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DIFFUSION MODELS FOR JUPITER'S RADIATION BELT*

by

S. A. Jacques and Leverett Davis, Jr.
Physics Department
California Institute of Technology

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Abstract

Solutions in three cases are given for the diffusion of trapped particles in a planetary magnetic field in which the first and second adiabatic invariants are preserved but the third is not, using as boundary conditions a fixed density at the outer boundary (the magnetopause) and a zero density at an inner boundary (the planetary surface). Losses to an orbiting natural satellite are included and an approximate evaluation is made of the effects of the synchrotron radiation on the energy of relativistic electrons. The three cases considered have diffusion coefficients proportional to L^{10}, L^{6}, and $L^{2}(L-1)$, respectively. The first two derive from familiar mechanisms, the last from a speculative mechanism in which the diffusion is driven by ionospheric winds. Choosing parameters appropriate to Jupiter, the first two cases fail completely to explain the electrons required to produce the observed synchrotron radiation; the third can explain the electrons using a large, but not unreasonable, value of the diffusion coefficient if the mechanism is acceptable. Only if a mechanism of this type is the true explanation of the electrons producing the synchrotron emission can one reliably conclude that Jupiter's inner magnetosphere should be occupied by an energetic proton flux that would be a serious hazard to spacecraft.

I. Introduction

The only accepted explanation of the decimetric radiation observed near Jupiter is that it is due to the synchrotron radiation of relativistic electrons. The most commonly accepted mechanism for supplying the electrons is inward diffusion of solar wind electrons from the magnetosheath by a process that conserves the first and second adiabatic invariants. If the diffusion hypothesis is indeed correct then it seems almost certain that protons will diffuse inward by the same mechanism and that their flux density and energy can be deduced from the flux density and energy of the electrons required to produce the synchrotron radiation. Almost all analyses (Beck, 1972) along these lines lead to high fluxes of energetic protons that will be a serious hazard to all spacecraft that venture within a few Jupiter radii of its surface. If the diffusion hypothesis should not be correct, the relativistic electrons could perhaps be explained by some more local acceleration hypothesis and might have but little connection with energetic protons. In this case, no reliable estimate of the flux density of energetic protons could be made in advance of in situ measurements, but it would be quite plausible to assume that it would be considerably lower than in the diffusion case.

In this report, we investigate the diffusion process to learn what characteristics it must have if it is to be able to supply the electrons needed to explain the synchrotron radiation. We find that the diffusion mechanisms usually used to explain the particles in the earth's radiation belts are far too slow for Jupiter, but that diffusion driven by winds and turbulence in the ionosphere or upper atmosphere could produce the relativistic electrons provided the diffusion coefficient is rather, but not unreasonably, large.

II. The Fokker-Planck Equation

Assume that Jupiter's field is approximately a dipole and that the particles conserve the first and second adiabatic invariants, μ and J, but violate the third, the flux invariant. The resulting radial diffusion of the particles is described by a Fokker-Planck equation. (Davis and Chang, 1962, and Fälthammer, 1966)

First, some definitions. Let

R_J = the radius of Jupiter = 7×10^4 km

r = (radial distance)/R_J

L = value of r for a field line at the equator

θ = polar angle from Jupiter's magnetic axis

n = number density in (μ,J,L) space

R_I = the radius of Io = 1/40 R_J

Note that L and r are dimensionless.

Consider the diffusion of particles having μ in a small range $d\mu$ and J in dJ. Then the Fokker-Planck equation is

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial L}\left(n \left\langle\frac{\Delta L}{\Delta t}\right\rangle\right) + \frac{1}{2}\frac{\partial^2}{\partial L^2}\left(n \left\langle\frac{(\Delta L)^2}{\Delta t}\right\rangle\right) - A_n \tag{1}$$

where $\left\langle\frac{\Delta L}{\Delta t}\right\rangle$ and $\left\langle\frac{(\Delta L)^2}{\Delta t}\right\rangle$ are the mean and mean square change of L per unit time, and $-A_n$ is the contribution to $\partial n/\partial t$ due to the absorbtion of particles by a satellite such as Io.

For much of the discussion it is more convenient to replace the dependent variable n by f, where it will turn out that $f = kL^2 n$ and k is a suitable normalization constant that drops out of the equations. Since we assume μ and J to remain constant, we drop them in the subsequent discussion. In thinking of n and f we suppose them to refer only to the particles in some particular range of μ and J and ignore the possible presence of other particles. In a sense now to be discussed, f may be defined as the fraction of full flux tubes at any L or, equivalently, as the filling fraction of any one flux tube. We first suppose that diffusion takes place by the convection, random walk, or interchange of flux tubes, including their entire contents of both cold plasma and energetic particles. We suppose that all tubes at the magnetopause, where $L = L_1$, have been filled to the standard level with particles in the ranges of μ and J of interest, and hence that $f(L_1) = 1$. We suppose that at L_o, which is at or near the surface of Jupiter, all tubes are emptied of these particles and $f(L_o) = 0$. Between L_o and L_1 some tubes will be empty, those that had random walked to L_o since walking to L_1, and some will be full; f(L) is the fraction that are full.

The connection between n and f is easily deduced. A shell that intersects the equatorial plane in a ring of radius $R_J L$ and width $R_J dL$ contains (n dμ dJ) dL particles; and the magnetic flux, or total number of flux tubes, in $R_J dL$ is $2\pi R_J^2 BL\ dL$. The number of full flux tubes is $f\cdot(\text{flux}) = fBR_J^2\ 2\pi L\ dL \propto f\,\frac{dL}{L^2}$. Since the number of particles is proportional to the number of full flux tubes we have $n \propto \frac{f}{L^2}$.

The differential equation for f, temporarily regarded as the fraction of the tubes that are full, is derived from (1), the connection between f and n, and the relation

$$\left\langle\frac{\Delta L}{\Delta t}\right\rangle = \frac{L^2}{2}\frac{\partial}{\partial L}\left(\frac{1}{L^2}\left\langle\frac{(\Delta L)^2}{\Delta t}\right\rangle\right) \tag{2}$$

derived by Fälthammer (1966). The result is

$$\frac{\partial f}{\partial t} = L^2\frac{\partial}{\partial L}\left[\frac{D}{L^2}\frac{\partial f}{\partial L}\right] - A \tag{3}$$

where $D = \langle(\Delta L)^2/\Delta t\rangle/2$ and $A = L^2 A_n$ is the rate at which f decreases due to the satellite.

Next suppose that in any L-shell the gradient and curvature drifts cause energetic particles to move from one tube to another with the same L at a rate that depends on energy and the mass/charge ratio. This stirring tends to equalize the degree of filling of all tubes in this shell and we say that all are filled to the fraction f of the particle density they would contain if brought without loss from the magnetopause. In considering the subsequent transport of particles to another L-shell, it does not matter, on the average, whether we move a tube filled to the fraction f or whether we make a random choice from a mixture of tubes of which the fraction f are completely filled and the remainder are empty. Thus, the entire analysis is valid for both meanings of f.

In the following, expressions for D and A are obtained, and equation (3) with these expressions is solved for f.

III. The Diffusion Coefficient

We will consider solutions of the Fokker-Planck equation with three different expressions for the diffusion coefficient D. First we will assume that diffusion is produced by the violation of the 3rd adiabatic invariant due to fluctuations in the magnetic field produced by variations in the solar wind pressure. The diffusion coefficient for this mechanism was found by Davis and Chang (1962) to be proportional to L^{10}. On the assumption that conditions near Jupiter are similar to those near earth, Mead and Hess (1972) estimate the proportionality constant. We use their value, rounding 0.13 upward to 0.2 since we are interested in an upper limit, and get

$$D_{SW} = 0.2\ L_1^{-8}\ L^{10}\ \mathrm{day}^{-1} \tag{4}$$

where L_1 is the value of L at Jupiter's magnetopause and the subscript SW means that this expression for D is appropriate for diffusion driven by fluctuations in solar wind pressure. The expected value of L_1 is 50.

Radial diffusion due to fluctuating electric fields in the magnetosphere has been considered by several authors as a mechanism for populating the earth's radiation belts (Fälthammer, 1965, Birmingham, 1969, and Cornwall, 1972). If ω_D is the azimuthal drift frequency, $P_m(m\ \omega_D)$ is the power in the Fourier component of E having frequency $m\ \omega_D$, and c is the velocity of light, the diffusion coefficient, measured in units of length^2/time for use in a diffusion equation in which the position of the diffusing particle is given by a coordinate with the dimensions of length, is

$$D_E = \frac{c^2}{8B^2} \sum_{m=1}^{\infty} P_m\ (m\omega_D)\ \mathrm{cm}^2\ \mathrm{sec}^{-1}$$

For the case of the earth, with B_{OE} the earth's surface equatorial field, τ_c the correlation time, and a, the mean square amplitude of the electric field fluctuations, this can be written (Birmingham, 1969) as

$$D_E = \frac{\sqrt{\pi}\, c^2\, \tau_c\, a\, L^6}{4\, B_{OE}^{\,2}\, R_E^{\,2}} \quad \sec^{-1}$$

where L refers to the earth's magnetosphere and the units are 1/time for use in a diffusion equation where the position of a particle is measured by L. Typical values for the earth are (see Birmingham) $\tau_c \approx 1$ hr., $a = (2 \times 10^{-4} \frac{V}{m})^2$. This gives $D_E(\text{Earth}) = 1.3 \times 10^{-4}\ \text{day}^{-1}$. If we assume that τ_c and a are about the same for Jupiter as for the earth, we get

$$\begin{aligned} D_E(\text{Jupiter}) &= \left(\frac{B_{OE}\ R_E}{B_{OJ}\ R_J} \right)^2 D_E(\text{Earth}) \\ &= 1.7 \times 10^{-9}\ L^6\ \text{day}^{-1} \end{aligned} \tag{5}$$

where this will be used in an equation where L refers to Jupiter.

In the inner magnetosphere, winds and turbulence in Jupiter's upper atmosphere and ionosphere could be (Brice, 1972) more effective in producing diffusion than the processes described above. Consider a field line emerging from the surface of Jupiter. The electric potential along this field line is constant since above the ionosphere electric fields cannot be maintained parallel to the magnetic field. In general, a neighboring field line will be at a different potential because of the electric fields produced by the motion of the upper atmosphere. These potential differences cause tubes of flux to be interchanged in cyclical processes, and the feet of these tubes of flux can be viewed as performing a random walk on the surface of Jupiter. It is shown in the Appendix that the dipole field is maintained during this interchange of flux tubes.

The result will be diffusion of particles by violation of the 3rd adiabatic invariant (since the magnetic shell of a tube, given by L, changes when tubes are interchanged), while μ and J will be conserved.

Let us discuss D_A, the diffusion coefficient for this process. For a dipole field, a line which emerges from the surface of Jupiter at colatitude θ crosses the equatorial plane at a radial distance LR_J, where

$$L = 1/\sin^2 \theta \tag{6}$$

If the foot of a flux tube at the surface moves a distance $R_J\Delta\theta$ in the θ-direction, the change in L is

$$\Delta L = \frac{-2 \cos \theta}{\sin^3 \theta} \Delta\theta = -2 \ (L^3 - L^2)^{1/2} \ \Delta\theta \tag{7}$$

This constitutes one step of length $s_1 = R_J \ \Delta\theta$ in the random walk of the foot of the flux tube. After N steps, which require a time Nt_1, if t_1 is the mean time per step, the mean displacement of the foot given by random walk theory is

$$R_J^2 \ \langle (\Delta\theta)^2 \rangle = N \ \langle s_1^2 \rangle$$

and the mean square change of L per unit time is

$$\frac{\langle (\Delta L)^2 \rangle}{\Delta t} = \frac{4L^2(L-1)\langle (\Delta\theta)^2 \rangle}{N \ t_1} = \frac{4L^2(L-1)\langle s_1^2 \rangle}{t_1 \ R_J^2} \tag{8}$$

To obtain a value for $D = (1/2)\langle \frac{(\Delta L)^2}{\Delta t} \rangle$, we need to choose reasonable values for the step length s_1 and for t_1. Differential rotation observed in the Jovian lower atmosphere suggests velocities of the order of 0.1 km/sec. If we assume that similar velocities extend into the ionosphere and if the wind blows in approximately the same direction for 10^4 sec ($\approx$ 3 hrs.), the distance the foot of a flux tube moves is $s_1 = 10^3$ km, and t_1 is 10^4 sec $= 10^{-1}$ day. Thus

$$D_A = 4 \times 10^{-3} L^2(L-1) \text{ day}^{-1} \tag{9}$$

where the subscript A implies that D is due to atmospheric motion.

With different assumptions on the distributions with elevation of wind velocity, conductivity, and coherence time, it would be plausible to propose considerably smaller values of this diffusion constant, but it seems difficult to propose a plausible model in which the constant would be increasedby an order of magnitude. We have assumed that the random velocity is horizontal in the ionosphere and is independent of latitude. If a different dependence on latitude were assumed, a somewhat different L dependence in (9) would result. Another alternative would be to consider the ionospheric electric field which may be regarded as driving the fluid motions above the ionosphere and which, we assume, combines with gravity and pressure gradients to produce our horizontal motions in the ionosphere. If the average value of the square of the strength of this field is independent of latitude, the factor $L^2(L-1)$ in (9) is replaced by L^3. This simplifies somewhat the solution of the diffusion equation but does not make an significant change in the results beyond about $L = 2$. Recent work by Coroniti, Kennel, and Thorne (private communication) examines the mechanisms that could drive the ionospheric winds whose existence we have postulated on the basis of Brice's conjecture. They conclude that such winds may well be possible and deduce a diffusion coefficient proportional to L^3 with a numerical value roughly an order of magnitude smaller than ours.

As shown in Figure 1, the diffusion coefficients are of the same general magnitude near the magnetopause in all three models. But D_A is very much larger than the others for $1.5 < L < 6$, and this makes drastic differences in the expected population of the inner radiation belts.

IV. Absorbtion by Io.

In this section we derive the loss term in equation (3) due to the collision of trapped particles with the satellite Io, whose orbit will be taken to be in the $L = 6$ shell. Similar terms for the other satellites are easily deduced but will be neglected here since the flux in the region $L \lesssim 6$ is dominated by the effects of Io.

To calculate the loss rate we need to know the properties of typical energetic particles near $L = 6$. For relativistic electrons, observations of synchrotron radiation indicate (Warwick, 1970) a typical kinetic energy of $T = 6.2$ MeV at $L = 1.8$. To compute the energy at $L = 6$, the relativistic adiabatic invariant $p_\perp^2/2mB$ must be used ($p_\perp$ is the electron momentum normal to $\underset{\sim}{B}$ and m is the rest mass of the electron) and the energy loss due to synchrotron radiation must be estimated. This is done below for a 10 gauss equatorial surface magnetic field strength, two values of the diffusion coefficient, and particles in very flat helices that mirror near the equatorial plane. We find T to be in the range 1 to 2 MeV at $L = 6$. For these 1 MeV electrons, the gyroradius is 1 km, the bounce period is $\tau_B = 5$ sec, and the drift period is $\tau_D = 30$ days (Hamlin et al., 1961). The variation of these parameters with pitch angle is unimportant.

For protons, our main interest is in those which, after thermalization in the bow shock and stagnation region, have energies of the order of 1 keV just inside the magnetopause and hence, by conservation of μ for particles in flat helices, energies of 0.58 MeV at $L = 6$. Thus, the gyroradius here is 22 km, $\tau_B = 120$ sec, and $\tau_D = 35$ days. The gyroradii of both electrons and protons is so small compared to the 1750 km radius of Io that we may use the guiding center approximation.

As a simple approximation, assume that Io moves in a circle at $L = 6$ in the Jovian magnetic equatorial plane with a siderial period of 42.5 hours. The period with respect to the tubes of force, which rotate with Jupiter, is 13 hours and at $L = 6$ the velocity of Io with respect to the tubes of force is 57 km/sec. In the model considered for the derivation of D_A as given by equation (9), a velocity of 0.1 km/sec at the foot of a flux tube produces a velocity of the tube of 1.5 km/sec if it is in the azimuthal direction and, by equation (6), of 2.7 km/sec if it is meridional. Thus, these velocities due to the non-static nature of the magnetic field can be ignored compared to the velocities of the field due to Jupiter's rotation. However, the trapped particles of interest to us do not move with the tubes of force; they drift around the L-shell with period τ_D. The values of τ_D found above are so large compared to the 13 hour orbital period of Io that the drift motion can also be neglected in making a first order approximation to the loss coefficient.

If Io blocks off a tube of force in the equatorial plane for half of the bounce period, the tube will be completely emptied of particles. For electrons with $\tau_B/2 = 2.5$ sec, Io moves only 143 km during this time. This is so much less than the 3500 km diameter of Io that all tubes through which Io passes are completely emptied of electrons except for a few whose L shells are nearly tangent to Io's surface. Thus, for L in a ring of width $\Delta L = 3500/70{,}000 = 0.05$ at $L = 6$, all tubes are emptied of whatever particles they may contain in the 13 hours it takes Io to make one revolution and the contribution to $\partial f/\partial t$ is $-A = -(1/13) f \text{ hr}^{-1}$. If we define α by $A = \alpha f$, we have

$$\alpha = \begin{cases} 2 \text{ day}^{-1} & \text{if } 6 \leq L \leq 6.05 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

This formula is accurate for all particles for which $\tau_B/2$ is much less than $\tau_M = 3500/57 = 61$ sec, the time required for Io to move a distance equal to its diameter, and thus is to be used for fast enough particles whether they are electrons or protons.

If $\tau_B/2 > \tau_M$, no particle that could be striking Io can have been lost during its previous bounce and the entire area of Io is effective in removing particles. If Io filled the entire ring of width ΔL, α would be $2/\tau_B$. Since actually the cross-sectional area of Io, S_{Io}, is very much less than the area of the ring, $S_R = 2\pi R_J^2 L\Delta L$, we have

$$\alpha = \begin{cases} \dfrac{2}{\tau_B}\dfrac{S_{Io}}{S_R} & \text{if } 6 \leq L \leq 6.05 \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

Strictly, this formula should vary with L over $6 \leq L \leq 6.05$; α being equal to $C(L)/\tau_B \pi R_J L$, where $C(L)$ is the chord of Io's equatorial plane cut off by an L-shell. However, the use of a constant, average value as in equation (11) should be a good approximation. Also, for $\tau_B/2$ only a bit less than τ_M, α is given by $C(L)/\tau_B \pi R_J L$ for those chords far from Io's center where this is less than 2 day^{-1}, the value for the chords nearer the center. This correction drops rapidly in importance as $\tau_B/2$ decreases below τ_M and for practical purposes it should be adequate to use (10) whenever $\tau_B/2 < \tau_M$; i.e., for energies above 300 eV for electrons and above 0.56 MeV for protons.

In a more complete and precise treatment, it would be necessary to consider a number of other effects. The roughly 10° angle between the magnetic equator and the orbit of Io means that particles mirroring at less than 21° from the equator (35% of an isotropic distribution) can strike Io during only part of their drift period. The center of Io's circular orbit is not at the dipole center of the magnetic field; thus the particle loss takes place over a somewhat larger interval than the ΔL assumed above, but with a lower value of α. Both the loss rate to Io and f vary with longitude and with time as Io moves in the orbit. Only if the average over longitude and time of the product αf is the product of the averages will our treatment in which α and f are such averages be completely valid. As the electrons move into the region of strong magnetic fields and become relativistic, they will lose energy because of synchrotron radiation. However, in this treatment, we postpone all such refinements for consideration after the characteristics of the simpler, approximate formulation have been worked out.

The effect of Io on diffusion can be estimated from the ratio

$$\beta_1 = \left[\frac{\alpha \Delta L}{\frac{1}{L} D}\right]_{L = 6} \tag{12}$$

since α is the fraction of particles lost per unit time and the average time a particle remains in the range ΔL in which $\alpha \neq 0$ is $\Delta L/|V_D|$, where V_D, the average convection velocity produced by the diffusion, may be estimated from eq. (23) to be of order D/L. For the 3 different values of D, the values of β_1 are:

1) When $D = D_{SW} = .2(50)^{-8}\ L^{10}\ \text{day}^{-1} = 5 \times 10^{-15}\ L^{10}\ \text{day}^{-1}$, $\beta_1 = 2 \times 10^6$. Since $\beta_1 >> 1$, the effect of Io is overwhelming, i.e., almost all particles will be lost to Io.
2) For $D = D_E = 1.7 \times 10^{-9}\ L^6\ \text{day}^{-1}$, $\beta_1 = 7 \times 10^3$, so $\beta_1 >> 1$ here also and almost all particles will be absorbed.
3) For $D = D_A = 4 \times 10^{-3}\ L^2(L-1)\ \text{day}^{-1}$, $\beta_1 \sim 1$, so in this case the effect of Io is neither overwhelming or negligible.

V. Solutions of the Fokker-Planck Equation

For a steady state, eq. (3) becomes

$$L^2 \frac{d}{dL} \left[\frac{D}{L^2}\frac{df}{dL}\right] - \alpha f = 0 \qquad f(L_1) = 1, \quad f(L_0) = 0. \tag{13}$$

To solve eq. (13) for an arbitrary diffusion coefficient D, define a new independent variable

$$y = D_0 \int_{L_0}^{L} \frac{L^2}{D}\, dL, \tag{14}$$

where D_0 is the constant factor in D. Note that we have chosen the constant of integration in y such that $y(L_0) = 0$. Substituting y for L in equation (13), we get

$$\frac{d^2 f}{dy^2} - \gamma(L)^2 f = 0, \qquad \gamma(L)^2 = \frac{\alpha D}{L^4 D_0^{\;2}} \tag{15}$$

Since $\alpha \neq 0$ only in $L \leq 6 \leq L + \Delta L$, where ΔL is small, by γ we always mean $\gamma(6)$ where the value of α is the typical or average value for this range. Also, define $y(L_1) = y_1$, $y(6) = y_6$, $y(6 + \Delta L) = y_{6+}$, and $\Delta = y_{6+} - y_6$. The solution of equation (15) that satisfies the boundary conditions $f = 0$ at L_0 and $f = 1$ at L_1 is

$$f = \begin{cases} Ay & \text{if } L \leq 6 \\ B_1\, e^{\gamma(y-y_6)} + B_2 e^{-\gamma(y-y_6)} & \text{if } 6 < L < 6 + \Delta L \\ 1 - C(y_1 - y) & \text{if } L \geq 6 + \Delta L \end{cases} \tag{16}$$

By requiring that f and df/dy be continuous at y equal to y_6 and y_{6+}, we find that the coefficients are

$$\begin{aligned}
A &= 2\gamma B_1/(y_6\gamma-1) \\
B_1 &= 2(y_6\gamma+1)/[\gamma(y_1-\Delta)\cosh\gamma\Delta + (1+\gamma^2 y_6(y_1-y_{6+}))\sinh\gamma\Delta] \\
B_2 &= B_1(y_6\gamma-1)/(y_6\gamma+1) \\
C &= 2\gamma B_1(y_6\gamma\sinh\gamma\Delta + \cosh\gamma\Delta)/(y_6\gamma+1)
\end{aligned} \tag{17}$$

Equations (14), (16) and (17) then give f(L). The results for various choices of D both neglecting and including the effect of Io are shown in figure 2. These solutions are valid for both electrons and protons. From this figure we conclude that practically no particles will get past Io for $D = D_{SW}$ and $D = D_E$, while if $D = D_A$ a large fraction of the particles will not be absorbed by Io and will diffuse in to the region $L < 6$. For example, when the effect of Io is included, $f \approx 10^{-60}$ for $L < 6$ when $D = D_{SW}$, and $f \approx 10^{-10}$ for $L < 6$ when $D = D_E$. Since synchrotron radiation from electrons is observed from $L < 6$ and D_{SW} and D_E cannot produce a significant number of particles in this region, we will neglect the effect of these mechanisms and will restrict further attention to $D = D_A$.

In the case $D = D_A$, $y = \ln\left(\frac{L-1}{L_0-1}\right)$ and it is not reasonable to apply a boundary condition at $L_0 = 1$. Accordingly we require that f = 0 at, say, $L_0 = 1.1$. This can either be applied as an arbitrary condition for particles in completely flat helices, or we can use the argument that in an isotropic distribution about half of the particles at L = 1.1 would mirror below the surface and hence this is a good mean value to use as a uniform cut-off for all particles.

The density per unit volume, N, can be obtained from f by recalling that $f \propto nL^2$ and ndL is the number of particles in dL per unit range of μ and J. If the density is independent of θ and φ near the equator, then $N \propto n/L^2$, or

$$N(L) = N(L_1)\ f(L)\ (L_1/L)^4 \tag{18}$$

(Hess, 1968, p. 229).

Figure 3 is a plot of the electron density N(L) for the three cases indicated, each normalized to $N(2) = 6.7 \times 10^{-4}\ cm^{-3}$, the densities required at $L \approx 2$ to give the observed synchrotron radiation (Warwick, 1970). If N is evaluated at $L = L_1 = 50$ for these cases assuming no loss mechanisms between L = 6 and L = 50, values of N ranging between 2×10^{-8} and $2 \times 10^{-7}\ cm^{-3}$ are obtained. Since the expected densities in the solar wind are of order $.3\ cm^{-3}$, there should be no difficulty in supplying the required particles by diffusion driven by ionospheric winds even if transfer across the magnetopause is impeaded and there are losses by other mechanisms as suggested by Kennel (1972) and by Thorne and Coroniti (1972). Similar calculations for the other two diffusion mechanisms, with their much smaller value of $f(2)/f(L_1)$ when losses at Io are included, show that in these cases N at the magnetopause would have to be larger than in the solar wind.

The differential flux, j(particles/cm^2 s sr MeV) is related to f by $j \propto f/L^3$ (Roederer, 1970, p. 122). To get the total integrated flux, Φ, one must integrate j over energy and solid angle just as to get the total particle density N must be integrated over μ and J.

VI. The Effects of Synchrotron Radiation

In this section we first consider the effect of synchrotron radiation on relativistic electrons and, secondly, we estimate the particle diffusion time and compare this with the electron radiation lifetime. Since the energy losses of the electrons has no effect on their density, the solutions for f in Section V remains valid even with synchrotron radiation if the ranges of μ and J occupied by particles of interest are suitably adjusted. For these order of magnitude estimates, we shall consider only electrons moving normal to the magnetic field; i.e. in the magnetic equatorial plane.

The rate of change of total energy, E, measured in MeV, by an electron moving normal to the field is (Rossi and Olbert, 1970, p. 39)

$$\frac{dE}{dt} = -K\, B^2 c^2 p^2, \text{ where } K = 3.8 \times 10^{-9} \text{ MeV}^{-1} \text{ G}^{-2} \text{ sec}^{-1} \tag{19}$$

$$\simeq -K\, B^2\, E^2 \qquad \text{if } E \gg mc^2 = 0.51 \text{ MeV}.$$

For the highly relativistic range, it follows that the time for a particle's energy to drop from E to E/2, or from infinity to E, is

$$\tau_{rad} = 1/K\, B^2\, E \tag{20}$$

For a field of $(10/1.8^3)$ Gauss and an energy of 6 MeV at L = 1.8, τ_{rad} = 175 days and it increases rapidly with L. For particles of any kinetic energy, $T = E - mc^2$, the time required for the kinetic energy to drop from T to T/2 is

$$\tau'_{rad} = \frac{1}{2mc^2\, K\, B^2}\, \ell n\, \frac{T + 4mc^2}{T + 2mc^2} \tag{21}$$

We would like to determine the amount of energy that the electrons near L = 2 have lost because of synchrotron radiation. This is a difficult problem to attack with any rigor because the electrons progress inward by a random walk and different electrons spend different amounts of time in each region. The ideal procedure would be to solve a three-dimensional Fokker-Planck equation for $f(L,\mu,J)$ (Farley and Walt, 1971)

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D}{L^2} \frac{\partial f}{\partial L}\right) - \frac{\partial}{\partial \mu} \left(\langle\frac{d\mu}{dt}\rangle f\right) - \frac{\partial}{\partial J} \left(\langle\frac{dJ}{dt}\rangle f\right) \tag{22}$$

Here $\langle\frac{d\mu}{dt}\rangle = \frac{\partial \mu}{\partial E} \frac{dE}{dt}$ and $\langle\frac{dJ}{dt}\rangle$ are the rates of change of μ and J due to the energy loss by synchrotron radiation. If we consider only particles in very flat helices, $J \approx 0$, the last term is

$$\frac{\partial}{\partial J} \left(\langle\frac{dJ}{dt}\rangle f\right)\Big|_{J=0} = \langle\frac{d\mu}{dt}\rangle \frac{f}{2\mu}$$

and (22) becomes two dimensional. Solving even this equation would require a substantial effort and would require a knowledge of the distribution $f(L_1, \mu)$ on the outer boundary. We will instead attempt a simpler, approximate solution of the problem in which we assume that all particles that start at $L = 50$ with a particular energy have lost the same energy by synchrotron radiation by the time they have diffused in to any particular L. This ignores the random walk process and aims at a simple approximation based on the average inward motion. Thus, we get an estimate of the effect of the synchrotron radiation in modifying the energy predicted from the conservation of the first two adiabatic invariants. We first consider estimates of τ_{dif}, the time to diffuse through the high field region, for comparison with τ_{rad}, the characteristic time for energy loss. If $\tau_{dif} \gg \tau_{rad}$, the electrons will not be able to reach small L values with relativistic energies.

One way to estimate the diffusion time is to introduce the concept of average diffusion velocity, V_D, which we take equal to the net flux of particles at a point divided by the particle density. First, write the Fokker-Planck equation in the form of a conservation equation (Roederer, 1970, p. 130)

$$\frac{\partial N}{\partial t} + \nabla \cdot (N\ \vec{V}_D) = \text{-Losses}$$

By equation (6), near the equatorial plane L-shells are essentially spherical and in the divergence operator in spherical polar coordinates we may replace r by L. Also N is proportional to f/L^4. Hence, we get

$$\frac{\partial f}{\partial t} + L^2 \frac{\partial}{\partial L} \left(V_D \frac{f}{L^2}\right) = -A \tag{23}$$

if we assume that V_D is essentially radial. When equation (23) is compared with equation (3), we see that

$$V_D = -D \frac{d\ \ell n\ f}{dL} \tag{24}$$

Because $B = 10/L^3$ and p^2 is proportional to B, the synchrotron radiation for $L > 3$ will be relatively small and can be ignored. The relativisitc electrons required to explain the observations are mostly in the range $1.5 < L < 3$. Hence the mean time to diffuse from $L = 3$ to 1.5 is an appropriate τ_{dif}, and our first estimate of this is

$$\tau_{dif} = \int_3^{1.5} \frac{dL}{V_D} \tag{25}$$

This assumes that all particles move in steadily at a rate determined by the gradient in f, i.e. by a solution that depends on all the boundary conditions and on the losses at Io. Alternatively, one could argue that to random walk a distance ΔL should take a mean time of the order of $\Delta L^2/2D$ and hence that a plausible estimate that emphasizes the random motion is

$$\tau^*_{dif} = \left[1.5^2/2D\right]_{L = 2} \tag{26}$$

where D is evaluated for L = 2 rather than L = 2.25 because diffusion is slower at small L. For L < 6 where equation (16) gives f = Ay and equation (14) gives y in terms of D, it is easy to evaluate τ_{dif} and τ^*_{dif} for each of the diffusion models considered in Section III. The results of this calculation are shown below in Table 1.

<u>Table 1</u>. Values of τ_{dif}, τ^*_{dif} (equations 25 and 26), and β_1 (equation 12), the parameters that measure the effects of synchrotron radiation on electrons and the absorbtion of Io on all particles, for the diffusion mechanisms and coefficients under consideration.

Mechanism responsible for diffusion	Diffusion coefficient, D $(day)^{-1}$	τ_{dif} (days)	τ^*_{dif} (days)	$\beta_1 = \frac{\alpha \Delta L}{D/L}\Big\vert_{L=6}$
Deformation of the magnetic field by the solar wind	$5 \times 10^{-15} L^{10}$	10^{13}	2×10^{11}	2×10^6
Randomly fluctuating electric fields	$1.7 \times 10^{-9} L^6$	6×10^7	10^7	7×10^3
Interchange of flux tubes by ionospheric motions	$4 \times 10^{-3} L^2(L-1)$ $4 \times 10^{-4} L^2(L-1)$	300 3×10^3	70 700	1 10

When these values of τ_{dif} and τ^*_{dif} are compared with $\tau_{rad} \approx 175$ days, and when the values of β_1 are compared with unity, it is obvious that the two well-known diffusion mechanisms, the first two, are unable to balance the drain due to synchrotron radiation in order to maintain electrons at relativistic energies and are unable to transport enough particles past Io to produce either the observed synchrotron radiation or a radiation hazard to spacecraft. These failures are by very large factors and cannot be repaired by reasonable changes in the parameters. In the case of diffusion driven by ionospheric motion, the example with the smaller diffusion constant would probably be in some difficulty but the case with the larger constant would probably be satisfactory since only a small fraction of the electrons potentially available from the solar wind are needed to produce the synchrotron radiation. The major problem with this model is that it is based on conjectures whose validity we have not investigated. A less serious problem is that the numerical parameters must be pushed toward the highest reasonable values. If this model is not acceptable, we see no other way to supply the relativistic electrons by diffusion from the magnetopause or to produce high fluxes of MeV protons for L in the range 1.5 to 4.

If the values of τ_{dif} and τ^*_{dif} in Table 1 are compared, one based on the average flow and the other on a random walk, we see that in the third case they are of the same order of magnitude. This suggests that in this case it may not be unreasonable to use the average flow model to make rough estimates of the typical electron energy as a function of L. Since an accurate treatment based on a solution of equation (22) appears very difficult, we proceed with the approximation. For dL/dt, we use V_D as given by equation (24) with $D = D_A$. Write

$$\begin{aligned} \frac{dE}{dL} &= \left.\frac{dE}{dL}\right|_{ad} + \left.\frac{dE}{dL}\right|_{sync} \\ &= \left.\frac{dE}{dL}\right|_{ad} + \left.\frac{dE}{dt}\right|_{sync} V_D^{-1} \end{aligned} \tag{27}$$

where the subscript "ad" refers to terms produced by adiabatic changes when μ is conserved and "sync" refers to effects produced by synchrotron radiation. The relativistically exact form of equation (19) with $c^2p^2 = E^2 - m^2c^4$ and $\mu = p^2/2mB$ for the case of electrons in very flat helices yields

$$\frac{dE}{dL} = - \frac{3(E^2 - m^2c^4)}{2EL} + \frac{K\ B^2(E^2 - m^2c^4)}{-V_D} \tag{28}$$

where the first term on the right is the contribution due to the conservation of μ and the last term gives the effect of the synchrotron radiation.

Equation (28) gives the energy, including the effect of synchrotron radiation, as a function of L. The equation is solved by numerical integration for various values of the parameters and the results shown in Figure 4 as plots of $T = E - mc^2$, the kinetic energy, vs L. In the case in which synchrotron radiation is neglected, i.e. $K = 0$ in equation (27), the solution,

$$T = \left[(L_1/L)^3\ (E_1^2 - m^2c^4) + m^2c^4\right]^{1/2} - mc^2 \tag{29}$$

where $E_1 = T(L_1) + mc^2$ is obtained directly from the conservation of μ. For L larger than about 4, this solution is essentially the same as the corresponding solutions with finite K and V_D derived from third model. However, for smaller values of L, the curves are very different. If D_{SW} or D_E had been used to determine V_D, the much smaller value of V_D would keep the electrons from ever reaching relativistic velocities. For solutions with the electron energy normalized near $L \approx 2$ to the values used in Figure 4, the electron kinetic energy at $L = 50$, the assumed magnetopause, is 3.75 keV for curves 1 and 10.7 keV for curves 2, which is much larger than the approximately 0.2 keV usually assumed (Brice, 1972). This suggests that diffusion models have some difficulty in supplying electrons of the energy needed to produce the synchrotron radiation unless the magnetopause is placed at $L_1 = 100$ or 200 or unless the electrons in the magnetosheath are very hot indeed.

The total power radiated per unit volume is $P_T \propto N(L)\,\frac{dE}{dt} \propto fp^2/L^{10}$. Figure 5 is a plot of $P_T(L)/P_T(2)$ expected on the basis of this analysis for the same cases as in figure 4. Note that curves 1c and 2c have their maxima near L = 2 in rough agreement with observations.

Now consider the energy lost by particles with pitch angles $\beta \neq \pi/2$. The power radiated by these particles $\dot{E}_\beta$ is given by

$$\frac{\dot{E}_\beta}{\dot{E}_{eq}} = \frac{B^2 \sin^2 \beta}{B_{eq}^2} ,$$

where the subscript eq refers to equatorial particles. Since $\mu \simeq$ const over a bounce period, $\sin^2 \beta = B/B_m$ where B_m is the value of B at the mirror point. Thus, for particles with the same energy but different mirror points,

$$\frac{\dot{E}_\beta}{\dot{E}_{eq}} = \frac{B^3}{B_{eq}^2 B_m} = \begin{cases} \dfrac{B_{eq}}{B_m} & \text{at } \theta = \pi/2 \\[2ex] \dfrac{B_m^2}{B_{eq}^2} & \text{at } \theta = \theta_m \end{cases}$$

$$\text{For } \theta_m = 60^\circ, \quad \frac{B_m}{B_{eq}} = \frac{[3\cos^2\theta_m + 1]^{1/2}}{\sin^6\theta_m} \simeq 3.1$$

Since particles spend more of their time near the mirror points, it is clear that for particles of a given energy, the smaller β is, the more energy is lost to synchrotron radiation. Thus, for small L values, the electrons with the largest energy would have pitch angles near $\pi/2$.

VII. Summary

The conclusions derived from the preceeding analysis are:

1) When electron energy is computed, the effect of synchrotron radiation must be included. This results in a higher energy required at $L_1 = 50$ if the energy at $L = 2$ is to be close to 6 MeV, which we have taken to be the characteristic electron energy there. A reasonable estimate of the energy of an electron at $L = 50$ is .2 KeV, while the required energy is in the range 2 - 10 KeV. This could be remedied by extending the magnetopause, or assuming that the electrons in the magnetosheath are very hot, or by assuming that the electrons gain energy in some way besides conservation of the first adiabatic invariant (e.g. by disturbances near Io).

2) One specific model of diffusion due to the wind-driven interchange of tubes of flux is shown to have a diffusion coefficient of the form $D = D_0 L^2(L-1)$. This diffusion mechanism succeeds where the diffusion mechanisms usually used for the earth's magnetosphere fail for Jupiter, i.e. it is large enough in the region $L \lesssim 6$ to (1) get sufficient numbers of electrons past Io and (2) diffuse electrons inward fast enough to supply the energetic electrons required for synchrotron radiation. If this general mechanism is not acceptable, the electrons must be accelerated near Io and the energetic proton flux there is very difficult to estimate. If the particle flux matches the predictions of the diffusion model, this will give powerful support for Brice's mechanism.

Appendix

In this section we will show that electric fields $\vec{E}$ at the feet of dipole field lines will cause the plasma above the ionosphere to move such that field lines (whose motion is defined by the motion of the plasma) move into other dipole field lines. In other words, the magnetic field maintains its dipole configuration during the diffusion process.

We will consider two cases: 1) $\vec{E} = E_\varphi \hat{\varphi}$ and 2) $\vec{E} = \vec{E}_r \hat{r} + \vec{E}_\theta \hat{\theta}$ where $\vec{E}\cdot\vec{B} = 0$.

Case 1

Consider 2 neighboring field lines, both emerging from the surface at the same value of θ but one at φ and one at $\varphi + \Delta\varphi$, where $\Delta\varphi$ is small. The existence of E_φ implies that they differ in potential by some amount ΔV. Since the potential V = const. along a field line, ΔV = constant as we map the electric field out into the magnetosphere. The distance between the field lines in the φ direction is $\Delta S_\varphi = r \sin\theta \, \Delta\varphi$, so $E_\varphi(r,\theta) = \frac{\Delta V}{\Delta S_\varphi} = \frac{1}{r \sin\theta} \frac{\Delta V}{\Delta\varphi}$. The velocity of the field lines $\vec{u} = c \frac{\vec{E}\times\vec{B}}{B^2}$ is in the r, θ plane perpendicular to B. If a field line is to move into another dipole field line we must have $u \propto \Delta S_{r\theta}$, where $\Delta S_{r\theta}$ is the distance between two field lines in the $r\theta$ plane which are separated by $\Delta\theta$ at the surface. The expression for $\Delta S_{r\theta}$ can be obtained as follows.

Consider a small surface perpendicular to B. Let its dimensions be ΔS_φ in the φ direction and $\Delta S_{r\theta}$ in the $r\theta$ plane. The flux through it is $\Delta\Phi = B \, \Delta S_\varphi \, \Delta S_{r\theta} = [B \, \Delta S_\varphi \, \Delta S_{r\theta}]_{\text{surface}}$ = const. Thus $\Delta S_{r\theta} \propto \frac{1}{B\Delta S_\varphi}$, and

$$u = c \frac{E}{B} = \frac{c \, \Delta V}{B\Delta S_\varphi} \propto \Delta S_{r\theta}$$

so the field remains a dipole.

<u>Case 2</u>

This is similar to case 1. Here the field lines emerge from the surface at the same φ but one at θ and one at $\theta + \Delta\theta$, so E is in the $r\theta$ plane, and $\vec{u} = c\,\frac{E}{B}\,\hat{\varphi}$. If the field lines are to remain dipole, we must have $u \propto \Delta S_{\varphi}$. If the field lines differ in potential by ΔV, then $E = \frac{\Delta V}{\Delta S_{r\theta}} \propto B\ \Delta S_{\varphi}$, and $u \propto \Delta S_{\varphi}$ as required.

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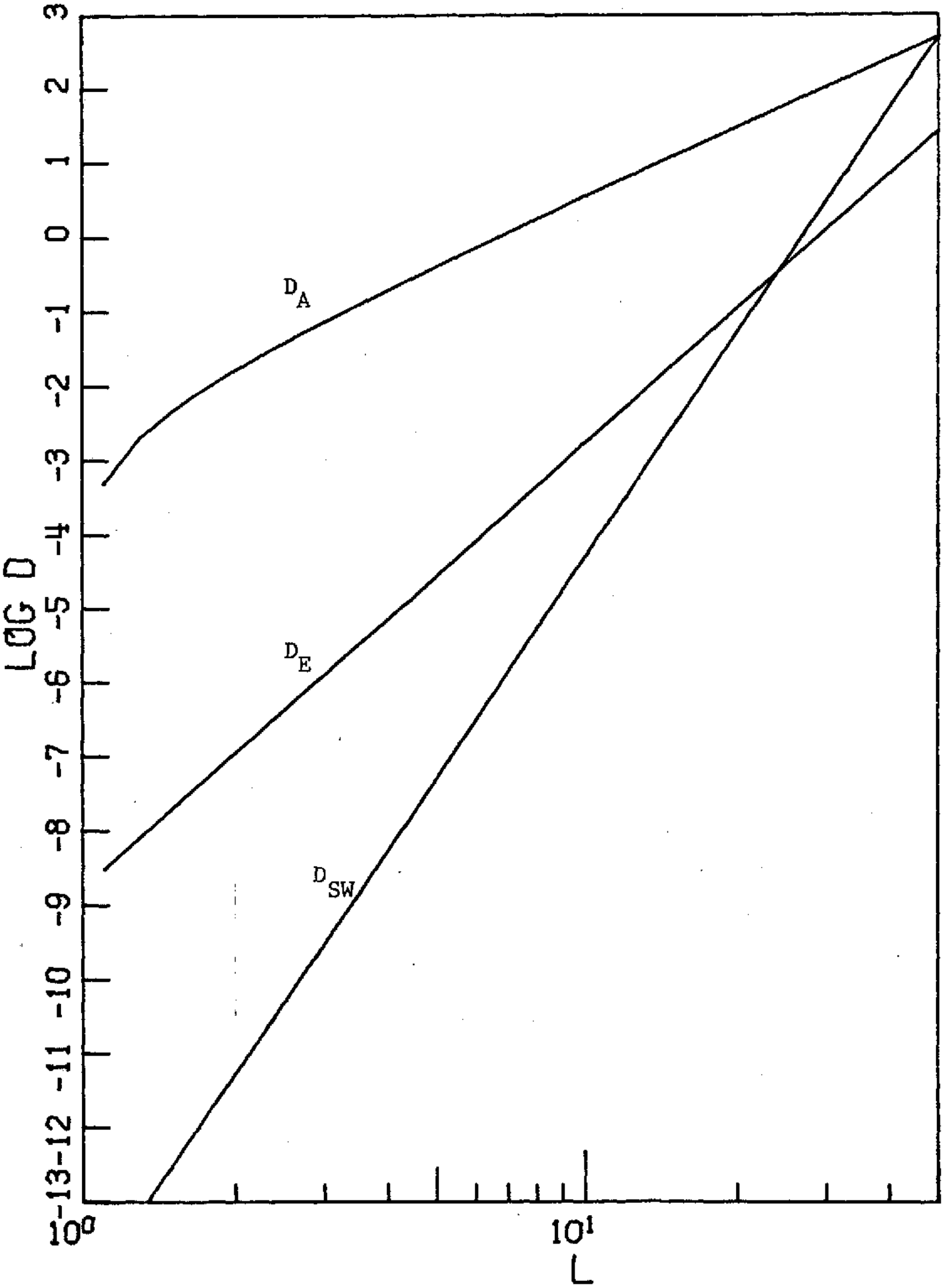


Figure 1

The diffusion coefficients D(L) for the three cases $D_A = 4 \times 10^{-3}\ L^2(L-1)\ \text{day}^{-1}$, $D_E = 1.7 \times 10^{-9}\ L^6\ \text{day}^{-1}$, and $D_{SW} = 5 \times 10^{-15}\ L^{10}\ \text{day}^{-1}$.

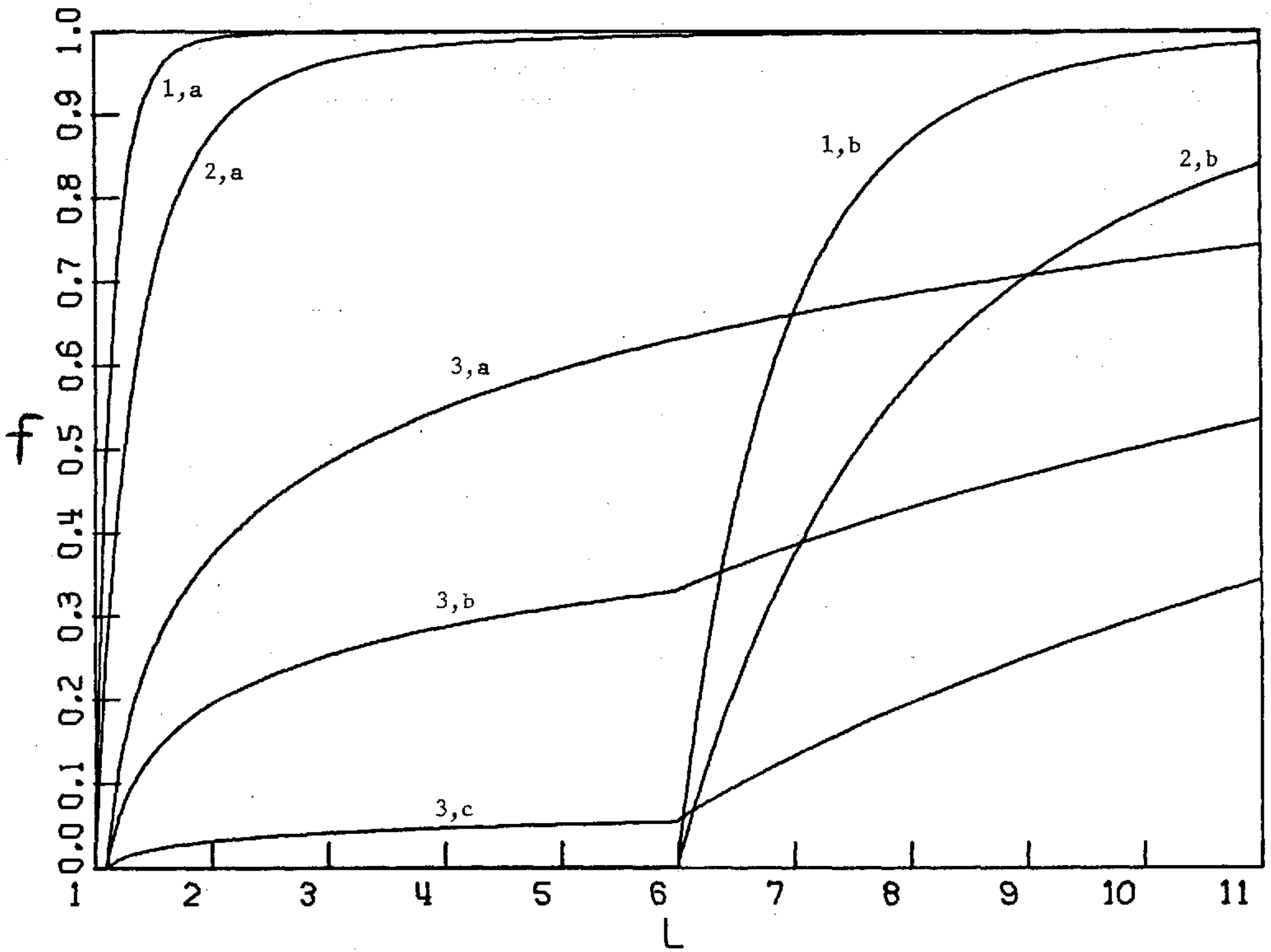
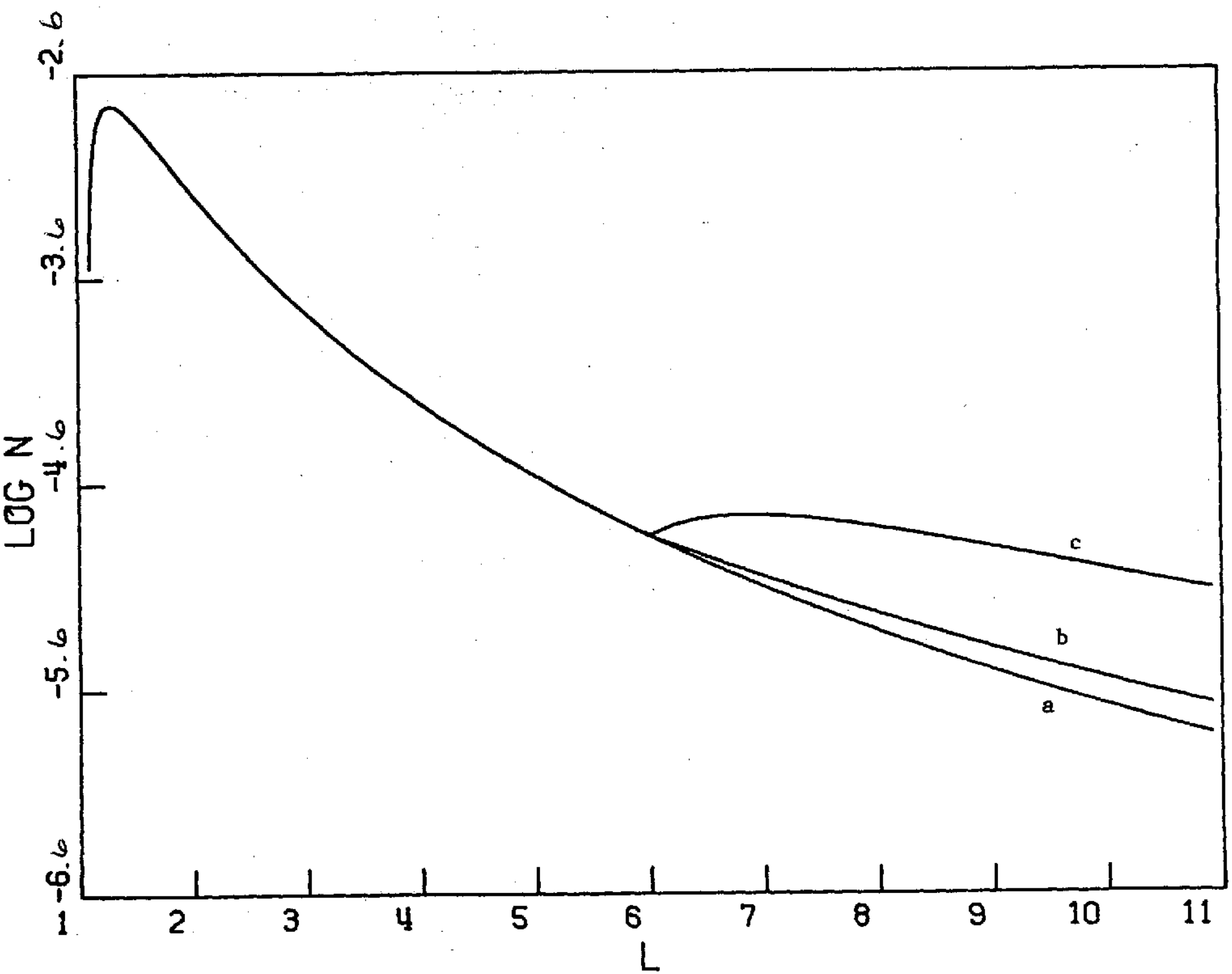


Figure 2

The fraction of full flux tubes, f, as a function of L for $D = D_{SW}$ (curves 1), $D = D_E$ (curves 2), and $D = D_A$ (curves 3). The labels a, b, c, identify, respectively, the curves for the cases in which α/D_o is zero (the effect of Io is neglected), α/D_o has its nominal value, and α/D_o has ten times its nominal value (presumably because of a lower diffusion rate). The curves 1b and 2b are essentially zero for $L < 6$.



<u>Figure 3</u>

The density, N, in particles cm^{-3} as a function of L for the case $D = D_A$ using as normalization $N = 6.7 \times 10^{-4}\ cm^{-3}$ at L = 2. As in Figure 2, the labels a, b, c, identify, respectively, the curve for $\alpha/D_o = 0$, $\alpha/D_o = 2/(4 \times 10^{-3})$, the nominal value, and $\alpha/D_o = 2/(4 \times 10^{-4})$.

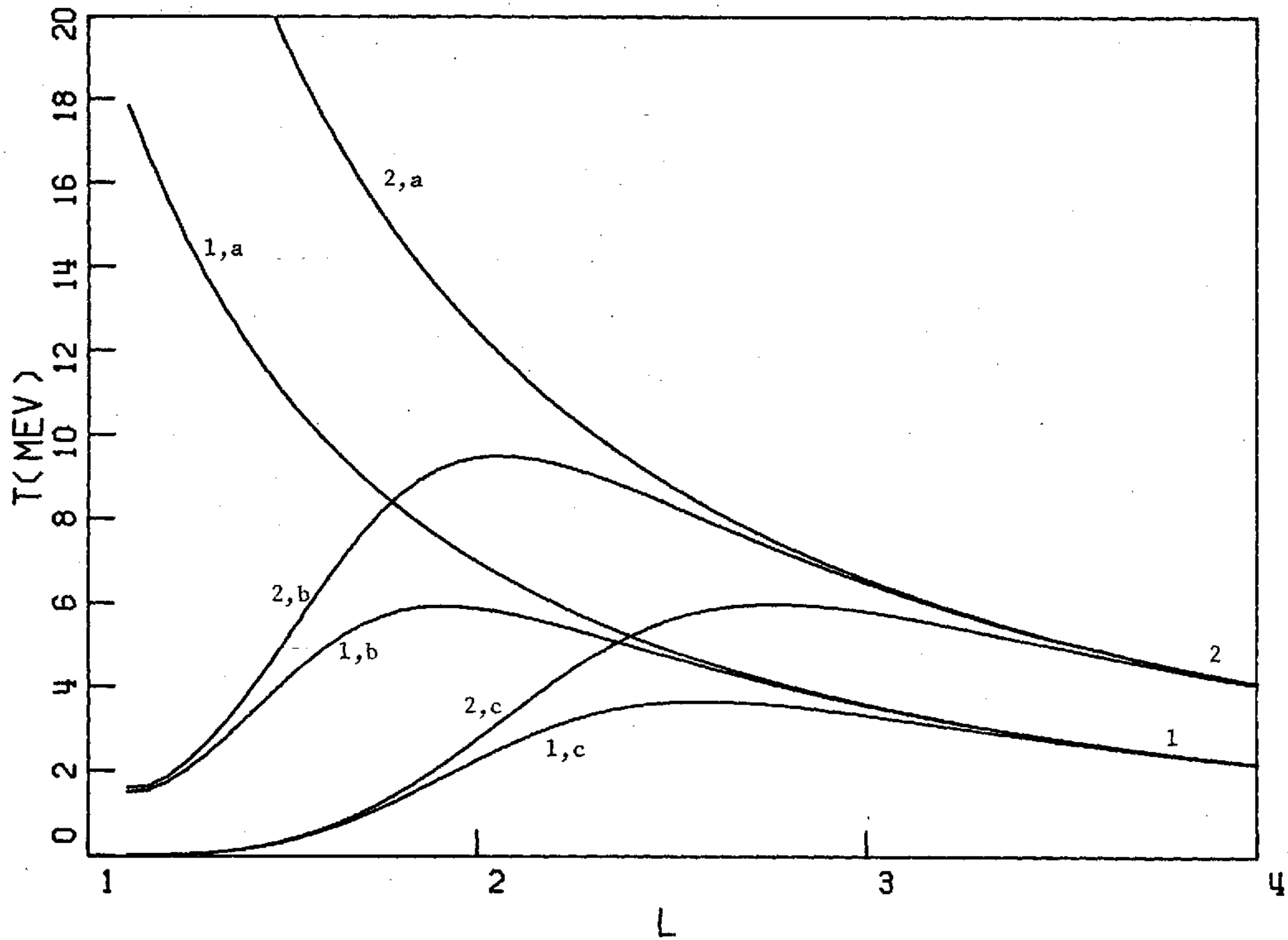


Figure 4

Electron kinetic energy T(MeV) in the synchrotron radiation zone for 6 cases. The label 2 implies $T(L = 6) = 2$ MeV, while 1 implies $T(L = 6) = 1$ MeV. Curves labeled a are the solutions neglecting the loss of energy by synchrotron radiation, and those labeled b and c are the solutions including the energy loss. Curves labeled b use $D = D_A$, while curves labeled c use $D = 1/10\ D_A$.

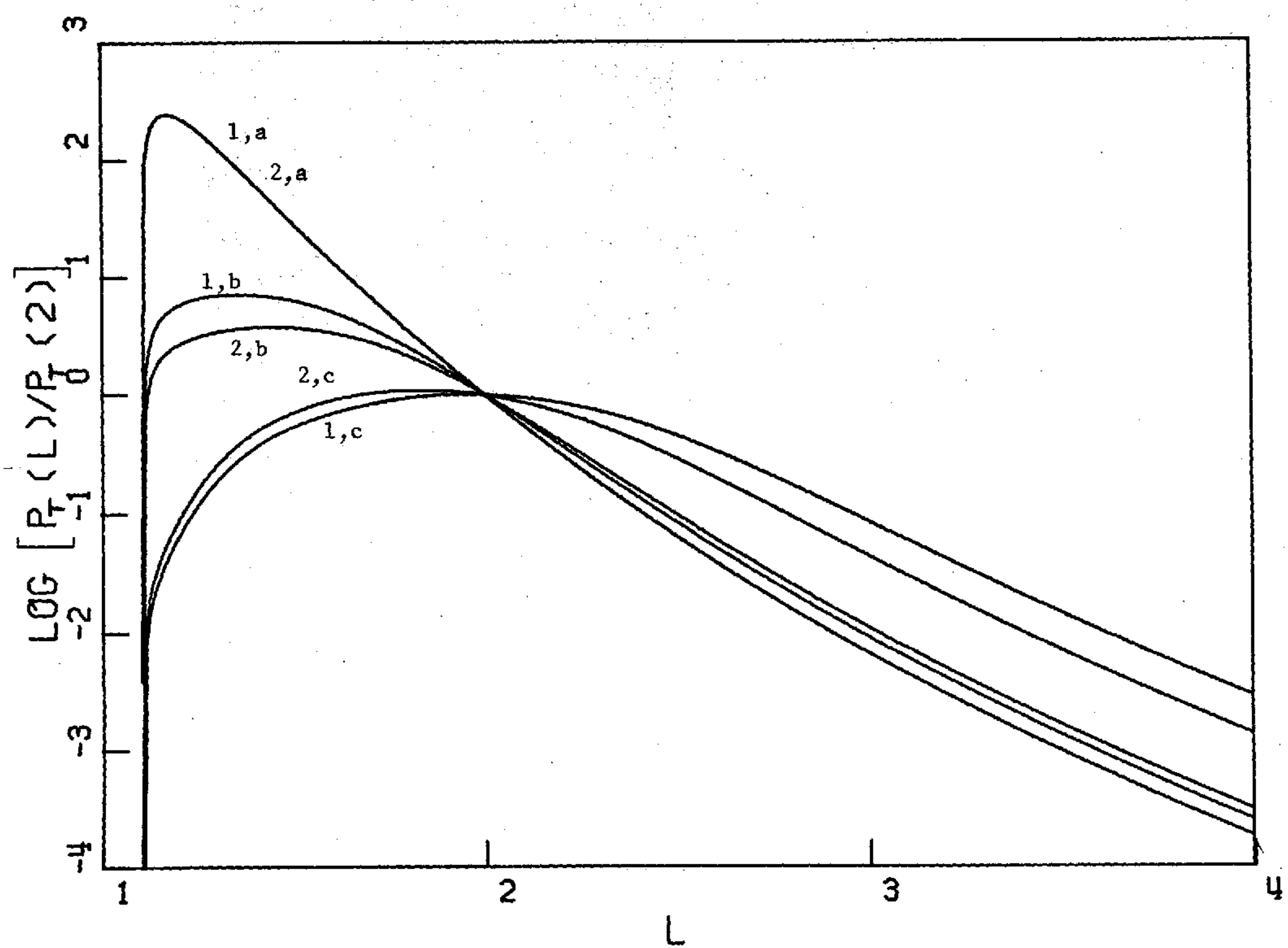


Figure 5

The total power radiated per unit volume, P_T, divided by $P_T(2)$. The labels have the same meaning as those in figure 4.