FINAL REPORT

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RESEARCH STUDY ON STABILIZATION AND CONTROL

MODERN SAMPLED-DATA CONTROL THEORY

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PREPARED FOR GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

FINAL REPORT

RESEARCH STUDY ON STABILIZATION AND CONTROL – MODERN SAMPLED-DATA CONTROL THEORY

SUBTITLE: DESIGN OF THE LARGE SPACE TELESCOPE SYSTEM

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ABSTRACT

The objective of this investigation is to study the conditions of self-sustained oscillations in a two-axis model of the nonlinear LST system.

The describing function of the CMG frictional nonlinearity of the LST system is used for the analysis, and both the continuous-data and the discrete-data models of the simplified LST control system are used.

When two nonlinear systems are coupled together, the conventional graphical method using describing function can no longer by used for the prediction of limit cycles. A numerical-iterative method is described and used for the analysis of the two-axis system. Approximation methods as well as the direct plotting of the stability equation are also used in the study.

It is shown that although the dynamics of the two axes are identical, the amplitudes of self-sustained oscillations in the two axes may in principle be different. Analysis shows that for the LST system, most of the LST systems are of equal amplitudes but with 180-degree phase shift.

The techniques described in this report can be extended to nonlinear systems with more than two axes without too much difficulty.

1. Prediction by Numerical Methods of Self-Sustained Oscillations in a Two-Axis Model of the Nonlinear Continuous-Data LST System

1-1. Introduction

It has been demonstrated in [1] that the methods of continuous and discrete describing function analysis can be applied to predict the existence of self-sustained oscillations in the single-axis model of the LST system with nonlinear CMG friction characteristics. Furthermore, it has been shown in [2] that the stability equations as a result of the describing function analysis may be solved by a numerical-iterative technique instead of the usual graphical methods. With an appropriate guess of the initial condition, the numerical method is found to be quite effective in leading to a convergent solution rapidly.

In the two-axis model of the LST system, the system contains two nonlinearities, and the general form of the stability equation in the continuous-data case is

$$1 + \hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A) = 0 \qquad (1\text{-}1)$$

where $\hat{G}_B(j\omega)$ and $\hat{G}_C(j\omega)$ denote transfer functions which depend on the linear elements of the coupled systems, and $N(A)$ represents the describing function of the CMG nonlinearity. In general, $\hat{G}_B(j\omega)$ and $\hat{G}_C(j\omega)$ are functions of frequency ω, and $N(A)$ is a function of the amplitude A of the assumed sinusoidal input to the nonlinearity.

The usual graphical method cannot be used to solve for the values of ω and A for self-sustained oscillations in Eq. (1-1), due to the $N^2(A)$ term in the equation. However, if the system parameters are such that the linear term $\hat{G}_B(j\omega)N(A)$ dominates over the quadratic term, or vice versa, then by neglecting the smaller quantity in Eq. (1-1), an approximate solution may still be obtained graphically.

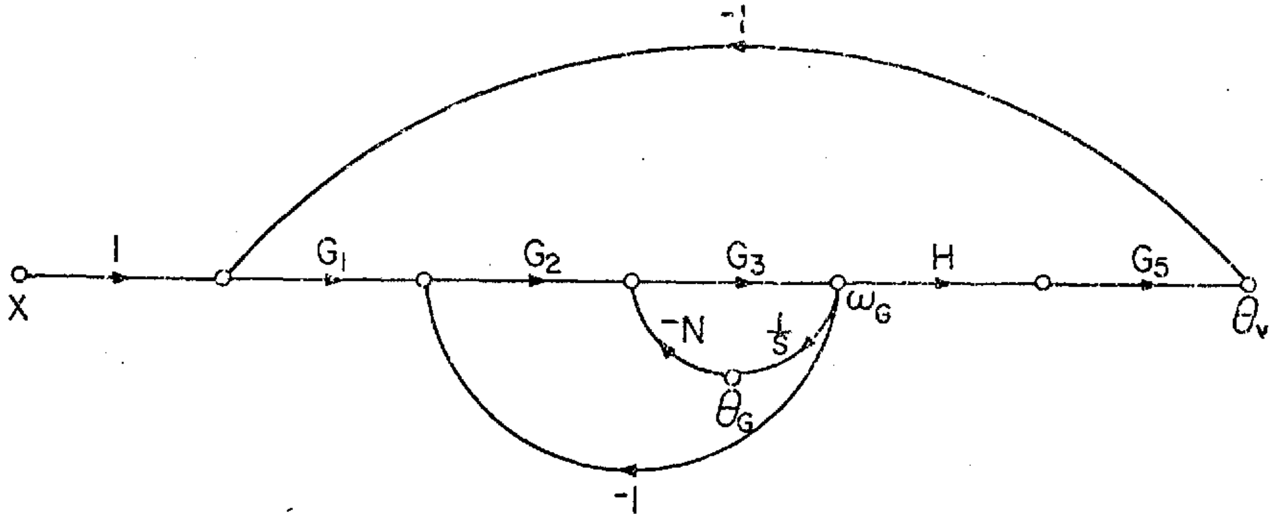
A direct but more time consuming approach of solving Eq. (1-1) would be to calculate the terms in Eq. (1-1) for a wide range of values of ω and A until all combinations which satisfy the equation are found.

Still a third alternative of solving Eq. (1-1) is to use the numerical-iterative method reported in [2]. If an appropriate initial guess can be made, the numerical-iterative method should lead to the exact solutions in an efficient manner.

All three of the above-mentioned methods have been applied to the two-axis continuous-data LST system. It is found that the graphical method with approximation and the direct method can provide useful information to the final solutions, and the numerical-iterative method is effective in arriving at the exact solution on a digital computer. The results of these studies are reported in the ensueing sections.

1-2. <u>The Continuous-Data Two-Axis LST System Model and Its Stability Equation</u>

Figure 1-1 Shows the signal flow graph representation of the continuous-data single-axis LST system. The nonlinear friction of the CMG is modeled by the branch with the gain N. If two such models are coupled together at the output stage through output torsional coupling, the two-axis model of Fig. 1-2 results. This representation of a continuous-data two axis LST system may not be a rigorous one from the



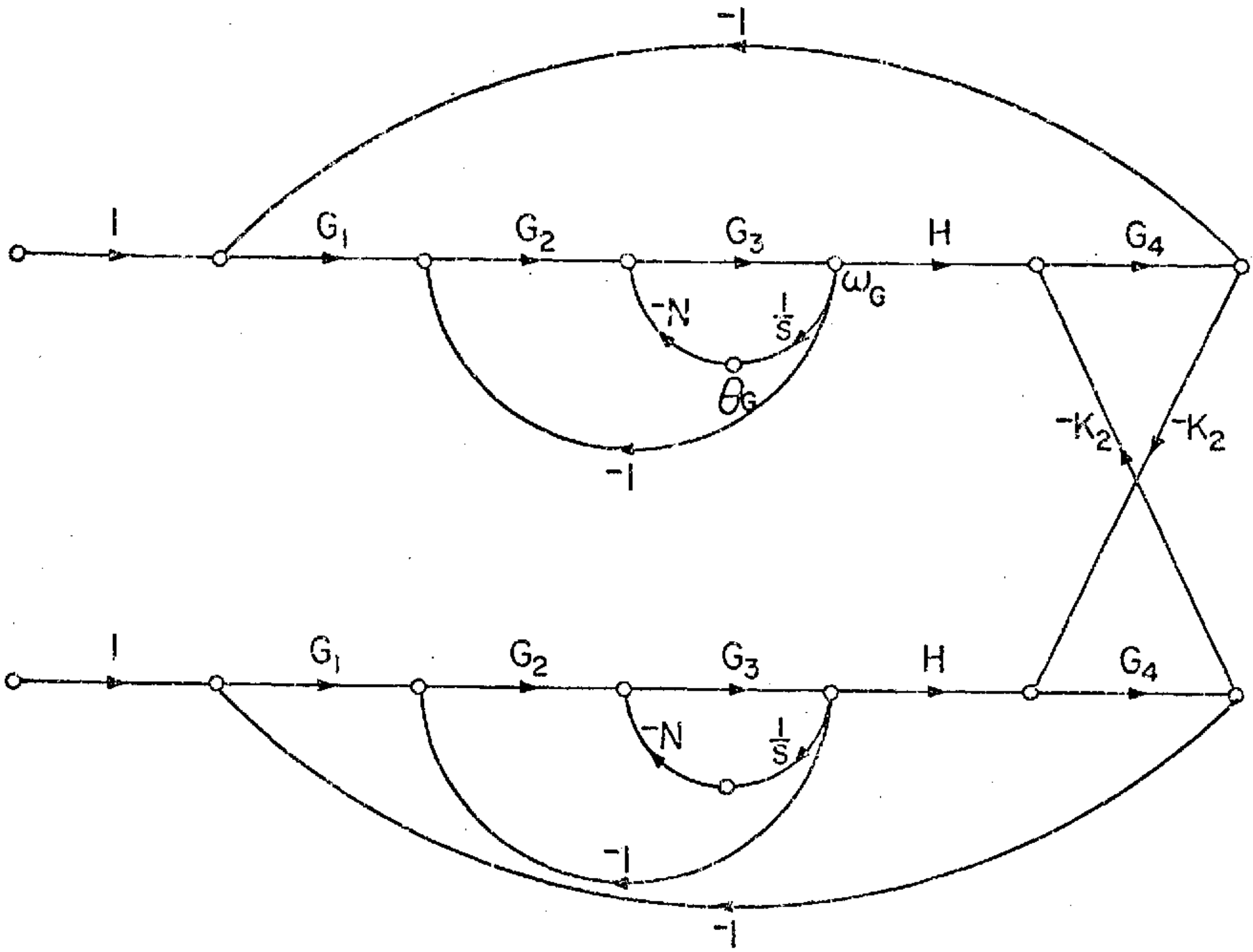
$$G_1 = (K_o + K_1 s)\,\frac{K_I}{(K_p s + K_I)}$$

$$G_2 = \frac{K_p s + K_I}{s}$$

$$G_3 = \frac{1}{J_G s}$$

$$G_5 = \frac{1}{J_v s^2}$$

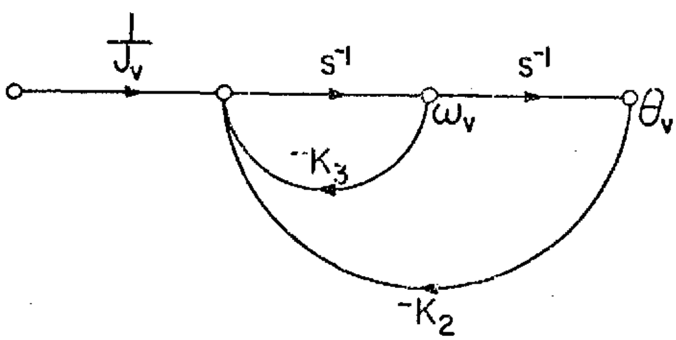
Figure 1-1. The simplified single-axis continuous-data LST system model.



$$G_1 = \frac{(K_o+K_1 s)K_I}{K_p s + K_I}$$

$$G_2 = \frac{K_p s + K_I}{s}$$

$$G_3 = \frac{1}{J_G s}$$



$$\longleftarrow \quad G_4 = \frac{1/J_v}{s^2 + K_3 s + K_2} \quad \longrightarrow$$

Figure 1-2. The simplified two-axis continuous-data LST system model.

structural standpoint. However, the purpose of the present study is to develop analytical techniques which are applicable to multi-coupled nonlinear systems. It is conjectured that for a two-axis nonlinear system, the describing function analysis will generally lead to a stability equation of the form of Eq. (1-1). Therefore, for these above-mentioned reasons, the model shown in Fig. 1-2 is considered to be adequate. The techniques developed in this report can be applied to the prediction of self-sustained oscillations in any continuous-data two-axis nonlinear system that is amenable to the describing function method.

With reference to Fig. 1-2, it can be seen that there are seven individual loops, with the following loop gains:

$$A_1 = -\frac{NG_3}{s} \tag{1-2}$$

$$A_2 = -\frac{NG_3}{s} \tag{1-3}$$

$$B_1 = -G_2G_3 \tag{1-4}$$

$$B_2 = -G_2G_3 \tag{1-5}$$

$$C_1 = -G_1G_2G_3G_4H \tag{1-6}$$

$$C_2 = -G_1G_2G_3G_4H \tag{1-7}$$

$$D = K_2^2G_4^2 \tag{1-8}$$

where G_1, G_2, G_3, and G_4 are transfer functions as defined in Fig. 1-2. All system parameters and variables, including the nonlinear describing function N, are consistent with those defined in the single-axis LST model reported previously [1,2]. The describing function N(A) is a function of the amplitude of the input sinusoid of the CMG nonlinearities. In the present case, since the two axes are identical, and the couplings are symmetric, it is assumed that the amplitudes of the input signals at the two nonlinearities are identical.

The characteristic equation of the coupled system in Fig. 1-2 is

$$\Delta = 0 \tag{1-9}$$

where

$$\begin{aligned}\Delta = 1 &- (A_1 + A_2 + B_1 + B_2 + C_1 + C_2 + D) \\ &+ (A_1A_2 + A_1B_2 + A_1C_2 + A_1D + B_1A_2 + B_1B_2 + B_1C_2 + B_1D \\ &\quad + C_1A_2 + C_1B_2 + C_1C_2 + A_2D + B_2D) \\ &- (A_1A_2D + A_1B_2D + B_1A_2D + B_1B_2D)\end{aligned} \tag{1-10}$$

The last equation is simplified if we define

$$\begin{aligned}A &= A_1 = A_2 \\ B &= B_1 = B_2 \\ C &= C_1 = C_2\end{aligned} \tag{1-11}$$

Then, Eq. (1-10) becomes

$$\begin{aligned}\Delta = 1 - 2(A + B + C) - D + (A^2 + B^2 + C^2 + 2AB + 2AC + 2BC \\ + 2AD + 2BD) - (A + B)^2D\end{aligned} \tag{1-12}$$

or

$$\Delta = 1 - 2(A + B + C) - D + (A + B + C)^2 - (A + B)^2 D \tag{1-13}$$

Substituting Eqs. (1-2) through (1-8) in Eq. (1-13) yields

$$\begin{aligned}\Delta = 1 &+ \frac{2G_3N}{s} + 2G_2G_3 + 2G_1G_2G_3G_4H - K_2^2G_4^2 \\ &+ \frac{G_3^2}{s^2}N^2 + G_2^2G_3^2 + (G_1G_2G_3G_4H)^2 + \frac{2NG_2G_3^2}{s} \\ &+ \frac{2NG_1G_2G_3^2G_4H}{s} + 2G_1G_2^2G_3^2G_4H - \frac{2NG_3G_4^2K_2^2}{s} \\ &- 2G_2G_3G_4^2K_2^2 - \frac{N^2G_3^2G_4^2K_2^2}{s^2} - \frac{2NG_2G_3^2G_4^2K_2^2}{s} \\ &- G_2^2G_3^2G_4^2K_2^2\end{aligned} \tag{1-14}$$

The characteristic equation in Eq. (1-9) can be written as

$$\Delta = 1 + G_A + G_BN + G_CN^2 = 0 \tag{1-15}$$

where

$$\begin{aligned}G_A = 2G_2G_3 &+ 2G_1G_2G_3G_4H - K_2^2G_4^2 + G_2^2G_3^2 + (G_1G_2G_3G_4H)^2 \\ &+ 2G_1G_2^2G_3^2G_4H - 2G_2G_3G_4^2K_2^2 - G_2^2G_3^2G_4^2K_2^2\end{aligned} \tag{1-16}$$

$$\begin{aligned}G_B = \frac{2G_3}{s} &+ \frac{2G_2G_3^2}{s} + \frac{2G_1G_2G_3^2G_4H}{s} - \frac{2G_3G_4^2K_2^2}{s} \\ &- \frac{2G_2G_3^2G_4^2K_2^2}{s}\end{aligned} \tag{1-17}$$

$$G_C = \frac{G_3^2}{s^2} - \frac{G_3^2 G_4^2 K_2^2}{s^2} \tag{1-18}$$

For stability analysis, the characteristic equation of Eq. (1-15) is written as

$$1 + \hat{G}_B N + \hat{G}_C N^2 = 0 \tag{1-19}$$

where

$$\hat{G}_B = \frac{G_B}{1 + G_A} \tag{1-20}$$

$$\hat{G}_C = \frac{G_C}{1 + G_A} \tag{1-21}$$

Equation (1-19) is a function of the system frequency ω and the input amplitude A. Since this equation is defined in the complex plane, it represents a set of two nonlinear equations with two unknowns. When these nonlinear equations have a solution for ω and A, it represents a condition of self-sustained oscillations for the system. The oscillations may be stable or unstable; therefore, the solutions must always be checked for stability.

1-3. Prediction of Self-Sustained Oscillations By The Approximation Method

It was mentioned in Sec. 1-1 that the graphical method of predicting self-sustained oscillations can still be applied to the two-axis LST system if one of the last two terms in Eq. (1-1) can be neglected. In other words, the following two conditions may exist:

1. $|\hat{G}_B(j\omega)N(A)| \gg |\hat{G}_C(j\omega)N^2(A)|$

Then Eq. (1-1) may be approximated by

$$1 + \hat{G}_B(j\omega)N(A) = 0 \tag{1-22}$$

and the condition of self-sustained oscillations is found from the following equation:

$$\hat{G}_B(j\omega) = - \frac{1}{N(A)} \tag{1-23}$$

2. $|\hat{G}_B(j\omega)N(A)| \ll |\hat{G}_C(j\omega)N^2(A)|$

Then Eq. (1-1) is approximated by

$$1 + \hat{G}_C(j\omega)N^2(A) = 0 \tag{1-24}$$

and the condition of self-sustained oscillations is found from

$$\hat{G}_C(j\omega) = - \frac{1}{N^2(A)} \tag{1-25}$$

Therefore, the graphical solutions involve the plotting of the curves for $\hat{G}_B(j\omega)$ and $-1/N(A)$, or $\hat{G}_C(j\omega)$ and $-1/N^2(A)$, as the case may be.

Figure 1-3 illustrates the plots of $|\hat{G}_B(j\omega)N(A)|$ and $|\hat{G}_C(j\omega)N^2(A)|$ versus ω for various values of A, and $|N(A)|$ versus A. The following parameters are used for the LST system:

$$J_V = 10^5 , \quad J_G = 2.1 , \quad K_p = 216 , \quad K_I = 9700$$

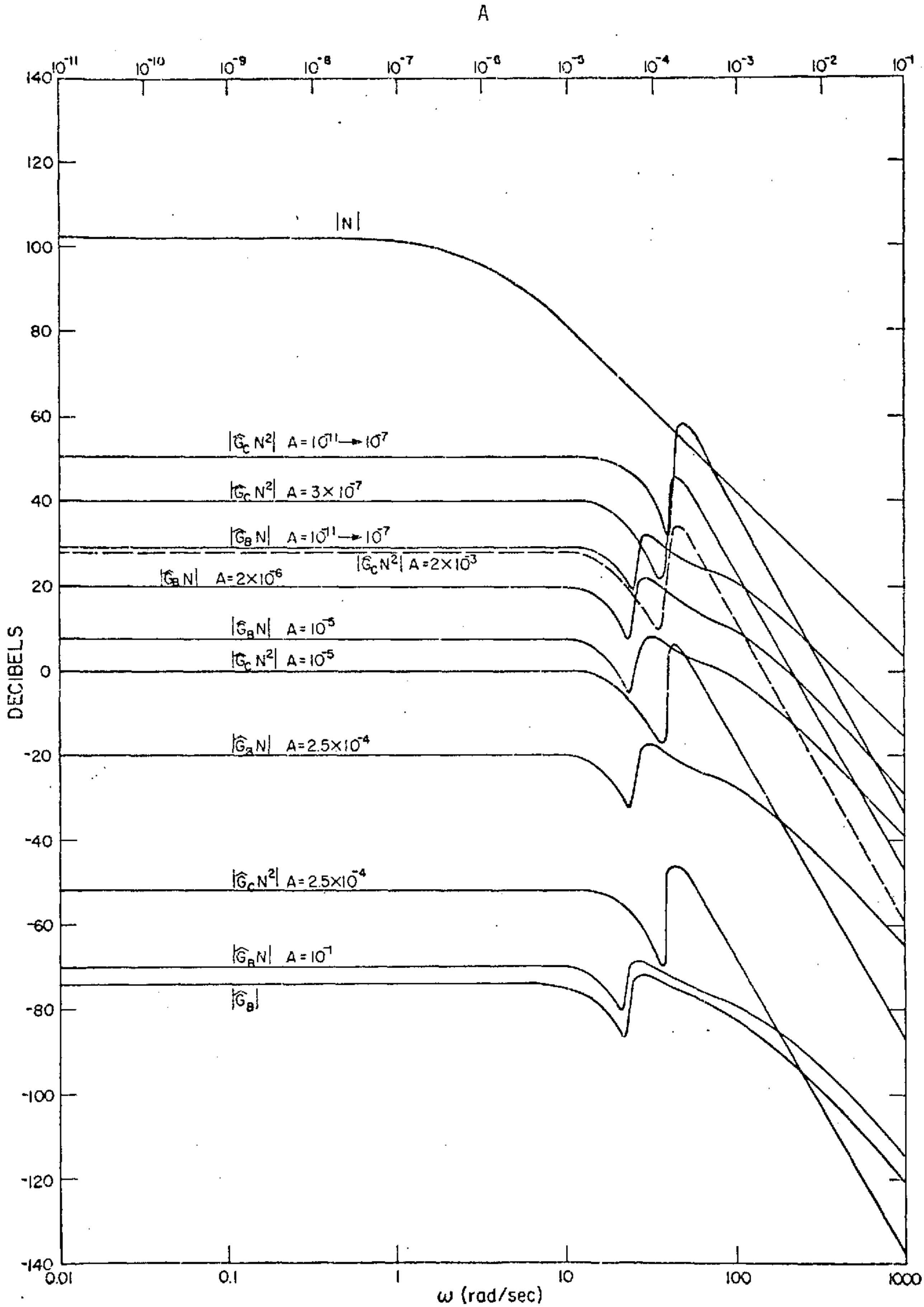

A
DECIBELS
ω (rad/sec)
|N|
|Ĝ_C N²| A = 10⁻¹¹ → 10⁻⁷
|Ĝ_C N²| A = 3×10⁻⁷
|Ĝ_B N| A = 10⁻¹¹ → 10⁻⁷
|Ĝ_C N²| A = 2×10⁻³
|Ĝ_B N| A = 2×10⁻⁶
|Ĝ_B N| A = 10⁻⁵
|Ĝ_C N²| A = 10⁻⁵
|Ĝ_B N| A = 2.5×10⁻⁴
|Ĝ_C N²| A = 2.5×10⁻⁴
|Ĝ_B N| A = 10⁻¹
|Ĝ_B|

Figure 1-3.

$H = 600$, $K_0 = 5758.35$, $K_1 = 1371.02$, $K_2 = 100$, $K_3 = 3$

$\gamma = 1.38 \times 10^7$

The last two parameters, K_2 and K_3 are the coefficients of coupling between the two axes.

The curves in Fig. 1-3 give information on the ranges of ω and A in which the approximation of Eq. (1-22) or Eq. (1-24) is valid. Figure 1-4 illustrates the regions in the ω versus A plane in which the two approximations are valid. The cross-hatched area represents the region in which no approximation can be made, and the graphical method cannot be used. The criterion of ≥ 20 db and ≤ -20 db is used for magnitude comparison for significance.

The results of Fig. 1-4 show that the graphical approximation method is valid for the following ranges of ω and A:

1. Use $1 + \hat{G}_C N^2 = 0$:

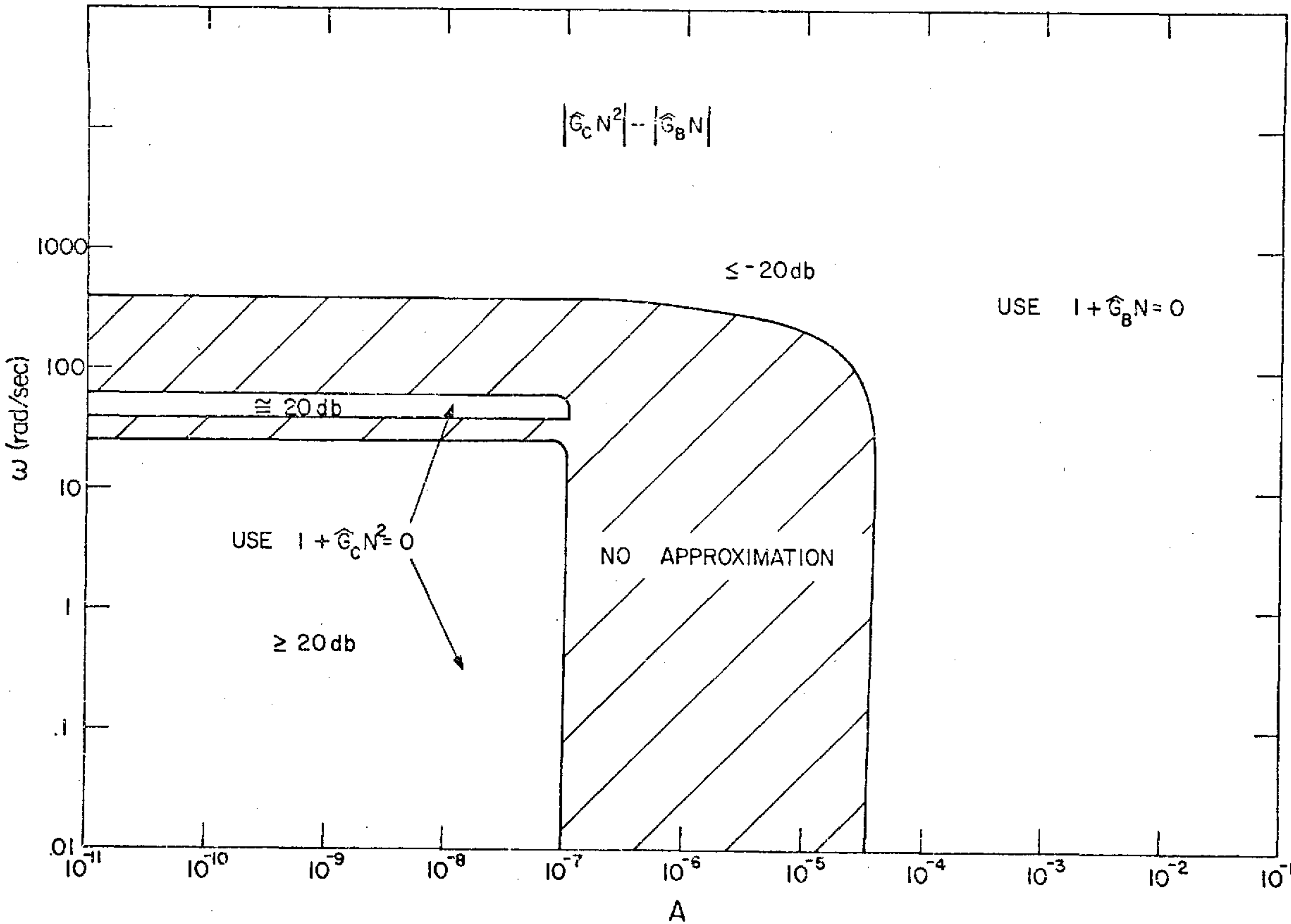
   $A \leq 10^{-7}$ $\qquad$ $\omega \leq 40$ rad/sec

   and $\quad 60 \leq \omega \leq 80$ rad/sec

2. Use $1 + \hat{G}_B N = 0$:

   $A \geq 6 \times 10^{-5}$ $\qquad$ $\omega \geq 600$ rad/sec.

For the region of validity of case 1 above, Fig. 1-5 shows the plots of $\hat{G}_C$ and $-1/N^2$. The heavy portions of the curves indicate the parts which are valid for stability analysis. Similarly, Fig. 1-6 shows the case when the equation $1 + \hat{G}_B N = 0$ may be used for approximation. Again, the intersection between the heavy portions of the $\hat{G}_B$ and the $-1/N$ curves would indicate the possibility of self-sustained oscillation in the two-axis LST system. Since there are no intersections



|Ĝc N²| -- |ĜB N|
≤ -20db
USE 1 + ĜB N = 0
≅ 20db
USE 1 + Ĝc N² = 0
≥ 20db
NO APPROXIMATION
ω (rad/sec)
1000
100
10
1
.1
.01
10⁻¹¹
10⁻¹⁰
10⁻⁹
10⁻⁸
10⁻⁷
10⁻⁶
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
A

Figure 1-4.

between the valid curves in Figs. 1-5 and 1-6, the system under study is stable for the parameter values used. This result will be substantiated by the two other methods which are discussed in the following sections.

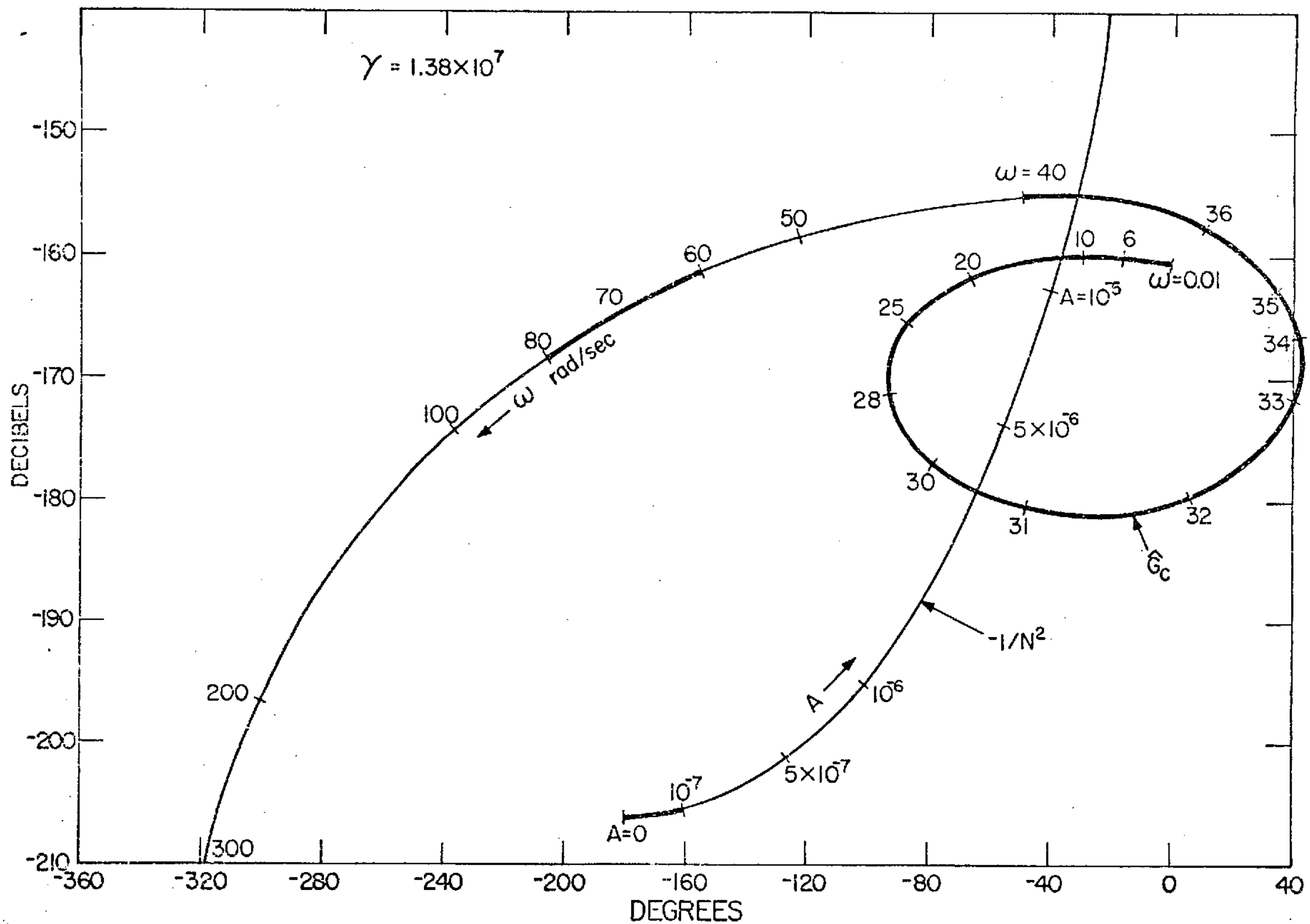
1-4. A Direct Method of Predicting Self-Sustained Oscillations in The Two-Axis LST System

A direct but time-consuming method of predicting self-sustained oscillations in the two-axis continuous-data LST system is to calculate all the terms in the following stability equation for a wide range of values of ω and A:

$$1 + \hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A) = 0 \tag{1-26}$$

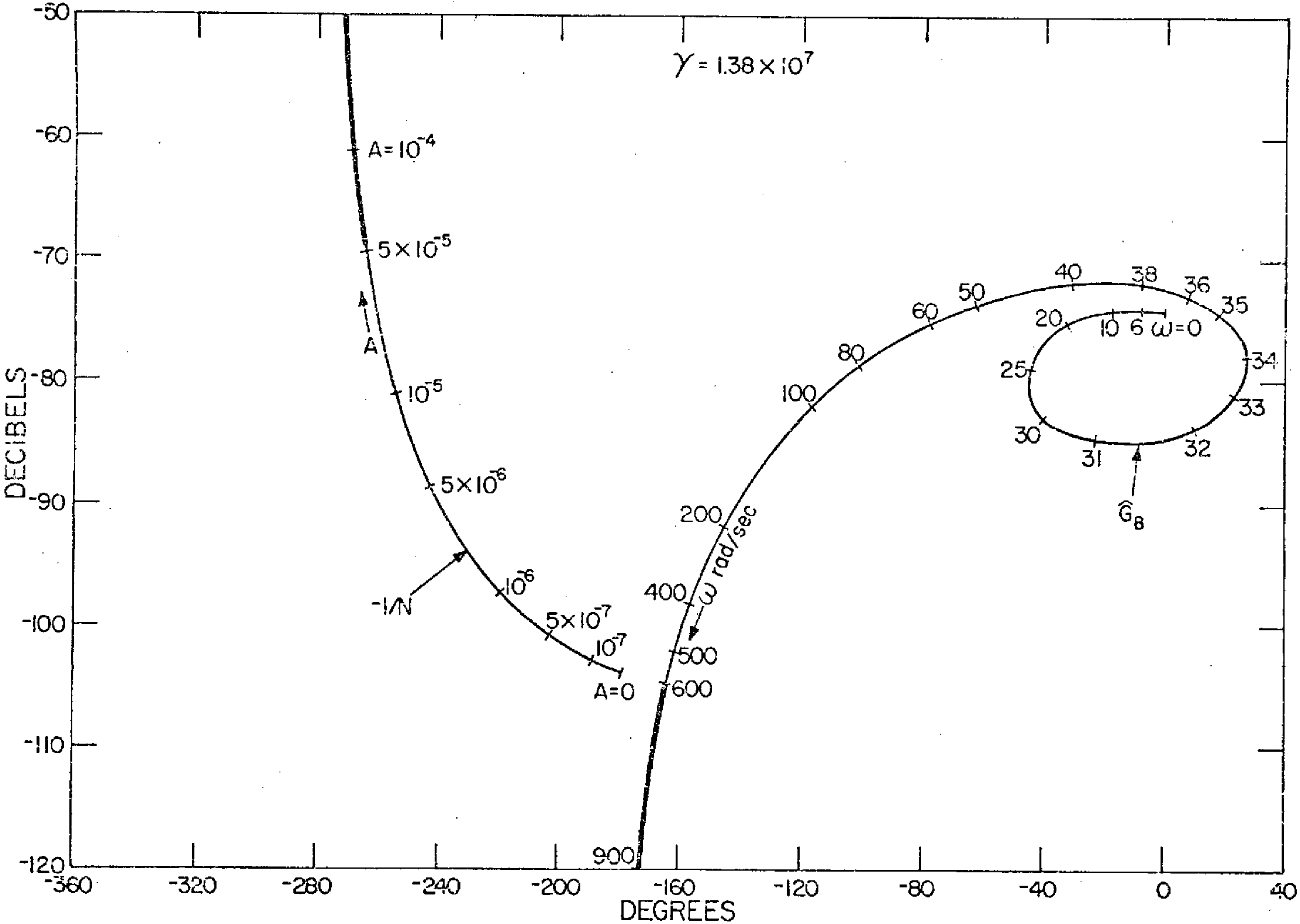
Although the direct method would be quite tedious, and it is possible that with a given selected increment of variation for the values of ω and A, the solution runs may still miss the exact solutions, the results in general will give insight to the appropriate guess on the initial values for the numerical-iteration method. Therefore, it is enlightening to investigate this direct approach at this point.

Figures 1-7 through 1-9 show the plots of $\hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A)$ for $\gamma = 1.38 \times 10^7$, and all the system parameters as given previously, and for three different sets of values for the coupling coefficients K_2 and K_3. Note that for the three cases illustrated, all the trajectories for various combinations of A and ω do not intersect the critical point which is at 0 db and -180 degrees. Thus the system is stable. However, the results show that as the value of K_2 is decreased, the trajectories



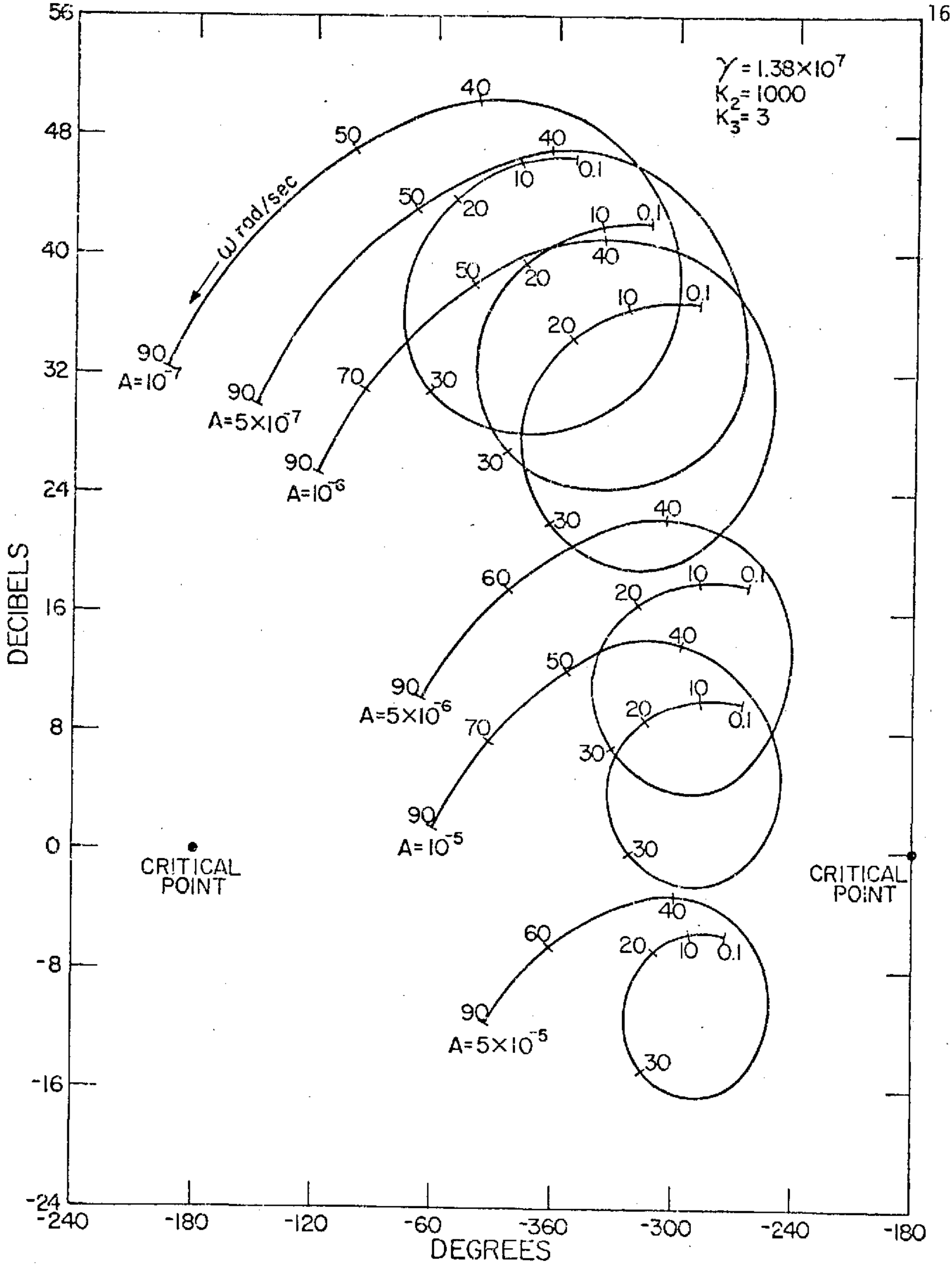
γ = 1.38×10^7
-150
-160
-170
-180
-190
-200
-210
DECIBELS
-360
-320
-280
-240
-200
-160
-120
-80
-40
0
40
DEGREES
ω = 40
50
60
70
80
100
200
300
ω rad/sec
36
35
34
33
32
31
30
28
25
20
10
6
ω=0.01
A=10^-5
5×10^-6
10^-6
5×10^-7
10^-7
A=0
A
-1/N²
Ĝc

Figure 1-5.



γ = 1.38 × 10^7
A = 10^-4
5 × 10^-5
A
10^-5
5 × 10^-6
-1/N
10^-6
5 × 10^-7
10^-7
A = 0
200
400
500
600
900
ω rad/sec
100
80
60
50
40
38
36
35
20
10
6
ω = 0
34
25
33
30
31
32
Ĝ_8
DECIBELS
DEGREES
-50
-60
-70
-80
-90
-100
-110
-120
-360
-320
-280
-240
-200
-160
-120
-80
-40
0
40

Figure 1-6.



$\gamma = 1.38 \times 10^7$
$K_2 = 1000$
$K_3 = 3$
ω rad/sec
A=10^-7
A=5×10^-7
A=10^-6
A=5×10^-6
A=10^-5
A=5×10^-5
CRITICAL POINT
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
16
-240
-180
-120
-60
-360
-300
-240
-180

Figure 1-7.

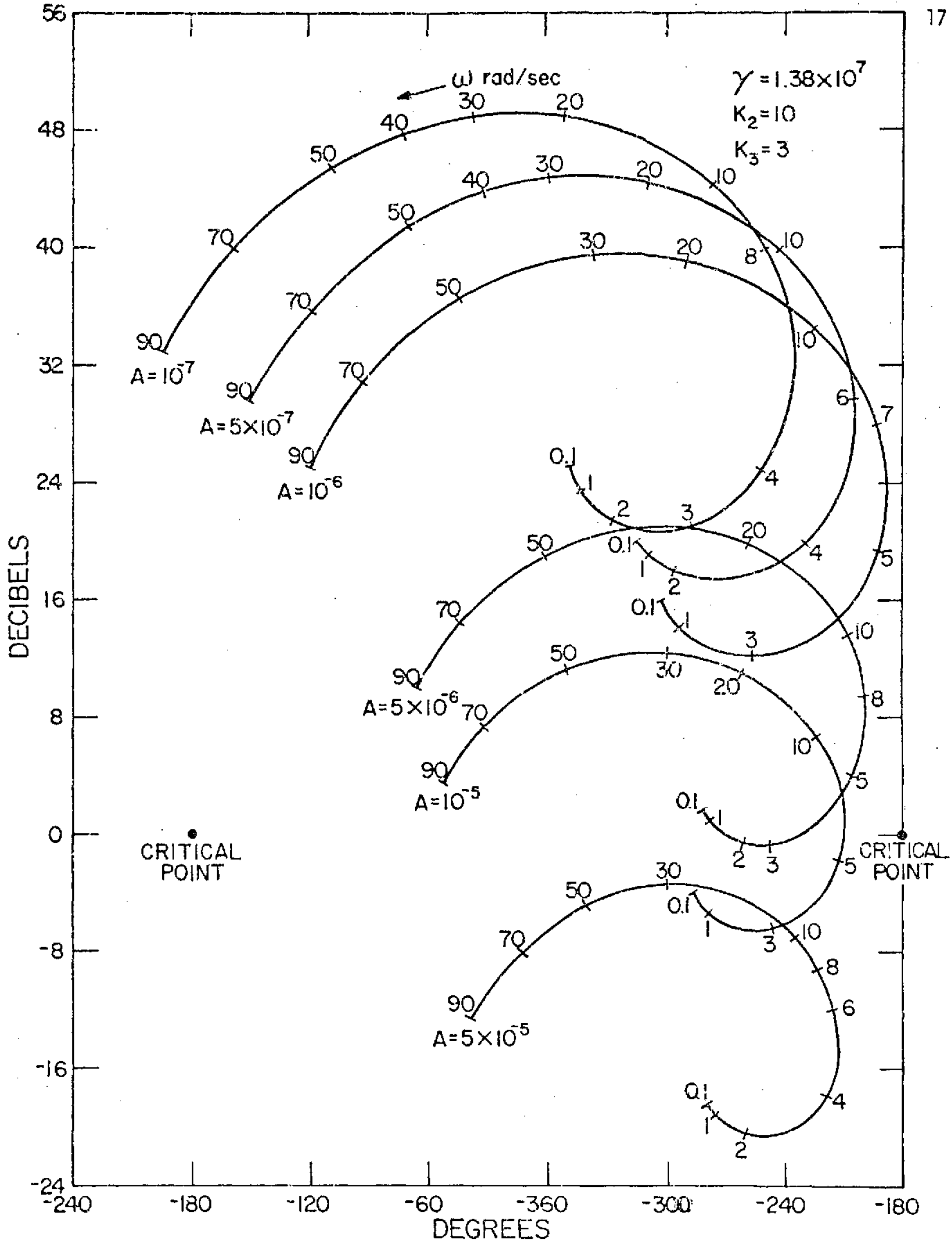
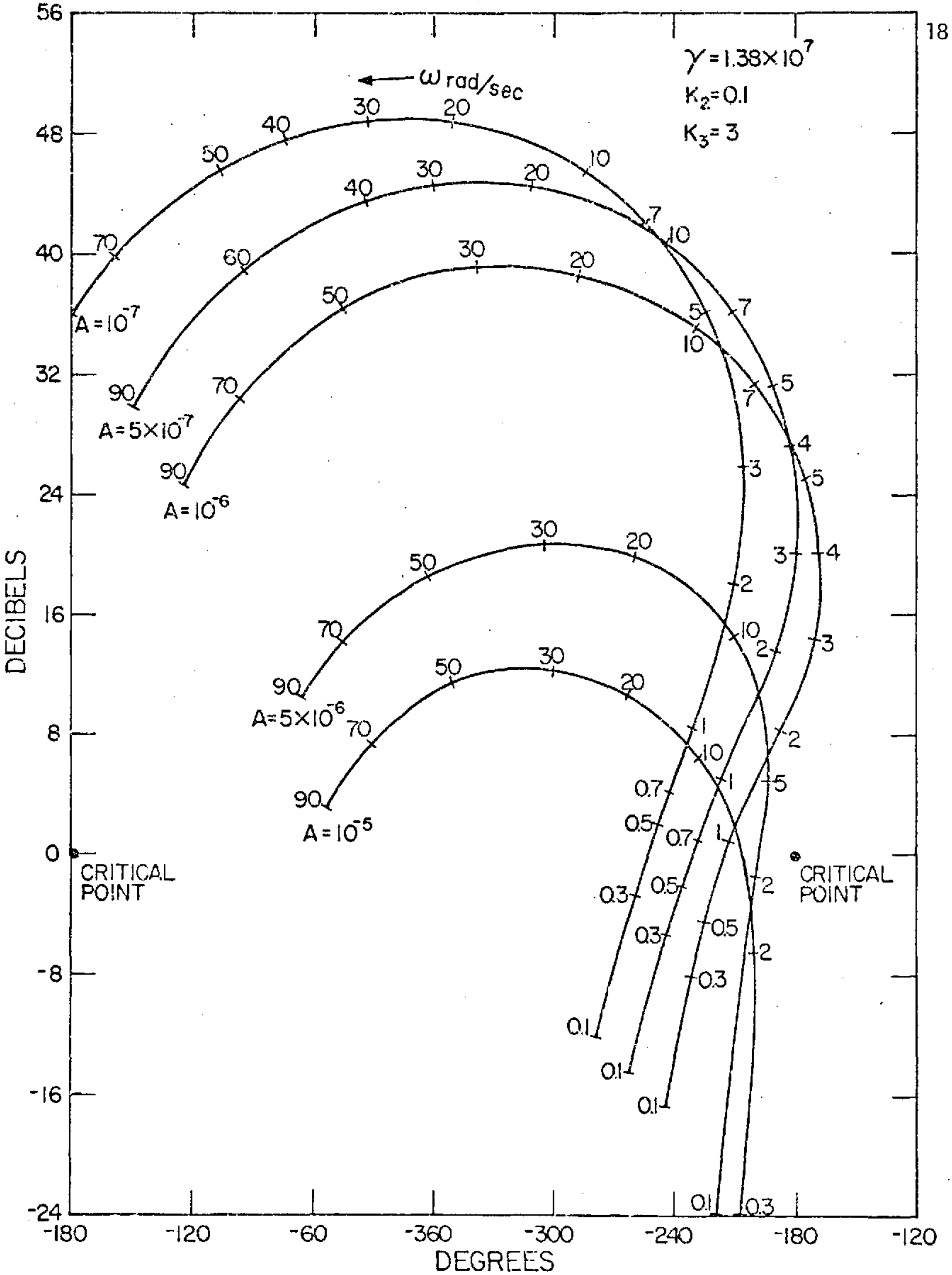

ω rad/sec
γ = 1.38×10^7
K2 = 10
K3 = 3
A = 10^-7
A = 5×10^-7
A = 10^-6
A = 5×10^-6
A = 10^-5
A = 5×10^-5
CRITICAL POINT
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
-240
-180
-120
-60
-360
-300
-240
-180

Figure 1-8.



ω rad/sec
γ = 1.38×10^7
K2 = 0.1
K3 = 3
A = 10^-7
A = 5×10^-7
A = 10^-6
A = 5×10^-6
A = 10^-5
CRITICAL POINT
CRITICAL POINT
DECIBELS
DEGREES

Figure 1-9.

get closer to the critical point. Figure 1-10 shows the trajectories when $K_2 = 0.1$ and $K_3 = 0.1$. It is noticed that several trajectories are very close to the critical point. Figure 1-11 shows a magnified version of Fig. 1-10 around the critical point. The figure shows that the trajectories for $A = 5 \times 10^{-6}$, 10^{-6}, 5×10^{-6}, are all very close to the critical point. This gives indications that there may be more than one solution. It will be shown in the next section by the numerical-iterative method that there are indeed two solutions for the stability equation. One is at $A = 5.9867 \times 10^{-7}$ and $\omega = 1.88$ rad/sec, and the other is at $A = 5.07397 \times 10^{-6}$ and $\omega = 4.1086$ rad/sec. These solutions must still be checked for stable or unstable equilibrium solutions.

1-5. Exact Solution of the Stability Equation by Numerical-Iterative Techniques

In this section the stability equation developed in Sec. 1-2 is solved numerically for its exact solutions. The numerical method utilized has been described in [2] and is found to be quite effective for the two-axis LST system.

The stability equation, Eq. (1-19), can be written as

$$1 + \hat{G}_B(j\omega)N(A) + \hat{G}_C(j\omega)N^2(A) = 0 \qquad (1\text{-}27)$$

Define

$$\hat{G}_B(j\omega) = G_{R1} + jG_{I1} \qquad (1\text{-}28)$$

$$\hat{G}_C(j\omega) = G_{R2} + jG_{I2} \qquad (1\text{-}29)$$

$$N(A) = N_{R1} + jN_{I1} \qquad (1\text{-}30)$$

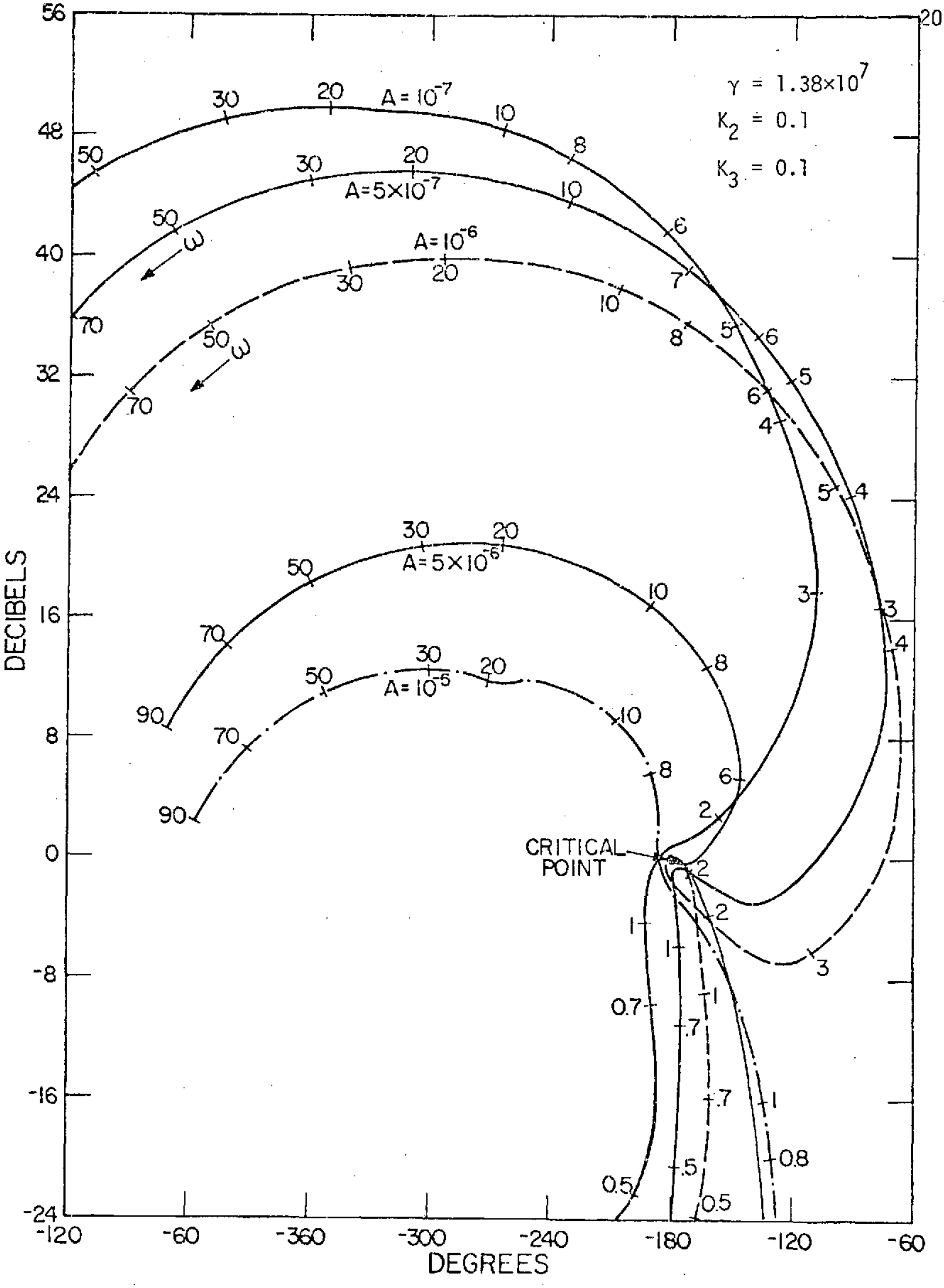
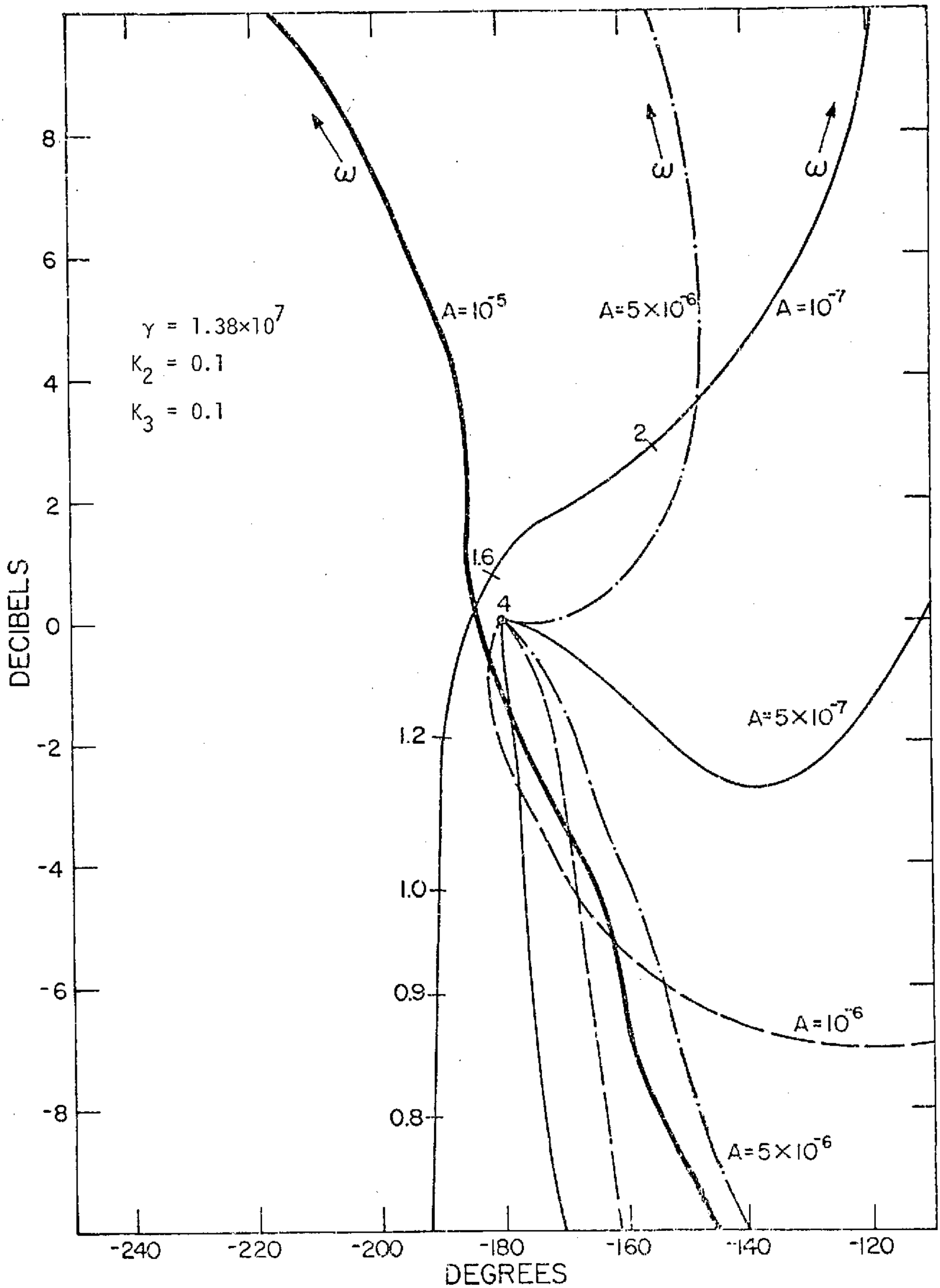

γ = 1.38×10^7
K2 ≐ 0.1
K3 = 0.1
A = 10^-7
A=5×10^-7
A=10^-6
A=5×10^-6
A=10^-5
ω
CRITICAL POINT
DECIBELS
DEGREES
56
48
40
32
24
16
8
0
-8
-16
-24
20
-120
-60
-360
-300
-240
-180
-120
-60

Figure 1-10.



8
6
4
2
0
-2
-4
-6
-8
DECIBELS
-240
-220
-200
-180
-160
-140
-120
DEGREES
γ = 1.38×10^7
K2 = 0.1
K3 = 0.1
ω
ω
ω
A=10^-5
A=5×10^-6
A=10^-7
2
1.6
4
A=5×10^-7
1.2
1.0
0.9
A=10^-6
0.8
A=5×10^-6

Figure 1-11.

$$N^2(A) = N_{R2} + jN_{I2} \tag{1-31}$$

where G_{R1}, G_{I1}, G_{R2}, G_{I2}, N_{R1}, N_{I1}, N_{R2}, and N_{I2} are all real quantities.

If Eqs. (1-28) through (1-31) are substituted into Eq. (1-27), it becomes

$$1 + (G_{R1} + jG_{I1})(N_{R1} + jN_{I1}) + (G_{R2} + jG_{I2})(N_{R2} + jN_{I2}) = 0 \tag{1-32}$$

When the real and imaginary parts are separated, Eq. (1-32) yields

$$\Delta = \Delta_R + j\Delta_I = 0 \tag{1-33}$$

with

$$\Delta_R = 1 + G_{R1}N_{R1} - G_{I1}N_{I1} + G_{R2}N_{R2} - G_{I2}N_{I2} = 0 \tag{1-34}$$

and

$$\Delta_I = G_{I1}N_{R1} + G_{R1}N_{I1} + G_{I2}N_{R2} + G_{R2}N_{I2} = 0 \tag{1-35}$$

Further simplification is possible in Eqs. (1-34) and (1-35) if we recognize that $N_{R2} + jN_{I2} = (N_{R1} + jN_{I1})^2$ but is not necessary since the solution of Eqs. (1-34) and (1-35) will be performed on a digital computer.

Equations (1-33) and (1-34) represent two equations in the two variables ω and A. As before [2], let us define

$$\underline{x} = \begin{Bmatrix} \omega \\ A \end{Bmatrix} \tag{1-36}$$

$$\underline{F} = \begin{Bmatrix} \Delta_R \\ \Delta_I \end{Bmatrix} \tag{1-37}$$

Then, Eqs. (1-34) and (1-35) can be written as

$$\underline{F}(\underline{x}) = \underline{0} \tag{1-38}$$

The Newton-type quadratically convergent numerical method described in [2] can now be directly applied to this two-variable system.

To initiate the iterations, an initial solution is needed. The results of the direct calculation method, presented in the previous section provide an adequate initial solution. As in that section, the parameters which need to be varied are K_2 and K_3. Once a solution for a given value of K_2 and K_3 is known, K_2 and/or K_3 could be changed slightly and the new solution is obtained by using the old solution as the initial guess for the new system.

With K_2 and K_3 set to very small values, the system is decoupled, and the solution of the single-axis LST system could be used as an initial guess for that of the two-axis LST system.

If $K_2 = 0.1$ and $K_3 = 0.1$, the results from the direct calculation method shown in Figs. 1-10 and 1-11 indicate that two possible solutions exist. The approximate amplitudes and frequencies of the solutions are

Solution No. 1	$\omega = 5$ rad/sec
	$A = 5 \times 10^{-6}$ rad
Solution No. 2	$\omega = 2$ rad/sec
	$A = 1 \times 10^{-6}$ rad

With these approximate solutions as initial conditions, the results of the numerical iteration procedure are obtained and tabulated in the first set of iterations in Fig. 1-12 and Fig. 1-13. These iterations indicate that the actual solutions are

Solution No. 1 $\omega = 4.1086$ rad/sec

$A = 5.0739 \times 10^{-6}$ rad

Solution No. 2 $\omega = 1.88$ rad/sec

$A = 5.99 \times 10^{-7}$ rad

Figures 1-12 and 1-13 also show the change in the solutions when K_2 and K_3 are reduced to $K_2 = K_3 = 0.01$ and $K_2 = K_3 = 0.001$, respectively. As mentioned previously, the initial guess for a new set of values of K_2 and K_3 is the solution with the previous values of K_2 and K_3.

With K_3 fixed at 0.001, if K_2 is increased (Figs. 1-14 and 1-15), the solutions will change, until $K_2 = 8$ for solution no. 1 and $K_2 = 7$ for solution no. 2, beyond which no solution is obtained. It is interesting to note that the two solutions move closer together with increasing K_2 and almost merge into each other before disappearing altogether. The result is analogous to lowering the $G(j\omega)$ curve or raising the $-1/N$ curve in the single-axis case.

It can be concluded that for $K_3 = 0.001$, K_2 should be less than 8 for a sustained oscillation to occur. In fact, with $K_2 = 8$ (solution 1) and $K_2 = 7$ (solution 2) the roots of the stability equations do not appear to have converged adequately, and well defined roots occur only for lower values of K_2. A maximum of 20 iterations are attempted before the numerical process is terminated.

If K_3 is kept fixed at 0.2 and K_2 is varied, the iterations in

Figs. 1-16 and 1-17 show that K_2 can be increased only up to $K_2 = 3$ for solutions to be obtained.

On the other hand, K_2 can be kept fixed and K_3 varied; these results are shown in Figs. 1-18 through 1-25. In Figs. 1-18 and 1-19 K_2 is fixed at 0.001, and the maximum value of K_3 for a solution to the stability equation is approximately $K_3 = 0.4$ for solution no. 1 and $K_2 = 0.3$ for solution no. 2. With $K_2 = 0.1$, 1.0 and 5.0, the maximum values of K_3 are 0.4 (solution 1 and 2), 0.3 (solution 1 and 2) and <0.1 (solution 1 and 2), respectively. These results are shown in Figs. 1-20 through 1-25.

Table 1-1 summarizes all the results. In the table, the solutions for $K_2 = K_3 = 0$ are the solutions with the single-axis LST system. It is seen that when K_2 and K_3 are very small, the solutions of the two-axis system approach those of the single-axis system.

The data in Table 1-1 can be used to plot the region in the $K_2 - K_3$ plane in which sustained oscillation can exist. Figure 1-26 shows this region. For values of K_2 and K_3 outside the crosshatched area, no sustained oscillations should exist.

The motion of the roots with variable K_2 and K_3 are plotted in Fig. 1-27, using the data in Table 1-1. The points marked $K_2 = 0$, $K_3 = 0$, represent the single-axis LST system solutions. The solid closed regions represent the regions of root movements where reliable convergence was obtained. The dotted lines include the regions where reliable convergence of the iterative scheme was not obtained with 20 iterations, and, which can be considered as a boundary area between existence and non-existence of solutions. Note that in Fig. 1-27 the two solutions are farthest apart when $K_2 = K_3 = 0$. Figure 1-27 also shows how the two solutions tend to merge just before the solutions disappear.

Table 1-1

K_2	K_3	ROOT #1 FREQUENCY	ROOT #1 AMPLITUDE	ROOT #2 FREQUENCY	ROOT #2 AMPLITUDE
0.1	0.1	4.1	5.07×10^{-6}	1.8	5.99×10^{-7}
0.01	0.01	4.26	5.52×10^{-6}	1.76	4.58×10^{-7}
0.001	0.001	4.27	5.56×10^{-6}	1.75	4.45×10^{-7}
0.0	0.0	4.27	5.57×10^{-6}	1.75	4.45×10^{-7}
0.01	0.001	4.27	5.57×10^{-6}	1.75	4.46×10^{-7}
0.1	0.001	4.28	5.55×10^{-6}	1.79	4.59×10^{-7}
1.0	0.001	4.31	5.24×10^{-6}	2.11	5.79×10^{-7}
2.0	0.001	4.34	4.91×10^{-6}	2.43	7.17×10^{-7}
3.0	0.001	4.36	4.56×10^{-6}	2.73	8.69×10^{-7}
4.0	0.001	4.38	4.20×10^{-6}	3.01	1.04×10^{-6}
5.0	0.001	4.37	3.81×10^{-6}	3.29	1.25×10^{-6}
6.0	0.001	4.34	3.35×10^{-6}	3.58	1.53×10^{-6}
7.0	0.001	4.19	2.6×10^{-6}	3.97**	2.06×10^{-6}**
8.0	0.001	3.95**	1.50×10^{-6}**	-	-
1.0	0.001	4.31	5.24×10^{-6}	2.11	5.79×10^{-7}
1.0	0.01	4.29	5.19×10^{-6}	2.23	5.93×10^{-7}
1.0	0.1	4.13	4.74×10^{-6}	2.12	7.71×10^{-7}
1.0	0.2	3.91	4.18×10^{-6}	2.39	1.03×10^{-6}
1.0	0.3	3.62	3.47×10^{-6}	2.62	1.43×10^{-6}
1.0	0.4	-	-	3.2**	2.57×10^{-6}**
5.0	0.001	4.37	3.81×10^{-6}	3.29	1.25×10^{6}
5.0	0.01	4.34	3.72×10^{-6}	3.31	1.30×10^{6}
5.0	0.1	5.46**	7.13×10^{-6}**	3.75**	2.25×10^{6}**
1.0	0.2	3.91	4.17×10^{-6}	2.39	1.04×10^{-6}
2.0	0.2	3.85	3.64×10^{-6}	2.79	1.37×10^{-6}
3.0	0.2	3.69	2.86×10^{-6}	3.28	1.95×10^{-6}
0.001	0.001	4.27	5.57×10^{-6}	1.75	4.45×10^{-7}
0.001	0.01	4.26	5.54×10^{-6}	1.76	4.56×10^{-7}

Table 1-1 (cont'd)

K_2	K_3	ROOT #1		ROOT #2	
		FREQUENCY	AMPLITUDE	FREQUENCY	AMPLITUDE
0.001	0.1	4.11	5.12×10^{-6}	1.84	5.79×10^{-7}
0.001	0.2	3.92	4.63×10^{-6}	1.95	7.52×10^{-7}
0.001	0.3	3.7	4.08×10^{-6}	2.1	9.82×10^{-7}
0.001	0.4	3.41	3.42×10^{-6}	-	-
0.001	0.5	2.94**	2.39×10^{-6}	-	-
0.1	0.1	4.1	5.08×10^{-6}	1.88	5.98×10^{-7}
0.1	0.2	3.92	4.59×10^{-6}	2.0	7.79×10^{-7}
0.1	0.3	3.7	4.03×10^{-6}	2.15	1.02×10^{-6}
0.1	0.4	3.40	3.34×10^{-6}	2.38	1.39×10^{-6}
0.1	0.5	-	-	2.82**	2.22×10^{-6}**

** Solution did not converge in 20 iterations and is not considered reliable.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-01 K3= 1.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.5000000000D 01	0.5000000000D-05
1	0.4733762972D 01	0.5433071349D-05
2	0.4526562584D 01	0.5454333473D-05
3	0.4375323004D 01	0.5360015536D-05
4	0.4270975962D 01	0.5259907533D-05
5	0.4203232573D 01	0.5183187332D-05
6	0.4161323855D 01	0.5132000856D-05
7	0.4136226451D 01	0.5100190831D-05
8	0.4121313005D 01	0.5080665473D-05
9	0.4111573450D 01	0.5066187720D-05
10	0.4125227288D 01	0.5118030600D-05
11	0.4115488437D 01	0.5099437398D-05
12	0.4104483976D 01	0.5075156770D-05
13	0.4108951055D 01	0.5075090383D-05
14	0.4108776583D 01	0.5074449931D-05

ND= 1 FREQUENCY= 4.10860D 00RAD/SEC AMPLITUDE= 5.07397D-06 NIT= 14

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-02 K3= 1.00000D-02

ITERATION	FREQUENCY	AMPLITUDE
0	0.4108597203D 01	0.5073966584D-05
1	0.4168820145D 01	0.5248288860D-05
2	0.4205531785D 01	0.5364353532D-05
3	0.4226335066D 01	0.5432259787D-05
4	0.4237936472D 01	0.5470663581D-05
5	0.4244508200D 01	0.5492764659D-05
6	0.4246721288D 01	0.5477545295D-05
7	0.4253298522D 01	0.5511028750D-05
8	0.4253818862D 01	0.5517540846D-05
9	0.4253573182D 01	0.5522607930D-05
10	0.4262696521D 01	0.5520588880D-05
11	0.4255858006D 01	0.5512477452D-05
12	0.4255246471D 01	0.5516839666D-05

ND= 1 FREQUENCY= 4.25522D 00RAD/SEC AMPLITUDE= 5.52114D-06 NIT= 12

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-03 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.4255217824D 01	0.5521142485D-05
1	0.4260676424D 01	0.5542795818D-05
2	0.4256534816D 01	0.5327396968D-05
3	0.4252613070D 01	0.5420675622D-05
4	0.4261846964D 01	0.5496517630D-05
5	0.4266693459D 01	0.5536135509D-05
6	0.4269530544D 01	0.5557835647D-05
7	0.4269042818D 01	0.5561545739D-05

ND= 1 FREQUENCY= 4.26902D 00RAD/SEC AMPLITUDE= 5.56645D-06 NIT= 7

Figure 1-12.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01     K3= 1.00000D-01

  ITERATION     FREQUENCY               AMPLITUDE
       0     0.2000000000D 01     0.1000000000D-05
       1     0.1849233024D 01     0.6242848602D-06
       2     0.1852950575D 01     0.5882981170D-06
       3     0.1867139237D 01     0.5930626214D-06
       4     0.1874829545D 01     0.5962272806D-06
       5     0.1878890231D 01     0.5979138586D-06
       6     0.1880129011D 01     0.5979328084D-06
       7     0.1880201346D 01     0.5986153910D-06
       8     0.1880334350D 01     0.5986705996D-06

ND= 1   FREQUENCY= 1.88042D 00RAD/SEC   AMPLITUDE= 5.98812D-07   NIT=  8
```

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-02     K3= 1.00000D-02

  ITERATION     FREQUENCY               AMPLITUDE
       0     0.1880421807D 01     0.5988118404D-06
       1     0.1816575499D 01     0.5204743608D-06
       2     0.1787791116D 01     0.4874090332D-06
       3     0.1774071480D 01     0.4720024797D-06
       4     0.1767310255D 01     0.4645333492D-06
       5     0.1763726929D 01     0.4605878423D-06
       6     0.1758874217D 01     0.4525793409D-06
       7     0.1758812030D 01     0.4613877605D-06
       8     0.1758882166D 01     0.4583095382D-06
       9     0.1759606827D 01     0.4577612065D-06
      10     0.1759617287D 01     0.4581187707D-06

ND= 1   FREQUENCY= 1.75963D 00RAD/SEC   AMPLITUDE= 4.58129D-07   NIT= 10
```

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-03     K3= 1.00000D-03

  ITERATION     FREQUENCY               AMPLITUDE
       0     0.1759634012D 01     0.4581286762D-06
       1     0.1754793059D 01     0.4515838346D-06
       2     0.1752096824D 01     0.4481194314D-06
       3     0.1743716093D 01     0.4323366512D-06
       4     0.1744785219D 01     0.4403176104D-06
       5     0.1746624895D 01     0.4423270886D-06
       6     0.1747172576D 01     0.4436675832D-06
       7     0.1747595835D 01     0.4442578922D-06
       8     0.1747828656D 01     0.4446765861D-06

ND= 1   FREQUENCY= 1.74799D 00RAD/SEC   AMPLITUDE= 4.44921D-07   NIT=  8
```

Figure 1-13.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07  TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-02      K3= 1.00000D-03

  ITERATION     FREQUENCY              AMPLITUDE
       0      0.4269023768D 01     0.5566448250D-05
       1      0.4260430902D 01     0.5563841178D-05
       2      0.4267484262D 01     0.5564859360D-05
       3      0.4272363751D 01     0.5564759926D-05
       4      0.4272512787D 01     0.5571703289D-05
       5      0.4272700920D 01     0.5571487431D-05

ND= 1   FREQUENCY= 4.27286D 00RAD/SEC   AMPLITUDE= 5.57162D-06   NIT=  5

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07  TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01      K3= 1.00000D-03

  ITERATION     FREQUENCY              AMPLITUDE
       0      0.4272860404D 01     0.5571617068D-05
       1      0.4277637363D 01     0.5561946869D-05
       2      0.4275891190D 01     0.5550585436D-05
       3      0.4275463586D 01     0.5545271994D-05

ND= 1   FREQUENCY= 4.27574D 00RAD/SEC   AMPLITUDE= 5.54293D-06   NIT=  3

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07  TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D 00      K3= 1.00000D-03

  ITERATION     FREQUENCY              AMPLITUDE
       0      0.4275736610D 01     0.5542934874D-05
       1      0.4335235297D 01     0.5481663112D-05
       2      0.4324421026D 01     0.5365104085D-05
       3      0.4316383079D 01     0.5303160811D-05
       4      0.4312372981D 01     0.5272793022D-05
       5      0.4310288669D 01     0.5257432430D-05
       6      0.4309114710D 01     0.5249302855D-05
       7      0.4307622250D 01     0.5243785475D-05
       8      0.4302486615D 01     0.5236904245D-05
       9      0.4307060453D 01     0.5237003046D-05
      10      0.4306932848D 01     0.5236915323D-05

ND= 1   FREQUENCY= 4.30676D 00RAD/SEC   AMPLITUDE= 5.23683D-06   NIT= 10
```

Figure 1-14a.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D 00     K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.4300000000D 01     0.5200000000D-05
       1      0.4305281886D 01     0.5228303901D-05
       2      0.4305283850D 01     0.5222453439D-05
       3      0.4308744870D 01     0.5245748728D-05
       4      0.4308634353D 01     0.5243241015D-05

ND= 1    FREQUENCY= 4.30865D 00RAD/SEC    AMPLITUDE= 5.24185D-06    NIT=  4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 2.00000D 00     K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.4308645193D 01     0.5241850229D-05
       1      0.4375960005D 01     0.5185863190D-05
       2      0.4360033500D 01     0.5047381412D-05
       3      0.4349924772D 01     0.4976486582D-05
       4      0.4344905380D 01     0.4941913249D-05
       5      0.4342313852D 01     0.4924475958D-05
       6      0.4340863107D 01     0.4915247605D-05
       7      0.4339884306D 01     0.4909947550D-05
       8      0.4339062983D 01     0.4906586223D-05

ND= 1    FREQUENCY= 4.33810D 00RAD/SEC    AMPLITUDE= 4.90415D-06    NIT=  8

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 3.00000D 00     K3= 1.00000D-03

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.4338100273D 01     0.4904152108D-05
       1      0.4407947330D 01     0.4861009128D-05
       2      0.4387383490D 01     0.4711098352D-05
       3      0.4375506949D 01     0.4636301815D-05
       4      0.4369644072D 01     0.4599948927D-05
       5      0.4366618831D 01     0.4581578394D-05
       6      0.4364918225D 01     0.4571778521D-05
       7      0.4363815507D 01     0.4566133105D-05
       8      0.4362939491D 01     0.4562548417D-05

ND= 1    FREQUENCY= 4.36199D 00RAD/SEC    AMPLITUDE= 4.55997D-06    NIT=  8
```

Figure 1-14b.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 4.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.4361988175D 01	0.4559965684D-05
1	0.4432440367D 01	0.4525930616D-05
2	0.4405631554D 01	0.4360942028D-05
3	0.4391262016D 01	0.4280822026D-05
4	0.4384243852D 01	0.4242108964D-05
5	0.4380628732D 01	0.4222494599D-05
6	0.4378590830D 01	0.4211923921D-05
7	0.4377394674D 01	0.4205893989D-05
8	0.4376650705D 01	0.4202218261D-05

ND= 1 FREQUENCY= 4.37615D 00RAD/SEC AMPLITUDE= 4.19983D-06 NIT= 8

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 5.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.4376148613D 01	0.4199830027D-05
1	0.4446268690D 01	0.4172529128D-05
2	0.4410723393D 01	0.3986239376D-05
3	0.4392316296D 01	0.3897312851D-05
4	0.4383497914D 01	0.3854826622D-05
5	0.4378966024D 01	0.3833214180D-05
6	0.4376399041D 01	0.3821392659D-05
7	0.4374898308D 01	0.3814536904D-05
8	0.4374036350D 01	0.3810350253D-05
9	0.4373567032D 01	0.3807689161D-05

ND= 1 FREQUENCY= 4.37333D 00RAD/SEC AMPLITUDE= 3.80595D-06 NIT= 9

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 6.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.4373325369D 01	0.3805950594D-05
1	0.4443688083D 01	0.3786225692D-05
2	0.4393716014D 01	0.3563944711D-05
3	0.4366906568D 01	0.3456015384D-05
4	0.4354547527D 01	0.3405764070D-05
5	0.4348162746D 01	0.3379916365D-05
6	0.4344485644D 01	0.3365382272D-05
7	0.4342266743D 01	0.3356649144D-05
8	0.4341147983D 01	0.3351365017D-05
9	0.4340833114D 01	0.3348264128D-05

ND= 1 FREQUENCY= 4.34081D 00RAD/SEC AMPLITUDE= 3.34647D-06 NIT= 9

Figure 1-14c.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N◆G1+N◆N◆G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 7.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.4340812381D 01	0.3346473373D-05
1	0.4408645049D 01	0.3325789758D-05
2	0.4325039146D 01	0.3018597257D-05
3	0.4262701797D 01	0.2828294047D-05
4	0.4233182205D 01	0.2738989086D-05
5	0.4216530616D 01	0.2688925065D-05
6	0.4204935306D 01	0.2655545194D-05
7	0.4197896293D 01	0.2634100150D-05
8	0.4188751327D 01	0.2612619774D-05
9	0.4189724044D 01	0.2606441657D-05
10	0.4191211536D 01	0.2604446450D-05

ND= 1 FREQUENCY= 4.19137D 00RAD/SEC AMPLITUDE= 2.60430D-06 NIT= 10

LST SYSTEM-NUMERICAL SOLUTION OF 1+N◆G1+N◆N◆G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 9.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.4191366586D 01	0.2604300965D-05
1	0.4320830567D 01	0.2755449920D-05
2	0.3885654674D 01	0.1537227546D-05
3	0.3992688964D 01	0.1777098203D-05
4	0.4101211254D 01	0.2018197669D-05
5	0.4049068157D 01	0.2133752715D-05
6	0.4218650868D 01	0.2442808016D-05
7	0.5328541039D 01	0.5121542808D-05
8	0.4932374097D 01	0.4090396161D-05
9	0.4580742990D 01	0.2899763479D-05
10	0.4272683117D 01	0.2338930959D-05
11	0.4726012700D 01	0.3527236013D-05
12	0.4642462032D 01	0.2206214287D-05
13	0.4102385161D 01	0.1712607528D-05
14	0.4511000940D 01	0.2807657101D-05
15	0.4185049431D 01	0.2143995240D-05
16	0.4337921541D 01	0.2566385783D-05
17	0.4584719291D 01	0.3172396064D-05
18	0.4410313136D 01	0.2595417616D-05
19	0.3901533720D 01	0.1457090202D-05
20	0.3951355389D 01	0.1500036479D-05

ND= 1 FREQUENCY= 3.95136D 00RAD/SEC AMPLITUDE= 1.50004D-06 NIT= 20

Figure 1-14d.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-02     K3= 1.00000D-03

  ITERATION    FREQUENCY            AMPLITUDE
       0     0.1747994849D 01    0.4449210692D-06
       1     0.1750970353D 01    0.4459628356D-06
       2     0.1751578924D 01    0.4458610524D-06
       3     0.1751663855D 01    0.4463090840D-06
       4     0.1751762687D 01    0.4463992173D-06

ND= 1   FREQUENCY= 1.75183D 00RAD/SEC   AMPLITUDE= 4.46504D-07   NIT=  4


LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D-01     K3= 1.00000D-03

  ITERATION    FREQUENCY            AMPLITUDE
       0     0.1751834176D 01    0.4465041105D-06
       1     0.1769294195D 01    0.4510101543D-06
       2     0.1778904699D 01    0.4546263062D-06
       3     0.1784009536D 01    0.4567644431D-06
       4     0.1786666712D 01    0.4577447044D-06
       5     0.1786879223D 01    0.4579707749D-06

ND= 1   FREQUENCY= 1.78701D 00RAD/SEC   AMPLITUDE= 4.58193D-07   NIT=  5


LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03    K2= 1.00000D 00     K3= 1.00000D-03

  ITERATION    FREQUENCY            AMPLITUDE
       0     0.1787010012D 01    0.4581926081D-06
       1     0.1866463182D 01    0.3314511173D-06
       2     0.1953999120D 01    0.4148348970D-06
       3     0.2021290853D 01    0.4825286553D-06
       4     0.2064528709D 01    0.5264398004D-06
       5     0.2089101053D 01    0.5514356079D-06
       6     0.2101978469D 01    0.5644300305D-06
       7     0.2108266521D 01    0.5706690202D-06
       8     0.2110689098D 01    0.5735103622D-06
       9     0.2111477387D 01    0.5755678257D-06
      10     0.2112135359D 01    0.5764854872D-06
      11     0.2112491813D 01    0.5772192262D-06
      12     0.2112768819D 01    0.5776432494D-06

ND= 1   FREQUENCY= 2.11296D 00RAD/SEC   AMPLITUDE= 5.77976D-07   NIT= 12
```

Figure 1-15a.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.000000D-01 JV= 1.000000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2100000000D 01	0.5800000000D-06
1	0.2105819298D 01	0.5764408218D-06
2	0.2110683445D 01	0.5777939437D-06
3	0.2113400637D 01	0.5784885398D-06
4	0.2113458576D 01	0.5787551958D-06

ND= 1 FREQUENCY= 2.11352D 00RAD/SEC AMPLITUDE= 5.78829D-07 NIT= 4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.000000D-01 JV= 1.000000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 2.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2113522368D 01	0.5788294272D-06
1	0.2198783766D 01	0.4745734256D-06
2	0.2286273095D 01	0.5541505311D-06
3	0.2351691717D 01	0.6233706268D-06
4	0.2392340682D 01	0.6677152962D-06
5	0.2414543718D 01	0.6919494671D-06
6	0.2425574373D 01	0.7037403848D-06
7	0.2430351540D 01	0.7088785615D-06
8	0.2431880854D 01	0.7120244100D-06
9	0.2432839304D 01	0.7138996007D-06
10	0.2433482653D 01	0.7151060090D-06
11	0.2433935486D 01	0.7159386678D-06
12	0.2434270538D 01	0.7165388026D-06

ND= 1 FREQUENCY= 2.43453D 00RAD/SEC AMPLITUDE= 7.16991D-07 NIT= 12

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.000000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 3.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2434527302D 01	0.7169908360D-06
1	0.2530703725D 01	0.6753917942D-06
2	0.2615049668D 01	0.7473016901D-06
3	0.2671818957D 01	0.8057580249D-06
4	0.2703849298D 01	0.8395671944D-06
5	0.2719819820D 01	0.8557534471D-06
6	0.2726656058D 01	0.8617656173D-06
7	0.2728173461D 01	0.8639006049D-06
8	0.2728919117D 01	0.8658976396D-06
9	0.2729603251D 01	0.8669499962D-06
10	0.2730045451D 01	0.8679340734D-06
11	0.2730421824D 01	0.8685866208D-06

ND= 1 FREQUENCY= 2.73071D 00RAD/SEC AMPLITUDE= 8.69150D-07 NIT= 11

Figure 1-15b.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 4.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2730711241D 01	0.8691498890D-06
1	0.2832761272D 01	0.8690562992D-06
2	0.2914072369D 01	0.9404417447D-06
3	0.2965207509D 01	0.9931230333D-06
4	0.2992799595D 01	0.1021781452D-05
5	0.3006306516D 01	0.1034869396D-05
6	0.3011143382D 01	0.1038001938D-05
7	0.3011605306D 01	0.1040285461D-05
8	0.3012294614D 01	0.1040916602D-05
9	0.3012666960D 01	0.1041986665D-05
10	0.3013041149D 01	0.1042589602D-05

ND= 1 FREQUENCY= 3.01333D 00RAD/SEC AMPLITUDE= 1.04321D-06 NIT= 10

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 5.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.3013329729D 01	0.1043207423D-05
1	0.3115885948D 01	0.1065992225D-05
2	0.3195716654D 01	0.1142568839D-05
3	0.3245957530D 01	0.1198719877D-05
4	0.3273574471D 01	0.1230191890D-05
5	0.3286982993D 01	0.1244194422D-05
6	0.3289964384D 01	0.1245681245D-05
7	0.3290445550D 01	0.1248731532D-05
8	0.3291333009D 01	0.1249376945D-05
9	0.3291765982D 01	0.1250826172D-05
10	0.3292244767D 01	0.1251583502D-05

ND= 1 FREQUENCY= 3.29261D 00RAD/SEC AMPLITUDE= 1.25244D-06 NIT= 10

Figure 1-15c.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 6.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.3292608903D 01	0.1252441749D-05
1	0.3393118204D 01	0.1288015344D-05
2	0.3476648004D 01	0.1382122654D-05
3	0.3532419145D 01	0.1455593528D-05
4	0.3564585758D 01	0.1499204802D-05
5	0.3578292446D 01	0.1515064339D-05
6	0.3579959517D 01	0.1517891216D-05
7	0.3581068138D 01	0.1520970971D-05
8	0.3582153412D 01	0.1523094025D-05
9	0.3582999113D 01	0.1525178838D-05
10	0.3583776544D 01	0.1526827869D-05
11	0.3584436003D 01	0.1528349560D-05

ND= 1 FREQUENCY= 3.58503D 00RAD/SEC AMPLITUDE= 1.52965D-06 NIT= 11

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 7.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.3585029455D 01	0.1529650116D-05
1	0.3683549918D 01	0.1575826583D-05
2	0.3782302317D 01	0.1715782565D-05
3	0.3860384989D 01	0.1845847172D-05
4	0.3910084363D 01	0.1930617649D-05
5	0.3921557362D 01	0.1947309300D-05
6	0.3927955010D 01	0.1959014848D-05
7	0.3931729440D 01	0.1990015234D-05
8	0.3949752319D 01	0.2014021481D-05
9	0.3950920814D 01	0.2024242421D-05
10	0.3954783767D 01	0.2027433804D-05
11	0.3956024832D 01	0.2033021197D-05
12	0.3957889057D 01	0.2035757353D-05
13	0.3959142055D 01	0.2039619541D-05
14	0.3960536245D 01	0.2042465906D-05
15	0.3961719070D 01	0.2045558664D-05
16	0.3962898757D 01	0.2048246234D-05
17	0.3963979662D 01	0.2050905635D-05
18	0.3965020801D 01	0.2053361281D-05
19	0.3966000409D 01	0.2055723118D-05
20	0.3966936161D 01	0.2057952294D-05

ND= 1 FREQUENCY= 3.96694D 00RAD/SEC AMPLITUDE= 2.05795D-06 NIT= 20

Figure 1-15d.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D 00     K3= 2.00000D-01

  ITERATION    FREQUENCY              AMPLITUDE
      0      0.3900000000D 01     0.4200000000D-05
      1      0.3911764448D 01     0.4198154369D-05
      2      0.3910445465D 01     0.4185610296D-05
      3      0.3907288862D 01     0.4175505017D-05
      4      0.3909715946D 01     0.4174222859D-05

ND= 1   FREQUENCY= 3.91004D 00RAD/SEC   AMPLITUDE= 4.17378D-06    NIT=  4

LST SYSTEM-NUMERICAL SOLUTAON OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 2.00000D 00     K3= 2.00000D-01

  ITERATION    FREQUENCY              AMPLITUDE
      0      0.3910038835D 01     0.4173781422D-05
      1      0.3944858612D 01     0.4034291927D-05
      2      0.3902493997D 01     0.3834529223D-05
      3      0.3880015397D 01     0.3738445839D-05
      4      0.3869325567D 01     0.3692951635D-05
      5      0.3863963301D 01     0.3670144295D-05
      6      0.3861018723D 01     0.3657865665D-05
      7      0.3859279764D 01     0.3650781946D-05
      8      0.3858194339D 01     0.3646410253D-05
      9      0.3857484154D 01     0.3643544224D-05

ND= 1   FREQUENCY= 3.85700D 00RAD/SEC   AMPLITUDE= 3.64157D-06    NIT=  9

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 3.00000D 00     K3= 2.00000D-01

  ITERATION    FREQUENCY              AMPLITUDE
      0      0.3856997882D 01     0.3641569608D-05
      1      0.3868151877D 01     0.3446356451D-05
      2      0.3795140139D 01     0.3178207222D-05
      3      0.3744743850D 01     0.3023885812D-05
      4      0.3720285047D 01     0.2950694083D-05
      5      0.3708526518D 01     0.2914399703D-05
      6      0.3700756579D 01     0.2891913462D-05
      7      0.3696893587D 01     0.2879208811D-05
      8      0.3689410851D 01     0.2864645745D-05
      9      0.3692449829D 01     0.2862905116D-05

ND= 1   FREQUENCY= 3.692(4D 00RAD/SEC   AMPLITUDE= 2.86133D-06    NIT=  9
```

Figure 1-16.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 2.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2400000000D 01	0.1000000000D-05
1	0.2409479441D 01	0.1046712564D-05
2	0.2399290724D 01	0.1038804169D-05
3	0.2394536278D 01	0.1035725356D-05
4	0.2390864967D 01	0.1031368943D-05
5	0.2395076850D 01	0.1046663017D-05
6	0.2398404185D 01	0.1051367019D-05
7	02400539326D 01	0.1042485398D-05
8	0.2392532346D 01	0.1031818835D-05
9	0.2392904613D 01	0.1037398336D-05
10	0.2393401118D 01	0.1040674411D-05
11	0.2393603834D 01	0.1040720365D-05

ND= 1 FREQUENCY= 2.39389D 00RAD/SEC AMPLITUDE= 1.04135D-06 NIT= 11

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 2.00000D 00 K3= 2.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2393886185D 01	0.1041351791D-05
1	0."545583278D 01	0.1115190202D-05
2	0.2658949567D 01	0.1222449668D-05
3	0.2730043552D 01	0.1298874812D-05
4	0.2768736905D 01	0.1341388924D-05
5	0.2787645362D 01	0.1361207870D-05
6	0.2794552219D 01	0.1366690838D-05
7	0.2795382620D 01	0.1368716008D-05
8	0.2796186906D 01	0.1370021105D-05

ND= 1 FREQUENCY= 2.79679D 00RAD/SEC AMPLITUDE= 1.37127D-06 NIT= 8

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 3.00000D 00 K3= 2.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2796785954D 01	0.1371271994D-05
1	0.2957809532D 01	0.1502914789D-05
2	0.3082893020D 01	0.1654250751D-05
3	0.3172400981D 01	0.1779866320D-05
4	0.3229810908D 01	0.1865488514D-05
5	0.3255267109D 01	0.1900803739D-05
6	0.3260443833D 01	0.1908847559D-05
7	0.3264018726D 01	0.1916707271D-05
8	0.3266798778D 01	0.1923509780D-05
9	0.3269286411D 01	0.1928721353D-05
10	0.3271371276D 01	0.1933362076D-05
11	0.3273230827D 01	0.1937306384D-05
12	0.3274855980D 01	0.1940826760D-05
1[illegible]	0.327[illegible]D 01	0.1946[illegible]D-05
15	0.3278802721D 01	0.1949234815D-05
16	0.3279876254D 01	0.1951518427D-05
17	0.3280857769D 01	0.1953599601D-05
18	0.3281758233D 01	0.1955508065D-05

ND= 1 FREQUENCY= 3.28259D 00RAD/SEC AMPLITUDE= 1.95726D-06 NIT= 18

Figure 1-17.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-03     K3= 1.00000D-03

  ITERATION     FREQUENCY           AMPLITUDE
      0      0.4300000000D 01    0.5500000000D-05
      1      0.4271086058D 01    0.5508161796D-05
      2      0.4265786694D 01    0.5527424953D-05
      3      0.4268750642D 01    0.5552197589D-05
      4      0.4290753071D 01    0.5645724078D-05
      5      0.4278015463D 01    0.5604179382D-05
      6      0.4271073196D 01    0.5582480418D-05
      7      0.4272476170D 01    0.5579216017D-05

ND= 1   FREQUENCY= 4.27381D 00RAD/SEC   AMPLITUDE= 5.57884D-06   NIT=  7

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-03     K3= 1.00000D-02

  ITERATION     FREQUENCY           AMPLITUDE
      0      0.4273811433D 01    0.5578838989D-05
      1      0.4265165280D 01    0.5561562337D-05
      2      0.4256891908D 01    0.5539854734D-05
      3      0.4258571915D 01    0.5537666565D-05

ND= 1   FREQUENCY= 4.25947D 00RAD/SEC   AMPLITUDE= 5.53740D-06   NIT=  3

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-03     K3= 1.00000D-01

  ITERATION     FREQUENCY           AMPLITUDE
      0      0.4259473371D 01    0.5537397133D-05
      1      0.4196212759D 01    0.5385323809D-05
      2      0.4154989128D 01    0.5265320509D-05
      3      0.4130148775D 01    0.5190949316D-05
      4      0.4115216330D 01    0.5146566295D-05
      5      0.4107031778D 01    0.5123226637D-05
      6      0.4109286422D 01    .5120752653D-05

ND= 1   FREQUENCY= 4.11028D 00RAD/SEC   AMPLITUDE= 5.12006D-06   NIT=  6
```

Figure 1-18a.

```
LST SYSTEM-NUMERIAAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-03     K= 2.00000D-01

  ITERATION    FREQUENCY            AMPLITUDE
      Ø0      0.4110284514D 01    0.5120057257D-05
       1      0.4032019881D 01    0.4940948570D-05
       2      0.3980474639D 01    0.4799138365D-05
       3      0.3948897805D 01    0.4710444853D-05
       4      0.3929525533D 01    0.4656795911D-05
       5      0.3920240815D 01    0.4631418674D-05
       6      0.3922604706D 01    0.4628602863D-05

ND= 1   FREQUENCY= 3.92377D 00RAD/SEC   AMPLITUDE= 4.62766D-06   NIT=  6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.0000D-03     K3= 3.00000D-01

  ITERATION    FREQUENCY            AMPLITUD

       0      0.3923766777D 01    0.4627662693D-05
       1      0.3832689293D 01    0.4430973619D-05
       2      0.3770977701D 01    0.4271935832D-05
       3      0.3731825309D 01    0.4170216481D-05
       4      0.3706733818D 01    0.4106969040D-05
       5      0.3702042847D 01    0.4090408277D-05
       6      0.3701950079D 01    0.4084753797D-05
       7      0.3704511777D 01    0.4084614961D-05

ND= 1   FREQUENCY= 3.70495D 00RAD/SEC   AMPLITUDE= 4.08573D-06   NIT=  7

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-03     K3= 4.00000D-01

  ITERATION    FREQUENCY            AMPLITUDE
       0      0.3704948126D 01    0.4085726824D-05
       1      0.3590600302D 01    0.3852326005D-05
       2      0.3508515778D 01    0.3657206541D-05
       3      0.3451606441D 01    0.3523805145D-05
       4      0.3412420645D 01    0.3436756348D-05
       5      0.3416493548D 01    0.3433169185D-05
       6      0.3408329008D 01    0.3418746939D-05
       7      0.3412284461D 01    0.3418436857D-05
       8      0.3412294387D 01    0.3417129272D-05

ND= 1   FREQUENCY= 3.41234D 00RAD/SEC   AMPLITUDE= 3.41641D-06   NIT=  8
```

Figure 1-18b.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-03 K3= 5.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.3412341331D 01	0.3416405203D-05
1	0.3241064820D 01	0.3083542987D-05
2	0.3077393747D 01	0.2738920401D-05
3	0.2804995297D 01	0.2210792560D-05
4	0.2920590092D 01	0.2394592414D-05
5	0.2947896527D 01	0.2746799724D-05
6	0.3223599934D 01	0.3158399913D-05
7	0.3039734314D 01	0.2693238377D-05
8	0.2821487226D 01	0.2247034327D-05
9	0.3001552640D 01	0.2542816655D-05
10	0.3097700472D 01	0.2720433834D-05
11	0.2749506483D 01	0.1990339067D-05
12	0.2948088164D 01	0.2408982921D-05
13	0.3295723942D 01	0.3130139548D-05
14	0.3159659222D 01	0.3069275282D-05
15	0.3022611977D 01	0.2679898335D-05
16	0.2858930557D 01	0.2324232956D-05
17	0.468415503D 01	0.3383591544D-05
18	0.3320392030D 01	0.3057606152D-05
19	0.3141388572D 01	0.2765634620D-05
20	0.2941220353D 01	0.2394984045D-05

ND= 1 FREQUENCY= 2.94122D 00RAD/SEC AMPLITUDE= 2.39498D-06 NIT= 20

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-03 K3= 6.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2941220353D 01	0.2394984045D-05
1	0.3168814268D 01	0.3627195910D-05
2	0.2703307611D 01	0.2052990628D-05
3	0.3160507042D 01	0.2863909229D-05
4	0.2695124944D 01	0.2013043232D-05
5	0.3115544834D 01	0.2763671945D-05
6	0.2633809736D 01	0.1937538081D-05
7	0.2999865321D 01	0.2567990764D-05
8	0.1973349554D 01	0.6483543094D-06
9	0.2053560184D 01	0909766304 8D-06
10	0.2256811360D 01	0.1229528999D-05
11	0.2485081417D 01	0.1592105676D-05
12	0.2952122537D 01	0.2199870999D-05
13	-0.5763678769D 01	-0.1319216559D-04
1 -5.76368D 00	-1.31922D-05	
14	0.9999999996D 00	0.9999999996D 00
15	0.1000000000D 01	0.1000000000D 01

ND= 1 FREQUENCY= 1.00000D 00RAD/SEC AMPLITUDE= 1.00000D 00 NIT= 15

Figure 1-18c.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-03 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.1700000000D 01	0.4500000000D-06
1	0.1719691894D 01	0.4377777686D-06
2	0.1733817210D 01	0.4401730766D-06
3	0.1741656390D 01	0.4426018322D-06
4	0.1745891631D 01	0.4441635449D-06
5	0.1748065537D 01	0.4447812824D-06
6	0.1748141543D 01	0.4451726996D-06

ND= 1 FREQUENCY= 1.74823D 00RAD/SEC AMPLITUDE= 4.45254D-07 NIT= 6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-03 K3= 1.00000D-02

ITERATION	FREQUENCY	AMPLITUDE
0	0.1748227280D 01	0.4452536775D-06
1	0.1752126778D 01	0.4502796250D-06
2	0.1753650190D 01	0.4529774839D-06
3	0.1754421161D 01	0.4544379007D-06
4	0.1754888859D 01	0.4552138844D-06
5	0.1755172026D 01	0.4556845266D-06
6	0.1755362634D 01	0.4559779957D-06

ND= 1 FREQUENCY= 1.75550D 00RAD/SEC AMPLITUDE= 4.56178D-07 NIT= 6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-03 K3= 1.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.1755497191D 01	0.4561784994D-06
1	0.1791779346D 01	0.5101612925D-06
2	0.1813712215D 01	0.5420184214D-06
3	0.1826081811D 01	0.5595830487D-06
4	0.1832621721D 01	0.5687897904D-06
5	0.1835348001D 01	0.5738808560D-06
6	0.1836420820D 01	0.5768742238D-06
7	0.1837388002D 01	0.5780171524D-06
8	0.1837823626D 01	0.5789107407D-06
9	0.1838174721D 01	0.5793698723D-06

ND= 1 FREQUENCY= 1.83840D 00RAD/SEC AMPLITUDE= 5.79735D-07 NIT= 9

Figure 1-19a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03     K2= 1.00000D-03     K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0      0.1838400034D 01     0.5797354291D-06
        1      0.1886569754D 01     0.6511742156D-06
        2      0.1917483403D 01     0.6957220257D-06
        3      0.1936546542D 01     0.7211874011D-06
        4      0.1927945480D 01     0.7499418438D,06
        5      0.1938822886D 01     0.7468010454D-06
        6      0.1947515177D 01     0.7500348081D-06
        7      0.1952344239D 01     0.7521033896D-06
        8      0.1954077725D 01     0.7522564047D-06

ND= 1    FREQUENCY= 1.95414D 00RAD/SEC    AMPLITUDE= 7.52932D-07    NIT=  8

 LST SYSTEM-NUMERICAL SOLUTIOF OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
 KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02      K0= 5.75835D 03
 K1= 1.37102D 03     K2= 1.00000D-03     K3= 3.00000D-01

   ITERATION     FREQUENCY              AMPITUDE
        0      0.1954136225D 01     0.7529324744D-06
        1      0.2015669259D 01     0.8437249890D-06
        2      0.2059812684D 01     0.9034212199D-06
        3      0.2063822449D 01     0.9620807453D-06
        4      0.2084573235D 01     0.9719225440D-06
        5      0.209641411LD 01     0.9788230989D-06
        6      0.2102542141D 01     0.9822800761D-06
        7      0.2104071397D 01     0.9823954219D-06

ND= 1    FREQUENCY= 2.10420D 00RAD/OEC    AMPLITUDE= 9.83224D-05    NIT=  7
```

Figure 1-19b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03    H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01    K3= 1.00000D-01

   ITERATION    FREQUENCY            AMPLITUDE
       0     0.4000000000D 01    0.5000000000D-05
       1     0.4069348489D 01    0.5060816759D-05
       2     0.4096002875D 01    0.5075146970D-05
       3     0.4105852505D 01    0.5077069171D-05
       4     0.4109947410D 01    0.5076838934D-05

ND= 1   FREQUENCY= 4.10996D 00RAD/SEC   AMPLITUDE= 5.07630D-06   NIT=  4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03    H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01    K3= 2.00000D-01

   ITERATION    FREQUENCY            AMPLITUDE
       0     0.4109963752D 01    0.5076302789D-05
       1     0.4031709712D 01    0.4897281315D-05
       2     0.3980008704D 01    0.4755356981D-05
       3     0.3948237513D 01    0.4666438611D-05
       4     0.3928651596D 01    0.4612520582D-05
       5     0.3920000370D 01    0.4588512642D-05
       6     0.3922292760D 01    0.4585636143D-05

ND= 1   FREQUENCY= 3.92343D 00RAD/SEC   AMPLITUDE= 4.58466D-06   NIT=  6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03    H= 6.00000D 02      K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01    K3= 3.00000D-01

   ITERATION    FREQUENCY            AMPLITUDE
       0     0.3923434006D 01    0.4584662591D-05
       1     0.3830573188D 01    0.4383986105D-05
       2     0.3767396170D 01    0.4221711154D-05
       3     0.3727073921D 01    0.4117495178D-05
       4     0.3701052335D 01    0.4052404157D-05
       5     0.3697334293D 01    0.4037389204D-05
       6     0.3696451845D 01    0.4030788857D-05
       7     0.3700278587D 01    0.4031793320D-05
       8     0.3700684880D 01    0.4034395196D-05

ND= 1   FREQUENCY= 3.70157D 00RAD/SEC   AMPLITUDE= 4.03611D-06   NIT=  8
```

Figure 1-20a.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01       JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01      K3= 4.00000D-01

   ITERATION     FREQUENCY               AMPLITUDE
        0     0.3701567171D 01     0.4036108745D-05
        1     0.3582816527D 01     0.3793426426D-05
        2     0.3496558168D 01     0.3589545285D-05
        3     0.3435511973D 01     0.3447780717D-05
        4     0.3392930038D 01     0.3354372236D-05
        5     0.3398193057D 01     0.3352297996D-05
        6     0.3392147326D 01     0.3340631474D-05
        7     0.3395177657D 01     0.3339691379D-05

ND= 1   FREQUENCY= 3.39539D 00RAD/SEC   AMPLITUDE= 3.33865D-06   NIT=  7

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01       JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01      K3= 5.00000D-01

   ITERATION     FREQUENCY               AMPLITUDE
        0     0.3395365263D 01     0.3338654674D-05
        1     0.3209113520D 01     0.2976214976D-05
        2     0.3008614935D 01     0.2560313865D-05
        3     0.1185413321D 01    -0.8497375904D-06
  1    1.18541D 00    -8.49738D-07
        4     0.1000000098D 01     0.9999999038D 00
        5     0.1000000000D 01     0.1000000000D 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT=  5
```

Figure 1-20b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01     K3= 1.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.1800000000D 01     0.6000000000D-06
       1      0.1829985273D 01     0.5796322434D-06
       2      0.1854650683D 01     0.5871916572D-06
       3      0.1868379408D 01     0.5930486354D-06
       4      0.1875561288D 01     0.5963240748D-06
       5      0.1879253743D 01     0.5978864735D-06
       6      0.1879958027D 01     0.5978609512D-06

ND= 1   FREQUENCY= 1.88008D 00RAD/SEC   AMPLITUDE= 5.98399D-07   NIT=  6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01     K3= 2.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.1880077917D 01     0.5983992937D-06
       1      0.1929412767D 01     0.6723251343D-06
       2      0.1961241662D 01     0.7186786352D-06
       3      0.1981174432D 01     0.7452828246D-06
       4      0.1979638157D 01     0.7715334579D-06
       5      0.1990078720D 01     0.7747937864D-06
       6      0.1996130859D 01     0.7774268861D-06
       7      0.1999099796D 01     0.7783768185D-06
       8      0.1999207661D 01     0.7786887285D-06

ND= 1   FREQUENCY= 1.99932D 00RAD/SEC   AMPLITUDE= 7.78826D-07   NIT=  8

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D-01     K3= 3.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.1999323876D 01     0.7788262524D-06
       1      0.2062672759D 01     0.8734319104D-06
       2      0.2109122091D 01     0.9362249601D-06
       3      0.2115867393D 01     0.1007920039D-05
       4      0.2132853221D 01     0.1009006204D-05
       5      0.2145665678D 01     0.1016009078D-05
       6      0.2152649862D 01     0.1020051049D-05
       7      0.2155018071D 01     0.1020575884D-05
       8      0.2155123139D 01     0.1021438689D-05

ND= 1   FREQUENCY= 2.15534D 00RAD/SEC   AMPLITUDE= 1.02156D-06   NIT=  8
```

Figure 1-21a.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-01 K3= 4.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2155339261D 01	0.1021562747D-05
1	0.2244225191D 01	0.1153988517D-05
2	0.2085370885D 01	0.1261870534D-05
3	0.2180247480D 01	0.1225745261D-05
4	0.2271122614D 01	0.1290986319D-05
5	0.2328959390D 01	0.1343089153D-05
6	0.2361075624D 01	0.1373743172D-05
7	0.2377184815D 01	0.1388691120D-05
8	0.2382499328D 01	0.1392166003D-05
9	0.2383050692D 01	0.1393893795D-05
10	0.2383771714D 01	0.1394708494D-05

ND= 1 FREQUENCY= 2.38425D 00RAD/SEC AMPLITUDE= 1.39567D-06 NIT= 10

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 K0= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D-01 K3= 5.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2384254008D 01	0.1395666760D-05
1	0.2535609988D 01	0.1627192817D-05
2	0.2630189741D 01	0.1909884571D-05
3	0.2806366902D 01	0.2179832984D-05
4	0.2979318718D 01	0.2469654213D-05
5	0.2937010575D 01	0.2512698384D-05
6	0.2589337751D 01	0.1773952983D-05
7	0.2692365763D 01	0.1941448335D-05
8	0.2790768007D 01	0.2105677939D-05
9	0.2816230986D 01	0.2292847674D-05
10	0.3214144945D 01	0.2985565780D-05
11	0.3015994159D 01	0.2575097196D-05
12	0.1819751304D 01	0.3275667481D-06
13	0.1844100614D 01	0.5327149873D-06
14	0.1997997856D 01	0.7845170266D-06
15	0.2160355299D 01	0.1036885407D-05
16	0.2316437477D 01	0.1281658930D-05
17	0.2472457163D 01	0.1520150314D-05
18	0.2503493328D 01	0.1807034130D-05
19	0.2643365118D 01	0.1953910480D-05
20	0.2819086976D 01	0.2217526558D-05

ND= 1 FREQUENCY= 2.81909D 00RAD/SEC AMPLITUDE= 2.21753D-06 NIT= 20

Figure 1-21b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D 00     K3= 1.00000D-03

  ITERATION     FREQUENCY                AMPLITUDE
      0      0.4300000000D 01      0.5200000000D-05
      1      0.4305281886D 01      0.5228303901D-05
      2      0.4305283850D 01      0.5222453439D-05
      3      0.4308744870D 01      0.5245748728D-05
      4      0.4308634353D 01      0.5243241015D-05

ND= 1   FREQUENCY= 4.30865D 00RAD/SEC   AMPLITUDE= 5.24185D-06   NIT=  4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D 00     K3= 1.00000D-02

  ITERATION     FREQUENCY                AMPLITUDE
      0      0.4308645193D 01      0.5241850229D-05
      1      0.4298590514D 01      0.5220253018D-05
      2      0.4287294964D 01      0.5194472604D-05
      3      0.4291913505D 01      0.5194433798D-05
      4      0.4291762417D 01      0.5193867749D-05

ND= 1   FREQUENCY= 4.29161D 00RAD/SEC   AMPLITUDE= 5.19344D-06   NIT=  4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 1.00000D 00     K3= 1.00000D-01

  ITERATION     FREQUENCY                AMPLITUDE
      0      0.4291607580D 01      0.5193440699D-05
      1      0.4222646483D 01      0.5026200492D-05
      2      0.4177094770D 01      0.4895733868D-05
      3      0.4149095004D 01      0.4814071540D-05
      4      0.4131739544D 01      0.4764607870D-05
      5      0.4126277778D 01      0.4746822755D-05
      6      0.4128102003D 01      0.4743764169D-05

ND= 1   FREQUENCY= 4.12899D 00RAD/SEC   AMPLITUDE= 4.74269D-06   NIT=  6
```

Figure 1-22a.

```
 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 2.00000D-01

   ITERATION     FREQUENCY               AMPLITUDE
        0      0.4128986601D 01     0.4742691103D-05
        1      0.4038338318D 01     0.4534458733D-05
        2      0.3977078042D 01     0.4369916734D-05
        3      0.3938140899D 01     0.4264565926D-05
        4      0.3913052885D 01     0.4198877967D-05
        5      0.3909417445D 01     0.4183451086D-05
        6      0.3908686365D 01     0.4176817051D-05
        7      0.3910473948D 01     0.4175575370D-05

ND= 1   FREQUENCY= 3.91085D 00RAD/SEC   AMPLITUDE= 4.17559D-06   NIT=  7


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 3.00000D-01

   ITERATION     FREQUENCY               AMPLITUDE
        0      0.3910850693D 01     0.4175586358D-05
        1      0.3796023096D 01     0.3926734444D-05
        2      0.3712803035D 01     0.3720541479D-05
        3      0.3654173534D 01     0.3577665941D-05
        4      0.3613985577D 01     0.3485187376D-05
        5      0.3619420497D 01     0.3483188329D-05
        6      0.3611493035D 01     0.3468979466D-05
        7      0.3615626748D 01     0.3468727258D-05
        8      0.3615619356D 01     0.3467415619D-05

ND= 1   FREQUENCY= 3.61565D 00RAD/SEC   AMPLITUDE= 3.46667D-06   NIT=  8


 LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
 GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
 KP= 2.16000D 02    KI= 9.70000D 03      H= 6.00000D 02       K0= 5.75835D 03
 K1= 1.37102D 03    K2= 1.00000D 00      K3= 4.00000D-01

   ITERATION     FREQUENCY               AMPLITUDE
        0      0.3615649056D 01     0.3466672496D-05
        1      0.3434297949D 01     0.3093089012D-05
        2      0.3240677649D 01     0.2669980182D-05
        3      0.1809721258D 01    -0.1572573193D-06
  1    1.80972D 00    -1.57257D-07
IHC900I EXECUTION TERMINATING DUE TO ERROR COUNT FOR ERROR NUMBER  261
```

Figure 1-22b.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 1.00000D-03

ITERATION	FREQUENCY	AMPLITUDE
0	0.2100000000D 01	0.5800000000D-06
1	0.2105819298D 01	0.5764408218D-06
2	0.2110683445D 01	0.5777939437D-06
3	0.2113400637D 01	0.5784885398D-06
4	0.2113458576D 01	0.5787551958D-06

ND= 1 FREQUENCY= 2.11352D 00RAD/SEC AMPLITUDE= 5.78829D-07 NIT= 4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 1.00000D-02

ITERATION	FREQUENCY	AMPLITUDE
0	0.2113522368D 01	0.5788294272D-06
1	0.2118091876D 01	0.5850930129D-06
2	0.2119895218D 01	0.5887325060D-06
3	0.2120905168D 01	0.5907279733D-06
4	0.2121552444D 01	0.5918624186D-06
5	0.2121967930D 01	0.5926023104D-06
6	0.2122261884D 01	0.5930924560D-06

ND= 1 FREQUENCY= 2.12248D 00RAD/SEC AMPLITUDE= 5.93446D-07 NIT= 6

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 1.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2122476046D 01	0.5934460004D-06
1	0.2167383373D 01	0.6662858378D-06
2	0.2195828091D 01	0.7114007488D-06
3	0.2212832342D 01	0.7372180210D-06
4	0.2224826790D 01	0.7496457755D-06
5	0.2227666826D 01	0.7599491087D-06
6	0.2230133199D 01	0.7862067825D-06
7	0.2226734026D 01	0.7706783119D-06
8	0.2230362791D 01	0.7714213880D-06
9	0.2231377553D 01	0.7708596286D-06

ND= 1 FREQUENCY= 2.23141D 00RAD/SEC AMPLITUDE= 7.71332D-07 NIT= 9

Figure 1-23a.

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 2.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2231405519D 01	0.7713323182D-06
1	0.2291896995D 01	0.8703784724D-06
2	0.2333268636D 01	0.9360260647D-06
3	0.2367492400D 01	0.9747466296D-06
4	0.2375837684D 01	0.1001508909D-05
5	0.2379792987D 01	0.1013459034D-05
6	0.2382208495D 01	0.1021661878D-05
7	0.2382672134D 01	0.1026021014D-05
8	0.2384497135D 01	0.1026574826D-05
9	0.2384769959D 01	0.1027982124D-05
10	0.2385228987D 01	0.1028302341D-05

ND= 1 FREQUENCY= 2.38548D 00RAD/SEC AMPLITUDE= 1.02888D-06 NIT= 10

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.

GAMMA= 1.38000D 07 TO= 1.00000D-01 JV= 1.00000D 05 JG= 2.10000D 00
KP= 2.16000D 02 KI= 9.70000D 03 H= 6.00000D 02 KO= 5.75835D 03
K1= 1.37102D 03 K2= 1.00000D 00 K3= 3.00000D-01

ITERATION	FREQUENCY	AMPLITUDE
0	0.2385483461D 01	0.1028881516D-05
1	0.2467801350D 01	0.1165047963D-05
2	0.2539852885D 01	0.1266348362D-05
3	0.2557845090D 01	0.1292374398D-05
4	0.2577636773D 01	0.1344201029D-05
5	0.2588264309D 01	0.1369898918D-05
6	0.2594811438D 01	0.1384321546D-05
7	0.2599548808D 01	0.1395250181D-05
8	0.2602932943D 01	0.1402758971D-05
9	0.2605653351D 01	0.1410399939D-05
10	0.2605424268D 01	0.1425499818D-05
11	0.2612458009D 01	0.1429024937D-05
12	0.2615196373D 01	0.1429353498D-05
13	0.2615238207D 01	0.1430138978D-05

ND= 1 FREQUENCY= 2.61534D 00RAD/SEC AMPLITUDE= 1.43025D-06 NIT= 13

Figure 1-23b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.380000 07 TO= 1.00000D-01        JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02      KI= 9.70000D 03      H= 6.00000D 02       KO= 5.75835D 03
K1= 1.37102D 03      K2= 1.00000D 00      K3= 4.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0      0.2615338329D 01      0.1430252227D-05
        1      0.2749145177D 01      0.1657689676D-05
        2      0.2715500806D 01      0.1833191963D-05
        3      0.2856867045D 01      0.1992147664D-05
        4      0.3018124428D 01      0.2243113599D-05
        5      0.3236273102D 01      0.2630059278D-05
        6      0.2985502150D 01      0.2252579941D-05
        7      0.3176883118D 01      0.2567539012D-05
        8      0.1914112309D 01      0.4284134675D-07
        9      0.2083386846D 01      0.6430288460D-06
       10      0.2288243203D 01      0.9357461008D-06
       11      0.2457510229D 01      0.1195439114D-05
       12      0.2599150032D 01      0.1428264308D-05
       13      0.2719769617D 01      0.1635828728D-05
       14      0.2824434339D 01      0.1820999738D-05
       15      0.2922033575D 01      0.1993800623D-05
       16      0.2880418289D 01      0.2071625349D-05
       17      0.3015374900D 01      0.2259080778D-05

       18      0.3296672556D 01      0.2765230531D-05
       19      0.3005244254D 01      0.2217956259D-05
       20      0.3207792607D 01      0.2574309121D-05

ND= 1    FREQUENCY= 3.20779D 00RAD/SEC    AMPLITUDE= 2.57431D-06    NIT= 20

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01        JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02      KI= 9.70000D 03      H= 6.00000D 02       KO= 5.75835D 03
K1= 1.37102D 03      K2= 1.00000D 00      K3= 5.00000D-01

   ITERATION     FREQUENCY              AMPLITUDE
        0      0.3207792607D 01      0.2574309121D-05
        1      0.9630065600D 00     -0.1820196181D-05
  1     9.63007D-01     -1.82020D-06

IHC900I EXECUTION TERMINATING DUE TO ERROR COUNT FOR ERROR NUMBER  261
```

Figure 1-23c.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 5.00000D 00     K3= 1.00000D-03

   ITERATION     FREQUENCY              AMLITUDE
       0      0.4400000000D 01     0.3800000000D-05
       1      0.4362207906D 01     0.3738621826D-05
       2      0.4366626986D 01     0.3770396609D-05
       3      0.4371378655D 01     0.3796664763D-05
       4      0.4371379386D 01     0.3796971020D-05

ND= 1    FREQUENCY= 4.37138D 00RAD/SEC    AMPLITUDE= 3.79739D-06    NIT=  4

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 5.00000D 00     K3= 1.00000D-02

   ITERATION     FREQUENCY              AMPLITUDE
       0      0.4371378655D 01     0.3797390715D-05
       1      0.4394449719D 01     0.3922771641D-05
       2      0.4394847293D 01     0.3806676407D-05
       3      0.4351626143D 01     0.3715020961D-05
       4      0.4308108330D 01     0.3600618889D-05
       5      0.4326020053D 01     0.3670741887D-05
       6      0.4328794391D 01     0.3684325061D-05
       7      0.4328135618D 01     0.3612592391D-05
       8      0.4327365248D 01     0.3646707945D-05
       9      0.4337145059D 01     0.3694037842D-05
      10      0.4340696990D 01     0.3715332488D-05
      11      0.4340665599D 01     0.3717126124D-05

ND= 1    FREQUENCY= 4.34039D 00RAD/SEC    AMPLITUDE= 3.71822D-06    NIT= 11
```

Figure 1-24a.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03    K2= 5.00000D 00     K3= 1.00000D-01

  ITERATION     FREQUENCY            AMPLITUDE
       0     0.4340393741D 01    0.3718223726D-05
       1     0.4190085991D 01    0.3353391990D-05
       2     0.4044315491D 01    0.2984572933D-05
       3     0.3747709579D 01    0.2300337882D-05
       4     0.3884955159D 01    0.2566378093D-05
       5     0.3894792652D 01    0.2583707284D-05
       6     0.3899572181D 01    0.2662049162D-05
       7     0.3855020759D 01    0.2532341129D-05
       8     0.3721527710D 01    0.2238764305D-05
       9     0.3807352703D 01   0.2394332821D-05
      10     0.3820202516D 01    0.2411078840D-05
      11     0.3826854927D 01    0.2418213325D-05
      12     0.3833505168D 01    0.2441533335D-05
      13     0.3828836738D 01    0.2494032042D-05
      14     0.3810807139D 01    0.2423709029D-05
      15     0.4478769421D 01    0.3745416146D-05
      16     0.4302404333D 01    0.3434707118D-05
      17     0.4155898188D 01    0.3134172196D-05
      18     0.4002947916D 01    0.2793637519D-05
      19     0.6631278884D 01    0.9435432741D-05
      20     0.5460803221D 01    0.7121848480D-05

ND= 1   FREQUENCY= 5.46080D 00RAD/SEC   AMPLITUDE= 7.12185D-06   NIT= 20

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01     JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02    KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03    K2= 5.00000D 00     K3= 2.00000D-01

  ITERATION     FREQUENCY            AMPLITUDE
       0     0.5460803221D 01    0.7121848480D-05
       1     0.4774497461D 01    0.4994817969D-05
       2     0.4351151367D 01    0.3777846126D-05
       3     0.3983327133D 01    0.2825519782D-05
       4     0.2617985486D 01   -0.3957335850D-06
  1     2.61799D 00    -3.95734D-07
       5     0.1000000281D 01    0.9999997211D 00
       6     0.1000000000D 01    0.1000000000D 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT=  6
```

Figure 1-24b.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 0%      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02       K0= 5.75835D 03
K1= 1.37102D 03     K2= 5.00000D 00     K3= 1.00000D-03

  ITERATION     FREQUENCY                AMPLITUDE
      0      0.3300000000D 01      0.1300000000D-05
      1      0.3281481592D 01      0.1244882028D-05
      2      0.3291320054D 01      0.1252997737D-05
      3      0.3293122419D 01      0.1252620382D-05
      4      0.3293284790D 01      0.1254202959D-05
      5      0.3293602555D 01      0.1254376232D-05

ND= 1    FREQUENCY= 3.29381D 00RAD/SEC    AMPLITUDE= 1.25492D-06    NIT=  5

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TO= 1.00000D-01      JV= 1.00000D 05      JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02    -  K0= 5.75835D 03
K1= 1.37102D 03     K2= 5.00000D 00     K3= 1.00000D-02

  ITERATION     FREQUENY                 AMPLITUDE
      0      0.3293810983D 01      0.1254923947D-05
      1      0.3301772175D 01      0.1268076601D-05
      2      0.3306494197D 01      0.1277011784D-05
      3      0.3307227351D 01      0.1293558294D-05
      4      0.3315635781D 01      0.1300553084D-05
      5      0.3316192298D 01      0.1301379606D-05

ND= 1    FREQUENCY= 3.31660D 00RAD/SEC    AMPLITUDE= 1.30238D-06    NIT=  5
```

Figure 1-25a.

```
LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02     K0= 5.75835D 03
K1= 1.37102D 03     K2= 5.00000D 00     K3= 1.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0      0.3316597982D 01     0.1302379026D-05
      1      0.3413836922D 01     0.1504480331D-05
      2      0.3497137166D 01     0.1675209538D-05
      3      0.3650983247D 01     0.1863445424D-05
      4      0.3822585581D 01     0.2333538259D-05
      5      0.4025859796D 01     0.2869226076D-05
      6      0.4306719963D 01     0.3600958328D-05
      7      0.4163884584D 01     0.3293571013D-05
      8      0.4010876222D 01     0.2903848972D-05
      9      0.3552083028D 01     0.1874565879D-05
     10      0.3642164398D 01     0.2031512541D-05
     11      0.3688151376D 01     0.2110027783D-05
     12      0.3705009393D 01     0.2138959098D-05
     13      0.3711639221D 01     0.2090685945D-05
     14      0.3790756437D 01     0.2319209448D-05
     15      0.3810400446D 01     0.2385603934D-05
     16      0.3814455882D 01     0.2511116680D-05
     17      0.3993908443D 01     0.2866222875D-05
     18      0.3631616890D 01     0.2051966841D-05
     19      0.3705705816D 01     0.2176188941D-05
     20      0.3752088065D 01     0.2253631425D-05

ND= 1   FREQUENCY= 3.75209D 00RAD/SEC   AMPLITUDE= 2.25363D,06   NIT= 20

LST SYSTEM-NUMERICAL SOLUTION OF 1+N*G1+N*N*G2=0.
GAMMA= 1.38000D 07 TD= 1.00000D-01     JV= 1.00000D 05     JG= 2.10000D 00
KP= 2.16000D 02     KI= 9.70000D 03     H= 6.00000D 02     K0= 5.75835D 03
K1= 1.37102D 03     K2= 5.00000D 00     K3= 2.00000D-01

  ITERATION     FREQUENCY              AMPLITUDE
      0      0.3752088065D 01     0.2253631425D-05
      1      0.4209056132D 01     0.3272927164D-05
      2      0.3834035192D 01     0.2560546135D-05
      3      0.5380608385D 01     0.6152138662D-05
      4      0.4758738637D 01     0.4678815369D-05
      5      0.4342781489D 01     0.3681723091D-05
      6      0.3980004715D 01     0.2815716924D-05
      7      0.2544538760D 01    -0.5643857587D-06
  1     2.54454D 00     -5.64386D-07
      8      0.9999998771D 00     0.1000000125D 01
      9      0.1000000000D 01     0.1000000000D 01

ND= 1   FREQUENCY= 1.00000D 00RAD/SEC   AMPLITUDE= 1.00000D 00   NIT=  9
```

Figure 1-25b.

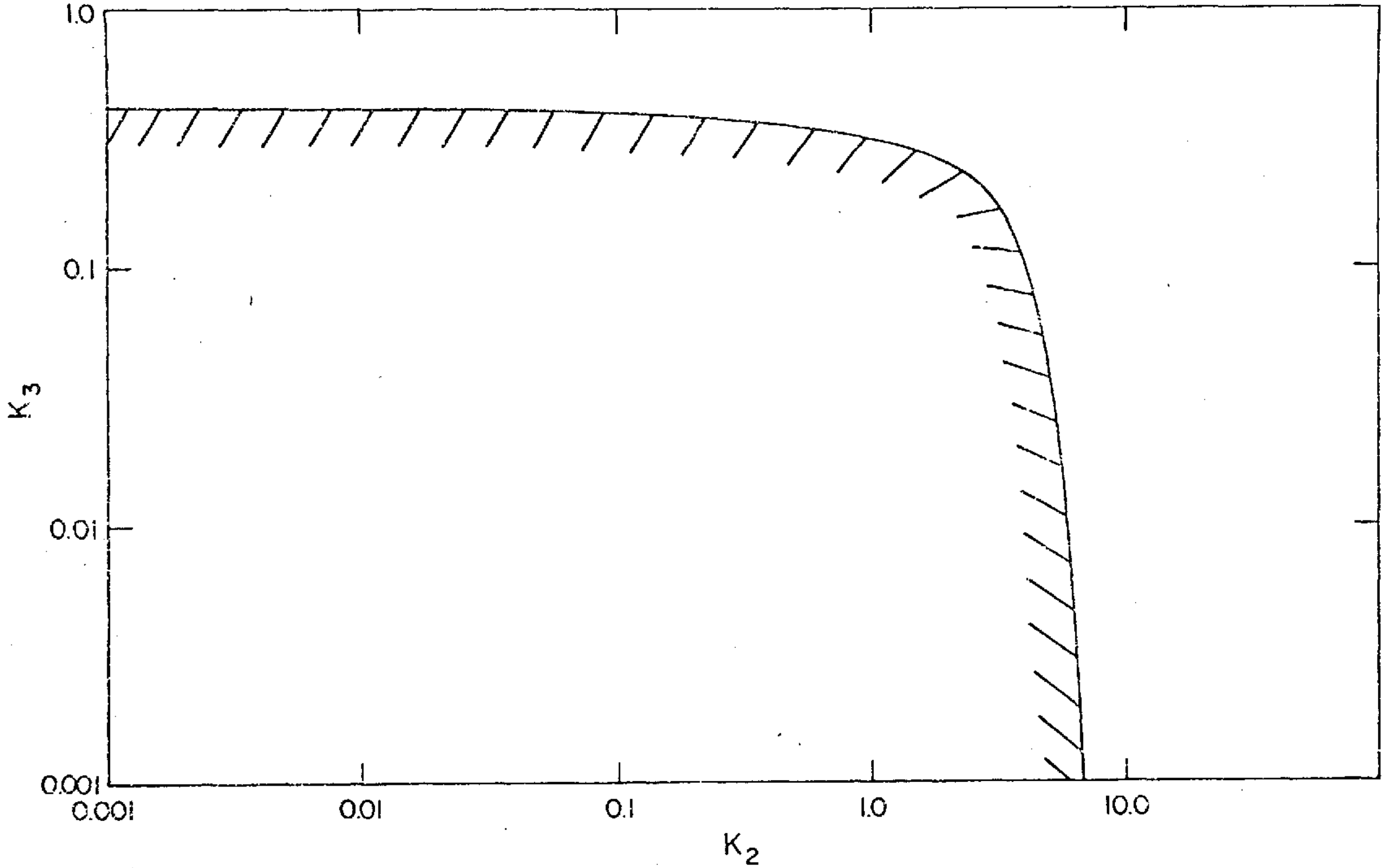


Figure 1-26. Region of self-sustained oscillations in the K_2 - K_3 plane for the two-axis LST system.

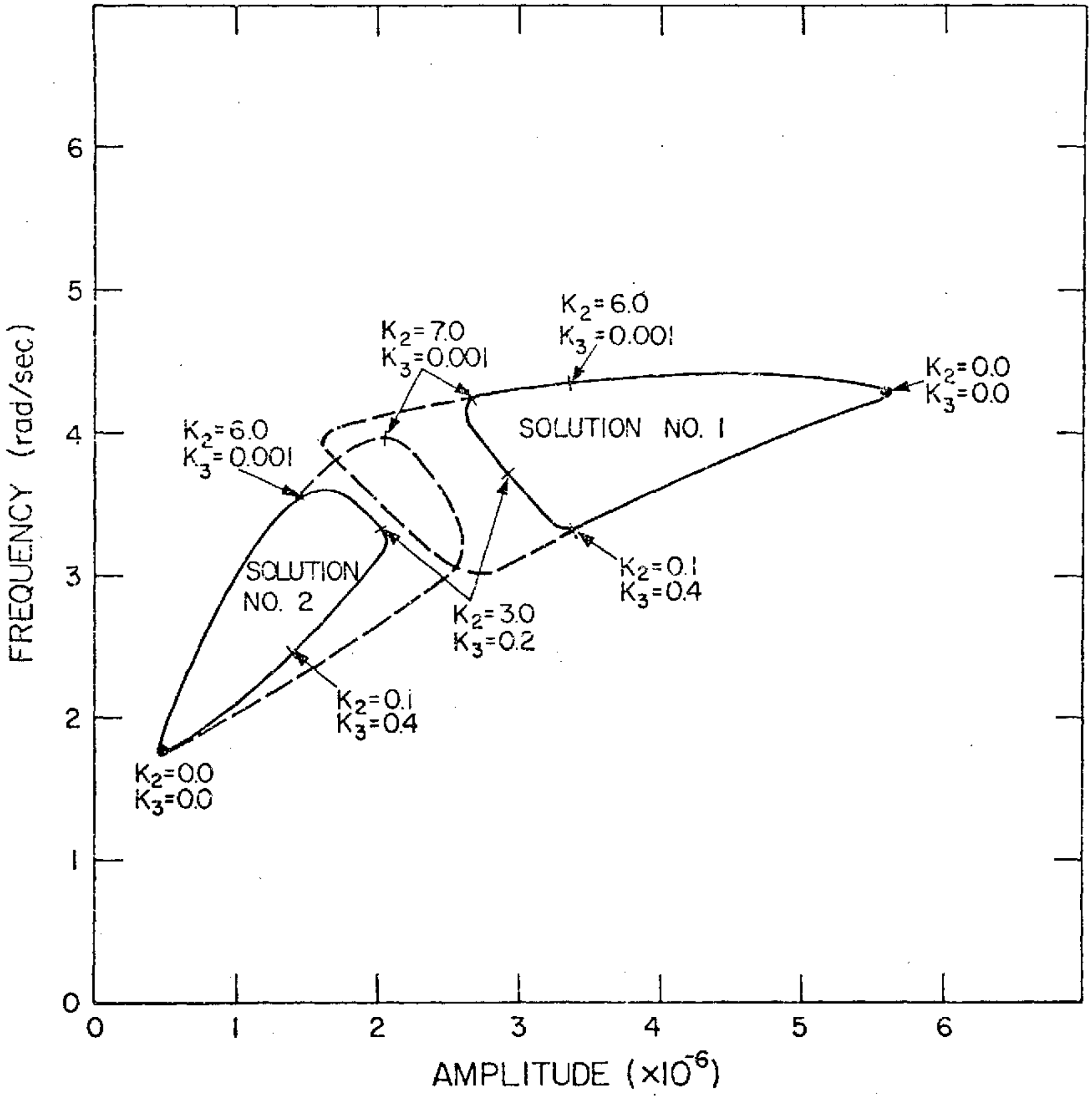


Figure 1-27. Region of possible solution points for the two-axis LST system.

REFERENCES

1. B. C. Kuo and G. Singh, Design of the Large Space Telescope System, Final Report, for NAS8-29853, Systems Research Laboratory, Champaign, Illinois, July 1, 1974.

2. B. C. Kuo and G. Singh, Continuous and Discrete Describing Function Analysis of the LST System, Final Report, for NAS8-29853, Systems Research Laboratory, Champaign, Illinois, January 1, 1974.

2. Prediction by Numerical Methods of Self-Sustained Oscillations in a Two-Axis Model of the Nonlinear Sampled-Data LST System

2-1. Introduction

It has been demonstrated in [2] that the stability equations as a result of the describing function analysis may be solved by a numerical-iterative technique. In Chapter 1, the numerical technique is applied to the prediction of self-sustained oscillations in a two-axis model of the nonlinear LST system with continuous-data. In this chapter the same numerical technique will be extended to the prediction of self-sustained oscillations in a two-axis model of the nonlinear LST system with sampled-data.

In the two-axis model of the LST system, the system contains two nonlinearities, and the general form of the stability equation in the sampled-data case is

$$1 + \hat{G}_B(T,n)N(A,n,\phi) + \hat{G}_C(T,n)N^2(A,n,\phi) = 0 \qquad (2\text{-}1)$$

where $G_B(T,n)$ and $G_C(T,n)$ are transfer functions which describe the linear elements of the coupled systems, and $N(A,n,\phi)$ represents the discrete describing function of the CMG nonlinearity. The amplitude of the sinusoidal signal at the input of the nonlinearities is denoted by A; T is the sampling period in second, and n is the integer which relates the period of the sustained oscillation to the sampling period; ϕ is the phase angle of the input sinusoid. Therefore,

$$T_C = nT = \text{period of sustained oscillation} \tag{2-2}$$

or

$$\omega_C = \frac{2\pi}{nT} = \text{frequency of sustained oscillation in rad/sec} \tag{2-3}$$

2-2. The Sampled-Data Two-Axis LST System Model and Its Stability Equation

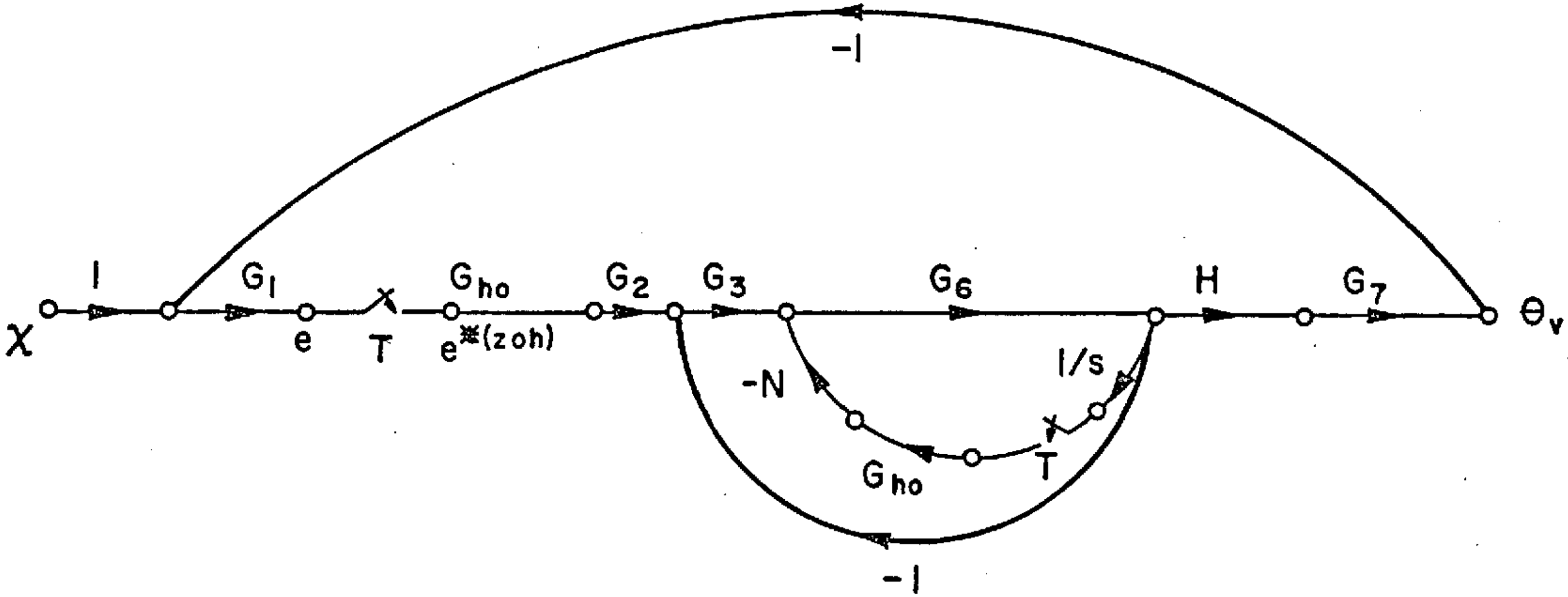
Figure 2-1 illustrates the signal flow graph representation of the sampled-data single-axis LST system. The nonlinear friction of the CMG is modeled by the branch with the gain N, where strictly, N denotes the discrete describing function $N(A,n,\phi)$. The sample-and-hold at the input of the nonlinearity is added for analytical purposes, so that the overall system may be treated by the z-transform method. Since the actual system does not have the sample-and-hold preceding N, the system configuration of Fig. 2-1 should be considered as an approximation to the sampled-data single-axis LST system.

Figure 2-2 shows the two-axis model of the LST system, with the coupling made at the output stages. The coupling coefficients K_2 and K_3 may be varied for various degrees of coupling between the two single-axis systems.

For the study of self-sustained oscillations, we set $\chi_1 = \chi_2 = 0$. From the signal flow graph of Fig. 2-2, the following cause-and-effect relationships are written for the variables at the outputs of the four samplers:

$$E_1^* = -A_1^* E_1^* + A_3^* N^* X_1^* + A_2^* E_2^* - A_4^* N^* X_2^* \tag{2-4}$$

$$E_2^* = -A_1^* E_2^* + A_3^* N^* X_2^* + A_2^* E_1^* - A_4^* N^* X_1^* \tag{2-5}$$



$$G_3(s) = \frac{K_p s + K_I}{s}$$

$$G_6(s) = \frac{1}{J_G s}$$

$$G_7(s) = \frac{1}{J_v s^2}$$

$$G_{ho}(s) = \frac{1 - e^{-Ts}}{s}$$

$$G_1(s) = K_o + K_1 s$$

$$G_2(s) = \frac{K_I}{K_p s + K_I}$$

Figure 2-1. The single-axis sampled-data LST system model.

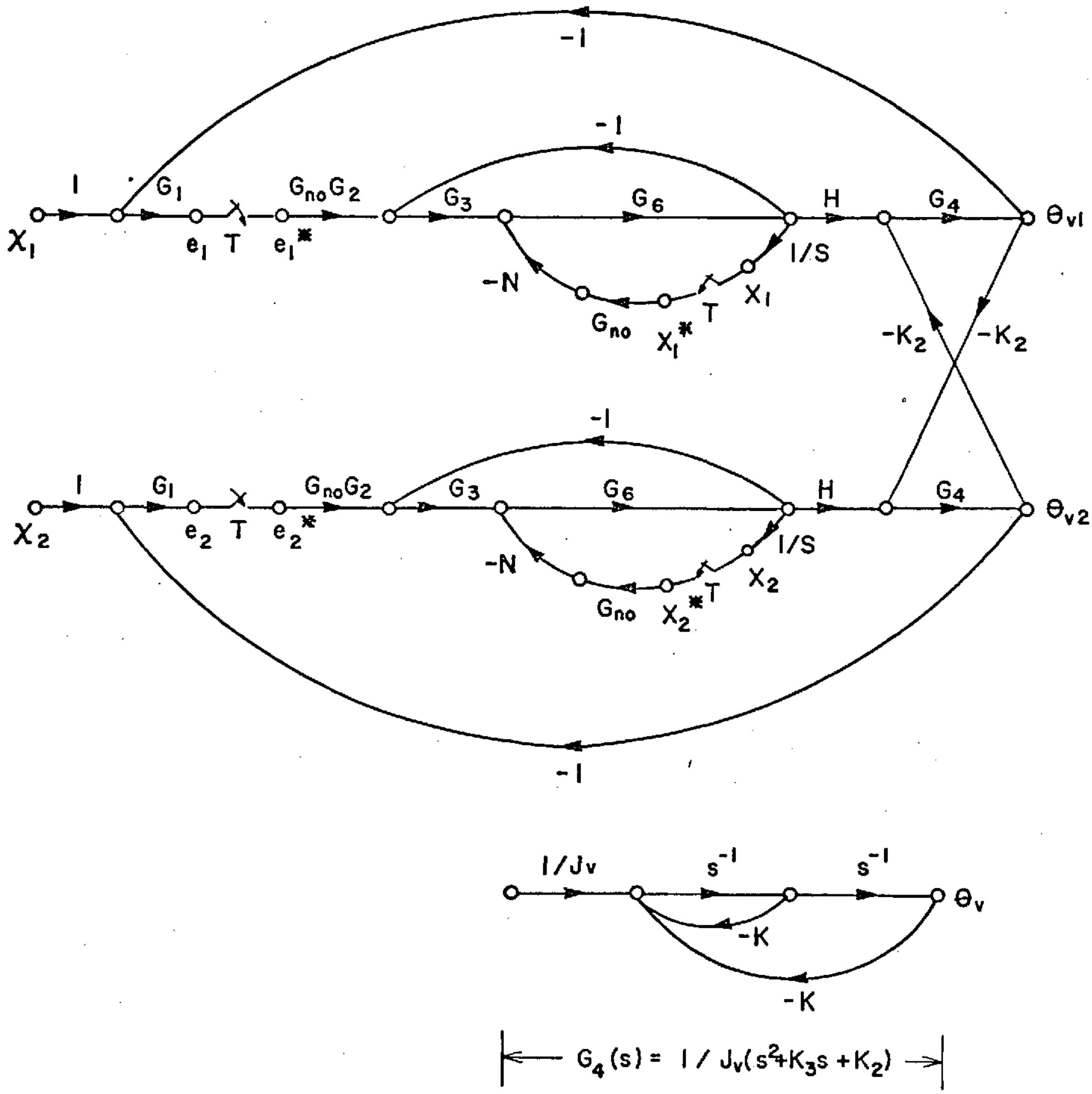


Figure 2-2. The two-axis sampled-data LST system model.

$$X_1^* = -B_1^*N^*X_1^* + B_2^*E_1^* \tag{2-6}$$

$$X_2^* = -B_1^*N^*X_2^* + B_2^*E_2^* \tag{2-7}$$

where

$$A_1 = \frac{G_{ho}G_1G_2G_3G_4G_6H\Delta_1}{\Delta_o} \tag{2-8}$$

$$A_2 = \frac{G_{ho}G_1G_2G_3G_4^2G_6HK_2\Delta_1}{\Delta_o} = A_1G_4K_2 \tag{2-9}$$

$$A_3 = \frac{G_{ho}G_1G_4G_6H\Delta_1}{\Delta_o} \tag{2-10}$$

$$A_4 = \frac{G_{ho}G_1G_4^2G_6HK_2\Delta_1}{\Delta_o} = A_3G_4K_2 \tag{2-11}$$

$$B_1 = \frac{G_{ho}G_6\Delta_2}{s\Delta_o} \tag{2-12}$$

$$B_2 = \frac{G_{ho}G_2G_3G_6\Delta_2}{s\Delta_o} = B_1G_2G_3 \tag{2-13}$$

$$\Delta_1 = 1 + G_3G_6 \tag{2-14}$$

$$\Delta_o = 1 + 2G_3G_6 - K_2^2G_4^2 + G_3^2G_6^2 - 2G_3G_4^2G_6K_2^2 - G_3^2G_4^2G_6^2K_2^2 \tag{2-15}$$

$$\Delta_2 = 1 + G_3G_6 - G_3G_6K_2^2G_4^2 - K_2^2G_4^2 \tag{2-16}$$

In the last nine equations all functions are functions of s. The sampled signal flow graph of the system is drawn in Fig. 2-3 using Eqs. (2-4) through (2-7).

The determinant of the sampled signal flow graph is obtained from Fig. 2-3 by use of Mason's gain formula. The expression is given in terms of the z-transform functions:

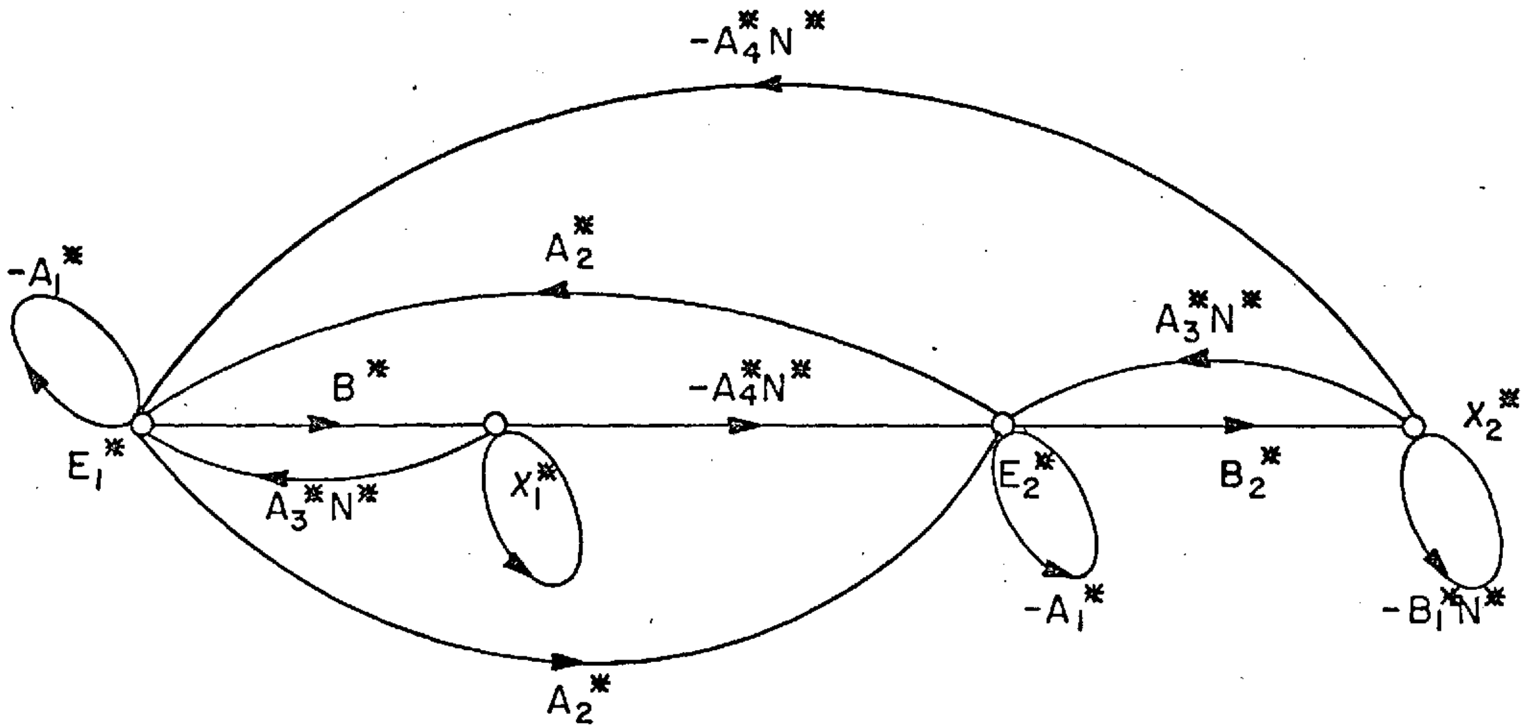


Figure 2-3. Sampled signal flow graph of the two-axis sampled-data LST system.

$$\Delta(z) = [1 + 2A_1(z) - A_2^2(z) + A_1^2(z)] + 2N(A,n,\phi)[B_1(z) - B_2(z)A_3(z) + A_2(z)B_2(z)A_4(z) + 2A_1(z)B_1(z) - A_1(z)A_3(z)B_2(z) + A_1^2(z)B_1(z) - A_2^2(z)B_1(z)] + N^2(A,n,\phi)[B_1^2(z) - A_4^2(z)B_2^2(z) - 2B_1(z)B_2(z)A_3(z) + A_3^2(z)B_2^2(z) + 2A_2(z)A_4(z)B_1(z)B_2(z) + 2A_1(z)B_1^2(z) - 2A_1(z)A_3(z)B_1(z)B_2(z) + A_1^2(z)B_1^2(z) - A_2^2(z)B_1^2(z)] \tag{2-17}$$

The components of $\Delta(z)$ are determined as follows:

$$A_1(z) = (1 - z^{-1})\mathfrak{Z}\left\{\frac{K_I H J_V(K_o + K_1 s)(J_G s^2 + K_p s + K_I)(s^2 + K_3 s + K_2)}{s\Delta_{op}}\right\} \tag{2-18}$$

$$A_2(z) = (1 - z^{-1})\mathfrak{Z}\left\{\frac{K_I K_2 H(K_o + K_1 s)(J_G s^2 + K_p s + K_I)}{\Delta_{op}}\right\} \tag{2-19}$$

$$A_3(z) = (1 - z^{-1})\mathfrak{Z}\left\{\frac{H J_V(K_o + K_1 s)(J_G s^2 + K_p s + K_I)(s^2 + K_3 s + K_2)}{\Delta_{op}}\right\} \tag{2-20}$$

$$A_4(z) = (1 - z^{-1})\mathfrak{Z}\left\{\frac{H K_2(K_o + K_1 s)(J_G s^2 + K_p s + K_I)}{\Delta_{op}}\right\} \tag{2-21}$$

$$B_1(z) = (1 - z^{-1})\mathfrak{Z}\left\{\frac{(J_G s^2 + K_p s + K_I)[J_V^2(s^2 + K_3 s + K_2)^2 - K_2^2]}{s\Delta_{op}}\right\} \tag{2-22}$$

$$B_2(z) = (1 - z^{-1})\mathfrak{Z}\left\{\frac{K_I(J_G s^2 + K_p s + K_I)[J_V^2(s^2 + K_3 s + K_2)^2 - K_2^2]}{s^2\Delta_{op}}\right\} \tag{2-23}$$

$$\Delta_{op} = [J_G^2 s^4 + 2K_p J_G s^3 + (K_p^2 + 2K_I J_G)s^2 + 2K_p K_I s + K_I^2][J_V^2(s^2 + K_3 s + K_2)^2 - K_2^2] \tag{2-24}$$

The expressions for the discrete describing function of the CMG nonlinearity, $N(A,n,\phi)$, may be found in [2]. In the present case, since the two axes are considered to be identical, and the couplings are symmetric, it is assumed that the amplitudes of the input signals at the two nonlinearities are identical.

The stability equation is obtained by setting $\Delta(z)$ of Eq. (2-17) to zero. It is apparent that $\Delta(z) = 0$ is of the form:

$$\Delta(z) = 1 + G_A(z) + G_B(z)N(A,n,\phi) + G_C(z)N^2(A,n,\phi) = 0 \tag{2-25}$$

which can be written as

$$1 + \hat{G}_B(T,n)N(A,n,\phi) + \hat{G}_C(T,n)N^2(A,n,\phi) = 0 \tag{2-26}$$

by dividing both sides of the equation by $1 + G_A(z)$, and setting

$$z = e^{j2\pi/n} \tag{2-27}$$

Equation (2-26) shows that in the sampled-data system, there are four variable parameters in the stability equation in T, n, A, and ϕ, whereas in the continuous-data system there are only two variables in A and ω.

2-3. Exact Solution of the Stability Equation of the Two-Axis Sampled-Data LST System by Numerical-Iterative Techniques

The numerical-iterative method described in Chapter 1 is now used to solve the stability equation of the two-axis sampled-data LST system, Eq. (2-26).

In Eq. (2-26) we define

$$\hat{G}_B(T,n) = G_{R1} + jG_{I1} \tag{2-28}$$

$$\hat{G}_C(T,n) = G_{R2} + jG_{I2} \tag{2-29}$$

$$N(A,n,\phi) = N_{R1} + jN_{I1} \tag{2-30}$$

$$N^2(A,n,\phi) = N_{R2} + jN_{I2} \tag{2-31}$$

where G_{R1}, G_{I1}, G_{R2}, G_{I2}, N_{R1}, N_{I1}, N_{R2}, and N_{I2} are all real quantities.

When Eqs. (2-28) through (2-31) are substituted into Eq. (2-26), and after grouping the real and imaginary parts, we have,

$$\Delta = \Delta_R + j\Delta_I = 0 \tag{2-32}$$

where

$$\Delta_R = 1 + G_{R1}N_{R1} - G_{I1}N_{I1} + G_{R2}N_{R2} - G_{I2}N_{I2} = 0 \tag{2-33}$$

$$\Delta_I = G_{I1}N_{R1} + G_{R1}N_{I1} + G_{I2}N_{R2} + G_{R2}N_{I2} = 0 \tag{2-34}$$

These two expressions represent two equations in four variables in A, T, ϕ, and n. It is necessary to assign values to two of these variables and solve for the remaining two using the numerical-iterative method. In general, it is practical to assign values to T and n, and Eqs. (2-33) and (2-34) are solved to yield A and ϕ. Alternatively, we may assign values to n and ϕ, and solve for A and T. The latter case has been carried out in the current study.

As in Chapter 1, we define

$$\underline{x} = \begin{bmatrix} T \\ A \end{bmatrix} \tag{2-35}$$

$$\underline{F} = \begin{pmatrix} \Delta_R \\ \Delta_I \end{pmatrix} \tag{2-36}$$

Then, Eqs. (2-33) and (2-34) can be written as

$$F(\underline{x}) = \underline{0} \tag{2-37}$$

The Newton-type quadratically convergent numerical method described in Chapter 1 can now be directly applied to this two-variable system.

The following system parameters are used:

$T_{GFO} = 0.1$ ft-lb

$H = 600$ ft-lb-sec

$J_G = 2.1$ ft-lb-sec^2

$K_o = 5758.35$

$K_1 = 1371.02$

$K_p = 216$ ft-lb/rad/sec

$K_I = 9700$ ft-lb/rad

$J_V = 10^5$ ft-lb-sec^2

$\gamma = 1.38 \times 10^7$

Three sets of coefficients of coupling are used for this study:

(1) $K_2 = 0.1$ $K_3 = 0.1$

(2) $K_2 = 1.0$ $K_3 = 0.1$

(3) $K_2 = 5.0$ $K_3 = 0.1$

For each integral n, and a fixed value of ϕ, a solution of Eq. (2-37) is attempted with an initial guess on T and A. A convergent solution for T and A together with the assigned n and ϕ describes a sustained oscillation in the system. One of the characteristics of the numerical method is that the solution depends on the initial conditions set by the analyst. As shown in the earlier studies, it is quite possible that a system may have a multiple number of solutions for the same set of n and ϕ. An exhaustive search can only be conducted by a systematic scan of all the possible combinations of the initial values in the iterative process, unless one has general information at the outset on the number and the proximity of the solutions.

Figure 2-4 shows the plots of the two solutions of T versus A for $K_2 = K_3 = 0.1$, and for relative large values of n. The results are shown only for $\phi = 0°$. When all the values of ϕ are used, the solutions of T and A trace out a loop in the T versus A domain. For large values of n, these loops are very small, as the solutions are relatively insensitive to n. As n decreases, these loops become more pronounced, as indicated in Fig. 2-5. As n decreases, one of the solutions disappears, as the amplitude of A becomes exceedingly small.

Figure 2-6 gives the computer results for $K_2 = K_3 = 0.1$ that correspond to one set of solutions for $\phi = 0°$. Figure 2-7 tabulates the other set of results for $K_2 = K_3 = 0.1$ with $\phi = 0°$. Figure 2-8 gives the tabulations of results for $K_2 = K_3 = 0.1$, $n = 4$, and variable ϕ.

Figures 2-9 and 2-10 give the results with $K_2 = 1.0$ and $K_3 = 0.1$. Figure 2-11 shows the solutions for $K_2 = 5.0$ and $K_3 = 0.1$ with $\phi = 0°$. In this case, the amplitude of oscillation decreases as n decreases, so that there is less chance for the low-frequency oscillations to occur.

The results indicate that harder coupling (larger values of K_2 and K_3) tends to reduce the chance of self-sustained oscillations.

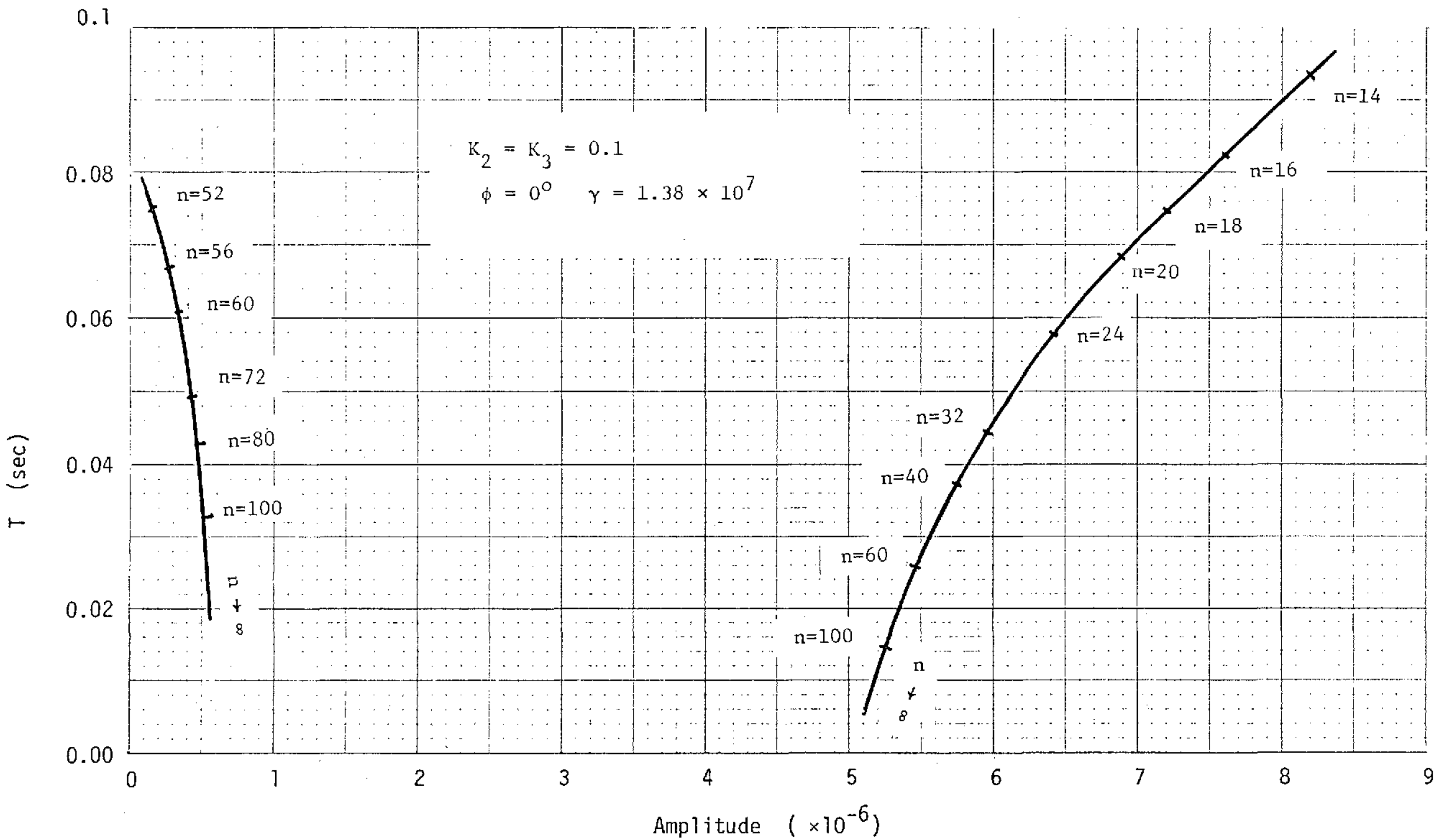


Figure 2-4. Amplitude and frequency ($2\pi/nT$) of self-sustained oscillations for various sampling periods in the discrete-data two-axis LST system; $\gamma = 1.38\times10^{7}$.

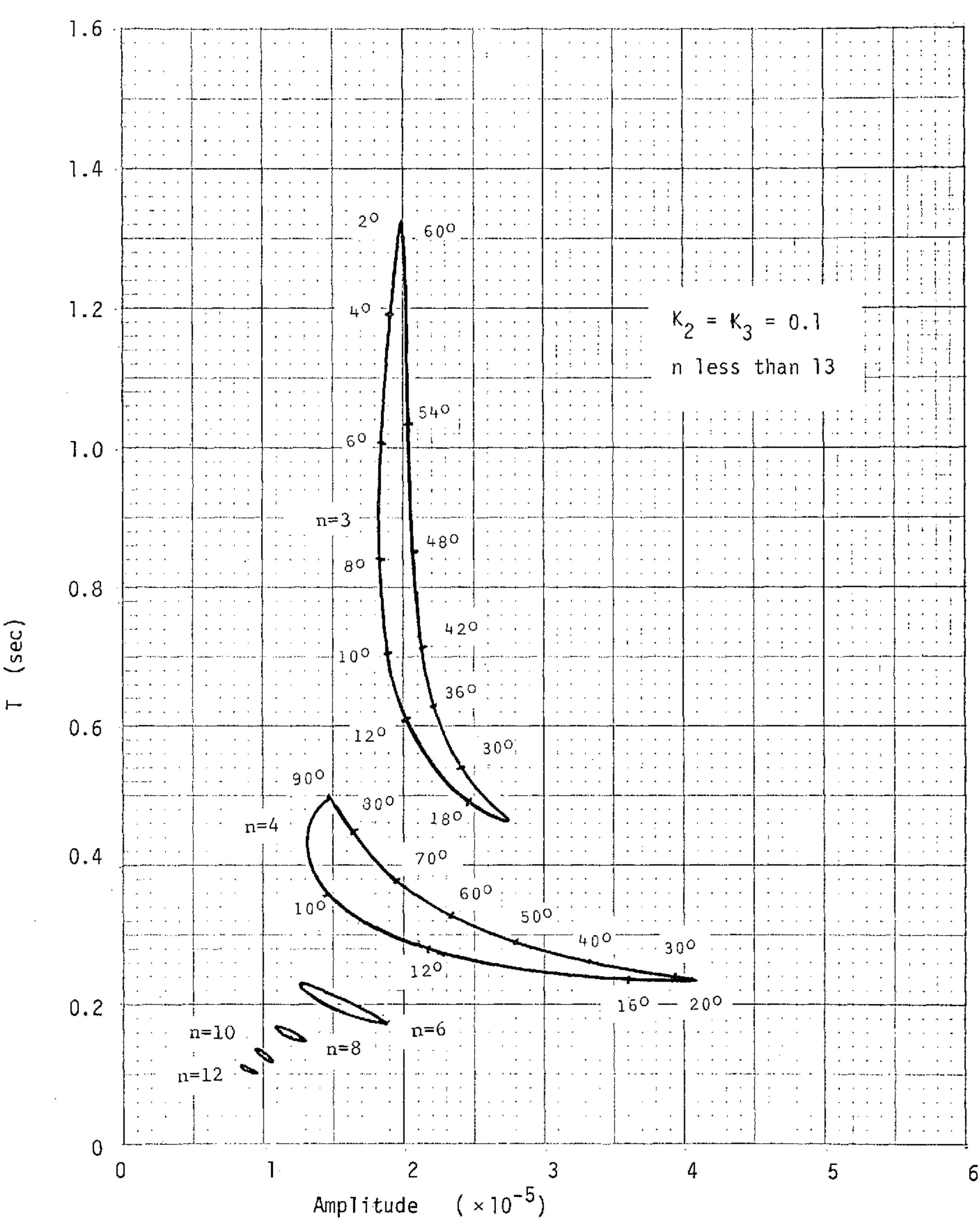


Figure 2-5. Amplitude and frequency ($2\pi/nT$) of self-sustained oscillations for various sampling periods in the discrete-data two-axis LST system; $\gamma = 1.38 \times 10^7$.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 100 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3320000000D-01	0.6000000000D-06
1	0.3342229250D-01	0.5950679807D-06
2	0.3359786023D-01	0.5747976449D-06
3	0.3366351585D-01	0.5668210713D-06
4	0.3369566512D-01	0.5625329043D-06
5	0.3369587215D-01	0.5627405691D-06
6	0.3369532634D-01	0.5435030290D-06
7	0.3364046622D-01	0.5571699439D-06
8	0.3367687123D-01	0.5598125454D-06
9	0.3369855892D-01	0.5609805886D-06
10	0.3366940604D-01	0.5629427540D-06
11	0.3369803223D-01	0.5630963918D-06

ND= 1 SAMPLING PERIOD= 3.36972D-02SEC AMPLITUDE= 5.63342D-07 NT=11

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 96 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3369720849D-01	0.5633423550D-06
1	0.3474291538D-01	0.5443359999D-06
2	0.3502375402D-01	0.5470937221D-06
3	0.3511512195D-01	0.5512909117D-06
4	0.3515743863D-01	0.5536089799D-06
5	0.3518081455D-01	0.5546943617D-06
6	0.3515967466D-01	0.5562991873D-06
7	0.3518635462D-01	0.5566505449D-06

ND= 1 SAMPLING PERIOD= 3.51849D-02SEC AMPLITUDE= 5.56679D-07 NT= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 92 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3518492122D-01	0.5566790381D-06
1	0.3632633789D-01	0.5366432487D-06
2	0.3663329016D-01	0.5393705578D-06
3	0.3673534759D-01	0.5435543568D-06
4	0.3678294190D-01	0.5458594565D-06
5	0.3680897407D-01	0.5469412658D-06
6	0.3675900402D-01	0.5494039884D-06
7	0.3679886044D-01	0.5492251820D-06
8	0.3680191726D-01	0.5613414977D-06
9	0.3686631866D-01	0.5506174407D-06
10	0.3684513633D-01	0.5495252589D-06
11	0.3683223886D-01	0.5491315696D-06
12	0.3698436993D-01	0.5346254689D-06
13	0.3691543458D-01	0.5378992242D-06
14	0.3684653330D-01	0.5439682800D-06
15	0.3672725204D-01	0.5512047322D-06

ND= 1 SAMPLING PERIOD= 3.67273D-02SEC AMPLITUDE= 5.51205D-07 NT=15

Figure 2-6a.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 88 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3672725204D-01	0.5512047322D-06
1	0.3804951071D-01	0.5276558662D-06
2	0.3840576574D-01	0.5297954747D-06
3	0.3852469444D-01	0.5339405723D-06
4	0.3858000219D-01	0.5362532059D-06
5	0.3860978892D-01	0.5373461539D-06
6	0.3880907639D-01	0.5325967023D-06
7	0.3868869863D-01	0.5378718864D-06
8	0.3862592890D-01	0.5407750867D-06
9	0.3864946584D-01	0.5376668762D-06
10	0.3865443397D-01	0.5369972470D-06
11	0.3865827663D-01	0.5367328089D-06

ND= 1 SAMPLING PERIOD= 3.86637D-02SEC AMPLITUDE= 5.36429D-07 NT=11

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 84 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3866366421D-01	0.5364285381D-06
1	0.4002364976D-01	0.5150849745D-06
2	0.4039100134D-01	0.5180707747D-06
3	0.4052223920D-01	0.5221609257D-06
4	0.4058490375D-01	0.5243717423D-06
5	0.4061860445D-01	0.5254083819D-06
6	0.4071084231D-01	0.5239274439D-06
7	0.4059663023D-01	0.5309992476D-06
8	0.4063733861D-01	0.5264303961D-06
9	0.4063732542D-01	0.5264214375D-06

ND= 1 SAMPLING PERIOD= 4.06373D-02SEC AMPLITUDE= 5.26414D-07 NT= 9

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 80 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.4063731418D-01	0.5264143904D-06
1	0.4219079242D-01	0.5014704472D-06
2	0.4261349518D-01	0.5035421161D-06
3	0.4276607502D-01	0.5073101923D-06
4	0.4283895084D-01	0.5093689227D-06
5	0.4287790411D-01	0.5103291743D-06
6	0.4294168292D-01	0.5097411227D-06
7	0.4295739576D-01	0.5084227662D-06
8	0.4299599510D-01	0.5061352377D-06
9	0.4293233241D-01	0.5120656117D-06
10	0.4289974529D-01	0.5131564094D-06
11	0.4297234920D-01	0.5063036490D-06
12	0.4212358522D-01	0.5310503269D-06
13	0.4259046767D-01	0.5186986412D-06
14	0.4277253918D-01	0.5147065691D-06
15	0.4284118828D-01	0.5137414959D-06

ND= 1 SAMPLING PERIOD= 4.28412D-02SEC AMPLITUDE= 5.13741D-07 NT=15

Figure 2-6b.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 76 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.4284118828D-01	0.5137414959D-06
1	0.4464118167D-01	0.4841942659D-06
2	0.4513404690D-01	0.4850800623D-06
3	0.4531456953D-01	0.4883825027D-06
4	0.4540097269D-01	0.4902121173D-06
5	0.4544690240D-01	0.4910561502D-06
6	0.4550158575D-01	0.4908031820D-06
7	0.4550362167D-01	0.4906108580D-06

ND= 1 SAMPLING PERIOD= 4.55056D-02SEC AMPLITUDE= 4.90508D-07 NT= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 72 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.4550564878D-01	0.4905077202D-06
1	0.4748492582D-01	0.4602676060D-06
2	0.4803270735D-01	0.4611052828D-06
3	0.4824690167D-01	0.4639223444D-06
4	0.4835158441D-01	0.4654279102D-06
5	0.4840710028D-01	0.4661008946D-06
6	0.4845896027D-01	0.4660126242D-06
7	0.4845913823D-01	0.4659959617D-06

ND= 1 SAMPLING PERIOD= 4.84594D-02SEC AMPLITUDE= 4.65983D-07 NT= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 68 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.4845942328D-01	0.4659832194D-06
1	0.5077128819D-01	0.4305541220D-06
2	0.5141496277D-01	0.4300316467D-06
3	0.5167697185D-01	0.4319612733D-06
4	0.5180659395D-01	0.4329628233D-06
5	0.5187519910D-01	0.4333733303D-06
6	0.5192792338D-01	0.4333555693D-06
7	0.5192794877D-01	0.4333381285D-06

ND= 1 SAMPLING PERIOD= 5.19280D-02SEC AMPLITUDE= 4.33328D-07 NT= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 64 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.5192797142D-01	0.4333278588D-06
1	0.5467724495D-01	0.3910672890D-06
2	0.5544071605D-01	0.3889380378D-06
3	0.5576780112D-01	0.3896501958D-06
4	0.5593237218D-01	0.3899343046D-06
5	0.5601946040D-01	0.3899684217D-06
6	0.5607441769D-01	0.3899899126D-06

ND= 1 SAMPLING PERIOD= 5.60744D-02SEC AMPLITUDE= 3.90010D-07 NT= 6

Figure 2-6c.

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LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07    N=  60    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
       0      0.5607440724D-01     0.3900104301D-06
       1      0.5943855184D-01     0.3377828184D-06
       2      0.6034357899D-01     0.3340925505D-06
       3      0.6075649615D-01     0.3332972694D-06
       4      0.6096880899D-01     0.3326451744D-06
       5      0.6108148602D-01     0.3321774347D-06
       6      0.6113998602D-01     0.3321931013D-06
       7      0.6113993477D-01     0.3289034850D-06
       8      0.6109176163D-01     0.3321856964D-06
       9      0.6113616294D-01     0.3324425205D-06
      10      0.6113606465D-01     0.3377813726D-06
      11      0.6120186225D-01     0.3313524294D-06
      12      0.6120897519D-01     0.3301622286D-06
      13      0.6121337436D-01     0.3297677155D-06
      14      0.6121811619D-01     0.3295921224D-06

ND= 1   SAMPLING PERIOD= 6.12248D-02SEC   AMPLITUDE= 3.29351D-07   NT=14

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07    N=  56    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
       0      0.6122479469D-01     0.3293509878D-06
       1      0.6543301761D-01     0.2639577502D-06
       2      0.6645173242D-01     0.2608253382D-06
       3      0.6695797940D-01     0.2589494244D-06
       4      0.6722604406D-01     0.2574374605D-06
       5      0.6737019848D-01     0.2564397595D-06
       6      0.6743896903D-01     0.2562879806D-06
       7      0.6743928847D-01     0.2557157062D-06
       8      0.6744092388D-01     0.2559094333D-06

ND= 1   SAMPLING PERIOD= 6.74417D-02SEC   AMPLITUDE= 2.55843D-07   NT= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07    N=  52    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
       0      0.6744166977D-01     0.2558430579D-06
       1      0.7328507727D-01     0.1592166005D-06
       2      0.7518829983D-01     0.1180376352D-06
       3      0.7544197006D-01     0.1414855224D-06
       4      0.7548893187D-01     0.1539881111D-06
       5      0.7540087284D-01     0.1577965747D-06
       6      0.7534052323D-01     0.1600971463D-06
       7      0.7534675854D-01     0.1589113884D-06
       8      0.7656361713D-01     0.1052268639D-06
       9      0.7609634676D-01     0.1297537881D-06
      10      0.7572642564D-01     0.1448123698D-06
      11      0.7551859241D-01     0.1531497593D-06
      12      0.7541613162D-01     0.1573705387D-06
      13      0.7535353854D-01     0.1597008993D-06
      14      0.7537343098D-01     0.1579077493D-06
      15      0.7533086135D-01     0.1600055409D-06

ND= 1   SAMPLING PERIOD= 7.53309D-02SEC   AMPLITUDE= 1.60006D-07   NT=15
```

Figure 2-6d.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07 N= 100 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1530000000D-01	0.5000000000D-05
1	0.1518129565D-01	0.5123194566D-05
2	0.1512363339D-01	0.5186988252D-05
3	0.1509538498D-01	0.5218608809D-05
4	0.1507480998D-01	0.5240422024D-05
5	0.1507577419D-01	0.5231844048D-05
6	0.1508043353D-01	0.5225764830D-05
7	0.1505980197D-01	0.5253128272D-05
8	0.1505978776D-01	0.5253562257D-05

ND= 1 SAMPLING PERIOD= 1.50598D-02SEC AMPLITUDE= 5.25387D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07 N= 96 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1505977177D-01	0.5253865498D-05
1	0.1524686473D-01	0.5424210948D-05
2	0.1545679048D-01	0.5350687791D-05
3	0.1556856306D-01	0.5306547099D-05
4	0.1562697461D-01	0.5282550091D-05
5	0.1565694849D-01	0.5270912439D-05
6	0.1566106863D-01	0.5268612074D-05

ND= 1 SAMPLING PERIOD= 1.56629D-02SEC AMPLITUDE= 5.26612D-06 N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= K3=0.1
GAMMA= 1.38000D 07 N= 92 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1566289858D-01	0.5266116798D-05
1	0.1586928307D-01	0.5447574799D-05
2	0.1610269510D-01	0.5367998314D-05
3	0.1622686675D-01	0.5320407881D-05
4	0.1629157791D-01	0.5294671135D-05
5	0.1632500326D-01	0.5281905680D-05
6	0.1633084641D-01	0.5279681922D-05

ND= 1 SAMPLING PERIOD= 1.63327D-02SEC AMPLITUDE= 5.27664D-06 N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= K3=0.1
GAMMA= 1.38000D 07 N= 88 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1633272606D-01	0.5276636683D-05
1	0.1655487224D-01	0.5469159244D-05
2	0.1681022992D-01	0.5384743931D-05
3	0.1694596034D-01	0.5334456511D-05
4	0.1701655283D-01	0.5307374841D-05
5	0.1705310040D-01	0.5293798727D-05
6	0.1706059616D-01	0.5291479667D-05

ND= 1 SAMPLING PERIOD= 1.70625D-02SEC AMPLITUDE= 5.28804D-06 N= 6

Figure 2-7a.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07   N= 84   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1706249398D-01	0.5288035305D-05
1	0.1730216910D-01	0.5492965186D-05
2	0.1758264220D-01	0.5403218271D-05
3	0.1773159629D-01	0.5349960075D-05
4	0.1780891451D-01	0.5321397585D-05
5	0.1784898970D-01	0.5306963522D-05
6	0.1785857215D-01	0.5304398319D-05

ND= 1   SAMPLING PERIOD= 1.78605D-02SEC   AMPLITUDE= 5.30055D-06   N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07   N= 80   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1786045844D-01	0.5300553416D-05
1	0.1811982227D-01	0.5519351613D-05
2	0.1842922889D-01	0.5423709952D-05
3	0.1859341236D-01	0.5367165464D-05
4	0.1867847392D-01	0.5336964779D-05
5	0.1872256983D-01	0.5321613915D-05
6	0.1873473958D-01	0.5318625613D-05

ND= 1   SAMPLING PERIOD= 1.87366D-02SEC   AMPLITUDE= 5.31440D-06   N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07   N= 76   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1873659666D-01	0.5314404685D-05
1	0.1901820814D-01	0.5548749727D-05
2	0.1936116657D-01	0.5446568116D-05
3	0.1954300378D-01	0.5386375032D-05
4	0.1963703870D-01	0.5354355062D-05
5	0.1968575521D-01	0.5338016228D-05
6	0.1970106583D-01	0.5334412132D-05

ND= 1   SAMPLING PERIOD= 1.97029D-02SEC   AMPLITUDE= 5.32987D-06   N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07   N= 72   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1970289817D-01	0.5329874321D-05
1	0.2000983375D-01	0.5581692582D-05
2	0.2039198082D-01	0.5472228691D-05
3	0.2059442799D-01	0.5407968980D-05
4	0.2069893590D-01	0.5373920400D-05
5	0.2075300819D-01	0.5356507849D-05
6	0.2077205228D-01	0.5352094185D-05

ND= 1   SAMPLING PERIOD= 2.07739D-02SEC   AMPLITUDE= 5.34734D-06   N= 6

Figure 2-7b.

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LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07    N=  68    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
      0      0.2077389646D-01    0.5347338290D-05
      1      0.2110990716D-01    0.5618848722D-05
      2      0.2153819175D-01    0.5501243897D-05
      3      0.2176488804D-01    0.5432432679D-05
      4      0.2188171440D-01    0.5396112375D-05
      5      0.2194205081D-01    0.5377523077D-05
      6      0.2196545462D-01    0.5372120519D-05
      7      0.2196738711D-01    0.5367283776D-05

ND= 1    SAMPLING PERIOD= 2.19698D-02SEC    AMPLITUDE= 5.36633D-06    N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07    N=  64    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
      0      0.2196979444D-01    0.5366325166D-05
      1      0.2233797565D-01    0.5660864357D-05
      2      0.2282065817D-01    0.5534189623D-05
      3      0.2307590015D-01    0.5460323816D-05
      4      0.2320721900D-01    0.5421483097D-05
      5      0.2327489029D-01    0.5401612738D-05
      6      0.2330325793D-01    0.5395087322D-05
      7      0.2330541216D-01    0.5390331951D-05

ND= 1    SAMPLING PERIOD= 2.33090D-02SEC    AMPLITUDE= 5.38932D-06    N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07    N=  60    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
      0      0.2330800876D-01    0.5389317022D-05
      1      0.2371546467D-01    0.5709270125D-05
      2      0.2426386869D-01    0.5572273900D-05
      3      0.2455357389D-01    0.5492624723D-05
      4      0.2470238392D-01    0.5450894726D-05
      5      0.2477887551D-01    0.5429575152D-05
      6      0.2481300177D-01    0.5421785319D-05
      7      0.2481560381D-01    0.5417289555D-05

ND= 1    SAMPLING PERIOD= 2.48185D-02SEC    AMPLITUDE= 5.41611D-06    N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2=K3=0.1
GAMMA= 1.38000D 07    N=  56    PHI= 0.0
  ITERATION  SAMPLING PERIOD        AMPLITUDE
      0      0.2481845775D-01    0.5416109190D-05
      1      0.2527222590D-01    0.5765341923D-05
      2      0.2590037116D-01    0.5616616810D-05
      3      0.2623180446D-01    0.5530395043D-05
      4      0.2640177621D-01    0.5485381470D-05
      5      0.2648890472D-01    0.5462434531D-05
      6      0.2652964524D-01    0.5453286148D-05
      7      0.2653307741D-01    0.5449211264D-05

ND= 1    SAMPLING PERIOD= 2.65363D-02SEC    AMPLITUDE= 5.44773D-06    N= 7
```

Figure 2-7c.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= K3=0.1
GAMMA= 1.38000D 07   N= 52   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.2653626294D-01	0.5447726057D-05
1	0.2704516365D-01	0.5831066474D-05
2	0.2777126829D-01	0.5668929305D-05
3	0.2815380132D-01	0.5575201503D-05
4	0.2834965647D-01	0.5526440168D-05
5	0.2844975878D-01	0.5501653871D-05
6	0.2849817152D-01	0.5491072646D-05
7	0.2850309567D-01	0.5487526844D-05

ND= 1   SAMPLING PERIOD= 2.85067D-02SEC   AMPLITUDE= 5.48557D-06   N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= K3=0.1
GAMMA= 1.38000D 07   N= 48   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.2850665382D-01	0.5485568768D-05
1	0.2908182190D-01	0.5909195409D-05
2	0.2992997663D-01	0.5731615644D-05
3	0.3037586916D-01	0.5629278377D-05
4	0.3060375828D-01	0.5576224764D-05
5	0.3071987517D-01	0.5549348977D-05
6	0.3077733074D-01	0.5537268220D-05
7	0.3078489658D-01	0.5534252531D-05

ND= 1   SAMPLING PERIOD= 3.07888D-02SEC   AMPLITUDE= 5.53163D-06   N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= K3=0.1
GAMMA= 1.38000D 07   N= 44   PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3078877141D-01	0.5531631338D-05
1	0.3144439053D-01	0.6003648010D-05
2	0.3244709861D-01	0.5808159804D-05
3	0.3297263070D-01	0.5695914173D-05
4	0.3324066935D-01	0.5637937052D-05
5	0.3337680368D-01	0.5608681697D-05
6	0.3344512981D-01	0.5595031020D-05
7	0.3345727029D-01	0.5592361838D-05

ND= 1   SAMPLING PERIOD= 3.34613D-02SEC   AMPLITUDE= 5.58889D-06   N= 7

Figure 2-7d.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 20 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.6000000000D-01	0.1000000000D-04
1	0.6480879984D-01	0.7617629854D-05
2	0.6678026019D-01	0.7172493189D-05
3	0.6769282120D-01	0.6996822188D-05
4	0.6813042341D-01	0.6918483150D-05
5	0.6834250429D-01	0.6882531139D-05
6	0.6843089594D-01	0.6868190612D-05
7	0.6844645543D-01	0.6858162908D-05
8	0.6846072173D-01	0.6854744930D-05

ND= 1 SAMPLING PERIOD= 6.84679D-02SEC AMPLITUDE= 6.85169D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 24 PHI= 0:0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.6846787904D-01	0.6851692663D-05
1	0.6789688908D-01	0.3849797353D-05
2	0.6337381658D-01	0.4653644434D-05
3	0.5831592193D-01	0.5118555517D-05
4	0.5725147888D-01	0.6136508163D-05
5	0.5736972579D-01	0.6487885453D-05
6	0.5792383875D-01	0.6435582888D-05
7	0.5820889924D-01	0.6406344573D-05
8	0.5835654751D-01	0.6390808096D-05
9	0.5842629652D-01	0.6385681095D-05
10	0.5842870922D-01	0.6381238265D-05

ND= 1 SAMPLING PERIOD= 5.84322D-02SEC AMPLITUDE= 6.38106D-06 N=10

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 28 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.5843215529D-01	0.6381063121D-05
1	0.5820296047D-01	0.4079361184D-05
2	0.5508804677D-01	0.4770388789D-05
3	0.5318242804D-01	0.5298937351D-05
4	0.5210219143D-01	0.5650467227D-05
5	0.5152852903D-01	0.5858912969D-05
6	0.5124344808D-01	0.5972604748D-05
7	0.5111741517D-01	0.6030052909D-05
8	0.5106046578D-01	0.6056801388D-05
9	0.5095802379D-01	0.6100937101D-05
10	0.5095684164D-01	0.6038873942D-05
11	0.5095120443D-01	0.6074084692D-05
12	0.5096937825D-01	0.6080501700D-05
13	0.5096940058D-01	0.6077955158D-05

ND= 1 SAMPLING PERIOD= 5.09695D-02SEC AMPLITUDE= 6.07877D-06 N=13

Figure 2-7e.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 32 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.5096945201D-01	0.6078770868D-05
1	0.5102497922D-01	0.4119164140D-05
2	0.4843472824D-01	0.4757019016D-05
3	0.4689683602D-01	0.5226284689D-05
4	0.4604573283D-01	0.5527945196D-05
5	0.4560202105D-01	0.5702210947D-05
6	0.4538356113D-01	0.5795155652D-05
7	0.4527921299D-01	0.5841551842D-05
8	0.4521944665D-01	0.5866212585D-05
9	0.4547300879D-01	0.5727916838D-05
10	0.4526315608D-01	0.5814044194D-05
11	0.4537422954D-01	0.5854147556D-05
12	0.4524681043D-01	0.5881198353D-05
13	0.4564413164D-01	0.5608709870D-05
14	0.4633465363D-01	0.5827421616D-05
15	0.4573028770D-01	0.5870448911D-05
16	0.4545788829D-01	0.5873782757D-05
17	0.4530423077D-01	0.5883699879D-05
18	0.4519352759D-01	0.5908390757D-05
19	0.4518646046D-01	0.5885245069D-05
20	0.4518756513D-01	0.5872280096D-05

ND= 1 SAMPLING PERIOD= 4.51876D-02SEC AMPLITUDE= 5.87228D-06 N=20

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 36 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.4518756513D-01	0.5872280096D-05
1	0.4548441205D-01	0.4079529734D-05
2	0.4319492135D-01	0.4704818647D-05
3	0.4187535337D-01	0.5152752542D-05
4	0.4116129664D-01	0.5433852030D-05
5	0.4079400964D-01	0.5593423376D-05
6	0.4061516882D-01	0.5678091430D-05
7	0.4054580647D-01	0.5719556013D-05
8	0.4051715401D-01	0.5738437457D-05
9	0.4052024514D-01	0.5733467174D-05

ND= 1 SAMPLING PERIOD= 4.05284D-02SEC AMPLITUDE= 5.72936D-06 N= 9

Figure 2-7f.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 40 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.4052841794D-01	0.5729359139D-05
1	0.4105375766D-01	0.3993945099D-05
2	0.3886407364D-01	0.4642514785D-05
3	0.3754964350D-01	0.5096797069D-05
4	0.3794399220D-01	0.5485304103D-05
5	0.3726602911D-01	0.5596246507D-05
6	0.3696647229D-01	0.5629638876D-05
7	0.3681466579D-01	0.5646601267D-05
8	0.3671742546D-01	0.5669053607D-05
9	0.3671836255D-01	0.5637763821D-05
10	0.3674581125D-01	0.5613664939D-05
11	0.3671290779D-01	0.5635281029D-05
12	0.3674022718D-01	0.5616052818D-05
13	0.3670940690D-01	0.5636684469D-05
14	0.3672410413D-01	0.5624874991D-05
15	0.3669172621D-01	0.5645132778D-05
16	0.3669257664D-01	0.5640180348D-05

ND= 1 SAMPLING PERIOD= 3.66940D-02SEC AMPLITUDE= 5.63892D-06 N=16

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 44 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3669404019D-01	0.5638919976D-05
1	0.3740138530D-01	0.3908915533D-05
2	0.3491010532D-01	0.4588934634D-05
3	0.3481136753D-01	0.5336708821D-05
4	0.3411195900D-01	0.5483498604D-05
5	0.3379425697D-01	0.5541220300D-05
6	0.3363829381D-01	0.5568211972D-05
7	0.3354367470D-01	0.5593421704D-05
8	0.3354678742D-01	0.5560533574D-05
9	0.3369878931D-01	0.5457110214D-05
10	0.3358042361D-01	0.5524775541D-05
11	0.3354143435D-01	0.5555863009D-05
12	0.3348766726D-01	0.5589398120D-05
13	0.3348753557D-01	0.5678668056D-05
14	0.3352719081D-01	0.5526348349D-05
15	0.3352337686D-01	0.5596966864D-05
16	0.3352119786D-01	0.5575070099D-05
17	0.3352224713D-01	0.5566019732D-05
18	0.3352487997D-01	0.5563343263D-05

ND= 1 SAMPLING PERIOD= 3.35304D-02SEC AMPLITUDE= 5.56041D-06 N=18

Figure 2-7g.

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LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 20 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.6000000000D-01	0.1000000000D-04
1	0.6480879984D-01	0.7617629854D-05
2	0.6678026019D-01	0.7172493189D-05
3	0.6769282120D-01	0.6996822188D-05
4	0.6813043341D-01	0.6918483150D-05
5	0.6834250429D-01	0.6882531139D-05
6	0.6843089594D-01	0.6868190612D-05
7	0.6844645543D-01	0.6858162908D-05
8	0.6846072173D-01	0.6854744930D-05

ND= 1 SAMPLING PERIOD= 6.84679D-02SEC AMPLITUDE= 6.85169D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 18 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.6846787904D-01	0.6851692663D-05
1	0.7043256707D-01	0.7494237996D-05
2	0.7274274026D-01	0.7358573278D-05
3	0.7391256151D-01	0.7270807448D-05
4	0.7449677609D-01	0.7224620222D-05
5	0.7478967216D-01	0.7201219589D-05
6	0.7494085861D-01	0.7189269556D-05
7	0.7500155196D-01	0.7187936355D-05

ND= 1 SAMPLING PERIOD= 7.50031D-02SEC AMPLITUDE= 7.18146D-06 N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 16 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.7500313876D-01	0.7181455222D-05
1	0.7757207225D-01	0.7904102565D-05
2	0.8036526528D-01	0.7783556832D-05
3	0.8177101684D-01	0.7699329912D-05
4	0.8247175785D-01	0.7654031462D-05
5	0.8282203857D-01	0.7631003421D-05
6	0.8300181793D-01	0.7619062586D-05
7	0.8309100145D-01	0.7615679741D-05
8	0.8309249707D-01	0.7610629302D-05

ND= 1 SAMPLING PERIOD= 8.30950D-02SEC AMPLITUDE= 7.61094D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 14 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.8309501073D-01	0.7610937609D-05
1	0.8671839029D-01	0.8403642551D-05
2	0.9017647994D-01	0.8318391106D-05
3	0.9190691383D-01	0.8246307498D-05
4	0.9276881881D-01	0.8205510559D-05
5	0.9319861204D-01	0.8184558899D-05
6	0.9341750696D-01	0.8173588624D-05
7	0.9353498942D-01	0.8168723849D-05
8	0.9353941607D-01	0.8167792823D-05

ND= 1 SAMPLING PERIOD= 9.35426D-02SEC AMPLITUDE= 8.16689D-06 N= 8

Figure 2-7h.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 12 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.9354264774D-01	0.8166885799D-05
1	0.9916118222D-01	0.8990156255D-05
2	0.1035439723D 00	0.8984162026D-05
3	0.1057325363D 00	0.8944119276D-05
4	0.1068237479D 00	0.8916745965D-05
5	0.1073668119D 00	0.8902207034D-05
6	0.1076409935D 00	0.8894513616D-05
7	0.1077889937D 00	0.8890091772D-05
8	0.1078160524D 00	0.8894539016D-05

ND= 1 SAMPLING PERIOD= 1.07818D-01SEC AMPLITUDE= 8.88944D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 10 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1078178113D 00	0.8889444839D-05
1	0.1177100597D 00	0.9621415477D-05
2	0.1232336219D 00	0.9801365517D-05
3	0.1260503243D 00	0.9840692922D-05
4	0.1274639680D 00	0.9849211926D-05
5	0.1281670380D 00	0.9852245354D-05
6	0.1285188980D 00	0.9853611699D-05
7	0.1287044745D 00	0.9853366902D-05
8	0.1288196333D 00	0.9855235553D-05

ND= 1 SAMPLING PERIOD= 1.28820D-01SEC AMPLITUDE= 9.85437D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 8 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1288195875D 00	0.9854365774D-05
1	0.1497371337D 00	0.1019829650D-04
2	0.1558208532D 00	0.1080467416D-04
3	0.1593738457D 00	0.1102825825D-04
4	0.1612007188D 00	0.1112635578D-04
5	0.1621135626D 00	0.1117426051D-04
6	0.1625677190D 00	0.1119856757D-04
7	0.1628001760D 00	0.1121022683D-04
8	0.1629475305D 00	0.1121329115D-04

ND= 1 SAMPLING PERIOD= 1.62948D-01SEC AMPLITUDE= 1.12223D-05 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE K2=K3=0.1
GAMMA= 1.38000D 07 N= 6 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1629483722D 00	0.1122229409D-04
1	0.2218967506D 00	0.1065971960D-04
2	0.2236741739D 00	0.1198790559D-04
3	0.2275103182D 00	0.1256521127D-04
4	0.2295949249D 00	0.1286281146D-04
5	0.2306573515D 00	0.1301572755D-04
6	0.2311896028D 00	0.1309374339D-04
7	0.2314570519D 00	0.1313313639D-04
8	0.2315967783D 00	0.1315256779D-04
9	0.2317193751D 00	0.1315821728D-04

ND= 1 SAMPLING PERIOD= 2.31695D-01SEC AMPLITUDE= 1.31578D-05 N= 9

Figure 2-7i.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 19 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.6900000000D-01	0.6850000000D-05
1	0.6982002868D-01	0.7104879786D-05
2	0.7067198796D-01	0.7062188727D-05
3	0.7111813172D-01	0.7032306300D-05
4	0.7134776361D-01	0.7015467044D-05
5	0.7146958603D-01	0.7006501293D-05
6	0.7149620972D-01	0.7007741175D-05

ND= 1 SAMPLING PERIOD= 7.14990D-02SEC AMPLITUDE= 7.00315D-06 N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 17 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.7149897424D-01	0.7003148764D-05
1	0.7359621221D-01	0.7702531492D-05
2	0.7613679043D-01	0.7565776232D-05
3	0.7741360351D-01	0.7477910174D-05
4	0.7804933352D-01	0.7431759498D-05
5	0.7836715321D-01	0.7408489682D-05
6	0.7853062384D-01	0.7396573734D-05
7	0.7860385230D-01	0.7394488910D-05

ND= 1 SAMPLING PERIOD= 7.86051D-02SEC AMPLITUDE= 7.38821D-06 N= 7

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 15 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.7860513935D-01	0.7388205125D-05
1	0.8131859728D-01	0.8191990949D-05
2	0.8442897636D-01	0.8070950868D-05
3	0.8598065958D-01	0.7987256391D-05
4	0.8675165745D-01	0.7942301507D-05
5	0.8713578826D-01	0.7919630257D-05
6	0.8733187734D-01	0.7907942364D-05
7	0.8743345623D-01	0.7903856553D-05
8	0.8743538238D-01	0.7900274883D-05

ND= 1 SAMPLING PERIOD= 8.74380D-02SEC AMPLITUDE= 7.90034D-06 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 13 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.8743796585D-01	0.7900338702D-05
1	0.9117515478D-01	0.8823611040D-05
2	0.9508524084D-01	0.8739792180D-05
3	0.9702350587D-01	0.8668990230D-05
4	0.9798596860D-01	0.8628730973D-05
5	0.9846407971D-01	0.8608334827D-05
6	0.9870591642D-01	0.8597856687D-05
7	0.9883613977D-01	0.8592761851D-05
8	0.9884468557D-01	0.8594229618D-05

ND= 1 SAMPLING PERIOD= 9.88479D-02SEC AMPLITUDE= 8.59222D-06 N= 8

Figure 2-7j.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 11 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.9884794368D-01	0.8592219278D-05
1	0.1044747499D 00	0.9640300450D-05
2	0.1095187279D 00	0.9645574868D-05
3	0.1120148145D 00	0.9609851254D-05
4	0.1132569073D 00	0.9583728478D-05
5	0.1138726419D 00	0.9570155354D-05
6	0.1141811415D 00	0.9563325304D-05
7	0.1143452988D 00	0.9559276510D-05
8	0.1143988886D 00	0.9564527102D-05
9	0.1143991287D 00	0.9554879230D-05
10	0.1144006480D 00	0.9558310577D-05

ND= 1 SAMPLING PERIOD= 1.14401D-01SEC AMPLITUDE= 9.55743D-06 N=10

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 9 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1144013081D 00	0.9557425297D-05
1	0.1241494017D 00	0.1067754392D-04
2	0.1306587769D 00	0.1091301385D-04
3	0.1339507820D 00	0.1097420956D-04
4	0.1356040142D 00	0.1099044938D-04
5	0.1364238133D 00	0.1099746791D-04
6	0.1368311416D 00	0.1100135561D-04
7	0.1370419319D 00	0.1100264376D-04
8	0.1371781879D 00	0.1100269404D-04

ND= 1 SAMPLING PERIOD= 1.37178D-01SEC AMPLITUDE= 1.10042D-05 N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 7 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1371781828D 00	0.1100419929D-04
1	0.1588942126D 00	0.1188690438D-04
2	0.1661359636D 00	0.1274324753D-04
3	0.1704188526D 00	0.1307466534D-04
4	0.1726362836D 00	0.1322051006D-04
5	0.1737443075D 00	0.1329200577D-04
6	0.1742927971D 00	0.1332879126D-04
7	0.1745694438D 00	0.1334713151D-04
8	0.1747289928D 00	0.1335402056D-04
9	0.1745581227D 00	0.1337072875D-04
10	0.1747405965D 00	0.1336926525D-04
11	0.1747407951D 00	0.1336251723D-04

ND= 1 SAMPLING PERIOD= 1.74742D-01SEC AMPLITUDE= 1.33682D-05 N=11

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 5 PHI= 0.0

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.1747423365D 00	0.1336817555D-04
1	0.2495296658D 00	0.1275924454D-04
2	0.2467069249D 00	0.1494332451D-04
3	0.2510819205D 00	0.1595232873D-04
4	0.2535581725D 00	0.1649747219D-04
5	0.2548456164D 00	0.1678316421D-04
6	0.2554959218D 00	0.1693025549D-04
7	0.2558226254D 00	0.1700500828D-04
8	0.2559894316D 00	0.1704250683D-04
9	0.2560881662D 00	0.1706015366D-04
10	0.2560833384D 00	0.1707718967D-04

ND= 1 SAMPLING PERIOD= 2.56084D-01SEC AMPLITUDE= 1.70779D-05 N=10

Figure 2-7k.

```
LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07    N=   4   PHI= 9.00000D 01
  ITERATION  SAMPLING PERIOD        AMPLITUDE
       0      0.5000000000D 00    0.1500000000D-04
       1      0.5154240480D 00    0.1377904361D-04
       2      0.5395948070D 00    0.1446567295D-04
       3      0.5480038734D 00    0.1474781509D-04
       4      0.5501232936D 00    0.1486946298D-04
       5      0.5498353305D 00    0.1493377898D-04
       6      0.5496153256D 00    0.1496763755D-04
       7      0.5495519111D 00    0.1498535914D-04
       8      0.5495572657D 00    0.1498126090D-04

ND= 1    SAMPLING PERIOD= 5.49637D-01SEC    AMPLITUDE= 1.49913D-05    N= 8

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07    N=   4   PHI= 8.00000D 01
  ITERATION  SAMPLING PERIOD        AMPLITUDE
       0      0.5496365534D 00    0.1499127478D-04
 2     5.49637D-01    -1.91174D-05
 2     5.50186D-01    -1.95688D-05
       1      0.4234186796D 00    0.8454562303D-04
 2     4.23419D-01    -7.38180D-05
 2     4.23842D-01    -7.45811D-05
       2      0.3730698891D 00    0.1692422145D-04
       3      0.4128434144D 00    0.1637625694D-04
       4      0.4312546179D 00    0.1654907656D-04
       5      0.4397401524D 00    0.1665122919D-04
       6      0.4438301096D 00    0.1670404294D-04
       7      0.4458433195D 00    0.1673064847D-04
       8      0.4468524793D 00    0.1674390833D-04
       9      0.4473906822D 00    0.1675010719D-04
      10      0.4475205608D 00    0.1675157026D-04

ND= 1    SAMPLING PERIOD= 4.47512D-01SEC    AMPLITUDE= 1.67488D-05    N=10

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07    N=   4   PHI= 7.00000D 01
  ITERATION  SAMPLING PERIOD        AMPLITUDE
       0      0.4475120178D 00    0.1674881540D-04
       1      0.4216082457D 00    0.1861170429D-04
       2      0.4051047902D 00    0.1900082144D-04
       3      0.3938394079D 00    0.1918318637D-04
       4      0.3867604154D 00    0.1928180740D-04
       5      0.3825407502D 00    0.1933525459D-04
       6      0.3801177764D 00    0.1936336477D-04
       7      0.3787622054D 00    0.1937772905D-04
       8      0.3780160288D 00    0.1938492295D-04
       9      0.3776076550D 00    0.1938848190D-04
      10      0.3773823844D 00    0.1939023413D-04

ND= 1    SAMPLING PERIOD= 3.77255D-01SEC    AMPLITUDE= 1.93911D-05    N=10
```

Figure 2-8a.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 4 PHI= 6.00000D 01

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3772549942D 00	0.1939110275D-04
1	0.3599882449D 00	0.2122313759D-04
2	0.3474431151D 00	0.2199493076D-04
3	0.3389745293D 00	0.2246394718D-04
4	0.3336445753D 00	0.2275171224D-04
5	0.3304527295D 00	0.2292115436D-04
6	0.3286099425D 00	0.2301675458D-04
7	0.3275734451D 00	0.2306894923D-04
8	0.3270001735D 00	0.2309677767D-04
9	0.3266852982D 00	0.2311134342D-04

ND= 1 SAMPLING PERIOD= 3.26511D-01SEC AMPLITUDE= 2.31189D-05 N= 9

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 4 PHI= 5.00000D 01

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3265114680D 00	0.2311885352D-04
1	0.3134437038D 00	0.2496959109D-04
2	0.3043395095D 00	0.2615638510D-04
3	0.2983602071D 00	0.2692653836D-04
4	0.2946473529D 00	0.2740556060D-04
5	0.2924408293D 00	0.2768659223D-04
6	0.2911717454D 00	0.2784892157D-04
7	0.2904579278D 00	0.2793712255D-04
8	0.2900603812D 00	0.2798493040D-04
9	0.2898360556D 00	0.2801133966D-04

ND= 1 SAMPLING PERIOD= 2.89699D-01SEC AMPLITUDE= 2.80285D-05 N= 9

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07 N= 4 PHI= 4.00000D 01

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.2896990149D 00	0.2802847803D-04
1	0.2794890678D 00	0.3001133103D-04
2	0.2729917247D 00	0.3146410314D-04
3	0.2689976369D 00	0.3242627184D-04
4	0.2666235114D 00	0.3301513461D-04
5	0.2652524683D 00	0.3335574996D-04
6	0.2644772430D 00	0.3354573933D-04
7	0.2640421182D 00	0.3365027538D-04
8	0.2637896353D 00	0.3371108549D-04
9	0.2635933842D 00	0.3377144366D-04
10	0.2635372810D 00	0.3378429577D-04
11	0.2634699260D 00	0.3382358768D-04
12	0.2628642669D 00	0.3435228752D-04
13	0.2633101399D 00	0.3394466571D-04
14	0.2635730871D 00	0.3370494245D-04
15	0.2635730069D 00	0.3368040171D-04

ND= 1 SAMPLING PERIOD= 2.63573D-01SEC AMPLITUDE= 3.36804D-05 N=15

Figure 2-8b.

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07   N=   4   PHI= 3.00000D 01

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.2635730069D 00	0.3368040171D-04
1	0.2557200863D 00	0.3568292423D-04
2	0.2512815177D 00	0.3709936920D-04
3	0.2488011387D 00	0.3799526518D-04
4	0.2474297532D 00	0.3851949943D-04
5	0.2466773243D 00	0.3881190135D-04
6	0.2462623409D 00	0.3897256217D-04
7	0.2460172960D 00	0.3907037428D-04
8	0.2456114935D 00	0.3934850366D-04
9	0.2459718360D 00	0.3864238696D-04
10	0.2458310873D 00	0.3891659330D-04
11	0.2457483856D 00	0.3904801434D-04
12	0.2460093270D 00	0.3911037421D-04
13	0.2457639879D 00	0.3924292727D-04
14	0.2453306847D 00	0.3981897911D-04
15	0.2456430421D 00	0.3930859796D-04

ND= 1   SAMPLING PERIOD= 2.45643D-01SEC   AMPLITUDE= 3.93086D-05   N=15

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07   N=   4   PHI= 2.00000D 01

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.2456430421D 00	0.3930859796D-04
1	0.2403415437D 00	0.4039844502D-04
2	0.2382262614D 00	0.4068682338D-04
3	0.2374380085D 00	0.4064485820D-04
4	0.2371315275D 00	0.4054959708D-04
5	0.2369781829D 00	0.4050046427D-04
6	0.2368591609D 00	0.4052036897D-04

ND= 1   SAMPLING PERIOD= 2.36794D-01SEC   AMPLITUDE= 4.05412D-05   N= 6

LST SYSTEM-NUMERICAL SOLUTION OF DISCRETE DESCRIBING FUNCTION
TWO AXIS-EQUAL AMPLITUDE CASE, K2= 1.000D-01, K3= 1.000D-01
GAMMA= 1.38000D 07   N=   4   PHI= 1.00000D 01

ITERATION	SAMPLING PERIOD	AMPLITUDE
0	0.3223799858D 00	0.1674694093D-04
1	0.3217327985D 00	0.2144602416D-04
2	0.3479304133D 00	0.1476851826D-04
3	0.3560044890D 00	0.1469518706D-04
4	0.3593430003D 00	0.1468487728D-04

ND= 1   SAMPLING PERIOD= 3.60729D-01SEC   AMPLITUDE= 1.46858D-05   NT= 4

Figure 2-8c.

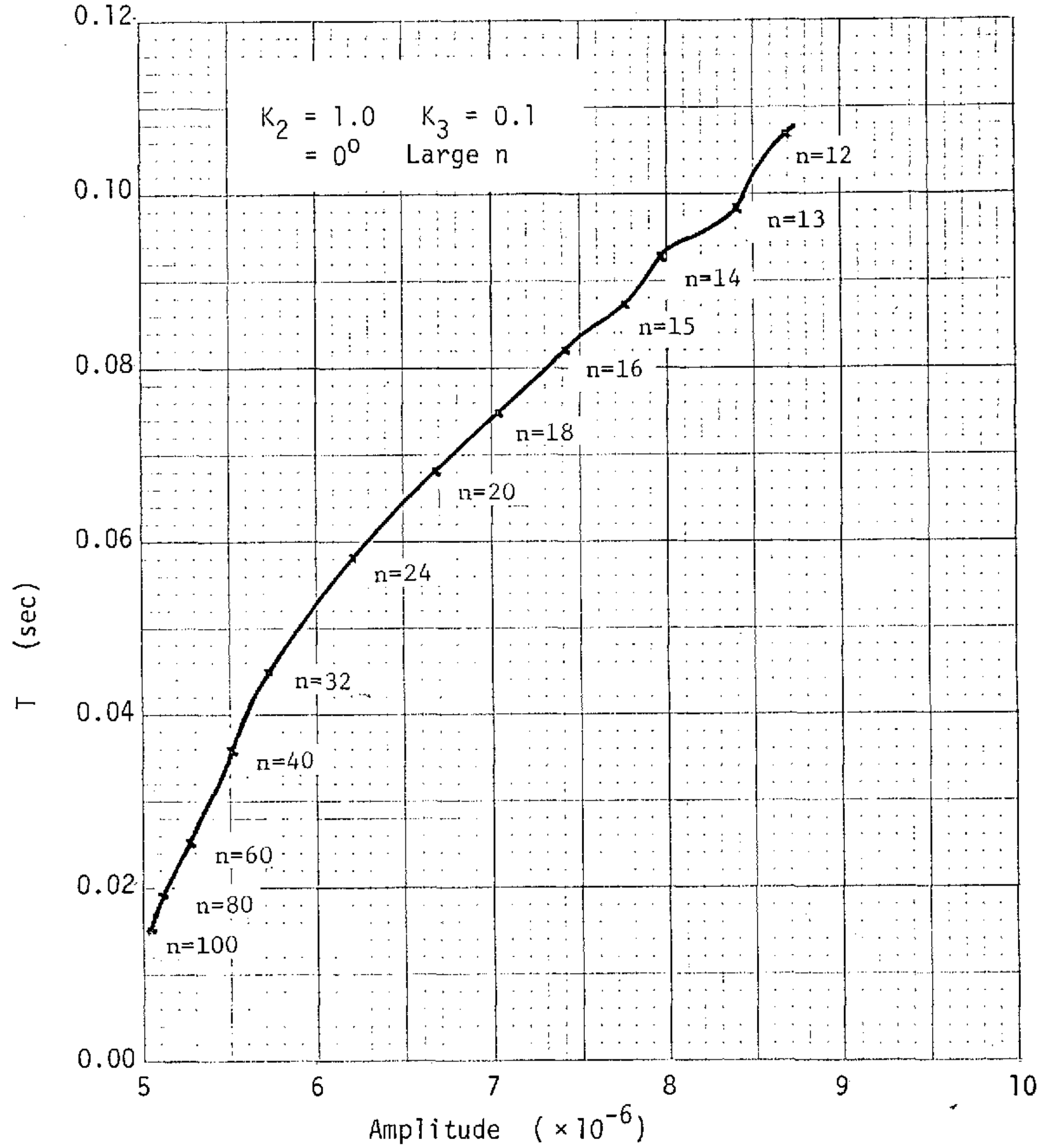


Figure 2-9. Amplitude and frequency ($2\pi/nT$) of self-sustained oscillations for various sampling periods in the discrete-data two-axis LST system; $\gamma = 1.38\times10^{7}$.

2-2

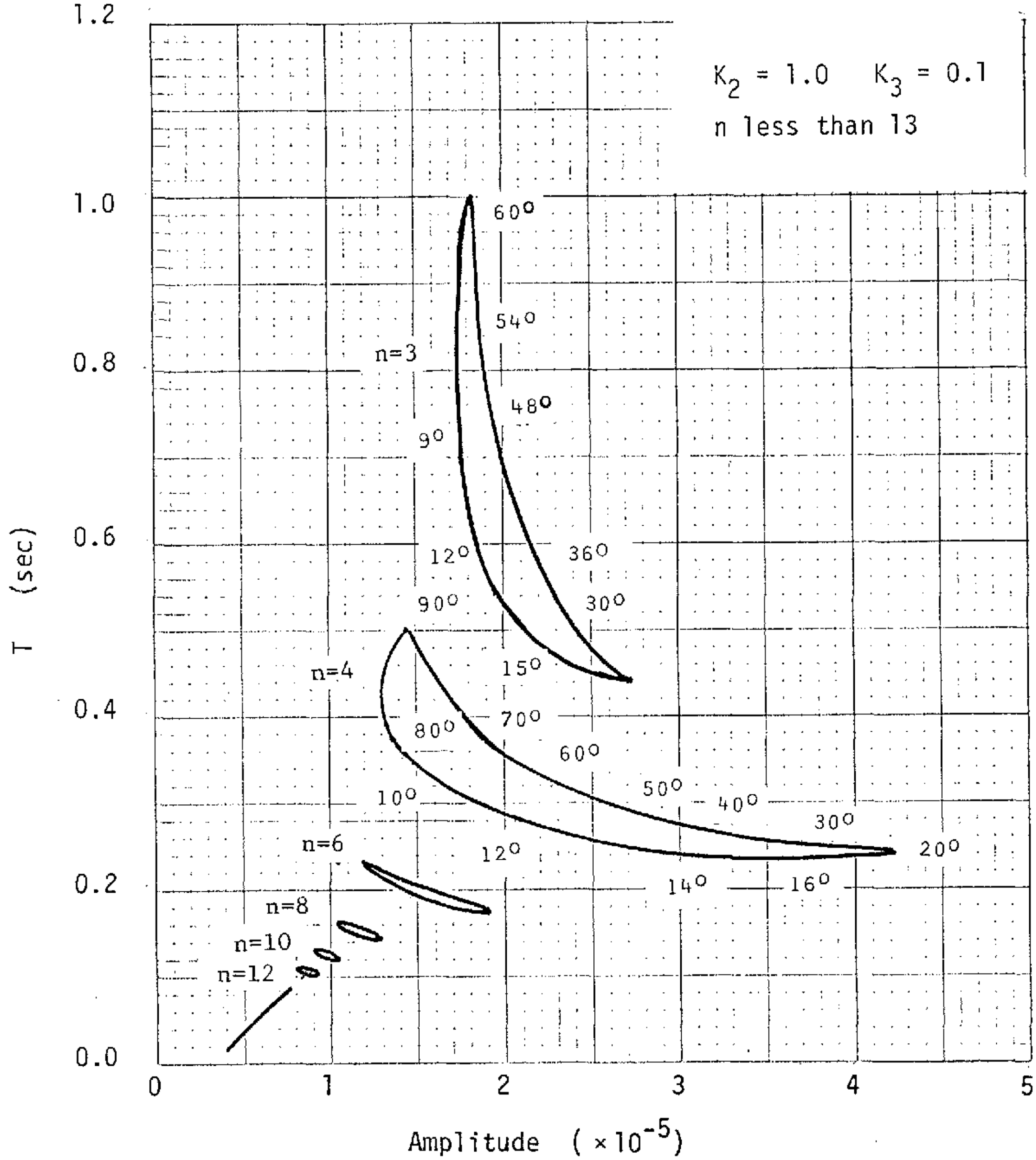


Figure 2-10. Amplitude and frequency ($2\pi/nT$) of self-sustained oscillations for various sampling periods in the discrete-data two-axis LST system; $\gamma = 1.38\times10^{7}$.

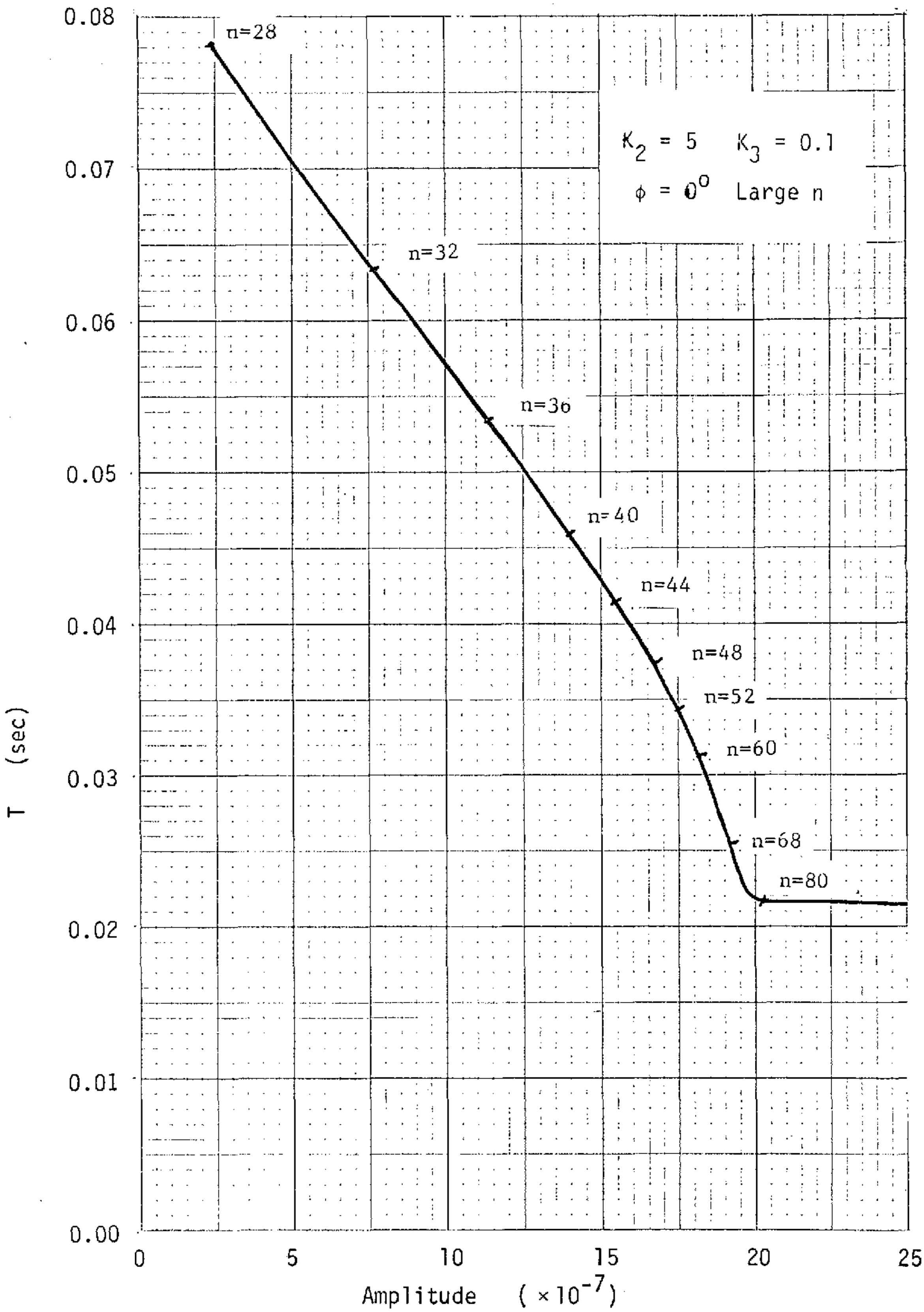


Figure 2-11. Amplitude and frequency ($2\pi/nT$) of self-sustained oscillations for various sampling periods in the discrete-data two-axis LST system; $\gamma = 1.38\times10^{7}$.

REFERENCES

1. B. C. Kuo and G. Singh, Design of the Large Space Telescope System, Final Report, for NAS8-29853, Systems Research Laboratory, Champaign, Illinois, July 1, 1974.

2. B. C. Kuo and G. Singh, Continuous and Discrete Describing Function Analysis of the LST System, Final Report, for NAS8-29853, Systems Research Laboratory, Champaign, Illinois, January 1, 1974.

3. Study of Unequal-Amplitude Oscillations in Two-Axis Coupled Nonlinear Systems

3-1. Introduction

In the preceding two chapters conditions of self-sustained oscillations in the two-axis LST system have been studied by a numerical-iterative technique using the describing function. It is demonstrated that the numerical-iterative technique can be applied to the continuous-data as well as the discrete-data LST system. The difference between these two types of systems lies only in the stability equations, and these made no difference to the iterative algorithm.

It is important to note that since the dynamics of the two axes are identical, it is assumed a priori that the amplitudes of oscillations, if there are any, are of equal amplitude. Certainly, all the solutions of the self-sustained oscillations obtained in the previous chapters are sinusoidal signals with equal amplitude in both axes. However, in this chapter we shall show that, in general, it is possible to have unequal magnitudes of oscillations in the coupled system even if the dynamics of the two axes are identical. Furthermore, when the amplitudes of oscillations are equal, the phase can still be 180 degrees apart.

Since we have reasoned that the numerical-iterative technique does not distinguish a continuous-data system from a discrete-data system, only models of the former version are discussed here.

3-2. Conditions of Unequal-Amplitude Oscillations in The Continuous-Data Two-Axis Coupled LST System

Figure 3-1 shows the signal flow graph of the continuous-data two-axis model of the LST system. The inputs to the two nonlinearities are designated as $E_1(j\omega)$ and $E_2(j\omega)$, respectively. Notice that these two variables are complex quantities in general, and each represents a magnitude and phase in the sinusoidal steady state. Since the describing function of the CMG nonlinearity is a function of the input amplitude and frequency, Fig. 3-1 shows that the describing functions which correspond to $E_1(j\omega)$ and $E_2(j\omega)$ are $N(E_1,\omega)$ and $N(E_2,\omega)$, respectively. For simplicity, the following notations are used:

$$E_1(j\omega) = E_1$$

$$E_2(j\omega) = E_2$$

$$N(E_1,\omega) = N_1$$

$$N(E_2,\omega) = N_2$$

The transfer functions in Fig. 3-1 are defined as

$$G_1 = \frac{(K_o + K_1 s)K_I}{K_p s + K_I} \quad (3\text{-}1)$$

$$G_2 = \frac{K_p s + K_I}{s} \quad (3\text{-}2)$$

$$G_3 = \frac{1}{J_G s} \quad (3\text{-}3)$$

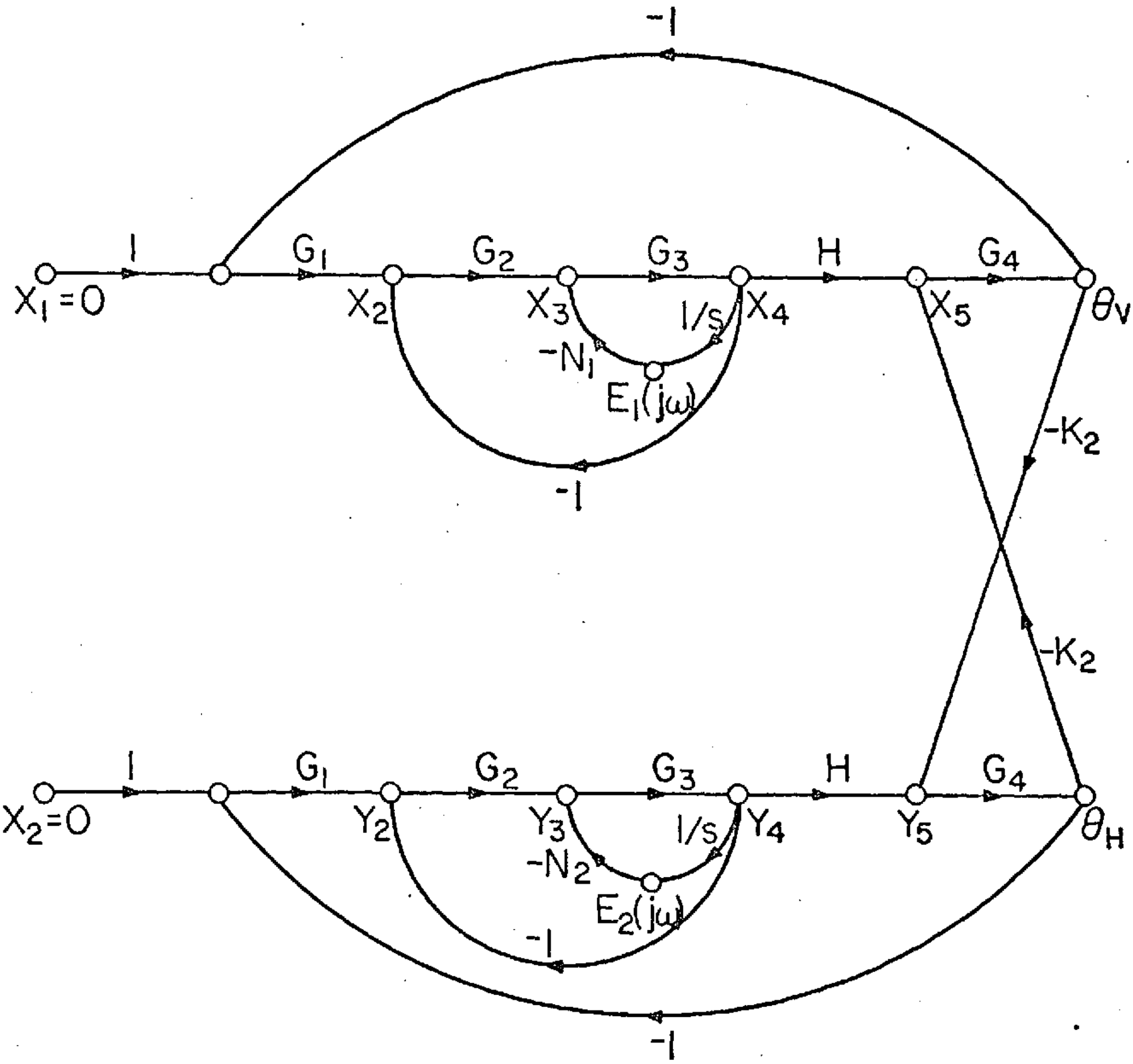


Figure 3-1. The simplified continuous-data two-axis LST system model.

$$G_4 = \frac{1/J_v}{s^2 + K_3 s + K_2} \tag{3-4}$$

The determinant (characteristic equation) of the coupled system of Fig. 3-1 is written

$$\Delta = 1 + G_A + G_{B1}N_1 + G_{B2}N_2 + G_C N_1 N_2 = 0 \tag{3-5}$$

The last expression is also known as the stability equation of the system.

The linear transfer functions in Eq. (3-5) are

$$G_A = 2G_2G_3 + 2G_1G_2G_3G_4H - K_2^2G_4^2 + G_2^2G_3^2 + (G_1G_2G_3G_4H)^2$$
$$+ 2G_1G_2^2G_3^2G_4H - 2G_2G_3G_4^2K_2^2 - G_2^2G_3^2G_4^2K_2^2 \tag{3-6}$$

$$G_{B1} = G_{B2} = \frac{G_3}{s} + \frac{G_2G_3^2}{s} + \frac{G_1G_2G_3^2G_4H}{s} - \frac{G_3G_4^2K_2^2}{s} - \frac{G_2G_3^2G_4^2K_2^2}{s} \tag{3-7}$$

$$G_C = \frac{G_3^2}{s^2} - \frac{G_3^2G_4^2K_2^2}{s^2} \tag{3-8}$$

Equation (3-5) represents a necessary condition for self-sustained oscillations in the system of Fig. 3-1. However, in general, if the amplitudes of E_1 and E_2 are not equal, Eq. (3-5) represents a set of two nonlinear equations with four unknowns in E_1, E_2, the frequency ω, and ϕ, the phase between E_1 and E_2. Therefore, in addition to the two equations which are obtained from the real and imaginary parts of Eq. (3-5), a third complex equation must come from the relation between E_1 and E_2 which is derived as follows.

The following equations are written directly from Fig. 3-1:

$$
\begin{aligned}
E_1 &= \frac{1}{s}x_4 \\
x_4 &= G_3x_3 \\
x_3 &= -N_1E_1 + G_2x_2 \\
x_2 &= -x_4 - G_1\theta_V \\
\theta_V &= G_4x_5 \\
x_5 &= Hx_4 - K_2\theta_H
\end{aligned}
\tag{3-9}
$$

$$
\begin{aligned}
E_2 &= \frac{1}{s}y_4 \\
y_4 &= G_3y_3 \\
y_3 &= -N_2E_2 + G_2y_2 \\
y_2 &= -y_4 - G_1\theta_H \\
\theta_H &= G_4y_5 \\
y_5 &= Hy_4 - K_2\theta_V
\end{aligned}
\tag{3-10}
$$

From these equations it is observed that

$$\frac{E_1}{E_2} = \frac{x_4}{y_4} = \frac{x_3}{y_3} = \frac{1 + \frac{N_2G_3}{s} + G_2G_3}{1 + \frac{N_1G_3}{s} + G_2G_3} \cdot \frac{x_5}{y_5} \tag{3-11}$$

Eliminating E_1, E_2, θ_V, θ_H, x_4 and y_4 from Eqs. (3-9) and (3-10), we have

$$x_3 = -\frac{N_1 G_3}{s} x_3 + G_2 x_2$$

$$x_2 = -G_3 x_3 - G_1 G_4 x_5 \qquad (3\text{-}12)$$

$$x_5 = HG_3 x_3 - K_2 G_4 y_5$$

$$y_3 = -\frac{N_2 G_3}{s} y_3 + G_2 y_2$$

$$y_2 = -G_3 y_3 - G_1 G_4 y_5 \qquad (3\text{-}13)$$

$$y_5 = HG_3 y_3 - K_2 G_4 x_5$$

These equations are solved to yield

$$\frac{x_5}{y_5} = \frac{1 + A_2 + B + C}{-K_2 G_4 (1 + A_2 + B)} \qquad (3\text{-}14)$$

or

$$\frac{x_5}{y_5} = \frac{-K_2 G_4 (1 + A_1 + B)}{1 + A_1 + B + C} \qquad (3\text{-}15)$$

where

$$A_1 = \frac{N_1 G_3}{s} \qquad (3\text{-}16)$$

$$A_2 = \frac{N_2 G_3}{s} \qquad (3\text{-}17)$$

$$B = G_2 G_3 \qquad (3\text{-}18)$$

$$C = G_1 G_2 G_3 G_4 H \qquad (3\text{-}19)$$

Thus, from Eq. (3-11),

$$\frac{E_1}{E_2} = \frac{(1 + A_2 + B)x_5}{(1 + A_1 + B)y_5} = \frac{1 + A_2 + B + C}{-K_2G_4(1 + A_1 + B)} \tag{3-20}$$

or

$$\frac{E_1}{E_2} = \frac{-K_2G_4(1 + A_2 + B)}{1 + A_1 + B + C} \tag{3-21}$$

It is interesting to point out that the two equations for E_1/E_2 in Eqs. (3-20) and (3-21) should be equal to each other. Thus,

$$\frac{E_1}{E_2} = \frac{-K_2G_4(1 + A_2 + B)}{1 + A_1 + B + C} = \frac{1 + A_2 + B + C}{-K_2G_4(1 + A_1 + B)} \tag{3-22}$$

When the last equation is expanded, it can be shown that it is identical to the condition of $\Delta = 0$ given in Eq. (3-5).

Multiplying Eqs. (3-20) and (3-21) yields

$$\left(\frac{E_1}{E_2}\right)^2 = \frac{(1 + A_2 + B)(1 + A_2 + B + C)}{(1 + A_1 + B)(1 + A_1 + B + C)} \tag{3-23}$$

When $|E_1| = |E_2|$, $N_1 = N_2$, and $A_1 = A_2$; Eq. (2-23) gives

$$\left(\frac{E_1}{E_2}\right)^2 = 1 \tag{3-24}$$

Equation (3-24) implies that E_1 and E_2 can be of the same phase or they can be 180 degrees out of phase. Instead, when the numerical-iterative technique is applied to the $|E_1| = |E_2| = |E|$

situation, it is found that the case, $E_1 = -E_2$, is true.

Returning to the stability problem, Eqs. (3-5) and (3-23) form the necessary conditions of self-sustained oscillations for the general case of the system in Fig. 3-1.

Since Eqs. (3-5) and (3-23) are independent mathematical relations, it is conceivable that in general there may be $E_1 \neq E_2$ solutions which may satisfy these equations simultaneously, thus signifying the possibility of self-sustained oscillations with unequal amplitudes in the two axes of the coupled system.

Now since there are three unknown parameters in the magnitudes of E_1 and E_2, and ω, it is difficult to arrive at a sustained-oscillation solution via the numerical-iterative method for the complex LST system. For the given system with the specific nonlinearity, even if a sustained oscillation exists, one has to be able to give a proper set of initial guesses for E_1, E_2, ϕ, and ω in order to arrive at the convergent solution. This has been proven to be quite difficult for the LST system. In order to gain further insight to the problem, two systems of lesser complexity are considered in the following sections. Through these simpler systems we are able to draw general conclusions on the possibility of having unequal-amplitude oscillations with the CMG nonlinearity.

3-3. A Simple Two-Axis Coupled Nonlinear System

Consider the simple two-axis coupled system shown in Fig. 3-2. The characteristic equation of the system is

$$\Delta = 1 - K^2G^2N_1N_2 \tag{3-25}$$

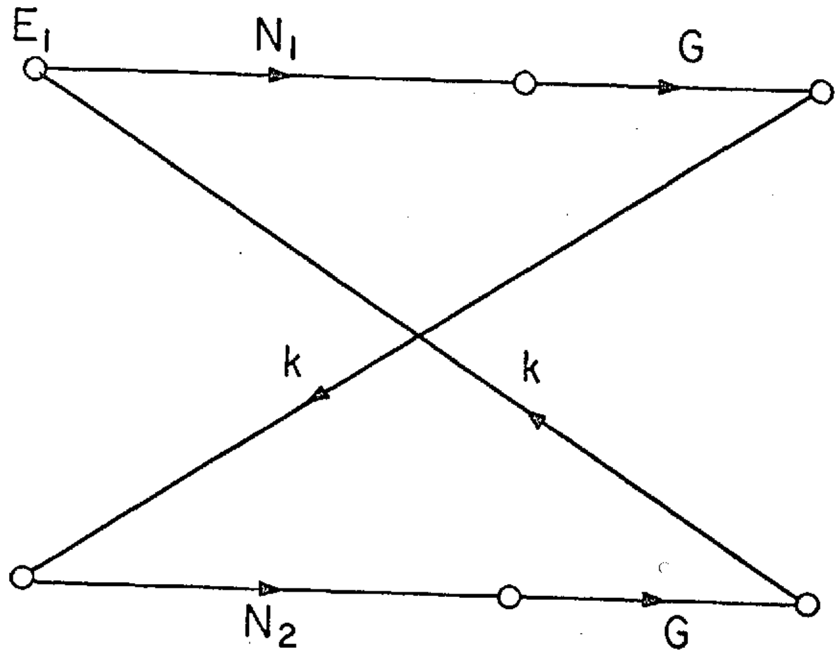


Figure 3-2. A simple two-axis coupled nonlinear system.

The input signals of the nonlinear elements are written from Fig. 3-2.

$$E_1 = kx_2 = N_2GkE_2 \tag{3-26}$$

Thus,

$$\frac{E_1}{E_2} = N_2Gk \tag{3-27}$$

Also,

$$E_2 = kx_1 = N_1GkE_1 \tag{3-28}$$

from which we have

$$\frac{E_1}{E_2} = \frac{1}{N_1Gk} \tag{3-29}$$

Equations (3-27) and (3-29) are multipled to give

$$\left(\frac{E_1}{E_2}\right)^2 = \frac{N_2}{N_1} \tag{3-30}$$

In order for the system to exhibit self-sustained oscillations, it is necessary that $\Delta = 0$ be satisfied, along with any one of Eqs. (3-27), (3-29), and (3-30). These provide two complex or four real equations with four unknowns in $|E_1|$, $|E_2|$, ω, and ϕ.

In general, if appropriate initial guesses are available for $|E_1|$, $|E_2|$, ω, and ϕ, the numerical-iterative method will seek out the solution to the system of nonlinear equations represented by the equations discussed above. This is useful for the case when $|E_1| = |E_2| = E$, and $\phi = 0$, since the initial guess for these solutions is available from the single-axis system solutions.

In the case when $|E_1| \neq |E_2|$ and $\phi \neq 0$, a general procedure is needed to solve the nonlinear equations.

One method of solution is to span the ω, $|E_1|$, and $|E_2|$ space and plot Δ (or $\Delta - 1$) or Eq. (3-25) in magnitude versus phase coordinates, and seek out the $\Delta = 0$ solutions. These solutions then become candidates for possible solutions of Eq. (3-27), (3-29) or (3-30), with the variable parameter ϕ free.

An alternative is to solve Eq. (3-27), (3-29) or (3-30) first and then use the solution or solutions to check with Eq. (3-25). Along this direction, a necessary condition for the existence of limit cycles is given by Eq. (3-30) which is interpreted as follows.

Equation (3-30) is written

$$\frac{|E_1^2|}{|E_2^2|} = \frac{|N_2|}{|N_1|} \tag{3-31}$$

Since $N_2 = N(E_2)$ and $N_1 = N(E_1)$, Eq. (3-31) will be satisfied if E^2 and $K/N(E)$, where K is an arbitrary constant, have the same magnitude for two different values of E. This means that if the curves of E^2 and $K/N(E)$ are plotted with respect to E, there must be two intersections between these two curves in order to satisfy

Eq. (3-31). Figure 3-3 shows the plots of E^2 and $|1/N|$ in db versus E in db, with N being the describing function of the CMG nonlinearity with $\gamma = 1.38 \times 10^6$. It is apparent that these two curves intersect at at most one point for any K. Therefore, we may conclude that with the CMG nonlinearity in the system configuration of Fig. 3-2, self-sustained oscillations of unequal amplitudes cannot occur with any transfer function G.

Let us define

$$f = |N| \tag{3-32}$$

$$h = \frac{1}{f} = \frac{1}{|N|} \tag{3-33}$$

Also, let

$$g = \log_{10} h \tag{3-34}$$

and

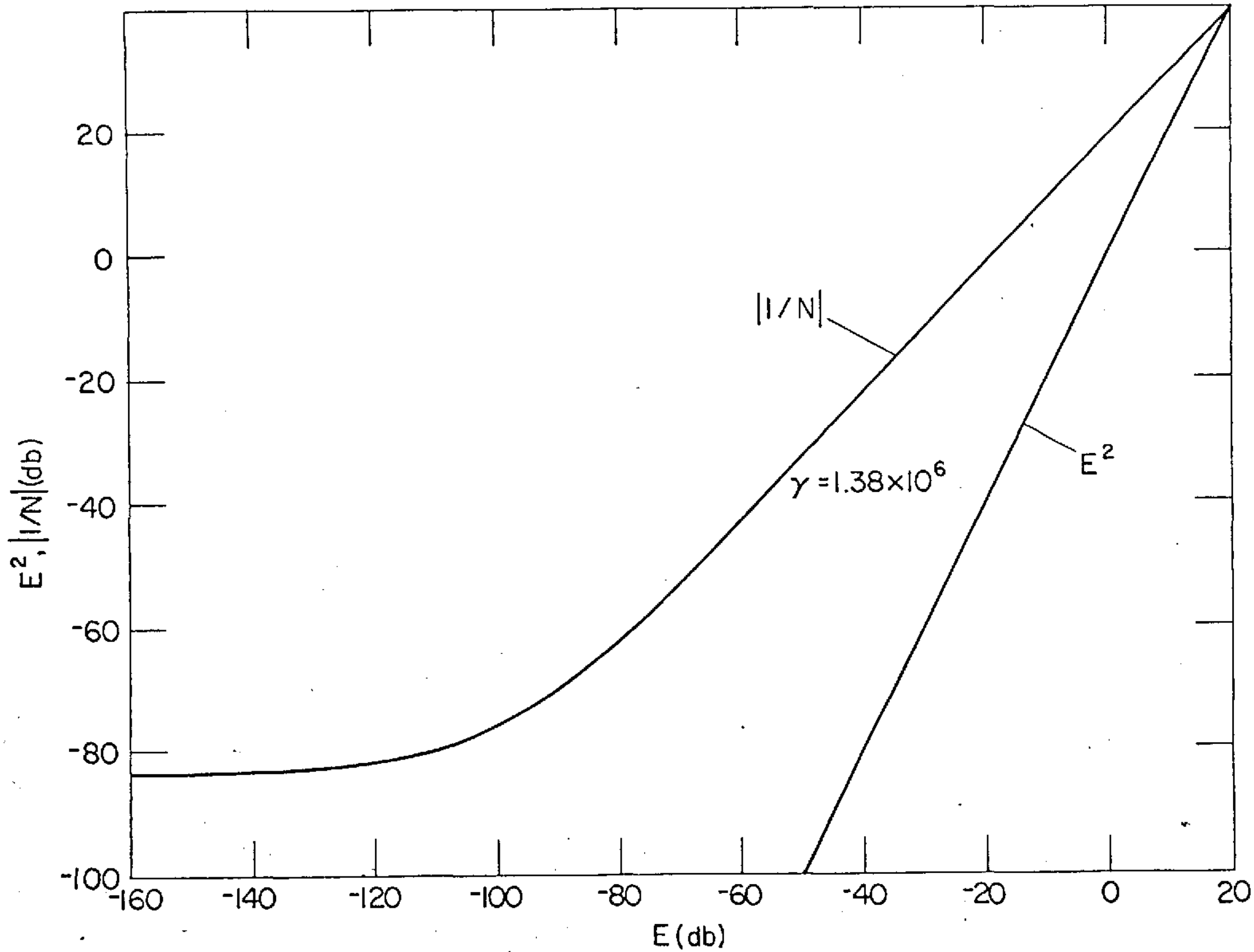
$$x = \log_{10} E \tag{3-35}$$

Then, the slope of the curve for $|1/N|$ in Fig. 3-3 is given by

$$\frac{dg}{dx} = \frac{1}{h}\frac{dh}{df}\frac{df}{dE}\frac{dE}{dx} = -Eh\frac{df}{dE} \tag{3-36}$$

or

$$\frac{dg}{dx} = -Eh\frac{d|N|}{dE} \tag{3-37}$$



|1/N|
γ = 1.38×10^6
E^2
E^2, |1/N| (db)
E (db)
20
0
-20
-40
-60
-80
-100
-160
-140
-120
-100
-80
-60
-40
-20
0
20

Figure 3-3.

With

$$f = E^2 \tag{3-38}$$

Eq. (3-36) gives

$$\frac{dg}{dx} = 2 \tag{3-39}$$

Thus, the necessary condition for Eq. (3-31) to be satisfied is that the slope of the $\log_{10}|1/N|$ versus $\log_{10}E$ curve exceeds the value 2 at at least one point. This will ensure two intersections between the E^2 and $|1/N|$ curves.

3-4. A Simplified Multiple-Loop Two-Axis Coupled Nonlinear System

Figure 3-4 shows the signal flow graph of a two-axis coupled system which is simpler than the LST system and yet still retains the essential characteristics of the system.

The transfer functions of the system are defined as

$$G_1 = \frac{1}{s+1}$$

$$G_2 = \frac{1}{s^2} \tag{3-40}$$

$$H = s + 10^{-3}$$

These transfer functions and parameters are selected so that the single-axis system has two limit cycles, one stable and one unstable, with $\gamma = 1.38 \times 10^6$ and $T_o = 0.1$ for the CMG nonlinearity.

The determinant of the coupled system is

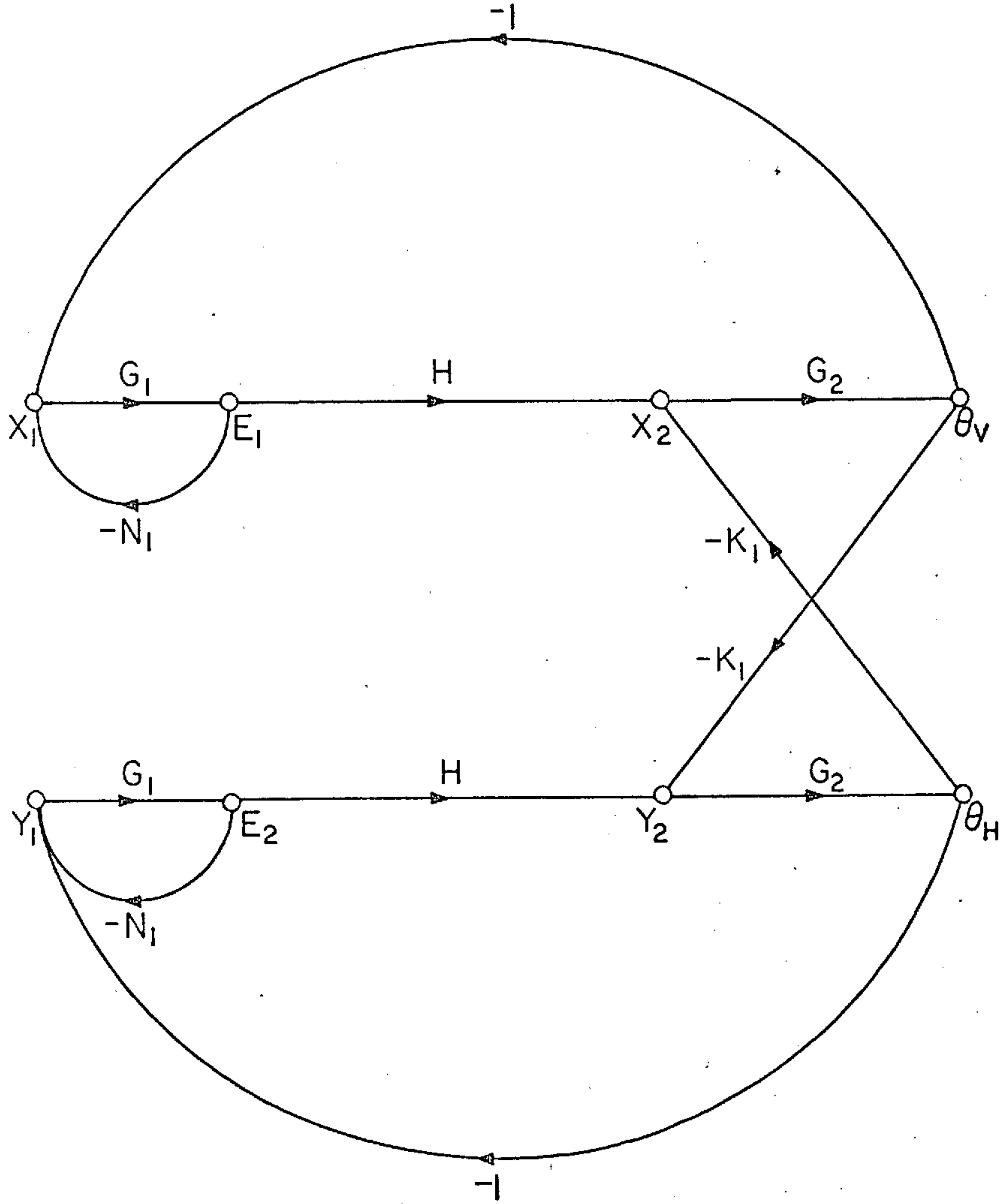


Figure 3-4.

$$\Delta = [(1 + HG_1G_2)^2 - K_1^2G_2^2] + (N_1 + N_2)G_1(1 + HG_1G_2 - K_1^2G_2^2)$$
$$+ N_1N_2G_1^2(1 - K_1^2G_2^2) \qquad (3\text{-}41)$$

The relationships between E_1 and E_2 are determined from the following equations:

$$\begin{aligned} x_1 &= -N_1E_1 - G_2x_2 \\ x_2 &= HG_1x_1 - K_1G_2y_2 \\ y_1 &= -N_2E_2 - G_2y_2 \\ y_2 &= HG_1y_1 - K_1G_2x_2 \end{aligned} \qquad (3\text{-}42)$$

Thus,

$$\frac{E_1}{E_2} = \frac{x_1}{y_1} = \frac{1 + N_2G_1 + HG_1G_2}{-K_1G_2(1 + N_1G_1)} \qquad (3\text{-}43)$$

or

$$\frac{E_1}{E_2} = \frac{-K_1G_2(1 + N_2G_1)}{1 + N_1G_1 + HG_1G_2} \qquad (3\text{-}44)$$

Again, if we equate Eq. (3-43) to Eq. (3-44), we would get Δ as in Eq. (3-41).

From Eqs. (3-43) and (3-44) we have

$$\left(\frac{E_1}{E_2}\right)^2 = \frac{(1 + N_2G_1)(1 + N_2G_1 + HG_1G_2)}{(1 + N_1G_1)(1 + N_1G_1 + HG_1G_2)} \qquad (3\text{-}45)$$

The numerical-iterative method was first used to solve for the conditions of limit cycles for $|E_1| = |E_2|$ given by Eqs. (3-41) and one of the equations in Eqs. (3-43), (3-44), and (3-45). It turned out that when $|E_1| = |E_2|$, $\phi = 180°$; that is, the two sinusoidal signals at the inputs of N_1 and N_2 are 180 degrees out of phase. Using appropriate initial guesses for $|E_1| = |E_2|$, ω, and with $\phi = 180°$, the numerical-iterative method yielded limit cycle solutions shown in Fig. 3-5 for various degrees of coupling coefficient K_1. Figure 3-6 shows the numerical iterations for the various values of K_1.

A graphical procedure was carried out for the solution of limit cycles when $E_1 \neq E_2$, since the numerical-iterative method yielded no convergent solutions.

Figure 3-7 shows the plots of Δ of Eq. (3-41) for various combinations of $|E_1|$ and $|E_2|$. For limit cycles to exist the plot of Δ must go through the (0 db, -180°) critical point. Figure 3-7 shows that the $|E_1| = |E_2| = 0.9163 \times 10^{-3}$ locus does go through the critical point. However, most of the $|E_1| \neq |E_2|$ trajectories are far from intersecting the critical point. Although, it is difficult to span the $|E_1|$, $|E_2|$ space completely, these plots do give indication of possible existence of unequal-amplitude oscillations.

The other necessary condition that needs to be checked is given by Eq. (3-45). Let

$$f = (1 + NG_1 + HG_1G_2)(1 + NG_1) \qquad (3\text{-}46)$$

Then, for a limit cycle solution, it is necessary that the E^2 versus

E curve intersects the K/f (K = arbitrary constant) versus E curve at two points. Figure 3-8 shows the trajectories of these curves in the decibel coordinates. Since the curves for $20\log_{10}K/f$ is raised or lowered for different values of K, the following possible solutions are indicated from Fig. 3-8:

ω	Range of $\vert E_1\vert$ and $\vert E_2\vert$
10^{-3}	10^{-5} - 10^{-4}
10^{-2}	10^{-3} - 10^{-5}
10^{-1}	10^{-5} - 10^{-1}
1	10^{-4} - 10^{-2}
10	10^{-4} - 10^{-3}

However, checking with the Δ curves in Fig. 3-6, they show that Δ cannot be zero in the ranges of amplitudes indicated above.

The conclusion of this study is that although theoretically sustained oscillations with unequal amplitudes can occur in a two-axis coupled system with identical dynamics, the nature of the CMG nonlinearity most likely precludes this possibility. Another interesting discovery is that while a great majority of the limit cycles are of equal amplitude in the two axis, the phases are 180 degrees apart.

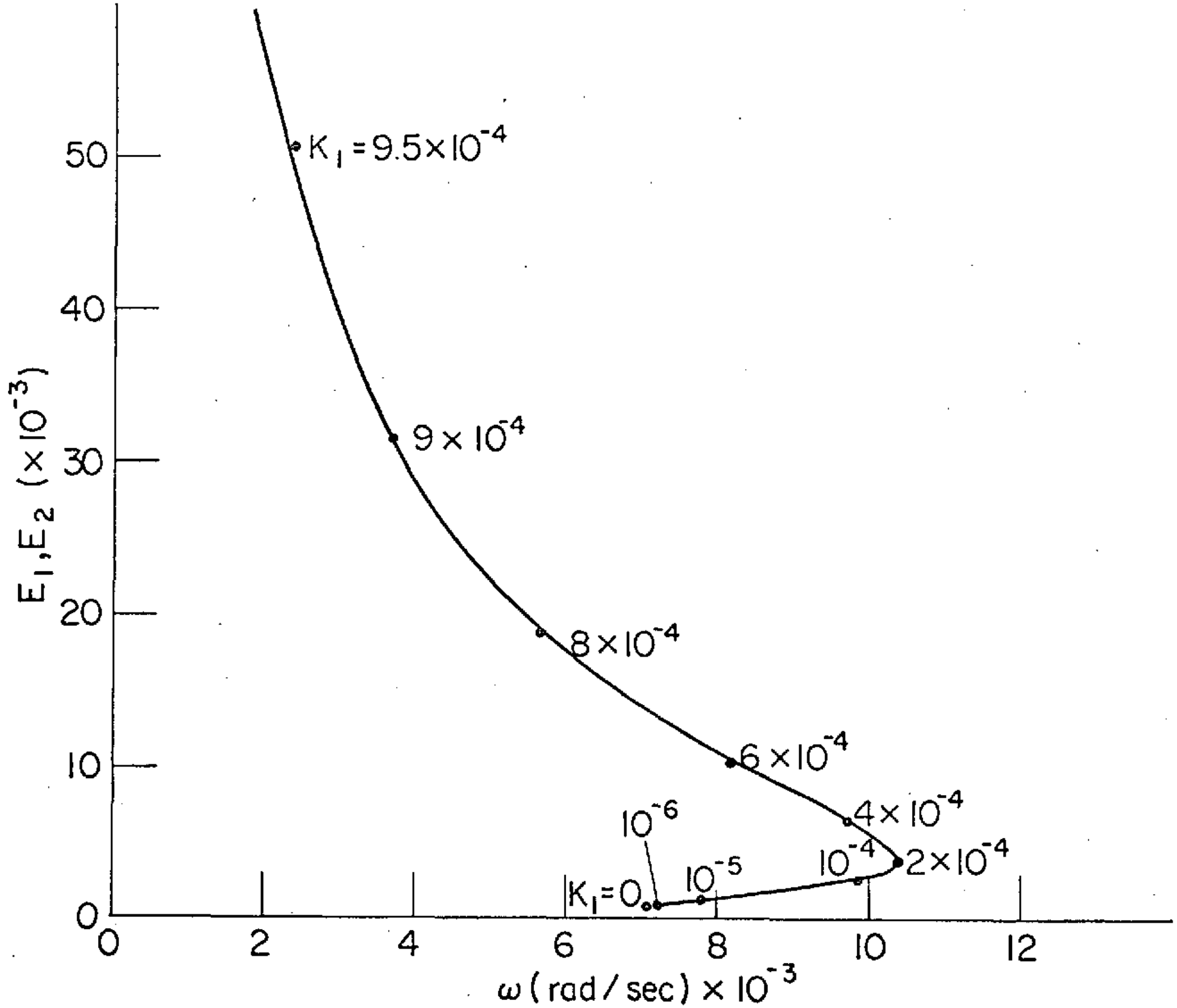

E_1, E_2 ($\times 10^{-3}$)
50
40
30
20
10
0
0
2
4
6
8
10
12
ω (rad/sec) $\times 10^{-3}$
$K_1 = 9.5 \times 10^{-4}$
9×10^{-4}
8×10^{-4}
6×10^{-4}
4×10^{-4}
2×10^{-4}
10^{-4}
10^{-5}
10^{-6}
$K_1 = 0$

Figure 3-5.

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 1.00000D-06

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.7000000D-02	0.9000000D-03	0.9000000D-03	0.1800000D 03
1	0.7301941D-02	0.9299407D-03	0.9252809D-03	0.1819683D 03
2	0.7279211D-02	0.9214010D-03	0.9212873D-03	0.1813883D 03
3	0.7246809D-02	0.9182832D-03	0.9162762D-03	0.1804199D 03
4	0.7241904D-02	0.9165880D-03	0.9165417D-03	0.1800975D 03
5	0.7239953D-02	0.9163476D-03	0.9163381D-03	0.1800225D 03
6	0.7239505D-02	0.9162839D-03	0.9162818D-03	0.1800069D 03

IND= 1 FREQUENCY= 7.23940D-03RAD/SEC E1= 9.16269D-04 E2= 9.16268D-04
THETA= 1.80003D 02 NIT= 6

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 1.00000D-05

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.7239395D-02	0.9162686D-03	0.9162681D-03	0.1800031D 03
1	0.8372262D-02	0.1112805D-02	0.1044858D-02	0.1846639D 03
2	0.8923282D-02	0.1260735D-02	0.1243920D-02	0.1854688D 03
3	0.6489651D-02	0.1020344D-02	0.9576581D-03	0.1746883D 03
4	0.7054192D-02	0.1038280D-02	0.1053084D-02	0.1763719D 03
5	0.7518007D-02	0.1097584D-02	0.1101126D-02	0.1783785D 03
6	0.7789838D-02	0.1130505D-02	0.1130854D-02	0.1796059D 03
7	0.7869107D-02	0.1139882D-02	0.1139915D-02	0.1799691D 03
8	0.7876078D-02	0.1140722D-02	0.1140728D-02	0.1800010D 03
9	0.7876270D-02	0.1140748D-02	0.1140748D-02	0.1800019D 03

IND= 1 FREQUENCY= 7.87627D-03RAD/SEC E1= 1.14075D-03 E2= 1.14075D-03
THETA= 1.80002D 02 NIT= 9

Figure 3-6a.

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 1.00000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.9941000D-02	0.2520000D-02	0.2520000D-02	0.1800000D 03
1	0.9941336D-02	0.2520555D-02	0.2520555D-02	0.1800019D 03
2	0.9941336D-02	0.2520555D-02	0.2520555D-02	0.1800019D 03

IND= 1 FREQUENCY= 9.94134D-03RAD/SEC E1= 2.52056D-03 E2= 2.52056D-03
THETA= 1.80002D 02 NIT= 2

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 2.00000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.9941336D-02	0.2520555D-02	0.2520555D-02	0.1800019D 03
1	0.8826681D-02	0.3492381D-02	0.3775372D-02	0.1787370D 03
2	0.9573039D-02	0.3637139D-02	0.3846694D-02	0.1793218D 03
3	0.1012638D-01	0.3725995D-02	0.3774828D-02	0.1797817D 03
4	0.1036430D-01	0.3744250D-02	0.3749602D-02	0.1799755D 03
5	0.1039656D-01	0.3746356D-02	0.3746405D-02	0.1800016D 03
6	0.1039692D-01	0.3746376D-02	0.3746375D-02	0.1800019D 03

IND= 1 FREQUENCY= 1.03969D-02RAD/SEC E1= 3.74638D-03 E2= 3.74638D-03
THETA= 1.80002D 02 NIT= 6

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 4.00000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.1039691D-01	0.3746375D-02	0.3746375D-02	0.1800019D 03
1	0.5863165D-02	0.7227177D-02	0.7451501D-02	0.1768248D 03
2	0.6955595D-02	0.6060971D-02	0.8070780D-02	0.1773450D 03
3	0.8078298D-02	0.6229025D-02	0.7205775D-02	0.1785639D 03
4	0.9031302D-02	0.6315208D-02	0.6724968D-02	0.1793910D 03
5	0.9617574D-02	0.6383023D-02	0.6483201D-02	0.1798492D 03
6	0.9800658D-02	0.6409813D-02	0.6416998D-02	0.1799869D 03
7	0.9821735D-02	0.6409441D-02	0.6409438D-02	0.1800018D 03
8	0.9821825D-02	0.6409420D-02	0.6409420D-02	0.1800019D 03

IND= 1 FREQUENCY= 9.82182D-03RAD/SEC E1= 6.40942D-03 E2= 6.40942D-03
THETA= 1.80002D 02 NIT= 8

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 6.00000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.9821825D-02	0.6409420D-02	0.6409420D-02	0.1800019D 03
1	0.5735355D-02	0.1144923D-01	0.1160452D-01	0.1777600D 03
2	0.6465793D-02	0.1071898D-01	0.1248778D-01	0.1782281D 03
3	0.7323528D-02	0.1057956D-01	0.1119825D-01	0.1792237D 03
4	0.7914920D-02	0.1040805D-01	0.1059804D-01	0.1797632D 03
5	0.8162418D-02	0.1034690D-01	0.1036832D-01	0.1799723D 03
6	0.8198133D-02	0.1033763D-01	0.1033778D-01	0.1800015D 03
7	0.8198588D-02	0.1033746D-01	0.1033746D-01	0.1800019D 03

IND= 1 FREQUENCY= 8.19858D-03RAD/SEC E1= 1.03375D-02 E2= 1.03375D-02
THETA= 1.80002D 02 NIT= 7

Figure 3-6b.

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 8.00000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.8198585D-02	0.1033746D-01	0.1033746D-01	0.1800019D 03
1	0.3482485D-02	0.1839274D-01	0.2190817D-01	0.1753581D 03
2	0.3121926D-02	0.2720710D-01	0.3050192D-01	0.1758942D 03
3	0.3694573D-02	0.2440467D-01	0.2737973D-01	0.1771812D 03
4	0.4333109D-02	0.2224205D-01	0.2334315D-01	0.1784748D 03
5	0.4908505D-02	0.2043765D-01	0.2107548D-01	0.1792209D 03
6	0.5335999D-02	0.1943281D-01	0.1963290D-01	0.1797269D 03
7	0.5546392D-02	0.1899701D-01	0.1903136D-01	0.1799537D 03
8	0.5590206D-02	0.1891363D-01	0.1891418D-01	0.1800006D 03
9	0.5591465D-02	0.1891121D-01	0.1891118D-01	0.1800019D 03
10	0.5591450D-02	0.1891123D-01	0.1891123D-01	0.1800019D 03

IND= 1 FREQUENCY= 5.59145D-03RAD/SEC E1= 1.89112D-02 E2= 1.89112D-02
THETA= 1.80002D 02 NIT= 10

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 8.50000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.5591451D-02	0.1891123D-01	0.1891123D-01	0.1800019D 03
1	0.4560251D-02	0.2399517D-01	0.2316868D-01	0.1797862D 03
2	0.4661299D-02	0.2368992D-01	0.2378736D-01	0.1799267D 03
3	0.4713075D-02	0.2354609D-01	0.2354633D-01	0.1799992D 03
4	0.4715453D-02	0.2353809D-01	0.2353802D-01	0.1800019D 03
5	0.4715421D-02	0.2353818D-01	0.2353818D-01	0.1800019D 03

IND= 1 FREQUENCY= 4.71542D-03RAD/SEC E1= 2.35382D-02 E2= 2.35382D-02
THETA= 1.80002D 02 NIT= 5

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 9.00000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.4715422D-02	0.2353818D-01	0.2353818D-01	0.1800019D 03
1	0.3485092D-02	0.3142799D-01	0.3107595D-01	0.1791651D 03
2	0.3532983D-02	0.3254688D-01	0.3280586D-01	0.1797728D 03
3	0.3659170D-02	0.3172735D-01	0.3173334D-01	0.1799660D 03
4	0.3684810D-02	0.3155989D-01	0.3155995D-01	0.1800010D 03
5	0.3685423D-02	0.3155622D-01	0.3155619D-01	0.1800019D 03
6	0.3685410D-02	0.3155631D-01	0.3155631D-01	0.1800019D 03

IND= 1 FREQUENCY= 3.68541D-03RAD/SEC E1= 3.15563D-02 E2= 3.15563D-02
THETA= 1.80002D 02 NIT= 6

TEST SYSTEM-NUMERICAL SOLUTION OF DELTA(W,E1,E2,THETA)=0.
GAMMA= 1.38000D 06 TD= 1.00000D-01 ASM= 1.00000D-03 K1= 9.50000D-04

ITERATION	FREQUENCY	E1	E2	PHASE-E1
0	0.3685410D-02	0.3155630D-01	0.3155630D-01	0.1800019D 03
1	0.1976666D-02	0.4159336D-01	0.5582929D-01	0.1726228D 03
2	0.1657416D-02	0.6596966D-01	0.7001921D-01	0.1770600D 03
3	0.1800327D-02	0.6581877D-01	0.6669853D-01	0.1790101D 03
4	0.2020104D-02	0.5954712D-01	0.5879754D-01	0.1795288D 03
5	0.2196704D-02	0.5491439D-01	0.5484045D-01	0.1797066D 03
6	0.2323826D-02	0.5217919D-01	0.5216096D-01	0.1798891D 03
7	0.2381586D-02	0.5107308D-01	0.5107025D-01	0.1799836D 03
8	0.2391558D-02	0.5089335D-01	0.5089260D-01	0.1800019D 03
9	0.2391499D-02	0.5089476D-01	0.5089477D-01	0.1800019D 03

IND= 1 FREQUENCY= 2.39150D-03RAD/SEC E1= 5.08947D-02 E2= 5.08947D-02
THETA= 1.80002D 02 NIT= 9

Figure 3-6c.

(n,m) represents

$(|E_1|,|E_2|) = (10^{-n},10^{-m})$

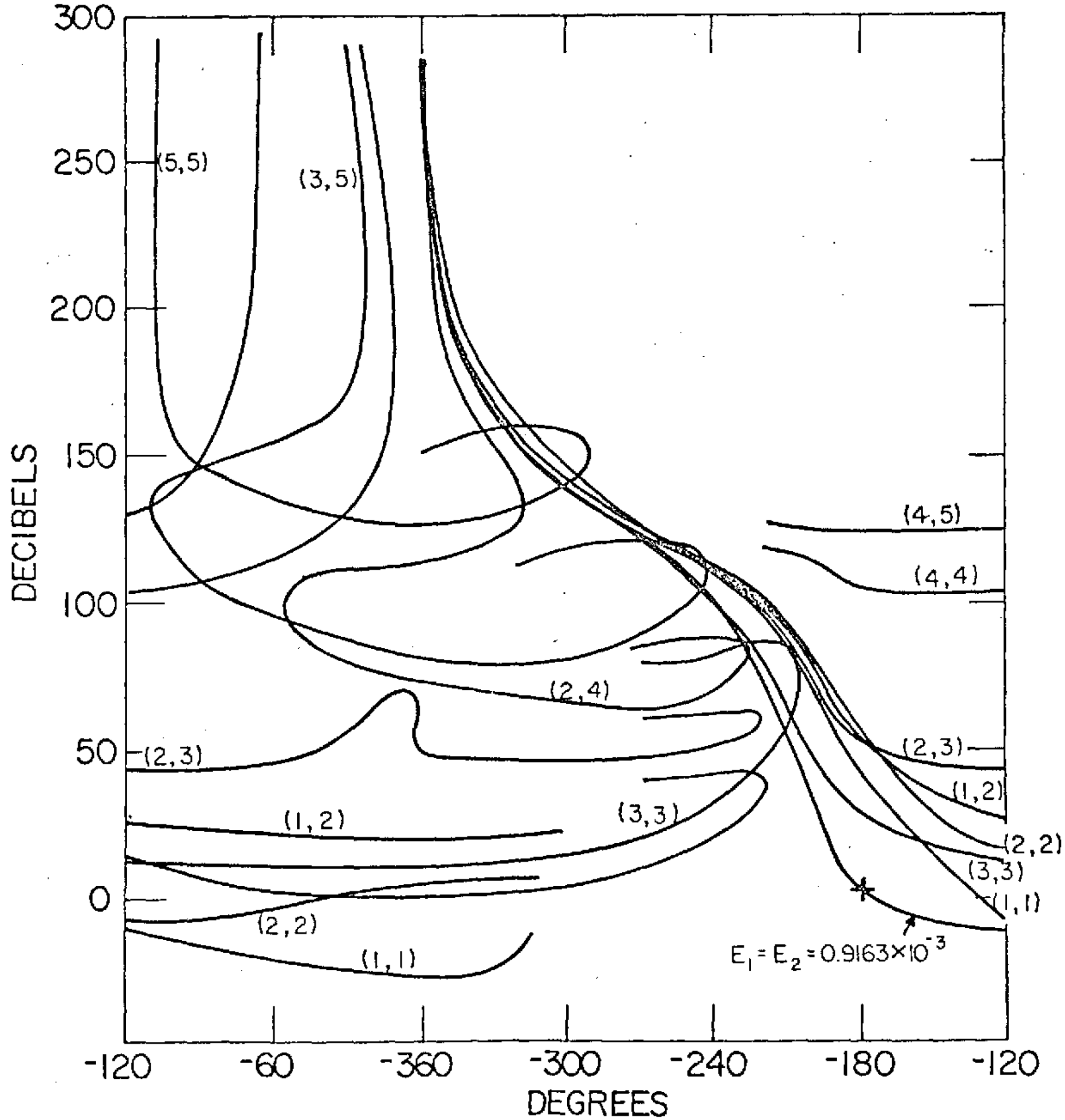
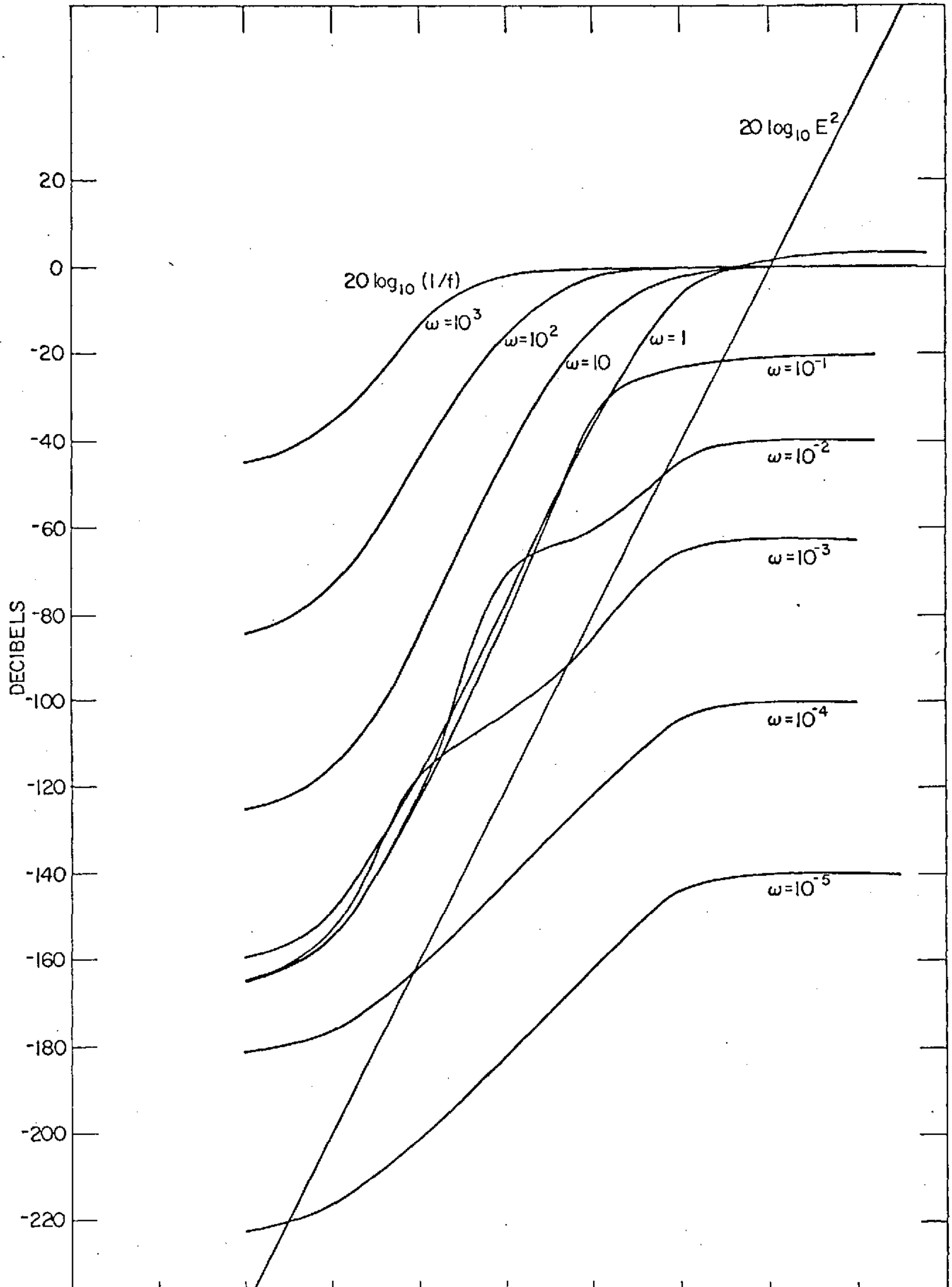


Figure 3-7.


20 log10 E²
20 log10 (1/f)
ω=10³
ω=10²
ω=10
ω=1
ω=10⁻¹
ω=10⁻²
ω=10⁻³
ω=10⁻⁴
ω=10⁻⁵
DECIBELS
20 log10 E (db)
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
-220
-240
-160
-140
-120
-100
-80
-60
-40
-20
0
20

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