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STRUCTURE OF SATURN'S RINGS: OPTICAL AND DYNAMICAL CONSIDERATIONS

What I would like to do is to consider some properties of Saturn's rings from two rather distinct aspects. First, I would like to talk a little about some of the photometry — specifically, photometry in the visual region — leading to photometric phase curves of Saturn's rings. Then I should like to point out to you some of the past interpretations that people have brought forth from these curves to indicate what information is available and try to be somewhat critical.

Second, I want to speak on what information can be provided by certain dynamical arguments. It has been a long-standing conviction, I think, that the perturbations of the inner satellites on the ring particles are to a large extent responsible for many of the features of the radial profile of the ring. I would like, therefore, to rediscuss with you briefly some of the dynamical arguments and indicate what seems to me one area of conflict between dynamical and photometric models of the rings and then, in the last few minutes, try to resolve aspects of this discrepancy by presenting to you a new observation which may indirectly aid in the resolution of this discrepancy.

What I am mainly going to be concerned with is ring B, the bright interior ring, and the outer one, ring A. As you know, there are other rings closer to Saturn that introduce quite exciting new problems, but I am going to ignore them until discussion later. What I would like to call your attention to is the Cassini's division, a conspicuous gap, which is situated very close to a distance from Saturn corresponding to one-half the period of Mimas. The Cassini's division is some 4000 km across, and it presents a real challenge that any comprehensive dynamical ring model must explain.

I want to speak only for the moment about the dependence of ring brightness of angular separation of the Earth and Sun as viewed from Saturn; that is, upon the solar phase angle α. Figure 1 is a photometric scan along a portion of the major axis of the Saturn system taken from a photograph exposed near quadrature when

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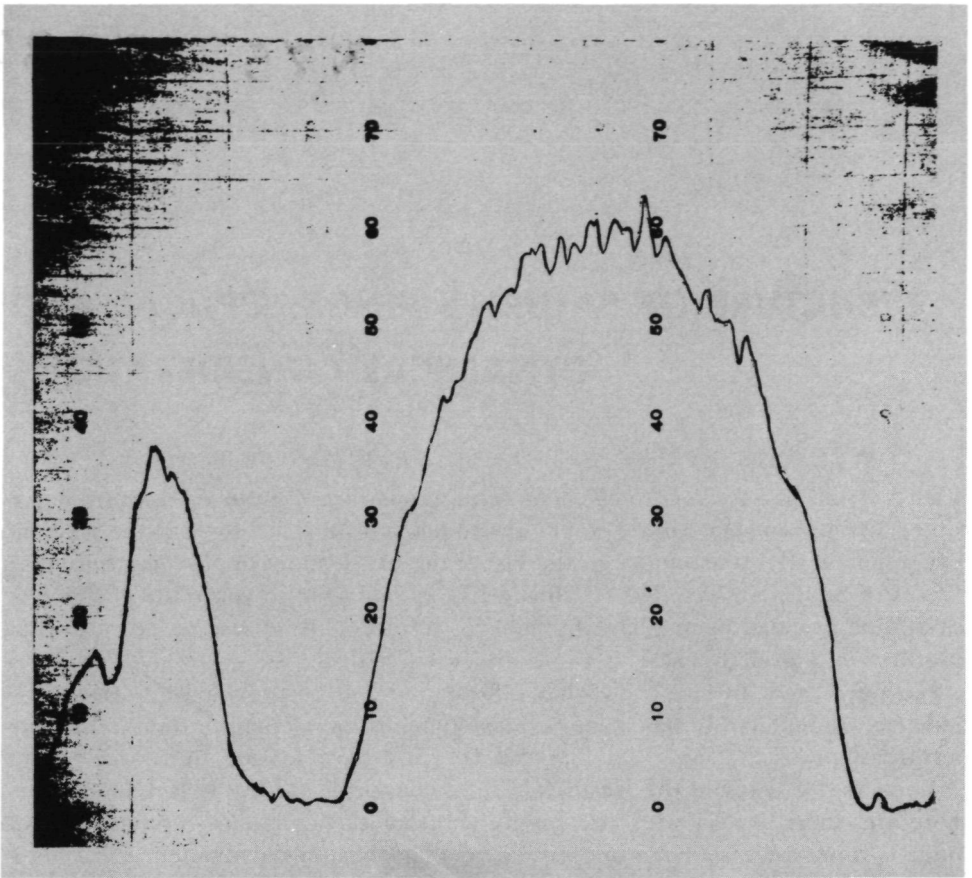
FIGURE 1. — *An unrectified tracing of one ring ansa and of the disk of Saturn obtained from a photograph (in V) taken near quadrature,* $\alpha = 6°$.

$\alpha \cong 6°$. On this unrectified tracing, in which only the disk and one ansa are shown, it is clear that ring B is much fainter than the brightest part of the disk and that ring A is fainter still. At opposition — that is, when the phase angle has fallen to zero so that the Earth and the Sun are in a direct line with Saturn — a similar photographic scan, figure 2, shows that ring B and ring A have both brightened enormously with respect to the disk. This behavior has been known for some time and has been measured reasonably accurately. Measurements, not just at quadrature or opposition but throughout the entire range $0° < \alpha < 6°$, give results similar to those of figure 3, which is a plot of magnitude in the visual (that is, the broad band V of the Johnson-Morgan System) against phase angle α. It is unquestionable that the ring does change its brightness with time, depending on solar phase angle, in this way.

The curve of figure 3 has two distinct portions. From about 1° or 1.5° to 6° or somewhat more, it is quite linear. But shortwards of about 1.5°, it becomes quite steep and nonlinear. This latter portion is the one of particular interest. The right-

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hand part ($\sim 1.5° < \alpha < 6°$), usually labeled in the literature as the "phase varia-
tion," can be accounted for by the scattering of individual particles. One knows,
for instance, that many objects in the solar system — the Moon at large phase
angles, asteroids, and some satellites — show a behavior similar to this one. A
steep slope of the sort shown in figure 3 for the particles of Saturn's ring, that is,
of about 0.030 or 0.035 magnitude per degree, is characteristic of an object with
rough and pitted surfaces.

The nonlinear portion or "pip" in the region $0° < \alpha \lesssim 1.5°$ is referred to in
the literature as the opposition effect and is quite marked in the case of Saturn's
rings. To my knowledge, it was the first object in the solar system to show such
behavior. Most of the planets tend to show linear or nearly linear phase curves,
all the way to $\alpha = 0°$. Now it is well known that both the Moon and most of the
asteroids do show an opposition effect. But Saturn's ring remains, I think, some-
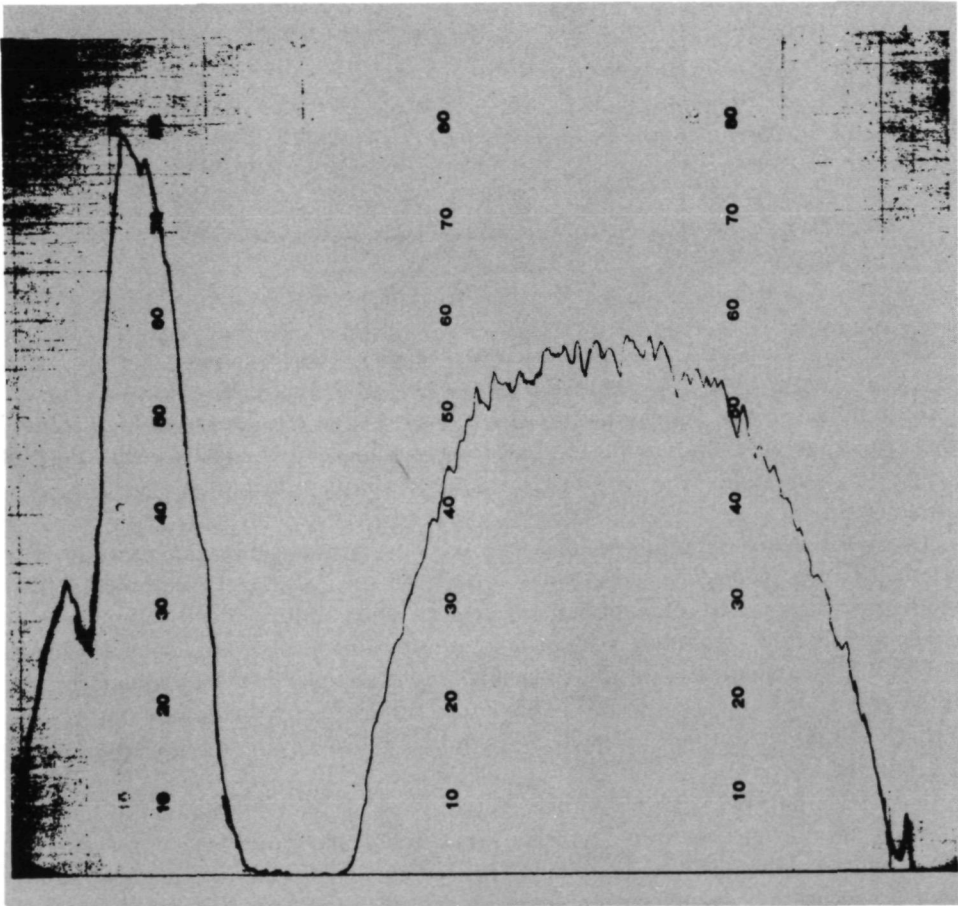
FIGURE 2. — *Similar to figure 1 except that photograph was taken near opposition, $\alpha = 0.1°$. Note the
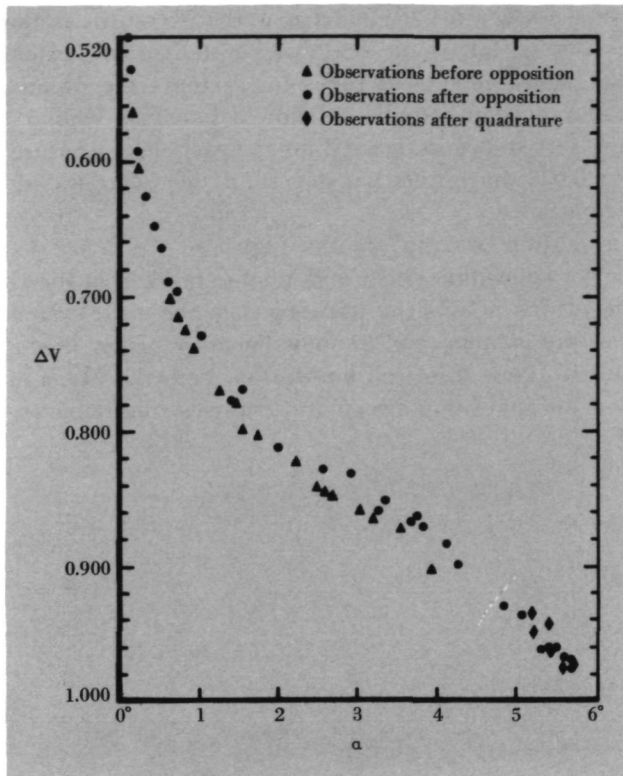brightness increase of rings A and B relative to the disk of Saturn.*

FIGURE 3. — *Visual (V) phase curve of Saturn's B ring in a plot of magnitude vs solar phase angle.*

what unusual, in that it exhibits this effect even though it possesses a high albedo. One would expect that multiple scattering would tend to destroy this type of brightness surge, but the ring unquestionably shows a brightness surge that is pronounced.

The interpretation of this nonlinear brightness surge, which amounts in V to about 0.2 magnitude, has led to what I shall call the "classical ring model." This model provides a ready explanation for the opposition effect; indeed, the model was suggested by it. To elaborate very briefly: at opposition, when $\alpha = 0°$, each particle covers its own shadow, and the ring as a whole appears correspondingly bright. However, as α increases toward its maximum value of $\sim 6°$, shadows of foreground particles impinge on those in the background, and the ring consequently undergoes a brightness decrease.

It is clear that the brightness changes will depend upon the amount of material in the ring. Various workers, some in great detail, have derived expressions for this brightness dependence upon α, and, I believe, Dr. Irvine (see contribution by Irvine) will present some very nice recent work along this line. In general, one finds that it is possible to fit a theoretical curve to the observed opposition effect and thereby derive one parameter of particular interest, the so-called volume density, D. This parameter is the fraction of the total ring volume occupied by particles,

and one finds in the literature values of D that range between 10^{-2} and 10^{-3}, with perhaps a few that are even a bit smaller. Inasmuch as $D < 10^{-2}$ appears to be well established observationally, the view that the ring is a medium many-particle radii in vertical extent has found wide support. This conception of the ring is what I refer to as the classical ring model. In some sense, the observational work that supports this model also suggests that it may require at least a slight revision. It seems quite well established, for example, that the opposition effect is wavelength-dependent in a rather complex fashion—a fact that might be accounted for by the albedos of the ring particles. Photometry also provides other parameters for the ring system: geometric albedos, Bond albedos (by inference), and optical thicknesses.

The basic point I wish to stress here is that the only observational support for the classical ring model is the opposition effect. It seems, therefore, important to ask whether the opposition brightening could result, at least in part, from any other mechanism. Studies by Oetking (1966) and Hapke (1966) indicate that this may indeed be the case. Their laboratory studies have shown that a wide variety of different substances do exhibit a brightness surge when the phase angle drops below about 1°. This behavior appears to be largely independent of particle size, roughness, and, to some extent, even albedo. One would expect that as the single scattering albedo rises, an opposition effect would diminish, owing to the increasing role of multiple scattering. To some extent laboratory samples did show this anticipated behavior, but even from albedos of 0.7 to 0.8 a brightness surge of ~ 10 percent was measured. Because the laboratory measures indicated that this phenomenon is very common (although ice particles were not examined), one is strongly led to the suggestion that a part of (and maybe the entire) opposition effect presented by the rings can be traced to the particles themselves and therefore is not the result of interparticle shadowing. Should this prove completely true, then the main observational prop of the classical model is removed, and the rings might, therefore, be simply a medium that is (nearly) a particle diameter in vertical thickness.

Let me leave the subject of photometry and survey for you some work that three colleagues—Drs. G. Colombo, M. Lecar, and A. Cook—and I have been doing on the dynamics of the ring system and related topics. There are clearly certain approximations involved in the course of this work that I don't mean to obscure, and please feel free to criticize if you are unhappy. I have to confess that I began with the naive hope that all one had to do was to take a planet, put a large number of massless ring particles around it, introduce the inner satellites with their appropriate masses (they are quite well known but might be increased so that things would happen faster in a sample calculation), and one just turned on a machine and after a while discovered that the satellite perturbations, particularly ones near resonance, had sculptured the ring, so that from an initially featureless uniform ring Saturn's ring was formed. I was set to lease the movie rights so that one could watch a nice uniform ring quickly being sculptured into Saturn's ring. Well, the movie has yet to be made and I am afraid my hopes were terribly naive. The problem is simply that things happen on an enormously slow time scale, not

slow cosmologically, but very slow when it comes to the allowances made by administrators of computing facilities.

I would like to digress for just a moment to introduce some quantitative calculations we have carried out for bodies in and near the present asteroidal belt. These calculations include the planets Mars, Jupiter, and Saturn — with their observed masses — and a swarm of massless bodies, with the objective of seeing whether planetary perturbations alone could produce the presently observed distribution from a uniform belt of asteroids extending from Mars to Saturn. Results (Lecar and Franklin, 1973) show that if gravitational perturbations by the planets are responsible for the details of the asteroidal distribution — particularly in the region between the resonances at $\frac{1}{2}$ and $\frac{2}{3}$ of the period of Jupiter — then they require times much greater than $\sim 10^3$ Jovian periods to be effective. By inference, the inner satellite Mimas, with a mass $\sim 7 \times 10^{-8}$ that of Saturn, would require, to produce ring features, a time much too long to be accessible by direct computational methods. One has therefore to examine this problem indirectly.

Before doing so, however, I would like to call attention to figure 4 which shows, for a primary to secondary mass ratio, μ of 10^{-3}, the region near the 1:2 resonance. Here we have the oscillations in the semimajor axis a of a body near the 1:2 resonance plotted as a function of time measured in periods of the perturber (secondary). There are, it seems to me, two features to be mentioned in the present context. First, it is clear that the oscillations in a are substantially
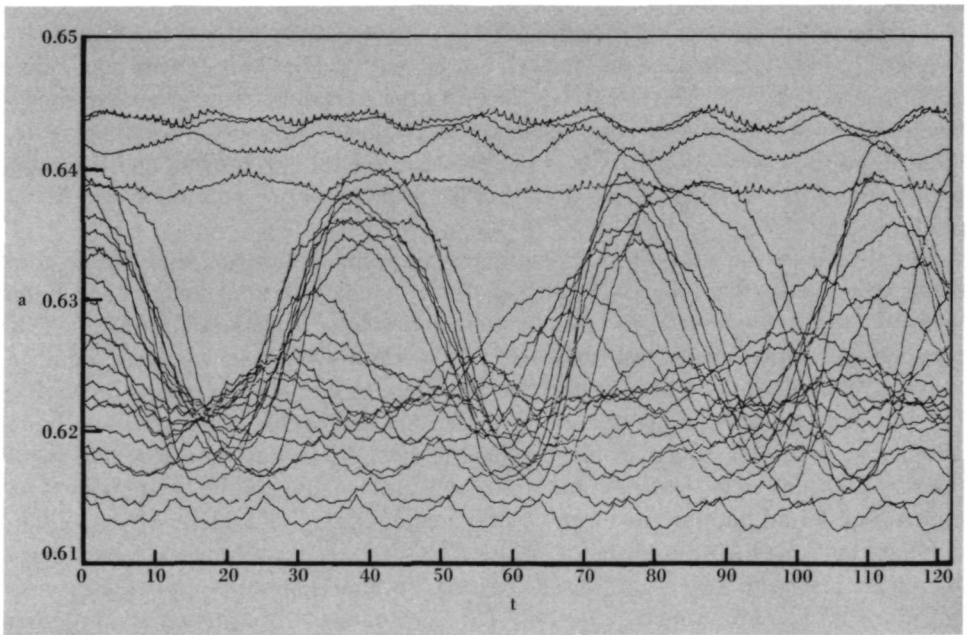

FIGURE 4. — *Semimajor axis* a *vs time* t *measured in periods of Jupiter for bodies at and near the* 1:2 *resonance* (1:2 *being the ratio of the periods of an asteroid and Jupiter*).

greater inside than outside of resonance. This is also true of the similar oscillations in the eccentricity e; both are of the order $\mu^{1/2}$. The second point is the absence of coherence in this type of motion: one is not dealing with a running density wave that follows the perturber around, but rather a longer period oscillation for which, at a point in space and time, two particles may have different velocity vectors.

With these remarks in mind, let us return to the specific dynamical problems presented by the rings. In view of the difficulty of carrying out a complete dynamical calculation, with realistic values for the satellite masses and with the inclusion of collisions, we have tried to begin to understand whether the perturbations of the satellites on ring particles could be sufficient to have a marked effect on the ring—to produce, for example, the Cassini division or to define the outer boundary of ring A—by a simple and approximate argument. Please have patience here, because I do not want to stress the argument itself so much as to apply it gently a little later. What we have done is to suppose that the ring has evolved, owing probably to particle collisions, to a state such that ring particles are now separated from one another by just such an amount that mutual collisions no longer occur. It is then a straightforward matter to calculate, from the formalism of the restricted three-body problem, a set of periodic orbits (in the rotating frame) that fulfills this condition, i.e., that are so spaced that objects moving in such orbits do not collide. Effectively, then, we have calculated the area, in this planar model, of the epicycle executed by a particle, with a given semimajor axis, owing to the perturbations of the inner satellite, Mimas. We then make the assumption of a collisionless ring (at the present time) so that the area of these epicycles must be inversely proportional to the surface density of the particles. We make the further reasonable assumption that the observed brightness is proportional to the surface density, and we are therefore able to plot a profile of the radial structure of the ring under the influence of a perturbing satellite. Near a resonance, for example, the epicyclic areas increase rapidly, leading to a much reduced brightness.

Results are shown in figure 5 for the case in which the perturber, Mimas, with a mass 6.7×10^{-8} that of Saturn, moves in a circular orbit. In this figure, pairs of vertical lines mark the resonances; the right-hand one of any set includes the effect of the oblateness of Saturn. It effectively shifts the position of a resonance slightly outwards. Also marked near the bottom of this figure are measures of the width of the Cassini division, where the upper set of arrows defines the most probable value. The inner boundary of ring B, the outer boundary of ring A, and the Encke "division" are also indicated by vertical arrows. The horizontal scale of this figure plots distance from Saturn, where 1.0 corresponds to the semimajor axis of Mimas' orbit. The vertical scale is the inverse of the epicyclic area; or, in other words, ΔR is the radial excursion executed by a particle in a periodic orbit with a given semimajor axis owing to the perturbations of Mimas. The quantity $R\Delta\theta$ is the corresponding tangential excursion.

There are, it seems, both encouraging and discouraging features to the approach depicted in figure 5. The first encouraging one is that ring B is predicted to be the brighter of the two major rings. Secondly, it is clear that, although this
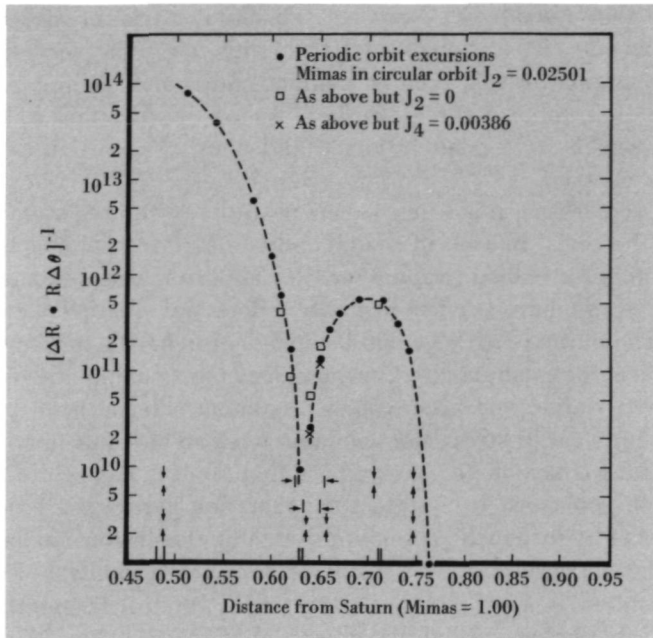
FIGURE 5. — *Predicted density profile of the ring obtained with the perturber, Mimas, in a circular orbit. Arrows mark measured ring features. Both scales are drawn such that $R_{mimas} = 1.0$. The quantity ΔR, for example, is the minimum radial separation of adjacent particles such that collisions will not occur.*

model does not offer a precise definition for the extent of the Cassini division width, it has produced, in a region whose size is comparable to the observed Cassini division, a surface density that is low relative to its value in rings A and B. Finally, note that the outer boundary of ring A has a close association with the 2:3 resonance with Mimas.

There are, however, discouraging features associated with this model as well. First, no inner boundary to ring B is predicted while a sharp one is observed. The Encke division, which apparently lies near the 3:5 resonance, is also absent. Finally, the position of the predicted and observed locations of the Cassini division do not agree; the gap predicted by this model lies closer to Saturn than the observed one by about 0.2 second of arc. This latter point has been used to support the claim that resonances with Mimas are insufficient to account for the details of the ring structure. The first two discouraging features can to some extent be removed by including in the calculations the eccentricity of Mimas' orbit. This serves to excite other (higher-order) resonances, e.g., at 1:3 and 3:5. It also serves to broaden slightly the predicted Cassini division width. These results are presented in figure 6.

I do wish again to direct your attention to the displacement of the "predicted" location of the Cassini division from the observed one. Now to suggest a possible way to account for this shift: if ring B is sufficiently massive, such a displacement

must occur. To put it briefly, if the mass of ring B is large enough, then the local orbital frequency of a particle near or beyond its outer boundary will be markedly increased. Thus, the distance from Saturn corresponding to the 1:2 resonance with Mimas is displaced outwards (when compared to the case of negligible mass in ring B). This approach leads us to a revision of the work presented in figures 5 and 6, for now we wish to recalculate those curves but to include as a parameter the mass of ring B. Franklin, Colombo, and Cook (1971) have examined these ideas quantitatively and have shown that the required shift of the resonance can be produced if the mass of ring B lies near 6×10^{-6} that of Saturn, give or take 30 percent. If we adopt a vertical ring thickness of 2 km, this value of the mass leads to a mean ring density of about 0.1 gm/cm³. Although this value is rather large, it does not conflict with stability criteria; densities larger by a factor of 2 or 3 would still be stable against self-gravitating condensations.

These results conflict with our earlier remarks concerning the volume density derived from the classical ring model. From dynamical considerations, we require the mean density for ring B to lie near 0.1 gm/cm³. From the classical model, we obtained a volume density less than 10^{-2}. This figure, of course, coupled with a particle density of ~ 1 gm/cm³, leads to average densities that are at least an order of magnitude smaller.

It is this dilemma that needs to be resolved. Perhaps I did drop a hint at the
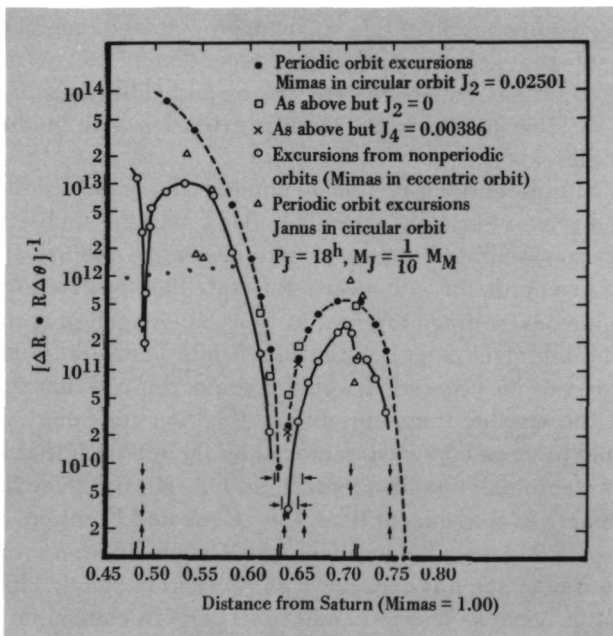
FIGURE 6.—*Predicted density profile obtained with Mimas in an eccentric orbit (e = 0.02). Some features resulting from the recently discovered satellite, Janus, are also indicated. For further details, see Franklin and Colombo (1970).*

outset as to a possible resolution; in any event, I would like to suggest that the trouble lies in the rather tacit assumption that the opposition surge is produced solely (or largely) by interparticle shadowing. Inasmuch as Oetking (1966) and Hapke (1966) have both suggested that a nonlinear opposition surge is a phenomenon presented by numerous laboratory samples, I would like now to suggest that this phenomenon is also presented by the particles of Saturn's ring. To test this suggestion we require a determination of the phase curve of an individual ring particle — something that is not, alas, even possible for the MJS mission. However, if we broaden our requirement slightly and ask for the phase curve of a single ice-covered body in the Saturnian environment, the prospect is not so bleak. Saturn is, after all, attended by an abundance of satellites, and the evidence is rapidly growing that particularly the inner ones are ice coated if not solid ice. By way of evidence, let me just remind you that it is impossible to reconcile the brightnesses with the densities of the inner satellites, unless their albedos lie near 0.6 and their densities are near unity. My hope, then, is to substitute the phase curve of a satellite for the phase curve of a ring particle by making the claim that an icy surface structure is common to both. Unfortunately, this sub-stitution is not quite so easy or immediate as one might wish. The problem here is twofold. First, as any observer will attest, the scattered light from the disk and rings of Saturn makes photometry of the inner satellites very, very difficult. Second, existing photometry suggests rather decidedly that the brightness of the inner satellites depends upon orbital phase in a pronounced manner. This makes a determination of the solar phase angle dependence more complex and time consuming. Dr. D. Morrison informs me that he and colleagues are hard at work in the fine skies of Hawaii to provide us with extensive high-quality photometry of these inner satellites.

Although it sounds curious at first encounter, the best candidate for a single ice-covered and photometrically accessible body may be the bright hemisphere of the more distant satellite, Iapetus, which lies at 60 Saturnian radii from the planet. As you are no doubt well aware, this satellite shows enormous light varia-tions, being some seven times brighter at western elongation than at eastern. This very large amplitude itself suggests that the bright hemisphere has a high albedo, i.e., that it may be ice covered. I would remind you too that it is the "trailing" hemisphere of the satellite that is bright and the "leading" one dark. This suggests that an erosional process may have removed all the ice from that hemisphere upon which, say, meteoroidal flux is greatest and is slowly removing ice from the trailing hemisphere at the present time. (See Cook and Franklin, 1970, for details.)

During the past 2 years, I have been making observations of the bright face of Iapetus in order to see if it does show an opposition effect. This project seemed especially fruitful because last December the western elongation of Iapetus (when the satellite is at maximum light) nearly coincided with opposition. Since I began these observations, Drs. D. Morrison and R. Murphy have carried out infrared measurements of Iapetus that argue for an albedo of ~ 0.3 for the bright hemi-sphere — rather lower than what one might expect for ice. Thus it may well be that only a fraction of the entire hemisphere is ice coated. With these caveats and

reservations in mind, let me still proceed with the tentative claim that a phase curve of Iapetus might have a resemblance to the phase curves of individual ring particles. Figure 7 is a plot of the variation of the brightness (maximum) of Iapetus at western elongation (vertical scale) versus solar phase angle α (horizontal scale). It is at once clear that Iapetus does brighten substantially at opposition—by some 0.3 magnitude over its brightness at $\alpha \cong 6°$. However, it is also apparent that figure 7 is as yet incomplete; one requires a knowledge of the satellite's brightness in the range $1° < \alpha < 3°$. I would add too that the point at 3° rests upon too few observations to be too much relied upon. Note the date at the top of the figure; on January 10, 1974, a brightness maximum falls at $\alpha \cong 2°$, and observations at that time will be crucial in deciding whether the slope of the phase curve is steep but linear or whether it is less steep but shows an opposition surge.

Let me just conclude by saying that I think there begins to be some evidence that the chief underpinnings of the classical ring model requiring that the rings be a medium many-particle radii in vertical thickness may be somewhat less than secure. It seems to me, therefore, that a model satisfying both dynamical and
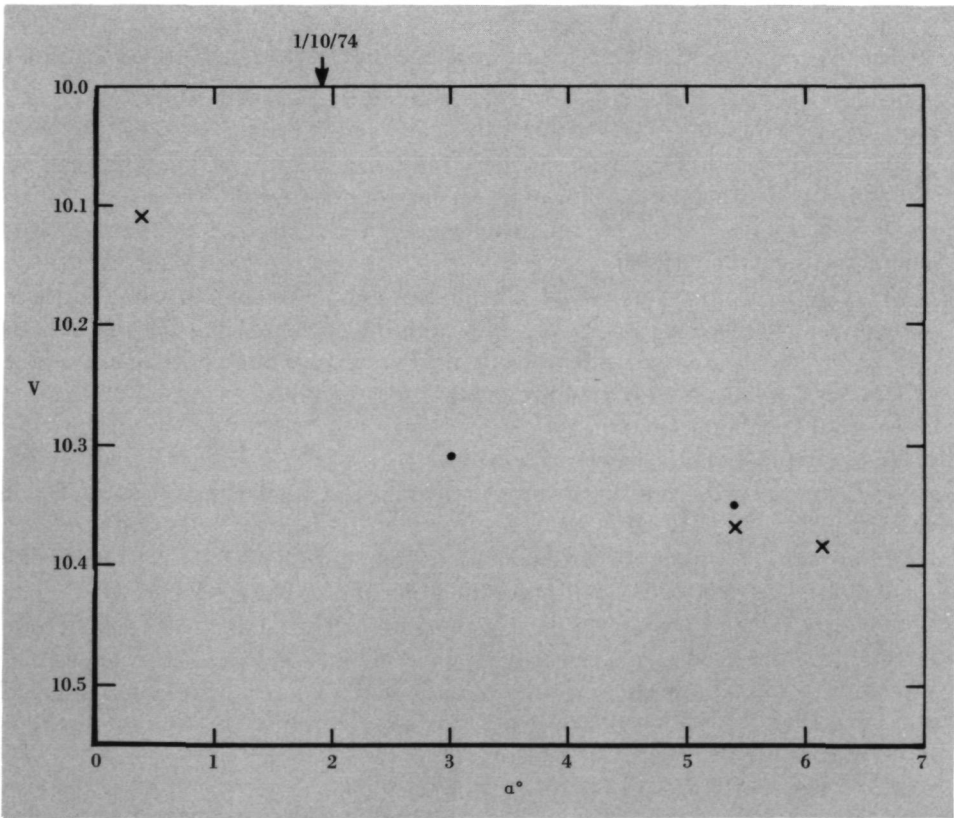
FIGURE 7.—*Brightness (in V) of Iapetus at its western (or bright) elongation as a function of solar phase angle, indicating that this satellite displays either an unusually steep phase variation or an opposition effect. Dots are observations of R. Millis; crosses are those of the author.*

photometric constraints is one in which the rings are nearly a monolayer of large bodies.

DISCUSSION

James Pollack My question relates to the type of opposition effect you might expect from an ice particle. In Bobrov's review article (1970) he addresses himself to this question and he points out that the Galilean satellites, for which there is some evidence that they are ice covered and quite bright, do not show as big an opposition effect. What is your reaction to that sort of evidence?

Fred Franklin Well, one very trivial reaction. The Jovian environment is a very strange one. In fact, even the models that suggest Jovian satellites have ice also often introduce an overlying layer of something else. I am somewhat happier doing observations of this sort in the Saturnian environment, but what you say is a very good point.

William Irvine Isn't it also true that the strongest opposition effect, or perhaps the only real indication of an opposition effect, is for the darkest Galilean satellites?

Pollack That's right. Bobrov's point was that if you take something of comparable albedo to the rings it doesn't show the same large opposition effect.

Robert Murphy In my work with Cruikshank and Morrison we also observed Rhea, which definitely has an albedo on the order of 0.5 or 0.6. I think Rhea would be an ideal satellite to look at for an opposition effect.

Franklin Rhea does show an orbital phase angle variation of a rather variable character; isn't that right?

Murphy Morrison has derived an albedo and radius for Dione, using infrared radiometry and visual photometry. His preprint gives a value for the density of 1.2g cm^{-3}, and a visual albedo, with an average over the entire visual region of 0.6. That would suggest we have another icy satellite.

Irvine That is a Bond albedo?

Murphy That is a visual geometric albedo.

Gordon Pettengill Do you think you can actually get a solar phase curve, David (Morrison)?

David Morrison Yes, and in particular I have one here for Rhea. We have had a photometric project this last opposition at Mauna Kea Observatory. The photometry is good enough to clearly separate orbital phase from solar phase effects. It is not good enough to make a clear distinction between an opposition pip and a gradual increase, but the average for Rhea has a wavelength dependence and runs from about 0.025 magnitude per degree in the blue up to about 0.040 magnitude per degree at 8000 Å.

Franklin The increase with phase angle is fairly linear.

Morrison A linear fit was consistent with the data which, as I say, is not good enough to make a clear distinction.

Franklin But you will look for opposition effects, in time, or could you put limits on any opposition effect at the present time?

Morrison   I don't have a plot here that would let me put limits on it, but I think we can.

Franklin   That would be very nice.

Murphy   In deriving a phase curve for Iapetus, we have to bear in mind that the surface has some very dark material and some very bright material. The observing geometry changes with time, and we may get very strong variations in the fraction of the observed area covered by each material type.

Franklin   I can only defend myself by saying I was aware of that and did measure successive western elongations to minimize any change, but it certainly is there.

Hugh Kieffer   I would like to change the subject a bit and question you about the dynamical model. In deriving these reasonably good fits of brightness versus radius from the planet, you assumed that each little epicycle was basically clean except for one particle.

Franklin   That's right.

Kieffer   It seems to me as though that in itself immediately places very strong constraints on particle size, which must be kilometers.

Franklin   I would never use any argument like this to get a particle size. In fact, I dwelt on it today to such an extent only to have a "zero-order" model in which I could estimate how much mass would have to be put into ring B to shift the Cassini division. I can only say that the problem is difficult, and one really has to consider collisions and one has to introduce an elasticity parameter. For the present I don't know how to handle the problem except in the rather simple way that I have outlined. The model is therefore incomplete, certainly, but I do not see that it is inconsistent. The real question that it addressed was, "How particles would arrange themselves, given the perturbing force of Mimas (and Titan), if no mutual collisions were allowed?"

Kieffer   I don't know what the right answer is. I just wondered what your thoughts were.

Franklin   The minimum epicycle diameter is, by the way, of the order of 200 meters. This value would apply near the center of ring B, at large distances from any strong resonance. In this model, then, a figure of this order would formally correspond to an average particle diameter.

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