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16. Abstract

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SUMMARY

A general solution using an elliptic integral approximation which relates flight time, aircraft airspeed, and ground distance on straight-line and circular-arc trajectory segments is developed in this paper. The solution procedure is applicable to both constant and accelerating aircraft flight. A number of acceleration profiles can be considered; the two profiles developed in the paper are for constant and exponential acceleration. In addition, wind shear including both magnitude and heading change is incorporated in the solution.

The solution concept is utilized and tested in a four-dimensional (4-D) control algorithm where both flight time and final velocity are specified. The results show that the algorithm converges both rapidly and accurately. In addition, the effects of wind gusts and flight control system dynamics are investigated in order to demonstrate the adaptability of the algorithm to 4-D guidance.

INTRODUCTION

Increased capacity of air traffic can be obtained through more precise sequencing of aircraft in both terminal and en route environments. To improve longitudinal spacing and sequencing, the four-dimensional (4-D) control concept has been proposed. (See ref. 1.) In this concept the aircraft must not only fly a prescribed ground route and altitude profile but must also traverse segments of the route in prescribed time intervals in order to sequence and space properly with other aircraft in the vicinity.

In investigating aircraft 4-D control systems, one of the primary requirements is to determine either the time required to fly a fixed route with a prescribed velocity profile or, alternatively, to determine the airspeed profile needed to fly the route in a specified elapsed time. Some of the research on 4-D control has incorporated specific closed form solutions to this problem, generally based upon solutions of the elliptic integral function over curved flight portions of the trajectory. (See refs. 2 to 5.) With the exception of Erzberger and Pecsvaradi (ref. 3), who used only first-order approximations, the distance-time-velocity relationships utilized have been only for constant airspeed.

The research reported in this investigation extends the capability by developing accurate closed form solutions which define the relationship between flight time and ground distances covered for accelerating (decelerating) flight on both straight-line and circular-arc segments of a ground trajectory and for the inclusion of wind shear during constant-airspeed flight. The procedure indicates that a number of acceleration profiles can be handled; the two profiles derived in this report are for constant and exponential acceleration. The latter is representative of a response of a first-order lag controller to a step command. Wind shear includes linear changes in both windspeed and direction and can be keyed to the aircraft ascent rate. It has been incorporated during the straight-line portions of the flight at constant airspeed.

The research utilizes and tests these solutions in an algorithm for defining the flight profile for an aircraft in a 4-D control situation. The aircraft is required to traverse a trajectory in a specified time and arrive at the final point with a specified velocity. Although the algorithm must be solved by a computer, the process is much simpler than one which would solve the aircraft equations of motion as a two-point boundary-value problem. Hence, the procedures developed here will find considerable use in implementing velocity-command systems for ground-based or onboard aircraft flight control, especially when 4-D control is required. (See ref. 4.)

SYMBOLS

Values are given in both SI and U.S. Customary Units in the text and in SI units in the figures. The measurements and calculations were made in U.S. Customary Units.

a acceleration (or deceleration) constant

$b = v_w \sin z$

$c = v_c{}^2 - v_w{}^2 \sin z$

d ground distance

d_i ground distance on ith segment

d_r ground distance remaining

$d_s(v_1,v_2)$ ground distance traversed during acceleration from v_1 to v_2

$f(\ )$	functional notation
g	constant, $2v_c$
K	velocity control servo gain
k	ratio of windspeed to airspeed
k_o	k at beginning of wind shear
q	instantaneous velocity during acceleration
R	ground turn radius
s	Laplace variable
T	variable of integration, $v_c - v_u$
T_D	time required to traverse a K-segment trajectory
t	time
t_a	time to accelerate
t_f	final velocity stabilization time
t_i	time to fly ith straight-line segment
t_i'	time to fly ith circular-arc segment
t_j	time of jth instant
v_c	commanded airspeed
v_f	final airspeed
v_g	ground velocity
v_u	airspeed

v_w windspeed

v_1 airspeed at start of acceleration

v_2 airspeed at end of acceleration

X function, $c + gT + T^2$

x,y general distance variables

z ground heading wind direction, $\psi_g - \psi_w$

Δz incremental change in z

z_i value of z on ith straight-line segment

z_j value of z at t_j

z_o value of z at beginning of wind shear

β time constant

$\beta_o = k_o - \frac{\chi_2}{\chi_1} z_o$

$\beta_1 = \frac{\chi_2}{\chi_1}$

σ_w wind standard deviation

ψ aircraft heading

ψ_g ground heading

$\psi_{g,i}$ ground heading in ith segment

ψ_w wind direction

χ_1 wind shear heading gradient

χ_2 wind-airspeed ratio gradient

Subscript:

max maximum

Superscript:

k denotes kth iteration solution

Abbreviations:

C.A. circular arc

S.L. straight line

Dots over symbols denote derivatives with respect to time and an arrow over a symbol denotes a vector.

BASIC TIME-DISTANCE RELATIONSHIPS FOR AIRCRAFT FLIGHT

It is desired to determine the basic relationship for the ground distance traversed by an aircraft flying at airspeed $\vec{v}_u$ and perturbed by a steady wind v_w at heading ψ_w. The ground trajectory in the horizontal plane consists of straight-line (S.L.) segments and connecting circular arcs (C.A.) of constant radius.[1]

The time to fly the ith straight-line portion is

$$t_i = \frac{d_i}{v_g}$$

where d_i is the ith straight-line segment distance and v_g is the ground velocity. The ground velocity v_g can be calculated from

$$\vec{v}_g = v_g(\cos \psi_g, \sin \psi_g) = \vec{v}_u + \vec{v}_w = v_u(\cos \psi, \sin \psi) + v_w(\cos \psi_w, \sin \psi_w) \tag{1}$$

where v_u and ψ are the airspeed magnitude and heading, respectively, v_w and ψ_w are the windspeed magnitude and heading, respectively, and ψ_g is the ground heading angle. Solving for v_g gives

---

[1]The radius R is usually selected on the basis of maximum operating velocity during the flight regime of interest and maximum bank angle, passenger comfort being considered.

$$v_g = v_u\left[1 + k^2 + 2k \cos\left(\psi - \psi_w\right)\right]^{1/2} \tag{2}$$

where $k = v_w/v_u$. Since the variable ψ_g is independent of flight conditions, it is desirable to rewrite equation (2) to eliminate ψ. By the law of cosines,

$$\cos\left(\psi - \psi_w\right) = \frac{v_g}{v_u}\cos\left(\psi_g - \psi_w\right) - k \tag{3}$$

Substituting equation (3) into equation (2) and performing additional algebraic manipulation result in

$$v_g = v_u\left\{\left[1 - k^2 \sin^2\left(\psi_g - \psi_w\right)\right]^{1/2} + k \cos\left(\psi_g - \psi_w\right)\right\} \tag{4}$$

(Note that Erzberger and Lee (ref. 2) uses the $0(k)$ approximation.) The time to fly the circular-arc segment can be determined by considering the angular velocity as

$$\dot{\psi}_g = \frac{v_g}{R} \tag{5}$$

By use of equation (4), equation (5) can be written as

$$\int_0^{t_i'} \frac{v_u}{R}\,dt = \int_{\psi_{g,i}}^{\psi_{g,i+1}} \frac{d\psi_g}{\left[1 - k^2 \sin^2\left(\psi_g - \psi_w\right)\right]^{1/2} + k \cos\left(\psi_g - \psi_w\right)} \tag{6}$$

The right-hand side of equation (6) can be evaluated as

$$f_1(k,z_{i+1},z_i) = \int_{z_i}^{z_{i+1}} \frac{\left(1 - k^2 \sin^2 z\right)^{1/2} - k \cos z}{1 - k^2}\,dz \tag{7}$$

where

$$z = \psi_g - \psi_w$$

For k constant, the first term of equation (7) is the incomplete elliptic integral of the second kind. By using the approximation

$$(1 - x)^{1/2} \approx 1 - \frac{x}{2} - \frac{x^2}{8} \tag{8}$$

the integral can be evaluated as

$$f_1\left(k,z_{i+1},z_i\right) = \frac{1}{\left(1 - k^2\right)}\left[\left(1 - \frac{k^2}{4} - \frac{3}{64}k^4\right)z - k \sin z + \frac{1}{8}\left(k^2 + \frac{k^4}{4}\right)\sin 2z - \frac{k^4}{256}\sin 4z\right]\Bigg|_{z_i}^{z_{i+1}} \tag{9}$$

A previously reported study (ref. 4) has shown equation (9) to be accurate to within 0.02 percent for all reasonable wind conditions.

Equations (4) and (6) form the basis for developing solutions to determine time-distance relationships for both constant-velocity and accelerating-velocity flight on straight-line and circular-arc trajectories.

Constant-Airspeed Flight

The time-distance relationship for constant-airspeed flight for a straight-line trajectory is given by

$$t_i = \frac{d_i}{v_u\left[\left(1 - k^2 \sin^2 z\right)^{1/2} + k \cos z\right]} = \frac{d_i}{v_u} f_2(k,z) \tag{10}$$

For the circular-arc segment, the left-hand side of equation (6) is integrated to give

$$t_i' = \frac{R}{v_u} f_1(k,z_{i+1},z_i) \tag{11}$$

Constant-Acceleration Flight

It is assumed that the aircraft is accelerating (decelerating) at the constant rate a. Usually, this acceleration will take place when the aircraft must reach some final velocity constraint, for example, high-altitude cruise or final approach velocity. Generally, the percentage velocity change made by the aircraft during any one maneuver, except takeoff and touchdown, will be small.

For a straight-line segment of flight,

$$v_u = v_1 + at \tag{12}$$

Substituting equation (12) into equation (4) results in

$$v_g = \frac{dx}{dt} = \left[\left(v_1 + at\right)^2 - v_w^2 \sin^2 z\right]^{1/2} + v_w \cos z$$

This function can be integrated in closed form to obtain

$$d = \left(\frac{v_w}{a} \cos z\right)q + \frac{1}{2a}\left\{q\left(q^2 - b^2\right)^{1/2} - b^2 \log\left[q + \left(q^2 - b^2\right)^{1/2}\right]\right\}\Bigg|_{v_1}^{v_2} \tag{13}$$

where

$$q = v_1 + at$$

$$b = v_w \sin z$$

For the circular-arc segment of flight, equation (12) is substituted into the left-hand side of equation (6) and integrated to obtain

$$\frac{a}{2R} t'^2 + \frac{v_1}{R} t' \approx f_1(k,z+\Delta z,z) \tag{14}$$

Note that equation (14) is an approximation since k in the evaluation of f_1 was assumed to be constant but in the accelerating case is a function of time t'. The resulting error can be kept small if during the maneuver, the change in v_u is small. Here, this means choosing small increments Δz and averaging the velocity v_u over the interval τ. The value k becomes

$$k \approx \frac{v_w}{v_1 + \frac{a\tau}{2}}$$

where τ can be calculated for Δz by

$$\tau = \frac{|\Delta z| R}{v_g}$$

and by using equation (4) to calculate v_g. In the case of equation (14), the time-distance (in terms of heading change) relationship is obtained by solving the quadratic function for t subject to the approximations noted previously.

Exponential-Acceleration Flight

To obtain exponentially accelerated (decelerated) flight, it is assumed that the airplane velocity change (probably with an autothrottle) can be approximately represented by a first-order-lag system; that is

$$\dot{v}_u = -\beta(v_u - v_c)$$

where

v_c commanded velocity

β system time constant

The velocity relationship is

$$v_u = v_c - (v_c - v_1)e^{-\beta t} \tag{15}$$

For the straight-line segment of flight, equation (15) can be substituted into equation (4) in a manner similar to the previous section. The resultant equation after algebraic manipulation becomes

$$d_s - v_w t \cos z = \int_{v_c - v_u}^{v_c - v_1} \frac{\left(v_c^2 - 2v_c T + T^2 - v_w^2 \sin^2 z\right)^{1/2}}{\beta T} \, dT$$

This integral can be found in the tables in the form

$$\int \frac{\sqrt{X}}{x} \, dx = \sqrt{X} + \frac{b}{2} \int \frac{dx}{\sqrt{X}} + a \int \frac{dx}{x\sqrt{X}} \tag{16}$$

where

$$X = c + gT + T^2$$

$$c \doteq v_c^2 - v_w^2 \sin^2 z$$

$$g = -2v_c$$

The solution can be evaluated in general in the form of equation (16). If the approximation $\frac{g}{2} \approx -\sqrt{c}$ is used, then the final two integrals cancel and result in the first-order wind shear solution

$$d_s - v_w t \cos z = \frac{1}{\beta}\left(c + gT + T^2\right)^{1/2}$$

The circular-arc segment for the exponential acceleration can be evaluated in a manner similar to the constant acceleration. Substituting equation (15) into the left-hand side of equation (6) and integrating gives

$$\left. \frac{v_c t}{R} + \frac{v_c - v_1}{R\beta} e^{-\beta t} \right|_{t_j}^{t_{j+1}} \approx f_1\left(k, z_j + \Delta z, z_j\right) \tag{17}$$

for small incremental changes in heading. In this case for small velocity changes

$$k \approx \frac{v_w}{v_c - \left(v_c - v_1\right) e^{-\beta t_j}}$$

where the velocity at the beginning of the segment Δt and not the average is used. In order to find the time-distance relationship, the left-hand side of equation (17) must be solved for t_{j+1} in an iterative fashion for each increment Δz until the heading change is traversed or the velocity nearly equals v_c.

Equations Including Wind Shear

Wind shear can be a significant factor especially when the aircraft is in ascending or descending flight where changes in wind character can take place over relatively short distances.[2] The form of wind shear changes is assumed to be linear and this linearity does not appear to be a significant restriction because the character of wind shear is not well defined and, as will be shown later, random gusts (which can be used to simulate higher order wind shear effects) have little overall effect on the equation accuracy.

During the straight-line segment of flight,

$$v_g = \frac{dx}{dt} = v_u\left[\left(1 - k^2 \sin^2 z\right)^{1/2} + k \cos z\right]$$

Let

$$z(t) = \psi_g - \psi_w = z_o + \chi_1 t \qquad (0 \leqq t \leqq t_1)$$

Hence

$$k = k_o + \chi_2 t \qquad (0 \leqq t \leqq t_1)$$

where χ_1 and χ_2 are the wind shear heading and wind airspeed ratio gradients, respectively. In ascent or descent χ_1 and χ_2 may be related to $\dot{h}$, the altitude rate. By using z as the basic variable,

$$t = \frac{z - z_o}{\chi_1}$$

and

$$k = k_o + \frac{\chi_2}{\chi_1}\left(z - z_o\right) = \beta_o + \beta_1 z$$

By using only the first two terms of the expansion equation (8), the right-hand side can be approximated as

$$d_s \approx \frac{v_u}{\chi_1}\left[\left(1 - \frac{\beta_o^2}{4}\right)z - \frac{\beta_o\beta_1 z^2}{4} - \frac{\beta_1^2 z^3}{12} + \left(\beta_o + \beta_1 z\right) \sin z + \beta_1 \cos z\right.$$

$$\left. - \left(\frac{\beta_o^2}{8} + \frac{\beta_o\beta_1 z}{8} + \frac{\beta_1^2 z^2}{8} - \frac{\beta_1^2}{16}\right) \sin 2z + \left(\frac{\beta_o\beta_1}{8} + \frac{\beta_1 z^2}{8}\right) \cos 2z\right]\Bigg|_{z_1}^{z_2} \qquad (18)$$

[2]Wind changes which are encountered over long distances can be most easily incorporated by abrupt changes in $\vec{v}_W$ at waypoints. Additional waypoints may be added in order to define the wind vector more accurately.

On circular-arc segments of the trajectories, wind shear is more easily taken into account by processing the equation in a manner similar to the acceleration procedures, that is, processing the turn in small segments of Δz and changing k and z on each segment. This procedure is comparable to breaking the trajectory in a sequence of short distance waypoints, as suggested earlier.

ALGORITHM FOR FLIGHT REQUIREMENTS TO TRAVERSE A GROUND TRAJECTORY

The basic equations developed in the previous section can be used in various problems. For example, time of flight or distance traversed can be predicted when velocity profile is provided. In this paper an algorithm has been developed to test the equations which are compatible with the 4-D guidance requirement. Here, as in reference 4 trajectory and flight time are defined and a velocity profile is to be determined.

The trajectory and flight time requirements are

(1) The flight time for a K segment trajectory is T_D.

(2) The final velocity v_f starts at time $t = T_D - t_f$. (This feature allows a final velocity stabilization time $t_f \geqq 0$.)

(3) The deceleration (acceleration) profile is selected.

The velocity profile characteristics to be defined cover three distinct phases of flight for which various conditions need to be determined. For phase 1 (final velocity phase v_f for t_f), determine the point in the trajectory where this phase begins. For phase 2 (deceleration phase, using either a constant or an exponential condition), determine the point at which the deceleration phase begins. For phase 3 (constant velocity phase), determine the constant velocity to fly the remainder of the trajectory so that the flight time constraint T_D is attained.

The algorithm first determines the distance traveled over the phase 1 part of the trajectory and subtracts this segment and time from further consideration. Since the velocity is constant, equation (10) and/or (11) are used to find the distance or heading change necessary to complete this phase.

The remaining two phases are solved in an iterative fashion. The steps are as follows:

(1) The velocity v_u for phase 3 is determined by assuming that no deceleration (phase 2) is required. This velocity is determined by using equations (9) and (10) in the form:

$$v_u^{(k+1)} = \frac{1}{T_D - t_f} \sum_i \left[d_i f_2\left(z_i, k^{(k)}\right) + R f_1\left(z_{i+1}, z_i, k^{(k)}\right) \right] \tag{19}$$

Equation (19), summed over the i segments of the trajectory, is solved iteratively, where the kth iteration of v_u, or $v_u^{(k)}$, is used in

$$k^{(k)} = \frac{v_w}{v_u^{(k)}}$$

to find the (k + 1)th iteration. This simple method has been found to be extremely effective in solving equation (19). If wind shear exists, then equation (18) is used over that part of the trajectory.

(2) The deceleration (acceleration) time t_a is determined. For example, for the constant-deceleration condition,

$$t_a = \left|\frac{v_u - v_f}{a}\right|$$

Two conditions for the deceleration phase need to be considered depending upon where the deceleration takes place on a straight-line or circular-arc segment. On the straight-line part, either equation (13) or (16) is used to determine the distance $d_s(v_u,v_f)$ traversed. The final distance segment is reduced to

$$d_r = d_i - d_s(v_u,v_f)$$

by assuming that $d_i \geqq d_s$. If this condition is not satisfied, then the velocity v_1 must be determined so that $d_s(v_1,v_f) = d_i$ and the remaining portion of phase 2 must be accomplished on a circular-arc portion of the trajectory. On the circular-arc portion of the trajectory, equation (14) or (17) are used to determine the elapsed time to turn a heading increment Δz starting at z_i. The turn is incremented with $\Delta z \leqq \Delta z_{max}$ until either the time t_a is reached or the turning segment ends. In this latter case, the remainder of the trajectory must be incorporated on a straight-line segment.

The deceleration (phase 2) is continued until t_a is reached. However, the phase reduces the distance which is traveled on the constant-velocity phase 1. Hence, with this reduced distance, step 1 is repeated. This step adds additional velocity and requires a return to step 2. This iterative process continues until the change in constant velocity v_u resulting from step (1) is negligible and the process is converged.

SIMULATION EXAMPLE

The algorithm developed earlier for flight requirements to traverse a ground trajectory has been employed in a simulation in order to determine the suitability of the method to the 4-D control of an aircraft. In this example, a number of features are tested, including the capability of the algorithm to converge to the desired constant-velocity condition, the accuracy of the algorithm, the effect of wind gust disturbances, and the capability of a simple autothrottle to fly a typical aircraft to the commanded velocity profile. The algorithm includes only constant deceleration and no wind shear. Results using other deceleration and wind shear conditions may differ from those presented.

The algorithm for determining the flight requirements has been incorporated in a digital simulation of a 4-D aircraft control problem. In this simulation, a one-dimensional model (along the trajectory only) of a jet transport is used. The velocity commands from the algorithm input to an autothrottle system were used to test both the algorithm and the ability of an aircraft to fly the required trajectory.

The algorithm to generate flight commands, shown in figure 1, is incorporated as a single subroutine. The subroutine takes the present aircraft conditions on the trajectory and determines the commanded constant velocity v_u and the points along the trajectory where the deceleration or acceleration commands start and end.

A five-segment trajectory used for a test case is shown in table I. Each point along the trajectory where the flight changes between straight line and circular arc is included.

TABLE I.- TEST TRAJECTORY

Segment	Type	Distance along trajectory where condition ends, m (ft)	Heading on straight-line segment, deg
1	Straight line	3 921.8 (12 866.7)	153.4
	Circular arc	8 551.4 (28 055.6)	
2	Straight line	16 008.8 (52 519.5)	240.5
	Circular arc	16 327.1 (53 566.5)	
3	Straight line	18 512.8 (60 737.3)	234.5
	Circular arc	19 064.5 (62 547.5)	
4	Straight line	20 891.9 (68 542.8)	224.1
	Circular arc	23 334.1 (76 555.4)	
5	Straight line	23 643.4 (77 570.0)	270.0

The flight requirements and wind conditions are as follows:

$T_D = 250$ sec

$v_f = 94.49$ m/sec (310 ft/sec)

$t_f = 10$ sec

$a = \pm 0.3048$ m/sec (± 1 ft/sec)

$v_w = 3.048$ m/sec (10 ft/sec)

$\psi_w = 0^{\circ}$

Convergence of Algorithm to Desired Command Conditions

The algorithm is based upon the closed form solution equations which must be iterated until the velocity continuity is obtained at the junction between the constant velocity (phase 3) and the acceleration (phase 2) portions of the flight. In order for the concept to be useful, this convergence must be rapid.

The test trajectory (table I) is used to illustrate this feature. The sequence of steps for the algorithm working backward from the end of the trajectory is shown in figure 2. The constant final velocity phase, which takes 10 seconds at an airspeed of 94.49 m/sec (310 ft/sec) consumes 922.0 m (3025 ft). Hence, deceleration ends on the curve segment 4 at the point 22 721.4 m (74 545 ft) along the trajectory.

The algorithm must now iterate over the remaining portion of the trajectory to define phases 2 and 3. The first iteration determines the constant-velocity v_u to fly to the 22 721.4-m (74 545-ft) point as 105.52 m/sec (346.19 ft/sec). At a constant deceleration of 0.3048 m/sec (1 ft/sec) to 94.49 m/sec (310 ft/sec), the time required is 36.19 seconds. The computer determines that a 15.74-second time elapse and a speed of 100.72 m/sec (330.45 ft/sec) occur at the beginning of the circular-arc segment 4 and the remainder must be accomplished on the straight-line segment 4. The first iteration finds the starting point for the deceleration at 384.63 m (1261.9 ft) after the beginning of the straight-line segment 4 as shown in figure 2. By using this point, the second iteration finds a velocity v_u of 106.43 m/sec (349.19 ft/sec). This condition can be met by an additional deceleration distance; hence, the new deceleration starting point is at 100.77 m (330.63 ft) after straight-line segment 4 or at a distance along the trajectory of 19 165.3 m (62 878 ft). A final check of constant velocity shows a change to 106.47 m/sec (349.32 ft/sec). This change is within the velocity error for convergence of the algorithm.

The flight commands to fly the required conditions are as follows: airspeed of 106.47 m/sec (349.32 ft/sec), deceleration to start at 19 165.3 m (62 878 ft), and deceleration to end at 22 721.4 m (74 545 ft). Alternate forms of command in terms of time are also available as deceleration starts at 200.8 seconds and deceleration ends at 240 seconds.

The illustrative example used shows convergence after the second iteration. Other cases show that similar convergence usually within two to three iterations for accuracies of 0.01 second, 0.3048 m (1 ft), and 0.003048 m/sec (0.01 ft/sec) in time, distance, and velocity, respectively. In a few cases four iterations have been necessary.

Accuracy of Solution Algorithm

The accuracy of the solution algorithm was tested by using the commands generated at an initial trajectory point as input into a perfect aircraft control system; that is, the aircraft airspeed instantaneously assumed the command value. With this condition, the aircraft equations of motion along the trajectory were solved by using a fourth-order Runge-Kutta routine.

The errors on the final miss distance for the various wind magnitudes for a wind heading $\psi_W = 270^{\circ}$ are shown in figure 3. For other conditions, similar accuracies were obtained, which verified that the approximation using the closed form solution and the iteration procedures used in the algorithm were more than sufficient to maintain an excellent and usable solution accuracy. (Note that the results in fig. 3 cannot be accepted as absolute because of rounding and other errors in the fourth-order Runge-Kutta simulation.)

Effect of Wind Gusts on Flight Accuracy

Perturbations in windspeed will affect the accuracy with which the final trajectory conditions are obtained. Hence, a Dryden model of wind gusts was added to the motion simulation. As in the previous case, the control system was ideal; that is, the true airspeed including wind gusts was instantaneously established at the commanded value. Hence, a positive wind gust would instantaneously reduce the aircraft airspeed relative to the steady wind and, equivalently, the aircraft ground speed.

A Monte Carlo simulation of 20 runs was made for a gust standard deviation of 3.048 m/sec (10 ft/sec). The final miss distance standard deviation was found to be 89 m (292 ft). Hence one can conclude that wind perturbations should cause only small errors in final miss distance for the ideally controlled aircraft.

Effect of Airspeed Control System on Flight Accuracy

In the previous cases an ideal airspeed control system was used. Since the commands can include both constant, ramp (constant acceleration), and step changes in velocity, a real controller can cause additional error in the final position. The autothrottle system used in this study, shown in the block diagram of figure 4, is similar to the one in references 4 and 6.

Two features of the real airspeed controller were examined. The first is the error caused by a ramp velocity command when a type 1 servo system (zero steady-state position error) of figure 4 is used. Figure 5 shows the velocity error for a ramp command from 96.62 m/sec (317 ft/sec) to 91.44 m/sec (300 ft/sec) at a deceleration of 0.3048 m/sec^2 (1 ft/sec^2). Integrating the area under the curve (fig. 5(b)) results in a 15.76-m (51.7-ft) error. Increasing the deceleration period would increase the error at the rate of approximately 0.914 m (3 ft) for each additional second (the steady-state airspeed error, fig. 5). For acceleration commands occurring early in the trajectory, this control error would be removed during subsequent redetermination of the constant-velocity commands; for commands late in the trajectory, it is small enough to be ignored. Different feedback and forward loop gains in the control system would affect the magnitude of the error.

The second feature of the airspeed controller illustrates the effect of gusts on the type of airspeed sensing system available in the aircraft. One system uses the instantaneous true airspeed which includes wind gusts and the second uses inertially filtered airspeed typical of the signal obtained from a complementary filter (ref. 6) or an inertial platform. The results are shown in figure 6, for the test trajectory discussed previously. The wind conditions were v_W = 15.24 m/sec (50 ft/sec) and $\psi_W = 0^{\circ}$. An identical random process generated with a gust standard deviation σ_W = 3.048 m/sec (10 ft/sec) was used to excite both systems. Figure 6 demonstrates that the system using inertially filtered airspeed is capable of providing a much smoother flight. The difference in final error for these two systems, however, is only 0.79 m (2.6 ft), a negligible value.

CONCLUDING REMARKS

The major contribution of this paper is to demonstrate that the basic time-airspeed-distance relationship for aircraft flight on a trajectory consisting of straight-line and circular-arc segments can be obtained in a closed form which is readily adaptable into a useful algorithm for four-dimensional (4-D) aircraft control. The flight profile can contain both constant and accelerating velocity segments and wind shear conditions. The resultant algorithm can be solved in an iterative fashion by using a computer.

The results demonstrate that the algorithm solution tested in this paper converges rapidly, usually within three to four iterations, and accurately, within 15.24 m (50 ft) for 24.38 m/sec (80 ft/sec) wind velocity, for the test trajectory used. Similar convergence rates have been obtained for other trajectory conditions. When the algorithm is incorporated into an autothrottle system to execute the commanded velocity profile, the effects of wind gusts and control system errors cause only small additional final position errors. Hence, 4-D control using the closed form time-distance-airspeed relationships developed in this paper is shown to be feasible.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., January 7, 1975.

REFERENCES

1. FAA/Industry RNAV Task Force: Application of Area Navigation in the National Airspace System. FAA, Feb. 1973.
2. Erzberger, Heinz; and Lee, Homer Q.: Terminal-Area Guidance Algorithms for Automated Air-Traffic Control. NASA TN D-6773, 1972.
3. Erzberger, Heinz; and Pecsvaradi, Thomas: 4-D Guidance System Design With Application to STOL Air Traffic Control. Paper presented at Joint Automatic Control Conference, Aug. 1972.
4. Foudriat, Edwin C.: Aircraft 4-D Constant Velocity Control System. J. Aircraft, vol. 11, no. 6, June 1974, pp. 326-333.
5. Farrington, F. D.; and Goodson, R. E.: Simulated Flight Tests of a Digitally Autopiloted STOL-Craft on a Curved Approach With Scanning Microwave Guidance. Fourteenth Joint Automatic Control Conference of the American Automatic Control Council, IEEE, c.1973, pp. 437-448.
6. Popik, M. J.; and Gaylor, R.: Development of an Automatic Throttle System for Automatic Landing Application. Paper presented at FAA International Aviation Research and Development Symposium (Atlantic City, N.J.), Sept. 1963.

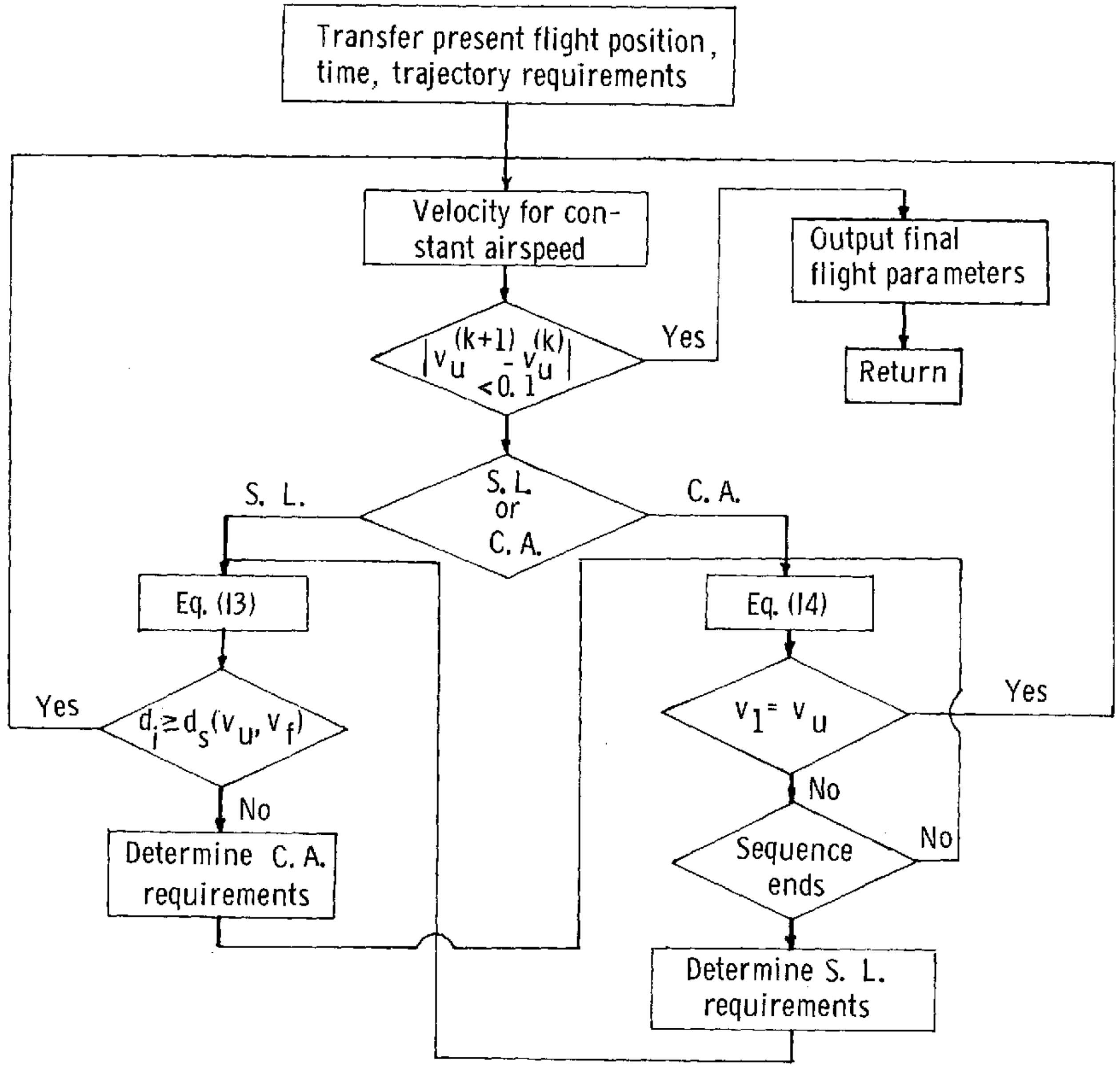

Figure 1. - Subroutine for flight parameter determination. (S.L. denotes straight line; C.A. denotes circular arc.)

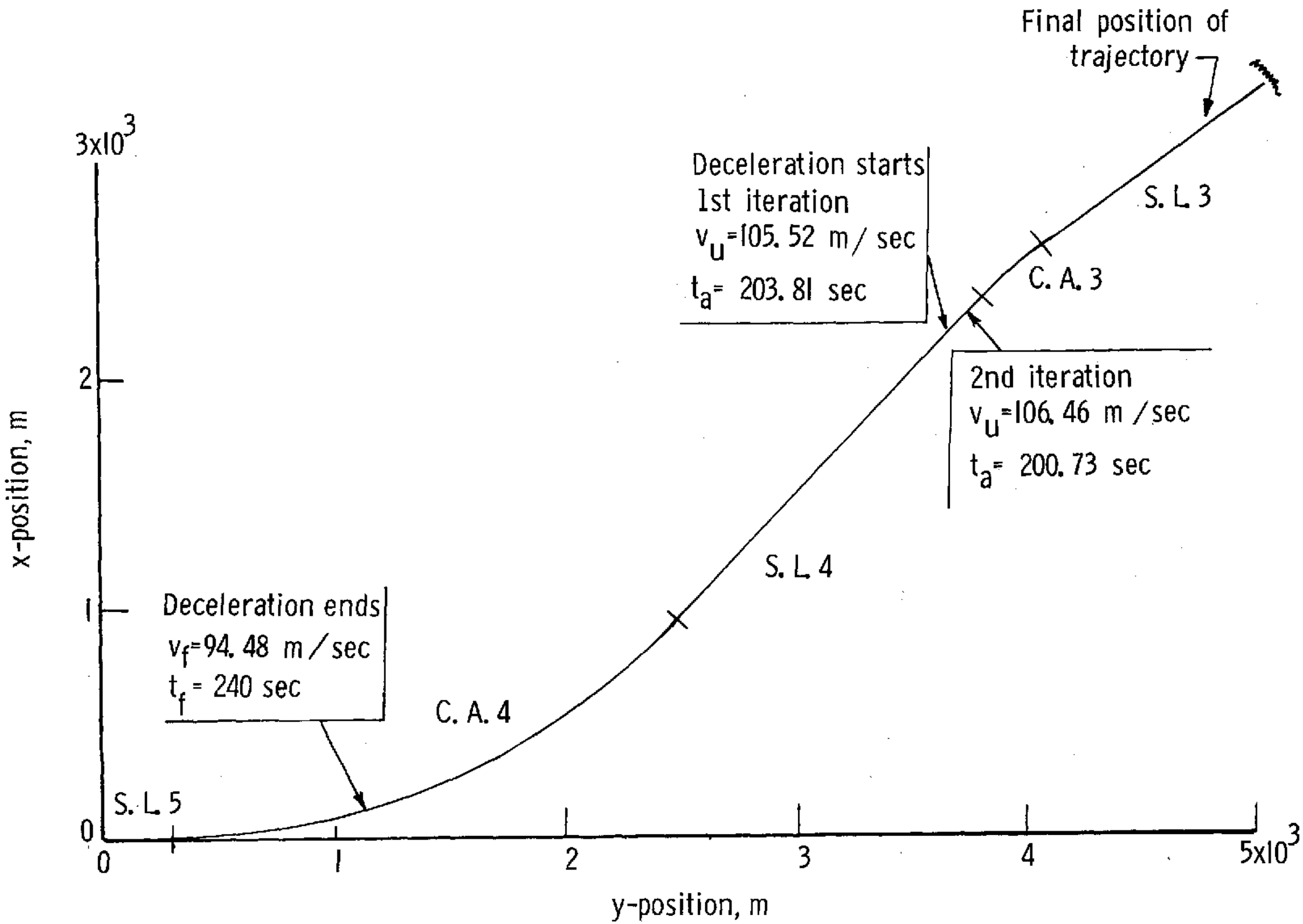

Figure 2.- Convergence of algorithm for example trajectory. (S.L. denotes straight line; C.A. denotes circular arc.)

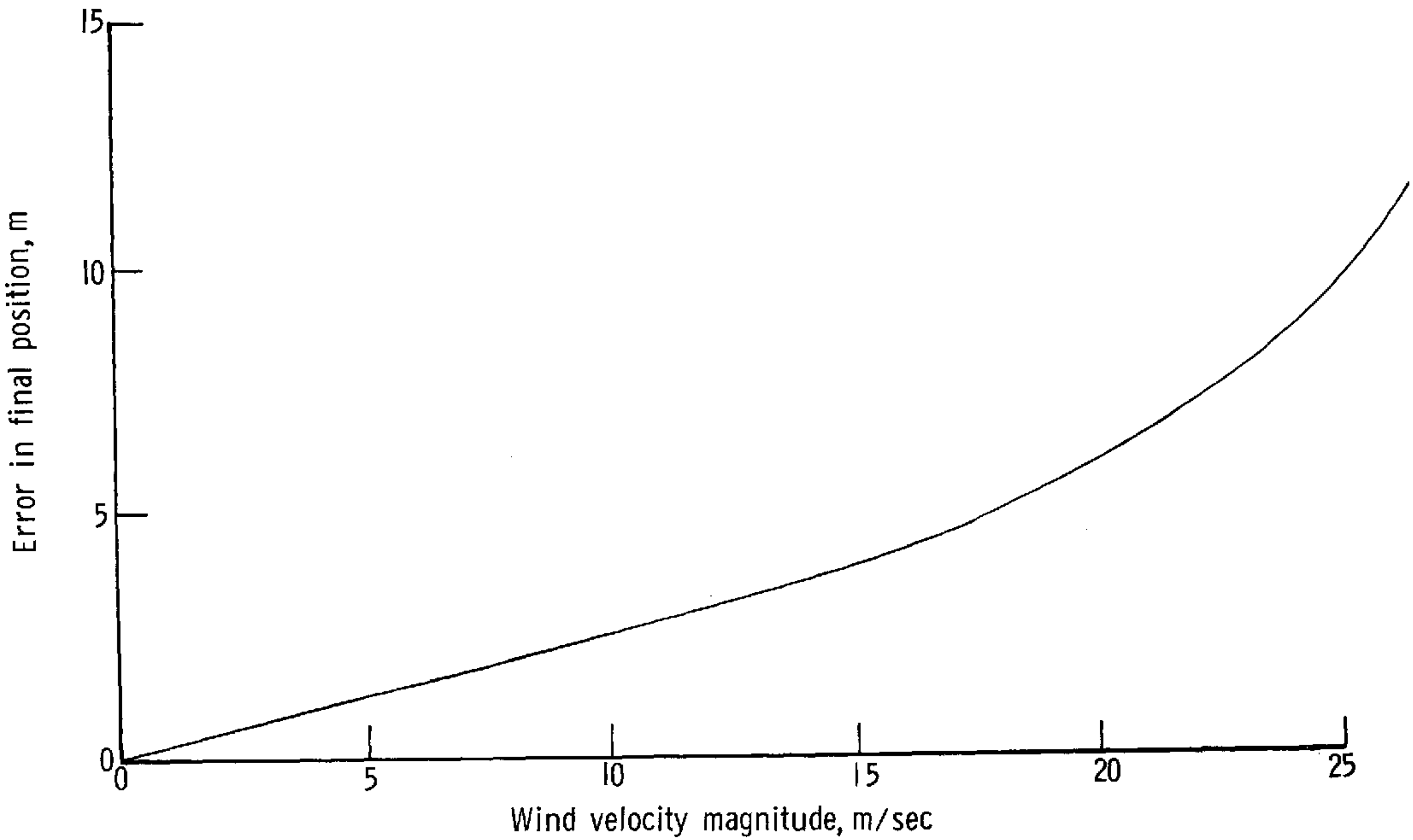

Figure 3.- Errors in time-distance-airspeed algorithm as a function of windspeed. $\psi_w = 270^{\circ}$.

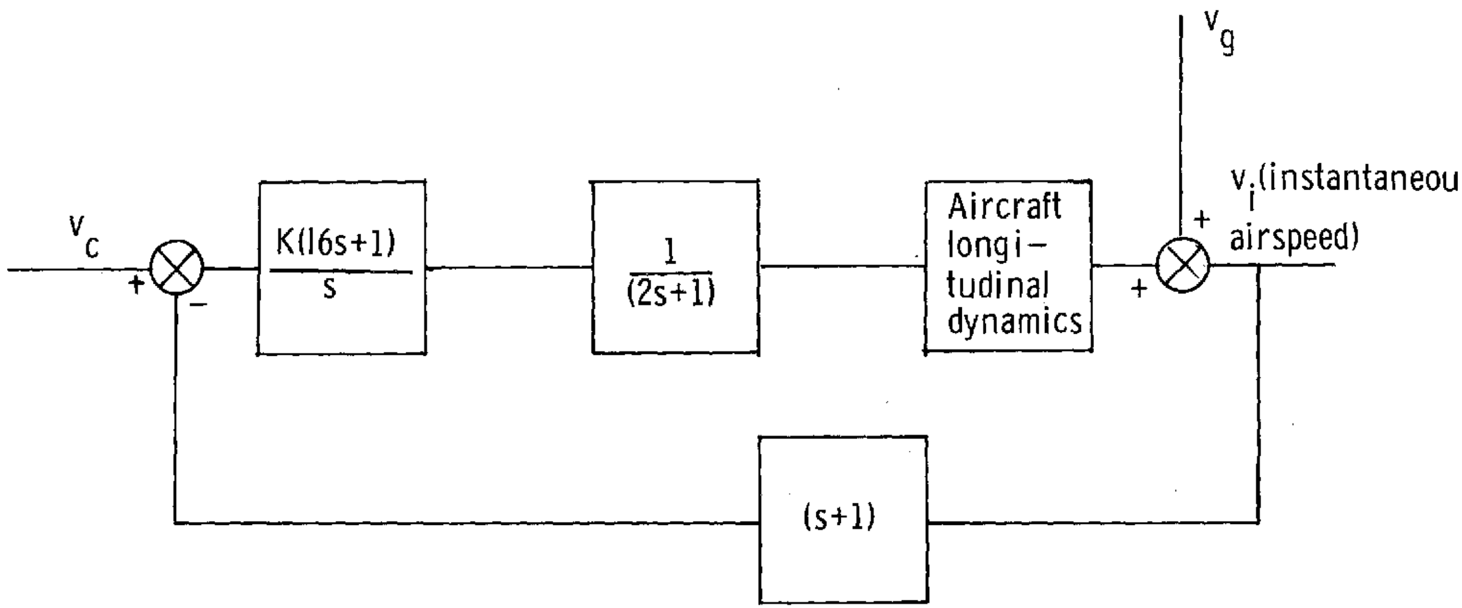

Figure 4. - Block diagram of autothrottle system.

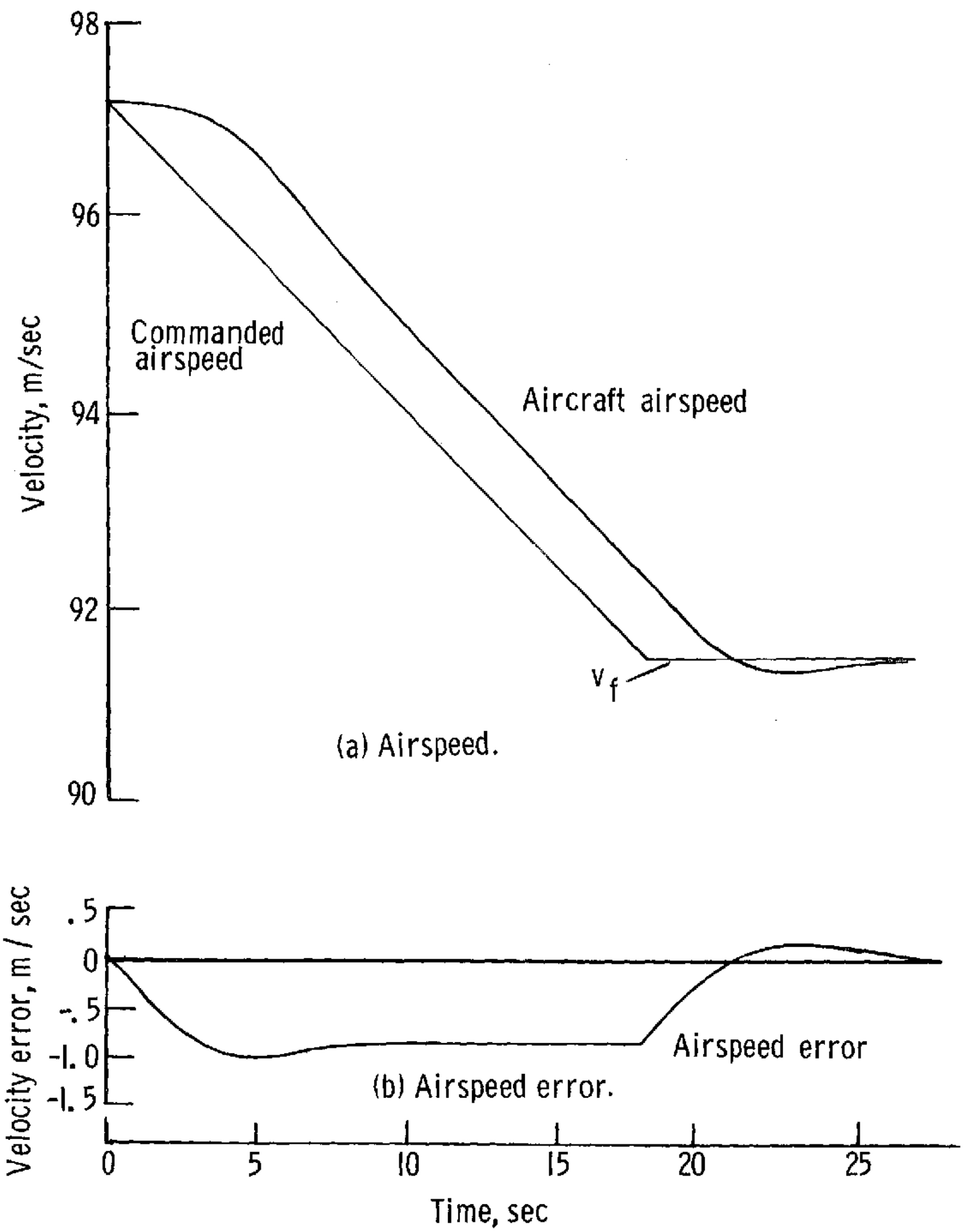

Figure 5.- Airspeed control for a constant acceleration command. Servo gain K = 400.

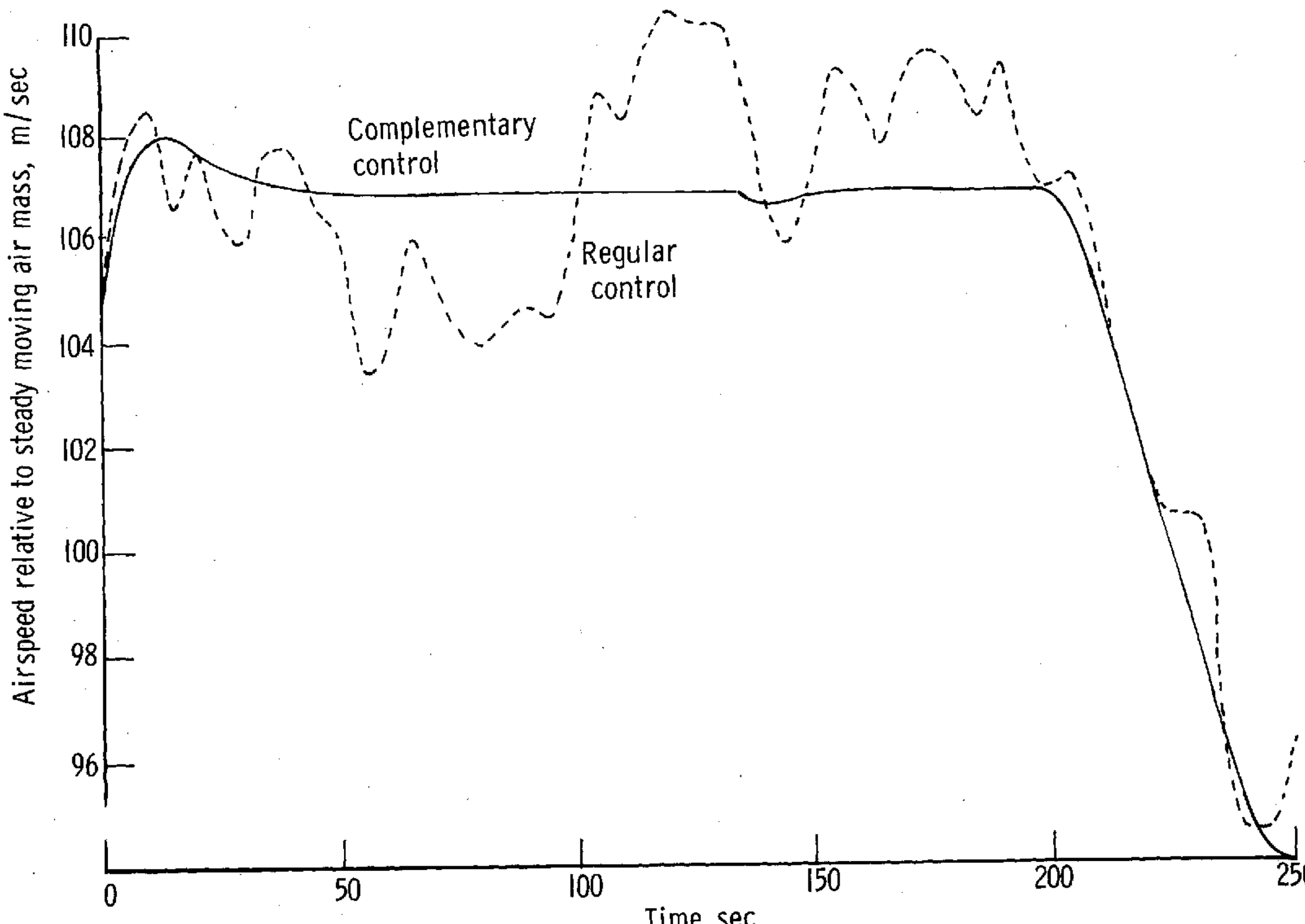

Figure 6.- Comparison between complementary and regular airspeed control. $v_W = 15.24$ m/sec; $\psi_W = 0^{o}$; $\sigma_W = 3.048$ m/sec.

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