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LASER PULSE ANALYSIS

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1. INTRODUCTION

In a pulsed laser ranging system, the range error for a single-electron return is equal to the standard deviation of a probability distribution having the shape of the transmitted pulse. A typical Q-switched ruby laser transmits a pulse whose shape is closely approximated by a normal-density distribution

$$\phi(t) = \sigma^{-1}(2\pi)^{-1/2} \exp(-t^2/2\sigma^2) \quad , \tag{1}$$

where the center of the pulse is at $t = 0$ and the pulse's standard deviation is σ. For a typical pulse, the value of τ, the time between half-intensity points, is about 20 ns, and the corresponding σ value is $\tau/2(2\ln 2)^{1/2}$ or $\sigma \doteq 8.5$ ns. When a single photo-electron is received, the range error is this σ value times half the velocity of light, 0.15 m/ns, or 1.3 m. If, instead of a single electron, the return contains N electrons, the returned pulse gives N independent measurements of the range. The average of these measurements has a range error that is smaller than the single-electron measurement by a factor of $N^{1/2}$. Hence, it can be seen that the range error of a 20-ns laser can be less than 0.5 m when there are 7 or more electrons in the return and less than 0.1 m when the number of electrons is 163 or more.

It is seldom practical to resolve each electron in the return, determine its respective range, and compute the average. Instead, the digital range counter stops at a point on the pulse's leading edge, this point being determined by the threshold-voltage setting of the input discriminator of the counter's STOP circuit. A series of incremental ranges referenced from this threshold point are then measured to various points on the pulse, and these points are referred to the center of the pulse. If there is a significant pulse-to-pulse variation in the shape or size of the transmitted pulse, a similar correction for the time between the START threshold and the transmitted pulse center can be obtained by the same procedure.

At the observing stations of the Smithsonian Astrophysical Observatory (SAO), oscilloscopic presentations of the transmitted and received pulses have been photographed in a study of the laser system's ranging accuracy. Figure 1 is a block diagram of the range-measuring equipment. When the counter's START and STOP thresholds are reached, the sweep circuits of oscilloscope channels 1 and 2, respectively, are triggered, and the transmitted and received pulses are displayed and photographed.

Figure 2 shows typical transmitted and received pulses on a time axis, with the amplitude of the transmitted pulse greatly attenuated. The counter measures the time T_c between the threshold points T_{TT} and T_{RT}. Since the corrected range is the time T_R between the pulse centers T_{TC} and T_{RC}, a correction Δ must be added to T_C, where

$$\Delta = (T_{RC} - T_{RT}) - (T_{TC} - T_{TT}) \quad . \tag{2}$$

This correction is known once the threshold points and pulse centers have been determined. Figure 3 shows a typical pair of transmitted and received pulses as displayed on an oscilloscope photograph, measured, and plotted by a digital computer.

This paper is concerned with methods for locating the points T_{TT}, T_{TC}, T_{RT}, and T_{RC} on presentations like those of Figure 3. There are errors involved in the determination of each these quantities, and an attempt is made to separate their effects on the overall range correction. Several series of corrected range measurements for fixed reflectors and satellites were obtained. Residuals were computed by fitting the range measurements to either fixed-reflector distances or short arcs of satellite orbits. Rms values of these residuals are presented but not enough data have yet been analyzed to be sure that the results are either typical or optimum.

2. THE EXPERIMENTAL DATA

The oscilloscope photographs of the transmitted and received pulses are obtained with a Tektronix model 454 oscilloscope, whose bandwidth is 150 MHz. The pulses were originally photographed on Polaroid film; later, 35-mm cameras were used. At first, the x-axis sweep was 50 ns, but this was later changed to 100 ns to avoid the truncation of the leading and trailing edges of the pulses. The Polaroid films were measured by hand at 50 points uniformly spaced along the x axis. The 35-mm films were measured at 100 time points by projecting the film on a screen and using a measuring machine. The measurements were punched on cards, read into a digital computer, and stored on magnetic tape. Two sets of the data* used in this report were obtained from only 30 to 40 measured points per pulse. These were converted to 100 data points by constructing a complete Fourier series of 15 harmonics. Tests showed that data so obtained are nearly identical to those obtained by the other method. The data for each frame are stored on magnetic tape as 200 amplitudes for 100 equally spaced time points on the transmitted and received pulses. For the 50-ns sweep, the points were 0.5 ns apart, whereas for the 100-ns sweep, they were 1 ns apart.

3. ANALYSIS OF THE EXPERIMENTAL DATA

In computing the range correction Δ, the transmitted and received pulses must be located in time with respect to the starting and the stopping of the range counter. These locations must be determined to within a small fraction of the pulse durations. Hence, some point on the received pulse must be identified with a similar point on the transmitted pulse. The nearly normal shape of the transmitted pulse makes its "center" an obvious choice for this point. However, as will be shown below, the corresponding center of the irregular received pulse can be specified in several different ways. It is also possible to align the 100-point sets of the transmitted and received pulses by using a correlation technique that does not require the pulse center to be identified.

One way to find the center of the pulse is to compute its centroid. If the pulse shape defines a density distribution on the time axis, then the centroid is the mean

*The fixed-reflector returns at the station in Arizona and the BE-C satellite returns at the station in Peru.

value of this distribution. If RP(t) is a continuous function representing the received pulse and t_c is its centroid, then the probability density distribution is

$$f(t) = RP(t) \Big/ \int_{-\infty}^{\infty} RP(t)\, dt$$

and

$$t_c = \int_{-\infty}^{\infty} t\, f(t)\, dt \quad . \tag{3}$$

When the integral is evaluated on the computer, summation, rather than integration, is used.

The centroid determination does not explicitly use the fact that the transmitted pulse and the probability-density function producing the received pulse are symmetric* about their mean values. To find this symmetric center for an irregularly shaped received pulse, we set its time origin ($t' = 0$) at successive points $t_1, t_2, \ldots, t_i, \ldots, t_{100}$. For each of these origins, we write f(t) as a sum of an even and an odd function, viz,

$$f(t') = f_e(t') + f_0(t') \quad , \tag{4}$$

where $f_e(t')$ and $f_0(t')$ can be determined as follows:

$$f_e(t') = \frac{1}{2}\,[f(t') + f(-t')] \quad , \tag{5a}$$

$$f_0(t') = \frac{1}{2}\,[f(t') - f(-t')] \quad . \tag{5b}$$

The symmetric center, t_s, is defined as that value of t_i that minimizes the odd function in the sense that

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*Here we are neglecting the distortion that is due to the nonsymmetric configurations of retroreflectors on satellites currently in orbit. Since the amount of this distortion varies as the satellite transits, it cannot readily be taken into account by the methods we are now using.

$$\int_0^\infty |f_0(t')| \, dt' = \min \quad . \tag{6}$$

In practice, the t_i values are taken at 0.5- or 1-ns intervals (depending on whether an oscilloscope sweep of 50 ns or 100 ns is used), and a linear interpolation near the minimum is used to obtain t_s precisely.

Another method uses the symmetry of the received pulse by finding the point t_h that divides the area under RP(t) into two equal halves.

The correlation method does not give a center of the received pulse, but instead determines the time T_{CL} that the received pulse must be shifted back toward the transmitted pulse to achieve the best correlation between the two pulses. Since $T_{CL} = T_{RC} - T_{TC}$ and it is this difference in pulse centers, rather than their individual values, that is used in determining the range correction Δ, the method is applicable to the purpose at hand. The value of T_{CL} is determined as that value that maximizes the cross-correlation function, i.e., the value that gives

$$\int_{-\infty}^{\infty} TP(t) \cdot RP(t + T_{CL}) = \max \quad . \tag{7}$$

In practice, the integration is replaced by summation, and T_{CL} is interpolated between two discrete values.

Once T_{RC} and T_{TC} or their difference has been determined, it remains to determine either the threshold values T_{TT} and T_{RT}, or $T_{TT} - T_{RT}$. Since the oscilloscope sweeps are started by trigger pulses from the counter's START and STOP circuits, $T_{TT} - T_{RT}$ is constant to within the jitter of the electronic circuits involved. If this constant has been included in the calibration of the system's electronic delay, the range correction becomes $T_{RC} - T_{TC}$.

If the jitter is appreciable, it can be eliminated from the range correction by determining T_{TT} and T_{RT} from the corresponding threshold voltages V_T and V_R.

Although it is difficult to measure these voltages directly because they are somewhat dependent on the slope of the pulse's leading edge, they can be obtained experimentally by ranging to a reflector in a fixed position. The method, which works best when there are large pulse-to-pulse variations in the shape of the leading edge, is the following: a series of threshold voltages are assumed, and the corresponding threshold times are determined from the photographs for each pulse in the series. Then the sample mean and the sample variance of the threshold time are computed for each assumed voltage. The threshold corresponding to the smallest sample variance corresponds to the actual threshold. Without jitter, the threshold should occur at a fixed position on the x axis of the oscilloscope and the variance should be zero. Since there is jitter, the sample variance will not be zero at the actual threshold voltage, but equal to the rms value of the jitter between the threshold activation of the counter and the start of the sweep.

When the values of V_T and V_R are precisely known, T_{TT} and T_{RT} can be determined from the oscilloscope photographs, and the jitter between counter and oscilloscope does not enter into the measurement of the range correction. There is a method that can be used to determine V_T and V_R without assuming that T_{TT} and T_{RT} have fixed positions on the oscilloscope photographs. This method uses range measurements made to a reflector in a fixed position. A series of threshold voltages are again assumed for the transmitted and received pulses. For the measurements corresponding to each pair* of assumed thresholds, the range correction Δ is calculated and added to the counter reading. Then, the sample mean and the sample variance are calculated for all the fixed-reflector measurements. Because this range is fixed, the variance is a minimum when the assumed thresholds are equal to the actual thresholds. This minimum variance is a measure of the range-correction error for the received signal strength (in photoelectrons) that was used for the experiment.

4. RESULTS

Tables 1 and 2 summarize initial results obtained from range measurements made in accordance with the methods described in previous sections. These results are

*In practice, the transmitter threshold voltage is determined with sufficient accuracy by the previously described procedure, and only variations in the receiver threshold are assumed.

illustrative of the procedures involved. They show that the corrections made from the oscilloscope photographs do improve the accuracy of the counter readings. The results are consistent with those expected from the equipment in current use at the SAO observing stations, but they are not representative of the ultimate accuracy to be expected eventually with improved equipment and procedures. It can be seen from Table 1 that σ_1, the jitter between counter and oscilloscope, is about 1 ns, with most of this error associated with the received pulse. Although the counter's threshold discrimination and trigger-generating circuits are identical in both START and STOP channels, they respond differently to the smooth leading edge of the transmitted pulse than they do to the irregular leading edge of the received pulse. Hence, it appears reasonable that the jitter associated with the received pulse is larger than that associated with the transmitted pulse.

The range error σ_1 applies to range corrections obtained by setting $\Delta = T_{RC} - T_{TC}$. These corrections are based on the assumption that $T_{TT} - T_{RT} = \text{const.}$ Hence, σ_1 includes the effect of the synchronization jitter. Table 1 shows that σ_1 is about 1.5 ns. The quantity σ_2 applies to measurements made by using the correction $\Delta = (T_{RC} - T_{RT}) - (T_{TC} - T_{TT})$, with V_T and V_R selected to minimize σ_2. The σ_2 values in Table 1 are about 1 ns, a value that appears to be reasonable for a return pulse of 500 electrons. For a return of this strength, we would expect an error of $8.5/500^{1/2} \doteq 0.4$ ns in determining the center of the pulse and an additional error of $1/12^{1/2} \doteq 0.3$ ns for a counter with a uniform probability density over its 1-ns resolution interval.

Table 2 shows that the residuals of three satellite transits were reduced by applying the corrections obtained from oscilloscope photographs. Range corrections for several other satellite transits were also computed, but improvements in the short-arc residuals could not be realized because in some cases the variance of the uncorrected range residuals was considerably larger than that of the range corrections. In one case, the average number of electrons per pulse was too small to make an effective determination of the center of the received pulse.

The satellite BE-C shows the greater decrease in residual (0.7 to 0.4 m), a result that might be expected because the retroreflectors on this satellite are arranged in such a way that they produce less of a random variation in range with changes in the satellite's aspect angle than do those on Geos 1. For the measured pulse width of the BE-C returns, the corresponding σ of a normal pulse is about 8.5 ns; hence, the error in determining the center of a 38-electron pulse is $8.5/38^{1/2} = 1.4$ ns (0.2 m). This error combined with the 1.7-ns synchronization jitter in Peru gives $(1.4^2 + 1.7^2)^{1/2} = 2.2$ ns (0.33 m), a value comparable to the 0.39 m shown in Table 2.

In computing the short-arc residuals shown in Table 2, a 4- to 5-day orbit was first obtained by using laser data from all stations. Then, best-fitting short arcs for the given pass were obtained by varying inclination, eccentricity, mean anomaly, and mean motion. All range observations used in the orbit computations were corrected for the nonrandom displacements between pulse center and the center of mass of the satellite. These corrections, which depend on the angle between the laser beam and the axis of the satellite's retroreflector, were computed by Arnold (1972).

5. ACKNOWLEDGMENTS

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6. REFERENCE

Arnold, D. A.

1972. Calculation of retroreflector array transfer functions. Final Tech. Rep. NASA grant NGR 09-015-196, 57 pp., December.

Table 1. Summary of range measurements to fixed reflectors.

Station	Oscilloscope sweep duration (ns)	Number of pulses	Average number of electrons in received pulse	Measured pulse durations		Synchronization jitter		Range error		
				Transmitted (ns)	Received (ns)	Transmitted pulse (ns)	Received pulse (ns)	σ_c (ns)	σ_1 (ns)	σ_2 (ns)
Brazil	50	17	560	22	26	0.1	1.3	2.1	2.1 (CT)	0.7 (CL)
Peru	50	18	463	26	32	0.2	1.3	1.7	1.5 (CT)	1.1 (CT)
Arizona	100	20	416	21	26	0.3	0.9	1.7	1.3 (S)	1.3 (CL)

The σ's are sample standard deviations of the series of pulses: σ_c represents the uncorrected counter readings, and σ_1 and σ_2 represent corrected counter readings. For σ_1, the counter and oscilloscope were assumed to be in exact synchronism. For σ_2, the receiver threshold was determined by using the fact that the range is constant. The σ_1 and σ_2 values given above are the lowest for the four methods of pulse-center determination: centroid (CT), symmetry (S), half-area (A), or correlation (CL).

Table 2. Summary of satellite-range corrections. Method used for pulse-center determination: centroid (CT), symmetry (S).

Date	Station location	Satellite	Oscilloscope sweep duration (ns)	Number of observations	Average number of electrons in received pulse	Measured pulse widths		Short-arc residuals	
						Transmitted (ns)	Received (ns)	Uncorrected observations (m)	Corrected observations (m)
7 June 1972	Brazil	BE-C	50	9	38	24	22	0.73	0.45 (CT)
17 June 1972	Brazil	Geos 1	50	35	85	20	21	0.71	0.57 (S)
17 March 1973	Peru	BE-C	100	19	276	23	19	0.74	0.39 (CT)

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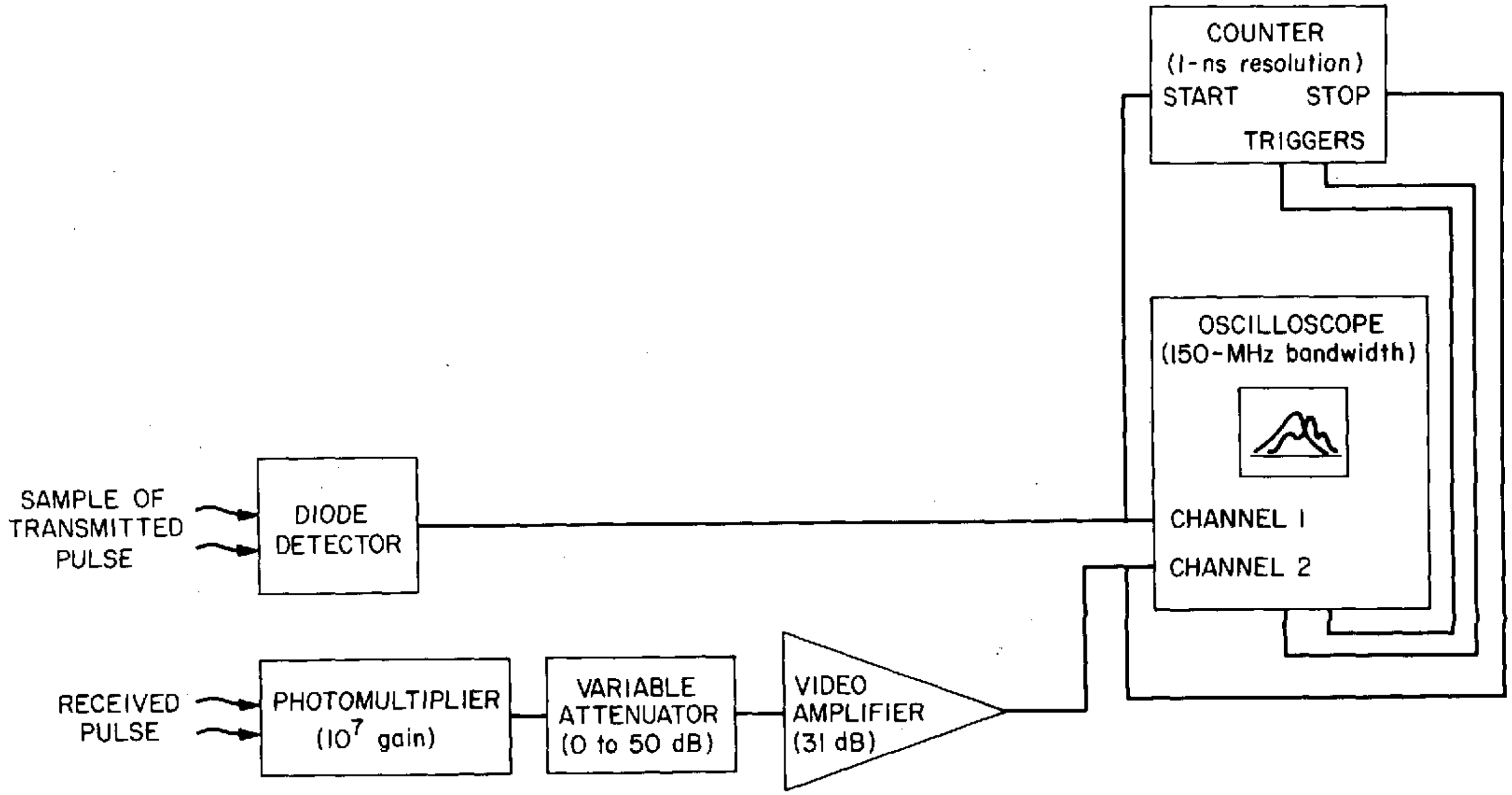

Figure 1. Block diagram for pulse detection.

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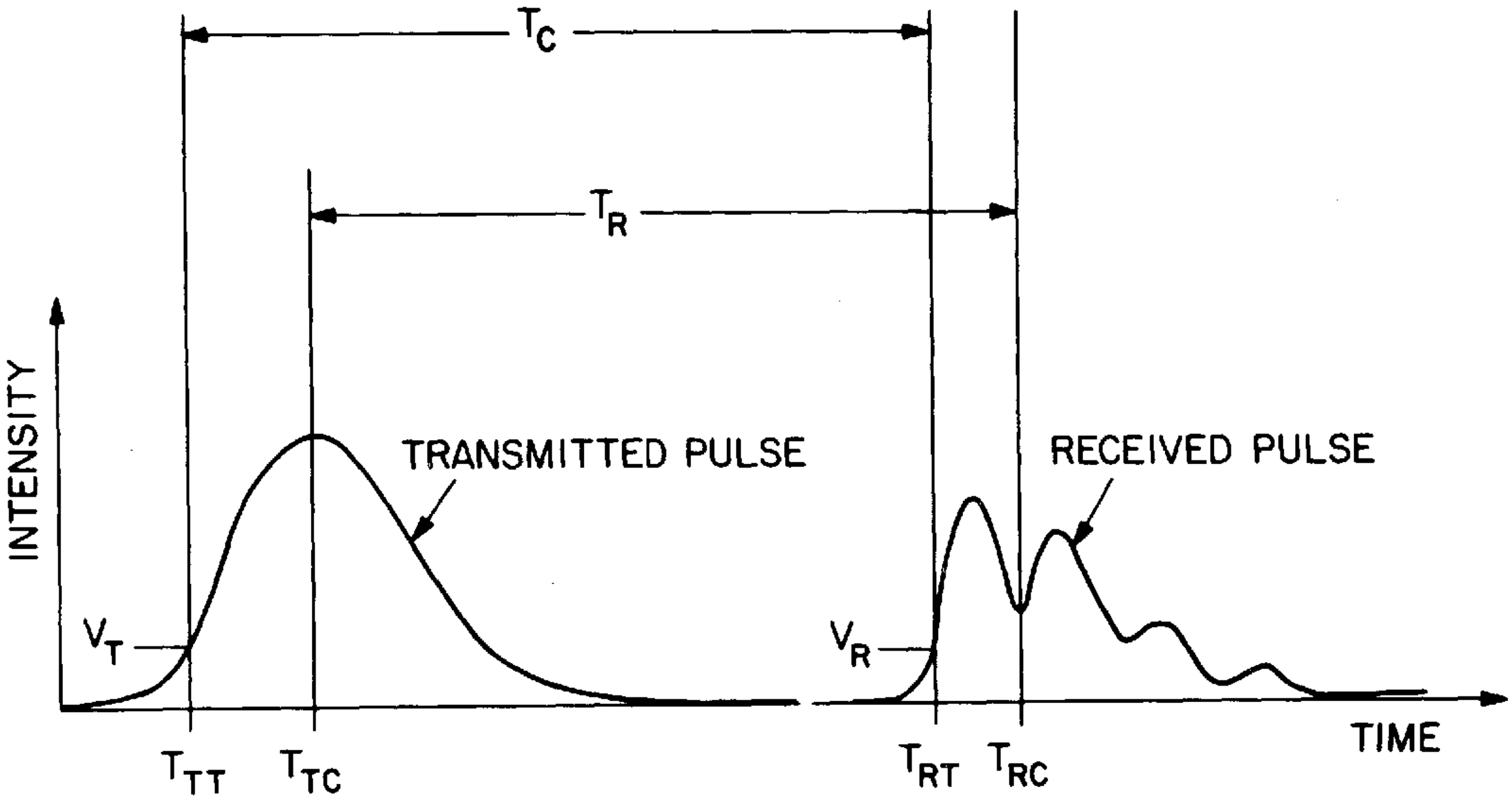

Figure 2. Diagram for the range correction Δ, where $T_R = T_C + \Delta$ and $\Delta = (T_{RC} - T_{RT}) - (T_{TC} - T_{TT})$.

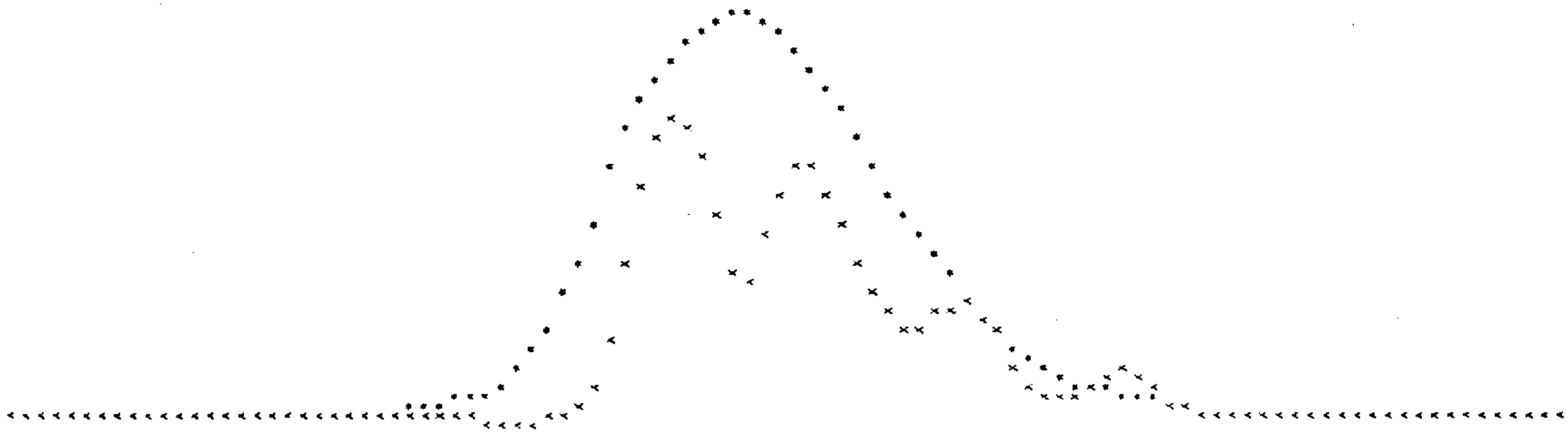

Figure 3. Digitized representation of the photographed pulses: * = transmitted pulse; x = received pulse. Sweep duration = 100 ns.