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VECTOR MAGNETOMETER AS AN ATTITUDE DETERMINING INSTRUMENT

Research Report

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Preface

The following report is based on work performed as a part of NASA Grant 05-017-031 by the University of Santa Clara, Department of Electrical Engineering and Computer Science.

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INTRODUCTION

An essential requirement of an aircraft attitude control system is that deviation of the body axes relative to a reference axes frame must be sensed. In addition, to overcome the ever-present possibility of errors or failure of the sensors, various configurations of redundant sensors are usually employed to assist in detection and correction of errors. To this end, there has been a continuing effort to improve existing sensors, to develop new sensor configurations and to develop new sensor devices.

This paper discusses the role of a vector magnetometer[1] as a new instrument for aircraft attitude determination. Although magnetometers have played a role in the attitude measurement of missiles and satellites [1], there is an apparent lack of application in aircraft systems. By providing independent measures of attitude, the solid state vector magnetometer sensor system can not only assist in improving accuracy and reliability of existing systems, but can also reduce component count with obvious benefits in weight and cost. Additionally, since a large number of aircraft heading reference systems depend on measurement of earth's magnetic field, it can be shown that by substituting a 3-axis magnetometer for the remote sensing unit, both heading and attitude measurement functions can be derived using common elements thereby further reducing the component count.

To investigate the feasibility of the above system, the paper will proceed by developing a technique to determine attitude given magnetic field components. Sample calculations are then made using earth's magnetic field data acquired during actual flight conditions. Results of these calculations are

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[1]Aviation use to date has been essentially scalar magnetometry.

then compared graphically with measured attitude data acquired simultaneously with the magnetic data. The role and possible implementation of various reference angles is then discussed along with other pertinent considerations. Finally, it is concluded that earth's magnetic field as measured by modern vector magnetometers can play a significant role in attitude control systems.

ATTITUDE DETERMINATION

A. APPROACH

Coordinate systems are usually defined by orthogonal right-handed sets of three unit vectors. An example of such a set is illustrated in figure 1 where the orientation of the body fixed frame used in this paper is delineated.

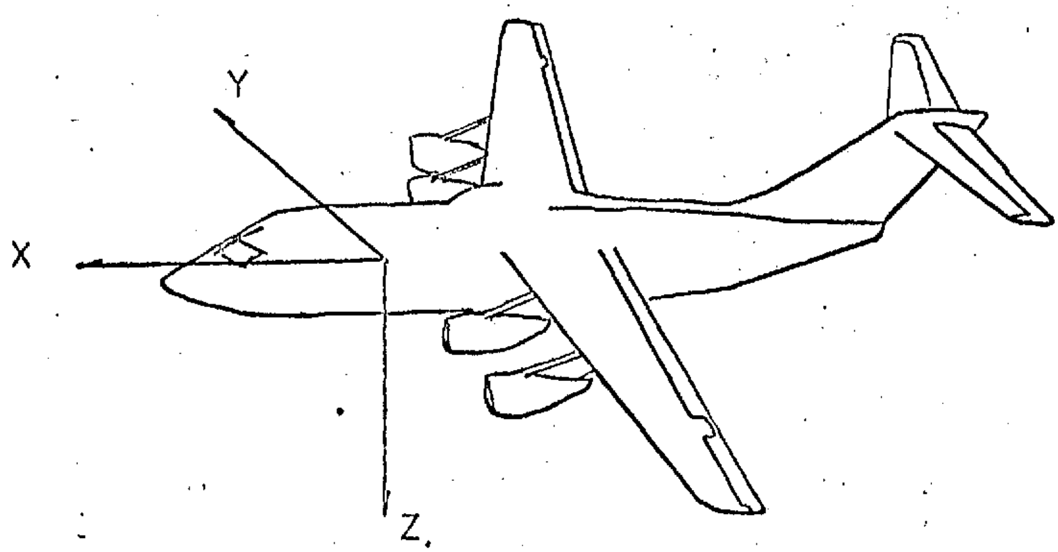

FIGURE 1

Angular rotations are conventionally defined as rotations in the plane normal to a unit vector with the positive sense of rotation defined by the right-hand rule [2].

To derive relationships of attitude variations as a function of magnetic vector component variation we can proceed by considering matrix representations of an orthogonal transformation. If Hx, Hy and Hz are the magnetic components measured at a desired airframe attitude and Hx', Hy' and Hz' are the components measured after any rotation of the body, vector $H' = [Hx' \; Hy' \; Hz']^T$ can be related to vector $H = [Hx \; Hy \; Hz]^T$ by an orthogonal linear transformation $H' = AH$. Here A must satisfy the orthogonality condition $A A^T = I$, where A^T is the transpose of A additionally, the determinant of A must be unity [3], [4].

Rotations about the z axis in figure 1 result in yaw deviations (ψ) and result in new components (H') as shown below.

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{1}$$

Similarly independent rotations about the y axis and x axis result in pitch (θ) and roll (ϕ) dependent variations in the measured H components and are shown below.

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} Hz' \\ Hy' \\ Hz' \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{3}$$

The effect of a combined rotation can be expressed by using the product of the transformation matrices. In addition, if the rotations are small, the total rotation experienced by applying sequential rotations is independent of the order in which the rotations are performed [3], [4].

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} \tag{4a}$$

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\phi + \sin\theta\cos\psi\sin\phi & \sin\phi\sin\psi - \sin\theta\cos\psi\cos\phi \\ -\sin\psi\cos\theta & \cos\psi\cos\phi - \sin\phi\sin\psi\sin\theta & \cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ \sin\theta & -\cos\theta\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} Hz \\ Hy \\ Hz \end{bmatrix} \tag{4b}$$

Assume that the angular variations θ, ψ and ϕ are small enough so that the small angle approximations

$$\sin\theta \approx \theta$$

$$\sin\psi \approx \psi$$

$$\sin\phi \approx \phi \quad \text{and}$$

$$\cos\theta \approx \cos\psi \approx \cos\phi \approx 1$$

can be made. Then, if the products of small angles (in radians) can be assumed to be much smaller than the angles alone, the expression reduces to

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} 1 & \psi & \theta \\ -\psi & 1 & \phi \\ \theta & \phi & 1 \end{bmatrix} \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} . \tag{5}$$

Further modifications in the form of the matrices result in

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} = \begin{bmatrix} -Hz & Hy & 0 \\ 0 & -Hz & Hz \\ Hx & 0 & -Hy \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} + \begin{bmatrix} Hx \\ Hy \\ Hz \end{bmatrix} . \qquad (6)$$

By subtracting, we arrive at an expression for the difference in H components as functions of angular deviation.

$$\begin{bmatrix} Hx' \\ Hy' \\ Hz' \end{bmatrix} - \begin{bmatrix} Hz \\ Hy \\ Hz \end{bmatrix} = \begin{bmatrix} \Delta Hx \\ \Delta Hy \\ \Delta Hz \end{bmatrix} = \begin{bmatrix} -Hz & Hy & 0 \\ 0 & -Hx & Hz \\ Hx & 0 & -Hy \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} \qquad (7)$$

It is significant to note at this point that the transformation matrix is singular implying that solutions for θ, ψ and ϕ are not independently available.

ATTITUDE DETERMINATION EMPLOYING MAGNETIC FIELD COMPONENTS

A given orthogonal set of three unit vectors can be displaced in Euclidean space by rotating the system through any angle δ about a directed rotation axis. It is also customary to represent this rotation vectorially as a directed line-segment whose length is proportional to the rotation angle. This rotation is analagous to the rotation experienced by the body fixed frame of figure 1 as the aircraft experiences combined pitch, yaw and roll variation. During flight the body fixed set rotates about this rotation axis assuming new (possibly erroneous) attitudes in space. The task of the attitude sensing system is to provide measures of compounded pitch, yaw and roll that would result in this same attitude assuming that the rotations occured sequentially about the x, y and z axes rather than the actual rotation axis.

It was shown in the previous section that a compounded rotation of an orthogonal set can be described by a product of respective transformation matrices, Additionally it was noted that for small angular rotations the order of multiplication is unimportant. Using the relationships of (7) expressions for the angular deviations in terms of measured magnetic vector components can be derived.

$$\Delta Hx = -Hz\ \theta + Hy\ \psi \text{ yields} \tag{8a}$$

$$\theta = (Hy\ \psi - \Delta Hx)/Hz \text{ and} \tag{8b}$$

$$\psi = (\Delta Hx + Hz\ \theta)/Hy \quad . \tag{8c}$$

Similarly,

$$\Delta Hy = -Hx\ \psi + Hz\ \phi \tag{9a}$$

$$\psi = (Hz\ \phi - \Delta Hy)/Hx \tag{9b}$$

$$\phi = (\Delta Hy + Hx\ \psi)/Hz \quad ; \tag{9c}$$

$$\Delta Hz = Hx\ \psi - Hy\ \phi \tag{10a}$$

$$\theta = (\Delta Hz + Hy\ \phi)/Hx \tag{10b}$$

$$\phi = (Hx\ \theta - \Delta Hz)/Hy \quad . \tag{10c}$$

Assuming that Hx, Hy and Hz are nominal vector components as measured in a reference attitude and that Hx', Hy' and Hz' are new field components at the new attitude, then $\Delta Hx = Hx' - Hx$, $\Delta Hy = Hy' - Hy$, $\Delta Hz = Hz' - Hz$ are expressions of the incremental changes in field components. Additionally before using (8), (9) or (10) to solve for attitude variations (pitch, yaw or roll) one additional angle from an auxilliary sensor[2] must be supplied. Using one additional angle of rotation (about any one axis) the remaining two rota-

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[2]It was noted following (7) above that a unique solution for attitude variation is not possible using magnetic field data alone.

tions can then be calculated.

To illustrate this point, flight data acquired during the flight of a NASA flown Convair 990 instrumented with a three axis magnetometer and a Litton inertial navigation system was used to calculate roll, pitch and yaw. Attitude variation about each of the three axes was calculated using measured magnetic field components supported by one angle from the inertial system. The results of these calculations are plotted in figures 2, 3, and 4.

It is significant to note that the rotations shown occurred simultaneously (i.e., time base is the same for all three figures). The flight was at an altitude of approximately 5,000 feet at an airspeed of approximately 250 nautical miles per hour.

Although the data used to plot the attitudes shown in figures 2, 3, and 4 was not acquired specifically for this purpose, the correlations in measured and calculated attitude clearly show that, within the limits of instrument accuracy, signals proportional to attitude variation can be derived using flight data.

A POSSIBLE SYSTEM CONFIGURATION

Since the intent of this paper is to introduce the notion that magnetometer technology has advanced to the point where 3-axis magnetometers can be incorporated in aircraft attitude sensing systems on a cost effective basis, the system discussion will be limited in scope to describing a possible combined heading and attitude measurement method.

Heading references fall into three classes; (1) those that depend on earth's magnetic field, (2) those that depend on the use of a low-drift gyroscope to retain a preset azimuth, and (3) those (gyrocompasses) that depend on sensing earth's rotation [5]. By far the greatest number of aircraft heading systems depend on earth's magnetic field, although many of these in-

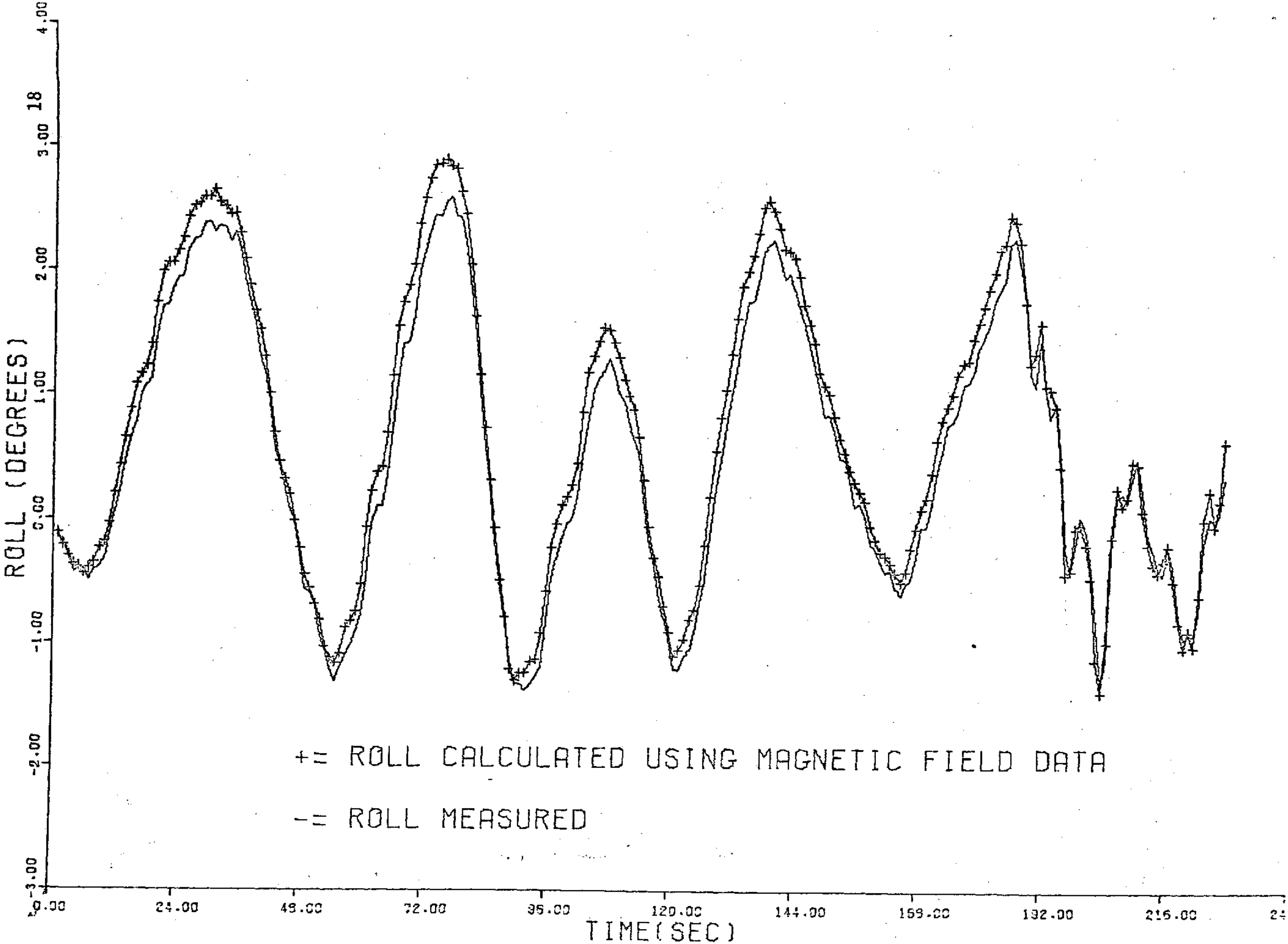

FIGURE 2 ROLL AXIS

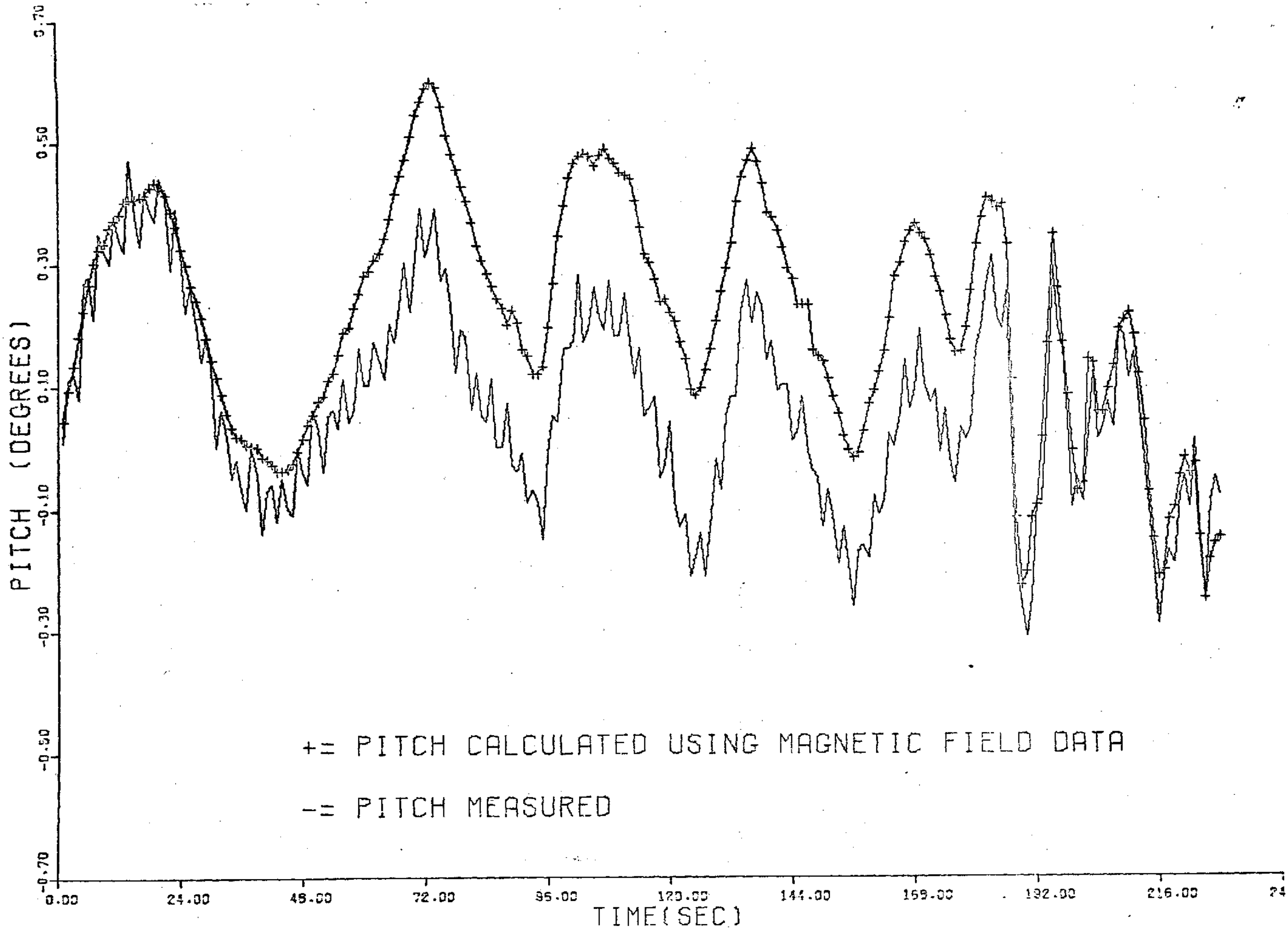

FIGURE 3 PITCH AXIS

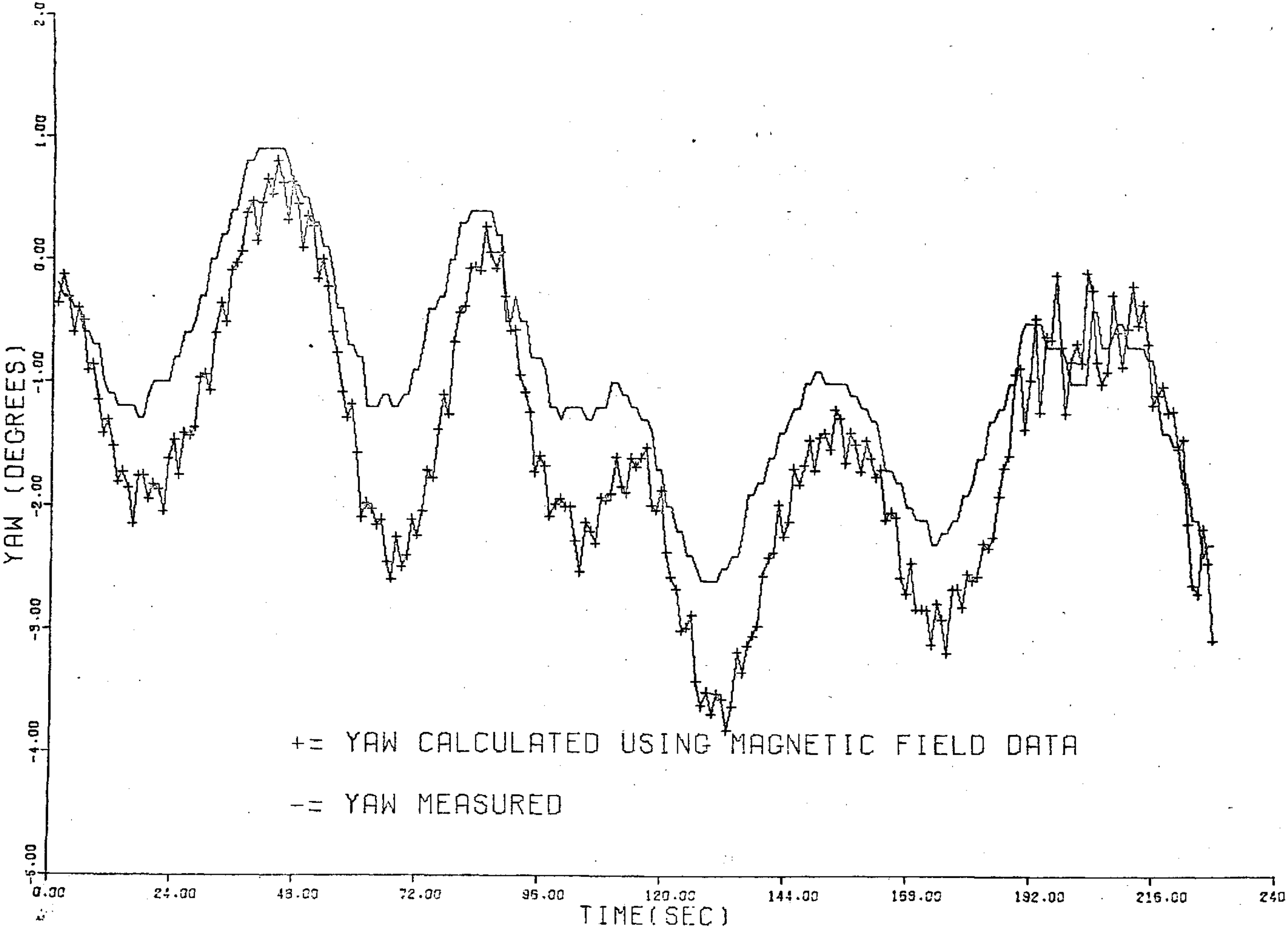

FIGURE 4 YAW AXIS

clude gyroscopes to improve the performance characteristics.

A popular system combination (with no gyro) is to combine a pendulous remote magnetic sensor and a synchro receiver in a null seeking circuit. The philosophy being to attempt to measure only the horizontal component of earth's magnetic field and to swing the receiver into alignment with it. Under acceleration, departures of the sensor unit from the horizontal result in angular heading errors [5] ε

$$\varepsilon = \frac{aH}{g} \tan \gamma \sin \theta \quad .$$

Where aH is the horizontal acceleration

g is the acceleration due to gravity

θ is the angle between the acceleration vector and magnetic north

γ is the magnetic field dip angle; arctan (verticle field/horizontal field) .

Accuracy of this system can be improved by incorporating a strapped-down solid state magnetic sensing unit (free of acceleration errors) that measures and displays the angle of earth's horizontal magnetic component relative to the aircraft. This system can be implemented as follows:

(a) Determine the direction of the magnetic vector (F) relative to the sensors (and the airframe), by measuring x, y and z components (figures 1 and 5). The direction cosines $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the cosines of the angles α, β, γ between the magnetic vector and the positive x, y and z axes. Additionally,

$$\cos\alpha = \frac{x^2}{(x^2 + y^2 + z^2)^{1/2}}$$

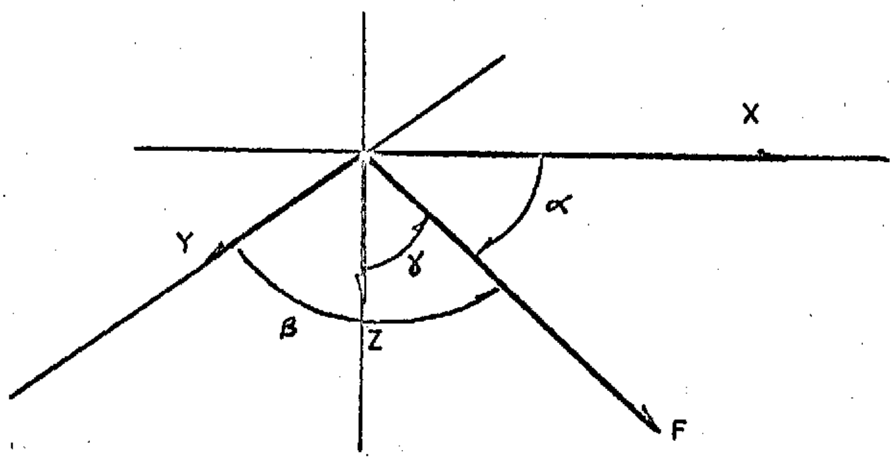

FIGURE 5

$$\cos \beta = \frac{y^2}{(x^2 + y^2 + z^2)^{1/2}}$$

$$\cos \gamma = \frac{z^2}{(x^2 + y^2 + z^2)^{1/2}}$$

(b) Using either a vertical reference[3] or knowledge of aircraft attitude, we can effectively rotate the body axes such that the x-y plane is horizontal (yielding corrected values for measured x and y magnetic components).

(c) Simple application of direction cosines will yield the direction of magnetic north in the aircraft's x-y plane.

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[3]Not necessarily derived inertially [11].

Although the above discussion implies that heading can be determined by using a strapped-down magnetometer, there remains the problem of attitude determination. Another widely used system for obtaining a heading reference is to combine the relatively excellent short term stability of a directional gyroscope with the long term stability of magnetic field measurements. By slaving the directional gyroscope to the magnetic field [5,section 10.4.7], gyroscopes with relatively large free drift error can be used to provide an excellent heading reference.

Replacement of the pendulous remote sensing unit of this type of system with a strapped-down vector magnetometer would result in both heading and attitude information on a continuous basis. This combination would operate as follows:

(a) The system is initialized by determining a reference attitude (perhaps by using a primary inertial attitude system).

(b) The angular position of the horizontal magnetic field component is computed as above and used to slave the directional gyroscope.

(c) The directional gyroscope, with relatively good short term stability (devices with free drift of less than 0.5 degrees per hour have been designed), is used to determine yaw (ψ) errors.

(d) For small angle deviations, equations (8), (9) and (10) can be employed to recalculate aircraft attitude. The process loops back to step (b) above closing the loop on a combined attitude and heading reference system.

The sampling frequency required to maintain an acceptable level of error is of course determined by the aircraft performance expected (angular rates) and by the gyro error (drift rate plus errors due to additional sources such as gyroscope tilt from vertical). The overall system is such that heading can be determined as before with errors due to sensor departures from horizontal substituted for long term accumulation of attitude uncertainty (this can be corrected by looping to step (a) at a frequency dependent on error rates). Additionally one gains measurements of attitude with minimal computation and replacement of a mechanical remote sensing unit with a solid state strapped-down magnetometer sensor.

OTHER CONSIDERATIONS

The characteristics of earth's magnetic field and its variations have long been established [6]-[10]. Since the field is to be used as a reference in the attitude measurement scheme, there is a need here to discuss its adverse characteristics. Although the field does experience variation, most of the variation is either in amplitude (ionospheric contributions) or has time constants that make the variation negligible (secular variation).

In traversing local anomolies, there will however be deflections in the ambient field due to the additive effect of local dipoles or monopoles. The effect of local terrain caused anomolies can be visualized by picturing the main field vector oriented in space with a second modulating vector rotating at its tip. Maximum angular error would occur when this modulating vector has maximum magnitude and is positioned at right angles to the main vector.

To illustrate the effect of local anomolies one can calculate the level of anomoly required to cause an error. Since earth's main field is typically in the order of 0.50 gauss it is readily apparent that a local anomoly of approximately 0.01 gauss at right angles to the local field is required to cause an error of one degree. Furthermore, the local anomoly would have to be aligned with one of the aircraft body axes to result in one degree of attitude error in any one axis. Fortunately, anomolies with components of this magnitude positioned at right angles to the main field are extremely rare. In addition the anomolies are localized over ore bodies or other geophysical irregularities, have magnitudes that diminish as the cube of altitude and tend to average to zero over relatively short distances. In summary, the probability of encountering an anomoly that would cause as much as one degree error is relatively small. The error, if introduced, will be short lived and unlike drift error will average to zero.

Fundamental to a magnetic field referenced system is the ability to measure orthogonal components of the field vector. Precision and accuracy of measurement of the components is of course specified by the desired control specifications.

Since the earth's magnetic field varies in magnitude on a global basis between 0.3 gauss and 0.6 gauss (30,000 gamma to 60,000 gamma) it is apparent that full scale measurements of 0.6 gauss can be expected. Sensors mounted at right angles to the field will monitor no measureable field and thus define the lower limit of measurement to be zero. For the continental U.S.A. the declination varies between 60 and 80 degrees resulting in a range in horizontal component of 0.15 to 0.25 gauss with vertical component in the range of 0.4 to 0.55 gauss. Heading variations (yaw) result in changes of the horizontally sensed field components and would specify the maximum precision required. In addition, flight at $45^{\circ} \pm (n \times 90^{\circ})$ (where n is any whole number) with respect to magnetic north results in minimum sensitivity of the x and y axes measurements. In this case sensor inputs would range between 0.106 and 0.177 gauss with minimum field at the north. Assuming the above ambient measurements, variations in component magnitude of approximately 0.0180 to 0.0305 gauss per degree for small angle variations can be expected.

A brief survey of commercial magnetometer manufacturers reveals that triaxial magnetometers that measure from zero to 0.6 gauss with linearities of 0.5%, noise less than ±1 milligauss and sensitivities of at least 2.5 volts per 600 milligauss are currently available. In addition, these devices have a bandwidth of d.c. to at least 500 Hz and are rated to have less than 1 degree error in orthogonality.

From a precision standpoint, it is apparent that variations in yaw for this worst case situation can be sensed to better than 0.1 degrees with currently available magnetometer technology. The sensor technology required to implement an attitude sensing system of reasonable specifications is available.

Although the calculations above indicated that for small angular variations attitude can be calculated using measured magnetic data, there is a need to consider the effects of larger finite rotations. In this case the small angle assumptions would not be valid and an Euler transformation would have to be made. Measurement of three axes of field components could be used to develop the direction cosines required to determine the orientation of the axis of rotation, the angular rotation about it and the three angular rotations of pitch, roll and yaw.

For the special case where the axis of rotation aligns with the magnetic vector, there would of course be no measured component changes[4]. By measuring the attitude of a second vector (not in alignment with the magnetic vector), we could resolve the ambiguous situation cited above and provide additional redundancy.

The optimum auxilliary vector would be one that could be sensed without using inertial devices. Earth's electric field is considered in this paper.

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[4]An example of this would be yaw rotation while flying straight and level over the magnetic poles or roll rotation while flying towards a pole at the magnetic equator.

The main reason for considering this field as a means of providing an auxilliary angular reference is that the resultant system has potential of being completely solid state. The electric field vector can be used to determine attitude variation in a manner analagous to the magnetic vector system. Inherent limitations of each single vector system can be obviated if the vectors are not coincident.

Although Hill [11] reported success in controlling pitch and roll using the electrostatic field alone, comments by Markson [12] indicate that the electrostatic field is not always a reliable vertical reference. Employment of the electrostatic field for this attitude measurement system are limited to augmenting the magnetic field measurements by eliminating ambiguity of motion around the magnetic vector. The requirement of vertical electrostatic field is thus removed and replaced by a requirement that the field direction is relatively stable.

By using two independently derived vectors we have sufficient data to obviate the ambiguity cited above and have the potential of providing redundancy as well.

CONCLUSION

This paper has identified a novel method of measuring aircraft attitude using relatively inexpensive, well developed instrumentation. It has recognized that magnetic field sensing systems have been used to some extent in attitude sensing and control of space vehicles; it has also suggested however, that with appropriate support, magnetometers can find increased application in aircraft attitude measurement systems.

The above claim is corroborated by actual flight test data. Magnetometers have evolved to a point where three axis measurements of earth's magnetic field can be made with sufficient precision and accuracy enabling measurement of small angle attitude variations.

The paper has also discussed a possible system configuration combining heading determination and attitude measurement functions. By replacing the conventional remote sensing unit with a 3-axis magnetometer, it has been suggested that both functions can be obtained with the hardware required previously for heading measurement alone.

As with any system, there are limitations imposed. The main limitation for a vector magnetometer system seems to be the inability to sense rotations around the magnetic vector itself. This problem is not unlike the ambiguity experienced by magnetic heading systems at high latitudes. By judiciously incorporating auxilliary instruments, not only can the ambiguities be removed but a degree of redundancy can be added while still maintaining a cost and weight advantage over comparable systems.

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