DESIGN TRADEOFF STUDY FOR REFLECTOR ANTENNA SYSTEMS

for the

SHUTTLE IMAGING MICROWAVE SYSTEM

Final Report 636-1

on

Contract 954026

by

R. C. HANSEN

(NASA-CR-140915) DESIGN TRADEOFF STUDY FOR REFLECTOR ANTENNA SYSTEMS FOR THE SHUTTLE IMAGING MICROWAVE SYSTEM Final Report (Hansen (R.C.), Inc., Encino, Calif.) CSCL 20N G3/32 N75-14940 Unclas 17483

Prepared for

DR. JOE WATERS

JET PROPULSION LABORATORY

30 August 1974

R. C. HANSEN, INC.

Suite 218

17100 Ventura Blvd.

Encino. Calif. 91316

N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM THE BEST COPY FURNISHED US BY THE SPONSORING AGENCY. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE.

DESIGN TRADEOFF STUDY FOR REFLECTOR ANTENNA SYSTEMS

for the

SHUTTLE IMAGING MICROWAVE SYSTEM

Final Report 636-1

on

Contract 954026

by

R. C. HANSEN

Prepared for

DR. JOE WATERS

JET PROPULSION LABORATORY

30 August 1974

This work was performed for the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration under Contract NAS7-100.

i

TABLE OF CONTENTS

1.0 SUMMARY

A general tradeoff is made of the symmetric Cassegrain antenna particularly with regard to the possibility of meeting a 90% beam efficiency. The factors that affect beam efficiency are aperture taper and blockage, feed cross polarization and spillover, and reflector cross polarization and backlobes. Such tradeoffs have not appeared before due to the large number of variables involved - 4 to specify a Cassegrain and several more for the feed. To allow a meaningful yet simple tradeoff, the effects of aperture taper and blockage are calculated using an adjustable sidelobe circular distribution, the Modified $J_1(x)/x$. Numerical integration is used. For the feed spillover calculation, a low sidelobe symmetric feed pattern is used with the equivalent parabola and numerical integration. Reflector cross polarization is calculated using double numerical integration. Reflector back lobes are estimated from radiation pattern envelopes of commercial common carrier dish antennas. To maintain dish and feed back radiation (90 to 180 deg.) and feed cross-pol power each below 1/2% will require careful design. The main reflector will need a heavy edge taper and may need absorber at the edge. The feed horn will also need special design treatment and may need some absorber on the horn exterior.

The curves allow a range of f/D to be determined for a specified edge taper and blockage diameter ratio. With a table of Cassegrain geometric parameters, a range of possible designs that meet the 90% beam efficiency is obtained. Use of a low magnification (M = 1.5) allows smaller feed horn diameters (typically 1.25λ), but the feed center is close to the sub-reflector. A larger magnification (M = 3 to 5) will move the feed center back toward the main reflector apex but requires larger (typically 3.4λ) feed diameters. In the latter case, satisfactory designs are feasible with f/D from at least .3 to .5. The actual antenna design should include GTD calculations so that the reflector and sub-reflector diffraction may be included.

Thus a symmetric Cassegrain can be designed to yield 90% beam efficiency, but the feed and reflector design and implementation must be carefully done.

2.0 SYMMETRIC CASSEGRAIN ANTENNA

2.1 Introduction

The objective of this part of the study is to determine whether a symmetrical Cassegrain antenna with appreciable blockage can provide a beam efficiency (power between main beam nulls) of 90% or more with cross polarization below -20 db. Components of energy outside the main beam are:

aperture taper and blockage (these affect sidelobes)
feed spillover including back lobes
feed cross-pol
reflector back lobes
reflector cross-pol

Specification of a Cassegrain requires 4 parameters, and typical feeds require 2 to 3 additional parameters. Thus the major task in any tradeoff is to reduce the number of variables. This is accomplished by studying the effects of aperture illumination taper and aperture blockage on an aperture distribution, where the main reflector has been simulated by the distribution. For this purpose a new, highly efficient, and low Q circular aperture distribution is used. Feed spillover and reflector cross-polarization are evaluated using the equivalent parabola, and a single parameter feed. Thus for the first time a set of general tradeoffs for beam efficiency, and as a by-product (directivity) efficiency, are obtained, giving general capabilities of the Cassegrain system.

In the next section the new distribution is discussed. Following sections cover the taper and blockage beam efficiency calculation.

2.2 Modified $J_1(\pi u)/\pi u$ Aperture

Circular aperture distributions used in texts and papers are almost always chosen because they can be readily integrated. For example, the $(1-\rho^2)^n$ on a pedestal is common.[1,2,3,4] The Gaussian is of use only for very heavily tapered apertures where the truncation is not serious, but represents a very inefficient distribution. All of these distributions suffer two major disadvantages. First, there is no simple way of choosing optimum parameters, e.g. maximum efficiency for a given sidelobe level. And second, there is no simple way of finding the parameters to yield a given sidelobe level. Both these limitations are removed in a new circular distribution, the Modified $J_1(\pi u)/\pi u$. This is related to the Modified $\sin \pi u/\pi u$ line source developed by Taylor.[5]

Uniform amplitude over a circular aperture gives a $J_1(\pi u)/\pi u$ pattern, where $u = D \sin \theta/\lambda$. Throughout constant phase is assumed to exclude supergain. Sidelobes are - 17.6 db, and the sidelobe envelope decays as $1/u$. This latter item is important as the far out sidelobes (large u) must decay as $1/u$ to allow a well behaved aperture distribution and low Q. Otherwise the distribution tends to have edge peaks, and the energy storage becomes large. Taylor, in developing the Modified $\sin(\pi u)/\pi u$ line source, adjusted the close in zeroes of the pattern function to reduce the corresponding sidelobes, thereby increasing the sidelobe ratio, while leaving alone the farther out zeroes which produce the $1/u$ envelope. A similar procedure has been followed by Hansen[6] for the circular aperture. The pattern function

---

1. R. C. Hansen, "Microwave Scanning Antennas," Vol. 1, Academic Press, 1964, p. 64.
2. A. F. Sciambi, "The Effect of the Aperture Illumination on the Circular Aperture Antenna Pattern Characteristics," report, 1964, RCA
3. J. P. Grantham, "The Secondary Characteristics of the Circular Aerial for a Range of Theoretical Uniphase Aperture Distributions," TN NX-53-2, January 12, 1954, ASRE, England.
4. S. Silver, "Microwave Antenna Theory and Design," McGraw-Hill, 1949.
5. R. C. Hansen, op cit, Vol. 1, p. 58.
6. Unpublished data.

is

$$E(\theta) = \frac{J_1\left(\pi\sqrt{u^2 - H^2}\right)}{\pi\sqrt{u^2 - H^2}}$$

where again $u = D \sin \theta/\lambda$ and H is a constant that adjusts the sidelobe ratio. For $u < H$ the pattern (part of the main beam) is given by

$$E(\theta) = \frac{I_1\left(\pi\sqrt{H^2 - u^2}\right)}{\pi\sqrt{H^2 - u^2}}$$

and the sidelobe ratio is

$$SLR = 20 \log \frac{I_1(\pi H)}{\pi H} + 17.58 \text{ db}$$

This Modified $J_1(\pi u)/\pi u$ distribution has the proper $1/u$ sidelobe envelope behavior, and because it is chosen to give the proper pattern zeroes, it 'fits the physics'. It is thus an efficient and easily used distribution. Parameters are given in the tables. The Modified $J_1(\pi u)/\pi u$ distribution itself is a monotonic function with an edge pedestal.

MODIFIED $J_1(\pi u)/\pi u$ PARAMETERS

SLR	H	taper efficiency	relative beamwidth
17.58 db	0	1	1
20	.4872	.980	1.0483
25	.8899	.870	1.1408
30	1.1977	.762	1.2252
35	1.4708	.668	1.3025
40	1.7254	.595	1.3741

PATTERN ZEROES AND PEAKS

$\pi\sqrt{n^2 - H^2}$	type
3.8317	zero
5.1356	peak
7.0156	zero
8.4172	peak
10.1735	zero
11.6198	peak
13.3237	zero
14.7960	peak
16.4706	zero

2.3 Beam Efficiency vs Taper and Blockage

This parameter, η_{beam}, is defined as the fraction of radiated power that is contained in the main beam, null-to-null. To avoid a multi-dimensional tradeoff, the equivalent parabola concept[1] is used. The figure shows the geometry. This equivalent parabola has the same diameter as the main reflector, and the same focal angle as the sub-reflector. Calculations using the equivalent parabola do not exactly match experimental results for two reasons: the sub-reflector edge diffraction is omitted; the sub-reflector may not be large in wavelengths. Thus for actual design of a Cassegrain antenna, more accurate methods are used. However, for capability tradeoffs, the equivalent parabola is adequate, and it reduces the 4 parameters to just two: D/λ and f_e/D. For the main reflector

$$\tan \frac{\theta_e}{2} = \frac{1}{4 f/D}$$

and for the equivalent parabola

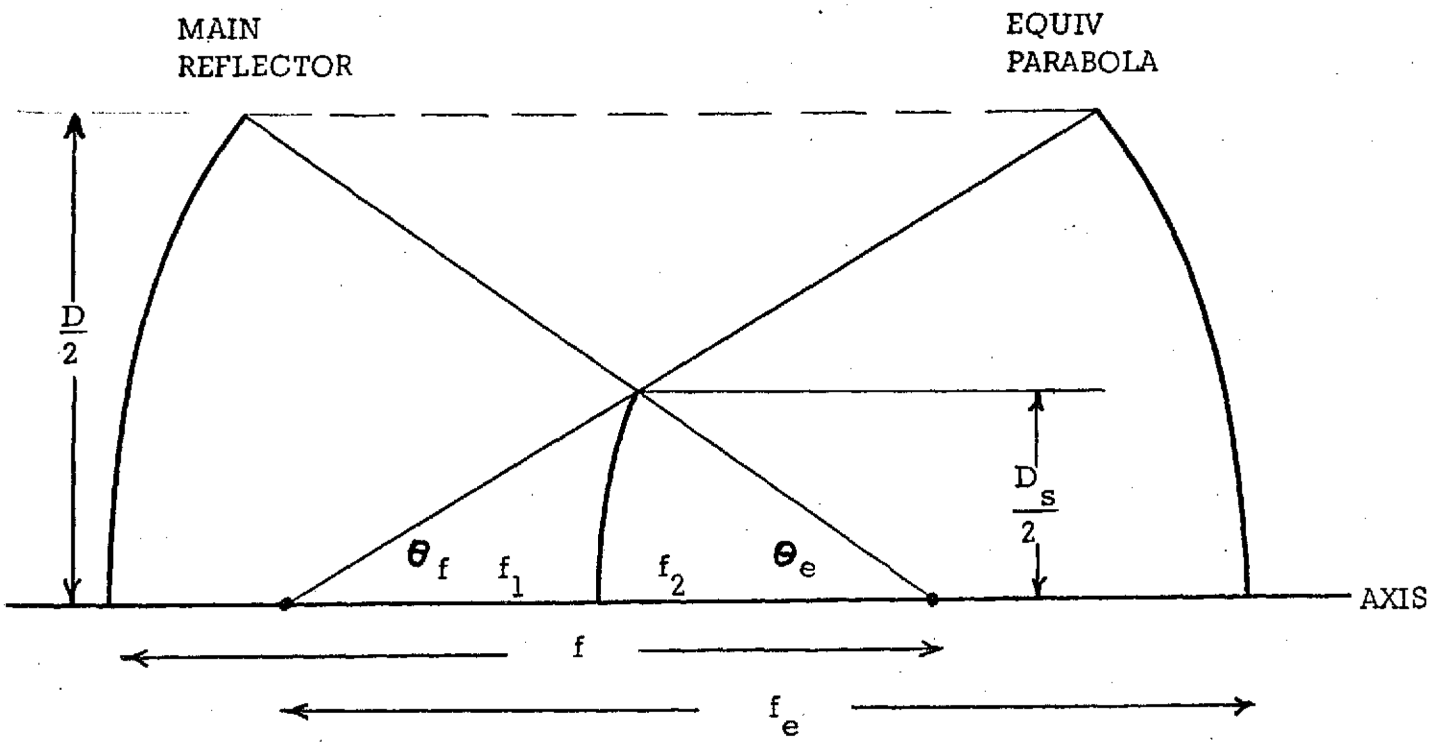
$$\tan \frac{\theta_f}{2} = \frac{1}{4 f_e/D}$$

The magnification M is the ratio of focal lengths:

$$M = \frac{f_1}{f_2} = \frac{f_e}{f}$$

---

1. P. W. Hannan, "Microwave Antennas Derived From the Cassegrain Telescope," Trans. IRE, Vol. AP-9, March 1961, pp. 140-153.



Cassegrain Reflector

After the sub-reflector diameter D_s is chosen, the hyperbola focal lengths can be calculated:

$$f_1 = \frac{f_e D_s}{D}\left[1 - \frac{\tan\frac{\theta_e}{2} + \tan\frac{\theta_f}{2}}{\cot\frac{\theta_e}{2} + \cot\frac{\theta_f}{2}}\right]$$

$$f_2 = \frac{f D_s}{D}\left[1 - \qquad '' \qquad\right]$$

This equivalent parabola will be used in the feed spillover calculations of a later section. For taper and blockage effects, the equivalent parabola is replaced by a circular aperture distribution, the Modified $J_1(x)/x$ of Section 2.2. Now the beam efficiency vs sidelobe ratio and the effects of blockage can be calculated using the circular aperture. The taper efficiency η_t is

$$\eta_t = \frac{2 E_0^2}{\frac{\pi^2 D^2}{\lambda^2}\int_0^{\pi/2} E^2 \sin\theta \, d\theta}$$

and the beam efficiency is

$$\eta_{beam} = \frac{\int_0^{\theta_e} E^2 \sin\theta \, d\theta}{\int_0^{\pi/2} E^2 \sin\theta \, d\theta}$$

Circularly symmetric patterns are assumed. Blockage is usually a small part of the aperture diameter D, so the aperture distribution is nearly constant over the blockage diameter $D_{b\ell}$. This diameter is that of the sub-reflector increased to include support blockage. An often used and good approximation is for uniform amplitude blockage, which gives a pattern

$$\frac{2 D_{b\ell}^2 \, J_1(\pi u_b)}{D^2 \, \pi u_b} \, , \qquad u_b = \frac{D_{b\ell}}{\lambda} \sin\theta$$

This pattern is subtracted from the aperture pattern to include blockage effects.

Now the first null is shifted to a smaller value of u, called u_o, where u_o is the solution of:

$$\frac{J_1\left(\pi\sqrt{u_o^2 - H^2}\right)}{\pi\sqrt{u_o^2 - H^2}} = \frac{E_o \, D_{b\ell}^2 \, J_1(\pi u_b)}{D^2 \, \pi u_b}$$

and the main beam peak E_o is given by:

$$E_o = \frac{I_1(\pi H)}{\pi H}$$

For low sidelobe designs, the blockage can change the null position appreciably, and thus the perturbation method first tried was discarded. In the computer code, the equation above is solved for u_o using a Wegstein root finding subroutine for each blockage and sidelobe ratio case. Null shifts will be given later.

Unfortunately, integrals of the type above can only be solved by numerical integration. And because of many sidelobes with interspersed zeroes, either a high order integration process or many points are needed. After some experimentation a Romberg 6th order integration was used,* with the aperture size increased until the results converged. Library subroutines were used for Romberg and for the Bessel functions. The code is given in the Appendix.

For a non-blocked aperture the beam and taper efficiencies are given in the figure vs sidelobe ratio, which is the ratio of main beam peak to first sidelobe. It can be seen that a 20 db SLR gives 91% beam efficiency and 25 db SLR gives 97% beam efficiency. With circular blocking, the beam efficiency of course drops, as shown in the second figure. Here η_{beam} is plotted against blocking diameter/aperture diameter. These results allow the fraction of energy outside the main beam due to blockage and taper to be determined.

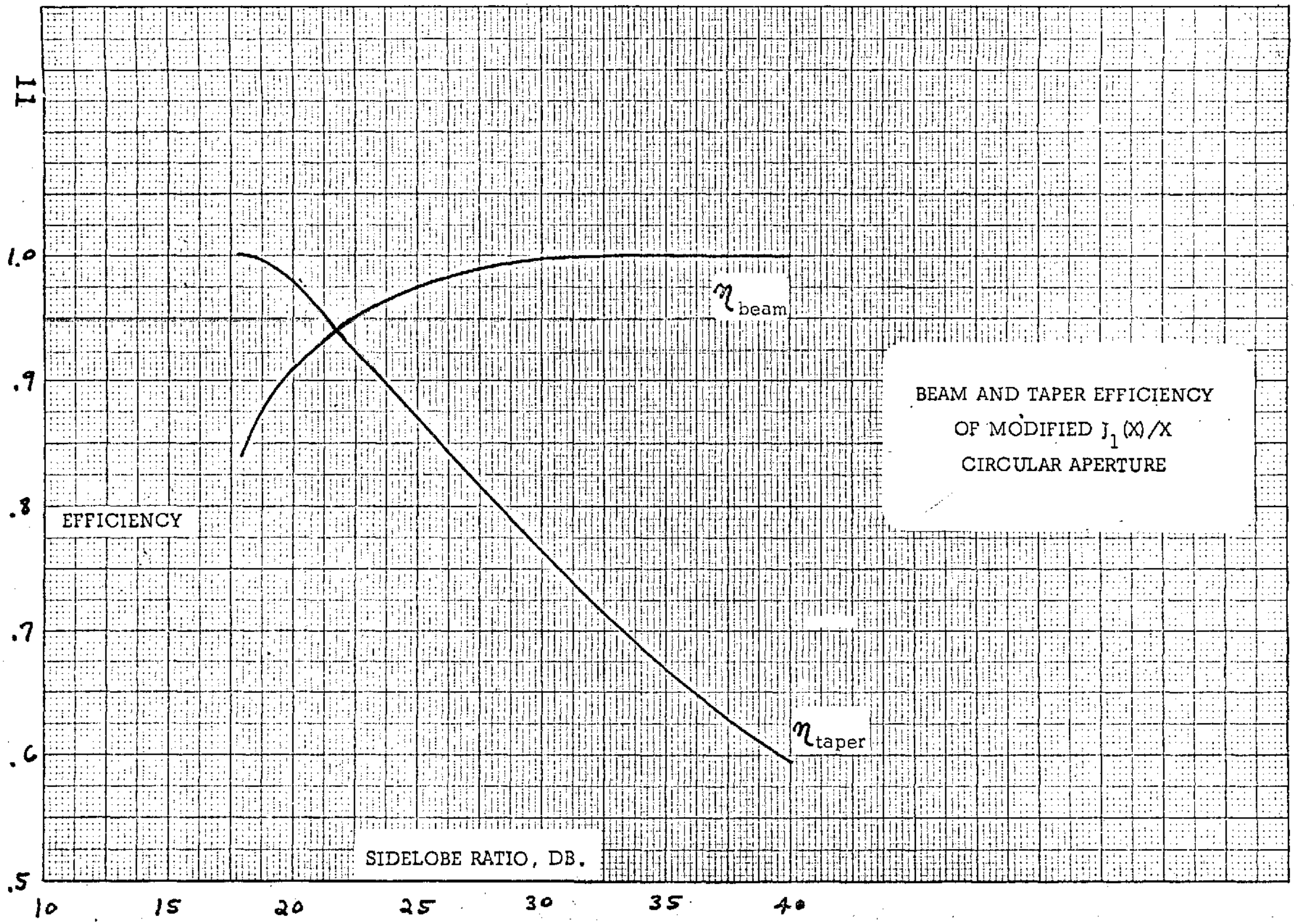
Taper efficiency, which gives directivity when multiplied by $(\pi D/\lambda)^2$, and the null shift are given in tables. Note that the blockage moves the first null closer in, and that the taper efficiency actually increases with blockage for low sidelobe levels. This is because the main beam is narrowing faster than the peak field squared is decreasing. Since the narrower main beam and higher sidelobes make the pattern more like that of a uniform aperture, it is not surprising that the η_t increases.

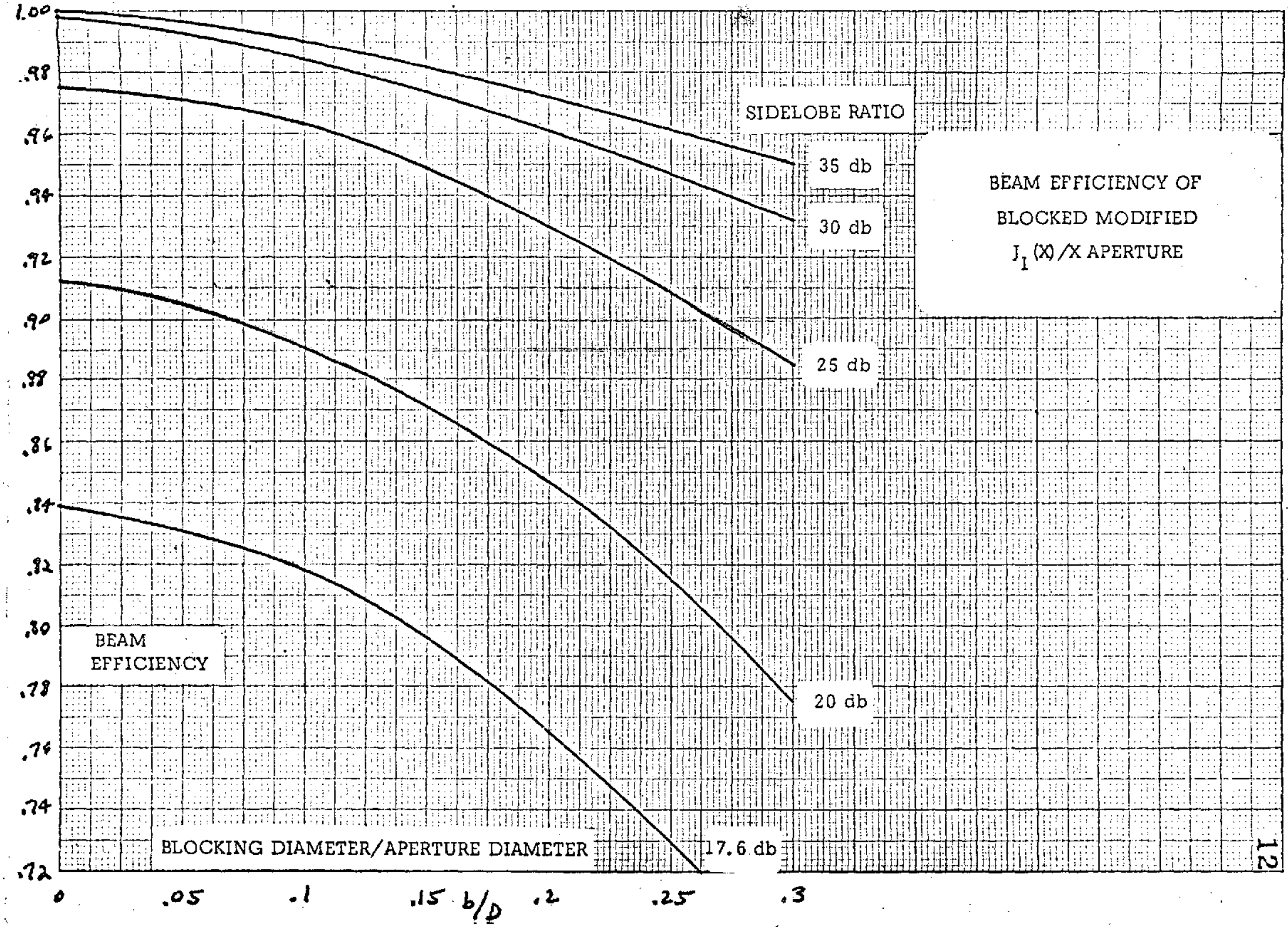
Because the Modified $J_1(x)/x$ aperture distribution is not yet available in functional form, the relation between sidelobe ratio and edge taper is obtained from another distribution, the $(1-\rho^2)^n$ on a pedestal. Using available raw data,[1] the optimum n and pedestal are picked for each SLR. Results are shown in the figure. An estimate

---

* Romberg is an adaptive trapezoidal integrator.

1. A. F. Sciambi, op cit.


BEAM AND TAPER EFFICIENCY
OF MODIFIED $J_1(X)/X$
CIRCULAR APERTURE
EFFICIENCY
SIDELOBE RATIO, DB.
η_{beam}
η_{taper}
1.0
.9
.8
.7
.6
.5
10
15
20
25
30
35
40



BEAM EFFICIENCY OF
BLOCKED MODIFIED
$J_1(X)/X$ APERTURE
SIDELOBE RATIO
35 db
30 db
25 db
20 db
17.6 db
BEAM
EFFICIENCY
BLOCKING DIAMETER/APERTURE DIAMETER
1.00
.98
.96
.94
.92
.90
.88
.86
.84
.82
.80
.78
.76
.74
.72
0
.05
.1
.15
b/D
.2
.25
.3

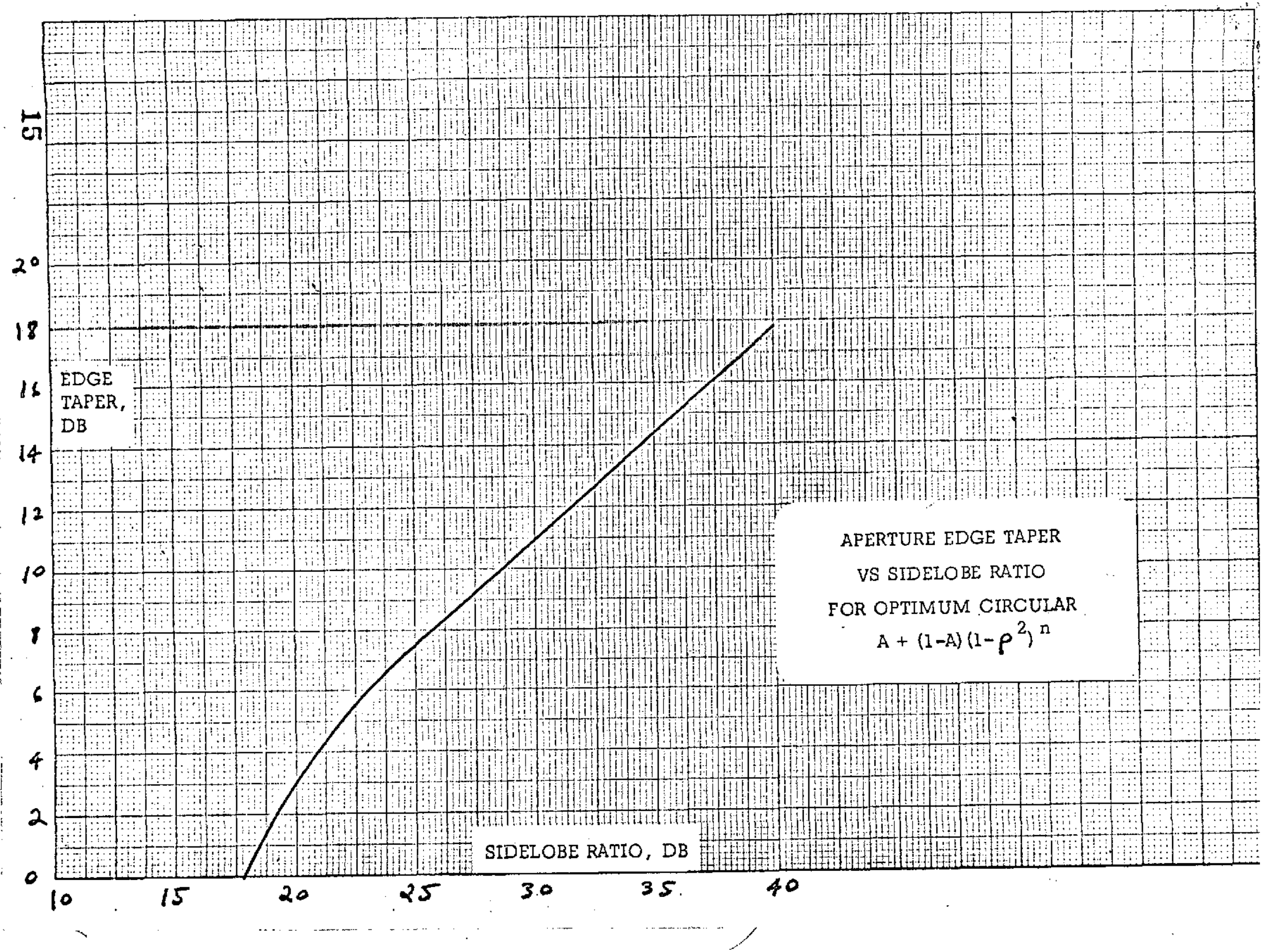
APPROXIMATE TAPER EFFICIENCY

SLR	D_s/D			
	0	.1	.2	.3
17.6	1.000	.99	.96	.91
20	.98	.97	.95	.90
25	.87	.87	.88	.89
30	.76	.76	.78	.80
35	.67	.67	.69	.71

FIRST NULL REDUCTION

D_{bl}/D

SLR	.1	.2	.3
17.6	1.2%	4.4%	8.6%
20	1.6	5.6	10.9
25	2.7	9.3	17.2
30	4.7	15.1	26.8
35	7.9	24.0	41.4



15
20
18
16
14
12
10
8
6
4
2
0
EDGE
TAPER,
DB
10
15
20
25
30
35
40
SIDELOBE RATIO, DB
APERTURE EDGE TAPER
VS SIDELOBE RATIO
FOR OPTIMUM CIRCULAR
A + (1-A) (1-ρ²)ⁿ

of the errors can be obtained by comparing these results with the Taylor circular source[1] with $\bar{n}$ equal level sidelobes.

SLR	$A + (1-A)(1-\rho^2)^n$ edge taper	Taylor edge taper
25 db	- 7.5 db	- 8.1 db
30	-11.0	-11.5
35	-14.4	-14.6

The $(1-\rho^2)^n$ values are used as it is expected to be closer to the $J_1(x)/x$ values.

Now through the previous curves, giving η_{beam} vs SLR, and this curve, the η_{beam} is related to edge taper, a convenient parameter for characterizing feed patterns.

Shaping of the sub-reflector and consequent phase correction of the main reflector is sometimes used to obtain higher efficiency.[2,3] However, this is accomplished by reducing spillover and making the illumination more nearly uniform. This increases the sidelobes and correspondingly does not result in a good beam efficiency. Thus shaping is not recommended for radiometer applications.

---

1. R. C. Rudduck, D. C. F. Wu, and R. F. Hyneman, "Directive Gain of Circular Taylor Patterns," Rad. Sci., Vol. 6, No. 12, Dec. 1971, p. 1117.
2. W. F. Williams, "High Efficiency Antenna Reflector," The Microwave Journal, July 1965, pp. 79-82.
3. V. Galindo, "Design of Dual-Reflector Antennas with Arbitrary Phase and Amplitude Distributions," IEEE Trans., Vol. AP-12, July 1964, pp. 403-408.

2.4 Reflector Back Lobes

Back lobes (90 to 180 deg.) of any reflector are largely controlled by the edge diffraction and by other diffracting structure such as support members. Here the edge illumination plays a key role in that a heavier taper yields lower back lobes. Typical back lobe power is obtained from radiation pattern envelopes; these are available[1] for both conventional focus fed parabolic antennas, and those with edge 'blinders'. The latter have a much lower back lobe level. An example is taken of a 12 ft. dish at 11.2 ghz, with efficiency of 52%. The measured gain of 49.8 db ($D/\lambda = 136.6$) gives a total integral of E^2 of:

$$\int_0^{\pi} E^2 \sin\theta \, d\theta = 2.09 \times 10^{-5}$$

Since the RPE consists of straight lines, an approximate integration from 90 to 180 deg. is easy. For the conventional and edge treated dishes the results are:

DISH	RPE no.		power in back lobes
ordinary	3111	$.38 \times 10^{-5}$	18%
edge shield	3177	1.02×10^{-7}	.5%

Although these are not Cassegrain, the results are expected to be the same. See RPE's in Appendix. Thus the conclusions are that a heavy edge taper, or an edge shield, is necessary to reduce back lobe power below 1%. Of course when the antenna is pointed away, at most only part of the back lobes see the hot earth so this mitigates the back

1. Andrew Corp. radiation pattern envelopes (RPE).

lobe problem. For the actual antenna design, a GTD* calculation of back lobe energy should be made.

---

* Geometric Theory of Diffraction.

2.5 Feed Horn Pattern and Spillover Efficiency

The ideal feed horn should have a circularly symmetric pattern to give equal E and H plane patterns and should have a parameter controlling beamwidth. A representative pattern[1] is

$$E_f = (1 + \cos\theta)\,\frac{J_1(x)}{x}\quad,\quad x = \frac{\pi D_f}{\lambda}\sin\theta$$

where the feed diameter D_f controls the beamwidth. The $(1 + \cos\theta)/2$ factor is the reflector path loss function; $2J_1(x)/x$ is unity at $x = 0$. This feed pattern will be used to calculate feed spillover and cross-pol losses. A convenient way of characterising the feed is by edge taper. Values of D_f and θ_f giving specific values of edge taper are obtained using a root finder to determine x (in the equation above) for a given θ_f and E_f; see appendix. The table gives D_f/λ for θ_f = 20 (5) 90 and for E_f = -10 (2) -18 db, which covers the practical range of edge tapers. Using this feed pattern to determine spillover efficiency (and cross-pol efficiency) vs edge taper (relatable to sidelobe ratio) and reflector angle θ_f again requires numerical integration:

$$\eta_{spill} = \frac{\displaystyle\int_0^{\theta_f} E^2 \sin\theta\, d\theta}{\displaystyle\int_0^{\pi/2} E^2 \sin\theta\, d\theta}$$

---

1. A. W. Rudge and M. Shirazi, "Investigation of Reflector-Antenna Radiation," Final Report on Contract No. SC/11/73/HQ, Univ. of Birmingham, June 1974.

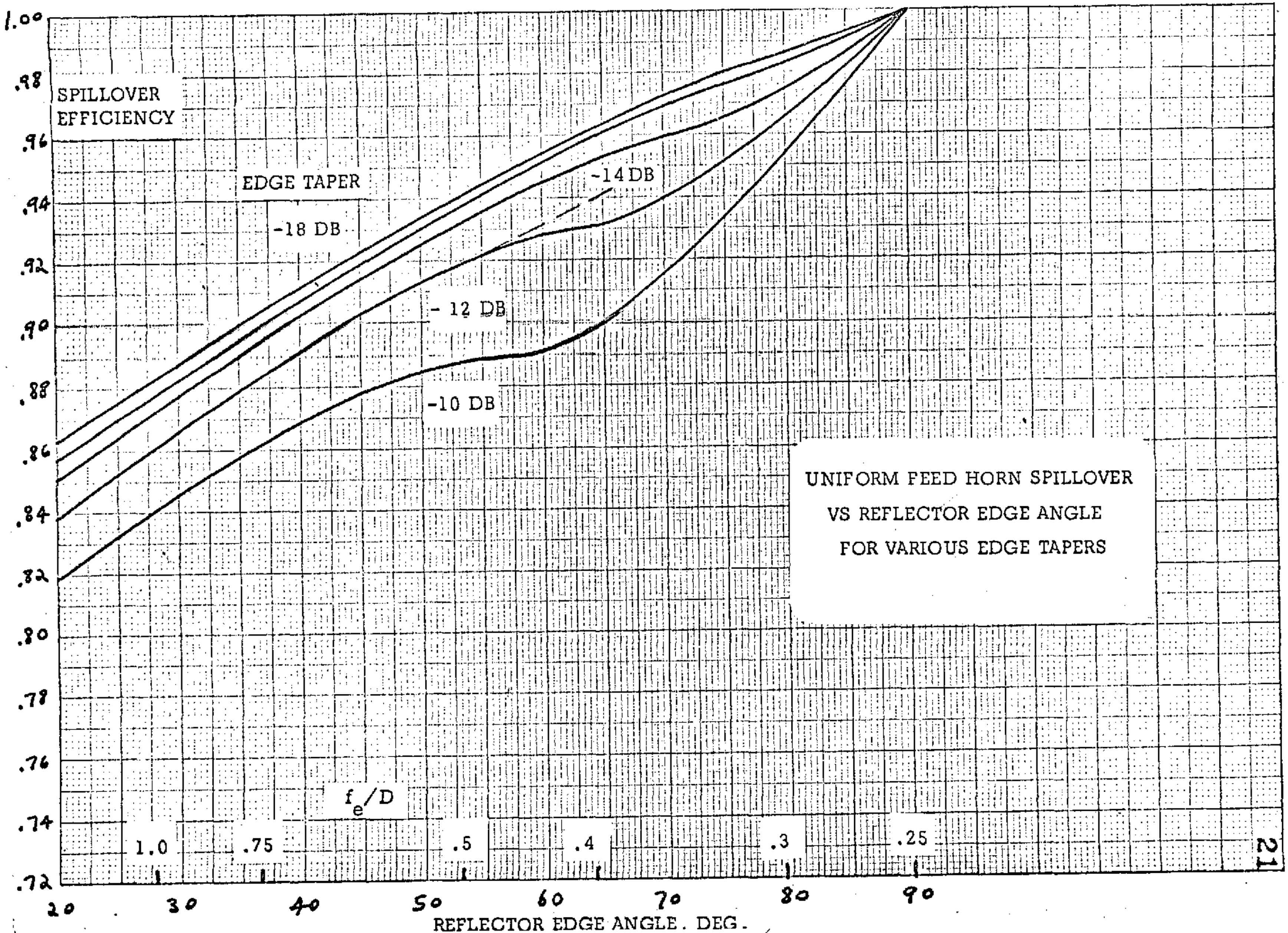
Feed energy past 90 deg. will be estimated separately, as the external structure strongly affects this part of the feed pattern. Now the results will contain oscillations; for a fixed dish edge angle θ_f as D_f changes the number and fraction of feed pattern sidelobes included changes, giving a fine structure which tends to obscure the results. Since feed sidelobe structures are highly variable, the effects of these particular ones can be eliminated by using the integrated sidelobe envelope.

The sidelobes cannot be integrated, even without the sin $\theta d\theta$ which makes the result dependent upon D. However, the power of a single sidelobe can be approximated by an envelope, provided sin $\theta \simeq$ constant. The single sidelobe power is approximated by the envelope power:

$$\int_{x_1}^{x_2} \left(\frac{2 J_1(x)}{x} \right)^2 d\psi = \alpha \int_{x_1}^{x_2} \frac{8 \, d\psi}{\pi x^3}$$

Numerical (Romberg) integration is used to determine the value of the constant α. From the table it appears that $\alpha \rightarrow .5$, and it is suspected that use of the asymptotic form of the Bessel function in the integral above (without sin θ) can be directly integrated to give $\alpha = .5$. Now that α is known, the envelope as a function of θ, together with sin θ, can be numerically integrated. This somewhat devious process allows more rapid numerical integration, and more important, will remove oscillations from the spillover data.

Using the feed horn pattern just described and the envelope thereof, the spillover is calculated at increments in reflector edge angle of 5% with the appropriate feed diameters used for each angle to give the edge taper values of -12 (to) -18 db. Romberg integration is again used to calculate the total energy and the energy impinging upon the reflector. The latter integration is divided into two parts where the feed pattern weighted sidelobe envelope meets the main beam. In the figure is shown the spillover efficiency vs reflector edge angle for the five different edge taper cases. The change


1.00
.98
.96
.94
.92
.90
.88
.86
.84
.82
.80
.78
.76
.74
.72
SPILLOVER EFFICIENCY
EDGE TAPER
-18 DB
-14 DB
- 12 DB
-10 DB
UNIFORM FEED HORN SPILLOVER VS REFLECTOR EDGE ANGLE FOR VARIOUS EDGE TAPERS
f_e/D
1.0
.75
.5
.4
.3
.25
20
30
40
50
60
70
80
90
REFLECTOR EDGE ANGLE. DEG.

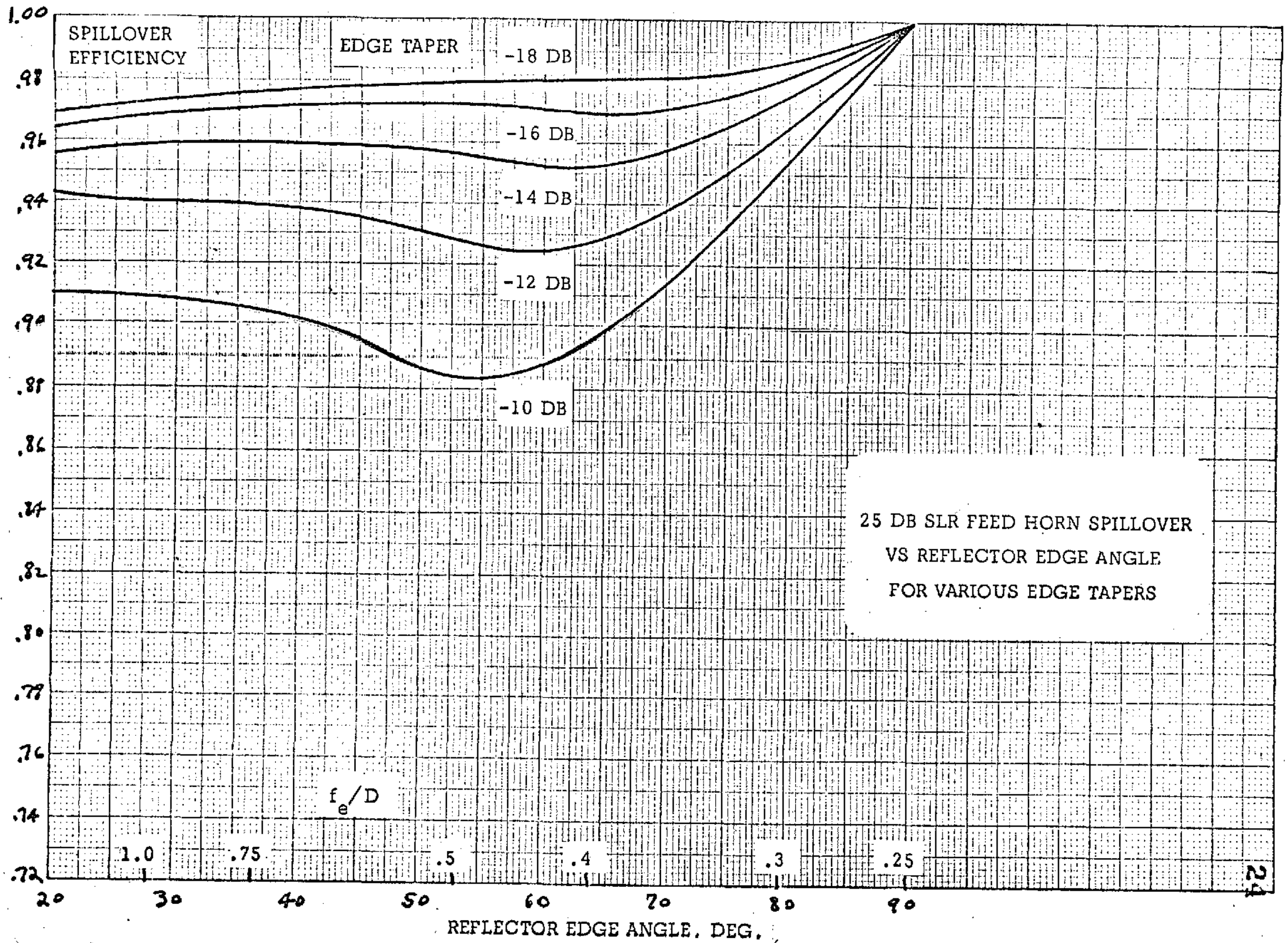
ENVELOPE COEFFICIENT

n	α	error
1	.48256	3.4 %
2	.49263	1.4
3	.49599	.74
4	.49749	.44
5	.49828	.28
6	.49875	.19
10	.49950	.04

in shape of the curves occurs when the feed pattern main beam to weighted envelope transition occurs at the edge of the dish. This occurs for u = .968; see code in Appendix. It should be recalled that actual feed patterns with discrete sidelobes will produce curves containing considerable fine structure. However, these envelope curves are more useful for design tradeoffs.

In a subsequent section, it appears that the achievement of 90% efficiency is limited by the feed spillover. Thus a feed with low sidelobes must be used. An approximate calculation of the performance of a typical feed of this type is obtained by using the $J_1(x)/x$ distribution for the feed horn. Here the constant H is chosen for 25 db feed SLR and for each reflector edge angle the feed diameter is chosen to provide the specified reflector edge taper as before. The new feed pattern envelope has a different intersection with the main beam, which is also recalculated. The only approximation occurs wherein the constant alfa which relates the power in a sidelobe to the integral of the envelope power is taken from the uniform feed case. For large feed horns, this represents a small error and in any case gives a rapid and useful answer. Again the spillover integrals are calculated with the new feed pattern and envelope and the results are shown in the figure. Since the sidelobe envelope now contributes a small amount of power, one might expect the spillover efficiency to be somewhat constant until the reflector edge angle is sufficiently large that the only remaining sidelobe starts disappearing. This is in fact the case for edge tapers of 15 db or more. However, for smaller edge tapers, the main beam power changes with reflector angle also, because of the square root argument that now appears in the $J_1(x)/x$. Thus the important conclusion that for this type of low sidelobe feed, and probably for all conventional low sidelobe feeds, edge tapers in the 15 db or more range are indicated. So called 'high efficiency' feeds, where a large feed aperture with many modes is used to provide nearly uniform reflector illumination with very low feed sidelobes, are not of interest here, as their beam efficiency is close to that of a uniformly illuminated aperture, i.e. 84% or worse, depending upon blockage.

The calculations for beam efficiency and spillover have included the feed pattern out to 90 deg. This is because the back lobes of a feed are not only dependent



1.00
.98
.96
.94
.92
.90
.88
.86
.84
.82
.80
.78
.76
.74
.72
SPILLOVER
EFFICIENCY
EDGE TAPER
-18 DB
-16 DB
-14 DB
-12 DB
-10 DB
25 DB SLR FEED HORN SPILLOVER
VS REFLECTOR EDGE ANGLE
FOR VARIOUS EDGE TAPERS
f_e/D
1.0
.75
.5
.4
.3
.25
20
30
40
50
60
70
80
90
REFLECTOR EDGE ANGLE, DEG.

upon the specific feed but also upon the physical structure including supports and all material in the vicinity of the feed. Thus calculation of feed back lobe is both difficult and somewhat irrelevant until a final feed and reflector geometry, including all supports, cables, dielectric members, etc., are fixed. A satisfactory procedure is to estimate the percentage of power in the back lobes of a reasonable feed. Using multimode or corrugated horn feeds, the back lobe energy can be expected to be of the order of .5%. However, dipole array feeds typically will have much higher back lobe energy due to diffraction around the ground plane edges. Feed cross polarized energy must be considered, of course, but has not been calculated here, as it is completely dependent upon the particular type of feed.

2.6 Cross Polarization Efficiency

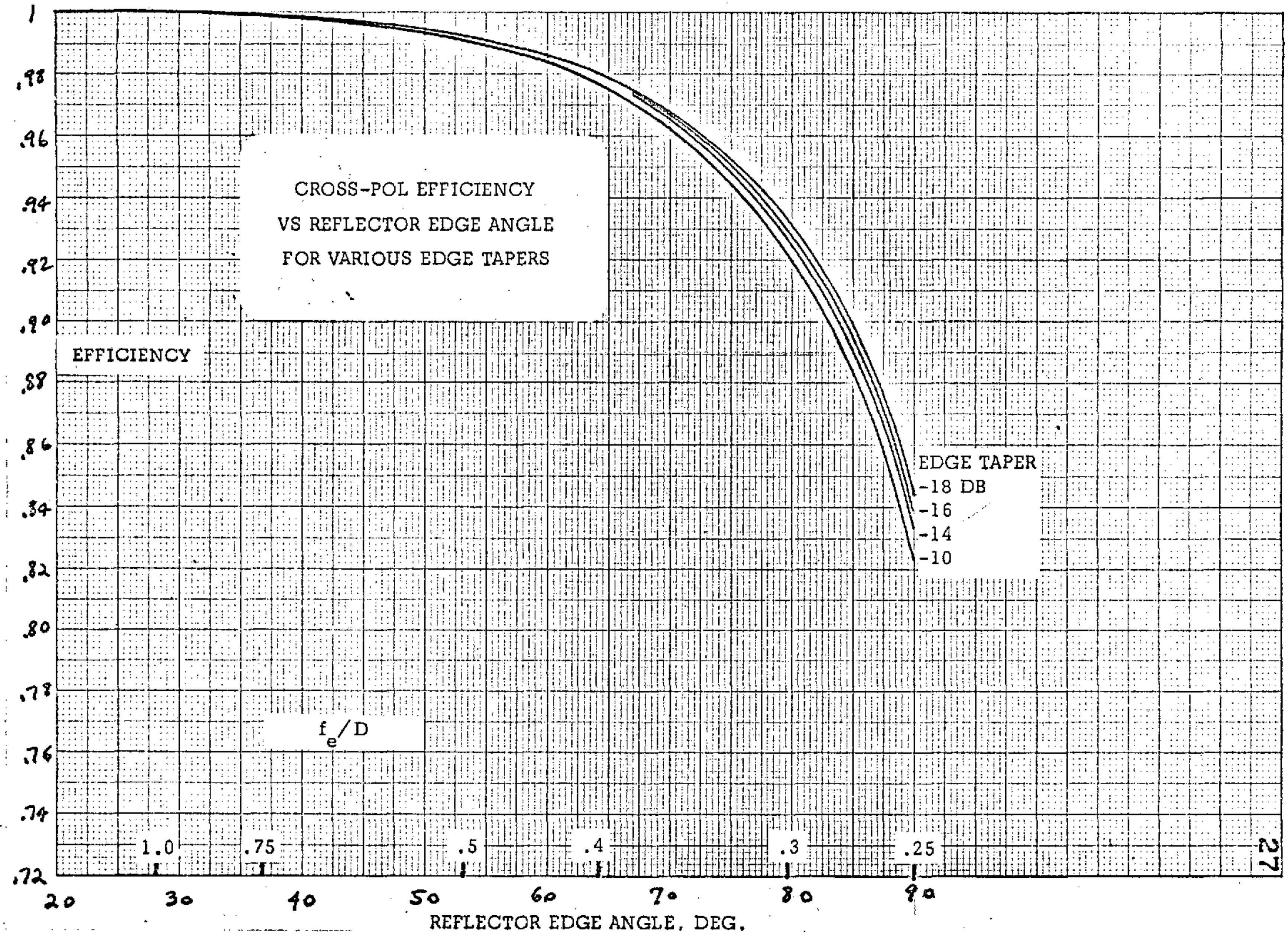
The definition used here for cross polarization power is that using spherical coordinates.[1] The integral is[2]

$$\eta_x = \frac{\left| \int_0^{\pi/2} \int_0^{\theta_f} E(\theta) \frac{\cos\theta \sin^2\varphi + \cos^2\varphi}{\sqrt{1 - \sin^2\theta \sin^2\varphi}} \tan\frac{\theta}{2} \, d\theta \, d\varphi \right|^2}{\left| \frac{\pi}{2} \int_0^{\theta_f} E(\theta) \tan\frac{\theta}{2} \, d\theta \right|^2}$$

Here the same feed pattern used for spillover calculations is used. Since reflector cross polarization is zero in the principal planes, the double integration is required; and this is performed with a double Simpson. The co-pol power is obtained with a Romberg integration. Since only the main beam of the feed pattern illuminates the reflector, the question of feed pattern sidelobe vs envelope is not germane. The appendix shows the computer code used for this calculation. The set of equivalent feed diameters and reflector edge angles corresponding to edge tapers of -10, -12, -14, -16, and -18 db were inserted into the program. The Figure shows the cross polarization efficiency as a function of feed edge angle for these several edge tapers. It is interesting to note that for an equivalent parabola f_e/D of .5, the loss is about 1%; whereas for f_e/D = .35, the loss is about 4%. These results compare favorably with those calculated for a variety of feed patterns by Rudge.[3,4,5]

---

1. A. C. Ludwig, "The Definition of Cross Polarization," Trans. IEEE, Vol. AP-21, January 1973, pp. 116-119.
2. R. E. Collin and F. J. Zucker, "Antenna Theory, Part II," McGraw-Hill, 1969, Chap. 17.
3. A. W. Rudge and M. Shirazi, "Investigation of Reflector-Antenna Radiation," Interim Report on Contract No. SC/11/73/HQ, Univ. of Birmingham, October 1973.
4. A. W. Rudge and M. Shirazi, "Investigation of Reflector-Antenna Radiation," Final Report on Contract No. SC/11/73/HQ, Univ. of Birmingham, June 1974.


CROSS-POL EFFICIENCY
VS REFLECTOR EDGE ANGLE
FOR VARIOUS EDGE TAPERS
EFFICIENCY
1
.98
.96
.94
.92
.90
.88
.86
.84
.82
.80
.78
.76
.74
.72
EDGE TAPER
-18 DB
-16
-14
-10
f_e/D
1.0
.75
.5
.4
.3
.25
20
30
40
50
60
70
80
90
REFLECTOR EDGE ANGLE, DEG.

2.7 Cassegrain Design Tradeoffs

Due to the multidimensionality of the design problem, it is not feasible to have a single tradeoff expression or curve. Rather a beam efficiency budget is constructed from the several components and the parameters are varied until the beam efficiency reaches (or does not reach) a satisfactory level. Note that there is no mention of optimizing beam efficiency. It is probably more effective from a systems standpoint to maximize a combination of gain and inverse beamwidth while maintaining a satisfactory beam efficiency.

The major constituents to beam efficiency as previously discussed are:

aperture taper
blockage
feed spillover
polarization loss

To start, a combined reflector back lobe power, feed back lobe power, and cross polarization power of 1% is assumed. Meeting this stringent requirement involves either a low edge taper or edge treatment for both the reflector and the horn and careful attention to polarization purity in the horn. A cut and try process using the curves previously presented is now performed for several edge tapers and for several blockage diameter ratios. The results are quite pessimistic and show that there is a very narrow range of edge angles that gives an overall 90% beam efficiency. For example, the following table shows taper, spillover, and cross-pol values for a blockage ratio of .1 for the three tapers shown.

Blockage Diameter Ratio = .1

edge taper	taper efficiency	spillover efficiency	cross-pol efficiency	edge angle
-16 db	.993	.920	.996	46 deg
-14	.989	.925	.995	50
-12	.988	.930	.990	59

With a blockage ratio of .15, a solution is just possible for a -16 db, 50 deg. edge and for a -14 db, 56 deg. edge. For a blockage ratio of .2, the -16 db, 54 deg. edge is just possible. Since none of the calculations herein have included reflector and sub-reflector edge diffraction, there does not seem to be a feasible Cassegrain solution using the uniform feed model.

As mentioned before, the low sidelobe feed considerably improves the situation. Because of the considerably better spillover efficiencies, a range of angles is now feasible. In each case, the design is possible down to the lowest edge angle calculated, 20 deg. The maximum edge angle is shown in the following table. Because of this latitude, an intermediate value can be chosen to allow for the unincluded diffraction effects. Also the heavier edge tapers will give less power in the back lobes. The correspondingly large feeds, however, may interfere with each other since feeds are needed for several frequencies, and in all cases the feed should not be bigger than the equal blockage feed size.

Maximum Edge Angle

	Blockage Diameter Ratio		
Edge Taper	.1	.15	.2
-16 db	77 deg.	76	73
-14	72	66	61
-12	53	46	~ 30

Thus for modest blockage (.1) or less, there is a range of edge angles of from 20 deg. (or even less) to about 60 deg., with edge taper in the -14 to -16 range. To allow the smallest diameter feed, the -14 db number is chosen here.

Next the Cassegrain parameters are examined to see if a practical design will fit the restrictions. The feed diameter (including all feeds) should be no larger than the 'minimum blockage' criterion[1] wherein the feed blockage equals the sub-reflector blockage. Calling the feed diameter D_f, the minimum blockage value is

$$\frac{D_f}{D_s} = \frac{(f_1+f_2)/f}{\frac{1}{2}+\frac{1}{2}\sqrt{1-D_s^2/f^2}} \simeq \frac{f_1+f_2}{f}$$

The frequencies of primary concern are:

channel 5	freq. =	20.5 ghz	D/λ = 273
6		22.2	296
7		31.4	419
8		52.9	705
9		94	1253
10		116	1547

Typically channels 5 and 6 will have a single horn, with the channels 7 and 8 horns adjacent. The last two will be either coaxial with a lower frequency horn or located among the others.* So the net feed diameter will be roughly 1.5 times that of channel 1. Taking the -14 db edge taper, and edge angles of 20 and 60 deg. as design extremes gives:

θ_f	D_f/λ ch. 5	D_f/D ch. 5	D_f/D total	f_e/D
20	3.35	.01226	.0184	1.418
60	1.24	.00454	.00681	.433

1. P. W. Hannan, op cit

* This would minimize the offset (in wavelengths) for any one feed.

To assist in determining the blockage diameter ratio and $(f_1 + f_2)/f$, a table of parameters is given in the Appendix. The parameter $(f_1 + f_2)/f$ is important as it determines where the feed phase center is located between the main reflector vertex and the sub-reflector. If $(f_1 + f_2)/f = 1$, the feed center is at the main reflector vertex. The feed diameter is readily obtained by multiplying together the first and last columns:

$$\frac{D_f}{D} = \frac{D_s}{D} \cdot \frac{f_1 + f_2}{f}$$

Several examples will illustrate the possible sets of parameters:

Case 1

Take the largest edge angle, $\theta_f \simeq 60$ deg. or $\frac{f_e}{D} = .45$.

For this case, small magnification is needed, say M = 1.5. Then f/D = .3

$$\frac{D_f}{D} \simeq .007$$

Values that provide feed blockage $\leq$ sub-reflector blockage are:

$\frac{D_s}{D}$	$\frac{f_1 + f_2}{f}$	minimum blockage D_f/D
.075	.100	.007
.1	.134	.0134
.15	.201	.0302

In this case, the main reflector f/D = .3 but the feed point is located close to the sub-reflector.

Case 2

Take the smallest edge angle, $\theta_f \simeq 20$ deg., $\frac{f_e}{D} = 1.5$.

Actually smaller angles can be used but require even larger feed diameters. Take M = 5, so f/D = .3.

$$\frac{D_f}{D} \simeq .02$$

Acceptable values are:

$\frac{D_s}{D}$	$\frac{f_1 + f_2}{f}$	minimum blockage D_f/D
.065	.335	.02
.1	.517	.05
.15	.775	.12
.2	1.033	.21

A blockage of .1 and feed point .517f is a reasonable design.

Case 3

Again the small edge angle, $f_e/D = 1.5$ but M = 3. f/D = .5. $\frac{D_f}{D} \simeq .02$

Acceptable values are:

$\frac{D_s}{D}$	$\frac{f_1 + f_2}{f}$	minimum blockage D_f/D
.075	.275	.02
.1	.367	.014
.15	.550	.08
.2	.733	.15

Here are also reasonable designs, with the main reflector f/D larger than Case 2 but with a feed center closer to the sub-reflector for a given blockage ratio. For example, a blockage of .15 and feed point .550 f is practical.

These three cases are now compared in terms of beam efficiency, using in each case the sub-reflector diameter representing minimum blockage even though the feed distance is small. The following table gives the pertinent parameters.

	Case 1	Case 2	Case 3
f_e/D	.45	1.5	1.5
M	1.5	5	3
f/D	.3	.3	.5
D_s/D	.075	.065	.075
$(f_1 + f_2)/f$	.100	.335	.275
θ_e	58 deg.	19 deg.	19 deg.
η taper	.993	.994	.993
η spill	.953	.955	.955
η xρ	.987	1.000	1.000
η beam	.925	.940	.939

edge taper = - 14 db

dish and feed back lobes and feed cross-pol = 1%

It can be seen that the performance with large feed (f_e/D large) is somewhat better, and the feed distance is also more manageable. The tradeoff between main reflector f/D appears to be more nearly that of space and construction.

If the band 5-6, band 7 and band 8 feeds are clustered in a triangular configuration, with the focus of the reflector located just outside each feed horn edge (between the 3 horsn), the offset of each beam varies depending on the f/D, D_s/D, etc. In all three cases, the offset is 4 beamwidths or less, and this allows good sidelobe control. The Case 3 with f/D = .5 is better, and Case 1 with low magnification is better yet. However, all are acceptable.

3.0 OFFSET PARABOLOID ANTENNAS

3.1 Polarization Loss

The offset reflector can have negligible polarization loss if the feed axis is maintained parallel to the paraboloid axis. But of course the feed must be tipped, with the feed angle approximately half the edge angle, to reduce spillover and to provide the proper aperture taper. Polarization loss has been calculated for offset reflectors by Dijk et al.[1] For an edge angle of 60 deg. (f/D = .433), the cross-pol efficiency is .98 for polarization parallel to the offset direction and .97 for polarization perpendicular. These numbers compare with a cross-pol efficiency of 1.00 for the symmetric Cassegrain cases. Thus for the offset reflector to be a candidate, the spillover and taper efficiencies together must be higher than for the Cassegrain.

---

1. J. Dijk, et al, "The Polarization Losses of Offset Paraboloid Antennas," IEEE Trans., Vol. AP-22, July 1974.

3.2 Asymmetric Amplitude Taper

The offset reflector, of course, has no blockage; for the Cassegrain cases considered, the blockage accounts for less than 1% degradation of beam efficiency. On the other hand, the offset reflector has a serious difficulty, the asymmetric amplitude distribution produced by the geometry. The limited scope of this investigation did not permit the extensive two-dimensional numerical integration analysis necessary to quantify this tradeoff, so a simple calculation is made to indicate the magnitude of the problem. A plane through the reflector and feed center in the offset direction is taken as a one-dimensional aperture. Over this aperture should be heavy edge taper (at least 14 db) to reduce edge diffraction, sidelobes, etc., and a cosine function is used to simulate this. However, in addition there is the asymmetric geometry taper which has maximum amplitude at the edge of the reflector closest to the reflector axis and minimum amplitude at the top edge. For example, with f/D = .25, the amplitude ratio is 2:1. Larger f/D's give a ratio closer to 1. The worst case is taken, f/D = .25. The basic integral is

$$F(u) = \frac{3}{5}\int_{-\pi/2}^{\pi/2} \cos^2\frac{\pi-2p}{8}\cos p \; e^{j2pu}\, dp$$

where $u = (D/\lambda)\sin\theta$, and p is the aperture variable. The symmetric taper is given by $\cos p$ and the path length asymmetric taper by $\cos^2(\pi - 2p)/8$. The result is

$$F(u) = \frac{.6\cos\pi u}{1-4u^2} + \frac{1.2(\sin\pi u + \cos\pi u)}{3+8u-16u^2}$$

The normal cosine taper pattern is

$$F_o(u) = \frac{\cos \pi u}{1 - 4u^2}$$

Calculation of the two patterns shows that the asymmetric path loss leaves the sidelobe peaks essentially unchanged, broadens the null-to-null beamwidth by about 8%, and fills in all nulls to roughly 10 to 20 db below the sidelobe peaks. None of these effects are serious, and for larger f/D will be even less.[1]

With the higher polarization loss, the offset reflector appears to offer lower beam efficiency than the symmetric Cassegrain; but the 90% figure for the offset reflector seems possible.

---

1. L. S. Wagner and K. W. Morin, "Performance of a Parabolic Antenna with an Offset Feed," memo report 2375, Dec. 1971, NRL.

APPENDIX

Preceding page blank

/J1BEAM/

```
C       MODIFIED J1(X)/X BEAM EFFICIENCY & GAIN
C       WITH SYMMETRIC CIRCULAR HOLE BLOCKAGE
C       H IS MOD J1 PARAMETER, U0 IS FIRST NULL
        COMMON H2,P,D,DBL,E
        COMMON /W/ PI,HH,EB,B
        P=PI=3.1415926536
        ZN=3.83171
        DISPLAY /
        ACCEPT "M,H,XS    ",M,H,XS
        H2=HH=H*H
C       CALC BEAM PEAK
        CALL BESSEL(P*H,1,BFI0,IF,3)
        E=2*BFI0/(P*H)
        IF (P*H.LT..001) E=1
        E2=E*E
100     ACCEPT "D/WV,BLOCK DIA/D    ",D,B
C       CALC BLOCKED NULL POSIION
        EB=E*B*B
        CALL ROOTW(U0B,F0,XS,1.E-5,50,IFL)
        X0=P*SQRT(U0B*U0B-H2)
        DISPLAY X0,F0,IFL
        TH0B=ARCSIN(U0B/D)
        DISPLAY "% NULL SHIFT    ",#
        WRITE (1,110) 100*ABS(X0-ZN)/ZN
 10     FORMAT(F8.2)
        DBL=D*B
C       CALC WITH BLOCKED FEED
        CALL ROMBER(0.,TH0B,M,Y1)
        CALL ROMBER(TH0B,.5*P,M,Y2)
        W=2*E2/(P*D)**2
        ETAT=W*(1-B*B)**2/(Y1+Y2)
        ETAB=Y1/(Y1+Y2)
        WRITE (1,200) ETAT,ETAB,/
200     FORMAT (2F8.3)
        GO TO 100
        END
        FUNCTION FCT(X)
        COMMON HF,PI,DF,BF,E
        S=SIN(X)
        U2=(DF*S)**2
        IF (U2.LT.HF) GO TO 40
        ARG=PI*SQRT(U2-HF)
        IF (ARG.LT..001) GO TO 30
        CALL BESSEL(ARG,1,BFJ,IF,1)
        FCT=2*BFJ/ARG
        GO TO 50
30      FCT=1
        GO TO 50
40      ARG=PI*SQRT(HF-U2)
```

/J1BEAM/

```
      IF (ARG.LT..001) GO TO 70
      CALL BESSEL(ARG,1,BFI,IF,3)
      FCT=2*BFI/ARG
      GO TO 50
70    FCT=E
50    AR=PI*S*BF
      IF (AR.LT..001) GO TO 10
      CALL BESSEL(AR,1,BFJ0,IF,1)
      FCT0=2*BFJ0/AR
      GO TO 20
10    FCT0=1
20    FCT0=FCT0*E*(BF/DF)**2
      FCT=S*(FCT-FCT0)**2
      RETURN
      END
      FUNCTION WFUN(X)
      COMMON /W/ PI,HH,A,B
      ARG=PI*SQRT(X*X-HH)
      CALL BESSEL(ARG,1,BFJM,K,1)
      AR=PI*B*X
      CALL BESSEL(AR,1,BFJB,L,1)
      WFUN=BFJM/ARG-A*BFJB/AR+X
      RETURN
      END
```

42

RADIATION PATTERN ENVELOPE

ANTENNA TYPE NUMBER P12-107C, P12-107D
PL12-107C, PL12-107D

ANDREW

RPE 3111
S. A. Walker
Approved
August 4, 1971

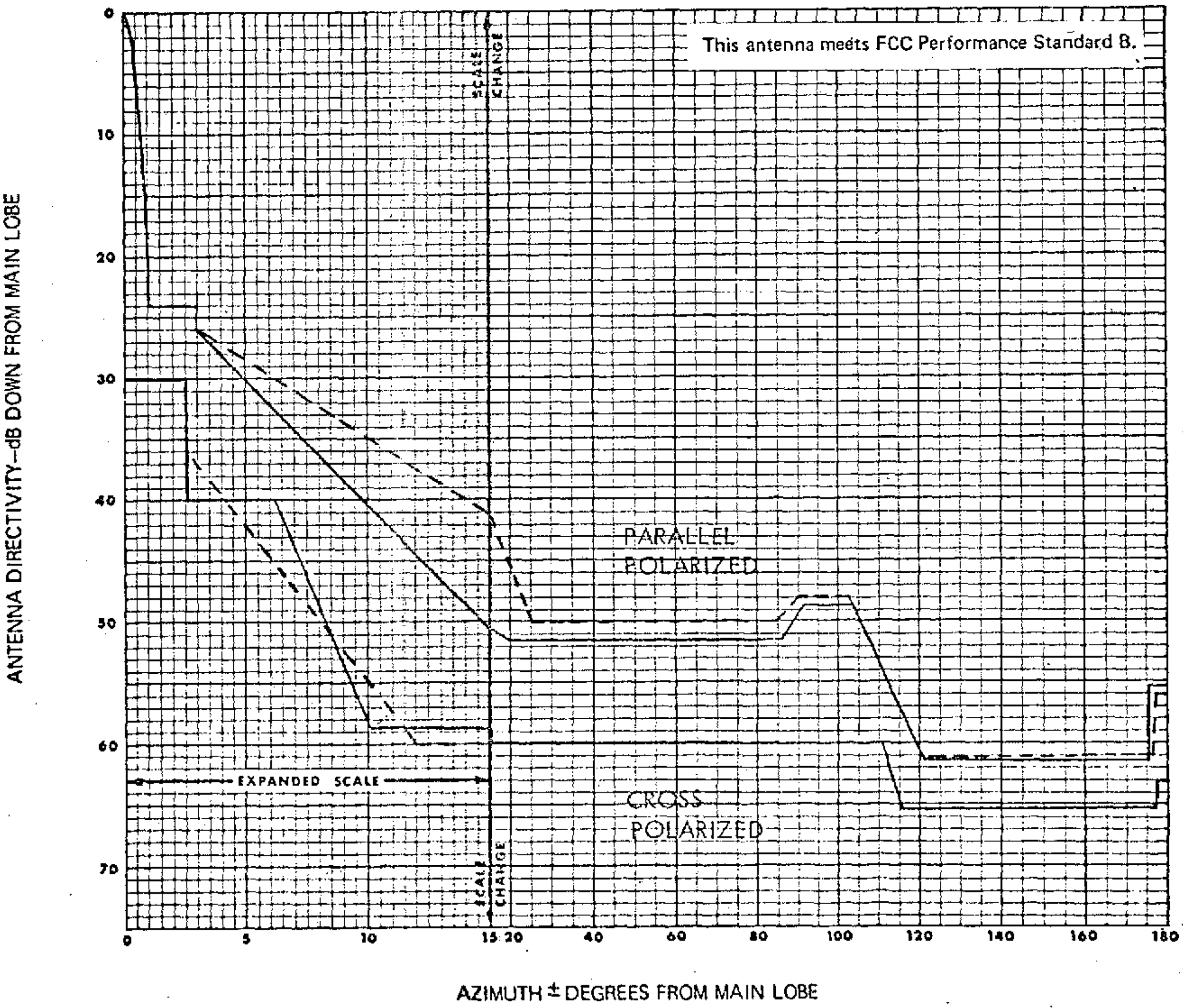
12 FOOT ANTENNA
10.7 - 11.7 GHz
PLANE POLARIZED

——— Envelope for a Horizontally Polarized Antenna
- - - - Envelope for a Vertically Polarized Antenna

Gain: 49.8 ± 0.2 dBi at 11.2 GHz

For reference to a half wave dipole subtract 2.15 dB.

See Andrew Bulletin 1032, "Radiation Pattern Envelopes," for further information.



Andrew Corporation
10500 W. 153rd Street
Orland Park, Ill., U.S.A. 60462

Andrew Antenna Company Ltd.
606 Beech Street
Whitby, Ontario, Canada

Andrew Antenna Systems
Lochgelly, Fife
Great Britain

Andrew Antennas
171 Henty Street
Reservoir, Victoria, Australia 3073

RADIATION PATTERN ENVELOPE

ANTENNA TYPE NUMBER HP12-107D

ANDREW

43

RPE 3177

Approved

November 17, 1971

12 FOOT ANTENNA

10.7 - 11.7 GHz

PLANE POLARIZED

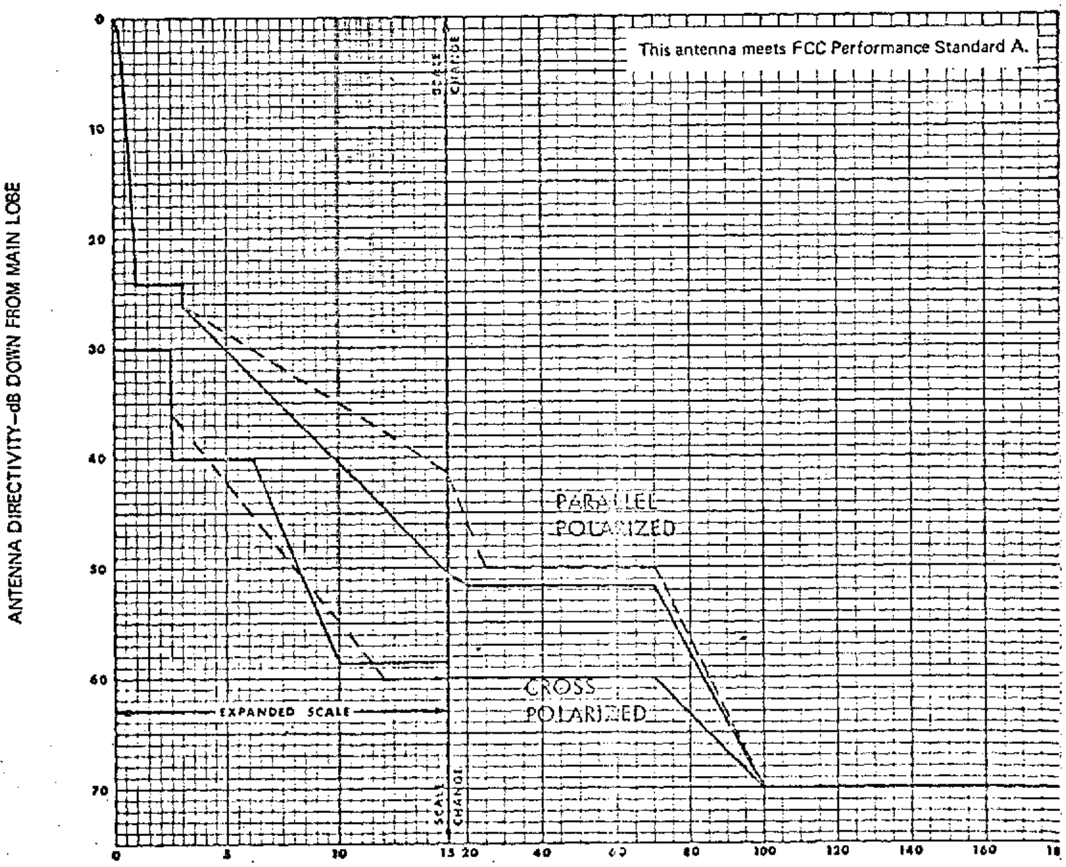
——————— Envelope for a Horizontally Polarized Antenna

– – – – – – – Envelope for a Vertically Polarized Antenna

Gain: 49.8 ± 0.2 dBi at 11.2 GHz

For reference to a half wave dipole subtract 2.15 dB.

See Andrew Bulletin 1032, "Radiation Pattern Envelopes," for further information.



Andrew Corporation
10500 W. 153rd Street
Orland Park, Ill., U.S.A. 60462

Andrew Antenna Company Ltd.
606 Beech Street
Whitby, Ontario, Canada

Andrew Antenna Systems
Lochgelly, Fife
Great Britain

Andrew Antennas
171 Henty Street
Reservoir, Victoria, Australia 3073

```
OLD FILE?
12 LINES
*XTRAN
+COM
OUTPUT:
OPTIONS: PRO

+R
OPTIONS:
SPROG:
XLIBE   JAN 25


X1,X2    19.61586,22.76008

     4.230522494E-04

X1,X2    22.76008,25.90367

     2.789364966E-04

X1,X2    25.90367,29.04683

     1.935447211E-04

X1,X2    29.04683,32.18968

     1.397537889E-04

X1,X2    32.18968,35.33231

     1.041928841E-04

X1,X2    --

ESC:    ($MAIN$)0
+Q
*P
C       BESSEL J1(X)/X SIDELOBE INTEGRATION
        DISPLAY
100     ACCEPT "X1,X2    ",X1,X2
        CALL ROMBER(X1,X2,6,R)
        DISPLAY /,R,/
        GO TO 100
        END
        FUNCTION FCT(X)
        CALL BESSEL(X,1,BF,I,1)
        FCT=(2*BF/X)**2
        RETURN
        END
*
```

44

```
-COP /BES/ TO TEL



C       BESSEL J1(X)/X SIDELOBE INTEGRATION
C       ENVELOPE
        COMMON P
        DISPLAY
        P=3.1415826536
100     ACCEPT "N    ",N
        CALL ROMBER((N+.25)*P,(N+1.25)*P,5,R)
        DISPLAY /,R,/
        GO TO 100
        END
        FUNCTION FCT(X)
        COMMON P
        FCT=8/(P*X**3)
        RETURN
        END
```

-COP /EDGE/ TO TPT

```
C       EDGE TAPER VS DIAM OF J1(X)/X FEED
C       USES WEGSTEIN ROOT FINDER
        DIMENSION D(5)
        COMMON C,E,H,P
        DISPLAY /
        P=3.1415926536
        H=.8899
        XS=1
        DO 100 TH=20,90,5
        THR=TH*P/180
        S=SIN(THR)
        C=COS(THR)
        J=0
        DO 200 EDB=2.58,10.58,2
        J=J+1
        E=10**(-.05*EDB)
        CALL ROOTW(X0,F0,XS,1E-6,50,IFL)
        IF (IFL.EQ.0.AND.F0.LT.1E-05) GO TO 200
        DISPLAY "ERROR FLAG & RESIDUE    ",IFL,F0
200     D(J)=X0/S
100     WRITE (1,110) TH,(D(I),I=1,5)
110     FORMAT (F6.0,5F8.3)
        DISPLAY /
        END
        FUNCTION WFUN(X)
        COMMON C,E,H,P
        AR=P*SQRT(ABS(X*X-H*H))
        IF (X.LT.H) GO TO 10
        CALL BESSEL(AR,1,BF,I,1)
        WF=(1+C)*BF/AR
        GO TO 20
10      CALL BESSEL(AR,1,BF,I,3)
        WF=(1+C)*BF/AR
20      WFUN=WF-E+X
        RETURN
        END
```

25 sh SCR

```
*WRI /EDGE/
OLD FILE?
36 LINES
*XTRAN
+COM
OUTPUT:
 PTIONS: PRO

+R
OPTIONS:
SPROG:
XLIBE  JAN 25
```

aperture diameters

25 db SLR

θ_F	-10	-12	-14	-16	-18 db edge
20.	2.921	3.151	3.346	3.514	3.659
25.	2.349	2.537	2.697	2.835	2.953
30.	1.969	2.131	2.268	2.386	2.487
35.	1.700	1.843	1.965	2.069	2.159
40.	1.499	1.629	1.740	1.835	1.917
45.	1.343	1.465	1.568	1.656	1.732
50.	1.219	1.335	1.432	1.516	1.588
55.	1.117	1.229	1.324	1.404	1.473
60.	1.032	1.142	1.235	1.313	1.381
65.	0.960	1.069	1.161	1.239	1.305
70.	0.895	1.006	1.098	1.177	1.244
75.	0.837	0.951	1.045	1.125	1.193
80.	0.782	0.901	0.999	1.081	1.151
85.	0.728	0.854	0.957	1.043	1.116
90.	0.671	0.809	0.918	1.009	1.086

```
STOP*
($MAIN$)0
+
```

```
COP /SPILL/ TO TPT

C       REFLECTOR SPILLOVER USING A SYMMETRIC J1(PU)/PU FEED
C       FEED PATTERN PAST 9Ø DEG NOT INCLUDED
C       FEED SIDELOBE ENVELOPE = MAIN LOBE AT Z
C       INSERT D(IAMETER) VECTOR FOR DESIRED EDGE TAPER
        DIMENSION D(15)
        COMMON P,DF,Z
        P=3.1415926536
        Z=3.Ø4126
        TS=2Ø
        TD=5
        I=Ø
        M=4
        DISPLAY /
2ØØ     DATA D/2.516,2.Ø24,1.697,1.466,1.293,1.16Ø,1.Ø54,
          .967,.894,.832,.777,.728,.681,.634,.586/
        DO 1ØØ TH=TS,9Ø,TD
        I=I+1
        DF=D(I)
        THR=TH*P/18Ø
        ARG=P*DF*SIN(THR)
        CALL BESSEL(ARG,1,BFJ,IF,1)
        CALL ROMBER(Ø.,THR,M,R1)
        CALL ROMBER(THR,.5*P,M,R2)
        ETAS=R1/(R1+R2)
        FOD=.25*COS(.5*THR)/SIN(.5*THR)
        WRITE (1,3ØØ) TH,FOD,ETAS
1ØØ     CONTINUE
3ØØ     FORMAT (F7.1,2F8.3)
        END
        SUBROUTINE FCT(X)
        COMMON P,F,Z
        S=SIN(X)
        AR=P*F*S
        IF (AR.GT.Z) GO TO 3Ø
        IF (AR.LT..ØØ1) GO TO 1Ø
        CALL BESSEL(AR,1,BF,I,1)
        FCTØ=(1+COS(X))*BF/AR
        GO TO 4Ø
1Ø      FCTØ=1
        GO TO 4Ø
3Ø      FCTØ=(1+COS(X))/(AR*SQRT(P*AR))
 Ø      FCT=S*FCTØ*FCTØ
        RETURN
        END
```

```
*WRI /INTEH/
NEW FILE?
16 LINES
*XTRAN
+TEST


START VAL   3
    3.Ø41263871   -1.Ø18634Ø66E-1Ø
```

$\alpha = \frac{1}{2}$ (power)

```
START VAL   2.5
    3.Ø41263869   -5.966285244E-1Ø


START VAL   4
    3.Ø41263871    Ø
```

for $\frac{2J_1(X)}{X} = \frac{2}{X\sqrt{\pi X}}$ this is

-13.44 db pt.

```
START VAL
ESC:  ($MAIN$)2ØØ+1
+COM
OUTPUT:
OPTIONS: SYM
CORRECTED SYMBOLIC:/INTER/
OLD FILE?

-COP /INTER/ TO TEL


C       INTERSECTION OF J1(X)/X & WEIGHTED ENVELOPE
C       USES WEGSTEIN ROOT FINDER
2ØØ     DISPLAY /
        ACCEPT "START VAL    ",XS
        CALL ROOTW(XØ,FØ,XS,1.E-5,5Ø,IFL)
        IF (IFL.EQ.Ø) GO TO 1ØØ
        DISPLAY "ERROR FLAG    ",IFL
1ØØ     DISPLAY XØ,FØ,/
        GO TO 2ØØ
        END
        FUNCTION WFUN(X)
        P=3.1415926536
        CALL BESSEL(X,1,BFJ,IF,1)
        WFUN=BFJ-1/SQRT(P*X)+X
        RETURN
        END
```

-COP /POL/ TO TPT

```
C       REFLECTOR POLARIZATION LOSS
C       J1(PU)/PU FEED PATTERN
C       USES DOUBLE SIMPSON AND ROMBERG INTEGRATIONS
C       INSERT D(IAMETER) VECTOR FOR DESIRED EDGE TAPER
        DIMENSION D(8)
        COMMON W
        DISPLAY /
200     DATA D/2.516,1.697,1.293,1.054,.894,.777,.681,.586/
        P=3.1415926536
        A=B=0
        N=6
        M=2
        MS=8
        L=3
        I=0
        TS=20
        TD=10
        TSR=TS*P/180
        TDR=TD*P/180
        U=0
        V=TSR
        DO 100 TH=TS,90,TD
        I=I+1
        W=P*D(I)
        THR=TH*P/180
        MM=M
        IF (TH.EQ.TS) MM=MS
        CALL SIMPDB(0.,.5*P,N,U,V,MM,AA)
        A=A+AA
        CALL ROMB(U,V,L,BB)
        U=U+TDR
        IF (TH.EQ.TS) U=TSR
        V=V+TDR
        B=B+.5*P*BB
        ETA=(A/B)**2
        DB=10*ALOG10(ETA)
        FOD=.25*COS(.5*THR)/SIN(.5*THR)
100     WRITE (1,105) TH,FOD,ETA,DB
        DISPLAY /
105     FORMAT (F6.0,3F7.3)
        END
        FUNCTION FCT(X,Y)
        COMMON W
        AR=W*SIN(Y)
        IF (AR.LT..001) GO TO 30
        CALL BESSEL(AR,1,BF,IF,1)
        F=2*BF/AR
        GO TO 40
30      AR=1
```

```
-COP /CASS/ TO TPT


C       CASSEGRAIN PARAMETERS
        DISPLAY /
100     ACCEPT "MAG     ",XM
        DO 110 FD=.25,.5,.05
        DISPLAY
        FE=XM*FD
        A=16*XM*FD*FD
        DSD0=A/((XM+1)*(A-1))
        DISPLAY "FOD,FED,DSD0     ",FD,FE,DSD0,/
        DO 110 DSD=.05,.3,.05
        F2=DSD*(A-1)/A
        F1=XM*F2
        FP=F1+F2
110     WRITE (1,150) DSD,F1,F2,FP
150     FORMAT (4F8.3)
        DISPLAY /,/
        GO TO 100
        END

-
```

9

Ds/D for f1+f2=F

MAG 1 F/D fe/D

FOD,FED 0.25 0.25 ∞

Ds/D	f1/f	f2/f	(f1+f2)/F
0.050	0.000	0.000	0.000
0.100	0.000	0.000	0.000
0.150	0.000	0.000	0.000
0.200	0.000	0.000	0.000
0.250	0.000	0.000	0.000
0.300	0.000	0.000	0.000

FOD,FED 0.3 0.3 1.636

0.050	0.015	0.015	0.031
0.100	0.031	0.031	0.061
0.150	0.046	0.046	0.092
0.200	0.061	0.061	0.122
0.250	0.076	0.076	0.153
0.300	0.092	0.092	0.183

FOD,FED 0.35 0.35 1.021

0.050	0.024	0.024	0.049
0.100	0.049	0.049	0.098
0.150	0.073	0.073	0.147
0.200	0.098	0.098	0.196
0.250	0.122	0.122	0.245
0.300	0.147	0.147	0.294

FOD,FED 0.4 0.4 .8205

0.050	0.030	0.030	0.061
0.100	0.061	0.061	0.122
0.150	0.091	0.091	0.183
0.200	0.122	0.122	0.244
0.250	0.152	0.152	0.305
0.300	0.183	0.183	0.366

FOD,FED 0.45 0.45 .7232

0.050	0.035	0.035	0.069
0.100	0.069	0.069	0.138
0.150	0.104	0.104	0.207
0.200	0.138	0.138	0.277
0.250	0.173	0.173	0.346
0.300	0.207	0.207	0.415

FOD,FED 0.5 0.5 .6667

0.050	0.037	0.037	0.075
0.100	0.075	0.075	0.150
0.150	0.113	0.113	0.225
0.200	0.150	0.150	0.300
0.250	0.187	0.187	0.375
0.300	0.225	0.225	0.450

MAG ←

*WRI/CASS/
Old File?
16 Lines
*Xtran
+Test

MAG 1.5 f/D fe/D

Ds/D for $(f_1+f_2)=f$

FOD,FED 0.25 0.375 1.200

Ds/D	f_1/f	f_2/f	$(f_1+f_2)/f$
0.050	0.025	0.017	0.042
0.100	0.050	0.033	0.083
0.150	0.075	0.050	0.125
0.200	0.100	0.067	0.167
0.250	0.125	0.083	0.208
0.300	0.150	0.100	0.250

FOD,FED 0.3 0.45 .7448

0.050	0.040	0.027	0.067
0.100	0.081	0.054	0.134
0.150	0.121	0.081	0.201
0.200	0.161	0.107	0.269
0.250	0.201	0.134	0.336
0.300	0.242	0.161	0.403

FOD,FED 0.35 0.525 .6062

0.050	0.049	0.033	0.082
0.100	0.099	0.066	0.165
0.150	0.148	0.099	0.247
0.200	0.198	0.132	0.330
0.250	0.247	0.165	0.412
0.300	0.297	0.198	0.495

FOD,FED 0.4 0.6 .5408

0.050	0.055	0.037	0.092
0.100	0.111	0.074	0.185
0.150	0.166	0.111	0.277
0.200	0.222	0.148	0.370
0.250	0.277	0.185	0.462
0.300	0.333	0.222	0.555

FOD,FED 0.45 0.675 .5036

0.050	0.060	0.040	0.099
0.100	0.119	0.079	0.199
0.150	0.179	0.119	0.298
0.200	0.238	0.159	0.397
0.250	0.298	0.199	0.496
0.300	0.357	0.238	0.596

FOD,FED 0.5 0.75 .4800

0.050	0.062	0.042	0.104
0.100	0.125	0.083	0.208
0.150	0.187	0.125	0.312
0.200	0.250	0.167	0.417
0.250	0.312	0.208	0.521
0.300	0.375	0.250	0.625

MAG —

9

MAG 2

FOD,FED,DSD0 0.25 0.5 0.666666667

0.050	0.050	0.025	0.075
0.100	0.100	0.050	0.150
0.150	0.150	0.075	0.225
0.200	0.200	0.100	0.300
0.250	0.250	0.125	0.375
0.300	0.300	0.150	0.450

FOD,FED,DSD0 0.3 0.6 0.510638298

0.050	0.065	0.033	0.098
0.100	0.131	0.065	0.196
0.150	0.196	0.098	0.294
0.200	0.261	0.131	0.392
0.250	0.326	0.163	0.490
0.300	0.392	0.196	0.587

FOD,FED,DSD0 0.35 0.7 0.447488584

0.050	0.074	0.037	0.112
0.100	0.149	0.074	0.223
0.150	0.223	0.112	0.335
0.200	0.298	0.149	0.447
0.250	0.372	0.186	0.559
0.300	0.447	0.223	0.670

FOD,FED,DSD0 0.4 0.8 0.414239482

0.050	0.080	0.040	0.121
0.100	0.161	0.080	0.241
0.150	0.241	0.121	0.362
0.200	0.322	0.161	0.483
0.250	0.402	0.201	0.604
0.300	0.483	0.241	0.724

FOD,FED,DSD0 0.45 0.9 0.394160584

0.050	0.085	0.042	0.127
0.100	0.169	0.085	0.254
0.150	0.254	0.127	0.381
0.200	0.338	0.169	0.507
0.250	0.423	0.211	0.634
0.300	0.507	0.254	0.761

FOD,FED,DSD0 0.5 1 0.380952381

0.050	0.087	0.044	0.131
0.100	0.175	0.087	0.262
0.150	0.262	0.131	0.394
0.200	0.350	0.175	0.525
0.250	0.437	0.219	0.656
0.300	0.525	0.262	0.787

MAG 3

FOD,FED,DSDØ Ø.25 Ø.75 Ø.375

Ø.Ø5Ø	Ø.1ØØ	Ø.Ø33	Ø.133
Ø.1ØØ	Ø.2ØØ	Ø.Ø67	Ø.267
Ø.15Ø	Ø.3ØØ	Ø.1ØØ	Ø.4ØØ
Ø.2ØØ	Ø.4ØØ	Ø.133	Ø.533
Ø.25Ø	Ø.5ØØ	Ø.167	Ø.667
Ø.3ØØ	Ø.6ØØ	Ø.2ØØ	Ø.8ØØ

FOD,FED,DSDØ Ø.3 Ø.9 Ø.3253Ø12Ø5

Ø.Ø5Ø	Ø.115	Ø.Ø38	Ø.154
Ø.1ØØ	Ø.231	Ø.Ø77	Ø.3Ø7
Ø.15Ø	Ø.346	Ø.115	Ø.461
Ø.2ØØ	Ø.461	Ø.154	Ø.615
Ø.25Ø	Ø.576	Ø.192	Ø.769
Ø.3ØØ	Ø.692	Ø.231	Ø.922

FOD,FED,DSDØ Ø.35 1.Ø5 Ø.3Ø12295Ø8

Ø.Ø5Ø	Ø.124	Ø.Ø41	Ø.166
Ø.1ØØ	Ø.249	Ø.Ø83	Ø.332
Ø.15Ø	Ø.373	Ø.124	Ø.498
Ø.2ØØ	Ø.498	Ø.166	Ø.664
Ø.25Ø	Ø.622	Ø.2Ø7	Ø.83Ø
Ø.3ØØ	Ø.747	Ø.249	Ø.996

FOD,FED,DSDØ Ø.4 1.2 Ø.28742515

Ø.Ø5Ø	Ø.13Ø	Ø.Ø43	Ø.174
Ø.1ØØ	Ø.261	Ø.Ø87	Ø.348
Ø.15Ø	Ø.391	Ø.13Ø	Ø.522
Ø.2ØØ	Ø.522	Ø.174	Ø.696
Ø.25Ø	Ø.652	Ø.217	Ø.87Ø
Ø.3ØØ	Ø.783	Ø.261	1.Ø44

FOD,FED,DSDØ Ø.45 1.35 Ø.278669725

Ø.Ø5Ø	Ø.135	Ø.Ø45	Ø.179
Ø.1ØØ	Ø.269	Ø.Ø9Ø	Ø.359
Ø.15Ø	Ø.4Ø4	Ø.135	Ø.538
Ø.2ØØ	Ø.538	Ø.179	Ø.718
Ø.25Ø	Ø.673	Ø.224	Ø.897
Ø.3ØØ	Ø.8Ø7	Ø.269	1.Ø77

FOD,FED,DSDØ Ø.5 1.5 Ø.272727273

Ø.Ø5Ø	Ø.137	Ø.Ø46	Ø.183
Ø.1ØØ	Ø.275	Ø.Ø92	Ø.367
Ø.15Ø	Ø.412	Ø.138	Ø.55Ø
Ø.2ØØ	Ø.55Ø	Ø.183	Ø.733
Ø.25Ø	Ø.687	Ø.229	Ø.917
Ø.3ØØ	Ø.825	Ø.275	1.1ØØ

MAG

R

MAG 5

FOD,FED Ø.25 1.25 .2083

Ø.Ø5Ø	Ø.2ØØ	Ø.Ø4Ø	Ø.24Ø
Ø.1ØØ	Ø.4ØØ	Ø.Ø8Ø	Ø.48Ø
Ø.15Ø	Ø.6ØØ	Ø.12Ø	Ø.72Ø
Ø.2ØØ	Ø.8ØØ	Ø.16Ø	Ø.96Ø
Ø.25Ø	1.ØØØ	Ø.2ØØ	1.2ØØ
Ø.3ØØ	1.2ØØ	Ø.24Ø	1.44Ø

FOD,FED Ø.3 1.5 .1935

Ø.Ø5Ø	Ø.215	Ø.Ø43	Ø.258
Ø.1ØØ	Ø.431	Ø.Ø86	Ø.517
Ø.15Ø	Ø.646	Ø.129	Ø.775
Ø.2ØØ	Ø.861	Ø.172	1.Ø33
Ø.25Ø	1.Ø76	Ø.215	1.292
Ø.3ØØ	1.292	Ø.258	1.55Ø

FOD,FED Ø.35 1.75 .1856

Ø.Ø5Ø	Ø.224	Ø.Ø45	Ø.269
Ø.1ØØ	Ø.449	Ø.Ø9Ø	Ø.539
Ø.15Ø	Ø.673	Ø.135	Ø.8Ø8
Ø.2ØØ	Ø.898	Ø.18Ø	1.Ø78
Ø.25Ø	1.122	Ø.224	1.347
Ø.3ØØ	1.347	Ø.269	1.616

FOD,FED Ø.4 2 .1808

Ø.Ø5Ø	Ø.23Ø	Ø.Ø46	Ø.277
Ø.1ØØ	Ø.461	Ø.Ø92	Ø.553
Ø.15Ø	Ø.691	Ø.138	Ø.83Ø
Ø.2ØØ	Ø.922	Ø.184	1.1Ø6
Ø.25Ø	1.152	Ø.23Ø	1.383
Ø.3ØØ	1.383	Ø.277	1.659

FOD,FED Ø.45 2.25 .1776

Ø.Ø5Ø	Ø.235	Ø.Ø47	Ø.281
Ø.1ØØ	Ø.469	Ø.Ø94	Ø.563
Ø.15Ø	Ø.7Ø4	Ø.141	Ø.844
Ø.2ØØ	Ø.938	Ø.188	1.126
Ø.25Ø	1.173	Ø.235	1.4Ø7
Ø.3ØØ	1.4Ø7	Ø.281	1.689

FOD,FED Ø.5 2.5 .1754

Ø.Ø5Ø	Ø.238	Ø.Ø47	Ø.285
Ø.1ØØ	Ø.475	Ø.Ø95	Ø.57Ø
Ø.15Ø	Ø.712	Ø.143	Ø.855
Ø.2ØØ	Ø.95Ø	Ø.19Ø	1.14Ø
Ø.25Ø	1.187	Ø.237	1.425
Ø.3ØØ	1.425	Ø.285	1.71Ø

MAG ←

```
+Q
*A /CS↑ASS/
*XTRAN
+TEST


MAG    7

FOD.FED.DSD0          0.25     1.75     0.145833333

   0.050    0.300    0.043    0.343
   0.100    0.600    0.086    0.686
   0.150    0.900    0.129    1.029
   0.200    1.200    0.171    1.371
   0.250    1.500    0.214    1.714
   0.300    1.800    0.257    2.057

FOD.FED.DSD0          0.3     2.1     0.13876652

   0.050    0.315    0.045    0.360
   0.100    0.631    0.090    0.721
   0.150    0.946    0.135    1.081
   0.200    1.261    0.180    1.441
   0.250    1.576    0.225    1.802
   0.300    1.892    0.270    2.162

FOD.FED.DSD0          0.35     2.45     0.134827044

   0.050    0.324    0.046    0.371
   0.100    0.649    0.093    0.742
   0.150    0.973    0.139    1.113
   0.200    1.298    0.185    1.483
   0.250    1.622    0.232    1.854
   0.300    1.947    0.278    2.225

FOD.FED.DSD0          0.4     2.8     0.132387707

   0.050    0.330    0.047    0.378
   0.100    0.661    0.094    0.755
   0.150    0.991    0.142    1.133
   0.200    1.322    0.189    1.511
   0.250    1.652    0.236    1.888
   0.300    1.983    0.283    2.266

FOD.FED.DSD0          0.45     3.15     0.130765683

   0.050    0.335    0.048    0.382
   0.100    0.669    0.096    0.765
   0.150    1.004    0.143    1.147
   0.200    1.338    0.191    1.529
   0.250    1.673    0.239    1.912
   0.300    2.007    0.287    2.294

FOD.FED.DSD0          0.5     3.5     0.12962963

   0.050    0.337    0.048    0.386
   0.100    0.675    0.096    0.771
   0.150    1.013    0.145    1.157
   0.200    1.350    0.193    1.543
   0.250    1.687    0.241    1.929
   0.300    2.025    0.289    2.314
```

MAG 1Ø

FOD,FED,DSDØ Ø.25 2.5 Ø.1Ø1Ø1Ø1Ø1

Ø.Ø5Ø	Ø.45Ø	Ø.Ø45	Ø.495
Ø.1ØØ	Ø.9ØØ	Ø.Ø9Ø	Ø.99Ø
Ø.15Ø	1.35Ø	Ø.135	1.485
Ø.2ØØ	1.8ØØ	Ø.18Ø	1.98Ø
Ø.25Ø	2.25Ø	Ø.225	2.475
Ø.3ØØ	2.7ØØ	Ø.27Ø	2.97Ø

FOD,FED,DSDØ Ø.3 3 Ø.Ø97693351

Ø.Ø5Ø	Ø.465	Ø.Ø47	Ø.512
Ø.1ØØ	Ø.931	Ø.Ø93	1.Ø24
Ø.15Ø	1.396	Ø.14Ø	1.535
Ø.2ØØ	1.861	Ø.186	2.Ø47
Ø.25Ø	2.326	Ø.233	2.559
Ø.3ØØ	2.792	Ø.279	3.Ø71

FOD,FED,DSDØ Ø.35 3.5 Ø.Ø95796676

Ø.Ø5Ø	Ø.474	Ø.Ø47	Ø.522
Ø.1ØØ	Ø.949	Ø.Ø95	1.Ø44
Ø.15Ø	1.423	Ø.142	1.566
Ø.2ØØ	1.898	Ø.19Ø	2.Ø88
Ø.25Ø	2.372	Ø.237	2.61Ø
Ø.3ØØ	2.847	Ø.285	3.132

FOD,FED,DSDØ Ø.4 4 Ø.Ø946Ø4582

Ø.Ø5Ø	Ø.48Ø	Ø.Ø48	Ø.529
Ø.1ØØ	Ø.961	Ø.Ø96	1.Ø57
Ø.15Ø	1.441	Ø.144	1.586
Ø.2ØØ	1.922	Ø.192	2.114
Ø.25Ø	2.4Ø2	Ø.24Ø	2.643
Ø.3ØØ	2.883	Ø.288	3.171

FOD,FED,DSDØ Ø.45 4.5 Ø.Ø938Ø4285

Ø.Ø5Ø	Ø.485	Ø.Ø48	Ø.533
Ø.1ØØ	Ø.969	Ø.Ø97	1.Ø66
Ø.15Ø	1.454	Ø.145	1.599
Ø.2ØØ	1.938	Ø.194	2.132
Ø.25Ø	2.423	Ø.242	2.665
Ø.3ØØ	2.9Ø7	Ø.291	3.198

FOD,FED,DSDØ Ø.5 5 Ø.Ø9324ØØ93

Ø.Ø5Ø	Ø.487	Ø.Ø49	Ø.536
Ø.1ØØ	Ø.975	Ø.Ø97	1.Ø73
Ø.15Ø	1.463	Ø.146	1.6Ø9
Ø.2ØØ	1.95Ø	Ø.195	2.145
Ø.25Ø	2.437	Ø.244	2.681
Ø.3ØØ	2.925	Ø.292	3.217

MAG ←

ESC: ($MAIN$)1ØØ
+