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LIQUID FILM DEMONSTRATION EXPERIMENT
SKYLAB SL-4

By Wesley Darbro
Space Sciences Laboratory

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16. ABSTRACT The liquid film demonstration experiment performed on Skylab 4 by Astronaut Gerald Carr, which involved the construction of water and soap films by boundary expansion and inertia, is discussed. Results include a 1-ml globule of water expanded into a 7-cm-diameter film as well as complex film structures produced by inertia whose lifetimes are longer in the low-g environment. Also discussed are 1-g acceleration experiments in which the unprovoked rupture of films was photographed and film lifetimes of stationary and rotated soap films were compared. Finally, there is a mathematical discussion regarding minimal surfaces, an isoperimetric problem, and liquid films.					
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LIQUID FILM EXPERIMENT – SKYLAB SL-4

I. INTRODUCTION

Given a closed contour, the problem of finding the smallest surface bounded by it was proposed by Euler in the 18th century. It involves solving a **boundary valued, nonlinear partial differential equation problem**. Riemann, Schwarz, Weierstrass, and others of the 19th century gave solutions for certain contours. General existence proofs have been published in the 20th century by Rado, Douglas, McShane, Garnier, and Courant. Neglecting gravitational effects, a soap film assumes the shape of a (minimal) surface of least area spanning the given contour. J. Plateau, a blind Belgian physicist of the mid-19th century, performed and published the results of numerous experiments involving the construction of soap films on a wide variety of wire contours. This "minimal surface" problem has subsequently become known as Plateau's Problem. This report then concerns revisiting an old problem – one which captured the attention and energies of the great mathematicians and experimenters who have gone before.

Though now the question of existence of a solution has been solved, the uniqueness is another problem. One may observe from a soap film demonstration that one contour may have more than one minimal surface (Fig. 1).

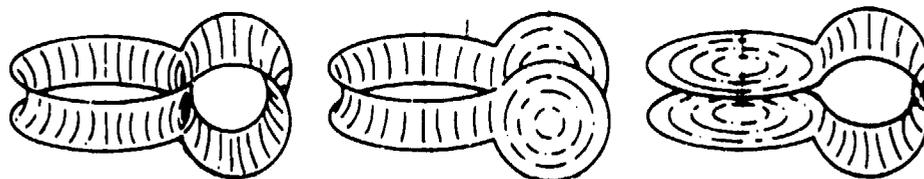


Figure 1. Diagram of one boundary with three minimal surfaces.

Besides the obvious existence and uniqueness problem of a minimal surface on a given boundary, other interesting problems have been studied. One may start with a circular contour enclosing a planar film, Figure 2(a), and deform it continuously toward the new configuration of Figure 2(b). The film

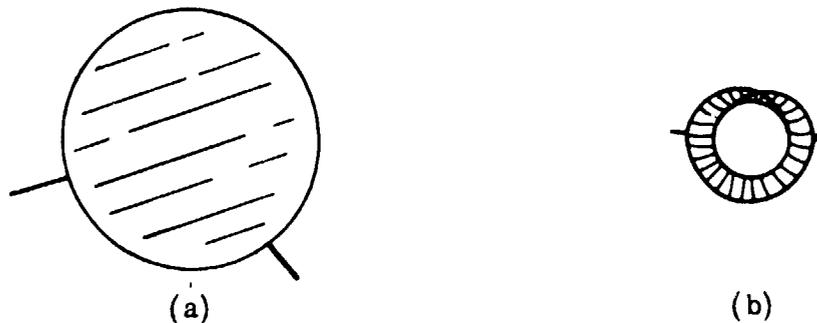


Figure 2. Two topological types.

will continuously change for a period of time, then it will jump to the totally new minimal surface, the one-sided Moebius strip in Figure 2(b). This demonstrates that the topological structure of the film may change even though that of the contour remains the same.

Today Plateau-type problems find themselves a long way from the mainstream of mathematics. In fact, the last major results were published in the 1930's, and few significant mathematical publications on the subject exist for the last 30 years. The situation is quite the contrary for the interesting soap films. Experimentalists continue to publish literature regarding such topics as life of soap films, film thickness laws, temperature effects on soap films, film elasticity, film stability, etc. Apparently there is much yet to be learned regarding liquid films. Adamson [1] states, "The actual mechanism of the rupture process itself is not well understood; it is not known whether rupture in an undisturbed film usually initiates at a border (as seems likely) or more or less randomly." In earth environment, the rupture of liquid films takes place under the influence of the forces of gravitation and surface tension. Generally speaking, surface tension is the more dominant of the two forces. Not much is known about the relation between the persistence of a film and the drainage rate, though numerous experiments have sought to determine it. Perhaps research in liquid films has become more important since the use of thin films in the semiconductor industry and the wide use of foams today.

Recently a liquid film demonstration experiment was approved by NASA and performed during January 1974 by Astronaut Jerry Carr on Skylab SL-4. The type of experiment known as a "demonstration," is not really a full-fledged experiment. It is not scheduled but is used to fill in if a time period occurs when the astronaut has nothing pressing to do. Ten such demonstrations were accomplished during the 3-month Skylab SL-4 mission. Generally the guidelines were that the experiment be short, simple, and involve only equipment already

on board. The goal of the liquid film demonstration experiment was to construct liquid films in a low-g environment from water and from a soap solution by two different methods and to photograph and observe their formation, their characteristics, and their time to rupture.

In the first method, a globule of liquid was placed on a closed, expandable wire frame which was expanded, stretching the spherical shape of the globule into a lens-like thin film. In a one-g environment this cannot be done with water because the gravitational force immediately strips most of it from the wire. The second method of film construction consisted of accelerating a closed wire frame submersed in liquid. The inertial forces on the liquid allowed the frame and bulk liquid to separate and left a liquid film bounded by the frame.

II. LABORATORY EXPERIMENTS

Several preliminary experiments in the laboratory were performed in connection with the Skylab experiment. High-speed movie film was taken of the rupture of soap films on circular hoops. Also soap films on a hoop and a tetrahedron were rotated, the results giving a clear indication that gravitational effects on films could not be neglected in understanding and predicting their lifetimes. The frames in Figure 3 were those used for rotating in the one-g environment. Some of the results are shown in Table 1. The frames were made from safety wire and are about 2.54 cm (1 in.) in diameter. The soap used was the shower soap of the Skylab mission, Miranol Jem Concentrate (MJC). According to Table 1, increasing the rotation rate or the soap concentration increases the film lifetime. This effect, however, does not continue, as eventually an increase of either will decrease the lifetime. Rotating the film has the effect of alternating the direction of the gravitational field and produces an average zero-g force. One must realize, however, that this is different from the orbital situation because the bulk liquid between the two film faces is in motion, alternating from one direction to the other under the one-g force. This motion is visually apparent at first, but after a couple of minutes, evaporation has thinned the film and no motion can be perceived.

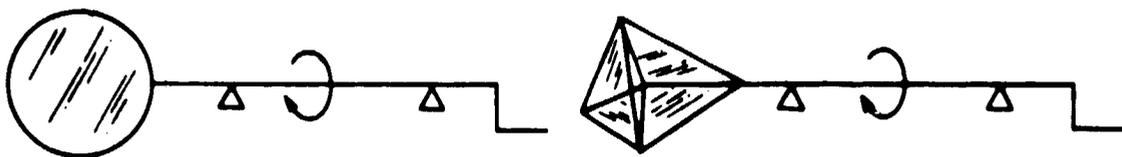


Figure 3. Laboratory apparatus for rotational experiment.

TABLE 1. SOAP FILMS ROTATED IN ONE-g

Wire Frame	Soap Conc.	Rotation Rate (Hz)	Avg. Lifetime (s)
Vertical Hoop	120:1	0	2.0
		1	4.3
	60:1	0	5.4
		1	13.1
		2.5	177.5
	40:1	0	6.0
1		290.0	
Tetrahedron	120:1	0	0.0
		0	5.0
	60:1	1	74.4
		2.5	245.0

The results of this experiment, as well as others, do not support the statement from [1] that "the main cause of bursting is evaporation."

One of the goals of our high-speed photography experiments was to observe where the film rupture started, whether it initiated on the edge, near the center, or randomly. Previous experiments [2,3,4] had initiated rupture by an electric spark. Our approach was to allow it to occur passively. Photographs were taken at 2000 and 3000 frames per second, and the total duration of each filming was about 5 seconds. It was possible to predict correctly, with a probability of 0.6, a 5-second interval in which the film on the hoop would rupture.

Three hoops were then made on a common frame, and this increased the probability of photographing at least one rupture to $1 - (0.4)^3 = 0.936$. This was demonstrated by the fact that each trial produced two or three ruptures. Some of the hoops were held horizontally and others at a 30-degree angle. All those on an angle broke from the top edge downward. Although initially a 40 to 1 ratio of water to soap (MJC) solution was tried, it may well have been twice that high for some of the pictures. The heat from the lamps was so

intense that often before the experiment was finished, the solution would begin to boil. As the film broke, the leading edge had an average velocity of a few meters per second (Fig. 8), having a higher velocity at the beginning of the tear, then slowing as it went across the hoop.

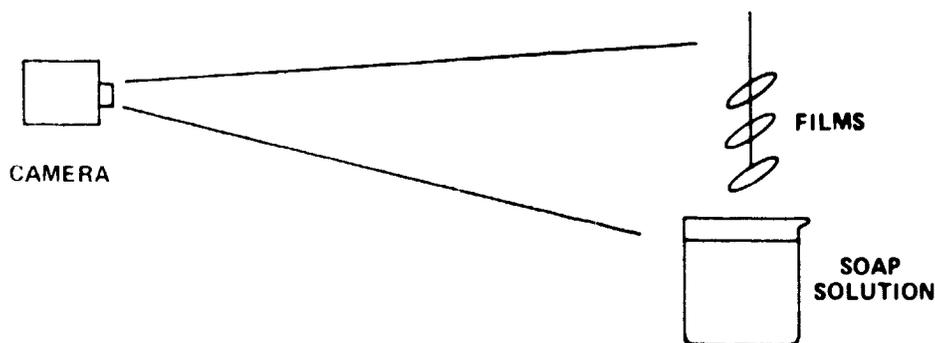


Figure 4. High-speed photography experiment setup.

The first configuration used three approximately circular frames (about 2.5 cm in diameter), tilted about 30 degrees. The three hoops were raised from the beaker as shown in Figure 4. In no case did it appear that the breaking of one film had any effect on the breaking of another. This was probably because rupture takes place in the plane of the hoop so there was little normal acceleration. Three such experiments were run and photographed at 3000 frames per second. Fortunately, each time all three soap films broke before the high-speed film was depleted, giving nine ruptures to observe. All of the soap films began breaking from the top edge of the frame and proceeded in line across the frame to the opposite edge. The main phases are shown in Figure 5. The leading edges of the breaks for one set of three appear in graph form in Figure 6.

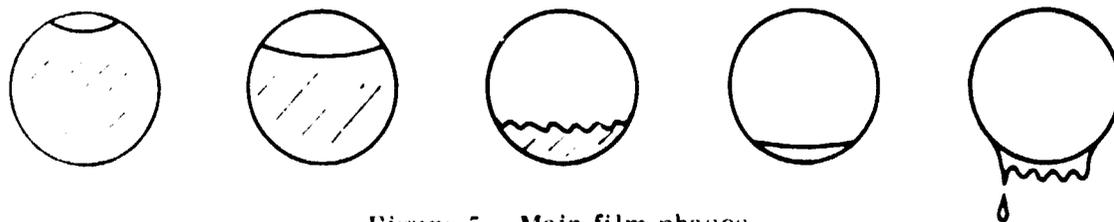


Figure 5. Main film phases.

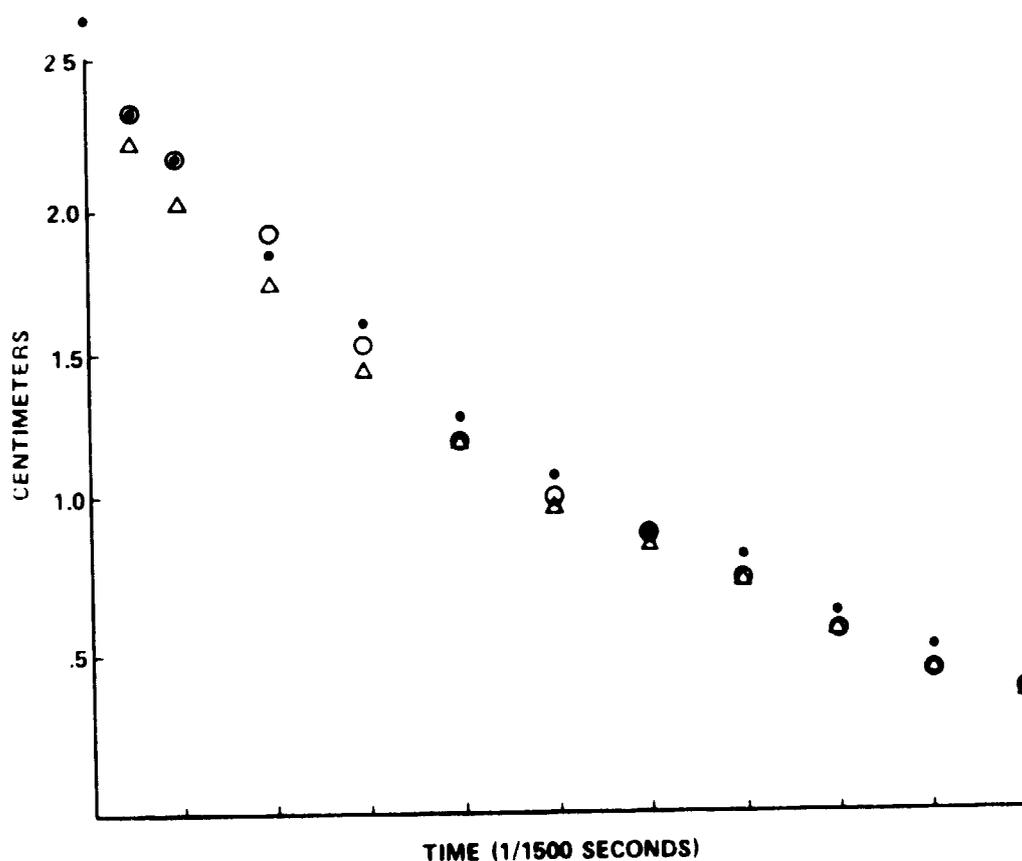


Figure 6. Film diameter versus time (three trials).

Since all the soap films broke from the top to the bottom in the first configuration, another set of experiments was performed with a horizontal configuration. This involved the photographing of rupture of horizontal soap films. A soap film on a horizontal frame has little drainage and so persists longer. This tends to spread the probable time of rupture over a longer interval and makes it much more difficult to catch it while breaking. The soap concentration ratios were adjusted, and the horizontal films were photographed at 2000 frames per second, which gives 50 percent more photographing time. Also, the frames were not mounted directly above each other but radially about the stem and normal to it (Fig. 7). Unfortunately, these frames were only as circular and as horizontal as the hand and eye could make them, therefore, in the strictest sense they were not horizontal planar films. One could easily observe the varying curvature over these minimal surfaces. It is interesting to note that



Figure 7. Horizontal configuration for high-speed photography experiment.

in no case (here or in the nonhorizontal cases previously discussed) did the rupture take place near a point of large Gaussian curvature, nor did it occur at the edge of a frame near a point where the frame was not roughly circular, nor where the wire was twisted together.

Three experiments were performed using the horizontal frames and, in each case, two of the three films broke. Three of the films broke from back edge to front, one from the left edge to the right, and, surprisingly enough, two from the interior, about halfway from the left-back edge to the center. The holes appeared not at all circular but elliptical and quickly reached the nearest edge, spreading across to the opposite side in a manner not perceptibly different from the films which broke from the edge.

Figure 8 shows two averaged velocity graphs of the leading edge of the break, one of horizontal films and the other from the 30-degree tilt experiment. A tentative explanation of the higher velocity for the horizontal film is that because of the slower film speed and the reduced lighting requirements, less heating occurred. The surface tension and, consequently, the velocity were greater. The decreasing velocity of both curves may be explained by the circular frame which forces the film to thicken as the rupture proceeds. Such a conclusion may be drawn from Figure 7 of Reference 2.

III. THE SPACE EXPERIMENT

The first part of the demonstration made use of a wire loop [Fig. 9(a)] which could be expanded from an oval with diameter of perhaps 1 cm to about 20 cm, much as one would expand a lasso. The astronaut could not smoothly expand the loop because of the resilience of the wire, and the shape it took could be

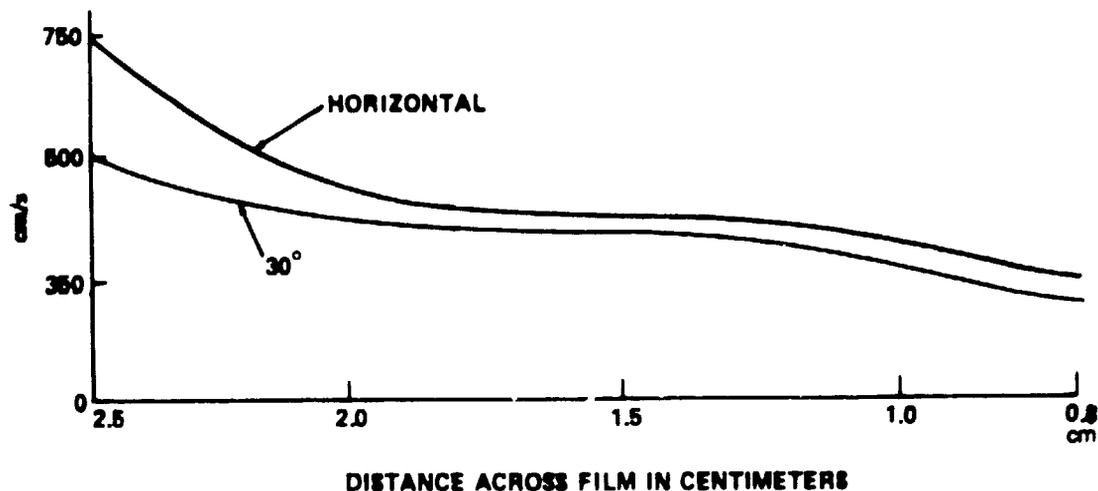


Figure 8. Velocity curves for horizontal and nonhorizontal soap films.

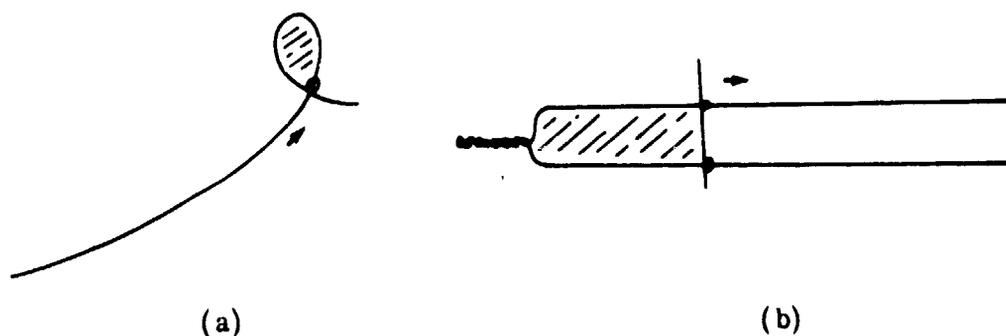


Figure 9. Apparatus for Skylab SL-4 experiment (boundary expansion).

described as slightly elliptical and planar. There were two attempts to draw a water film. These films were made from 1 ml of water and expanded to diameters of 6 and 7.5 cm, respectively. They both broke (from the edge) immediately after being ticked by a finger in the process of further expansion. According to Astronaut Carr, these films were as large or larger than any he had attempted in practice. The second film appeared to break loose from one edge first but then came completely free from the wire loop. It appeared to retain its film-like structure until it passed from the field of view (Fig. 10), but this may have been an illusion produced by TV video tape processing.



(a)



(b)

Figure 10. Expansion of plain water film on Skylab SL-4:
(a) before rupture, (b) after rupture.

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Astronaut Carr next tried to fill the loop with 1 ml of the soap solution, but the liquid globule stuck tighter to the syringe than it did to the wire, so that the camera had to be shut off while he practiced the transfer. Eventually he did get a globule to transfer onto the wire loop, but remarked that it was not a full ml and estimated that it may have been about 0.25 ml. The loop was expanded to about 7- or 8-cm diameter **before the film broke.** A second try with a full ml produced a 15-cm-diameter film, which is very close to the results obtained in a one-g environment.

Next Astronaut Carr used the sliding rectangle [Fig. 9(b)] and he estimated that he had 1/8 ml of soap solution transferred onto it. He then moved the sliding side to form a 2 cm by 20 cm rectangle. Once the film broke, the liquid concentrated on the wire was sufficient to be redrawn. Again, nothing different from a similar one-g experiment was observed.

In the next portion of the demonstration, the soap solution was contained in a used food can, and the three wire frames (Fig. 11) were secured by a piece of tape. Carr set the clock in view of the camera, dipped the hoop into the solution and pulled it quickly out. As expected, a film formed, and the frame was then stuck to the tape. Then the tetrahedron frame was pulled from the tape (this noticeably shook the film on the hoop), placed in the soap solution, and jerked out with apparently the usual characteristic film for this type bounding frame. However, Astronaut Carr describes it as having films on the four faces. Although the video tape picture is not clear enough to be certain, **it is assumed that this comment was unintentional since it is mathematically impossible.** While moving the frame about, the film popped, so he redrew it. He noted then that the hoop film had ruptured in 1 1/2 minutes. Generally speaking, one would not expect the film on the hoop held stationary to last over a minute in one-g, even if it were held horizontally. The extra time to bursting may be partially due to it being a thicker film than that which could be drawn in one-g. Film thickness was an important variable in the analysis of this demonstration and we were unable to measure it.

The cubical frame was submerged several times in the soap solution then jerked from it with films forming in various configurations. One lasted no longer than 5 seconds, but other lifetimes were on the order of 1 minute. One may have expected the film lifetimes in space to be much longer, since any sagging and drainage could not be the results of a gravitational force. In one-g one would expect, for a stationary film, a lifetime of a few seconds and for one rotated, a lifetime of a few minutes. So it does appear that for a near zero gravitational force the lifetime is increased some, but perhaps no more than one order of magnitude for our particular solution. On the other hand, the much longer lifetime of the rotated film seems to be due to the sloshing about **of the bulk liquid, which perhaps replenishes and patches the weak and rupture-prone areas of the film.**

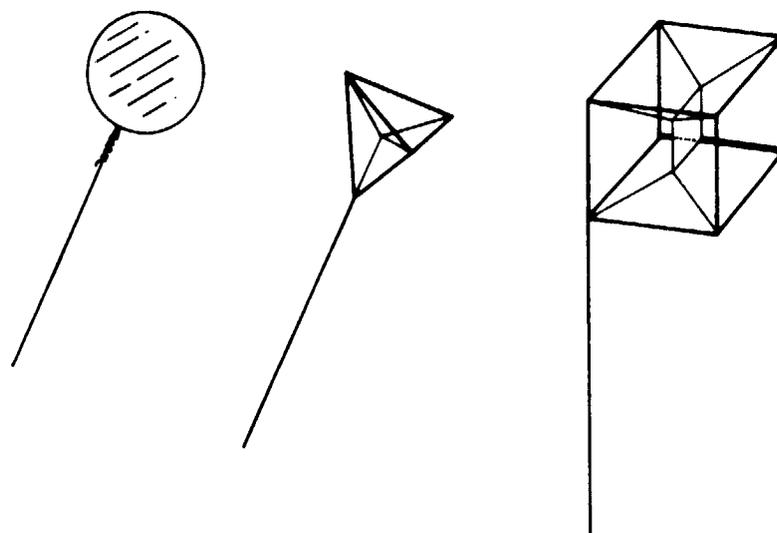


Figure 11. Apparatus for Skylab SL-4 experiment
(inertial acceleration).

Astronaut Carr demonstrated that he could pull the frame out so that a full cube of liquid was bounded by the frame (Fig. 12). The sight of such an "open" container is rather unusual and is brought about because the amount of the liquid which clings to the cubical frame depends on how fast the frame is jerked from the solution. He then emptied the liquid from the frame by accelerating (shaking) it. Some liquid left the frame with each shake until only a very thin set of films remained. He described the resulting structure as having a small cube in the center, held by thin films attached to the wire frame (Fig. 13). This is not inconsistent with theoretical studies, even though the structure formed in one-g does not form an internal cube. In one-g the structure consists of 13 film faces, one square in the center held in position by 12 bounding films (see Section IV-B on isoperimetric problems for a discussion).

The cube of liquid held only by the wire frame demonstrates the careful attention that liquids must be given when handled in space. They can easily be caught by edges, take up shapes dictated by surface tension, and hence form **surfaces of minimal area. In one-g the shapes that liquids take are dominated by the gravitational force.** We take this property into consideration in making liquid containers. For instance, a cylinder with one end open and the other closed makes a good container if it is not to be accelerated too greatly since it can be oriented so that the gravitational force holds the liquid away from the



Figure 12. Full cube of soap solution.

open end. In space, all the usual design and technology for building liquid containers is of no use, whether it be a "space" coffee cup or a goldfish bowl. The primary consideration should be given to surface effects, wetting characteristics, surface tension, and viscosity.

IV. MATHEMATICAL BACKGROUND

A. Minimal Surfaces

The study of surfaces is rather fascinating and involved and has a most colorful history. A short collection of facts should serve our purposes here [5].

Considering a point P in a surface S , construct a normal \bar{n} to S through P . Pass a plane through the normal \bar{n} so that it may be rotated about \bar{n} through any angle θ (Fig. 14). For each θ , the intersection of the plane with S is a line and its curvature $k(\theta)$ at P may be computed. Where $k(\theta)$



Figure 13. Film structure formed by accelerating wire cube.

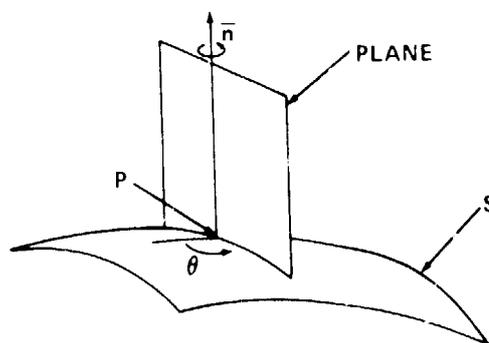


Figure 14. Curvature illustration.

attains a maximum, say at θ_1 , it is known that $k(\theta_1 + \pi/2)$ is a minimum [except where $k(\theta)$ is a constant]. The reciprocals of $k(\theta_1)$ and $k(\theta_1 + \pi/2)$ are denoted by R_1 and R_2 and are known as the principal radii of curvature. The mean curvature H is defined by $H = 1/2 (1/R_1 + 1/R_2)$, the Gaussian curvature K is $1/R_1 R_2$. Those special surfaces where $H = 0$ at every point are called "minimal surfaces." The term "minimal" is somewhat justified by the property that for each simple closed curve C which bounds a surface S of minimal area, it is the case that $H = 0$ at each point of S . It has further been shown that for any simple closed Jordan curve there exists at least one minimal surface bounded by it. In many cases, there is more than one (Fig. 1) and, in fact, there may be infinitely many.

The condition $H = 0$ implies that unless both R_1 and R_2 are ∞ , $R_1 = -R_2$, and this gives the saddle shaped appearance that a minimal surface has (at each point). An example minimal surface is given by

$$\bar{x}(u, v) = (v \cos u, v \sin u, cu)$$

which is known as a right helicoid. Another minimal surface (known as Enneper's) is given by

$$\bar{x}(u, v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + u^2v, u^2 - v^2 \right)$$

Catenoids (which are rotations of catenaries) are the only surfaces of revolution which are minimal surfaces, and these may be physically realized by suspending a soap film between two hoops. Soap films form minimal surfaces because their surface tension forces them into a configuration of least area bounded by the wire frames. Those films which also bound volumes of air do not form minimal surfaces even though they tend to a least area surface. Since every possible wrinkle of a bounding wire produces another minimal surface, there is no shortage of such surfaces even though they are so special.

One should be a bit cautious for fear of reading too much into the foregoing discussion. If a curve is given by $y = y(x)$, then the curvature at x is $k(x) = y'' / [1 + y'^2]^{3/2}$. The curve $y = x^2$ has a curvature of 2 at zero. However, the function $y = x^4$, which appears to be bending through zero, has a curvature of zero

from the definition of curvature. It is this phenomenon which keeps one from constructing a surface whose maximal and minimal curvature is less than $\pi/2$ radians apart. Obviously a surface with a ridge and a valley can be made to meet at any angle, but at the intersection (unless the angle is $\pi/2$), the curvature is zero in all directions.

B. Isoperimetric and Steiner Problems

In many cases soap films are not just single surfaces but may involve several surfaces connected together. They have some volume and sometimes surround air masses (bubbles and foams), so the following discussion may be in order.

The circle encloses the largest area among all closed curves with a fixed length. The sphere encloses the largest volume among all surfaces with a fixed area — which is self-evident, but a rigorous proof is difficult. Problems of this type are known as isoperimetric problems.

"Steiner's problem" refers to a problem solved by Jacob Steiner, University of Berlin, in the early 1800's. How should one join three cities by a system of roads whose total length is the smallest possible? The solution, generally speaking, involves constructing a road from each city to a single point so that the angle between adjacent roads is 120 degrees (Fig. 15). The problem and solution may be generalized to any number of cities. The solution obtained does not allow more than three roads to intersect and in each intersection of three, again the angles must be equal.

To help explain soap film formation, we shall consider an isoperimetric problem together with a condition on the boundary which in the limit reduces to a Steiner problem. The particular formation we wish to consider is that of a cubical wire frame. The structure in one-g is shown in Figure 16, consisting of a small "square" film centrally held by twelve other films which connect to the cubical frame. In space, the shape is totally dependent on the volume of

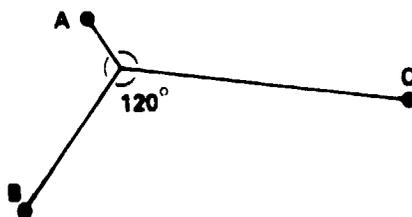


Figure 15. Steiner's three-city problem.

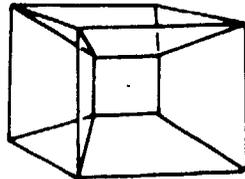


Figure 16. Soap film structure on cubical boundary.

fluid. As was demonstrated in the Skylab experiment, if the amount is less than the volume of the wire cube, the liquid adheres to the wires and is spherically concave in each face. As the liquid volume is reduced, adjacent spherical faces become tangent, eventually forming planar films between each other and a concave "cube" centrally held by these films. If this were the configuration as the liquid approached zero, one would expect the formation to consist of twelve planar films coming together in the center at a point. This does not happen (as mentioned above in the one-g case), as the center is not a point but consists of a film which is approximately square. Is there some discontinuity, and what is the nature of this rather unusual situation?

Although it is tricky to infer three-dimensional results from a two-dimensional solution, the following result is helpful. Consider four points in a square arrangement of two units on a side. Interior to the points is a rectangle whose corners are each connected to the adjacent point (see Fig. 17). The problem: For a given rectangular area A , what is the ratio of adjacent sides of the rectangle when the total perimeter is minimized? Equivalently, find α and d (for instance, if the rectangle is square, $\alpha = \pi/4$)? The perimeter is given by

$$P(\alpha, d) = 4 \sec \alpha (2 - 2d + \cos \alpha - \sin \alpha + d \cos \alpha + d \sin \alpha) \quad ,$$

and for any value of area, A , the function $g(\alpha, d) = 0$ shows the relation which must exist between α and d :

$$g(\alpha, d) = 4 \sin \alpha \, d^2 + 4(\cos \alpha - \sin \alpha) d - A \cos \alpha = 0 \quad .$$

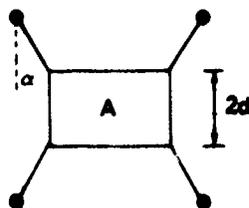


Figure 17. Diagram of an isoperimetric mathematical problem.

Formulated in this manner, this problem of minimizing P subject to g is a natural for the method of Lagrange multipliers. One writes the Lagrangian $L = P(\alpha, d) + \lambda g(\alpha, d)$ and solves the three equations:

$$\frac{\partial L}{\partial d} = 0, \quad \frac{\partial L}{\partial \alpha} = 0, \quad \frac{\partial L}{\partial \lambda} = 0.$$

Finally, this leads to finding the zeros of $f(\alpha)$:

$$f(\alpha) = \frac{2(2 \sin \alpha - 1)}{\cos \alpha (2 - \cos \alpha - \sin \alpha)} + \frac{x^2 \cos \alpha - 2x(\cos \alpha + \sin \alpha) + A \sin \alpha}{(2 - x)(x \sin \alpha + \cos \alpha - \sin \alpha)},$$

where

$$x = 1 - \cot \alpha + \sqrt{(1 - \cot \alpha)^2 + A \cot \alpha}.$$

One can see immediately that for $A = 0$, $f(\pi/6) = 0$ (this corresponds to Steiner's solution of four cities), but for other values one needs an iterative scheme to find the solutions. The graph (Fig. 18) is a plot of α versus the ratio of the area of the interior rectangle to the exterior square. This indicates that the interior rectangle is a square until its area is small, approximately 5 percent, then it flattens into a true rectangle and finally a straight line at $A = 0$. Although this may seem surprising, it may be seen to be the expected thing when one considers the following. A square has least perimeter of all rectangles enclosing a given area, but on the other hand, the configuration of Steiner's problem of four cities gives the minimal perimeter for a zero area

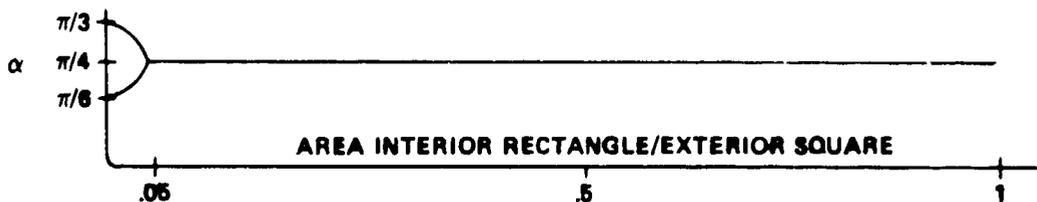


Figure 18. Graph of the solution to isoperimetric problem presented in Fig. 17.

rectangle. It is the first that dominates our perimeter until the square is very small and the "holding arms" are very long, then Steiner's solution enters the picture and dominates near zero. The double valued graph between 0 and 5 percent indicates that the **square may alternately flatten vertically or horizontally.**

Although a one-g experiment demonstrating this (except for $A = 0$) seems impossible, it could easily be done in space. Figure 19 shows two sheets of Plexiglass held horizontally apart by four small pillars placed in a square. This structure submersed in a soap solution in low-g should demonstrate results given above when viewed from the top. The cross-sectional area could be reduced by shaking the solution from the structure a little at a time. The only difference one should expect is that the central "rectangle" will necessarily have concave sides which will slightly adjust the point at which it departs from a "square."

We now return to the three-dimensional problem, where our one-g experiment produced a "square" in the cubical frame as opposed to Astronaut Carr's reporting of a "cube" in the center. In view of the above 2-dimensional problem, perhaps the following explanation holds in this case. Since the cube contains the least surface area for any rectangular-piped of a given volume, this is the configuration taken as the liquid is shaken out. Eventually we expect the cube to get so small and the "holding films" so large that they begin to dominate; the cube gives up its shape in the interest of reducing the surface area of these films and flattens into a square. Presumably this is the case in one-g, and the only reason it was not observed in space was because too much liquid remained in the frame.

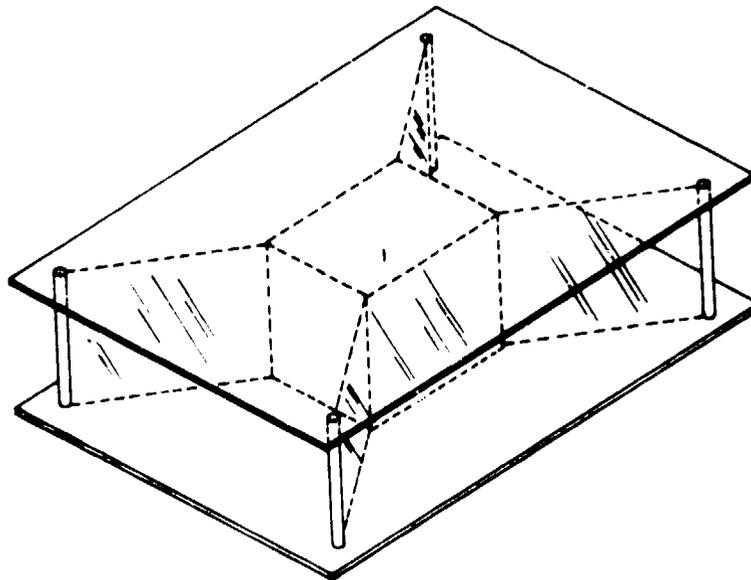


Figure 19. Diagram of a space experiment solution to the isoperimetric problem.

V. CONCLUSION

In the preceding pages a view of the fascinating study of liquid films has been given, both from a mathematical side and an experimental side. Plateau's Problem, the finding of a surface of least area bounded by a closed contour, minimal surfaces, Steiner's Problem, and isoperimetric problems have been discussed and connected to the structure formation of soap films. A new mathematical result concerning film structure was derived which seems to be verified by the space experiment.

Apparently new results occurred from the experimental work done relative to the gravitational effects on liquids and their surfaces. These include rotating soap films in one-g, photographing the unprovoked rupture of soap films, together with results from the Skylab space experiment. Astronaut Carr produced a 7-cm-diameter liquid film from 1 ml of plain water by expanding the wire loop which bounded it. He demonstrated another experiment, impossible in one-g, by submersing a cubical wire frame in a soap solution and retracted a full cube of liquid in the frame. Then by shaking (applying an inertial force), he emptied most of the liquid, leaving the resulting films. The

film lifetime was longer for some of the space films than for similar ones in one-g, as was expected, but none as long as those rotated in one-g. The extra lifetime in space could be due to the lack of gravitational induced drainage, or, more indirectly, absence of gravity allowed thicker films to be made which increased film lifetime. A more advanced experiment would be necessary to determine the precise cause of this effect.

In future space flights it is suggested that due to the preliminary results from Skylab, an experiment be proposed to make a film by boundary expansion using, instead of water, materials with higher surface tension, perhaps metals. Moldless casts may also be tried where the liquid film is hardened into a solid by a chemical or temperature change. As an example, due to chemical change, one observes that a film of paint over a nail hole often hardens before it ruptures if the hole is not too large or the paint too thin. As for making a solid film by temperature change, we have frozen soap films over liquid nitrogen dewars but generally found cracks in the frozen film due to the volume change of our solution caused when the temperature was quickly lowered. Foams, which are nothing more than a large film structure, are yet to be studied in any depth in space but because of their wide use will receive attention both from a basic theory and an application viewpoint in the future.

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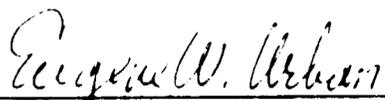
APPROVAL

LIQUID FILM DEMONSTRATION EXPERIMENT — SKYLAB SL -4

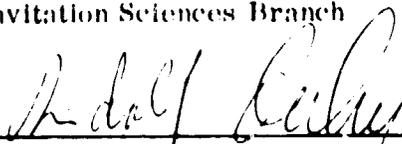
By Wesley Darbro

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EUGENE W. URBAN
Chief, Low Temperature and
Gravitation Sciences Branch



RUDOLF DECHER
Chief, Radiation and Low Temperature
Sciences Division



CHARLES A. LUNDQUIST
Director, Space Sciences Laboratory