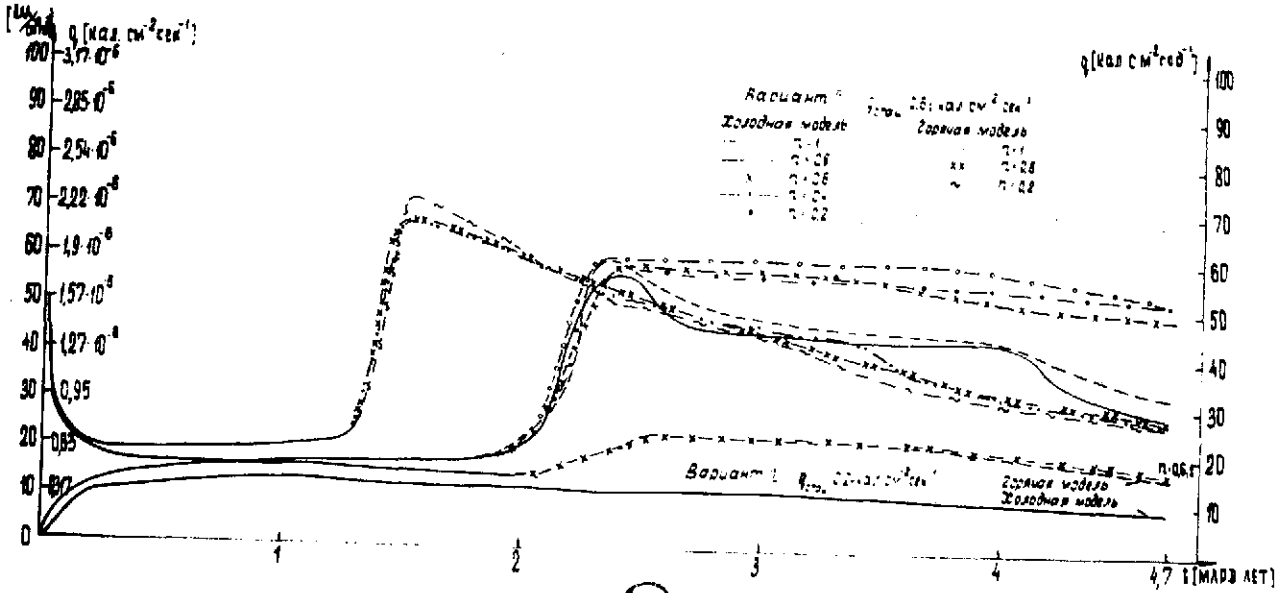


Рис. 12.

Fig. 12.