Mission Planning and Analysis Division NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

LYNDON B. JOHNSON SPACE CENTER
houston, TEXAS 77058

REPLY TO
ATTN OF: FIM84 (74-143)

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## MEMORANDUM

TO: Informal Distribution
FROM: FMB/Mathematical. Physics Branch
SUBJECT: Pattern Recognition Algorithm Using Temporal Data

At the present time, classification is limited almost exclusively to the use of spectral information. This memorandum presents a method for classification using spectral and temporal information.

The value of a previously classified image may be seen by means of the following example: Suppose two images are taken of a scene at different times, "then" and "now." Suppose no classification errors were made in classifying the image "then." Further, suppose no change in class occurred for any pixel between "then" and "now." Finally, suppose the images are registered perfectly; every element in each image is associated exactly with its appropriate counterpart. In this case, the second image is already classified. No additional classification effort needs to be performed. Since these three error sources are, in fact, always present, their effects should be modeled: to estimate appropriate a priori probabilities for the second image.

Probability of a Signal X Belonging to Class $\pi_{i}$
For the discussion, assume a signal $X$ is to be classed into one of two classes, $\pi_{1}$ or $\pi_{2}$. The generalization to any number of classes will be obvious. Suppose $f_{1}(X)$ and $f_{2}(X)$ are the probability distributions of $\pi_{1}$ and $\pi_{2}$, respectively, evaluated at $X$, and that $N_{1}$ and $N_{2}$ are the numbers of elements in the image in $\pi_{1}$ and $\pi_{2}$, respectively. Then, there exists $N_{1} f_{1}(X)$ elements in $\pi_{l}$ with signal value of $X$, and similarly for $\pi_{2}$. This means the probability of $X$
 belonging to $\pi_{i}$ is

$$
P\left(\pi_{i} \mid X\right)=\frac{N_{i} f_{i}(X)}{N_{1} f_{1}(X)+N_{2} f_{2}(X)}
$$



The a priori probability of $\pi_{i}$ is

$$
\alpha_{i}=\frac{N_{i}}{N_{1}+N_{2}}
$$

Hence,

$$
N_{1}=\alpha_{i}\left(N_{I}+N_{2}\right)
$$

or

$$
\begin{align*}
& P\left(\pi_{i} \mid X\right)=\frac{\alpha_{i}\left(N_{1}+N_{2}\right) f_{i}(X)}{\alpha_{1}\left(N_{1}+N_{2}\right) f_{1}(X)+\alpha_{2}\left(N_{1}+N_{2}\right) f_{2}(X X)} \\
& P\left(\pi_{i} \mid X\right)=\frac{\alpha_{i} f_{i}(X)}{a_{1} f_{1}(X)+\alpha_{2} f_{2}(X)} \tag{I}
\end{align*}
$$

This is the probability that a signal belongs to class $\pi_{i}$ given that the value of the signal is X .

Probability that a Pixel V Changed Class
Suppose a pixel $V$ in the first image was a vacant lot and was correctly classified "then." However, in the time interval between the two images, a house was built on pixel $V$. Clearly, the probability of this happening must be considered in computing appropriate a prioriprobabilities for the second image. The probability of change of pixel from $\pi_{i}$ to $\pi_{j}$ will be denoted by $P\left(\pi_{i} \rightarrow \pi_{j}\right)(V)$. Note, the probability of change depends on $\pi_{i}$ and $\pi_{j}$ as well as the location $V$.
Assigning a value to the probability of change will require a model. Experience may be sufficient. For example, a county agricultural agent would have a knowledge of when harvest for a particular crop $\pi_{\text {i }}$ might occur, and would assign a high probability for $P\left(\pi_{i} \rightarrow \pi_{j}\right)(V)$ at harvest time, where $V$ was associated with the crop $\pi_{i}$, and $\pi_{j}$ is the classification bare soil.

## Probability of Misregistration

Denote by $g(V, U)$ the probability of registering $V$ in the old image with $U$ in the new image. This means that in classifying pixel $U$ in the new image, and attempting to use information from the old image response of pixel $U$, there is the probability $g(V, U)$ that the information from pixel $V$ will be used instead. Consequently, in calculating the a priori probability of pixel $U$, the effect of the surrounding pixels, weighted by their respective probabilities, must be considered.

Putting It All Together
Recall $P\left(\pi_{j} \mid X\right)$ denotes the probability of class $\pi_{j}$ given the signal $X$ ． Suppose a pixel V had a signal response of $X$＂then．＂The probability that pixel V＂now＂belongs to class $\pi_{i}$ is

$$
\sum_{j} P\left(\pi_{j} \mid X\right) \cdot P\left(\pi_{j} \rightarrow \pi_{i}\right)(V)
$$

Finally，since pixel $U$ in the new image is not necessarily registered exactily to its corresponding point in the old image，a weighted average of the above probabilities of $V$ must be made，where the weight is the probability that $U$ and $V$ are registered．Hence，the probability that $U$ in the new image is in class $\pi_{i}$ is

$$
P\left(U_{\varepsilon} \pi_{j} \text { now }\right)=\sum_{V}\left\{g(V, U)\left[\sum_{j} P\left(\pi_{j} \mid X\right) \cdot P\left(\pi_{j} \rightarrow \pi_{j}\right)(V)\right]\right\}
$$

Notice，no assumption is made so far concerning the definition of $f_{i}(X), P\left(\pi_{j} \rightarrow \pi_{i}\right)(V)$ ，or $g(V, U)$ ．The method is entirely general．In fact，defining $P\left(\pi_{j} \rightarrow \pi_{i}\right)(V)$ will probably requixe knowledge of the specific application．However，some reasonable probability distri－ butions may be suggested for $f_{i}(X)$ and $g(V, U)$ ．For example，$f_{i}(X)$ may be assumed to be a discretized transformation of a normal distri－ bution，where the mean and covariance data are defined from training sets．The distribution function for registration $g(V, U)$ may be assumed normal with expected value $U$ ．The covariance matrix may be estimated using the registration residuals of known points．

This algorithm will be coded in the near future，and will be tested using real and simulated data．


Arland L．Actkinson
APPROVED BY：

それまい<br>Emil R．Schiesser<br>Chief＇，Mathematical Physics Branch<br>Distribution：<br>Gee attached list<br>PMB：ALActkinson：sas：5／13／74：3001

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