INVESTIGATION OF MATTER-ANTIMATTER INTERACTION
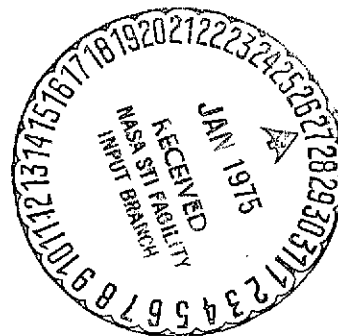## FOR
POSSIBLE PROPULSION APPLICATIONS*

David L. Morgan, Jr.
Lawrence Livermore Laboratory
Livermore, California

for

Jet Propulsion Laboratory**

California Institute of Technology

Pasadena, California

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CONTENTS

I. INTRODUCTION

The most important feature of the matter-antimatter interaction for rocket propulsion is matter-antimatter annihilation. The occurrence of annihilation is necessary for the release of energy for propulsion, but the avoidance of annihilation is necessary for antimatter storage. In both cases a knowledge of annihilation rates under various circumstances is important. The principal purpose of this report is to consider how the annihilation rates are affected by the various circumstances and how this, in turn, affects the choice of the means of antimatter storage.

Only a very limited number of accurate calculations that may be used to determine accurate annihilation rates have been carried out, and it is beyond the scope of this study to carry out such calculations. Therefore, in considering the feasibility of various means of antimatter storage, upper and lower limits for annihilation rates were employed. For a given means of storage, a sufficiently high lower limit on the annihilation rate renders the means unfeasible, or a sufficiently low upper limit on the annihilation rate renders the means feasible.

To determine limits on the annihilation rates, upper and lower bounds to atom-antiatom interatomic potential energies and rearrangement-annihilation cross sections were obtained. The upper bounds to the interatomic potential energies were obtained by use of a simple atomic model that was also used to obtain bounds on other important quantities. The lower bounds to the rearrangement cross sections turned out to be so low that other means could be employed to obtain lower bounds to the annihilation rates. Consequently, the lower bounds for the cross sections were not employed explicitly in the determination of annihilation rates for various means of storage.

Section III of this report contains a general description of the processes that are involved in matter-antimatter annihilation. In Section IV, the important features of atom-antiatom interatomic potential energies for various types of atoms and antiatoms are described, and a model is developed that allows the approximate calculation of the upper bound to the interatomic potential energy for any atom-antiatom pair. The lower bounds are also obtained. Based on these potential energies, formulae for the upper and lower bounds for atom-antiatom annihilation cross sections are obtained in Section V.

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Equations for the annihilation energy production rate and the annihilation degradation rate for matter-antimatter solids in contact and mixed solids are obtained in Section VI by using the model of Section IV. These equations are applied in Section VII in considering the possible existence of kinds of matter and antimatter that could exist as a stable, solid, homogeneous mixture, and are applied in Section VIII to the problem of solid matter and antimatter in surface contact. Lower bounds to annihilation energy production rates for a gaseous matter-antimatter mix are obtained in Section IX; and in Section X, the annihilation problems associated with high vacuum, no-contact antimatter storage are considered. In Section XI, suggestions for future studies are made.

It was not the purpose of this study to consider means for the production of the requisite amounts of antimatter. Consequently, no specific composition or physical state was assumed for the antimatter.

Atomic units, in which $e = a_o = \hbar = m_e = 1$, are used in this report, except where stated otherwise.

II. LITERATURE AND OTHER INVESTIGATIONS

Past and present research on the aspects of the matter-antimatter interaction that are relevant to rocket propulsion is very limited.

The most important publications are as follows:

(1)   A paper by Morgan and Hughes (Ref. 1) that gives detailed general and specific consideration to most important atom-antiatom interactions at thermal energies.

(2)   An earlier paper by Morgan and Hughes (Ref. 2) that includes consideration of the matter-antimatter interaction at very high energies.

(3)   A paper by Junker and Bardsley (Ref. 3) that presents the best calculations to date on the thermal energy hydrogen-atom (H) antihydrogen-atom ($\overline{H}$) interaction.

(4)   A paper by Omidvar (Ref. 4) that gives the results of a Born approximation calculation for the H-$\overline{H}$ rearrangement and annihilation cross section that is important for kinetic energies somewhat above the normal thermal energy range.

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The past work of G. Steigman on the positronium formation cross section in an H-$\bar{\text{H}}$ collision at energies above normal thermal energies appears in his dissertation (Ref. 5). The work of H. Alfvén and O. Klein (Ref. 6), Nauenberg and Ruderman (Ref. 7), and J. L. Puget (Ref. 8) as it applies to this study is superseded by References 1 through 4.

Positron scattering, which is a subject of considerable theoretical (Ref. 9) and experimental investigation, may be of some relevance in future studies to the use of antimatter annihilation for rocket propulsion.

III. ANNIHILATION PROCESSES

The processes and circumstances that affect the annihilation rate when matter and antimatter come into contact occur on three scales:

(1) Large-scale processes and circumstances that involve the composition, physical state, degree of mixing, density, pressure, and temperature of the matter and antimatter. Large-scale dynamic processes may involve all the complexities of plasma physics and thermodynamics, and are not considered in this report. Annihilation rates are determined for certain conditions of degree of mixing, kinds of atomic species, physical state, density, and temperature, usually to see if a given set of conditions is capable of stationary or nearly-stationary existence.

(2) Atomic scale processes that have a direct bearing on the probability of contact between the individual particles and antiparticles of which the matter and antimatter are composed (Ref. 2). Atom-antiatom interactions are of paramount importance here, and their consideration forms a large part of this report. Atom-antiatom interactions, for general considerations, may be taken to mean interactions of neutral or ionized atoms or molecules with neutral or ionized antiatoms or antimolecules, although, in specific considerations in the following sections, molecules and anti-molecules will not be included.

(3) Nuclear scale interactions wherein the actual, direct particle-antiparticle annihilations occur. For cases involving single particles or simple nuclei and antinuclei, the annihilation cross sections and rates for these processes are fairly accurately known, either theoretically or from experiments (Ref. 2).

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For gaseous matter and antimatter, mixed or in contact, or for individually colliding atoms and antiatoms, the types of processes that determine the annihilation rates depend principally on the temperature of the gases or equivalently on the atom-antiatom collision energy. At temperatures and energies in considerable excess of the total atomic binding energy of the electrons (e^-) of an atom and the positron (e^+) of an antiatom, the particle-antiparticle annihilations occur directly and the annihilation rates are determined from the particle-antiparticle annihilation cross sections (Ref. 2). This is true irrespective of the state of ionization of the atoms and antiatoms. At temperatures and energies around the value of the binding energy down to zero the processes affecting annihilation depend on the degree of ionization of the atoms and antiatoms. If they are wholly ionized, the particles of the nuclei and the antiparticles of the antinuclei annihilate directly. The electrons and positrons annihilate directly or indirectly by first undergoing radiative capture to form an atom of positronium (bound e^+e^-) and then annihilating. When the atoms and antiatoms are not wholly ionized at these temperatures and energies, annihilation occurs nearly always in the atoms of nucleonium (bound nucleus and antinucleus) and positronium that are formed in atom-antiatom rearrangement collisions. Under nearly all circumstances in which atoms of nucleonium are formed, the atoms will not subsequently break up before annihilation occurs. Because of this and the fact that the rearrangement cross sections are considerably greater than the direct particle-antiparticle annihilation cross sections, an atom-antiatom annihilation cross section is equal to the rearrangement cross section.

The physics of the atom-antiatom rearrangement reaction is described in detail in Ref. 1. Briefly, for each kind of atom-antiatom pair there exists a critical radius R_c, such that, in a collision, rearrangement will occur if and only if the distance of closest approach between the atom and antiatom is less than R_c. The rearrangement always involves the formation of nucleonium. It may not always involve the formation of positronium since the electrons and positrons may be emitted as free particles. This latter fact is not important for most considerations here where annihilation resulting directly from collisions is important because the atom-antiatom annihilation energy comes nearly entirely from the nucleus-antinucleus annihilation.

Consideration of the interaction of solid or liquid matter and antimatter is important for solid or liquid surface contact of the two, or for a hypothetical homogenous solid or liquid mixture of the two. Here somewhat different annihilation processes occur. In a solid the atoms and antiatoms are held in fairly fixed positions and do not collide. In a liquid the relative positions are not fixed, but the distances between atoms and/or antiatoms do not vary greatly and are always greater than R_c. If a homogenous solid or liquid mixture could exist, or if the surfaces of separate pieces of matter and antimatter in contact would not interpenetrate, there would have to be a strong atom-antiatom repulsion, at least at small interatomic distances. This repulsion would keep the nuclei and antinuclei sufficiently far apart to avoid annihilation. However, there would always be some degree of overlap between the electron and positron wave functions. Annihilation of an atom and antiatom would then occur first through direct electron-positron annihilation or by positronium formation. Once this occured, the atom and antiatom would have opposite charges. The resulting attraction between them or with other neighbors that they would polarize would bring them close enough together or close enough to others to permit the formation of nucleonium and probably more positronium, both of which would subsequently annihilate.

Because the critical radius R_c is defined as the point where two nearly parallel exact energy curves join (Ref. 1), it is very difficult to determine. It is known to be $0.639\ a_o$ for an antiproton and a hydrogen atom (Ref. 1) and to be slightly less than $1\ a_o$ for a hydrogen atom and an antihydrogen atom (Ref. 3). In this study R_c was taken to be $1\ a_o$ for all neutral atom-neutral antiatom pairs and $0.5\ a_o$ for all atom-antiatom pairs in which one or both of the atom and antiatom are ionized. In many and perhaps all cases where the atom-antiatom interatomic potential energy is negative for all R, the exact value of R_c need not be known to calculate atom-antiatom rearrangement-annihilation cross sections (Ref. 1).

IV. ATOM-ANTIATOM INTERATOMIC POTENTIAL ENERGIES

Consideration is given here and in the following sections to atoms and antiatoms that are spherically symmetric and in the ground state. This limits the consideration to atoms and antiatoms with one, two, or eight electrons or positrons in their outermost shell. The complexities that arise from the multipole moments of the charge distributions of nonspherical atoms are too

great to consider here. Nevertheless, most of the formulae derived in this and the following sections may be applied to such cases, although with less accuracy. Magnetic and other relativistic effects, which have only a small effect on the interatomic potential energies, are also not considered.

The atom-antiatom interatomic potential energy, V, is the total energy of atom and antiatom with their nuclei held fixed at a distance R apart minus the same quantity for $R = \infty$.

There are two features of V that are present in all atom-antiatom pairs. First, there is no interatomic exchange energy as is present in the V of an atom-atom pair. Second, for sufficiently small R, where there is no screening, $V = -Z\bar{Z}/R$, the coulomb potential energy between the positive nucleus with Z protons and the negative antinucleus with $\bar{Z}$ antiprotons.

To describe other features of V it is necessary to distinguish three types of atom-antiatom pairs:

(1)   Neutral atom-neutral antiatom

(2)   Neutral atom-antiion (or ion-neutral antiatom)

(3)   Ion-antiion, where the ions are positive and the antiions are negative.

It is helpful to consider a perturbation expansion for V in which the perturbation potential energy is the sum of the coulomb energies between the particles of the atom and the particles of the antiatom (Ref. 1). In such an expansion the lowest (first) order term, E_1, is just the electrostatic potential energy between the nucleus and the undistorted electron charge distribution of the atom and the antinucleus and the undistorted positron charge distribution of the antiatom. The second and higher order terms represent the effects of distortions of the charge distributions caused by the other member of the pair.

For type 3 pairs, E_1 and E_2 (the second order energy term), and probably the sum of the higher order terms, are negative for all values of R. Here, V may be described by the equation:

$$V = -C(R)/R \tag{1}$$

For $R \geq 2a_o$, C(R) is equal to the product of the absolute values of the net charges of the ion and antiion. For small values of R, C(R) increases to a limiting value of the product of $Z\bar{Z}$. There are no accurate calculations available for V for type 3 pairs, although the foregoing knowledge of C(R) is

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sufficient to allow good estimates of ion-antiion rearrangement-annihilation cross sections to be made (Ref. 1) (see Section V).

For type 2 pairs, E_1 and E_2 are again negative for all values of R as is V. For R $\gtrsim$ $4a_o$, V is dominated by the monopole-induced dipole energy from E_2, which is given by

$$V = -\alpha/2R^4 \qquad (2)$$

where α is the polarizability of the neutral member of the pair. Values of α for antiatoms are the same as the values of α for the corresponding atoms which, in most cases, are known. The only accurate calculation of V for a type 2 pair is that of Bates (Ref. 10) for $\bar{p}$(antiproton)-H (or p-$\bar{H}$). For this case the long range form of V given by Eq. (2) is sufficient to allow accurate approximations of the rearrangement-annihilation cross section to be obtained (Ref. 1) (see Section V).

As discussed in Ref. 1, the character of V for type 2 and 3 pairs, principally the fact that it is negative and attractive for all values of R, allows simple reliable formulae to be obtained for the rearrangement cross sections of these pairs. The formulae provide accurate approximations or at least reasonable estimates for the cross sections. Such is not the case for type 1 pairs. Here E_1 is positive and repulsive for R $\gtrsim$ $2a_o$, while E_2 (which includes the R^{-6}, R^{-8}, R^{-10}, etc., long range dispersion energy terms) is negative and attractive for all values of R. The relative magnitudes of E_1, E_2, and the higher order terms and their dependence on R determine the sign of V as a function of R, but accurate calculations of these quantities are difficult for all type 1 pairs. The only accurate calculation of V for a type 1 pair is the calculation of V for H-$\bar{H}$ by Junker and Bardsley (Ref. 3). Their results indicate that for this pair, at least, the effects of distortion as represented by E_2 and higher order terms are sufficient to overcome E_1 where it is positive and render V negative for all values of R, although only barely so for R $\simeq$ $3a_o$.

Type 1 pairs are of principal importance to considerations involving antimatter storage, so it is necessary to obtain at least some knowledge of V for pairs of this type for use in the following sections. This may be accomplished through the use of upper and lower bounds to V. For the upper bound, E_1 may be employed since it is a variational upper bound to V. For the lower bound, the long-range van der Waals dispersion energy, which is the leading

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term in the expansion of E_2 for large R,

$$V = -C_6/R^6 \tag{3}$$

along with any reasonable, smooth interpolation between the value of this at $R \simeq 5a_0$ and the nucleus-antinucleus potential energy at small values of R may be employed. A somewhat better lower bound could be obtained by adding to Eq. (3) the R^{-8} and R^{-10} terms, but their presence is of limited value due to their relatively small values, and it would greatly complicate subsequent calculations.

Values of C_6 for an atom-antiatom pair are equal to those for the corresponding atom-atom pair, which are known or may be obtained from simple formulae (Ref. 11). The upper bound, E_1, cannot be evaluated as easily. E_1 is the negative of E_1 for the corresponding atom-atom pair, and hence in principle could be obtained approximately from a knowledge of Hartree-Fock wave functions of neutral atoms. Since the amount of work required to do this is too great for a study such as this, a simple atomic model was found that allows reasonably accurate values of E_1 to be obtained.

In this model only the outer shell electrons or positrons are considered and their wave function is taken to be a product of exponential functions. The inner electrons and positrons are placed at the positions of the nucleus and antinucleus to form the cores of the atom and antiatom. Hence, the electron and positron charge distributions are, respectively,

$$Q(r) = -\frac{n\beta^3}{8\pi} e^{-\beta r} \quad \text{and} \quad \overline{Q}(\overline{r}) = \frac{\overline{n}\overline{\beta}^3}{8\pi} e^{-\overline{\beta}\overline{r}} \tag{4}$$

where r is the distance between the nucleus and a position in the electron charge distribution of the atom, and $\overline{r}$ is similarly defined for the antiatom; n and $\overline{n}$ are the numbers of outer shell electrons and positrons; and β and $\overline{\beta}$ are twice the exponent factors in the exponential wave functions. The exponent factors in the wave functions are taken to be $\sqrt{2E_I}$ and $\sqrt{2\overline{E_I}}$, as they are in the hydrogenic case, where E_I and $\overline{E_I}$ are the ionization energies of the atom and antiatom. Hence,

$$\beta = \sqrt{8E_I} \quad \text{and} \quad \overline{\beta} = \sqrt{8\overline{E_I}} \tag{5}$$

This model is equivalent or similar to atomic models that have been employed by many authors (particularly in the early years of the quantum theory of atoms) to determine the effects on atoms of external fields and other

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atoms. The accuracy of results obtained from the use of the model depends on the specific application and is then usually somewhat uncertain. In this study the model was used to obtain reasonably valid bounds to various quantities.

In this model E_1 is the sum of the atom core-antiatom core, atom core-$\bar{Q}$, Q-antiatom core, and Q-$\bar{Q}$ potential energies. The result for $\beta \neq \bar{\beta}$ is

$$E_1 = \frac{n\bar{n}(\beta + \bar{\beta})/2}{32R'} \ e^{-R'} \left[e^{-vR'} \left(\left(1 + \frac{1}{v}\right)^4 (v - 4v^2 + v^3) \right. \right.$$

$$\left. + \left(1 + \frac{1}{v}\right)^3 (v - 6v^2 + v^3) \ R' \right)$$

$$+ e^{vR'} \left(-\left(\frac{1}{v} - 1\right)^4 (v + 4v^2 + v^3) \right.$$

$$\left. \left. + \left(\frac{1}{v} - 1\right)^3 (v + 6v^2 + v^3) \ R' \right) \right] \tag{6}$$

where

$$R' = \frac{\beta + \bar{\beta}}{2} R \quad \text{and} \quad v = \left(\frac{\beta}{\bar{\beta}} - \frac{\bar{\beta}}{\beta}\right) \bigg/ \left(\frac{\beta}{\bar{\beta}} + 2 + \frac{\bar{\beta}}{\beta}\right) \tag{7}$$

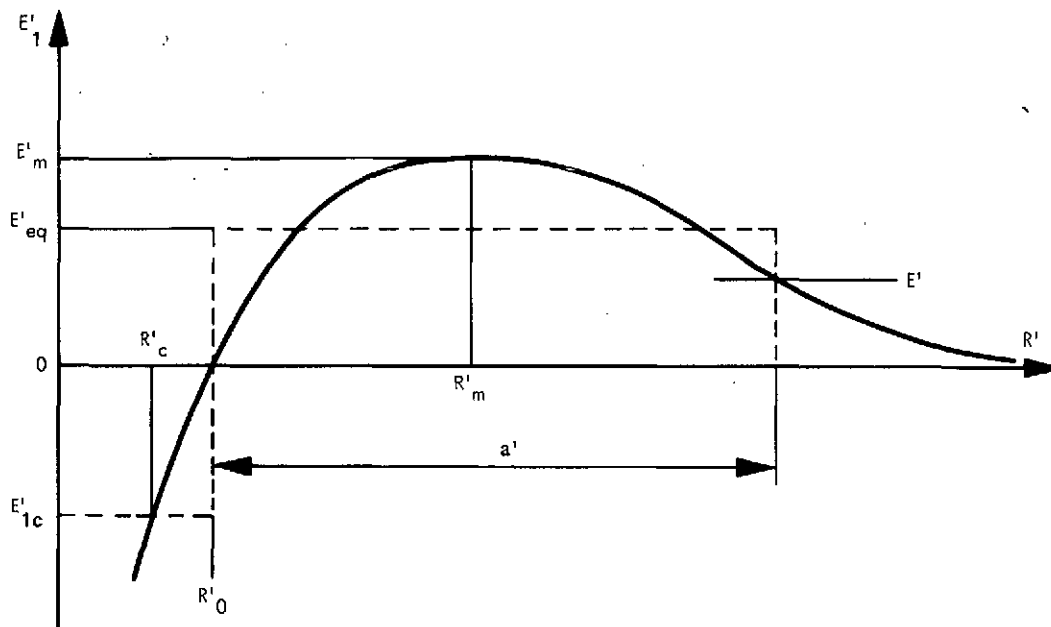and for $\beta = \bar{\beta}$ is

$$E_1 = \frac{n\bar{n}\beta}{R'} \ e^{-R'} \left[-1 - \frac{5}{16} R' + \frac{3}{16} R'^2 + \frac{1}{48} R'^3 \right] \tag{8}$$

E_1 may be expressed as

$$E_1 = \frac{n\bar{n}(\beta + \bar{\beta})}{2} \ E_1' \tag{9}$$

E_1' depends only on the variables v and R' and the characteristics of E_1 may be readily determined from the characteristics of E_1'.

The principal characteristics of E_1' are described in terms of the quantities depicted in the figure below:

The quantities E_{1c}', E_{eq}', a', and E' are related to the replacement of E_1' by an equivalent square potential energy barrier. E' is $\overline{nn}(\beta + \overline{\beta})/2$ times E, the energy of a particle incident on this barrier; E_{eq}' is $\overline{nn}(\beta + \overline{\beta})/2$ times E_{eq}, the equivalent height of the barrier; E_{1c}' is $\overline{nn}(\beta + \overline{\beta})/2$ times E_{1c}, the height of the inner portion of this barrier; and a' is $(\beta + \overline{\beta})/2$ times a, the width of the barrier. $R_c' = \beta + \overline{\beta}/2 R_c$ (R_c is the critical radius), and the other quantities are self-explanatory.

The following tables and formulae give these quantities as functions of v and the relations between them:

v	R_o'	R_m'	E_m'
0.0	2.7	3.74	9.8052×10^{-3}
0.2	2.8	3.80	8.8938×10^{-3}
0.4	2.9	3.98	6.4443×10^{-3}
0.6	3.2	4.36	3.3037×10^{-3}
0.8	3.9	5.23	7.6309×10^{-4}

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E'_{1c}:

R' \ v	0	0.2	0.4	0.6	0.8
0.25	-3.3211	-3.3072	-3.2668	-3.2033	-3.1225
0.4	-1.8328	-1.8202	-1.7835	-1.7260	-1.6531
0.6	-1.0203	-1.0098	-0.97905	-0.93080	-0.86826
1.0	-0.40620	-0.39976	-0.38084	-0.35075	-0.31237
1.5	-0.14527	-0.14275	-0.13504	-0.12186	-0.10363
2.5	-0.0093201	-0.010150	-0.012114	-0.013724	-0.013027
4.0	0.0095394	0.0087440	0.0064431	0.0031275	0.00021887

a':

E' \ v	0	0.2	0.4	0.6	0.8
9.8×10^{-3}	0				
8.9×10^{-3}		0			
6.4×10^{-3}			0		
3.3×10^{-3}				0	
$1.0 \times 10^{-2.5}$	3.5	3.8	3.1	1.7	
$1.0 \times 10^{-3.0}$	5.0	5.1	5.2	5.0	
7.6×10^{-4}					0
$1.0 \times 10^{-3.5}$	6.4	6.7	7.1	7.9	6.3
$1.0 \times 10^{-4.0}$	7.9	8.2	9.2	10.8	12.2
$1.0 \times 10^{-4.5}$	9.2	9.7	11.1	13.7	18.0
$1.0 \times 10^{-5.0}$	10.5	11.1	13.0	16.6	23.7
$1.0 \times 10^{-5.5}$	11.8	12.6	15.0	19.5	29.5
$1.0 \times 10^{-6.0}$	13.1	14.1	17.0	22.3	35.3
$1.0 \times 10^{-6.5}$	14.4	15.5	19.0	25.2	42.0
$1.0 \times 10^{-7.0}$	15.7	17.0	20.9	27.9	46.8

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The quantity E'_{eq} is determined by requiring that the integral of the square root of the potential energy of the real and equivalent barriers be the same. It is given to a good approximation by the formula,

$$E'_{eq} = \left[\frac{R'_m - R'_o}{a'} \sqrt{E'_m} \right.$$

$$\left. + \frac{1}{a'} \sqrt{\frac{\overline{B}(v)}{8}} \frac{1}{1-v} \left(e^{-1/2(1-v)R'_m} - e^{-1/2(1-v)(a'+R'_o)} \right) \right]^2,$$

$$\text{for } v \geq 0.2$$

and

$$E'_{eq} = \left[\frac{R'_m - R'_o}{a'} \sqrt{E'_m} \right.$$

$$\left. + \frac{0.39}{a'} \left((R'_m + 2) e^{-R'_m/2} - (a' + R'_o + 2) e^{-(a'+R'_o)/2} \right) \right]^2,$$

$$\text{for } v \leq 0.2$$

$$\tag{10}$$

where

$$\overline{B}(v) = \left(\frac{1}{v} - 1 \right)^3 (v + 6v^2 + v^3)$$

For use in Section VII, for purposes of consistency, it is necessary to obtain E_2 in the form given by Eq. (3) but with C_6 calculated by use of the above model. This may be done by using the wave functions of this model and by using the closure approximation as is done in Refs. 1 and 2. The result is

$$E_2 = - \frac{768 \, n\overline{n}}{(n\beta^2 + \overline{n}\overline{\beta}^2) \beta^2 \overline{\beta}^2 R^6} \tag{11}$$

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The simple atomic model is also employed in Section VI to calculate the e^- - e^+ annihilation rate in solid or liquid atomic scale mixtures of matter and antimatter.

V. ATOM-ANTIATOM ANNIHILATION CROSS SECTIONS

The rearrangement-annihilation cross sections in collisions involving the members of type 2 and 3 pairs and an upper bound to the cross sections involving the members of a type 1 pair may be obtained by use of the semiclassical method employed in Refs. 1, 2, and 3, which is valid for collision energies in the thermal range from about 0.001 eV to a few eV.

As discussed in Ref. 1, the nature of the lower bound to V for the type 1 pair (corresponding to the upper bound to the rearrangement cross section) and of V for the type 2 pair is such that only the form of V at large values of R is important in determining the rearrangement cross section. Hence the forms of V given by Eqs. (2) and (3), which are valid for large values of R, may be employed.

The upper bound to the cross section in the collision of a type 1 pair is

$$\sigma = 3\pi \left(\frac{C_6}{4E}\right)^{1/3} \tag{12}$$

and the cross section in the collision of a type 2 pair is (Ref. 1)

$$\sigma = \pi \left(\frac{2\alpha}{E}\right)^{1/2} \tag{13}$$

Note that these two results are independent of the value of R_c. These formulae and Eq. (14) are valid for values of E in the thermal energy range (roughly 1 K to 10,000 K)(Ref. 1).

For a type 3 pair, V for all values of R is important. Taking V to be given by Eq. (1) with $C(R) = q \bar{q}$, the product of the charge numbers of the ion and antiion, the semiclassical method yields (Ref. 1)

$$\sigma = (1 + q \bar{q}/R_c E) \pi R_c^2 \tag{14}$$

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for the rearrangement cross section in the collision of a type 3 pair. Due to the uncertainty of V for this case and the uncertainty of R_c ($\approx 0.5\ a_o$ here), Eq. (14) must be regarded as an estimate of the cross section. In Eqs. (12), (13), and (14), E is the collision kinetic energy in the center of mass coordinate system of the pair.

A lower bound to the rearrangement cross section in the collision of a type 1 pair may be obtained by using the upper bound to V, which is E_1 as determined from the atomic model calculation in Section IV, and treating the scattering as a quantum mechanical barrier penetration problem.

To do so it is first necessary to determine the effect of the centrifugal potential energy barrier on the problem. The centrifugal potential energy is given by $U = \ell(\ell + 1)/2MR^2$ where ℓ is the angular momentum quantum number of the pair and M is the reduced mass of the pair. For small values of ℓ the value of U at $R = R_c$ will be less than E_m, the maximum value of E_1, and will not substantially reduce the amount of the incoming wave that reaches R_c. For large values of ℓ, the value of U at $R = R_c$ will be greater than E_m and the centrifugal potential energy barrier will substantially reduce the penetration. The value of the mean square of the total angular momentum L that results in U being equal to V_m at $R = R_c$ is given by

$$L^2 = \ell_c(\ell_c + 1) = 2ME_m \tag{15}$$

It will be assumed the all incoming waves with $\ell > \ell_c$ will not penetrate to $R = R_c$ whereas incoming waves with $\ell < \ell_c$ may penetrate to $R = R_c$, subject to the effects of the E_1 barrier only. If a wave penetrates to $R = R_c$ it will undergo rearrangement.

The value of L given by Eq. (15) corresponds to an impact parameter, R_1, equal to $L/\sqrt{2ME} = \sqrt{E_m/E}$. Hence, in the classical picture, all collisions with $R_1 > \sqrt{E_m/E}$ ($\equiv R_2$) do not involve penetration to R_c whereas collisions with $R_1 < \sqrt{E_m/E}$ will involve penetration to R_c if the B_1 barrier is passed through. The cross section for rearrangement and subsequent annihilation is therefore given by

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$$\sigma = \pi R_2^2 T = \pi \frac{E_m}{E} T = \pi \frac{E'_m}{E'} T \qquad (16)$$

where T is the coefficient of transmission of the incoming waves through the E_1 barrier.

Second, it is necessary to replace the E_1 barrier by an equivalent square barrier to calculate T. This barrier is shown as the dashed line in the figure in the preceding section, and the quantities in terms of which it is defined are given there. It is equivalent in the sense that it is the best choice of a square barrier to represent the E_1 barrier that can be made from simple principles. It will not reproduce the effects of E_1 exactly.

The transmission coefficient T for the square barrier may be determined by following the derivation given by Margenau and Murphy (Ref. 12) without making the approximation involved in their final result. Doing so, the result is,

$$T = \frac{4 K_1/K_3}{\left(1 + \frac{K_1^2}{k_2^2}\right)\left(1 + \frac{k_2^2}{K_3^2}\right) \cosh^2 k_2 a - \left(\frac{K_1}{k_2} - \frac{k_2}{K_3}\right)^2} \qquad (17)$$

for $E \leq E_{eq}$ and

$$T = \frac{4K_1/K_3}{\left(1 - \frac{K_1^2}{k_2^2}\right)\left(1 - \frac{K_2^2}{K_3^2}\right) \cos^2 K_2 a - \left(\frac{K_1}{K_2} + \frac{K_2}{K_3}\right)^2} \qquad (18)$$

for $E \geq E_{eq}$ where

$$
\left.
\begin{aligned}
K_1 &= \sqrt{2M(E - E_{1c})} & a &= 2a'/(\beta + \bar{\beta}) \\[2ex]
k_2 &= \sqrt{2M(E_{eq} - E)} & E_{1c} &= 2E'_{1c}/n\bar{n}(\beta + \bar{\beta}) \\[2ex]
K_2 &= \sqrt{2M(E - E_{eq})} & E_{eq} &= 2E'_{eq}/n\bar{n}(\beta + \bar{\beta}) \\[2ex]
K_3 &= \sqrt{2ME} & E &= 2E'/n\bar{n}(\beta + \bar{\beta})
\end{aligned}
\right\}
\quad (19)
$$

Equation (16) along with Eqs. (17), (18), (19), and (10) and the tables of Section IV constitute a means of obtaining an upper bound for the rearrangement-annihilation cross section for a given atom-antiatom pair. Employing this means, the lower limit to the H-$\bar{\text{H}}$ rearrangement cross section is found to be 4.18×10^{-13} (in units of $a_o^2 = 1$ in atomic units) for a collision energy of 2×10^{-3} (room temperature, in units of $e^2/a_o = 1$ in atomic units). For the same energy and $C_6(\text{H-}\bar{\text{H}}) = 6.5$, Eq. (12) gives 87.9 for the upper bound, whereas Junker and Bardsley (Ref. 3) find 27.2, which is the best value to date. The very low value of the lower bound demonstrates the extreme sensitivity of the cross section to the presence of a positive maximum in the interatomic potential energy for collision energies in the thermal range. The values of the lower bounds to σ for H-$\bar{\text{H}}$ and other pairs are so low that in places in the following sections where these lower bounds might be employed to give lower bounds on annihilation rates, other means were found that gave higher lower bounds on the annihilation rates.

VI. ANNIHILATION RATES IN THE SOLID OR LIQUID STATE

As was stated in Section III, the first step in atom-antiatom annihilation in the solid or liquid state is the annihilation of the electrons and positrons by direct contact or through positronium formation. Positronium formation is difficult to calculate here, and it is energetically impossible

for all but a few atoms and antiatoms in a nonscattering situation. In those cases it will probably not increase the rate substantially; hence, positronium formation is neglected.

The direct contact annihilation rate may be determined by employing the atomic model of Section IV to obtain the total probability for any electron of the atom being at the same position in space as any positron of the antiatom when the atom and antiatom are a distant R apart. This probability is then multiplied by the appropriate factors (Ref. 13) to obtain the total $e^+ - e^-$ annihilation rate in the atom and antiatom.

If the result is multiplied by the number of nearest neighbors of the opposite character (taken to be 6) and divided by 2 to make it the rate per atom or antiatom the result is

$$\mathscr{R} = 1.20 \times 10^{10} \, \frac{n\bar{n}\beta\bar{\beta}}{\left(\frac{\beta}{\bar{\beta}} - \frac{\bar{\beta}}{\beta}\right)^3} \left(\left(-\frac{4}{R} + \left(\frac{\beta}{\bar{\beta}} - \frac{\bar{\beta}}{\beta}\right)\beta \right) e^{-\bar{\beta}R} \right.$$

$$\left. + \left(\frac{4}{R} + \left(\frac{\beta}{\bar{\beta}} - \frac{\bar{\beta}}{\beta}\right)\bar{\beta} \right) e^{-\beta R} \right) \tag{20}$$

for $\beta \neq \bar{\beta}$, and

$$\mathscr{R} = 1.51 \times 10^9 \, n\bar{n}\beta^3 (1 + \beta R + \beta^2 R^2/3) e^{-\beta R} \tag{21}$$

for $\bar{\beta} = \beta$, where $\mathscr{R}$ is the annihilation rate in number of $e^+ - e^-$ annihilations alone per second per atom or antiatom.

The corresponding energy production rates for $e^+ - e^-$ annihilation alone are given by

$$W = 4.44 \times 10^{11} \, \mathscr{R} \tag{22}$$

where W is in calories per mole per second. If the energy from the subsequent nucleus-antinucleus annihilation (as discussed in Section III) is to be included, W must be multiplied by 1836 times the mean atomic number of the nucleus and antinucleus times the fraction of the nucleus-antinucleus mass energy that is released in the nucleus-antinucleus annihilation.

For use in the following section it is important to have a knowledge of what R would be in a solid or liquid mixture of atoms and antiatoms. An upper bound to R may be found in the following way. The atomic model may be used to calculate E_1 and the long range, R^{-6}, form for E_2. If this is done and it is assumed that $V = E_1 + E_2$ it is indeed found that there is a minimum in V for R equal to several Bohr radii where the R^{-6} form is a good approximation to E_2. Thus, if $V = E_1 + E_2$ were a good approximation to V, solid and liquid mixtures of atoms and antiatoms (neglecting the effects of annihilation) could indeed exist at least momentarily with their mean atom-antiatom separation being given by this value of R at the "van der Walls minimum" of their interatomic potential energy.

There are three important facts relevant to the accuracy of a V of this form. First, in the atomic model it is assumed that $\beta = 2\sqrt{2E_I}$ and $\bar{\beta} = 2\sqrt{2\bar{E}_I}$, where E_I and $\bar{E}_I$ are the atom and antiatom ionization energies. This assumption is expected to be valid when there is only one electron or positron in the outer shell, but when there are more than one it will underestimate the value of $\bar{\beta}$ or β by a few percent, as may be shown to be true of helium and argon at least. This in turn results in E_1 being overestimated. Second, from known values of C_6 (Ref. 11) it may be shown that the values of C_6 predicted by this model may be incorrect by up to about 50%. Third, the neglect of perturbation terms in V of higher order than E_2 almost certainly will yield higher values of V than the exact ones, as the calculations of Junker and Bardsley (Ref. 3) indicate for $H - \bar{H}$. Taken together these facts mean that there may be no van der Walls minimum as predicted by the model or, if there is, the model will probably over estimate the value of R at which it occurs.

Using the model for $\overline{\beta} = \beta$ it is found that the value of R for which the minimum occurs, R_{min}, is given to a good approximation by

$$R_{min} = 15.2 \, \beta^{-0.65} \tag{23}$$

for closed shell atoms and antiatoms and is somewhat less for atoms and antiatoms with open shells. For $\beta \neq \overline{\beta}$ the equation corresponding to Eq. (23) has not been derived. However, it may be shown that replacing β by the lesser of β and $\overline{\beta}$ will result in an overestimate of R_{min}. Hence the equation,

$$R_{min} = 15.2 \, (\text{lesser of } \beta, \, \overline{\beta})^{-0.65} \tag{24}$$

will, relative to these and the preceding considerations give an upper bound to R_{min} if an R_{min} does indeed exist. (R_{min} does not exist only when V is negative and attractive for all R. In this case R = 0 gives V = $-\infty$, so, in a sense, $R_{min} = 0.$)

VII. THE STABILITY OF A SOLID OR LIQUID MIXTURE OF MATTER AND ANTIMATTER

An ideal means of storage of antimatter would be in a homogenous solid or liquid mixture with antimatter, if a particular mixture could be found that was stable over a reasonable time period. If the conditions in such a mixture were only somewhat on the side of producing stability, annihilation could be commenced by an achievable amount of compression or heating.

Equations (20) through (22) may be used to determine the values of R at which the rate of annihilation energy production by $e^+ - e^-$ annihilation alone is one calorie per mole per second (a value that might conceivably be tolerable in storage). The values of R along with lifetimes of the mixtures for H - $\overline{H}$ and helium (He)-antihelium ($\overline{He}$) are

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Mixture	R	Lifetime
H - $\bar{H}$	28.5	14,203 years
He - $\overline{He}$	22.0	14,203 years

The above values of R are, however, considerably greater than the values of R_{min}. Using Eq. (24) to give R_{min} for substitution into Eqs. (20) through (22), the following results are obtained. For upper limits on R_{min}, lower limits on W, and upper limits on the lifetime,

Lesser of β, $\bar{\beta}$	R_{min}	W/n$\bar{n}$, cal/ (mole · s)	n$\bar{n}$· lifetime	
1	15.2	1.56×10^{16}	2.8×10^{-5} s	
2	9.7	2.932×10^{15}	1.5×10^{-4} s	(25)
3	7.4	7.734×10^{14}	5.8×10^{-4} s	

Since all elements have β's between 1 and 3, it is therefore clearly impossible on this basis to store antimatter in a solid or liquid mixture with matter. The addition of the annihilation energy of the nuclei and antinuclei would greatly increase the energy production rates.

VIII. MATTER-ANTIMATTER SURFACE CONTACT

Another possible means of storage of antimatter would be to store it in contact with solid matter, such as liquid antimatter in a matter bottle, or solid antimatter in contact at certain points on its surface with matter.

To consider this possibility, Eqs. (20) and (21) may be applied to the atoms and antiatoms in contact at the surface and may be regarded as degradation rates. One annihilation of an e^+ - e^- pair is rapidly followed by annihilation of the nucleus and antinucleus of the atom and antiatom that will take much less time to be completed than the e^+ - e^- pair annihilation.

Hence, Eqs. (20) and (21) with n and $\bar{n}$ set equal to 1 give directly the number of monatomic atom or antiatom layers that are degraded while contact is maintained at the same value of R. It is therefore possible to determine the velocity with which the two surfaces must approach the point of contact to maintain contact in the face of degradation as well as the rate of annihilation energy production per square centimeter.

Using Eq. (23) for the relation between β and R_{min}, the results are

β	R_{min}	Atomic Layers Degraded/s	Velocity, cm/h	Annihilation Energy, $cal/(cm^2 \cdot s)$	
1	15.2	35165	10.4	4.47×10^{14}	(26)
2	9.7	6599	1.23	5.29×10^{13}	
3	7.4	1741	0.244	1.05×10^{13}	

In calculating the annihilation energy it has been assumed that the densities of the matter and the antimatter are one gram per cubic centimeter and that the nuclei and antinuclei annihilate completely. In addition the formation of a "leidenfrost" layer between the matter and antimatter has been neglected. The leidenfrost layer would consist of positronium gas and gasses of the matter and antimatter, and its presence would lower the annihilation rate. If these effects were taken into account and the smallest possible area of contact that could support the antimatter were used, it is, nevertheless, very doubtful that the rate of energy production could be reduced to a tolerable level. It therefore appears impossible to store solid or liquid antimatter in contact with solid matter.

IX. MATTER-ANTIMATTER CONTACT INVOLVING THE GASEOUS STATE

Two possible means of antimatter storage involve the gaseous state. The first is a gaseous mixture of matter and antimatter in a magnetic bottle or in a matter container. The second is gaseous antimatter in a matter container.

In the first case we will assume a mixture of equal numbers of atoms and antiatoms. If a significant amount of antimatter is to be stored by this means, the annihilation occurring on the walls of any container that may be present will be insignificant compared to the annihilation within the gaseous mixture. At thermal temperatures, the annihilation will come from two processes, direct electron positron annihilation as in the solid or liquid state followed by rearrangement annihilation involving the resulting ions and antiions, and neutral atom-neutral antiatom rearrangement annihilation.

The atom-antiatom interatomic potential energy is taken to be given by $V = E_1 + E_2$, where E_1 and E_2 are determined by the atomic model calculation. This gives an upper bound to V for all but large R and a lower limit to the annihilation rate. Another consequence of this choice of V is that the annihilation due to the first process will greatly exceed that due to the second process. An upper bound to the lifetime of a gaseous mixture can then be obtained by making the simplifying assumption that no annihilation will take place when the distance between an atom and antiatom less than $R_{min} - 2a_o$ or is greater than $R_{min} + 2a_o$ and by using Eq. (21) to obtain a mean value for $\mathscr{R}$. The results may be presented in terms of the ratio of the density to gas density at STP required to give a lifetime of one hour to the mixture, the diameter of a sphere that would hold one gram of the mixture at STP, and the rate of annihilation energy produced per cubic meter assuming two nuclei or antinuclei per electron or positron, complete nuclei-antinuclei annihilation, and complete trapping of the energy within the container:

β	ρ/ρ_{STP}	Diameter, m	cal/(meter$^3 \cdot$ s)
1	3.4×10^{-7}	50	8.97×10^4
2	4.4×10^{-6}	11	9.31×10^6
3	2.9×10^{-5}	6	6.26×10^7

(27)

Even if the above annihilation energy rates were reduced by the fraction of nucleons annihilated and the fraction of the energy trapped within the container or in its walls, they would probably still be prohibitive. If

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the volumes were increased (along with a proportionate increase in storage time) to make the annihilation energy production rates tolerable, the system would be so large that it would probably be only barely competitive with chemical propellants in terms of the total mass to energy stored ratio. It therefore appears that this form of antimatter storage is not practical.

If the scattering annihilation rates were employed, either with the upper limit to the rearrangement cross section for thermal energies (Eq. (12)) or with the direct particle-antiparticle cross sections that would apply at temperatures of a few thousand Kelvins or greater (Ref. 2), then the results would be considerably worse.

In the second means of storage, annihilation occurs only between the atoms of the antimatter gas and the atoms of the container, and the treatment is otherwise essentially the same as above. If we assume $V = E_1 + E_2$ with E_1 and E_2 being given by the model, then annihilation will occur only within a shell about $2a_o$ thick which is located a distance of R_{min} from the inside of the container. For various diameters of spherical containers and the three values of β, upper limits to the lifetime of the antimatter gas and lower limits to the annihilation energy production rate for $e^+ - e^-$ annihilation alone are:

$\overline{nn}$ x lifetime (days), energy produced (cal/mole $\cdot$ s)/$\overline{nn}$

β \ D	1 cm	1 m	10 m
1	0.020, 2.5 x 10^8	2.0, 2.5 x 10^6	20.0, 2.5 x 10^5
2	0.11 , 4.7 x 10^7	11.0, 4.7 x 10^5	110 , 4.7 x 10^4
3	0.42 , 1.2 x 10^7	42.0, 1.2 x 10^5	420 , 1.2 x 10^4

$$(28)$$

These energy production rates are prohibitive and would be more so if scattering annihilation had been considered with the cross-section given by Eq. (12), or by the high-energy direct particle-antiparticle annihilation cross sections (Ref. 2).

X. ANNIHILATION ENERGY PRODUCTION RATES FOR ANTIMATTER IN A VACUUM

Another means of possible antimatter storage is electromagnetic suspension of solid antimatter in a vacuum. Annihilation would occur here due to vaporization of the antimatter and the surrounding matter structure. The composition and positioning of the matter could undoubtedly be chosen to make its vaporization insignificant, but the composition of the antimatter would, most likely, be determined by other factors. Hence, it is likely in a practical situation that significant annihilation would occur only in the interaction of vaporized antiatoms with atoms at the surfaces of the matter.

An upper limit to the annihilation and energy production rates may be obtained by assuming that each antiatom interacts with the matter surface atoms in the same way that it interacts with an individual atom, that Eq. (12) for the cross section applies, and that complete nucleus-antinucleus annihilation occurs following the rearrangement reactions. This cross section and the corresponding interaction are such that for all temperatures in the ordinary thermal range, except a few Kelvins or less, several angular momentum waves take part in the rearrangement, and the scattering is dominated by the inelastic rearrangement process. Under these circumstances, the atom-antiatom elastic scattering cross section is about equal to the rearrangement cross section (Ref. 1). Thus, when an antiatom strikes a matter surface, it will undergo annihilation with a surface atom with a probability of about one half. Since the elastic scattering will be predominantly in a forward direction, most of the elastically scattered antiatoms will enter the matter and subsequently annihilate. It may therefore be assumed that all antiatoms striking a matter surface will be annihilated.

It follows that there is a very simple relation between the lifetime against evaporation in years, τ, of the antimatter, and the upper bound to the annihilation energy production rate, W. If it is assumed that the fraction of the annihilation energy that is deposited in the antimatter is one half (an upper limit since roughly one half of the energy goes into neutrinos and most of the energy could be deposited elsewhere) the relation is:

$$W = 6.83 \times 10^5 \, \frac{\overline{A}}{\top} \, \frac{cal}{mole \cdot s} \tag{29}$$

where $\overline{A}$ is the atomic number of the antimatter. Considering the nature of W as an upper bound and small values of $\overline{A}$, this form of storage would be practical $[W \lesssim 1 \, cal/(mole \cdot s)]$ if the antimatter has a lifetime against evaporation of about one million years or greater, and may be practical for smaller lifetimes. Such lifetimes can be readily achieved with matter. They could, in principle, be achieved with antimatter; but whether they could be achieved or not, in a practical situation, is difficult to determine.

XI. SUGGESTIONS FOR FURTHER STUDY

The results of this study were based on the interactions of spherically symmetric atoms and antiatoms that possess no electric or significant magnetic multiple moments. A better knowledge of the interatomic potential energy for such species would lower the gap between the upper and lower bounds determined for various quantities. This would be particularly important in evaluating the possible means of transferral of the antimatter to the combustion chamber of a rocket and its annihilation in the combustion chamber.

The interactions relevant to annihilation of atoms and molecules that possess electric or magnetic multipole moments may differ significantly from the interactions considered in this study. Multipole moments may result in different annihilation rates, and they may conceivably alter the atom-antiatom interactomic potential energy to the extent that the mean atom-antiatom separation in a solid, atomic scale matter-antimatter mixture is sufficiently great to make such a mixture stable over a long period of time.

Electric and magnetic multipole moments are possessed by some atoms and molecules. For these or other atoms and molecules moments could be enhanced or induced by the application of external electric and/or magnetic fields. The effects of multipole moments on annihilation rates and atom-antiatom separations deserve further study.

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The means used for and the feasibility of suspending antimatter in a vacuum depend on the physical state and properties of the antimatter. The means and feasibility depend on whether the antimatter is in the form of a plasma, gas, liquid, or solid, and whether it is magnetized, magnetic, paramagnetic, diamagnetic, electrically conductive, or electrically polarizable. Means exist for the complete or partial suspension of anti-matter in any of the above states and with any of the above properties, but a study should be made to ascertain the practical feasibility of suspension under the various possible combinations of states and properties.

In addition it would be worthwhile to study the effects of annihilation rates on various means to transfer the antimatter from storage to the combustion chamber, and on the design of the combustion chamber.

REFERENCES

1. D. L. Morgan, Jr. and V. W. Hughes, Phys. Rev. A, 7, 1811 (1973).

2. D. L. Morgan, Jr. and V. W. Hughes, Phys. Rev. D, 2, 1389 (1970).

3. B. R. Junker and J. N. Bardsley, Phys. Rev. Lett., 28, 1227 (1972).

4. Preprint by K. Omidvar, Code 641, Goddard Space Flight Center, Greenbelt, Maryland.

5. G. Steigman, Ph. D. dissertation, New York University, New York, N. Y., 1968 (unpublished).

6. H. Alfvén and O. Klein, Ark. Fys., 23, 187 (1963);
    H. Alfvén, Rev. Mod. Phys., 37, 652 (1965);
    O. Klein, Nature (London), 211, 1337 (1966);
    H. Alfvén and A. Elvius, Science, 164, 911 (1969).

7. M. Nauenberg and M. A. Ruderman, Phys. Lett., 22, 512 (1966).

8. S. M. V. Aldrovandi and J. L. Puget, Astron. and Astrophys., 12, 126 (1971).

9. See for instance: Y. F. Chan, The Scattering of Positrons by Hydrogen Atoms, Ph. D. dissertation. Dept. of Physics, The University of Western Ontario, London, Ontario, Canada, Feb. 1972.

10. D. R. Bates, private communication to R. Drachman (1968) (see Ref. 1).

11. A. Dalgarno and A. E. Kingston, Proc. Phys. Soc. Lond., 73, 455 (1959); Proc. Phys. Soc. Lond., 78, 607 (1961); A. E. Kingston, Phys. Rev., 135, A1018 (1964); F. London, Z. Phys. Chem. (Leipzig) B, 11, 222 (1931); L. Salem, Mol. Phys., 3, 441 (1960); K. S. Pitzer, Adv. Phys., 2, 59 (1959); H. Margenau, J. Chem. Phys., 6, 896 (1938); R. Heller, J. Chem. Phys., 9, 154 (1941); P. R. Fontana, Ph. D. dissertation. Yale University, New Haven, Conn., 1960 (unpublished).

12. H. Margenau and G. M. Murphy, <u>The Mathematics of Chemistry and Physics,</u> Second Edition, pp. 356 - 358. D. Van Nostrand, New York, N. Y., 1956.

13. M. Deutsch, <u>Progr. Nucl. Phys.</u>, <u>3</u>, 131 (1953).